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Activism, Costly Participation, and Polarization

Raghul S Venkatesh

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CRETA
Centre for Research in Economic Theory and its Applications

Department of Economics
University of Warwick, Coventry,
CV4 7AL, United Kingdom

warwick.ac.uk/fac/soc/economics/research/centres/creta/papers
Activism, Costly Participation, and Polarization*

Raghul S Venkatesh†

May 31, 2017

Abstract

I develop a model of activism and polarization in the context of electoral competition. Two candidates simultaneously announce policy platforms and seek the support of ideologically inclined activists. Activists compete to influence electoral outcomes by expending costly support for their respective candidates. The presence of activists always moderates the platform choice of candidates, compared to the case of no activism. The main finding is to provide conditions under which as activists’ ideological partisanship increases (decreases), polarization of candidate platforms reduces (widens) - meaning candidates may compromise even though their supporters become more extreme. I precisely characterize the conditions under which the presence of activism and increasing partisanship among activists are both welfare-improving for voters. Finally, I identify a novel crowding out effect of big money on the demand for activism. My analysis suggests public funding of elections as an important institutional reform that could mitigate the pernicious effects of high polarization.

Keywords: activism, electoral participation, downsian competition, influence seeking, public funding

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†Department of Economics, University of Warwick. Email: raghul.venkatesh@warwick.ac.uk
1 Introduction

“People at the top might devote time and resources to supporting a political party strongly opposed to redistribution. People at the bottom would have an opposite response.” McCarty, Poole and Rosenthal (Polarized America, 2006)

Activism is an important channel of political participation in representative democracies. Activists belonging to either sides of the political spectrum participate in campaigns to influence voters’ preferences about the platforms of candidates and therefore affect the electoral prospects of candidates. Further, this form of political activism is mostly ideological and partisan in nature (e.g., Aldrich (1983a)). Activists follow the ideological agenda set by their preferred candidate (or party) and provide their support during elections, without trying to directly set the platforms of candidates. However, by indirectly affecting the prospects of candidates in this way, activists influence their choice of platforms. Therefore, the mere presence of activists changes the nature of political competition and the extent of platform polarization.

Several instances of political activism can be cited. For example, Obama’s 2008 and 2012 campaigns were propelled by grass-roots activists and mobilization. Activists belonging to the Tea Party movement played a crucial role in the Republicans winning back the House of Representatives in the 2010 mid-term elections. Outside the US context, the 2015 state elections in New Delhi saw a fledgling party AAP win 95% of the seats by promising a platform of ending governmental corruption and nepotism. The campaign witnessed mass participation by grass roots activists and volunteers who were able to influence voters’ preferences towards supporting the newly formed party.

Given the importance and relevance of activism for political competition, it is surprising that there is little theoretical literature on political activism. Most of the literature has extensively studied the role of interest groups and lobbies (e.g., Grossman and Helpman (1996, 1999, 2001)). However, there is a fundamental distinction that has been ignored. Unlike lobbies, activists do not commit to policy-contingent (implicit) contracts. Rather, activists support a candidate taking as given the policy platforms announced by the candidate. This inability to commit to platform contingent support changes the nature of incentives and provides for novel trade-offs. Aldrich (1983a,b), in his seminal work, provides a rational-choice explanation for participation in activism. Though Aldrich’s work provides a framework for studying the motivations for becoming an activist, questions of how partisan activism influences political platforms and what affects the extent of activism remain pertinent, yet unanswered.

1The typical role of partisan activists include persuading voters, donating money, helping with door-to-door canvassing and leafleting, attending the national party convention, and mobilizing voters through grass-roots campaigning. See Norris (2007).
2More recently, Bernie Sanders’ campaign against the Democratic party’s front runner Hillary Clinton witnessed grass-roots participation from a wide array of activists.
3In the 1990’s, a similar activism led campaign by right-wing groups led to the formation of the first successful non-Congress national government in India, almost 50 years after its Independence.
My paper attempts to bridge this gap in the literature by developing a model of activism to shed light on the relationship between activism and platform polarization. I consider a Downs-Hotelling setup that captures the role of activists, the voting decisions of voters, and candidates’ platform selection in an unified framework. My model incorporates three key features: i) candidates care about ideology and benefits of office (they are “responsible”, in the spirit of Calvert (1985) and Wittman (1983)); ii) activists are ideological price-takers; and iii) activists persuade voters but face participation costs.

The political process proceeds as follows: candidates simultaneously announce platforms, party activists expend effort to influence voters given the set of platforms, and (median) voter decides whom to vote for; in that order. Candidates, when announcing platforms, and activists, when deciding on levels of participation, are unaware of the median voter’s preferred policy, which is drawn from a uniform distribution. Activist participation plays a role of direct influence and their effort affects the median voter’s preferences.\(^4\) Two important trade-offs emerge in this setup. Candidates trade off their ideology to elicit greater participation from activists, and activists trade off benefits from participation and the costs of doing so. Together, these twin trade-offs deliver a novel set of results.

First, I find that political polarization decreases in the presence of party activists, compared to settings with no activists. The result is driven by the fact that when activists are price-takers, they punish both their own candidate (by reducing effort) and the other candidate (increasing effort) for polarizing. As a result, competition between activists decreases equilibrium political polarization, irrespective of the activists’ ideological preferences. This suggests that unlike organized lobbies and interest groups (see Grossman and Helpman (2001, 1996)), the presence of activists moderates platforms and brings about political compromise. An important implication of this result is that activism disciplines the platform choice of candidates in electoral democracies. They do so by not only restricting the extent to which their preferred candidate polarizes, but by how much the other side polarizes as well.\(^5\)

Second, I characterize the relationship between increased partisanship between activists and platform polarization of candidates. This relationship is important given the current nature of the polarization debate in US politics.\(^6\) The main contribution of my work is the finding that increased partisanship between activists does not necessarily increase platform polarization of candidates. The nature of this relationship is determined by the activists’ willingness to engage in the political

\(^4\)See Norris (2002), Chapter 6, Table 6.4 for empirical evidence of a persuasive effect of campaign activism. More recently, Madestam et al. (2013) find evidence from the tea party activism of 2010 for a similar persuasive role. In this paper, I treat this kind of persuasive activism as analogous to influence seeking behavior in organizations (Milgrom (1988) and Milgrom and Roberts (1988)).

\(^5\)In some sense, this kind of punishment for polarization is the opposite of what Virág (2008) uncovers.

\(^6\)A cursory look at electoral campaigns in the US, especially from the 1960’s onwards, suggests a widening polarization between the elected representatives of the two major parties. For example, McCarthy, Poole and Rosenthal (2006), henceforth MPR, record this increasing polarization by looking at the roll-call votes in both the US Senate and House of representatives. They find a growing divide between Democrats and Republicans across issues, and a decrease in moderates in both chambers of the US congress.
process. Specifically, when activists’ willingness to engage in the political process exceeds a critical threshold, increased partisanship leads to greater convergence in candidate platforms, resulting in reduced political polarization.

The intuition for this result is the following. Activists trade off the marginal effect of higher participation and the marginal costs of doing so. As activists become more extreme and the willingness exceeds a threshold, the marginal costs of participation increases but is concave. Candidates understand these trade-offs. Specifically, when one candidate polarizes, the activist supporting that candidate reduces participation. Further, the opposing activist increases participation since the marginal benefit of increased activism is positive. When the elasticity exceeds the critical threshold, these effects are stronger. This decreases the benefits accruing to a candidate from polarizing, resulting in reduced political polarization in equilibrium.

Next, I investigate the effect of activist polarization on voters’ ex-ante welfare. There are two important sets of results. The first pertains to the introduction of activists and its effect on welfare. The second is the relationship between voter welfare and the level of activist polarization. From a normative angle, my analysis indicates that as long as the level of divergence in the absence of activism exceeds a threshold (the ex-ante welfare maximizing level of polarization), the introduction of activism can improve overall voter welfare. However, the presence of activists could also hurt voters: When the candidates already offer similar political choices, introducing activists makes the platforms converge further, to the point that it hurts voters’ welfare.

On a similar vein, as activists themselves polarize, the platforms of candidates could either diverge or converge. Depending on the extent of the resulting polarization, this may increase or decrease voter welfare. I precisely characterize the conditions under which increasing (decreasing) activist polarization decreases (increases) the welfare of voters. The intuition is the following. When activists’ elasticity of engagement is high enough, increased partisanship between activists decreases polarization of platforms by candidates. However, when the prevailing levels of polarization are already below the socially optimum level, as candidates moderate their platforms even further, the overall (ex-ante) welfare of voters decreases. This result suggests that democratic societies with greater barriers to political participation could actually benefit from increased partisanship among the political activists, when the existing choices provided by candidates are highly similar.

In Section 5, I consider two extensions. First, I investigate the role of activism in a noisy campaign, in the sense of Austen-Smith (1987). Activism, instead of influencing voters’ utility, plays an informative role. The median voter observes an imperfect (noisy) signal of the actual platform, and greater activism reduces the variance of this noise, rendering platforms more informative. The elasticity of engagement in this case is dependent on the efficiency of activism (in reducing the noise of platforms) and the participation-cost aversion of activists. The results in this modified setup are similar to the original game. Specifically, the noisy campaign game yields a unique equilibrium and the comparative statics results do not change.
Finally, I study the role of big money in the electoral competition game. Apart from seeking the support of party activists, suppose candidates are also endowed with campaign money (e.g. big donors, organized PACs or super PACs), which can be used as alternate resources for influencing voter preferences. That is, suppose that money and activist participation both perform similar roles and are substitutable goods. In this setting, my analysis suggests a novel crowding-out effect of big money on activism: a greater pool of resources decreases dependence on activism and reduces participation of activists, resulting in more polarized platforms. That is, the inability of activists to commit to policy-contingent contracts in exchange for resources is critical for achieving greater compromise.

My analysis suggests that introducing public funding of elections could be useful as a potential policy intervention. Public funding restricts the resources available to a candidate, limiting the crowding-out effect of big money. Capping the extent of campaign finance and restricting the influence of big money would help shorten the length of the campaign cycle, and increase the demand for grass-roots political activism. This way, the political process restores the dependence of candidates on activists and engenders greater consensus in the polity, resulting in reduced platform polarization.

The paper is organized as follows. In Section 2, I discuss related literature. Section 3 presents the benchmark model and characterizes the equilibrium of the electoral game. Section 4 analyzes the main result. Section 5 details the welfare results and Section 6 presents two extensions to the model. Concluding remarks follow in Section 7. All proofs are confined to the appendix.

2 Related Literature

Median voter convergence with office-motivated candidates, as propagated by Downs (1957) and Black (1986), has been widely considered a benchmark for the analysis of political competition. Subsequent work has however shown, under different conditions, the emergence of platform divergence in equilibrium. My paper is related to models of electoral competition that induce platform separation. Specifically, my paper looks into electoral activism as a possible channel for divergence.

My results on the welfare effects of polarization have a similar foundation to the work of Bernhardt, Duggan and Squintani (2009) (henceforth BDS). BDS consider the case for responsible parties in the presence of uncertainty around the median voter’s ideal policy. They present an important normative result – a small level of polarization actually improves voter welfare by increasing the platform choice available to voters. I introduce political participation (through activists) to this setup and consider the impact of activist polarization and participation on candidate platforms.

The role of activism in my work is similar to the models of influence studied by Milgrom (1988), Milgrom and Roberts (1988), and Meyer, Milgrom and Roberts (1992). While they study influence seeking in organizations, I apply analogous ideas to model political competition with activism. In my work, activists engage with voters in order to influence their choice of candidate, and further, this engagement in influence seeking is costly for the activists. This brings about a novel trade-off between the activists’ preferences and the extent of participation.

This paper is also related to the work on political participation of voters. On the theoretical side, this strand of literature could be broadly categorized into two classes of turnout models – turn-out driven by costly voting (Riker and Ordeshook (1968), Palfrey and Rosenthal (1983, 1985), Morton (1991), and Feddersen and Sandroni (2006b,a)) and turnout driven by candidates or activists (Shachar and Nalebuff (1999), Herrera and Martinelli (2006), Herrera, Levine and Martinelli (2008), Feddersen and Gul (2014)). Of these, the models closest to my setup are those by Herrera and Martinelli (2006) and Herrera, Levine and Martinelli (2008). My model of activism differs from this in two ways. Firstly, my paper is not a model of turnout but one of costly influence. Secondly, the effort is borne not by candidates but by activists who belong to their party. The main question I address is the relationship between partisan activism and candidate platforms.

My model is also closely related to the work on direct informative role of campaign spending, notably Austen-Smith (1987). In Austen-Smith (1987), candidates simultaneously announce policy, and elicit contributions from two firms. While Austen-Smith considers an informative role of campaign contributions, I instead focus on the persuasive role of activism. The motivation for activism is purely ideological and therefore, activists in my setup are committed to supporting only one of the parties.

My paper is also related to the vast literature on campaign contributions and influence seeking by interest groups or lobbies (Baron (1994, 1989), Grossman and Helpman (1994, 1999, 1996)), Bernheim and Whinston (1986), and Austen-Smith (1995)). Activists are different from lobbies in the sense that they do not exert direct control over platforms, but rather act as intermediaries in the political competition process. Further, political activism is partisan in nature whereas lobbies typically donate to both sides of the political spectrum.

3 Model

Candidate Preferences. Two candidates $L$ and $R$, who care about ideology and benefits of office, contest elections on an unidimensional policy space $[-1, 1]$. Candidate $L$ has an ideal point $p_L = -\alpha$ and candidate $R$ has an ideal point $p_R = \alpha$, where $\alpha \in (0, 1)$. The candidates simultane-

8 See Herrera, Morelli and Nunnari (2015) and Herrera, Morelli and Palfrey (2014) for work that relates turnout to institutional arrangements.

9 Coate (2004) presents an alternate model of informative campaign spending.

10 We extend our model to include noisy campaigns, and show that the fundamental predictions on equilibrium polarization and activist participation holds. See Section 6.
ously announce policy $X_i$ (where $i \in \{L, R\}$), and the winning candidate enjoys benefits from office, $b > 0$. The winner implements the ex-ante chosen policy. The candidates’ utility as a function of the policy pair $X = (X_L, X_R)$ is given by,

$$U_i(X, p_i) = \begin{cases} - (X_i - p_i)^2 + b & \text{if } i \text{ wins} \\ - (X_i - p_i)^2 & \text{otherwise} \end{cases}$$

**Activist Preferences.** After platforms are announced, the candidates seek the support of activists $A_L$ and $A_R$, each supporting candidate $L$ and $R$ respectively. I assume that the ideological preferences of activists are unknown when candidates choose their platforms. The two activists have symmetric ideological preferences given by $-\beta$ and $\beta$ respectively, where $\beta \in [\underline{\beta}, 1]$ is drawn from a distribution with cdf $F$ and a differentiable density $f$ with full support.\(^\text{11}\) Activists contribute to the electoral process by making a costly effort/participation decision. This is captured by a convex cost function, $m(c_i)$, such that $m'(c_i) > 0, m''(c_i) > 0, m(0) \geq 0, m'(0) > 0$. Let $\gamma_m(c_i) = c_i \frac{m''(c_i)}{m'(c_i)}$ be defined as the elasticity of marginal cost of participation for activists.\(^\text{12}\)

Candidates are uncertain about the ideological distance between activists, $2\beta$. For example, a greater $\beta$ could be interpreted as more extreme views (on the right and left) on tax policy, gay rights, minimum wages, and so on. Given $\beta$, the mobilization $c_i$ is very loosely defined to capture any form of contribution by activists. Broadly, any measure of time, effort, or money spent on endorsing and campaigning for the candidate could be accounted for by the variable $c_i$. Let $C = (c_L, c_R)$ be a pair of activist contributions. The utility of an activist with bliss point $p_{A_i}$ is given by,

$$U_{A_i}(X, c_i, p_{A_i}) = \begin{cases} - (X_L - p_{A_i})^2 - m(c_i) & \text{if } L \text{ wins} \\ - (X_R - p_{A_i})^2 - m(c_i) & \text{if } R \text{ wins} \end{cases}$$

**Voter Preferences.** There is a continuum of voters $v \in [-1, 1]$ distributed uniformly. After candidates announce platforms and activists decide on contributions, the voters experience a shock $\mu$ distributed uniformly\(^\text{14}\) on $[-\sigma, \sigma]$ that shifts their ideal points. The final bliss point of a voter with an initial ideology $v$ becomes $\theta_v = v + \mu$. The utility function of voters consists of two

\(^\text{11}\)The assumption of symmetry in activist ideologies is a simplifying one. It ensures that the equilibrium is symmetric and unique, and helps enunciating the main result clearly. Relaxing the symmetry assumption by allowing activists to be drawn from different distributions with different support needlessly muddles the analysis without providing much additional value. However, introducing asymmetry is potentially an important direction in which future research could be directed.

\(^\text{12}\)This provides a measure analogous to risk aversion, except that it measures the elasticity of the marginal costs involved with participation.

\(^\text{13}\)Aldrich refers to this partisan identification as “party cleavages”.

\(^\text{14}\)The results hold for any generic symmetric distributions around the ex-ante median.
components - a purely ideological component of policy announcement and an influence component driven by activism. Specifically, effort spent by activists provides a positive utility (additively separable from ideology) according to an influence function $v(c_i)$, where $v'(.) > 0$, $v'' \leq 0$. Given this formulation, I can focus on solely the problem of the median voter. The utility of the median voter is:

$$U_{mv}(X, C, \mu) = \begin{cases} -(X_L - \mu)^2 + \eta_v(c_L) & \text{if } L \text{ wins} \\ -(X_R - \mu)^2 + \eta_v(c_R) & \text{if } R \text{ wins} \end{cases}$$

This influence function $v(.)$ could be interpreted in different ways. For example, it could be thought of as a preference shock that is induced by activists on voters, similar to the effect identified by Madestam et al. (2013). Alternatively, the influence activity may be interpreted as a direct utility benefit that voters derive from activism.

The salience of activism is captured by the $\eta > 0$ parameter. A greater $\eta$ implies that activist participation is weighed more significantly by the median voter, thereby increasing their relevance in the campaign process. Thus, $\eta$ represents the “relative importance of activism” to candidates.

It is useful to define $\gamma_v(c_i) = -c_i \frac{v''}{v'}$ as the elasticity of marginal influence. $\gamma_v(c_i)$ describes the curvature of the influence function and measures the effectiveness of activism in the electoral process. For sake of exposition, I make the following assumptions.

**Assumption 1.** $\beta > \sigma$

**Assumption 2.** $\gamma_p(c_i) = \gamma_p$, $\gamma_m(c_i) = \gamma_m$

The first assumption implies that the activist is always more extreme than the median voter. The second states that the elasticities of participation costs and influence are independent of the activist contribution $c_i$.

**Timing and Equilibrium concept:**

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15The National Election Survey (NES) data from the US between 1952-2000 shows evidence for this kind of a persuasive role. Specifically, the survey data finds that a significant proportion (around 30-40%) of the electorate in the US indulged in persuasive activism – engaging with potential voters about the candidates’ policy – over these years. Please refer to Norris (2002) for more.

16This may be a form of informative benefit or persuasion utility that is similar to the role of influence seeking (see Milgrom and Roberts (1988)) or advertising (see Dixit and Norman (1978)) in the organizational literature. Notice that the additive separable form of voters’ utility is similar to models that study the role of valence in political competition. See Aragones and Palfrey (2002), Ashworth and Bueno de Mesquita (2009), Bernhardt, Cámara and Squintani (2011), Groseclose (2001) for more on models with a valence component.

17When $\eta = 0$, the median voter is unaffected by activism, and the game resembles a variant of the BDS paper in which candidates with mixed motivations compete for an electoral office, in the presence of uncertainty about median voter’s ideal preference.

18This phenomenon has been widely documented in the political science literature. See Aldrich (2011) for more on this.

19For a broad class of power functions of the form $c^\rho$, the elasticities are constant.
1. Candidates L and R simultaneously announce policy platforms $X = (X_L, X_R)$

2. Nature draws activists’ ideology $\beta \sim [\beta, 1]$ from a cdf $F(.)$

3. Activists observe platforms, and simultaneously choose contributions $C = (c_L, c_R)$ respectively

4. Nature draws the median voter’s bliss point $\mu$ from an uniform distribution $[-\sigma, \sigma]$ 

5. The median voter observes $(X, C)$ and decides the winner

All exogenous parameters $(\alpha, \beta, \eta, \sigma, b)$ and the functional forms of $m(.)$ and $v(.)$ are common knowledge. The players maximize expected-utility, and the notion of equilibrium is Subgame Perfect Nash Equilibrium in symmetric pure strategies.

4 Optimal Activism

To characterize the activists’ contribution schedules, the behavior of the median voter first needs to be pinned down. The ideology of the median voter affects the winnability of either candidate. In particular, the median voter chooses the party which gives a higher payoff, i.e., she prefers candidate L over candidate R iff,

$$-(X_L - \mu)^2 + \eta v(c_L) \geq -(X_R - \mu)^2 + \eta v(c_R)$$

Therefore the cutoff ideology $\mu$, below which the median voter will vote for party L is,

$$\hat{\mu}(X, C) = \frac{(X_R + X_L)}{2} + \frac{\eta v(c_L) - v(c_R)}{2(X_R - X_L)}$$

Let $\lambda(X, C)$ denote the probability with which candidate L wins when $X_L \neq X_R$. Given the distribution of $\mu$, the probability that candidate L wins is,

$$\lambda(X, C) = \frac{1}{2} + \frac{(X_R + X_L)}{4\sigma} + \frac{\eta}{4\sigma} \frac{v(c_L) - v(c_R)}{(X_R - X_L)} \equiv \lambda \tag{4.1}$$

Consider the contribution decision of an activist, say $A_R^{20}$. Given a policy pair $X$ and a realization of ideology $\beta$, $A_R$ maximizes the following,

$$\mathbb{E}U_{A_R} = -(X_L \beta)^2 + (1 - \lambda)(X_R - \beta)^2 - m(c_R)$$

It is straightforward to observe from the equation above that when platforms of candidates are not differentiated, the activists do not have incentives to participate in the process, implying that $c_i = 0$. However, when the platforms diverge, interesting trade-off’s emerge.

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20 The decision problem is symmetric for $A_L$. 

The relationship between the contribution of activist $A_R$ and platform $X$ exhibits two key features. As $X_R$ increases, it means that the candidate $R$’s policy is closer to that of the activist. However, by taking a more extreme position, the candidate can differentiate her platform and soften the effect of activists.\footnote{This softening argument plays a crucial role for the main result in Section 6.} As a result, the marginal effect of activism ($c_R$) on winnability of $L$, $\lambda(X,C)$, is softened by more polarized platforms, reducing the marginal benefits from increased activism.\footnote{This is given by $\lambda' = \frac{\eta}{4\sigma} \frac{v'(c_R)}{(X_R-X_L)}$. Notice that as $(X_R-X_L)$ increases, the marginal influence of activism decreases. Ashworth and Bueno de Mesquita (2009) find a similar trade off in a model of electoral competition in which candidates invest (costly) in valence. Also, see Groseclose (2001) for a similar trade-off between divergent platforms and valence advantage.} In the same way, when $X_L$ becomes more extreme, these incentives switch. Now even though $A_R$ cares about preventing $L$ from winning, the fact that $X_L$ is more extreme reduces the marginal influence of activism on $L$’s chances of winning.

**Lemma 1.** When $X_L \neq X_R \neq 0$, each activist chooses a level of contribution given by,

$$\frac{m'(c_L)}{v'(c_L)} = \frac{\eta}{4\sigma} [2\beta + (X_L + X_R)]$$

and

$$\frac{m'(c_R)}{v'(c_R)} = \frac{\eta}{4\sigma} [2\beta - (X_L + X_R)]$$

**Proof.** See Appendix A.1

The trade-offs discussed earlier reflects in the equilibrium supply of activism. Two effects are at play: i) **preference-for-moderation effect**, and ii) **counter-mobilization effect**. Specifically, when one of the candidates becomes more extreme, the party activist supporting that candidate reduces contributions to the campaign ($\frac{\partial c_R}{\partial X_R} < 0$ and $\frac{\partial c_L}{\partial X_L} > 0$). Activists dislike more extreme platforms, and a willingness to compromise by a candidate -by moving closer to the other candidate’s platform- increases the marginal influence of activism, thereby increasing participation from one’s own activist.

In addition, activists also impose a **disciplining effect**. When one candidate becomes more extreme, the activist supporting the other candidate counter-mobilizes and increases participation ($\frac{\partial c_R}{\partial X_L} < 0$ and $\frac{\partial c_L}{\partial X_R} > 0$). This stems from the fact that when a candidate polarizes, the expected ideological loss is higher for the other party’s activist, and the marginal benefits of participation rises. Thus, activists care about whether the other candidate becomes more extreme. In this sense, the preferences of party activists are such that it favors greater moderation and compromise from candidates during the campaign process.

When platforms are symmetric, meaning $X_R = -X_L$, the contributions by activists are independent of the announced platforms $(X_L,X_R)$, since $X_R + X_L = 0$. This implies that equilibrium
supply of activism is purely a function of the exogenous parameters of the model – \((\eta, \sigma)\) – and is independent of the extent of polarization in platforms.\footnote{This property is due to the fact that there is no strategic interaction between ideology and the activists’ influence function \(v()\). This additive separability in the median voter’s preferences implies that as long as the two platforms are symmetric on either side of the political spectrum, the optimal contributions of the party activists are unaffected by the extent of platform polarization.}

**Lemma 2.** When candidate platforms are symmetric, i.e., \(X_R = -X_L\), supply of activism is given by \(c^*_L(\beta) = c^*_R(\beta) = c^*(\beta)\) that solves \(m'(c) = \frac{\eta \beta}{2\sigma} \cdot v'(c)\). Moreover, the following holds: \(\frac{\partial c^*}{\partial \eta} > 0\), \(\frac{\partial c^*}{\partial \sigma} < 0\).

Notice that the equilibrium supply of activism has a simple structure. The characterization equates marginal costs and marginal benefits of contribution, resulting in an unique equilibrium of the activist subgame. Any increase in the relative importance of activism or reduced variance in median voter’s ideal point shifts the marginal benefit curve up (see Figure 2), thereby increasing the contributions in equilibrium. However, when party activists diverge, participation increases as the stakes are now higher for activists. Therefore, any increase in \(\eta\) and \(\beta\), or a decrease in \(\sigma\), leads to greater participation in equilibrium.\footnote{Although participation in equilibrium is positive when platforms are polarized, it is still wasteful, in the sense that both the activists’ contributions are equal and therefore do not have any relative impact on the winnability of the candidate. However, the reason why they are positive is precisely that if one activist were to reduce contributions, it would decrease their candidate’s probability of winning. The other activist, as a consequence, has a greater incentive to contribute, since the marginal benefits of contributing exceed the marginal costs. This interdependence between winnability and activism prompts both party activists to contribute a positive level in the campaign, even though in equilibrium the two contributions cancel each other out, resulting in zero net effect of activism.}

## 5 Symmetric Candidate Equilibrium

Candidates anticipate contributions and the winning probability as a function of their chosen platforms. A (subgame perfect) Nash equilibrium strategy for a candidate is a policy platform that maximizes their payoff, given the other candidate’s platform choice and the subsequent play of the game. I restrict attention to symmetric candidate platforms. Before characterizing equilibrium platforms with party activists, it would be useful to consider the case when there is no demand for activists, meaning \(\eta = 0\). This describes a political environment devoid of activists, and the
Marg Benefit

\[ K = \frac{\eta}{4\sigma} [2\beta \pm (X_R + X_L)] \]

**Figure 2:** The intersection of the marginal benefit and marginal cost curves determine equilibrium activism. At a symmetric equilibrium, \( K = \frac{\eta \beta}{2\sigma} \). Any increase in \( \beta \) or \( \eta \), or a decrease in \( \sigma \), pushes the marginal benefit curve upwards, increasing activism in equilibrium. Notice that the curvature of the marginal cost function affects the equilibrium activism (points \( a \) and \( a' \)).

The following proposition characterizes the equilibrium platforms in the absence of activism.

**Proposition 1.** The electoral game without activism has a symmetric equilibrium \((\bar{x}, \bar{x})\) such that, if \( \alpha > \frac{b}{4\sigma} \) then \( \bar{x} = \frac{4\sigma \alpha - b}{4(\alpha + \sigma)} \); and if \( \alpha \leq \frac{b}{4\sigma} \) then \( \bar{x} = 0 \).

**Proof.** See Appendix A.2

An important point to note in the above proposition is that \( \bar{x} < \alpha \). This implies that responsible candidates never choose their ideal policy and always moderate in equilibrium. Given this, introducing ideologically risk averse party activists further changes the incentives for candidates. On top of targeting the median voter’s ideal policy, candidates also have to cater to the preferences of activists. As argued earlier, party activists prefer moderation in platforms. This indicates that candidates may further trade off ideology in order to elicit greater participation from activists by moderating platforms in equilibrium, irrespective of the extent of the partisan gap between party activists.

**Proposition 2.** The electoral game with activism has an unique symmetric pure strategy equilibrium in candidate platforms \((x^*, x^*)\) that solves,

\[
4(\alpha + \sigma)x^2 - [4\alpha(\sigma - \frac{1}{2} \int_{\beta}^{1} D(\beta, c^*(\beta))dF(\beta)) - b]x + \frac{b}{2} \int_{\beta}^{1} D(\beta, c^*(\beta))dF(\beta) = 0
\]

\[25 \text{The existence and uniqueness of a symmetric equilibrium in such a set-up has been shown by an earlier work of Bernhardt, Duggan and Squintani (2009). The same arguments can be extended to my analysis with activism.} \]
where \( D(\beta, c^* (\beta)) = \frac{\eta}{\beta} \frac{c^*(\beta) \cdot v'(c^*(\beta))}{\gamma_m + \gamma_v} \), such that if \( \alpha > \frac{b}{4(\sigma - \frac{1}{2} \int_{\hat{\beta}}^1 D(\beta, c^*(\beta))dF(\beta))} \) then \( x^* > 0 \); if \( \alpha \leq \frac{b}{4(\sigma - \frac{1}{2} \int_{\hat{\beta}}^1 D(\beta, c^*(\beta))dF(\beta))} \), then \( x^* = 0 \). Furthermore, \( \bar{x} \geq x^* \).

**Proof.** See Appendix A.3

The equilibrium with activists is more moderate than in the absence of them. This is an interesting finding since it illustrates an important role for political activism. Political activists in representative democracies help constrain extreme platforms and build consensus. In the absence of activists, candidates with mixed motivations would tend to move away from each other and closer to their preferred platform, causing greater polarization. In polarized societies like the present day US, activism is indeed good. The inability of activists, unlike lobbies and organized interest groups, to commit to (implicit) policy contingent contracts\(^{26}\) implies that candidates find common ground by moderating their platforms.

### 6 Partisan gap and candidate polarization

The result presented in Proposition 2 implies that the mere presence of ideologically inclined activists reduces the polarization of candidate platforms, irrespective of the extent of partisanship between the activists. An important question that arises then is how does equilibrium polarization react to increasing levels of partisanship between the two sides’ activists.

In order to capture the notion of partisanship in preferences of activists, I use the idea of first-order stochastic dominance (FOSD) shift in the distribution of activist ideologies. Specifically, suppose the activist bliss points are drawn from an alternate distribution with CDF \( G(.) \) and differentiable density \( g \), such that \( \forall t \in [\hat{\beta}, 1] : G(t) \leq F(t) \).\(^{27}\)

Consequently, candidates while choosing their platforms, face the following trade off. Moving closer to the ex-ante median is preferred by activists and this increases -in expectation- the extent of activism. On the other hand, if the candidate chooses to move closer to her preferred ideology, it increases the payoff conditional on winning. However, by polarizing, the candidate faces the possibility of lower activism from her own activists, and greater counter-activism from the other side. The nature of balance between these two forces -increased conditional payoff and lower chances of winning- depends crucially on the activists’ willingness to engage in the electoral process.

To make the exposition clearer, I will define the activists’ willingness to engage as \( WTE = \frac{1}{\gamma_m + \gamma_v} \). That is, willingness to engage is simply the inverse of the sum of elasticities of marginal cost of participation and marginal influence. This gives us an intuitive way to think about the participation decision of activists in the electoral process.


\(^{27}\)This way, the activists drawn from \( G \) are ex-ante more partisan in a FOSD sense.
Proposition 3. When the activists’ ideology is drawn from a distribution $G$, and further if $G \text{ FOSD } F$, the following holds:

(i) equilibrium platforms are less polarized if $\gamma_v \leq \frac{1-\gamma_m}{2}$; or $\text{WTE} \geq \frac{1}{1-\gamma_v}$

(ii) equilibrium platforms are more polarized if $\gamma_p > \frac{1-\gamma_m}{2}$; or $\text{WTE} < \frac{1}{1-\gamma_v}$

Proof. See Appendix A.4

There is a non-monotonic relationship between activist polarization and political polarization of candidates, and the nature of this relationship is captured by the willingness to engage of activists. To understand this result more deeply, I will present a simple parameterized example. Suppose the influence function of activism is linear $v(c_i) = c_i$ and the cost function of activists is a convex power function of the form $m(c_i) = c_i^\rho$.

Figure 3 illustrates the equilibrium activism under symmetric platforms, and when one of the candidates (say $R$) polarizes and shifts away from the symmetric equilibrium ($X_R = x + \Delta x$). Notice that this polarization brings about an increase in activism from $A_L$ and a decrease in activism from own activist $A_R$. Further, the increase in activism from $A_L$ is greater when the marginal cost function has a lower elasticity (concave in the left side figure).

This implies two things. First, as candidates move away from a symmetric equilibrium, they risk alienating their own activists, while at the same time mobilizing the opposite side’s activists. Second, the extent of this change in activism-gap depends on the willingness to engage (in this case, $\text{WTE} = (\rho - 1)$). Specifically, when $\rho \leq 2$, the marginal cost function (on the left) is concave and the increase in activism-gap is high. This in turn hurts $R$’s winnability to the extent where the marginal benefits of moving closer to his preferred ideology $\alpha$ is offset by the marginal losses incurred by polarizing (reduced winnability). Therefore, in the case when $\rho \leq 2$ (or $\text{WTE} \geq 1$), as one of the candidate polarizes, it pushes the two effects – counter-mobilization and preference-for-moderation – more strongly in favor of the less polarized candidate. The candidates therefore choose to moderate when activists have a high $\text{WTE}$.

The opposite is true when the willingness to engage is low (the right side graph of figure 3). In this case, candidates find it optimal to polarize since doing so does not change the activism-ga by enough to compensate for the increased gains of moving closer to their preferred platform ($-\alpha, \alpha$). Candidates understand this trade-off while announcing their platforms. They recognize that a lower $\text{WTE}$ means that activists do not react to polarization as strongly, and this provides them with incentives to move closer to their ideal points.

\[^{28}\text{This kind of political compromise was witnessed in the recent US primary battle between Hillary Clinton and Bernie Sanders. Clinton, during the course of the primaries, adopted platforms far more progressive than her perceived ideological moorings (as a centrist candidate). Platforms that resulted in compromise were, e.g., college tuition and } \$15 \text{ minimum wage. A similar compromise was witnessed in the 2015 New Delhi elections in which both the main competing parties decided to adopt a variant of the anti-corruption bill, which remained the main campaign issue.}\]
Figure 3: In the above figure, when $R$ is more extreme (polarizes by $\Delta x$ from a symmetric equilibrium), the marginal benefit curve of activist $A_R$ moves down (points $d''$ & $e''$) while that of $A_L$ moves up (points $d'$ and $e'$). Clearly, the greater $\Delta x$ is, the greater is the difference in activism (activism-gap), $d''-d''$ and $e'-e''$, between the two sides. Notice that this gap is greater (lower) when the marginal cost function is concave (convex). This illustrates the trade-off between polarization and the elasticity parameter.

Implications. These results are of fundamental importance in understanding the intricate relationship between party polarization and candidates’ platform polarization. An important implication of this result is that a widening partisan gap is neither necessary nor sufficient for causing increased polarization of platforms. Take, for example, the Pew Research Center’s study\footnote{See \url{http://www.people-press.org/2014/06/12/political-polarization-in-the-american-public}.} in 2014 that documents this partisanship. To quote, “Today, 92% of Republicans are to the right of the median Democrat, and 94% of Democrats are to the left of the median Republican”, and further, “But on every measure of engagement, political participation is strongly related to ideology and partisan antipathy; those who hold consistently liberal or conservative views, and who hold strongly negative views of the other political party, are far more likely to participate in the political process than the rest of the nation. This results in a consistent “U-shaped” pattern, with higher levels of engagement on the right and left of the ideological spectrum, and lower levels in the center.” My finding suggests that polarization in US politics is not driven purely by participation of increasingly partisan supporters. What is important is the interaction between partisan gap and the willingness to engage of activists in the electoral process, meaning, a combination of $(\beta, WTE)$ is critical in determining whether polarization increases.

For example, consider the issue of rising income inequality. This may push political activists towards supporting policies that are more progressive on one side and more conservative on the other. However, this does not necessarily guarantee that candidates representing their interests
would further polarize their platforms to reflect this partisanship. In fact, as my analysis suggests, a compromise could be reached in equilibrium if both parties’ willingness to engage in the political process is high enough. In this case, any increase in the \(WTE\) implies that party activists react more severely to polarization by either candidate, and this precludes them from polarizing in the first place, even though activists belonging to either side share more disparate views on the issues.

This result helps refocus attention on the role of party structure in electoral campaigns and democratic polities. If party activists show a greater willingness to engage with, and persuade, ordinary voters, then even if their own policy preferences diverge, the fact that there are imminent risks associated with electoral competition would imply that candidates may adopt more moderate stances reflecting the risky nature of campaigns. Parties and activists in democratic states could then provide a natural barrier against polarization as long as they remain actively engaged in the political process.

**Proposition 4.** When \(x^* > 0\), candidate platforms become more extreme if i) candidates’ ideological polarization \(\alpha\) increases; ii) benefits of office \(b\) decrease; iii) the relative importance of activism \(\eta\) decreases; iv) variance in median voter’s ideological preference \(\sigma\) increases.

**Proof.** See Appendix A.5

The relation between platform divergence and \(\eta\) is along expected lines. The rationale is the following. As the demand for activism decreases, it implies that voters weight candidate ideology more heavily compared to activist engagement. This implies activists end up decreasing their participation in the political process. Besides, the increased weight on candidate platforms means that the candidates rely less (in marginal terms) on activist participation. This decreased dependency on activists, therefore, translates into more divergent platforms in equilibrium.\(^{30}\)

Lastly, platform divergence increases when there is greater uncertainty regarding voters’ preferences. This makes intuitive sense in that, ceteris paribus, candidates in the model are trying to locate the median voter’s bliss point. Remember that greater uncertainty reduces activist participation because the possibility of more extreme median platforms reduces the marginal benefits for the activists. As this uncertainty or the variance increases, candidates adjust their platform in a way so as to account for this reduced participation from party activists and move more closer to their ideal policy.

7 Welfare implications of activism

In this section, I study the welfare effects of activism and polarization on voters. I address two questions. First, when does the presence of activists welfare-improving for voters, compared to

\(^{30}\)As \(\eta\) goes to zero, notice that the equilibrium platform is the same as the case with no activism. That is, as \(\eta \to 0, x^* \to \bar{x}\). This can be gleaned by substituting \(\eta = 0\) into the equilibrium equation in Proposition 2.
polities without their presence? Second, under what conditions does increased (ex-ante) polarization among activists adversely affect the welfare of voters?

Consider the ex-ante welfare of voters first under no activism. Let $W_v(-x,x)$ be the welfare of any voter $v$ under the symmetric equilibrium platforms of the two candidates. Then,

$$W_v(-x,x) = -\frac{1}{2\sigma} \left[ \int_{-\sigma}^{0} (x + v + \mu)^2 d\mu - \int_{0}^{\sigma} (x - v - \mu)^2 d\mu \right]$$

The sum of all voters’ welfare is given by,

$$W_{tot}(-x,x) = -\frac{1}{4\sigma} \int_{-1}^{1} \left[ \int_{-\sigma}^{0} (x + v + \mu)^2 d\mu + \int_{0}^{\sigma} (x - v - \mu)^2 d\mu \right] dv$$

**Lemma 3.** Under no activism, the welfare of voters is maximized at $x = \frac{\sigma}{2}$.

This result follows from Proposition 7 in BDS. When there is uncertainty about the median voter’s bliss point, voters are better off with a small degree of divergence in platforms, compared to Downsian convergence on the ex-ante median.

When there are activists engaging voters, they generate a positive welfare effect on voters. Since in any symmetric equilibrium, the two activists’ contributions cancel each other out and activism is independent of the symmetric platform choice of candidates, the first best levels of polarization remain unchanged in the presence of activism.

Given this, it is imperative to characterize the conditions under which the introduction of activists is actually welfare improving. From the previous analysis, we know that the presence of activists moderates platforms of candidates. In this case, any welfare improvement is possible only if two conditions hold: i) $\bar{x} > \frac{\sigma}{2}$, and ii) the equilibrium with activists $(-x^*,x^*)$ is not low enough that $W_{tot}(-\bar{x},\bar{x}) > W_{tot}(-x^*,x^*)$. The first condition ensures that in the absence of activism, the level of polarization is above the social optimal and the second implies that the presence of activists must not moderate the platforms to below a threshold (given by $(\sigma - \bar{x})$).

**Proposition 5.** When $b < 2\sigma(\alpha - \sigma)$ and $x^* > \frac{4\sigma^2 + b}{4(\alpha + \sigma)}$, activism improves welfare of voters. When either $b > 2\sigma(\alpha - \sigma)$ or, $b < 2\sigma(\alpha - \sigma)$ and $x^* < \frac{4\sigma^2 + b}{4(\alpha + \sigma)}$, the presence of activists hurts the welfare of voters.

**Proof.** See Appendix A.6

The presence of activists does not always improve welfare. The reason is that by their preference for compromise, activists may constrain the choices of candidates to the point where they are too similar. One way by which this compromise occurs is when one of the candidates adopts some features of the platform offered by the other, therefore compromising on their ideological preferences. By doing so, the candidates may end up providing very little choice to the electorate, harming their welfare.
Lemma 4. Given an equilibrium level of polarization \( x^* \in (0, \frac{\sigma}{2}) \), any increase in the partisan gap between activists reduces total welfare when activists’ WTE > \( \frac{1}{1-\gamma_p} \) and improves welfare when WTE < \( \frac{1}{1-\gamma_p} \); and vice versa for \( x^* > \frac{\sigma}{2} \).

Proof. See Appendix A.7

The welfare effects of activist polarization are ambiguous. That is, the impact of activist divergence on overall welfare depends on the extent of prevailing polarization in the political process. As we observed earlier, when the WTE of activists is high (condition i) of Proposition 3), the equilibrium level of polarization decreases as activists diverge more. Now, when the level of polarization is in the interval \([0, \frac{\sigma}{2}]\) (see figure 4), a high WTE implies that activist divergence results in greater moderation, and this reduces overall welfare.

The implication of this result is that to achieve first best levels of polarization calls for either lowering or increasing the WTE of activists. A lowering of WTE could be seen as introducing barriers to participation, or increasing the noise such that activism is rendered ineffective. On the other hand, an increase in WTE could be interpreted as a way in which the marginal costs are decreased for activism, and/or increasing the effectiveness of activists. Therefore, by varying the barriers to participation, the campaign process with activism can indeed lead to welfare improving outcomes, by varying the exogenous parameters \( b \) and \( \eta \).

Lemma 5. Consider a social planner trying to implement the socially optimal level of polarization.  
1. As long as \( b < 2\sigma(\alpha - \sigma) \), there exists a \( \eta^* \) such that the first best levels of polarization can be achieved.
2. As long as \( \int_1^\beta D(\beta, c(\beta))dF(\beta) < \frac{\sigma}{\alpha}(\alpha - \sigma) \), there exists a \( b^* \) such that the social planner can implement the first best levels of polarization.

Proof. See Appendix A.8

8 Extensions

8.1 Activism in Noisy campaigns

Suppose the policy platforms of candidates are observed with noise by the median voter, and activists play an informative role (see Austen-Smith (1987)). That is, activists inform the median voter of the precise position of their candidate’s platform. If \( X_i \) is the true position of the candidate, the policy observed by the median voter is \( \tilde{X}_i = X_i + \eta_i \), where \( \eta_i \) is a random variable (noise term) with expectation zero and variance \( \sigma_i^2 \). Contributions from activists reduces the variance of the noise term. If \( c_i \) is the contribution from the activist, then \( \sigma_i^2 = a(c_i) \). To simplify analysis, I assume that the median voter’s ideology is drawn from an uniform distribution on \([-1, 1]\). Further, activist ideologies are known at the time of candidates announcing platforms, and it is fixed at \((-\beta, \beta)\).

Additionally, following assumptions are made on the functional form of \( a(.)^{31} \):
\[ a(\cdot) < 0, \quad a''(\cdot) > 0, \quad a'''(\cdot) < 0 \text{ and } a(0) > 0. \]
The first two conditions ensure that as activists contribute more, the variance of noise function is decreasing, and convex. The subsequent condition ensures the concavity of the marginal variance of noise reduction, and the last condition states that, in the absence of activism, there is a positive level of noise in platforms, meaning voters imperfectly observe the platform of candidates.

This formulation naturally implies that greater activist participation is beneficial for candidates since it reduces the variance of the platforms, and since the voter is risk-averse, less variance is preferred. Activists or volunteers, then, have an important role in conveying - through door-to-door canvassing or phone calls - the true policy stance of their candidate.

Before presenting the results, it is important to glean the role of noise reduction function. Remember, activism is now not a persuasive tool, but restricted to only reducing the variance of the noisy platform. I introduce the parameter \( \gamma_n(c) \) to define the efficiency of activism in reducing the noisiness of platforms. That is, Efficiency of Marginal Noise Reduction, \( \gamma_n(c) = -c \frac{a}{a'} \). As in the baseline model, the willingness to engage is defined by \( WTE_N = \frac{1}{\gamma_m + \gamma_n} \).

Proposition 6. In a noisy electoral campaign with activism, there exists an unique symmetric equilibrium in candidate platforms. Furthermore,
\[ \text{i. } \frac{\partial x}{\partial \beta} < 0 \text{ if } \gamma_n < \frac{1 - \gamma_m}{2}; \quad \text{ii. } \frac{\partial x}{\partial \beta} > 0 \text{ if } \gamma_n > \frac{1 - \gamma_m}{2}. \]

\(^{31}\)We additionally assume that the noise reduction mechanism \( a(\cdot) \), is the same for both candidates.
Proof. See Appendix B.1

Proposition 8 shows that the main equilibrium and comparative statics result holds. The details can be found in Appendix B.

8.2 The role of big money

Though the reliance on grassroots activists is an important avenue of campaigning, big money campaign donations, in the form of PACs or super PACs, have played an increasingly important role. McCarthy, Poole and Rosenthal (2006) find that large contributions and contributors on average were more extreme\(^{32}\) (on either side of the political spectrum), and moreover, they tended to favor extreme candidates. The presence of such contributions provides the candidates with alternate resources with which to engage and influence voters - say, through direct political communication and advertisements targeting voters. Therefore, big money contributions act as a substitute for activism, reducing the dependence of candidates on grass-roots activists. Therefore, when candidates have access to big money, their reliance on activists goes down. This in turn may provoke candidates to polarize away from each other, and towards more extreme ideologies.

To see this mechanism, I will modify the model to consider the role of big money.\(^{33}\) Suppose \(S\) is the available contributions for either candidate. Then, the candidates have two goods that are employed for influence in elections – activist participation \(c_i\) and money \(S\). Moreover, I modify the influence function to include the money parameter. That is, \(v(S, c_i)\) is the total influence generated by campaigning, such that \(v_2(\cdot) > 0, v_{22}(\cdot) < 0, v_1(\cdot) > 0, v_{11}(\cdot) < 0,\) and \(v_{12}(\cdot) < 0\). The concavity assumption holds as before, and the last assumption states that \(S\) and \(c_i\) are strategic substitutes. This makes intuitive sense. Candidates need money to spend on advertisements, hiring campaign staff, on personalized communication to voters, and so on. Therefore, money supplements the traditional grassroots campaign of activists.

What this suggests is that the presence of big money may crowd out the role of activism, thereby decreasing the level of participation. This crowding out of activists may then drive candidates to polarize for much the same reasons as discussed earlier. Any decrease in the marginal effect of activism reduces the candidates’ incentive to compromise. This leads to greater polarization in equilibrium.

Lemma 6. In the presence of big money in campaigns, the equilibrium polarization and participation are such that, i) \(\frac{dc^*_i}{dS} < 0\) and ii) \(\frac{dx^*_i}{dS} > 0\).

Proof. See Appendix B.2

\(^{32}\)Please refer to chapter 5 of MPR.

\(^{33}\)I specifically abstract away from the strategic interaction of big donors. Instead, I assume that candidates are given a fixed sum of (big money) contributions exogenously by these donors. This can be interpreted as a pure electoral motive of giving campaign contributions (see Jacobson (1985)).
Lemma 3 shows how big money has a crowding out effect on activism and pushes platforms to further extremes as a result. The fact that this kind of money reaches more extreme candidates in the first place would only exacerbate its effect on political polarization \( \frac{dx^*}{dS}, \frac{dx^*}{d\alpha} > 0 \). As a result, this suggests a rationale for curbing this kind of big money spending by individual contributors or organized groups\(^{34}\). My analysis suggests curbing the use of money and increasing the dependence on activists. This could help counter balance the current trend of the excessive role of big money in the political process and potentially reduce platform polarization.

9 Discussion and concluding remarks

I have analyzed a model of political competition that addresses the question of whether, and how, the participation of activists in the electoral process affects political polarization. I find that when activists are price-takers in Aldrichian sense, candidate platforms are always more moderate than the case without activists. That is, the political process with activists yields a greater compromise between the two candidates. The inability of activists to make policy contingent (implicit) contributions and commit to *quid pro quo contracts*\(^{35}\) implies that candidates take into account the preferences of activists when announcing policy platforms, and this leads to moderation in platforms.

Moreover, I show that when the distribution of activists become more polarized, and their willingness to engage is above a threshold, the equilibrium platforms of candidates tend to converge. Therefore, political polarization decreases as partisanship between activists increases. This finding provides a clear testable threshold on the willingness to engage of activists. Below this threshold, any increase in divergence between activists must impact the candidate platforms by polarizing them. The implication of this result is that activist polarization, on its own, is insufficient to explain political polarization. The combination of activist partisanship and the willingness to engage in the political process together determine the extent of political polarization. The other testable result of my analysis is that polarization reduces as the demand for activism increases. That is, there is a negative correlation between polarization and the salience of activism in campaigns.

The willingness to engage can be reconciled by looking at the supply side factors that affect activism. On the supply front, one possible interpretation of the willingness to engage parameter is that it captures the costs incurred by activists during the electoral process. Specifically, modern day campaigns, especially in the US, happen over a long period of time. Electoral cycles are a continuous and arduous process, taking up a lot of time and resources on the part of candidates and activists, starting with the announcement of platforms up until election day. In some sense, the elasticity parameter reflects the marginal costs involved, and captures the length of the electoral

\(^{34}\)The so called “527 group” spending, e.g., places no upper bounds on how much and to whom to contribute. Some prominent ones include Club for Growth, MoveOn.org, New Democrat Network, among others.

\(^{35}\)For more on organized interest groups and lobbies that are able to commit to such policy contingent contracts, see Grossman and Helpman (1996), and Morton, Myerson et al. (1992).
campaign cycle. If this is indeed the case, then local (municipal bodies or city councils) elections where stakes are lower and the length of the campaign is substantially shorter, may have lower levels of polarization compared to state or federal elections, controlling for activist ideologies.

Some important welfare implications emerge from my analysis. Specifically, I establish that when political polarization is very high, increased divergence among activists can actually be welfare improving for voters as long as the willingness to engage for the activists is high. That is, highly polarized polities may benefit from activism in campaigns, irrespective of the ideological inclinations of the activists. Even when activists become more diverged, they could end up moderating platforms further, and increase voter welfare. However, on the downside, democratic societies with high levels of participation and less polarization may indeed provide lesser choice (decreased polarization) to the voters, making them worse off in welfare terms.

My welfare analysis also raises an important point that is related to the issue of party strength and polarization in democracies. Specifically, several studies (e.g., Norris (2002, 2007), Dalton and Wattenberg (2002)) have pointed out the phenomenon of decreasing political engagement and weakening party structures in advanced economies. My work suggests that this may be due to lack of incentives to associate with political outfits (high marginal costs, say), or alternatively, a result of reduced demand for them (crowding out effect of big money). The empirical evidence suggests that the willingness to engage of the wider electorate in activism has decreased over time in industrialized nations. In the case of the US, it may be that the decline of traditional participation is closely linked to the demand for such activism. For example, modern communication and messaging techniques employed by candidates preclude the need for a more grass-roots campaign. The candidates rely instead on big campaign expenditure to communicate directly to the electorate. Activists, who associate with parties and act as intermediaries, get crowded out in the political process. The polarization in US politics, therefore, can be pinned down to such a mix of supply-side and demand-side considerations of activism.

Finally, my theoretical findings suggests a possible link between publicly funded elections and political polarization. Public funding of campaigns could potentially have two effects. First, it reduces the crowding out effect of big money on activism. This would increase the reliance of candidates on activists and tilt the balance of demand towards smaller volunteers and a grass-roots campaign. Second, public funding may automatically reduce the length of campaign cycle, encouraging supporters with high marginal costs of participation to contribute effectively. These two effects, according to the predictions of my model, would decrease the levels of polarization

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36 Refer to Dalton and Wattenberg (2002), Schmitt and Holmberg (1995), and Mair and Van Biezen (2001) for a comprehensive account of the decline in partisanship in western democracies.

37 See Whiteley and Seyd (2002); Seyd and Whiteley (2004) for evidence concerning the reduced demand for activists in Britain’s political process.

38 Some other prominent advantages of public funding are that it reduces candidates’ dependence on pernicious interest group considerations, and possibly decrease the incumbency advantage. In US, the states of Maine and Arizona have had publicly funded state legislature elections. See the report by Government Accountability Office (GAO) for more on this, http://www.gao.gov/assets/310/305079.pdf.
and improve welfare of voters. Therefore, curbing independent expenditures (e.g., restrictions on PACs and super-PACs spending) by introducing stricter laws, akin to ones previously articulated by Prat (2002) and Gul and Pesendorfer (2012), and simultaneously introducing publicly funded campaigns would act as a useful institutional reform to encourage wider participation and bringing down political polarization.
A  Proofs - Benchmark model

All the proofs are carried out for candidate $R$ and activist $A_R$, and are symmetric for candidate $L$ and activist $A_L$.

A.1  Proof of Lemma 1

Consider activist $A_R$ and their contribution decision. Given candidate $R$’s win probability, the expected utility of activist $A_R$ whose ideological bliss point is at $\beta$ is,

$$
\mathbb{E}[U_{A_R}(X, c_R, \beta)] = -\lambda (X_L - \beta)^2 - (1 - \lambda) (X_R - \beta)^2 - m(c_R)
$$

Taking the first order condition and rearranging yields,

$$
\frac{m'(c_R)}{v'(c_R)} = \frac{\eta}{4\sigma} [2\beta - (X_L + X_R)] \quad (A.1)
$$

An analogous argument holds for the activist $A_L$.

$$
\frac{m'(c_L)}{v'(c_L)} = \frac{\eta}{4\sigma} [2\beta + (X_L + X_R)] \quad (A.2)
$$

A.2  Proof of Proposition 1

When $\eta = 0$, the win probability of candidate $R$ is just $1 - \lambda = \frac{1}{2} - \frac{(X_R + X_L)}{4\sigma}$. Each candidate chooses a platform to maximize their payoffs, taking as given the platform chosen by the other candidate. The SPNE is such a pair of platform choices that maximizes the expected utility of both the candidates in the first stage. Let us consider candidate $R$.

$$
\mathbb{E}[U^C_R(X, \alpha)] = -\lambda (X_L - \alpha)^2 - (1 - \lambda) (X_R - \alpha)^2 + (1 - \lambda) b
$$

Supposing that candidate $L$ chooses $X_L = -x$. Taking the FOC and evaluating the expression at $(-x, x)$,

$$
\lambda'_{x_R} [4\alpha x + b] - (\alpha - x) = 0
$$

where $\lambda'_{x_R} \mid (-x,x) = \frac{1}{4\sigma}$

Solving this gives us $\bar{x} = \frac{4\alpha - b}{4(\alpha + \sigma)}$. If $\alpha > \frac{b}{4\sigma}$, then $\bar{x} > 0$ follows from the expression.
A.3 Proof of Proposition 2

In the case of $\eta > 0$, the win-probabilities of both candidates are affected by activism. Specifically, $Pr(L\ wins) = \lambda(X, C)$ where $X = (X_L, X_R)$ and $C = (c_L(X, \beta), c_R(X, \beta))$. Candidate $R$’s expected utility can then be expressed as the following:

$$E_\beta[U^C_R(X, \alpha)] = -\int_\beta^1 [\lambda(X, C)(X_L - \alpha)^2 + (1 - \lambda(X, C))(X_R - \alpha)^2 - b] dF(\beta)$$

A SPNE in pure strategies is a pair of policies $X^* = (X^*_L, X^*_R)$ such that,

$$\max_{X_R} E_\beta[U^C_R(X, \alpha)] \text{ subject to }$$

$$\lambda(X, C) = \frac{1}{2} + \frac{(X_R + X_L)}{4\sigma} + \frac{\eta}{4\sigma} \frac{v(c_L(X, \beta)) - v(c_R(X, \beta))}{(X_R - X_L)}$$

$c_L(X, \beta)$ solves A.2

c_R(X, \beta)$ solves A.1

For sake of exposition, I will refer to $\lambda = \lambda(X, C)$. The FOC of candidate $R$’s problem is then given by the equation,

$$-\int_\beta^1 \left[ \lambda'_{X_R} \left( (X_L - \alpha)^2 - (X_R - \alpha)^2 + b \right) + 2(1 - \lambda) \left( X_R - \alpha \right) \right] dF(\beta) = 0 \quad (A.3)$$

where,

$$\lambda'_{X_R} = \frac{1}{4\sigma} + \frac{\eta}{4\sigma} \frac{(X_R - X_L)(v'(c_L) \frac{dc_L}{dX_R} - v'(c_R) \frac{dc_R}{dX_R} - (v(c_L) - v(c_R))}{(X_R - X_L)^2}$$

Applying implicit function theorem to equations A.1 and A.2, we can compute $\frac{dc_R}{dX_R}$ and $\frac{dc_L}{dX_R}$ respectively.

$$\frac{dc_R}{dX_R} = \frac{\eta}{4\sigma} \frac{v'(c_R)}{\eta \frac{4\sigma}{2\beta - (X_L + X_R))v''(c_R) - m''(c_R)}}$$

$$\frac{dc_L}{dX_R} = \frac{\eta}{4\sigma} \frac{v'(c_L)}{\eta \frac{4\sigma}{2\beta + (X_L + X_R))v''(c_L) - m''(c_L)}}$$

Since I am interested in a symmetric equilibrium of the form $X = (-x, x)$, the equations A.3, A.4 and A.5 simplify to,

$$\int_\beta^1 \left[ \lambda'_{X_R}(-x, x) \left[ 4\alpha x + b \right] + 2(1 - \lambda)(x - \alpha) \right] dF(\beta) = 0 \quad (A.6)$$
\[
\frac{dc_R}{dX_R} \bigg|_{(-x,x)} = \eta \frac{4\sigma}{\eta \beta} \frac{\nu'(c)}{\nu''(c) - m''(c)} = -\frac{dc_L}{dX_R} \bigg|_{(-x,x)}
\]

The change in win-probability \( \lambda \) at \((-x,x)\) is,

\[
\lambda'_{X_R} = \frac{1}{4\sigma} \left[ 1 - \eta \frac{2}{\eta \beta} \left( \frac{\nu'(c)}{\nu''(c) - m''(c)} \right)^2 \right]
\]

(A.7)

At \((-x,x)\), the supply of activism is \(c(\beta)\) that solves \(m'(c) = \frac{\eta \beta}{\gamma m} \cdot \nu'(c)\). Simplifying the above expression and using the fact that \(\gamma_m = \frac{c}{m} m''\) and \(\gamma_v = -\frac{c}{v} v''\),

\[
\Rightarrow \lambda'_{X_R} = \frac{1}{4\sigma} \left[ 1 + \frac{\eta}{2\beta x} \frac{cv'(c)}{\gamma_m + \gamma_v} \right] > 0
\]

Defining \(D(\beta, c(\beta)) = \frac{\eta}{\beta} \cdot \frac{c(\beta) \cdot \nu'(c(\beta))}{\gamma_m + \gamma_v}\), we can rewrite the above as \(\lambda'_{X_R} = \frac{1}{4\sigma} \left[ 1 + \frac{1}{2x} D(\beta, c(\beta)) \right]\).

Further, we also know that at \((-x,x)\), \(\lambda(X, C) = \frac{1}{2}\). Substituting this into equation A.6, we get:

\[
\frac{1}{4\sigma} \int_{\beta}^{1} \left( 1 + \frac{1}{2x} D(\beta, c(\beta)) \right) (4\alpha x + b) dF(\beta) = (\alpha - x)
\]

(A.8)

This yields us the required condition,

\[
4(\alpha + \sigma)x^2 - \left[ 4\alpha(\sigma - \frac{1}{2} \int_{\beta}^{1} D(\beta, c(\beta))dF(\beta)) - b \right] x + \frac{b}{2} \int_{\beta}^{1} D(\beta, c(\beta))dF(\beta) = 0
\]

(A.9)

To simplify exposition, I will hereafter refer to \(\int_{\beta}^{1} D(\beta, c(\beta))dF(\beta)\) as \(D_F\). Notice that \(x = 0\) if \(4\alpha\left( \sigma - \frac{1}{2} D_F \right) - b < 0\). This implies that \(\alpha \leq \frac{b}{4(\sigma - \frac{1}{2} D_F)}\) for \(x^* = 0\).

Further, \(x\) is decreasing in \(D_F\). As a result, when \(\eta = 0\) and \(D_F = 0\) the resulting equilibrium \(\bar{x} > x_{D_F > 0}\). This proves proposition 2.

### A.4 Proof of Proposition 3

To prove that when the ideology of activists \((-\beta, \beta)\) increases in a FOSD sense, the equilibrium platforms converge, we only need to show that under a distribution \(G\) that FOSD \(F\), the following holds:

\[
\int_{\beta}^{1} D(\beta, c(\beta))dG(\beta) \geq \int_{\beta}^{1} D(\beta, c(\beta))dF(\beta)
\]

But, from the definition of FOSD, the above expression is true only if \(D(\beta, c(\beta))\) is weakly in-
creasing in $\beta$. That is $D_G \geq D_F$ iff,

$$\frac{dD(\beta, c(\beta))}{d\beta} = \frac{\partial D}{\partial \beta} + \frac{\partial D}{\partial c} \cdot \frac{\partial c}{\partial \beta} \geq 0 \quad \text{(A.10)}$$

Given $D(\beta, c(\beta)) = \frac{\eta}{\beta} \cdot \frac{c(\beta) \cdot v'(c(\beta))}{\gamma_m + \gamma_v}$ and $c(\beta)$ solves $m'(c) = \frac{\eta}{\beta} \cdot v'(c)$, applying implicit function theorem to the above expressions gives us,

$$\frac{\partial D}{\partial \beta} = -\frac{\eta}{\beta^2} \cdot \frac{c \cdot v'(c)}{\gamma_m + \gamma_v} = -\frac{1}{\beta} D(\beta, c(\beta))$$

$$\frac{\partial c}{\partial \beta} = -\frac{\eta}{\sigma} \cdot \frac{c \cdot v'(c)}{\gamma_m + \gamma_v}$$

$$\Rightarrow \frac{\partial c}{\partial \beta} = \frac{D(\beta, c(\beta))}{\eta v'(c)}$$

$$\frac{\partial D}{\partial c} = \frac{\eta}{\beta} \cdot \frac{c v''(c) + v'(c)}{(\gamma_m + \gamma_v)}$$

$$\Rightarrow \frac{\partial D}{\partial c} = \frac{\eta \cdot v'(c) \cdot (1 - \gamma_v)}{\beta \cdot (\gamma_m + \gamma_v)}$$

Combining these expressions and substituting back into equation A.10, we get,

$$\frac{dD}{d\beta} = D \left[ \frac{1}{\eta \cdot v'(c)} \cdot \frac{\partial D}{\partial c} - \frac{1}{\beta} \right]$$

Therefore, $\frac{dD}{d\beta} \geq 0$ if

$$\frac{\partial D}{\partial c} \geq \frac{\eta \cdot v'(c)}{\beta}$$

$$\Rightarrow \frac{(1 - \gamma_v)}{(\gamma_m + \gamma_v)} \geq 1$$

The above condition simplifies to $\gamma_v \leq 1 - \frac{\gamma_m}{2}$. A similar argument follows for the other case. This completes the proof.

**A.5 Proof of Proposition 4**

The comparative statics result with respect to $\alpha$ and $b$ follows from Bernhardt, Duggan and Squintani (2009). I will therefore concentrate on the parameters $\eta$ and $\sigma$.

From the equilibrium equation A.9, let

$$\Phi(x; \alpha, \eta, \sigma, b) = 4(\alpha + \sigma)x^2 - [4\alpha(\sigma - \frac{1}{2}D_F) - b]x + \frac{b}{2}D_F \quad \text{(A.11)}$$
From implicit function theorem, \(\frac{dx}{d\eta} = -\frac{\partial \phi}{\partial \eta}\).

The following holds: \(\frac{d\phi}{dx} > 0\) at the equilibrium \((-x,x)\). To see this,

\[
\frac{d\phi}{dx} = 8(\alpha + \sigma)x - [4\alpha(\sigma - \frac{1}{2}D_F) - b]
\]

That is, \(\frac{d\phi}{dx} > 0\) iff \(x > \frac{[4\alpha(\sigma - \frac{1}{2}D_F) - b]}{8(\alpha + \sigma)}\). But,

\[
x = \frac{1}{8(\alpha + \sigma)} \left[ (4\alpha(\sigma - \frac{1}{2}D_F) - b) + \sqrt{(4\alpha(\sigma - \frac{1}{2}D_F) - b)^2 - 8b(\alpha + \sigma)D_F} \right]
\]

\[\Rightarrow x = \frac{[4\alpha(\sigma - \frac{1}{2}D_F) - b]}{8(\alpha + \sigma)} + \frac{\sqrt{(4\alpha(\sigma - \frac{1}{2}D_F) - b)^2 - 8b(\alpha + \sigma)D_F}}{8(\alpha + \sigma)}
\]

Therefore it holds that \(\frac{d\phi}{dx} > 0\). The sign of \(\frac{d\phi}{d\eta}\) determines the sign of \(\frac{dx}{d\eta}\). But \(\frac{d\phi}{d\eta} = \frac{d\phi}{dx} \cdot \frac{dx}{d\eta}\).

Further, it is straightforward to observe that \(\frac{dD}{dF} > 0\). Therefore, we can conclude that \(\frac{dx}{d\eta} < 0\) if \(\frac{dDx}{d\eta} > 0\). We check for the sign of the derivative \(\frac{dD(\beta,c(\beta))}{d\eta}\).

\[
\frac{dD(.)}{d\eta} = \frac{\partial D(.)}{\partial c} \cdot \frac{\partial c}{d\eta} + \frac{\partial D(.)}{d\eta}
\]

Since \(\frac{\partial c}{d\eta}, \frac{\partial D(.)}{d\eta} > 0\), it must be that \(\frac{dD(\beta)}{d\eta} > 0\) and \(\frac{dDx}{d\eta} > 0\) which further implies that \(\frac{dx}{d\eta} < 0\).

By a similar argument, the sign of \(\frac{dx}{d\sigma}\) depends on \(\frac{dD(.)}{d\sigma}\). But \(\frac{dD(.)}{d\sigma} = \frac{\partial D(.)}{\partial c} \cdot \frac{\partial c}{d\sigma}\). Since \(\frac{\partial c}{d\sigma} < 0\), this implies that \(\frac{dD(.)}{d\sigma} < 0\) and \(\frac{dx}{d\sigma} > 0\). This completes the proof.

### A.6 Proof of Proposition 5

Notice that the total welfare of voters is symmetric around \(\frac{\sigma}{2}\) such that \(W_{tot}(0,0) = W_{tot}(-\sigma, \sigma)\). This means that the welfare function is increasing in the interval \([0, \frac{\sigma}{2}]\) and decreasing in the interval \((\frac{\sigma}{2}, \sigma]\). This, combined with the fact that activism reduces platform divergence, implies that for the presence of activism to be welfare improving, a necessary condition is that the candidate platforms in the absence of activism must be above the social optimum. This is guaranteed when \(\bar{x} = \frac{4\alpha\alpha-b}{4(\alpha+\sigma)} > \frac{\sigma}{2}\). Simplifying this gives us the required condition. However, this condition is not sufficient for welfare improvement. The reason is that activism could potentially moderate the platforms to an extent that it reduces the overall welfare. For this to not happen, it must be that the platforms under activism must be above a threshold. This threshold is easily calculated using the symmetry property of the welfare functions. Specifically, as long as \(x^* > (\sigma - \frac{4\alpha\alpha-b}{4(\alpha+\sigma)})\), the presence of activists improves welfare compared to the no activists case. This gives us the required conditions. It is easy to observe that when either or both of these fail, the introduction of activists reduces welfare of voters unambiguously. This completes the proof.
A.7 Proof of Lemma 5

When the levels of polarization induced by activism is in the interval \((0, \sigma^2)\), the welfare function is increasing in \(x\). This implies that as \(\beta\) increases, and \(WTE\) is above the threshold \(\frac{1}{(1-\gamma)}\), the candidate platforms in equilibrium decrease (or converge towards each other). This reduces the welfare of voters as a result. The opposite holds when \(WTE < \frac{1}{(1-\gamma)}\), as in this case the polarization of activists induces more divergence in candidate platforms, leading to improved welfare. Similar arguments hold when the levels of polarization is above \(\sigma^2\).

A.8 Proof for Lemma 6

To prove the first part, notice that as \(\eta \to 0\), the equilibrium polarization of the electoral game converges to \(\tilde{x}\). When \(b < 2\sigma(\alpha - \sigma)\), the equilibrium platform is \(\tilde{x} > \frac{\sigma}{2}\). But, since \(\frac{dx}{d\eta} < 0\), it is always possible to find a \(\eta^* > 0\) such that the equilibrium polarization falls to the social optimal \(\frac{\sigma}{2}\).

In a similar way, as \(b \to 0\), the equilibrium polarization is such that \(\lim b \to 0 x = \frac{a(\sigma - \frac{1}{2}D_F)}{(\alpha + \sigma)}\). And, as long as this is above the social optimal levels, the social planner can always increase the benefits of office and achieve the social optimum levels of polarization (since \(\frac{dx}{db} < 0\)). This completes the proof.

B Proofs - Extensions

B.1 Activism and Noisy Campaigns

Suppose, for sake of exposition, \(\sigma = 1\). The solution to the electoral model is solved backwards, as previously. As before, we present the results for candidate (and activist) \(R\).

The median voter observes the platforms with a noise: \(\tilde{X}_L = X_L + \eta_L, \tilde{X}_R = X_R + \eta_R\).

Voter prefers candidate \(L\) if, \[E[-(\tilde{X}_L - \mu)^2] \geq E[-(\tilde{X}_R - \mu)^2] \]
\[\iff E[(X_L - \mu)^2 + \eta_L^2 + 2\eta_L(X_L - \mu)] \leq E[(X_R - \mu)^2 + \eta_R^2 + 2\eta_R(X_R - \mu)] \]
\[\iff E(\eta_L^2) - E(\eta_R^2) \leq (X_R - \mu)^2 - (X_L - \mu)^2 \]
\[\iff a(c_L) - a(c_R) \leq (X_R - \mu)^2 - (X_L - \mu)^2 \]
\[\mu \leq \frac{(X_R + X_L)}{2} + \frac{a(c_R) - a(c_L)}{2(X_R - X_L)} \]

The win-probability for Candidate \(L\) is \(\lambda = \frac{1}{2} + \frac{(X_R + X_L)}{4} + \frac{a(c_R) - a(c_L)}{4(X_R - X_L)}\). As in the original model, the equilibrium condition for participation is

\[\frac{m'(c_R)}{d'(c_R)} = \frac{[X_R + X_L - 2\beta]}{4} \quad (B.1)\]
By symmetry, the equivalent condition for activist \( L \) is,

\[
\frac{m'(c_R)}{a'(c_R)} = -\frac{[X_R + X_L + 2\beta]}{4} \tag{B.2}
\]

At the symmetric equilibrium \((-x, x)\), equations B.1 and B.2 reduce to \( \frac{m'(c)}{a'(c)} = -\frac{\beta}{2} \). Let \( D(\beta, c(\beta)) = -\frac{1}{\beta} \frac{c a'(c)}{\gamma_m + \gamma_p} \approx D \). Then, the equation that solves for the symmetric equilibrium is given by the following:

\[
4(\alpha + 1)x^2 - (4\alpha(1 - \frac{1}{2}D) - b)x + \frac{b}{2}D = 0 \tag{B.3}
\]

\[
\frac{m'(c)}{a'(c)} = -\frac{\beta}{2} \tag{B.4}
\]

Together, the above two equations determine the symmetric equilibrium platform of candidates, and mobilization by activists. The dependence of equilibrium platform \( x \) on \( \alpha \) and \( b \) are along the lines of the main model. To derive comparative statics with respect to \( \beta \), let \( \Phi = 4(\alpha + 1)x^2 - (4\alpha - b - 2\alpha D)x + \frac{b}{2}D \).

\[
\frac{dx}{d\beta} = -\frac{\frac{d\Phi}{dx}}{-\frac{d\Phi}{d\beta}} = +ve
\]

\[
\frac{d\Phi}{d\beta} = \frac{d\Phi}{dx} \frac{\partial D}{\partial \beta} + \frac{\partial D}{\partial c} \frac{\partial c}{\partial \beta}
\]

The rest of the proof follows from arguments similar to one in Proposition 3. The sign of \( \frac{\partial \Phi}{\partial \beta} \) depends on the expression \((-2 - \frac{1\gamma_n}{\gamma_m + \gamma_n})\), which is greater than zero when \( \gamma_n < \frac{1 - \gamma_n}{2} \). When this is satisfied, \( \frac{\partial \Phi}{\partial \beta} > 0 \) and further, \( \frac{dx}{d\beta} < 0 \). When the sign of this inequality is reversed, that is \( \gamma_n > \frac{1 - \gamma_n}{2} \), then \( \frac{dx}{d\beta} > 0 \). This concludes the analysis.

**B.2 The role of big money in campaigns**

The participation in equilibrium is modified to

\[
m'(c) = \frac{\eta_1 \beta}{2\sigma} \nu_2(S, c)
\]

Applying implicit function theorem yields \( \frac{dc}{dS} = -\frac{\eta_1 \beta}{2\sigma} \nu_2'(S, c) - m''(c) < 0 \). Similarly, the equilibrium is just the modified equation where,

\[
D(c, \eta, \beta, S) = \frac{\eta_1 \beta}{\gamma_m(c) + \gamma_p(c)}
\]
The sign of \( \frac{dD}{dS} \) determines the sign of \( \frac{dx}{dS} \), as before.

\[
\frac{dD}{dS} = \frac{\partial D(\cdot)}{\partial c} \frac{dc}{dS} + \frac{\partial D(\cdot)}{\partial S} \frac{dS}{dS}
\]

Since \( \frac{dc}{dS} < 0 \) and \( \frac{dS}{dS} < 0 \), and from our earlier assumption on \( WTE \), it is true that \( \frac{dD}{dS} < 0 \). This further implies that \( \frac{dx}{dS} > 0 \).

References


