Practical Synthesis of Ternary Sequences for System Identification

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***Abstract*—Several issues related to the practical synthesis of ternary sequences with specified spectra are addressed in this paper. Specifically, sequences with harmonic multiples of two and three suppressed are studied, given their relevance to system identification applications. In particular, the effect of non-uniform Digital to Analog Converter (DAC) levels on the spectral properties of the generated signal is analyzed. It is analytically shown that the DAC non-uniform levels result in degraded harmonic suppression performance. Moreover, a new approach is proposed for designing ternary sequences, which is flexible and can be adapted to suit different requirements. The resulting sequences, denoted as randomized constrained sequences, are compared to direct sequences already proposed in the literature. The approach is validated by numerical simulations and experimental results, showing the potential to achieve harmonic suppression performance of approximately 100 dB.**

***Index Terms*—System identification, ternary sequences, digital-to-analog converters, spectral analysis.**

I. Introduction

The system identification process typically consists in providing an excitation signal to the input of the system under test and measuring the response of the system at the output. Thus, the design of excitation signals is of fundamental importance for the effectiveness and performance of the system identification activity. Specifically, for frequency-domain system identification, periodic signals with a specified power spectrum provide considerable advantages over other types of excitation signals. By properly choosing the spectral components of the periodic signal, in fact, it is possible to isolate, detect, and analyze quantitatively the non-linear distortions, while from the same measurements also a nonparametric noise model is retrieved. This mitigates the impact of nonlinear distortions on frequency response function (FRF) measurement [1]. In the class of periodic excitation signals, ternary sequences are particularly useful, since they are the simplest multi-level signals that allow suppression of harmonics multiples of two and three. Hence, they facilitate the analysis of even and odd non-linearities [2].

The aim of this paper is twofold. On the one hand, we provide an analysis of the implementation issues related to the generation and acquisition of ternary sequences. On the other hand, we propose a new method to design ternary sequences with harmonic multiples of two and three suppressed that will be more robust with respect to the effect of non-ideal levels in the Digital to Analog Converter (DAC) generator. The method is based on numerical optimization, and is tunable to meet a wide range of requirements for the excitation signal spectrum.

II. Synthesis of Ternary Sequences

When measuring the FRF of a linear system in the presence of nonlinear distortions, the primary goal is to minimize the impact of such distortions on the measurement results. In fact, nonlinear distortions create additional harmonics at the output that were not present at the input. Specifically, for periodic signals, a nonlinearity of degree n generates additional harmonics at frequencies given by all the possible combinations of the harmonics of the input signal, taken n at a time. In the particular case of n even, the generated harmonics are always even. Therefore, if a signal consisting only of odd harmonics is applied at the input of a nonlinear system, even-order nonlinearities do not influence the FRF measurement, because they do not overlap with the harmonics originally present in the input signal [1].

Instead, for an input signal consisting only of odd harmonics, it is impossible to completely eliminate the impact of odd-order nonlinearities. However, their effect can be mitigated by providing an input signal whose harmonic multiples of three are suppressed. This reduces the impact of third-order nonlinearities on FRF measurement [3].

For these reasons, it is advisable to design excitation signals with harmonic multiples of both two and three suppressed. Such suppression can only be achieved using signals having more than two levels. Using binary signals, in fact, it is only possible to suppress even-order harmonics [3]. In this context, pseudorandom ternary sequences are interesting, since they represent the simplest form of multi-level signal and are easily applied to transducers and actuators [2]. Moreover, such signals, when compared to signals with a larger number of levels, normally provide better dispersion performance, as quantified by the performance index for perturbation signals introduced in [4], while being easier to design.

Therefore, in this paper we focus our attention to ternary pseudorandom sequences. In particular, in the following, we provide an analysis of the spectral properties of practical ternary sequences. To do so, we start by presenting an analysis of the ideal case, already provided in the literature in [3] and [5], among others. We then proceed to study the case where the ternary sequences are generated by a practical DAC with nonuniform levels.

A. Ideal Case

Following the derivations in [3] and [5], define $u(n)$ as a ternary sequence taking values in $\{-1, 0, 1\}$ when $n = 1, \ldots, N$, with N being a multiple of 6. The condition $U(\cdot) = \sum_{i=1}^{N} u(i) \exp\left(-\frac{j2\pi ik}{N}\right)$, being zero at even frequencies and at multiples of $3m$, m integer, requires:

$$u(i)+u\,(i+N/2)=0;\; u(i)+u\,(i+N/3)+u\,(i+2N/3)=0 \quad (1)$$

In the direct synthesis technique described in [5], $u(n)$ is obtained by multiplying a basic binary sequence $u_{basic}(\cdot)$ taking values in $(-1, 1)$ by the special sequence $[1\ 1\ 0\ -1\ -1\ 0]$. By periodizing the product sequence, the 0's in $u(n)$ are exactly located at $n = 3m$.

If the signal is generated by an ideal DAC with uniform levels, the condition (1) ensures that harmonic multiples of two and three are entirely suppressed. However, if the signal is generated by a real DAC with non-uniform levels, the suppression is not perfect and undesired harmonic components are present, as we show in the following subsection.

B. Effect of DAC non-uniform levels on the spectrum of ternary sequences

In order to quantify the effect of non-ideal levels in the DAC, the sequence $u(i)$ is assumed as being generated by a nonuniform DAC that maps the input values $(-1, 0, 1)$ to the output voltages a_{-1}, a_0, a_1, with the only constraint $a_{-1} < a_0 < a_1$. Thus the DAC output sequence y_{DAC} is equal to

$$y_{DAC}(n) = \sum_{k=-1}^{1} I(u(n) = k)a_k$$

where $I(A)$ is the indicator function of the event A. By defining $\beta = \frac{a_{-1}+a_1}{2}$, and $\alpha = a_1 - \beta$, then

$$y_{DAC}(n) = \alpha u(n) + \beta + (a_0 - \beta)I(u(n) = 0)$$

that shows that apart from a constant gain α and a constant offset β, $y_{DAC}(\cdot)$ differs form $u(\cdot)$ by a nonlinear error sequence $e_{DAC}(n) = (a_0 - \beta)I(u(n) = 0)$. Observe that when the DAC is uniform $a_{-1} = -a_1 = -1$ and $a_0 = 0$, so that $e_{DAC}(\cdot)$ is identically zero. While the problem of spectral distortion due to nonuniform DACs is conventionally solved by resorting to *dynamic element matching* techniques or by using *mismatch shaping* DACs, e.g. in the case of $\Sigma\Delta$ ADCs [6], the synthesis of $u(\cdot)$ through off–the–shelf waveform synthesizers prevents from using compensating methods at the very large scale integration level.

Next it will be shown how the nonlinear error contribution due to $e_{DAC}(\cdot)$ destroys the properties (1) and results in frequency components not present in $U(k)$. In fact, the error sequence $e_{DAC}(n)$ produces a spectral contribution that can be evaluated as:

$$E_{DAC}(k) = \sum_{m=1}^{N} e_{DAC}(m) \exp\left(-\frac{j2\pi mk}{N}\right) \quad (2)$$

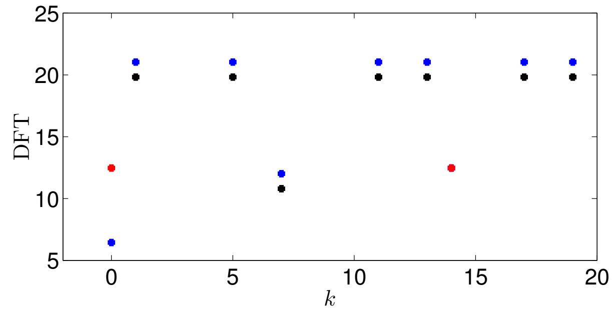

Fig. 1: Spectra associated to the synthesis of a ternary signal using a nonuniform DAC, with $a_{-1} = -1.3, a_0 = 0.15, a_1 = 1.0$ and $N = 42$: uniform DAC – ideal behavior (black); nonlinear error (red); nonuniform DAC (blue).

Since $e_{DAC}(i)$ is identically zero when $i \neq 3m$, m integer, (2) becomes

$$E_{DAC}(k) = (a_0 - \beta) \sum_{i=1}^{N/3} \exp\left(-\frac{j2\pi 3ik}{N}\right)$$

Given the orthogonality of the complex exponential basis functions the only nonnegative values of k that result in $E_{DAC}(k) \neq 0$ are $k = 0, N/3$ for which $E_{DAC}(k) = (a_0 - \beta)\frac{N}{3}$. Thus the effect of nonuniform DAC output levels results in a nonlinear contribution that adds a mean value and an harmonic component at an *even* frequency $k = \frac{N}{3}$ that should ideally be zero.

A simulation was run with $a_{-1} = -1.3, a_0 = 0.15, a_1 = 1.0$ and $N = 42$. Spectra associated to the synthesized signal are shown in Fig. 1. In this figure blue dots represent the spectrum generated by the nonuniform DAC, red dots show the nonlinear error spectrum, and black dots denote the spectrum synthesized by the ideal sequence $u(\cdot)$. The contributions at DC and at $k = N/3 = 14$ are clearly visible.

III. Other implementation issues

In a practical implementation, DAC non-uniformity is not the only aspect that influences the properties of the synthesized sequences. In particular, another implementation issue is related to the duration of each sequence element (denoted as *chip*). For theoretical results to be applicable, in fact, the uniformity of such duration should be ensured. Non-uniformity may arise due to the principle of operation of Direct Digital Synthesis (DDS) instrumentation [7]. Specifically, when there is a non-integer ratio between waveform memory depth and sequence length, the DDS device performs waveform "stretching" to fill the internal waveform memory. It can be mitigated by acquiring only one sample per chip, or by repeating the sequence multiple times for a more effective memory usage.

Furthermore, synchronization between the signal generation device and the acquisition device should be ensured. If synchronization is not possible, the effect of leakage can be mitigated by the methods in [8].

Moreover, when generating a continuous-time signal from the digital sequence using the zero-order hold approach, the discrete spectrum is multiplied by a sinc function, as discussed in [3]. This effect should be taken into account when designing the acquisition system.

Since the ternary signal contains sharp transitions, jitter of the sampling frequency should be taken into account. In fact, the higher the maximum slope of the signal, the more sensitive to jitter the acquisition system becomes. To mitigate the effect of the sharp edges, a hardware low-pass filter should be used, providing also the benefit of reducing overshoot.

Fundamental limitations are given by noise in the acquisition system. Wideband additive noise can be mitigated by coherent averaging over multiple periods. Observe also that the finite resolution of the DAC and ADC used for generating and acquiring the sequence is a fundamental limitation for the attainable spurious free dynamic range (SFDR). In the remainder of this paper, SFDR is defined as the ratio between the highest desired harmonic and the highest undesired harmonic. Furthermore, we define the signal-to-noise-and-distortion ratio (SINAD), as the ratio of the root-mean-square (RMS) amplitude of the signal to the RMS amplitude of the noise and distortion components [9], i.e. SINAD $\triangleq 20\log_{10}\frac{V_{SI}}{V_{NAD}}$, where V_{SI} denotes the RMS value of the desired harmonics, i.e. $V_{SI}=\frac{1}{N}\sqrt{\sum X_{SI}^2}$, and V_{NAD} that of all other components discarding DC, i.e. $V_{NAD} = \frac{1}{\sqrt{N(N-N_{SI}-1)}}\sqrt{\sum X_{NAD}^2}$. Here, X_{SI} denotes the vector containing the spectral magnitude values of the desired harmonics and X_{NAD} that of the undesired components discarding DC, N is the total number of samples, and N_{SI} is the number of desired components.

IV. Randomized constrained sequences with harmonic multiples of two and three suppressed

A. *Construction of the sequences*

The two constraints in (1) may suggest that it is possible to generate sequences of length that is a multiple of 6 and that – at the same time – suppress harmonic multiples of two and three. This can be done by starting from the vector $b_0 = [-1\ 0\ 1]$ and constructing the 3 subsequences r_1, r_2 and r_3 as follows:

$$r_1 = \begin{bmatrix} r_{11} \\ r_{12} \\ \vdots \\ \vdots \\ r_{1\frac{N}{6}} \end{bmatrix} \quad r_2 = \begin{bmatrix} r_{21} \\ r_{22} \\ \vdots \\ \vdots \\ r_{2\frac{N}{6}} \end{bmatrix} \quad r_3 = \begin{bmatrix} r_{31} \\ r_{32} \\ \vdots \\ \vdots \\ r_{3\frac{N}{6}} \end{bmatrix} \left.\vphantom{\begin{bmatrix} r \\ r \\ \vdots \\ \vdots \\ r \end{bmatrix}}\right\} \frac{N}{6} \tag{3}$$

such that for every $i = 1, \ldots, \frac{N}{6}$ the (1×3) vector $[r_{1i}\ r_{2i}\ r_{3i}]$ is obtained by permuting, at random, the elements in the vector b_0. Then the final sequence is obtained by juxtaposition of the vectors r_1, r_2, r_3 as follows:

$$x = [\ r_1^T \quad -r_2^T \quad r_3^T \quad -r_1^T \quad r_2^T \quad -r_3^T\ ]$$

where the superscript T denotes the transpose operation.

The sequence x has length N and, by construction, its elements satisfy both constraints in (1). Observe also that by swapping any two values in different subsequences, e.g. in r_1 and r_3, the harmonic suppression properties still hold. Swapping can be applied to eventually improve the performance of the generated sequence against given criteria, e.g. RMS value or standard deviation of amplitudes in the generated harmonics (see [5]). Thus, once a performance criterion is established, both swapping and new sequences can be generated to improve the obtained performance.

Algorithm 1 Generate a ternary sequence with harmonic multiples of two and three suppressed. The parameters K_{max} and J_{max} are selected by the user so to limit processing time.

procedure GENERATE RCS x
    $N \leftarrow$ length of sequence
    $b_0 \leftarrow [-1\,0\,1]$
    Define performance criterion $V(x)$
    $V_{min} \leftarrow V_0$
    $x \leftarrow x_0$
    for $k = 1$ to K_{max} **do**
        Randomly construct r_1, r_2, r_3, as in (3)
        $x_k \leftarrow [\ r_1^T \quad -r_2^T \quad r_3^T \quad -r_1^T \quad r_2^T \quad -r_3^T\ ]$
        if $V(x_k) < V_{min}$ **then**
            $V_{min} \leftarrow V(x_k)$
            $x \leftarrow x_k$
        end if
    end for
    for $j = 1$ to J_{max} **do**
        $x_j \leftarrow$ Randomly swap two values in r_1, r_2, r_3
        if $V(x_j) < V_{min}$ **then**
            $V_{min} \leftarrow V(x_j)$
            $x \leftarrow x_j$
        end if
    end for
    return x
end procedure

In the following, any sequence synthesized using the alternative generation approach proposed here will be referred to as *Randomized Constrained Sequence* (RCS). Conversely, any sequence generated using the direct generation approach in [5] will be denoted as *Direct Sequence* (DS). The operation of the proposed generation method is summarized in Algorithm 1.

With respect to the DS, this approach:

- does not present a low–amplitude harmonic content in the generated sequence that is present in the DS (see figures 2 and 3);
- does not concentrate the effect of non–ideal DAC levels in two single components (at 0 frequency and at $N/3$) but spreads the non–ideal error over a larger set of harmonic resulting in an better range free of unwanted harmonic contributions (see Fig. 3);
- allows for the generation of several different sequences possibly used to source multiple–input single–output system while few uncorrelated direct sequences are available (stated in [5]);
- but shows a larger spread in the amplitudes of the generated harmonics for long sequences.

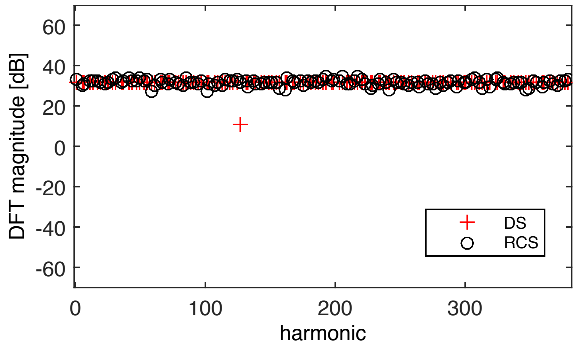

Fig. 2: Simulation results. Spectra associated to the synthesis of one period of a ternary sequence of length 762 using the DS method (red crosses) and the RCS approach described here (black circles).

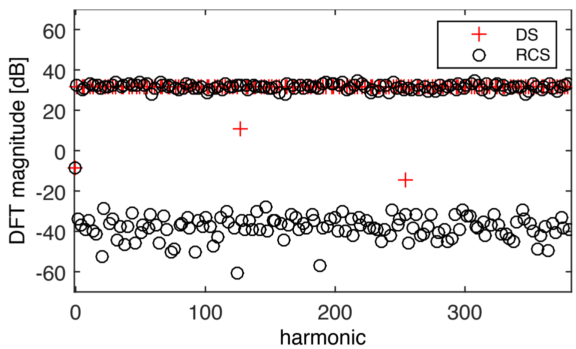

Fig. 3: Simulation results. Spectra associated to the synthesis of one period of a ternary sequence of length 762 using the DS method (red crosses) and the RCS approach described here (black circles). Effect of non–ideal DAC levels: a large harmonic appears at $N/3$ in the red spectrum.

B. Illustration in simulations

In Fig. 2, the spectra of the DS is plotted (red crosses) together with that of the RCS (black circles), assuming ideal DAC levels and $N = 762$. Both sequences have components where expected only. The DS shows a smaller standard deviation in the amplitude of the generated harmonics. However, there is one frequency in the DS for which the amplitude is rather small. Thus the largest deviation from the maximum amplitude is larger in the DS than in the RCS.

When the DAC levels are assumed non ideal (-1.0005, 0.001, 1.001) the same spectra appear as in Fig. 3. There are the additional expected components at 0 and $N/3$ while the black spectrum shows an overall increase in the wideband noise but a smaller component at $N/3$ and the same component at 0 frequency (points overlap). In Fig. 3 the red harmonic at $n = 127$ is wanted but has a rather small amplitude, while the component at $N/3$ is unwanted and has a rather large amplitude. Hence a reduction by about 20 dB of the largest unwanted component is obtained.

V. Experimental results

The theoretical results described in Section II and IV have been validated experimentally. The following subsections describe two versions of the employed experimental setup and related results.

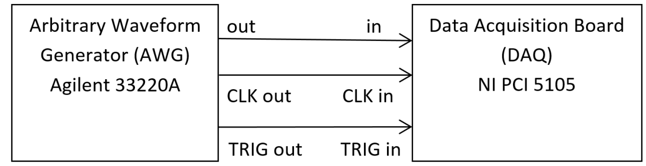

Fig. 4: Diagram of the experimental setup used for the characterization of the AWG generating the ternary sequence.

A. Experimental Setup 1 - DDS AWG system

A first set of experiments was conducted using the setup shown in Fig. 4. A 14-bit Arbitrary Waveform Generator (AWG), 33220A by Agilent, operating according to the direct digital synthesis (DDS) principle, was used to generate a designed DS of length 42, according to [5], which was loaded in the arbitrary waveform memory. The repetition period of the sequence was set to 10 ms. The signal generated by the AWG was synchronously sampled at a rate of 10 MSa/s by an acquisition board (12-bit PCI 5105 by National Instruments). A total of $4 \cdot 10^6$ samples were acquired, corresponding to 40 periods of the generated sequence.

Results using such a setup are shown in Fig. 5. In this case the ratio between waveform memory depth (65536) and sequence length (42) is not an integer. Results show that, when loading only one repetition of the sequence in memory, the automatic stretching operated by the instrument causes non-uniform duration of the sequence elements (*chip duration*). Then, this non-uniform chip duration results in undesired harmonic content. This can be observed in Fig. 5(a), where undesired components at 300, 900, 1500, and 2100 Hz are higher than the other undesired harmonics by about 30 dB. Conversely, using a repeated sequence to fill the waveform memory, this effect is mitigated and an SFDR of 80 dB is obtained, as shown in Fig. 5(b) where the effect of the DC component is neglected.

This experimental setup can potentially operate at frequencies up to 20 MHz. However, it is prone to some of the implementation issues described in Section III, mainly non-uniform duration of elements within the sequence and limited ADC resolution. Therefore, a different experimental setup that addresses such issues, albeit operating only up to audio frequencies, was employed for further validation. Such setup is described in the following subsection.

B. Experimental Setup 2 - PC sound card

1) Setup: Periodic sequences of length 42 were generated by means of the DAC in the sound card (ALC887 by Realtek) of a personal computer. The acquisition was performed using the ADC on the same sound card. The DAC and ADC were set at 42 kSa/s sample rate, with a resolution of 16 bits. The sequence period was 10 ms, its repetition rate was set to 10^2 Hz, resulting in a chip rate of 4.2 kHz. Coherent sampling was performed, with the DAC and ADC using the same internal clock source. The duration of the acquisition was 10^2 s, corresponding to 10^4 periods. Initial DAC and ADC transients of duration 0.1 s were discarded.

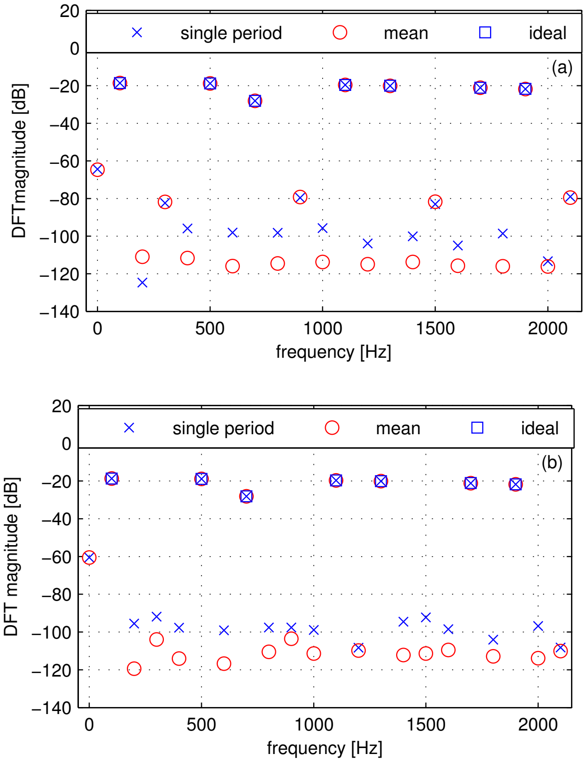

Fig. 5: Estimated spectra obtained with the DS using the setup in Fig. 4: (a) single repetition of the sequence in the AWG memory; (b) multiple repetition to fill the AWG memory (sequence repeated a fractional number of times). The mean spectrum is obtained by coherent averaging over 40 periods.

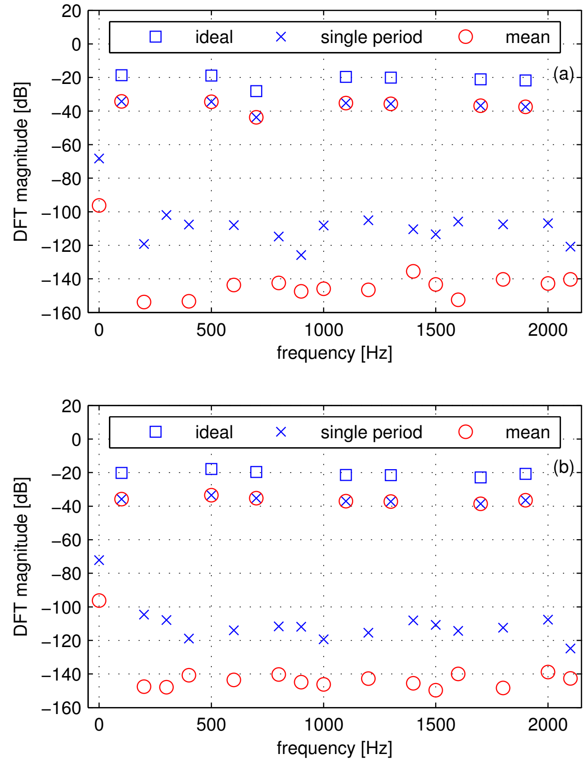

Fig. 6: Estimated spectra related to the PC sound card acquisition of: (a) the DS in [5]; (b) the proposed RCS. The mean spectrum is obtained by coherent averaging over 10^4 periods.

2) Direct connection results: Results are presented in Fig. 6(a) and 6(b), for the DS generated according to [5] and for the RCS proposed in Section IV, respectively. The discrepancy between the ideal levels of the desired harmonics and the actual measured levels is due to a scale factor related to the amplitude range of the ADC. Such scale factor can be compensated by calibration. From Fig. 6, it is possible to observe that, by averaging over 10^4 periods, an SFDR of approximately 100 dB and a SINAD of 104 dB are obtained using both methods. We also stress that it is not possible to discriminate whether the undesired harmonics are caused by the DAC or by the ADC. The DC component is excluded from the calculation of the SFDR because it can be relatively easily attenuated by means of a series capacitor or by averaging. Moreover, it can be observed that the difference between the highest desired harmonic and the lowest desired harmonic is approximately 5 dB for the RCS and 9 dB for the DS.

3) DAC non-uniformity emulation: A significant DAC non-uniformity was emulated by substituting a value of 10^{-3} to the zero values, both in the DS and in the RCS. Acquisition results for the DS are shown in Fig. 7(a). The component at 1400 Hz, corresponding to the harmonic number $N/3 = 14$ is considerably higher, resulting in an SFDR of 53 dB and a SINAD of 59 dB. This validates the theoretical derivations in Section II-B, where it is shown that non-uniformity in the DAC levels causes the power of the harmonic at $N/3$ to increase when using the DS. This phenomenon has a negative effect on performance, because it contradicts the design objective stated in Section II. In fact, since the even-order harmonics are not entirely suppressed, it is not possible to eliminate the effect of even-order nonlinearities on the frequency response measurement. However, the DS may be conveniently used to highlight the presence of DAC non-uniformity, just by observing one specific harmonic. On the other hand, an acquisition using the RCS is shown in Fig. 7(b). The highest undesired harmonic is at a lower level with respect to Fig. 7(a), namely 58 dB below the highest desired harmonic, illustrating that the proposed method mitigates the effect of DAC non-uniformity, by about 5 dB with respect to DS performance. Therefore, the RCS can be used in those cases where insensitivity to non-uniformity is required, while the DS may be beneficial when the detection of DAC non-uniformity is necessary.

4) Analog filter effect: Tests in the presence of a hardware first-order RC low-pass filter at the input of the ADC were conducted. The filter is necessary in those applications where sharp edges and non-uniform signal amplitude distributions are problematic, e.g. for ADC testing. Without it, in fact, intermediate ADC levels are not excited. Results from this test are shown in Fig. 8, using the DS. The filter is realized with $R = 1000\Omega$ and $C = 80$ nF, yielding a 3-dB cutoff frequency of approximately 2 kHz. The spectral properties of the sequence are similar to those of Fig. 6(a). However,

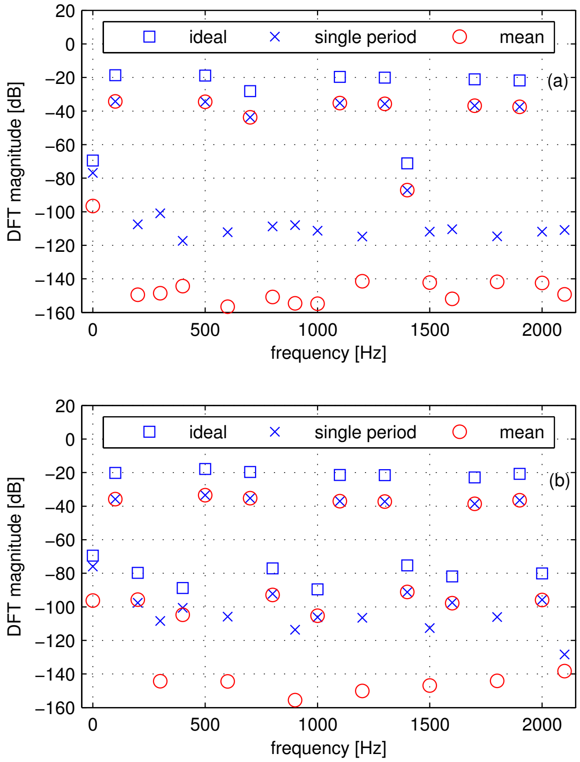

Fig. 7: Experimental emulation of DAC non-uniformity. Periodic sequence of length 42 with zeros replaced by 10^{-3}: (a) DS; (b) RCS. Notice that in (a) the 14th harmonic at frequency 1400 Hz is relatively high, approximately 55 dB higher than the other undesired harmonics.

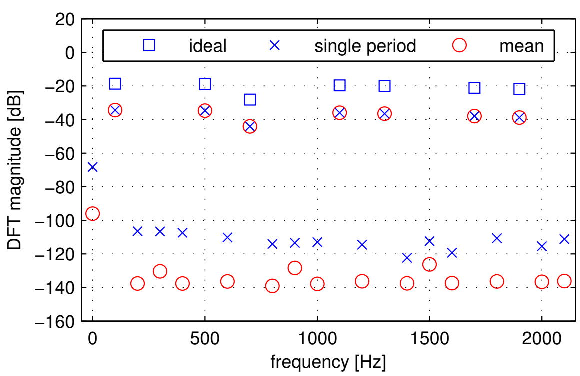

Fig. 8: Estimated spectra related to an acquisition in the presence of a 2-kHz RC low pass filter.

the time-domain behavior in Fig. 9 illustrates that the filter smooths the sharp edges and reduces high-frequency noise. Similar results for RCS are not presented here for brevity. An analysis of the spectrum in Fig. 8 allows the detection of a nonlinear behavior of the system. Specifically, it can be noticed that the 3rd, 9th, and 15th harmonics, at 300 Hz, 900 Hz, and 1500 Hz respectively, are approximately 10 dB higher than the average level of the other undesired harmonics. The presence of this nonlinear component is probably due to an increased loading of the DAC by the filter. An impedance matching circuit could mitigate this behavior.

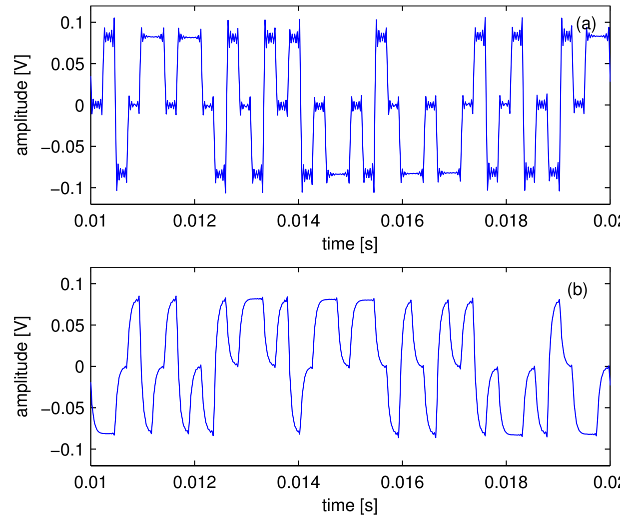

Fig. 9: Time-domain behavior of one period of the acquired ternary sequence: (a) direct connection; (b) with hardware RC low pass filter.

VI. Conclusion

The effect of DAC non-uniform levels on the spectral properties of synthesized ternary sequences was analyzed, together with other practical implementation issues, both theoretically and experimentally. Furthermore, a low cost method was developed to generate signals with a very high spectral purity. Specifically, a new constrained randomized approach for synthesizing ternary sequences with harmonic multiples of two and three suppressed was proposed, compared with the direct synthesis approach from the literature, and validated with experimental tests, demonstrating harmonic suppression of the order of 100 dB.

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