Revisiting the Relationship between Price Stickiness and the Non-Neutrality of Money

by

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Abstract

By lots of economists and central banks, price stickiness is believed to be the main factor which brings about the non-neutrality of money. Based on the belief, most of the New Keynesian models are developed to feature price stickiness in order to make the real effect of money. Among those, the Calvo pricing has been the most popular framework in featuring the sticky price. This thesis investigates whether the non-neutrality of money is always guaranteed by the Calvo-type price stickiness or not. In particular, the focus lies on the effect of volatility of firms' optimal prices on the relationship between price stickiness and the non-neutrality of money. Chapter 1 presents the theoretical possibility of the non-relationship between the two phenomenons in such case that repricing firms' optimal prices are very volatile, and the following two chapters propose more micro-founded endogenous frameworks to deliver the results which support the argument in Chapter 1.

It is shown in Chapter 1 that high volatility of reset prices has the same effect as that of lowering the degree of price stickiness and increasing the future discount factor in the standard Calvo framework. Due to the effect, it can be illustrated that the aggregate price level can be flexible even when some firms maintain the previous price level if the other repricing firms' prices respond very elastically to monetary shocks. Chapter 2 proposes a model in which repricing firms behave as in collusion and exploit the information on aggregate price dynamics by taking the aggregate price as a function of their own price at the process of optimization. It is shown that the colluding firms set much higher prices for monopoly gains against positive monetary shocks, and therefore, the aggregate price level can be very responsive even with price stickiness of the firms. Lastly, Chapter 3 presents the case where firms have no information on other firms' pricing behaviours and have expectations on average reset price with bounded rationality. The model of this chapter demonstrates that the realized level of average reset price of the firms can be much higher than that of the standard model when their expectations are heterogeneous.

All the results of the chapters imply that the monetary policy might not be able to have the real effect even with price stickiness if firms' reset prices show very volatile movements. Therefore, economists and central banks should research more on the volatility of firms' reset prices when analysing monetary policy and also try to find other factors which might have direct relationship with the rigidity of aggregate price, rather than price stickiness which focuses just on individual prices, when developing a monetary model.

Chapter 1

Volatility of Optimal Price and Inflation Rigidity

1.1 Introduction

In most countries, central banks implement various monetary policies with the purpose of attenuating the fluctuation of the economy based on the belief that money has real effects on the economy. At least in the short-run, the central banks' belief of the non-neutrality of money seem to be credible since we can observe that aggregate price does not respond quickly to monetary shocks, and therefore, output in the economy changes significantly. Also, it is not hard to find an economist who supports the belief; a decent number of economists accept the inflation rigidity against monetary shocks.[1] For example, Romer and Romer (2004) show the effectiveness of monetary policy using the data of narrative in the Federal Open Market Committee (FOMC), and Christiano, Eichenbaum, and Evans (1999) present the econometric evidence of the non-neutrality of money using the VAR with the data of federal funds rate.

However, even though we accept the belief that money has a real effect on the

[1] In this chapter, "real effect of money" and "non-neutrality of money" are used in much the same sense. Also, "inflation rigidity" and "rigid aggregate price level" have the same meaning in describing one side of the economy when money is non-neutral.

economy, it can be still asked what makes the non-neutrality of money. Regarding this issue, many economists seem to find the reason for the real effect of money in price stickiness of individual firms, though the cause of price stickiness is another problem. For example, Ball and Mankiw (1994), who are considered as one of the representatives of those economists, say: "We believe that price stickiness is the best explanation for monetary non-neutrality."[2] Another representative, Woodford (2003), says: "... taking account of delays in the adjustment of wages and prices provides a clear justification for an approach to monetary policy that aims at price stability." Based on these firm beliefs, economists have been making models which feature price stickiness to make the rigid aggregate price level and the non-neutrality of money for the purpose of monetary policy analysis.

In this chapter, the above beliefs are challenged. Even though many economists take price stickiness as a main factor of the non-neutrality of money, this chapter shows that money can be almost neutral even with price stickiness. In other words, it can be shown that the sticky price of individual firm does not always guarantee the rigid aggregate price level. However, this is not meant to argue that money is neutral and that monetary policy is ineffective in the real world. Money may have the real effect as seen in various data, but what is shown here is that price stickiness may have nothing to do with the real effect of money. This chapter aims to demonstrate the theoretical possibility of the non-relationship between the sticky individual price and the rigid aggregate price level. In other words, the simple model in this chapter is suggested as an illustration for the weak position of price stickiness as the main factor of the non-neutrality of money, not for the claim that money is neutral.

For the purpose mentioned above, this chapter develops a model in which price stickiness is featured and then shows that this model does not guarantee the non-neutrality of money. The first issue is how price stickiness should be embodied into

[2] Clarida, Gali, and Gertler (1999) indicate that this belief is the main factor of the so-called New Keynesian perspective, saying: "... we should make clear that the approach we take is based on the idea that temporary nominal price rigidities provide the key friction that gives rise to nonneutral effects of monetary policy... For this reason, we append "New Keynesian Perspective" to the title."

a model. Many economists have tried to find micro evidences of price stickiness, and most of them found that some prices often remain unchanged for some months and that only a fraction of firms reset their prices in any given period.[3] In other words, price stickiness has been measured mainly by the infrequency of price changes, and many economists have developed models featuring this characteristic. Taylor (1980) and Calvo (1983), in particular, pose technical restrictions on firms' price decisions so that the price setting is staggered. Mankiw and Reis (2002) assume that information is costly and disseminated slowly throughout the economy so that some firms may use outdated information for their price settings. While these models have the frequency of price adjustment as given exogenously,[4] some other models known as menu cost model have the pricing frequency decided endogenously assuming that firms reset their prices only when the benefits by doing so is greater than the costs of changing prices.[5] Though the menu cost models are known to be more intuitively appealing, the above-mentioned models have been more popular in featuring price stickiness for their simplicity.[6] Especially, among those, Calvo pricing has been the workhorse model for its analytical tractability. Also, as for its performance, many economists argue that it is a good approximation to a fully-specified menu cost model.[7] The popularity of the Calvo model can be seen in the fact that most central banks use it as a basic framework for their own models for monetary policy analysis.[8] In other words, the Calvo pricing model is the most representative tool for price stickiness and the real effect of money in most literature on monetary policy. In this sense, this chapter uses the Calvo pricing as a basic framework for price stickiness.

[3]This kind of survey paper includes Bils and Klenow (2004), Angeloni et al. (2006), Nakamura and Steinsson (2008), and Klenow and Malin (2010).

[4]For the reason, these models are called the time-dependent model.

[5]The menu cost models are also called the state-dependent model, and the examples are Caplin and Spulber (1987), Caballero and Engel (1991), Dotsey, King, and Wolman (1999), Golosov and Lucas (2007), Gertler and Leahy (2008), and Midrigan (2011).

[6]Recently, some papers develop models which use both the time- and state-dependent pricing framework. See Bonomo, Carvalho, and Garcia (2010), Alvarez et al. (2011), and Demery (2012).

[7]See Gertler and Leahy (2008) and Midrigan (2011).

[8]For example, Kara (2011) indicates that the models which are used for analysing monetary policy in the European Central Bank (ECB) are Christoffel, Coenen and Warne (2008) and Motto, Rostagno and Christiano (2008), and the main frameworks of these two models are based on the Calvo-type pricing mechanism.

Then does the Calvo pricing model which features price stickiness guarantee the real effect of money? In the standard Calvo model, a certain fraction of firms are not allowed to change their prices while the rest reset their prices. In response to positive monetary shock, some firms are stuck to the price level of the previous period and the other firms raise their prices up to the new optimal level maximizing their expected profits. In this case, due to price stickiness, the aggregate price level does not rise enough to offset the monetary shock. This mechanism seems to show that sticky individual price always produces rigid aggregate price and the non-neutrality of money. However, we should note that even though the aggregate price level is fettered by the previous price level due to price stickiness, it still depends on the optimal prices of repricing firms. What would happen if the firms facing the positive monetary shock set much higher prices than in the standard model? Even in this situation, can price stickiness make the real effect of money on the economy? As is well known, in the standard model, the optimization processes of firms are implemented under many assumptions which are adopted for the simplicity of the model. For example, in the standard model, it is assumed that all firms are homogeneous and that they make rational expectations on future economic status. What would happen if there is any breakdown of those assumptions? It may be possible that this breakdown might make the repricing firms choose another optimal price which is different from that of the firms in the standard Calvo model against monetary shocks. The question is whether or not even in such case we can find a solid causal connection between price stickiness and the non-neutrality of money.

To answer the above question, this chapter sets up a model which is the same as the standard Calvo pricing framework, except that the repricing firms are assumed to set different prices from the optimal price of the standard model. More precisely, in the proposed model, firms' reset prices are assumed to be more volatile. In other words, the repricing firms set much higher and lower prices than the optimal price against positive and negative monetary shocks, respectively. This is the reason why the model is called 'Volatile Prices Model' in this chapter. However, the assumption

of volatile optimal price is very *ad hoc*. In other words, the model gives no explanation on what makes firms set volatile prices. Therefore, it cannot be said that the model reflects the real world very accurately. However, the main focus of this chapter is placed on the effect of volatility of firms' optimal prices on the role of price stickiness in relation to the non-neutrality of money. In this sense, the discussion on the factor which brings about the volatility of firms' prices is put aside for the next two chapters, and this chapter just focuses on the model only as a simple illustration of the possibility that price stickiness does not lead to the monetary non-neutrality in the case where firms' optimal prices are volatile. The simple illustration in this chapter will be augmented by more micro-founded models developed in Chapters 2 and 3.

The modified Calvo model of this chapter still features price stickiness because some fraction of firms are not allowed to reset their prices as in the standard Calvo framework. However, this model shows that, in spite of the sticky price, the real effect of monetary policy is very small compared to the standard model when the reset prices are volatile. It is shown in the model that increase in the volatility of reset price has the effect of lowering the degree of price stickiness and increasing the future discount factor in the standard Calvo framework. Also, this effect leads to more flexible aggregate price level and less output response against monetary shocks. This result means that price stickiness of individual firm which is generated by the Calvo pricing may not always imply the non-neutrality of money.[9] In other words, even though the price of an individual firm is sticky, the aggregate price level may not be rigid under certain circumstances like the case where the repricing firms respond very elastically to the monetary shock as in the proposed model. That is, depending on the volatility of the optimal price, the inflation rigidity may not be guaranteed even if there is price stickiness among individual firms. This result implies that

[9]The economy in the model of this chapter is cashless as in many of the standard New Keynesian models following Woodford (2003) in which monetary policy is performed by the direct adjustment of nominal interest rate. In other words, money does not exist in a strict sense of the word. Rather it is just used as a unit of account. In that sense, the term, "non-neutrality of money", should be understood as "the real effect of monetary policy" rather than that of the literal meaning of "money".

economists and central banks need to pay more attention to the volatility of the reset prices when analysing the effect of monetary policy. Also, they need to try to find another factor which can make the non-neutrality of money regardless of the price volatility in developing monetary models.

The structure of this chapter is as follows. Section 1.2 introduces the review of the literature related to this chapter. Section 1.3 outlines the Calvo framework with volatile prices and explains how it differs from the standard model. Section 1.4 describes the simple Dynamic Stochastic General Equilibrium (DSGE) model in which the modified Calvo pricing framework is embodied. In section 1.5, the results of simple simulations conducted with this model are reported, and the discussions of the results with further issues for future research are presented. The last section summarizes the main arguments of this chapter and draws the main conclusion.

1.2 Literature Review

As stated above, it is a common convention that price of individual firm is sticky, and many survey papers present micro evidences of price stickiness. However, as for the relationship between price stickiness and the non-neutrality of money, it still remains a topic of controversy. Even though many economists, especially the New Keynesians, believe that sticky price is the main factor which brings about the non-neutrality of money, there are still many papers which argue that price stickiness does not always imply the real effect of money.

Head et al. (2012) is the most representative paper which insists that money can be neutral in spite of price stickiness. They set up the model in which money is fully neutral and the equilibrium involves a non-degenerate price distribution using search friction approach of Burdett and Judd (1983). Since all firms have the same profit within a certain support of the price distribution, firms do not have to adjust their prices so long as they remain within the support in spite of monetary shock. Suppose that the price distribution in equilibrium is given by $F(p)$ with the support, $\mathcal{F} = \left[\underline{p}, \bar{p}\right]$. With positive monetary shock, aggregate price (P) goes

up and the price distribution shifts to the right. That is, $\mathcal{F}_t \left(= \left[\underline{p}_t, \bar{p}_t\right]\right)$ shifts to $\mathcal{F}_{t+1} \left(= \left[\underline{p}_{t+1}, \bar{p}_{t+1}\right]\right)$ where $\underline{p}_t < \underline{p}_{t+1} < \bar{p}_t < \bar{p}_{t+1}$. However, because money is neutral by construction, the distribution of real price does not change. That is, we have the invariant $F(p/P)$. With this monetary shock, firms with their prices (p_t) out of the new support ($\mathcal{F}_{t+1}$) must reset the prices. However, firms with $p_t \in \mathcal{F}_{t+1}$ do not have to do so. Therefore, we can observe sticky price in this framework because some fraction of firms may not adjust their prices. However, money is neutral because there is neither menu cost nor Calvo fairy in this model. With these results, the authors argue that price stickiness does not imply the existence of any real resource cost of changing prices ("menu costs") or technological constraint (e.g. Calvo pricing) to adjusting prices and that price stickiness has nothing to do with the real effect of money. However, there are some criticisms on these arguments. Kryvtsov (2010) points out that the meaning of price stickiness in Head et al. (2012) is somewhat different from what is commonly used.[10] It is commonly accepted that price becomes sticky because adjusting prices always cause decrease in profit due to the so-called menu cost. That is, price stickiness is attributed to the fact that firms are not free in changing prices. However, in Head et al. (2012), firms are absolutely free to adjust prices. The reason why firms do not change prices is because they do not have to do so, not because they cannot do so. In other words, sticky price is not an inevitable consequence of menu costs, but the result of a firm's discretionary choice.[11] Kryvtsov (2010) also argues that the model of Head et al. (2012) cannot predict individual price behaviour. Since each firm has the same profit within a certain support of price distribution, the individual price after a shock cannot be predicted without any assumption on a policy function which shows how each firm sets its own price.

Golosov and Lucas (2007) is another paper which shows that price stickiness does

[10] The paper Kryvtsov (2010) discussed is the previous version of Head et al. (2012), "Equilibrium Price Dispersion and Rigidity: A New Monetarist Approach" (Head et al., 2010).

[11] Similarly, Gorodnichenko and Weber (2013) say that the neutrality of money in the model of Head et al. (2012) is because sticky prices are not costly. However, they show that menu costs are the main factor of price stickiness using the data on stock market returns supporting the New Keynesian interpretation of price stickiness.

not guarantee the non-neutrality of money. Unlike Head et al. (2012), in order to produce price stickiness, they develop the state-dependent menu cost model in which firms face a fixed cost for changing prices. However, they demonstrate that money is approximately neutral in spite of price stickiness across individual firms when there are idiosyncratic productivity shocks as well as aggregate monetary shocks. This result is based on the "selection effect" which means repricing firms are not selected at random but are those whose prices are furthest from the optimal reset price level. Suppose that a positive monetary shock is applied. Then the relative price distribution which is generated by the idiosyncratic productivity shock shifts to the left, and more firms in the left-hand tail of the distribution will feel the need for increasing prices. However, because the positive monetary shock offsets the negative idiosyncratic shocks at the same time, the firms in the right-hand tail that would otherwise have decreased prices will not change their prices. Hence, most price adjustments in this economy are taken by the firms in the left-hand tail, and therefore, the aggregate price goes up quickly with these large positive adjustments. As a result, under the menu cost framework with idiosyncratic productivity shock, price stickiness does not imply the real effect of money. However, Golosov and Lucas (2007) are criticized for the inconsistency with micro evidences on the size of price changes. In the menu cost framework, firms change their prices only when doing so gives more profit than the costs, and therefore, we cannot see the small price changes in this model. But the micro data shows that high fraction of price changes are very small.[12] Midrigan (2011) also argues that the standard deviation of the size of price changes in absolute value in the Golosov and Lucas (1.2%) is too small compared to the data (8.2%). It is shown in numerous literature that the non-neutrality of money can be obtained when the wide dispersion of price changes are featured in the menu cost model.[13]

As can be seen above, although some papers argue that observation of sticky

[12]For example, Klenow and Kryvtsov (2008) analyse that 44.3% of price changes are distributed within 5% in absolute value.

[13]See Klenow and Kryvtsov (2008), Nakamura and Steinsson (2010) and Midrigan (2011).

price in the micro data does not imply the non-neutrality of money, there are still lots of criticism on these papers. Therefore, the conventional view still seems to be that price stickiness is the main factor that brings about the non-neutrality of money and the real effect of monetary policy. In addition, the most popular framework which represents this conventional thought is the Calvo pricing model which is most widely used among economists and central banks for the purpose of monetary policy analysis. Moreover, within the scope of what we have researched, though some literature makes issues of its performance and inconsistency with micro data, there is no paper which brings the Calvo framework's ability to produce the non-neutrality of money into question. Therefore, this chapter would be the first to bring attention to the relationship of the Calvo pricing, price stickiness generated by the Calvo framework, and the non-neutrality of money.

1.3 Calvo Framework with Volatile Prices

Under the standard Calvo pricing framework, each firm adjusts its price by maximizing the present discounted value of expected profits as

$$E_t \sum_{s=0}^{\infty} \theta^s Q_{t,t+s} P_{t+s} \left(\frac{p_{it}}{P_{t+s}} - mc_{t+s} \right) y_{it+s},$$

where θ is the probability of not changing prices, $Q_{t,t+s}$ is the stochastic discount factor, P_t is the aggregate price level, p_{it} is the individual price of firm i, mc_t is the marginal cost, and y_{it} is the output demand which firm i faces. Given the demand curve, $y_{it} = (P_{it}/P_t)^{-\epsilon} y_t$, we have

$$E_t \sum_{s=0}^{\infty} \theta^s Q_{t,t+s} P_{t+s} \left[\left(\frac{p_{it}}{P_{t+s}} \right)^{1-\epsilon} - mc_{t+s} \left(\frac{p_{it}}{P_{t+s}} \right)^{-\epsilon} \right] y_{t+s},$$

where ϵ is the price elasticity of demand and y_t is the aggregate output. The first order condition with respect to p_{it} is given by

$$E_t \sum_{s=0}^{\infty} \theta^s Q_{t,t+s} \left(\frac{p_{it}}{P_{t+s}} \right)^{-\epsilon} y_{t+s} \left[1 - \frac{\epsilon}{\epsilon - 1} mc_{t+s} \left(\frac{p_{it}}{P_{t+s}} \right)^{-1} \right] = 0. \tag{1.1}$$

Therefore, firm i chooses p_{it} satisfying (1.1) when it receives the Calvo signal. Since all firms that receive the signal at time t face the same optimization problem, each firm changing its price chooses the same optimal price denoted by p_t^*. Then we have the following dynamics of the aggregate price level.

$$P_t^{1-\epsilon} = \theta P_{t-1}^{1-\epsilon} + (1-\theta)(p_t^*)^{1-\epsilon}. \tag{1.2}$$

Facing the monetary expansion, $m_t = \alpha m_{t-1} (\alpha > 1)$, $(1-\theta)$ fraction of firms choose $p_t^* > p_{it-1}$. However, since θ fraction of firms are stuck to p_{it-1} (price stickiness), the aggregate price level does not fully go up ($P_t < \alpha P_{t-1}$), and therefore, the aggregate output increases ($y_t > y_{t-1}$). This is the mechanism by which the Calvo pricing produces the non-neutrality of money.

However, as mentioned above, not all economists agree to the relationship between price stickiness and non-neutrality of money. For example, Head et al. (2012) show that individual price can be sticky even if firms are free to adjust prices and aggregate price is flexible. In other words, they demonstrate the non-relationship between the sticky price and the rigidity of aggregate price level. Then what drives them to the different interpretation on price stickiness? Gorodnichenko and Weber (2013) indicate that sticky price is not costly in the model of Head et al. (2012), which is not the case in other New Keynesian models, and this difference leads to the opposing result. However, we attribute the result to the difference of volatility in optimal price rather than the cost of sticky price between the two kinds of models. In other words, the reset prices in Head et al. (2012) are much more volatile than those in the standard Calvo model. According to the result of quantitative exercise in the model of Head et al. (2012), average absolute value of price changes is 9%

with the inflation rate of 3% and the possibility of not changing price around 0.9. However, in the standard Calvo model, the optimal price changes by less than 1% under a similar condition.[14] Then can we get the fully neutral money as in Head et al. (2012) if we set more volatile optimal prices even under the Calvo framework? We can find a clue from the dynamics of the aggregate price level. As can be seen in (1.2), P_t is fettered by P_{t-1} due to price stickiness. However, we focus on the fact that P_t still depends on p_t^*. We can anticipate that the volatility of p_t^* will affect that of P_t in any way, and therefore, the degree of non-neutrality of money will change.

For the purpose of verification of the above conjecture, we develop the 'Volatile Prices' model in which the optimal price, p_t^*, is very volatile. In the standard Calvo model, p_t^* is obtained under the assumption that repricing firms behave rationally with full information in pricing. However, in the real economy, firms do not always show rational behaviours, and information is usually imperfect. In this sense, we suppose that firms, for whatever reason, set more volatile prices than the optimal price, p_t^*. More precisely, we assume that repricing firms set higher and lower prices than p_t^* against positive and negative monetary shocks, respectively. In this case, we can expect that the volatility of individual firms' prices makes the aggregate price level more flexible than that in the standard model. With the 'Volatile Prices' model, we can check whether the expectation is correct and the Calvo pricing can guarantee the non-neutrality of money even under the volatile reset prices of individual firms.

1.4 Model

The framework used in this chapter is the standard simple New Keynesian model based on Calvo (1983), except that firms set a volatile price rather than an optimally driven price.

[14] We firstly set the size of monetary shock so that the annual inflation rate would be around 3% under the flexible economy (θ=0), and then simulate the model with $\theta = 0.9$.

1.4.1 Firms

There is a continuum of firms $i \in [0, 1]$. Each firm hires labour, h_{it}, and produces its own goods, y_{it}, through the production technology which takes form

$$y_{it} = h_{it}.$$

Each firm i sets its price by optimizing its decision-making through minimizing costs of production and maximizing its expected profit with the production. The cost minimization is the same as in the standard model as in Appendix 1.A. However, the profit maximization shows that the reset price of firm i is more volatile than the optimal price of the standard model as shown in the following.

Profit Maximization

As in the standard mechanism of Calvo framework, a fraction θ of firms are not allowed to change their prices at time t. The remaining fraction $(1 - \theta)$ of firms reset their prices, p_{it}. Since all firms which receive Calvo signal at time t face the same decision problem of profit maximization, each repricing firm sets the same price that we denote by p_t^v. Therefore, using the definition of the aggregate price index $\left(P_t^{1-\epsilon} = \int_0^1 (p_{it})^{1-\epsilon}\, di\right)$,[15] we have the following dynamics of the aggregate price level,

$$P_t^{1-\epsilon} = \int_{\omega_t} P_{it-1}^{1-\epsilon} di + (1-\theta)(p_t^v)^{1-\epsilon} = \theta P_{t-1}^{1-\epsilon} + (1-\theta)(p_t^v)^{1-\epsilon}, \tag{1.3}$$

where ω_t is the set of firms which are not allowed to change their prices at time t. In our framework, the repricing firms are assumed to set more volatile prices (p_t^v) than the optimal prices (p_t^*) in the standard Calvo pricing model. More precisely, the volatile price is assumed to be given by

$$p_t^v = \alpha(p_t^* - P_t) + P_t,$$

[15] See Appendix 1.B for the derivation.

where $\alpha > 1$ is the coefficient which displays a degree of volatility of individual firm's price.[16] Dividing both sides of the equation by P_t, we can have

$$\tilde{p}_t^v = \alpha \tilde{p}_t + (1 - \alpha) \tag{1.4}$$

as an equivalence relation, where $\tilde{p}_t^v (= p_t^v / P_t)$ and $\tilde{p}_t (= p_t^* / P_t)$ denote the relative value of volatile and optimal price, respectively. The optimal price, p_t^*, can be obtained by satisfying the first order condition of firms' profit maximization problems as in equation (1.1) which can be rearranged as

$$\tilde{p}_t = \frac{\epsilon}{\epsilon - 1} \frac{f_{1t}}{f_{2t}}, \tag{1.5}$$

where

$$f_{1t} = \lambda_t y_t mc_t + \beta \theta E_t (\pi_{t+1})^{\epsilon} f_{1t+1} \tag{1.6}$$

$$f_{2t} = \lambda_t y_t + \beta \theta E_t (\pi_{t+1})^{\epsilon - 1} f_{2t+1}, \tag{1.7}$$

and $\pi_t (= P_t / P_{t-1})$ denotes the gross inflation rate.

As mentioned above, the assumption of volatile optimal price is very *ad hoc* without any micro-foundation. In other words, this model gives no explanation as to why the firms set such volatile prices. Therefore, it cannot be said that the model with this assumption is fit for purpose for reflecting the real economy. However, considering the main interest of this chapter which is the investigation on how the relationship between price stickiness and the non-neutrality of money is affected by the volatility of firms' reset prices, the model should be understood only as a tool for the simulation of the effect of volatile reset price on the economy. Therefore, we can take the above assumption as the simplest and most efficient method for enabling the model to illustrate how the volatility of reset price functions on the real effect of money when firms face price stickiness.[17]

[16] We can easily find that p_t^v is nothing but p_t^* when α is equal to one.

[17] The missing part of the assumption, which is about how and why the firms' prices are so volatile, will be made up for by more micro-founded models in the next two chapters.

1.4.2 Rest of the Model

The optimization problems of households, the policy of monetary authority, and the market clearing conditions are the same as those in the standard model (see Appendix 1.B). The representative household chooses the aggregate consumption (c_t), the amount of hours of labour supply (h_t), and the real bond holding (b_t) by minimizing expenditure and maximizing utility. The monetary authority sets interest rate (R_t) by following the simple standard rule against monetary shock (ν_t) and all the markets are assumed to be cleared.

The conditions for equilibrium, the steady state, the log-linearized form, and the calibration of the model are described in Appendix 1.C.

1.5 Simulation Results

1.5.1 Impulse Responses

We simulate the model and compare it to the benchmark model which is the standard Calvo model with the same parameters.[18] The impulse responses of the main variables of the two models are given as in Figure 1.1. In this simulation, the degree of volatility (α) is set to a purely arbitrary value of 1.2. The simulation result shows that the 'Volatile Prices' model produces bigger inflation response and smaller output response to the expansionary money shocks than the benchmark model. This means that the real effect of money on the economy is much lessened with volatile reset prices. In our model, since the repricing firms set higher prices than that of the standard model against positive monetary shocks, we can easily expect that the aggregate price level will be also higher than that of the benchmark model. Due to this highly elevated price level, the aggregate output response is not so high. This result demonstrates that even though the price of an individual firm is sticky, the aggregate price level may not be so rigid if the repricing firms adjust their prices fully enough to absorb the shocks for whatever reason.

[18] Dynare with Matlab is used for the simulation.

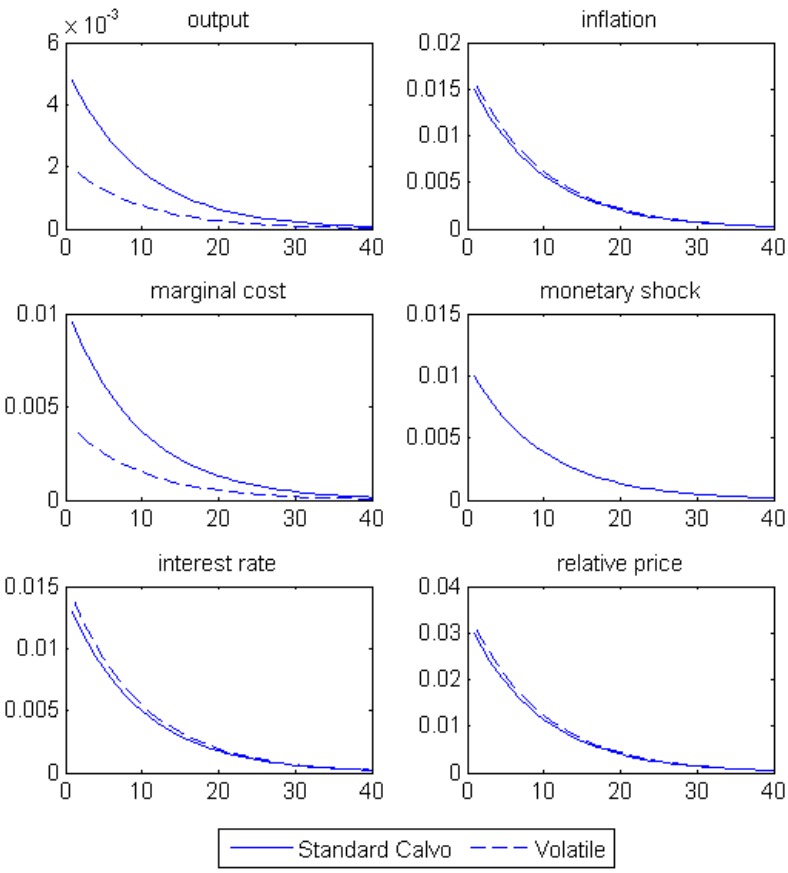

Figure 1.1: Impulse Response Functions to the Monetary Shocks

1.5.2 Pricing Volatility and Price Stickiness

Overall, above impulse responses of the 'Volatile Prices' model resemble those of the standard Calvo model with lower level of θ. In this sense, we need to check what the meaning of the volatility of our model (α) is with respect to its relationship with the degree of price stickiness (θ) in the standard model. Firstly, from the log-linearized forms of (1.3)$\sim$(1.7),[19] we can get the following NKPC (New Keynesian Phillips Curve).

$$\hat{\pi}_t = \frac{\alpha(1-\beta\theta)(1-\theta)}{\theta}\hat{mc}_t + \beta\left\{\alpha + (1-\alpha)\theta\right\} E_t\hat{\pi}_{t+1} \tag{1.8}$$

[19]See Appendix 1.C

Then we can re-express this NKPC as a standard form of NKPC using new variables of $\tilde{\beta}$ and $\tilde{\theta}$ as follows.

$$\hat{\pi}_t = \frac{(1-\tilde{\beta}\tilde{\theta})(1-\tilde{\theta})}{\tilde{\theta}}\hat{mc}_t + \tilde{\beta}E_t\hat{\pi}_{t+1}, \tag{1.9}$$

where

$$\frac{(1-\tilde{\beta}\tilde{\theta})(1-\tilde{\theta})}{\tilde{\theta}} = \frac{\alpha(1-\beta\theta)(1-\theta)}{\theta} \tag{1.10}$$

and

$$\tilde{\beta} = \beta\left\{\alpha + (1-\alpha)\theta\right\}. \tag{1.11}$$

We can interpret this standard NKPC to show the supply side of an economy in which a discount factor of household is given by $\tilde{\beta}$ and a degree of price stickiness firms face is expressed by $\tilde{\theta}$. Putting (1.11) into (1.10), we can get the expression for $\tilde{\theta}$ with α and θ as

$$\tilde{\theta} = \frac{\theta}{\alpha + \theta - \alpha\theta}. \tag{1.12}$$

From (1.12), we have

$$\frac{\Delta\tilde{\theta}}{\Delta\alpha} = -\frac{\theta(1-\theta)}{(\alpha+\theta-\alpha\theta)^2} < 0, \tag{1.13}$$

which means that $\tilde{\theta}$ decreases as α increases.[20] In other words, increased volatility in the model has the meaning of decrease in stickiness of individual firms' prices in the standard Calvo framework.

1.5.3 Sticky Individual Price and Rigid Aggregate Price

The above negative relationship between α and $\tilde{\theta}$ can also be confirmed in the following result of recalibration exercise as shown in Figure 1.2. In this exercise, the degree of price stickiness of the standard model (θ_s) is recalibrated to a lower level in order to get the same impulse responses as of the 'Volatile Prices' model. When

[20] Actually, we have another solution for the equations of (1.10) and (1.11) which is $\tilde{\theta} = 1/(\beta\theta)$. However, it is discarded with the consideration that $\tilde{\theta}$ can be regarded as the probability of not changing prices in the standard form of NKPC, (1.9), and therefore, it should be between zero and one.

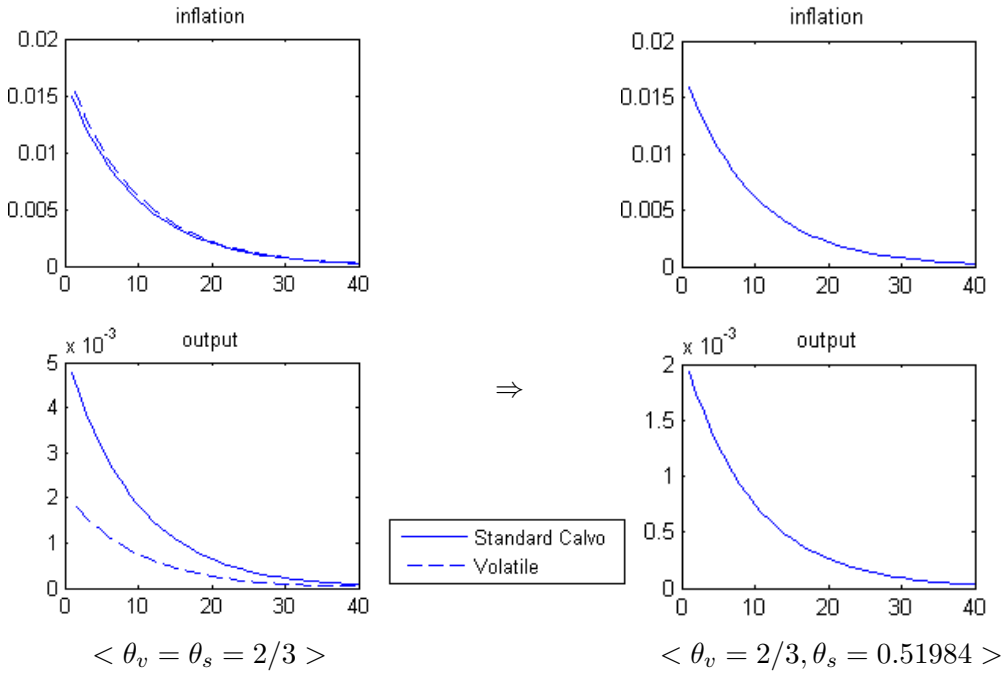

Figure 1.2: Recalibration Exercise

we lower θ_s from 2/3 which is very standard in the literature to 0.51984, we can get the same inflation and output responses as of the 'Volatile Prices' model with the degree of price stickiness (θ_v) fixed to 2/3 and the degree of volatility (α) of 1.2. As stated above, the result of this exercise shows that the proposed model can be regarded as another standard Calvo model which has a much lower θ. However, the more interesting and important meaning of this result is that we can get some hints on the relationship between the sticky individual price and the rigid aggregate price. As can be seen on the right panel of Figure 1.2, we set different θ with two models but we get the same inflation response. The probability of not changing price which is denoted by θ deals with the degree of price stickiness, and the inflation response shows the rigidity of aggregate price. Therefore, we can say that with different price stickiness we can get the same rigidity of aggregate price. This result makes us cast doubt on the relationship between price stickiness of individual firms and the rigidity of aggregate price level. We can guess that these two things may not be related, and therefore, price stickiness may not bring about the non-neutrality of

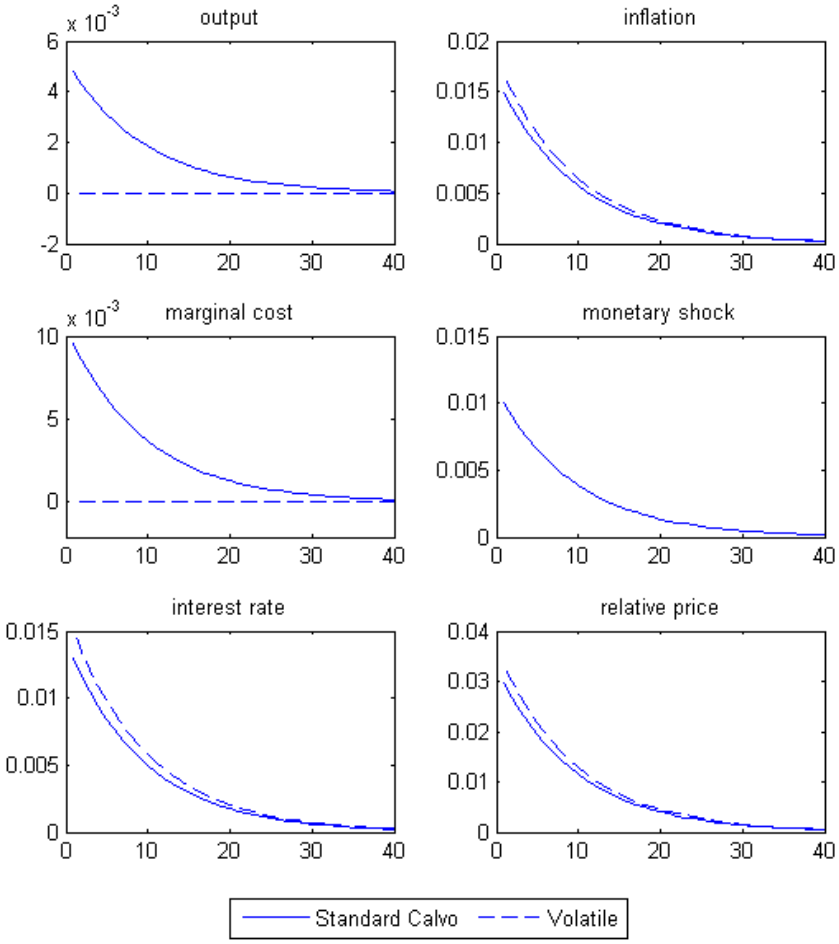

Figure 1.3: Neutral Money with Positive Value of θ

money. These hypotheses are supported by the next simulation result as in Figure 1.3. In this simulation, we set θ equal to be 2/3 for both the models but give more volatility by setting α to be 1.367 rather than 1.2 for the 'Volatile Prices' model. As can be seen, the model shows fully neutral money. That is, in this model, a small fraction of repricing firms raise their prices fully enough to absorb the whole monetary shocks. As is well known, in the standard Calvo model, we should set θ to be zero in order to get fully neutral money. However, in the proposed model, even with a very positive value of θ, that is even with very considerable degree of price stickiness, money has no real effect. Therefore, with this result, we can say that price stickiness does not always guarantee the non-neutrality of money.

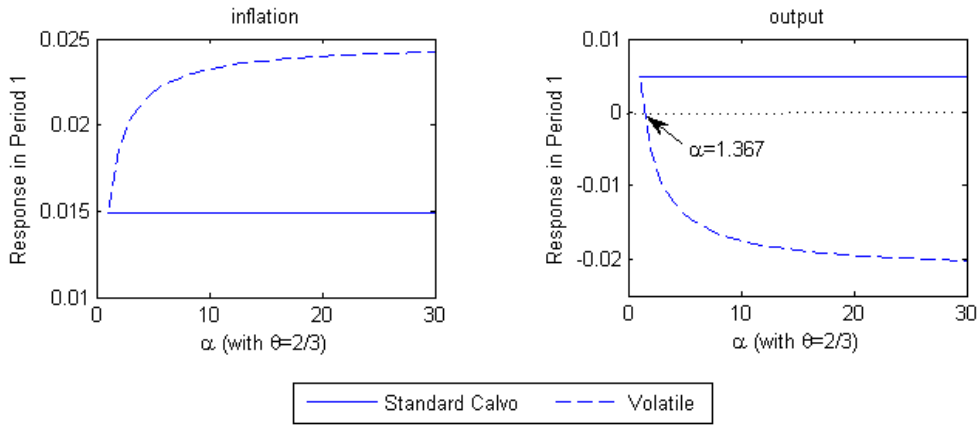

Figure 1.4: Impulse Responses with Different Value of α

1.5.4 Pricing Volatility and Discount Factor

As mentioned above, in the 'Volatile Prices' model, the main factor which leads to the above disconnection between price stickiness and the aggregate price rigidity is the assumption that repricing firms set more volatile prices than those in the standard Calvo model. It is also shown that increased volatility of pricing by this assumption has the effect of decreasing the degree of price stickiness in the standard Calvo framework. However, the result as shown in Figure 1.3 suggests that there is another factor which brings about the above result. If the only effect that the assumption of the volatile prices has in the proposed model is just to lower the degree of price stickiness, then it is expected that the economy expressed by the NKPC as in (1.9) should have no price stickiness. In other words, $\tilde{\theta}$ in equation (1.12) should be zero with α of 1.367. However, $\tilde{\theta}$ is calculated at 0.594 with the value of α.

Impulse response as shown in Figure 1.4 is another evidence that the effect of lowering $\tilde{\theta}$ is not everything. Figure 1.4 shows the responses of inflation and output in the first period after positive monetary shock with different levels of α.[21] In

[21] As is well known, the responses of main variables in the standard framework are expressed as increasing or decreasing linear functions of their own values in the previous period. In other words, the response function of $\hat{x}_t$ in equilibrium is given by

$$\hat{x}_t = \rho \hat{x}_{t-1},$$

where ρ is the persistence parameter of the monetary shocks. The model in this chapter shows the same dynamics of responses. Therefore, even though the numerical differences will change, it is

particular, with α above 1.367, it can be verified that the real effect of money occurs in the opposite direction. That is, with the expansionary money shock and the resulting price increase, the aggregate output drops down, which is beyond our common sense. This eccentric result is due to inflation rising up higher than the level where the effect of shocks are fully absorbed as the degree of volatility in repricing firm's pricing, α, becomes extremely big, and therefore, the effect of rising prices of the firms overwhelms the economy. In the standard Calvo framework, this result cannot be obtained by only lowering the degree of individual price stickiness. Then what is the factor which makes this result possible? It can be found in equation (1.11) which shows the functional relationship between $\tilde{\beta}$ and α. From this, we have

$$\frac{\Delta\tilde{\beta}}{\Delta\alpha} = \beta(1-\theta) > 0, \tag{1.14}$$

which means that $\tilde{\beta}$ increases as α gets larger. This is another effect that increasing α brings about besides lowering $\tilde{\theta}$. In other words, increasing volatility in individual firm's pricing has the effect of not only decreasing the degree of price stickiness but also increasing the discount factor in the standard Calvo model. Generally speaking, the bigger the discount factor is, the more weight economic agents put on the future profits. Therefore, within the standard Calvo framework, repricing firms with a high value of discount factor are very much concerned about the possibility of not changing prices next period and, consequently, losing profits. Therefore, firms set very high prices even with very small amount of expansionary money shocks.

Figure 1.5 shows that this effect on $\tilde{\beta}$ is greater than that on $\tilde{\theta}$ in achieving much higher inflation response against positive money shock. In the figure, we can find that the inflation responses in the first period after the shock are plotted in $\left(\tilde{\theta}, \tilde{\beta}\right)$ space. Firstly, the dotted line is for the 'Volatile Prices' model with different value of α. In other words, the dotted line shows the sets of the size of inflation responses,

expected that the characteristic differences of the responses in the first period after shocks between the models will continue until the shocks finally disappear. In that sense, we can grasp the whole picture of dynamics even just through the first period after the shock.

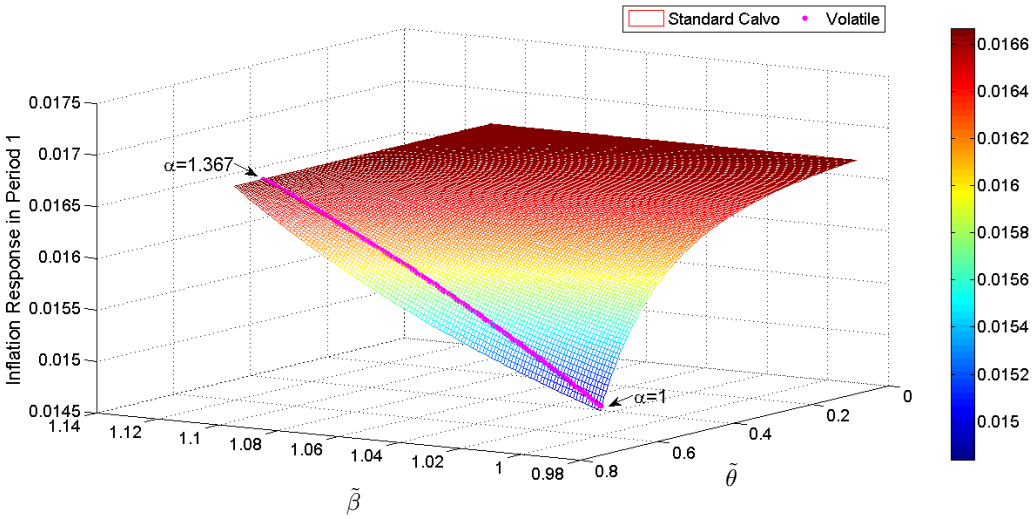

Figure 1.5: Inflation Response with Different Value of $\tilde{\beta}$ and $\tilde{\theta}$

$\tilde{\beta}$, and $\tilde{\theta}$, which are corresponding to the value of α between 1 to 1.367.[22] We can easily find that as α increases, inflation response gets larger with $\tilde{\beta}$ increasing and $\tilde{\theta}$ decreasing. The three-dimensional shape with a curved surface in this figure is for the standard Calvo model with the form of NKPC as in (1.9). The surface shows all the levels of inflation responses which can be obtained with every possible sets of $\tilde{\beta}$ and $\tilde{\theta}$. As can be seen, decreasing $\tilde{\theta}$ and increasing $\tilde{\beta}$ bring about the increase of inflation responses.[23] However, this figure shows that the major driving force behind the rising level of inflation response in the 'Volatile Prices' model is the increase of $\tilde{\beta}$ rather than the decrease of $\tilde{\theta}$. As summarized in Table 1.1, $\tilde{\beta}$ increases from 0.99 to 10/9 and $\tilde{\theta}$ decreases from 2/3 to 0.594 as α increases from 1 to 1.367 with β equal to be 0.99 and θ equal to be 2/3. These movements raise the level of inflation

[22]The value of α is limited to the range between 1 and 1.367 because the minimum value of α is one by construction and the values over 1.367 brings about the real effect in the opposite direction which is beyond our interest.

[23]However, the effect of decreasing $\tilde{\theta}$, given the value of $\tilde{\beta}$, is not positive any more in raising the level of inflation responses when α (or $\tilde{\beta}$) is greater than 1.367 (or 10/9). Even though the decrease of $\tilde{\theta}$ in the economy expressed by the NKPC of (1.9) means that more firms can adjust their prices, and therefore, the aggregate price level rises even higher, we should note that there is another meaning that the probability of not changing prices next period is decreasing. With the range of high $\tilde{\beta}$, the repricing firms in this economy of (1.9) take much account of future profit and, therefore, worry about the possibility of not changing prices next period which would bring about the loss of profit as stated above. However, in this case, decreasing $\tilde{\theta}$ lowers the possibility of not changing prices, offsetting the effect of $\tilde{\beta}$. Even though this offsetting effect is very small with a low value of $\tilde{\beta}$, it becomes much larger as $\tilde{\beta}$ increases.

$\tilde{\beta}$ \ $\tilde{\theta}$	$2/3$ ($\alpha = 1$)	$\longrightarrow$	0.594 ($\alpha = 1.367$)
0.99 ($\alpha = 1$)	0.0149	$\longrightarrow$	0.0155
$\downarrow$	$\downarrow$	$\searrow$	$\downarrow$
$10/9$ ($\alpha = 1.367$)	0.0167	$\longrightarrow$	0.0167

Table 1.1: Size of Inflation Response with Different Values of $\tilde{\beta}$ and $\tilde{\theta}$

response from 0.0149 to 0.0167. However, while the increase of $\tilde{\beta}$ could raise the response to the maximum level even if there would have been no movement of $\tilde{\theta}$, the decrease of $\tilde{\theta}$ could raise the response only to the level of 0.0155 without any changes in $\tilde{\beta}$. Therefore, it can be concluded that most of the increase in inflation response in the model is due to the increase of $\tilde{\beta}$.

1.5.5 Limitations

The above results show that volatile reset price can make even neutral money through the decrease of $\tilde{\theta}$ and the increase of $\tilde{\beta}$. However, as mentioned before, this model is not for the reflection of the real world but for a simple illustration of the theoretical possibility of non-relationship between price stickiness and the real effect of money. Therefore, we cannot say the above result demonstrates that money can be neutral in the real world. However, we can claim that monetary policy can be less powerful than we expect when firms' reset prices are very volatile and that price stickiness may not be a sufficient condition for the non-neutrality of money.

Also, for the purpose of simplicity, this model leaves out some important factors to be considered. Firstly, the monetary authority in this model is assumed to implement the same policy rule as in the standard model. However, we can easily expect that the monetary policy will vary with different structure of economy. Secondly, this model does not say anything about the change in welfare of the economy when firms' reset prices are very volatile. As shown in the model equations above, firm's reset price (p_t^v) deviates from the optimal level (p_t^*) in terms of firm's profit

maximization. Therefore, welfare in the economy might be worsened by the higher volatility of the reset price compared to that of the standard model. Finally, as mentioned several times above, the model in this chapter does not explain what causes such volatility in firms' reset prices. Any change or relaxation of the strict assumptions in the standard model might be able to bring about the volatility. Colluding firms or boundedly rational pricing behaviours as in the frameworks proposed in the next two chapters would be one of the examples.

Therefore, all the simulation results of this model should be understood with the consideration of the limitations above, and future research should be focused on solving those limitations.

1.6 Conclusion

The simple Calvo model with volatile prices of individual firms shows that it is possible to have very small amount of real effect of money or fully neutral money in spite of the fact that there exists price stickiness in the pricing behaviour of individual firms. In this 'Volatile Prices' model, the volatility of reset price has the same effect as that of lowering the degree of price stickiness and increasing the future discount factor in the standard Calvo framework. This effect makes the aggregate price level much more flexible than the standard model, and therefore, price stickiness cannot fully generate the non-neutrality of money. There are two implications from the results. Firstly, the model shows that there is a need to pay more attention to the repricing firms and the volatility of their prices when analysing monetary policy. Price stickiness featured by the Calvo mechanism mainly focuses on the firms which cannot adjust prices and does not say much about the volatility of the reset price. However, the results of the model in this chapter demonstrate that the effect of unadjusted prices can be cancelled out by the volatile movements of the adjusted prices. The second implication which is more important can be obtained from the fact that the result of the model challenges the conventional view that the main factor which produces the non-neutrality of money is price stickiness. According to

the results, money can be neutral even though the price of individual firm is sticky under certain circumstances like the case where the repricing firms respond very elastically to the monetary shocks as in the proposed model. Is it possible to give a categorical assurance that the reset price in the real world is not more volatile than the standard Calvo model suggests? If it is not, it is clear that the Calvo-type price stickiness is not sufficient in producing the non-neutrality of money. Also, we need to note that what is important for the non-neutrality of money is the stickiness of the aggregate price level itself rather than that of the individual price. In other words, neither searching micro evidences on the stickiness of individual prices nor modelling such stickiness is sufficient to argue that monetary policy has the real effect on the economy. Instead, effort should be made to find out which other factors make the aggregate price level sticky and to make models which feature such factors.

Appendix 1.A

Taking the real wage as given, firm i hires an optimal level of labour by minimizing the production costs as

$$\min_{h_{it}} \frac{W_t}{P_t} h_{it} \qquad \text{s.t.} \quad y_{it} = h_{it},$$

where W_t is the nominal wage. The first order condition with respect to h_{it} yields

$$\gamma_t = \frac{W_t}{P_t},$$

where γ_t is the Lagrangian multiplier. Since γ_t means the additional real cost of producing an extra unit, we can interpret γ_t as the real marginal cost, mc_t, as

$$mc_t = \frac{W_t}{P_t} = w_t, \tag{1.15}$$

where w_t denotes the real wage.

Appendix 1.B

Households

It is assumed that there are a large number of infinitely-lived households of measure 1. The representative household maximizes the expected present value of utility by choosing the aggregate consumption (c_t), the amount of hours of labour supply (h_t), and the bond holding (B_t). This utility maximization problem is given by

$$\max E_t \sum_{k=0}^{\infty} \beta^k \left(\frac{c_{t+k}^{1-\sigma} - 1}{1-\sigma} - \frac{h_{t+k}^{1+\varrho}}{1+\varrho} \right) \quad \text{s.t.} \quad \int_0^1 p_{it} c_{it} di + Q_t B_t = B_{t-1} + W_t h_t + \Phi_t,$$

where β is the discount factor, σ is the inverse of the inter-temporal elasticity of substitution, ϱ is the inverse of the Frisch elasticity of labour supply, p_{it} is the price of differentiated good of firm i, c_{it} is a consumption of differentiated goods of firm i, Q_t is the price of bonds, W_t is the nominal wage, and Φ_t is the nominal dividends from firms.

(Expenditure Minimization)

The aggregate consumption, c_t, is defined as

$$c_t = \left(\int_0^1 c_{it}^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}}, \tag{1.16}$$

where $\epsilon > 1$ is the price elasticity of demand. Given any level of aggregate consumption, c_t, each household minimizes consumption expenditure as

$$\min_{c_{it}} \int_0^1 p_{it} c_{it} di \qquad \text{s.t.} \quad c_t = \left(\int_0^1 c_{it}^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}}.$$

Then the first order condition with respect to c_{it} gives

$$p_{it} - \mu_t \left[\left(\int_0^1 c_{it}^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{1}{\epsilon-1}} c_{it}^{-1/\epsilon} \right] = 0, \tag{1.17}$$

where μ_t is the Lagrangian multiplier. Using (1.16), we can simplify this into

$$c_{it} = c_t \left(\frac{p_{it}}{\mu_t} \right)^{-\epsilon}. \tag{1.18}$$

Putting this into (1.16), we get

$$\mu_t = \left[\int_0^1 (p_{it})^{1-\epsilon} \, di \right]^{\frac{1}{1-\epsilon}}. \tag{1.19}$$

Since the Lagrangian multiplier (μ_t) is the marginal cost needed for getting one unit of the aggregate consumption (c_t), μ_t can be interpreted as the aggregate price index (P_t) as

$$P_t = \left[\int_0^1 (p_{it})^{1-\epsilon} \, di \right]^{\frac{1}{1-\epsilon}}. \tag{1.20}$$

Then, from (1.18) and (1.20), we have the demand for each differentiated good as

$$c_{it} = c_t \left(\frac{p_{it}}{P_t} \right)^{-\epsilon}. \tag{1.21}$$

(Utility Maximization)

Using (1.20) and (1.21), the utility maximization problem of the representative household can be re-expressed by

$$\max E_t \sum_{k=0}^{\infty} \beta^k \left(\frac{c_{t+k}^{1-\sigma} - 1}{1-\sigma} - \frac{h_{t+k}^{1+\varrho}}{1+\varrho} \right) \quad \text{s.t.} \quad c_t + Q_t b_t = \frac{b_{t-1} P_{t-1}}{P_t} + \frac{W_t}{P_t} h_t + \phi_t,$$

where $b_t (= B_t/P_t)$ is the real bond and $\phi_t (= \Phi_t/P_t)$ is the real profit. The first order conditions with respect to c_t, b_t, and h_t are respectively given by

$$c_t^{-\sigma} = \lambda_t \tag{1.22}$$

$$Q_t = \beta E_t \left(\frac{P_t}{P_{t+1}} \frac{\lambda_{t+1}}{\lambda_t} \right) \tag{1.23}$$

$$\lambda_t \frac{W_t}{P_t} = \lambda_t w_t = h_t^{\varrho}, \tag{1.24}$$

where λ_t is the Lagrangian multiplier and $w_t\,(= W_t/P_t)$ denotes real wage at time t. Equation (1.22) is the marginal utility of consumption, equation (1.23) is the Euler equation which shows the condition for consumption optimality, and equation (1.24) represents the optimality condition for labour supply.

Monetary Authority

The central bank as the monetary authority is assumed to implement monetary policy by following the simple interest rate rule.

$$\frac{R_t}{R} = \left(\frac{\pi_t}{\pi}\right)^{\eta_\pi} \left(\frac{y_t}{y}\right)^{\eta_y} (\nu_t)^{-1}, \tag{1.25}$$

where R_t is the gross interest rate which is the same as the inverse of the bond price $(R_t = 1/Q_t)$, and ν_t is the monetary policy shock which follows[24]

$$\frac{\nu_t}{\nu} = \left(\frac{\nu_{t-1}}{\nu}\right)^{\rho} \exp(e_t), \qquad e_t \sim N(0, \varsigma), \tag{1.26}$$

where ρ is the persistence parameter of the shocks. The coefficients, η_π and η_y, show the degree of responsiveness of the monetary authority on inflation and aggregate, respectively.

Market Clearing

Firstly, the demand for each good (c_{it}) should be equal to supply (y_{it}) so that we have $c_{it} = y_{it}$. Let the aggregate output (y_t) be defined as $y_t = \left(\int_0^1 y_{it}^{\frac{\epsilon-1}{\epsilon}} di\right)^{\frac{\epsilon}{\epsilon-1}}$. Then the market clearing condition in the goods market is given by

$$y_t = c_t. \tag{1.27}$$

The labour market clears with the following condition.

$$\int_0^1 h_{it} di = h_t.$$

[24]Note that from (1.25), the positive value of ν_t means the expansionary monetary policy shocks.

From the demand function (1.21) and goods market's clearing condition (1.27), we have

$$y_{it} = y_t \left(\frac{p_{it}}{P_t}\right)^{-\epsilon}.$$

Since firm's production technology is given by $y_{it} = h_{it}$, we have

$$h_{it} = y_t \left(\frac{p_{it}}{P_t}\right)^{-\epsilon}.$$

Integrating both sides of the equation, we can get

$$h_t = y_t s_t,$$

where $s_t \left(= \int_0^1 \left(\frac{p_{it}}{P_t}\right)^{-\epsilon} di\right)$ is a measure of price dispersion. Since s_t is equal to one up to a first order approximation, the market clearing condition becomes

$$h_t = y_t. \tag{1.28}$$

Appendix 1.C

Equilibrium

Equilibrium of this model can be obtained when all the endogenous variables, which are the stationary processes of c_t, λ_t, h_t, w_t, y_t, R_t, π_t, mc_t, $\tilde{p}_t^v$, $\tilde{p}_t$, f_{1t}, f_{2t}, and the exogenous variable, which is the stochastic process of $\{\nu_t\}_{t=0}^{\infty}$, satisfy the equations of (1.3)∼(1.7), (1.15) and (1.22)∼(1.28).

Steady State

The steady states of variables which are calculated by removing the time subscripts from the above equations are presented in Table 1.2.

Variable	Steady State
$\pi, \nu, \tilde{p}^v, \tilde{p}$	1
R	$1/\beta$
mc	$(\epsilon - 1)/\epsilon$
y	$mc^{\frac{1}{\sigma+\varrho}}$
h, c	y
λ	$c^{-\sigma}$
w	mc
f_1	$\lambda ymc/(1-\beta\theta)$
f_2	$\lambda y/(1-\beta\theta)$

Table 1.2: Steady State Values of Variables of Volatile Prices Model

Log-linearization

All the equations of (1.3)∼(1.7), (1.15) and (1.22)∼(1.28), which are needed for the equilibrium, are log-linearized as

$$\hat{\tilde{p}}_t^v = \frac{\theta}{1-\theta}\hat{\pi}_t \tag{1.29}$$

$$\hat{\tilde{p}}_t^v = \alpha\hat{\tilde{p}}_t \tag{1.30}$$

$$\hat{\tilde{p}}_t = \hat{f}_{1t} - \hat{f}_{2t} \tag{1.31}$$

$$\hat{f}_{1t} = (1-\beta\theta)\left(\hat{\lambda}_t + \hat{y}_t + \hat{mc}_t\right) + \beta\theta E_t\left[\epsilon\hat{\pi}_{t+1} + \hat{f}_{1t+1}\right] \tag{1.32}$$

$$\hat{f}_{2t} = (1-\beta\theta)\left(\hat{\lambda}_t + \hat{y}_t\right) + \beta\theta E_t\left[(\epsilon-1)\hat{\pi}_{t+1} + \hat{f}_{2t+1}\right] \tag{1.33}$$

$$\hat{mc}_t = \hat{w}_t \tag{1.34}$$

$$\hat{\lambda}_t = -\sigma\hat{c}_t \tag{1.35}$$

$$\hat{\lambda}_t = E_t\left[\hat{\lambda}_{t+1}\right] - E_t\left[\hat{\pi}_{t+1}\right] + \hat{R}_t \tag{1.36}$$

$$\varrho\hat{h}_t = \hat{\lambda}_t + \hat{w}_t \tag{1.37}$$

$$\hat{R}_t = \eta_\pi\hat{\pi}_t + \eta_y\hat{y}_t + \hat{\nu}_t \tag{1.38}$$

$$\hat{\nu}_t = \rho\hat{\nu}_{t-1} + e_t \tag{1.39}$$

$$\hat{y}_t = \hat{c}_t \tag{1.40}$$

$$\hat{y}_t = \hat{h}_t. \tag{1.41}$$

Calibration

The parameters used in this model are calibrated following the standard assumptions in the literature. Basically, the frequency in the model is assumed to be quarterly. With this assumption on the frequency, the constant discount factor, β, is set to be 0.99 following the fact that the average nominal annual interest rate in the US is around 4%. The inverse of the inter-temporal elasticity of substitution, σ, is set to be 1 following the literature. The measure of the inverse of the Frisch elasticity of labour supply, ϱ, is also set to be 1 as in numerous literature. The price elasticity of demand, ϵ, is set to be 11 to match the data that average mark-ups are around 10%

in the steady state. Also, the Calvo parameter, θ, is set to be 2/3 to match the micro data that firms re-set their prices once every three quarters on average. Following Taylor (1993), the coefficients on inflation and output, η_π and η_y, are set to be 1.5 and 0.125, respectively.[25] Lastly, the persistence parameter of the monetary shocks, ρ, is set to be 0.9.

[25]Taylor (1993) estimates η_y as 0.5 under the annual frequency. In this chapter, since the model is based on the quarterly frequency, a quarter of 0.5 (=0.125) is used instead.

Chapter 2

Colluding Firms with Price Stickiness and the Non-neutrality of Money

2.1 Introduction

Many economists and central banks believe that money is non-neutral at least in the short-run. That is, they believe that monetary policy of central bank or any monetary shocks can make significant changes on the real economy. The belief seems to be supported by many evidences showing that monetary shocks are not followed by the quick response of aggregate price and, therefore, give rise to the change of aggregate output.[1] Then what makes the non-neutrality of money? Many economists supporting the above belief seem to attribute the real effect of money to firm's price stickiness. Ball and Mankiw (1994) regard price stickiness as the best answer for the non-neutrality of money, and Woodford (2003) claims that monetary policy for stabilization of an economy is justified by the sluggish adjustment of price. Such beliefs have made the economists and central banks establish models for the monetary policy analysis with the micro-foundation of price stickiness by giving

[1] Christiano, Eichenbaum, and Evans (1999) is the representative example showing the evidence of the non-neutrality of money.

restrictions on firm's pricing either exogenously as in Taylor (1980), Calvo (1983), and Mankiw and Reis (2002), or endogenously as in many menu cost models.[2] In particular, the Calvo framework has been the most popular for its simplicity and tractability, and therefore, the majority of monetary models of central banks are based on the Calvo mechanism.[3]

The phenomenon of sticky price is easily observed in the real economy in the sense that we can find many firms whose prices are not changed for a period of time. Furthermore, there are many survey papers providing micro evidences of the phenomenon such as Bils and Klenow (2004), Angeloni et al. (2006), Nakamura and Steinsson (2008), and Klenow and Malin (2010). However, there seems to be a lack of consensus on whether such price stickiness can always lead to the non-neutrality of money. Many different opinions are addressed and discussed, but the discussion has been restricted to only some frameworks of price stickiness such as menu cost model, e.g., in Midrigan (2011). As for the Calvo framework which is the most popular tool for price stickiness, the relationship between price stickiness and the real effect of money has rarely been questioned. The aim of this chapter is to examine whether price stickiness featured by the Calvo framework can always give rise to the non-neutrality of money. In association with Chapter 1 which shows that the volatility of optimal price can weaken the ability of price stickiness to produce the real effect of money, this chapter presents more micro-founded endogenous mechanism for the volatile optimal price and checks whether the result of Chapter 1 is augmented with the mechanism. In particular, this chapter investigates how different treatments of information on aggregate price affect firms' pricings and inflation response. In the standard Calvo model, in response to a positive monetary shock, some firms are stuck to the price level of the previous period and other firms raise their prices up to the new optimal level maximizing their expected profits. However, due to price

[2]Examples of the menu cost models include Caplin and Spulber (1987), Caballero and Engel (1991), Dotsey, King, and Wolman (1999), Golosov and Lucas (2007), Gertler and Leahy (2008) and Midrigan (2011).

[3]For example, as Kara (2011) indicates, the models of Christoffel, Coenen, and Warne (2008) and Motto, Rostagno, and Christiano (2008), which are the main tools for monetary policy analysis of the European Central Bank (ECB), are based on the Calvo system.

stickiness, the aggregate price level does not rise enough to offset the monetary shock. This chapter focuses on the optimization processes of the firms which are allowed to change their prices under this framework. In the standard model, the processes are implemented under the assumption that the aggregate price level is just given and that each firm treats it as an exogenous factor when choosing its optimal price. However, as explained in Section 2.2, the aggregate price level comes to contain the information on the link between each firm's optimal price and the aggregate price level as long as the optimal price is the same for all firms. In other words, so long as the homogeneous firms treat the level of aggregate price in the same way, and thus, their optimal prices are the same, the aggregate price level can be expressed by a function of the optimal price. This means that the level of aggregate price cannot be an exogenous variable anymore in each firm's price adjustment. Therefore, each firm might have enough incentive to use the information of aggregate price level when adjusting its price, though the condition should be met that all repricing firms use the information equally. What would happen if firms behave as in "collusion" so as to take the aggregate price level as a function of their optimal prices? If firms recognize that their choices of optimal prices can affect the aggregate price level and know the relationship between the optimal price and the aggregate price level exactly, we can expect that the new optimal prices of the "colluding" firms will be higher than those of firms in the standard Calvo model as we can anticipate that a monopoly's price is higher than that of a perfectly competitive market.

In order to investigate whether the link between the Calvo-style price stickiness and the real effect of money can be firmly maintained even in the above-explained framework of colluding firms, this chapter proposes a model which is the same as the standard Calvo pricing framework, except that the repricing firms behave as in collusion so as to exploit the dynamics of the aggregate price level. Since some fraction of firms are still not allowed to reset their prices as in the standard Calvo framework, price stickiness is still featured in this modified Calvo model. However, it is shown that, in spite of the sticky price, the real effect of monetary policy is

very small compared to the standard model. This is because the colluding firms set their prices very high to achieve the monopolistic gains. This result means that price stickiness of an individual firm which is generated by the Calvo framework may not always imply the non-neutrality of money. Even though the price of an individual firm is sticky, the aggregate price level may not be sticky under certain circumstances like the case where the repricing firms respond very elastically to the monetary shock as in the proposed model.

This chapter relates to some menu cost models which argue the neutrality of money. Caplin and Spulber (1987) show no relationship between price stickiness and the aggregate price level in response to monetary shocks, using an Ss framework with an assumption of uniformly distributed prices. Golosov and Lucas (2007) also claim that money can be approximately neutral in spite of price stickiness, when idiosyncratic productivity shocks coexist with aggregate monetary shocks under the menu cost framework. These two papers are very closely related to this chapter in the sense that they cast doubt on the relationship between firm's sticky price and the aggregate price level. However, the biggest difference from this chapter is that they are based on the menu cost framework under which firms reset prices only when the benefits of changing price is greater than the menu costs. Under such framework, size of price changes cannot be dispersed, which is not consistent with micro evidences. Klenow and Kryvtsov (2008), Nakamura and Steinsson (2010), and Midrigan (2011) argue that even the menu cost framework can produce the non-neutrality of money when the wide dispersion of price changes are featured in the model.

Another related paper is Head et al. (2012) which claims that sticky price can be observable even in a flexible economy, using the approach of the *New Monetarist Economics*.[4] Even though they also show that price stickiness has nothing to do with the rigidity of aggregate price, their framework is far from the New Keynesian model, not to mention the Calvo framework. The consequence of such a different framework is that, in Head et al. (2012), price stickiness is just an outcome of firms'

[4]Refer to Williamson and Wright (2010a, 2010b) for detailed explanations on the *New Monetarist Economics*.

discretionary choices under the circumstance where they do not have to change prices within a certain range of price in which firms have the same profits. However, as is well known in the Calvo framework as well as other New Keynesian models, sticky price of individual firm is not just a firm's decision, but an unavoidable consequence of the costs which come with adjustment of price.[5]

The rest of this chapter is organized as follows. Section 2.2 gives an idea on firms which behave as in collusion, and explains how they are incorporated in the Calvo framework. Section 2.3 presents the detailed feature of the simple New Keynesian Calvo model in which the colluding firms are embodied. Section 2.4 shows the simulation results on how the colluding firms react to monetary shocks and how inflation and output response are different from the standard model, and discusses the meaning of the results with respect to the relationship between price stickiness and the non-neutrality of money. Finally, section 2.5 summarizes the main arguments of this chapter and concludes with further issues to be handled in future research.

2.2 Calvo Pricing of Colluding Firms

As expressed in Chapter 1, in the standard Calvo model, the dynamics of the aggregate price level (P_t) is given by

$$P_t^{1-\epsilon} = \theta P_{t-1}^{1-\epsilon} + (1-\theta)(p_t^*)^{1-\epsilon}, \tag{2.1}$$

where ϵ is the price elasticity of demand, θ is the probability with which firms cannot change their prices, and p_t^* denotes the average of each firm i's optimal price (p_{it}) which is defined by

$$(p_t^*)^{1-\epsilon} = \int p_{it}^{1-\epsilon} di, \qquad i \in (1-\theta). \tag{2.2}$$

[5]See Kryvtsov (2010) and Gorodnichenko and Weber (2013) for the discussion on the meaning of price stickiness in Head et al. (2012).

If there is no heterogeneity among firms such as idiosyncratic shocks, all firms will have the same optimization problem, and therefore, the optimal prices of all repricing firms should be the same. In other words, we have to have

$$p_{it} = p_t^* \tag{2.3}$$

for all $i \in (1-\theta)$. Hence, for each firm i, the dynamics of the aggregate price level can be given as in

$$P_t^{1-\epsilon} = \theta P_{t-1}^{1-\epsilon} + (1-\theta) p_{it}^{1-\epsilon}. \tag{2.4}$$

Since, in the standard model, all firms are assumed to have perfect information on the economy, we can interpret that firm i has already the information of (2.4) which shows the relationship between the aggregate price level and its own optimal price at the moment of price setting. This means the firm knows that P_t is the endogenous variable which is affected by its optimal price, p_{it}. Therefore, the equation (2.4) should be one of the main considerations in the optimization process of the firm. However, firms do not use this information in deriving their optimality condition in the standard framework. They just take P_t as given in the sense that there are so many firms in the economy and that any individual firm cannot affect the aggregate price level. In other words, firms treat P_t as an exogenous factor in solving their optimization problems. More precisely, taking P_t as given and using the demand curve[6] which is given by $y_{it} = (p_{it}/P_t)^{-\epsilon} y_t$, the optimization problem of each firm i, which is the profit maximization as

$$\max_{p_{it}} E_t \sum_{s=0}^{\infty} \theta^s Q_{t,t+s} P_{t+s} \left(\frac{p_{it}}{P_{t+s}} - mc_{t+s} \right) y_{it+s}, \tag{2.5}$$

yields the first order condition as in

$$E_t \sum_{s=0}^{\infty} \theta^s Q_{t,t+s} \left(\frac{p_{it}}{P_{t+s}} \right)^{-\epsilon} y_{t+s} \left[1 - \frac{\epsilon}{\epsilon - 1} mc_{t+s} \left(\frac{p_{it}}{P_{t+s}} \right)^{-1} \right] = 0, \tag{2.6}$$

[6]This can be derived by the expenditure minimization of households and the market clearing condition. See Appendix 1.B of Chapter 1 for details.

where $Q_{t,t+s}$ is the stochastic discount factor, mc_t is the marginal cost, y_{it} is the individual demand of output which the firm i faces, and y_t is the aggregate level of output. Therefore, at a glance, it seems questionable that firms do not use the information on the dynamics of the aggregate price level and just take P_t as given when they set their optimal prices. However, we need to check the nature of the information of (2.4). The equation is based on the fact that all firms have the same optimal price as in (2.3). The reason we can have the same optimal price is because all firms are equally assumed to take P_t as given in their price settings. In other words, the same optimal price among firms is the ex-post notion which is valid only if all firms have the same concept on P_t when choosing optimal prices. In the standard framework, P_t is assumed to be given for all firms, and only then, the same optimal price and the information on the dynamics of P_t as in (2.4) can be obtained. If any of the firms deal with P_t differently, then (2.3) and (2.4) do not hold any longer. The two equations are just outcomes of pricing with the assumption of given P_t, not the information which can be used in the process of pricing. In this sense, it is very natural that firms do not use the information on the dynamics of P_t at the stage of deriving the first order condition.

However, in spite of the discussion above, what would happen if firms actively use the information of (2.4)? Is there any possibility that firms use the knowledge on the relationship between their optimal prices and the aggregate price level when adjusting prices? As we have seen above, the same optimizing behaviour of maximizing (2.5) for a given P_t yields the same optimal price for all firms. However, taking P_t as given is not the only way of generating the same p_t^*. As long as all firms maximize their profits in the same way as in (2.5), all that is needed to achieve the same optimal price is for the firms to deal with P_t equally regardless of how P_t is treated. Since all firms are assumed to be rational and have all the information on the economy, they already know the fact mentioned above at the moment of price setting. If it is the case, it might be possible that firms want to exploit the information on the dynamics of P_t as in (2.4) with common consent when setting

the price. In other words, if all firms treat P_t equally as a function of (2.4) when deriving their optimality conditions, their optimal prices would be the same, and therefore, the information on P_t they used in price setting proves to be correct and model-consistent. In this sense, the model of this chapter assumes that all repricing firms take P_t as a function of their own optimal prices in anticipation of the same behaviours of other firms. That is, they treat P_t equally as an endogenous variable and exploit the relationship between the optimal price and the aggregate price level as in (2.4) by using the equation in deriving their optimality condition. In such case, we can interpret that the repricing firms behave as in collusion in the sense that the treatment of P_t as a function of optimal price is possible only when they have a strong conviction that all the resetting firms will deal with P_t in the same manner. Also, it can be said that the firms behave as if they form a monopoly because they recognize that they can exercise an influence on the aggregate price level by the treatment of information on the dynamics of P_t. Therefore, we may be able to expect that the optimal price of the colluding firms would be different from that of the standard model as we do with the monopolistic and competitive markets. In the next section, a model is developed by incorporating the above assumption onto the base of standard Calvo pricing framework, and it is checked whether the Calvo pricing can guarantee the non-neutrality of money with the colluding firms.

2.3 Model

This chapter uses the standard Calvo framework in which households and firms make decisions by maximizing utility and profit, respectively, and the monetary authority carries out the policy following a standard interest rate rule. However, as explained below, the difference from the standard model comes from the assumption that firms behave as in collusion.

2.3.1 Firms

A continuum of firms $i \in [0, 1]$ produces its own good, y_{it}, with hired labour, h_{it}, as in

$$y_{it} = h_{it}.$$

Given real wage, w_t, the firm i's intra-temporal problem of minimizing expenditure gives the following condition.[7]

$$mc_t = w_t \tag{2.7}$$

Inter-temporal Problem

As in the standard Calvo framework, a fraction θ of firms cannot adjust their prices, while the remaining firms reset their optimal prices, p_{it}. Therefore, the aggregate price index, $P_t^{1-\epsilon} = \int_0^1 p_{it}^{1-\epsilon} di$, gives the following dynamics of aggregate price level as

$$\begin{aligned} P_t^{1-\epsilon} &= \int_{\omega_t} p_{it-1}^{1-\epsilon} di + \int_{1-\omega_t} p_{it}^{1-\epsilon} di \\ &= \theta P_{t-1}^{1-\epsilon} + (1-\theta)(p_t^*)^{1-\epsilon}, \end{aligned} \tag{2.8}$$

where ω_t denotes the set of firms which have to stick to their previous levels of prices, and p_t^* is the average optimal price at time t. In our framework, all the repricing firms have the same profit maximization problem as in (2.5) and assumed to use the above information on the dynamics of P_t as if they were in collusion, which means that they face the same optimization problem including the way of dealing with the information on P_t. Therefore, their optimal prices are the same, and we can have $p_{it} = p_t^*$ for all repricing firms. In this case, the information on P_t which firms use for their price settings turns to be

$$P_t^{1-\epsilon} = \theta P_{t-1}^{1-\epsilon} + (1-\theta)(p_{it})^{1-\epsilon}, \tag{2.9}$$

[7] See Appendix 1.A of Chapter 1 for the derivation.

where p_{it} is equal to the average optimal price of the economy. This means that the repricing firms take the aggregate price (P_t) as a function of their own optimal prices ($p_{it} = p_t^*$). That is, the repricing firms exploit the dynamics of the aggregate price level, (2.8) or equivalently (2.9), in their optimization problems. Thus, given mc_t and the demand for goods, each firm's optimization problem of profit maximization as in (2.5) can be re-expressed by the following value functions.

$$\bar{V}(p, P_t) = \phi(p, P_t) + E_t\left[\theta Q_{t,t+1}\bar{V}(p, P_{t+1}) + (1-\theta) Q_{t,t+1}\tilde{V}(P_t)\right] \tag{2.10}$$

$$\tilde{V}(P_{t-1}) = \max_{p_t^*} \bar{V}(p_t^*, P_t), \tag{2.11}$$

where P_t satisfies

$$P_t^{1-\epsilon} = \theta P_{t-1}^{1-\epsilon} + (1-\theta)(p_t^*)^{1-\epsilon}, \tag{2.12}$$

and $\bar{V}(p, P_t)$ denotes the value function for a firm that is not allowed to change its price and, therefore, inherits a price p from a previous period and now faces an aggregate price P_t. The notation of $\phi(p, P_t)$ is the real profit of a firm charging price p and facing an aggregate price P_t as in

$$\phi(p_t, P_t) = \left(\frac{p_t}{P_t} - mc_t\right) y_{it}. \tag{2.13}$$

$Q_{t,t+1}\left(= \beta\frac{\lambda_{t+1}}{\lambda_t}\right)$ denotes the stochastic discount factor where λ_t is a Lagrangian multiplier of consumer's optimization problem. $\tilde{V}(P_{t-1})$ is the value function for a firm that is allowed to reset its price exploiting the dynamics of the aggregate price level which is expressed as in the transition equation, (2.12), with P_{t-1} given. Following the dynamic programming technique explained in Appendix 2.A, we have the Euler equation for the optimality of pricing as

$$f_{1t} - f_{2t} + f_{3t} + f_{4t} = f_{5t} + f_{6t}, \tag{2.14}$$

where

$$f_{1t} = (\epsilon - 1)\left(\tilde{p}_t\right)^{1-\epsilon} y_t + E_t \beta\theta^2 \frac{\lambda_{t+1}}{\lambda_t}\left(\frac{\tilde{p}_t}{\tilde{p}_{t+1}}\right)^{1-\epsilon} \pi_{t+1}^{2\epsilon-2} f_{1t+1} \tag{2.15}$$

$$f_{2t} = \epsilon mc_t(\tilde{p}_t)^{-\epsilon} y_t + E_t \beta\theta^2 \frac{\lambda_{t+1}}{\lambda_t}\left(\frac{\tilde{p}_t}{\tilde{p}_{t+1}}\right)^{-\epsilon} \pi_{t+1}^{2\epsilon-1} f_{2t+1} \tag{2.16}$$

$$f_{3t} = \frac{1-\epsilon}{1-\theta} y_t + E_t \beta\theta \frac{\lambda_{t+1}}{\lambda_t} \pi_{t+1}^{\epsilon-1} f_{3t+1} \tag{2.17}$$

$$f_{4t} = \frac{\epsilon}{1-\theta} mc_t(\tilde{p}_t)^{-1} y_t + E_t \beta\theta \frac{\lambda_{t+1}}{\lambda_t}\left(\frac{\tilde{p}_t}{\tilde{p}_{t+1}}\right)^{-1} \pi_{t+1}^{\epsilon} f_{4t+1} \tag{2.18}$$

$$f_{5t} = E_t(1-\theta)\beta\theta \frac{\lambda_{t+1}}{\lambda_t} f_{3t+1} \pi_{t+1}^{\epsilon-1} + \beta\theta^2 \frac{\lambda_{t+1}}{\lambda_t} \pi_{t+1}^{\epsilon-1} f_{5t+1} \tag{2.19}$$

$$f_{6t} = E_t(1-\theta)\beta\theta \frac{\lambda_{t+1}}{\lambda_t} f_{4t+1} \pi_{t+1}^{\epsilon-1} + \beta\theta^2 \frac{\lambda_{t+1}}{\lambda_t} \pi_{t+1}^{\epsilon-1} f_{6t+1}, \tag{2.20}$$

and $\tilde{p}_t (= p_t^*/P_t)$ denotes the relative price, and $\pi_t (= P_t/P_{t-1})$ denotes the gross inflation rate.

2.3.2 Rest of the Model

Aside from firms' optimization problems, all other parts of the model are the same as the standard Calvo model.[8] Consequently, the optimality conditions of representative household for the aggregate consumption (c_t), the labour supply (h_t), and the real bond holding (b_t) are given by

$$c_t^{-\sigma} = \lambda_t \tag{2.21}$$

$$Q_t = \beta E_t \left(\frac{P_t}{P_{t+1}} \frac{\lambda_{t+1}}{\lambda_t}\right) \tag{2.22}$$

$$\lambda_t \frac{W_t}{P_t} = \lambda_t w_t = h_t^{\varrho}, \tag{2.23}$$

where λ_t is the Lagrangian multiplier of the household's utility maximization problem, Q_t is the price of bonds which yields Q_t^{-1} as a nominal return, and W_t denotes

[8] See Appendix 1.B of Chapter 1 for details.

the nominal wage. Also, the rule of monetary authority for interest rate $\left(R_t = Q_t^{-1}\right)$ is given by

$$\frac{R_t}{R} = \left(\frac{\pi_t}{\pi}\right)^{\eta_\pi} \left(\frac{y_t}{y}\right)^{\eta_y} (\nu_t)^{-1}, \tag{2.24}$$

where ν_t is the monetary shock which follows

$$\frac{\nu_t}{\nu} = \left(\frac{\nu_{t-1}}{\nu}\right)^{\rho} \exp(e_t), \qquad e_t \sim N(0, \varsigma), \tag{2.25}$$

with the persistence of the shocks captured by ρ. Lastly, the market clears with the following conditions.

$$y_t = c_t \tag{2.26}$$

$$h_t = y_t \tag{2.27}$$

2.3.3 Equilibrium and Simulation

In the equilibrium, all the endogenous variables of c_t, λ_t, h_t, w_t, y_t, R_t, π_t, mc_t, $\tilde{p}_t$, f_{1t}, f_{2t}, f_{3t}, f_{4t}, f_{5t}, and f_{6t}, and the exogenous variable of ν_t satisfy the set of equations of (2.7), (2.8) and (2.14)$\sim$(2.27). For the solution of the model, the equations are log-linearized with the steady state values of variables, which are presented in Appendix 2.B. The simulation of the model is performed with the very standard calibration[9] using Dynare through Matlab.

2.4 Simulation Results

2.4.1 Impulse Responses with Collusion

The model is simulated with expansionary money shocks, and the results are compared to the benchmark model which is the standard Calvo model with the same parameters. The impulse responses of the main variables of the model are given with the comparison to those of the standard model as in Figure 2.1. Firstly, as can be seen in the bottom left panel, the optimal price of the colluding firms is higher

[9] See Appendix 1.C of Chapter 1 for details.

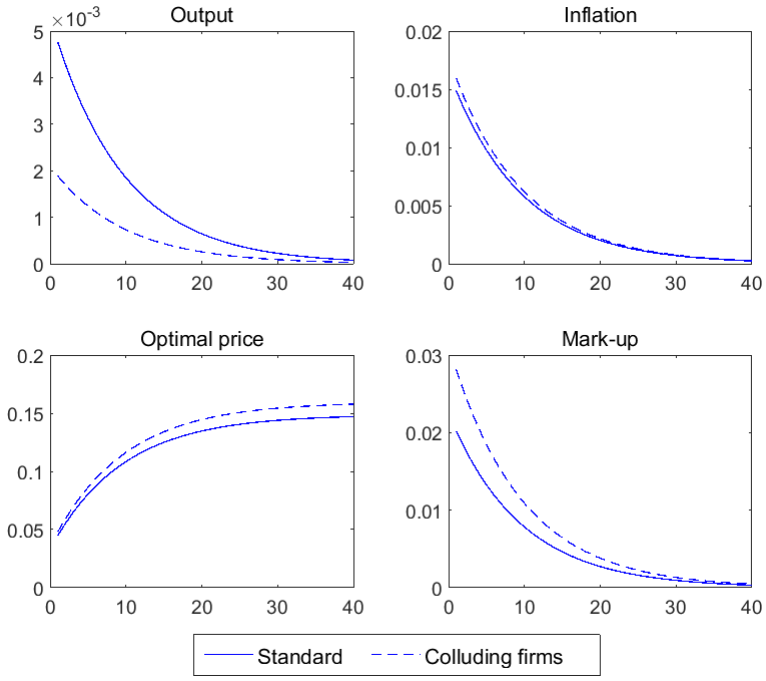

Figure 2.1: Impulse Response Functions to the Monetary Shocks

than that of the non-colluding firms in the standard model. Consequently, we can find that the inflation response is bigger and the output response is smaller than that of the benchmark model. In particular, the size of output response is less than half of that in the standard model. This implies that we have a different slope for the New Keynesian Phillips Curve (NKPC) compared to the standard model. The NKPC can be derived using the log-linearized equations, (2.58)∼(2.67), (2.69), and (2.72)∼(2.73) in Appendix 2.B as in

$$\hat{\pi}_t = A\hat{y}_t + BE_t\hat{\pi}_{t+1}$$

where

$$A = \frac{-(1-\beta\theta)(1-\theta)(1-\beta\theta^2)(\sigma+\varrho)}{(1-\beta\theta)(1-\theta)-(1-\beta\theta^2)(\beta\theta+\epsilon(1-\beta\theta))}$$

$$B = \frac{(1-\beta\theta)(1-\theta)(\beta\theta+1-\beta\theta^2)-\beta(1-\beta\theta^2)}{(1-\beta\theta)(1-\theta)-(1-\beta\theta^2)(\beta\theta+\epsilon(1-\beta\theta))}.$$

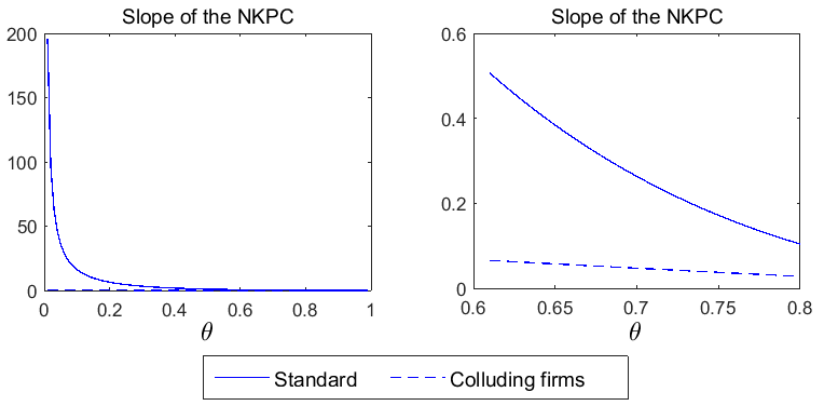

Figure 2.2: Slope of the New Keynesian Phillips Curve

Figure 2.2 shows that slope of the NKPC in this model, A, is much gentler than that of the benchmark model[10] for all θ with the standard calibration of $\beta = 0.99$, $\epsilon = 11$, and $\sigma = \varrho = 1$.

With the gentler NKPC, the model generates much lower output response against positive monetary shocks. This means that the real effect of money on the economy in this model is much lessened compared to the benchmark model. In the proposed model, since the repricing firms behave as in collusion, they have a more monopolistic status than the firms following the standard Calvo framework. With this monopolistic power, they can raise their prices higher in response to the positive monetary shocks. This pricing behaviour can be confirmed in the bottom right panel of Figure 2.1 which shows that mark-up of the colluding firm's reset price is much higher than that of the standard model. The increased mark-up leads to the higher level of individual reset price and aggregate price as well. Since much more shocks are absorbed by the higher response of the aggregate price level, the output response naturally becomes much lower compared to the standard model. This result shows that even though the price of individual firm is sticky, the aggregate price level may not be so rigid if the firms which are allowed to change by the Calvo signal adjust their prices fully enough to absorb the shocks.

[10] In the standard model, the slope of the NKPC is given by $(1 - \beta\theta)(1 - \theta)(\sigma + \varrho)/\theta$.

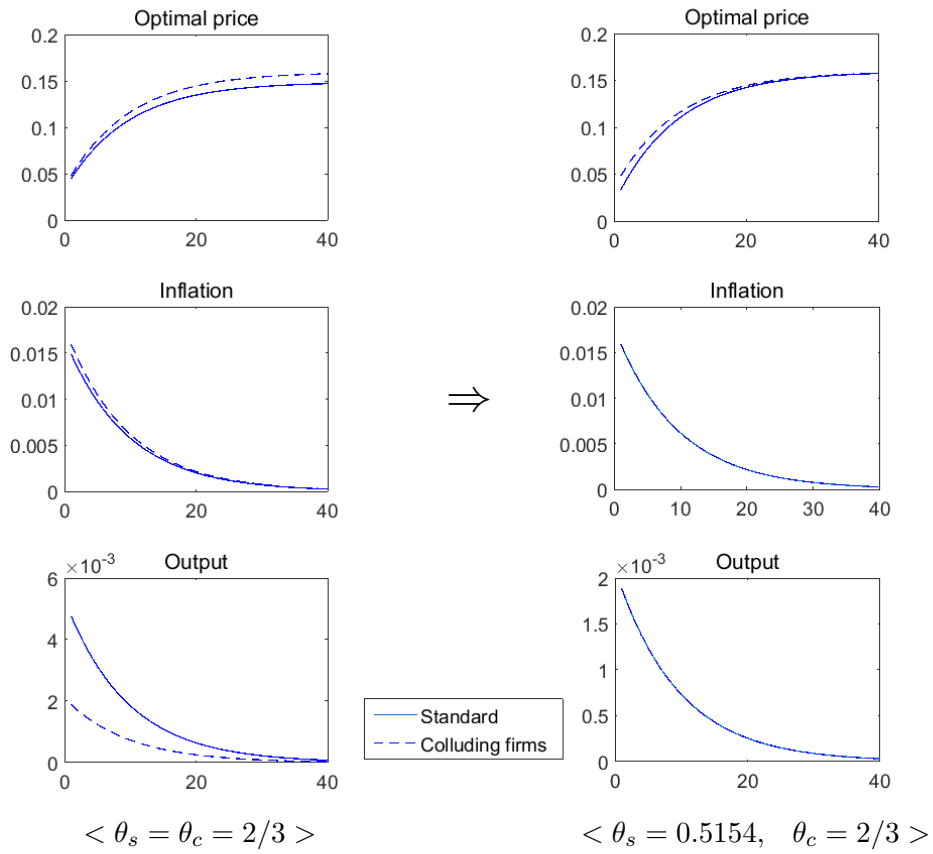

Figure 2.3: Recalibration Exercise

2.4.2 Collusion and Price Stickiness

In order to check if there is another factor bringing about the decrease in output response, the recalibration exercise is implemented as conducted in Chapter 1. That is, θ of the standard model is adjusted to get the same amount of inflation response as the colluding firms model, and then it is checked how large the output response is compared to that of the colluding firms model. The left panels of Figure 2.3 show the same impulse responses as we have seen in the previous section in which the degree of price stickiness of the benchmark model (θ_s) is the same as that of the colluding firms model (θ_c) at the level of 2/3. However, the right panels represent the results of the case where θ_s is recalibrated to 0.5154, which assures the same amount of inflation response as the colluding firms model in which θ_c is still maintained at 2/3.

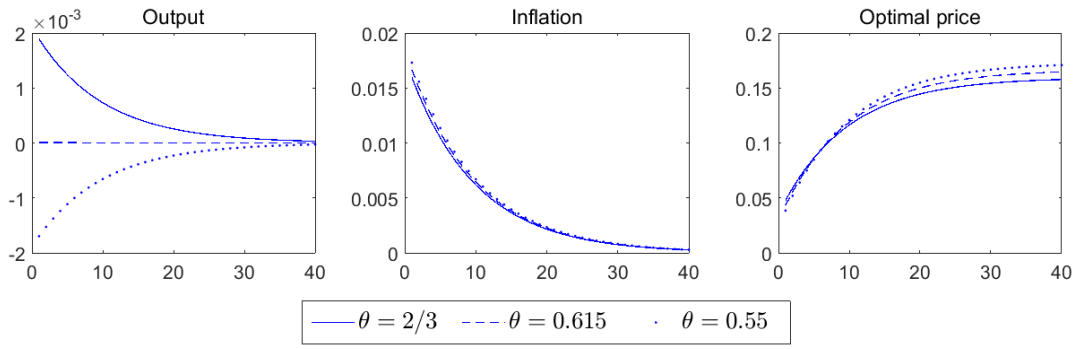

Figure 2.4: Impulse Response Functions with Different θ

As can be seen, the output response of the recalibrated standard model shows the same amount of that of the colluding firms model. This result seems to indicate that the model with colluding firms is nothing but the standard Calvo model with a lower value of θ. In other words, the only thing we do by assuming that there is collusion between repricing firms in the model seems to be just lowering the fraction of firms which are stuck to their current prices in the benchmark model.

However, this is not the whole story. As can be seen in Figure 2.4, the impulse responses of the model with various levels of θ are very different from those of the standard Calvo model. As stated above, with θ equal to be 2/3, the model produces real effect of money that is about 50% as large as in the benchmark model. However, with $\theta = 0.615$, the output response is around zero which means that money is almost neutral. This is not just cutting the real effect of the benchmark model by half. As is well known, in the standard Calvo model, we should set θ to be zero in order to get the fully neutral money. Therefore, the fact that we can produce the neutrality of money even with very high value of θ is the most interesting feature of the proposed model. If θ gets much smaller, we can find that the real effect of money occurs in the opposite direction. Figure 2.4 shows that the aggregate output drops down in response to the expansionary money shock when θ is equal to 0.55. This is because the aggregate inflation, even with such degree of price stickiness, goes up higher than the level where the effects of shocks are fully absorbed. Figure 2.5 demonstrates that the optimal price of colluding firms with

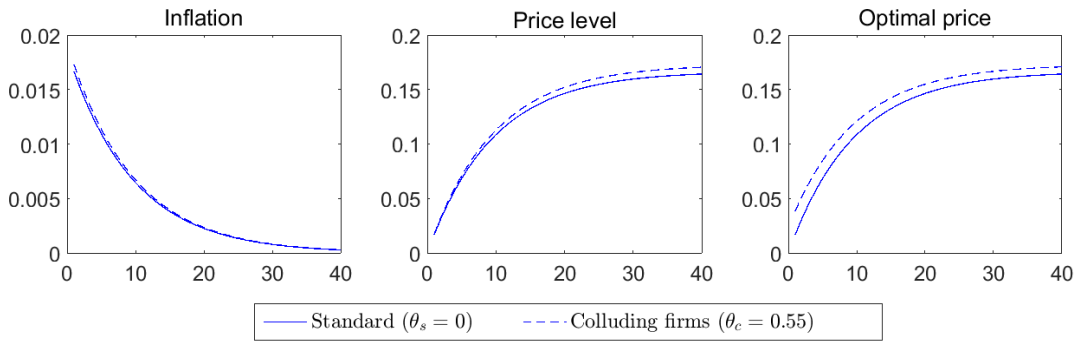

Figure 2.5: Comparison to the Flexible Economy in the Standard Model

$\theta_c = 0.55$ is much higher than that of the standard flexible economy with $\theta_s = 0$ so that the aggregate price level, even dampened by the non-repricing firms as of half of the economy, goes up beyond the limit which non-colluding firms can make in the standard framework. This result shows that the fraction of repricing firms which are in collusion get bigger as θ gets smaller, and therefore, the effect of rising prices by the colluding firms overwhelm the economy so that we can have negative response of output even with the positive monetary shocks.

2.4.3 Collusion in Extreme Cases of Price Stickiness

In the above section, we have seen that the monopolistic power of colluding firms makes much bigger response of inflation to monetary shocks compared to the standard Calvo model. Furthermore, the effect of collusion gets bigger as the degree of price stickiness gets smaller. This section examines whether the above results still hold with the extreme cases of price stickiness. In other words, it is checked in this section if the effect of collusion can be found even with extremely high degree of price stickiness and if the effect keeps growing as the degree of sticky price approaches zero. Firstly, given the previous result which shows a greater effect of collusion with smaller price stickiness, it can be anticipated that the opposite case will occur in which the effect of collusion is reduced as price stickiness approaches the maximum level. Such anticipation also matches our intuition. When θ gets larger and approaches one, the fraction of colluding firms becomes infinitely small.

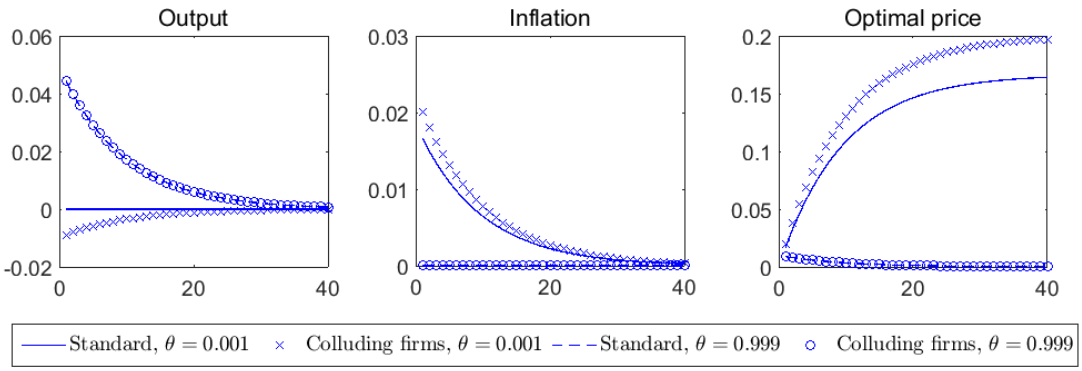

Figure 2.6: Extreme Cases of Price Stickiness

Therefore, no matter how high they set their optimal prices, they cannot affect the aggregate price level. Figure 2.6 confirms such anticipation showing that the impulse responses of colluding firms model are the same as those of the standard one when the degree of price stickiness is extremely high ($\theta = 0.999$). Secondly, would the inference based on the results in above section also match our intuition in another extreme case of price stickiness? We have seen that the collusion effect gets bigger as θ gets smaller so that the inflation response could reach above the level of the standard flexible economy. The extension of logical reasoning on this result is that inflation goes up infinitely high when θ approaches zero. However, the reasoning does not entirely coincide with our intuition. On the one hand, firms know that their optimal prices can affect the aggregate price level, and therefore, they have incentive to have the monopolistic gains by adjusting the aggregate levels of price and output. Moreover, with $\theta = 0$, even a slightly higher reset price can bring about an infinitely high inflation response. In this sense, it seems that the colluding effect would be extremely big when θ is very close to zero. However, on the other hand, colluding firms also gain the monopoly profit through the adjustment of the relative price (p_t^*/P_t) using the information on the dynamics of the aggregate price level as of (2.12). When θ is equal to zero, the dynamics of P_t^* turns to just $P_t = p_t^*$, which means that all firms in the economy have the same price, and therefore, the relative price is fixed to one. Hence, in this case, there is no room for firms to get extra profit by adjusting the relative price. Therefore, the increase in the inflation response will

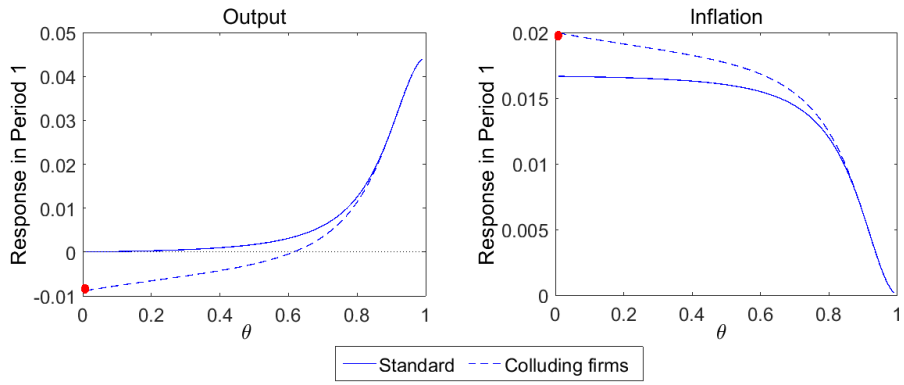

Figure 2.7: Impulse Responses with Different Price Stickiness

not be continued forever. Figure 2.6 supports this reasoning by demonstrating that the colluding firms give much higher response of inflation compared to the standard model when θ is very close to zero ($\theta = 0.001$), but the response does not show an extremely steep rise. Therefore, we can expect that the colluding effect, which yields higher inflation response compared to the standard model, gets bigger as θ gets smaller but is finite even when θ becomes closer to zero. For the confirmation of such expectation, it is examined how the inflation and output responses vary as θ goes from zero to 0.999 in the colluding firms model. Figure 2.7 shows the results of the impulse responses in the first period after monetary shocks. As can be seen, the results generally coincide with our expectations. We can see that the responses are almost the same as the standard model when θ is very high ($\theta > 0.9$). However, unless θ is so high ($\theta < 0.9$), colluding firms set higher optimal prices compared to that of the standard model, and the inflation response gets bigger as θ gets smaller. Also, we can observe that though the inflation goes high when θ approaches zero, it does not go to infinity. However, Figure 2.7 shows that the model has no equilibrium at the point where θ is equal to 0 which is marked by a bullet point in the figure. This unexpected result seems to be due to the extraordinary property of the model structure. As presented in Appendix 2.B, the steady state values of many variables are expressed as functions of θ, and they affect the equilibrium of the model. In particular, we need to focus on the steady state value of marginal cost (mc) which

is calculated by solving (2.14) with the steady state values of $f_{1t} \sim f_{6t}$ ($f_1 \sim f_6$). With $\theta = 0$, we have $f_1 = (\epsilon - 1)y$, $f_2 = \epsilon mcy$, $f_3 = (1 - \epsilon)y$, $f_4 = \epsilon mcy$, $f_5 = 0$, and $f_6 = 0$. Therefore, (2.14) is expressed by

$$\begin{aligned} & f_1 - f_2 + f_3 + f_4 = f_5 + f_6 \\ \Leftrightarrow \quad & (\epsilon - 1)y - \epsilon mcy + (1 - \epsilon)y + \epsilon mcy = 0. \end{aligned} \tag{2.28}$$

As can be seen, (2.28) is an identical equation, and therefore, mc cannot be defined, which makes the model have no stable equilibrium. Considering that the level of θ is very high in most economies as shown in many micro evidences, zero degree of price stickiness is out of our main interest. However, the indeterminacy of the model with $\theta = 0$ is still problematic in terms of model compatibility and should be solved in future researches.

2.5 Conclusion and Further Research

Colluding firms have incentives to set high prices in order to gain monopolistic profits. We have seen that such incentive of repricing firms in the modified Calvo framework gives rise to a much higher optimal price and, therefore, a much bigger inflation response to monetary shocks, compared to the standard model. Furthermore, the model shows that, unless the degree of price stickiness is extremely low or high, the effect of firms' collusion gets bigger as economy becomes more flexible, and therefore, we could get even fully neutral money with a certain level of price stickiness.

These results show that the real effect of money in the economy of colluding firms can be very small or even none with the existence of price stickiness among individual firms which is generated by the Calvo framework. In other words, even if a certain fraction of firms maintain their prices at the level of the previous period, that is, even if individual prices are sticky, the aggregate price level may be rather flexible under certain circumstances like the case where firms which are to adjust prices

respond very elastically to monetary shock as in the proposed model. Consequently, the conventional belief that price stickiness is the main driving force of rigid inflation and monetary non-neutrality is challenged with this model.

Also, the results of the model provide some implications with respect to monetary policy. This chapter shows that when firms behave as in collusion the effect of monetary policy can be very small. Therefore, central banks need to make a close investigation on market structures of the economy, particularly with respect to the degree of monopoly before establishing a macro model for monetary policy analysis. Furthermore, they need to research how to incorporate the information on market power into the monetary model.

Even though the argument of this paper is very clear, there are still some further researches necessary. Firstly, as discussed in the previous section, this model cannot show the responses of the main variables when the economy is fully flexible. For the fineness of the model, this issue should be made up for in future research. Secondly, in this chapter, the model with relatively high fraction of repricing firms reveals a negative output response to positive monetary shocks, which is contrary to our conventional knowledge and empirical evidences. More analyses on this result need to be carried out from the viewpoint of monetary policy. Third, the framework which is used in this paper for simulation is a very simple Dynamic Stochastic General Equilibrium model. It is required to check whether the results of the model with colluding firms are robust even under more augmented framework which, for example, includes wage rigidity, indexation, capital adjustment costs, and habit in consumption. Finally, empirical studies should be carried out to support the model of this chapter. Through empirical assessments, we need to check whether the theoretical predictions of this model lead to actual occurrences. In other words, it is necessary to check empirically whether monetary policy actually has less effect in an economy in which firms have more monopoly power, rather than in an economy with relatively more competitive markets.

Appendix 2.A

In this appendix, the Euler equation for a firm's optimal price is derived. From the Bellman equation which solves the value function for colluding firms, (2.11), we have the First Order Condition (FOC) as in

$$\text{FOC}: \qquad \frac{\partial \bar{V}\left(p_t^*, P_t\right)}{\partial p_t^*} + \frac{\partial \bar{V}\left(p_t^*, P_t\right)}{\partial P_t}\frac{\partial P_t}{\partial p_t^*} = 0. \tag{2.29}$$

The Envelope Condition for this Bellman equation ($\widetilde{\text{EC}}$) is given by

$$\widetilde{\text{EC}}: \qquad \frac{\partial \tilde{V}\left(P_{t-1}\right)}{\partial P_{t-1}} = \frac{\partial \bar{V}\left(p_t^*, P_t\right)}{\partial P_t}\frac{\partial P_t}{\partial P_{t-1}}. \tag{2.30}$$

From the Bellman equation which solves the value function for firms which are not allowed to change prices, (2.10), we get the derivative of $\bar{V}$ evaluated at p_t^* with respect to P_t as

$$\begin{aligned}\frac{\partial \bar{V}\left(p_t^*, P_t\right)}{\partial P_t} =& \frac{\partial \phi\left(p_t^*, P_t\right)}{\partial P_t} + \theta E_t\left[Q_{t,t+1}\frac{\partial \bar{V}\left(p_t^*, P_{t+1}\right)}{\partial P_{t+1}}\frac{\partial P_{t+1}}{\partial P_t}\right] \\ &+ (1-\theta)E_t\left[Q_{t,t+1}\frac{\partial \tilde{V}\left(P_t\right)}{\partial P_t}\right].\end{aligned} \tag{2.31}$$

Substituting (2.31) into FOC (2.29) gives

$$\begin{aligned}0 =& \frac{\partial \bar{V}\left(p_t^*, P_t\right)}{\partial p_t^*} + \frac{\partial \phi\left(p_t^*, P_t\right)}{\partial P_t}\frac{\partial P_t}{\partial p_t^*} + \theta E_t\left[Q_{t,t+1}\frac{\partial \bar{V}\left(p_t^*, P_{t+1}\right)}{\partial P_{t+1}}\frac{\partial P_{t+1}}{\partial P_t}\right]\frac{\partial P_t}{\partial p_t^*} \\ &+ (1-\theta)E_t\left[Q_{t,t+1}\frac{\partial \tilde{V}\left(P_t\right)}{\partial P_t}\right]\frac{\partial P_t}{\partial p_t^*} \\ \Rightarrow (1-\theta)E_t&\left[Q_{t,t+1}\frac{\partial \tilde{V}\left(P_t\right)}{\partial P_t}\right] = -\left(\frac{\partial \bar{V}\left(p_t^*, P_t\right)}{\partial p_t^*}\Big/\frac{\partial P_t}{\partial p_t^*}\right) - \frac{\partial \phi\left(p_t^*, P_t\right)}{\partial P_t} \\ &- \theta E_t\left[Q_{t,t+1}\frac{\partial \bar{V}\left(p_t^*, P_{t+1}\right)}{\partial P_{t+1}}\frac{\partial P_{t+1}}{\partial P_t}\right],\end{aligned} \tag{2.32}$$

and putting (2.31) into $\widetilde{\mathrm{EC}}$ (2.30) yields

$$\begin{aligned}\frac{\partial \tilde{V}\left(P_{t-1}\right)}{\partial P_{t-1}} =& \frac{\partial \phi\left(p_t^*, P_t\right)}{\partial P_t} \frac{\partial P_t}{\partial P_{t-1}} + \theta E_t\left[Q_{t,t+1} \frac{\partial \bar{V}\left(p_t^*, P_{t+1}\right)}{\partial P_{t+1}} \frac{\partial P_{t+1}}{\partial P_t}\right] \frac{\partial P_t}{\partial P_{t-1}} \\ &+ (1-\theta) E_t\left[Q_{t,t+1} \frac{\partial \tilde{V}\left(P_t\right)}{\partial P_t}\right] \frac{\partial P_t}{\partial P_{t-1}}. \end{aligned} \tag{2.33}$$

With (2.32) and (2.33), we have

$$\begin{aligned}\frac{\partial \tilde{V}\left(P_{t-1}\right)}{\partial P_{t-1}} =& \frac{\partial \phi\left(p_t^*, P_t\right)}{\partial P_t} \frac{\partial P_t}{\partial P_{t-1}} + \theta E_t\left[Q_{t,t+1} \frac{\partial \bar{V}\left(p_t^*, P_{t+1}\right)}{\partial P_{t+1}} \frac{\partial P_{t+1}}{\partial P_t}\right] \frac{\partial P_t}{\partial P_{t-1}} \\ &- \left(\frac{\partial \bar{V}\left(p_t^*, P_t\right)}{\partial p_t^*} \Big/ \frac{\partial P_t}{\partial p_t^*}\right) \frac{\partial P_t}{\partial P_{t-1}} - \frac{\partial \phi\left(p_t^*, P_t\right)}{\partial P_t} \frac{\partial P_t}{\partial P_{t-1}} \\ &- \theta E_t\left[Q_{t,t+1} \frac{\partial \bar{V}\left(p_t^*, P_{t+1}\right)}{\partial P_{t+1}} \frac{\partial P_{t+1}}{\partial P_t}\right] \frac{\partial P_t}{\partial P_{t-1}} \\ =& -\left(\frac{\partial \bar{V}\left(p_t^*, P_t\right)}{\partial p_t^*} \Big/ \frac{\partial P_t}{\partial p_t^*}\right) \frac{\partial P_t}{\partial P_{t-1}}. \end{aligned} \tag{2.34}$$

Iterating (2.34) forward one period, we have

$$\frac{\partial \tilde{V}\left(P_t\right)}{\partial P_t} = -\left(\frac{\partial \bar{V}\left(p_{t+1}^*, P_{t+1}\right)}{\partial p_{t+1}^*} \Big/ \frac{\partial P_{t+1}}{\partial p_{t+1}^*}\right) \frac{\partial P_{t+1}}{\partial P_t}. \tag{2.35}$$

Substituting (2.35) into (2.32) gives

$$\begin{aligned}&(1-\theta) E_t\left[Q_{t,t+1}\left(\frac{\partial \bar{V}\left(p_{t+1}^*, P_{t+1}\right)}{\partial p_{t+1}^*} \Big/ \frac{\partial P_{t+1}}{\partial p_{t+1}^*}\right) \frac{\partial P_{t+1}}{\partial P_t}\right] \\ &= \left(\frac{\partial \bar{V}\left(p_t^*, P_t\right)}{\partial p_t^*} \Big/ \frac{\partial P_t}{\partial p_t^*}\right) + \frac{\partial \phi\left(p_t^*, P_t\right)}{\partial P_t} \\ &\quad + \theta E_t\left[Q_{t,t+1} \frac{\partial \bar{V}\left(p_t^*, P_{t+1}\right)}{\partial P_{t+1}} \frac{\partial P_{t+1}}{\partial P_t}\right]. \end{aligned} \tag{2.36}$$

From (2.31), we have

$$\begin{aligned}\frac{\partial \bar{V}\left(p_t^*, P_{t+1}\right)}{\partial P_{t+1}} =& \frac{\partial \phi\left(p_t^*, P_{t+1}\right)}{\partial P_{t+1}} + \theta E_{t+1}\left[Q_{t+1,t+2} \frac{\partial \bar{V}\left(p_t^*, P_{t+2}\right)}{\partial P_{t+2}} \frac{\partial P_{t+2}}{\partial P_{t+1}}\right] \\ &+ (1-\theta) E_{t+1}\left[Q_{t+1,t+2} \frac{\partial \tilde{V}\left(P_{t+1}\right)}{\partial P_{t+1}}\right]. \end{aligned} \tag{2.37}$$

Putting (2.37) into (2.36), we get

$$
\begin{aligned}
&(1-\theta)E_t\left[Q_{t,t+1}\left(\frac{\partial\bar{V}\left(p_{t+1}^*,P_{t+1}\right)}{\partial p_{t+1}^*}\Big/\frac{\partial P_{t+1}}{\partial p_{t+1}^*}\right)\frac{\partial P_{t+1}}{\partial P_t}\right]\\
&=\left(\frac{\partial\bar{V}\left(p_t^*,P_t\right)}{\partial p_t^*}\Big/\frac{\partial P_t}{\partial p_t^*}\right)+\frac{\partial\phi\left(p_t^*,P_t\right)}{\partial P_t}+\theta E_t\left[Q_{t,t+1}\frac{\partial\phi\left(p_t^*,P_{t+1}\right)}{\partial P_{t+1}}\frac{\partial P_{t+1}}{\partial P_t}\right]\\
&\quad+\theta^2E_t\left[Q_{t,t+2}\frac{\partial\bar{V}\left(p_t^*,P_{t+2}\right)}{\partial P_{t+2}}\frac{\partial P_{t+2}}{\partial P_{t+1}}\frac{\partial P_{t+1}}{\partial P_t}\right]\\
&\quad+\theta(1-\theta)E_t\left[Q_{t,t+2}\frac{\partial\tilde{V}\left(P_{t+1}\right)}{\partial P_{t+1}}\frac{\partial P_{t+1}}{\partial P_t}\right].
\end{aligned}
\tag{2.38}
$$

Iterating (2.35) forward one period, we have

$$
\frac{\partial\tilde{V}\left(P_{t+1}\right)}{\partial P_{t+1}}=-\left(\frac{\partial\bar{V}\left(p_{t+2}^*,P_{t+2}\right)}{\partial p_{t+2}^*}\Big/\frac{\partial P_{t+2}}{\partial p_{t+2}^*}\right)\frac{\partial P_{t+2}}{\partial P_{t+1}}.
\tag{2.39}
$$

Putting (2.39) into (2.38) yields

$$
\begin{aligned}
&(1-\theta)E_t\left[Q_{t,t+1}\left(\frac{\partial\bar{V}\left(p_{t+1}^*,P_{t+1}\right)}{\partial p_{t+1}^*}\Big/\frac{\partial P_{t+1}}{\partial p_{t+1}^*}\right)\frac{\partial P_{t+1}}{\partial P_t}\right]\\
&=\left(\frac{\partial\bar{V}\left(p_t^*,P_t\right)}{\partial p_t^*}\Big/\frac{\partial P_t}{\partial p_t^*}\right)+\frac{\partial\phi\left(p_t^*,P_t\right)}{\partial P_t}+\theta E_t\left[Q_{t,t+1}\frac{\partial\phi\left(p_t^*,P_{t+1}\right)}{\partial P_{t+1}}\frac{\partial P_{t+1}}{\partial P_t}\right]\\
&\quad+\theta^2E_t\left[Q_{t,t+2}\frac{\partial\bar{V}\left(p_t^*,P_{t+2}\right)}{\partial P_{t+2}}\frac{\partial P_{t+2}}{\partial P_{t+1}}\frac{\partial P_{t+1}}{\partial P_t}\right]\\
&\quad-\theta(1-\theta)E_t\left[Q_{t,t+2}\left(\frac{\partial\bar{V}\left(p_{t+2}^*,P_{t+2}\right)}{\partial p_{t+2}^*}\Big/\frac{\partial P_{t+2}}{\partial p_{t+2}^*}\right)\frac{\partial P_{t+2}}{\partial P_{t+1}}\frac{\partial P_{t+1}}{\partial P_t}\right].
\end{aligned}
\tag{2.40}
$$

Iterating forward by repeating the same procedures and rearranging, we have

$$
\begin{aligned}
&E_t\sum_{s=0}^{\infty}\theta^sQ_{t,t+s}\frac{\partial\phi\left(p_t^*,P_{t+s}\right)}{\partial P_{t+s}}\Gamma_{t+s}+\left(\frac{\partial\bar{V}\left(p_t^*,P_t\right)}{\partial p_t^*}\Big/\frac{\partial P_t}{\partial p_t^*}\right)\\
&=E_t\sum_{k=0}^{\infty}(1-\theta)\theta^kQ_{t,t+1+k}\left(\frac{\partial\bar{V}\left(p_{t+1+k}^*,P_{t+1+k}\right)}{\partial p_{t+1+k}^*}\Big/\frac{\partial P_{t+1+k}}{\partial p_{t+1+k}^*}\right)\Gamma_{t+1+k},
\end{aligned}
\tag{2.41}
$$

where

$$
\Gamma_{t+n}=\begin{cases}\prod_{i=1}^{n}\frac{\partial P_{t+i}}{\partial P_{t+i-1}} & \text{for } n>0\\ 1 & \text{for } n=0\end{cases}
\qquad\text{and}\qquad
Q_{t,t+s}=\beta^s\frac{\lambda_{t+s}}{\lambda_t}.
\tag{2.42}
$$

From (2.10), we have the derivative of $\bar{V}$ evaluated at p_t^* with respect to p_t^* as

$$\frac{\partial \bar{V}\left(p_t^*, P_t\right)}{\partial p_t^*} = \frac{\partial \phi\left(p_t^*, P_t\right)}{\partial p_t^*} + \theta E_t \left[Q_{t,t+1} \frac{\partial \bar{V}\left(p_t^*, P_{t+1}\right)}{\partial p_t^*} \right]. \tag{2.43}$$

Iterating this forward repeatedly, we get

$$\frac{\partial \bar{V}\left(p_t^*, P_t\right)}{\partial p_t^*} = E_t \sum_{s=0}^{\infty} \theta^s \left[Q_{t,t+s} \frac{\partial \phi\left(p_t^*, P_{t+s}\right)}{\partial p_t^*} \right]. \tag{2.44}$$

A firm's real profit, ϕ, is given by

$$\phi\left(p_t^*, P_t\right) = \left(\frac{p_t^*}{P_t} - mc_t \right) y_{it}. \tag{2.45}$$

Since an individual firm faces the following demand curve as

$$y_{it} = y_t \left(\frac{p_t^*}{P_t} \right)^{-\epsilon}, \tag{2.46}$$

we can re-express (2.45) as

$$\phi\left(p_t^*, P_t\right) = \left[\left(\frac{p_t^*}{P_t} \right)^{1-\epsilon} - mc_t \left(\frac{p_t^*}{P_t} \right)^{-\epsilon} \right] y_t. \tag{2.47}$$

Then the derivatives of ϕ evaluated at p_t^* with respect to p_t^* and P_{t+s} are given by

$$\frac{\partial \phi\left(p_t^*, P_{t+s}\right)}{\partial p_t^*} = \left[(1-\epsilon) \left(\frac{p_t^*}{P_{t+s}} \right)^{-\epsilon} + \epsilon mc_{t+s} \left(\frac{p_t^*}{P_{t+s}} \right)^{-\epsilon-1} \right] \frac{y_{t+s}}{P_{t+s}} \tag{2.48}$$

and

$$\frac{\partial \phi\left(p_t^*, P_{t+s}\right)}{\partial P_{t+s}} = \left[(\epsilon-1) \left(\frac{p_t^*}{P_{t+s}} \right)^{1-\epsilon} - \epsilon mc_{t+s} \left(\frac{p_t^*}{P_{t+s}} \right)^{-\epsilon} \right] \frac{y_{t+s}}{P_{t+s}}. \tag{2.49}$$

From the transition equation, (2.12), we have

$$\Gamma_{t+n} = \begin{cases} \theta^n \prod_{i=1}^{n} \pi_{t+i}^{\epsilon} & \text{for } n > 0 \\ 1 & \text{for } n = 0 \end{cases} \tag{2.50}$$

and

$$\frac{\partial P_t}{\partial p_t^*} = (1-\theta)\left(\frac{p_t^*}{P_t}\right)^{-\epsilon}. \tag{2.51}$$

Therefore, using (2.44) and (2.48)∼(2.51), we can re-express (2.41) as

$$\begin{aligned}
&E_t \sum_{s=0}^{\infty} \left(\beta\theta^2\right)^s \frac{\lambda_{t+s}}{\lambda_t}(\epsilon-1)\left(\frac{p_t^*}{P_{t+s}}\right)^{1-\epsilon} \frac{y_{t+s}}{P_{t+s}} \prod_{i=1}^{s} \pi_{t+i}^{\epsilon} \\
&- E_t \sum_{s=0}^{\infty} \left(\beta\theta^2\right)^s \frac{\lambda_{t+s}}{\lambda_t}\epsilon mc_{t+s}\left(\frac{p_t^*}{P_{t+s}}\right)^{-\epsilon} \frac{y_{t+s}}{P_{t+s}} \prod_{i=1}^{s} \pi_{t+i}^{\epsilon} \\
&+ E_t \sum_{s=0}^{\infty} (\beta\theta)^s \frac{\lambda_{t+s}}{\lambda_t}\frac{1-\epsilon}{1-\theta}\left(\frac{p_t^*}{P_{t+s}}\right)^{-\epsilon} \frac{y_{t+s}}{P_{t+s}}\left(\frac{p_t^*}{P_t}\right)^{\epsilon} \\
&+ E_t \sum_{s=0}^{\infty} (\beta\theta)^s \frac{\lambda_{t+s}}{\lambda_t}\frac{\epsilon}{1-\theta} mc_{t+s}\left(\frac{p_t^*}{P_{t+s}}\right)^{-\epsilon-1} \frac{y_{t+s}}{P_{t+s}}\left(\frac{p_t^*}{P_t}\right)^{\epsilon} \\
&= E_t \sum_{k=0}^{\infty} (1-\theta)(\beta\theta)^{k+1}\theta^k \frac{\lambda_{t+1+k}}{\lambda_t}\left(\mathcal{A}+\mathcal{B}\right) \prod_{i=1}^{k+1} \pi_{t+i}^{\epsilon},
\end{aligned} \tag{2.52}$$

where $\mathcal{A}$ and $\mathcal{B}$ denote

$$\begin{aligned}
\mathcal{A} &= \sum_{s=0}^{\infty} (\beta\theta)^s \frac{\lambda_{t+1+k+s}}{\lambda_{t+1+k}}\frac{1-\epsilon}{1-\theta}\left(\frac{p_{t+1+k}^*}{P_{t+1+k+s}}\right)^{-\epsilon} \frac{y_{t+1+k+s}}{P_{t+1+k+s}}\left(\frac{p_{t+1+k}^*}{P_{t+1+k}}\right)^{\epsilon} \\
\mathcal{B} &= \sum_{s=0}^{\infty} (\beta\theta)^s \frac{\lambda_{t+1+k+s}}{\lambda_{t+1+k}}\frac{\epsilon}{1-\theta} mc_{t+1+k+s}\left(\frac{p_{t+1+k}^*}{P_{t+1+k+s}}\right)^{-\epsilon-1} \frac{y_{t+1+k+s}}{P_{t+1+k+s}}\left(\frac{p_{t+1+k}^*}{P_{t+1+k}}\right)^{\epsilon},
\end{aligned}$$

and $\prod_{i=1}^{s} \pi_{t+i}^{\epsilon} = 1$ for $s = 0$. Multiply both sides of (2.52) by P_t, and let f_{1t}, f_{2t}, f_{3t} and f_{4t} denote the followings as in

$$f_{1t} = E_t \sum_{s=0}^{\infty} \left(\beta\theta^2\right)^s \frac{\lambda_{t+s}}{\lambda_t}(\epsilon-1)\left(\frac{p_t^*}{P_{t+s}}\right)^{1-\epsilon} \frac{y_{t+s}}{P_{t+s}} P_t \prod_{i=1}^{s} \pi_{t+i}^{\epsilon} \tag{2.53}$$

$$f_{2t} = E_t \sum_{s=0}^{\infty} \left(\beta\theta^2\right)^s \frac{\lambda_{t+s}}{\lambda_t}\epsilon mc_{t+s}\left(\frac{p_t^*}{P_{t+s}}\right)^{-\epsilon} \frac{y_{t+s}}{P_{t+s}} P_t \prod_{i=1}^{s} \pi_{t+i}^{\epsilon} \tag{2.54}$$

$$f_{3t} = E_t \sum_{s=0}^{\infty} (\beta\theta)^s \frac{\lambda_{t+s}}{\lambda_t}\frac{1-\epsilon}{1-\theta}\left(\frac{p_t^*}{P_{t+s}}\right)^{-\epsilon} \frac{y_{t+s}}{P_{t+s}}\left(\frac{p_t^*}{P_t}\right)^{\epsilon} P_t \tag{2.55}$$

$$f_{4t} = E_t \sum_{s=0}^{\infty} (\beta\theta)^s \frac{\lambda_{t+s}}{\lambda_t}\frac{\epsilon}{1-\theta} mc_{t+s}\left(\frac{p_t^*}{P_{t+s}}\right)^{-\epsilon-1} \frac{y_{t+s}}{P_{t+s}}\left(\frac{p_t^*}{P_t}\right)^{\epsilon} P_t. \tag{2.56}$$

Then (2.52) can be re-expressed as

$$
\begin{aligned}
&f_{1t} - f_{2t} + f_{3t} + f_{4t} \\
&= E_t \sum_{k=0}^{\infty} (1-\theta)(\beta\theta)^{k+1}\theta^k \frac{\lambda_{t+1+k}}{\lambda_t} \left(f_{3t+1+k} + f_{4t+1+k}\right) \prod_{i=1}^{k+1} \pi_{t+i}^{\epsilon-1}.
\end{aligned}
$$

This can be expressed in recursive form as follows.

$$
f_{1t} - f_{2t} + f_{3t} + f_{4t} = f_{5t} + f_{6t}, \tag{2.57}
$$

where

$$
\begin{aligned}
f_{1t} &= (\epsilon - 1)\left(\tilde{p}_t\right)^{1-\epsilon} y_t + E_t \beta\theta^2 \frac{\lambda_{t+1}}{\lambda_t} \left(\frac{\tilde{p}_t}{\tilde{p}_{t+1}}\right)^{1-\epsilon} \pi_{t+1}^{2\epsilon-2} f_{1t+1} \\
f_{2t} &= \epsilon mc_t (\tilde{p}_t)^{-\epsilon} y_t + E_t \beta\theta^2 \frac{\lambda_{t+1}}{\lambda_t} \left(\frac{\tilde{p}_t}{\tilde{p}_{t+1}}\right)^{-\epsilon} \pi_{t+1}^{2\epsilon-1} f_{2t+1} \\
f_{3t} &= \frac{1-\epsilon}{1-\theta} y_t + E_t \beta\theta \frac{\lambda_{t+1}}{\lambda_t} \pi_{t+1}^{\epsilon-1} f_{3t+1} \\
f_{4t} &= \frac{\epsilon}{1-\theta} mc_t (\tilde{p}_t)^{-1} y_t + E_t \beta\theta \frac{\lambda_{t+1}}{\lambda_t} \left(\frac{\tilde{p}_t}{\tilde{p}_{t+1}}\right)^{-1} \pi_{t+1}^{\epsilon} f_{4t+1} \\
f_{5t} &= E_t (1-\theta)\beta\theta \frac{\lambda_{t+1}}{\lambda_t} f_{3t+1} \pi_{t+1}^{\epsilon-1} + \beta\theta^2 \frac{\lambda_{t+1}}{\lambda_t} \pi_{t+1}^{\epsilon-1} f_{5t+1} \\
f_{6t} &= E_t (1-\theta)\beta\theta \frac{\lambda_{t+1}}{\lambda_t} f_{4t+1} \pi_{t+1}^{\epsilon-1} + \beta\theta^2 \frac{\lambda_{t+1}}{\lambda_t} \pi_{t+1}^{\epsilon-1} f_{6t+1} \\
\tilde{p}_t &= p_t^* / P_t.
\end{aligned}
$$

Appendix 2.B

Log-linearization

The equations of (2.7), (2.8) and (2.14)∼(2.27) which constitute the model are respectively log-linearized as in

$$\hat{mc}_t = \hat{w}_t \tag{2.58}$$

$$\hat{\tilde{p}}_t = \frac{\theta}{1-\theta}\hat{\pi}_t \tag{2.59}$$

$$f_1\hat{f}_{1t} = f_2\hat{f}_{2t} - f_3\hat{f}_{3t} - f_4\hat{f}_{4t} + f_5\hat{f}_{5t} + f_6\hat{f}_{6t} \tag{2.60}$$

$$\begin{aligned}\hat{f}_{1t} =& (1-\beta\theta^2)\left((1-\epsilon)\hat{\tilde{p}}_t + \hat{y}_t\right) \\ &+ \beta\theta^2 E_t\left[\hat{\lambda}_{t+1} - \hat{\lambda}_t + (1-\epsilon)(\hat{\tilde{p}}_t - \hat{\tilde{p}}_{t+1}) + (2\epsilon-2)\hat{\pi}_{t+1} + \hat{f}_{1t+1}\right]\end{aligned} \tag{2.61}$$

$$\begin{aligned}\hat{f}_{2t} =& (1-\beta\theta^2)\left(\hat{mc}_t - \epsilon\hat{\tilde{p}}_t + \hat{y}_t\right) \\ &+ \beta\theta^2 E_t\left[\hat{\lambda}_{t+1} - \hat{\lambda}_t - \epsilon(\hat{\tilde{p}}_t - \hat{\tilde{p}}_{t+1}) + (2\epsilon-1)\hat{\pi}_{t+1} + \hat{f}_{2t+1}\right]\end{aligned} \tag{2.62}$$

$$\hat{f}_{3t} = (1-\beta\theta)\hat{y}_t + \beta\theta E_t\left[\hat{\lambda}_{t+1} - \hat{\lambda}_t + (\epsilon-1)\hat{\pi}_{t+1} + \hat{f}_{3t+1}\right] \tag{2.63}$$

$$\begin{aligned}\hat{f}_{4t} =& (1-\beta\theta)\left(\hat{mc}_t - \epsilon\hat{\tilde{p}}_t + \hat{y}_t\right) \\ &+ \beta\theta E_t\left[\hat{\lambda}_{t+1} - \hat{\lambda}_t - (\hat{\tilde{p}}_t - \hat{\tilde{p}}_{t+1}) + \epsilon\hat{\pi}_{t+1} + \hat{f}_{4t+1}\right]\end{aligned} \tag{2.64}$$

$$\begin{aligned}\hat{f}_{5t} =& (1-\beta\theta^2)E_t\left[\hat{\lambda}_{t+1} - \hat{\lambda}_t + \hat{f}_{3t+1} + (\epsilon-1)\hat{\pi}_{t+1}\right] \\ &+ \beta\theta^2 E_t\left[\hat{\lambda}_{t+1} - \hat{\lambda}_t + (\epsilon-1)\hat{\pi}_{t+1} + \hat{f}_{5t+1}\right]\end{aligned} \tag{2.65}$$

$$\begin{aligned}\hat{f}_{6t} =& (1-\beta\theta^2)E_t\left[\hat{\lambda}_{t+1} - \hat{\lambda}_t + \hat{f}_{4t+1} + (\epsilon-1)\hat{\pi}_{t+1}\right] \\ &+ \beta\theta^2 E_t\left[\hat{\lambda}_{t+1} - \hat{\lambda}_t + (\epsilon-1)\hat{\pi}_{t+1} + \hat{f}_{6t+1}\right]\end{aligned} \tag{2.66}$$

$$\hat{\lambda}_t = -\sigma\hat{c}_t \tag{2.67}$$

$$\hat{\lambda}_t = E_t\left[\hat{\lambda}_{t+1}\right] - E_t\left[\hat{\pi}_{t+1}\right] + \hat{R}_t \tag{2.68}$$

$$\varrho\hat{h}_t = \hat{\lambda}_t + \hat{w}_t \tag{2.69}$$

$$\hat{R}_t = \eta_\pi\hat{\pi}_t + \eta_y\hat{y}_t + \hat{\nu}_t \tag{2.70}$$

$$\hat{\nu}_t = \rho\hat{\nu}_{t-1} + e_t \tag{2.71}$$

$$\hat{y}_t = \hat{c}_t \tag{2.72}$$

$$\hat{y}_t = \hat{h}_t. \tag{2.73}$$

Steady State

The steady state values of all the endogenous and exogenous variables are presented in the following Table 2.1.

Variable	Steady State
$\pi, \nu, \tilde{p}$	1
R	$1/\beta$
mc	$(\epsilon - 1)/\epsilon$
y	$mc^{\frac{1}{\sigma+\varrho}}$
h, c	y
λ	$c^{-\sigma}$
w	mc
f_1	$(\epsilon - 1)y/(1 - \beta\theta^2)$
f_2	$\epsilon mcy/(1 - \beta\theta^2)$
f_3	$(1 - \epsilon)y/\{(1 - \beta\theta)(1 - \theta)\}$
f_4	$\epsilon mcy/\{(1 - \beta\theta)(1 - \theta)\}$
f_5	$(1 - \theta)\beta\theta f_3/(1 - \beta\theta^2)$
f_6	$(1 - \theta)\beta\theta f_4/(1 - \beta\theta^2)$

Table 2.1: Steady State Values of Variables of Colluding Firms Model

Chapter 3

Expectations on Other Firms' Pricing Behaviours and the Real Effect of Money

3.1 Introduction

The New Keynesian model has long been a workhorse in analysing monetary policy. In the framework of a dynamic stochastic general equilibrium, the model generates the non-neutrality of money with rational firms facing nominal price stickiness on a monopolistically competitive market. In particular, price stickiness is regarded as one of the main driving forces of the real effect of money which supports the effectiveness of monetary policy on the real activity in the economy. To feature price stickiness, it is usually assumed that some firms are not allowed to change prices as in Calvo (1983) or that the update process of information is not quite frequent because of the costs involved as suggested by Mankiw and Reis (2002).[1] With price stickiness featured as above, the optimal prices which the rational firms choose by optimizing the profit-maximization problems are not so responsive to

[1] Menu cost models as in Caplin and Spulber (1987), Caballero and Engel (1991), Dotsey, King, and Wolman (1999), Golosov and Lucas (2007), Gertler and Leahy (2008), and Midrigan (2011) suggest another way of featuring price stickiness by assuming that firms adjust prices only when the benefits is greater than the costs of resetting prices.

monetary shocks in the New Keynesian model.

We can easily find that prices of individual firms are sticky in the sense that they are not reset at every period. Also, numerous papers like Bils and Klenow (2004), Angeloni et al. (2006), Nakamura and Steinsson (2008), and Klenow and Malin (2010) provide the micro evidences of the phenomenon of sticky price. However, it is open to debate whether such price stickiness can always generate the non-neutrality of money. Caplin and Spulber (1987) and Golosov and Lucas (2007) assert that money can be neutral even if the price of each individual firm is not flexible due to the menu costs when adjusting price. However, many papers such as Klenow and Kryvtsov (2008), Nakamura and Steinsson (2010), and Midrigan (2011) argue that the non-neutrality of money can be obtained even in such menu cost framework if the model features the wide dispersion of price changes which is observed in the real economy. Also, Head et al. (2012) claims that price stickiness has nothing to do with the real effect of money showing that price can be sticky even in a flexible economy. However, there has been criticism that price stickiness in the framework of the *New Monetarist Economics*,[2] which Head et al. (2012) use, has a different meaning from that of the standard New Keynesian model.[3]

As seen above, various opinions have been stated and debated on the relationship between price stickiness and the real effect of money. However, if we confine our interest to the New Keynesian framework, the area of debate has been primarily focused on the menu cost models rather than other popular frameworks such as Taylor (1980) and Calvo (1983). Even though the most popular and the most commonly used tool in featuring price stickiness is the Calvo framework, no question has been raised on whether the Calvo-style price stickiness can always produce the non-neutrality of money. The purpose of this chapter is to test the degree of interrelationship between the two phenomenons. Particularly, this chapter gives attention to the rationality assumption of the standard Calvo framework and tests whether we can still find the close relationship between price stickiness and the real effect of

[2]See Williamson and Wright (2010a, 2010b) for detailed overviews.
[3]See Kryvtsov (2010) and Gorodnichenko and Weber (2013) for detailed discussions.

money even if the basic assumption of rationality is relaxed. In this chapter, the relaxed rationality provides the endogenous mechanism which brings about the high volatility of firms' reset prices as is shown in Chapter 1 resulting in a more flexible inflationary response even with price stickiness, and it is checked whether the result of Chapter 1 can be replicated.

The assumption of rationality is commonly used in macroeconomic models for its usefulness. In the standard New Keynesian model, it has two different meanings. Firstly, all agents' behaviours are decided by maximizing or minimizing their objective functions within their information sets. The second one is that they have 'rational expectations', which means agents' expectations coincide with the realized one in the model.[4] Under such assumption, the model can be simplified such that we can easily understand the sophistication of the economic agents' behaviours.[5] However, the assumption of rational agents requires two additional implicit assumptions. One is that agents have perfect knowledge about the model and share it as a common information set. The other is that they have such a strong ability of calculation as to solve extremely complicated optimization problems with infinite horizon. However, these requirements are nearly impossible to meet in the real world. Many experiments such as in Kahneman, Knetsch, and Thaler (1991) demonstrate the biases of decisions made by people. Also, Mankiw, Reis, and Wolfers (2004) argue that rationality cannot be supported for the inflation forecasts by showing that expectations on median inflation have been biased and inefficient.[6] In this sense, there have been many attempts to relax the strong assumption of rationality. Since Simon (1955, 1957), numerous literature assumes that agents are 'boundedly rational' due to their limited knowledge and restricted ability of calculation so that their behaviours can lead to a systematic bias and a discrepancy between the agents' expectations and

[4]In other words, the expectations are model-consistent.

[5]For example, with the rational expectation hypothesis, all we need to take into consideration is just the agent's expectation for only the next period, not for the infinitely long horizon, due to the law of iterated expectations.

[6]Similarly, Souleles (2004) shows that households' expectations are biased and inefficient, and Capistran and Timmermann (2009) claim that the assumption of rationality cannot explain the tendency of under- and over-prediction of inflation observed in the US survey data.

the realized one. The main objective of this chapter is to examine how such relaxation of agent's rationality affects the relationship between price stickiness and the non-neutrality of money.

Therefore, this chapter basically builds on the assumption of irrational agents so as to belong to the stream of literature modelling bounded rationality. However, this chapter differs from the literature in the sense that the relaxation of rationality is at its minimal level. Firstly, the bounded rationality is confined to only firms. While the literature assumes that all agents in an economy are irrational, households and monetary authority in this chapter are still assumed to be fully rational as in the standard model. Secondly, the degree of restriction to the knowledge which firms can access is also set to a minimum. Unlike most of the literature,[7] this chapter assumes that the boundedly rational firms have most of the information on the economic structure of the model, and the only missing information is the dynamics of the aggregate price level which is explained in detail below. Also, this chapter maintains the assumption that firms have enough computational capability so that they can solve the optimization problems with the same degree of complexity as in the standard model with rational firms. This enables the firms in the model to solve their own optimization problems and choose the reset prices as the rational firms do in the standard model, once the gap in the information set is filled with their subjective expectations.

As stated above, firms in the model of this chapter have all the information about the economic structure except for the dynamics of the aggregate price level. The formulation of Calvo pricing is also in the information set given to firms. In other words, all firms know that only a certain fraction of firms can adjust their prices *á la* Calvo (1983) and that the rest maintains the previous level of price. This means it is known to firms that the aggregate price level is decided by the weighted combination of the price in the previous period and the average of reset prices in

[7] Most literature with bounded rationality assumes that agents have no information on any structure of economic system so that they have to have their own expectations on the law of motion for the economy or on the main variables such as inflation and output.

the current period. That is, firms have information on how the dynamics of the aggregate price level is structured. However, the incompleteness of the information comes from the assumption that firms are not aware of what the average reset price would be. If firms are all rational and have the information that all firms face the same optimization problem, they will also know that their reset prices would be the same. This enables each individual firm in the standard model to have exact information that the average reset price is the same as its own optimal price. However, in the model of this chapter, firms are assumed to have no information on how other firms' prices would be decided, and consequently, they cannot know what the average of firms' reset prices would be. This makes the information on the dynamics of the aggregate price level incomplete to each individual firm.

Without full knowledge on how the aggregate price level varies, firms cannot optimize their reset prices. Therefore, firms might try to fill in the missing part of the information. In this sense, each individual firm of this model is assumed to have its own expectation on the average reset price in order to make up for the incompleteness of the information on the dynamics of the aggregate price level. In having expectation on the average reset price, firms are assumed to be boundedly rational in the sense that they use simple heuristics due to the limited knowledge on other firms' pricing behaviours. With the heuristics on average reset price, firms come to have their own subjective information on the dynamics of the aggregate price level. Hence, given the subjective expectations, firms can choose their optimal prices in the same way as the rational firms do because they are assumed to have the same high level of computing ability as addressed above. However, since the expectation on average reset price is established in an irrational way,[8] which might lead to systematic biases, firms' ex-ante expectations on the future variables as solutions of their perceived model cannot coincide with the ex-post realized equilibrium of the model.

The reset prices chosen by the individual firms behaving in the above-mentioned

[8]This means that firms do not know the true feature of the dynamics of the aggregate price level.

manners are different from those in the standard model. Most of all, the simulation results of the model with positive monetary shocks show that, when each firm's subjective expectation on the average reset price of all repricing firms is smaller than its own, the price chosen by the individual firm is higher than that of the fully rational firms. Consequently, the realized response of inflation is higher even though the firm's expectation is much lower than that of the standard model. A firm's expectation of lower aggregate price naturally leads to an expectation of higher output level, which means that the firm comes to anticipate a higher demand against which it has an incentive to raise its price. A lower aggregate price level also means a higher relative price, and therefore, the firm would have an incentive to lower its price. However, the effect of higher demand is much greater than that of the higher relative price when shocks are persistent as in the proposed model. This is because the persistent shocks lead to an anticipation of continuous increases of demand in the future, and firms have much incentive to raise prices in advance in consideration of not being able to adjust their prices in subsequent periods due to price stickiness.

This model also shows that heterogeneity of firms' subjective expectations on the average reset price gives rise to higher aggregated level of firms' chosen prices, even if the average of firms' various expectations is the same as that of the rational firms. In other words, even if a *median firm*[9] believes that the average of all firms' reset prices would be the same as its own as the rational firms do in the standard model, the realized level of average reset price is higher than the median firm's reset price which is the same as in the standard model, once the expectations of individual firms are different from one another and dispersed enough. This is because the deviation of firm's price from the standard model is not the same between the two opposite cases of expectations: lower and higher average reset price. By the same reasoning as in the above case of lower expectation, a firm which expects higher average reset price sets its price at a lower level compared to the standard model. However, the difference from the standard model is smaller than that of the opposite case.

[9]The *median firm* here denotes the firm whose expectation is the same as the average of all firms' expectations.

Even though the firm's incentive to raise price is reduced due to the anticipation of lower output response which comes from the higher expectation on the average reset price and inflation response, the firm still has concerns about the possibility of not receiving the Calvo signal to adjust its price next period. Hence, with such fear, the firm needs to hold its price at a certain high level in advance. In other words, price stickiness which the firm faces props up the level of reset price. Due to these reasons, the difference of a firm's reset price from the standard model in this case is smaller than that of the opposite case, and therefore, the average of the two cases is higher than the optimal price of the standard model. Consequently, this indicates that the heterogeneous expectations can make the response of inflation higher.[10] Furthermore, in the model, such responsive inflation is maintained unless the update speed of expectation is so high.

All these results indicate that if firms have different expectations on the average reset price and if some of their expectations are inclined to a lower level than their own, the average of the repricing firms' reset prices can be higher than that of the standard model due to the elastic response of firms' pricing against the expected change in the future demand. As a result, the real effect of money in this model can be very small even with price stickiness featured in each firm's price setting. This means that price stickiness cannot guarantee the non-neutrality of money when repricing firms have subjective expectations on the average reset price with bounded rationality due to the limited knowledge about other firms' pricing behaviours. In the real world, firms are not fully rational and their information is not complete. The model shows that price stickiness cannot ensure the effectiveness of monetary policy under such circumstance and implies that central banks and economists need to try to find another factor which brings about the non-neutrality of money other than price stickiness of individual firms.

This chapter consists of the following sections. Section 3.2 addresses the litera-

[10]This result is consistent with the empirical evidence that there is a positive relationship between the dispersed inflation expectation and the level of realized inflation, which can be seen in Cukierman and Wachtel (1979), Mankiw, Reis and Wolfers (2004), and Souleles (2004).

ture which can be related to this chapter. Section 3.3 presents the main assumptions on the information set firms have regarding the dynamics of the aggregate price level and explains how the firms with bounded rationality deal with the information. Section 3.4 describes the simple New Keynesian Calvo model which the firms perceive and demonstrates how the economy is realized. In section 3.5, it is reported how the boundedly rational firms respond to monetary shocks and how the aggregate price and output level are different from those in the standard model. Last section summarizes the main results of this chapter and concludes with limitations and weaknesses of the model which can be made up for in future research.

3.2 Literature Review

This chapter can belong to a set of literature on bounded rationality in the sense that firms in the model are assumed to be boundedly rational in expecting average reset price. The literature on models with the bounded rationality has been led with the popularity of 'adaptive learning' over the past two decades.[11] Evans and Honkapohja (2001), Bullard and Duffy (2002), Bullard and Mitra (2002), and Evans and Honkapohja (2003) show the determinacy and convergence to rational expectation under adaptive learning. Orphanides and Williams (2004, 2005a, 2005b), Adam (2005), Milani (2011), and Eusepi and Preston (2011) demonstrate that learning can make enough persistence of the economic variables as seen in the real economy, while Sargent and Williams (2005), McGough (2006), Cho and Kasa (2008), Ellison and Scott (2013), and Kolyuzhnov, Bogomolova, and Slobodyan (2014) argue that learning can yield large deviation from the rational expectation equilibrium. Even though the economic knowledge known to the learning agents in the literature is incomplete as in this chapter, the learning agents are still assumed to share the information. That is, most of the learning models assume that there exists the representative agent who holds imperfect information on the economy, but the model in

[11] Overview of the adaptive learning models can be seen in Sargent (1993) and Evans and Honkapohja (2001).

this chapter opens the possibility of heterogeneous expectations. Also, the representative agents in those literature take expectations for the reduced form of the law of motion of the main aggregate variables in equilibrium and calculate the coefficients of the expected form using econometric and statistical methods, while the model of this chapter use the standard optimization process.

Therefore, this chapter relates more to another stream of literature with bounded rationality which deals with heterogeneous expectations with imperfect information. The examples in this category include Evans and Honkapohja (2003, 2006), Berardi (2007), Dennis and Ravenna (2008), Branch and McGough (2009, 2010), Grauwe (2010), Anufriev, Assenza, Hommes, and Massaro (2008), and Massaro (2013). These papers assume that agents use simple but different heuristics for their own expectations with the support of empirical evidences for heterogeneous forecasts provided by various literature such as Carroll (2003), Mankiw, Reis, and Wolfers (2004), Branch (2004), and Pfajfar and Santoro (2010). However, the difference from the model in this chapter comes from the fact that the agents in those literature establish direct expectations on inflation or output which are the main variables of the economy; whereas, in this chapter, firms' heuristics are for just average reset price, and the expectations on inflation or output are derived through firms' optimization processes as in the standard model, even though the expectations are not model-consistent. This is due to the differences in assumptions made on firms' information sets between this model and the ones in literature. As mentioned above, the model of this chapter assumes that all information other than the average reset price is given to firms, which means firms know how the aggregate variables are determined. Therefore, firms can derive the equilibrium level of inflation and output once their missing information is filled with their own subjective expectations. However, in most literature, the only thing that firms know is their own optimization problems and the corresponding constraints. All other information needed for the derivation of aggregate variables is not given. Consequently, in such setup, firms must have expectations on inflation or output level. Also, the liter-

ature in this stream mainly deals with the determinacy and stability of equilibrium in relation to the design of monetary policy. However, this chapter focuses on the real effect of money under price stickiness with firms' irrational and heterogeneous expectations in comparison to the standard model with the rational expectation.

If we broaden the category of our interest to rational expectations, we can find numerous literature focusing on inflation dynamics under price stickiness with imperfect and heterogeneous information. The literature relates to this chapter in the sense that they assume the information on economic structure is not complete to each individual firm. Demery and Duck (2001), Nimark (2008), Angeletos and La'O (2009), and Lorenzoni (2009) show gradual response of inflation when idiosyncratic or noisy factors are mixed with aggregate shocks. However, these are for marginal cost, demand, or productivity shocks which are not appropriate for analysing of the non-neutrality of money. Even though Hellwig and Venkateswaran (2009)[12] deal with monetary shocks, they assume a fully flexible economy in which firms are free to adjust their prices, which is inadequate for analysing the effect of price stickiness on the non-neutrality of money. However, the biggest difference between these papers and the model of this chapter is that firms are able to have exact information on how the other firms reset their prices. In the literature, firms know that all firms have the same optimization problem[13] even though their information on the shocks is incomplete and different from one another. Also, the idiosyncratic or noisy parts which give rise to uncertainty or incompleteness of the information are assumed to be drawn from certain distributions which are known to firms exactly. Therefore, firms can calculate the probability of distribution of other firms' prices, and the predictions are consistent with the realized one. However, in this chapter, firms neither have an idea on the other firms' optimization problems nor on the distributions of the idiosyncratic or noisy shocks.

The model in this chapter shows that the real effect of money on the economy

[12]This paper can also be classified into the category explained in the next paragraph in the sense that the authors demonstrate the possibility of nearly neutral money.

[13]This means that all firms know that they choose their reset prices in the same way of maximizing the present value of expected future profits through infinitely long horizons.

can be reduced when repricing firms set their reset prices with high volatility due to, for example, incomplete information and limited calculation ability. In this sense, this chapter shares the implications on the effect of price stickiness on the non-neutrality of money with the literature[14] such as Caplin and Spulber (1987), Golosov and Lucas (2007) and Head et al. (2012), though their models are under state-dependent menu cost framework or search friction while this chapter is based on the standard Calvo-style sticky price model.

3.3 What Firms Know about the Aggregate Price level

As expressed in the previous chapters, an individual firm i in the standard Calvo model chooses its optimal price following the optimality condition[15] which is given by

$$E_t \sum_{s=0}^{\infty} \theta^s Q_{t,t+s} \left(\frac{p_{it}}{P_{t+s}} \right)^{-\epsilon} y_{t+s} \left[1 - \frac{\epsilon}{\epsilon - 1} mc_{t+s} \left(\frac{p_{it}}{P_{t+s}} \right)^{-1} \right] = 0, \tag{3.1}$$

where θ is the probability of not changing prices, $Q_{t,t+s}$ is the stochastic discount factor, P_t is the aggregate price level, p_{it} is the optimal price of firm i, mc_t is the marginal cost, ϵ is the price elasticity of demand, and y_t is the aggregate output. Since the firm is assumed to have all the information on the economy including households' optimality condition and the policy of monetary authority, it can have its optimality condition as the following function.[16]

$$p_{it} = f(P_t, E_t \Omega_{t+s}), \tag{3.2}$$

where $\Omega_{t+s} = \{P_{t+s}, \nu_{t+s}\}$ and $s > 0$.

As can be seen, for the adjustment of price, an individual firm i needs the information on P_t and its expectation on the future aggregate price level and the monetary shocks. In the standard model, P_t is assumed to be given to all firms in the sense that the individual firm's price cannot affect the aggregate price level because

[14] See Chapter 1 for the review over the literature in this category.
[15] See Appendix 3.A for the derivation.
[16] See Appendix 3.B.

there are so many firms in the economy. However, this does not mean that all firms know the exact level of P_t at the moment of price setting because the aggregate price level is decided just after all firms' price settings are completed. Therefore, a firm cannot have information on P_t when it resets its optimal price. However, this cannot be a problem under the standard framework under which all firms are assumed to be rational with perfect information and face the same decision problem. In the standard Calvo model, only a fraction $(1-\theta)$ of firms can adjust their prices, and an individual firm j in the fraction $(1-\theta)$ resets its optimal price (p_{jt}) maximizing its expected profit. Since all repricing firms are assumed to face the same optimization problem, their optimal prices would be the same $(p_{jt} = p_t^*, j \in (1-\theta))$. Therefore, from the definition of the aggregate price level $\left(P_t = \left[\int_0^1 p_{it}^{1-\epsilon} di\right]^{\frac{1}{1-\epsilon}}\right)$, [17] we can get the dynamics of the aggregate price level as

$$\begin{aligned} P_t^{1-\epsilon} &= \int_{\omega_t} p_{it-1}^{1-\epsilon} di + \int_{1-\omega_t} p_{it}^{1-\epsilon} di \\ &= \theta P_{t-1}^{1-\epsilon} + (1-\theta) \int p_{jt}^{1-\epsilon} dj \\ &= \theta P_{t-1}^{1-\epsilon} + (1-\theta) \left(p_t^*\right)^{1-\epsilon}, \end{aligned} \tag{3.3}$$

where $\omega_t \subset [0,1]$ denotes the set of firms that are unable to reset prices at time t. In the standard model, the information on this dynamics of P_t is implicitly assumed to be known to the repricing firms like any other information on the economy. In other words, any individual firm in the standard model knows that all the optimal prices of other firms will be the same as its own and the aggregate price level will be as in (3.3). Therefore, at the moment of price setting, firm i can have the information on P_t as a function of P_{t-1} which is already known and p_t^* which is the same as its optimal price. If equation (3.3) is substituted into the function (3.2), we can have

$$p_{it} = p_t^* = f(E_t \Omega_{t+s}), \qquad s > 0 \tag{3.4}$$

given that P_{t-1} is known in period t. This equation shows that only the expectations

[17] See Appendix 1.B of Chapter 1 for the derivation.

on future variables matter in setting optimal price.

We need to note that the main assumption which enables firms to have such information on P_t is that firms acknowledge that their optimal prices are the same. As seen before, knowledge on the definition of the aggregate price level gives the firms information on how P_t is set, which is expressed by

$$P_t^{1-\epsilon} = \theta P_{t-1}^{1-\epsilon} + (1-\theta)\left(p_t^*\right)^{1-\epsilon}, \tag{3.5}$$

where p_t^* is the average reset prices of all the repricing firms as follows.

$$\left(p_t^*\right)^{1-\epsilon} = \int p_{jt}^{1-\epsilon} dj, \qquad j \in (1-\theta) \tag{3.6}$$

With the knowledge of the same optimal price, each firm i can replace p_t^* by p_{it} which is its own optimal price and, therefore, get the exact information on P_t.

However, in the real world, it is difficult to think of a case where all firms have the same optimal prices and also all firms know such information at the moment of price setting. Rather, it would be more common to think that firms have no precise information on what other firms' prices would be.[18] There are many factors which prevent firms from obtaining information on the same optimal prices in the real world. It might be that there are idiosyncratic productivity shocks, or that each firm faces different price stickiness. In those cases, optimal prices of the firms cannot be the same. However, if the rational firms have full information on such idiosyncratic shocks or different degree of price stickiness, they can easily get the information on P_t even without knowledge of the same optimal price. This is because they know how p_t^* is decided. For example, given the information on the distribution of the idiosyncratic shocks, firms have expectations on how other firms set their optimal prices against the shocks and, therefore, can have the information on the average of

[18] Based on such circumstance of limited information, *competitor analysis* (see, for example, Smith, Grimm, and Gannon (1992) and Czepiel and Kerin (2012) for detailed explanations) becomes one of the main fields in marketing theory. Also, many economists, e.g., Goic and Montgomery (2011), suggest econometric models to predict competitors' pricings with such imperfect information.

the optimal prices.[19] In addition, in such a case, firms' beliefs on the other firms' prices are consistent with each other because the information they have on the shocks is *true*, and consequently, their subjective probability distributions can be the same as the true ones of the aggregate variables such as inflation and output. As can be seen in this example, what is important for the information on P_t is not whether firms have the same optimal prices, but whether they can have information on the average optimal prices, p_t^*. In other words, if all firms know exactly how the other firms' optimal prices are set, they can have exact information on the aggregate price level; but if not, they cannot. In above cases, if firms have no information on the other firms' idiosyncratic shocks or degree of price stickiness, they cannot know how other firms set their optimal prices, and therefore, they cannot have information on P_t.

In this sense, firms in this chapter are modelled not to know how other firms' optimal prices are set. More precisely, even though all firms in the model of this chapter face the same optimization problems as in the standard model,[20] they cannot be sure that they are setting the same optimal price because they have no information on other firms' pricing behaviours. This means that there is no common knowledge about the process of choosing reset price. As in the standard model, this chapter assumes that all firms adjust their prices maximizing the present discounted value of the expected profits. However, unlike the standard rationality framework, the information about the same pricing behaviour is not common to all firms.[21] If the information is common, each firm would have resolute confidence that the other firms have the same rational optimization problem, and therefore, that they will set the same optimal price as its own. However, in the model of this chapter, firms do not know whether the other firms optimize in the same way as their own.[22] Even

[19]Lots of papers featuring this kind of idiosyncratic shock solve the models with an algorithm using higher order expectations. See Nimark (2008) for example.

[20]This means there is no factor which makes any heterogeneity among firms such as idiosyncratic shocks and difference in individual price stickiness.

[21]This is reminiscent of the concept in game theory; Lack of Common Knowledge of Rationality (LCKR). However, the difference is that LCKR still assumes the rationality of each player in the game while this chapter does not.

[22]In the real world, firms' pricing strategies are not confined to the profit maximization used in

if they happen to know that their pricing behaviours are based on the same way of profit maximization, they have no idea whether the other firms have idiosyncratic shocks, whether the shocks are noisy, or which distribution the idiosyncratic or noisy factor of the shocks follow.[23] Therefore, each individual firm in the model cannot be sure of other firms' pricing behaviours and, thus, cannot have any information on the average reset price (p_t^*) of the economy. Hence, the firms cannot have such information on the dynamics of the aggregate price level as in (3.3).

Then, what can firms do without any information on p_t^*? As we have seen in (3.5), if p_t^* is unknown, it means that there is no information on P_t either. However, equation (3.2) shows that an individual firm needs information on P_t for its optimal price.[24] Therefore, before price setting, all firms are anticipated to establish their expectations on p_t^* using the information they have. For the purpose of estimating p_t^*, firms might want to know how many different pricing strategies exist within firms, what they look like, and what other firms' expectations on p_t^* would be. However, in this model, it is assumed that firms cannot have perfect information on those factors due to their restricted availability and/or limited processing ability of the information. Hence, their expectations on p_t^* are far from rational expectation in the sense that they do not align with the realized one after all firms' pricing processes are over. In this model, it is assumed that the expectation on p_t^* of an individual firm i is decided using a simple heuristic which is expressed by

$$\tilde{E}_{it} p_t^* = p_{it}^{\gamma_i} P_{t-1}^{1-\gamma_i}, \tag{3.7}$$

where $\tilde{E}_{it}$ is the subjective expectation operator of the firm i and γ_i is the weight which the firm i puts on p_{it} and P_{t-1}. There are several things to note in the

the standard macro model. See Monroe (2001) and Kotler and Armstrong (2010) for the examples of various pricing behaviours. Firms in the model of this chapter might have the variety of pricing patterns in minds.

[23] Most literature such as Nimark (2008) and Lorenzoni (2009), which models the incomplete and dispersed information on the shocks with idiosyncratic or noisy components, assumes that the information on the shocks is exactly given to all firms.

[24] More precisely, in the viewpoint of model solution, each firm needs to know the relationship between the aggregate price level and its own reset price.

above heuristic. Firstly, the heuristic is expressed as a function of firm i's reset price and the aggregate price level in the previous period. However, most literature with bounded rationality uses constant heuristics for agents' predictors, and one of the predictors coincides with the equilibrium value of rational expectation. For example, in most literature, each agent chooses one of the given set of constant numbers for the expectation on future inflation like $\tilde{E}_{it}\hat{\pi}_{t+1} = a$, and if the agent has rational expectation, then a would be equal to zero which is the steady state value of equilibrium in the standard model. In our case, firms with rational expectation would have the belief, $p_t^* = p_{it}$, which cannot be expressed by any certain constant, and this shows that the plausible form of expectation on p_t^* should be a function of p_{it}. Furthermore, by the functional form of p_{it}, we can incorporate into the model the implicit assumption that the value of p_t^* which firms are interested in is the relative value with respect to their own reset prices rather than the absolute value.[25] On the other hand, inclusion of P_{t-1} in the function is to prevent extremely wild irrationality. In other words, by using information on P_{t-1}, we can avoid the irrational expectation on p_t^* which deviates too far from the level of recent aggregate price level. The second point to be given attention is that the main factor which decides the firm's expectation is the constant, γ_i, in the heuristic. Since the variables in the function of heuristic are the control variable of the firm (p_{it}) or the information already given to all firms (P_{t-1}), practical factor which has significant effect on $\tilde{E}_{it}p_t^*$ is γ_i. For example, if a firm believes that all the other firms have the same optimization problem as its own and has confidence that all the other firms have the same beliefs as well, γ_i would be one, which is the same case as in the standard model with rational firms. However, in other cases, γ_i can have any numbers. Suppose that firm i observes expansionary monetary shock at the beginning of a period. If it believes that there are many firms with different optimization problems from its own and that their pricing behaviours are overwhelmed by pessimistic outlook on

[25] This is parallel to the phenomenon that the 'competition-based pricing', which refers to the strategy of setting price at the similar value of competitor's price, is one of the popular pricing strategies in the modern economy. For example, Raju and Zhang (2010) say "Competition-based pricing is the second-most-popular price-setting approach".

the economy, it might expect that the average reset price of the economy would be less than its own price and, consequently, have γ_i much smaller than one but greater than zero.[26] However, if it expects a much higher average reset price, γ_i would be greater than one. Therefore, we can say that γ_i represents all the subjective information of firm i on the other firms' pricing behaviours.[27] Lastly, the heuristic decides firm i's expectation on the aggregate price level (P_t) as in the same way of the standard model. As seen above, the heuristic is for the average reset price (p_t^*) of the economy. However, each individual firm knows the dynamics of P_t as of (3.5), and therefore, they can have their expectations on P_t as well based on the expectation on p_t^* obtained by using the heuristic. In the sense that the heuristic is a function of a firm's own choice (p_{it}), the expectation on P_t is also a function of its reset price, not a fixed value. This applies to the rational firms in the standard model as well. They also use (3.5) for the information on P_t, and the information on p_t^* is given by a function of their own optimal prices ($p_{it} = p_t^*$). The only difference is that their information on p_t^* is true while the information of the firms in this model is based on their subjective expectations.

3.4 Model

As in the previous chapters, the basic framework of the model in this chapter is the standard new Keynesian Calvo model. Rational households optimize their decisions and monetary authority follows a certain interest rule for its policy as in the standard model. However, as shown below, firms' pricing behaviours are affected by their non-rational expectations on the average reset price due to the limited information on

[26] The positive γ_i in this example is due to the implicit assumption that each firm does not expect the price cutting of the adjusting firms, including itself, against expansionary monetary shocks.

[27] Due to its subjectivity, γ might not be able to be observable in any published data. This can be an obstacle that hinders empirical analyses on firms' expectations on the average reset price represented by γ. One possible way of getting the information on γ is to use the equations (3.5) and (3.7) jointly. If we could get the information of an individual firm's ex-ante expectation on P_t, we would be able to estimate γ indirectly with the published data on P_{t-1} and p_{it} using the equations. However, even though there are many kinds of survey data on inflation expectations, it is another problem whether we can gather as many data sets as we need in order to secure the representativeness of the data for all repricing firms.

other firms' optimization problems.

3.4.1 Households

As explained in Appendix 1.B of Chapter 1, the rational representative household chooses the aggregate consumption (c_t), the amount of hours of labour supply (h_t), and the real bond holding (b_t) following the optimality conditions as

$$c_t^{-\sigma} = \lambda_t \tag{3.8}$$

$$Q_t = \beta E_t \left(\frac{P_t}{P_{t+1}} \frac{\lambda_{t+1}}{\lambda_t} \right) \tag{3.9}$$

$$\lambda_t \frac{W_t}{P_t} = \lambda_t w_t = h_t^{\varrho}, \tag{3.10}$$

where λ_t is the Lagrangian multiplier of utility maximization problem, Q_t is the price of bonds with a nominal return rate of $Q_t^{-1}(= R_t)$, and $w_t\,(= W_t/P_t)$ denotes real wage.

3.4.2 Monetary Authority and Market Clearing

The policy rule of monetary authority for interest rate (R_t) against monetary shock (ν_t) and market clearing conditions (as in Appendix 1.B of Chapter 1) are given as

$$\frac{R_t}{R} = \left(\frac{\pi_t}{\pi}\right)^{\eta_\pi} \left(\frac{y_t}{y}\right)^{\eta_y} (\nu_t)^{-1} \tag{3.11}$$

$$\frac{\nu_t}{\nu} = \left(\frac{\nu_{t-1}}{\nu}\right)^{\rho} \exp(e_t), \qquad e_t \sim N(0, \varsigma) \tag{3.12}$$

$$y_t = c_t \tag{3.13}$$

$$h_t = y_t, \tag{3.14}$$

where ρ is the persistence parameter of the shocks.

3.4.3 Firms

Each of a continuum of firms $i \in [0, 1]$ produces its own goods, y_{it}, hiring labour, h_{it}, through the production technology as

$$y_{it} = h_{it}.$$

Each firm i minimizes costs and maximizes its expected profit regarding the production to choose its reset price (p_{it}). As explained in Appendix 3.A, the decision makings on the cost minimization and the profit maximization yields the conditions as

$$mc_t = w_t \tag{3.15}$$

and

$$\tilde{p_t}\left(= \frac{p_{it}}{P_t}\right) = \frac{\epsilon}{\epsilon - 1}\frac{f_{1t}}{f_{2t}}, \tag{3.16}$$

where

$$f_{1t} = \lambda_t mc_t y_t + E_t \theta \beta \pi_{t+1}^{\epsilon} f_{1t+1} \tag{3.17}$$

$$f_{2t} = \lambda_t y_t + E_t \theta \beta \pi_{t+1}^{\epsilon-1} f_{2t+1}, \tag{3.18}$$

and $\tilde{p_t}\,(= p_{it}/P_t)$ is the relative optimal price of firm i. However, the information which firm i has on P_t (and on π_t as well) is not complete as shown in Section 3.3. Hence, the dynamics of the aggregate price level perceived by the firm is modelled as described in the subsection below.

Dynamics of Aggregate Price Level

Following the standard Calvo framework, only a fraction $(1 - \theta)$ of firms are allowed to change their prices, and the rest of the firms are assumed to remain at their previous level of prices. Therefore, as mentioned above, the definition of the aggregate price index gives the dynamics of the aggregate price level as in (3.5) and (3.6). However, because of the lack of knowledge on the other firms' optimization

problems, firm i cannot have precise information on the average optimal price, p_t^*. Even though all firms optimize in the same way of profit maximization as rational firms in the standard model and, therefore, have the same optimality conditions as in (3.15)∼(3.18), they cannot be sure of this because they are assumed not to know how the other firms reset their prices. Therefore, they cannot be confident that they would have the same reset price against monetary shocks, and consequently, each firm has to have its own expectation on p_t^* which is assumed to be established as in (3.7). With the expectation on p_t^*, individual firm i can have its subjective information on the dynamics of the aggregate price level as a form of

$$\tilde{E}_{it} P_t^{1-\epsilon} = \theta P_{t-1}^{1-\epsilon} + (1-\theta)\tilde{E}_{it} \left(p_t^*\right)^{1-\epsilon}. \tag{3.19}$$

Dividing both sides of (3.19) and (3.7) by P_t, firm i can get the following equations in its information set.

$$1 = \theta \tilde{E}_{it} \pi_t^{\epsilon-1} + (1-\theta)\tilde{E}_{it} \left(\tilde{p_t}^*\right)^{1-\epsilon} \tag{3.20}$$

and

$$\tilde{E}_{it} \tilde{p_t}^* = \tilde{E}_{it} \left[\tilde{p_t}^{\gamma_i} \pi_t^{\gamma_i - 1}\right], \tag{3.21}$$

where $\tilde{p_t}^* \left(= p_t^*/P_t\right)$ is the relative average reset price. It is necessary to note that firm i's subjective expectation operator, $\tilde{E}_{it}$, is put in front of P_t and π_t as well as p_t^*. This means that information on P_t and π_t is not given from the market anymore and that it is obtained through the subjective expectation on p_t^*.

3.4.4 Information Set and Price Setting

Each individual firm i is assumed to have all the information on the economy except for other firms' reset prices. In other words, from the market, all firms receive exact information on households' optimality conditions, (3.8)∼(3.10), policy rule

of monetary authority, (3.11), monetary shocks, (3.12), and the market clearing conditions, (3.13)∼(3.14). However, it is assumed that the pricing behaviours of other firms are unknown, which leads to the lack of information on the average optimal price, p_t^*. This missing information makes each firm have expectation on p_t^* as in (3.7). Also, as we have seen in (3.5), only after the set of expectation $\left(\tilde{E}_{it}p_t^*\right)$, the subjective information on P_t comes to each firm i. With such information added to the existing knowledge on the economy, individual firm i is then able to solve its optimization problem as the rational firms do in the standard model. More precisely, for firm i, the two equations of (3.20) and (3.21) which are obtained by the expectation on p_t^* and the other above-mentioned information are incorporated into an economic model to be solved. The model perceived by firm i consists of the log-linearized equations of

$$\tilde{E}_{it}\hat{y}_t = \tilde{E}_{it}\left[\hat{y}_{t+1} - \frac{1}{\sigma}\left(\hat{R}_t - \hat{\pi}_{t+1}\right)\right] \tag{3.22}$$

$$\tilde{E}_{it}\hat{R}_t = \tilde{E}_{it}\left[\eta_\pi\hat{\pi}_t + \eta_y\hat{y}_t - \hat{\nu}_t\right] \tag{3.23}$$

$$\hat{\nu}_t = \rho\hat{\nu}_{t-1} + e_t \tag{3.24}$$

$$\tilde{E}_{it}\hat{\tilde{p}}_t = \tilde{E}_{it}\left[(1-\beta\theta)(\sigma+\varrho)\hat{y}_t + \beta\theta\left(\hat{\pi}_{t+1} + \hat{\tilde{p}}_{t+1}\right)\right] \tag{3.25}$$

$$\tilde{E}_{it}\hat{\pi}_t = \frac{1-\theta}{\theta}\tilde{E}_{it}\hat{\tilde{p}}_t^* \tag{3.26}$$

$$\tilde{E}_{it}\hat{\tilde{p}}_t^* = \tilde{E}_{it}\left[\gamma_i\hat{\tilde{p}}_t - (1-\gamma_i)\hat{\pi}_t\right]. \tag{3.27}$$

The first equation, (3.22), represents information on the demand side of the economy which firm i can obtain from (3.8), (3.9), (3.13), and the fact of $\left(Q_t^{-1} = R_t\right)$. The information on the interest rule of monetary authority, (3.11), and the dynamics of shocks, (3.12), are given as in (3.23) and (3.24) respectively. Firm i's pricing behaviour can be expressed as in (3.25) using (3.10) and (3.13)∼(3.18). Equation (3.26) represents the subjective information on the dynamics of the aggregate price level, (3.20), and the irrational expectation on the average reset price, (3.21), is shown in (3.27).

As can be seen, all of the above equations except for the dynamics of shocks have the subjective expectation operator, $\tilde{E}_{it}$, in front of all the variables regardless of whether they are for current or future period. Also, the rational expectation operator, E_t, disappears even in the equation about the rational household's behaviour, (3.22). As mentioned above, the equations above construct the model which firm i perceives. In other words, firm i chooses its reset price by solving its perceived model which is the system of above equations. Even though the direct effect of firm i's irrational expectation on p_t^*, (3.27), is on the aggregate price level as in (3.26), the dynamics of the aggregate price level indirectly affects all the other variables in the process of solving the model. That is, all the variables in the equilibrium of the perceived model are based on firm i's expectation on the average reset price and, therefore, is affected by the firm's bounded rationality. In this sense, the subjective expectation operator, $\tilde{E}_{it}$, reflects firm i's expectation on the aggregate variables based on the belief on the average reset price. However, there is a common aspect between $\tilde{E}_{it}$ and E_t. It is that expectations are established 'rationally'. In other words, the expectations of both operators coincide with the outcome of the economic model perceived by the agents.[28] However, the difference between the two operators is that the economic structure comprehended by the agent with E_t is the real one, which leads to an agreement between the agent's expectation and the realized one, while the economy perceived by the agent with $\tilde{E}_{it}$ is just based on the subjective belief of the agent, which gives rise to biases of the agent's expectation on the aggregate economic variables. Therefore, in the model of this chapter, firm i's expectations on aggregate variables do not coincide with the realized one even though $\tilde{E}_{it}$ ensures the consistency between agents' expectations within the firm's perceived model.

[28]For example, $\tilde{E}_{it}\hat{\pi}_{t+1}$ has the same value as the equilibrium level of $\hat{\pi}_{t+1}$ as a solution of the perceived model.

3.4.5 Calculation of Realized Inflation and Output Response

The solution of the perceived model which consists of (3.22)∼(3.27) gives firm i not only its reset price, but also the equilibrium levels of aggregate price (P_t) and the output (y_t) within the model. However, as mentioned in the above section, the equilibrium levels do not align with the realized ones of the economy because the model perceived by firm i might not reflect the true structure of the economy but is based on its expectation on average reset price as in (3.7). Therefore, even though firm i expects that the aggregate price level would be decided as in (3.19), the expected price level can coincide with the realized one only when its expectation on the average optimal price comes true. Aggregate output level which comes as an equilibrium of the perceived model is also different from the realized one. By substituting (3.23) into (3.22), we can see that the equilibrium level of output which firm i anticipates is given by

$$\tilde{E}_{it}\hat{y}_t = \tilde{E}_{it}\left[\frac{\sigma}{\sigma+\eta_y}\hat{y}_{t+1} - \frac{1}{\sigma+\eta_y}\left(\eta_\pi\hat{\pi}_t - \hat{\pi}_{t+1} - \hat{\nu}_t\right)\right]. \tag{3.28}$$

As can be seen, y_t is affected by π_t and π_{t+1} which are based on firm i's expectation on P_t as in (3.20)∼(3.21). Therefore, the equilibrium level of y_t in the model perceived by firm i cannot coincide with the realized one as long as the expected level of P_t based on $\tilde{E}_{it}p_t^*$ does not match the ex-post aggregate price level after all of the firms' price settings are over.

The realized levels of P_t are decided by (3.5) and (3.6), not by (3.7) nor (3.19). In other words, once each firm sets its price, the average reset price of the economy is decided as in (3.6), and then, the aggregate price level is calculated as in (3.5). Consequently, the log-linearized expression for the realized inflation is given by

$$\hat{\pi}_t = \frac{1-\theta}{\theta}\hat{\tilde{p}}_t^*, \tag{3.29}$$

where we can find no expectation operator, which means that the variables in the equation are ex-post facto. The corresponding level of aggregate output is decided

by the representative rational household whose demand is given by

$$\hat{y}_t = E_t \left[\frac{\sigma}{\sigma + \eta_y} \hat{y}_{t+1} - \frac{1}{\sigma + \eta_y} \left(\eta_\pi \hat{\pi}_t - \hat{\pi}_{t+1} - \hat{\nu}_t \right) \right], \tag{3.30}$$

which is the log-linearized expression for the combination of (3.8), (3.9), (3.11), and (3.13). The expectation operator, E_t, in the equation is for the rational households, not for firms. Since the households are assumed to have perfect information on the economy including firms' boundedly rational pricing behaviours, they can have correct expectations on π_{t+1} and y_{t+1}.

3.5 Simulation Results

The model is simulated through two stages. In the first stage, the reset prices of individual firms are obtained by the simulation of the model perceived by each firm i, as in (3.22)∼(3.27), against positive monetary shocks.[29] The second stage is for the calculation of the realized outcome of the economy as in Section 3.4.5 above. Once the reset prices, p_{it}, are obtained in the first stage, we can calculate the average of the reset price, p_t^*, using (3.6). And then, equation (3.5) gives the ex-post aggregate price level. Consequently, we can obtain the realized value of inflation rate and aggregate output using (3.29) and (3.30), respectively.

The following sections show various simulation results with different assumptions on γ_i which plays a decisive role in firm i's expectation on p_t^*: the same γ among firms, the distributed γ_i across firms, and the updated distribution of γ_i. Firstly, the assumption of the same γ allows us to have a simple understanding on how the directions of firms' expectations affect the economy. However, in the real world, the expectations are usually not the same.[30] Some firms can have lower expectations while others have higher ones. What would happen if the average of the different expectations is the same as that of the fully rational firms? Can we find any differ-

[29] Dynare with Matlab is used for the simulation.

[30] For example, Mankiw, Reis, and Wolfers (2004) present the micro evidence that the interquartile range of inflation expectations is 1.5%∼2.5% among experts and 0%∼5% among the populace.

ence from the standard model even in such a case? The simulation with the second assumption of distributed γ_i enables us to answer the question. We can check what the aggregate effect would be in the case where there are different expectations and their directions cancel one another out. In other words, we can show whether the heterogeneity itself makes a difference even if the aggregate level of expectation is the same as in the standard model. However, the simulations with the first and second assumptions do not allow an adjustment of a firm's expectation. If a firm finds any discrepancy between its expectation and the outcome of the economy, it might want to adjust its expectation. The simulation with the last assumption of updated γ_i shows how the model is affected in such a case where firms reflect the realized level of aggregate price in the previous period.

3.5.1 The Same γ among Firms

The first simulation is with the assumption that γ_i is the same among all firms[31] for the simplicity of the model, which means all the repricing firms set the same optimal price. Under this assumption, the model is simulated with arbitrary values of γ, which are 0.8 and 1.2, and compared to the benchmark standard Calvo model. The impulse responses of the aggregate output, inflation and firm's reset price are given as in Figure 3.1.[32] The simulation result shows that γ equal to 0.8, which is less than one, makes a higher reset price, higher inflation, and a lower output response compared to the cases of $\gamma = 1.2$ and the standard model.[33]

Mechanism of γ on Firm's Optimal Price

From (3.7), we can find that an individual firm with lower γ expects a lower average optimal price and, therefore, anticipates a lower aggregate price level compared to the case of higher γ. Also, the firm will expect a much higher output with the

[31] With this assumption, the subscript of γ_i is removed in this section.

[32] The inflation and output responses shown in this and below sections are all realized outcome of the true model, not the ex-ante equilibrium level of the model perceived by each firm.

[33] Note that the model with $\gamma = 1$ is nothing but the standard model. As can be seen in (3.7), with $\gamma = 1$, we can have $\tilde{E}_{it} p_t^* = p_{it}$ which gives the same dynamics of aggregate price level, $P_t^{1-\epsilon} = \theta P_{t-1}^{1-\epsilon} + (1-\theta) p_{it}^{1-\epsilon}$, as in the standard model.

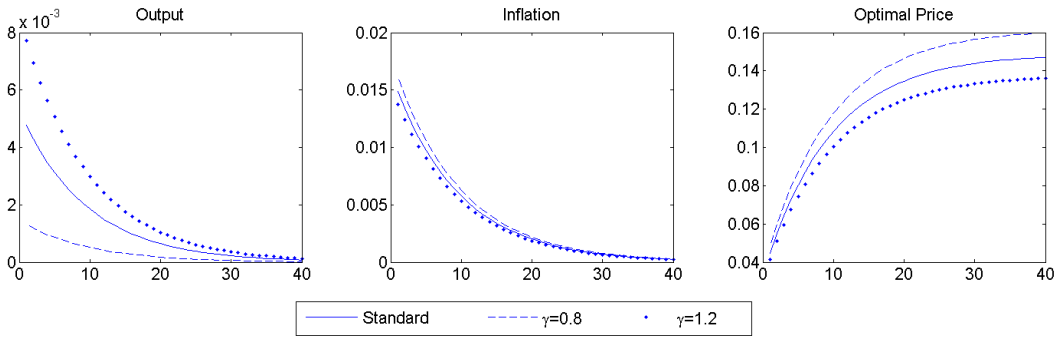

Figure 3.1: Impulse Response Functions with Different γ to Monetary Shocks

expectation of lower price level. This can be easily confirmed by rearranging some equations in the firm's information set. Putting (3.27) into (3.26), we can have the following equation.

$$\begin{aligned}\tilde{E}_{it}\hat{\pi}_t &= \frac{\gamma(1-\theta)}{1-\gamma(1-\theta)}\tilde{E}_{it}\hat{\tilde{p}}_t \\ &= \frac{1-\tilde{\theta}}{\tilde{\theta}}\tilde{E}_{it}\hat{\tilde{p}}_t, \end{aligned} \tag{3.31}$$

where $\tilde{\theta}$ is defined by $1-\gamma(1-\theta)$. Substituting (3.31) into (3.25) yields the following NKPC.

$$\tilde{E}_{it}\hat{\pi}_t = \tilde{E}_{it}\left[\frac{(1-\beta\theta)(1-\tilde{\theta})(\sigma+\varrho)}{\tilde{\theta}}\hat{y}_t + \frac{\beta\theta}{\tilde{\theta}}\hat{\pi}_{t+1}\right] \tag{3.32}$$

For comparison with the standard model, we can express the above equation as

$$\tilde{E}_{it}\hat{\pi}_t = \tilde{E}_{it}\left[\frac{(1-\beta'\tilde{\theta})(1-\tilde{\theta})(\sigma+\varrho)}{\tilde{\theta}}\hat{y}_t + \beta'\hat{\pi}_{t+1}\right], \tag{3.33}$$

where β' is defined by $\beta\theta/\tilde{\theta}$. Above NKPC is the same as that of the standard model with $\tilde{\theta}$ and β' except that there are subjective expectation operators ($\tilde{E}_{it}$) instead of the rational expectation operator (E_t), which means that the equation comes from firm i's perception on the economy, not from the true economic model. In the case of $\gamma < 1$, we have $\tilde{\theta} > \theta$ and $\beta' < \beta$. This indicates that the above NKPC has a gentler slope and a lower intercept when γ is less than one, compared to that of the standard model. This shows that when a firm expects a lower average

reset price than its own, the economy which the firm perceives is expected to have a lower aggregate price level and a higher output level as an equilibrium compared to that in the standard model.

The expectation of a lower price level leads to the anticipation of a higher relative price (p_{it}/P_t) at a specific level of reset price, which gives a firm an incentive to lower its price. At the same time, however, the firm with lower γ expects a higher level of output as we have seen above.[34] The higher level of output means that the firm faces higher demand, and this gives the firm an incentive to raise its price. Therefore, we can think of the two effects of lowering γ on the reset price which have clearly opposite directions. Whether which effect is greater depends on the size of parameters of the model. For a better understanding, suppose a simple case where there is no price stickiness ($\theta = 0$).[35] Using (3.8), (3.10) and (3.13)∼(3.18), we can have firm i's reset price for the simple case of the flexible economy as in

$$p_{it} = \frac{\epsilon}{\epsilon - 1} P_t y_t^{\sigma+\varrho}. \tag{3.34}$$

As can be seen, firm's reset price depends on the level of aggregate price and output of the economy which it expects. When an expectation on P_t is very low, a firm lowers its price, but the price is raised when the firm expects a high level of y_t. In particular, we can find that the effect of higher expectation on y_t is subject to the level of σ and ϱ.

If the economy is not so sensitive with the parameters having a high value, the effect of change in demand is usually greater than that of the relative price when the shocks are persistent as in the model of this chapter with $\rho = 0.9$. It is because, with the persistent shocks, the change in demand is extended to the far future, which means the forward-looking firm should take account of a much larger amount of change in demand in advance. Furthermore, the incentive to raise price

[34] For example, in the economy where Fisher's quantity theory of money applies, agent A with expectation of higher inflation will naturally expect lower output level than agent B who expects relatively lower inflation, given the same observation of monetary expansion.

[35] Even in this case, as long as γ is less than one, we have $\tilde{\theta} = 1 - \gamma > 0$ which means the economy a firm perceives is the same as the standard one with a certain positive level of price stickiness.

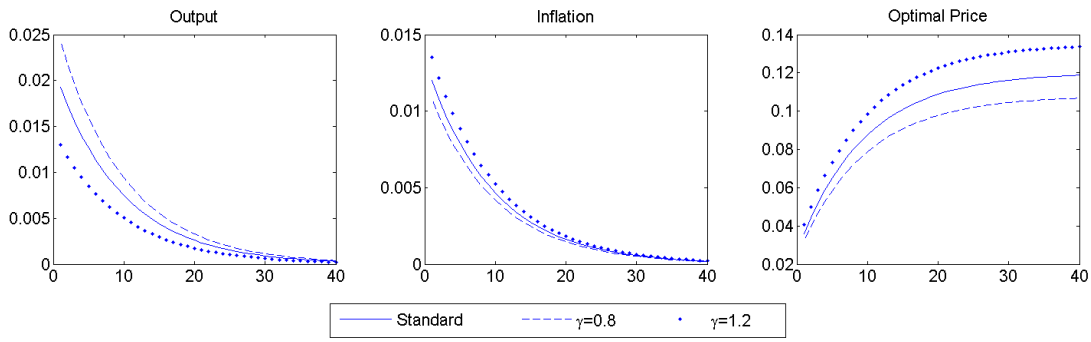

Figure 3.2: Impulse Response Functions with $\sigma = \varrho = 0.2$

against the expected increase in demand is amplified with price stickiness. Since firms are concerned with the possibility of not being able to adjust their prices in subsequent periods under the Calvo-type price stickiness, they want to set much higher prices in advance than needed in the current period. Therefore, the above incentive to raise price becomes greater to the extent to which the firm sets a higher price than the standard model as we have seen in Figure 3.1. However, if σ and ϱ have extremely small values like $\sigma = \varrho = 0.2$, for example, the effect of increase in demand gets smaller compared to the relative price effect, and therefore, firms' reset prices become lower than the standard model as shown in Figure 3.2 which demonstrates the opposite results to Figure 3.1. Firstly, lower σ represents the elastic responsiveness of consumption growth to the change of real interest rate. Since the real interest rate declines with the increase in price level, we can expect that the rise in a firm's price leads to a much smaller increase in future consumption compared to the current one. In a firm's viewpoint, this means that the importance of future demand gets smaller, and hence, the firm does not have such high incentive to raise price in advance even with the possibility of not being able to change the price in subsequent periods. Secondly, the lower level of ϱ means that household's labour supply responds elastically to the change in real wage. Therefore, with the rise in firms' prices which leads to a lower level of real wage, the labour supply decreases sharply. Since the level of labour supply is the same as the aggregate demand in the equilibrium of the model ($h_t = c_t$), the decrease in labour supply

means lower demand level, and therefore, firms will hesitate in raising their prices. After all, because of these two effects of low σ and ϱ, a firm's incentive to raise its price becomes very low compared to the case of higher values of the parameters. Therefore, in the case of low level of σ and ϱ, the effect of higher relative price dominates that of the expected increase in demand. However, as addressed above, as long as the parameters do not deviate too far from the standard value as in numerous literature, the latter overwhelms the former as shown in Figure 3.1 in which the model is simulated with $\sigma = \varrho = 1$.

3.5.2 Different γ with Different Firms

In the previous section, we assumed the same γ for all repricing firms. As mentioned above, the value of γ_i represents the information which firm i has regarding the expectation on the average reset price. Therefore, the same γ means that all firms have the same information about p_t^*. However, if each firm has a different ability in calculating the complicated problem, and therefore, if each firm has different information in expecting p_t^*, γ_i will be different among the repricing firms. For example, some firms have lower expectations on the average reset price, whereas others have higher ones compared to their own reset prices. In some cases, all the directions of different expectations can cancel one another out, and the average level of expectations can be the same as that of the rational firms in the standard model in which γ is equal to one.

We have seen in the section above that firms with $\gamma < 1$ set much higher reset prices compared to the standard model. However, we have also seen the opposite result with the firm which has $\gamma > 1$. Therefore, we can expect that a mixture of firms' expectations with opposite directions makes the significance of the results in the above section less meaningful. If a firm's response is linearly proportional to the level of γ, there might not be any difference from the standard model in the above mentioned case where the average expectation is the same as that of the rational firms due to the differences in expectations being offset. However, if the effect of γ

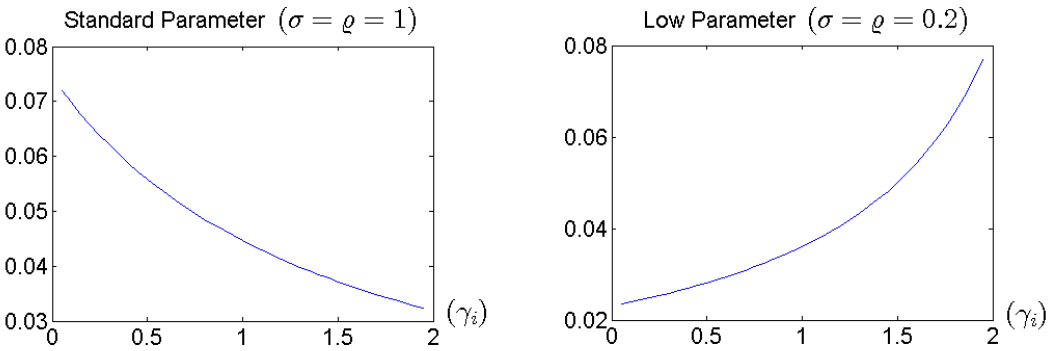

Figure 3.3: Reset Price with Different γ in the First Period after Shocks

on a firm's reset price is not linear, the heterogeneity of firms' expectations itself can make a meaningful difference from the standard model. In order to check whether there is a significant effect of the heterogeneity in expectations and how the effect, if any, varies in accordance with the distribution of expectations, this section assumes the above mentioned case. That is, γ in this section reflects the circumstance where all firms have different expectations but the same average level as the standard model. For this purpose, each firm i's γ_i is assumed to be drawn from the following uniform distribution as

$$\gamma_i \sim \mathcal{U}\left(a,\ \ 2-a\right), \tag{3.35}$$

where $0 < a < 1$ is the lower limit of the interval.[36] The selection of uniform distribution is for simplification of the model, which means that γ_i is evenly distributed among all firms. Also, we can find that the mean of the distribution is set to be equal to one for the assumption that the aggregate level of expectations makes no difference from the standard model.

Figure 3.3 shows the level of reset price of firm i with γ_i in the first period after expansionary monetary shocks. As we have seen in the previous section, with the standard values of σ and ϱ, a firm with lower γ_i sets a higher price as in the left panel of Figure 3.3. However, as also discussed above, the right panel shows higher

[36]This does not mean that each individual firm knows the distribution. The distribution is just a tool for the assumption that γ_i of an individual firm is not the same for all firms.

reset price with higher γ_i when σ and ϱ have very low values. Despite the opposite results of the two panels, it is common that the level of reset price is convex with respect to γ_i, which means that Jensen's inequality holds as

$$g\left(\int_a^{2-a} \gamma_i di\right) \leq \int_a^{2-a} g(\gamma_i) di, \tag{3.36}$$

where g is the function which relates γ_i to the reset price of firm i. In other words, both the panels of Figure 3.3 show that the size of increase in the reset price gets bigger as γ moves to one side. This means, in the case with the standard values of σ and ϱ for example, that lower expectation on the average reset price ($\gamma_i < 1$) makes a bigger difference to a firm's reset price from the standard model than when expecting a higher average reset price ($\gamma_j > 1$), even if the distances of the two opposite expectations from that of the rational firms in the standard model are the same $\left(\frac{\gamma_i+\gamma_j}{2} = 1\right)$.

As mentioned in the previous section, a firm with $\gamma < 1$ has an incentive to raise its price in anticipation of demand increase. Also, we have seen that the incentive is amplified by price stickiness which makes the firm have concerns about having no chance to change price in subsequent periods and forces it to set its price high in advance. However, these two mechanisms operate differently when a firm expects a higher average reset price with $\gamma > 1$. Firstly, the incentive to raise price against the expected increase in demand becomes very small. It is because the firm's higher expectation on average reset price leads to the anticipation of higher inflation response which is expected to absorb a majority of shocks. After all, the firm will expect just a slight increase in demand by the expansionary money shocks and, consequently, will not need to set a high price.[37] Thus, the firm sets a much lower price compared to that of the rational firms in the standard model. However, the difference is not as big as that of the case where a firm expects a lower average

[37]Expectation of high inflation response leads to an anticipation of low relative price which allows a further raising of reset price. However, as mentioned in the section above, this effect cannot dominate the effect from the expected change in demand when the shocks are expected to be persistent as in this model.

reset price. It is because the second mechanism of price stickiness still operates in the same direction as in the case of $\gamma < 1$. In other words, the firm still has to worry about not being able to adjust price in the future, and this concern forces the firm not to have its price at a very low level. As a result, the circumstance of sticky price backs up the level of the firm's reset price and does not allow it to be further away from that of the standard model compared to the case of $\gamma < 1$.

As seen above, price stickiness augments the incentive to raise price when a firm expects a lower average reset price, while it dampens the tendency of price to fall in the opposite case. Such different operation of price stickiness produces the convex line as in Figure 3.3 and makes the inequality of (3.36) hold. Since the mean of γ_i is set to be one, which gives the same level of reset price as in the standard model, the above inequality means that the average reset price (p_t^*) in the economy with distributed γ is higher than that of the standard model, regardless of the parameters, σ and ϱ. In other words, if firms' expectations on p_t^* are not homogeneous, even if the average of expectations is the same as in the standard model, firms' reset prices are much more responsive against monetary shocks, compared to those of the standard rational firms.

In the far right panel of Figure 3.4, it is shown that the impulse response of the average reset price of the firms which have different γ_i is higher than that of the standard model when simulated with $a = 0.05$ against positive monetary shocks. The two left panels demonstrate the realized inflation and output responses calculated with the average reset price, which shows that the real effect of money in the economy with distributed γ is much less than that of the economy of rational firms with perfect information as in the standard model. Also, we can find that this result holds with any level of parameters as there is little difference between the two cases of standard parameters ($\sigma = \varrho = 1$, upper panels) and low parameters ($\sigma = \varrho = 0.2$, lower panels), which confirms the discussion above.

The degree of deviation of the model with distributed γ from the standard model depends on the shape of the distribution of γ. Though we have a considerable drop

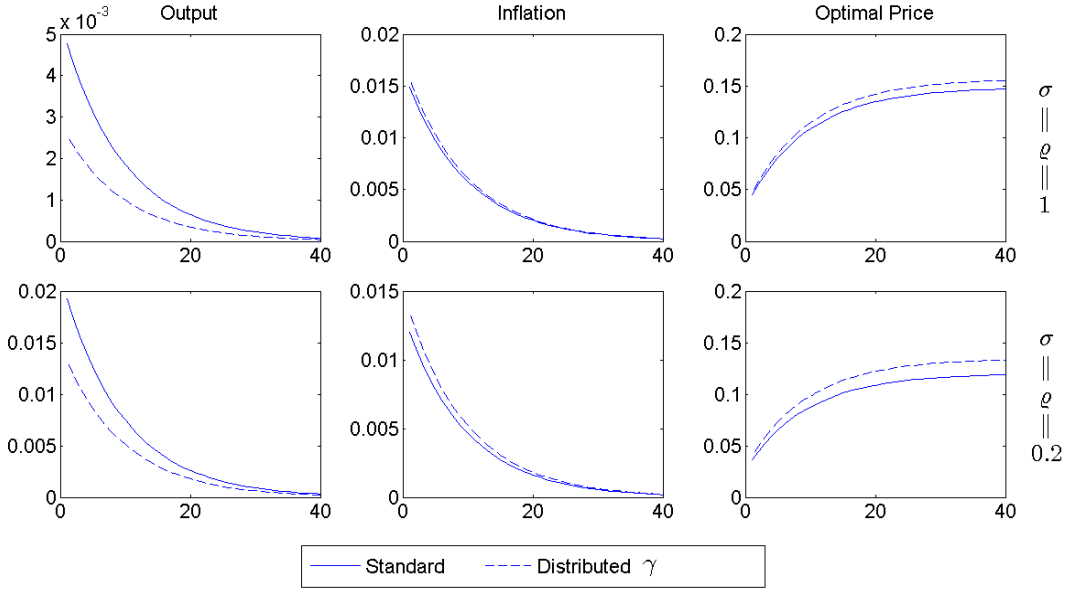

Figure 3.4: Impulse Response Functions with Distributed γ

in the output response compared to the standard model in the above simulation with the assumption of uniform distribution, the size of the drop can be very small if γ_i is concentrated around the mean, which is one in our case, like the normal distribution. Also, if the probability density function is much higher around both ends of the distribution curve, we will have a much larger drop in the output response. However, no matter what the distribution is, it is always true that the output response is smaller than that of the standard model as long as the distribution has a positive variance with mean equal to one. This is because Jensen's inequality still holds with any probability density function (f) as long as the function (g), which relates γ_i to firm i's reset price, is convex as

$$g\left(\int_a^{2-a} \gamma_i f(\gamma_i) di\right) \leq \int_a^{2-a} g(\gamma_i) f(\gamma_i) di, \tag{3.37}$$

and the convexity of the function g is guaranteed by the Calvo-type price stickiness as reviewed above. This result implies that the effect of monetary policy can be very small when analysed with any price stickiness model featured by the Calvo mechanism under the circumstance of heterogeneity in firms' expectations on other

firm's pricing behaviours and average level of reset prices.

3.5.3 Adjustment of γ

In the above sections, it is assumed that once a firm i sets its expectation on p_t^* with a certain level of γ_i, the γ_i is held constant. In other words, individual firms do not adjust their γ even after they observe that the realized p_t^* is different from their expectations. For example, in the above simulation in which all firms have different γ evenly distributed from 0.05 to 1.95, it can be shown that the firm with the smallest γ ($= 0.05$) sets its optimal price as 0.072[38] in the first period after the shock. This means that the firm expects the average reset price would be 0.0036 ($= 0.05 \times 0.072$). However, the realized average of all firms' reset prices is calculated to be 0.047 and can be observed before the price setting for the next period. It is not so realistic to assume that the firm ignores the discrepancy between its ex-ante expectation and ex-post observed value of p_t^*. Therefore, in this sense, it is assumed in this section that firms update their expectations on p_t^* by adjusting γ after observing the realized values of the main variables at the end of each period.

Even though we cannot know exactly how γ_i is adjusted using the information on the realized outcome of the economy because we have not modelled how firm i sets its γ_i at the moment of price setting, this section assumes that firms follow the very basic learning scheme as

$$\gamma_{i,t} = \gamma_{i,t-1} + \phi \left(\gamma_{i,t-1}^R - \gamma_{i,t-1} \right), \tag{3.38}$$

where ϕ is the parameter deciding the speed of adjustment, and $\gamma_{i,t}^R$ denotes the realized value of γ_i which can be observed just after the price setting in period t. In the case above for example, after price setting of $\ln p_{i,1} = 0.072$ with $\gamma_{i,1} = 0.05$ at the end of period $t = 1$, the firm i observes $\ln p_1^* = 0.047$. Then using the following

[38] From now onward, the values given are for the logged variables, which means $\ln p_{it} = 0.072$ in this case.

equation as in

$$\tilde{E}_{it} \ln p_t^* = \tilde{E}_{it} \left[\gamma_i \ln p_{it} + (1 - \gamma_i) \ln P_{t-1} \right], \tag{3.39}$$

which can be obtained by taking logarithms of both sides of (3.7), the firm can calculate the ex-post value of $\gamma_{i,1}^R$ at 0.653.[39] Therefore, according to (3.38), the firm's updated γ at the beginning of period $t = 2$, $(\gamma_{i,2})$, becomes $0.05 + 0.603\phi$. As can be seen, the updated level of γ is affected by the parameter, ϕ. In other words, depending on the speed of adjustment, the above $\gamma_{i,2}$ can be any number within the interval of $(0.05, 0.653)$.[40]

We can think of two extreme cases with regard to the adjustment speed. If the value of $\gamma_{i,t}^R$ contains lots of information on the true value of γ, firms will have ϕ very close to one. However, if firms consider that there is much more information on the true γ outside the economic model they perceive, then ϕ would be closer to zero. As mentioned in the previous sections, γ contains the subjective information of individual firms' views on the economy. More precisely, each firm i might have its own perception on how many firms have different optimization problem from its own which is the maximization of expected profit, how the different pricing behaviours look like, and what other firms' perceptions on those factors would be. The γ_i represents a subjective perception of firm i on such information in connection with the average reset price. Such nature of γ shows that there is quite a psychological influence in firms' setting of γ_i. Furthermore, as long as each individual firm thinks that there are many firms with different pricing behaviours which might be irrational, each firm will not think that a fixed true value of γ exists. In other words, firms might believe that lots of irrational firms in the economy may set their prices differently every period relying on psychological factors, and accordingly, each firm might try to set an optimum period-by-period $\gamma_{i,t}$ thinking that it best explains the relationship between the average reset price and the individual firm's own price. In this case, firms will not think that $\gamma_{i,t}^R$ reflects the true economy because $\gamma_{i,t}^R$ could be plausible

[39] The value of aggregate price level in the previous period is already known as $\ln P_0 = 0$ with the assumption that steady state value of price before the shock is one. Therefore, $\gamma_{i,1}^R$ is obtained by solving the equation, $0.047 = \gamma_{i,1}^R \times 0.072 + (1 - \gamma_{i,1}^R) \times 0$.

[40] Note that ϕ is assumed to be between zero and one.

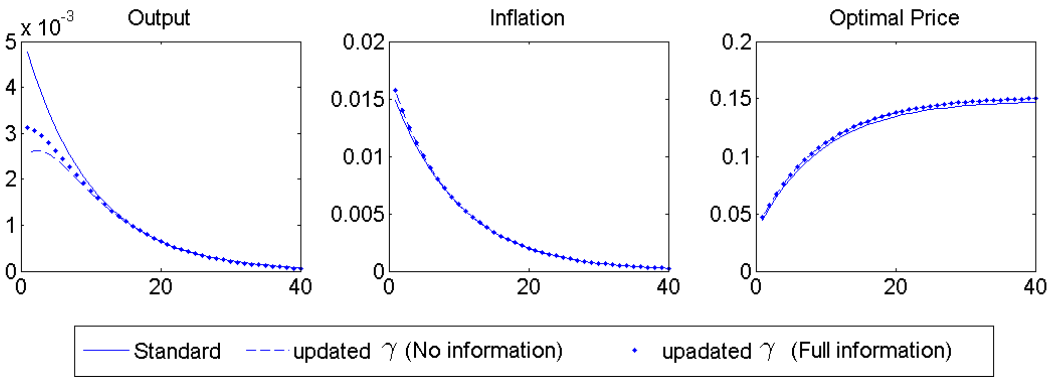

Figure 3.5: Impulse Response Functions with Updated Distribution of γ

only under the assumption that true value of γ exists and that all information on the true value lie in the perceived model. In this sense, firms might put much more weight on the information out of their perceived model rather than on the realized value of $\gamma_{i,t}^R$. In the extreme case, firms might not reflect the outcome of $\gamma_{i,t}^R$ at all and have ϕ equal to zero.[41]

Following such reasoning, the model is simulated with a very low value of $\phi = 0.1$. Figure 3.5 shows that the real effect of money is still smaller than that of the standard model with the low adjustment speed of distribution of γ. Compared to Figure 3.4 which is for the constant distribution of γ, we can see that all the variables converge fast to those of the standard model, which is the natural outcome of the adjustment of γ. As long as firm i is assumed to use the learning scheme, (3.38), all the information used for the update of γ_i is from the inside of the model and entirely reflected into γ_i^R. Therefore, even though ϕ is set very low for the psychological aspects of γ_i, γ_i^R naturally converges to one in the end unless ϕ is equal to zero because all firms in the model are assumed to be the same by construction. This makes the responses of the model closer to those of the standard model as seen in Figure 3.5.

[41] Even in this case, it need not be the case where each firm maintains its initial γ. Rather, it can be that firms adjust their γ, but the whole distribution of γ does not change. For example, the firm with $\gamma = 0.05$ might change to $\gamma = 1.95$ after observing much higher average reset price than its expectation, while another firm adjusts its initial γ equal to 1.95 to 0.05 for reason to the contrary, which makes the distribution of γ invariable.

The interesting result is that a hump-shaped output response is shown. As discussed in the previous section, the realized output level is decided by the representative rational household. Even though the household is assumed to have perfect information on the true economy, it can be an issue whether it has knowledge on firms' behaviours in updating γ as well. If the household has no information on the adjustment of γ in advance, the output level would also be adjusted every period according to the update of γ, which leads to the hump shape of the output response as shown in the dashed line of Figure 3.5. However, if full information on the update schedule of γ is assumed to be given, the household demands much more in advance because it anticipates the fast convergence of inflation in the future, as shown in the dotted line of the figure.

3.6 Conclusion

Various versions of the standard New Keynesian models presented in great amounts of literature generate the non-neutrality of money by featuring price stickiness of individual firms with the classic assumption of rational agents who have perfect information on the economy. However, in the real world, it is almost impossible to find such agents with omniscience about the economy. This chapter examines whether the real effect of money can still be guaranteed by price stickiness even in the circumstance where the requirement of full information for rational agents is not satisfied. Specifically, when firms face the limited information on other firms' pricing behaviours, it is shown that firms' reset prices against monetary shocks can be much more volatile than that of the standard model depending on their subjective expectations on the average reset price. Consequently, it is demonstrated that the real effect of money could be very small even with price stickiness of individual firms. Furthermore, as long as the firms' subjective expectations are fully heterogeneous, even if they match the true feature of the economy on average as the rational firms do in the standard model, it is shown that the interaction of heterogeneity in expectations with price stickiness can make the price much more

volatile, and therefore, the real effect of money is reduced. This implies that price stickiness can lose its position as the main factor of the non-neutrality of money just with the firms' heterogeneous expectations on the economy. Therefore, economists and central banks need to research more on the distribution of the economic agents' expectations, and they should try to find another factor which brings about the non-neutrality of money directly rather than price stickiness.

In spite of some implications above, this chapter has several limitations. First of all, the model in this chapter is based on the assumption of boundedly rational firms. As in numerous literature, the wilderness of bounded rationality can be an issue. Even though the very basic learning mechanism is introduced in this chapter, the psychological nature of heuristics, which firms have for their expectations, are vulnerable to criticism on the wilderness of heuristics. The best way to avoid the criticism would be to establish a model based on rational firms. In this sense, the very next attempt for future research would be to set up a model in which fully rational firms, facing imperfect information on other firms' optimization problems, set much more volatile optimal prices compared to the standard model. Secondly, aside from the issue of wilderness of bounded rationality, this model lacks the micro-foundation on firms' subjective expectations. Even though it is assumed that firms have simple heuristics in expecting the average reset price, there is no further explanation on how the parameter, γ, which is the critical factor of the heuristic, is decided. As a result, the update mechanism of the parameter could not be more exquisite. If future research gives more detailed micro-foundation on the parameter, this model would be much richer than now with the equipment of much elaborate learning mechanism for the update of heuristics, which would be helpful in limiting the wilderness of bounded rationality as well.

Appendix 3.A

Cost Minimization

Taking the real wage as given, firm i hires an optimal level of labour by minimizing the production costs as

$$\min_{h_{it}} \frac{W_t}{P_t} h_{it} \qquad \text{s.t.} \quad y_{it} = h_{it},$$

where W_t is the nominal wage. The first order condition with respect to h_{it} yields

$$\xi_t = \frac{W_t}{P_t},$$

where ξ_t is the Lagrangian multiplier. Since ξ_t means the additional real cost of producing an extra unit, we can interpret ξ_t as the real marginal cost, mc_t, as

$$mc_t = \frac{W_t}{P_t} = w_t, \tag{3.40}$$

where w_t denotes the real wage.

Profit Maximization

Each firm adjusts its price by maximizing the present discounted value of expected profits as

$$E_t \sum_{s=0}^{\infty} \theta^s Q_{t,t+s} P_{t+s} \left(\frac{p_{it}}{P_{t+s}} - mc_{t+s} \right) y_{it+s}.$$

Given the demand curve, $y_{it} = (P_{it}/P_t)^{-\epsilon} y_t$, we can re-express this optimization problem as

$$E_t \sum_{s=0}^{\infty} \theta^s Q_{t,t+s} P_{t+s} \left[\left(\frac{p_{it}}{P_{t+s}} \right)^{1-\epsilon} - mc_{t+s} \left(\frac{p_{it}}{P_{t+s}} \right)^{-\epsilon} \right] y_{t+s}.$$

Taking the derivative with respect to p_{it} gives the first order condition as.

$$E_t \sum_{s=0}^{\infty} \theta^s Q_{t,t+s} \left(\frac{p_{it}}{P_{t+s}}\right)^{-\epsilon} y_{t+s} \left[1 - \frac{\epsilon}{\epsilon - 1} mc_{t+s} \left(\frac{p_{it}}{P_{t+s}}\right)^{-1}\right] = 0, \tag{3.41}$$

which can be re-expressed by

$$E_t \sum_{s=0}^{\infty} \theta^s Q_{t,t+s} \left(\frac{p_{it}}{P_{t+s}}\right)^{-\epsilon} y_{t+s} = \frac{\epsilon}{\epsilon - 1} E_t \sum_{s=0}^{\infty} \theta^s Q_{t,t+s} mc_{t+s} \left(\frac{p_{it}}{P_{t+s}}\right)^{-\epsilon-1} y_{t+s}. \tag{3.42}$$

Using $Q_{t,t+s} = \beta^s E_t \left(\frac{P_t}{P_{t+s}} \frac{\lambda_{t+s}}{\lambda_t}\right)$ and multiplying both sides by $p_{it}^{\epsilon+1}/P_t$, we can rearrange this as

$$\tilde{p_t} \left(= \frac{p_{it}}{P_t}\right) = \frac{\epsilon}{\epsilon - 1} \frac{f_{1t}}{f_{2t}}, \tag{3.43}$$

where $\tilde{p_t}$ is the relative optimal price, and

$$f_{1t} = E_t \sum_{s=0}^{\infty} \theta^s \beta^s \lambda_{t+s} mc_{t+s} y_{t+s} \left(\frac{P_{t+s}}{P_t}\right)^{\epsilon} \tag{3.44}$$

$$f_{2t} = E_t \sum_{s=0}^{\infty} \theta^s \beta^s \lambda_{t+s} y_{t+s} \left(\frac{P_{t+s}}{P_t}\right)^{\epsilon-1}. \tag{3.45}$$

Expanding the summation notation in (3.44), we get

$$\begin{aligned} f_{1t} = \lambda_t mc_t y_t + E_t \theta \beta \lambda_{t+1} mc_{t+1} y_{t+1} \left(\frac{P_{t+1}}{P_t}\right)^{\epsilon} \\ + E_t \theta^2 \beta^2 \lambda_{t+2} mc_{t+2} y_{t+2} \left(\frac{P_{t+2}}{P_t}\right)^{\epsilon} + \cdots . \end{aligned} \tag{3.46}$$

Shifting one period forward yields

$$\begin{aligned} f_{1t+1} = \lambda_{t+1} mc_{t+1} y_{t+1} + E_{t+1} \theta \beta \lambda_{t+2} mc_{t+2} y_{t+2} \left(\frac{P_{t+2}}{P_{t+1}}\right)^{\epsilon} \\ + E_{t+1} \theta^2 \beta^2 \lambda_{t+3} mc_{t+3} y_{t+3} \left(\frac{P_{t+3}}{P_{t+2}}\right)^{\epsilon} + \cdots . \end{aligned} \tag{3.47}$$

Multiplying both sides by $\theta\beta\left(\frac{P_{t+1}}{P_t}\right)^{\epsilon}$, we can have

$$\begin{aligned}\theta\beta\left(\frac{P_{t+1}}{P_t}\right)^{\epsilon} f_{1t+1} = \theta\beta\lambda_{t+1}mc_{t+1}y_{t+1}\left(\frac{P_{t+1}}{P_t}\right)^{\epsilon} \\ + E_{t+1}\theta^2\beta^2\lambda_{t+2}mc_{t+2}y_{t+2}\left(\frac{P_{t+2}}{P_t}\right)^{\epsilon} \\ + E_{t+1}\theta^3\beta^3\lambda_{t+3}mc_{t+3}y_{t+3}\left(\frac{P_{t+3}}{P_t}\right)^{\epsilon} + \cdots .\end{aligned} \tag{3.48}$$

Taking expectation, E_t, on both sides and using the law of iterated expectations, we can get

$$\begin{aligned}E_t\theta\beta\left(\frac{P_{t+1}}{P_t}\right)^{\epsilon} f_{1t+1} = E_t\theta\beta\lambda_{t+1}mc_{t+1}y_{t+1}\left(\frac{P_{t+1}}{P_t}\right)^{\epsilon} \\ + E_t\theta^2\beta^2\lambda_{t+2}mc_{t+2}y_{t+2}\left(\frac{P_{t+2}}{P_t}\right)^{\epsilon} \\ + E_t\theta^3\beta^3\lambda_{t+3}mc_{t+3}y_{t+3}\left(\frac{P_{t+3}}{P_t}\right)^{\epsilon} + \cdots .\end{aligned} \tag{3.49}$$

From (3.46) and (3.49), we have

$$f_{1t} = \lambda_t mc_t y_t + E_t\theta\beta\pi_{t+1}^{\epsilon} f_{1t+1}, \tag{3.50}$$

where $\pi_t\,(= P_t/P_{t-1})$ is the inflation rate. Following the same logic, (3.45) can be re-expressed by

$$f_{2t} = \lambda_t y_t + E_t\theta\beta\pi_{t+1}^{\epsilon-1} f_{2t+1}. \tag{3.51}$$

Appendix 3.B

From (3.1), we know

$$p_{it} = f\left(P_t, y_t, mc_t, E_t Q_{t,t+s}, E_t P_{t+s}, E_t y_{t+s}, E_t mc_{t+s}\right), \tag{3.52}$$

where $s > 0$. Using (1.15) and (1.22)∼(1.24) of Chapter 1, we can have

$$Q_t(\equiv Q_{t,t+1}) = \beta E_t \left(\frac{P_t}{P_{t+1}} \frac{y_{t+1}^{-\sigma}}{y_t^{-\sigma}} \right) \tag{3.53}$$

$$mc_t = y_t^{\sigma+\varrho}. \tag{3.54}$$

Also, with (1.25) of Chapter 1 and $R_t = 1/Q_t$, we get

$$y_t = \left(Q y^{\eta_y} \frac{\nu_t}{Q_t} \left(\frac{\pi}{\pi_t} \right)^{\eta_\pi} \right)^{\frac{1}{\eta_y}}. \tag{3.55}$$

Substituting (3.53) into (3.55) and using $\pi_t = P_t/P_{t-1}$, we can find that y_t is a function of $P_{t-1}, P_t, P_{t+1}, y_{t+1}$, and ν_t as

$$y_t = f\left(P_{t-1}, P_t, E_t P_{t+1}, E_t y_{t+1}, \nu_t\right). \tag{3.56}$$

If we shift the above equation one period forward, we obtain

$$y_{t+1} = f\left(P_t, P_{t+1}, E_t P_{t+2}, E_t y_{t+2}, E_t \nu_{t+1}\right), \tag{3.57}$$

and putting this equation into (3.56) gives

$$y_t = f\left(P_{t-1}, P_t, E_t P_{t+1}, E_t P_{t+2}, E_t y_{t+2}, \nu_t, E_t \nu_{t+1}\right). \tag{3.58}$$

Iteration of this process yields

$$y_t = f\left(P_{t-1}, P_t, E_t P_{t+s}, \nu_t, E_t \nu_{t+s}\right), \tag{3.59}$$

where $s > 0$. With (3.53), (3.54), and (3.59), we can re-express (3.52) as

$$p_{it} = f\left(P_{t-1}, P_t, E_t P_{t+s}, \nu_t, E_t \nu_{t+s}\right), \tag{3.60}$$

where $s > 0$. Given that P_{t-1} and ν_t can be observable at the moment of price setting in period t, we can have

$$p_{it} = f(P_t, E_t \Omega_{t+s}), \tag{3.61}$$

where $\Omega_{t+s} = \{P_{t+s}, \nu_{t+s}\}$ and $s > 0$.

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