Essays on Bidding with Securities

Andrés Fioriti

PhD in Economics

University of Warwick

Department of Economics

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Contents

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Declaration

This thesis is submitted to the University of Warwick in support of my application for the degree of Doctor of Philosophy. It has been composed by myself and has not been submitted in any previous application for any degree.

The work presented (including data generated and data analysis) was carried out by the author except in the cases outlined below:

Chapter 2 was done in collaboration with Allan Hernández-Chanto.

Chapter 3 was done in collaboration with Allan Hernández-Chanto.

Abstract

Chapter 1 partially surveys auctions with contingent contracts, i.e., contracts in which payments are allowed to depend on an ex-post verifiable variable, such as revenues. The review starts with the seminal paper of DeMarzo et al. (2005) and partially departs from Skrzypacz (2013) by analyzing on externalities and risk aversion concerns. A partial ranking of auction revenues for auctions that differ in terms of contract forms, pricing rules and seller commitment are described. Models incorporating adverse selection, moral hazard, competition between auctioneers, externalities and risk aversion are discussed.

In Chapter 2 we study second price auctions, where buyers compete for the allocation of a project, by bidding securities over project's realized value. In addition, we allow for negative externalities, which are suffered by the losers in case the winner implements the project. Under this environment, we introduce two payment instruments: the Fixed-Equity Hybrid -which embeds cash- and the Fixed-Cash Hybrid -which embeds equity. As our main result, we rank the instruments in terms of revenue, and show that the fixed-equity hybrid is the best instrument whereas equity is the worst despite of being the most sensitive instrument to bidders' true type.

Finally, in Chapter 3 second-price auctions, where buyers compete for the allocation of a project, by bidding securities over project's realized value are studied. In addition, bidders are allowed to be asymmetric not only with respect to their underlying distribution of payoffs but also with respect to their risk aversion. Under this environment, it is shown that steeper securities provide higher insurance. As main result, the instruments are ranked in terms of efficiency, and shows that the steepest security minimizes the efficiency loss when bidders are indeed asymmetric. Moreover, steeper securities are shown to increase revenue for the seller as in DeMarzo et al. (2005).

Chapter 1

Survey of Auctions with Contingent Payments

1.1 Introduction

In recent years a flourishment of auctions with contingent payments has taken place both in practice and in theoretical analysis. Most of these auctions involve the selling of an asset/contract whose value is at least partially observed. For example, in oil-leased auctions, if the winner explores the field, the government can measure revenue obtained from the exploration. It is a common practice around the world for the government selling the rights to drill for oil or natural gas to collect additional revenue in the form of royalties. Other examples may include the 3G auction that took place in Hong Kong where bidders submitted bids on equity.

Auctions with contingent payments refer to cases in which the auctioneer/seller allows bidders to compete for the allocation of an asset by means on an auction where payoffs are at least partially tied to the asset realized value. The theoretical analysis of such auctions has received great attention lately. In this chapter, I provide a selected survey of literature on this topic to frame the remaining chapters of the dissertation. I will use DeMarzo et al. (2005) as a focal point –henceforth DKS– since it is crucial to understand auctions with contingent payments.

In this survey I depart from Skrzypacz (2013) as it covers real life situations in which auctions with contingent payments are relevant and focus strictly on the theoretical analysis. Hence, the review is structured as follows. I first describe the benchmark model of DKS with independent private values and explain why in such auctions revenue is higher than in cash auctions. Then I discuss the ranking of auctions with different types of contracts if the seller restricts bidders to a single-dimensional type of contracts. In section 1.3, I review papers that enrich the benchmark model with important real-life considerations. These features shed light on some tradeoffs that could change the predictions of the basic model. In sections 1.4 and 1.5, I extend the survey to externalities and risk aversion to provide a broad picture of the relevant literature to the current dissertation.

This chapter is not a comprehensive survey, however, it provides some extensions to Skrzypacz (2013) by including the role of securities as means of risk sharing -under the presence of risk averse bidders- and looks at the impact of externalities on auction design.

1.2 The Model

The benchmark model with contingent payments follows DKS. There is a seller and N ex-ante symmetric risk-neutral bidders. Bidders have independent private values. The seller runs an auction for a project that requires the winner to make an up-front investment $c > 0$. If bidder i wins the project, it generates verifiable revenue/cashflow Z_i. Each bidder has private information about his expected cashflow z_i. The types z_i are distributed independently and symmetrically according to some distribution $f(z_i)$ over the interval $[\underline{z}, \overline{z}]$. Conditional on z_i, bidder i cashflow is distributed according to an atomless distribution $h(Z_i|z_i)$.

The authors assume $h(Z_i|z_i)$ has full support and satisfies the strict Monotone Likelihood Ratio Property (SMLRP). This assumption implies that a higher estimate represents a stronger distribution of cashflow realizations.

The model considers a bid as a contingent payment offer as a function of the future cashflow, $S(Z_i)$. They restrict the attention to bids that satisfy that $S(Z)$ and $S(Z) - Z$ are increasing and $S(Z) > 0$, implying the seller cannot subsidize the bidders. Moreover, $S(Z) \leq Z$, representing limited liability on the side of the bidders. They define $ES(z) \equiv E[S(Z)|z)$.

On this survey, formal auctions are only addressed. A formal auction is described by an ordered set of contracts/securities and an auction format. The set of allowed contracts $\mathbf{S}$ is indexed by $s \in [s_0, s_1]$. This notation allows us to represent $S(s, Z)$ as the ex-post payment to the seller of contract with index s if the realized revenue is Z. Denote $ES(s, z) = E[S(s, Z_i)|z_i = z)$.

The only requirement the author imposes on $ES(s, z)$ to consider it an order is that it is increasing on s for every z. In other words, conditional on fixing the type, the payment to the seller should be increasing on the index. There are several types of contracts that fit on this description: cash, royalty/equity, fixed royalty plus cash bids, fixed contract plus cash bids, debt, allowance plus royalty contract, royalty contract with a cost deduction and call option.

In an auction the bidder that submitted the highest index wins and pays according to the auction format (for example first-price or second-price).

Hansen (1985) was the first one to consider auctions with contingent contracts comparing cash to royalty contracts in second-price auctions. He showed that royalty auctions accrue a higher revenue because the winner is the same but instead of paying the reservation value of the second highest type now he has to compute the royalty payment on his distribution, increasing the sensitivity of the payment to the type of the winner. Riley (1988) showed similar results.

Even though DKS extends Hansen result to a comparison between any security and cash, their main concern is how to rank different securities. For example, does a royalty contract auction or debt contract auction yields higher revenue? In this case the slopes of $S(Z)$ are ranked differently for different levels of Z: debt has a higher slope than equity for low realizations of Z while the opposite ranking is true for high realizations of Z.

DKS have shown that many standard sets of contracts can be ranked under the SMLRP assumption. The crucial condition needed to rank different securities is as follows.

Definition 1 *An ordered set of contracts/securities* $\mathbf{S}_A$ *is steeper than an ordered set* $\mathbf{S}_B$ *if, for all indices* s_A *and* s_B *from the two sets,* $ES_A(s_A, z^*) = ES_B(s_B, z^*)$ *implies that* $ES_2^A(s_A, z^*) > ES_2^B(s_B, z^*)$. *If that is true we say that "*$S_A(s_A, z)$

strictly crosses $S_B(s_B, z)$ from below."

Lemma 2 *(Lemma 5 in DKS) If $h(Z|z)$ satisfies SMLRP then a sufficient condition for $S_A(s_A, z)$ to strictly cross $S_B(s_B, z)$ from below is that there exists a Z^* such that $S_A(s_A, z) \leq S_B(s_B, z)$ for $Z < Z^*$ and $S_A(s_A, z) \geq S_B(s_B, z)$ for $Z > Z^*$.*

This lemma implies that equity is steeper than debt and a call option is steeper than either of them, as seen in Fig. 1.1.

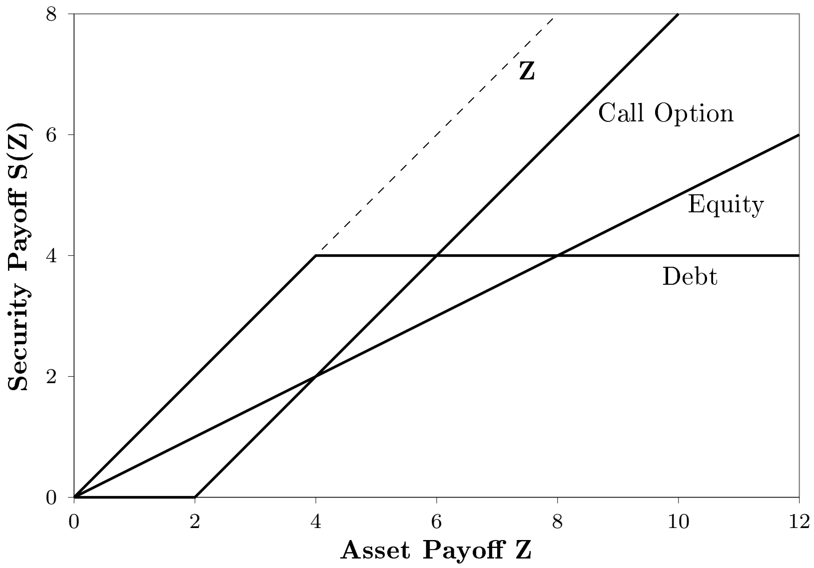

Figure 1.1: **Payoff Diagrams for Call Options, Equity, and Debt.**

Proposition 3 *(Proposition 1 in DKS) Suppose the ordered set of contracts/securities $\mathbf{S}_A$ is steeper than $\mathbf{S}_B$. Then for either a first-price or a second-price auction, for any realization of types (almost surely), the seller's revenues are higher using $\mathbf{S}_A$ than $\mathbf{S}_B$.*

As a corollary of this proposition and the previous lemma, debt auctions yield a lower revenue than royalty/equity and both are dominated by auctions with call options.

This result constitutes the main contribution of DKS since it allows to rank in terms of revenue any security that could be ranked in terms of steepness. Moreover, they show that this result could be extended to informal auctions, which are beyond the scope of this review.

1.3 Extensions

DKS main result goes in one direction: Steeper securities are better for the seller and worse for the bidders. Crémer (1987) makes an even stronger point: if in the Hansen (1985) environment the seller subsidized most of the up-front cost c, he would extract arbitrarily close to the full surplus.

The problem is that in practice we observe a rich variety of contracts, ranging from cash to equity mostly. One plausible explanation has to do with bargaining power at the time of deciding the payment method. Another alternative could be risk aversion and the risk sharing allowed by different securities. There could also be externalities or competition among sellers.

Adverse selection: Che and Kim (2010) consider the case in which higher z are related to higher c. For example a firm may obtain higher revenues from a project because they will spend more on marketing. On a second-price auction it is still a weakly dominant strategy to bid according to your reservation value. In this case cash is still efficient but securities need not be. Under equity if $c(z)/z$ is increasing then the winner will be the one with the lowest z.

The adverse selection concern pointed by Che and Kim (2010) could also take place if c and z are two-dimensional private information of the bidders, which are independent across bidders. In this case using securities could lead to an inefficient allocation of the asset.

Moral hazard: In those cases where the revenue realization depends on the effort exerted by the winner, using steeper contracts could harm revenue. Using steeper securities have the benefit that they extract a higher fraction of the revenue but the main drawback is that they decrease the incentives to exert effort. There is a tradeoff between extracting surplus and providing incentives. Kogan and Morgan (2010) consider a model where effort enters multiplicatively while Jun and Wolfstetter (2014) consider it additively. Both articles highlight that depending on the cost of effort and the number of bidders is the degree of steepness that maximizes surplus for the seller.

Even in this case, it could be argued that the relationship between cash and securities remains: McAfee and McMillan (1986) show that it is possible under moral hazard to find auctions with contingent payments that dominate cash. For example, asking for a small fixed royalty and letting bidders compete on cash introduces a second-order loss in terms of efficiency, but a first-order gain in terms of surplus, thus dominating pure cash.

Competition between auctioneers: Gorbenko and Malenko (2011) provide a rationale for using somewhat flat instruments. If there are many sellers and a fixed pool of bidders then one way in which sellers could attract bidders is by lowering the steepness, which is similar to a surplus transfer. Their paper is related to bargaining power since when sellers should compete for bidders the latter get more power (in relative terms) and force sellers to move away from steep securities.

Budget constraints: Debt contracts induce the same environment as the one analyzed by Che and Gale (1998). By imposing a budget constraint bidders bid the minimum between the budget and the valuation (broadly speaking). The analysis is not exactly the same because DKS considers securities with limited liability, which solves the problem for budget constrained bidders.

If bidders have budget constraints then DKS recovers bids in terms of the true type since bidders will pay with the proceeds of the project. DKS show one way of moving away from Che and Gale (1998) budget constraint analysis.

Rhodes-Kropf and Viswanathan (2005) try to solve the budget constraint issue by allowing bidders to access a financial market. They show that the financial market is not efficient thus the inefficiencies of the auction remain.

Bankruptcy: DKS assumes that keeping the type of the winner fixed (and post-auction actions fixed in case of moral hazard), the overall surplus generated by the project is independent of the contract. Board (2007) changes this assumption pointing out that bankruptcy costs are often non-negligible, creating new trade-offs between division of surplus and surplus creation. He shows that a first-price auction may generate higher revenue for the seller than a second-price auction when bankruptcy costs are sufficiently high.

1.4 Externalities

Auctions with externalities have been studied since the seminal papers of Jehiel et al. (1996, 1999). They use a model of identity-dependent externalities and solve the optimal mechanism when bidders type is multidimensional –bidders have valuations and identity-dependent externalities, resulting in type vectors with $N+1$

entries.

Under the optimal mechanism the seller extracts surplus even from bidders that do not get the object. They show that if the externalities are sufficiently high the seller is better off by keeping the object for himself even though he gives value zero to it. Lastly they argue that participation constraints are endogenous. The intuition is that if a firm sees that its main competitor in the downstream market is participating in an auction for a patent, it may have incentives to participate as well because by not doing so its market share could be severely reduced.

Securities pose a higher threat to revenue under negative externalities. When bidders compete on the downstream market they may go to the auction to prevent his competitors from getting the asset. This protective strategy is easier to be carried out with securities since no implementation leads to no payment. The second chapter of the dissertation tries to understand the implications of negative externalities when bidding with securities.

Positive externalities have a free rider problem since bidders may be better off not participating in the auction but enjoying the externalities. Even though this setup is difficult to motivate it is worth being considered.

1.5 Risk Aversion

Abhishek et al. (2015) is the first paper that introduces risk aversion to DKS environment. They have risk averse bidders although their utility functions are the same –bidders are homogeneous in terms of their utility functions. This setup is closer to reality because bidders may be competing for a technology whose returns are unknown, involving risk. When bidders are risk averse then securities provide a channel to share risk. For example, equity allows bidders to pay less under low realizations and more when realizations are high, representing some kind of insurance.

The authors focus on revenue and show that DKS order prevails with homogeneous risk averse bidders as long as the SMLRP holds. Moreover, they show that relaxing the signals ordering to FOSD breaks the revenue ordering. Lastly, they characterize Strong Steepness which is the condition needed to recover DKS ranking under FOSD.

Another possible case where risk aversion becomes relevant is the one of heterogeneous bidders in terms of the utility function. In this case the insurance plays an asymmetric role since the more risk averse benefit the most out of it. The third chapter of the dissertation tries to understand the implications of heterogeneous risk aversion when bidding with securities.

1.6 Concluding Remarks

In recent years auctions with contingent payments have increased their popularity. Researchers started developing models to understand the extent of the practice pros and cons. Since no solution fits all, many papers have been written after the benchmark model proposed by DKS.

This survey reviews the benchmark model and some of the modifications that were proposed afterwards, dealing with adverse selection, moral hazard, competition between auctioneers, budget constraints, externalities and risk aversion. The last two topics will be extended on this dissertation.

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Chapter 2

Bidding Securities in Projects with Externalities

2.1 Introduction

Over the last two decades, the sector of technological firms have witnessed a flourishment without precedence, boosted among others, by the presence of internet and a robust market of patents. The role of this market has been twofold. From one hand, it has allowed companies to monetize their inventions by auctioning them to a pool of interested firms, but at the same time has permitted the same companies to acquire patents to develop their own products. Such environment has made possible for start-up companies -unlike in any other market- to evolve into strong competitors, with a large market capitalization, in short time. Remarkable examples include Uber -which reached a capitalization of $41 billion in less than six years, the fastest in history- WhatsApp and Spnapchat.[1]

Therefore, if a competitor acquires "the right" portfolio of patents, an operating firm in a specific niche might promptly see its market share reduced, because it would enable the competitor to develop its own innovation. For this reason, many large firms acquire patents as a protective strategy: to preclude the development of nascent companies that may change the *status quo* of its market participation. Examples here include Facebook, Yahoo and Microsoft.[2] In addition, Hall and Ziedonis (2001) find that after 1982, the US semiconductor firms started *patent portfolio races*, not to appropriate R&D revenue, but to prevent other firms from getting these patents.

This scenario raises many interesting questions. First, if a start-up is selling its project -or innovation- through a standard second price auction and wants to maximize revenue, we could ask what the optimal method of payment is. Should the seller conduct the auction in cash, or should he use a security, contingent on project's return? This dichotomy has relevance, because if the innovation is allotted to a firm that intends not to implement the project, the seller would receive a payoff of zero if he uses a security. On the other hand, if the project only has value for the winner when he implements it, the seller might be better off using a contingent payment as it is more sensitive to bidder's true valuation (c.f. DeMarzo, Kremer and Skrzypacz, 2005).

A related question is how bidders' optimal strategies behave under the presence of a negative externality, given the method of payment. Here, the key observation

[1] For more details see http://www.wsj.com/articles/uber and http://www.wsj.com/articles/snapchat.

[2] Recently Facebook acquired a portfolio of 750 patents to defend itself from a lawsuit from Yahoo and other companies. See http://techcrunch.com/2012/03/23/facebook

is that the presence of a negative externality increases the eagerness of the bidders to win the auction, even when they attach a very low valuation to the project.

To answer these questions, we build a model where the seller sells the rights of a project through a standard second price auction, but where he can utilize two hybrids as methods of payment: (i) a fixed-equity hybrid where the seller fixes the fraction of equity requested, and let bidders compete in cash, and (ii) a fixed-cash hybrid, where the seller fixes the amount of cash the winner has to pay, and let bidders compete in equity. Notice that the former embeds pure cash whereas the latter embeds pure equity.

The reason for which including a fixed payment in the instruments may be beneficial for the seller, resides in the problem of adverse selection associated with the incentives of a buyer to participate in the auction. Specifically, a buyer may want to participate in the auction either to try to implement the project (because it is profitable to do so), or just to attempt to block the allocation of his rival. If the seller is paid upon the implementation of the project, allocating it to a buyer of the second class (i.e. "the bad type") would be detrimental for his revenue. In the absence of a fixed payment, the bad type have always an incentive to participate in the auction to try to destroy the equilibrium where the project is implemented. Thus, the fixed payment acts as a screening device among bidders. However, the seller faces a clear trade-off in his aim, because introducing a fixed payment decreases the profitability of the project for all buyers, which in turn leads to a lower probability of implementation. The goal of the present chapter is to determine the optimal fixed payment for both hybrids, and rank them with respect to seller's expected revenue.

Certainly, our article is not the first interested in exploring the relation between revenue and the method of payment used in an auction. In fact, De Marzo, Kremer and Skrzypacz (2005) has shown that if there are no externalities, the methods of payment can be ranked in revenue by their "steepness", or the sensitivity of bidder's true type to the instrument utilized. An insight first hinted by Hansen (1985) and Riley (1988). They also show that the auction format has only an impact on revenue by its ability of modifying the steepness of the particular instrument utilized. Nonetheless, to arrive to their conclusions it is crucial that bidders operate in an environment free of negative externalities. When we incorporate them into the model, their main result does not hold anymore, precisely because a winner of the auction may acquire the project not to implement it.

In order to isolate the effect produced by the interaction of the externalities with the method of payment, we focus on a simple model of two bidders, where the loser of the auction suffers a commonly known negative externality if the winner implements the project.[3] This framework arises naturally in industries where bidding firms are similar ex-ante, and the project gives a comparative advantage in the downstream market to the winner. Surprisingly, many of the insights can be captured with this simple version. First, we consider a simple model where both, externalities and valuations are public information. Even in this simple framework the characterization of equilibria is not trivial, because it depends on the interaction of the externality, the cost of the project and bidder's own valuation.

Our main result, stated in theorem 6, shows that under some mild technical conditions, the following is satisfied. First, the optimal fixed-equity requires a

[3] One important clarification is that we use the word implementation, because if an agent wins the object but does not implement it, no agent suffer any externality.

strictly positive equity payment, and the optimal fixed-cash hybrid involves a strictly positive payment in cash. Second, the optimal fixed-equity hybrid is the instrument that yields the highest expected revenue, followed by cash, which in turn is followed by the optimal fixed-cash hybrid. Equity is the worst instrument in the menu, despite of being the steepest.

The intuition of the latter results lies in the fact that with equity, a bidder will pay zero if he does not implement the project, but his bid will affect the profitability of implementing the project for his opponent. In that sense, a particular buyer can effectively use the threatening-power equity equips bidders with, to destroy the equilibria when the other buyer finds profitable to implement the project. When the seller uses cash as the instrument, this problem is mitigated by the fact that all payments are made upfront, rather than conditional on the implementation of the project. Therefore, the optimal instrument for the seller would be one that simultaneously features the screening benefits offered by cash, and the ability of equity to extract surplus. This design is precisely at the heart of the fixed equity hybrid. On the other hand, when the seller sets the fixed payment in terms of cash, and let buyers use equity to screen themselves, buyers conserve part of their power to block the implementation of the project, and so the adverse selection motive dominates. Surprisingly, this effect is so powerful that the optimal fixed-cash hybrid performs worse than cash for a sufficiently low implementation cost and a sufficiently high negative externality.

The ranking of the instruments is robust to the structure of information, as it is preserved for a large class of log concave distributions over private buyers' valuation. In particular, equity continues to deliver zero revenue despite of being the steepest instrument in the menu. Even though, we cannot deliver a general theorem as in the case of public information, we obtain very similar results via a simulation.

Finally, in a comparative statics exercise, theorem 7 finds that the fixed portion of both instruments is weakly increasing with respect to an improvement in the distributions, in the sense implied by the Monotone Likelihood Ratio (MLR) property. This result is clearly intuitive: as the probability of drawing higher valuation increases, the seller is less concerned of inducing participation, and can commit himself to extract a higher portion of revenue before the competition in the auction takes place.

Related Literature Our article is related to the literature of auctions with securities and to the literature of auctions with externalities. Nonetheless, as far as we know this is the first article connecting both strands of literature, to analyze how the interaction of negative externalities and securities impact bidding strategies and seller's expected revenue. Moreover, as we discussed before, due to the implementability incentives of buyers, our model can also be framed in the literature of auctions under adverse selection.

The literature of auctions with securities started with the seminal articles of Hansen (1985) and Riley (1988), who basically showed that a second price auction run in equity yields higher expected revenue to the seller than one run in cash. More recently, De Marzo, Kremer and Skrzypacz (2005) -hereafter DKS- generalize this framework by providing a methodology to rank securities with respect to revenue. Specifically, they characterize the "steepness" or sensitivity of several instruments via a single crossing property argument, and show that steeper instruments yield a higher revenue for the seller. Furthermore, they argue that the auction format is only relevant as long as it modifies the steepness of the instrument utilized.[4] Although DKS analyze a larger class of securities than what we do in this article, the main essence of their analysis is retained, because the distinction of the payment condition (i.e. contingent vs non-contingent) is the key ingredient to obtain our main result. As mentioned before, we focus in two hybrid instruments that are used in practice, and which include cash and equity as particular cases.[5]

Following endeavors to DKS include Gorbenko and Malenko (2011) and Che (2010). The former analyzes the predictions of a DKS model when the set of bidders is finite and many sellers compete for them. Their main result shows that sellers will not use the steepest instrument because they would not attract enough bidders. We also obtain the same result but for different reasons. In our case, using a pure security is detrimental for seller's revenue because it allows buyers, who do not intend to implement the project, to destroy the equilibria where good-type buyers would have implemented it otherwise.

Meanwhile, Che and Kim (2010) modify DKS framework by assuming that buyers with higher valuations also have a higher cost to implement the project. This simple modification leads to an adverse selection problem when the seller uses a security, because buyers with high valuation would bid a lower amount, and

[4]In particular, they prove that when the seller uses securities the Revenue Equivalence Theorem may not hold.

[5]Our fixed-equity hybrid resembles the way writers sell the rights of their books because there is a fixed royalty rate and publishers compete on cash. On the other side our fixed-cash hybrid captures the main feature of the oil rights auction in Mexico where buyers pay a fixed amount and compete on equity.

therefore, more often such buyers will win the auction. As the revenue of the seller is tied to bidders' true type when he uses a security, this adverse selection problem cause the revenue to decrease. We found that using securities when externalities are present can lead to the same result. Here, the low-valuation buyers would bid more aggressively because they want to avoid the negative externality, and can block implementation at no cost when the seller uses pure securities. Nonetheless, whereas Che and Kim (2010) makes assumptions on the cost structure of the model, we make assumptions on the after-market behavior of firms, which we consider more significant in many patent auctions where securities are normally utilized.

Our article also contributes -in minor extent- to the literature of auctions with externalities, initiated by Jehiel, Moldovanu and Stacchetti (1996, 1999). However, rather than proposing an optimal mechanism exercise under an environment with externalities, we analyze a small but widely used class of instruments, which unlike Jehiel et al. also incorporates securities as a method of payment. We are able to show that under negative externalities, a second price auction in cash is no longer an optimal mechanism, because in our model we find that the best instrument is a fixed-equity hybrid.

Organization of the chapter The rest of the chapter is structured as follows. Section 2.2 states the environment of the model. In section 2.3 we introduce the case of complete information, derive the equilibrium bidding strategies, and rank the instruments with respect to revenue. Section 2.4 presents a robustness exercise for the case of private information. Section 2.5 concludes. Some of the proofs are relegated to the appendix.

2.2 The Environment

A seller is interested in allocating an indivisible asset -which can be thought as the rights of a project or innovation- among two different buyers. The winner is required to pay a cost of $c > 0$ in order to implement the project, which is considered as the initial investment to run the project, and is commonly known. We index buyers by $i = 1, 2$ whereas the seller is designated as player $i = 0$. Buyer i's valuation v_i is drawn identically and independently from $[\underline{v}, \bar{v}]$, according to the distribution F which corresponding non-atomic density f.

If the project is implemented by a competitor, buyer i suffers a negative externality of $e \in [\bar{e}, 0]$, which we assume is symmetric and publicly known among buyers. One important aspect of our model is that externalities are contingent to the implementation of the allotted buyer. Second, private valuation refers to the gross return of the project, and so a rational winner i will only implement it if $v_i - c > 0$. Third, even if a buyer does not want to implement the project it would be beneficial for him to acquire it to preclude the implementation by other competitors, and thus avoiding the potential negative externality he might suffer.

The seller commits to use a second price auction to sell the project, but we assume he can utilize two different instruments: a fixed-equity hybrid and a fixed-cash hybrid. In the former the seller fixes the equity over project's return requested from the winner, and let bidders to compete in cash. The winner is the buyer who submits the highest bid in cash but pays the bid of his opponent. Clearly, a standard second price auction with cash corresponds to the case when the seller request zero equity. On the other hand, when the seller uses a fixed-cash hybrid, he fixes an amount in cash the winner of the auction has to pay, and let buyers to compete in equity. As before, the winner of the auction is the buyer who submits the highest equity bid, but pays the lowest bid. In this case, when the seller asks a fixed cash of zero, the auction is conducted in pure equity.

All players are risk neutral, and buyers' utility is additively separable. Let z_i be the return buyer i derives from the project after his implementation decision. That is, $z_i = v_i - c$ if he implements the project and zero otherwise. Thus, if buyer i, with type v_i, wins the auction his payoff is given by $z_i - t_i(v_i)$, where $t_i(v_i)$ represents the payment to the seller, which potentially depends on his valuation. On the other hand, if the seller allocates the object to buyer j, then buyer i's payoff corresponds to e, provided his competitor implements the project; and zero otherwise. The value of the project for the seller is zero, and hence in any trade with buyer i his utility is $t_i(v_i)$.[6] If no trade occurs, the payoff is zero for all players.

Figure 2.1 depicts the timing of the game. First, seller chooses a payment instrument and commits to run a second price auction under this format. Then, buyers learn their valuations and submit their bids to the seller, who determines the winner of the auction. Next, the winner determines if he wants to implement or not the project. Finally, payoffs are realized contingent on the implementation decision.

[6] Think for example in a seller who owns a patent over a specific productive process that by itself cannot be monetized, but can potentially enhance the productivity of the current technology held by the buyers.

Figure 2.1: Timing of the auction

2.3 Public Buyer's Valuation

In this section we assume that before participating in the auction, each buyer learns his own valuation as well as the valuation of his opponent. Without loss of generality we will assume $v_1 > v_2$. The seller, on the other hand, only knows the distribution where buyers' valuations come from. Nonetheless, the negative externality is public information for all players. This setting plausibly corresponds to a situation where both buyers have been operating in a market for long time and have learned the technology of each opponent, but where a seller is an outsider of the industry who has developed an innovation that can enhance the technology of both buyers, but cannot evaluate to which extent.

The seller wants to maximize the ex-ante revenue and for that purpose has to choose which instrument to utilize. Once the seller chooses an instrument he commits to it. Thus, bidders are engaged in a game of public information, where they have to choose their bid b_i in the correspondent security space. In the case of the fixed-equity hybrid $b_i \in \mathbb{R}_+$, whereas in the case of the fixed-cash hybrid $b_i \in [0, 1]$.

A Motivating Example In this section we will go through an easy example that will highlight the main results of the chapter.

A. No Externalties Consider an auction in which two buyers, Alice and Bob, compete for a project. The project requires an initial fixed investment of $c > 0$ which can be interpreted as the minimum up-front cash payment required by the seller. Alice expects that if she undertakes the project, it would yield her a return of v_a, whereas Bob expects a cash flow of v_b. Without loss of generality, $c < v_b < v_a$. We assume that both valuations are common knowledge to both buyers. As the seller commits to use a second price auction, the weakly dominant strategy for both buyers is to bid their reservation value. As a result, Alice would bid $b_a(v_a) = v_a - c$ and Bob would bid $b_b(v_b) = v_b - c$. Hence, Alice wins the auction and pays Bob's bid, which implies seller's revenue would be $\Pi^{ca} = v_b - c$.

Now, suppose that rather than bidding with cash, the buyers compete by offering equity over the return of the project. As we discuss later, in this case it is also a weakly dominant strategy for both buyers to bid their reservation value.[7] Thus, Alice would make aa equity bid of $b_a(v_a) = \frac{v_a - c}{v_a}$, whereas Bob would make an equity bid of $b_b(v_b) = \frac{v_b - c}{v_b}$. As a result, Alice wins the auction and pays according to Bob's bid. Seller's revenue would be $\Pi^{eq} = \frac{v_b - c}{v_b} v_a$. By an easy algebraic manipulation, it is possible to see that sellers revenue under equity is higher than under cash, as

$$\Pi^{eq} = \frac{v_b - c}{v_b} v_a = (v_b - c)\frac{v_a}{v_b} > v_b - c = \Pi^{ca}$$

B. Externalities Consider the same auction as before but now with the modification that if buyer i wins the auction and implements the project, the payoff of

[7] The reservation value of buyer i is when his payoff equals 0: $(1 - b_i(v_i))v_i - c = 0$ thus $b_i(v_i) = \frac{v_i - c}{v_i}$.

buyer $-i$ will be $e < 0$.

Cash. When the payment instrument is cash, bidding the reservation value continue to be a weakly dominant strategy for both buyers. Nonetheless, it now should include the externality. Thus, $b_a(v_a) = v_a - c - e$ and $b_b(v_b) = v_b - c - e$. Seller's revenue becomes $\Pi^{ca} = v_b - c - e$.

Equity. If buyers compete by offering equity the analysis is more interesting. Here, Alice knows that if she bids $b_a(v_a) = \frac{v_a - c}{v_a}$ then Bob has no incentives to implement the project in case he wins, because $(1 - \frac{v_a - c}{v_a})v_b - c < 0$.[8] This implies that Alice will be willing to make the same offer as without externalities. For Bob, the incentives in the auction change. On one hand, he can bid his reservation value, lose the auction, let Alice implement the project, and obtain a payoff of $e < 0$. On the other hand, he can bid higher than Alice, win the auction, shut down the project, and obtain a payoff of 0. By comparing both scenarios, it is clear that Bob's optimal strategy is to bid anything on the interval $(b_a(v_a), 1]$ and secure for himself a payoff of 0. Seller's revenue becomes $\Pi^{eq} = 0$ in this case.

Fixed-Equity and Fixed-Cash. To conclude the example we will provide a rationale for introducing a fixed-equity and a fixed-cash hybrid as methods of payment. By definition, the revenue collected by both instruments depends on the selection of the fixed component. The challenge for the seller resides in choosing such fixed components when he only knows the distribution of valuations. For instance, if the seller sets a very high fixed equity $\bar{\alpha}$, buyers may lose the incentive to participate in the auction. Likewise, if he sets a very low fixed cash $\bar{b}$, he would not be extracting as much surplus as possible from the winner.

[8] As $(1 - \frac{v_a - c}{v_a})v_a - c = 0$ and $v_b < v_a$.

The following table shows the values of $\bar{\alpha}^*$, Π^{fe}, $\bar{b}^*$, Π^{fc}, Π^{ca} and Π^{eq} for different distributions of types when the cost of implementing the project is $c = 0.1$ and the externality is $e = -0.2$.

Table 2.1: Seller expected revenue under optimal securities: Public Info

			Expected Seller Revenue			
Distribution	$\bar{\alpha}^*$	$\bar{b}^*$	$\Pi^{fe}(\bar{\alpha}^*)$	Π^{ca}	$\Pi^{fc}(\bar{b}^*)$	Π^{eq}
$U[0,1]$	0.51	0.53	0.56	0.44	0.32	0
$B[2,2]$	0.49	0.43	0.57	0.47	0.3	0
$B[2,7]$	0.1	0.16	0.24	0.23	0.09	0
$IB[2,7]$[9]	0.73	0.63	0.90	0.80	0.56	0

By looking at the distributions and the revenues some facts can be highlighted:

- When comparing symmetric distributions $\bar{\alpha}^*$, $\bar{b}^*$, $\Pi^{fe}(\bar{\alpha}^*)$, Π^{ca}, and $\Pi^{fc}(\bar{b}^*)$ are very similar.
- When the relative likelihood of high types to low types increases, $\bar{\alpha}^*$, $\bar{b}^*$, $\Pi^{fe}(\bar{\alpha}^*)$, Π^{ca}, and $\Pi^{fc}(\bar{b}^*)$ increase as well.
- Given $c = 0.1$ and $e = -0.2$ the rank of the instruments with respect to revenue is as follows $\Pi^{fe}(\bar{\alpha}^*) > \Pi^{ca} > \Pi^{fc}(\bar{b}^*) > \Pi^{eq}$.

In the succeeding section we will formally introduce the instruments, characterize the equilibrium bidding strategies, and obtain seller's expected revenue.

[9] The Inverse-Beta distribution is computed from a former Beta distribution. If $f(x)$ represents the PDF of a Beta then the PDF of an Inverse-Beta would be $g(y) = f(-x+1)$. If the former Beta distribution had a right tail then the Inverse-Beta associated to it will have a left tail. When the former Beta is symmetric then the Inverse-Beta is exactly the same.

2.3.1 Fixed-Equity Hybrid

In the fixed-equity hybrid, seller fixes the equity $\bar{\alpha}$ the winner of the auction has to pay over the return of the project. Knowing this information buyers compete in cash for the allocation of the project. Thus, winner's payment to the seller consists of the lowest bid in cash, plus the fixed-equity fraction over projects' return.

Proposition 4 *The dominant-strategy equilibrium of the second price auction under a fixed-equity hybrid is characterized as follows:*

i) If $(1-\bar{\alpha})v_1 - c < 0$*, then* $b_1 = b_2 = 0$*.*

ii) If $(1-\bar{\alpha})v_1 - c > 0$ *and* $(1-\bar{\alpha})v_2 - c < 0$*, then* $b_1 = (1-\bar{\alpha})v_1 - c$ *and* $b_2 = -e$*.*

iii) If $(1-\bar{\alpha})v_2 - c \geq 0$*, then* $b_1 = (1-\bar{\alpha})v_1 - c - e$ *and* $b_2 = (1-\bar{\alpha})v_2 - c - e$*.*

Proof. In case *i)* the project is not profitable to implement for any of the buyers, and thus, their best strategy is to submit a bid of zero. On the other hand, in case *ii)* the project is profitable to implement for buyer 1 but not for buyer 2; hence, the best strategy for buyer 1 is to bid his reservation value, and implement the project if he is allocated. Given buyer 1's strategy, the best response of buyer 2 is to bid his reservation value, which in this case is the negative externality he knows will suffer if buyer 1 wins the auction. Finally, if the project is profitable for both bidders, both will bid their reservation value, which includes the avoidance of the externality. ∎

There are several interesting observations that can be highlighted from proposition 4. First, the likelihood of allocations and payments are not necessarily weakly increasing in buyer's type. For instance, if buyer 2 -the one with the lowest valuation- bids the absolute value of the externality, wins the auction, and pays the reservation value of buyer 1. Moreover, if both buyers find profitable to implement the project, there cannot be an equilibrium in which buyer 2 implements the project, and therefore, his incentives to participate in the auction reside in avoiding the externality if he can win the auction at a price lower than the value of the externality e.

Figure 2.2 shows the bidding strategy of bidder 1 as a function of the valuation of bidder 2, given that $v_1 > \frac{c}{1-\bar{\alpha}}$, and thus when only cases *ii)* and *iii)* are possible.[10]

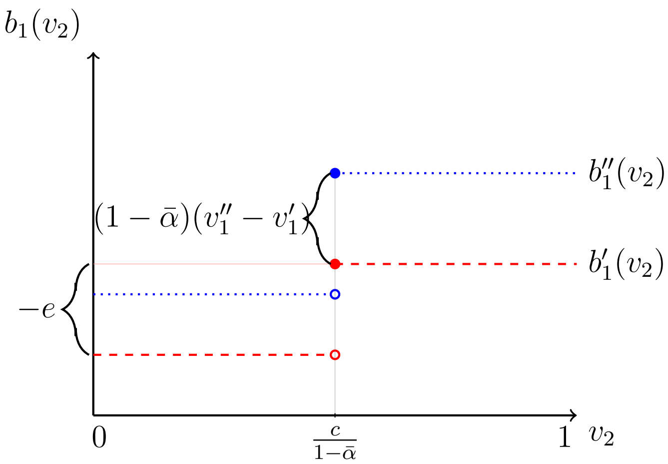


Figure 2.2: Bidding strategies with fixed-equity for buyer 1

It can be observed from figure 2.2 that as soon as the project becomes profitable for buyer 2 (i.e. when $v_2 \geq \frac{c}{1-\alpha}$) buyer 1 increases his bid by $-e$, to reflect the fact that he would suffer the externality in case he loses the auction.

The expected revenue generated by the fixed-equity hybrid under these equilibrium strategies correspond to

[10]If $v_1 < \frac{c}{1-\bar{\alpha}}$ then he will bid $b_1(v_1) = 0$ when $v_2 < \frac{c}{1-\bar{\alpha}}$, otherwise $b_1(v_1) = -e$.

$$
\begin{aligned}
\Pi^{fe}(\bar{\alpha}) &= 2F(\frac{c}{1-\bar{\alpha}})\int_{\frac{c}{1-\bar{\alpha}}}^{\frac{c-e}{1-\bar{\alpha}}}((1-\bar{\alpha})v_1 - c)f(v_1)dv_1 \qquad (2.1)\\
&+ 2F(\frac{c}{1-\bar{\alpha}})\int_{\frac{c-e}{1-\bar{\alpha}}}^{\bar{v}}(\bar{\alpha}v_1 - e)f(v_1)dv_1\\
&+ \int_{\frac{c}{1-\bar{\alpha}}}^{\bar{v}}\int_{\frac{c}{1-\bar{\alpha}}}^{\bar{v}}[(1-\bar{\alpha})\min\{v_1,v_2\} - c - e + \bar{\alpha}\max\{v_1,v_2\}]f(v_1)f(v_2)dv_1dv_2
\end{aligned}
$$

First, notice that if the project is not profitable for any buyer, the auction will generate zero revenue. In the case it is profitable for buyer 1 but not for buyer 2, we need to identify two sub-cases: one when $0 < (1-\bar{\alpha})v_1 - c < -e$, and the other one when $-e < (1-\bar{\alpha})v_1 - c$. In the former, buyer 2 wins the auction but does not implement the project, therefore the seller does not collect revenue from the equity portion of the hybrid, but will get a transfer of $(1-\bar{\alpha})v_1 - c$, which is the lowest bid in cash. This case corresponds to the first term in equation (1). Now, in the other case, buyer 1 will win and implement the project, which means the seller will collect a contingent revenue of $\bar{\alpha}v_1$ plus a transfer in cash of $-e$. This corresponds to the second term. Finally, when both buyers find profitable to implement the project, the seller collects the lowest reservation value in cash, plus the fraction of equity corresponding to the highest type. This is precisely the third term.

2.3.2 Fixed-Cash Hybrid

When the seller uses a fixed-cash hybrid he fixes the amount in cash the winner of the auction has to pay, $\bar{b}$. Knowing this information, bidders compete in equity for the allocation of the project, and it is allocated to the buyer with the highest bid in equity. Therefore, winner's final payment to the seller corresponds to the lowest bid in equity, times the return of the project when it is implemented by him, plus the fixed-amount in cash.

Proposition 5 *The Nash Equilibrium of the second price auction under a fixed-cash hybrid is characterized as follows:*

i) If $v_1 - c - \bar{b} < 0$*; then* $b_1 = b_2 = 0$.

ii,a) If $v_1 - c - \bar{b} > 0$ *and* $-\bar{b} \leq e$*; then* $b_1 = \frac{v_1 - c - \bar{b}}{v_1}$ *and* $b_2 = 0$.

ii,b) If $v_1 - c - \bar{b} > 0$ *and* $-\bar{b} > e$*; then* $b_1 = \frac{v_1 - c - \bar{b}}{v_1}$ *and* $b_2 = (\frac{v_1 - c - \bar{b}}{v_1}, 1]$.

Proof. In the first case the project is not profitable for any of the buyers and then no one will suffer the externality in case the project is allocated to his opponent. Moreover, bidding a positive equity will give the buyers a positive probability of winning the auction, which will force them to pay the amount $\bar{b}$ to the seller. Therefore, the best strategy for both buyers is to stay out of the auction. If the project is profitable for buyer 1 but not for buyer 2, and the fixed amount of cash $\bar{b}$ is higher or equal to the value of avoiding the externality $-e$, buyer 2 prefers to stay out of the auction and suffer the externality. On the other hand, if $-\bar{b} > e$, buyer 2 has an incentive to participate in the auction to bid high enough in order to destroy the incentives of buyer 1 to implement the project in case he wins the auction. In both cases, the best response of buyer 1 is to bid his reservation value, which does not take into account the avoidance of the externality, because he knows buyer 2 never will implement the project if he has the opportunity to do so. Finally if the project is profitable for both, there is no equilibrium in which buyer 2 wins the auction and implements the project. The reason is that as the reservation value of buyer 2 is lower than the one of buyer 1, if buyer 1 is not the winner then there is a profitable deviation in which he offers a slightly higher bid than buyer 2, wins the auction, and avoid the negative externality. Given that situation, the best response for buyer 2 is to bid 0 if $-\bar{b} \leq e$, or otherwise bid high enough to destroy the incentive of buyer 1 to implement the project in case he wins the project. Following the strategy of buyer 2, the best strategy for buyer 1 is to submit his reservation value. ■

Equity represents the particular case in which $\bar{b} = 0$. In this case implementation never takes place and blocking is always the best response of the weak buyer. Notice that this is true for any $e < 0$ and moreover this is one of the possible equilibrium for $e = 0$, being this equilibrium particularly robust.

In figure 2.3 we present the bidding strategy of bidder 1 as a function of the valuation of bidder 2, given that it is profitable for him to implement (i.e. when $v_1 > c + \bar{b}$). In other words, we restrict attention to cases *ii,a)* and *ii,b*.[11]

Figure 2.3 shows that as v_1 increases $b_1(v_1, \cdot)$ increases as well ($v_1'' > v_1'$), which implies that the region of parameters under which buyer 1 just block the allocation decreases.[12]

The revenue generated by these equilibrium strategies corresponds to

$$\Pi^{fc}(\bar{b}) = (1 - F(c + \bar{b})^2)\bar{b} \tag{2.2}$$

[11] If $v_1 < c + \bar{b}$ then he will bid $b_1(v_1) = 0$ when $v_2 < c + \bar{b}$ or $\bar{b} > -e$, otherwise he will bid anything between 1 and the reservation value of buyer 2.

[12] Under v_1' he will block the allocation at the dashed plus dotted region whereas under v_1'' he will block only at the dotted region.

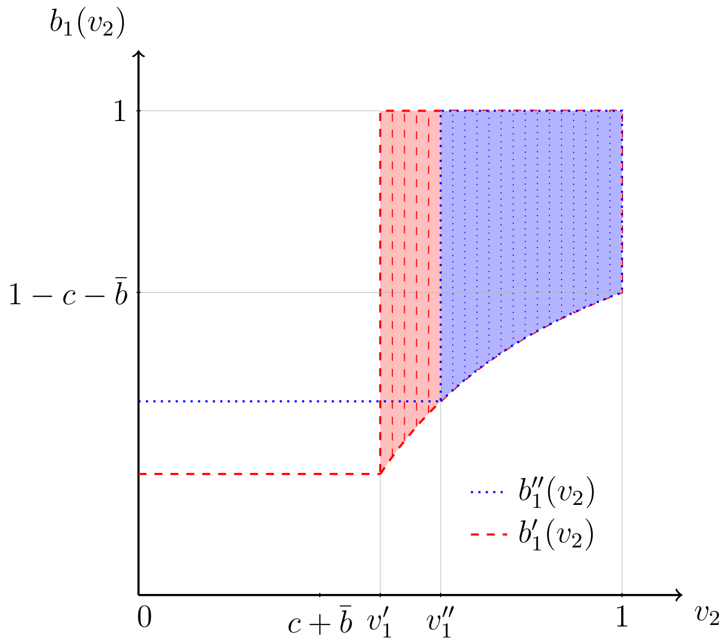


Figure 2.3: Bidding strategies with fixed-cash for buyer 1

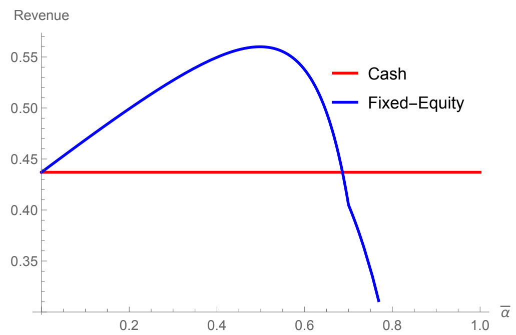
It states that the seller will collect the fixed amount of cash $\bar{b}$ as long as at least one of the buyers find profitable to implement the project. The clear tradeoff for the seller is that increasing $\bar{b}$ diminishes the probability of implementation, but increases the surplus extracted conditional on implementation.

Once we considered the expected revenues of both instruments given by expressions (2.1) and (2.2), the natural following step is to determine how do they rank. This is precisely the matter of the following theorem.

Theorem 6 *For any log-concave density f, there exists a cutoff values $\bar{c}$ and $\underline{e}$, such that if $c \in (0, \bar{c})$ and $e < \underline{e}$ the instruments can be ranked in expected revenue as follows:*

$$\Pi^{fe}(\bar{\alpha}^*) > \Pi^{fe}(0) > \Pi^{fc}(\bar{b}^*) > \Pi^{fc}(0) \tag{2.3}$$

Theorem 6 states that if the cost of the project and the negative externality are sufficiently low, the seller is globally better off using a fixed-equity hybrid. The reason of this result is that, as we discussed before, when payments are made upfront in cash buyers have to face a sunk cost if they want to block the implementation of his opponent. Therefore, the willingness of a “bad type” to pay is bounded above by the absolute value of the negative externality. If the externality is so large that the bad type wins the auction, the seller secures for himself the reservation value of the highest type; otherwise, he receives the fixed equity from the good type, plus the value of the externality in cash. On the other hand, when buyers can bid in equity they can destroy more often the equilibrium in which the project is implemented. The effect is particularly dramatic when the seller uses a pure security, because blocking can be done at no cost. This problem can be mitigated by incorporating a fixed cash component $\bar{b}$; however as the theorem shows, its presence is not sufficient to offset the perverse incentives of the “bad type” buyers. The result holds for any log concave density, which suggests that the interaction between buyers is strong enough to hold under different distributional assumptions.



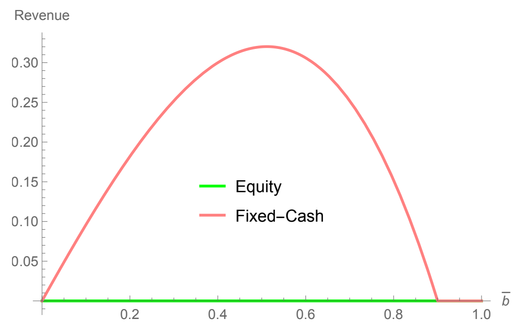


Figure 2.4: Revenue as a function of $\bar{\alpha}$ and $\bar{b}$ for $U[0, 1]$

Figure 2.4 illustrates the result of theorem 6 when valuations are drawn independently and identically from a uniform distribution with support $[0, 1]$, with a negative externality of $e = -0.2$ and a cost of $c = 0.1$. As it can be seen, there is a large range for the parameter $\bar{\alpha}$ such that the fixed-equity hybrid renders a higher revenue than cash, which in turn yields a higher revenue than the fixed-cash hybrid. Noticeably, the revenue obtained by cash is 50% higher than the one yielded by the optimal fixed-cash hybrid.[13]

Monotone Comparative Statics Now we will inspect what happens to the optimal fixed parameters $\bar{b}^*$ and $\bar{\alpha}^*$ when the distribution improves in the sense of the Monotone Likelihood Ratio property. This analysis will provide an insight of how different distributions affect the design of both hybrids

Theorem 7 *Suppose f_1 dominates f_0 in the Monotone Likelihood Ratio (MLR), then $\bar{b}_1^* \geq \bar{b}_0^*$. If additionally, $\frac{F_1(y)}{F_0(y)} > \frac{f_1(y)}{f_0(y)}$ for all $y \in [c, \bar{v}]$, then $\bar{\alpha}_1^* \geq \bar{\alpha}_1^*$*

Proof. See the appendix. ■

Theorem 7 says that for a fixed cost and an externality, if the likelihood of getting higher values improve in the sense of MLR, the optimal fixed cash amount in its respective hybrid cannot decrease. If in addition the ratio of the densities is majorized by the ratio of the distributions for all values greater than the cost, the optimal fixed equity portion in its respective hybrid cannot decrease. It implies that when the seller is using the fixed-equity hybrid he will apply a higher equity portion over a higher expected return of the project. Likewise, when the seller uses a fixed-cash hybrid, it means that now the barrier a bad type has to surpass

[13] Similar figures for different distributions are presented in the appendix.

to enter the auction and block implementation is higher. Naturally, in both cases expected seller's revenue increases.

2.3.3 Other Variations

Deposit Insurance Notice that depending on the security design, the seller can collect a payment from buyers in two stages: after a buyer wins the auction, and after a winner implements the project. As mentioned before, the idea of introducing a payment in cash was a device to screen the low type buyers who otherwise would always have an incentive to enter into the auction to destroy the implementation incentives of the high type buyers. In particular, the fixed-cash hybrid forces the winner to make a payment in cash right after winning the auction. A variant of this instrument, is to introduce a cash deposit (or insurance). This device would work as follows. The seller fixes an amount each buyer has to deposit to participate in the auction. Then, a second price auction in equity is run. The loser gets the deposit back. If the winner implements the project, he has to pay the correspondent equity over project's return but the seller gives back the cash deposit. On the other hand, if the winner does not implement the project the seller retains the cash deposit.[14] Although the cash transfer is determined in a different stage, it can be shown that bidding strategies are the same as in the fixed-cash hybrid, and therefore the revenue for the seller does not change. In other words: If the cash deposit is below $-e$ then the "bad type" will block implementation, otherwise his bid will be zero thus revenue is the same as in the fixed-cash hybrid.

[14] Another way of doing the same is by fixing the size of the deposit the winner should pay upon winning the auction (only the winner pays) and he can claim it back upon implementation.

Unconstrained Bids The two hybrids presented before share the characteristic that the seller determines ex-ante the bid in one of the securities. For instance, in the fixed-equity hybrid, the seller fixes the fraction of equity asked but let buyers to compete in cash. Meanwhile, in the fixed-cash hybrid, the seller fixes the possible bids of cash but let buyers to compete in equity. Alternatively, one can think in a format where the seller decides to run a second price auction but without imposing any restriction on buyers' bids. Thus, each player bid consists on a tuple $b_i = (\alpha_i, \beta_i) \in \mathbb{R} \times [0,1]$, where α_i represents the equity promised on the return of the project and β_i corresponds to an upfront payment in cash. The critical difference of this approach with respect to the former is that now there is no trivial way to rank bids and determine the winner of the auction. Suppose the seller uses an order ψ such that $(\mathbb{R} \times [0,1], \psi)$ constitutes a linearly ordered set.

Proposition 8 *Fix an arbitrary ψ. The dominant strategy equilibrium of the second price auction under unconstrained bids corresponds to: $b_1 = (0, v_1 - c - e)$ if $v_1 - c > 0$ and $b_1 = (1, 0)$ otherwise; $b_2 = (1, 0)$.*

Proof. If the project is not profitable for buyer 2 his dominant strategy is to bid the whole equity and nothing in cash. Following this strategy, he makes sure the project is never implemented at no cost, and so, he never suffers the externality. When the project is profitable for both, bidder 1 offers his reservation value in the cheapest way, which involves only cash, as he is the highest type and any marginal fraction he bids in equity is only valued by the seller with respect to the expected type. Given this strategy, bidder 2 offers the whole equity and no cash, to block the allocation in which buyer 1 wins the auction and implements the project. ■

Notice that this equilibrium is obtained irrespectively of the order ψ the seller uses to rank the bids. The result follows because equity is the instrument that permits to avoid the externality without paying any cost. This is the worst case scenario for the seller, as the revenue under unconstrained bids is $\Pi^{ub} = 0$. The critical assumption is that the auction is a second price, because the buyer is forced to pay each component of his opponent bid.

2.4 Robustness: Private Buyer's Valuations

In this section we will analyze the set of securities under the assumption that buyers do not longer know the valuation of his opponent, which turns our model into a standard private values auction model. We will analyze how the information structure affects our main result.

2.4.1 Fixed-Equity Hybrid

Analogously to section 2.3.1 we characterize the equilibrium under private information. We use the Bayes-Nash equilibrium as the solution concept.

Proposition 9 *Bayes-Nash equilibrium bidding strategies of the second price auction when the seller uses fixed-equity, $\bar{\alpha}$, are characterized by*

i) $b_i(v_i) = 0$ *if* $(1-\bar{\alpha})v_i - c < 0$.

ii) $b_i(v_i) = (1-\bar{\alpha})v_i - c + (1 - F(\frac{c}{1-\bar{\alpha}}))(-e)$ *if* $(1-\bar{\alpha})v_i - c > 0$.

Proof. As the seller utilizes a second price auction, and buyers bid in cash, the best strategy for a buyer who finds profitable to implement the project is to bid their reservation value. Now, buyer i's reservation value depends on the implementation decision of his opponent. Thus, with probability $F(\frac{c}{1-\bar{\alpha}})$ it is not profitable for the other buyer to implement the project, and so buyer i's reservation value is equal to the net payoff of implementing the project: $(1-\bar{\alpha})v_i - c$. On the other hand, with probability $1-F(\frac{c}{1-\bar{\alpha}})$ it is profitable for the other buyer to implement the project, which implies that in case buyer i loses the auction he will suffer the externality e, and thus, such expected loss has to be added to his bid. For the buyer who does not want to implement the project his reservation value is given by $1 - F(\frac{c}{1-\bar{\alpha}})(-e)$. If he bids his reservation value, he will lose with probability one if he faces an opponent who wants to implement the project. In such case his payment will be e. On the other hand, when he faces an opponent who does not want to implement the project neither, his expected payoff will be $\frac{1}{2}(1 - F(\frac{c}{1-\bar{\alpha}}))(e)$. Hence, there is clearly a profitable deviation to zero. By doing this the buyer will continue losing the auction when facing an opponent who wants to implement the project, and then will obtain the same payoff, but now will obtain a zero payoff if he faces an opponent who does not want to implement. $\blacksquare$

Following the same reasoning as with the public case, seller's ex-ante revenue is given by

$$
\begin{aligned}
\Pi^{fe}(\bar{\alpha}) &= 2F(\frac{c}{1-\bar{\alpha}})\int_{\frac{c}{1-\bar{\alpha}}}^{\bar{v}} (\bar{\alpha}v_1)f(v_1)dv_1 \\
&+ \int_{\frac{c}{1-\bar{\alpha}}}^{\bar{v}}\int_{\frac{c}{1-\bar{\alpha}}}^{\bar{v}} [(1-\bar{\alpha})\min\{v_1,v_2\} - c + (1-F(\frac{c}{1-\bar{\alpha}}))(-e) + \bar{\alpha}\max\{v_1,v_2\}]f(v_1)f(v_2)dv_1dv_2
\end{aligned}
$$

The first term in the integral corresponds to the case when one buyer finds profitable to implement the project and the other does not. In this scenario, the seller collects the fixed equity portion from the highest type and receives zero in cash. Meanwhile, the second term represents the expected revenue when both buyers want to implement the project. Here the seller receives the equity portion from the highest type, plus the cash embedded in the lowest bid.

2.4.2 Fixed-Cash Hybrid

In a similar fashion to section 2.3.2 we characterize the equilibrium under private information. We show that the existence of a Bayes-Nash equilibrium in pure strategies depends on the relationship between the fixed amount of cash $\bar{b}$ and the value of the externality e.

Proposition 10 *There are no equilibria in pure strategies in the fixed-cash hybrid if:*

i) $-\bar{b} > e$ *and* $(1 - F(c + \bar{b}))e < -\bar{b}$.

ii) $-\bar{b} > e$ *and* $(1 - F(c + \bar{b}))e > -\bar{b}$.

Proof. The problem to reach an equilibrium on case (i) resides in the optimal strategy of the buyer's type who does not want to implement the project: the "bad type." If both buyers of such type bid zero, any of them would find profitable to deviate and bid the smallest amount that guarantees him to be the winner of the auction. In such case the deviant buyer would get a payoff of $-\bar{b}$ which is greater than $(1 - F(c + \bar{b}))e$. For the same reason, the other buyer also deviates to the same bid, which yields a payoff of $F(c+\bar{b})(-\frac{\bar{b}}{2})(1 - F(c+\bar{b}))(-\bar{b})$ to both buyers. Notice that in such situation both buyers block the implementation with certainty and share the cost. Nonetheless, as soon as both buyers bid the same amount, any of them -say buyer i- has an incentive to bid an arbitrarily lower amount. It guarantees to suffer the externality with a very low probability in case he faces the good type of buyer $-i$, but saves his portion of the fixed amount of cash when faces his fellow bad type. The moment buyer i deviates, buyer $-i$ has two possible deviations, either to bid lower than buyer i, or returning to the initial bid. The former deviation is more profitable. Continuing with this analysis, some buyer will reach a level at which there is no downward deviation for his rival. That is, a point where if his opponent submits a lower bid, he will suffer a payoff lower than $-\bar{b}$. Or in other words, a bid $k \in (0, 1)$ such that $F(b^{-1}(k))(e) = \bar{b}$. Under this scenario, if buyer i bids k, the best deviation for buyer $-i$ is to return to the initial bid, which will start again the cycle of deviations. In order to prove case (ii) it is worth noting that "bad buyers" will make a bid of zero. Now the problem resides on the "good buyers". Consider the type $v_i = c + \bar{b}$. If he bids $b_i(v_i) = \frac{v_i - c - \bar{b}}{v_i}$ he wins against all the types that do now want to implement the project getting a payoff of zero but loses against all the other types that want to implement the project (it is clear that no bidder who wants to implement the project has incentives to bid below his reservation value without considering the externality). Whenever he loses, he gets e for sure (he only loses against types that are willing to implement at his reservation value) which is worse than paying $\bar{b}$ and not implementing. Hence, he is better off blocking every possible implementation: bidding the smallest amount that guarantees him to be the winner of the auction. Sufficiently many types will deviate to this bid as long as $-\bar{b} > e$, because they can block potential implementation. At this point the cyclical logic of case (i) comes into place, not for the "bad types" now but for the "good types", and no equilibrium is reached in pure strategies. ■

Proposition 11 *Bayes-Nash equilibrium bidding strategies of the second price auction under the fixed-cash hybrid, when* $-\bar{b} \leq e$, *are characterized by*

i) $b_i(v_i) = \frac{v_i - c - \bar{b}}{v_i}$, *if* $v_i - c - \bar{b} > 0$.

ii) $b_i(v_i) = 0$ *if* $v_i - c - \bar{b} < 0$.

Proof. In case (ii) the project is not profitable for the buyer, and moreover, the negative externality e is lower than the loss he would get by winning the auction and not implementing the project, $-\bar{b}$. As there is no way to prevent the implementation of the project by his competitor without winning the auction, the best strategy of the "bad type" is to bid zero in equity. In case (i), the buyer finds profitable to implement the project, and his best strategy is to bid his reservation value -which does not depends on the implementation decision of his opponent. If $b_i(v_i) > \frac{v_i - c - \bar{b}}{v_i}$ he will win whenever $b_i(v_i) > b_{-i}(v_{-i})$ but there are two different situations. When $b_{-i}(v_{-i}) < \frac{v_i - c - \bar{b}}{v_i}$ buyer i will win the auction and implement the project, guaranteeing for himself a payoff of at least zero. When $b_i(v_i) > b_{-i}(v_{-i}) > \frac{v_i - c - \bar{b}}{v_i}$ buyer i will win the auction but cannot implement the project, thus his payoff is $-\bar{b}$. By deviating to $b_i(v_i) = \frac{v_i - c - \bar{b}}{v_i}$ he keeps the positive payoffs (wins and implements in all the cases he wants to do so) and at most suffers a payoff of e upon losing which is better than $-\bar{b}$. ■

Once we have derived the equilibrium strategies we can state the expression for seller's expected revenue. Given we have equilibrium whenever $-\bar{b} < e$ we can state the revenue just for this particular case.

$$
\begin{aligned}
\Pi^{fc}(\bar{b}) &= 2(1-F(c+\bar{b}))F(c+\bar{b})\bar{b} \\
&+ \int_{c+\bar{b}}^{\bar{v}}\int_{c+\bar{b}}^{\bar{b}}(\min\{\frac{v_1-c-\bar{b}}{v_1},\frac{v_2-c-\bar{b}}{v_2}\}\max\{v_1,v_2\}+\bar{b})f(v_1)f(v_2)dv_1dv_2
\end{aligned}
$$

2.4.3 Equity

To analyze equity, we cannot simply take bidding strategies as particular cases of the fixed-cash hybrid, because now even for very low valuations the buyer can bid sufficiently high, and still avoid a positive payment to the seller.

Proposition 12 *Equilibrium bidding strategy when the seller uses equity is uniquely characterized by* $b_i(v_i) = 1$*.*

Proof. Clearly, if $v_i - c < 0$ buyer i will not implement the project if he wins, so winning the project only has value as long as it prevents the other agent to win and implement the project, because in this case buyer i avoids the negative externality it would entail. Now if $v_i - c > 0$, in principle buyer i optimal strategy is to bid his reservation value, as now he has the normal trade-off any buyer faces in an auction: increasing the bid increases the probability of winning but decreases the surplus. However, the presence of the externality biases buyer's incentives towards winning the auction. In concrete, if $v_i - c > 0$ but small, the buyer might be better off by bidding one in equity and avoiding the externality with certainty, than gambling on winning the auction and suffering the externality with positive probability.

This behavior may give room for the possibility of having a cut-off strategy. If this were the case, there would exist a value $\tilde{v}$ such that if $v_i < \tilde{v}$ then $b_i(v_i) = 1$ and

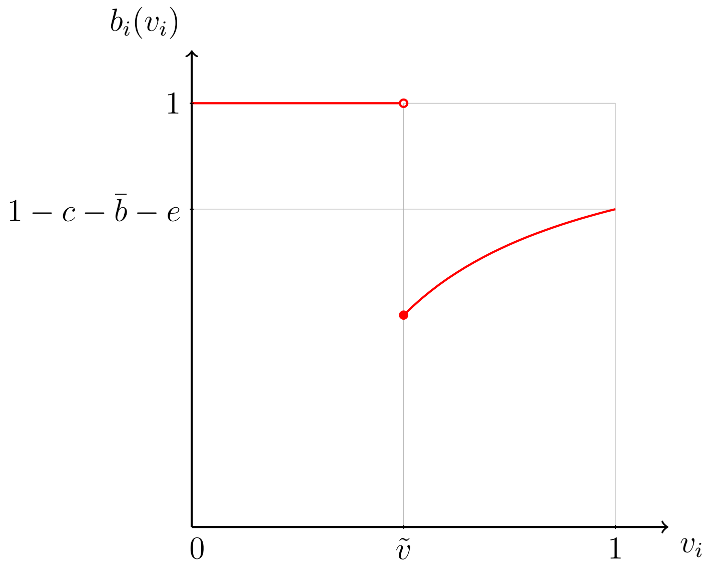


Figure 2.5: Threshold equilibrium under Equity

if $v_i \geq \tilde{v}$ then buyers bid their reservation value -which includes the externality he would suffer in case of his opponent implements the project. In such case the strategy of buyer i would have a discontinuity at $\tilde{v}$, as shown in figure 2.5. However, if it were the case, at $\tilde{v}$ the bid of the agent will be the lowest possible, which implies he loses the auction for sure and will suffer the externality with positive probability. Thus, bidding one is a profitable deviation. This observation holds for any value $\tilde{v} < 1$. Therefore, both agents will bid one in equilibrium and the project is never implemented. ■

2.4.4 Example Revisited

Following the example presented in section 2.3 we show the values of $\bar{\alpha}^*$, Π^{fe}, $\bar{b}^*$, Π^{fc}, Π^{ca} and Π^{eq} for different distributions of types when the cost of implementing the project is $c = 0.1$ and the externality is $e = -0.2$.

Table 2.2: Seller expected revenue under optimal securities: Private Info

			Expected Seller Revenue			
Distribution	$\bar{\alpha}^*$	$\bar{b}^*$	$\Pi^{fe}(\bar{\alpha}^*)$	Π^{ca}	$\Pi^{fc}(\bar{b}^*)$	Π^{eq}
$U[0,1]$	0.67	0.42	0.54	0.39	0.35	0
$B[2,2]$	0.58	0.32	0.55	0.45	0.34	0
$B[2,7]$	0.23	$**$	0.18	0.17	$**$	0
$IB[2,7]$	0.75	0.2	0.91	0.80	0.68	0

**: $\bar{b}$ is in the no equilibrium range

Even though the table computes Π^{fc} only for the case where we have an equilibrium in pure strategies, our results are robust: Most of the entries on table 2.2 are similar to the ones presented on table 2.1. The only difference is $\bar{b}$ for the $IB[2,7]$ because now the seller can force bidders to bid in equity without considering the externality and he seems willing to do so.[15] However the payoff he obtains is similar to the case of public information.

2.5 Concluding Remarks

We analyzed a simple two-buyer second price auction, where the seller can use two different hybrids and the buyers suffer negative externalities upon the implementation of the project by their opponent. In particular, we consider a fixed-equity hybrid, where the seller fixes a portion of equity over project's return and buyers compete in cash; and a fixed-cash hybrid, where the buyers compete in equity and the winner has to pay an amount in cash predetermined by the seller.

Our main observation lies in the fact that pure securities equip low-valuation buyers (those who do not want to implement the project, or bad types) with a

[15] 0.2 is the $\bar{\alpha}^*$ that maximizes revenue on $\bar{\alpha}^* \in [0.2, 1]$ but it is still possible (although unlikely) that $\bar{\alpha}^* < 0.2$.

powerful tool to block the implementation from the good types, which impacts revenue negatively. Then, we find that in order to circumvent this problem the seller has to incorporate a fixed payment in the instruments to be used as a device to prevent "bad types" from blocking. However, mitigating this adverse selection problem poses a tradeoff on the seller: by increasing the fixed portion of the hybrid utilized, the project becomes less profitable for buyers, and thus, induces less participation.

The fixed-equity hybrid conducts the screening in cash, whereas the fixed-cash hybrid conducts the screening in equity. If the seller decides to use the latter, buyers retain the power of blocking the implementation, conditional on the fact that they decide to participate in the auction -which now depends on the fixed-amount of cash requested by seller to the winners. On the other hand, when the seller uses the former, the screening is realized in cash, which is the cheapest way "good types" can use to distinguish themselves. Therefore, the screening realized is more effective, and the seller ends up trading with the good types more often. This is the intuition that justifies the preeminence of the fixed-equity hybrid as the best instrument in the menu. At the same time, that is the reason why equity is the worst. More surprisingly is the result that the optimal fixed-cash hybrid performs worse than cash, if the value of the externality is sufficiently high (in absolute value). However, it reflects the fact that when buyers want to avoid a sufficiently high negative externality, their willingness to pay upfront more than offsets the potential extraction through equity.

An interesting feature of our result is that it seems to be robust to the structure of the information. That is, even when buyer's valuations are private information, the fixed-equity hybrid continues to be the best, and equity continues to be the worst. However, the fixed-equity instrument now does not always have an equilibrium in pure strategies, which increases the uncertainty over seller's revenue in the more general case.

Finally, we analyze what would happen to the optimal fixed-payment portion in both hybrids when the distribution improves in the Monotone Likelihood Ratio property. Intuitively, we obtain that the amount of cash in the fixed-cash hybrid is non-decreasing, and that under some condition of the distributions, the equity portion in the fixed-equity hybrid is also non-decreasing. These results state that when buyers draw better valuations, the seller is less concerned about inducing participation, and can extract a higher surplus from the winner of the auction.

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Appendix

A. Omitted Proofs

Proof of Theorem 6.

We will prove the theorem following three steps. First we will prove that the optimal fixed-equity hybrid involves a portion of equity $\bar{\alpha}^* \in (0,1)$, which immediately implies that the hybrid dominates pure cash in revenue. Analogously, in the second step we will show that the optimal fixed-cash hybrid involves a positive amount of cash (i.e. $\bar{b}^* > 0$), which in turn implies that it dominates pure equity in revenue. Finally, we will prove that the revenue under cash is higher than the revenue under the optimal fixed-cash hybrid.

Step 1 We take first order conditions by applying Leibniz' rule to the three different terms in (2.1). First derivative $D_1(\bar{\alpha}, c, e)$ corresponds to:

$$\begin{aligned} & 2F(\frac{c}{1-\bar{\alpha}})[\frac{(c-e)e}{(1-\bar{\alpha})^2} f(\frac{c-e}{(1-\bar{\alpha})}) - \int_{\frac{c}{1-\bar{\alpha}}}^{\frac{c-e}{1-\bar{\alpha}}} v_1 f(v_1) dv_1] \\ - \quad & 2f(\frac{c}{1-\bar{\alpha}}) \frac{c}{(1-\bar{\alpha})^2} \int_{\frac{c}{1-\bar{\alpha}}}^{\frac{c-e}{1-\bar{\alpha}}} ((1-\bar{\alpha})v_1 - c) f(v_1) dv_1 \end{aligned}$$

Likewise, second derivative $D_2(\bar{\alpha}, c, e)$ is given by

$$
\begin{aligned}
&2F(\frac{c}{1-\bar{\alpha}})[\frac{(\bar{\alpha}c-e)}{1-\bar{\alpha}}\frac{(c-e)}{(1-\bar{\alpha})^2}f(\frac{c-e}{(1-\bar{\alpha})})\\
+ \quad &\int_{\frac{c-e}{1-\bar{\alpha}}}^{\bar{v}} v_1 f(v_1)dv_1] - 2f(\frac{c}{1-\bar{\alpha}})\frac{c}{(1-\bar{\alpha})^2}\int_{\frac{c}{1-\bar{\alpha}}}^{\frac{c-e}{1-\bar{\alpha}}}(\bar{\alpha}v_1 - e)f(v_1)dv_1
\end{aligned}
$$

Finally, applying Leibniz rule twice in the third term and using the fact that valuations are independently and identically distributed, the third derivative $D_3(\bar{\alpha}, c, e)$ becomes:

$$
\begin{aligned}
&2[\int_{\frac{c}{1-\bar{\alpha}}}^{\bar{v}}(\bar{\alpha}v_1 - e)f(v_1)dv_1]\frac{c}{(1-\bar{\alpha})^2}f(\frac{c}{1-\bar{\alpha}})\\
+ \quad &\int_{\frac{c}{1-\bar{\alpha}}}^{\bar{v}}\int_{\frac{c}{1-\bar{\alpha}}}^{\bar{v}}(\max\{v_1, v_2\} - \min\{v_1, v_2\})f(v_1)f(v_2)dv_1dv_2
\end{aligned}
$$

Letting $\tilde{D}(\alpha, c, e) = D_1(\alpha, c, e) + D_2(\alpha, c, e) + D_3(\alpha, c, e)$ we have that

$$
\begin{aligned}
\tilde{D}(0, c, e) &= 2F(c)[\int_{c-e}^{\bar{v}} v_1 f(v_1)dv_1 - \int_{c}^{c-e} v_1 f(v_1)dv_1]\\
&- 2f(c)c[\int_{c}^{c-e}(v_1 - c)f(v_1)dv_1] - e(1 - 2F(c) + F(c-e))]\\
&+ \int_{c}^{\bar{v}}\int_{c}^{\bar{v}}(\max\{v_1, v_2\} - \min\{v_1, v_2\})f(v_1)f(v_2)dv_1dv_2
\end{aligned}
$$

Now, we will explore the behavior of the first order condition when the cost tends to zero.

$$\lim_{c \downarrow 0} \tilde{D}(0,c,e) = \int_0^{\bar{v}} \int_0^{\bar{v}} (\max\{v_1, v_2\} - \min\{v_1, v_2\}) f(v_1) f(v_2) dv_1 dv_2 > 0 \;\; \forall e < 0$$

Notice that for a given e, as $\tilde{D}(0,c,e))$ is continuous, there exists a cut-off in the cost

$$\bar{c}_1 := \sup\{\tilde{c} > 0 : \tilde{D}(0,c,e) > 0 \;\text{ for all }\; c \in (0,\tilde{c})\}$$

Moreover,

$$\lim_{\bar{\alpha} \to 1} \Pi^{fe}(\bar{\alpha}) = 0$$

and,

$$\begin{aligned} \Pi^{fe}(0) &= 2F(c)[\int_c^{c-e} (v-c) f(v) dv + \int_{c-e}^{\bar{v}} (-e) f(v) dv] \\ &+ \int_c^{\bar{v}} \int_c^{\bar{v}} (\min\{v_1, v_2\} - c - e) f(v_1) f(v_2) dv_1 dv_2 > 0 \end{aligned}$$

Therefore, because revenue is strictly increasing at $\bar{\alpha} = 0$ and $\Pi^{fe}(0) > \Pi^{fe}(1)$ for all e, the optimal fraction of equity $\bar{\alpha}^* \in (0,1)$.

Step 2 Now we will prove that the optimal portion of cash in the fixed-cash hybrid is positive. That is, $\bar{b}^* > 0$.

Taking first order conditions of (2.2) with respect to $\bar{b}$ we have that

$$\bar{b}^* = \frac{1 - F(c + \bar{b}^*)}{f(c + \bar{b}^*)} \frac{1 + F(c + \bar{b}^*)}{2F(c + \bar{b}^*)} = \frac{1}{\lambda(c + \bar{b}^*)} \frac{1 + F(c + \bar{b}^*)}{2F(c + \bar{b}^*)} \tag{2.4}$$

where $\lambda(\cdot)$ is the hazard ratio associated with f.

Now, as the density f is log-concave, by theorem 3 in Bagnoli and Bergstrom (2005) the hazard rate λ of F is an increasing function. Therefore, the second derivative of (2.2) is negative for all $\bar{b}$, and the expression in (2.4) corresponds to its unique global solution. Intuitively, if the seller raises marginally the fixed amount $\bar{b}$, his revenue increases by this amount only with probability $1 - F(c + \bar{b})$, which is the likelihood that the project is profitable for a particular buyer. On the other hand, $f(\bar{b})$, measures the loss in implementation the seller will cause by rising the fixed amount of cash requested. That is, the seller will gain the marginal amount in the cash requested except in those cases where the winner was already indifferent between implementing or not the project. In those cases, if the seller raises $\bar{b}$ now the project is not profitable for the winner, and the seller will reduce participation. This expression is scaled by the factor at the right.

Step 3 In the last step we will show that the revenue under cash is higher than the revenue under the optimal fixed-cash hybrid.

Let $\bar{b}^*$ be the optimal fixed-cash amount when the cost is zero, and thus $\bar{b}^* =$

$\frac{1-F(\bar{b}^*)}{f(\bar{b}^*)}\frac{1+F(\bar{b}^*)}{2F(\bar{b}^*)}$. Hence,

$$\begin{aligned}\lim_{c\downarrow 0}\Pi^{fe}(0,c,e) &= -e+\int_0^{\bar{v}}\int_0^{\bar{v}}\min\{v_1,v_2\}f(v_1)f(v_2)dv_1dv_2\\ &> -e+\int_{\bar{b}^*}^{\bar{v}}\int_{\bar{b}^*}^{\bar{v}}\min\{v_1,v_2\}f(v_1)f(v_2)dv_1dv_2\end{aligned}$$

That is, when the cost approaches to zero from above, the expected revenue when the seller uses cash is higher than the expected revenue under the best fixed-cash hybrid.

Now, fix $c \in (c, \bar{c})$. Using the expressions of revenue for fixed-equity (2.1) and fixed-cash (2.2) hybrids, we need to show that

$$\begin{aligned}\Pi^{fe}(0,c,e) &= 2F(c)[\int_c^{c-e}(v_1-c)f(v_1)dv_1+\int_{c-e}^{\bar{v}}(-e)f(v_1)dv_1]\\ &+ \int_c^{\bar{v}}\int_c^{\bar{v}}[\min\{v_1,v_2\}-c-e]f(v_1)f(v_2)dv_1dv_2\end{aligned}$$

is greater than

$$\begin{aligned}\Pi^{fc}(\bar{b}^*,c,e) &= 2\int_0^{c+\bar{b}^*(c)}\int_{c+\bar{b}^*(c)}^{\bar{v}}\bar{b}^*f(v_1)f(v_2)dv_1dv_2\\ &+ \int_{c+\bar{b}^*(c)}^{\bar{v}}\int_{c+\bar{b}^*(c)}^{\bar{v}}\bar{b}^*f(v_1)f(v_2)dv_1dv_2\end{aligned}$$

Or rearranging terms, we need that

$$2F(c)[\int_{c}^{c-e}(v_1-c)f(v_1)dv_1+\int_{c-e}^{\bar{v}}(-e)f(v_1)dv_1]+(1-F(c))^2(-c-e)$$
$$+\int_{c+\bar{b}^*(c)}^{\bar{v}}\int_{c+\bar{b}^*(c)}^{\bar{v}}[\min\{v_1,v_2\}-\bar{b}^*(c)]f(v_1)f(v_2)dv_1dv_2$$
$$\int_{0}^{c+\bar{b}^*(c)}\int_{0}^{c+\bar{b}^*(c)}\min\{v_1,v_2\}f(v_1)f(v_2)dv_1dv_2$$

be greater than

$$\frac{(1-F(c+\bar{b}^*(c))^2)}{f(c+\bar{b}^*(c))}(1+F(c+\bar{b}^*(c)))$$

where the last expression is obtained by replacing the functional form of $\bar{b}^*(c)$. Hence, to show that $\Pi^{fe}(0)>\Pi^{fc}(\bar{b}^*(c))$ is sufficient that

$$-e>\frac{1+F(c+\bar{b}^*(c))}{f(c+\bar{b}^*(c))}+c-\frac{2}{(1-F(c))^2}\int_{0}^{c+\bar{b}^*(c)}(1-F(v_1))f(v_1)v_1dv_1$$

Therefore we can define

$$-\underline{e}=\arg\max_{c\in(0,\bar{c})}\{\frac{1+F(c+\bar{b}^*(c))}{f(c+\bar{b}^*(c))}+c-\frac{2}{(1-F(c))^2}\int_{0}^{c+\bar{b}^*(c)}(1-F(v_1))f(v_1)v_1dv_1\}$$

Figure 2.6 shows the behavior of $\bar{c}$ as a function of $|e|$. If $c<\bar{c}$ then theorem 6 holds thus $\Pi^{fc}(0)>\Pi^{fc}(\bar{b}^*)$, otherwise the reverse is true.

On table 2.3 we explore theorem 6 by showing the value of $\underline{e}$ for different distributions. Alongside, we present the values of $\Pi^{fe}(0)$ and $\Pi^{fc}(\bar{b}^*)$ for different values of e, to confirm why the bound is needed although it is rather low.

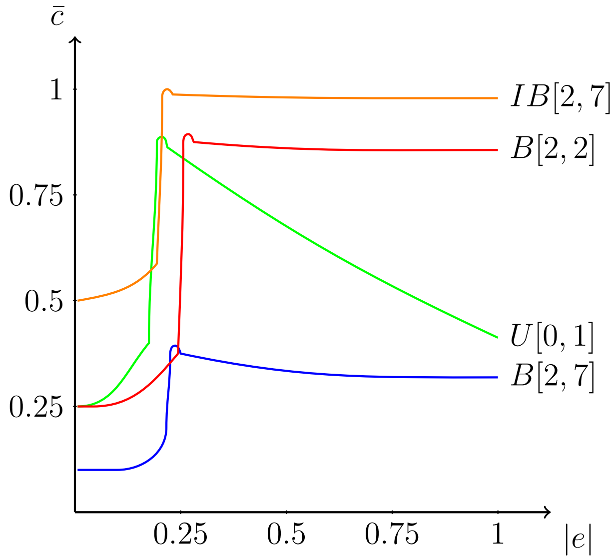

Figure 2.6: Upper bound of the cost for different distributions

Table 2.3: Revenue in Cash and Fixed-Cash as a function of e

		Expected Seller Revenue			
Distribution	e	$\Pi^{fe}(0)$	$\Pi^{fc}(\bar{b}^*)$	$\bar{b}^*$	$\underline{e}$
	-0.001	0.334333			
$U[0,1]$	-0.01	0.343333	0.3849	0.57735	-0.0512
	-0.1	0.433333			
	-0.001	0.372429			
$B[2,2]$	-0.01	0.381429	0.375	0.5	-0.0036
	-0.1	0.471429			
	-0.001	0.15002			
$B[2,7]$	-0.01	0.15902	0.152539	0.222329	-0.0035
	-0.1	0.24902			
	-0.001	0.705575			
$IB[2,7]$	-0.01	0.714575	0.656547	0.718398	$\nexists$
	-0.1	0.804575			

■

Proof of Theorem 7. We will prove the result using techniques of monotone comparative statics on lattice programming, for which we need to introduce some the definitions and results of this theory.

Definition 13 (Milgrom and Shannon (1994)) *Let X and T be non-empty subsets of $\mathbb{R}$ and let $g : X \times T \to \mathbb{R}$. We say g satisfies the* **strict single crossing property** *(SSCP) in (x, t) if for every x'', x' in X and t'', t' in T, with $x'' > x'$ and $t'' > t'$*

$$g(x'', t') \geq g(x', t') \text{ implies } g(x'', t'') > g(x', t'') \tag{2.5}$$

and we write $g(\cdot, t'') \succeq_{SSCP} g(\cdot, t')$.

Definition 14 (Quah and Strulovici (2009)) *Let X and T be non-empty subsets of $\mathbb{R}$, and let $\{g(\cdot, t)\}_{t \in T}$ be a family of real valued functions defined on X, we say that $g(\cdot, t')$ is interval order dominated by $g(\cdot, t'')$ -with the notation $g(\cdot, t'') \succeq_{IDO} g(\cdot, t'')$- if equation (2.5) holds for all $x' < x''$ whenever $g(x, t') < g(x'', t'')$ for all $x \in [x', x'']$.*

Proposition 15 (Quah and Strulovici (2009)) *Let X and T be respectively an interval and a non-empty subsets of $\mathbb{R}$, and suppose that $\{g(x, \cdot)\}_{t \in T}$ is a family of real valued functions, which are also absolutely continuous in intervals of X; and that there is a positive an increasing function $h : X \to \mathbb{R}$ such that $g'(x, t'') > h(x)g'(x, t')$ a.e. Then, $g(\cdot, t'') \succeq_{IDO} g(\cdot, t')$*

Theorem 16 (Quah and Strulovici (2009)) *Let X and T be non-empty subsets of $\mathbb{R}$ and let $g(\cdot, t''), g(\cdot, t')$ be two real valued functions defined on X, with*

$t'', t' \in T$ such that $t'' > t'$. If $g(\cdot, t'') \succeq_{IDO} g(\cdot, t')$ then

$$argmax_{x \in J} g(\cdot, t'') > argmax_{x \in J} g(\cdot, t') \quad \textit{for any interval } J \textit{ of } X. \tag{2.6}$$

Furthermore, if (2.6) is satisfied then $g(\cdot, t'') \succeq_{IDO} g(\cdot, t')$

Suppose f_1 dominates f_0 in the monotone likelihood ratio (MLR) and rewrite (**??**) as

$$\begin{aligned} \Pi^{fe}(\bar{\alpha}, t) &= 2F_t(\frac{c}{1-\bar{\alpha}}) \int_{\frac{c}{1-\bar{\alpha}}}^{\frac{c-e}{1-\bar{\alpha}}} ((1-\bar{\alpha})v_1 - c) f_t(v_1) dv_1 \\ &+ 2F_t(\frac{c}{1-\bar{\alpha}}) \int_{\frac{c-e}{1-\bar{\alpha}}}^{\bar{v}} (\bar{\alpha} v_1 - e) f_t(v_1) dv_1 \\ &+ \int_{\frac{c}{1-\bar{\alpha}}}^{\bar{v}} \int_{\frac{c}{1-\bar{\alpha}}}^{\bar{v}} [(1-\bar{\alpha}) \min\{v_1, v_2\} - c - e + \bar{\alpha} \max\{v_1, v_2\}] f_t(v_1) f_t(v_2) dv_1 dv_2 \end{aligned} \tag{2.7}$$

with $t \in \{0, 1\}$. It is sufficient to show that there exists a positive and increasing function $h(\alpha)$ such that $\Pi^{fe}_{\alpha}(\alpha, 1) > h(\alpha)\Pi^{fe}_{\alpha}(\alpha, 0)$ to show that $\alpha^*_1 \geq \alpha^*_0$, in virtue of proposition 15 and theorem 16.

Define $h(\alpha, c) = \frac{f_1(\frac{c-e}{1-\bar{\alpha}})}{f_0(\frac{c}{1-\bar{\alpha}})}$ and $g(\alpha, c) = \frac{F_1(\frac{c}{1-\bar{\alpha}})}{F_0(\frac{c}{1-\bar{\alpha}})}$. Notice that $h(\alpha, c)$ is increasing in α for all c, and hence, if we show that $\Pi^{fe}_{\alpha}(\alpha, 1) - h(\alpha, c)\Pi^{fe}_{\alpha}(\alpha, 0) > 0$ we can conclude that $\Pi^{fe}(\alpha, 1) \succeq_{IDO} \Pi^{fe}(\alpha, 0)$ In order to show that, we can proceed separately as we did with the derivative the proof of theorem 6.

Thus, for the first term we have

$$\frac{2(c-e)e}{(1-\bar{\alpha})^2} f(\frac{c}{(1-\bar{\alpha})}) \left[g(\alpha,c) - h(\alpha,c)\right] - 2\int_{\frac{c}{1-\bar{\alpha}}}^{\frac{c-e}{1-\bar{\alpha}}} v_1[g(\alpha,c)\frac{f_1(v_1)}{f_0(v_1)} - h(\alpha,c)]dv_1 \tag{2.8}$$

Likewise, the second term corresponds to

$$\frac{2(\bar{\alpha}c-e)}{1-\bar{\alpha}} \frac{c-e}{(1-\bar{\alpha})^2}[g(\alpha,c) - h(\alpha,c)] + 2\int_{\frac{c}{1-\bar{\alpha}}}^{\bar{v}} v_1[g(\alpha,c)\frac{f_1(v_1)}{f_0(v_1)} - h(\alpha,c)]dv_1 \tag{2.9}$$

The third term is equal to

$$\frac{2c}{(1-\bar{\alpha})^2}\int_{\frac{c}{1-\bar{\alpha}}}^{\bar{v}} (\bar{\alpha}v_1 - e)[\frac{f_1(v)}{f_0(v_1)} - h(\alpha,c)]dv_1 \tag{2.10}$$
$$+\int_{\frac{c}{1-\bar{\alpha}}}^{\bar{v}}\int_{\frac{c}{1-\bar{\alpha}}}^{\bar{v}} (\max\{v_1,v_2\} - \min\{v_1,v_2\})[\frac{f_1(v_1)f_1(v_2)}{f_0(v_1)f_0(v_2)} - h(\alpha,c)]dv_1dv_2$$

Grouping the first terms in (2.8) and (2.9), respectively, we get

$$\frac{2\bar{\alpha}(c-e)^2}{(1-\bar{\alpha})^3}[g(\alpha,c) - h(\alpha,c)] \tag{2.11}$$

Likewise, adding the second terms in (2.8) and (2.9) we obtain

$$2\int_{\frac{c-e}{1-\bar{\alpha}}}^{\bar{v}} v_1[g(\alpha,c)\frac{f_1(v_1)}{f_0(v_1)} - h(\alpha,c)]dv_1 \tag{2.12}$$

Terms (2.10)-(2.12) imply the result because we assume that $g(\alpha, c) > h(\alpha, c)$, f_0 is dominated in MLR by f_1, and the inferior limit of all the integrals involved is greater than or equal to $\frac{c}{1-\bar{\alpha}}$.

Applying the same argument, we can see that $\bar{b}_1^* > \bar{b}_0^*$ if and only if

$$-2\bar{b} + \frac{1}{\lambda_0(c+\bar{b})}\frac{1+F_0(c+\bar{b})}{F_0(c+\bar{b})} > h(\bar{b})[-2\bar{b} + \frac{1}{\lambda_1(c+\bar{b})}\frac{1+F_1(c+\bar{b})}{F_1(c+\bar{b})}] \tag{2.13}$$

for $h(\cdot)$ increasing and positive.

Notice that as f_1 dominates f_0 in MLR then the hazard ratio is decreasing (i.e $\lambda_0 < \lambda_1$). Moreover, it implies that F_1 dominates F_0 in first stochastic dominance order (FOSD), which in turn implies that $\frac{1+F_0(c+\bar{b})}{F_0(c+\bar{b})} < \frac{1+F_1(c+\bar{b})}{F_1(c+\bar{b})}$. Therefore the condition in (2.13) is satisfied for $h(\cdot)$. ■

B. Simulation for Different Distributions

Following the results presented on figure 2.4, here we show the behavior of revenue as a function of $\bar{\alpha}$ and $\bar{b}$ for the main distributions considered in this chapter. Figure 2.7 has the functions for a $Beta[2,2]$, figure 2.8 has the functions for a $Beta[2,7]$ and figure 2.9 has the functions for an $InverseBeta[2,7]$

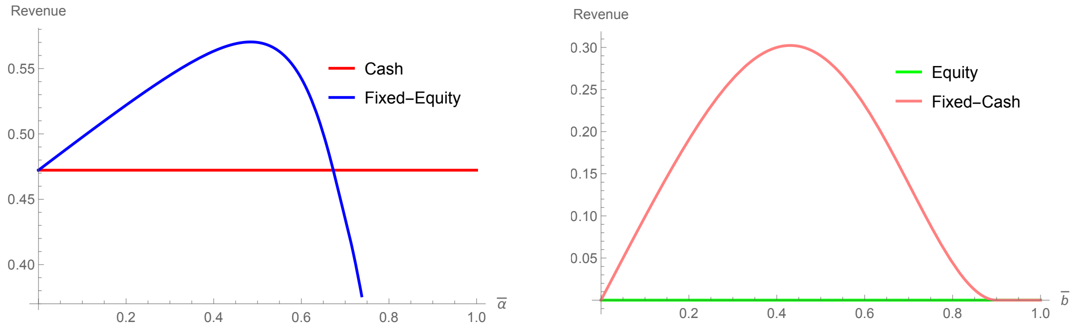

Figure 2.7: Revenue as a function of $\bar{\alpha}$ and $\bar{b}$ for $B[2, 2]$

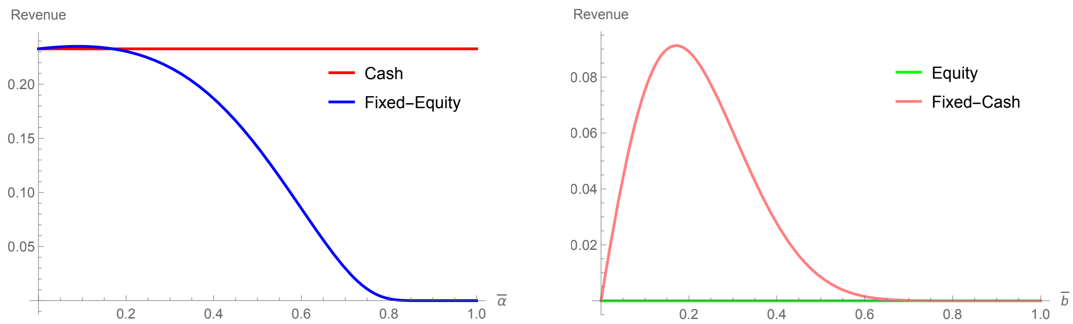

Figure 2.8: Revenue as a function of $\bar{\alpha}$ and $\bar{b}$ for $B[2, 7]$

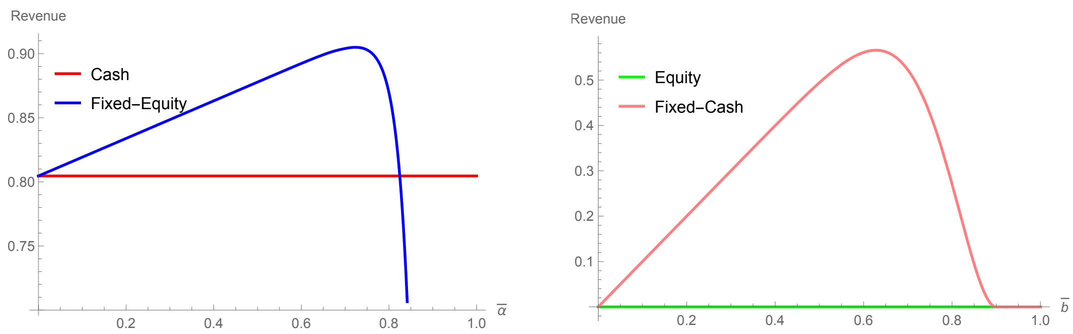

Figure 2.9: Revenue as a function of $\bar{\alpha}$ and $\bar{b}$ for $IB[2, 7]$

Chapter 3

Bidding with Securities: Insurance and Efficiency

3.1 Introduction

In a seminal work, DeMarzo, Kremer and Skrzypacz (2005) (henceforth DKS) introduces a general framework to analyse security-bid auctions—auctions where the bidder's payment to the seller is securitized by the underlying return of the asset being auctioned. Their central finding is that securities that are more sensitive to the winner's true type (steeper securities) yield the seller higher expected revenue. DKS formalizes Hansen (1985) insight. Moreover, they show that the auction format to which the seller commits only affects revenues via its ability to modify the steepness of the security utilized.

DKS analyze a setting with risk-neutral, ex-ante homogeneous bidders who receive i.i.d. signals about the private value of the asset. Our contribution is to introduce ex-ante heterogeneously risk-averse bidders to their setting. In practice, security auctions are often used to sell the rights to the stream of payoffs from long-term projects, and there is extensive uncertainty over what those future cash flows will be.[1] In such environments, it is paramount to understand how the heterogeneous attitudes of different bidders toward risk affects their bidding and the auction efficiency.

Greater risk aversion causes bidders to discount bids by more, because they suffer greater disutility when asset revenues turn out to be low. The key consequence of this heterogeneity is that bids' face values and bidders' types cease to be aligned: a more risk-averse bidder may lose an auction even when its underlying distribution of payoffs stochastically dominates that of the winning bidder.[2] The inefficiencies that result resemble those that arise when some bidders are financially constrained (e.g. Che and Gale (1998)). There, bidders with budget constraints experience handicaps that limit their competitiveness in an auction, even when they have better valuations. In a similar fashion, Board (2007) shows that a second-price auction yields higher revenue for the seller than a first-price auction when bankruptcy represents a concern.

In our realm, we show that steeper securities both alleviate these inefficiencies and increase seller's expected revenue. The central force underlying this result is that steeper securities provide bidders more insurance, because they ask for lower payments when the realizations of the project are low, and vice versa when they are high. This insurance *levels the field* for more risk-averse bidders, inducing them

[1] Examples of such auctions include coal leases in the US, 3G telecommunication rights in Hong Kong; see DKS for additional examples.

[2] A similar effect can be observed in takeover auctions when bidders have heterogeneous standalone values and exhibit different synergies when merged with the target firm (cf. Liu (2016)).

to bid relatively more aggressively, and improving the alignment of private signals and bids.

We show that when the seller switches from a flatter to a steeper security, the valuation type of the winning bidder never decreases. This result has a clear effect on both efficiency and expected seller revenue: not only does the distribution of project payoffs of the winner weakly improve, but, as DKS shows, steeper securities also allow the seller to extract a higher share of the surplus.

Finally, we deliver a result on efficiency that is non-classical in the auction literature: we show that if the auction environment is sufficiently rich, the only security that guarantees ex-post Pareto efficiency is the steepest possible one—the call option. Intuitively, the only dimension on which bidders and seller interests could possibly be aligned is the steepness of the security, because steeper securities allow the seller to extract a higher surplus and provide more insurance to the bidders. Therefore, if bidders are sufficiently risk averse, using a locally steeper security might be mutually beneficial.

Our findings have direct policy implications for the design of auctions where a government is interested in both, procuring efficiency and maximizing revenue, as in selling the right to exploit public resources. In our model, we abstract from other possible schemes the seller might use to provide insurance or induce higher participation.

We provide two cases where our setup is relevant. The first one took place in the US, during the coal lease carried out by the government. The second example comes from Hong Kong, where it was decided to perform the 3G auction on equity.[3] The Hong Kong auction was a reaction to the concerns raised at the 3G European auction. Binmore and Klemperer (2002) state that during the British 3G auction, run with cash, one of the biggest concerns was to increase the number of bidders; namely, how to attract entrants –the incentives for incumbents to participate is always higher. Klemperer (2002) states that revenue in other 3G European auctions was lower than the British one mostly because of a failure on attracting entrants. We show that the policy followed by Hong Kong was the correct one since it cannot do any harm, but it can also lead to discrete increases on surplus by allowing risk averse bidders more likely to win the auction, thus making it more attractive to them.

Another related paper corresponds to Gorbenko and Malenko (2011). They argue that reserve prices are detrimental for efficiency while securities are not. We add that securities are beneficial for efficiency, representing a stronger result. Moreover, we extend Abhishek et al. (2015) result to heterogeneous risk averse bidders since they extend DKS for the case of homogeneous risk averse bidders.

Lastly, Abhishek et al. (2015) analyses the case of homogeneous risk averse bidders and shows that DKS revenue result still holds.

Empirical Evidence. In this section, we cover some particular applications for our results. They show cases where bidders were risk averse, there was no reason to believe the less risk averse bidder had a better distribution and using some kind of security was allowed.

The first case comes from the US. In the 20th century, the US conducted many

[3] Retrieved from http://archives.ofca.gov.hk/en/3g-auction/rules.pdf

coal leases under different formats.[4] There were two main options to lease the contracts: cash bonus and royalties. Cash bonus implied a fixed amount to be paid for the lease, usually when it was issued, or on a fixed payment schedule. A royalty is a payment made as production occurs based on the amount or value of production. In terms of our model, cash is a flatter instrument than royalty (equity).

One concern about coal leases was the ownership patterns that have resulted from the history of public land disposal. Often, specific firms had the lease of the land prior to the auction taking place (incumbent), given them more information about the mine that any other firm (entrant). This information gap created more risk for the entrants.

In order to close this gap, the U.S Geological Survey (GS) estimated the value of each tract to be offered. The estimates obtained were disclosed to bidders prior to the sale. In doing so, they were reducing the uncertainty of the project, reducing the impact that risk aversion may have had.

In 1969, it was decided that the GS estimates would be confidential, thus risk aversion became a prominent issue again.

Since it is not clear that the most efficient firm to operate a coal tract is the incumbent because there could be firms that are better for the specific tract, we would suggest that using royalty is a better way of mitigating the negative effect of risk aversion because they not only improve efficiency, but also revenue for the government.

Our second case comes from Hong Kong. The 3G telecommunication auction

[4] The information comes from the report of the Linowes commission: Commission on Fair Market Value Policy for Federal Coal Leasing (1984)

in Hong Kong was conducted on equity and many economists were in favour of such an idea (see for example Ure (2002)).

There were concerns about the value of the technology since it was not fully known and there was no information available about alternative technologies at the time, such as the 802.11 standard. Another concern in terms of valuing the licenses was the growth of internet. The future market value of internet was difficult to be forecasted, thus the value of 3G was uncertain since it allowed accessing the internet. As Ure argued at that time "Still, no one knows what services will be available, which of them will sell, who will buy them or how the revenue will be collected along the value chain".

Looking at the uncertainty plus the arguments of several CEOs from the UK complaining about the high price they paid to get a license, Ure proposed using royalties in order to tie the payoff of the license to the auction payment. As he stated "If no one could know the sensible value to place on a 3G licence, then an up-front money auction was totally inappropriate".

The lack of information increased the risk of the project. This environment is similar to the setup of our model where bidders are averse as a consequence of such risk.

Organization of the chapter The rest of the chapter is structured as follows. Section 3.2 presents an example with the main result of the chapter, Section 3.3 introduces the model and presents the main results regarding asymmetric bidders and Section 3.4 presents the conclusions.

3.2 A Motivating Example

Consider two bidders participating in a second-price, security-bid auction. Bidders compete for the rights to a project that has a stochastic revenue and requires an initial investment of X. We assume that X is commonly known and equal to 0.2. Each bidder has private information about the distribution of the project's payoffs if they undertake it.

We assume that the risk-neutral seller runs a second price auction using one of four security types: cash, debt, equity or call option. The seller commits to a security design and announces it to bidders, who then submit bids. A winner is then determined, project revenues are realized, and payoffs are made according to the security chosen by the losing bidder.

We assume that the project's revenue Z can attain two values, Z^H and Z^L –that stand for high and low. The high realization happens with probability p_i^H and the low realization with probability $1-p_i^H$. In this example we fix $Z^H = 1$ and $Z^L = 0.1$. Given this simplification, it can be argued that the auction is efficient if the winner is the bidder with the highest p_i^H.

Bidders are heterogeneously risk averse and seek to maximize expected utility. We assume that the functional form for utility over money is the same for both bidders, $u(m) = 1 - e^{(-\lambda_i m)}$, but that bidders differ in the parameter λ: the less risk averse bidder a has $\lambda_a = 1$, whereas the more risk averse b has $\lambda_b = 4$. Before submitting bids, each bidder receives a signal of the parameter p_i^H, which can be regarded as his type. We further fix the probability of a high realization of the less risk averse bidder to $p_a^H = 0.5$, and analyze the implications for efficiency as we vary the parameter p_b^H of the more risk averse bidder.

Due to the second price structure of the auction, it is a weakly dominant strategy for both bidders to bid their reservation values σ_i, as we will see later. A bidder's reservation value depends on the security used:

i) Cash: $\sigma_i^{ca} := s_i : \mathbb{E}[u_i(Z_i - X - s_i)|p_i^H] = 0$.

ii) Debt: $\sigma_i^{de} := s_i : \mathbb{E}[u_i(Max(0, Z_i - s_i) - X)|p_i^H] = 0$.

iii) Equity: $\sigma_i^{eq} := s_i : \mathbb{E}[u_i((1 - s_i)Z_i - X)|p_i^H] = 0$.

iv) Call Option: $\sigma_i^{co} := s_i : \mathbb{E}[u_i(Min(Z_i, s_i) - X)|p_i^H] = 0$.

The auction would be efficient in the classical way if the winner is the bidder with the highest p_i^H, since for the same security the expect revenue for the seller is increasing on p^H.

We begin by noting that the auction will always be efficient whenever $p_a^H > p_b^H$. The interesting case is where the more risk averse bidder also has the highest p^H. When this is so, the steepness of the security matters because the more risk averse bidder benefits more from having insurance, bidding more aggressively in relative terms as a result.

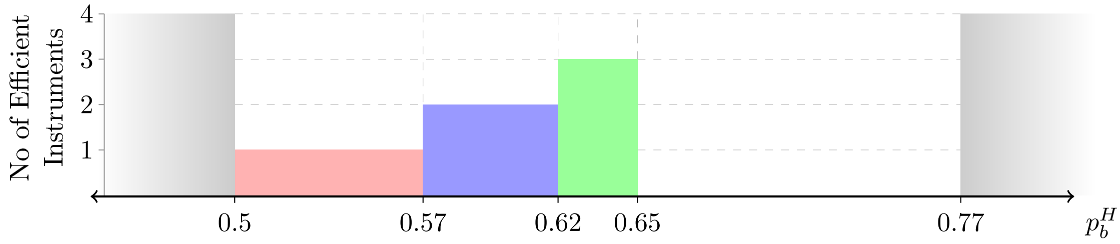

Figure 3.1: **Efficiency Range.**

Figure 3.1 illustrates how efficiency is affected by changes in the distribution of the more risk averse bidder for the four different types of securities. The gray area denotes the ranges of p_b^H for which the auction is efficient for the four instruments. To the right, it is efficient because the more risk averse bidder wins—regardless of the insurance—since his probability of obtaining Z^H is considerably higher. To the left, it is efficient because the less risk averse has the highest p^H and wins. In the green area only debt, equity and call option are efficient, implying that some insurance is needed to allow the more risk averse bidder to win. In the blue area equity and call option are efficient. In the pink area only call option is efficient. Finally, when $p_b^H \in (0.65, 0.77)$, no instrument is efficient—even with the maximum insurance that call option provides, the more risk averse cannot compensate for the risk aversion gap.

An important feature of the example is that the more risk averse bidder might prefer a steeper security for two reasons: (1) he may win the auction in cases he would otherwise lose it with a flatter one; and (2) even if he wins in both cases, the insurance provided by the steeper security may offset the surplus extraction done by the seller.[5] This is the main reason why a sufficiently risk averse bidder may not prefer *ex post* the flattest security conditional on winning.

3.3 The Model

The structure of the model follows DKS, but it is slightly modified to allow for heterogeneous risk aversion. There is a risk neutral seller interested in allocating an indivisible project among a set of N different buyers. The project is valuable for

[5] In the example, when the more risk averse bidder has a coefficient $p_b^H = 0.62$, he wins under equity and call option, but prefers call option because with equity he has to pay his reservation value, thus his expected utility is zero, while with call option he pays strictly less than his reservation value (he will only pay his reservation value if $p_b^H = 0.57$) implying a positive expected utility.

all buyers but is useless if the seller undertakes it. To implement the project, any buyer needs to make a non contractible investment of $X > 0$, which is considered as the initial fixed cost to implement the project and it is common knowledge to all agents. If buyer i acquires the project, and makes the required investment, then it will yield a (contractible) stochastic revenue of Z_i.

Before buyers participate in the auction, they receive a private signal V_i of the stochastic revenue Z_i, which are identically and independently distributed according to the positive everywhere density f on the support $\mathcal{V} := [\underline{v}, \overline{v}]$. Likewise, the conditional payoff Z_i, on the signal $V_i = v_i$, has a positive and continuous density $h(Z_i|v_i)$ everywhere on the support $\mathcal{Z} := [0, \infty)$. We assume that the parametrized family $\{h(\cdot|v)\}$ satisfies the Strict Monotone Likelihood Ratio (sMLRP). That is, the likelihood ratio $h(z|v)/h(z|v')$ is increasing in z if $v > v'$. All densities are common knowledge.

All buyers are expected utility maximizers, but they are ex-ante heterogeneous in their level of risk aversion, which is captured by the private parameter $\theta_i \in [\underline{\theta}, \overline{\theta}]$. We assume that if $\theta_i > \theta_j$ then buyer i is more risk averse than buyer j Each buyer i has a utility function over money m denoted by $u_i(m) := u(m, \theta_i)$ which is jointly continuous, and concave and increasing in m. Furthermore, it is normalized so that $u_i(0) = 0$.

Bids are expressed by derivative securities in which the underlying asset is the project's revenue Z_i. Formally, a security is a function $S : \mathcal{Z} \to \mathbb{R}$, where $S(z)$ represents buyer's payment to the seller when the realized revenue of the project is equal to z.

As in DKS, buyers choose their bids from a linearly ordered family of securities

$\mathcal{S}$, which can be written as $\mathcal{S} := \{S(\cdot, s)|s \in [s_0, s_1]\}$. Here, if buyer i submits a bid of s, he expects to make a payment equal of $S(z_i, s)$ to the seller in case he wins the auction. The interval $[s_0, s_1]$ can be used to parametrize all families of securities without loss of generality.[6] We assume that an ordered family of securities satisfies the following two conditions.

Assumption 1 *For all s, (i) $S(z, s)$ and $z - S(z, s)$ are continuous and increasing and (ii) $0 \leq S(z, s) \leq z$ for all z.*

Assumption 2 *For all bidder i and all signal v_i*

i) $\mathbb{E}U_i[S(s, v_i)] := \mathbb{E}[u_i(Z_i - X - S(Z_i, s))|V_i = v_i]$ is continuous and decreasing in s, nonnegative for $s = s_0$, and nonpositive for $s = s_1$.

ii) $\mathbb{E}S(s, v_i) := \mathbb{E}[S(Z_i, s|V_i = v_i)]$ is continuous and increasing in s.

Assumption 1 states that the payment for the seller and the net payoff for the buyer are increasing in the revenue of the project for all security bids. Furthermore, it says that securities have to be feasible: buyers cannot promise to pay more than the revenue of the project, and the seller cannot finance its implementation.[7] Meanwhile, assumption 2 merely says that securities are completely ordered from the perspective of the buyer and the seller.

Following DKS, we rank securities using the notion of steepness.

[6] The interval can be normalized to any close interval independent of the security S by rescaling and translating the parameter s in $S(\cdot, s)$. For instance, if the security S is equity, a bid s can be expressed as $s = (\hat{s} - s_0)/s_1$ for some $\hat{s} \in [s_0, s_1]$.

[7] This last assumption is crucial to rule out Crémer (1987) critique, who claims that if the seller could finance the implementation cost of the project, he would be able to extract the whole surplus.

Definition 17 *Let* $\mathbb{E}S_s(s,v)$ *and* $\mathbb{E}S_v(s,v)$ *be the partial derivatives of* $\mathbb{E}S(s,v)$*. The family of securities* $\mathcal{S}'$ *is steeper than the family* $\mathcal{S}''$ *if for all* $S' \in \mathcal{S}'$ *and all* $S'' \in \mathcal{S}''$*,* $\mathbb{E}S'_v(s',v) > \mathbb{E}S''_v(s'',v)$ *whenever* $\mathbb{E}S'(s',v) = \mathbb{E}S''(s'',v)$*.*

Steeper securities are then more sensitive to the true bidder's type at the point where the expected payment to the seller is the same.

In particular, debt represents the flattest instrument and call option the steepest.[8] Indeed, for realizations below the debt value, the payment to the seller grows at the same rate as the return of the project, whereas for realizations above the debt value, the payment to the seller remains constant. Since it is impossible for a security to generate a payment that grows faster than the return of the project –in virtue of the liability constraint–, it implies that for realizations below the debt value, debt cannot be crossed from below by any security. Once the payment becomes flat, it can only be crossed from below. Thus, debt cannot cross any instrument from below, implying it is the flattest. By a similar argument, call option is the steepest instrument. When the return is below the strike price, the payment to the seller is constant and equal to zero, hence it cannot be crossed from below. For realizations above the strike price, the payment to the seller grows at the same rate as the realization of the project, and thus it can only cut other securities from below. Therefore, since it cannot be crossed from below and it can only cut other securities from below, it is the steepest instrument.[9]

[8] This figure can be seen in chapter 1 figure 1.

[9] The figure for debt, equity and call option in terms of steepness is provided in chapter 1.

3.3.1 Equilibrium

In order to solve the game, denote $\sigma_i(v_i, S)$ as the security payment that makes buyer i indifferent between implementing or not the project, conditional on the fact that his signal is v_i and the seller is using security S. That is, $\sigma_i(v_i, S)$ is defined as the security bid s such that $\mathbb{E}U_i[S(s, v_i)] = 0$. The existence and uniqueness of s is guaranteed by assumption 2.

Lemma 18 *The profile $(\sigma_1(v,S), \cdots, \sigma_N(v_N, S))$ constitutes an equilibrium in dominant strategies to the game induced by a the second-price auction under security S.*

Proof. Suppose that $(s_j)_{j \neq i}$ are the bids submitted by the opponents of bidder i and let $s_{-i}^{(1)} = \max\{s_j,\ j \neq i\}$. Then, bidder i has to choose the security bid s_i that maximizes his expected utility,

$$\mathbb{E}[U_i(Z_i - X - S(Z_i, s_{-i}^{(1)}))\mathbb{1}(s_i > s_{-i}^{(1)})]$$

By the law of iterated expectations, the last expression can be rewritten as

$$\mathbb{E}[\mathbb{E}[U_i(Z_i - X - S(Z_i, s_{-i}^{(1)}))|V_i = v_i]\mathbb{1}(s_i > s_{-i}^{(1)})]$$

Using assumption 2, and by the definition of $\sigma_i(v_i, S)$ it is immediate to conclude that $s_i = \sigma_i(v_i, S)$ uniquely maximizes bidder's expected utility. ■

Given $\sigma_i(v_i, S)$, we can compute $R_i(v_i, S) = \mathbb{E}S(\sigma_i(v_i, S), v_i)$ as the revenue for the seller from the equilibrium bid of bidder i, given the signal v_i. The function $B(v_i, S, S')$ maps the revenue $R_i(v_i, S)$ into the bid $s' \in S'$ such that $\mathbb{E}S(\sigma_i(v_i, S), v_i) = \mathbb{E}S'(s', v_i)$.

3.3.2 Insurance

Let $\phi_S : \mathcal{Z} \to \mathcal{Y} \equiv [0, \bar{y}]$ be the function that maps the revenue of the project into the space of net return to the bidder, under security S.[10] Thus, $y_S = \phi_S(z) \equiv z - S(z)$. For easiness in the notation, we write y_S simply as y when there is no risk of confusion. An example of the net return for standard securities is presented in figure 3.2.

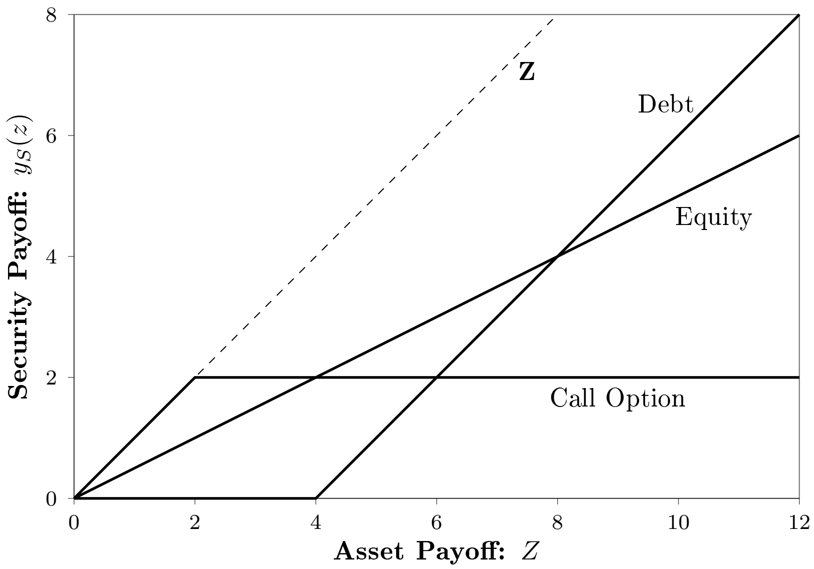

Figure 3.2: **Payoff Diagrams for Call Options, Equity, and Debt.** The monetary payoff to the bidder exhibits the reverse single crossing property as in DKS. A steeper instrument crosses from below a flatter one for the seller, whereas for the bidder, the crossing pattern is reversed.

Furthermore, we let G_S denote the lottery over the set of net payoffs $\mathcal{Y}$, induced

[10]We call net return to the realized return of the project after subtracting the payment to the seller, but without subtracting the implementation cost.

by the security S. We denote its density by g_S.

Proposition 19 *Let S' and S'' be two different feasible securities, and suppose that there exists a z^* such that $S'(z) \leq S''(z)$ for all $z \leq z^*$, and $S'(z) > S''(z)$ for all $z > z^*$. Then, if*

$$\int_0^{\bar{y}} y g_{S'}(y)dy = \int_0^{\bar{y}} y g_{S''}(y)dy \tag{3.1}$$

any risk averse individual would prefer the lottery $G_{S'}$ to the lottery $G_{S''}$.

In other words, $\mathbb{E}U_i[S'(B(v_i, S'', S'), v_i)|V_i = v_i] > 0$.

Proof. Notice that by the definition of net return and the single crossing property in the first part of the theorem, it follows immediately that $\phi_{S'}(z) \geq \phi_{S''}(z)$ for all $z \leq z^*$ and $\phi_{S'}(z) < \phi_{S''}(z)$ for all $z > z^*$. That is, under low realizations of the project the security bid S' yields a higher net return than S'', and vice-versa for lower realizations.

Moreover, it also implies that the induced distributions also satisfy a single crossing property. That is,

$$G_{S'}(z) \leq G_{S''}(z) \text{ for all } z \leq z^* \text{ and } G_{S'}(z) > G_{S''}(z) \text{ for all } z > z^*$$

Since net returns are non-negative, we can use integration by parts in (3.1) to show that

$$\int_0^{\bar{y}} G_{S'}(y)dy = \int_0^{\bar{y}} G_{S''}(y)dy$$

Then, we have that

$$\int_0^{\tilde{y}} [G_{S'}(y) - G_{S''}(y)]dy \leq 0$$

for all $\tilde{y} \in \mathcal{Y}$. Otherwise, condition (3.1) would be violated.

Hence, we conclude that $G_{S'}$ dominates $G_{S''}$ in Second Order Stochastic Dominance (*viz.* $G_{S'} \succeq_{SOSD} G_{S''}$). Therefore, by theorem 2 in Rothschild and Stiglitz (1970), every risk averse buyer prefers $G_{S'}$ to $G_{S''}$. In other words, for each concave utility function $u(\cdot)$, we have that

$$\int_0^{\bar{y}} u(y) g_{S'}(y) dy \geq \int_0^{\bar{y}} u(y) g_{S''}(y) dy$$

■

A particular example of proposition 19 is shown in figure 3.3.

The single crossing condition in the statement of proposition 19 implies that any risk averse bidder would prefer the lottery induced by a steeper security. In other words, steeper securities provide higher insurance to risk averse buyers.

3.3.3 Efficiency and Revenue

In this section, we provide the two core results on efficiency. The first is more standard to the auction literature, and says that the signal of the winning bidder is weakly increasing in the steepness of the instrument utilized. The second one, states that the only security that guarantees ex-post Pareto efficiency is call option.

Definition 20 *Let $V^{(n)}(S)$ be the signal of the nth highest bid in a second-price auction under security S, auction $\mathcal{A}(S)$. We say that $\mathcal{A}(S')$ is less inefficient than $\mathcal{A}(S'')$ (viz. $\mathcal{A}(S') \succeq_{LIN} \mathcal{A}(S'')$) if*

$$V^{(1)}(S') \geq V^{(1)}(S'')$$

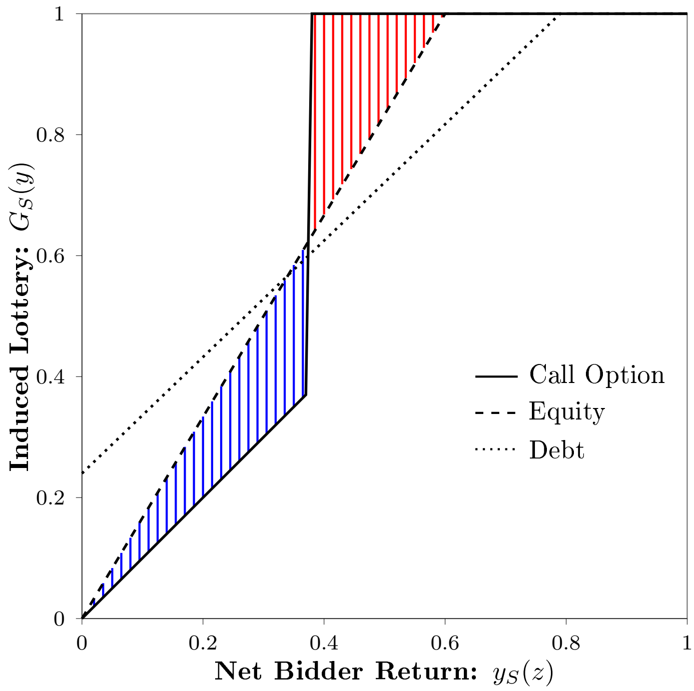

Figure 3.3: **Distribution Function of the Payoff for Call Options, Equity, and Debt.** When the revenue distribution function is $U[0, 1]$ and $\mathbb{E}S = 0.4$ then $G_S(y)$ follows the single crossing pattern of figure 3.2. If we compare call option and equity, the blue area is equal to the red area since the mean of both distributions is the same, implying call option second order stochastically dominates equity. The same result holds true when comparing a steeper instrument with a flatter one.

Notice that definition 20 does not rule out the fact that $\mathcal{A}(S)$ could be inefficient in the classical sense; that is, $V^{(1)}(S) < \max\{V_i : 1 \leq i \leq N\}$. However, if the auction is efficient under S'', then it has to be efficient under S'.

Proposition 21 *If security S' is steeper than security S'' then $\mathcal{A}(S') \succeq_{LIN} \mathcal{A}(S'')$.*

Proof. First, we notice that for any two buyers i and j, such that $\theta_i > \theta_j$, if $v_i < v_j$, then $\sigma_i(v_i, S) < \sigma_j(v_j, S)$ under any feasible security S. That is, for two given buyers, the individual with a higher signal and lower risk aversion will always submit a higher bid in equilibrium.

The interesting case corresponds to the situation when $\theta_i > \theta_j$ and $v_i > v_j$.

Suppose the seller switches from the security S'' to a steeper security S'. As commented before, there are two effects that come into play: the insurance effect and the extraction effect. The former positive effect helps to alleviate the latter negative effect, and it is larger as more risk averse is the buyer. Let $\sigma_i(S'', v_i)$ and $\sigma_j(S', v_j)$ the equilibrium bids under the security S'' and S', respectively. Now, because both buyers are risk averse, u_i and u_j are concave. Moreover, u_i can be represented by a strict concave transformation of u_j. Therefore, in virtue of our previous discussion, $\mathbb{E}U_i[S'(B(v_i, S'', S'), v_i)|V_i = v_i] > 0$ and by the concavity and monotonicity we can obtain

$$\mathbb{E}U_i[S'(B(v_i, S'', S'), v_i)|V_i = v_i] > \mathbb{E}U_j[S'(B(v_j, S'', S'), v_i)|V_j = v_j]$$

since buyer i is more risk averse and the insurance provided by the steeper security is more valuable.

Then, by assumption (2) we have that

$$|R_i(v_i, S') - R_i(v_i, S'')| \geq |R_j(v_j, S') - R_j(v_j, S'')|$$

Therefore, the more risk averse buyers become relatively more aggressive at the time to submit their bids. But then, it implies that the bid ranking for the buyer with higher risk aversion cannot decrease when switching from S'' to S'. ■

Corollary 22 *Let $\mathcal{S}'$ be a family of securities steeper than $\mathcal{S}''$. Then, the expected revenue for the seller generated by any security of the family $\mathcal{S}'$ is at least as high* *as the expected revenue generated by any security of $\mathcal{S}''$.*

Proof. Let $\sigma^{(n)}(S)$ be the nth highest bid when the auction is run under security S. Suppose that S' is steeper than S'', then by proposition 19 $\mathbb{E}^{(n)}S'(\sigma^{(n)}(S'), v_n) > \mathbb{E}^{(n)}S''(\sigma^{(n)}(S''), v_n)$ since all bidders become more aggressive because of higher insurance. Furthermore, by proposition 21, $V^{(1)}(S') \geq V^{(1)}(S'')$. Therefore, by assumption 2 and the sMLRP condition, we have that

$$\mathbb{E}S'(\sigma^{(2)}(S'), V^{(1)}(S')) - \mathbb{E}S''(\sigma^{(2)}(S''), V^{(1)}(S'')) > 0$$

■

We turn to our second result: that call option is the only security that ex-ante guarantees ex-post Pareto efficiency. In order to do so, we need to introduce first a notion of local steepness.

Definition 23 *Let S be a security flatter than call option. We say that S' is an (ϵ, z^*)-steeper security of S if $S'(z,s) = (1-\epsilon)S(z,s)$ for all $z \leq z^*$ and $S'(z,s) = (1+\epsilon)S(z,s)$ for all $z > z^*$.*

Notice that the only direction in which there might be an ex-post Pareto improvement in the auction is if the steepness of the security increases, since the seller would extract more revenue and the bidders would benefit from having more insurance. The effect for the seller is unambiguous. Nonetheless, for bidders it is necessary to provide conditions such that the higher insurance more than offsets, the higher surplus extraction, conditional on the fact that the winner remains the same.

Proposition 24 *For all securities S flatter than call option, if $\underline{\theta}$ is sufficiently large, there exists a (ϵ, z^*)-steeper security of S, called S', and constants δ_1 and δ_2, such that if $\overline{\theta} - \underline{\theta} < \delta_1$, and $\overline{v} - \underline{v} < \delta_2$, S' ex-post Pareto dominates S.*

Proof. Let $w(S)$ be the identity of the winner under security S. Hence, by the continuity of $u(\theta, \cdot)$ in θ, the continuity of $h(z|v)$ in v, and by assumption 1, there exists δ_1 and δ_2, such that if $\overline{\theta} - \underline{\theta} < \delta_1$, and $\overline{v} - \underline{v} < \delta_2$, $w(S) = w(S')$ for some (ϵ, z^*)-steeper security of S, S'.

Now, we have to prove that the higher insurance provided to the winner, more than offsets the higher bid he has to pay under the steeper security. Indeed, if bidders are sufficiently risk averse (i.e. if the lower bound $\underline{\theta}$ is sufficiently large), then we have that for all $\theta \in [\underline{\theta}, \overline{\theta}]$

$$\int_0^{y^*} u(\theta, y_{S'}) g_{S'} dy_{S'} - \int_0^{y^*} u(\theta, y_S) g_S dy_S > \int_{y^*}^{\bar{y}} u(\theta, y_S) g_S dy_S - \int_{y^*}^{\bar{y}} u(\theta, y_{S'}) g_{S'} dy_{S'}$$

where y^* corresponds to z^* in the $\mathcal{Y}$ space. ■

Remarkably, if the environment is sufficiently rich, the only security that guarantees ex-post Pareto efficiency is call option, which also happens to be the security that delivers the highest revenue to the seller.

3.4 Concluding Remarks

In this chapter, we incorporate ex-ante heterogeneous risk averse bidders into the model of DKS to analyse the implications of using steeper securities for efficiency and seller's revenue. We show that steeper securities provide higher insurance to risk averse bidders because they induce payoff distributions that dominate in second order stochastic dominance the ones derived from flatter securities. The higher level of insurance *levels the field* for risk averse bidders and allows them to be more aggressive in their bids. This increase in the aggressiveness has two effects: (i) the signal of the winner under a steeper security is weakly higher, and (ii) the expected revenue for the seller increases.

We also show that unlike standard auctions, the interest of the seller and the bidders might be aligned if the seller utilizes a steeper security to run the auction. The seller is better off because it is extracting a higher surplus, whereas bidders benefit from having higher insurance, provided they are sufficiently alike and risk averse. This alignment makes it possible to derive Pareto improvements for any security flatter than call option if the environment is sufficiently rich. Therefore, call option not only maximizes seller's expected revenue, but also increases classical efficiency in the sense that the winner tends to have a higher signal. Moreover, it is the only security that guarantees ex-post Pareto efficiency.

We present two applications to back up our results. The first application comes from a decision of the US government affecting coal lease at the end of the 60s. After deciding to stop providing an estimate of the mine value before the auction the information across bidders became asymmetric, thus bidders with higher risk aversion became less interested in the auction. Since royalty was a plausible scheme to run the auction, we concluded that it would have been wiser to use it more often. Afterwards, we analyse the 3G Hong Kong auction and argue that their decision to conduct it on equity was the appropriate one.

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