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1 Introduction

Since the seminal work by Mitchell (1927), considerable effort has been devoted to examine non-linearities in macroeconomic time series. Graham (1930), Keynes (1936) and Friedman and Schwartz (1963) have then stimulated a vast debate on the asymmetric effects of monetary policy. Widespread empirical evidence has been produced in support of the view that monetary policy exerts asymmetric effects on output and prices with respect to the economic conditions as well as the direction and size of the policy action. Such effects have important implications not only for the way we think about the macroeconomy, but also for the conduct of economic policy.

Lo and Piger (2005) account for different forms of asymmetry in the monetary transmission mechanism. According to their empirical analysis, the most pervasive form of non-linearity is represented by the asymmetric transmission of monetary policy over contractions and expansions in the business cycle. In this respect, the econometric evidence available to date has mostly focused on the state-dependent responsiveness of output and inflation, reporting two coexisting regularities (see also Weise, 1999 and Peersman and Smets, 2005). On one hand, monetary policy innovations have greater impact on output during contractions. On the other hand, changes in the monetary policy stance do not induce statistically different responses of prices during different cyclical phases. Our objective is to provide a parsimonious explanation of these facts. To that effect, we first document some asymmetries in the cyclical behavior of key macroeconomic variables, as well as their responses to monetary policy shocks. We then present a tractable macroeconomic model in which households display reference-dependent preferences of the type popularized by Kahneman and Tversky (1979) as ‘prospect theory’. The modeling strategy consists of assuming that households’ utility partly depends on the deviation of their consumption from a habit-based reference level of consumption below which loss aversion is displayed. In line with the key tenet of prospect theory, losses in consumption utility resonate more than gains.

The behavioral mechanism underlying loss-averse preferences has found wide empirical and experimental support in the literature (Thaler, Tversky, Kahneman, and Schwartz, 1997). Benartzi and Thaler (1995) and Barberis, Huang, and Santos (2001) show that prospect theory may help to explain the behavior of asset returns and resolve a number of quantitative asset pricing puzzles. Koszegi and Rabin (2006, 2009) assume that households care about gains and losses in consumption, a hypothesis that finds empirical support in Yogo (2008) and Rosenblatt-Wisch (2008). However, none of these approaches takes the analysis to a general equilibrium perspective.\(^1\) The novelty of this paper is to embed prospect theory in a dynamic general equilibrium framework and focus on the

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\(^1\)So far little effort has been made to explore the relevance of prospect theory for the dynamics of macroeconomic aggregates. Some applications to price-setting (Heidhues and Koszegi, 2005) and consumption theory (Bowman et al., 1999 and Koszegi and Rabin, 2009) have been proposed.
transmission of monetary policy to output and inflation. In this respect, two key mechanisms are characterized. First, during contractions changes in the real rate of interest exert stronger impact on output through an increased elasticity of intertemporal substitution between current and future consumption. This feature has been extensively examined in the literature on asset pricing (Yogo, 2008). Second, embedding loss-averse preferences in a general equilibrium setting implies a state-dependent marginal rate of substitution between consumption and leisure that can be related to firms’ real marginal cost, so as to impose the labor market equilibrium. The labor supply schedule retains the key property of being flatter below the reference point, so that real wages feature downward stickiness in contractions. Both features of the model are compatible with output being more adversely affected by monetary policy innovations during contractions. Concurrently, inflation responses are attenuated through an increased degree of real rigidity in the labor market. As a result, no difference can be appreciated between inflation responses over different cyclical phases. State-dependent degrees of intertemporal substitutability in consumption and intratemporal substitutability between consumption and leisure induce relevant non-linearities with respect to the economic conditions as well as the direction of the policy action. The model predicts stronger output responses when monetary policy is restrictive, as compared with expansive policy actions, while inflation displays nearly symmetric responses to monetary shocks with opposite signs.\(^2\) In addition, the cyclical movements of real activity as implied by the model are manifestly asymmetric, with statistical evidence of both ‘deepness’ (troughs are deeper than peaks are tall) and ‘steepness’ (contractions are steeper than expansions).\(^3\)

It is important to acknowledge that the macroeconomic literature has proposed a variety of mechanisms acting from both the supply and the demand side of the economy and capable to take account of different forms of non-linearity.\(^4\) For instance, Peersman and Smets (2005) suggest that the financial accelerator theory may explain why the effects of money on output are stronger in contractions. However, this mechanism implies an analogous amplification (attenuation) of monetary policy innovations on both prices and real activity during contractions (expansions). To overcome such a discrepancy with the existing empirical evidence, the balance-sheet channel needs to be complemented with a mechanism capable of producing competing effects on prices, so as to obtain the de-

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\(^2\)These properties are in line with the evidence reported by Cover (1992), Morgan (1993), Karras and Stokes (1999), Weise (1999) and Dufrenot, Mignon, and Peguin-Feissolle (2004).

\(^3\)These features have been extensively documented, among others, by Neftci (1984), Hamilton (1989), Sichel (1993) and, more recently, Morley and Piger (2012).

\(^4\)The list of mechanisms that may give rise to asymmetries in the monetary transmission mechanism includes: non-linearities in investment (Bertola and Caballero, 1994), patterns of entry and exit from a given market under uncertainty about profit perspectives (Dixit, 1989), nominal rigidities in the labor and the goods market (Ball and Mankiw, 1994), learning and information aggregation (Chalkley and Lee, 1998), state-dependent pricing and convex aggregate supply (Devereux and Siu, 2007). However, none of these mechanisms is per se capable to take account of cyclical asymmetries in the joint reaction of output and prices to monetary policy innovations.
sired non-linearity in the response of output, while generating symmetric price responses. In this respect, models with inverse ‘L-shaped’ or convex aggregate supply curves that belong to the Keynesian tradition are plausible candidates. A convex aggregate supply retains the property to be steeper for price levels above expected prices (see, e.g., Ball and Mankiw, 1994), so that it ensures a stronger (lower) reaction of output (prices) in contraction. Therefore, reconciling the macroeconomic theory with the evidence of no asymmetry in the response of prices typically calls for the coexistence of multiple driving forces. This paper provides an alternative explanation based on a simple and well-established behavioral mechanism.

Once it is recognized that loss aversion induces various types of asymmetry in the transmission of shocks to the economy, it seems relevant to provide some guidance as to how monetary policy should be designed to cope with such non-linearities. To this end, we derive the optimal monetary policy from the perspective of the Ramsey planner. In this context, the policy maker faces a non-trivial trade-off, as she needs to weigh the distortion stemming from price rigidity with the one induced by external habits in consumption. In fact, with only one policy instrument available the Central Bank cannot simultaneously ensure that output is at its efficient level and inflation is eliminated. The resulting policy is state-dependent and, due to loss averse preferences, it imposes the policy maker to attach greater importance to the consumption externality during contractions as compared with expansions.

The remainder of the paper is laid out as follows: Section 2 documents some evidence on a range of asymmetries that are consistent with the non-linear mechanisms that characterize the model we put forward; Section 3 presents the theoretical framework; Section 4 details the model solution technique; Section 5 discusses the key mechanisms that generate non-linear responses of output and inflation to monetary innovations; Section 6 discusses the normative implications of embedding loss-averse consumption preferences in a general equilibrium setting; the last section concludes.

2 Empirical Evidence

There is widespread evidence that monetary policy shocks exert a different impact on macroeconomic aggregates over different stages of the business cycle. Prior to providing a theoretical explanation to these findings, we add to the existing evidence by documenting cyclical asymmetries in the behavior of some macroeconomic variables that assume a central role in the theoretical framework we design. In this respect, it is important to stress that most of the empirical studies available to date have focused on the state-dependent responsiveness of output and inflation. We enlarge the picture by exploring

\footnote{See, e.g., Weise (1999), Peersman and Smets (2005) and Lo and Piger (2005).}
the cyclical behavior of real wages.\textsuperscript{6}

\section*{2.1 Asymmetries in the Transmission of Monetary Policy}

The responses of key variables to monetary policy shocks are computed over different phases of the business cycle. Specifically, contractions (expansions) are intended as periods in which the cycle moves from its peak (trough) to the trough (peak).\textsuperscript{7} The monetary policy unforecasted innovations are taken from Coibion, Gorodnichenko, Silvia, and Kueng (2012), who have extended the time series of shocks originally obtained by Romer and Romer (2004). In order to identify the transmission of these shocks to the output gap, inflation, the real wage and the monetary policy instrument, the projection method proposed by Jorda (2005) is employed. This can easily accommodate non-linearities in the data and has the advantage of not imposing the dynamic restrictions implicitly embedded in standard VARs. We run a series of single-equation regressions, one for each of the four variables of interest, which are generically denoted by \( x_t \):

\[ x_{t+h} = d_t^c c_h + \left[ a^E_h (L) x_{t-1} + \beta^E_h \mu_t \right] [1 - F(z_t)] + \left[ a^C_h (L) x_{t-1} + \beta^C_h \mu_t \right] F(z_t) + \varepsilon_t, \]

where \( d_t \) is a vector of deterministic covariates (including a constant and, whenever necessary, a linear and a quadratic trend),\textsuperscript{8} while \( a^i_h (L) \) is a polynomial in the lag operator (with \( i = \{E, C\} \), where \( E \) stands for ‘Expansion’ and \( C \) stands for ‘Contraction’).\textsuperscript{9} Expansions (contractions) are traditionally identified as periods of positive (negative) growth of a smooth indicator of the level of the economic activity. This indicator is denoted by \( y^g_t \). Following Granger and Terasvirta (1993), a logistic transformation of \( z_t \equiv \Delta y^g_t \) is then applied: \( F(z_t) = \exp \left( -\alpha z_t \right) \left[ 1 + \exp \left( -\alpha z_t \right) \right]^{-1} \). This can be thought of as a measure of the likelihood of being in a contraction, with \( \alpha \) measuring how quickly the economy switches from a contraction to an expansion (and vice versa). The monetary policy shock is denoted by \( \mu_t \). Therefore, the coefficient \( \beta^E_h (\beta^C_h) \) captures the average effect of a monetary policy innovation on a specific series of interest during expansions (contractions) and at a specific horizon \( h \).

Quarterly time series data over the 1982:Q3-2008:Q4 period have been selected.\textsuperscript{10}

\textsuperscript{6}It should also be noted that most of the recent studies exploring cyclical asymmetries in the labor market are silent about the role of real and nominal wages, while just focusing on the non-linear behavior of hours and employment.

\textsuperscript{7}Zarnowitz (1992) refers to these cyclical phases as ‘growth cycles’.

\textsuperscript{8}Specifically, we allow for a linear and a quadratic trend when dealing with the real wage. Otherwise, we only include a constant.

\textsuperscript{9}The baseline specification includes two lags for each variable. The evidence we report is robust to including more lags, as well as to considering a regime-specific coefficient for the deterministic component.

\textsuperscript{10}The start date is warranted by vast empirical evidence documenting marked differences in the conduct of monetary policy between the pre- and post-Volcker era (e.g., Clarida et al., 2000, Castelnovo and Surico, 2010). The end date is chosen so as to exclude the period in which the policy rate has reached the zero lower bound.
Focusing on this time window allows us to exclude the great inflation of the 1970s, thus insulating the analysis from an important monetary policy regime change. The output gap is computed as the (log) difference between the real GDP and the potential level of output calculated by the Congressional Budget Office (CBO hereafter). Inflation is the quarter-to-quarter change in the logarithm of the GDP deflator. The real wage is proxied by the real compensation per hour in the Nonfarm Business Sector. The monetary policy instrument is proxied by the Federal Funds rate.

Figure 1 graphs the impulse response functions from our regression exercise.\textsuperscript{11} The monetary policy shock results into a transitory increase in the Federal Funds rate which is not substantially different across opposite stages of the cycle. By contrast, we appreciate a more pronounced response of the output gap during contractions, thus confirming the evidence reported by previous studies.\textsuperscript{12} What is perhaps more surprising is that, compared with the output gap responses, both inflation and the real wage do not display marked asymmetries, as the confidence intervals of their responses tend to overlap. Therefore, both series are somewhat insulated by the stronger reactivity of the output gap during contractions. This finding is incompatible with the mechanism underlying the standard formulation of a model economy with nominal frictions. In fact a linear New Keynesian Phillips curve (NKPC hereafter) implies that, holding constant future expectations, output gap changes should linearly transmit to inflation dynamics through their equilibrium relationship with the real wage. Taken together, the responses of the output gap and the real wage point to the presence labor market asymmetries. Section 3 presents a framework that may help to rationalize these facts. One of its peculiarities is to generate a lower slope of the NKPC in contractions, due to an endogenous form of real wage rigidity. The next two subsections support the empirical plausibility of these non-linearities and discuss the empirical literature that has addressed them in the past.

\section*{2.2 Asymmetries in the Slope of the Phillips Curve}

Daly and Hobijn (2013) have recently shown that the Phillips curve (PC hereafter) has flattened during all the recession episodes taking place over the 1986-2012 time window.\textsuperscript{13}

\textsuperscript{11}For robustness purposes Section A in the online appendix replicates this figure under alternative definitions of the business cycle turning points.

\textsuperscript{12}Tenreyro and Thwaites (2013) have recently shown that monetary policy shocks tend to have greater effects in expansions than in recessions. However, their analysis differs from ours in a number of respects. First and foremost, while we focus on phases of acceleration/deceleration in the cycle (i.e., "growth cycles"), they account for situations in which real activity is above/below potential (i.e., "classical cycles"). In addition, their sample includes the Volcker disinflation, while we exclude it so as to avoid accounting for a major regime change (Goodfriend and King, 2005; Sims and Zha, 2006).

\textsuperscript{13}Laxton, Clark, and Rose (1996) and Debelle and Laxton (1997) represent some earlier examples of empirical works supporting the convexity of the PC.
In this subsection further evidence is provided in support of the inherent non-linearity of the supply schedule, showing how this flattens during the contractionary episodes. In doing so, we follow the approach pursued by Coibion and Gorodnichenko (2013), running some (reduced form) expectations-augmented PC regressions:

\[
\pi_t - E_t \pi_{t+1} = \delta_0 + \delta_1 y_t + \delta_2 F(z_t) y_t + u_t,
\]

where \( \pi_t \) is the rate of inflation, \( E_t \pi_{t+1} \) denotes expectations of inflation and \( y_t \) is the output gap. A negative estimate of \( \delta_2 \) would support the PC becoming flatter during contractions. Once again, the 1982:Q3-2008:Q4 period is examined, with inflation measured by the GDP deflator and \( y_t \) measured by the CBO output gap. Two proxies for inflation expectations are considered: (i) first, forward looking expectations are obtained from the survey of professional forecasters (SPF); (ii) second, a backward looking measure of inflation expectations is computed as an exponentially weighted moving average (EWMA hereafter) of past inflation, as advocated by Cogley (2002). Table 1 reports both OLS and IV estimates.\(^{15}\)

Insert Table 1 here

Both estimators lead to very similar results. However, the coefficient attached to the output gap is slightly larger when using the EWMA measure of inflation expectations, as compared with the estimates obtained by employing the SPF forecast. What we find more relevant for the sake of our analysis is that the interaction terms always enter with a negative sign. Furthermore, the estimated \( \delta_2 \) turns out to be statistically different from zero across all specifications.\(^{16}\) Altogether, these facts point to the possibility that the PC flattens during contractions. As such, this finding provides an alternative interpretation of the evidence reported by Ball and Mazumder (2011) on the missing deflation during the Great Recession. The next subsection reports evidence supporting the view that such non-linearity may stem from real wage asymmetries.

### 2.3 Real Wage Asymmetries

Recent years have borne witness to a growing literature supporting the hypothesis that real wages encounter some resistance to adjusting in the face of shocks to the economy. Using a survey of US firms, Holzer and Montgomery (1993) lend support to this view,\(^{14}\) In the computation of this proxy we follow Cogley (2002) and set the smoothing parameter to 0.125. However, the results are robust to sensible departures from this value.\(^{15}\) In the second case we use a lag of the output gap and the interaction term as instruments.\(^{16}\) Laxton, Rose, and Tambakis (1999) warn on the difficulty to identify convexities if policymakers are successful in avoiding large boom and bust cycles, as in the sample we look at. In light of this, one should interpret evidence in support of the PC asymmetry as potentially being downward biased during the Great Moderation period we examine.
showing that wage adjustments are fairly small, compared with employment adjustments. Most importantly, wage changes are asymmetric, with significant adjustments in response to positive demand shifts, but not negative shifts. Also Dickens et al. (2007) document wage-setting asymmetries for a number of countries, using data from the International Wage Flexibility Project. Their results support both nominal and real downward wage rigidities. While the former appears relatively more important for the US, both forms of rigidity are necessary to match the cross-sectional distribution of wages. A number of other studies deliver evidence in support of downward real wage rigidity, using large panels of OECD countries (e.g., Fabiani et al., 2006; Bauer et al., 2007; Du Caju et al., 2009; Messina et al., 2009, 2010; Babecky et al., 2010). Along with this substantial body of empirical evidence based on micro data, we also find contributions that reach similar conclusions by examining experimental data (e.g., Goette et al., 2004 and Farber, 2008), industry-level data (Holden and Wulfsberg, 2009) and macro data (Benigno et al., 2012).

To illustrate the behavior of the real wage over different phases of the cycle, Figure 2 graphs the cumulative log-deviation of the real wage and output from their trend at different turning points in the sample under examination. Abbritti and Fahr (2013) produce similar evidence – though their focus is restricted to recessionary episodes – and document a fundamental disconnection between employment and the real wage. We show that similar evidence emerges after a peak in the cycle. Only the contraction including the 2001 recession is associated with a drop in the detrended real wage, whereas in the other two contractionary episodes this variable tends to remain positive, notwithstanding a negative output gap. Further evidence in support of the view that real wages find it harder to adjust in a downturn comes from observing the correlation between annualized changes in the real wage and labor productivity. As a matter of fact, this is greater in expansions (0.26), as compared with contractions (0.17).

3 A Model of Loss-averse Consumption

This section sets out the structure of the model we use to explain asymmetries in the responses of output and prices to monetary innovations. The supply side is populated by monopolistically competitive firms that produce intermediate goods, indexed by \( j \in [0, 1] \), and a perfectly competitive sector of production that sells a composite of consumption

\[17\] Along with these studies, there are works that rely on surveys of managers and firm owners (e.g., Bewley, 1999 and Agell and Lundborg, 2003).

\[18\] Also Abbritti and Fahr (2013) appreciate this fact.

\[19\] Analogous considerations can be made when looking at alternative measures of productivity, such as the TFP (0.24 vs. 0.11) and the utilization-adjusted TFP (0.25 vs. 0.13).
goods. As to the demand side, there is a continuum of atomistic consumers, indexed by \( i \in [0, 1] \).

### 3.1 Demand Side

Households have preferences defined over leisure \((1 - N_t)\), consumption \((C_t)\) and gains and losses in consumption relative to its reference level \((X_{it})\). They maximize the expected present discounted value of their utility:

\[
W_{it} = E_t \sum_{s=0}^{\infty} \beta^s \left[ U(C_{it+s}, X_{it+s}) - \chi \frac{N_{it+s}^{1+\eta}}{1 + \eta} \right] ; \quad \chi > 0,
\]

where \( \beta \) is the intertemporal discount factor and \( \eta \) is the inverse of the Frisch elasticity of labor supply. Following Koszegi and Rabin (2006) and Yogo (2008), a general class of reference-dependent preferences is considered:

\[
U(C; X) = V(C) + (1 - \alpha) G(V(C) - V(X)) ; \quad \alpha \in [0, 1],
\]

where \( V(C) \) is a neoclassical utility function: this is assumed to be continuously differentiable, strictly increasing, and concave for all \( C > 0 \). The term \( G(\cdot) \) is a gain-loss function (Kahneman and Tversky, 1979), that is, utility derived from the deviation of consumption utility from its reference level, \( V(X) \). Preferences that depend on a reference level of consumption have psychological foundations in hedonic adaptation (see Frederick and Loewenstein, 1999). We assume that \( G(\cdot) \) satisfies certain properties. Specifically:

(i) \( G(Z) \) is continuous for all \( Z \)'s, twice differentiable for \( Z \neq 0 \) and \( G(0) = 0 \); (ii) \( G(Z) \) is strictly increasing; (iii) \(-G(-Z) > G(Z) \) and \( G'(Z) > G'(Z) \), \( \forall Z > 0 \); (iv) \( G''(Z) \leq 0 \) for \( Z > 0 \) and \( G''(Z) \geq 0 \) for \( Z < 0 \). Properties (i) and (ii) imply monotonicity, i.e. utility is strictly increasing in the magnitude of the gain. Property (iii) embodies the notion of loss aversion, i.e. the impact of a loss is greater than that of an equally-sized gain. This implies that the representative consumer becomes more sensitive to deviations from her relative consumption when she is in a bad state, compared with a good state. Finally, property (iv) is referred to as diminishing sensitivity, i.e. the marginal effect of a gain or a loss diminishes with its magnitude. Specifically, a strong form of diminishing sensitivity will be imposed. This amounts to assume that the curvature of the gain-loss function approaches zero as \( Z \to \pm \infty \).

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To take account of these properties, an exponential gain-loss utility is considered (Köbberling and Wakker, 2005):

\[
G(Z) = \begin{cases} 
\frac{1-\exp(-\theta Z)}{\theta} & \text{iff } Z \geq 0 \\
-\lambda \left[1-\exp\left(\frac{\theta}{\lambda} Z\right)\right] & \text{otherwise}
\end{cases}
; \quad \theta \geq 0, \quad \lambda > 1,
\] (4)

where \(\theta\) governs diminishing sensitivity and \(\lambda\) is a parameter that indexes the degree of loss aversion. Note that for \(\theta = 0\) a linear gain-loss function is obtained. Otherwise, (4) retains the property to be smooth at the reference point.\(^{22}\) To gain further intuition on the structure of reference-dependent preferences over consumption, Figure 3 plots the exponential gain-loss function and its first order derivative for different values of \(Z\) (x-axis) and \(\lambda\). As predicted by Kahneman and Tversky (1979), loss aversion reflects the widely observed behavior that agents are more sensitive to losses than gains, resulting in a gain-loss function that is steeper in the first case (see the left-hand panel of Figure 3). Moreover, the right-hand panel of Figure 3 captures the essence of diminishing sensitivity in consumer preferences, according to which marginal departures from the reference point are more (less) important the less (more) away they are from it.

Insert Figure 3 here

As to the reference consumption level, it is assumed that consumers evaluate the distance between consumption utility and a function of the average consumption in the previous period: \(X_{it} = C_{t-1}^\gamma\), where \(\gamma \in [0,1]\) indexes the importance of external habit formation.\(^{23,24}\)

The \(i^{th}\) consumer, whose labor is remunerated at the real wage \(W_t\), enters period \(t\) with \(B_{it-1}\) one-period nominal bonds that pay \(R_{it-1} = 1 + i_{t-1}\) gross interest. Moreover, behavior for large gambles. Loss aversion refers to the magnitude of marginal utility for losses relative to gains, which determines the household’s behavior for small gambles. Tversky and Kahneman (1991) extend their treatment of choice under uncertainty (see, e.g., Kahneman and Tversky, 1979) to the problem of facing a riskless choice.

\(^{22}\)This property is particularly useful in the perspective of linearizing the model economy.

\(^{23}\)Gill and Prowse (2012) report experimental evidence that supports the role of endogenous choice-acclimating reference points in economic decisions. In the present context external habits allow us to establish a direct link between the empirical evidence on the transmission of monetary policy during contractionary/expansionary phases of the cycle and the state-dependent model we build up. In principle, internal habit formation could have been considered. However, along with being computationally prohibitive, such a modelling option would lead to no specific insight into the problem under examination.

\(^{24}\)In line with Yogo (2008), we embed external habit formation in a model of reference-dependent consumption preferences. Since the work of Abel (1990), external habit formation has become known as ‘catching up with the Joneses’. External habit formation in consumption is usually introduced to account for the empirical persistence in the consumption process (Smets and Wouters, 2007). Unlike internal habit formation, this mechanism implies that households fail to internalize the externality of their own consumption on the utility of other households.
she pays a lump sum tax $T_{it}$ to the government and receives the flow of dividends from a continuum of monopolistically competitive producers, $D_{ijt}$:

$$P_tC_{it} + B_{it} \leq R_{t-1}B_{it-1} + P_tW_{it}N_{it} + D_{it} - T_{it},$$

(5)

where $P_t$ is the nominal price level, $D_{ijt} = \int_0^1 D_{ijt}d$ and $D_{ijt}$ denotes the dividends of firm $j$ paid to the $i^{th}$ household.

Differentiating the Lagrangian with respect to individual consumption ($C_{it}$) and taking the consumption reference level as external to the $i^{th}$ household returns the following Euler equation:

$$1 = \beta E_t \left\{ \frac{R_t}{\Pi_{t+1}} \frac{U_C(C_{it+1}, X_{it+1} | \xi_{it+1})}{U_C(C_{it}, X_{it} | \xi_{it})} \right\},$$

(6)

where $\Pi_t \equiv 1 + \pi_t$ denotes the gross rate of inflation and $\xi_{it+1}$ is a discrete valued random variable that equals one if consumption utility is above its reference level (i.e., $V(C_{it}) \geq V(X_{it})$) and zero otherwise. Therefore, the marginal utility of consumption depends on the gain-loss profile and its shape changes depending on whether consumption is above or below the reference level.

The expected marginal rate of substitution between $C_{it}$ and $N_{it}$ reads as:

$$\frac{\chi N_{it}^\gamma}{U_C(C_{it}, X_{it} | \xi_{it})} = W_t,$$

(7)

Equations (6) and (7) are key to understanding how cyclical asymmetries in the transmission of monetary policy may arise in our model. Equation (6) regulates intertemporal substitution between current and future consumption. A closer look at this relationship allows us to provide some intuition on the key mechanism governing consumption dynamics. The curvature of the gain-loss function is lower when consumption is below its reference level, implying higher elasticity of intertemporal substitution during negative growth cycles. Concurrently, equation (7) governs the intratemporal substitution between consumption and leisure. For a given $|V(C) - V(X)|$, households are more willing to cut on their leisure when consumption utility lies below its reference level. This implies a flatter labor supply schedule, *ceteris paribus*. Section 5.1 details the key implications of this feature for the labor market equilibrium.
3.2 Supply Side

The supply side of the model conforms to the standard treatment of frameworks with nominal price rigidities. The final good is produced by perfectly competitive firms and requires the assembly of a continuum of intermediate goods via the following technology:

$$Y_t = \left( \int_{0}^{1} (Y_t^{j+})^{-\varepsilon} dj \right)^{1/\varepsilon},$$

where $\varepsilon$ denotes the elasticity of substitution between differentiated goods in the production composite. Profit maximization leads to the demand function $Y_{jt} = (P_{jt}/P_t)^{-\varepsilon} Y_t$ for the $j$th type of good, where $P_{jt}$ denotes its price. Thus, $P_t = \left( \int_{0}^{1} (P_{jt})^{1-\varepsilon} dj \right)^{1/\varepsilon}$ is the price index consistent with the final good producer earning null profits.

Each firm in the intermediate goods sector produces a unique good using only labor as input according to

$$Y_{jt} = Z_t N_{jt},$$

where $Z_t$ is the total factor productivity. Firms choose the amount of labor that minimizes the cost of production. The resulting real marginal cost is $\tau_t = W_t = Z_t$. Following Rotemberg (1982), we allow for sluggish nominal price adjustment by assuming that firms face a quadratic resource cost for adjusting prices:

$$\frac{\varphi}{2} \left( \frac{P_{jt,t}}{P_{jt,t-1}} - 1 \right)^2 Y_t, \varphi \geq 0.$$

The Lagrangian representation of the firm’s problem is:

$$\max_{P_{jt,t+s}} E_t \sum_{s=0}^{\infty} \left( 1 + \tau \right) \left( \frac{P_{jt,t+s}}{P_{jt,t+s}} \right)^{1-\varepsilon} - \Omega_{t+s} \left( \frac{P_{jt,t+s}}{P_{jt,t+s}} \right)^{-\varepsilon} - \frac{\varphi}{2} \left( \Pi_{jt,t+s} - 1 \right)^2,$$

where $M_{t,t+s} \equiv \beta^s U_C \left( C_{t+s} N_{jt} \mid \xi_{t+s} \right) U_C \left( C_{t+s} N_{jt} \mid \xi_{t+s} \right)$ is the stochastic discount factor, $\tau$ is a subsidy received from the government (this is financed by the lump-sum taxes on the household) and $\Pi_{jt,t+s} \equiv P_{jt,t+s}/P_{jt,t-1+s}$. Firms can change their price in each period, subject to the adjustment cost. They all face the same problem, ultimately setting the same price and employing the same amount of labor, so that the aggregate production function is $Y_t = Z_t N_t$. Therefore, we retrieve the following first order condition:

$$(1 + \tau) (1 - \varepsilon) + \varepsilon \Omega_t - \varphi (\Pi_t - 1) = 0. \quad (9)$$

In turn, (9) gives rise to the conventional NKPC, according to which current inflation depends on the discounted sum of current and expected future real marginal costs. Finally, the market clearing condition accounts for the price adjustment cost, so that $Y_t = C_t + \frac{\varphi}{2} (\Pi_t - 1)^2 Y_t$.

3.3 Government

The government consists of two authorities. First, there is a monetary authority which controls the nominal interest rate on short-term nominal bonds through open market operations. Since we deal with a cashless economy, seigniorage revenue is not considered,
as open market operations are implicitly assumed to be infinitesimally small. Second, there is a fiscal authority that collects lump sum taxes from households and rebates them to firms as production subsidies. There is no government spending per se. For the time being, we assume that the Central Bank implements a standard instrument rule:\textsuperscript{25}

\[
\frac{R_t}{R} = \left( \frac{\Pi_t}{\Pi} \right)^{\gamma n} \exp (\mu_t),
\]

where \(\mu_t\) captures non-systematic monetary policy responses.\textsuperscript{26} Assuming a symmetric policy function to stabilize inflation represents a convenient way to close the model and focus on the effects of introducing reference-dependent preferences into an otherwise standard framework. Section 6 departs from this hypothesis and examines optimal monetary policy-making from the perspective of the Ramsey planner.

4 Model Solution

In the framework set out above households’ utility is reference-dependent, i.e. its functional form depends on whether individual consumption is above or below the reference level (which is itself determined by aggregate past consumption). At this stage of the analysis we need to specify a mechanism that governs switching in consumers’ preferences, which in turn depend on their consumption profile with respect to the reference level. Given the intertemporal dimension of households’ decisions, accounting for the expectations of future consumption is paramount. To solve for endogenous (consumption) regime switching we use the monotone map algorithm that finds fixed point in decision rules (Coleman, 1991).\textsuperscript{27}

As a preliminary step to solve the model, its quasi-linear representation is derived. In the absence of sector-specific shocks or other forms of heterogeneity, households are symmetric and make identical consumption-saving decisions. Therefore, the equilibrium conditions and the policy reaction function can be linearized in the neighborhood of the steady state consistent with \(C/X = 1.\textsuperscript{28}

\textsuperscript{25}Section E in the online appendix reports additional exercises under an instrument rule according to which the Central Bank adjusts the policy rate in response to both the inflation rate and the output term. The qualitative and quantitative predictions of the model are not affected by this type of extension.

\textsuperscript{26}In the remainder variables without time subscript will denote the steady state values of their time-indexed counterparts.

\textsuperscript{27}The problem we tackle is isomorphic to the preemptive policy behavior examined by Davig and Leeper (2008). To initialize the algorithm, we start from the solution of the framework without loss-averse preferences, but also check that the final solution is not sensitive to initial conditions by perturbing these initial conditions. The final solution is invariant to perturbing the initial rules, which suggests that the solution is locally unique.

\textsuperscript{28}The difference between the logarithm of a generic variable \(Z_t\) and that of its steady state counterpart \(Z\) is denoted by \(z_t\). We also assume, without loss of generality, logarithmic consumption utility. For
\[ y_t = \phi_1 \left( E_t \xi_{t+1}, \xi_t \right) E_t y_{t+1} + \phi_2 \left( E_t \xi_{t+1}, \xi_t \right) y_{t-1} - \phi_3 \left( E_t \xi_{t+1}, \xi_t \right) (i_t - E_t \pi_{t+1}) \]

\[ \pi_t = \beta E_t \pi_{t+1} + \psi_1 \left( E_t \xi_{t+1}, \xi_t \right) y_t + \psi_2 \left( E_t \xi_{t+1}, \xi_t \right) y_{t-1} - \varepsilon \chi \varphi^{-1} (1 + \eta) z_t, \]

\[ i_t = r \pi_t + \mu_t. \] (13)

In every period the model can generate four states, depending on whether \( E_t \xi_{t+1} = \{1, 0\} \) and \( \xi_t = \{1, 0\} \). Therefore, we deal with a Markov Switching Rational Expectations (MSRE) model (see Davig and Leeper, 2007 and Farmer, Waggoner and Zha, 2008, 2009). As such, some of the parameters in the equations describing the private sector’s behavior depend on the state of the economy and the probability of switching across different states is endogenously determined. In addition, the process of (rational) expectation formation necessarily accounts for the presence of switching across different consumption regimes. The solution is a function that maps the minimum set of state variables, \( \Theta_t = (z_t, \mu_t, y_{t-1}) \), into values for the endogenous variables, so that the rules for output and inflation can be expressed as \( h^\pi (z_t, \mu_t, y_{t-1}) = y_t \) and \( h^\pi (z_t, \mu_t, y_{t-1}) = \pi_t. \)

## 5 The Asymmetric Transmission of Monetary Policy

This section discusses the key mechanisms at work in the model and shows how different types of asymmetry can be generated in connection with the transmission of monetary policy.

### 5.1 Some Qualitative Insights

Embedding loss-averse preferences over consumption in a general equilibrium setting induces two major modifications in the equations accounting for the dynamics of real activity and prices. First, the elasticity of intertemporal substitution is state-dependent, being higher (lower) in contraction (expansion). Second, the state-dependent marginal rate of substitution between consumption and leisure dampens the impact of real activity on firms’ price-setting behavior during contractions. The first property has been widely explored and validated by Yogo (2008). The second property is intimately connected with the role of loss-averse preferences in a general equilibrium setting. A globally convex aggregate supply function can be envisaged in this context, which retains the property to be steeper (flatter) during expansionary (contractionary) episodes. Similar functional forms have been explored in the literature on the Phillips curve, emphasizing the role of large

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\[ \text{Section C in the online appendix reports additional details on the solution of the state-dependent model with endogenous switching.} \]
shocks relative to small ones for firms’ price-setting behavior. In this respect, the existence of menu costs can rationalize a convex aggregate supply schedule. Nonetheless, it is interesting to note that introducing loss-averse preferences in a general equilibrium setting allows us to provide a microfoundation that emphasizes the role of state-dependent degrees of real rigidity in the labor market equilibrium allocation, rather than nominal rigidities. Equation (7) accounts for the dynamics of labor supply: its key property is to be steeper at levels of total hours above the reference consumption/production point. Figure 4 portrays this function against a perfectly elastic labor demand. The two schedules intersect at \( V(C) = V(X) \). Starting from this point, a contraction in labor demand induces a larger (smaller) drop in equilibrium employment (wage), as compared with the responses induced by an equally-sized upward shift in the labor demand schedule. Therefore, the model generates downward real wage rigidity during contractionary episodes due to higher intratemporal substitutability between leisure and consumption opportunities, and not from asymmetric wage-setting frictions affecting the labor market (as in Kim and Ruge-Murcia, 2009 and Benigno and Ricci, 2011). As such, this mechanism marks the key difference between our framework and traditional Keynesian theories of macroeconomic fluctuations, which rather emphasize the influence of nominal and real rigidities on the shape of the labor demand schedule.

To elaborate further on these intuitions, the linearized relationships describing the behavior of demand and supply in different consumption regimes are inspected. In this context, expectation formation is non-trivial, as agents in the model economy do not know the realization of all future regimes. In turn, this sequence depends on the sequence of exogenous shocks that are realized and the serial correlation properties of those shocks. However, to provide a simple intuition of how output and inflation respond to a monetary policy shock over different cyclical phases, it is temporarily assumed that agents ‘naively’ expect the economy to permanently stay in either expansion or contraction. Furthermore, \( \gamma \) is set to zero, so that households consider the deviation of their consumption utility from the utility accruing from a constant reference level of consumption. This amounts to implicitly look at cyclical variations in output rather than expansions/contractions in the business cycle. These simplifying assumptions allow us to highlight the main structural differences between the state-dependent model with loss aversion and the standard

---

30 See Laxton et al. (1999) for a review of the literature and the analysis of the monetary policy implications of assuming a convex aggregate supply.

31 It should be noted that the degree of asymmetric reaction increases in the labor supply elasticity. In fact, at low levels of \( 1/\eta \) the state-dependent marginal consumption utility exerts a weak impact on the marginal rate of substitution between consumption and leisure, which is instead dominated by the marginal utility of leisure. In the limit (i.e., \( 1/\eta \to 0 \)) the income effect tends to fully compensate the substitution effect from a wage increase and the resulting labor supply function is close to anelastic.

32 Section C in the online appendix shows how to compute time-varying probabilities of regime switching.
New Keynesian setting. However, analogous implications carry over to the model with endogenous switching across consumption regimes, as we detail in the next section. Table 2 reports the state-dependent IS schedule and the NKPC.

Assuming loss-averse consumers implies state-dependent degrees of real rigidity and elasticity of intertemporal substitution in consumption that generate competing effects in the responses of output and inflation to monetary innovations. Specifically, during contractions the IS schedule displays higher elasticity of current consumption to the real rate of interest. As to the state-dependent NKPC, the elasticity of inflation to output deviations from its steady state level is higher in expansions. This results from the labor supply schedule being convex, which induces a dampened response of firms’ real marginal cost with respect to consumption when the latter lies below its reference point. Therefore, during contractions greater responsiveness of output to the real interest rate is counteracted by a flattening of the NKPC. The ultimate impact of a monetary policy shock on inflation depends on the relative magnitude of these competing forces.

5.2 Quantitative Analysis

To quantify the asymmetric impact of monetary policy over contractions and expansions, we compute the solution of the quasi-linear model economy. To this end, the model is calibrated at a quarterly frequency. The discount factor $\beta = 0.99$. The inverse of the Frisch elasticity of labor supply, $\eta$, is set to 0.25. As to households’ consumption preferences, $\alpha = 0.5$ and $\gamma = 0.9$. The literature on dynamic general equilibrium models does not provide us with any empirical reference on the coefficient that indexes the degree of loss aversion. Therefore, $\lambda$ is set to 2.25, in accordance with Tversky and Kahneman (1992). In line with Yogo (2008), $\theta = 1$. Nominal rigidity in price-setting is such that, on average, prices remain unchanged for three quarters. As to the policy reaction function, $r_t = 1.5$, while the non-systematic component $\mu_t = \rho_\mu \mu_{t-1} + \epsilon_\mu^t$, where $\rho_\mu = 0.5$ and $\epsilon_\mu^t \sim i.i.d. N(0, 0.02)$. As to the process governing the technology shock, $z_t = \rho_z z_{t-1} + \epsilon_z^t$, where $\rho_z = 0.9$ and $\epsilon_z^t \sim i.i.d. N(0, 0.06)$.

33 According to this mechanism, loss-averse consumption preferences should also be able to generate empirically relevant asymmetries in the transmission of fiscal shocks. Auerbach and Gorodnichenko (2012, 2013) show that the effects of a government spending shock on output are significantly larger and more persistent when the economy is characterized by underutilization of resources, as compared with a situation of full capacity.

34 Recall that $\eta$ measures the elasticity of the marginal disutility of labor with respect to hours worked. Rotemberg and Woodford (1998) report evidence of low values of this elasticity, generally between 0.25 and 0.4, while McCallum (2001) suggests values closer to the lower bound.

35 We assume higher volatility in the innovations to the technology process, so as to make it possible to generate contractionary and expansionary episodes that are predominantly driven by real rather than monetary innovations.
A quantitative assessment of the asymmetric impact of monetary innovations is an obvious goal for this study. To this end, contractions and expansions are generated by perturbing the system with a technology shock of the appropriate sign. Concurrently, a monetary policy shock is induced whose magnitude is not large enough to reverse the cyclical movement in output, so that it is not the Central Bank to determine the regime in place at any given point in time. Therefore, policy interventions are only "modest" in their scope (Leeper and Zha, 2003). Finally, the response to the technology shock is subtracted from the overall response of the system to both sources of exogenous perturbation, so as to isolate the effect of monetary innovations. Figure 5 displays greater reactiveness of output to monetary shocks during contractions as opposed to expansions - a result in line with the arguments of Section 5.1 - which confirms the robustness of the mechanism at work in generating asymmetric responses of real activity over positive and negative growth cycles. Otherwise, the difference between inflation responses over different stages of the cycle is negligible, suggesting that higher (lower) responsiveness of real activity in contractions (expansions) is attenuated (amplified) once output movements are passed through onto prices.

It is certainly important to assess the behavior of the model in response to sources of fluctuations other than monetary shocks, so as to highlight the distinctive features of contractionary and expansionary output movements. In this respect, the model displays elements of business cycle asymmetry that have been extensively documented, among others, by Neftci (1984), Hamilton (1989), Sichel (1993) and, more recently, Morley and Piger (2012). Figure 6 portrays equilibrium dynamics in response to positive and negative technology shocks of differing signs and magnitudes. In line with the evidence on the effects of monetary innovations, an adverse supply shock induces a deeper contraction, as compared with the reaction to an equally-sized expansionary shock (i.e., troughs are deeper than peaks are tall). Concurrently, contractionary movements tend to display higher steepness as compared with expansionary movements, given that in the first case output deviations from the steady state take longer to peter out. To investigate these properties in further detail, we implement a battery of asymmetry tests that aim at quantifying the statistical significance of deepness and steepness in the simulated business cycle series, conditional on different sources of exogenous disturbance. The results reported in Table 3 confirm that cyclical movements in the output series are manifestly asymmetric, with statistical evidence of both deepness and steepness. Moreover, deepness is primarily

\footnote{Monetary innovations are generated so as to induce a monetary tightening (loosening) during expansions (contractions). This is coherent with the empirical evidence of Romer and Romer (1994), who show that the Fed funds rate tends to decline (rise) right after the cyclical peak (trough).}
driven by monetary policy innovations, rather than by technology shocks. The intuition for this result is that monetary innovations induce greater deviations of the ex-ante real rate of interest from its steady state level, as compared with technology shocks, which make the nominal rate of interest and the rate of inflation move in the same direction. As a result, the non-linear mechanism of switching across different consumption regimes is magnified in the face of monetary policy innovations.

Unlike models based on exogenous mechanisms of switching across different states, our framework can generate non-linear responses to shocks with opposite signs and magnitudes. A vast empirical evidence has shown that monetary policy induces asymmetric responses of output and prices not only with respect to the economic conditions, but also depending on the size and direction of the policy action. A wide consensus has been reached on the view that money affects output strongly when monetary policy is restrictive, whereas it exerts little or no effect when it is expansive (Cover, 1992; Morgan, 1993; Dufrenot et al., 2004). Otherwise, the effect on prices is nearly symmetric (Karras and Stokes, 1999; Weise, 1999). There is also some evidence that shocks of different magnitudes have asymmetric effects on output. Weise (1999) shows that if the economy starts in a low-growth state, large negative shocks induce substantially larger contractionary (on impact) responses in output, though on a longer time horizon no asymmetry can be appreciated with respect to the size of the shock. It is possible to show that our model can account for empirically relevant non-linearities in the transmission of monetary shocks with opposite signs. Figure 7 shows how monetary contractions cause greater effects on output, as compared with the impact induced by positive monetary shocks of the same size. By contrast, inflation displays nearly symmetric responses. This can be intuitively explained by the fact that a rise in the nominal rate of interest increases the chances to trigger or deepen contractionary output movements, as compared with a loose monetary stance.\footnote{Concurrently, a non-linear labor supply schedule implies an attenuation of the pass-through from output to inflation during a contractionary stage of the cycle.} This is consistent with the evidence reported by Garcia and Schaller (2002), who show that an increase in the nominal rate of interest raises the probability of moving from an expansion to a recession. Similarly, an interest rate cut is typically associated with a higher probability of getting out of a recession.

6 Optimal Monetary Policy

Once it is recognized that loss aversion induces various types of asymmetry in the transmission of shocks to the economy, it becomes manifest that the policy maker should take
account of these facts. To this end, we derive the optimal monetary policy from the perspective of the Ramsey planner. Prior to set up the normative analysis, it is important to recall that there are two distortions affecting the model economy, namely nominal rigidity in price-setting and external habits in consumption. The presence of nominal rigidities would not *per se* be a problem in the presence of technology disturbances, as the monetary authority may always ensure that the efficient allocation is replicated, even in the presence of internal habits (see Amato and Laubach, 2004 and Leith et al., 2012). However when habits are external, such that one household does not take account of the impact their consumption has on the utility of others, then with one policy instrument available the monetary authority cannot simultaneously ensure output is at its efficient level and inflation is eliminated. Therefore, monetary policy faces an explicit trade-off when seeking to minimize the joint effect of these two distortions. If the monetary authority can credibly commit to following its policy plans, it then chooses the policy that maximizes households’ welfare subject to the equilibrium relationships that describe the competitive economy. The Lagrangian representation of the Ramsey problem reads as:

\[
\begin{align*}
\max_{\{C_t, X_t, N_t, R_t, Z_t\}} \min_{\{A_{1,t}, A_{2,t}, A_{3,t}, A_{4,t}\}} \sum_{t=0}^{\infty} \beta^t \left\{ \left[ U(C_t, X_t | \xi_t) - \frac{N_t^{1+\eta}}{1+\eta} \right] \right. \\
+ \frac{1}{R_t} - \beta \frac{U_C(C_{t+1}, X_{t+1} | \xi_{t+1})}{\Pi_{t+1} U_C(C_t, X_t | \xi_t)} \\
+ (1 + \tau)(1 - \epsilon) + \epsilon \frac{\chi N_t^2}{Z_t U_C(C_t, X_t | \xi_t)} - \varphi \Pi_t (\Pi_t - 1) + \varphi M_{t,t+1} \Pi_{t+1} (\Pi_{t+1} - 1) \frac{Z_{t+1} N_{t+1}}{Z_t N_t} \\
+ Z_t N_t - C_t - \varphi^2 (\Pi_t - 1)^2 Z_t N_t \\
\left. + X_t - C_{t-1} \right\} ,
\end{align*}
\]  

(14)

where \(A_{i,t}, i = 1, 2, 3, 4\) denotes the Lagrange multiplier associated with the constraints. Specifically, the first constraint is represented by the consumption Euler equation, the second one is the profit maximization condition, the third one is the aggregate resource constraint, while the fourth constraint accounts for the stock of consumption habits. After deriving the first order necessary conditions for the Lagrangian problem,\(^{38}\) equilibrium dynamics is examined by linearizing the model in the neighborhood of the steady state. To this end, we set \(\tau = [1 - (1 - \alpha) \varepsilon \gamma \beta] (\varepsilon - 1)^{-1}\). This value for the subsidy replicates the social planner’s steady state allocation by compensating for the net effect of the habits externality and the distortion due to imperfect competition.\(^{39}\)

The implications of implementing the optimal policy under commitment are examined in terms of impulse responses to the technology shock. We first consider the simplest

---

\(^{38}\)For further details, see Section D in the online appendix.

\(^{39}\)To avoid time-consistency problems and in line with the timeless perspective approach to optimal policy-making we set the time-zero value of the Lagrange multiplier associated with the price-setting equation at its (Ramsey) steady state level. Section D in the online appendix shows this value is zero.
scenario, in which consumers weigh their own consumption level with respect to an exoge- 
ous reference level (see Figure 8a, where $\gamma = 0$). This is done to isolate the role of loss 
aversion from the consumption externality due to external habit formation. In this case 
price rigidity is the only distortion the Central Bank is confronted with. Under these cir-
cumstances the rate of inflation may be fully stabilized (and so the output gap is closed), 
regardless of the sign of the technology shock. However, the policy response necessarily 
changes between contractions and expansions. Recall that the NKPC is relatively flatter 
during contractionary episodes. As a result, the policy maker tolerates a deeper output 
drop in this case, so as to stabilize inflation and the output gap. This property holds 
despite consumption is more responsive to interest rate changes during negative growth 
cycles. This result emphasizes the role of state-dependent non-linearities in the NKPC 
for the sake of monetary policy-making.

Insert Figure 8 here

The picture changes when external consumption habits are accounted for (see Figure 
8b). To see why, it is useful to recall the different determinants of monetary policy 
responses in a linear model with external (superficial) habits, as that envisaged by Leith, 
Moldovan, and Rossi (2012). In this context nominal inertia points to a loosening of policy 
in the face of a positive technology shock – so as to boost output – while the consumption 
externality stemming from external habit formation suggests that higher consumption 
entailed by the expansionary policy need not be desirable. In our model these incentives 
assume different weights depending on whether the shock induces a contraction or an 
exansion. On a priori grounds the net effect on the policy rate is not unambiguous, 
as during contractions loss averse preferences induce both a flattening of the NKPC and 
a stronger consumption externality. In this respect, a striking implication of assuming 
the presence of external habits is that not only inflation cannot be stabilized, but it also 
drops in the face of an adverse shock to technology. This property holds even though the 
Central Bank responds with a monetary tightening, thus attaching greater importance to 
price-setting distortions, as compared with consumption externalities. This result may be 
explained as follows. The negative shock shifts the aggregate supply backward. Due to 
the monetary tightening, also aggregate demand contracts. At this stage of the analysis 
we need to consider a key departure from the description of Figure 8a. In that case the 
monetary authority was only concerned with the distortion arising from price stickiness. 
The co-existence of endogenous habits in consumption and loss aversion now imposes 
the Central Bank to weigh the detrimental effects that a contractionary policy would 
induce in terms of declining real activity. This translates into a relatively more elastic 
aggregate demand, as compared with the one we appreciate during expansionary phases. 
As a result, a downward shift of the demand schedule tends to predominantly reflect into 
a drop of the inflation rate, ceteris paribus.
7 Concluding Remarks

A vast empirical evidence shows that output and prices react asymmetrically to monetary policy innovations over contractionary and expansionary phases of the business cycle. It is a well-established finding that monetary policy has stronger effects on the GDP during contractions, as compared with expansions. As to price responses, these are not statistically different across different stages of the cycle. This paper shows that embedding prospect theory into an otherwise standard dynamic general equilibrium model may rationalize these facts. Loss-averse consumption preferences imply state-dependent degrees of real rigidity and elasticity of intertemporal substitution in consumption that generate competing effects on the responses of output and inflation following a monetary innovation. The qualitative and quantitative analyses return predictions that are in line with the empirical evidence. Output responses to a monetary tightening are greater in contractions as compared with expansions. Despite the amplification of output responses, downward wage rigidity induced by loss-averse preferences tends to attenuate inflation responses during negative growth cycles. As a consequence, we cannot detect statistically relevant differences in inflation responses over alternative cyclical phases. In addition, the model can successfully reproduce empirically relevant non-linearities in the reaction of output and inflation to monetary innovations with opposite signs. A rise in the nominal rate of interest increases the chances to trigger or deepen a contractionary movement in output, as compared with a loose monetary stance. Therefore, unexpected monetary contractions have greater effects on output, as compared with the impact induced by positive shocks of the same absolute size. By contrast, inflation displays nearly symmetric responses. In this context the optimal monetary policy under Ramsey planning imposes to weigh the distortion stemming from price rigidity with the one induced by external habits in consumption. Importantly, loss averse preferences enhance the consumption externality during contractions, so that the Central Bank is challenged with a state-dependent policy trade-off.
References


Debelle, G., Laxton, D., 1997. Is the phillips curve really a curve? some evidence for canada, the united kingdom, and the united states. IMF Staff Papers 44 (2), 249–82.


Tables and Figures

<table>
<thead>
<tr>
<th>Table 1. Phillips curve regressions</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>SPF Inflation Forecasts</strong></td>
</tr>
<tr>
<td>δ₀</td>
</tr>
<tr>
<td>(0.0790)</td>
</tr>
<tr>
<td>δ₁</td>
</tr>
<tr>
<td>(0.0432)</td>
</tr>
<tr>
<td>δ₂</td>
</tr>
<tr>
<td>(0.0815)</td>
</tr>
</tbody>
</table>

| **EWMA Inflation Forecasts**       | OLS    | IV        |
| δ₀                                | -0.4957*** | -0.5078*** |
| (0.0670)                          | (0.0697)  |
| δ₁                                | 0.3109*** | 0.3315*** |
| (0.0366)                          | (0.0410)  |
| δ₂                                | -0.1441** | -0.1693** |
| (0.0691)                          | (0.0873)  |

Notes. P-values in parentheses. ***/**/* denote statistical significance at the 1%/5%/10% level, respectively.
<table>
<thead>
<tr>
<th>Expansion ($E_t \xi_{t+1} = \xi_t = 1$)</th>
<th>Contraction ($E_t \xi_{t+1} = \xi_t = 0$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>IS $y_t = E_t y_{t+1} - (1 + (1 - \alpha) \theta)^{-1} (i_t - E_t \pi_{t+1})$</td>
<td>$y_t = E_t y_{t+1} - \lambda (1 - \alpha) \theta^{-1} (i_t - E_t \pi_{t+1})$</td>
</tr>
<tr>
<td>NKPC $\pi_t = \beta E_t \pi_{t+1} + \kappa (1 + (1 - \alpha) \theta) y_t - \kappa (1 + \eta) z_t$</td>
<td>$\pi_t = \beta E_t \pi_{t+1} + \kappa \lambda^{-1} (1 + \eta) - \theta (1 - \alpha)) y_t - \kappa (1 + \eta) z_t$</td>
</tr>
</tbody>
</table>

Notes. The coefficient $\kappa$ equals $\varepsilon \chi \varphi^{-1}$. 
Table 3. Tests for asymmetry of the cycle.

<table>
<thead>
<tr>
<th></th>
<th>$Skew(y_t)$</th>
<th>$Skew(\Delta y_t)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Technology and Monetary Shocks</td>
<td>-0.1838</td>
<td>-0.2789</td>
</tr>
<tr>
<td></td>
<td>[0.00010]</td>
<td>[0.00005]</td>
</tr>
<tr>
<td>Technology Shocks</td>
<td>-0.1894</td>
<td>-0.2831</td>
</tr>
<tr>
<td></td>
<td>[0.00011]</td>
<td>[0.00005]</td>
</tr>
<tr>
<td>Monetary Policy Shocks</td>
<td>-0.5232</td>
<td>-0.2468</td>
</tr>
<tr>
<td></td>
<td>[0.00008]</td>
<td>[0.00005]</td>
</tr>
</tbody>
</table>

Notes. Table 3 reports some tests for asymmetry in the cyclical behavior of the simulated output series, conditional on different shock configurations. The first column reports the skewness of detrended output (a measure of deepness), whereas the second column reports the skewness of the growth rate of detrended output (a measure of steepness). Standard deviations are reported in square brackets. All statistics are significant at the 1% level.
Figure 1. Responses to a monetary tightening: contractions vs. expansions.

Notes. The green (light) line graphs the responses to a monetary tightening during an expansionary regime, while the blue (dark) line is associated with a contractionary regime. The dotted lines are one standard deviation confidence intervals.
Notes. Figure 2 graphs the output gap (continuous line), the real wage (dotted line), and the real wage in deviation from its HP(1600) trend (dashed line) during contractions (in percentage points).
Notes. Gain-loss function (LHS panel) and its first-order derivative (RHS panel), for $\theta = 1$ and different values of $\lambda$. 
Notes. The red-dashed-dotted line corresponds to the labor demand schedule; the blue-continuous line is the labor supply schedule under loss-averse preferences ($\alpha = 0.5$), while the blue-dashed line is the labor supply schedule under standard neoclassical preferences ($\alpha = 1$). We set $\eta = 0.25$, $\theta = 1$ and $\lambda = 2.25$. 
Figure 5 Impulse responses to monetary policy shocks in different cyclical phases.

Notes. The continuous line denotes the response to a (one standard deviation) negative monetary shock during a contractionary regime; the dashed line denotes the response to a (one standard deviation) positive monetary shock during an expansionary regime. To obtain these responses, we preliminary generate contractions and expansions by perturbing the system with a (one standard deviation) technology shock of the appropriate sign. Concurrently, we induce a (one standard deviation) monetary policy shock. Finally, we subtract the reaction to the technology shock from the overall response of the system to both sources of exogenous perturbation, so as to isolate the effects of monetary innovations.
Figure 6. Impulse responses to technology shocks of different signs and magnitudes.

Notes. We portray equilibrium dynamics in response to positive and negative technology shocks of different magnitudes, specifically: 1.5 standard deviation (continuous line), 1 standard deviation (dashed line), 0.5 standard deviation (dotted line). The green (light) line is associated with a positive technology shock, while the blue (dark) line is associated with a negative technology shock.
Notes. The blue-continuous line portrays cumulative responses to monetary policy innovations, while the green-dashed line accounts for impact responses. The magnitude of the monetary policy innovation is measured over the x-axis.
Notes. We portray equilibrium dynamics in response to positive and negative technology shocks under the optimal commitment policy. The green (light) line is associated with a positive technology shock, while the blue (dark) line is associated with a negative disturbance. We consider three different magnitudes for the technology shock: 1.5 standard deviation (continuous line), 1 standard deviation (dashed line), 0.5 standard deviation (dotted line). Figure 8a is obtained by excluding endogenous external habits from households’ preferences ($\gamma = 0$), while Figure 8b obtains under the baseline parameterization ($\gamma = 0.9$).
Technical Appendix

Appendix A: Contractions and Expansions in the U.S. Business Cycle

Figure A1. Turning points in the U.S. business cycle.

Notes. Upper panel: the blue (continuous) line represents the output gap, the green (dashed) line is a centered Moving Average (MA) of the output gap, the red (dotted) line is a Band Pass (BP) filtered log-real GDP. Bottom panel: transition probabilities corresponding to the two filtered series. Recessionary episodes as identified by the NBER are denoted by the vertical bands. We set $\kappa = 12$ to account for the speed of regime switching. The results are robust to alternative values of this coefficient.
Figure A2. Responses to a monetary tightening: contractions vs. expansions

Contractions/expansions indicator: MA filtered log-real GDP.

Notes. The green (light) line graphs the responses to a monetary tightening during an expansionary regime, while the blue (dark) line is associated with a contractionary regime. The dotted lines are one standard deviation confidence intervals.
Figure A3. Responses to a monetary tightening: contractions vs. expansions.
Contractions/expansions indicator: BP filtered log-real GDP (one period-lagged indicator).

Notes. The green (light) line graphs the responses to a monetary tightening during an expansionary regime, while the blue (dark) line is associated with a contractionary regime. The dotted lines are one standard deviation confidence intervals.
Appendix B: Log-linear State-Dependent System

This appendix reports the model linearized around the non-stochastic steady state. For clarity of exposition, we present the equations describing private sector’s behavior in each of the four possible states depending on consumption dynamics.

The IS Curve

We linearize the Euler equation in the neighborhood of the steady state consistent with $C/X = 1$, obtaining the following state-dependent system of linearized IS curves:

$$y_t = \begin{cases} \frac{1+(1-\alpha)\theta}{1+(1-\alpha)(1+\gamma)} E_i y_{t+1} + \frac{(1-\alpha)\gamma}{1+(1-\alpha)(1+\gamma)} y_{t-1} - \frac{i_t - E_i\pi_{t+1}}{1+(1-\alpha)(1+\gamma)} & \text{iff } E_i\xi_{t+1} = \xi_t = 1 \\ \frac{1-(1-\alpha)\theta}{1+(1-\alpha)\theta(1-\gamma)} E_i y_{t+1} + \frac{(1-\alpha)\gamma}{1+(1-\alpha)\theta(1-\gamma)} y_{t-1} - \frac{i_t - E_i\pi_{t+1}}{1+(1-\alpha)\theta(1-\gamma)} & \text{iff } E_i\xi_{t+1} = 0, \xi_t = 1 \\ \frac{1+(1-\alpha)\theta}{1-(1-\alpha)\theta(1+\gamma)^2} E_i y_{t+1} - \frac{(1-\alpha)\gamma}{1-(1-\alpha)\theta(1+\gamma)^2} y_{t-1} - \frac{i_t - E_i\pi_{t+1}}{1-(1-\alpha)\theta(1+\gamma)^2} & \text{iff } E_i\xi_{t+1} = 1, \xi_t = 0 \\ \frac{1-(1-\alpha)\theta}{1-(1-\alpha)(1+\gamma)^2} E_i y_{t+1} - \frac{(1-\alpha)\gamma}{1-(1-\alpha)(1+\gamma)^2} y_{t-1} - \frac{i_t - E_i\pi_{t+1}}{1-(1-\alpha)(1+\gamma)^2} & \text{iff } E_i\xi_{t+1} = \xi_t = 0 \end{cases}$$

where we have aggregated across individuals (imposing homogeneity) and used the goods market clearing condition, $Y_t = C_t$.

When it comes to linearize the model economy in the neighborhood of $C/X = 1$, it is important to note that $G'(Z)$ presents an ordinary double point at $Z = 0$. As such, $G'(Z)$ is not purely differentiable in that point, as also implied by property (i). Therefore, standard linear approximation techniques such as the Taylor expansion do not immediately apply in this case. However, we can resort to a first-order approximation of $G'(Z)$ by computing an affine global underestimator, thus determining the subgradients of the marginal utility function at $Z = 0$. A subgradient determines a support hyperplane to the graph of the function under scrutiny. In such a case the corresponding subdifferential is a direct generalization of the differentiable case. For a convex and non necessarily differentiable function $f : \mathbb{R}^n \to \mathbb{R}$, the subdifferential at $x_0$ is defined as $\partial f(x_0) = \{g \in \mathbb{R} : f(x) \geq f(x_0) + \langle g, x - x_0 \rangle\}$. Thus, $g \in f(x_0)$ is subgradient in $x_0$.

In our case it is straightforward to note that at $Z = 0$ there will be a single subgradient for each branch of the function under scrutiny. To gain intuition on this, we can re-write the marginal utility as $G'(Z) = \min \{G'_A(Z), G'_B(Z)\}$ for $Z \in \mathbb{R}$, where $G'_A(Z)$ and $G'_B(Z)$ are the functions that encompass the arms of marginal utility corresponding to $Z > 0$ and $Z < 0$, respectively. These functions are both convex. It is also easy to see that $G'_B(Z) > G'_A(Z)$ for $Z \in \mathbb{R}^+$ and $G'_B(Z) < G'_A(Z)$ for $Z \in \mathbb{R}^-$. Hence, our

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approach amounts to a piecewise linear approximation in the neighborhood of \( Z = 0 \). Note also that assuming a smooth gain-loss function \( G(Z) \) at \( Z = 0 \) allows us to obtain a continuous first derivative function, which improves the approximation around the point \( Z = 0 \), compared with what would happen, say, with a linear gain-loss function, which implies a discontinuity at \( G'(0) \).

**Inflation Dynamics**

After applying some trivial algebra we retrieve a log-linearized expression for the real marginal cost:

\[
\omega_t = \begin{cases} 
(\eta + 1 + (1 - \alpha) \theta) y_t - (1 - \alpha) \theta \gamma y_{t-1} & \text{iff } \xi_t = 1 \\
(\eta + 1 - (1 - \alpha) \frac{\theta}{\lambda}) y_t + (1 - \alpha) \frac{\theta}{\lambda} \gamma y_{t-1} & \text{otherwise}
\end{cases}
\]

Thus the piecewise linear NKPC reads as:

\[
\pi_t = \beta E_t \pi_{t+1} - \varepsilon \chi \varphi^{-1} (1 + \eta) z_t + \varepsilon \chi \varphi^{-1} \begin{cases} 
(\eta + 1 + (1 - \alpha) \theta) y_t - (1 - \alpha) \theta \gamma y_{t-1} & \text{iff } \xi_t = 1 \\
(\eta + 1 - (1 - \alpha) \frac{\theta}{\lambda}) y_t + (1 - \alpha) \frac{\theta}{\lambda} \gamma y_{t-1} & \text{otherwise}
\end{cases}
\]
Appendix C: Model Solution

The solution to the model is a function that maps the minimum set of state variables into values for the endogenous variables. Implementation of the map algorithm begins by taking the initial rules for inflation and the output gap, \( \hat{h}^\pi (z_t, \mu_t, y_{t-1}) = \pi_t \) and \( \hat{h}^y (z_t, \mu_t, y_{t-1}) = y_t \). We then substitute them, together with the interest rate rule, into the functions describing private sector behavior, yielding:

\[
y_t = \phi_1 (E_t \xi_{t+1}, \xi_t) E_t \left[ \hat{h}^y (z_{t+1}, \mu_{t+1}, y_t) \right] + \phi_2 (E_t \xi_{t+1}, \xi_t) y_{t-1} - \phi_3 (E_t \xi_{t+1}, \xi_t) \left( r_{\Pi} \pi_t + \mu_t - E_t \left[ \hat{h}^\pi (z_{t+1}, \mu_{t+1}, y_t) \right] \right),
\]

\[
\pi_t = \beta E_t \left[ \hat{h}^\pi (z_{t+1}, \mu_{t+1}, y_t) \right] + \psi_1 (E_t \xi_{t+1}, \xi_t) y_t + \psi_2 (E_t \xi_{t+1}, \xi_t) y_{t-1} - \varepsilon \chi \varphi^{-1} (1 + \eta) z_t,
\]

This system translates into:

\[
y_t = \phi_1 (E_t \xi_{t+1}, \xi_t) \int_{a_z}^{b_z} \phi \left( z; \sigma^2_z \right) \int_{a_\mu}^{b_\mu} \phi \left( \mu; \sigma^2_\mu \right) \hat{h}^y (z, \mu, y_t) d\mu \, dz + \phi_2 (E_t \xi_{t+1}, \xi_t) y_{t-1} - \phi_3 (E_t \xi_{t+1}, \xi_t) \left( r_{\Pi} \pi_t + \mu_t - \int_{a_z}^{b_z} \phi \left( z; \sigma^2_z \right) \int_{a_\mu}^{b_\mu} \phi \left( \mu; \sigma^2_\mu \right) \hat{h}^\pi (z, \mu, y_t) d\mu \, dz \right),
\]

\[
\pi_t = \beta \int_{a_z}^{b_z} \phi \left( z; \sigma^2_z \right) \int_{a_\mu}^{b_\mu} \phi \left( \mu; \sigma^2_\mu \right) \hat{h}^\pi (z, \mu, y_t) d\mu \, dz + \psi_1 (E_t \xi_{t+1}, \xi_t) y_t + \psi_2 (E_t \xi_{t+1}, \xi_t) y_{t-1} - \varepsilon \chi \varphi^{-1} (1 + \eta) z_t,
\]

where \( \phi (\cdot) \) is the normal density, \( a_i = -3\sigma_i^2 \) (\( i = z, \mu \)) and \( b_i = 3\sigma_i^2 \) (\( i = z, \mu \)). Expectations are evaluated using trapezoid integration. Linear interpolation is then used to evaluate \( \hat{h}^y (z_i, \mu_k, y_{t-1}) \) and \( \hat{h}^\pi (z_i, \mu_k, y_{t-1}) \) for \( i = 1, 2, ..., N_z \) and \( k = 1, 2, ..., N_\mu \), where \( N_i \) (\( i = z, \mu \)) denotes the number of nodes in each shock dimension. The state vector and the decision rules are taken as given when solving the system. The system is then solved for every set of state variables over a discrete partition of the state space. This procedure is repeated until the iteration improves the current decision rules at any given state vector by less than some convergence criterion, that we set to \( 1e^{-8} \). Note that to initialize the algorithm, we start from the solution of the framework without loss-averse preferences, but also check that the final solution is not sensitive to initial conditions by perturbing these initial conditions. The final solution is invariant with respect to
perturbations in the initial rules, suggesting the solution is locally unique.

In this setting, the probability of future regimes can be characterized. For instance, assume that we are interested in computing the probability that consumption expands above its reference level in the current period and it is expected to do so even in the next period. This amounts to compute the probability that shocks that buffet the system in the current period will not cause consumption to fall below the reference level, neither at time $t$ nor at time $t+1$, conditional on the information set available at time $t$. To provide an example, we rule out monetary policy non-systematic responses, so as to assume that endogenous switching is not influenced by monetary policy shocks. In this setting, the smallest innovation to the supply shock process necessary to induce $E_t \xi_{t+1} = \xi_t = 1$ is given by the solution to:

$$\min_{\epsilon_t^*} \left[ f(\epsilon_t^*), g(\epsilon_t^*) \right] \text{ s.t. } E_t y_{t+1} \geq \gamma y_t \text{ and } y_t \geq \gamma y_{t-1}, \quad (20)$$

where:

$$f(\epsilon_t^*) = E_t h^y (\rho_z (\rho_z z_{t-1} + \epsilon_t^*) + \epsilon_t^{*+1} h^y (\rho_z z_{t-1} + \epsilon_t^*; y_{t-1})) - \gamma h^y (\rho_z z_{t-1} + \epsilon_t^* h^y (z_{t-1}, y_{t-2})), \quad (21)$$

$$g(\epsilon_t^*) = h^y (\rho_z z_{t-1} + \epsilon_t^*; y_{t-1}) - \gamma h^y (z_{t-1}, y_{t-2}). \quad (22)$$

Therefore, the probability that both $E_t y_{t+1} \geq \gamma y_t$ and $y_t \geq \gamma y_{t-1}$ is:

$$\Pr \left[ E_t \xi_{t+1} = \xi_t = 1 \mid \Theta_t \right] = \int_{\epsilon_t^{*+}}^{\epsilon_t^{*}} \phi(\epsilon^*; \sigma^2) d\epsilon^*, \quad (24)$$

where $\epsilon^*$ is a positive truncation point and $\epsilon_t^{*+}$ is the solution to the minimization problem.
Appendix D: Ramsey Optimal Policy

We consider the optimal policy problem (15). The first order necessary condition with respect to \( \{ R_t \} \) is:

\[
\frac{\Lambda_1}{R_t} = 0, \quad t = 0, 1, 2, ...
\]  

(25)

From this it is immediately evident that \( \Lambda_1 = 0, \, t = 0, 1, 2, ... \) The remaining first order conditions are as follows:

\[
C_t : 0 = U_C (C_t, X_t | \xi_t) - \Lambda_{2,t} \frac{\epsilon \chi N^p U_{CC} (C_t, X_t | \xi_t)}{Z_t [U_C (C_t, X_t | \xi_t)]^2} \] 
\[
- \Lambda_{2,t} \beta \varphi E_t \left\{ \frac{U_C (C_{t+1}, X_{t+1} | \xi_{t+1}) U_{CC} (C_t, X_t | \xi_t)}{[U_C (C_t, X_t | \xi_t)]^2} Z_{t+1} N_{t+1} (\Pi_{t+1} - 1) \Pi_{t+1} \right\} 
\]
\[
+ \Lambda_{2,t-1} \frac{\varphi}{U_C (C_{t-1}, X_{t-1} | \xi_{t-1})} \frac{Z_t N_t}{Z_{t-1} N_{t-1}} (\Pi_t - 1) \Pi_t - \Lambda_{3,t} - \gamma \beta E_t A_{4,t+1} C_t^{-1},
\]

(26)

\[
X_t : 0 = U_X (C_t, X_t | \xi_t) - \Lambda_{2,t} \frac{\epsilon \chi N^p U_{CX} (C_t, X_t | \xi_t)}{Z_t [U_C (C_t, X_t | \xi_t)]^2} \] 
\[
- \Lambda_{2,t} \beta \varphi E_t \left\{ \frac{U_C (C_{t+1}, X_{t+1} | \xi_{t+1}) U_{CX} (C_t, X_t | \xi_t)}{[U_C (C_t, X_t | \xi_t)]^2} Z_{t+1} N_{t+1} (\Pi_{t+1} - 1) \Pi_{t+1} \right\} 
\]
\[
+ \Lambda_{2,t-1} \frac{\varphi}{U_C (C_{t-1}, X_{t-1} | \xi_{t-1})} \frac{Z_t N_t}{Z_{t-1} N_{t-1}} (\Pi_t - 1) \Pi_t + A_{4,t},
\]

(27)

\[
N_t : 0 = -\chi N^p_t + \Lambda_{2,t} \frac{\epsilon \eta \chi N^p_{t-1}}{Z_t U_C (C_t, X_t | \xi_t)} \] 
\[
- \Lambda_{2,t} \beta \varphi E_t \left\{ \frac{U_C (C_{t+1}, X_{t+1} | \xi_{t+1})}{U_C (C_t, X_t | \xi_t)} (\Pi_{t+1} - 1) \Pi_{t+1} \frac{Z_{t+1} N_{t+1}}{Z_{t+1} N_{t+1}^2} \right\} 
\]
\[
+ \Lambda_{2,t-1} \frac{\varphi}{U_C (C_{t-1}, X_{t-1} | \xi_{t-1})} \frac{Z_t}{Z_{t-1} N_{t-1}} (\Pi_t - 1) \Pi_t + \Lambda_{3,t} Z_t \left[ 1 - \frac{\varphi}{2} (\Pi_t - 1)^2 \right],
\]

(28)

\[
\Pi_t : 0 = - \left[ \Lambda_{2,t} - \Lambda_{2,t-1} \frac{U_C (C_t, X_t | \xi_t)}{U_C (C_{t-1}, X_{t-1} | \xi_{t-1})} \frac{Z_t N_t}{Z_{t-1} N_{t-1}} \right] [\varphi (2\Pi_t - 1)] - \Lambda_{3,t} \varphi (\Pi_t - 1) Z_t N_t.
\]

(29)

The first order conditions with respect to the Lagrange multipliers are given by:
\[ \lambda_{1,t} : 0 = \frac{1}{R_t} - \beta E_t \left\{ \frac{U_C(C_{t+1}, X_{t+1} | \xi_{t+1})}{\Pi_{t+1} U_C(C_t, X_t | \xi_t)} \right\}, \]  
\[ \lambda_{2,t} : 0 = (1 + \tau) (1 - \varepsilon) + \varepsilon \frac{\chi N_t}{Z_t U_C(C_t, X_t | \xi_t)} \]
\[- \varphi (\Pi_t - 1) \Pi_t + \varphi M_{t,t+1} (\Pi_{t+1} - 1) \Pi_{t+1} \frac{Z_{t+1} N_{t+1}}{Z_t N_t}, \]
\[ \lambda_{3,t} : 0 = Z_t N_t - C_t - Z (\Pi_t - 1)^2 Z_t N_t, \]
\[ \lambda_{4,t} : 0 = X_t - C_{t-1}. \]

**Ramsey Steady State**

It is immediate to verify that the Ramsey problem leads to the efficient (social planner’s) allocation for \( \lambda_{1,t} = \lambda_{2,t} = 0 \). Therefore, in the steady state the first order conditions (26)-(28) reduce to:

\[ 1 - \lambda_3 - \lambda_4 \gamma \beta = 0, \]  
\[ - (1 - \alpha) + \lambda_4 = 0, \]  
\[ - \chi + \lambda_3 = 0. \]

Equations (35) and (36) in this system provide us with a solution for \( \lambda_3 \) and \( \lambda_4 \):

\[ \lambda_3 = \chi, \]  
\[ \lambda_4 = 1 - \alpha. \]

These can be plugged into (34), so as to find a value of \( \chi \) that is consistent with the allocation \( C = X \):

\[ \chi = 1 - (1 - \alpha) \gamma \beta. \]

Equation (29) makes it clear that \( \Pi = 1 \), so that the steady state counterpart of (31) implies:

\[ 0 = (1 + \tau) (1 - \varepsilon) + \varepsilon \frac{\chi N_t}{U_C}. \]
where $U_C$ denotes the first-order derivative of $U$ with respect to $C_t$, evaluated at the steady state. Therefore, given (39), we find a coherent $\alpha$ value for the subsidy:

$$
\tau = \frac{1 - (1 - \alpha) \epsilon \gamma \beta}{\epsilon - 1}.
$$

Note that setting $\gamma = 0$ amounts to removing external habits and their inherent externality associated with consumption fluctuations. In this case the subsidy reduces to $1/(\epsilon - 1)$, as in the baseline New Keynesian model where the only distortion arises from monopolistic competition in the goods market.
Appendix E: Alternative Monetary Policy Rule

This appendix replicates Figures 5, 6, and 7 in the manuscript under the assumption that the Central Bank adjusts the policy rate in response to both the rate of inflation and the output term:

\[
\frac{R_t}{R} = \left( \frac{\Pi_t}{\Pi} \right)^{\rho_{\Pi}} \left( \frac{Y_t}{Y} \right)^{\rho_{Y}} \exp(\mu_t), \tag{40}
\]

where we set \( \rho_{Y} = 0.5 \), while the other coefficients are set in accordance with the baseline parameterization.

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Figure E1. Impulse responses to monetary policy shocks in different cyclical phases.

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Notes. The continuous line denotes the response to a (one standard deviation) negative monetary shock during a contractionary regime; the dashed line denotes the response to a (one standard deviation) positive monetary shock during an expansionary regime. To obtain these responses, we preliminary generate contractions and expansions by perturbing the system with a (one standard deviation) technology shock of the appropriate sign. Concurrently, we induce a (one standard deviation) monetary policy shock. Finally, we subtract the reaction to the technology shock from the overall response of the system to both sources of exogenous perturbation, so as to isolate the effects of monetary innovations.
Figure E2. Impulse responses to technology shocks of different signs and magnitudes.

Notes. We portray equilibrium dynamics in response to positive and negative technology shocks of different magnitudes, specifically: 1.5 standard deviation (continuous line), 1 standard deviation (dashed line), 0.5 standard deviation (dotted line). The green (light) line is associated with a positive technology shock, while the blue (dark) line is associated with a negative technology shock.
Notes. The blue-continuous line portrays cumulative responses to monetary policy innovations, while the green-dashed line accounts for impact responses. The magnitude of the monetary policy innovation is measured over the x-axis.