An ESPRIT-Based Approach for RF Fingerprint Estimation in Multi-Antenna OFDM Systems

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***Abstract*—Estimation of radio frequency fingerprint (RFF) of a transmitter from the received signal is often affected by the wireless communication channel. The existing approaches mainly rely on the channel reciprocity to obtain the RFF. In this letter, we propose a novel method to extract the RFF from the received signal by estimating the signal parameters using the rotational invariance technique (ESPRIT), which takes advantage of the multiple receiver antennas. We prove that the RFF estimation is independent of the wireless channel reciprocity. Numerical results show that the RFF can be accurately extracted from the received signal, and that the proposed algorithm has excellent performance.**

***Index Terms*—ESPRIT, multi-antenna, OFDM, RF fingerprint, reciprocity calibration.**

I. Introduction

RADIO frequency fingerprint (RFF) is the natural attribute of a RF device. It mainly refers to the physical characteristics integrated within the analog circuit. This fingerprint causes distortions to signals. In broadband systems, the spectrum of the in-band signal is barely flat. In addition, the RFFs for different devices are different. Even if the propagation channel is reciprocal, the channel considering RFF is not [1]. Therefore, RFF can reduce the accuracy of channel estimation and thus, degrade the system performance. On the other hand, RFF can be used as a means of device identification and authentication [2].

Estimation of RFF is important in improving the channel reciprocity of time division duplexing (TDD) systems to enhance the system throughput and security. In many works, the process of eliminating or obtaining RFF is known as reciprocity calibration [3]. For example, hardware calibration requires additional equipment or calibration modules. Over-the-air calibration extracts RFF using channel reciprocity. Using the channel reciprocity, for TDD systems, the RFF can be calculated from the feedback of channel state information [4], [5]. Further, pre-coding matrix can be designed to eliminate the RFF [6]. The reciprocity-based method is limited by coherence time, asymmetric interference and information feedback overhead. It can't be used for frequency division duplexing (FDD) systems directly either. Meanwhile, it is essential for wireless communication systems to obtain RFF for scaling gain.

This work was supported in part by National Natural Science Foundation of China 61571110. Dong Wang, Aiqun Hu, Yan Wang, Xiaohu You are with the Southeast University, China. Yunfei Chen is with the University of Warwick, UK. e-mail: {wd, aqhu, yanwang, xhyu}@seu.edu.cn, Yunfei.Chen@warwick.ac.uk. Aiqun Hu is the corresponding author.

In this letter, we propose a new method of estimating RFF by using array signal processing for multi-antenna orthogonal frequency division multiplexing (OFDM) systems. The ESPRIT is a classical algorithm for spectrum estimation, and recent works have focused on its use in the massive MIMO systems [7], [8]. However, its characteristic of signal subspace reconstruction has not been fully explored, especially in RFF estimation. We reconstruct the autocorrelation matrix of the signal subspace of each subcarrier, and then obtain the RFF by sorting the energy values on all subcarriers. The proposed method does not depend on channel reciprocity and is shown to be accurate.

II. System Model

Consider a transmitter (Tx) and a receiver (Rx) with one and M antennas, respectively, communicating over N_c subcarriers. Assume that the RF circuit of the receiver is calibrated. Thus, the RFF of the receiver can be ignored. This assumption does not lose any generality because the RFF of the receiver can be easily included in an equivalent filter. The terminal transmits a pilot sequence on different subcarriers as

$$\mathbf{x} = \begin{bmatrix} x(0) & x(1) & \cdots & x(N_c-1) \end{bmatrix}^T. \quad (1)$$

After a serial-to-parallel conversion and a N_c point-to-point inverse discrete Fourier transform (IDFT), the time domain OFDM symbol can be expressed as

$$s(n) = \frac{1}{\sqrt{N_c}} \sum_{k=0}^{N_c-1} x(k) e^{j\frac{2\pi}{N_c}kn}, n = 0, 1, \cdots, N_c - 1. \quad (2)$$

The cyclic prefix is then added, followed by a parallel-to-serial converter and digital-to-analog converter (DAC) to obtain the analog baseband signal $s'(n)$. The baseband signal is up-converted to a specified frequency band and transmitted through a RF circuit. The transmitting analog circuit (including the antenna) will induce the RFF. We model the RFF as a finite impulse response (FIR) filter [1] to study the amplitude distortion caused by the transmitter. The RFF performs convolution on transmitted signals in time domain. It is equivalent to the product of the spectra of RFF and transmitting signal in the frequency domain [9]. The transmitted signal with RFF can be expressed as

$$\begin{aligned} s_{tx}(n) &= s'(n) * h_{tx}(n) \\ x_{tx}(k) &= x(k) H_{tx}(k) \end{aligned} \quad (3)$$

where $h_{tx}(n)$ is the impulse response of the RFF, $H_{tx}(k)$ is the frequency response of the RFF, and $x_{tx}(k)$ is the IDFT of $s_{tx}(n)$. During transmission, each path introduces a time delay τ_l, and an attenuation of β_l. The received signal can be expressed as

$$r(n) = \sum_{l=1}^{L} \beta_l s_{tx}(n - \tau_l) + w(n) \tag{4}$$

where L is the number of multipath components and $w(n)$ denotes the additive white Gaussian noise. Note that the sub-channel gain is assumed constant for all subcarriers of the same path. This assumption is used in previous works [7], [8] and is also used here for fair comparison.

The receiver antenna array is a uniform linear array (ULA) with all antennas having omni-directional patterns. Assume that the source and the antenna array are in the same plane and the far field condition is satisfied.

III. Analysis of Received Signals

A. Phase rotation from delay

At the receiver, the OFDM signal is down-converted by the RF circuit and a digital baseband signal by analog-to-digital converter (ADC). After performing DFT transform and removing CP, the channel frequency response can be estimated. When the maximum channel delay spread is less than the CP length or $\max(\tau_l) < t_{CP}$, the multi-path delay does not introduce any inter-symbol interference (ISI) and only causes a subcarrier phase offset. The time domain signal of the l-th path after removing CP can be expressed as

$$\bar{r}_l(n) = \beta_l s_{tx}((n - \tau_l))_{N_c} G_{N_c}(n) \tag{5}$$

where $((.))_N$ indicates the N-point periodic extension, and $G_{N_c}(n)$ indicates the selection of the main sequence. The frequency domain signal of the l-th path can be expressed as

$$r_l(k) = DFT[\bar{r}_l(n)] = \beta_l g(k, \tau_l) r_1(k) \tag{6}$$

where $g(k, \tau_l) = e^{\varphi(k,l)} = \exp\left(-j\frac{2\pi}{N}k\tau_l\right)$ is the phase shift caused by the l-th path relative to the first arriving path, $g(k, \tau_l) = 1$ when l=1, and $r_1(k)$ is the frequency domain signal of the first path.

B. Phase rotation from AOA

In this letter, we assume that the received signal is a wideband signal. The time domain OFDM signal is a superposition of the frequency domain signals at all subcarriers. It is difficult to analyze the influence of the receiving array on each subcarrier from the combined signal. Therefore, we analyze each subcarrier in the frequency domain. The array manifolds of individual subcarrier are different, and the corresponding subcarrier sub-signal space is also different.

The phase shift of the k-th subcarrier on the m-th receiving antenna can be expressed as

$$\alpha_m(k, \theta_l) = \exp\left(-j2\pi(m-1)\frac{d}{\lambda_k}\sin\theta_l\right) \tag{7}$$

where θ_l is the AOA of the l-th path, d is the spacing between adjacent elements, and λ_k is the wavelength of the k-th subcarrier.

The phase difference between antennas is determined by the subcarrier frequency, signal's arrival angle, and the geometry of antenna array. Let $\omega(k, l) = -j2\pi\frac{d}{\lambda_k}\sin\theta_l$. The array response vector of the k-th subcarrier in the l-th path can be expressed as

$$\boldsymbol{a}(k, \theta_l) = \begin{bmatrix} 1 & e^{\omega(k,l)} & \cdots & e^{(M-1)\omega(k,l)} \end{bmatrix}^T. \tag{8}$$

The direction matrix $\boldsymbol{A}(k, \theta) \in \mathbb{C}^{M \times L}$ is defined as

$$\boldsymbol{A}(k, \theta) = \begin{bmatrix} \boldsymbol{a}(k, \theta_1) & \boldsymbol{a}(k, \theta_2) & \cdots & \boldsymbol{a}(k, \theta_L) \end{bmatrix}. \tag{9}$$

C. Decomposition of channel response

The frequency response of the k-th subcarrier on the m-th antenna can be expressed as

$$\begin{aligned} \hat{H}_m(k) &= H_m(k) + n_m(k) \\ H_m(k) &= \sum_{l=1}^{L} \alpha_m(k, \theta_l)\beta_l H_{tx}(k) g(k, \tau_l) H_{rx}(k, m) \end{aligned} \tag{10}$$

where $m = 1, 2, \cdots, M$, $n_m(k)$ is the additive white Gaussian noise on the m-th receiving antenna, and $H_{rx}(k, m)$ is the frequency response at the m-th receiving antenna. We have assumed that the receiver has been calibrated such that $H_{rx}(k, m) = 1$ [9]. Thus, $H_{rx}(k, m)$ is omitted in the following.

The frequency response of the k-th carrier can be expressed in the matrix form as

$$\begin{aligned} \hat{\boldsymbol{H}}(k) &= \boldsymbol{H}(k) + \boldsymbol{N}(k) \\ &= \boldsymbol{A}(k, \theta) H_{tx}(k)\boldsymbol{B}\boldsymbol{G}(k, \tau) + \boldsymbol{N}(k) \end{aligned} \tag{11}$$

$$\boldsymbol{B} = diag\left\{ \beta_1 \quad \beta_2 \quad \cdots \quad \beta_L \right\} \tag{12}$$

$$\boldsymbol{S}(k) = H_{tx}(k)\boldsymbol{B}\boldsymbol{G}(k, \tau) \tag{13}$$

$$\boldsymbol{G}(k, \tau) = \begin{bmatrix} g(k, \tau_1) & g(k, \tau_2) & \cdots & g(k, \tau_L) \end{bmatrix}^T \tag{14}$$

where $\hat{\boldsymbol{H}}(k), \boldsymbol{H}(k), \boldsymbol{N}(k) \in \mathbb{C}^{M \times 1}$, the complex gain matrix $\boldsymbol{B} \in \mathbb{C}^{L \times L}$, the path delay matrix $\boldsymbol{G}(k, \tau) \in \mathbb{C}^{L \times 1}$.

IV. RFF Estimation

A. Rank increase

The autocorrelation matrix $\boldsymbol{R}(k)$ of the k-th subcarrier in the frequency response is

$$\boldsymbol{R}(k) = \boldsymbol{A}(k, \theta)\boldsymbol{R}_S(k)\boldsymbol{A}(k, \theta)^H + \boldsymbol{R}_N \tag{15}$$

where $\boldsymbol{R}_S(k) = E\left\{\boldsymbol{B}H_{tx}(k)\boldsymbol{G}(k, \tau)\left[\boldsymbol{B}H_{tx}(k)\boldsymbol{G}(k, \tau)\right]^H\right\}$ and is a Hermitian matrix, $\boldsymbol{R}_N = \sigma^2\boldsymbol{I}$ is the autocorrelation matrix of the noise $\boldsymbol{N}(k)$, and σ^2 is the power of the additive white Gaussian noise.

Since the received signals are different copies of the same source, the coherent signal will lead to rank reduction of the covariance matrix. Since $\hat{\boldsymbol{H}}(k)$ is a one-dimensional vector, the rank of its autocorrelation matrix $\boldsymbol{R}(k)$ is 1. The eigen decomposition of $\boldsymbol{R}(k)$ has only one large eigenvalue. The rest are eigenvalues of noise. Therefore, it is necessary to increase the rank of the autocorrelation matrix $\boldsymbol{R}(k)$.

The forward and backward spatial smoothing algorithms can be used to remove the coherence between signals to increase the rank of $\boldsymbol{R}(k)$. Let subarray number P be greater than the multipath number L, so that the rank of $\boldsymbol{R}(k)$ is increased to P. The antenna number of subarray becomes M_0=M−P+1.

The direction matrix of subarray $\bar{\boldsymbol{A}}(k,\theta) \in \mathbb{C}^{M_0\times L}$ is defined as

$$\bar{\boldsymbol{A}} = \left[\begin{array}{cccc} \boldsymbol{a}(k,\theta_1) & \boldsymbol{a}(k,\theta_2) & \cdots & \boldsymbol{a}(k,\theta_L) \end{array}\right]. \quad (16)$$

The autocorrelation matrix is reconstructed by forward and backward spatial smoothing algorithm, and the smoothing array autocorrelation matrix $\bar{\boldsymbol{R}}(k) \in \mathbb{C}^{L\times L}$ is defined as

$$\bar{\boldsymbol{R}}(k) = \frac{1}{2P}\sum_{p=1}^{P}\left(\boldsymbol{R}_{k,p}^{f} + \boldsymbol{R}_{k,p}^{b}\right) = \bar{\boldsymbol{A}}\bar{\boldsymbol{R}}_S(k)\bar{\boldsymbol{A}}^H + \sigma^2\boldsymbol{I} \quad (17)$$

where $\boldsymbol{R}_{k,p}^{f}$ is the p-th forward autocorrelation matrix, $\boldsymbol{R}_{k,p}^{b}$ is the p-th backward autocorrelation matrix, and $\bar{\boldsymbol{R}}_S(k)$ is the smoothed version of $\boldsymbol{R}_S(k)$.

B. RFF estimation

The existing RFF estimation methods usually rely on channel reciprocity, and its efficiency will be greatly reduced when the channel reciprocity is not ideal. Therefore, we need a new method that does not depend on channel reciprocity. In this subsection, we perform total least squares (TLS) ESPRIT to calculate the multipath energy of each carrier, based on which the RFF can be obtained. The eigenvalue decomposition of the smoothing array autocorrelation matrix $\bar{\boldsymbol{R}}(k)$ is performed as

$$\begin{aligned} \bar{\boldsymbol{R}}(k) &= \sum_{i=1}^{M_0}\lambda_i\boldsymbol{e}_i{\boldsymbol{e}_i}^H = \sum_{i=1}^{L}\lambda_i\boldsymbol{e}_i{\boldsymbol{e}_i}^H + \sum_{i=1}^{M_0}\sigma^2\boldsymbol{e}_i{\boldsymbol{e}_i}^H \\ &= \sum_{i=1}^{L}\left(\lambda_i+\sigma^2\right)\boldsymbol{e}_i{\boldsymbol{e}_i}^H + \sum_{i=L+1}^{M_0}\sigma^2\boldsymbol{e}_i{\boldsymbol{e}_i}^H \\ &= \boldsymbol{U}_S\boldsymbol{\Sigma}_{\mathbf{S}}{\boldsymbol{U}_S}^H + \sigma^2\boldsymbol{U}_N{\boldsymbol{U}_N}^H \end{aligned} \quad (18)$$

where $\boldsymbol{\Sigma}_{\mathbf{S}}$ is a diagonal matrix consisting of the L largest eigenvalues, $\boldsymbol{e}_i$ is the eigenvector corresponding to the i-th eigenvalue λ_i. The eigenvectors corresponding to the L largest eigenvalues are expanded into the signal subspace $\boldsymbol{U}_S$. The eigenvectors corresponding to the $M_0 - L$ smallest eigenvalues are expanded into the noise subspace $\boldsymbol{U}_N$. The signal subspace $\boldsymbol{U}_S$ is orthogonal to the noise subspace $\boldsymbol{U}_N$. Since $span\{\boldsymbol{U}_S\} = span\{\boldsymbol{A}\}$, there exists a unique and nonsingular matrix that satisfies $\boldsymbol{U}_S = \bar{\boldsymbol{A}}\boldsymbol{T}$, $\boldsymbol{T} \in \mathbb{C}^{L\times L}$. The matrix $\boldsymbol{T}$ can be solved by using the TLS-ESPRIT algorithm from $\bar{\boldsymbol{R}}(k)$.

According to the properties of the signal subspace and the noise subspace, one has

$$\boldsymbol{U}_S{\boldsymbol{U}_S}^H + \boldsymbol{U}_N{\boldsymbol{U}_N}^H = \boldsymbol{I} \quad (19)$$

and,

$$\bar{\boldsymbol{R}}(k) = \boldsymbol{U}_S\boldsymbol{\Sigma}_{\mathbf{S}}{\boldsymbol{U}_S}^H + \sigma^2\left(\boldsymbol{I} - \boldsymbol{U}_S{\boldsymbol{U}_S}^H\right). \quad (20)$$

From Eq.(17) and Eq.(20),

$$\bar{\boldsymbol{A}}\bar{\boldsymbol{R}}_S(k)\bar{\boldsymbol{A}}^H = \boldsymbol{U}_S\left(\boldsymbol{\Sigma}_{\mathbf{S}} - \sigma^2\boldsymbol{I}\right)\boldsymbol{U}_S^H. \quad (21)$$

Combining $\boldsymbol{U}_S = \bar{\boldsymbol{A}}\boldsymbol{T}$ and Eq.(21), one has

$$\bar{\boldsymbol{R}}_S(k) = \boldsymbol{T}\left(\boldsymbol{\Sigma}_{\mathbf{S}} - \sigma^2\boldsymbol{I}\right)\boldsymbol{T}^H. \quad (22)$$

From Eq.(15) and Eq.(17), the diagonal elements of $\boldsymbol{R}_S$ and $\bar{\boldsymbol{R}}_S$ are equal, i.e.,

$$\boldsymbol{R}_S(i,j) = \bar{\boldsymbol{R}}_S(i,j), \;\; when \;\; i=j. \quad (23)$$

The $|\boldsymbol{B}h_{tx}(k)|$ part can be calculated from the square root of the diagonal elements. Since the subcarriers of the same path have the same channel gain β_l, the RFF of the transmitter can be obtained by considering all the subcarriers. Thus, the RFF estimate of the l-th path is

$$\boldsymbol{Rff}_l = \left[\begin{array}{cccc} |\beta_l H_{tx}(0)| & |\beta_l H_{tx}(1)| & \cdots & |\beta_l H_{tx}(N_c-1)| \end{array}\right]. \quad (24)$$

Eq.(19)-(24) give the solution of RFF estimation. In the eigenvalue decomposition of TLS-ESPRIT, the noise leads to abnormal values for some subcarriers that are significantly larger or smaller than values for adjacent subcarriers. In order to recover the authenticity of the data to improve the estimation performance, smoothing filter can be used to eliminate these abnormal values.

V. Numerical Result and Discussion

We present the Monte Carlo simulation to evaluate the RFF estimation performance of our algorithm. The main system parameters are given in Table I. Estimation performance are analyzed by normalizing the mean square error (MSE) and by the RFF successful identification percentage. Simulation results are averaged over 1000 independent simulation runs, with respect to normalizing the value to 0∼1. The smooth filter RLOWESS in Table I is a robust version of local regression using weighted linear least squares (LOWESS) that assigns lower weight to outliers in the regression.

TABLE I
Simulation Parameters

Parameters	Configurations
Path number	10
Receiver antennas	24 / 32 / 64 / 128
Receiver subarray number	11
Antenna spacing	one-half wavelength
Antenna array type	ULA
Subcarrier number	1024
Channel estimation method	minimum mean square error
Smooth filter	RLOWESS

We use a set of constrained equi-ripple FIR filters to simulate the non-flatness of the frequency response of transmitter RF circuit. The filter order is set to 30, the upper edge of the passband is 0.99 in the normalized frequency, and the stopband error is a constant of 0.01. We use different passband errors to represent 10 different filters. The passband error ranges from 0.01 to 0.055, with an interval of 0.005. In each simulation run, a filter is selected randomly from the 10 filters.

Fig.1 illustrates the average normalized MSE performance between the new ESPRIT based method and the conventional channel reciprocity based method.In the channel reciprocity based method, we assume that channel reciprocity is perfect. This is a benchmark for comparison when there is no

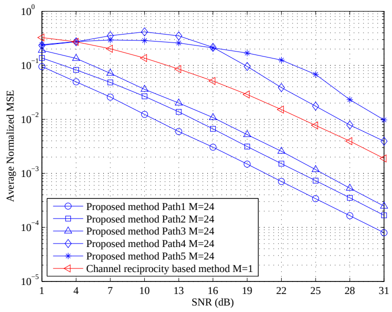

Fig. 1. The average normalized MSE vs. SNR.

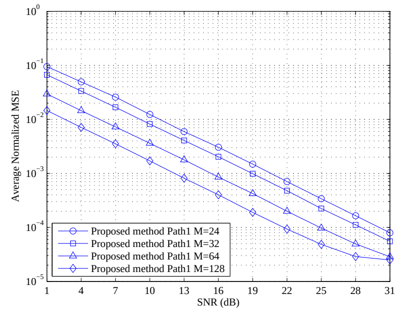

Fig. 2. The average normalized MSE of path 1 at four antenna configurations.

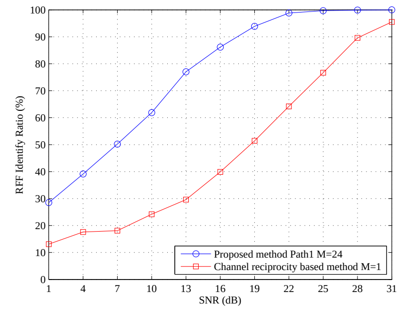

Fig. 3. The identification performance comparison against SNR.

asymmetric interference. Practical channel reciprocity based methods will be worse than this benchmark. For simplicity, only path 1 to path 5 of the ESPRIT based method are shown. Path 1 has the largest power, and the power of path decreases with the path index. Fig.1 demonstrates that the paths with large power are less affected by noise. When the signal-to-noise ratio (SNR) increases, the estimation performance is improved significantly. The proposed method has better MSE performance than that based on channel reciprocity for paths 1-3. It can be shown that the proposed method works well for other power delay profiles too, and that the path energy dominates the RFF estimation performance.

Fig.2 illustrates the average normalized MSE of path 1 at four antenna configurations. It is shown that our scheme works better for large numbers of antennas, such as massive MIMO, as the MSE decreases when M increases. Specifically, there is an SNR gain of 3dB at MSE=10^{-3} by increasing M from 64 to 128.

Fig.3 compares the identification performances of the E-SPRIT based method and channel reciprocity based method. The maximum correlation coefficient method is used to determine the filter used. The successful identification percentage of the proposed method can reach nearly 90% at SNR=16dB, while the channel reciprocity based method is only about 40%. When the number of antennas increases, the identification performance can be further improved.

Fig. 4 shows the RFF estimation performance in the presence of residual frequency offset. The frequency offset caused by the local oscillator offset and the Doppler spread will change the orthogonality of the subcarriers and thus degrades the RFF estimation performance. Therefore, frequency offset correction algorithm needs to be considered.

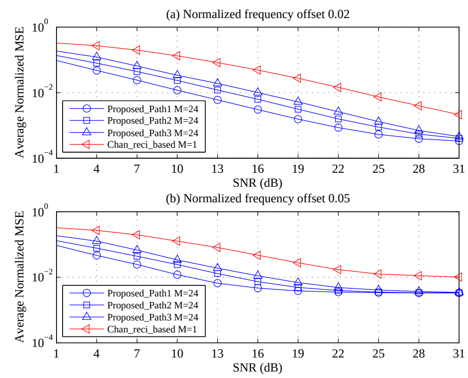

Fig. 4. RFF estimation performance in the presence of residual frequency offset, (a) Normalized frequency offset is 0.02, (b) Normalized frequency offset is 0.05.

VI. Conclusion

In this letter, an ESPRIT-based RF fingerprint estimation method for multi-antenna OFDM systems has been proposed, which does not depend on channel reciprocity. By analyzing the phase rotation caused by multipath delay and AOA, the decomposition of OFDM signal in antenna array has been derived. The proposed method takes advantage of the multiple antennas at receiver to decompose the received signal. Then, RFF can be obtained. The simulation results have shown that the proposed method can extract RFF from wireless channel.

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