Essays in Economics of Information and Optimal Contracting

by

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Spyridon Terovitis

April 2017

Declarations

I declare that this thesis is solely my own work.

Also, I declare that the material contained in this thesis has not been used or published before, and has not been submitted for another degree or at another university.

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Abstract

In the first chapter, I explore the problem of optimal contracting under delegation of information acquisition. I study a model where equity-holders of a fund delegate their portfolio allocation to a fund manager in an environment where: i) the expected return of the implemented portfolio depends on the ex-ante unknown future price of an asset, ii) the manager can acquire costly information about the future price, and iii) information acquisition is unobservable and unverifiable. I characterize the optimal contract which incentivizes the manager to obtain information and take the profit-maximizing position based on the available information. I show that the optimal contract implies a premium for positions *against* the publicly available information: a long position when an asset is considered overvalued, and vice versa. This premium leads the manager to adopt contrarian positions more often than the first best. I argue that this 'bias against the flow' is supported by empirical evidence.

In the second chapter, I explore the impact of Credit Rating Agencies (CRAs) on capital markets. I argue that the source for potential inefficiencies arising from CRAs might be more pathological than the literature

recognizes; even in the absence of conflicts of interest or other distortions resulting from players' behavior, a CRA might have an adverse effect on critical economics variables. I develop a model of investment financing which, similarly to capital markets, is characterized by information asymmetry and lack of commitment. In the benchmark setting, the CRA is capable of perfect monitoring and reveals its private information truthfully and without cost. I explore the impact of such an "ideal" CRA on the interest rate and the probabilities of project financing and default. I find that introducing such a CRA may lead to under-financing of projects with a positive net present value (NPV) that would otherwise be financed; a higher expected interest rate; and a higher expected probability of default. These findings relate to the feedback effect, which is inherent in capital markets, and its asymmetric impact on firms of different quality. I evaluate the policy of restricting CRAs to provide hard evidence with their ratings, and suggest that it might have an unfavorable effect on the probabilities of project financing and default.

In the third chapter, I explore the problem of security design with endogenous implementation choice. I study an economy where an entrepreneur raises capital to finance an investment project. My focus is on an environment where the entrepreneur shares the same characteristics as the representative entrepreneur in crowdfunding platforms: i) there is no record regarding her ability, ii) she might be associated with a negative-NPV project, and iii) she has limited liability. Asymmetric information regarding the entrepreneur's ability between the entrepreneur and potential investors gives rise to a signaling game when the former issues securities to

raise capital. I characterize the optimal security, and show that it is always optimal to reward the non-implementation of the project after financing takes place. I show that compared to a case where the entrepreneur is obliged to implement the project after raising capital, endogenizing the project implementation choice: i) prevents market breakdown, ii) leads to a more efficient allocation of resources, and iii) strengthens the incentive of an entrepreneur to invest in her productivity.

Abbreviations

CRA	Credit Rating Agency
NPV	Net Present Value
LEC	Low Expected Cost
HEC	High Expected Cost
CF	Crouwdfunding
VC	Venture Capital

Chapter 1

Motivating Information Acquisition Under Delegation

1.1 Introduction

The 'standard' principal-agent models focus on the problem of incentivizing an agent to take a particular action, which is in the principal's best interest.[1] This action usually includes exerting effort or not diverting resources from the company. There are many situations, however, in which the action which is in the principal's best interest is ex-ante unknown, i.e., it depends on the ex-ante unknown state of the world. One example is when equity holders delegate their portfolio allocation to a manager, where a long (short) position is profit-maximizing only if the ex-ante unknown future price falls (increases). Other examples include product design, marketing orientation, and the rating process by credit rating agencies.[2] The

[1]Seminal papers include Harris and Raviv (1978) and Hölmstrom (1979).

[2]In the product design problem, allocating company's resources to the production of a new product is a profit-maximizing action as long as it succeeds in accommodating

aim of this paper is to address the following question: how could a principal incentivize an agent to take a profit-maximizing action when neither he nor the agent knows ex-ante what the profit-maximizing action is?

The answer is straightforward. The principal should incentivize the agent to collect information regarding the state of the world, and subsequently take the profit-maximizing action, based on the available information. However, a problem which arises naturally in this environment is that information acquisition is costly, and in many cases, unobserved by the principal.[3] Consequently, the only way to motivate the agent to acquire information is through a contract contingent on the implemented action. Nevertheless, this contract – apart from affecting the agent's incentives to acquire information – also affects his implemented action after the information is obtained. This results in a trade-off between the cost of incentivizing information acquisition and the benefit of using the obtained information effectively. The key contribution of this paper is to show that the incentive scheme which optimally solves this trade-off, should reward contrarian actions, which subsequently leads an agent to adopt these actions more often than the first best.

We develop a model in which: i) a profit-maximizer principal delegates a binary action to a utility-maximizer expert, ii) the expected return of the implemented action depends on the ex-ante unknown binary state

the ex-ante unknown consumer' preference. Similarly, in the market orientation problem, adopting a new marketing technique over a conventional one is a profit-maximizing action as long as its ex-ante unknown productivity exceeds the productivity of the conventional technique. Finally, in the rating process, a bad rating over a good rating is a profit-maximizing action as long as the company defaults.

[3]This characteristic relates to the nature of the delegation problem – the principal, as opposed to the agent – has expertise in inferring the unknown state of the world. Thus, the principal unaware of the evidence the agent needs to collect to infer the unknown state of the world, and how costly it is for the agent to collect these evidence.

of the world, iii) the expert can acquire costly information about the state of the world, and iv) the action of information acquisition is not observable to the principal. In the benchmark model, the principal is a representative equity holder of a fund, and the agent is a fund manager. The action set consists of a short and a long position. Finally, the state of the world refers to the future price of the asset.

We differ from the conventional principal-agent setup in two dimensions. First, neither the principal nor the agent knows ex-ante the profit maximizing action. Second, we allow for an environment where neither the action of learning nor the learning outcome are observable to the principal. As a result, the compensation contract affects both the manager's incentives to acquire information and his investment decision after the information is obtained. Thus, the derivation of the optimal compensation contract consists of two parts: i) the characterization of the most cost-efficient contract which implements a given investment decision, and ii) the characterization of the investment decision which maximizes the equity holder's profit.

The first set of findings relates to the optimal compensation contract. We show that the promised payment is positive only: i) when the manager goes long, and the price eventually increases, and ii) when the manager goes short, and the price eventually decreases. In all other cases, the payment is zero. Besides, when the prior beliefs indicate that the asset is overvalued, the payment which corresponds to the long position exceeds the payment which corresponds to the short position, and vice versa. This premium leads to *over-investment against the flow*: when the asset is considered undervalued, a short position is implemented more often than the first

best, and the other way around. The *bias against the flow* is consistent with the empirical evidence on the behavior of financial analysts, whose problem is similarly to this setting: they need obtain private information before taking an observable action (issuing a forecast). For instance, Bernhardt and Kutsoati (2001) provide evidence that analysts issue biased contrarian forecasts. Similar evidence is provided by Pierdzioch et al. (2013), Laster et al. (1997), and Ehrbeck and Waldmann (1996).

The second set of findings relates to the consequences of the optimal compensation contract on the informational role of the investment decision. We show that the informational role of a given position differs depending on whether prior beliefs indicate that the asset is overvalued or undervalued. In particular, we find that when an asset is considered overvalued, the conditional probability that the asset price will drop, given a short position, exceeds the corresponding probability when there is no agency problem. In other words, a short position is a weak indicator that the asset price will eventually drop. This finding relates to the result that a short position might be driven by the premium of the corresponding payment, rather than the manager's belief that the price will fall. Following a similar logic, a long position is a strong indicator that the asset price will eventually increase. This finding relates to the result that the manager is willing to forgo the premium a short position entails, only if he is very confident that the price will increase. The opposite findings hold when the asset is considered undervalued. The previous mechanism could provide a theoretical foundation for the empirical evidence which indicates that investors over-react and/or under-react to financial analysts' forecasts (Elgers et al., 2001, Elliot et al., 1995, Elliot et al., 1995, Mendenhall, 1991,

Sloan, 1996). The rationale is that similar to the forecasts of financial analysts, the investment position of the manager is informative about his private information, and thus can be used by market participants to infer the future price of the asset. Our model predicts that rational investors should under-react when a manager takes a contrarian position, and over-react otherwise.

The *bias against the flow* is a key finding, which plays a critical role in the implications of the optimal contract. The intuition behind this bias relies on the interaction of unobservable information acquisition and the multi-tasking nature of the problem. In order to incentivize the agent to acquire information, the principal should promise a positive payment only if the (ex-post) revenue-maximizing position is implemented. However, the compensation contract also affects the manager's investment decision, which in turn, determines the portfolio's expected revenue. In this environment, the portfolio's revenue is maximized when the payments which correspond to a short and a long position are equal. For equal payments, the outside option of the manager is to follow the flow without acquiring information, which under the optimal contract, equals the manager's utility when information is obtained. The principal can thus worsen the manager's outside option by lowering the payment when the flow is followed. Worsening the outside option of the manager enables the principal to incentivize information acquisition at a lower cost. However, changing the ratio of payments comes at the cost of decreasing the portfolio's expected revenue; for a ratio of payments different than one, the first best is no longer implemented. The optimal contract thus solves the inherent trade-off between the cost of incentivizing learning and the benefit of

implementing an investment decision as close as possible to the revenue-maximizing one. This trade-off lies at the heart of our paper. We show that for small deviations from the first best, the decrease in the expected cost is greater than the decrease in the expected revenue.

In Appendix A.1 and A.2, we explore two extensions of the benchmark setting. Appendix A.1 analyzes the case where the state of the world is imperfectly observed. Appendix A.2 examines the case where equity holders allocate the tasks of information acquisition and portfolio allocation to two different individuals. We show that the optimal contract in these cases has the same features as the optimal contract in the benchmark setting, and the main finding of over-investment against the flow remains.

In Section 6 we discuss four alternative environments which share the same characteristics as the benchmark setting (delegation, uncertainty about the state of the world and unobservable information acquisition), and provide the main implications of the optimal contract in each setting. First, in a different portfolio allocation problem, where a fund manager invests in a risky or a safe asset, we argue that the optimal contract leads to under-investment or over-investment in the risky asset, depending on the prior beliefs. Second, in a product design problem, we argue that the optimal contract leads to product features which are more likely to fail to accommodate future demand compared to the first best. Third, we consider the rating process used by credit rating agencies, and claim that the optimal contract implies more frequent bad ratings than the first best when the market is optimistic about the company' creditworthiness, and vice versa. This finding could support extreme prior beliefs about a company's creditworthiness. Finally, in an environment where the principal

aims to motivate innovation, we claim that, depending on market beliefs, the optimal contract can capture both the under-implementation and over-implementation of innovative strategies.

The outline of the paper is as follows. Section 1.2 discusses the related literature. Section 1.3 describes the benchmark model. Section 1.4 characterizes the optimal compensation contract. Section 1.5 explores the implications of the optimal compensation contract. Section 1.6 discusses alternative applications and concludes. Appendix A.1 extends the model to the case where the state of the world is imperfectly revealed. Appendix A.2 explores the case where the principal allocates the tasks of information acquisition and portfolio allocation to different individuals.

1.2 Related Literature

This work pertains mainly to two strands of the literature. First, the literature which explores the problem of optimal contracting under delegation of information acquisition, and second, the literature which highlights the resulting distortion of information asymmetries in investment decisions and financial analysts' reports. Also, regarding its main implications, this paper relates to the literature which highlights the optimality of relative performance pay, and the literature which recognizes the emergence of potential distortions under multi-tasking.

1.2.1 Contracting and Information Acquisition

Similarly to our model, Inderst and Ottaviani (2009), Lewis and Sappington (1997), Gromb and Martimort (2007), Lambert (1986), and Chade and

Kovrijnykh (2016) explore the optimal contracting problem in a setting where a principal delegates information acquisition to an agent.

Inderst and Ottaviani (2009) consider a setup where a principal delegates a purchase recommendation to a seller representative. We differ from Inderst and Ottaviani (2009) because the principal can never observe whether the agent actually acquired information. This is a key departure, which critically affects not only the optimal compensation contract but also its main implications. Lewis and Sappington (1997) consider a setup, where an agent is optimally incentivized to acquire information about the state of the world, before choosing an unobservable level of cost-reducing effort. A critical difference from this paper is that, in our setting, the action that the principal prefers the agent to implement is state-dependent. This characteristic is responsible for an inherent trade-off between the cost of incentivizing information acquisition, and the benefit of implementing a decision as close as possible to the revenue-maximizing one. This trade-off, in turn, results in bias in the investment decision. In addition, as opposed to Lewis and Sappington (1997), allocating the tasks to two different agents, does not make the principal better-off.

The papers which consider environments which are closer to ours is Lambert (1986) and Gromb and Martimort (2007). Gromb and Martimort (2007) characterize the most cost-efficient contract of incentivizing an analyst – first, to acquire a costly binary signal about a project's quality, and second, to truthfully reveal the signal realization. We differ from Gromb and Martimort (2007) because our setup gives rise to the aforementioned trade-off, which results in the bias in the investment decision. Such trade-off, thus, bias does not appear in Gromb and Martimort (2007).

Lambert (1986) explores the optimal compensation contract which incentivizes the manager to obtain information before deciding to invest in a risky or a safe asset. As opposed to Lambert (1986), we are able to formally characterize a simple condition which determines the direction of the bias compared to the first best. This condition is whether the prior beliefs about the state being good exceed one-half. The simplicity of this condition allows us to generate testable implications about the informational role of investment decisions, and perform a series of comparative statics. Also, we explore a more general setup than Lambert (1986) by focusing on information acquisition regarding the realized state of the world, rather than a particular action. Hence, our setup could be used to explore alternative environments, like the ones we discuss in Section 1.6.

A recent paper which allows for delegation of information acquisition is Chade and Kovrijnykh (2016). The main focus of the two papers differ; in our paper, we focus on the investment decision, whereas Chade and Kovrijnykh (2016) focus on the quality of information available. Another critical difference is that in Chade and Kovrijnykh (2016) the signal realization (but not its precision) is observed by all parties, and the agents cannot misreport it. In contrast, we explore a setup where the principal cannot observe the realized signal, and as a result, the contract cannot be contingent on that.

The literature on delegation also includes, among others, Demski and Sappington (1987), Garicano and Santos (2001), Malcomson (2009), Szalay (2005), Eső and Szentes (2007).

1.2.2 Information asymmetry, investment decision and analysts reports

This work also relates to the literature which recognizes that information asymmetry might distort manager's investment decision or financial analysts' forecasts. This strand of the literature, which starts with the seminal work of Scharfstein and Stein (1990), and it includes Ottaviani and Sørensen (2006), Zwiebel (1995), and Levy (2004), highlights the impact of publicly available information on financial decisions. In particular, Scharfstein and Stein (1990) and Ottaviani and Sørensen (2006) provide models which give rise to managers mimicking the decisions of other managers (herding behavior), whereas, Zwiebel (1995), and Levy (2004) develop models where anti-herding behavior in the analysts' forecasts arises. The existing empirical evidence on the emergence of herding or anti-herding behavior is conflicting. For instance, Trueman (1990) provides evidence which indicates herding behavior. In contrast, Bernhardt and Kutsoati (2001) finds strong evidence that support anti-herding behavior. This finding is supported by Laster et al. (1997), Ehrbeck and Waldmann (1996), and Pierdzioch et al. (2013). More recently, Guerrieri and Kondor (2009) explores a setup where manager's career concerns can distort his investment decisions, which, in turn, magnify asset prices volatility, even in the absence of strategic complementaries among managers. In these papers, the main mechanism which leads to distortion compared to the first best is the agent's career concerns; the manager (analyst) uses his investment decision (forecast) as an instrument to signal his type, rather than to maximize the principal's profits.

We differ from this literature in three important directions. First, we are interested in the optimal compensation of an agent, a topic that is not examined in the previous papers. Second, we explore a different source of asymmetry, by recognizing that assuming that agents are endowed with a private signal is a significant shortcoming. In practice, the evaluation of a financial decision by a manager, or the forecasting of an asset's price by an analyst, involves a series of costly actions, such as collecting and evaluating information regarding industry competition, the economic climate, potential changes in regulation, etc. Thus, we are interested in exploring not only how to incentivize a manager to take a decision, or an analyst to disclose his private information truthfully, but also how to optimally incentivize the agent to acquire costly information before the action is taken. In terms of the model, the source of information asymmetry differs; our environment is not characterized by information asymmetry regarding the type (ability) of the manager, but by ex-ante moral hazard (information acquisition is a hidden action) and ex-post asymmetric information (the signal realization is the agent's private information). Hence, the mechanism that gives rise to the bias differs; the bias is a consequence of the principal's motive to optimally solve the trade-off between the cost of incentivizing information acquisition, and the benefit of implementing an action as close as possible to the portfolio's revenue-maximizing one.

1.2.3 Optimality of relative performance pay

An implication of our analysis is that under the optimal contract, both the agent's compensation and the bias in the agent's decision depend on

the prior beliefs. Since we do not impose any restriction on how prior beliefs are formed, our setup can allow for the case where prior beliefs capture actions of other agents, as long as these actions uncover some information about the state of the world. Based on this remark, our work relates to Hölmstrom (1979) , Holmstrom (1982), Lazear and Rosen (1979), McConnell et al. (1982) which highlight the benefit of evaluating agents on the basis of their relative performances in environments where agents' performance is affected by common shocks.

We differ from this literature in two directions. First, our setup explores the case where before taking a decision, the agent observes the actions of other agents. Hence, as opposed to Hölmstrom (1979), we allow for the agent to contingent his decision on other agents' actions. The second variation refers to the benefit of relative pay. In our model, the benefit of a relative pay comes from worsening the agent's outside option of not acquiring information, whereas in Hölmstrom (1979) the benefit stems from the fact that relative performance is a more informative indicator about whether an unobservable action is taken.

1.2.4 Contracting under multi-tasking

Finally, this paper pertains on the literature which highlights the misallocation consequences of the optimal contract. For instance, Holmstrom and Milgrom (1991) explore a principal-agent model, where an agent allocates his effort across multiple tasks, and the principal observes a performance measure for each of these tasks. The nature of this multi-tasking problem differs from Holmstrom and Milgrom (1991). In our setup, multi-tasking

emerges endogenously: the principal is only interested in information acquisition to the extent that it improves the information set, based on which the agent takes an action. In our paper, multi-taking arises endogenously. Inderst and Ottaviani (2009), and Lambert (1986) also find a resulting bias in one of the tasks. Also, Athey and Roberts (2001) point out that motivating effort in an efficient way may not necessarily coincide with incentivizing the agent to take the right investment choices.

1.3 Benchmark Model

Environment: We consider a setting with two risk neutral players: a representative equity holder of a fund (principal) and a fund manager (agent). The equity holder delegates an investment decision, denoted by d, to the fund manager. The investment decision consists of two options: buying a given asset ("going long", $d = L$) or short-selling a given asset ("going short", $d = S$).

The expected return of each position depends on the expected price movement of the asset, $P_1^\theta - P_0$, which in turn, depends on the ex-ante unknown quality (state of the world) of the asset, denoted by θ. The manager and the equity holder share common prior beliefs about the quality of the asset. In particular, they expect the asset to be of good quality ($\theta = G$), with probability p, and of bad quality ($\theta = B$), with probability $1 - p$. Following the binary nature of the quality, the value of p incorporates any publicly available information regarding the asset's quality.[4]

We assume that if the asset is of good quality, its price is expected to increase to $P_1^G = P_0 + \epsilon$, whereas if the asset is of bad quality, its price is expected to decrease to $P_1^B = P_0 - \epsilon$. This assumption captures the idea that the asset price moves towards its fundamental value.

Information Acquisition Technology: Before the investment decision is taken, the manager can acquire a signal about the quality of the asset. The manager's private information consists of the signal realization and signal

[4]The publicly available information, which is captured by p, can include analyst's forecasts, decisions of other managers, and credit ratings. We do not impose any structure on how these beliefs are formed.

acquisition, the latter of which incurs a cost c.[5] This cost can be interpreted a utility loss due to effort of acquiring a signal. Conditional on acquiring information, the manager receives a noisy signal $s \in [0, 1]$ about the quality of the asset. The signal s is distributed according to a continuous density function $f_\theta(.)$, with a distribution function $F_\theta(.)$, where $\theta = \{B, G\}$. Hence, for a signal realization s', the following hold:

$$f_\theta(s') \equiv Pr(s = s'|\theta)$$

$$F_\theta(s') \equiv Pr(s \leq s'|\theta)$$

After observing the signal realization s', the manager updates his beliefs as follows:

$$Pr(\theta = G|s = s') = \frac{f_G(s')p}{f_G(s')p + f_B(s')(1-p)} = 1 - Pr(\theta = B|s = s')$$

Assumption 1: Monotone Likelihood Ratio Property (MLRP)
For any signal realization $s' \in [0, 1]$, the ratio $\frac{f_G(s')}{f_B(s')}$ is increasing in s'.

A direct consequence of Assumption 1 is that the probability that the asset is of good quality is increasing in the signal realization. In addition, given that $f_G(s)$ and $f_B(s)$ represent probability density functions, Assumption 1 implies that $f_G(s)$ and $f_B(s)$ satisfy the single-crossing condition.

[5]This characteristic relates to the nature of the delegation problem – the manager, as opposed to the equity holder – has expertise in inferring the unknown state of the world. Thus, the equity holder is unaware of the evidence the agent needs to collect to infer the unknown state of the world, and how costly it is for the manager to collect these evidence.

Preferences and Actions: The equity holder designs the manager's compensation contract, which is denoted by $\hat{W}$ and specified below, in order to maximize his expected profits:

$$\mathbb{E}\,\Pi = \mathbb{E}[R|\hat{W}] - \mathbb{E}[C|\hat{W}] \tag{1.1}$$

where $\mathbb{E}[R|\hat{W}]$ stands for the expected revenue of the portfolio, whereas $\mathbb{E}[C|\hat{W}]$ stands for the manager's expected compensation, given a contract $\hat{W}$. Note that the compensation contract affects the expected portfolio return, $\mathbb{E}\,R$, through the investment decision of the manager.

The manager faces two kinds of decisions: the information acquisition decision, and the decision to go short or long. The objective of the manager is to maximize his expected utility:

$$\mathbb{E}\,V = \mathbb{E}[C|\hat{W}] - \mathbb{1}c \tag{1.2}$$

where $\mathbb{1}$ equals 1 if information is acquired, and zero otherwise.

Set of available contracts: We allow for contracts contingent on the implemented position, d, and the state realization, θ, i.e., $\hat{W} : d \times \theta \mapsto \mathbb{R}^+$, where $d = \{S, L\}$ and $\theta = \{B, G\}$. Thus, the contract is characterized by the following quadruple:

$$\hat{W} = \{w_{SG}, w_{SB}, w_{LG}, w_{LB}\}$$

where w_{SG} (w_{SB}) denotes the payment when the manager goes short and the asset quality is revealed to be good (bad). Likewise, w_{LG} (w_{LB}) denotes

the payment when the manager goes long, and the quality is revealed to be good (bad). For the sake of tractability, we assume that the agent is protected by limited liability, i.e., payment $w_{d,\theta}$ is non-negative. In section 1.4.2, however, we show that the limited liability assumption can be relaxed without affecting the main findings qualitatively.

In Appendix A.1, we explore the case where it is not feasible for the principal to offer contracts contingent on the realized state of the world. We show that, as long as there is a public signal, which is revealed after the decision is taken and it is informative about the actual state of the world, the main findings go through. Besides, in Appendix A.2, we allow for contracts contingent on: i) messages sent by the manager to the equity holder, and ii) the realized state of the world. We show that the optimal contract is effectively the same, independently of whether it is contingent on the implemented position or the agent's messages.

Timing: The sequence of events is the following:

1. The equity holder offers a compensation contract $\hat{W}$.
2. The manager decides whether to acquire information.
3. If information is obtained, the manager observes the signal realization.
4. The manager chooses the implemented position.
5. The state is realized, and the contract is executed.

1.4 Optimal Compensation Contract

In this section, we characterize the optimal compensation contract. Given that information acquisition is unobservable, the only way to motivate the manager to acquire information is through a contract contingent on the implemented position. As a result, the compensation contract affects both, the manager's incentives to acquire information, and his investment decision after the information is obtained. Thus, the derivation of the optimal compensation contract consists of two parts: i) the characterization of the most cost-efficient contract which implements a given investment decision, and ii) the characterization of the investment decision which maximizes the principal's profit.

Our analysis highlights the inherent trade-off between the cost of incentivizing information acquisition, and the benefit of implementing an investment decision as close as possible to the revenue-maximizing one. The goal of this section is to show that the contract which optimally solves this trade-off gives rise to over-investment against the flow. In other words, the manager takes the opposite position to the one that the equity holder would take, had he not hired a manager, more often than the first best.

Characterization of the optimal compensation contract

The derivation of the optimal contract can be analyzed in 4 steps:

1. Characterization of the investment decision rule, $\mathcal{DR} : s \mapsto d$, which maximizes equity holder's profits, given any compensation contract.

2. Characterization of the contract which minimizes the expected com-

pensation of the manager, subject to the constraint that a given decision rule $\mathcal{DR}$ is implemented. This step effectively derives the equity holder's expected cost of implementing a decision rule $\mathcal{DR}$, which we denote as $\mathbb{E}\, C(\mathcal{DR})$.

3. Derivation of the expected portfolio revenue of implementing a decision rule $\mathcal{DR}$, $\mathbb{E}\, R(\mathcal{DR})$.

4. Characterization of the optimal decision rule $\mathcal{DR}^*$, i.e., the decision rule $\mathcal{DR}$ which maximizes profits, $\mathbb{E}\,\Pi(\mathcal{DR}) = \mathbb{E}\, R(\mathcal{DR}) - \mathbb{E}\, C(\mathcal{DR})$.

Thus, the optimal compensation contract is the constrained-optimal compensation contract of step 2, subject to the constraint that $\mathcal{DR} \equiv \mathcal{DR}^*$, where $\mathcal{DR}^*$ is characterized in step 4.

1.4.1 Step 1: Investment Decision Rule $\mathcal{DR}$

In this step, we take the contract as given, as we are interested in finding the mapping from the signal realization to the position, i.e. $\mathcal{DR} : s \mapsto d$, which maximizes principal's profits. We show in Lemma 1 that, *independently* of the compensation contract, the principal's preference regarding the agent's position are characterized by a monotonic relation.

Lemma 1: *If the equity holder prefers the manager to go long for $s = s'$, then he also prefers the manager to go long for any $s > s'$. Likewise, if a short position is preferred for $s = s'$, then this also holds true for any $s < s'$. Also, for any compensation contract which incentivizes the manager to acquire information*

there always exists a unique s, denoted by $\hat{s}$, where the equity holder is indifferent between the manager going short or long.

Proof. See Appendix A.3. □

Lemma 1 is an implication of the MLRP, according to which the higher the signal realization, the higher the probability that going long is the profit-maximizing position. A direct consequence of Lemma 1 is that the decision rule is characterized by a threshold $\hat{s}$, such as:

$$\mathcal{DR} = \begin{cases} long & \text{if } s > \hat{s} \\ short & \text{if } s < \hat{s} \end{cases}$$

i.e., the principal aims to implement a decision rule, where the agent goes long when the signal realization is high enough (greater than $\hat{s}$) and goes short otherwise. Hence, a particular decision rule corresponds to a particular threshold $\hat{s}$, and vice versa.

1.4.2 Step 2: Constrained-optimal compensation contract that implements cut-off $\hat{s}$

The aim of this sub-section is to characterize the constrained-optimal compensation contract, i.e., the compensation contract which minimizes the expected compensation of the manager, subject to the constraint that cut-off $\hat{s}$ is implemented. We focus on the case where the equity holder finds it optimal to incentivize the manager to acquire information. In section 1.4.7, we characterize the conditions when incentivizing information acquisition is optimal.

Suppose that the equity holder designs a compensation contract such as the manager acquires information, and then implements $\hat{s}$. Then, the optimal contract should satisfy two sets of constraints. First, the manager should prefer acquiring to not acquiring a signal, and second, given that a signal is obtained, the manager should prefer implementing $\hat{s}$, to any other mapping from the signal realization to the implemented position.

Constraints for implementing $\mathcal{DR}$

Implementing $\hat{s}$ implies that the following two sets of constraints should be satisfied. The first set of constraints guarantees that the manager prefers going long for any $s' \in [\hat{s}, 1]$.

$$\mathbb{E}\, V[long|s'] \geq \mathbb{E}\, V[short|s'] \implies$$
$$Pr(G|s')[w_{LG} - w_{SG}] \geq Pr(B|s')[w_{SB} - w_{LB}].$$

The second set of constraints guarantees that the manager prefers going short for any $s'' \in [0, \hat{s}]$.

$$\mathbb{E}\, V[long|s''] \leq \mathbb{E}\, V[short|s''] \implies$$
$$Pr(G|s'')[w_{LG} - w_{SG}] \leq Pr(B|s'')[w_{SB} - w_{LB}].$$

Given that $Pr(G|\hat{s})$ is increasing in $\hat{s}$, the previous constraints are not mutually exclusive as long as:

$$w_{LG} - w_{SG} \geq 0 \tag{1.3}$$

$$w_{SB} - w_{LB} \geq 0. \tag{1.4}$$

In other words, during both states of the world, the payment which corresponds to the revenue-maximizing position should not be lower than the payment which corresponds to the opposite position. As long as (1.3) and (1.4) hold, the previous two sets of constraints pin down to a single constraint:

$$Pr(G|\hat{s})[w_{LG} - w_{SG}] = Pr(B|\hat{s})[w_{SB} - w_{LB}]. \tag{1.5}$$

Constraint (1.5) implies that for $s = \hat{s}$, the manager is indifferent between going long and going short. We assume, without loss of generality, that if the manager is indifferent, he goes short.

Information acquisition constraints

Relation (1.6) provides the manager's expected utility under his outside option of taking an investment position without acquiring information.

$$\mathbb{E}\, V[not\ signal] = max\{\mathbb{E}\, V[no\ signal + long], \mathbb{E}\, V[no\ signal + short]\} \tag{1.6}$$

where the expected utility of each position is given by:

$$\mathbb{E}\, V[no\ signal + long] = p \times w_{LG} + (1-p) \times w_{LB}$$
$$\mathbb{E}\, V[no\ signal + short] = p \times w_{SG} + (1-p) \times w_{SB}.$$

Note that the manager has no private information, thus, his expectations about the quality of the asset coincide with his prior.

We now derive the manager's expected utility when acquiring infor-

mation. In the derivation of the expected utility, the manager takes into account the decision rule that he anticipates to follow after the signal is obtained, i.e., going long for signals $s > \hat{s}$ and going short otherwise. The expected utility equals the expected compensation reduced by the cost of acquiring information, c:

$$\mathbb{E}\, V[signal|\hat{s}] = -c + \int_0^{\hat{s}} \mathbb{E}\, C[short|s] f(s) ds + \int_{\hat{s}}^1 \mathbb{E}\, C[long|s] f(s) ds \quad (1.7)$$

where $f(s) = pf_G(s) + (1-p)f_B(s)$, and the expected compensation of each position is given by:

$$\mathbb{E}\, C[long|s] = Pr(G|s) \times w_{LG} + Pr(B|s) \times w_{LB} \quad (1.8)$$

$$\mathbb{E}\, C[short|s] = Pr(G|s) \times w_{SG} + Pr(B|s) \times w_{SB}. \quad (1.9)$$

By substituting of (1.8) and (1.9) into (1.7), and applying Bayes rule, we obtain:

$$\mathbb{E}\, V[signal|\hat{s}] = -c +$$
$$pF_G(\hat{s})w_{SG} + (1-p)F_B(\hat{s})w_{SB} + p(1-F_G(\hat{s}))w_{LG} + (1-p)(1-F_B(\hat{s}))w_{LB} \quad (1.10)$$

Hence, the information acquisition constraints pin down to:

$$\mathbb{E}\, V[signal|\hat{s}] \geq \mathbb{E}\, V[no\ signal + long] \implies$$
$$pF_G(\hat{s})(w_{SG} - w_{LG}) + (1-p)F_B(\hat{s})(w_{SB} - w_{LB}) \geq c \quad (1.11)$$
$$\mathbb{E}\, V[signal|\hat{s}] \geq \mathbb{E}\, V[no\ signal + short] \implies$$

$$p(1 - F_G(\hat{s}))(w_{LG} - w_{SG}) + (1 - p)(1 - F_B(\hat{s}))(w_{LB} - w_{SB}) \geq c. \quad (1.12)$$

Cost Minimization Problem

Following the previous analysis, the cost minimization problem of incentivizing information acquisition, given a decision rule which is characterized by a threshold $\hat{s}$, pins down to:

$$\underset{w_{SG}, w_{SB}, w_{LG}, w_{LB}}{\text{Minimize}} \quad \mathbb{E}\, C(\hat{s}) \quad \text{s.t.}$$

$$pF_G(\hat{s})(w_{SG} - w_{LG}) + (1 - p)F_B(\hat{s})(w_{SB} - w_{LB}) \geq c \quad (1.11)$$

$$p(1 - F_G(\hat{s}))(w_{LG} - w_{SG}) + (1 - p)(1 - F_B(\hat{s}))(w_{LB} - w_{SB}) \geq c \quad (1.12)$$

$$f_G(\hat{s})p[w_{LG} - w_{SG}] = f_B(\hat{s})(1 - p)[w_{SB} - w_{LB}] \quad (1.13)$$

$$w_{LG} - w_{SG} \geq 0 \quad (1.14)$$

$$w_{SB} - w_{LB} \geq 0 \quad (1.15)$$

$$w_{SG} \geq 0,\ w_{SB} \geq 0,\ w_{LG} \geq 0,\ w_{LB} \geq 0 \quad (1.16)$$

where $\mathbb{E}\, C(\hat{s})$ is given by:

$$\mathbb{E}\, C(\hat{s}) =$$

$$pF_G(\hat{s})w_{SG} + (1 - p)F_B(\hat{s})w_{SB} + p(1 - F_G(\hat{s}))w_{LG} + (1 - p)(1 - F_B(\hat{s}))w_{LB} \quad (1.17)$$

Proposition 1 characterizes the optimal contract, subject to the constraint that the investment decision threshold is $\hat{s}$.

Proposition 1: Constrained Optimal Contract

For $\hat{s} \leq \hat{s}_{min}$ the constrained optimal contract is given by:

$$w^*_{SB}(\hat{s}) = \frac{f_G(\hat{s})}{(1-p)(F_B(\hat{s})f_G(\hat{s}) - F_G(\hat{s})f_B(\hat{s}))}c$$

$$w^*_{LG}(\hat{s}) = \frac{f_B(\hat{s})}{p(F_B(\hat{s})f_G(\hat{s}) - F_G(\hat{s})f_B(\hat{s}))}c$$

$$w^*_{LB}(\hat{s}) = w^*_{SG}(\hat{s}) = 0.$$

For $\hat{s} \geq \hat{s}_{min}$ the constrained optimal contract is given by:

$$w^*_{SB}(\hat{s}) = \frac{f_G(\hat{s})}{(1-p)[(1-F_G(\hat{s}))f_B(\hat{s}) - (1-F_B(\hat{s}))f_G(\hat{s})]}c$$

$$w^*_{LG}(\hat{s}) = \frac{f_B(\hat{s})}{p[(1-F_G(\hat{s}))f_B(\hat{s}) - (1-F_B(\hat{s}))f_G(\hat{s})]}c$$

$$w^*_{LB}(\hat{s}) = w^*_{SG}(\hat{s}) = 0,$$

where $\hat{s}_{min}$ solves $f_B(\hat{s}_{min}) = f_G(\hat{s}_{min})$.

Proof. See Appendix A.3. □

The underlying intuition behind the constrained optimal contract relates to three remarks.

1st Remark: Note that the incentive constraints in the minimization problem are expressed in terms of the difference between the payment which corresponds to the revenue-maximizing position and the payment which corresponds to the opposite position, i.e., $w_{SB} - w_{LB}$, if the realized quality of the asset is bad, and $w_{LG} - w_{SG}$, if the realized quality of the asset is good.

Following this remark, it is easy to show that under the optimal contract, the payment when the manager does not take the revenue-maximizing position is zero, i.e., $w^*_{LB} = w^*_{SG} = 0$. For $w^*_{LB} = 0$, the intuition is the following. By decreasing the payment which corresponds to the long (w_{LB}) and the short position (w_{SB}) by the same amount, the equity holder decreases the expected compensation of the manager without affecting the incentive constraints. Such a deviation is feasible until w_{LB} hits its low bound. Similar intuition applies for the optimality of $w^*_{SG} = 0$.

2nd Remark: By relation (1.13), the relative pay w_{SB}/w_{LG} should be equal to $Pr(G|\hat{s})/Pr(B|\hat{s})$, which is determined by the choice of $\hat{s}$. Note that, $Pr(G|\hat{s})/Pr(B|\hat{s})$ is increasing in $\hat{s}$. The intuition behind this remark is that as $\hat{s}$ increases, the probability that the manager attributes to the asset being of good quality for a signal realization $s = \hat{s}$, increases as well. Thus, the relative payment that the manager requires to go short for $s = \hat{s}$, increases with $\hat{s}$, in order to compensate for the low probability that this payment will be realized.

3rd Remark: The last remark relates to the information acquisition constraints. It can be shown that if $s < \hat{s}_{min}$, (1.12) becomes redundant, and under the optimal contract, (1.11) binds. The opposite holds when $s > \hat{s}_{min}$. The intuition is the following. First, recall that the take-it-or-leave-it nature of the contract implies that the manager's utility when acquiring information and implementing $\hat{s}$ should be equal to his outside option. Suppose now that the principal aims to implement a low value of $\hat{s}$. For this value of $\hat{s}$, the manager attributes a high probability to the asset being bad, and, fol-

lowing the second remark, requires a relatively high payment to go long, w_{LG}/w_{SB}. For this ratio of payments, the manager's utility when going long without acquiring information exceeds his utility when going short without acquiring information. Hence (1.11) is more restrictive than (1.12).

Relaxing Limited Liability

It is worth highlighting that relaxing the agent's limited liability assumption would not affect qualitatively the optimal contract. This is captured by the first remark. Hence, if the agent has an initial wealth of $\bar{w}$, and $\bar{w}$ is common knowledge, the payments of the optimal contract would be: $w'_{SG}(\hat{s}) = -\bar{w}$, $w'_{LB}(\hat{s}) = -\bar{w}$, $w'_{SB}(\hat{s}) = w^*_{SB}(\hat{s}) - \bar{w}$, and $w'_{LG}(\hat{s}) = w^*_{LG}(\hat{s}) - \bar{w}$.

The following lemma explores how the optimal payment $w^*_{SB}(\hat{s})$ and $w^*_{LG}(\hat{s})$ relates to $\hat{s}$. Lemma 2 is critical for the relationship between the expected compensation cost, $\mathbb{E}\, C(\hat{s})$, and $\hat{s}$, which will in turn, allow us to shed light on the characterization of the optimal value of $\hat{s}$. Figure 1.1 provides the graphical illustration of $w^*_{SB}(\hat{s})$ and $w^*_{LG}(\hat{s})$ for $p = 0.5$, and for the case where $f_G(s) = 2s$, $f_B(s) = 2(1-s)$ (linear signal structure), with $\hat{s}$ captured in the horizontal axis.

Lemma 2: Relationship between optimal payments and $\hat{s}$.

The optimal payments $w^*_{SB}(\hat{s})$ and $w^*_{LG}(\hat{s})$ are:

(i) *decreasing in $\hat{s}$, for $\hat{s} \leq \hat{s}_{min}$*

(ii) *increasing in $\hat{s}$, for $\hat{s} \geq \hat{s}_{min}$.*

(iii) *minimized for* $\hat{s}_{min}$ *such as* $f_G(\hat{s}_{min}) = f_B(\hat{s}_{min})$.

(iv) *convex in* $\hat{s}$.

Proof. See Appendix A.3. □

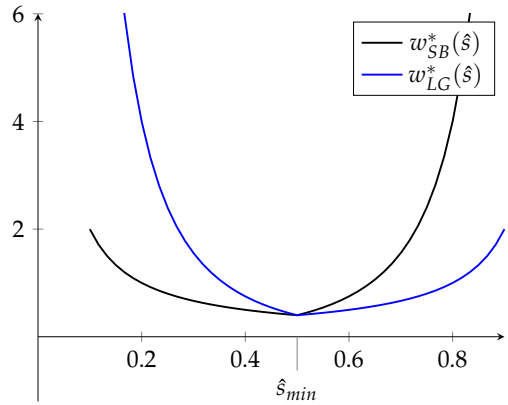


Figure 1.1. Relationship between optimal payments and $\hat{s}$.

Critical for the underlying mechanism in Lemma 2 is the role of the payments as opportunity cost. In fact, the opportunity cost of taking the "wrong" position coincides with w_{LG}, when the agent goes short and the revenue-maximizing position is long. In contrast, when the agent goes long and the revenue-maximizing position is short, the opportunity cost coincides with w_{SB}.

In order to capture the main mechanism, we explore the case where: i) the equity holder considers switching from an optimal contract which implements $\hat{s} = \hat{s}_{min}$ to an optimal contract which implements $\hat{s} = \hat{s}' > \hat{s}_{min}$, and ii) $p = 0.5$. For this value of p, the ratio of payments w_{SB}/w_{LG} that implements $\hat{s}_{min}$ equals one. Note that increasing $\hat{s}$ leads to a higher value of $\frac{Pr(G|\hat{s})}{Pr(B|\hat{s})}$. Thus, by (1.13), the increase in $\hat{s}$ should be accompanied by an increase in the payment ratio $\frac{w_{SB}}{w_{LG}}$. This implies that the equity holder,

in order to prevent the manager from going long for $s \in [\hat{s}_{min}, \hat{s}']$, should increase w_{SB}.[6] However, the increase in w_{SB} increases the manager's utility when going short without acquiring information, which now exceeds the manager's utility when acquiring information. Following that, the only way the equity holder can prevent the manager from going short without acquiring information is by increasing the opportunity cost of taking the wrong position, which is achieved by increasing w_{LG}. Hence, a deviation from $\hat{s} = \hat{s}_{min}$ to $\hat{s} = \hat{s}' > \hat{s}_{min}$ increases both w_{SB} and w_{LG}.[7]

The previous analysis refers to the part of Lemma 2 which explores deviations to values of $\hat{s}$ higher than $\hat{s}_{min}$. Similar intuition applies for the case where the principal considers switching from implementing $\hat{s} = \hat{s}_{min}$ to implementing $\hat{s} = \hat{s}'' < \hat{s}_{min}$.

From the constrained-optimal to the optimal contract.

Recall that the optimal contract is defined by the optimal compensation contract that implements a threshold $\hat{s} = \hat{s}^*$, where $\hat{s}^*$ is the value of $\hat{s}$ which maximizes the expected profit of the equity holder, $\mathbb{E}\,\Pi(\hat{s})$, with:

$$\mathbb{E}\,\Pi(\hat{s}) = \mathbb{E}\,R(\hat{s}) - \mathbb{E}\,C(\hat{s}). \tag{1.18}$$

where $\mathbb{E}\,C(\hat{s})$ denotes the expected compensation cost of implementing $\hat{s}$ (derived in subsection 1.4.3), and $\mathbb{E}\,R(\hat{s})$ denotes the expected revenue of implementing $\hat{s}$ (derived in subsection 1.4.4).

[6] We show in Appendix a.3 that decreasing w_{LG} instead of increasing w_{SB} is not feasible, because this would contradict with the optimality of the initial contract which implements $\hat{s} = \hat{s}_{min}$.

[7] The previous reasoning is only a pedagogical tool, which helps us capture the main mechanism; the steps presented above take place simultaneously.

1.4.3 Expected compensation cost of implementing $\hat{s}$, $\mathbb{E}\,C(\hat{s})$.

The constrained optimal contract, provided in Proposition 1, enables us to derive the expected cost of implementing $\hat{s}$, $\mathbb{E}\,C(\hat{s})$, which equals the manager's expected compensation:

$$\mathbb{E}\,C(\hat{s}) = (1-p)F_B(\hat{s})w^*_{SB}(\hat{s}) + p(1-F_G(\hat{s}))w^*_{LG}(\hat{s}) \tag{1.19}$$

Corollary 1: Expected cost of implementing $\hat{s}$, $\mathbb{E}\,C(\hat{s})$

For $\hat{s} \leq \hat{s}_{min}$, the expected cost is given by:

$$\mathbb{E}\,C(\hat{s}) \equiv \mathbb{E}\,C(\hat{s})^- = \left[\frac{F_B(\hat{s})f_G(\hat{s}) + (1-F_G(\hat{s}))f_B(\hat{s})}{F_B(\hat{s})f_G(\hat{s}) - F_G(\hat{s})f_B(\hat{s})}\right]c$$

For $\hat{s} \geq \hat{s}_{min}$, the expected cost is given by:

$$\mathbb{E}\,C(\hat{s}) \equiv \mathbb{E}\,C(\hat{s})^+ = \left[\frac{F_B(\hat{s})f_G(\hat{s}) + (1-F_G(\hat{s}))f_B(\hat{s})}{(1-F_G(\hat{s}))f_B(\hat{s}) - (1-F_B(\hat{s}))f_G(\hat{s})}\right]c$$

Proof. Corollary 2 is derived after substituting the optimal payments of Proposition 1 into (1.19). □

The first remark derived from Corollary 1 is that the expected cost of implementing $\hat{s}$ is linearly dependent on the cost of acquiring information, c. The second remark is that the expected cost of implementing a given threshold $\hat{s}$ does not depend on the prior beliefs, p - p is internalized by the optimal payment. The last remark is about the behavior of $\mathbb{E}\,C(\hat{s})$ with respect to threshold $\hat{s}$, which is summarized in Lemma 3.

Lemma 3: Relationship between $\mathbb{E}\,C(\hat{s})$ and $\hat{s}$.

(i) *For each* $\hat{s} \in [0, \hat{s}_{min})$, $\mathbb{E}\,C(\hat{s})$ *is decreasing in* $\hat{s}$.

(ii) *For each* $\hat{s} \in (\hat{s}_{min}, 1]$, $\mathbb{E}\,C(\hat{s})$ *is increasing in* $\hat{s}$.

(iii) $\mathbb{E}\,C(\hat{s})$ *is minimized for* $\hat{s}_{min}$ *such as* $f_G(\hat{s}_{min}) = f_B(\hat{s}_{min})$.

(iv) $\mathbb{E}\,C(\hat{s})$ *is convex in* $\hat{s}$.

One can easily notice the similarity of Lemma 3 with Lemma 2 – the behavior of $\mathbb{E}\,C(\hat{s})$ with respect to $\hat{s}$ follows a similar pattern with the behavior of $w_{SB}(\hat{s})$ and $w_{LG}(\hat{s})$ with respect to $\hat{s}$. This relates to the following remark. Under the optimal contract, the expected utility of the risk-neutral agent when implementing $\hat{s}$: i) coincides with his expected compensation reduced by the cost of acquiring information, c, and ii) equals his outside option, which is given by:

$$\begin{aligned}\mathbb{E}\,V(\hat{s}) &= max\{\mathbb{E}\,V(no\ signal + long), \mathbb{E}\,V(no\ signal + short)\} \\ &= max\{p \times w^*_{LG}(\hat{s}), (1-p) \times w^*_{SB}(\hat{s})\}. \end{aligned} \tag{1.20}$$

Hence, following Proposition 1, the expected cost $\mathbb{E}\,C(\hat{s})$ for $\hat{s} < \hat{s}_{min}$ equals:

$$\mathbb{E}\,C(\hat{s}) = p \times w^*_{LG}(\hat{s}) + c \tag{1.21}$$

whereas, the expected cost $\mathbb{E}\,C(\hat{s})$ for $\hat{s} > \hat{s}_{min}$ equals:

$$\mathbb{E}\,C(\hat{s}) = (1-p) \times w^*_{SB}(\hat{s}) + c. \tag{1.22}$$

Relations (1.21) and (1.22) capture the finding that $\mathbb{E}\,C(\hat{s})$ behaves similarly to the payment $w_{LG}(\hat{s})$, for $s < \hat{s}_{min}$, and similarly to the payment $w_{SB}(\hat{s})$,

for $s > \hat{s}_{min}$. Thus, the intuition behind the relation of $\mathbb{E}\,C(\hat{s})$ with $\hat{s}$ is identical to the underlying intuition in Lemma 2. Figure 1.2 provides the graphical illustration of $\mathbb{E}\,C(\hat{s})$, for the case where $f_G(s) = 2s$, $f_B(s) = 2(1-s)$ (linear signal structure), with $\hat{s}$ captured in the horizontal axis.

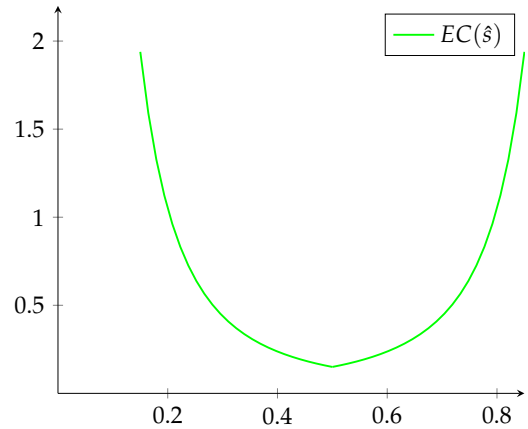


Figure 1.2. Relationship between $\mathbb{E}\,C(\hat{s})$ and $\hat{s}$.

1.4.4 Step 3: Expected revenue of implementing $\hat{s}$, $\mathbb{E}\,R(\hat{s})$

Suppose that the equity holder offers a contract which implements $\hat{s}$. When he forms his beliefs about the expected revenue, he anticipates the manager to go long when the signal is greater than $\hat{s}$, and to go short otherwise. Hence, the expected revenue of implementing a threshold $s = \hat{s}$ is:

$$\mathbb{E}\,R(\hat{s}) = \int_0^{\hat{s}} \mathbb{E}\,R[short|s]f(s)ds + \int_{\hat{s}}^1 \mathbb{E}\,R[long|s]f(s)ds \tag{1.23}$$

where $f(s) = pf_G(s) + (1-p)f_B(s)$. Also, the equity holder's expected revenue when the manager goes short or long, given a signal s, is given by:

$$\mathbb{E}\,R[short|s] = P_0 - \mathbb{E}[P_1|s] = \epsilon\{-Pr(G|s) + Pr(B|s)\} \tag{1.24}$$

$$\mathbb{E}\, R[long|s] = -P_0 + \mathbb{E}[P_1|s] = \epsilon\{Pr(G|s) - Pr(B|s)\} \tag{1.25}$$

By substituting (1.24) and (1.25) into (1.23), and applying the Bayes rule, we obtain:

$$\mathbb{E}\, R(\hat{s}) = \epsilon\{p(1 - 2F_G(\hat{s})) + (1-p)(2F_B(\hat{s}) - 1)\} \tag{1.26}$$

Lemma 4 provides the relationship between the expected revenue and the implemented threshold $\hat{s}$. Figure 1.3 provides the graphical illustration of $\mathbb{E}\, R(\hat{s})$ for the case where $f_G(s) = 2s$, $f_B(s) = 2(1-s)$ (linear signal structure), where $\hat{s}$ is presented in the horizontal axis.

Lemma 4 Relationship between $\mathbb{E}\, R$ and $\hat{s}$.

(i) *For each* $\hat{s} \in [0, \hat{s}^{FB})$, $\mathbb{E}\, R(\hat{s})$ *is increasing and concave in* $\hat{s}$.

(ii) *For each* $\hat{s} \in (\hat{s}^{FB}, 1]$, $\mathbb{E}\, R(\hat{s})$ *is decreasing and concave in* $\hat{s}$.

(iii) *The expected revenue* $\mathbb{E}\, R(\hat{s})$ *is single-peaked at* $\hat{s} = \hat{s}^{FB}$, *where* $\hat{s}^{FB}$ *solves:*

$$\frac{f_G(\hat{s}^{FB})}{f_B(\hat{s}^{FB})} = \frac{(1-p)}{p} \tag{1.27}$$

Proof. See Appendix A.3. □

1st Remark: Given the symmetry of the problem, the equity holder prefers the manager to go long when his updated beliefs are above 0.5, and to go short otherwise. For a signal realization where the manager's updated beliefs equal 0.5, the equity holder is indifferent. The signal realization which leads to an updated belief of 0.5 solves (1.27). We show in the Appendix

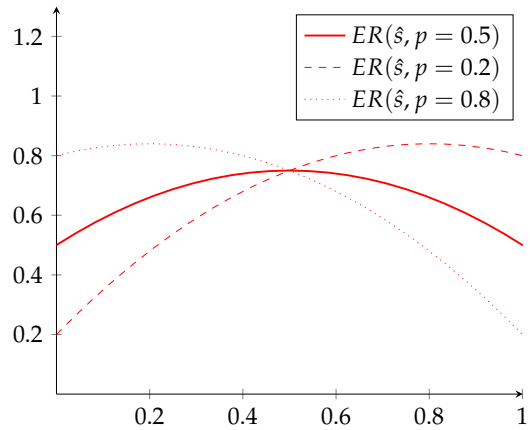


Figure 1.3. Relationship between $\mathbb{E}\, R(\hat{s})$ and $\hat{s}$.

that the signal realization which solves (1.27) coincides with the threshold that the equity holder would implement if there were no agency problem.

2nd Remark: The concavity of $\mathbb{E}\, R(\hat{s})$ is an implication of the MLRP. Recall that by applying Bayes rule, $\frac{Pr(G|s)}{Pr(B|s)}$ equals $\frac{f_G(s)}{f_B(s)} \times \frac{p}{1-p}$. Hence, the lower the value of $\hat{s}$ compared to $\hat{s}^{FB}$, the higher the expected distortion of going long in the area $s \in (\hat{s}, \hat{s}^{FB})$, as it is the more likely that going short is the revenue-maximizing position. Similar intuition applies for deviations to $\hat{s} > \hat{s}^{FB}$.

3rd Remark: A clear implication of (1.27) is that the peak of $\mathbb{E}\, R(\hat{s})$ moves towards lower $\hat{s}$ as the value of p increases. The intuition is straightforward; the higher the prior belief p, the higher the prior probability that the equity holder attributes to a long position being revenue-maximizing. Thus, for high values of p, the equity holder prefers the manager to go long, unless he receives strong evidence that going short is the revenue-maximizing position, i.e., a very low signal. The opposite holds for low values of p.

1.4.5 Step 4: Optimal threshold $\hat{s}^*$

The goal of this section is to characterize the optimal threshold $\hat{s}^*$, which maximizes the expected profit of the equity holder, $\mathbb{E}\,\Pi(\hat{s})$, where,

$$\mathbb{E}\,\Pi(\hat{s}) = \mathbb{E}\,R(\hat{s}) - \mathbb{E}\,C(\hat{s})$$

By Lemma 3, the expected cost $\mathbb{E}\,C(\hat{s})$ is minimized for $\hat{s} = \hat{s}_{min}$, such as:

$$\frac{f_G(\hat{s}_{min})}{f_B(\hat{s}_{min})} = 1 \tag{1.28}$$

Also, by Lemma 4, the expected revenue $\mathbb{E}\,R(\hat{s})$ is maximized for $\hat{s} = \hat{s}^{FB}$, such as:

$$\frac{f_G(\hat{s}^{FB})}{f_B(\hat{s}^{FB})} = \frac{(1-p)}{p} \tag{1.29}$$

Thus, as long as $p \neq 0.5$, the value of $\hat{s}$ which maximizes the expected revenue, $\hat{s}^{FB}$, differs from the value which minimizes the expected cost, $\hat{s}_{min}$. As a result, the optimal value of $\hat{s}$ relies on an *inherent trade-off* between minimizing the cost of incentivizing information acquisition, and the benefit of implementing a threshold $\hat{s}$, which is as close as possible to the revenue-maximizing threshold, $\hat{s}^{FB}$.

Optimality condition for $\hat{s}$.

The equity holder offers a contract that corresponds to a threshold $\hat{s}$ which optimally resolves this trade-off. Thus, for a threshold $\hat{s}$ to be optimal, the

following condition needs to hold:

$$\frac{\partial \mathbb{E}\,\Pi(\hat{s})}{\partial \hat{s}} = \frac{\partial \mathbb{E}\,R(\hat{s})}{\partial \hat{s}} - \frac{\partial \mathbb{E}\,C(\hat{s})}{\partial \hat{s}} = 0 \tag{1.30}$$

Condition (1.30) captures the idea that for the optimal value of $\hat{s}$, denoted by $\hat{s}^*$, the expected marginal revenue ($\partial \mathbb{E}\,R(\hat{s})/\partial \hat{s}$) should equal the expected marginal cost ($\partial \mathbb{E}\,C(\hat{s})/\partial \hat{s}$), where $\mathbb{E}\,R(\hat{s})$ is defined in Lemma 4, whereas $\mathbb{E}\,C(\hat{s})$ is defined in Lemma 3.

1.4.6 Optimal compensation contract

The optimal compensation contract $\hat{W}^*(\hat{s}^*)$ is the constrained-optimal compensation contract of Proposition 1, subject to the constraint that $\hat{s} \equiv \hat{s}^*$.

Proposition 2: Optimality Contract, $\hat{W}^*(\hat{s}^*)$
If $p \geq 0.5$, the optimal value of $\hat{s}$, $\hat{s}^*$ solves:

$$2\epsilon\{-pf_G(\hat{s}) + (1-p)f_B(\hat{s})\} = c\frac{F_B(\hat{s})[f_B'(\hat{s})f_G(\hat{s}) - f_B(\hat{s})f_G'(\hat{s})]}{(F_B(\hat{s})f_G(\hat{s})f_B(\hat{s})F_G(\hat{s}))^2} \tag{1.31}$$

and the optimal payments are given by:

$$w_{SB}^*(\hat{s}^*) = \frac{f_G(\hat{s}^*)}{(1-p)(F_B(\hat{s}^*)f_G(\hat{s}^*) - F_G\hat{s}^*)f_B(\hat{s}^*))}c$$

$$w_{LG}^*(\hat{s}^*) = \frac{f_B(\hat{s}^*)}{p(F_B(\hat{s}^*)f_G(\hat{s}^*) - F_G(\hat{s}^*)f_B(\hat{s}^*))}c$$

$$w_{LB}^*(\hat{s}^*) = w_{SG}^*(\hat{s}^*) = 0.$$

If $p \leq 0.5$, the optimal value of $\hat{s}$, $\hat{s}^*$ solves:

$$2\epsilon\{-pf_G(\hat{s}) + (1-p)f_B(\hat{s})\} = -c\frac{(f_B(\hat{s})f'_G(\hat{s}) - f'_B(\hat{s})f_G(\hat{s}))(-1+F_G(\hat{s}))}{[f_G(\hat{s}) - f_B(\hat{s})f_G(\hat{s}) + f_B(\hat{s})(-1+F_G(\hat{s}))]^2} \tag{1.32}$$

and the optimal payments are given by:

$$w^*_{SB}(\hat{s}^*) = \frac{f_G(\hat{s}^*)}{(1-p)[(1-F_G(\hat{s}^*))f_B(\hat{s}^*) - (1-F_B(\hat{s}^*))f_G(\hat{s}^*)]}c$$

$$w^*_{LG}(\hat{s}^*) = \frac{f_B(\hat{s}^*)}{p[(1-F_G(\hat{s}^*))f_B(\hat{s}^*) - (1-F_B(\hat{s}^*))f_G(\hat{s}^*)]}c$$

$$w^*_{LB}(\hat{s}^*) = w^*_{SG}(\hat{s}^*) = 0.$$

Proof. See Appendix A.3. □

1.4.7 When is information acquisition profitable?

Recall that the previous analysis refers to the case where the equity holder finds it optimal to incentivize the manager to acquire information. However, the equity holder has the option of offering a contract which does not impose the manager to obtain information. If the manager does not hold any private information, the equity holder would prefer the manager to go long if the asset is considered undervalued ($p > 0.5$), and to go short otherwise. We can show that the principal could implement this decision rule by offering a contract, denoted as $\hat{W}'$, which pays an arbitrarily small amount $\tilde{\eta} \to 0$, if the agent follows the aforementioned strategy, and zero otherwise. Following this remark, Lemma 7 characterizes the condition which needs to hold such as it is optimal for the equity holder to incentivize information acquisition.

Lemma 7

The equity holder finds it optimal to incentivize the manager to acquire information as long as:

$$2\epsilon\{(1-p)F_B(\hat{s}^*) - pF_G(\hat{s}^*) - 2p\} \geq \frac{F_B(\hat{s}^*)f_G(\hat{s}^*) + (1-F_G(\hat{s}^*))f_B(\hat{s}^*)}{F_B(\hat{s}^*)f_G(\hat{s}^*) - F_G(\hat{s}^*)f_B(\hat{s}^*)}c \tag{1.33}$$

$$\begin{aligned} &2\epsilon\{(1-p)F_B(\hat{s}^*) - pF_G(\hat{s}^*) - 2(1-p)\} \geq \\ &\frac{F_B(\hat{s}^*)f_G(\hat{s}^*) + (1-F_G(\hat{s}^*))f_B(\hat{s}^*)}{(1-F_G(\hat{s}^*)f_B(\hat{s}^*) - (1-F_B(\hat{s}^*))f_G(\hat{s}^*)}c \end{aligned} \tag{1.34}$$

where (1.33) ((1.34)) corresponds to the case where $p > 0.5$ ($p < 0.5$). Note that 2ϵ reflects the benefit of taking the "right" over the "wrong" position, where "right" is the revenue-maximizing position.[8] Note also that the term in the curly brackets captures the expected increase in the probability of taking the "right" position due to the signal acquisition. Thus, the LHS captures the expected benefit of acquiring information, where the RHS is nothing more than the expected compensation cost.

1.5 Implications of the Optimal Contract

In this section, we explore the main implications of the optimal contract. We focus on the most interesting case where the equity holder aims to incentivize the manager to acquire information.

[8] If the realized state is good and the manager goes long, equity holder's net return is $P_1^G - P_0 = \epsilon$. In contrast, if the manager goes short, equity holder's net return is $-P_1^G + P_0 = -\epsilon$. Likewise, if the realized state is bad and the manager goes short, equity holder's net return is $-P_1^B + P_0 = \epsilon$. In contrast, if the manager goes long, equity holder's net return is $P_1^B - P_0 = -\epsilon$. Thus, independently of the state of the world, the net benefit of taking the right over the wrong position is 2ϵ.

1.5.1 Premium for going against the flow & bias in the investment decision

Proposition 3 explores the relation between the optimal payment when the manager "goes against the flow" and the optimal payment when the manager "follows the flow". By "following the flow" we define the case where the manager takes the same position as the position that the equity holder would take, had he not hired the manager. In this setting, "going against the flow" means going long when the asset is overvalued ($p < 0.5$) and going short when the asset it undervalued ($p > 0.5$).

Proposition 3: Premium for going against the flow.

Under the optimal contract:

(i) *If* $p > 0.5$, *then* $w_{SB}(\hat{s}^*) > w_{LG}(\hat{s}^*)$.

(ii) *If* $p < 0.5$, *then* $w_{LG}(\hat{s}^*) > w_{SB}(\hat{s}^*)$.

Proof. See Appendix A.3. □

Hence, under the optimal contract, there is a premium for going against the flow. Proposition 4 explores how $\hat{s}^*$ relates to: i) the optimal value of $\hat{s}$ if there is no agency problem (first best), $\hat{s}^{FB}$, and ii) the value of $\hat{s}$ which minimizes the cost of incentivizing information acquisition, $\hat{s}_{min}$.

Proposition 4: Bias in the investment decision.

Under the optimal contract:

(i) *If* $p \geq 0.5$, *then* $\hat{s}^{FB} < \hat{s}^* \leq \hat{s}_{min}$.

(ii) *If* $p \leq 0.5$, *then* $\hat{s}^{FB} > \hat{s}^* \geq \hat{s}_{min}$.

Proof. See Appendix A.3. □

The proof of Proposition 3 and Proposition 4 can be captured in the following figures, which illustrates Lemma 3 and Lemma 4. Figure 1.4 presents the case where $p > 0.5$. Note that for these values of p, an investment threshold $\hat{s}' \in [0, \hat{s}^{FB}]$ cannot be optimal, because switching to $\hat{s}'' = \hat{s}' + \eta$, where η is a small positive number, decreases the expected compensation cost and increases the expected revenue. Likewise, an investment threshold $\hat{s}' \in [\hat{s}_{min}, 1]$ cannot be optimal, because switching to $\hat{s}'' = \hat{s}' - \eta$ decreases the expected compensation cost and increases the expected revenue. Thus, for $p > 0.5$, $\hat{s}^* \in (\hat{s}^{FB}, \hat{s}_{min}]$. Similar intuition applies for the case where $p < 0.5$, which is illustrated in Figure 1.5.

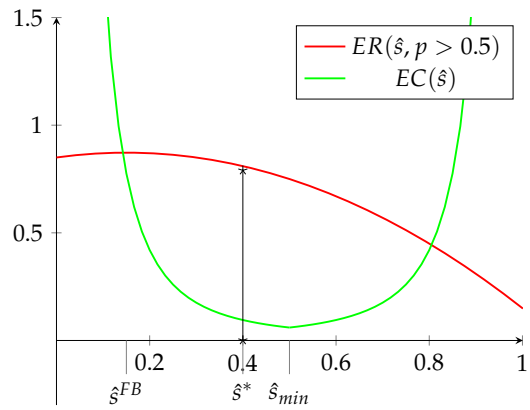


Figure 1.4. Bias in the investment decision - Case where $p > 0.5$, Flow: long

Proposition 4 captures the key feature of the optimal contract: the manager goes against the flow more often than the first best. Proposition 4 is a consequence of Proposition 3 and the MLRP property. The intuition behind the premium and the bias for going against the flow relies on the interaction of unobservable information acquisition and the multi-tasking nature of the problem. In order to incentivize the manager to acquire in-

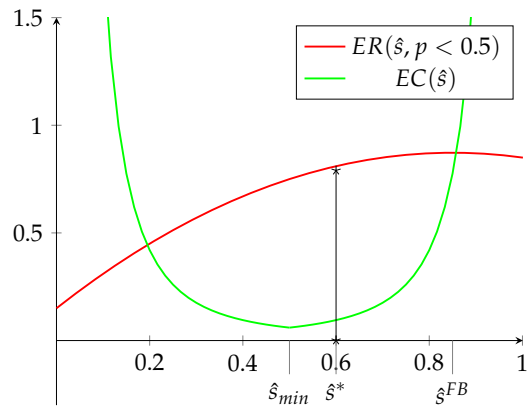


Figure 1.5. Bias in the investment decision - Case where $p < 0.5$, Flow: short

formation, the equity holder should promise a positive payment only if the ex-post right position is taken. However, the compensation contract – apart from affecting the incentive of the manager to acquire information – also affects his investment decision, which in turn, determines the portfolio's expected revenue.

In this environment, the portfolio's revenue is maximized when the payments which correspond to a short and long position are equal. A critical property is that under the optimal contract, the manager's utility should coincide with his outside option. For equal payments, the outside option of the manager is to follow the flow without acquiring information. The equity holder can thus worsen outside option of the manager by lowering the payment when the flow is followed. Worsening outside option of the manager enables the principal to incentivize information acquisition at a lower cost– however, changing the ratio of payments comes at the cost of decreasing the portfolio's expected revenue; for a ratio of payments different than one, the first best is no longer implemented. The optimal contract thus solves the inherent trade-off between the cost of incentivizing learning

and benefit of implementing an investment decision as close as possible to the revenue-maximizing one. We show that for small deviations from the first best, the decrease in the expected cost is greater than the decrease in the expected revenue.

Corollary 2 explores the main implications of the bias in the investment decision on: i) the probability of going short or long, ii) the probability that the implemented position is revenue-maximizing, and iii) the beliefs about the state of the world, after the position is implemented.

Corollary 2: Implications of the Optimal Contract compared to First Best. *If $p > 0.5$ ($p < 0.5$) and compared to the case where there is no agency problem, under the optimal contract:*

(i) *The manager is more (less) likely to go short.*

(ii) *Given that a short position is implemented, it is less (more) likely to be revenue-maximizing.*

(iii) *Given that a long position is implemented, it is more (less) likely to be revenue-maximizing.*

(iv) *Given the implemented position, the beliefs about the asset being good are higher (lower).*

Proof. See Appendix A.3. □

Part one implies that compared to the first best, it is less likely that the manager will invest in an asset where prior beliefs indicate that it is undervalued. This is a direct consequence of Proposition 4. The intuition behind part two is that under the optimal contract and for $p > 0.5$,

the manager is tempted by the premium to go short, even for signal realizations where he believes that it is more likely that going long is the revenue-maximizing position. Thus, for $p > 0.5$ and conditional on going short, the probability of going short being the right position is lower than in the case where the first best threshold is implemented. The intuition behind part three is similar: the manager is willing to go long and forgo the premium of the short position, only if he holds strong evidence that the price will increase. Part four is a direct consequence of part two and three.

1.5.2 Informational role of investment decision

Here we argue that the informational value of each position differs, depending on whether the asset is considered overvalued or undervalued. Note that an implication of part two and three of Corollary 2 is that, compared to the first best, if prior beliefs indicate that the asset is considered undervalued ($p > 0.5$): i) a short position is a weak indicator that the asset is of low quality,[9] whereas, ii) a long position is a strong indicator that the asset is of high quality. In contrast, if the asset is considered overvalued ($p < 0.5$), a short position is a strong indicator that the asset is of low quality, whereas a long position is a weak indicator that the asset is of high quality.

Another implication of Corollary 2 is that the resulting bias in the investment decision can sustain extreme prior beliefs about the asset's quality. For instance, if the market attributes a high probability to the asset

[9]This is because the conditional probability that the asset price will drop given a short position exceeds the corresponding probability, when there is no agency problem.

being of good quality (i.e., p has a high value), then the posterior beliefs that the asset is of good quality, conditional on both short and long position, are higher compared to the first best. Intuitively, this is because a high value of p leads to a high premium when the short position is chosen, which in turn, tempts the manager to go short. As a result, after observing a short position, investors do not radically downgrade their beliefs about the probability that the asset is of good quality. This is because the short position might be chosen not because of a low signal realization, but due to its high premium. Similarly, after observing a long position, investors upgrade radically their beliefs about the probability that the asset is good. This relies on the fact that investors believe that, if the manager is willing to forgo the premium that the short position entails, it must be that he attributes a high probability on the asset being good.

Following a similar reasoning, if the market attributes a low probability to the asset being of good quality, then the posterior beliefs that the asset is of good quality, conditional of both the short and long position, are lower compared to the first best.

1.5.3 Link with empirical evidence and empirical predictions

Similarly to this setting, financial analysts collect private information regarding the ex-ante unknown state of the world (profitability of a company), and issue a forecast regarding future earnings. By applying our framework to this environment, we would expect that the analysts' forecasts are biased against the publicly available information. The *bias against*

the flow is consistent with the empirical evidence on financial analysts. For instance, Bernhardt and Kutsoati (2001) and Pierdzioch et al. (2013) provide evidence that analysts issue biased contrarian forecasts. In fact, Bernhardt and Kutsoati (2001) show that the conditional probability that a forecast exceeds realized earnings, given that the forecasts exceeds the consensus forecast, is lower than the unconditional probability. Note that the underlying idea in Bernhardt and Kutsoati (2001) is similar to our setting: our model predicts that in an environment where prior beliefs indicate it is more likely that the price will drop, the conditional probability that the asset price increases given a long position, is lower than the corresponding probability when the first best threshold is implemented.

Also, Corollary 2 and the mechanism which was presented in the section 1.5.2 , could provide a theoretical foundation of the empirical evidence which indicate that investor over-react and/or under-react to financial analysts' forecasts (Elgers et al., 2001, Elliot et al., 1995, Elliot et al., 1995, Mendenhall, 1991, Sloan, 1996). Similarly to the forecasts of financial analysts, the position of the manager is informative about his private signal. Hence, the implemented position can be used by market participants, to infer the quality of the asset and the future price. Our model predicts that rational investors should under-react when the manager's position is against the flow, and over-react when the manager's position follows the flow.

1.5.4 Impact of information acquisition cost, c.

Proposition 5 summarizes the impact of the cost of acquiring information, c, on the optimal contract and its main implications. We consider values of c for which the equity holder finds it optimal to incentivize the manager to acquire information.

Proposition 5: Impact of an increase in c

If $p > 0.5$ ($p < 0.5$), an increase in the cost of information acquisition from c to c', leads to:

(i) *An increase (decrease) in the optimal threshold from $\hat{s}^*$ to $\hat{s}'^*$.*

(ii) *A higher bias $|\hat{s}'^* - \hat{s}^{FB}| > |\hat{s}^* - \hat{s}^{FB}|$.*

(iii) *A lower (higher) probability that an implemented short position is revenue-maximizing.*

(iv) *A higher (lower) probability that an implemented long position is revenue-maximizing.*

(v) *A lower expected probability of taking the revenue-maximizing position.*

Proof. See Appendix A.3. □

We provide the underlying intuition behind Proposition 5 for the case where $p > 0.5$. Similar intuition applies for the case where $p < 0.5$. The findings of Proposition 5 depend on the behavior of $\mathbb{E}\, C(\hat{s})$ as c increases. By Lemma 3, $\mathbb{E}\, C(\hat{s})$ is convex, decreasing in $\hat{s}$ for $\hat{s} \in [0, \hat{s}_{min})$, increasing in $\hat{s}$ for $\hat{s} \in (\hat{s}_{min}, 1]$ and linearly dependent in c. Thus, an increase in cost c shifts the entire $\mathbb{E}\, C(\hat{s})$ curve upwards, which leads to a

steeper-sloped U-shape. This is captured in Figure 1.6, where $\hat{s}$ is depicted in the horizontal axis, the red line represents the $\mathbb{E}\,R(\hat{s})$, the green line represents $\mathbb{E}\,C(\hat{s}, c)$ and the blue line represents $\mathbb{E}\,C(\hat{s}, c')$.

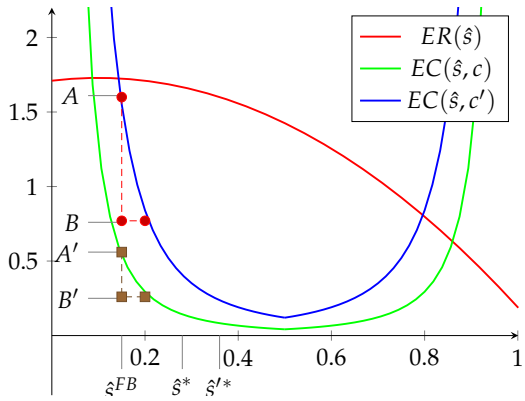


Figure 1.6. Impact of an increase in the information acquisition cost c

Recall that when $p \geq 0.5$, a deviation from $\hat{s}^{FB}$ to $\hat{s} = \hat{s}^{FB} + \eta$ has a positive and a negative effect. On the one hand, it decreases the expected compensation cost (Lemma 3), but on the other hand, it decreases the expected revenue (Lemma 4). Under the optimal value, $\hat{s}^*$, the two opposite forces cancel each other, i.e., the benefit of decreasing the expected cost coincides with the loss of decreasing the expected revenue. Notice that a steeper-sloped U-shape for $\mathbb{E}\,C(\hat{s})$, which follows an increase in c, strengthens the incentive to increase $\hat{s}$. This is because, for a given deviation η from the first best, the reduction in the expected cost is higher the steeper the slop of U-shape is. This can be seen in Figure 1.6, where the distance $A'B'$ (AB) captures the reduction in the expected cost before (after) the increase in c. Thus, $\hat{s}'^* > \hat{s}^*$, which combining with the fact that $\hat{s}^{FB}$ is unaffected by the change in c, leads to a higher bias compared to the first best.

Part three (four) is a consequence of part two of Proposition 5 and part two (three) of Corollary 2. Finally, part five stems from the monotonic

relation between the expected revenue and the probability of getting the revenue-maximizing position. Notice that the lower probability of taking the right position is not a consequence of the fact that the manager does not acquire information (recall that Proposition 5 refers to the case where incentivizing information acquisition is optimal). The underlined intuition is that the distortion in the investment decision increases, which in turn, diminishes the information which is incorporated into the implemented position.

1.5.5 Impact of market prior beliefs, p.

Proposition 6 summarizes the impact of market beliefs, p, on the implemented position. In particular, we explore the case where market prior beliefs become more extreme, i.e., $|p - 0.5|$ increases. We denote as $\hat{s}^*$ and $\hat{s}'^*$ the equilibrium value of $\hat{s}$ before and after the change in p. Similarly, $\hat{s}^{FB}$ and $\hat{s}'^{FB}$ correspond to the first best value of $\hat{s}$ before and after the change in p. We consider values of p for which incentivizing information acquisition is optimal.

Proposition 6: Impact of more extreme prior beliefs.
If $p > 0.5$ ($p < 0.5$), an increase (decrease) in beliefs p leads to:

(i) *A decrease (increase) in the optimal threshold from $\hat{s}^*$ to $\hat{s}'^*$.*

(ii) *Higher bias $|\hat{s}'^* - \hat{s}^{FB}| > |\hat{s}^* - \hat{s}^{FB}|$, when the signal structure is linear.*

Proof. See Appendix A.3. □

The intuition behind part one is straightforward. An increase (decrease)

in p decreases (increases) $\hat{s}^{FB}$ without affecting the threshold which minimizes the expected cost, $\hat{s}_{min}$. Hence, the second best $\hat{s}^*$ moves away from $\hat{s}_{min}$ towards $\hat{s}'^{FB}$.

Part two is less straightforward. We showed earlier that a deviation from the first best by a given η implies a loss due to the lower expected revenue, and a gain due to the lower expected compensation cost. As the prior beliefs, and the corresponding value of $\hat{s}^{FB}$, become more extreme, the convexity of the expected compensation cost implies that the gain due to the lower expected cost increases. However, the decrease in the expected revenue depends on the particular distribution of $f_\theta(s)$.

In Proposition 6, we focus on the linear signaling structure, i.e., $f_\theta(\hat{s})'$ is assumed to be constant. The advantage of this signaling structure is that the loss of a given deviation from the first best is independent of the value of p. Hence, as prior beliefs become more extreme, for a given deviation η, the decrease in the cost is larger, whereas the decrease in the revenue is unaffected. As a result, more extreme prior beliefs correspond to a higher distortion in the investment decision, compared to the first best.

1.6 Concluding Remarks & Further Discussion

This paper examines the case where equity holders of a fund delegate an investment decision to a fund manager. We explore an environment where the return of the investment decision depends on the unknown state of the world, and the information acquisition by the agent is unobservable and unverifiable. We show that the optimal contract which incentivizes the manager to acquire information promises a positive payment only the

decision of the agent is proven correct. Also, we find that a key feature of the optimal contract is that the manager takes contrarian actions more often than the first best. Furthermore, we find that both the direction and the extent of the distortion in the investment decision relates to market beliefs. Besides, we show how these beliefs affect the informational role of the investment decision. Finally, we show that the main findings are robust to: i) a setup where the state of the world is imperfectly observed, and ii) a setup where the equity holder allocates the tasks of information acquisition and investment decision to two different agents.

In this section, we attempt to make the connection between the benchmark setting and alternative environments. Recall that three are the critical characteristics of the benchmark model: i) delegation of a decision by a principal to an agent, ii) the revenue-maximizing action is state-dependent, and, iii) the agent can acquire costly and private information about the ex-ante unknown state of the world. In what follows, we discuss four environments which share, to some extent, these three characteristics, and present the implications of the optimal contract in each setting. Table 1 provides the analogy between the benchmark case and the environments we discuss.

Incentivizing Innovation

Similarly to Manso (2011), the benchmark model can also be used to characterize the optimal compensation contracts which incentivize an agent to innovate. For instance, suppose an environment where two strategies/techniques are available; a conventional one, which leads to a fixed return R, and an innovative one, which returns $R_S > R$, if it is successful,

and $R_F < R$ otherwise. By nature, the return of the innovative strategy is ex-ante unknown to the agents, who, however, hold prior beliefs about the return being R_S, denoted by p. The main difficulty which arises in this setup is that the principal and the agent cannot observe whether the innovative strategy is successful, unless this strategy is implemented. This difficulty is addressed in Appendix A.1.

In this environment, the optimal contract pays the agent only when he adopts an innovative strategy which is successful or a conventional strategy which is supported by the public signal. Besides, if prior beliefs indicate that the NPV of the innovative strategy is higher (lower) than the NPV of the conventional strategy, the optimal compensation contract results in under-implementation (over-implementation) of the innovative strategy, compared to the first best.

Credit Rating Agencies

We argue in the Online Appendix that the analysis presented in the previous sections can also be applied to the rating process used by credit rating agencies (CRAs). In practice, the rating process is characterized by delegation of the evaluation of a company's creditworthiness to an analyst, who can obtain information before issuing a rating. Consistently with the literature on credit rating agencies, which recognizes reputation concerns as the main objective of CRAs, we assume that the principal's objective is to maximize the probability of issuing a rating which corresponds to the actual type. In our setting, this objective translates into giving a good rating to a creditworthy company, and a bad rating otherwise, where the creditworthiness of a company corresponds to the ex-ante unknown state of the

world.

In this environment, the optimal compensation contract pays the analysts only when the rating matches the actual type of the company. Also, the optimal contract implies a premium for a rating against the flow. This premium gives rise to more frequent bad (good) ratings compared to the first best, when the company is ex-ante more (less) likely to be of good type. Moreover, if the company is ex-ante more likely to be of good type, then a bad rating is less (more) likely to be correct, whereas a good rating is more (less) likely to be correct. A direct implication of the bias in the issuance of ratings is that it can sustain extreme prior beliefs about a company's type.

Finally, we endogenize the company's borrowing interest rates after the rating is issued. We find that conditional on the rating, the interest rate is lower (higher) than the first best when prior beliefs indicate that the company is of good (bad) type. This finding could provide an explanation for the emergence of long periods of low or high interest rates.

Product Design

The main role of a product manager is, first, to analyze the market, and subsequently, decide about those product features which will accommodate the ex-ante unknown future demand. In this environment, the optimal contract pays the product manager only if he designs a product which eventually accommodates the demand. Also, the optimal contract leads to over-investment in products and product features which are ex-ante more likely to fail to accommodate the demand. Besides, conditional on adopting such product, it is less likely than the first best that the demand will be

addressed. In contrast, conditional on adopting a product which is ex-ante more likely to accommodate the demand, it is more likely than the first best that it will succeed in accommodating the demand. Thus, this model would predict an excessive supply of products which have a low ex-ante probability of accommodating the demand, and, in turn, are very likely not to accommodate the demand.

Portfolio Allocation II

Here we consider a different version of the portfolio allocation problem where the manager considers investing either in a risky project or a safe project. The return of the safe project is fixed, and normalized to zero, whereas the return of the risky project depends on its quality, which is unknown to both the equity holder and the manager. Also, the net return of the risky project is positive if its type is good, and negative if the type is bad. This environment is similar to Lambert (1986).

In this environment, the optimal contract pays the manager only in the case he invests on a safe (risky) asset, and the type of the risky asset is revealed to be bad (good). In any other case, the payment to the agent is zero. Besides, if prior indicate that the NPV of the risky asset is higher (lower) than the NPV of the safe asset, then the optimal compensation contract gives rise to under-investment (over-investment) to the risky asset.

The resulting over-investment against the flow has implications on the riskiness of the portfolio, which is captured by its variance. In particular, the riskiness is lower (higher) than the first best when the ex-ante NPV of the risky asset exceeds (is lower than) the NPV of the safe asset.

	Portfolio II	**CRA**	**Product Design**	**Innovation**
Principal	Equity holders	Manager	Equity holders	Venture Capitalist
Agent	Manager	Analyst	Manager	Researcher
State	Type of risky	Creditworthiness	Demand	Quality of innovation
Action	Risky/safe	Good/bad rating	Features	Conventional/ innovation
Objective	Profits	Rating precision	Profits	Profits

Table 1.1. Analogy

Chapter 2

The Impact of Credit Rating Agencies on Capital Markets

2.1 Introduction

The 2007-2008 financial crisis raised unprecedented scrutiny on credit rating agencies (CRAs). CRAs have been accused of failing to predict the financial crisis (Mason and Rosner, 2007), and of following investors' opinions rather than leading them (Richard and Steward, 2003). However, the main criticism regarding CRAs relates to the conflicts of interest, which might arise in an issuer-pays regime (Mathis et al. 2009, Bouvard and Levy 2012). In this paper, I shift the attention from the CRA's behavior to the framework in which CRAs operate. I explore the impact of a CRA in an environment which, similarly to capital markets, is characterized by information asymmetries and lack of commitment. I explore a model where i) there is no conflict of interest, ii) the CRA is truthful and issues ratings for free, and iii) these ratings are accurate. I argue that even in this environ-

ment, introducing a CRA might lead to i) under-financing of projects with positive NPV that would otherwise be financed, and ii) higher expected probability of default.

CRAs play a significant role in capital markets by affecting major economic variables, such as interest rate and probability of default. The main mechanism through which credit ratings affect such variables is by providing information which shapes investors' beliefs, and hence investment decisions. CRAs, however, differ from other information intermediaries in a critical direction: unlike meteorologists, who predict the weather without affecting it, CRAs produce ratings that do affect the quality of the asset they rate. Kuhner (2001) points out that a credit rating can take the form of a self-fulfilling prophecy. Similarly, Manso (2013), captures the self-fulfilling nature of ratings, by showing that a rating deterioration may trigger a "death spiral". The feedback effect, which is inherent in capital markets, lies at the heart of my model.

I develop a model of project's financing where an entrepreneur is privately informed about his cost of effort, and cannot commit to exerting effort in the implementation of the project. In particular, the entrepreneur's cost is drawn from a "good" or a "bad" distribution, where the good distribution dominates the bad distribution in the first order stochastic dominance sense. Thus, an entrepreneur whose cost is drawn from a good distribution is of low expected cost (LEC), and an entrepreneur whose cost is drawn from a bad distribution is of high expected cost (HEC). I focus on the case where the planner would finance both types. The information asymmetry about the entrepreneur's cost is partly resolved by a CRA. The combination of lack of commitment and information asymmetry allows

me to endogenize the probability of default and the interest rate. In this environment, a feedback effect arises: the interest rate is determined by the expected probability of default, which in turn, determines the realized probability of default.

I argue that introducing a truthful CRA which resolves, at zero cost, part of the information asymmetry, may not result in a better outcome from a social perspective. This is because the interaction information asymmetry and lack of commitment results in a trade-off when a CRA is introduced into the market. On the one hand, better information alleviates the information asymmetry problem; on the other hand, it exacerbates the adverse effect of non-commitment. The main findings suggest that the impact of introducing a CRA depends on the features of the environment. In particular, the introduction of a CRA: (i) leads to a higher expected probability of default when the entrepreneur is efficient enough to raise funds independently of the presence of a CRA (mild information asymmetry), (ii) leads to under-financing when financing of an HEC entrepreneur is feasible only if he is pooled with an LEC entrepreneur (moderate information asymmetry), and (iii) alleviates under-financing when financing of an LEC entrepreneur is feasible only if he can be differentiated from an HEC entrepreneur (severe information asymmetry). Besides, I characterize the level of accuracy of the CRA's ratings which leads to the best allocation of resources, and show that some degree of inaccuracy might be optimal. Also, I consider the case where the CRA charges a profit-maximizing fee. I find that this fee adversely affects the probability of default, but it does not influence the project's financing opportunities.

The intuition behind the findings presented in the previous para-

graph relates to the feedback effect in capital markets. In order to capture the feedback effect, some insights of the model are required. First, I assume that the project succeeds as long as effort is exerted. Thus, a critical role in our model is played by threshold $\hat{c}$, which denotes the maximum value of effort cost for which an entrepreneur exerts effort in the implementation of his project. This threshold is negatively related to the interest rate the entrepreneur is expected to pay. Suppose now that an investor considers financing an entrepreneur. First, he forms his beliefs about the probability of default, which in this setup coincides with the probability that the entrepreneur's cost is above $\hat{c}$. Based on his beliefs, the investor demands an interest rate which allows him to break even. This interest rate affects the threshold $\hat{c}$, which in turn, affects investors beliefs about the probability of default, and so on (feedback effect). We show that the increase in the interest rate and the corresponding probability of default, due to the feedback effect, is decreasing in the entrepreneur's ex-ante efficiency. Thus, for mild information asymmetry, when introducing a CRA, the negative effect on an HEC entrepreneur dominates the positive effect on an LEC entrepreneur. Along these lines, Kliger and Sarig (2000) use a natural experiment to show that credit ratings affect the cost of capital, and Kisgen (2006) shows that a firm's structural decision is directly affected by credit ratings.

The intuition underlying the finding that a CRA may lead to underfinancing of positive NPV projects is similar. Consider a case where an HEC entrepreneur has a bad rating and investors are not willing to finance him. Suppose now there is no CRA and an HEC entrepreneur is mixed with an LEC entrepreneur. As far as an HEC entrepreneur is concerned, investors are now more optimistic about the entrepreneur's creditworthi-

ness and require a lower interest rate. This lower interest rate decreases the probability of default of an HEC entrepreneur, which decreases the interest rate even more. This feedback effect can thus turn the financing of HEC entrepreneur to a credit-worthy investment.

Finally, I address the impact of restricting a CRA to provide hard evidence with its ratings. Notice that the hard-evidence assumption effectively restricts the rating policy to truthful disclosure, which subsequently diminishes the role of the CRA. Relaxing this assumption gives rise to a model similar to the Bayesian Persuasion setting developed by Kamenica and Gentzkow (2009). In this environment, I explore the optimal rating rule when the CRA can commit in advance to this rule. I find that the optimal rating rule is either a truthful revelation of the entrepreneur's type or a babbling equilibrium, where no information is revealed. Hence, both "rating inflation" and "rating deflation" can be part of the optimal rating policy. In addition, I show that restricting a CRA to provide hard evidence can only worsen financing opportunities. In contrast, "rating inflation" and "rating deflation" can improve the allocation of resources.

I argue that this paper has implications for the information disclosure policy implemented by a government. Note that the introduction of a CRA in capital markets can also be interpreted as a revelation of a signal about the creditworthiness of an agent or institution. For instance, a key question after the recent crisis is whether the results of stress tests for banks should be revealed (Goldstein and Leitner, 2013). The answer that my analysis suggests is that concealing these results could improve the allocation of resources, unless the market breaks down in the absence of additional information. Moreover, the implications of this paper can be

applied to the sovereign debt in euro zone countries. Finally, my work suggests that future research on the regulation of CRAs should take into account feedback effect, which is inherent in capital markets.

This paper is organized as follows: Section 2.2 reviews the related literature. Section 2.3 introduces the model. Section 2.4 explores the benchmark case. Section 2.5 presents the regime with and without a CRA. Section 2.6 explores the impact of introducing a CRA into the market. Section 2.7 relaxes the assumption of hard evidence and explores the optimal rating rule when the CRA can pre-commit to it. In Section 2.8, I explore an environment where a CRA can choose a profit-maximizing fee. Section 2.9 discusses and concludes.

2.2 Literature Review

This paper relates primarily to two strands of the literature: the literature dealing with the effect of CRAs on real economic variables, and the literature that recognizes the adverse effect of better information.

Much of the literature on CRAs focuses on the quality of the reported ratings, rather than the impact of CRAs. There are three main approaches in this strand. The first approach explores whether CRAs have incentive to inflate their ratings. The second approach examines the way CRAs choose to disclose their private information. The third approach highlights the role of rating shopping by issuers on the quality of ratings.

The most popular way of addressing whether a CRA has incentive to inflate its rating, is by testing the validity of the the reputation-concerns argument. According to this, inflating the ratings would harm a CRA's

reputation, and ultimately force it out of the market; hence, such conflict of interests does not exist. Mathis et al. (2009) develop a model with rating-contingent fees, and demonstrate that the reputation-concerns argument only works when a significant part of a CRA's income comes from sources other than rating complex projects. Bouvard and Levy (2012) also test the reputation concerns argument. They show that a higher reputation for transparency is not always desirable because it demotivates low-creditwothiness firms to ask for a rating.

The second approach links the quality of ratings with information disclosure. Lizzeri (1999) adopts a mechanism design setting. He shows that a monopolist CRA only reveals the minimum level of information, but if there are multiple CRAs, information is disclosed fully. In contrast, Faure-Grimaud et al. (2007) show that competition reduces information revelation.

The third approach attributes rating inflation to behavioral biases of investors and rating shopping by issuers. Bolton et al. (2012) develop a model with the interaction of sophisticated and naive investors, and the results show that a duopoly might be less efficient than a monopoly, because the entrepreneur has the opportunity to shop for a good rating to exploit naive investors. Skreta and Veldkamp (2009) obtain similar findings, where the direct implication of rating shopping is the systemic bias in disclosed ratings, even if each CRA produces unbiased ratings. Opp et al. (2013) show that rating inflation can emerge if the face value - not the informational value - of the rating that matters.

My central approach deviates from this literature; In the benchmark setting, I focus on a seemingly best-case scenario, where the CRA always

reports its private information truthfully, and hard evidence supports each rating. In this environment, I explore the impact of CRAs on capital markets. Save for Boot et al. (2006), Kuhner (2001), Manso (2013) and to some extent Mathis et al. (2009), the literature has ignored this effect. Boot et al. (2006) propose that credit ratings can serve as a coordination mechanism in situations where multiple equilibria exist. Mathis et al. (2009) show that the behavior of CRAs can lead to reputation cycles, with implications for credit spreads. Kuhner (2001) shows that when CRAs care about reputation, they are more likely to reveal their private information if their ratings cannot become self-fulfilling ex-post.

Manso (2013) deals with the feedback effect of credit ratings. He describes an environment where a single CRA repeatedly interacts with a firm that holds performance-sensitive debt, and whose payout flows are linked to its rating. This framework enables him to incorporate the feedback effect of credit ratings in a dynamic credit-rating model. He finds that, when forming its rating policy, the CRA should focus not only on ratings accuracy, but also on the effect of the ratings on the borrower's probability of survival.

This paper differs from Manso (2013) in three critical dimensions: (i) the nature of the feedback effect, (ii) the information available to the CRA and (iii) the main focus. First, this paper explores the feedback loop between the entrepreneur's decision and the investors' beliefs rather than the feedback loop between project's quality and credit rating. The second departure from Manso (2013) is that the CRA has an information advantage over potential investors. In Manso (2013) the cash flow process, which is the only parameter which determines firm's creditworthiness, is observed

by all market participants. In contrast, in this paper, the CRA obtains a private signal about the firm's creditworthiness. That characteristic enables me to provide micro-foundations for the impact of CRAs on the cost of capital. Another departure from Manso (2013) concerns the determination of capital cost. In Manso (2013) the capital cost depends on ratings exogenously, whereas in this paper, the capital cost is endogenously determined, capturing all the parameters of the model. Finally, regarding the main focus of the paper, Manso (2013) is interested in exploring the effect of the rating policy, i.e., the function which maps the cash flow into a rating, in the economy. This work focuses instead on the impact of providing information via a CRA on capital markets.

In addition, my work pertains to the literature on the adverse welfare consequence of information disclosure. In his seminal work, Hirshleifer (1971) argues that more information leads to welfare reduction because it destroys hedging opportunities. More recently, Amador and Weill (2010) show that the effect of releasing partial information about a monetary or productivity shock is two-fold: on the one hand, providing more information benefits the economy; but on the other hand, it forces households to value the newly released public information more and their private information less. As a result, the situation leads to reduction of the endogenous informational content of prices. Kondor (2013) shows that when the correlation between the private information of different groups is low, the release of public information increases disagreement among short-horizon traders about the expected selling price. Kurlat and Veldkamp (2012) argue that disclosure of information reduces an asset's risk and hence its return. As a result, high-risk, high-return investments disappear and investor wel-

fare falls.

The mechanism through which better information can be harmful differs from the existing literature. I suggest that the inefficient allocation of resources is a consequence of the coexistence of asymmetric information and lack of commitment. In this environment, resolving part of the information asymmetry amplifies the distortion arises from lack of commitment.

This paper also relates to the literature on Bayesian Persuasion. In Section 2.7 I show that if the CRA is not obliged to provide supporting evidence for its ratings, and can commit to a rating policy, then the emerging setup is similar to Kamenica and Gentzkow (2009).

2.3 Model

Environment: I consider a setting with three risk neutral players: an cashless entrepreneur, a CRA, and a representative investor. The entrepreneur seeks capital to finance an investment project. The project is non-divisible, and its implementation requires an investment equal to \$1.[1] The output of the project depends on whether the entrepreneur exerts effort in its implementation; if the entrepreneur exerts effort, the project succeeds with probability one. Otherwise, the project fails with probability one.[2] The project returns R, in case of success, and zero otherwise.[3][4] The outcome of the project is observed by all parties, and R is common knowledge. Exerting effort is costly, unobserved by investors, and the entrepreneur cannot commit to it.

Entrepreneur's types: The entrepreneur can be of two types, $i \in \{H, L\}$, where the type refers to the entrepreneur's cost of exerting effort, denoted by c. In particular, the cost of type i is drawn from a distribution where $f_i(c)$ and $F_i(c)$ denote the probability and density function, respectively. Throughout this paper, I assume the distribution which corresponds to type $i = L$ dominates the distribution which corresponds to type

[1]Raising funds in return for a security is the reduced form of a setup where the entrepreneur sells a security at a price P in order to finance a project of size 1. This is because a direct consequence of a pooling equilibrium in the contracting stage is that there is a loss for the efficient entrepreneur (LEC). Thus, the entrepreneur does not have the incentive to raise more than the capital required for the investment, i.e. $P = 1$.

[2]The main findings are robust if we instead assume that exerting effort increases the probability of success.

[3]This is without loss of generality as long as the return in case of failure, R_F is lower than R

[4]The main findings are robust to the case where R is drawn from a distribution which is known to the entrepreneur and investors.

$i = H$ in the first order stochastic dominance sense i.e., for each $c' \in [0, \bar{c}]$, $F_L(c') \leq F_H(c')$. Thereafter, I refer to type $i = L$ as low-expected-cost (LEC) entrepreneur, and type $i = H$ as high-expected-cost (HEC) entrepreneur. Thus, an LEC entrepreneur can be interpreted as an efficient (in expectation) entrepreneur, whereas an HEC entrepreneur as an inefficient (in expectation) entrepreneur. I restrict the analysis to the case where both types have positive ex-ante net present value, i.e., $R > 1 + \mathbb{E}[c_i]$ for each $i \in \{H, L\}$. Hence, if the aim of the planner is to maximize total surplus, both types would be financed.

Information sets: The entrepreneur has private information about his type. In contrast, investors hold prior beliefs about the entrepreneur's type. In particular, investors expect the entrepreneur to be of low expected cost (LEC) with probability λ, and of high expected cost (HEC) with probability $1 - \lambda$. These beliefs are common knowledge. Besides, I assume that the entrepreneur learns his realized cost, c, after carrying out the investment, and only then he takes the effort decision. The rationale of this assumption is that the cost of exerting effort in implementing a project depends not only on the entrepreneur's type, but also on the project itself. Thus, the cost of exerting effort is not known until the agent starts implementing the project.[5]

Unlike potential investors, the CRA can monitor the type of the entrepreneur. The intuition behind this assumption is the following. First, it captures the empirical observation that CRAs have better information than

[5]We show in Appendix B that the model is qualitatively similar to a setting where the entrepreneur observes a noisy signal of the realized cost, rather than the realized cost itself.

investors about the creditworthiness of an entrepreneur/firm. This is because they have access to critical economic indexes, and more experience in monitoring. Second, this assumption reflects the idea that regardless of the experience that CRAs may have, they cannot have better information about the creditworthiness of a firm, than what the firm itself has.[6]

It is worth highlighting that this model is qualitatively equivalent to a setting where: i) the role of the entrepreneur is played by a manager of a firm, ii) the cost of effort coincides with the cost of allocating production resources to the implementation of the project, and iii) the manager's objective is to maximize the firm's profitability.

Actions of each player: The entrepreneur faces three sets of actions. First, he decides whether to ask for a rating. Second, he decides about the structure of the security he issues. Finally, he decides whether to exert effort in the implementation of the project.

Investors' only action is to choose whether to finance an entrepreneur or not. We assume that capital markets are competitive, and that the interest rate investors demand is normalized to zero.

In order to disentangle the impact of a CRA, we allow for different sets of actions. In the benchmark setting we restrict the CRA to provide truthful rating at zero rating fee. In Section 2.7, the CRA chooses its rating

[6]The reason the CRA only knows the type of the entrepreneur/firm, i.e., the distribution from which the actual cost is derived and not the realized cost, relates to two particular characteristics of the rating industry: the monitoring method and the clustering of ratings. Regarding the monitoring method, CRAs usually base their ratings on a mix of indexes. Hence, two firms with similar index values are likely to receive a similar rating, even though the indexes do not reflect other parameters that are vital for a firm's profitability. Regarding the clustering of ratings, one of the characteristics of the industry is that CRAs issue ratings by following a specific rating scale, and refrain from giving predictions about the exact economic outcome.

policy, which maps the entrepreneur's type to a rating. In Section 2.8, the CRA chooses the rating fee.

Timing: The timing of the events is as follows:

1. Nature determines the type of the entrepreneur.
2. The entrepreneur chooses whether to ask for a rating.
3. Conditional on the entrepreneur asking for a rating, the CRA issues a rating.
4. The entrepreneur observes his rating (if any), and chooses the security design.
5. Investors observe the entrepreneur's rating (if any) and security, and then decide whether to invest.
6. Conditional on financing, the entrepreneur observes his realized cost, and chooses whether to exert effort in the project's implementation.
7. The output is realized, and the security is executed.

Equilibrium Concept: The equilibrium concept is *Perfect Bayesian Equilibrium*, where the CRA, the entrepreneur, and investors, choose their corresponding actions in order to maximize expected profits/utility. Finally, on-equilibrium beliefs are consistent.

2.4 Benchmark Case

In the benchmark case, I explore an environment where the CRA: i) can monitor the type of the entrepreneur perfectly, ii) reveals its private information truthfully, and iii) does not charge a rating fee. These assumptions

suppress any conflict of interests, and allow me to isolate the impact of better information, provided by a CRA, on capital markets. In Section 2.7, I relax the hard evidence assumption, whereas in Section 2.8, I relax the zero-fee assumption.

2.4.1 Entrepreneur's Problem

First, the entrepreneur decides whether to ask for a rating. In doing so, he takes into consideration: i) the expected rating, ii) what would be the security that he would issue with and without a rating, and iii) when he would exert effort.

A consequence of a rating fee equal to zero is that an LEC entrepreneur can costlessly differentiate himself, which allows him to promise a lower return to investors. An implication of this remark is that an HEC entrepreneur is indifferent between asking and not asking for a rating. This finding relates to the signaling component of the decision of asking for a rating. Note that if an HEC entrepreneur asks for a rating, his type is revealed by the CRA with certainty. Also, if he does not ask for a rating, the market will be able to infer his type, since anticipates that an LEC entrepreneur would always find it optimal to ask for a rating. Following the previous reasoning, we can assume without loss of generality that both types ask for a rating.

Second, the entrepreneur chooses the security design which maximizes his expected utility. It is shown in the Appendix that the optimal security promises a payment $(1 + r)$, if the project succeeds, and zero otherwise. This simple form of the optimal security is a consequence of non-

verifiability of effort cost.

Third, conditional on raising capital and starting implementing the project, the entrepreneur observes his actual cost of exerting effort c, and decides whether to exert effort. Recall that the project is successful only when the entrepreneur exerts effort. Thus, the entrepreneur's utility, depending on whether he exerts effort, is given by:

$$U(effort) = R - (1 + r) - c$$
$$U(not\ effort) = 0$$

Hence, the entrepreneur exerts effort as long as:

$$c \leq R - (1 + r) \equiv \hat{c} \tag{2.1}$$

Threshold $\hat{c}$ is a critical variable of the model. The intuition behind this threshold is straightforward: the entrepreneur exerts effort as long as the benefit from doing so, $R - (1 + r)$, exceeds the cost, c.

I define as "default" the event where the entrepreneur fails to pay back his loan. In this setting, a default occurs when the project returns zero, which coincides with the case where no effort is exerted. This leads to the following definition of the probability of default.

Definition 1

The probability of default is defined as the probability of financing an entrepreneur whose realized cost of effort exceeds $\hat{c}$:

$$Pr(default) \equiv Pr(c > \hat{c}(r)) \tag{2.2}$$

Similarly, the probability of success is defined as the probability of financing an entrepreneur whose realized cost of effort is below or equal to $\hat{c}$:

$$Pr(success) \equiv Pr(c \leq \hat{c}(r)) \tag{2.3}$$

Because $\hat{c}$ is negatively related to the payment $(1+r)$, the probability of default is positively related to $(1+r)$.[7] To avoid the trivial case where an entrepreneur can finance his project through risk-free debt, I restrict the probability of default to strictly positive values. This is true if there exists at least one value of c such that the entrepreneur does not exert effort even if the interest rate is zero, i.e., $\bar{c} > R - 1$.

2.4.2 The Investors' Problem

Investors form their beliefs about the probability of success, and subsequently require an interest rate that satisfies their participation constraint:

$$\mathbb{E}[Pr(success)|\Omega] \times (1+r) + \mathbb{E}[Pr(default)|\Omega] \times 0 \geq 1 \tag{2.4}$$

where the LHS of (2.4) is the investors' expected benefit, whereas the RHS of (2.4) is the capital they lend to the entrepreneur. More specifically, $(1+r)$ stands for the payment in case of success, whereas $E[Pr(success)|\Omega]$ stands for investors' *beliefs* about the probability of success, given their

[7] What matters for the main findings to go through, is that the effort threshold, which determines the probability of default, is negatively related to the payment $(1+r)$. Alternatively, this could be achieved by assuming that the project succeeds with certainty when the entrepreneur exerts costly effort and succeeds with probability q if there is no effort. In such a setup the entrepreneur chooses to exert effort if and only if $c \leq (R-(1+r))(1-q)$ which is associated to a probability of success equal to $Prob(c \leq \hat{c}) + qProb(c > \hat{c}) = q + (1-q)\,\mathbb{E}[Prob(c \leq \hat{c})|\Omega]$.

information set Ω. Similarly, $\mathbb{E}[Pr(default)|\Omega]$ stands for investors' *beliefs* about the probability of default. Following Definition 1, the investors's participation constraint becomes:

$$\mathbb{E}[Prob(c \leq \hat{c})|\Omega] \times (1+r) \geq 1 \tag{2.5}$$

Thus, investors are willing to finance an entrepreneur as long as $\mathbb{E}[Prob(c \leq \hat{c})|\Omega] \geq (1+r)^{-1}$. If the previous condition does not hold, the market collapses.

2.4.3 Equilibrium Condition

Recall that the entrepreneur exerts effort as long as:

$$c \leq R - (1+r) \equiv \hat{c}(r) \tag{2.6}$$

Note that investors' participation constraint is binding, as a consequence of the assumption that capital markets are perfectly competitive. Thus, the equilibrium interest rate is the *minimum* r which solves:

$$\mathbb{E}[Prob(c \leq \hat{c}(r))|\Omega] = (1+r)^{-1} \tag{2.7}$$

The inherent feedback effect can be seen in the previous fixed-point equation; the interest rate r which solves (2.7) depends on the threshold $\hat{c}$, which in turn, depends on the interest rate r via (2.6).

It is worth highlighting that it is the investors' beliefs about the probability of default - not the probability of default itself - that determine the

interest rate.

For notational convenience, I denote $\mathbb{E}[Prob(c \leq \hat{c}(r))|\Omega]$ as $\tilde{s}(\hat{c}(r))$. Thus, $\tilde{s}(\hat{c}(r))$ captures investors' beliefs about the probability of success, given their information set Ω. Note the there might be more than one combinations of r and $\hat{c}(r)$ which solve (2.7). The combination, however, which maximizes entrepreneur's expected utility is the one which corresponds to the lowest interest rate.

The previous analysis highlights the importance of investors' beliefs about the default probability on the determination of the equilibrium interest rate. The introduction of a CRA affects investors' beliefs as follows: when investors observe a bad rating, they anticipate that the entrepreneur is of HEC type. Consequently, they downgrade their beliefs about the probability of success compared to the case where there is no CRA. Similarly, when investors observe a good rating, anticipate that the entrepreneur is of LEC type. Subsequently, they upgrade their beliefs about the probability of success, compared to the case with no CRA. As a result, investors require a higher (lower) interest rate to finance an HEC (LEC) entrepreneur, compared to the regime without a CRA.

2.5 Regime with and without a CRA

In this section, I characterize the equilibrium in two different regimes: (i) a regime without a CRA, and (ii) a regime with a CRA. Recall that the only distributional assumption is First Order Stochastic Dominance, i.e., for each $c' \in [0, \bar{c}]$, $F_H(c') \leq F_L(c')$.

2.5.1 Regime without a CRA

The Investors' Problem: Investors' beliefs about the probability of success are given by:

$$\tilde{s}_{II}(\hat{c}(r)) =$$
$$Pr(c \leq \hat{c}(r)|i = L)\, \mathbb{E}[Pr(i = L)|\Omega] + Pr(c \leq \hat{c}(r)|i = H)\, \mathbb{E}[Pr(i = H)|\Omega] \tag{2.8}$$

Given that investors have no additional information, their beliefs about the entrepreneur's type coincide with their prior beliefs. Note that $Prob(c \leq \hat{c}(r)|i = L) = F_L(\hat{c}(r))$ and $Prob(c \leq \hat{c}(r)|i = H) = F_H(\hat{c}(r))$, where $F_L(.)$ is the c.d.f of the cost of an LEC entrepreneur, and $F_H(.)$ is the c.d.f of the cost of an HEC entrepreneur. Thus,

$$\tilde{s}_{II}(\hat{c}(r)) = \lambda F_L(\hat{c}(r)) + (1 - \lambda)F_H(\hat{c}(r)) \tag{2.9}$$

Market survival and equilibrium interest rate: I show in Appendix B that the market survives as long as there exists an interest rate, r, such as $\tilde{s}_{II}(\hat{c}(r)) \geq (1 + r)^{-1}$. Also, the equilibrium interest rate, which is denoted by r^*_{II}, is minimum interest rate which solves:

$$(1 + r)^{-1} = \lambda F_L(\hat{c}(r)) + (1 - \lambda)F_H(\hat{c}(r)) \tag{2.10}$$

2.5.2 Regime with a CRA

In this section, I introduce a CRA that has perfect monitoring, and its ratings are, by assumption, truthful. The CRA affects the equilibrium interest

rate by shaping investors' beliefs via changing their information set. As a result, the equilibrium interest rates are conditional on the rating.

The Entrepreneur's Problem: We argue in section 2.4 that an LEC entrepreneur always asks for a rating, whereas an HEC entrepreneur is indifferent between asking and not asking for a rating since, in both cases, his type is disclosed with certainty. Without loss of generality, we assume that when an LEC entrepreneur is indifferent, he asks for a rating.

Recall that the threshold $\hat{c}$ depends on the interest rate, which depends on investor's beliefs about the probability of success, which in turn, rely on the rating. Thus, as long as the interest rate differs depending on the rating, the resulting threshold also varies. Thus, there are two critical values: $\hat{c}(r_{GR})$, if the rating is good, and $\hat{c}(r_{BR})$, if the rating is bad.

The Investors Problem: Conditional on the rating, investors form their beliefs about the probability of success. $\tilde{s}(\hat{c}(r_{GR}))$ denotes investors' beliefs when the rating is good, and $\tilde{s}(\hat{c}(r_{BR}))$ when the rating is bad, where:

$$\tilde{s}(\hat{c}(r_{GR})) =$$
$$Pr(c \leq \hat{c}(r_{GR})|i = L)Pr(i = L|GR) + Pr(c \leq \hat{c}(r_{GR})|i = H)Pr(i = H|GR)$$
$$\tilde{s}(\hat{c}(r_{BR}) =$$
$$Pr(c \leq \hat{c}(r_{BR})|i = L)Pr(i = L|BR) + Pr(c \leq \hat{c}(r_{BR})|i = H)Pr(i = H|BR)$$

Note that $Pr(i = L|GR) = 1$ and $Pr(i = H|BR) = 1$, as the CRA has perfect monitoring and reports its private information truthfully. Hence, I

can re-write investors' beliefs as follows:

$$\tilde{s}(\hat{c}(r_{GR})) = Pr(c \leq \hat{c}(r_{GR})|i = L) = F_L(\hat{c}(r_{GR}))$$
$$\tilde{s}(\hat{c}(r_{BR})) = Pr(c \leq \hat{c}(r_{BR})|i = H) = F_H(\hat{c}(r_{BR}))$$

Market survival and equilibrium interest rate: Conditional on a good rating, the market survives as long as there exists an interest rate, r, such as $\tilde{s}(\hat{c}(r_{GR})) \geq (1+r)^{-1}$. Similarly, conditional on a bad rating, the market survives as long as there exists an interest rate, r, such as $\tilde{s}(\hat{c}(r_{BR})) \geq (1+r)^{-1}$. Also, the equilibrium interest rate which corresponds to a good rating, denoted by r^*_{GR}, is minimum interest rate which solves:

$$(1+r) = (F_L(\hat{c}(r)))^{-1} \tag{2.11}$$

Similarly, the equilibrium interest rate which corresponds to a bad rating, denoted by r^*_{BR}, is the minimum interest rate which solves:

$$(1+r) = (F_H(\hat{c}(r)))^{-1} \tag{2.12}$$

2.6 Impact of introducing a CRA

2.6.1 Comparison

In Section 2.5, I characterize the necessary and sufficient conditions for a market to survive. Besides, I characterized the equilibrium conditions for the regimes with and without a CRA. This section compares those regimes regarding three critical market variables: the probability of project financ-

ing, the expected probability of default, and the expected interest rate.

2.6.1.1 Impact on Project Financing

A fundamental aspect of capital markets is financing opportunities. The following Proposition explores the impact of introducing a CRA on the probability of raising capital. Recall that a social planner, whose objective is to maximize net surplus, would finance both types of entrepreneurs. This is because both types correspond to positive NPV projects.

Proposition 1 (Probability of Financing)

(i) *If there is r such as* $F_L(\hat{c}(r)) \geq (1+r)^{-1}$ *and* $\lambda F_L(\hat{c}(r)) + (1-\lambda)F_H(\hat{c}(r)) < (1+r)^{-1}$, *introducing a CRA alleviates under-financing.*

(ii) *If there is r such as* $F_H(\hat{c}(r)) < (1+r)^{-1}$ *and* $\lambda F_L(\hat{c}(r)) + (1-\lambda)F_H(\hat{c}(r)) \geq (1+r)^{-1}$, *introducing a CRA leads to under-financing.*

Proof. See Appendix. □

The intuition behind Proposition 1 is captured in Figure 2.1 and 2.2. These figures illustrate investors' beliefs about the probability of success, as a function of r, in three cases: i) when the entrepreneur holds a good rating (dash-dotted line), ii) when the entrepreneur holds a bad rating (dense-dotted line), and iii) when there is no rating/CRA (loose-dotted line).[8] The solid curve depicts the inverse of the payment $1+r$. Recall that $\hat{c} = R - (1-r)$. An implication of the First Order Stochastic Dominance assumption is that the graphical illustration of $F_L(\hat{c}(r))$ lies above

[8]We present the case where the entrepreneur is of an LEC or an HEC type with equal probability, and the cost of effort is uniformly distributed.

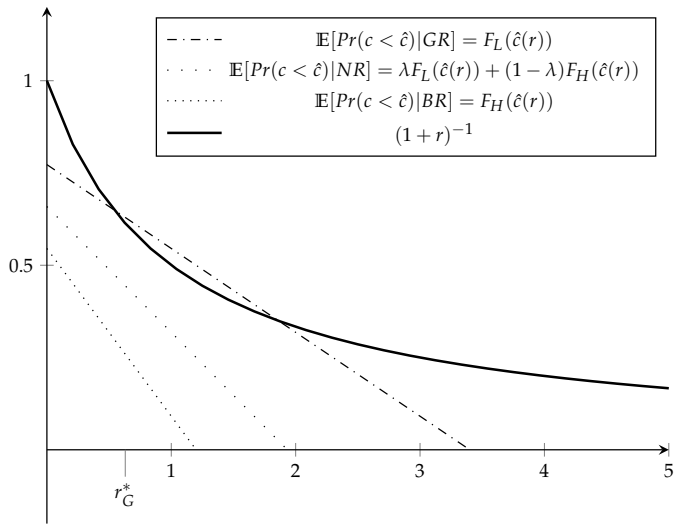


Figure 2.1. CRA alleviates under-financing

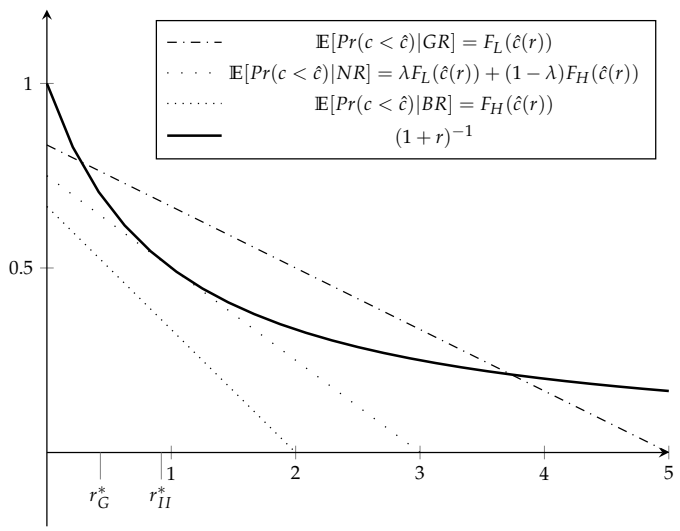


Figure 2.2. CRA leads to under-financing.

the graphical illustration of $F_H(\hat{c}(r))$. Recall that for the market to survive, there should be at least one value of r, such as the solid curve is above the line which corresponds to investors's beliefs.

Part one and two of Proposition 1 is depicted in Figure 2.1 and 2.2, respectively. Figure 2.1 illustrates the case where asymmetric information is severe, and an LEC entrepreneur can raise capital only if he can differentiate himself. Hence, introducing a CRA alleviates under-financing. Figure 2.2 illustrates the case where information asymmetry is moderate, and an HEC entrepreneur can raise capital only if he is pooled with an LEC entrepreneur. Thus, introducing a CRA prevents pooling, and leads to under-financing of an HEC entrepreneur, even though finance him is socially optimal.

The intuition behind the last observation relates to the fact that an entrepreneur cannot commit to exerting effort. Also, we argue later in this section, that the distortion due to lack of commitment is greater, the lower ex-ante efficiency is. Hence, pooling an HEC with an LEC entrepreneur alleviates the resulting distortion. This is evident in the case where an HEC entrepreneur cannot be funded in isolation, though he can be funded by being pooled with an LEC entrepreneur. The reason is that pooling leads investors to upgrade their beliefs about the probability of success, which in turn, reduces the interest rate they require. If the lower interest rate alters the choice of exerting effort from unprofitable to profitable, then investors anticipate this, and they are willing to finance the entrepreneur.

2.6.1.2 Impact on Probability of Default

Once the equilibrium interest rate is derived, I can compute the expected interest rate and the expected probability of default. The expected interest rate (probability of default) consists of the interest rate (probability of default) of each type of entrepreneur, i, weighted by the probability that

the entrepreneur is of type i. Recall that, as ratings are perfectly accurate, a good rating is associated with an LEC entrepreneur and a bad rating is associated with an HEC entrepreneur. I present the effect of introducing a CRA on these variables for both an LEC and an HEC entrepreneur; however, my main goal is the effect at the expected/market level.

Proposition 2 (Probability of Default)
If information asymmetry is mild, i.e., there exists r such as $F_H(\hat{c}(r)) \geq (1+r)^{-1}$ (the market survives independently of whether a CRA exists), then the relation among the equilibrium probabilities of default is:

$$1 - F_L(\hat{c}(r^*_{GR})) < 1 - F_L(\hat{c}(r^*_{II})) < 1 - F_H(\hat{c}(r^*_{II})) < 1 - F_H(\hat{c}(r^*_{BR})) \quad (2.13)$$

In addition, the expected probability of default in absence of a CRA is lower than the expected probability of default when a CRA exists:

$$\begin{aligned} &\lambda(1 - F_L(\hat{c}(r^*_{II}))) + (1-\lambda)(1 - F_H(\hat{c}(r^*_{II}))) < \\ &\lambda(1 - F_L(\hat{c}(r^*_{GR}))) + (1-\lambda)(1 - F_H(\hat{c}(r^*_{BR}))) \end{aligned} \quad (2.14)$$

Proof. See Appendix. □

Figure 2.3 illustrates Proposition 2 and 3. The distance of point G (B) from the vertical axis captures the equilibrium interest rate which corresponds to a good rating, and the distance of G (B) from the horizontal axis captures the probability of success of an LEC (HEC) entrepreneur. Also, the distance of point A from the vertical axis captures the expected equilibrium interest rate, and the distance of A from the horizontal axis captures the

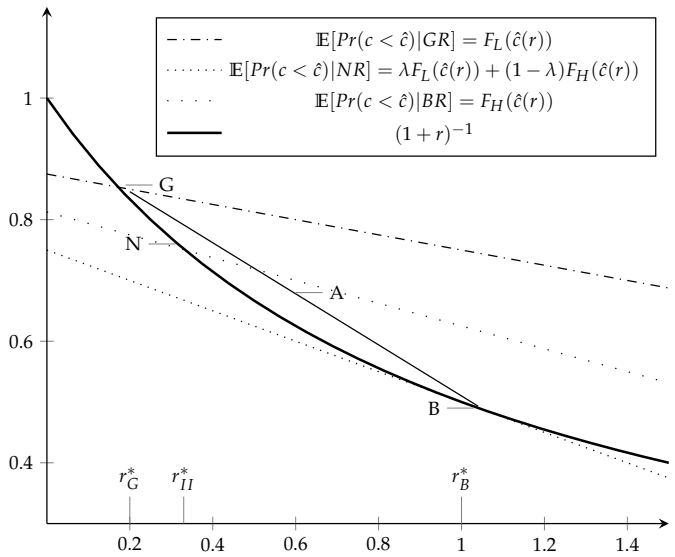


Figure 2.3. Interest rate and default probability with and without CRA.

probability of success, in the regime with a CRA. Similarly, point N indicates the *expected* equilibrium interest rate and the *expected* probability of success when there is no CRA. The first part of the proposition is a consequence of the FOSD assumption. The second part relates to the convexity of $(1+r)^{-1}$: in Figure 2.3, point N is always above point A.[9] Thus, the expected probability of success (default) is higher in the regime without a CRA.

2.6.1.3 Impact on Interest Rate

Proposition 3 (Equilibrium Interest Rates)

If information asymmetry is mild, i.e., there exists r such as $F_H(\hat{c}(r)) \geq (1+r)^{-1}$ *(the market survives independently of whether a CRA exists), then the relation*

[9] In G: $(1+r^*_{GR})^{-1} = F_L(\hat{c}(r^*_{GR}))$, in B: $(1+r^*_{BR})^{-1} = F_H(\hat{c}(r^*_{BR}))$ and in N: $(1+r^*_{GR})^{-1} = 0.5F_L(\hat{c}(r^*_{GR})) + 0.5F_H(\hat{c}(r^*_{BR}))$.

among the equilibrium interest rates is given by:

$$(1 + r^*_{GR}) < (1 + r^*_{II}) < (1 + r^*_{BR}) \tag{2.15}$$

In addition, the expected equilibrium interest rate in the absence of a CRA is always lower than the expected equilibrium interest rate when a CRA exists:

$$(1 + r^*_{II}) < \lambda(1 + r^*_{GR}) + (1 - \lambda)(1 + r^*_{BR}) \tag{2.16}$$

Proof. See Appendix. □

Similarly to proposition 2, part one follows directly from the FOSD assumption, whereas part two relates to the convexity of $(1 + r)^{-1}$.

Here I shed light on the intuition behind Proposition 2 and 3. The main message of Proposition 2 and 3 is that introducing a CRA has a negative effect on an HEC, and a positive effect on an LEC entrepreneur. I show that the former always offsets the latter in the case where asymmetric information is not severe enough to lead to a market breakdown. These findings arise from the feedback effect in capital markets and its asymmetric impact on entrepreneurs of different quality. In order to understand this asymmetry, it is crucial first to understand how the feedback effect works in practice.

After investors observe the rating and form their beliefs about the expected cost of effort and the probability of default, they demand an interest rate which allows them to break even. The raised funds adjusted by the interest rate (capital cost) need to be paid back by the entrepreneur. For an entrepreneur who was indifferent between exerting and not exert-

ing effort, adding the capital cost makes exerting effort unprofitable, and thus the effort threshold, $\hat{c}$, drops. The lower effort threshold coincides with a higher probability of default, which results in investors demanding a higher interest rate. This feedback loop continues until the interest rate converges to its equilibrium value.

Note that every loop leads to continuous updates in beliefs about the probability of default, and the demanded interest rate. The magnitude, however, of the increase in the probability of default and the interest rate after each loop, diminishes. This is a consequence of the always decreasing expected cost, conditional on exerting effort.

The previous reasoning implies that the change in the interest rate and the probability of default, due to this feedback mechanism, is smaller for more efficient entrepreneurs, i.e., the equilibrium interest rate is a convex function of the expected value of the cost of effort. A direct consequence of this remark is that the negative effect on an HEC entrepreneur dominates the positive effect on an LEC entrepreneur.

2.6.1.4 Optimal Level of Rating Precision

The main message of Propositions 1-3 is that better monitoring does not necessarily correspond to better allocation of resources. For example, consider an environment where CRA receives a signal about the entrepreneur's type, and the social planner can affect the precision of this signal. This could be achieved through, for instance, regulating the evidence that an entrepreneur should provide to the CRA during the evaluation process. A question which arises naturally is what would be the precision level that maximizes financing opportunities. To explore this, I allow for the

following modifications: I assume that the CRA receives a binary signal, $\sigma = \{GS, BS\}$, which reveals the true state with probability α, i.e.,

$$Prob(i = L|GS) = Prob(i = H|BS) = \alpha$$

where $\alpha \in (0.5, 1]$ is common knowledge.[10] To avoid any unnecessary complications, I assume, without loss of generality, that the CRA reveals his private signal truthfully, i.e., the CRA gives a good rating, if the signal is good, and a bad rating, if the signal is bad. The level of optimal precision is given in Proposition 4.

Proposition 4: (Optimal Level of Precision)
The level of precision α that maximizes an entrepreneur's financing opportunities does not necessarily coincide with 1 (perfectly precise signal). If an HEC entrepreneur's financing is feasible only if there is some pooling with an LEC entrepreneur, the financing opportunities are maximized for a value of α which is weakly smaller than one. The optimal level of precision, α^*, solves:

$$F_L(\hat{c}(\tilde{r}_{BR}))(1 - \alpha^*) + F_H(\hat{c}(\tilde{r}_{BR}))\alpha^* = (1 + \tilde{r}_{BR})^{-1} \tag{2.17}$$

Proof. See Appendix. □

The following Corollary summarizes the impact of introducing a CRA. Figure 2.4 and 2.5 illustrate findings of Corollary 1, for the case where the cost is uniformly distributed.

[10] For $\alpha = 1$ the signal is perfectly informative.

Corollary 1 (Impact of better Information)
The introduction of a CRA:

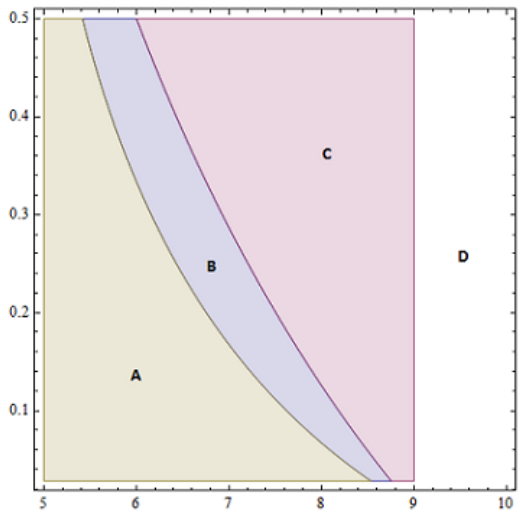
(i) *leads to a higher expected probability of default and interest rate, when both an LEC and an HEC entrepreneur can be financed independently of whether a CRA exists. (Area A)*

(ii) *leads to under-financing, when an HEC entrepreneur's financing is feasible only if he is pooled with an LEC entrepreneur. (Area B)*

(iii) *alleviates under-financing, when the LEC entrepreneur's financing is feasible only if he can be differentiated by an HEC entrepreneur. (Area C)*

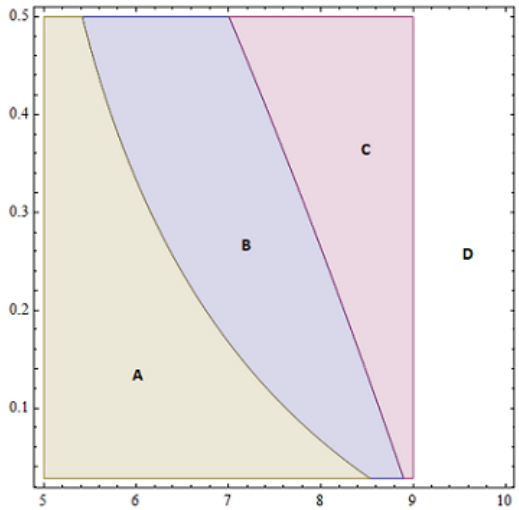
The intuition behind Corollary 1, which summarizes the main findings, relates to the co-existence of asymmetric information and a lack of commitment to exerting effort. In this environment, resolving part of the information asymmetry amplifies the impact of non-commitment. Thus, there is an inherent trade-off in introducing a CRA; on one hand, the CRA alleviates the information asymmetry problem, but on the other hand, exacerbates the adverse effect of non-commitment. It can be shown that, if the entrepreneur can commit to exerting effort, then better information always improves the allocation of resources, an outcome that may not be true if the entrepreneur is unable to commit. The net effect of a CRA on the allocation of resources depends on the relative extent of each problem. For instance, if the information asymmetry is severe (such the market collapses in the absence of a CRA), introducing a CRA improves the allocation of resources. Antithetically, if the information asymmetry is mild, the positive effect of the CRA in resolving part of the asymmetry is dominated by the negative

effect due to lack of commitment. This is captured in Figure 2.4 and 2.5, where, as the probability of the entrepreneur being LEC, λ, increases, area B spreads over area C.



$c_L \sim U[0, \bar{c}]$, $c_L \sim U[\gamma\bar{c}, \bar{c}]$, $R = 5$ & $\lambda = 0.5$
x-axis: $\bar{c}$, y-axis: γ

Figure 2.4. Equilibrium existence regions



$c_L \sim U[0, \bar{c}]$, $c_L \sim U[\gamma\bar{c}, \bar{c}]$, $R = 5$ & $\lambda = 0.8$
x-axis: $\bar{c}$, y-axis: γ

Figure 2.5. Equilibrium existence regions

2.7 Optimal Rating Policy

In the previous analysis, I assumed that the ratings should be accompanied by hard evidence. This assumption restricts the rating policy to truthful disclosure. Even though this assumption is useful in capturing the impact of better information, it diminishes the role of a CRA. In this section, I relax the hard-evidence assumption, and explore the CRA's optimal rating policy. The rationale behind relaxing the assumption of hard evidence is that, although it is true that CRAs provide supporting evidence with the ratings, there are cases where this evidence does not reveal perfectly the type of the firm.

The CRA's objective when designing the rating rule is to maximize expected profits,

$$\mathbb{E}[\Pi|\Omega_{CRA}] = \mathbb{E}[(P-d)D|\Omega_{CRA}]$$

where P, d and D stand for the rating fee, the cost of acquiring a signal, and the demand for ratings, respectively.

In order to disentangle the rating policy from the decision of acquiring a private signal and the rating fee determination, I assume that the CRA receives a perfect signal about the entrepreneur's type at cost $d = 0$, and that it is a price taker. In this environment, the objective of maximizing expected profit coincides with the objective of maximizing the expected demand for ratings:

$$\mathbb{E}[D|\Omega] = \mathbb{E}[\lambda I_L + (1-\lambda) I_H|\Omega]$$

where $I_L(I_H)$ equals one if an LEC (HEC) type ask for a rating, and zero otherwise. Note that the entrepreneur asks for a rating if he expects that having a rating will enable him to raise capital at a lower cost. Thus, the objective of maximizing demand pins down to designing a rating rule, such that the entrepreneur asks for a rating independently of his type.

This setup is similar to the persuasion setting developed in Kamenica and Gentzkow (2009), where a sender (CRA) observes a private signal (type of entrepreneur), and then sends a message (rating) to a receiver (investor). Subsequently, the receiver takes an action (decides whether to finance and the interest rate in case of financing), which affects the payoff of both the sender and receiver.

The persuasion game is relevant when the message is pivotal. In this setting, the message is pivotal if it can affect financing opportunities: an HEC entrepreneur cannot raise capital if his type is known.

To avoid any unnecessary complications, I assume that the signal can be of two values, good and bad ($\tilde{\sigma} = \{GS, BS\}$) and that it perfectly reveals the type of the entrepreneur, i.e. $Pr(i = L|GS) = 1$ and $Pr(i = H|BS) = 1$. The rating policy is a mapping from a signal realization to a rating, where the rating can take two values, good and bad ($\tilde{R} = \{GR, BR\}$). Thus, the rating policy consists two parameters, α_G and α_B, where $\alpha_G = Prob(GR|GS)$ and $\alpha_B = Prob(BR|BS)$.[11]

As long as both ratings are issued in equilibrium, the market survives if there exists an r, such that the following conditions hold:

$$Pr(i = L|GR)F_L(\hat{c}(r)) + Pr(i = H|GR)F_H(\hat{c}(r)) \geq (1 + r)^{-1} \tag{2.18}$$

[11]Truthful disclosure is implemented when $\alpha_G = \alpha_B = 1$.

$$Pr(i = L|BR)F_L(\hat{c}(r)) + Pr(i = H|BR)F_H(\hat{c}(r)) \geq (1+r)^{-1} \tag{2.19}$$

where condition (2.18) refers to the case where a good rating is issued, whereas condition (2.19) refers to the case where a bad rating is issued. By implementing the Bayes rule, I can re-write conditions (2.18) and (2.19) as follows:

$$\frac{\alpha_G\lambda}{\alpha_G\lambda + (1-\alpha_B)(1-\lambda)}F_L(\hat{c}(r)) + \frac{(1-\alpha_B)(1-\lambda)}{\alpha_G\lambda + (1-\alpha_B)(1-\lambda)}F_H(\hat{c}(r)) \geq (1+r)^{-1}$$

$$\frac{(1-\alpha_G)\lambda}{(1-\alpha_G)\lambda + \alpha_B(1-\lambda)}F_L(\hat{c}(r)) + \frac{\alpha_B(1-\lambda)}{(1-\alpha_G)\lambda + \alpha_B(1-\lambda)}F_H(\hat{c}(r)) \geq (1+r)^{-1}$$

Recall that by the FOSD assumption, $F_L(\hat{c}(r)) > F_H(\hat{c}(r))$ for any r. This property implies that the market survives as long as the probability that the investors attribute to the entrepreneur being of LEC type is sufficiently large. I define as $\hat{\lambda}$ the minimum probability that investors should attribute to the entrepreneur being of LEC type, such as financing takes place. $\hat{\lambda}$ solves:

$$\hat{\lambda}F_L(\hat{c}(r)) + (1-\hat{\lambda})F_H(\hat{c}(r)) = (1+r)^{-1}$$

Thus, a CRA can achieve project financing for both types ($I_{LEC} = I_{HEC} = 1$) as long as α_G and α_B are chosen in a way that investors' beliefs about the probability that the entrepreneur is of LEC type exceed $\hat{\lambda}$, independently of the rating. This translates into the CRA choosing a combination of α_G and α_B such that:

$$Min\{\frac{\alpha_G\lambda}{\alpha_G\lambda + (1-\alpha_B)(1-\lambda)}, \frac{(1-\alpha_G)\lambda}{(1-\alpha_G)\lambda + \alpha_B(1-\lambda)}\} \geq \hat{\lambda}$$

A question which arises naturatelly is to what extent the CRA can affect investor's beliefs, and more specifically, what is the maximum $\hat{\lambda}$, denoted as $\hat{\lambda}^{max}$, that could be achieved by a CRA when it can pre-commit to a rating policy. $\hat{\lambda}^{max}$ solves the following problem:

$$\underset{\alpha_G,\alpha_B}{\text{Maximize}}\ Min\{Pr(i = L|GR), Pr(i = H|BR)\}$$

$$\text{subject to } 0 \leq \alpha_G \leq 1\ ,\ 0 \leq \alpha_B \leq 1 \text{ and } \alpha_G \geq \alpha_B.$$

Note that the constraint $\alpha_G \geq \alpha_B$ relates the the signaling component of the decision to ask for a rating, and in particular to off-equilibrium beliefs. This condition guarantees that an LEC entrepreneur has stronger incentive than an HEC entrepreneur to ask for a rating. Otherwise, there could be an equilibrium when neither an LEC nor an HEC entrepreneur asks for a rating. The solution to the maximization problem satisfies the following equation:

$$\alpha_G^* = 1 - \alpha_B^* \tag{2.20}$$

Note that the equilibrium that maximizes financing opportunities is a babbling equilibrium. In a babbling equilibrium the updated beliefs equal the prior beliefs in a regime without CRA, i.e.,

$$\hat{\lambda}^{max} \equiv Pr(LEC|\alpha_G^*, \alpha_B^*, GR) = Pr(LEC|\alpha_G^*, \alpha_B^*, BR) = \lambda$$

Proposition 5: Optimal Rating Policy

The optimal rating policy when CRA's objective is to maximize its profit is:

i) $\alpha_G^* = \alpha_B^* = 0.5$ *(babbling equilibrium), if there is a value of* r *such that* $\lambda F_L(\hat{c}(r)) + (1-\lambda)F_H(\hat{c}(r)) \geq (1+r)^{-1}$,

ii) $\alpha_G^* = \alpha_B^* = 1$ *(truthful disclosure), otherwise.*

Rating Inflation, Rating Deflation and their Impact

A value of α_B^* smaller than one implies that the CRA *inflates* its rating, i.e., an HEC entrepreneur receives a rating which corresponds to an entrepreneur of a higher level of creditworthiness. In addition, a value of α_G^* smaller than 1 implies that the CRA *deflates* its rating, i.e., an LEC entrepreneur receives a rating which corresponds to an entrepreneur of a lower level of creditworthiness. The intuition is straightforward: by giving bad rating to an LEC type, the CRA prevents a significant downgrade in investors' beliefs when the observe a bad rating. Hence, the entrepreneur can raise capital even after a bad rating, thus, he has incentive to ask for a rating.

Also, regarding the allocation of resources, we show that once we allow for the feedback effect inherent in capital markets, rating inflation and deflation might lead to financing of an entrepreneur with positive NPV, that would not be financed if the rating were truthful. The message of this finding is that, if information asymmetry is not severe, restricting CRAs to provide hard evidence with their ratings might have a negative effect on the probabilities of project financing and default.

Corollary 2: Impact of Rating Inflation/Deflation

When the CRA can pre-commit to a rating rule, rating inflation/deflation can be part of the equilibrium even if the fee is not rating-contingent. Besides, rating inflation/deflation might lead to financing of positive NPV projects that would not otherwise be financed.

2.8 Profit-Maximizing Fee

In this section I allow the CRA to choose a profit-maximizing fee. A direct consequence of the positive rating fee is that it modifies the implementation threshold, $\hat{c}$. The comparison with the case where the rating fee is restricted to zero allows me to isolate the distortion that a rating fee incurs.

The CRA's Problem: The CRA anticipates that only an LEC entrepreneur has incentive to ask for a rating; thus, it chooses the rating fee P, such that an LEC entrepreneur is indifferent between having and not having a rating. The functional form of the profit-maximizing fee, denoted as P^{max} and specified below, is related to the entrepreneur's outside option, which is determined by whether financing is feasible without a rating. If having a rating is not necessary for raising capital, the CRA will charge a fee that makes an LEC entrepreneur indifferent between asking for a rating and differentiating himself, or not asking for a rating and pooled with an HEC entrepreneur. In contrast, when the absence of a good rating leads to no financing, the entrepreneur's outside option is zero. Hence the profit-maximizing CRA extracts all the surplus, i.e., it charges the maximum fee for which an equilibrium exists. The following conditions characterize the profit maximizing rating fee:[12]

$$P^{max} \equiv \begin{cases} \underline{P}^{max} = \frac{(r_{II} - \hat{r}_R)}{(1+\hat{r}_R)} & \text{If rating not necessary for financing} \\ \bar{P}^{max} : F(\hat{c}|P = \bar{P}^{max}) = 0.5 & \text{If rating necessary for financing} \end{cases} \tag{2.21}$$

[12] $\bar{P}^{max} = arg\max P \;\; \text{s.t.}(1+\hat{r}_R)^{-1} = F(\hat{c}|P = \bar{P}^{max})$

The Entrepreneur's Problem: The entrepreneur's problem is more complicated than before: now, before facing the problem of the security design or whether to exert effort, the entrepreneur has to choose whether to ask for a rating. An HEC entrepreneur has no incentive to ask for a rating because by doing so, he would reveal his type.

I first explore the incentives of an LEC entrepreneur in the case where the market does not collapse in the absence of a good rating. The decision of an LEC entrepreneur to ask for a rating depends on which of the following two forces dominates. On one hand, asking for a rating differentiates him from an HEC entrepreneur, and allows him to promise a lower interest rate, $\hat{r}_R$ instead of r_{II}. On the other hand, asking for a rating implies that he needs to borrow a higher amount to cover - apart from the investment cost- the rating fee, P^{max}. Thus, an LEC entrepreneur faces a trade-off between repaying a smaller loan (1, instead of $1 + P^{max}$) with higher interest rate, or a larger loan with a lower interest rate.[13] Thus, in an environment where a rating is not necessary for financing, the entrepreneur chooses the action which leads to the highest expected utility, where:

$$U(without\ rating) = max\{R - (1 + r_{II}) - c, 0\} \tag{2.22}$$

$$U(with\ rating) = max\{R - (1 + \hat{r}_R)(1 + P^{max}) - c, 0\} \tag{2.23}$$

where the max function reflects the fact that the entrepreneur always has the choice not to exert effort. Thus, an LEC entrepreneur asks for a rating

[13]Here I implicitly assume that there is a lag between the time that the CRA gives the rating and the time CRA is paid. Due to rational expectations of the CRA, such setup is feasible if the entrepreneur can commit to paying the fee after the loan is taken.

as long as:

$$(1+\hat{r}_R)(1+P^{max}) \leq (1+r_{II}) \tag{2.24}$$

and exerts effort as long as:

$$c \leq \hat{c} \equiv max\{R-(1+r_{II}), R-(1+\hat{r}_R)(1+P^{max})\} \tag{2.25}$$

I now proceed to the case where the market collapses in the absence of a good rating. In this case, the CRA will charge a fee which extracts all the surplus of the entrepreneur. Following the previous analysis, and the CRA's problem, I re-write the implementation threshold of an LEC entrepreneur, $\hat{c}_L$, as:

$$\hat{c}_L \equiv \begin{cases} \underline{\hat{c}}_L = R-1+r^*_{II} & \text{If rating \& rating not necessary} \\ \hat{\bar{c}}_L = R-(1+\bar{r}_R)(1+\bar{P}^{max}) & \text{If rating \& rating necessary} \end{cases} \tag{2.26}$$

The related threshold for an HEC entrepreneur is:

$$\hat{c}_H \equiv R-(1+r_{NR}) \tag{2.27}$$

The Investors' Problem: Investors know whether project financing is feasible without the rating, and update their beliefs after observing whether the entrepreneur holds a rating. Because the CRA reports its private information truthfully, and charges a fee that an LEC entrepreneur buys a rating in equilibrium: $Prob(i = L|GR) = 1$ and $Prob(i = H|NR) = 1$. Hence,

investors' beliefs are as follows:

$$\tilde{s(.)} = \begin{cases} \tilde{s}(r_{\underline{R}}) = F_L(\hat{c}(\underline{r}_R^*)) & \text{If GR \& rating not necessary for financing} \\ \tilde{s}(r_{\bar{R}}) = F_L(\hat{c}(\bar{r}_R^*)) & \text{If GR \& rating necessary for financing} \\ \tilde{s}(r_{NR}) = F_H(\hat{c}(r_{NR})) & \text{If No Rating} \end{cases} \tag{2.28}$$

Investors would be willing to finance an entrepreneur who holds a rating as long as $\tilde{s}(r_{\underline{R}}) \geq (1 + r_{\underline{R}})^{-1}$, if rating is necessary, and as long as $\tilde{s}(r_{\bar{R}}) \geq (1 + r_{\bar{R}})^{-1}$, if rating is not necessary for raising capital. Similarly, investors would be willing to finance an entrepreneur with no rating as long as $\tilde{s}_{NR} \geq (1 + r_{NR})^{-1}$.

2.8.1 Equilibrium Interest rates

The combined problems of the CRA, the entrepreneur, and investors, determine the equilibrium interest rates r_R^* and r_{NR}^*, for an entrepreneur with or without rating respectively.

$$(1 + r_R^*) \equiv \begin{cases} (1 + \underline{r}_R^*) = F_L(\hat{c}(\underline{r}_R^*))^{-1} & \text{If rating \& rating not necessary} \\ (1 + \bar{r}_R^*) = F_L(\hat{c}(\bar{r}_R^*))^{-1} = 2 & \text{If rating \& rating necessary} \end{cases} \tag{2.29}$$

$$(1 + r_{NR}^*) = F_H(\hat{c}(r_{NR}^*)) \tag{2.30}$$

Note that the interest rate r_R^* of an LEC entrepreneur is affected by the probability of the entrepreneur being LEC. A higher λ, or a lower cost of an HEC entrepreneur, reduces the interest rate r_{II}^*, improves the out-

side option of an LEC entrepreneur, and results in a lower rating fee and interest rate. Moreover, a higher the fee increases the probability of default.

Proposition 6: (Effect of Profit Maximizer CRA)
The profit-maximizing fee is negatively related to the probability λ and the cost of effort. Also, allowing the CRA to charge the profit-maximizing fee increases the interest rate and the probability of default of an LEC entrepreneur, but it does not affect the probability of project financing.

Proof. See Appendix. □

2.9 Concluding Remarks and Future Research

In this paper, I argue that the reason for potential inefficiencies emerging from credit rating agencies might be more pathological than the literature recognizes. I evaluate the impact of a CRA, in a setup of project financing, which is characterized by the coexistence of information asymmetry and lack of commitment. I show that even in an ideal environment, where a CRA has access to perfect monitoring and reveals its rating truthfully, introducing a CRA might lead to a higher probability of default and hurt financing opportunities of positive-NPV projects. Moreover, I evaluate the regulation policy of requiring CRAs to provide hard evidence with their ratings. I argue that this policy might have an adverse effect on project's financing opportunities. Finally, I show that rating inflation or deflation might lead to better allocation of resources.

My findings have implications for the optimal information rating policy of a government or a central bank. For instance, a key question after

the recent crisis is whether the results of stress tests for banks should be publicized. My analysis suggests that concealing these results might improve the allocation of resources. This is a consequence of the finding that the amplification in the probability of default of bad banks might dominate the beneficial effect on good banks. Goldstein and Leitner (2013) [14] arrive at a similar conclusion by adopting a different setting. Corollary 1 could also relate to the recent debate on the borrowing interests rates that countries in the eurozone face. Interest rates differ across countries due to differences in credit risk; this has resulted in some peripheral countries borrowing with high spreads, which kept rising over time and eventually led to some countries being close to default. This paper suggests that a policy that requires countries with low credit risk to guarantee for countries with high credit risks, could improve the allocation of resources and decrease the expected probability of default.

My analysis suggests that future research on CRAs' behavior should account for the inherent in capital markets feedback effect. Note that this feedback effect implies a *self-fulfilling effect* of ratings: keeping the efficiency of an entrepreneur fixed, the probability of default which corresponds to a bad rating exceeds the one which corresponds to a good rating. The literature has overlooked this self-fulfilling effect of ratings -a concept that could be applied in future work in the context of testing CRAs' arguments on reputation concerns.

The mechanism explained in the previous paragraph opens the door to policy considerations, and it raises concerns regarding CRAs' regulation.

[14] Disclosure of some information may be necessary to prevent a market breakdown, but disclosing too much destroys risk-sharing opportunities

A message from this paper is that regulation of CRAs has yet to consider their ability to affect crucial variables, such as the probability of default or project financing opportunities.

Chapter 3

Security Design with Endogenous Implementation Choice

3.1 Introduction

Crowdfunding (CF) is a new method for financing projects by raising capital from a large pool of investors, performed via an internet platform. The forecasts for capital raised in 2015 through CF platforms exceeds $34 billion, when the venture capital (VC) industry invests an average of $30 billion each year.[1] CF started as a method for raising capital from crowds whose contributions were driven mostly by non-monetary incentives (non-equity CF). However, since April 2012, it entered a new era: the Jump-start Our Business Startups (JOBS) Act legalized equity CF by relaxing a series of restrictions regarding the sale of securities. The aim of this paper is to shed light on the role of securities on the allocation of resources, and to show that rewarding the non-implementation of a project is always part of

[1] "2015 Crowdfunding industry report", Massolution (2015).

the optimal security.

A key characteristic of CF is the easier access to potential capital. This relates to the fundamental idea in crowdfunding: it is the crowd, rather than a venture capitalist or a bank that decides about the creditworthiness of a project. Simplifying the process of accessing potential capital has a critical impact on the type of entrepreneurs and projects that this method attracts. On the plus side, crowdfunding is more open to innovative ideas. On the downside, it might attract entrepreneurs who: i) are associated with negative-NPV projects, ii) have little experience, and as a result, no record regarding their ability, and iii) have limited liability.[2]

This paper is based on two observations. First, compared to VC, investors in CF do not participate in the determination of the terms of financing. In CF platforms, entrepreneurs offer a take-or-leave-it security to potential investors. Thus, the determination of the optimal security when raising capital is a signaling rather than a screening problem. This signaling problem has been studied thoroughly in the security design literature, where prominent examples are Myers and Majluf (1984) and Nachman and Noe (1994). This brings us to the second observation which regards the main characteristics of the capital seeking party. In particular, we recognize that the representative entrepreneur in CF platforms differs from a typical large company; hence, applying the main findings of the security design literature in this environment is not straightforward. Following the previous two observations, the goal of this paper is to characterize the optimal security issued by an entrepreneur which shares the same charac-

[2] Agrawal et al. (2013) provide a very detailed introduction to the incentives of the investors, entrepreneurs, and platforms in CF. Valanciene and Jegeleviciute (2013) highlight the main benefits and drawbacks of CF.

teristics as a representative entrepreneur in CF platforms, and explore its main implications on the allocation of resources.

We study a setup where a cashless entrepreneur seeks capital to finance an investment project. The entrepreneur is either of high-productivity (a good type) or of low productivity (a bad type). The entrepreneur is privately informed about her type whereas potential investors only hold beliefs about the entrepreneur's type. Asymmetric information between the entrepreneur and potential investors gives rise to a signaling game when the former issues securities to raise capital. We differ from the security design literature in two crucial dimensions. First, to capture better the crowdfunding example, we do not restrict our analysis to entrepreneurs which are associated with projects of positive net present value. In particular, we assume that the good type corresponds to a project of positive NPV, whereas the bad type corresponds to a project of negative NPV. Second, we relax the implicit or explicit assumption in the security design literature that the entrepreneur is obliged to implementing the project after raising funds; in our model, whether the project is implemented is determined endogenously. These two features not only enrich the security design problem by allowing the security to be also contingent on the implementation choice, but also enables us to explore the allocational impact of the optimal security.

The first set of findings refers to the characterization of the optimal security. We find that the unique equilibrium in the contracting game is a pooling equilibrium where the bad type offers the same security as the good type. The optimal security is characterized by two components: a payment scheme if the project is implemented, and a fixed payment if the

project is not implemented. The payment scheme is similar to the standard debt; the investors become the claimants of the return of the project if that fails, whereas they receive a predetermined amount otherwise. The payment in case of non-implementation is such as the bad type's expected utility when not implementing the project coincides with her expected utility when implementing the project. The intuition behind a debt-like contract is two-fold. First, offering a security where the entrepreneur's return is zero when the project fails -which is more likely when the entrepreneur is of bad type- is aligned with the incentive of the good type to separate from the bad type. This idea is similar to the intuition of Nachman and Noe (1994). Second, a debt-like security minimizes the expected utility of the bad type when implementing the project. Worsening the bad type's option of implementing the project effectively minimizes the cost of preventing the bad type from implementing her project, which in turn, minimizes the negative externality imposed by the bad to the good type.

The second set of findings relates to the implications of the optimal security in the allocation of resources. We show that, once we endogenize the choice of project implementation, the market survives, and a positive-NPV entrepreneur implements her projects, independently of the extent of information asymmetry. Besides, the optimal security achieves separation in the implementation of the project; the negative-NPV type never implements her project. These findings differ significantly from the case where the entrepreneur is obliged to project implementation, which leads to either under-implementation or over-implementation. Finally, we show that commitment to project implementation is never profitable for the entrepreneur. This is because preventing a negative-NPV type from implementation mit-

igates its negative externality to the positive-NPV type. These findings highlight that focusing on securities that do not allow for the possibility of non-implementing the project has negative ramifications on welfare.

In Section 5 we develop a richer environment where the entrepreneur chooses her productivity level in equilibrium. We derive the equilibrium productivity level, and explore how it relates to the assumption that the entrepreneur is obliged to project implementation. In particular, we are interested in examining whether the finding of rewarding an entrepreneur when not implementing her project could weaken her incentive to invest in increasing her productivity, given that productivity is irrelevant when the project is not implemented. To explore this, we allow the entrepreneur to take a costly action, which is unobservable to investors and increases her productivity. We find that allowing the project implementation to be at the entrepreneur's discretion increases the entrepreneur's expected productivity. This follows from the finding that high productivity is rewarded more due to the better allocation of resources, which, effectively, leads to a steeper incentive pay. This analysis indicates that allowing for non-implementation of projects is aligned with the CF platform's incentives of attracting high productivity projects.[3]

Relevant Literature

Our work pertains mainly to the literature on security design under asymmetric information, initiated by Myers and Majluf (1984) and Nachman and Noe (1994), and followed by DeMarzo and Duffie (1999) and De-

[3]The revenue of platforms comes from a commission of 4-5% on the amount raised.

Marzo (2005). Technically, our setup is closer to Nachman and Noe (1994), apart from two major modifications: first, the entrepreneur might be associated with a negative-NPV project, and second, we relax the (implicit) assumption that the entrepreneur is obliged to implement the project. Allowing for the project implementation to be at the entrepreneur's discretion differentiates our work from the papers in this strand of the literature.

Besides, this work relates to the literature which explores the question of security design in an environment which is characterized by moral hazard. Prominent examples include Innes (1990), Crémer et al. (1998), Hartman-Glaser et al. (2012). We differ from this strand of the literature with respect to the form of the hidden action. In our paper, the hidden action refers to the decision of the agent to invest in her productivity level. Regarding the finding of alleviating adverse selection, our paper relates to Brennan and Kraus (1987). In our model, alleviating adverse selection is achieved by endogenizing the choice of project implementation. In contrast, Brennan and Kraus (1987) is interested in financial settings which alleviate adverse selection.

This paper is also related to the literature which highlights the optimality of CEO's severance pay, known as "golden parachutes". "Golden parachutes" refers to the compensation of a CEO when her executive is terminated, as a result of a merger or takeover. Similarly to our paper, the idea behind the optimality of severance pay to an informed agent is that it might prevent her from taking an action, which results in an inefficient allocation of resources. In our setup, the inefficient allocation comes from implementing a negative-NPV project, whereas in the CEO example, comes from taking an investment decision which differs from the profit-

maximizing one. For example, Inderst and Mueller (2010) show that the rewarding an agent to quit could be optimal, as long as it is accompanied by a steep incentive pay. In a slightly different environment, Levitt and Snyder (1997) argue in favor of rewarding a CEO to reveal bad news which could, in turn, lead the principal to cancel an inefficient project. As opposed to Levitt and Snyder (1997), where the possibility of a project cancellation leads to lower effort and productivity, we find that rewarding non-implementation of the project results in higher productivity. This is a critical departure from the literature on "golden parachutes": in our setup, the optimality of this payment not only does not fade off when the productivity is endogenous, but it is reinforced. Also, in the technical part, a major difference of our work from the literature on CEO's compensation contracts is that, in our setup, it is the informed party who offers the contract. Hence, we explore a signaling rather than a screening problem.

A key finding of our paper is that allowing entrepreneurs to decide whether to implement their project, and to include this decision in the security, prevents market breakdown. Preventing market breakdown is also one of the main goals in Philippon and Skreta (2010), Tirole (2012) and Camargo et al. (2014), who focus on the role of the optimal government intervention, rather than financial securities.

This paper is organized as follows. Section 3.2 introduces the model. Section 3.3 explores the case where the entrepreneur is obliged to project implementation. Section 3.4 explores the case where the project implementation is at the entrepreneur's discretion. Section 3.5 presents the case where the productivity of the entrepreneur is endogenously determined. Section 3.6 discusses and concludes.

3.2 The Model

Environment: We consider an environment where a cashless, risk-neutral entrepreneur seeks capital to finance a project. The project is non-divisible, and its implementation requires an investment equal to I. The entrepreneur raises capital by potential investors, in exchange for securities. We assume that capital market is perfectly competitive, and that investors demand a net return normalized to zero. Once the necessary capital is raised, and the project is implemented, a flow x is generated. We allow for a binary cash flow, i.e., $x = \{S, F\}$, where $S > F$. Thus, $x = S$ can be interpreted as the cash flow when the project succeeds, and $x = F$ as the cash flow when the project fails. In order to be consistent with the crowdfunding example, we assume that the entrepreneur does not have any wealth other than the project's return. Also, both the entrepreneur and investors are protected by limited liability.

Entrepreneur's types & Information sets: The entrepreneur can be of two types, bad or good, i.e., $t \in T = \{B, G\}$. We assume that the good (bad) type generates a success with probability p_G (p_B), where $p_G > p_B$. A good type can be interpreted as a high-productivity entrepreneur, whereas a bad type can be interpreted as a low-productivity entrepreneur, where the productivity is determined by the probability of success of the project. The entrepreneur has private information regarding her type. In contrast, investors hold prior beliefs about the entrepreneur's type. In particular, investors expect the entrepreneur to be of good type, with probability λ_G, and of bad type, with the complementary probability $\lambda_B = 1 - \lambda_G$. Both

the probability of success of each type, and investors' beliefs regarding the entrepreneur's type are common knowledge.

It is worth highlighting a critical departure of our paper from other papers on the security design literature such as Nachman and Noe (1994). In this paper, we do not restrict the analysis to projects of positive net present value. Instead, we assume that the bad type corresponds to a negative-NPV project, whereas the good type corresponds to a positive-NPV project, i.e.,

$$p_G S + (1 - p_G)F > I > p_B S + (1 - p_B)F. \tag{3.1}$$

Finally, we do not impose any restriction on the ex-ante NPV of the project:

$$\lambda_G[(p_G S + (1 - p_G)F] + \lambda_B][p_B S + (1 - p_B)F] - I$$

i.e., conditional on investor's prior beliefs, the entrepreneur's project can have positive, negative or zero NPV. We explore each case separately.

A possible concern behind this assumption, is why an entrepreneur would choose to implement a negative-NPV project. The answer to this relates to the combination of two characteristics, which closely reflect what is documented in the crowd-funding industry: i) the project is funded fully by a third party, and ii) the entrepreneur has limited liability. A direct consequence of these two characteristics is that although the entrepreneur's project has negative expected net return, the entrepreneur's expected utility is always non-negative. Note that allowing for a negative-NPV project has implications for the planner's problem: if the aim of the planner is to

maximize total welfare, only type $t = G$ is worth financing.

Contracting game: The information asymmetry between the entrepreneur and investors, regarding the type of the former, turns the choice of the security design into a signaling game. A significant part of the literature on security design, such as Myers and Majluf (1984) and Nachman and Noe (1994) assumes, either implicitly or explicitly, that once financing takes place, the entrepreneur is obliged to project implementation.[4] The goal of this paper is to show that this assumption plays a critical role when the entrepreneur shares the same characteristics as the representative entrepreneur in crowdfunding platforms. In particular, we are going to show that this assumption leads to an inefficient allocation of resources, and if information asymmetry is severe, to a market breakdown. Hence, applying the main findings of Myers and Majluf (1984) and Nachman and Noe (1994) to a crowdfunding environment might be problematic.

Endogenizing the implementation choice enriches the contracting game; the security is contingent not only on the realized cash flow, but also on the implementation choice. We denote by g a security which consists of two sets of payments, $g(x)$ and $\bar{g}$. $g(x)$ denotes the payment when the project is implemented and the realized cash flow is x. In contrast, $\bar{g}$ denotes the payment if no implementation takes place. Thus, the security is defined by the triple: $g(S), g(F), \bar{g}$. Finally, given the price P_g of the secu-

[4]In fact, Nachman and Noe (1994) allow for securities which are contingent only on the realized cash flow.

rity, the assumption of limited liability imposes the following restrictions:

$$0 \leq g(x) \leq P_g - I + x, \text{ if the project is implemented}$$
$$0 \leq \bar{g} \leq P_g, \text{ if the project is not implemented}$$

where $g(x) \geq 0$ and $\bar{g} \geq 0$ capture investors' limited liability, whereas $g(x) \leq P_g - I + x$ and $\bar{g} \leq P_g$ capture the entrepreneur's limited liability. We denote by G the set of admissible securities, i.e., securities which are characterized by $g(x)$ and $\bar{g}$, and satisfy the limited liability assumption.

Entrepreneur's Maximization problem: An entrepreneur of type $t \in T$ issues a security g_t, which consists of $g_t(x)$ and $\bar{g}_t$, in order to maximize her expected utility:

$$V(t, g_t, P_{g_t}) = max\{\mathbb{E}_t[P_{g_t} - I + x - g_t(x)], P_{g_t} - \bar{g}_t\} \tag{3.2}$$

where P_{g_t} represents the price of security g_t. Note that $\mathbb{E}_t[P_{g_t} - I + x - g_t(x)]$ is the entrepreneur's expected utility when implementing her project, whereas $P_{g_t} - \bar{g}_t$ is her expected utility in the case where the project is not implemented. The *max* function indicates that the entrepreneur will choose the option which maximizes her expected return. Note that P_{g_t} cannot be lower than I, otherwise there would be insufficient funds for the implementation of the project. This remark, combined with the limited liability assumption, imply that $V(t, g_t, P_{g_t})$ is non-negative, independently of the type of the entrepreneur.

Timing: The sequence of events is as follows:

1. The entrepreneur of type $t \in T$ sells a security g_t at price P_{g_t}.
2. The entrepreneur decides whether to implement, by investing I.
3. If the project is implemented, its cash flow x is realized.
4. Contract is executed.

Equilibria characterization: A candidate for an equilibrium is a triple of functions $e^* = (g^*, \mu^*, P^*)$, where: i) $g^* : T \mapsto G$, where g_t^* is the security design chosen by the type t, ii) $\mu^* : G \mapsto \Delta_T$, and μ_g^* is the market's posterior beliefs given that the security g is offered by the entrepreneur, and iii) $P^* : G \mapsto R_+$. A Perfect Bayesian Equilibrium is a triple $e^* = (g^*, \mu^*, P^*)$, which satisfies the following conditions:

- *Sequential Rationality*: For each $t \in T$, g_t^* maximizes $V(t, g_t, P_{g_t})$, subject to the constraints that $g \in G$ and $P_g^* \geq I$.
- *Beliefs Consistency*: When security g is such that $g = g_t^*$ for some $t \in T$, g is "on the equilibrium" and μ is determined by Bayes' rule. When g is such that $g \neq g_t^*$ for every $t \in T$, g is "off the equilibrium", then it is only required that $\mu_g \in \Delta_t$.
- *Competitive Rationality*: $P_G^* = E_{\mu * g}[g]$, for all $g \in G$.

Regarding the "off-equilibrium beliefs", we adopt the D1 refinement criterion discussed in Cho and Kreps (1987). D1 places zero weight on a type $t = t'$ deviating to an off-equilibrium design if there exists a type $t = t''$ who has strong incentive to deviate, whenever type $t = t'$ has weak incentive to deviate.

3.3 Entrepreneur obliged to project implementation

In order to evaluate the impact of endogenizing the choice of project implementation, we use the case where the entrepreneur is obliged to implement the project, as a benchmark. To this end, we start by characterizing the optimal security, under the assumption that the entrepreneur is obliged to project implementation. This environment is consistent with the setup in Nachman and Noe (1994), except for the modification that we allow for a negative-NPV project. Note that, in this case, the security g is characterized only by the payment scheme $g(x)$; the payment $\bar{g}$ becomes irrelevant. To avoid any confusion, when the entrepreneur is obliged to implement the project, we denote the security as g' and the corresponding payment scheme as $g(x)'$. In contrast, when the choice of project implementation is endogenous, we denote the security as g, and the corresponding payment scheme as $g(x)$.

Lemma 1

In equilibrium, the bad type offers the same security as the good type (pooling equilibrium).

The intuition behind Lemma 1 is straightforward. If the bad type offered a different security, she would reveal herself, given that "on the equilibrium beliefs" should be correct. In that case, the bad type would not be able to raise capital, because investors anticipate that she corresponds

to a negative-NPV project. Hence, the only equilibrium in the contracting game, is an equilibrium where the bad type offers the same security as the good type. Consequently, investors posterior beliefs about the entrepreneur's type, coincide with their prior. Also, given that the bad type mimics the good type, we can focus on the maximization problem of the good type.

Lemma 2

In equilibrium, the price of a security g, P_g equals I.

Lemma 2 is a consequence of a pooling equilibrium, where the bad type mimics the good type. The rationale behind Lemma 2 is that the good type, when raising capital, suffers a negative externality from the bad type. This cross-subsidization implies that the good type ends up paying a higher capital cost, than the one that would correspond to her type. As a result, it is never optimal for the entrepreneur to raise more capital than the amount that is necessary for undertaking the project. Consequently, the entrepreneur would never offer a security g', whose corresponding price exceeds the cost of financing the project, i.e. $P_g \leq I$. Since the project is non-divisible, $P_g = I$. Given Lemma 1 and Lemma 2, the optimal security when the entrepreneur is obliged to project implementation, solves the following maximization problem:

$$\underset{g(S)',g(F)'}{\text{Maximize}} \quad p_G(S - g(S)') + (1 - p_G)(F - g(F)') \quad \text{s.t.}$$

$$\lambda_G[(p_G g(S)' + (1 - p_G)g(F)'] + \lambda_B[p_B g(s)' + (1 - p_B)g(F)'] = I \tag{3.3}$$

$$0 \leq g(S)' \leq S \tag{3.4}$$

$$0 \leq g(F)' \leq F \tag{3.5}$$

where (3.3) is the investors' participation constraint, which is binding, given that capital markets are assumed to be perfectly competitive. In particular, the LHS of (3.3) captures investors' expected return, whereas the RHS captures the amount investors pay for the security, denoted by P_g, which, following the Lemma 2, equals I. The last two constraints capture the limited liability of the entrepreneur and investors.

Before deriving the optimal security, we explore under what conditions the market survives. Given that the equilibrium in the contracting game is pooling, the market collapses and no financing takes place if there is no *feasible* security which satisfies investors' participation constraint:

$$\underbrace{\lambda_G[(p_G g(S)' + (1-p_G)g(F)'] + \lambda_B[p_B g(S)' + (1-p_B)g(F)']}_{\text{Expected Revenue}} \geq \underbrace{I}_{\text{Cost}}$$

Note that if the market collapses, the entrepreneur's expected utility is zero. In contrast, if the market survives, the expected utility of the entrepreneur is non-negative. Hence, the market collapses if, even for the maximum feasible payments ($g(S)' = S$, $g(F)' = F$), the cost of financing exceeds its expected revenue, i.e.,

$$\lambda_G[(p_G S + (1-p_G)F] + \lambda_B[p_B S + (1-p_B)F] < I \tag{3.6}$$

Condition (3.6) holds if the prior probability that investors attribute to the

entrepreneur being of good type is sufficiently small. Thus, the market collapses if:

$$\lambda_G < \lambda_G^{min} \equiv \frac{I - [p_B S + (1 - p_B)F]}{(p_G - p_B)(S - F)} \tag{3.7}$$

Proposition 1 characterizes the optimal security when the entrepreneur is obliged, by assumption, to project implementation.

Proposition 1

If $\lambda_G \geq \lambda_G^{min}$, financing takes place, and the optimal security is given by:

$$g^*(F)' = F \ \ , \ \ g^*(S)' = \frac{I - F}{\lambda_G p_G + \lambda_B p_B} + F$$

If $\lambda_G < \lambda_G^{min}$, no financing takes place (market collapses).

Proof. See Appendix C. □

The intuition behind the optimal security relates to the core idea in Nachman and Noe (1994). Namely, the good type tries to separate herself from the bad type, by offering a contract which promises the maximum feasible payment when the cash realization is low, because this cash flow is more likely to arise when the entrepreneur is of bad rather than good type.

Note that there are two types of distortions depending on the value of λ_G. If $\lambda_G < \lambda_G^{min}$, there is under-implementation compared to the first best: the positive-NPV type does not raise the necessary capital to implement her project. In contrast, if $\lambda_G \geq \lambda_G^{min}$, the market survives, and there is over-implementation: the negative-NPV type implements her project.

3.4 Endogenous project implementation

This section explores the case where the choice to implement the project is at the entrepreneur's discretion, and this choice is contractible. We showed that restricting the entrepreneur to implement the project leads to under-implementation, when the market collapses, and over-implementation, when the market survives. The goal of this section is to show that endogenizing the choice of project implementation: i) increases the entrepreneur's expected utility, independently of her type, ii) improves the allocation of resources, and iii) prevents the market from collapsing. Besides, we show in Section 3.5 that endogenizing project's implementation strengthens the incentive of the entrepreneur to invest in her productivity.

Recall that the entrepreneur issues a security g which consists of three payments; $g(S)$, if the project is implemented and it succeeds, $g(F)$, if the project is implemented and it fails, and $\bar{g}$, if the project is not implemented. Similarly to the case where the entrepreneur is obliged to project implementation, the bad type offers the same security as the good type, i.e., Lemma 1 goes through. Besides, following Lemma 1, Lemma 2 holds as well. Although there is pooling at the contracting stage, endogenizing the choice of project implementation might lead to separation at the stage of project implementation. Four are the possible scenarios regarding the implementation of the project:

- Scenario 1: Both types implement their project. As long as beliefs are consistent, this equilibrium is effectively the same as the equilibrium when the entrepreneur is obliged to project implementation.

- Scenario 2: Neither the good nor the bad type implements her project.
- Scenario 3: Only the bad type chooses to implement the project. This scenario can not emerge in equilibrium because it violates investors' participation constraint. Even for the maximum feasible payments, i.e., $g(F) = G$, $g(S) = S$, $\bar{g} = P_g$, investors would make zero profits if the entrepreneur is of good type, and negative profits otherwise.
- Scenario 4: Only the good type chooses to implement the project. This is the only case where separation can be achieved, in the sense that the implementation choice differs depending on the entrepreneur's type. The following analysis shows that this scenario can emerge in equilibrium.

We postpone exploring scenario 1 and 2 for the end of this section, and we start with analyzing scenario 4. We are interested in finding the security which maximizes the expected utility of the good type, subject to the constraint that only the good type implements the project. Compared to the case where the entrepreneur is obliged to project implementation, the maximization problem is augmented by one incentive compatibility condition for each type. The maximization problem of the good type is given by:

$$\underset{g(S),g(F),\bar{g}}{\text{Maximize}} \;\; EU(impl|t = G) \;\; \text{s.t.}$$

$$EU(impl|t = G) \geq EU(not\ impl|t = G) \qquad (ICC_G)$$

$$EU(impl|t = B) \leq EU(not\ impl|t = B) \qquad (ICC_B)$$

$$\lambda_G[p_G g(S) + (1 - p_G)g(F)] + \lambda_B \bar{g} \geq I \qquad (PC_I)$$

$$0 \leq g(S) \leq S \;\; , \;\; 0 \leq g(F) \leq F, \;\; 0 \leq \bar{g} \leq I \qquad (LL)$$

where,

$$EU(impl|t = G) = p_G(S - g(S)) + (1 - p_G)(F - g(F))$$
$$EU(impl|t = B) = p_B(S - g(S)) + (1 - p_B)(F - g(F))$$
$$EU(not\ impl|t = G) = EU(not\ impl|t = B) = I - \bar{g}$$

ICC_G (ICC_B) strands for the incentive compatibility constraint of the good (bad) type, whereas, PC_I stands for the investors' participation constraint.

We show in Appendix C that under the optimal security, ICC_B is binding, i.e. if the entrepreneur is of a bad type, her expected utility when not implementing the project equals her expected utility when implementing the project. If this is not the case, there is always a deviation to a higher level $\bar{g}$ and a lower level of $g(S)$, which does not violate the incentive constraints, and it is strictly preferred by the good type. Also, we show in Appendix C that ICC_G is slack. Finally, PC_I is binding, as an implication of the perfectly competitive capital market. Then, by substituting ICC_B into PC_I, and solving with respect to $g(S)$, we obtain:

$$g^*(S) = \frac{\lambda_B(-Fp_B + F + p_B S) + \lambda_G I}{\lambda_G p_G + \lambda_B p_B} - \frac{\overbrace{\lambda_G(1 - p_G) + \lambda_B(1 - p_B)}^{R}}{\lambda_G p_G + \lambda_B p_B} g^*(F) \tag{3.8}$$

Condition (3.8) implies that the entrepreneur could increase $g(F)$ by ϵ and decrease $g(S)$ by $R \times \epsilon$ without violating investors' participation constraint. Increasing $g(F)$ by ϵ decreases the entrepreneur's expected utility by $(1 - p_G) \times \epsilon$, whereas decreasing $g(S)$ by $R \times \epsilon$ increases the entrepreneur's expected utility by $p_G \times R \times \epsilon$. It can be shown that $p_G \times R \times \epsilon > (1 - p_G) \times \epsilon$

as long as $p_G > p_B$. Hence, under the optimal security, $g^*(F)$ reaches its maximum value, i.e. $g^*(F) = F$. Substituting $g^*(F) = F$ into (3.8), we derive the optimal value of $g^*(S)$, and subsequently, by substituting $g^*(F)$ and $g^*(S)$ into ICC_B, we derive $\bar{g}^*$. Note also that the security of Lemma 3 is the unique security which survives the Intuitive Criterion.

Lemma 3: Optimal security in scenario 4.

$$g^*_{s4}(F) = F$$

$$g^*_{s4}(S) = \frac{\lambda_G(I - F(1 - p_G)) + \lambda_B p_B S}{\lambda_G p_G + \lambda_B p_B}$$

$$\bar{g}^*_{s4} = I - p_B(S - g^*_{s4}(S))$$

Proof. See Appendix C. □

The intuition behind $g^*_{s4}(F) = F$ is two-fold. First, similarly to the intuition in Proposition 1, offering a security which promises the maximum feasible payment when the realized cash flow is low, is aligned with the entrepreneur's incentive to separate from the bad type. This is because a low cash flow is more likely to arise if the entrepreneur is of bad rather than good type. Second, compared to any other security which satisfies investors' participation constraint, $g(F) = F$ minimizes the expected utility of the bad type when implementing the project. Worsening the bad type's option of implementing the project, allows the good entrepreneur to minimize the negative externality imposed by the bad type. Subsequently, mitigating the cross-subsidization, enables the good entrepreneur to offer a lower payment when the project is implemented and generates a success.

Here, we show that scenario 1 cannot be an equilibrium. In scenario 1, similarly to the case where the entrepreneur is obliged to project implementation, both types implement their project. Thus, $\bar{g}$ is never realized. Although $\bar{g}$ does not affect the entrepreneur's utility directly, it plays a critical role when it comes to beliefs' formulation. Hence, for values of $\bar{g}$ for which the bad entrepreneur prefers implementing the project, the maximization problem in scenario 1 is effectively the same as the maximization problem when the entrepreneur is obliged to project implementation. As a result, $g^*_{s1}(F) = g^*(F)'$ and $g^*_{s1}(S) = g^*(S)'$. However, the good type is better-off under scenario 4 because $g^*_{s4}(S) \leq g^*(S)'$ and $g^*_{s4}(F) = g^*(F)'$. Thus, scenario 1 can not emerge in equilibrium: the good type has incentive to deviate to scenario 4, by choosing $\bar{g}$, such as the bad type prefers not implementing her project.

Similar intuition applies to scenario 2. In this scenario, the expected utility of investors equals $-P_g + \bar{g}$. Following Lemma 2 and the assumption of perfectly competitive markets, $P_g = \bar{g}$. In this equilibrium, assuming that such an equilibrium exists, the entrepreneur's utility is zero. However, the good type is better-off under scenario 4 because she achieves positive expected utility. Thus, scenario 2 can not emerge in equilibrium: the good type always has the incentive to deviate to scenario 4, by offering the security characterized in Lemma 3.

Corollary 1

Independently of her type, the entrepreneur is better-off when the project implementation is endogenous and contractible.

Proof. See Appendix C. □

Hence, the optimal security, provided in Lemma 3, is the unique optimal security. Note that the improvement in the entrepreneur's expected utility does not come at the cost of lower expected utility for investors; under both setups, the investors' participation constraint is binding. This Pareto improvement stems from a more efficient allocation of resources, due to the finding that the negative-NPV type does not implement her project.

Market breakdown when project implementation is endogenous

When the entrepreneur is allowed to choose whether to implement her project, and to offer securities contingent on this choice, the market survives if there is at least one *feasible* security $g \in G$, such as the investors' participation constraint is satisfied:

$$\underbrace{\lambda_B \bar{g} + \lambda_G [p_G g(S) + (1 - p_G) g(F)]}_{\text{Expected Revenue}} \geq \underbrace{I}_{\text{Cost}} \tag{3.9}$$

where the LHS of (3.9) denotes the expected revenue, whereas the RHS denotes the cost of investors. Recall that for a security to be feasible, it must satisfy limited liability, i.e., $0 \leq g(S) \leq S$, $0 \leq g(F) \leq F$, $0 \leq \bar{g} \leq I$. We now explore relation (3.9) for the optimal security of Lemma 3, which is, by construction, feasible. Hence, (3.9) becomes:

$$\lambda_B \bar{g}^*_{s4} + \lambda_G [p_G g^*_{s4}(S) + (1 - p_G) g^*_{s4}(F)] \geq I \tag{3.10}$$

By substituting the optimal values of $\bar{g}^*_{s4}$, $g^*_{s4}(S)$ and $g^*_{s4}(F)$ into (3.10), we obtain that the LHS of (3.10) equals I. Thus, (3.9) is satisfied for the security of Lemma 3. Note that if the investors' participation constraint is not satis-

fied (market breakdown), the entrepreneur's expected utility is zero. Recall also that the entrepreneur's expected utility under the security of Lemma 3 is non-negative. Hence, we can conclude that the market never collapses, because compared to that case, the entrepreneur is better-off when offering the security of Lemma 3, which always satisfies the investor's participation constraint.

Corollary 2

If the implementation choice is endogenous and contractible, there always exists a security which satisfies investors' participation constraint (market always survives), and the positive-NPV project is implemented.

The reason the market always survives is that, once we endogenize the implementation choice, the positive-NPV project is always implemented, whereas the negative-NPV project is not. The intuition behind this finding can be captured in the following example. To simplify the algebra, suppose an environment where the bad type always fails ($p_B = 0$). In this case, the best strategy for the good type is to offer a security which pays $g(F) = F$. Such a payment leaves no surplus to the bad type when implementing the project. Consequently, the bad type is willing to forgo implementation as long as $\bar{g} \leq I$. Note that the combination of $g(F) = F$ with $\bar{g} = I$ minimizes the loss of investors when financing a bad type, which effectively, minimizes the distortion imposed by the bad type to the good type. Eliminating the cross-subsidization enables the good type to offer a payment in case of success which equals the fair payment, i.e., the payment which corresponds to her type, $g(S) = \frac{(I-F(1-p_G))}{p_G}$.

It is worth highlighting that the extent of the decrease in cross-subsidization is negatively related to the probability of success of the bad type, p_B. This is because there is a monotonic relation between p_B and the cost of preventing the bad type from implementation. This relation is captured by the binding ICC_B.

The previous example captures the main idea of this paper; once the choice of project implementation is endogenous and contractible, the good entrepreneur can offer a security which, effectively, provides insurance to investors against the event of financing a bad entrepreneur. This insurance allows the entrepreneur to offer a lower payment in the case where the project is implemented and succeeds.

The combination of Lemma 3, Corollary 1 and Corollary 2 leads to Proposition 2, which characterizes the optimal security when the choice of implementing the project is endogenous and contractible.[5]

Proposition 2:

When the choice of project implementation is endogenous and contractible, project's financing always takes place, and the unique optimal security is given by:

$$g^*(F) = F$$

$$g^*(S) = \frac{\lambda_G(I - F(1 - p_G)) + \lambda_B p_B S}{\lambda_G p_G + \lambda_B p_B}$$

$$\bar{g}^* = I - p_B(S - g^*(S))$$

[5]Note that the optimal security resembles "Repurchase Agreements" (REPOs). REPOs is a form of borrowing where a party sells a security which agrees to buy it back after a given period. In the language of our model, the repurchase price would be equal to $\bar{g}$.

3.5 Endogenizing Entrepreneur's Productivity

The main source of revenues of crowdfunding platforms is a commission of 4-5% on the total capital raised. Thus, one of the main objectives of crowdfunding platforms is to attract high-productivity entrepreneurs.[6] We showed in the previous section that endogenizing the choice of project implementation implies that a subset of entrepreneur's types does not proceed with the implementation of the project. Hence, a question which arises naturally is whether this characteristic could demotivate entrepreneurs to invest in their productivity, given that it does not affect the expected return in case of non-implementation. If this concern is valid, it could potentially cancel out the benefits of preventing implementation of negative-NPV projects.

In this section, we shed light on this concern by allowing the entrepreneur to invests in her productivity, before seeking capital to finance her project. The final goal of this exercise is two-fold. First, to develop a richer environment, where the entrepreneur chooses her productivity in equilibrium. Second, to explore the impact of endogenizing the choice of project implementation on the entrepreneur's productivity.

Environment & Technology: For the sake of tractability, and in order to be consistent with the previous analysis, we allow for the simplest investment-in-productivity technology: the entrepreneur is of low productivity ($t = B$) unless she takes a costly action, which upgrades her to a high-productivity

[6]This is also consistent with the fact that it is a reputation-based industry.

entrepreneur ($t = G$).[7] This action can be interpreted as the acquisition of skills or relevant information, which would increase the probability of developing a successful project. In this setting, the probability that the entrepreneur invests in her productivity coincides with λ_G, which is now determined endogenously.

Endogenizing λ_G leads to a game consisting of two stages: the investment-in-productivity stage, and the contracting/implementation stage. The second stage is effectively the same as in the case where the probability λ_G is exogenous. The first stage refers to the entrepreneur's decision to take a costly action, which determines her productivity.

Information Sets: In order to maintain the setting of asymmetric information in the contracting stage, we assume that investors cannot observe whether the agent has invested in her productivity level. For instance, suppose that the entrepreneur's project is a new application for a smartphone. In this case, the entrepreneur can acquire costly information regarding similar applications, which will enable her to design a better application, and increase its probability of success. We assume that the cost of acquiring skills, c, is not verifiable, and as opposed to the entrepreneur who observes c, investors only hold beliefs about it. In particular, investors anticipate that c is drawn from an interval $[\underline{c}, \bar{c}]$, according to a continuous probability density function $\phi(c)$, with $\Phi(c)$ standing for the corresponding cumulative distribution function. The rationale behind this assumption is that the entrepreneur, mainly because of her expertise, knows the cost of

[7] The results are robust to any technology where taking the costly action increases the probability of becoming a high-productivity entrepreneur.

increasing the project's productivity, whereas investors have an imperfect estimate about it. Thus, the investment decision, which is denoted as d, is a mapping from c to the type $t \in T$.

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Timing: The sequence of events is as follows:

1. The entrepreneur observes her cost, and decides whether to invest in her productivity.
2. The type $t \in T$ of the entrepreneur is determined.
3. The entrepreneur of type $t \in T$ sells a security g_t at price P_{g_t}.
4. The entrepreneur decides whether to implement the project.
5. If the project is implemented, its cash flow x is realized.
6. Contract is executed.

Thus, compared to the benchmark model, the game is augmented by the investment-in-productivity stage.[8]

3.5.1 Derivation of probability λ_G.

A property of the equilibrium is that investors' beliefs about λ_G are correct. Note also that the probability that the entrepreneur invests in her productivity, i.e., λ_G, depends on the security, which in turn, depends on λ_G through investors' beliefs. Hence, the optimal security and the optimal value of λ_G, are determined jointly in equilibrium.

[8]Note the entrepreneur offers the security *after* investing in her productivity. However, as long as c is not verifiable, the equilibrium security is the same independently of whether the entrepreneur offers the security before or after investing in her productivity.

We solve the game backwards. We start from the contracting stage, by taking investors' beliefs about λ_G as given. Then, we proceed with the investment-in-productivity stage, by taking the security g as given.

3.5.1.1 Contracting stage

Recall that the entrepreneur's decision to invest in her productivity is unobservable. Thus, this stage is identical to the contracting stage at Section 3.4, apart from a critical modification: probability λ_G is now replaced by the *beliefs* of investors about the probability that the entrepreneur has invested in her productivity, denoted by $\tilde{\lambda}_G$. Hence, by Proposition 2, the security which maximizes the entrepreneur's expected utility, subject to the constraint that $\lambda_G = \tilde{\lambda}_G$, is captured in Lemma 4.

Lemma 4

$$g^*(F) = F \ \ , \ \ \bar{g}^* = I - p_B(S - g^*(S, \tilde{\lambda}_G))$$

$$g^*(S, \tilde{\lambda}_G) = \frac{\tilde{\lambda}_G(I - F(1 - p_G)) + \tilde{\lambda}_B p_B S}{\tilde{\lambda}_G p_G + \tilde{\lambda}_B p_B}$$

3.5.1.2 Investment-in-productivity stage

When the entrepreneur considers investing in her productivity, she anticipates that the payments in the financing stage will be given by Lemma 4. Thus the entrepreneur's expected utility in each case is:

$$EU(invest) = p_G(S - g^*(S, \tilde{\lambda}_G)) + (1 - p_G)(F - g^*(F)) - c$$

$$EU(not\ invest) = p_B(S - g^*(S, \tilde{\lambda}_G)) + (1 - p_B)(F - g^*(F))$$

Since $g^*(F) = F$, the entrepreneur invest in her productivity as long as:

$$\underbrace{c}_{\text{cost}} \leq \underbrace{(p_G - p_B)(S - g^*(S, \tilde{\lambda}_G))}_{\text{benefit}} \equiv \hat{c} \tag{3.11}$$

where $\hat{c}$ can be interpreted as the investment threshold. Figure 3.1 represents the relationship between $\tilde{\lambda}_G$ (on the horizontal axis) and the benefit of investing in productivity (dashed curve), for four different distributions of c. As expected, there is a negative relation between the probability $\tilde{\lambda}_G$, and the payment in case of success that investors are willing to accept to finance an entrepreneur. In addition, the higher the $\tilde{\lambda}_G$, the stronger the entrepreneur's incentive to invest in productivity. This is captured in equation (3.11).

3.5.1.3 Equilibrium existence and uniqueness of interior equilibrium

We start the analysis by focusing on equilibria for which $\tilde{\lambda}_G^* \in (0, 1)$, thereafter, "interior equilibria". We analyze the equilibrium conditions when the project implementation is at the entrepreneur's discretion- similar intuition applies when the entrepreneur is obliged to implement the project. We consider *monotone* or *threshold equilibria* in which the investment strategy is monotonic in c. In an interior threshold equilibrium, the following conditions need to hold:

$$\hat{c}^* = (p_G - p_B)(S - g^*(S, \tilde{\lambda}_G^*)) \tag{3.12}$$

$$Pr(c \leq \hat{c^*}) = \tilde{\lambda}_G^* \tag{3.13}$$

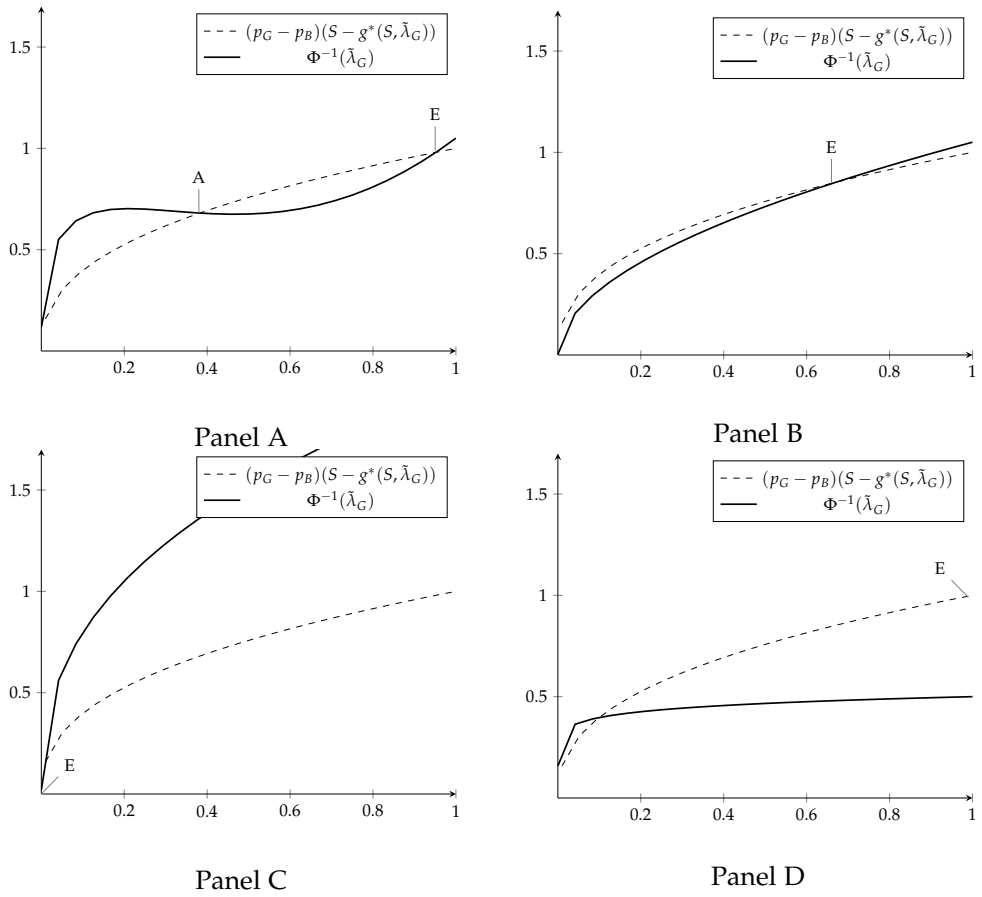


Figure 3.1. Cost and benefit of investing in productivity.

Condition (3.12) provides the critical value $\hat{c}$ for which the entrepreneur is indifferent between investing and not investing. Condition (3.13) relates to the fact that investor's beliefs should be correct. In particular, the probability that investors attribute to the entrepreneur being of good type coincides with the probability that the entrepreneur's cost is below $\hat{c}$. Besides, for investors' beliefs to be correct, the entrepreneur must prefer investing when $c \leq \hat{c}$, and not investing otherwise. An implication of a threshold equilibrium is that $\hat{c}$ depends on $g^*(S, \tilde{\lambda}_G)$, which in turn, depends on $\hat{c}$. Thus, the optimal values $\hat{c}^*$ and $g^*(S, \tilde{\lambda}_G^*)$, are determined jointly in equilibrium.

Recall that $Pr(c \leq \hat{c^*}) \equiv \Phi(\hat{c^*})$. In order to illustrate graphically the

equilibrium existence, it is more informative to use the inverse of $\Phi(\hat{c})$:

$$\Phi^{-1}(\tilde{\lambda}_G) = \hat{c}$$

where $\Phi(\hat{c})$ is invertible as a continuous and strictly increasing function. Hence, every interior equilibrium in the investment-in-productivity stage satisfies the following condition:

$$\Phi^{-1}(\tilde{\lambda}_G) = (p_G - p_B)(S - g^*(S, \tilde{\lambda}_G)) \tag{3.14}$$

where $\Phi^{-1}(\tilde{\lambda}_G)$ is depicted by the solid curve in Figure 3.1. Note that $\Phi^{-1}(\tilde{\lambda}_G)$ is strictly increasing in $\tilde{\lambda}_G$, as a consequence of the fact that $\Phi(\hat{c})$ is strictly increasing in $\hat{c}$. Thus, relation (3.14) characterizes value of $\tilde{\lambda}_G$ where the dashed and the solid curve intersect. Relation (3.14), however, is not a sufficient condition for an interior equilibrium to exist. For example, point A in panel A of Figure 3.1 satisfies (3.14) but it can not be an equilibrium; for cost $\hat{c}_A + \epsilon$, with $\epsilon > 0$, and as long as the beliefs are consistent, the expected benefit exceeds the cost. Hence, for cost $\hat{c}_A + \epsilon$ the entrepreneur has incentive to invest in productivity, which contradicts the definition of a threshold equilibrium: in a threshold equilibrium, the entrepreneur invests in her productivity only if $c \leq \hat{c}_A$. Thus, for an interior equilibrium to exist, the solid curve should cross the dashed curve from below. Following that, the unique interior equilibrium is given by point E in panel A of Figure 3.1.

Note that there could be more than one combinations of $\tilde{\lambda}_G$ and $g^*(S, \tilde{\lambda}_G)$ which satisfy (3.14) and the solid curve crosses the dashed curve

from below. If this the case, the unique interior equilibrium is the one which corresponds to the maximum $\tilde{\lambda}_G$. This is because the expected utility of the entrepreneur is increasing in $\tilde{\lambda}_G^*$, due to the impact of the latter on $g^*(S, \tilde{\lambda}_G^*)$.

We conclude this subsection with the case where there is no interior equilibrium. If it is very costly to invest in productivity, the only equilibrium in the investment stage is for $\tilde{\lambda}_G = 0$, for which the market collapses (Panel C). In contrast, if the cost of investing is very low, the unique equilibrium is for $\tilde{\lambda}_G = 1$ (Panel D). Lemma 5 presents the sufficient conditions for interior equilibrium to exist, where the payment $g^*(S, \tilde{\lambda}_G)$ is determined in Lemma 4.

Lemma 5: Interior and corner equilibrium - sufficient conditions

- *If there is no $c \in [\underline{c}, \bar{c}]$ which satisfies:*

$$c \leq (p_G - p_B)(S - g^*(S, \tilde{\lambda}_G = \Phi(c))) \tag{3.15}$$

  then, the unique equilibrium is for $\tilde{\lambda}_G = 0$, for which the market collapses.

- *If $\bar{c}$ satisfies:*

$$\bar{c} > (p_G - p_B)(S - g^*(S, \tilde{\lambda}_G = \Phi(\bar{c})) \tag{3.16}$$

  then, the unique equilibrium is for $\tilde{\lambda}_G = 1$.

- *If* (3.16) *is violated, and there exists $c \in [\underline{c}, \bar{c}]$ which satisfies* (3.15)*, then the unique interior equilibrium is characterized by the maximum $\tilde{\lambda}_G \in (0, 1)$*

which solves:

$$\Phi^{-1}(\tilde{\lambda}_G) = (p_G - p_B)(S - g^*(S, \tilde{\lambda}_G)) \tag{3.17}$$

Proof. See Appendix C. □

3.5.2 Optimal security when types are endogenously determined

In this subsection, we characterize the optimal security when the choice of project implementation is endogenous and the entrepreneur has to option to invest in her productivity level before financing takes place. In order to allow for an environment of asymmetric information between the entrepreneur and potential investors, we focus our analysis on the case where the type of the entrepreneur is uncertain, i.e., there exists an interior equilibrium in investment-in-productivity stage. The combination of Lemma 4 and Lemma 5, leads to Proposition 3.

Proposition 3

As long an interior equilibrium exists, the optimal security is given by:

$$g^*(F) = F$$

$$g^*(S, \tilde{\lambda}_G^*) = \frac{\tilde{\lambda}_G^*(I - F(1 - p_G)) + (1 - \tilde{\lambda}_G^*)p_B S}{\tilde{\lambda}_G^* p_G + (1 - \tilde{\lambda}_G^*)p_B}$$

$$\bar{g}^* = I - p_B(S - g^*(S, \tilde{\lambda}_G^*))$$

where $\tilde{\lambda}_G^*$ is the maximum $\tilde{\lambda}_G \in (0,1)$ which solves:

$$\Phi^{-1}(\tilde{\lambda}_G) = (p_G - p_B)(S - g^*(S, \tilde{\lambda}_G)) \tag{3.18}$$

For the sake of completeness we present in Proposition 4 the optimal security when the entrepreneur is obliged, by assumption, to the project implementation. Similarly to the previous case, we focus our analysis on the case where there is uncertainty about the type of the entrepreneur.

Proposition 4

As long as an interior equilibrium exists, the optimal security is given by:

$$g^*(F)' = F \ \ , \ \ g^*(S, \tilde{\lambda}_G^*)' = \frac{I - F}{\tilde{\lambda}_G^* p_G + (1 - \tilde{\lambda}_G^*) p_B} + F$$

where $\tilde{\lambda}_G^*$ is the the maximum $\tilde{\lambda}_G \in (0,1)$ which solves:

$$\Phi^{-1}(\tilde{\lambda}_G) = (p_G - p_B)(S - g^*(S, \tilde{\lambda}_G)') \tag{3.19}$$

Proof. See Appendix C. □

3.5.3 Impact of endogenizing project implementation on entrepreneur's productivity

In this subsection, we explore the impact of relaxing the assumption that the entrepreneur is obliged to project implementation on probability $\tilde{\lambda}_G$. This finding is summarized in Proposition 5.

Proposition 5:

Allowing for the project implementation to be endogenous and contractible, increases the probability that an entrepreneur invests in her productivity.

Proof. See Appendix C. □

The intuition behind this finding is straightforward: being productive is rewarded more when the project implementation is endogenous, due to the steeper incentive pay. Thus, for any given value of the cost c, the entrepreneur's incentive to invest in her productivity is stronger.

Figure 3.2 allows us to illustrate the main idea behind Proposition 5. Figure 3.2, similarly to Figure 3.1, represents the relationship between $\tilde{\lambda}_G$ (on the horizontal axis) and the benefit of investing in productivity. The dashed curve illustrates the benefit when the choice of project implementation is endogenous, whereas the dotted curve illustrates the benefit when the entrepreneur is obliged to the project implementation. Note that the dashed curve starts from the origin (excluding the origin point), highlighting the finding that the market survives as long as $\tilde{\lambda}_G > 0$. In contrast, the dotted curve starts from $\tilde{\lambda}_G^{min}$, which is the minimum value of $\tilde{\lambda}_G$ for which the market survives.[9] The dashed curve is always above the dotted curve, capturing the finding presented in Corollary 1, that for any given value of $\tilde{\lambda}_G$, $g^*(S, \tilde{\lambda}_G)' \geq g^*(S, \tilde{\lambda}_G)$.

Panels A to D illustrate four different distributions of c, where in each case, point "E" denotes the equilibrium when the project implementation is endogenous, whereas point "e" denotes the equilibrium when the entrepreneur is obliged to project implementation. We denote as $\tilde{\lambda}_G^*$ the

[9] $\tilde{\lambda}_G^{min} = \frac{I-[p_B S+(1-p_B)F]}{[(p_G S+(1-p_G)F]-[p_B S+(1-p_B)F]}$.

maximum value of $\tilde{\lambda}_G$ which solves (3.18) and as $\tilde{\lambda'}_G^*$ the maximum value of $\tilde{\lambda}_G$ which solves (3.19). Notice that only panels A and B represent environments where there is uncertainty about the type of the entrepreneur. The previous analysis gives rise to the following cases regarding the relationship between $\tilde{\lambda}_G^*$ and $\tilde{\lambda'}_G^*$.

- $1 > \tilde{\lambda}_G^* > \tilde{\lambda'}_G^* > \tilde{\lambda}_G^{min}$: *The probability that the entrepreneur invests in her productivity is higher when the project implementation is at the entrepreneur's discretion. (panel A).*

- $\tilde{\lambda}_G^* > \tilde{\lambda'}_G^* = 0$: *The market survives only in the environment where the choice of project implementation is endogenous. (panel B)*

- $\tilde{\lambda}_G^* = \tilde{\lambda'}_G^* = 0$: *The entrepreneur never invests in her productivity, and the market collapses independently of whether the choice of project implementation is endogenous (panel C).*

- $\tilde{\lambda}_G^* = \tilde{\lambda'}_G^* = 1$: *The entrepreneur invests in her productivity with certainty, independently of whether the choice of project implementation is endogenous (panel D).*

To conclude, we show in this section that allowing the entrepreneur to choose whether to implement her project, and to offer securities contingent on this choice, leads to a higher expected productivity. This finding has implications for the crowdfunding example. In particular, Proposition 5 suggests that allowing the entrepreneur to offer securities contingent on the implementation choice, is aligned with the objective of crowdfunding platforms is to attract productive entrepreneurs/projects.

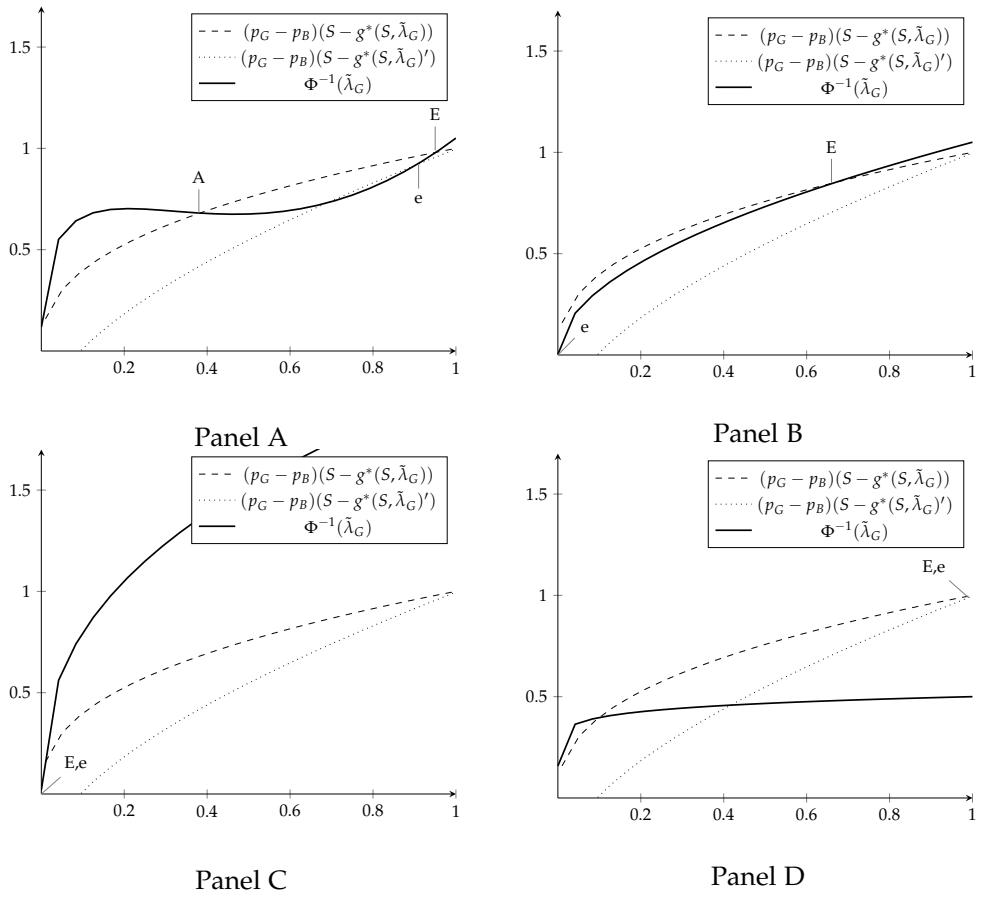


Figure 3.2. Equilibrium in the invest-in-productivity stage.

3.6 Conclusion and Further Discussion

In this paper, we develop a simple model of investment financing, where the entrepreneur shares the same characteristics as the representative entrepreneur in crowd-funding platforms. In particular, the entrepreneur has private information regarding her productivity, is protected by limited liability, and is associated with a negative-NPV project with positive probability.

The main message of this work is that, allowing the entrepreneur to

offer contracts contingent on the choice of project implementation, leads to a better allocation of resources, prevents market breakdown and strengthens the entrepreneur' incentive to invest in her productivity. Hence, this work indicates that crowd-funding platforms should promote the use of these securities.

The findings of this work can also be applied to the literature on venture capital financing. In venture capital financing, similar to our model, the entrepreneur is privately informed about the productivity of her project. A key difference between the two settings is that, in venture capital financing, it is the uninformed party (venture capitalist) who offers a security. This work suggests that, rewarding the non-implementation of a project, could prevent an entrepreneur from wasting the venture capitalist's resources in negative NPV projects.

Besides, Section 3.5 suggests that, by rewarding the non-implementation of a project, the venture capitalists would be able to offer a steeper incentive pay, in case of implementation. In turn, the steeper incentive pay could motivate the entrepreneur to undertake costly, hidden actions to increase the probability of success.

Lastly, our work has implication for the literature on the compensation contracts of CEOs, and more specifically, on the literature which highlights the optimality of severance pay. An insight of our paper is that allowing for a severance pay could be optimal, since it can prevent a CEO from taking an action which results in inefficient allocation of resources.

Appendix A

Motivating Information Acquisition Under Delegation

A.1 Imperfect State Realization

The previous analysis relies on the assumption that the state of the world is observable and contractible. There are many situations, however, where the state of the world is imperfectly revealed. For instance, suppose that the asset that the fund manager trades is a stock of Company "Z". In that case, rather than observing the realized quality of an asset, which in this example corresponds to the profitability of Company "Z", the equity holder and the manager might observe an imperfect public signal about the profitability of company "Z", such as earnings forecasts or credit rating announcements. Another case that the state of the world might not be contractible is when its realization takes place after the contract is terminated.

The goal of this section is two-fold. First, to derive the optimal compensation contract in an alternative setup where the payments can only be

contingent on an imperfect signal about the state of the world. Second, to examine whether the main features and implications of the optimal contract are similar to the ones in the benchmark model.

We explore an environment where neither the equity holder of a fund nor the fund manager observes the realized state of the world. Instead, we allow the principal and the agent to have access to a public signal which is revealed after the decision is taken, and imperfectly reveals the actual state of the world. In particular, we allow for a binary public signal i.e., $\sigma = \{b, g\}$, where:

$$Pr(\sigma = g|\theta = G) = q_G$$
$$Pr(\sigma = g|\theta = B) = q_B$$

with $q_G > q_B$. The intuition is straightforward; compared to the case where the state is $\theta = B$, when the state is $\theta = G$, it is more likely that the public signal will be $\sigma = g$. For instance, a good credit rating is more likely when Company "Z" is of good rather than bad quality. Likewise, compared to the case where the state is $\theta = G$, when the state is $\theta = B$, it is more likely that the public signal will be $\sigma = b$. Note that the benchmark model can be thought as a special case of the imperfect state realization case, if we restrict the values of q_G and q_B to be equal to one and zero, respectively.

The difference in the compensation contract is that instead of the payments being contingent on the realized state, θ, they are contingent on the realized public signal, σ, i.e., $\hat{W}' : d \times \sigma \mapsto \mathbb{R}^+$, where $d = \{S, L\}$ and

$\sigma = \{b, g\}$. The contract is characterized by the following quadruple:

$$\hat{W}' = \{w_{Sg}, w_{Sb}, w_{Lg}, w_{Lb}\}$$

In order to derive the optimal compensation contract, we follow the same four-step process we followed in the case where the state of the world is contractible.

A.1.1 Step 1: Decision Rule $\mathcal{DR}$

We show in Appendix A.1.3 that Lemma 1 extends to the case where the state of the world is imperfectly observed. Hence, similarly to the case where the state of the world is contractible, the decision rule that the equity holder aims to implement is given by a threshold $\hat{s}$, such as:

$$\mathcal{DR} = \begin{cases} long & \text{if } s > \hat{s} \\ short & \text{if } s < \hat{s} \end{cases}$$

Proof. See Lemma 1B in Appendix A.3. □

A.1.2 Step 2: Constrained-optimal compensation contract that implements $\hat{s}$

In this step, we derive the most cost-efficient contract of implementing a given threshold $\hat{s}$.

Proposition 1B: Constrained Optimal Contract

For $\hat{s} \in [0, \hat{s}_{min}]$ the constrained optimal contract is given by:

$$w^*_{Sb}(\hat{s}) = \frac{f_B(\hat{s})(1-p)q_B + f_G(\hat{s})pq_G}{(F_B(\hat{s})f_G(\hat{s}) - f_B(\hat{s})F_G(\hat{s}))(-1+p)p(q_B - q_G)}c$$

$$w^*_{Lg}(\hat{s}) = \frac{f_B(\hat{s})(-1+p)(-1+q_B) - f_G(\hat{s})p(-1+q_G)}{(F_B(\hat{s})f_G(\hat{s}) - f_B(\hat{s})F_G(\hat{s}))(-1+p)p(q_B - q_G)}c$$

$$w^*_{Lb}(\hat{s}) = w^*_{Sg}(\hat{s}) = 0.$$

For $\hat{s} \in [\hat{s}_{min}, 1]$ the constrained optimal contract is given by:

$$w^*_{Sb}(\hat{s}) = \frac{f_B(\hat{s})(-1+p)q_B - f_G(\hat{s})pq_G}{(f_B(\hat{s})(-1+F_G(\hat{s})) + f_G(\hat{s})(1-F_B(\hat{s})))(-1+p)p(q_B - q_G)}c$$

$$w^*_{Lg}(\hat{s}) = \frac{f_B(\hat{s})(-1+p)(1-q_B) + f_G(\hat{s})p(-1+q_G)}{((1-F_B(\hat{s}))f_G(\hat{s}) + f_B(\hat{s})(-1+F_G(\hat{s})))(-1+p)p(q_B - q_G)}$$

$$w^*_{Lb}(\hat{s}) = w^*_{Sg}(\hat{s}) = 0$$

where $\hat{s}_{min}$ solves $f_G(\hat{s}_{min}) = f_B(\hat{s}_{min})$.

Proof. See Appendix A.3. □

Proposition 1B is the analog of Proposition 1. The proof and the intuition is identical to ones in Proposition 1. Also, we show in Appendix A.3 that the relationship between the optimal payments and $\hat{s}$, which is captured in Lemma 2, extends to this setup. Lemma 3 also extends to this setup. Thus, $\mathbb{E}\, C(\hat{s})$ is U-shaped, and minimized for $f_G(\hat{s}) = f_B(\hat{s})$.

A.1.3 Step 3: Expected revenue $\mathbb{E}\, R(\hat{s})$

A critical remark is that the expected portfolio revenue of a given threshold $\hat{s}$ is determined by the actual state of the world, rather than the public

signal about the state of the world. Hence, the second step is identical to the corresponding step in the benchmark model. The behavior of $\mathbb{E}\,R(\hat{s})$ with respect to the implemented threshold $\hat{s}$ is given by Lemma 4 and is illustrated in Figure 1.3.

A.1.4 Step 4: Optimal Threshold $\hat{s}^*$

the optimal value of $\hat{s}$ denoted as $\hat{s}^*$ equates the expected marginal revenue ($\frac{\partial\,\mathbb{E}\,R(\hat{s})}{\partial\hat{s}}$) with the expected marginal cost ($\frac{\partial\,\mathbb{E}\,C(\hat{s})}{\partial\hat{s}}$), where $\mathbb{E}\,R(\hat{s})$ is defined in Lemma 2 whereas $\mathbb{E}\,C(\hat{s})$ is defined in Lemma 3B.

A.1.5 Optimal Compensation Contract

The optimal contract is given by the constrained optimal contract of Proposition 1B, subject to the constraint the $\hat{s}$ satisfies the optimality condition which is provided in Lemma A.4.

Proposition 2B: Optimality Contract, $\hat{W}^*(\hat{s}^*)$

If $p \geq 0.5$, the optimal value $\hat{s}$, $\hat{s}^*$ solves:

$$2\epsilon\{-pf_G(\hat{s}) + (1-p)f_B(\hat{s})\} = -\left[\frac{((-1+p)q_B - pq_G)((-1+p)(-1+q_B)F_B(\hat{s}) - p(-1+q_G)F_G(\hat{s})))\gamma(\hat{s})}{[F_B(\hat{s})f_G(\hat{s}) - F_G(\hat{s})f_B(\hat{s})(-1+p)p(q_B-q_G)]^2}\right]c \tag{A.1}$$

and the optimal payments are given by:

$$w^*_{Sb}(\hat{s}^*) = \frac{f_B(\hat{s}^*)(1-p)q_B + f_G(\hat{s}^*)pq_G}{(F_B(\hat{s}^*)f_G(\hat{s}^*) - f_B(\hat{s}^*)F_G(\hat{s}^*))(-1+p)p(q_B-q_G)}c$$

$$w^*_{Lg}(\hat{s}^*) = \frac{f_B(\hat{s}^*)(-1+p)(-1+q_B) - f_G(\hat{s}^*)p(-1+q_G)}{(F_B(\hat{s}^*)f_G(\hat{s}^*) - f_B(\hat{s}^*)F_G(\hat{s}^*))(-1+p)p(q_B - q_G)}c$$

$$w^*_{Lb}(\hat{s}^*) = w^*_{Sg}(\hat{s}^*) = 0$$

If $p \leq 0.5$, the optimal value $\hat{s}$, $\hat{s}^*$ solves:

$$2\epsilon\{-pf_G(\hat{s}) + (1-p)f_B(\hat{s})\} = \\ -\left[\frac{(1-(1-p)q_B - pq_G)((1-p)q_B(1-F_B(\hat{s})) + pq_G(1-F_G(\hat{s})))\gamma(\hat{s})}{[((-1+F_B(\hat{s})f_g(\hat{s}) + (1-F_G(\hat{s}))f_B(\hat{s})(-1+p)p(q_B - q_G))]^2}\right]c \tag{A.2}$$

and the optimal payments are given by:

$$w^*_{Sb}(\hat{s}^*) = \frac{f_B(\hat{s}^*)(-1+p)q_B - f_G(\hat{s}^*)pq_G}{(f_B(\hat{s}^*)(-1+F_G(\hat{s}^*)) + f_G(\hat{s}^*)(1-F_B(\hat{s}^*)))(-1+p)p(q_B - q_G)}c$$

$$w^*_{Lg}(\hat{s}^*) = \frac{f_B(\hat{s}^*)(-1+p)(1-q_B) + f_G(\hat{s}^*)p(-1+q_G)}{((1-F_B(\hat{s}^*))f_G(\hat{s}^*) + f_B(\hat{s}^*)(-1+F_G(\hat{s}^*)))(-1+p)p(q_B - q_G)}$$

where $\gamma(\hat{s}) \equiv (f_G(\hat{s})f'_B(\hat{s}) - f_B(\hat{s})f'_G(\hat{s})$.

Proof. See Appendix A.3. □

A.1.6 Bias in the investment decision

A crucial remark of the previous analysis is that $\mathbb{E}\,C(\hat{s})$ has similar behavior (U-shaped, and minimized for $f_G(\hat{s}) = f_B(\hat{s})$) independently of whether the principal and the agent observe the state of the world or a public signal which is informative about the state of the world. Recall also that $\mathbb{E}\,R(\hat{s})$ is unaffected by whether the state of the world or a public signal, is observed. Hence, the analysis of how the optimal threshold $\hat{s}^*$ relates to the

first best threshold $\hat{s}^{FB}$ is qualitatively the same with the benchmark model.

Proposition 4B: Bias in the investment decision.
Under the optimal contract:

(i) *For* $p \geq 0.5$, *then* $\hat{s}^{FB} < \hat{s}^* \leq \hat{s}_{min}$

(ii) *For* $p \leq 0.5$, *then* $\hat{s}^{FB} > \hat{s}^* \geq \hat{s}_{min}$

Proof. See Appendix C. □

The intuition is identical to the intuition of Proposition 4. Hence, the main finding that under the optimal contract, that the manager adopts contrarian actions more often than the first best, remains.

A.1.6.1 Impact on the investment decision

Proposition 7 explores the impact of of q_G and q_B the informativeness of the public signal, which is captured by q_G and q_B, on the investment decision and the emerging bias. Recall that q_G and q_B is the probability of observing a good public signal when the state is good and bad, respectively.

Proposition 7: Impact of q_G and q_B on investment decision.

(i) *If* $p > 0.5$, $\hat{s}^*$ *is decreasing in* q_B *and increasing in* q_G.

(ii) *If* $p < 0.5$, $\hat{s}^*$ *is increasing in* q_B *and decreasing in* q_G.

(iii) *The bias* $|\hat{s}^* - \hat{s}^{FB}|$ *is decreasing in* q_B *and increasing in* q_G.

Proof. See Appendix A.3. □

Proposition 7 relies on Lemma 6 and on the observation that the expected revenue of implementing $\hat{s}$ is independent of q_G or q_B. Thus, q_G and q_B affect $\hat{s}^*$ through their impact on expected cost $\mathbb{E}\,C(\hat{s})$. An implication of Lemma 6 is that an increase in q_G and/or a decrease in q_B moves the graphical illustration of $\mathbb{E}\,C(\hat{s})$ upwards, and leads to a steeper-sloped U-shape. The intuition behind Lemma 6 relies on the fact that as the difference $q_G - q_B$ increases, the quality of monitoring improves, and as a result, the equity holder can incentivize the manager to acquire information through lower payments. The intuition behind Proposition 7 is similar to the intuition behind Proposition 5. Namely, for a given deviation η from the first best $\hat{s}^{FB}$, a higher q_B and/or a lower q_G leads to a larger reduction in the expected cost, which in turn, strengthens the incentive to deviate. Proposition 6 is captured in Figure A.1 and A.1.

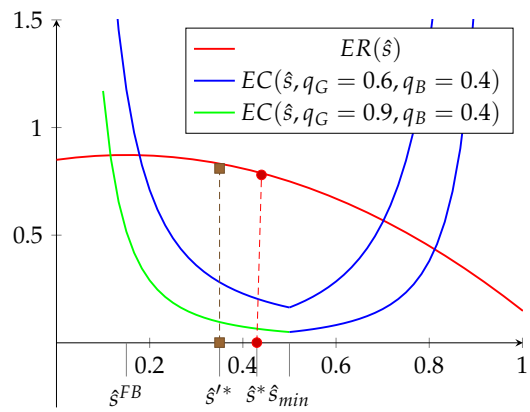


Figure A.1. Impact of higher q_G.

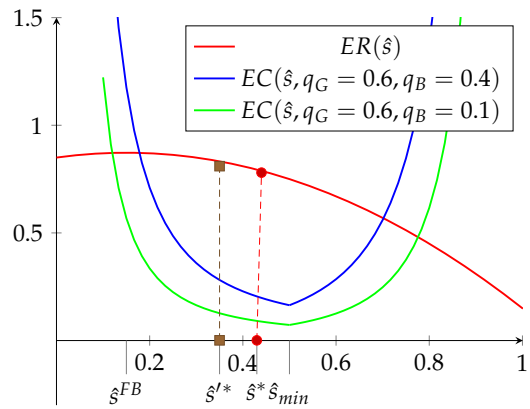


Figure A.2. Impact of lower q_B.

A.2 Allocating tasks to different agents

In Section 1.4, we argue that as long as $p \neq 0.5$, there is a trade-off because the threshold $\hat{s}$ which minimizes the cost of incentivizing the agent to acquire information differs from the threshold which maximizes the portfolio revenue. Two questions which arise naturally are whether allocating the tasks to two different individuals would increase equity holder's profits, and whether the optimal contract would exhibit the key feature of "bias against the flow".

To this end, we allow for two agents: an analyst and a manager, each of them performing one task. The analyst acquires information and sends a message to his manager. The manager receives the analyst's message and subsequently takes an investment decision. In this environment, the analyst's contract is contingent on the realized state of the world (θ) and his message, which we denote as $\tilde{s}$, i.e., $\hat{W}_A : \tilde{s} \times \theta \mapsto \mathbb{R}^+$. Regarding the manager's compensation contract, we can focus, without loss of generality, on contracts contingent on the analyst's message and the manager's

investment decision, i.e., $\hat{W_M} : \tilde{s} \times d \mapsto \mathbb{R}^+$.

Lemma 7: Equilibrium Messages.

Under the optimal contract, the analyst sends a message $\underline{s}$ if his private signal belongs in partition $[0, \hat{s}]$, and a message $\bar{s}$ if his private signal belongs in partition $(\hat{s}, 1]$.

Proof. Recall that what matters for taking the revenue-maximizing position is whether $\Pr(\theta = G|s)$ is greater or lower than one-half, given that the principal's best respond solely depends on that. Following the spirit of Lemma 1, for the incentives of the equity holder and the analyst to be aligned, it is sufficient for the equity holder to offer a contract where two messages are issued in equilibrium. Message $\underline{s}$ corresponds to partition $[0, \hat{s}]$, whereas message $\bar{s}$ corresponds to partition $(\hat{s}, 1]$, where $\hat{s} \in (0, 1)$. Notice that it is never optimal for the principal to offer a contract where more than two messages are issued in equilibrium; allowing for more messages would never lead to higher revenue, whereas it would increase the cost of incentivizing information acquisition. This is because more messages would imply a higher number of incentive compatibility constraints. □

Lemma 8 states that the analyst's optimal contract in this environment would coincide with the optimal contract of the manager in the benchmark model, which is characterized in Proposition 2.

Lemma 8: Analyst's optimal contract.

$w(\underline{s}, B)^* \equiv w_{SB}(\hat{s}^*)$, $w(\bar{s}, G)^* \equiv w_{LG}(\hat{s}^*)$, $w(\underline{s}, G)^* \equiv w_{SG}(\hat{s}^*)$ and $w(\bar{s}, B)^* \equiv$

$w_{LB}(\hat{s}^*)$

Proof. Lemma 8 is a consequence of Lemma 7. The main difference is that instead of incentivizing the manager to acquire information and follow an investment decision rule, which is characterized by a threshold $\hat{s}$, the principal incentivizes an analyst to acquire information and follow a disclosure rule, which is characterized by the same threshold $\hat{s}$. □

The main message of Lemma 8 is that the optimal compensation contract, and thus the cost of incentivizing an agent to acquire information, is the same independently of whether this agent is in charge of taking an investment decision. Thus, the feature of the "bias against the flow" emerges even when the principal allocates the tasks to two different individuals.
To complete the analysis of this environment, we should also characterize the optimal compensation contract of the manager. This contract is given by Lemma 9.

Lemma 9: Manager's optimal contract.
$w(\underline{s}, S)^* = w(\bar{s}, L)^* = \epsilon$ *and* $w(\underline{s}, L)^* = w(\bar{s}, S)^* = 0$, *where* $\epsilon \to 0$.

Proof. Since the manager has no private information, the incentives of the principal and the manager are aligned if the principal offers an arbitrarily small payment ϵ when the manager takes the revenue-maximizing position, and zero otherwise. □

To conclude, we show in this section that, as long as information acquisition is unobservable, delegating the tasks to different agents leads to the same investment decision, and it does not improve the profitability

of the equity holder: the equity holder also incurs the cost of incentivizing the manager to implement the revenue-maximizing investment decision.

A.3 Proofs

A.3.1 First Best

We start by analyzing the case where there is no agency problem, i.e., the equity holder is also the manager of the fund. First, we derive the expected profit of the equity holder when no information is acquired. Second, we derive the expected profit for the case where equity holder acquires information. Finally, we characterize the condition such as information acquisition is optimal.

Case where no information is acquired: In the absence of information acquisition the expected revenue of going short and long is given by:

$$\mathbb{E}\,\Pi[short] = p(P_0 - P_1^G) + (1-p)(P_0 - P_1^B) = (1-2p)\epsilon \tag{A.3}$$

$$\mathbb{E}\,\Pi[long] = p(P_1^G - P_0) + (1-p)(P_1^B - P_0) = (2p-1)\epsilon \tag{A.4}$$

Thus, short (long) is the revenue-maximizing action when the asset is considered overvalued (undervalued), i.e., when $p < 0.5$ ($p > 0.5$).

$$\mathbb{E}\,\Pi[no\ signal] = max\{(2p-1),(1-2p)\} \times \epsilon \tag{A.5}$$

Case where information is acquired: First, we characterize the optimal investment decision for each signal realization, s. The equity holder goes

long if:

$$\mathbb{E}\,\Pi[long|s] > \mathbb{E}\,\Pi[short|s] \implies$$
$$[Pr(G|s) - Pr(B|s)]\epsilon > [-Pr(G|s) + Pr(B|s)]\epsilon \tag{A.6}$$

whereas he goes short otherwise.[1] Since $Pr(G|s)$ is strictly increasing in s, there is a unique signal realization $s = \hat{s}^{FB}$ such that $E\Pi[long|\hat{s}^{FB}] = E\Pi[short|\hat{s}^{FB}]$. This is true for $\hat{s}^{FB}$ such that:

$$\frac{f_G(\hat{s}^{FB})}{f_B(\hat{s}^{FB})} = \frac{(1-p)}{p} \tag{A.7}$$

Hence, the expected profit of acquiring information is given by:

$$\mathbb{E}\,\Pi[signal|\hat{s}^{FB}] = \int_0^{\hat{s}^{FB}} \mathbb{E}\,\Pi[short|s]f(s)ds + \int_{\hat{s}^{FB}}^1 \mathbb{E}\,\Pi[long|s]f(s)ds - c \tag{A.8}$$

In section 1.4.4 we showed that the expected revenue of acquiring a signal, given a threshold $\hat{s}$, is:

$$\mathbb{E}\,R(\hat{s}) = \epsilon\{p(1-2F_G(\hat{s})) + (1-p)(2F_B(\hat{s}) - 1)\}$$

Thus, the expected profit of acquiring a signal, given the first best threshold $\hat{s}^{FB}$, is:

$$\mathbb{E}\,\Pi[signal|\hat{s}^{FB}] = \epsilon\{p(1-2F_G(\hat{s}^{FB})) + (1-p)(2F_B(\hat{s}^{FB}) - 1)\} - c \tag{A.9}$$

[1]The implicit assumption w.l.o.g. is that if the equity holder is indifferent, he goes short.

As a result, the equity holder acquires information as long as:

$$\mathbb{E}\,\Pi[signal|\hat{s}^{FB}] \geq \mathbb{E}\,\Pi[no\ signal]$$

which is satisfied for values of c such as:

$$c \leq [p(1-2F_G(\hat{s}^{FB})) + (1-p)(2F_B(\hat{s}^{FB})-1) - max\{(2p-1),(1-2p)\}] \times \epsilon \tag{A.10}$$

where the LHS of (A.10) captures the cost of acquiring information, whereas the RHS captures the expected benefit of acquiring information net of the equity holder's outside option.

A.3.2 Useful Lemmas

We provide Lemma A.1 and Lemma A.2, which are going to be useful for the remaining proofs.

A.3.2.1 Lemma A.1

Lemma A.1: *For each $\hat{s} \in S$, the following relation holds:*

$$\gamma_1 \equiv f'_B(\hat{s})f_G(\hat{s}) - f_B(\hat{s})f'_G(\hat{s}) < 0$$

Proof. Crucial for the determination of the sign of γ_1 is $f'_G(\hat{s})$ and $f'_B(\hat{s})$, where $f'_G(\hat{s}) = \frac{\partial f_G(\hat{s})}{\partial \hat{s}}$ and $f'_B(\hat{s}) = \frac{\partial f_B(\hat{s})}{\partial \hat{s}}$. Although we have made no assumption about $f'_G(\hat{s})$ and $f'_B(\hat{s})$, they are indirectly constrained by MLRP. For instance, a direct consequence of MLRP is that $f'_G(\hat{s}) > f'_B(\hat{s})$, otherwise $\frac{f_G(\hat{s})}{f_B(\hat{s})}$ is decreasing in $\hat{s}$. Thus, we explore the sign of γ_1 for all possible rela-

tions between $f'_G(\hat{s})$ and $f'_B(\hat{s})$, by taking into consideration the constraints imposed by MLRP.

(i) Case 1: $f'_G(\hat{s}) > f'_B(\hat{s}) > 0$

For this relation, $\gamma_1 < 0$, since $f_B(\hat{s}) > f_G(\hat{s})$ and $f'_G(\hat{s}) > f'_B(\hat{s}) > 0$.

(ii) Case 2: $f'_G(\hat{s}) > 0 > f'_B(\hat{s})$

For this relation, $\gamma_1 < 0$, since its first term is negative and its second term is positive.

(iii) Case 3: $0 > f'_G(\hat{s}) > f'_B(\hat{s})$

For this relation, $\gamma_1 < 0$. The proof is less straightforward since the sign of γ_1 depends on the relation between ratio $\frac{f'_G(\hat{s})}{f_G(\hat{s})}$ and $\frac{f'_B(\hat{s})}{f_B(\hat{s})}$. We show that $\gamma_1 < 0$ by following the method of contradiction. Suppose that $\gamma_1 > 0$, i.e.,

$$f'_B(\hat{s})f_G(\hat{s}) - f_B(\hat{s})f'_G(\hat{s}) > 0 \tag{A.11}$$

By dividing the LHS of (A.11) by $f_G(\hat{s})f_B(\hat{s})$ and rearranging, we obtain:

$$\frac{f'_B(\hat{s})}{f_B(\hat{s})} > \frac{f'_G(\hat{s})}{f_G(\hat{s})} \tag{A.12}$$

where $f'_B(\hat{s}) = \lim_{h\to 0} \frac{f_B(\hat{s}_0+h)-f_B(\hat{s}_0)}{h}$ and $f'_G(\hat{s}) = \lim_{h\to 0} \frac{f_G(\hat{s}_0+h)-f_G(\hat{s}_0)}{h}$.

Hence, (A.12) implies the following relation:

$$\frac{\lim_{h\to 0} \frac{f_B(\hat{s}_0+h)-f_B(\hat{s}_0)}{h}}{f_B(\hat{s}_0)} > \frac{\lim_{h\to 0} \frac{f_G(\hat{s}_0+h)-f_G(\hat{s}_0)}{h}}{f_G(\hat{s}_0)} \tag{A.13}$$

Thus, relation (A.13) implies that the percentage decrease in the $f_B(\hat{s})$ as $\hat{s}$ moves from $\hat{s}_0$ to $\hat{s}_1 = \hat{s}_0 + h$ is smaller than the percentage de-

crease in $f_G(\hat{s})$ as $\hat{s}$ moves from $\hat{s}_0$ to $\hat{s}_1 = \hat{s}_0 + h$. If this is true, then relation (A.13) gives rise to following relation

$$\frac{f_G(\hat{s}_0)}{f_B(\hat{s}_0)} > \frac{f_G(\hat{s}_1)}{f_B(\hat{s}_1)} \tag{A.14}$$

However, relation (A.14) contradicts with the MLRP assumption, according to which, $\frac{f_G(\hat{s})}{f_B(\hat{s})}$ is increasing in $\hat{s}$. Hence, $\gamma_1 < 0$.

Finally, for the sake of completeness, we list the remaining cases of the relationship between $f'_G(\hat{s})$ and $f'_B(\hat{s})$, which are irrelevant, since they violate the MLRP assumption.

(iv) Case 4: $f'_B(\hat{s}) > f'_G(\hat{s}) > 0$

The MLRP is violated since for this relation, the ratio $\frac{f_G(\hat{s})}{f_B(\hat{s})}$ is decreasing in $\hat{s}$.

(v) Case 5: $f'_B(\hat{s}) > 0 > f'_G(\hat{s})$

The MLRP is violated since for this relation, the ratio $\frac{f_G(\hat{s})}{f_B(\hat{s})}$ is decreasing in $\hat{s}$.

(vi) Case 6: $0 > f'_B(\hat{s}) > f'_G(\hat{s})$

The MLRP is violated since for this relation, the ratio $\frac{f_G(\hat{s})}{f_B(\hat{s})}$ is decreasing in $\hat{s}$

Thus, for the relevant cases (cases i - iii), γ_1 is negative independently of the value of $\hat{s}$.

□

A.3.2.2 Lemma A.2

Lemma A.2: *The monotone likelihood ratio property assumption implies that:*

$$\frac{1 - F_G(\hat{s})}{1 - F_B(\hat{s})} \geq \frac{f_G(\hat{s})}{f_B(\hat{s})} \geq \frac{F_G(\hat{s})}{F_B(\hat{s})}.$$

Proof. By the definition of the MLRP, for $s_1 \geq s_0$, the following condition holds:

$$\frac{f_G(s_1)}{f_B(s_1)} \geq \frac{f_G(s_0)}{f_B(s_0)}$$

which can be rearranged as:

$$f_G(s_1)f_B(s_0) \geq f_G(s_0)f_B(s_1) \tag{A.15}$$

Integrating both sides of (A.15) over s_0 from the lower bound of the distribution to s_1:

$$\int_0^{s_1} f_G(s_1)f_B(s_0)ds_0 \geq \int_0^{s_1} f_G(s_1)f_B(s_0)ds_0$$

which leads to:

$$\frac{f_G(s_1)}{f_B(s_1)} \geq \frac{F_G(s_1)}{F_B(s_1)}. \tag{A.16}$$

Next, we integrate both sides of (A.15) over s_1 from s_0 to the upper bound of the distribution:

$$\int_{s_0}^{1} f_G(s_1)f_B(s_0)ds_0 \geq \int_{s_0}^{1} f_G(s_1)f_B(s_0)ds_0$$

which leads to:

$$\frac{f_G(s_0)}{f_B(s_0)} \leq \frac{1 - F_G(s_0)}{1 - F_B(s_0)}. \tag{A.17}$$

Notice that s_0 and s_1 in (A.16) and (A.17) are arbitrary. Thus, combining these equation bu letting $s_0 = s_1 = \hat{s}$, we obtain:

$$\frac{1 - F_G(\hat{s})}{1 - F_B(\hat{s})} \geq \frac{f_G(\hat{s})}{f_B(\hat{s})} \geq \frac{F_G(\hat{s})}{F_B(\hat{s})}.$$

□

A.3.3 Benchmark Model

A.3.3.1 Proof of Lemma 1 (Monotonic investment rule)

The proof of Lemma 1 is identical to the proof of Lemma 1B once we set $q_G = 1$, $q_B = 0$, and we replace w_{Sb} with w_{SB}, w_{Sg} with w_{SG}, w_{Lb} with w_{LB}, and w_{Lg} with w_{LB}.

A.3.3.2 Proof of Proposition 1

The proof of Proposition 1 is identical to the proof of Proposition 1B once we set $q_G = 1$, $q_B = 0$, and we replace w_{Sb} with w_{SB}, w_{Sg} with w_{SG}, w_{Lb} with w_{LB}, and w_{Lg} with w_{LB}.

A.3.3.3 Proof of Lemma 2

The proof of Lemma 2 is identical to the proof of Lemma 2B once we set $q_G = 1$, $q_B = 0$, and we replace w_{Sb} with w_{SB}, w_{Sg} with w_{SG}, w_{Lb} with w_{LB}, and w_{Lg} with w_{LB}.

A.3.3.4 Proof of Lemma 4

In section 4.4 we showed that the expected revenue from acquiring a signal, given a threshold $\hat{s}$, is:

$$\mathbb{E}\, R(\hat{s}) = \epsilon\{p(1 - 2F_G(\hat{s})) + (1-p)(2F_B(\hat{s}) - 1)\}.$$

Thus, the first order condition is given by:

$$\frac{\partial\, \mathbb{E}\, R(\hat{s})}{\partial(\hat{s})} = 2\epsilon\{f_B(\hat{s})(1-p) - f_G(\hat{s})p\} = 0 \tag{A.18}$$

which is satisfied for a unique (due to MLRP) $\hat{s} = \hat{s}^{FB}$, which solves:

$$\frac{f_G(\hat{s})}{f_B(\hat{s})} = \frac{(1-p)}{p}. \tag{A.19}$$

The second derivative of $\mathbb{E}\, R(\hat{s})$ is given by:

$$\frac{\partial^2\, \mathbb{E}\, R(\hat{s})}{\partial(\hat{s})^2} = f_B'(\hat{s})(1-p) - f_G'(\hat{s})p. \tag{A.20}$$

For the second order condition to be satisfied, it must hold:

$$\left.\frac{\partial^2 ER(\hat{s})}{\partial(\hat{s})^2}\right|_{(1-p)=\frac{f_G(\hat{s}^{FB})p}{f_B(\hat{s}^{FB})}} = p\left[\frac{f_B'(\hat{s}^{FB})f_G(\hat{s}^{FB})}{f_B(\hat{s}^{FB})} - f_G'(\hat{s}^{FB})\right] < 0 \tag{A.21}$$

which implies that the following should hold:

$$f_B'(\hat{s}^{FB})f_G'(\hat{s}^{FB}) - f_G'(\hat{s}^{FB})f_B(\hat{s}^{FB}) < 0. \tag{A.22}$$

Recall that in Lemma A.1 we proved that $f_B'(\hat{s})f_G'(\hat{s}) - f_G'(\hat{s})f_B(\hat{s}) < 0$ for

any $\hat{s}$. Thus, (A.22) holds.

We now explore the behaviour of $\mathbb{E}\, R(\hat{s})$ for $\hat{s} > \hat{s}^{FB}$ and $\hat{s} < \hat{s}^{FB}$. We start by analysing the case where $\hat{s}' > \hat{s}^{FB}$. For $\hat{s}' > \hat{s}^{FB}$, and given (A.19), the MLRP implies that:

$$\frac{f_G(\hat{s}')}{f_B(\hat{s}')} > \frac{f_G(\hat{s}^{FB})}{f_B(\hat{s}^{FB})} = \frac{(1-p)}{p}.$$

Thus, $\frac{f_G(\hat{s}')}{f_B(\hat{s}')}$ can be expressed as:

$$\frac{f_G(\hat{s}')}{f_B(\hat{s}')} = \frac{(1-p)}{p} + \eta(\hat{s}') \tag{A.23}$$

where $\eta(\hat{s}') > 0$ and increasing in $\hat{s}'$ by the MLRP. Also, by rearranging the first derivative of the expected revenue with respect to $\hat{s}$, we obtain:

$$\frac{\partial\, \mathbb{E}\, R(\hat{s}')}{\partial(\hat{s}')} / f_B(\hat{s}) = \epsilon\{(1-p) - \frac{f_G(\hat{s}')}{f_B(\hat{s}')} p\} \tag{A.24}$$

and, by substituting (A.23) into (A.24):

$$\frac{\partial\, \mathbb{E}\, R(\hat{s}')}{\partial(\hat{s}')} / f_B(\hat{s}) = \epsilon\{(1-p) - (\frac{(1-p)}{p} + \eta(\hat{s}'))p\} = -\epsilon\eta(\hat{s}')p < 0. \tag{A.25}$$

Hence, for any $\hat{s}' > \hat{s}^{FB}$, $\mathbb{E}\, R(\hat{s})$ is decreasing and concave in $\hat{s}'$, since $\eta(\hat{s}') > 0$ and increasing in $\hat{s}'$.

Second, we explore the case where $\hat{s} < \hat{s}^{FB}$. For $\hat{s}' > \hat{s}^{FB}$, the MLRP implies

that:

$$\frac{f_G(\hat{s}')}{f_B(\hat{s}')} < \frac{f_G(\hat{s}^{FB})}{f_B(\hat{s}^{FB})} = \frac{(1-p)}{p}$$

where $\frac{f_G(\hat{s}')}{f_B(\hat{s}')}$ can be expressed as:

$$\frac{f_G(\hat{s}')}{f_B(\hat{s}')} = \frac{(1-p)}{p} - \eta'(\hat{s}') \tag{A.26}$$

where $\eta'(\hat{s}') > 0$ and decreasing in $\hat{s}'$ by the MLRP. Also, by substituting (A.26) into (A.24):

$$\frac{\partial \mathbb{E}\, R(\hat{s}')}{\partial(\hat{s}')} / f_B(\hat{s}) = \epsilon\{(1-p) - (\frac{(1-p)}{p} - \eta'(\hat{s}'))p\} = \epsilon\eta'(\hat{s}')p > 0. \tag{A.27}$$

Hence, for any $\hat{s}' > \hat{s}^{FB}$, $\mathbb{E}\, R(\hat{s})$ is increasing and concave in $\hat{s}$ since $\eta'(\hat{s}') > 0$ and decreasing in $\hat{s}'$.

A.3.3.5 Proof of Proposition 2

The proof of Proposition 2 is identical to the proof of Proposition 2B once we set $q_G = 1$, $q_B = 0$, and we replace w_{Sb} with w_{SB}, w_{Sg} with w_{SG}, w_{Lb} with w_{LB}, and w_{Lg} with w_{LB}.

A.3.3.6 Proof of Proposition 3

Notice that by (1.13), the ratio of the payments is given by:

$$\frac{w_{SB}(\hat{s}^*)}{w_{LG}(\hat{s}^*)} = \frac{\frac{f_G(\hat{s}^*)}{f_B(\hat{s}^*)}}{\frac{1-p}{p}}. \tag{A.28}$$

In addition, under the first best, the following condition should hold:

$$\frac{f_G(\hat{s}^{FB})}{f_B(\hat{s}^{FB})} = \frac{1-p}{p}$$

Part one: By Proposition 4, if $p > 0.5$, $\hat{s}^* > \hat{s}^{FB}$, which combined with MLRP implies:

$$\frac{f_G(\hat{s}^*)}{f_B(\hat{s}^*)} > \frac{1-p}{p} \equiv \frac{f_G(\hat{s}^{FB})}{f_B(\hat{s}^{FB})}$$

Thus,

$$\frac{w_{SB}(\hat{s}^*)}{w_{LG}(\hat{s}^*)} > 1 \implies w_{SB}(\hat{s}^*) > w_{LG}(\hat{s}^*)$$

Part two: By Proposition 4, if $p < 0.5$, $\hat{s}^* < \hat{s}^{FB}$, which combined with MLRP implies:

$$\frac{f_G(\hat{s}^*)}{f_B(\hat{s}^*)} < \frac{1-p}{p} \equiv \frac{f_G(\hat{s}^{FB})}{f_B(\hat{s}^{FB})}$$

Thus,

$$\frac{w_{SB}(\hat{s}^*)}{w_{LG}(\hat{s}^*)} < 1 \implies w_{SB}(\hat{s}^*) < w_{LG}(\hat{s}^*)$$

A.3.3.7 Proof of Proposition 4

The proof of Proposition 4 is identical to the proof of Proposition 4B once we set $q_G = 1$, $q_B = 0$, and we replace w_{Sb} with w_{SB}, w_{Sg} with w_{SG}, w_{Lb} with w_{LB}, and w_{Lg} with w_{LB}.

A.3.3.8 Proof of Corollary 2

We focus on the case where $p > 0.5$. Similar intuition applies for the case where $p < 0.5$. Recall that by Proposition 4, for $p > 0.5$, $\hat{s}^* > \hat{s}^{FB}$, whereas for $p < 0.5$, $\hat{s}^* < \hat{s}^{FB}$.

Part one: Recall that for a given $\hat{s}$, the expected probability that a short position is chosen, is:

$$\begin{aligned} Pr(short|\hat{s}) &= pPr(s \leq \hat{s}|G) + (1-p)Pr(s \leq \hat{s}|B) \\ &= pF_G(\hat{s}) + (1-p)F_B(\hat{s}) \end{aligned}$$

where $F_G(\hat{s})$ and $F_B(\hat{s})$ are increasing in $\hat{s}$. Thus, as $\hat{s}^* > \hat{s}^{FB}$,

$$Pr(short|\hat{s} = \hat{s}^*) > Pr(short|\hat{s} = \hat{s}^{FB})$$

Part two: The probability that a short position is revenue-maximizing is given by:

$$Pr(B|short) = \frac{Pr(short|B)(1-p)}{Pr(short|B)(1-p) + Pr(short|G)p} = \frac{F_B(\hat{s})(1-p)}{F_B(\hat{s})(1-p) + F_G(\hat{s})p}$$

Differentiating $Pr(B|short)$ with respect to $\hat{s}$, we obtain:

$$\frac{\partial Pr(B|short)}{\partial \hat{s}} = \frac{(-1+p)p[F_B(\hat{s})f_G(\hat{s}) - f_B(\hat{s})F_G(\hat{s})]}{[F_B(\hat{s})(-1+p) - F_G(\hat{s})p]^2}$$

where, by Lemma A.1, is negative for any value of $\hat{s}$. Recall that, if $p > 0.5$, $\hat{s}^* > \hat{s}^{FB}$, thus:

$$Pr(B|short, \hat{s} = \hat{s}^*) < Pr(B|short, \hat{s} = \hat{s}^{FB}) \tag{A.29}$$

whereas, if $p < 0.5$, $\hat{s}^* < \hat{s}^{FB}$, thus:

$$Pr(B|short, \hat{s} = \hat{s}^*) > Pr(B|short, \hat{s} = \hat{s}^{FB}) \tag{A.30}$$

Part three: The probability that a long position is revenue-maximizing, is given by:

$$\begin{aligned} Pr(G|long) &= \frac{Pr(long|G)p}{Pr(long|G)p + Pr(long|B)(1-p)} \\ &= \frac{(1-F_G(\hat{s}))p}{(1-F_G(\hat{s}))p + (1-F_B(\hat{s}))(1-p)} \end{aligned} \tag{A.31}$$

Differentiating $Pr(G|long)$ with respect to $\hat{s}$, we obtain:

$$\frac{\partial Pr(G|long)}{\partial \hat{s}} = \frac{(-1+p)p[(1-F_B(\hat{s}))f_G(\hat{s}) - f_B(\hat{s})(1-F_G(\hat{s}))]}{[F_B(\hat{s})(-1+p) - F_G(\hat{s})p + 1]^2}$$

where, by Lemma A.1, it is positive for any value of $\hat{s}$. Recall that, if $p > 0.5$, $\hat{s}^* > \hat{s}^{FB}$, thus:

$$Pr(G|long, \hat{s} = \hat{s}^*) > Pr(G|long, \hat{s} = \hat{s}^{FB}) \tag{A.32}$$

whereas, if $p < 0.5$, $\hat{s}^* < \hat{s}^{FB}$, thus:

$$Pr(G|long, \hat{s} = \hat{s}^*) < Pr(G|long, \hat{s} = \hat{s}^{FB}) \tag{A.33}$$

Part four: This part is a direct consequence of part two and three, and in particular, of relations (A.29) and (A.32) if $p > 0.5$, and (A.30) and (A.33) for $p < 0.5$.

A.3.3.9 Proof of Proposition 5

Part one: In this part we show that if $p > 0.5$ ($p < 0.5$), an increase (decrease) in the cost of acquiring information from c to c' increases (decreases) the optimal threshold from $\hat{s}^*$ to $\hat{s}'^*$. Similar intuition applies for

the case where $p < 0.5$. First, we explore the case where the cost of acquiring information is c. Given that the principal chooses $\hat{s}^*$, it must be the case that his expected profit of implementing $\hat{s}'^*$ is not higher than the expected profit of implementing $\hat{s}^*$, i.e.,

$$\mathbb{E}\,\Pi(\hat{s}'^*) \leq \mathbb{E}\,\Pi(\hat{s}^*) \implies$$
$$\underbrace{\mathbb{E}\,R(\hat{s}'^*) - \mathbb{E}\,R(\hat{s}^*)}_{DR} \leq \underbrace{\mathbb{E}\,C'(\hat{s}'^*) - \mathbb{E}\,C'(\hat{s}^*)}_{DC'} \tag{A.34}$$

where $\mathbb{E}\,C'(.)$ denotes the expected compensation *before* the increase in the cost of acquiring information, and $\mathbb{E}\,R(.)$ denotes the expected portfolio revenue.

Second, we explore the case where the cost of acquiring information is $c' > c$. Given that the principal chooses $\hat{s}'^*$, it must be the case that his expected profit of implementing $\hat{s}'^*$ is higher than the expected profit of implementing $\hat{s}^*$, i.e.,

$$\mathbb{E}\,\Pi(\hat{s}'^*) \geq \mathbb{E}\,\Pi(\hat{s}^*) \implies$$
$$\underbrace{\mathbb{E}\,R(\hat{s}'^*) - \mathbb{E}\,R(\hat{s}^*)}_{DR} \geq \underbrace{\mathbb{E}\,C''(\hat{s}'^*) - \mathbb{E}\,C''(\hat{s}^*)}_{DC''}. \tag{A.35}$$

where $\mathbb{E}\,C'(.)$ denotes the expected compensation *after* the increase in the cost of acquiring information. We prove part one by the method of contradiction. Suppose that $\hat{s}'^* < \hat{s}^*$. Then, by Lemma 2, $DR > 0$ and by Lemma 4, $DC' > 0$ and $DC > 0$. Relations (A.34) and (A.35) are not mutually exclusive as long as:

$$DC \geq DR \geq DC' \implies$$

$$\mathbb{E}\, C'(\hat{s}'^*) - \mathbb{E}\, C'(\hat{s}^*) \geq \mathbb{E}\, C''(\hat{s}'^*) - \mathbb{E}\, C''(\hat{s}^*). \tag{A.36}$$

Notice that (A.36) never holds because by $\mathbb{E}\, C(\hat{s})$ is convex in $\hat{s}$ and linearly dependent on the cost of acquiring information (Lemma 3). Thus, an increase in c leads to a steeper-sloped U-shape of $\mathbb{E}\, C(\hat{s})$. As a result, (A.34) and (A.35) are mutually exclusive, hence, the the initial hypothesis that $\hat{s}'^* < \hat{s}^*$ is not valid. Thus, $\hat{s}'^* > \hat{s}^*$.

Part two: Part one implies that as c increases, $\hat{s}^*$ moves towards $\hat{s}_{min}$. Recall that $\hat{s}^{FB}$ is not related to c. Thus, the bias $|\hat{s}^* - \hat{s}^{FB}|$ is increasing in c.

Part three and four: Part three and four are a direct implication of part one. The proof of part three and four is similar to the proof of part two and three of Corollary 2.

Part five: The expected precision is denoted as $\Phi(\hat{s})$, where:

$$\begin{aligned}\Phi(\hat{s}) &= Pr(short|\hat{s})Pr(B|short) + Pr(long|\hat{s})Pr(G|long) \\ &= F_B(\hat{s})(1-p) + (1-F_G(\hat{s}))p \end{aligned} \tag{A.37}$$

Recall that the expected portfolio return is given by:

$$\mathbb{E}\, R(\hat{s}) = \epsilon\{p(1-2F_G(\hat{s})) + (1-p)(2F_B(\hat{s})-1)\}.$$

Thus, we can express $\Phi(\hat{s})$ as a function $\mathbb{E}\, R(\hat{s})$,

$$\Phi(\hat{s}) = \frac{\mathbb{E}\, R(\hat{s})}{2\epsilon} + p. \tag{A.38}$$

Thus, an increase in c increases the bias $|\hat{s}^* - \hat{s}^{FB}|$, which corresponds to lower $\mathbb{E}\, R(\hat{s})$ and $\Phi(\hat{s})$.

A.3.3.10 Proof of Proposition 6

Part one: Note that $\mathbb{E}\, C(\hat{s})$ is not a function of p. Thus, only $\mathbb{E}\, R(\hat{s})$ is affected by a change in p. Lemma A.3 explores the impact of a change in p on $\mathbb{E}\, R(\hat{s})$.

Lemma A.3: Relationship between p and $\mathbb{E}\, R(\hat{s})$.

(i) $\frac{\partial\, \mathbb{E}\, R(\hat{s})}{\partial \hat{s}}$ *is decreasing in* p.

(ii) $\frac{\partial^2\, \mathbb{E}\, R(\hat{s})}{\partial \hat{s}^2}$ *is not a function of* p *when the signaling structure is linear.*

Proof.

$$\frac{\partial\, \mathbb{E}\, R(\hat{s})}{\partial \hat{s}} = 2\epsilon\{-p f_G(\hat{s}) + (1-p) f_B(\hat{s})\} < 0$$

$$\frac{\partial(\frac{\partial\, \mathbb{E}\, R(\hat{s})}{\partial \hat{s}})}{\partial p} = 2\epsilon\{-f_G(\hat{s}) - f_B(\hat{s})\} < 0$$

since both $f_G(\hat{s})$ and $f_B(\hat{s})$ are positive.

$$\frac{\partial^2\, \mathbb{E}\, R(\hat{s})}{\partial \hat{s}^2} = 2\epsilon\{-p(f'_G(\hat{s}) + f'_B(\hat{s})) + f'_B(\hat{s})\}$$

Note that for linear signaling structure, it holds that $f'_G(\hat{s}) + f'_B(\hat{s}) = 0$. $\square$

Coming back to the proof of the first part of Proposition 6, we denote $\frac{\partial \mathbb{E}\, R(\hat{s})}{\partial \hat{s}}$ as $\mathbb{E}\, MR(\hat{s})$, and $\frac{\partial \mathbb{E}\, C(\hat{s})}{\partial \hat{s}}$ as $\mathbb{E}\, MC(\hat{s})$. Given that $\mathbb{E}\, MR(\hat{s})$ decreases in p, the intersection of $\mathbb{E}\, MR(\hat{s})$ with $\mathbb{E}\, MC(\hat{s})$, which defines the equilibrium value $\hat{s}^*$, moves towards lower values of $\hat{s}$ as p increases.

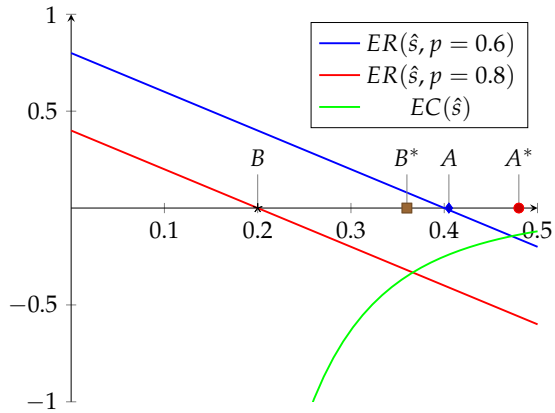


Figure A.3. Impact of more extreme prior beliefs

Part two: From Proposition 4, we know that if $p > 0.5$, $\hat{s}^{FB} < \hat{s}^* \leq \hat{s}_{min}$. For those values of $\hat{s}^*$, and conditional on a linear signaling structure, the following four properties hold: i) $\mathbb{E}\, MR(\hat{s})$ is linear in $\hat{s} \in [0, 1]$, ii) the slope of $\mathbb{E}\, MR(\hat{s})$ is independent of p iii) $\mathbb{E}\, MR(\hat{s})$ is negative for $\hat{s} > \hat{s}^{FB}$ and positive otherwise, iv) $\mathbb{E}\, MC(\hat{s})$ is increasing, and v) concave for $\hat{s} \in [0, \hat{s}_{min}]$. These properties are captured in the Figure **??**, where the blue and red line represent $\mathbb{E}\, MR(\hat{s})$ before and after the increase in p, whereas the red curve depicts the $\mathbb{E}\, MC(\hat{s})$. Point A (B) denotes the first best $\hat{s}$ before (after) the increase p, whereas point A^* (B^*) denotes the equilibrium value of $\hat{s}$ before (after) the increase p. Thus, the distance AA^* is the bias before

the increase in p, whereas the distance BB^* is the bias after the increase in p. Given that the aforementioned properties hold, it is always the case that the distance BB^* is greater than the distance AA^*.

A more intuitive approach is that for a given deviation η from the first best, the drop in $\mathbb{E}\,R(\hat{s})$, is independent of the value of $\hat{s}^{FB}$, thus the value of p, whereas the drop in $\mathbb{E}\,C(\hat{s})$ is decreasing in p. Hence, the incentive to deviate is stronger, the lower the value of p is.

A.3.4 Imperfect State Realization Case

A.3.4.1 Proof of Lemma 1B (Monotonic investment rule)

The expected profit of the equity holder when the manager goes short or long, given that the latter observes a signal s, is given by:

$$\mathbb{E}\,\Pi[short|s] =$$
$$Pr(G|s)(-\epsilon - w_{Sg}q_G - w_{Sb}(1-q_G)) + Pr(B|s)(\epsilon - w_{Sb}(1-q_B) - w_{Sg}q_B)$$
$$\mathbb{E}\,\Pi[long|s] =$$
$$Pr(G|s)(\epsilon - w_{Lg}q_G - w_{Lb}(1-q_B)) + Pr(B|s)(-\epsilon - w_{Lg}q_B - w_{Lb}(1-q_B))$$

given that $Pr(B|s) = 1 - Pr(G|s)$, we can rearrange the previous relations as:

$$\mathbb{E}\,\Pi[short|s] =$$
$$\underbrace{(-2\epsilon - w_{Sg}(q_G - q_B) + w_{Sb}(q_G - q_B)}_{A_S}\,Pr(G|s) + \underbrace{(\epsilon - w_{Sb}(1-q_B) - w_{Sg}q_B)}_{B_S}$$
$$\mathbb{E}\,\Pi[long|s] =$$

$$\underbrace{(2\epsilon - w_{Lg}(q_G - q_B) + w_{Lb}(q_G - q_B))}_{A_L} Pr(G|s) + \underbrace{(-\epsilon - w_{Lb}(1 - q_B) - w_{Lg}q_B)}_{B_L}$$

Suppose now that for $s = s'$ the principal prefers the agent to go long. Then, it must hold:

$$\mathbb{E}\,\Pi[long|s'] \geq \mathbb{E}\,\Pi[short|s'] \implies$$
$$A_L Pr(G|s') + B_L \geq A_S Pr(G|s') + B_S \tag{A.39}$$

where,

$$A_L - A_S = 4\epsilon - w_{Lg}(q_G - q_B) + w_{Lb}(q_G - q_B) + w_{Sg}(q_G - q_B) - w_{Sb}(q_G - q_B)$$
$$B_S - B_L = 2\epsilon - w_{Sb}(1 - q_B) + w_{Lb}(1 - q_B) - w_{Sg}q_B + w_{Lg}q_B \tag{A.40}$$

A useful remark is that the principal's net gain of taking the right position over taking the wrong position is 2ϵ.[2] Hence, the maximum value that a payment can reach is 2ϵ. A payment higher than 2ϵ would imply negative profits for the principal, as a result, the principal would not find profitable to incentivize the agent to acquire information. This remark implies that even in the extreme case where w_{Lb} and w_{Lg} reach their minimum value and w_{Sb} and w_{Sg} reach their maximum values, $B_S - B_L$ is positive. Hence, under the optimal contract, $B_S - B_L$, is always positive. Following the same

[2]If the realized state is good and the manager goes long, equity holder's net return is $P_1^G - P_0 = \epsilon$. In contrast, if the manager goes short, equity holder's net return is $-P_1^G + P_0 = -\epsilon$. Likewise, ff the realized state is bad and the manager goes short, equity holder's net return is $-P_1^B + P_0 = \epsilon$. In contrast, if the manager goes long, equity holder's net return is $P_1^B - P_0 = -\epsilon$. Thus, independently of the state of the world, the net benefit of taking the right over the wrong position is 2ϵ.

intuition, it can be shown that $A_L - A_S$ is also positive. Thus, (A.39) can be expressed as:

$$A_L - A_S \geq \frac{B_S - B_L}{Pr(G|s)}. \tag{A.41}$$

Besides, given that $Pr(G|s)$ is increasing in s, if relation (A.41) holds for $s = s'$, then it also holds for any $s \geq s'$, i.e., $\mathbb{E}\,\Pi[long|s'] - \mathbb{E}\,\Pi[short|s']$ is increasing in s. Also, by MLRP, there exists a unique s, denoted as $\hat{s}$, such as (A.41) binds. Thus, the decision rule that the equity holder prefers the manager to implement, is such as the latter goes short for $s \leq \hat{s}$ and long otherwise.

A.3.4.2 Proposition 1B: Minimization Problem when state imperfectly observed

Constraints for implementing $\mathcal{DR}$

Similarly to the analysis of section 1.4.2, the manager implements $\mathcal{DR}$ as long as his preferences are aligned with the preferences of the equity holder. This alignment pins down to the following three conditions, which are the analogue of (1.3), (1.4)and (1.5) in the benchmark model.

$$DG = w_{Lg} - w_{Sg} \geq 0 \tag{A.42}$$

$$DB = w_{Sb} - w_{Lb} \geq 0 \tag{A.43}$$

$$Pr(g|\hat{s})[w_{Lg} - w_{Sg}] = Pr(b|\hat{s})[w_{Sb} - w_{Lb}] \tag{A.44}$$

Information acquisition constraints

When the manager does not acquire information, his utility of going long

and short is given by:

$$\mathbb{E}\, V[long] = \{pq_G + (1-p)q_B\}w_{Lg} + \{p(1-q_G) + (1-p)(1-q_B)\}w_{Lb}$$

$$\mathbb{E}\, V[short] = \{pq_G + (1-p)q_B\}w_{Sg} + \{p(1-q_G) + (1-p)(1-q_B)\}w_{Sb}$$

Hence, the manager's expected utility if no information is acquired is given by:

$$\mathbb{E}\, V[no\ signal] = max\{\mathbb{E}\, V[long], \mathbb{E}\, V[short]\}$$

The manager's expected utility of acquiring information, given a threshold $\hat{s}$, is given by:

$$\mathbb{E}\, V[signal|\hat{s}] = \int_0^{\hat{s}} \mathbb{E}\, C[short|s]f(s)ds + \int_{\hat{s}}^1 \mathbb{E}\, C[long|s]f(s)ds - c. \quad \text{(A.45)}$$

where the expected compensation of going long and short given a signal realization s, is given by:

$$\mathbb{E}\, C[long|s] = Pr(g|s) \times w_{Lg} + Pr(b|s) \times w_{Lb} \quad \text{(A.46)}$$

$$\mathbb{E}\, C[short|s] = Pr(g|s) \times w_{Sg} + Pr(b|s) \times w_{Sb} \quad \text{(A.47)}$$

After substituting (A.46) and (A.47) into (A.45), and applying the Bayes rule, we obtain:

$$\begin{aligned}
&\mathbb{E}\, V[signal|\hat{s}] = -c + \\
&\int_0^{\hat{s}} \{(\frac{f_G(s)pq_G}{f(s)} + \frac{f_B(s)(1-p)q_B}{f(s)})w_{Sg}\}f(s)ds + \\
&\int_0^{\hat{s}} \{(\frac{f_B(s)(1-p)(1-q_B)}{f(s)} + \frac{f_G(s)p(1-q_G)}{f(s)})w_{Sb}\}f(s)ds +
\end{aligned}$$

$$\int_{\hat{s}}^{1}\{(\frac{f_G(s)pq_G}{f(s)}+\frac{f_B(s)(1-p)q_B}{f(s)})w_{Lg}\}f(s)ds+$$
$$\int_{\hat{s}}^{1}\{(\frac{f_B(s)(1-p)(1-q_B)}{f(s)}+\frac{f_G(s)p(1-q_B)}{f(s)})w_{Lb}\}f(s)ds \tag{A.48}$$

which simplifies to:

$$\begin{aligned}
&\mathbb{E}\,V[signal|\hat{s}] = -c+\\
&\{pq_GF_G(\hat{s})+(1-p)q_BF_B(\hat{s})\}w_{Sg}+\\
&\{(1-p)(1-q_B)F_B(\hat{s})+p(1-q_G)F_G(\hat{s})\}w_{Sb}+\\
&\{pq_G(1-F_G(\hat{s}))+(1-p)q_B(1-F_B(\hat{s}))\}w_{Lg}+\\
&\{(1-p)(1-q_B)(1-F_B(\hat{s}))+p(1-q_G)(1-F_G(\hat{s}))\}w_{Lb}
\end{aligned} \tag{A.49}$$

Hence, the information acquisition constraints lead to:

$$\begin{aligned}
&\mathbb{E}\,V[signal|\hat{s}] \geq \mathbb{E}\,V[long] \implies\\
&\{-pq_GF_G(\hat{s})-(1-p)q_BF_B(\hat{s})\}(w_{Lg}-w_{Sg})+\\
&+\{(1-p)(1-q_B)F_B(\hat{s})+F_G(\hat{s})p(1-q_G)\}(w_{Sb}-w_{Lb})) \geq c
\end{aligned} \tag{A.50}$$

$$\begin{aligned}
&\mathbb{E}\,V[signal|\hat{s}] \geq \mathbb{E}\,V[short] \implies\\
&\{pq_G(1-F_G(\hat{s}))+(1-p)q_B(1-F_B(\hat{s}))\}(w_{Lg}-w_{Sg})+\\
&+\{-(1-p)(1-q_B)(1-F_B(\hat{s}))-p(1-q_G)(1-F_G(\hat{s}))\}(w_{Sb}-w_{Lb}) \geq c
\end{aligned} \tag{A.51}$$

Cost Minimization Problem

Following the previous analysis, the cost minimization problem of incentivizing information acquisition, given an decision rule characterized by a

threshold $\hat{s}$, pins down to:

$$\underset{w_{Sg},w_{Sb},w_{Lg},w_{Lb}}{\text{Minimize}} \quad \mathbb{E}\, C(\hat{s})$$

subject to (A.42), (A.43), (A.44), (A.50), (A.51), $w_{Sg} \geq 0,\ w_{Sb} \geq 0,\ w_{Lg} \geq 0,\ w_{Lb} \geq 0$, where $\mathbb{E}\, C(\hat{s})$ is:

$$\begin{aligned}
&\mathbb{E}\, C(\hat{s}) = \\
&\{pq_G F_G(\hat{s}) + (1-p)q_B F_B(\hat{s})\}w_{Sg} + \\
&\{(1-p)(1-q_B)F_B(\hat{s}) + p(1-q_G)F_G(\hat{s})\}w_{Sb} + \\
&\{pq_G(1-F_G(\hat{s})) + (1-p)q_B(1-F_B(\hat{s}))\}w_{Lg} + \\
&\{(1-p)(1-q_B)(1-F_B(\hat{s})) + p(1-q_G)(1-F_G(\hat{s}))\}w_{Lb}
\end{aligned} \tag{A.52}$$

A.3.4.3 Proof of Proposition 1B

Proof that $w^*_{Sg}(\hat{s}) = 0$

Substituting constraints (A.43) and (A.42) into (A.44) implies:

$$DB = \frac{\{pq_G f_G(\hat{s}) + (1-p)q_B f_B(\hat{s})\}}{\{p(1-q_G)f_G(\hat{s}) + (1-p)(1-q_B)f_B(\hat{s})\}} DG \tag{A.53}$$

By substituting (A.53) into (A.50) and rearranging, we obtain:

$$DG \geq \frac{f_B(\hat{s})(-1+p)(-1+q_B) - f_G(\hat{s})p(-1+q_G)}{(F_B(\hat{s})f_G(\hat{s}) - f_B(\hat{s})F_G(\hat{s}))(-1+p)p(q_B - q_G)} c \tag{A.54}$$

where the RHS of (A.54) is positive, as an implication of Lemma A.2.

Substituting (A.53) into (A.51) and rearranging, we obtain:

$$DG \geq \frac{f_B(\hat{s})(-1+p)(-1+q_B) - f_G(\hat{s})p(-1+q_B)}{(-(1-F_B(\hat{s}))f_G(\hat{s}) + f_B(\hat{s})(1-F_G(\hat{s})))(1-p)p(q_B-q_G)}c \quad \text{(A.55)}$$

where the RHS of (A.55) is positive, as an implication of Lemma A.2.

The previous analysis shows that incentive constraints of the maximization problem are satisfied as long as DG satisfies (A.54) and (A.55). Hence, the equity holder would never find it optimal to offer a contract where w_{Sg} is positive, because, by decreasing w_{Lg} and w_{Sg} by the same amount, the incentive constraints are unaffected, and the expected compensation is lower.

Proof that $w^*_{Lb}(\hat{s}) = 0$

Substituting constraints (A.43) and (A.42) into (A.44) implies:

$$DG = \frac{\{p(1-q_G)f_G(\hat{s}) + (1-p)(1-q_B)f_B(\hat{s})\}}{\{pq_Gf_G(\hat{s}) + (1-p)q_Bf_B(\hat{s})\}}DB \quad \text{(A.56)}$$

By substituting (A.56) into (A.50) and rearranging, we obtain:

$$DB \geq \frac{f_B(\hat{s})(1-p)q_B + f_G(\hat{s})pq_G}{(F_B(\hat{s})f_G(\hat{s}) - f_B(\hat{s})F_G(\hat{s}))(-1+p)p(q_B-q_G)}c \quad \text{(A.57)}$$

where the RHS of (A.57) is positive, as an implication of Lemma A.2.

By substituting (A.56) into (A.51) and rearranging, we obtain:

$$DB \geq \frac{f_B(\hat{s})(1-p)q_B + f_G(\hat{s})pq_G}{(f_B(\hat{s})(1-F_G(\hat{s})) - (1-F_B(\hat{s}))f_G(\hat{s}))(-1+p)p(q_B-q_G)}c \quad \text{(A.58)}$$

where the RHS of (A.58) is positive, as an implication of Lemma A.2.

The previous analysis shows that incentive constraints of the maximization problem are satisfied as long as DB satisfies (A.56) and (A.57). Hence, the equity holder would never find it optimal to offer a contract where w_{Lb} is positive, because, by decreasing both w_{Sb} and w_{Lb} by the same amount, the incentive constraints are unaffected, and the expected payment is lower. Following the previous analysis, the minimization problem pins down to:

$$\underset{w_{Sg},w_{Sb},w_{Lg},w_{Lb}}{\text{Minimize}} \quad \mathbb{E}\, C(\hat{s}) \quad s.t.$$

$$\begin{aligned}
&\{-pq_G F_G(\hat{s}) - (1-p)q_B F_B(\hat{s})\}w_{Lg} + \\
&\{(1-p)(1-q_B)F_B(\hat{s}) + F_G(\hat{s})p(1-q_G)\}w_{Sb} \geq c && \text{(A.59)} \\
&\{pq_G(1-F_G(\hat{s})) + (1-p)q_B(1-F_B(\hat{s}))\}w_{Lg} - \\
&\{(1-p)(1-q_B)(1-F_B(\hat{s})) + p(1-q_G)(1-F_G(\hat{s}))\}w_{Sb} \geq c && \text{(A.60)} \\
&\{pq_G f_G(\hat{s}) + (1-p)q_B f_B(\hat{s})\}w_{Lg} = \\
&\{p(1-q_G)f_G(\hat{s}) + (1-p)(1-q_B)f_B(\hat{s})\}w_{Sb} && \text{(A.61)} \\
&w_{Sb} \geq 0\ ,\ w_{Lg} \geq 0
\end{aligned}$$

Redundant constraints

Simple algebra implies that (A.59) is redundant by (A.60) as long as:

$$f_G(\hat{s}) \geq f_B(\hat{s})$$

which holds for $s \in [\hat{s}_{min}, 1]$. Consequently, for $s \in [0, \hat{s}_{min}]$, (A.60) is re-

dundant by (A.59).

Case where $s \in [\hat{s}_{min}, 1]$

In this case, (A.59) becomes redundant. By substituting (A.61) into (A.60) and rearranging, we obtain:

$$w_{Lg} \geq \frac{f_B(\hat{s})(-1+p)(1-q_B) + f_G(\hat{s})p(-1+q_G)}{((1-F_B(\hat{s}))f_G(\hat{s}) + f_B(\hat{s})(-1+F_G(\hat{s})))(-1+p)p(q_B-q_G)}c.$$

Note that under the optimal contract, (A.60) binds, otherwise the equity holder could increase his profit by decreasing w_{Lg} until (A.60) is binding. Thus,

$$w^*_{Lg}(\hat{s}) = \frac{f_B(\hat{s})(-1+p)(1-q_B) + f_G(\hat{s})p(-1+q_G)}{((1-F_B(\hat{s}))f_G(\hat{s}) + f_B(\hat{s})(-1+F_G(\hat{s})))(-1+p)p(q_B-q_G)}c.$$

The last step is to substitute $w^*_{Lg}(\hat{s})$ into (A.61) to derive the optimal value of $w^*_{Sb}(\hat{s})$ which is:

$$w^*_{Sb}(\hat{s}) = \frac{f_B(\hat{s})(-1+p)q_B - f_G(\hat{s})pq_G}{(f_B(\hat{s})(-1+F_G(\hat{s})) + f_G(\hat{s})(1-F_B(\hat{s})))(-1+p)p(q_B-q_G)}c.$$

Case where $s \in [0, \hat{s}_{min}]$

In this case, (A.60) becomes redundant. By substituting (A.61) into (A.59) and rearranging, we obtain:

$$w_{Lg} \geq \frac{f_B(\hat{s})(-1+p)(-1+q_B) - f_G(\hat{s})p(-1+q_G)}{(F_B(\hat{s})f_G(\hat{s}) - f_B(\hat{s})F_G(\hat{s}))(-1+p)p(q_B-q_G)}c.$$

Note that under the optimal contract, (A.59) binds, otherwise the equity holder could increase his profit by decreasing w_{Lg} until (A.59) is binding.

Thus,

$$w^*_{Lg}(\hat{s}) = \frac{f_B(\hat{s})(-1+p)(-1+q_B) - f_G(\hat{s})p(-1+q_G)}{(F_B(\hat{s})f_G(\hat{s}) - f_B(\hat{s})F_G(\hat{s}))(-1+p)p(q_B - q_G)}c.$$

The last step is to substitute $w^*_{Lg}(\hat{s})$ into (A.61) to derive the optimal value of $w^*_{Sb}(\hat{s})$, which is:

$$w^*_{Sb}(\hat{s}) = \frac{f_B(\hat{s})(1-p)q_B + f_G(\hat{s})pq_G}{(F_B(\hat{s})f_G(\hat{s}) - f_B(\hat{s})F_G(\hat{s}))(-1+p)p(q_B - q_G)}c.$$

A.3.4.4 Proof of Lemma 2B

Part one: The derivative of $w_{Sb}(\hat{s})$ and $w_{Lg}(\hat{s})$ with respect to $\hat{s}$ (for $\hat{s} \in [0, \hat{s}_{min}]$), is given by:

$$\frac{\partial w_{Sb}(\hat{s})}{\partial \hat{s}} = c\frac{(pq_G F_G(\hat{s}) + (1-p)q_B F_B(\hat{s}))\gamma_1(\hat{s})}{(-1+p)p(q_B - q_G)(F_B(\hat{s})f_G(\hat{s}) - f_B(\hat{s})F_G(\hat{s}))^2}$$

$$\frac{\partial w_{Lg}(\hat{s})}{\partial \hat{s}} = -c\frac{((1-p)(-1+q_B)F_B + p(-1+q_G)F_G(\hat{s}))\gamma_1(\hat{s})}{p(-1+p)(q_B - q_G)(F_B(\hat{s})f_G(\hat{s}) - f_B(\hat{s})F_G(\hat{s}))^2}$$

where $\gamma_1(\hat{s}) \equiv [f'_B(\hat{s})f_G(\hat{s}) - f_B(\hat{s})f'_G(\hat{s})]$. Recall that $q_G \geq q_B$. Also, we show in Lemma A.1 that $\gamma_1 < 0$. Hence, both $w_{Sb}(\hat{s})$ and $w_{Lg}(\hat{s})$ are decreasing in $\hat{s}$ for $\hat{s} \in [0, \hat{s}_{min}]$.

Part two: The derivative of $w_{Sb}(\hat{s})$ and $w_{Lg}(\hat{s})$ with respect to $\hat{s}$ (for $\hat{s} \in [\hat{s}_{min}, 1]$), is given by:

$$\frac{\partial w_{Sb}(\hat{s})}{\partial \hat{s}} = c\frac{(pq_G(1-F_G(\hat{s})) + (-1+p)(F_B(\hat{s})-1))(-\gamma_1(\hat{s}))}{(-1+p)p(q_B - q_G)((1-F_G(\hat{s}))f_B(\hat{s}) - f_G(\hat{s})(1-F_B(\hat{s})))^2}$$

$$\frac{\partial w_{Lg}(\hat{s})}{\partial \hat{s}} =$$

$$-c\frac{\overbrace{(p(1-q_G)F_G(\hat{s})+(-1+p)(-1+q_B)F_B(\hat{s})+qb(1-p)+pq_B-1)}^{K_1}\gamma_1(\hat{s})}{p(-1+p)(q_B-q_G)(-(1-F_G(\hat{s}))f_B(\hat{s})+f_G(\hat{s})(1-F_B(\hat{s})))^2} \tag{A.62}$$

where $\frac{\partial w_{Sb}(\hat{s})}{\partial \hat{s}} \geq$, given that $q_G \geq q_B$ and $\gamma_1 < 0$. Defining the sign of $\frac{\partial w_{Lg}(\hat{s})}{\partial \hat{s}}$ is less straightforward. Notice that $\partial K_1/\partial q_G < 0$. Hence, in order to show that K_1 is negative, it suffices to show that $K_1 < 0$ for the minimum value of q_G, i.e., $q_G = q_B$. Replacing q_G with q_B in K_1 leads to negative K_1, thus, $K_1 < 0$ for any admissible value of q_G. Also, by Lemma A.1, $\gamma_1 < 0$, hence, both $w_{Sb}(\hat{s})$ and $w_{Lg}(\hat{s})$ are increasing in $\hat{s}$ for $\hat{s} \in [\hat{s}_{min}, 1]$.

Part three: Part three is a direct consequence of part one and two.

Part four: We first provide the intuition behind part one, which is going to act as a stepping stone for part four. The intuition behind part one (and part two) builds on three remarks. First, the manager is willing to implement $\hat{s}$, as long as:

$$Pr(G|\hat{s})/Pr(B|\hat{s}) = w_{Sb}/w_{Lg}. \tag{A.63}$$

Second, under the optimal contract, the manager's expected utility when acquiring information should be equal to his outside option of not acquiring information, given by:

$$max\{\mathbb{E}\,V(not\ expl + long), \mathbb{E}\,V(not\ expl + long)\} = max\{pw_{Lg}, (1-p)w_{Sb}\}$$

Third, the opportunity cost of not taking the revenue-maximizing position is w_{Lg}, when the manager goes short and the revenue-maximizing position is long, and w_{Sb}, when the manager goes long and the right position is short.

Given these remarks, in order to shed light on the intuition behind part one, we explore the ramifications of a deviation in the implementation threshold from $\hat{s}_{min}$ to $\hat{s}' < \hat{s}_{min}$.[3] Suppose that $p = 0.5$ and that the principal aims to implement $\hat{s} = \hat{s}_{min}$. Relation (A.63) holds, as long as $w_{Sb}(\hat{s}_{min}) = w_{Lg}(\hat{s}_{min})$. This implies that the manager's outside option is equal to $0.5w_{Lg}(\hat{s}_{min})$ and the cost in case of taking the wrong position is $w_{Sb}(\hat{s}_{min})$. Suppose now that the principal aims to implement a lower threshold, $\hat{s}' < \hat{s}_{min}$, for which $Pr(G|\hat{s}')/Pr(B|\hat{s}') < Pr(G|\hat{s}_{min})/Pr(B|\hat{s}_{min})$. For relation (A.63) to be satisfied, it must be that:

$$w_{Sb}(\hat{s}')/w_{Lg}(\hat{s}') = Pr(G|\hat{s}')/Pr(B|\hat{s}').$$

Note that $Pr(G|\hat{s}')/Pr(B|\hat{s}' < w_{Sb}(\hat{s}_{min})/w_{Lg}(\hat{s}_{min})$. There are four ways to achieve this.

(i) $w_{Lg}(\hat{s}') > w_{Lg}(\hat{s}_{min})$ & $w_{Sb}(\hat{s}') = w_{Sb}(\hat{s}_{min})$: In this case, the manager's outside option improves, whereas the cost of taking the wrong position is unaffected. Thus, the outside option now exceeds the manager's utility when he acquires information. Hence, this case is not feasible.

(ii) $w_{Sb}(\hat{s}') < w_{Sb}(\hat{s}_{min})$ & $w_{Lg}(\hat{s}') = w_{Lg}(\hat{s}_{min})$: In this case, the manager's utility from acquiring information drops, given that the reward

[3]Similar intuition applies for part two.

when the agent goes short drops, whereas the manager's outside option is unaffected. Thus, manager's utility when acquiring information falls below the manager's outside option. Hence, this case is not feasible.

(iii) $w_{Lg}(\hat{s}') < w_{Lg}(\hat{s}_{min})$ & $w_{Sb}(\hat{s}') < w_{Sb}(\hat{s}_{min})$: This case is not feasible. If this case were feasible, it would imply that the equity holder when implementing $\hat{s}_{min}$ would be able to increase his profits by decreasing both $w_{Lg}(\hat{s}_{min})$ and $w_{Lg}(\hat{s}_{min})$ without violating (A.63). However, such a profitable deviation would contradict with the initial hypothesis that $w_{Lg}(\hat{s}_{min})$ and $w_{Lg}(\hat{s}_{min})$ are the optimal payments.

(iv) $w_{Lg}(\hat{s}') > w_{Lg}(\hat{s}_{min})$ & $w_{Sb}(\hat{s}') < w_{Sb}(\hat{s}_{min})$: In this case, both the manager's outside option and the cost of taking the wrong position increase. For $w_{Lg}(\hat{s}')$ small enough and for $w_{Sb}(\hat{s}')$ large enough, the two opposing forces cancel each other. Hence, this is the only feasible case.

Summarizing the previous analysis, we show that as the value of $\hat{s}$ decreases, the value of $\frac{f_G(\hat{s})}{f_B(\hat{s})}$, and effectively of $\frac{Pr(g|\hat{s})}{Pr(b|\hat{s})}$, decreases as well. Hence, the equity holder, in order to incentivize the manager to take a long position for $s \in (\hat{s}, 1]$, should offer a high relative payment $\frac{w_{Sb}}{w_{Lg}}$. This can only happen if the equity holder increases w_{Lg}. This increase, however, improves the manager's outside option, and tempts him to go long without acquiring costly information. Hence, in order to prevent the manager from going long without acquiring information, the equity holder should increase the opportunity cost of not taking the revenue-maximizing position, i.e., offer a higher payment w_{Sb}.

Following the reasoning of the previous paragraph, the convexity of $w^*_{Lg}(\hat{s})$ and $w^*_{Sb}(\hat{s})$ can be seen in the following example. Suppose that we start from a case where the principal implements $\hat{s} = 0.5$, and the optimal payments are denoted by $w^*_{Lg}(0.5)$ and $w^*_{Sb}(0.5)$. Suppose now that the equity holder aims to implement $\hat{s} = 0.49$. In order to incentivice the manager to go long for $s \in (0.49, 0.50)$, the equity holder should increase w_{Lg} from $w^*_{Lg}(0.5)$ to w'_{Lg} such as $\frac{w'_{Lg}}{w^*_{Sb}(0.5)} = \frac{Pr(b|\hat{s}=0.49)}{Pr(g|\hat{s}=0.49)}$. At the same time, the increase in w_{Lg} triggers the manager to go long without acquiring information. Thus, in order to prevent the manager from going long without acquiring information, the equity holder should also increase w_{Sb} to w'_{Sb}. However, after this increase, $\frac{w'_{Lg}}{w'_{Sb}} < \frac{Pr(b|\hat{s}=0.49)}{Pr(g|\hat{s}=0.49)}$, thus the principal should also increase w'_{Lg} to w''_{Lg}, which will in turn, trigger another increase in w'_{Sb} to w''_{Sb} and so on. Notice that each loop leads to smaller increases in w_{Lg} and w_{Sb} because the opportunity cost of not taking the revenue-maximizing position w_{Sb} becomes very high. This process converges to the equilibrium, where the equilibrium payments are denoted by $w^*_{Lg}(0.49)$ and $w^*_{Sb}(0.49)$.

Suppose now that the equity holder aims to implement $\hat{s} = 0.48$. In order to incentivice the agent to go long for $s \in (0.48, 0.49)$, the equity holder should increase w_{Lg} from $w^*_{Lg}(0.49)$ to w^+_{Lg} such as $\frac{w^+_{Lg}}{w^*_{Sb}(0.49)} = \frac{Pr(B|\hat{s}=0.48)}{Pr(G|\hat{s}=0.48)}$. Recall that $\frac{f_G(\hat{s})}{f_B(\hat{s})}$, and effectively $\frac{Pr(g|\hat{s})}{Pr(b|\hat{s})}$, is increasing in $\hat{s}$ due to the MLRP assumption. Hence, the combination of MLRP and the finding that $w^*_{Sb}(0.49) > w^*_{Sb}(0.50)$ implies that the increase $w^+_{Lg} - w^*_{Lg}(0.49)$ will be higher than than the increase $w'_{Lg} - w^*_{Lg}(0.5)$. This implies that the incentive to go long without acquiring information is stronger than before, thus, w_{S_b} should increase significantly, which in turn, leads to a higher

w_{Lg}, such as the equity holder has incentive to go long for $s \in (0.48, 0.49)$. Thus, the equilibrium payments should increase more when the equity holder deviates from $\hat{s} = 0.49$ to $\hat{s} = 0.48$, compared to the deviation from $\hat{s} = 0.5$ to $\hat{s} = 0.49$, i.e., $w^*_{Lg}(0.49) - w^*_{Lg}(0.50) > w^*_{Lg}(0.48) - w^*_{Lg}(0.49)$, and $w^*_{Sb}(0.49) - w^*_{Sb}(0.50) > w^*_{Sb}(0.48) - w^*_{Sb}(0.49)$.

The last paragraphs refer to the part of Lemma 2B which explores deviations to lower values of $\hat{s}$. Similar intuition applies for the case where the principal considers shifting from implementing $\hat{s} = \hat{s}_{min}$ to implementing $\hat{s} = \hat{s}'' > \hat{s}_{min}$.

A.3.4.5 Expected cost of implementing $\hat{s}$

Corollary 1B derives $\mathbb{E}\,C(\hat{s})$ which arises from Proposition 1B. Lemma 3 extends to this setup.

Corollary 1B: Expected compensation cost of implementing $\hat{s}$, $\mathbb{E}\,C(\hat{s})$
For $\hat{s} \in [0, \hat{s}_{min}]$, the expected compensation cost is given by:

$$\mathbb{E}\,C(\hat{s}) \equiv \mathbb{E}\,C(\hat{s})^- = \left[\frac{\begin{array}{r}(f_B(\hat{s})(-1+p)(-1+q_B) - p(-1+q_G)f_G(\hat{s}))((-1+p)q_B(-1+F_B(\hat{s})) - pq_G(-1+F_G(\hat{s}))) \\ + (f_G(\hat{s})pq_G + q_B(1-p)f_B(\hat{s}))((-1+p)(-1+q_B)F_B(\hat{s}) - p(-1+q_G)F_G(\hat{s}))\end{array}}{(F_B(\hat{s})f_G(\hat{s}) - F_G(\hat{s})f_B(\hat{s}))(-1+p)p(q_B - q_G)}\right] c \tag{A.64}$$

For $\hat{s} \in [\hat{s}_{min}, 1]$, the expected compensation cost is given by:

$$\mathbb{E}\,C(\hat{s}) \equiv \mathbb{E}\,C(\hat{s})^+ =$$

$$\left[\frac{\begin{array}{c} pf_G(\hat{s})((-1+q_G)(q_B-pq_B+pq_G)+(1-p)(q_B-q_G)F_B(\hat{s})) \\ +(-1+p)f_B(\hat{s})((-1+q_B)((-1+p)q_B-pq_G)+p(q_B-q_G)F_G(\hat{s})) \end{array}}{((-1+F_G(\hat{s})f_B(\hat{s})+(1-F_B(\hat{s}))f_G(\hat{s}))(-1+p)p(q_B-q_G))}\right]c \tag{A.65}$$

A.3.4.6 Proof of Proposition 2B

The optimal contract consists of two parts: i) the optimal payment scheme, given $\hat{s}$, and, ii) an optimality condition for $\hat{s}$. The first part is characterized in Proposition 1B. The optimality condition for $\hat{s}$, given by (1.30), is characterized in Lemma A.4.

Lemma A.4: Optimality Conditions

For $\hat{s} \leq \hat{s}_{min}$, optimal value $\hat{s}$, $\hat{s}^*$ solves:

$$\underbrace{2\epsilon\{-pf_G(\hat{s})+(1-p)f_B(\hat{s})\}}_{\partial\,\mathbb{E}\,R(\hat{s})/\partial\hat{s}} = \underbrace{-\left[\frac{((-1+p)q_B-pq_G)((-1+p)(-1+q_B)F_B(\hat{s})-p(-1+q_G)F_G(\hat{s}))\gamma_1(\hat{s})}{[F_B(\hat{s})f_G(\hat{s})-F_G(\hat{s})f_B(\hat{s})(-1+p)p(q_B-q_G)]^2}\right]c}_{\partial\,\mathbb{E}\,C^-(\hat{s})/\partial\hat{s}} \tag{A.66}$$

For $\hat{s} \geq \hat{s}_{min}$, optimal value $\hat{s}$, $\hat{s}^*$ solves:

$$\underbrace{2\epsilon\{-pf_G(\hat{s})+(1-p)f_B(\hat{s})\}}_{\partial\,\mathbb{E}\,R(\hat{s})/\partial\hat{s}} =$$

$$\underbrace{-\left[\frac{(1-(1-p)q_B-pq_G)(1-p)q_B(1-F_B(\hat{s}))+pq_G(1-F_G(\hat{s})))\gamma_1(\hat{s})}{[((-1+F_B(\hat{s})f_g(\hat{s})+(1-F_G(\hat{s}))f_B(\hat{s})(-1+p)p(q_B-q_G))]^2}\right]c}_{\partial\,\mathbb{E}\,C^+(\hat{s})/\partial\hat{s}} \tag{A.67}$$

where $\gamma_1(\hat{s}) \equiv [f'_B(\hat{s})f_G(\hat{s}) - f_B(\hat{s})f'_G(\hat{s})]$.

We show in Proposition 4B, that for $p > 0.5$ ($p < 0.5$), it is never optimal to choose a threshold $\hat{s} \in (\hat{s}_{min}, 1]$ ($\hat{s} \in [0, \hat{s}_{min})$). Hence, for $p \geq 0.5$, the optimality condition is (A.66), whereas, for $p \leq 0.5$, the optimality condition is (A.67). The combination of this observation with Lemma A.4 and Proposition 1B, leads to Proposition 2B.

A.3.4.7 Proof of Proposition 4B

Proposition 4B emerges naturally from Lemma 3B and Lemma 4. For instance, if $p > 0.5$, we know that $\hat{s}^{FB} < \hat{s}_{min}$. Also, $\hat{s}^*$ cannot belong to $[0, \hat{s}^{FB}]$ as by switching to a higher threshold the expected revenue increases, and the expected cost decreases. Similarly, $\hat{s}^*$ cannot belong to $[\hat{s}_{min}, 1]$ as by switching to a lower threshold the expected revenue increases, and the expected cost decreases. Hence, $\hat{s}^* \in (\hat{s}^{FB}, \hat{s}_{min})$. A more formal proof is provided below.

Recall that the optimality conditions for the case where $p \geq 0.5$ and $p \leq 0.5$ are given by:

$$\partial\,\mathbb{E}\,R(\hat{s})/\partial\hat{s} = \partial\,\mathbb{E}\,C^-(\hat{s})/\partial\hat{s}$$

$$\partial\,\mathbb{E}\,R(\hat{s})/\partial\hat{s} = \partial\,\mathbb{E}\,C^+(\hat{s})/\partial\hat{s}$$

By Lemma 3B, $\partial \mathbb{E}\, C^{-}(\hat{s})/\partial\hat{s}$ is negative, and $\partial \mathbb{E}\, C^{+}(\hat{s})/\partial\hat{s}$ is positive. Also, by Lemma 4, $\partial \mathbb{E}\, R(\hat{s})/\partial\hat{s}$ is positive for $s < \hat{s}^{FB}$ and negative for $s > \hat{s}^{FB}$. Hence, for $p \geq 0.5$, for an equilibrium to exist it must be that $\partial \mathbb{E}\, R(\hat{s})/\partial\hat{s} = \mathbb{E}\, C^{-}(\hat{s})/\partial\hat{s}$. Since $\partial \mathbb{E}\, C^{-}(\hat{s})/\partial\hat{s}$ is negative, then $\partial \mathbb{E}\, R(\hat{s})/\partial\hat{s}$ has to be negative, which is true only if $s > \hat{s}^{FB}$. Similar intuition applies for the case where $p \leq 0.5$.

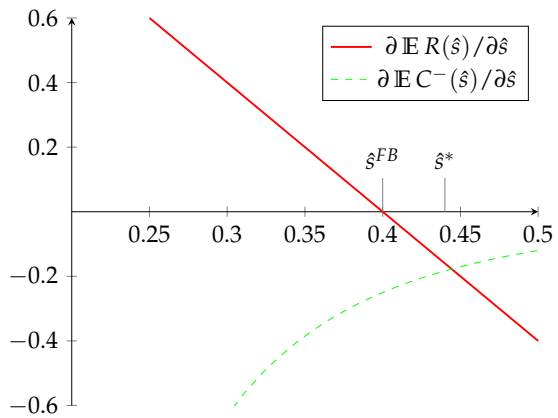


Figure A.4. Bias in the investment decision

A.3.4.8 Lemma 6

Lemma 6 explores the relationship between $\mathbb{E}\, C(\hat{s})$ and q_G and q_B.

Lemma 6: Relationship between $\mathbb{E}\, C(\hat{s})$ and q_G, q_B.

(i) $\mathbb{E}\, C(\hat{s})$ *is decreasing in* q_G.

(ii) $\mathbb{E}\, C(\hat{s})$ *is increasing in* q_B.

(iii) $\frac{\partial \mathbb{E}\, C(\hat{s})}{\partial\hat{s}}$ *is increasing in* q_G *for* $\hat{s} \in [0, \hat{s}_{min})$ *and decreasing in* q_G *for* $\hat{s} \in (\hat{s}_{min}, 1]$

(iv) $\frac{\partial \mathbb{E}\, C(\hat{s})}{\partial \hat{s}}$ *is decreasing in* q_B *for* $\hat{s} \in [0, \hat{s}_{min})$ *and increasing in* q_B *for* $\hat{s} \in (\hat{s}_{min}, 1]$

Proof. Part one: We explore the derivative of $\mathbb{E}\, C(\hat{s})$ with respect to q_G and q_B, first for $\hat{s} \in [0, \hat{s}_{min}]$, and second for $\hat{s} \in [\hat{s}_{min}, 1]$.

$$\frac{\partial \mathbb{E}\, C(\hat{s})^-}{\partial q_G} = c\frac{f_B(\hat{s})(-1+p)(-1+q_B)q_B + f_G(\hat{s})p(q_B(1-q_B) + p(q_G - q_B)^2)}{(F_B(\hat{s})f_G(\hat{s}) - F_G(\hat{s})f_B(\hat{s}))(-1+p)p(q_B - q_G)^2}$$

$$\frac{\partial \mathbb{E}\, C(\hat{s})^+}{\partial q_G} = c\frac{f_B(\hat{s})(-1+p)(-1+q_B)q_B + f_G(\hat{s})p(q_B(1-q_B) + p(q_G - q_B)^2)}{f_G(\hat{s})(1-F_B(\hat{s})) - f_B(\hat{s})(1-F_G(\hat{s}))(-1+p)p(q_B - q_G)^2}$$

which are negative as an implication of Lemma A.2.

$$\frac{\partial \mathbb{E}\, C(\hat{s})^-}{\partial q_B} =$$

$$-c\frac{f_B(\hat{s})(-1+p)(-1+q_B)q_B + f_G(\hat{s})p\overbrace{(q_B + (-1+p)q_B^2 - 2pq_Bq_G + pq_G^2)}^{e_1}}{f_G(\hat{s})(1-F_B(\hat{s})) - f_B(\hat{s})(1-F_G(\hat{s}))(-1+p)p(q_G - q_B)^2}$$

which is positive as an implication of Lemma A.2.

$$\frac{\partial \mathbb{E}\, C(\hat{s})^+}{\partial q_B} =$$

$$-c\frac{f_B(\hat{s})(-1+p)(-1+q_B)q_B + f_G(\hat{s})p\overbrace{(q_B + (-1+p)q_B^2 - 2pq_Bq_G + pq_G^2)}^{e_1}}{(F_B(\hat{s})f_G(\hat{s}) - F_G(\hat{s})f_B(\hat{s}))(-1+p)p(q_B - q_G)^2}$$

which is positive as an implication of Lemma A.2. Note that e_1 is increasing in q_B. Hence, it is easy to show that for the maximum feasible value of q_B, i.e., q_G, e_1 is negative.

Part three: First, we analyze the case where $\hat{s} \in [0, \hat{s}_{min})$.

$$\frac{\partial \mathbb{E}\, MC(\hat{s})^-}{\partial q_G} =$$

$$-c\frac{\gamma(\hat{s})\overbrace{((-1+p)(-1+q_B)q_B F_B(\hat{s}) + p(q_B + (-1+p)q_B^2 - 2pq_Bq_G + pq_G^2)F_G(\hat{s}))}^{L_1}}{(F_B(\hat{s})f_G(\hat{s}) - f_B(\hat{s})F_G(\hat{s}))^2(-1+p)p(q_B - q_G)^2}$$

where $\gamma(\hat{s}) \equiv (f_B(\hat{s})f'_G(\hat{s}) - f'_B(\hat{s})f_G(\hat{s}))$. Note that the denominator is always negative. Also, by Lemma A.1, $\gamma(\hat{s}) > 0$. Notice that $\partial L_1/\partial q_G > 0$. Hence, if L_1 is positive for the minimum feasible value of q_G, i.e. $q_G = q_B$, then L_1 is positive for any feasible value of q_G, where:

$$L1|_{q_G=q_B} = (F_B(\hat{s})(-1+p) - pF_G(\hat{s}))(-1+q_B)q_G > 0$$

Hence, $\frac{\partial \mathbb{E}\, MC(\hat{s})^-}{\partial q_G} > 0$. We now analyze the case where $\hat{s} \in (\hat{s}_{min}, 1]$:

$$\frac{\partial \mathbb{E}\, MC(\hat{s})^+}{\partial q_G} =$$

$$-c\frac{\gamma(\hat{s})\overbrace{(-q_B + q_B^2 + p^2q_B^2 + 2p^2q_Bq_G - p^2q_G^2 + (-1+p)(-1+q_B)q_BF_B(\hat{s}) + p(q_B + (-1+p)q_B^2 - 2pq_Bq_G + pq_G^2)F_G(\hat{s}))}^{L_3}}{((1-F_B(\hat{s}))f_G(\hat{s}) - f_B(\hat{s})(1-F_G(\hat{s})))^2(-1+p)p(q_B - q_G)^2} \tag{A.68}$$

where the denominator is always negative. Also, by Lemma A.1, $\gamma(\hat{s}) > 0$. Notice that $\partial L_3/\partial q_G < 0$. Hence, if L_3 is negative for the minimum feasible value of q_G, i.e. $q_G = q_B$, then $L3$ is negative for any feasible value of q_G, where:

$$L_3|_{q_G=q_B} = (1 + F_B(\hat{s})(-1+p) - F_G(\hat{s})p)(-1+q_G)q_B < 0$$

Hence, $\frac{\partial \mathbb{E}\, MC(\hat{s})^+}{\partial q_G} < 0$

Part four: First, we analyze the case where $\hat{s} \in [0, \hat{s}_{min})$.

$$\frac{\partial \mathbb{E}\, MC(\hat{s})^-}{\partial q_B} = c\frac{\gamma(\hat{s})\ \overbrace{((-1+p)((-1+p)q_B^2 - 2(-1+p)q_B q_G + q_G(-1+pq_G))F_B(\hat{s}) - p(-1+q_G)q_G F_G(\hat{s}))}^{L_2}}{(F_B(\hat{s})f_G(\hat{s}) - f_B(\hat{s})F_G(\hat{s}))^2(-1+p)p(q_B - q_G)^2}$$

where the denominator is always negative. Also, by Lemma A.1, $\gamma(\hat{s}) > 0$. Notice that $\partial L_2/\partial q_B < 0$. Hence, if L_2 is positive for the maximum feasible value of q_B, i.e. $q_B = q_G$, then L_2 is positive for any feasible value of q_B, where:

$$L_2|_{q_B=q_G} = (F_B(\hat{s})(-1+p) - F_G(\hat{s}))(-1+q_G)q_G > 0$$

Hence, $\frac{\partial \mathbb{E}\, MC(\hat{s})^-}{\partial q_G} < 0$. We now analyze the case where $\hat{s} \in (\hat{s}_{min}, 1]$.

$$\frac{\partial \mathbb{E}\, MC(\hat{s})^+}{\partial q_B} = -c\frac{\gamma L_4}{\kappa} \tag{A.69}$$

where

$$\gamma \equiv f_B(\hat{s})f_G'(\hat{s}) - f_B'(\hat{s})f_G(\hat{s})) \tag{A.70}$$

$$\begin{aligned} L_4 \equiv &(-q_B^2 + 2pq_B^2 - p^2q_B^2 - q_G + 2q_Bq_G - 4pq_Bq_G + 2p^2q_Bq_G + \\ &2pq_G^2 - p^2q_G^2 + (-1+p)((-1+p)q_B^2 - 2(-1+p)q_Bq_G + \\ &q_G(-1+pq_G))F_B(\hat{s}) - p(-1+q_G)q_GF_G(\hat{s})) \end{aligned} \tag{A.71}$$

$$\kappa \equiv ((1-F_B(\hat{s}))f_G(\hat{s}) - f_B(\hat{s})(1-F_G(\hat{s})))^2(-1+p)p(q_B - q_G)^2 \tag{A.72}$$

where the denominator is always negative. Also, by Lemma A.1, $\gamma > 0$. Notice that $\partial L_4/\partial q_B > 0$. Hence, if L_4 is negative for the maximum value

of q_B, i.e. $q_B = q_G$, then $L4$ is negative for any feasible value of q_B, where:

$$L_4|_{q_B=q_G} = (1 + F_B(\hat{s})(-1 + p) - F_G(\hat{s})p)(-1 + q_B)q_B < 0$$

Hence, $\frac{\partial \mathbb{E}\, MC(\hat{s})^+}{\partial q_G} > 0$. □

A.3.4.9 Proof of Proposition 7

The proof relies on five remarks: i) the expected return of of implementing $\hat{s}$, $\mathbb{E}\, R(\hat{s})$, is independent of q_G and q_B, ii) for $p > 0.5$, $\hat{s}^* \in (\hat{s}^{FB}, \hat{s}_{min}]$, and for $p < 0.5$, $\hat{s}^* \in [\hat{s}_{min}, \hat{s}^{FB})$, iii) by Lemma 4, the expected marginal revenue $\mathbb{E}\, MR(\hat{s}) \equiv \partial\, \mathbb{E}\, R(\hat{s})/\partial \hat{s}$ is positive for $\hat{s} < \hat{s}^{FB}$ and negative for $\hat{s} > \hat{s}^{FB}$, iv) $\frac{\partial\, \mathbb{E}\, C(\hat{s})}{\partial \hat{s}}$ is increasing in q_G for $\hat{s} \in [0, \hat{s}_{min})$, and decreasing in q_G for $\hat{s} \in (\hat{s}_{min}, 1]$, v)$\frac{\partial\, \mathbb{E}\, C(\hat{s})}{\partial \hat{s}}$ is decreasing in q_B for $\hat{s} \in [0, \hat{s}_{min})$, and increasing in q_B for $\hat{s} \in (\hat{s}_{min}, 1]$

Part one: For $p > 0.5$ where $\hat{s}^* \in (\hat{s}^{FB}, \hat{s}_{min}]$, an increase in q_G leads to a greater (still negative) expected marginal cost, thus the intersection with the expected marginal revenue corresponds to lower values of $\hat{s}$. Similarly, for $p < 0.5$ where $\hat{s}^* \in [\hat{s}_{min}, \hat{s}^{FB})$, an increase in q_G leads to a lower (still positive) expected marginal cost, thus the intersection with the expected marginal revenue corresponds to for higher values of $\hat{s}$.

Part two: For $p > 0.5$ where $\hat{s}^* \in (\hat{s}^{FB}, \hat{s}_{min}]$, an increase in q_B leads to a lower (still negative) expected marginal cost, thus the intersection with the expected marginal corresponds to higher values of $\hat{s}$. Similarly, for $p < 0.5$ where $\hat{s}^* \in [\hat{s}_{min}, \hat{s}^{FB})$, an increase in q_G leads to a higher (still positive)

expected marginal cost, thus the intersection with the expected marginal corresponds to lower values of $\hat{s}$.

Part three: Part three is a direct implication of part one and two.

Appendix B

The Impact of Credit Rating Agencies on Capital Markets

B.1 Optimal Security

A security, w, can be contingent on the realized value of c, and the final output of the project (R or 0). Recall that the cost of effort is not verifiable. An implication of lack of verifiability is that the optimal security pins down to two relevant-in-equilibrium payments. The intuition is the following. First, note that the entrepreneur's limited liability implies that the payment in the case where the output is zero, is also zero. We now consider the case where the project succeeds. Suppose that the payment which corresponds to the case where the project succeeds is contingent on c. Since c is non-verifiable, the entrepreneur would always find it optimal to report the value of cost which corresponds to the minimum payment, denoted by w_{min}. This implies that, in equilibrium, the only relevant payments would be 0 if the project's return is 0, and w_{min} if the return is R. In order to be

consistent with the benchmark model, we denote w_{min} as $(1+r)$. The previous analysis allows me to assume, without loss of generality, that the security is contingent on the project's outcome. Hence, the security is associated with the following return for potential investors:

$$\text{Investor's return} \begin{cases} (1+r) & \text{if project returns R} \\ 0 & \text{otherwise} \end{cases} \tag{B.1}$$

In addition, following the intuition Nachman and Noe (1994), asymmetric information regarding the entrepreneur's type implies the HEC entrepreneur mimics the LEC entrepreneur, by offering the same security.

Here I deal with the uniqueness of the optimal security. Since both HEC and LEC entrepreneurs offer the same security, the expected revenue of potential investors is $1+r$ times the probability of financing an entrepreneur who exerts effort, which coincides with the probability that $c < R-(1+r)$. This last point implies that the entrepreneur chooses the value of r by taking into consideration the probability of default, such that the investors' zero-profit condition is satisfied. Note that depending on the distribution of c, there could be multiple combinations of interest rate-default probabilities that satisfy the zero-profit condition. However, the only combination/equilibrium which survives is the one that corresponds to the lowest value of r, denoted as r^*. This is because that equilibrium dominates all other equilibria from the entrepreneur's perspective.

The question that emerges is whether such securities are observed in capital markets. Offering such securities is a method of financing similar to debt financing; the entrepreneur issues a bond that returns $min\{(1+r), V\}$,

where V is the firm's value and r the interest rate. Since the entrepreneur has no wealth other than what the project returns, the value of the firm will be either R if effort is exerted, or 0 otherwise. Note also that if $(1+r) \geq V$ the entrepreneur would choose not to exert effort because this would lead to negative profit. Hence, the return can only be $(1+r)$ if the project is implemented ($V = R$), or 0 if the project is not implemented ($V = 0$).

B.2 Equilibrium Existence Theorem

In order to find the conditions for the existence of an equilibrium I use the Bolzano Theorem: for two real numbers r_α and r_β, $r_\alpha < r_\beta$, if $G(r)$ is (i) a continuous function and (ii) $G(r_\alpha)$, $G(r_\beta)$ are of opposite signs, then there exists an $r^* \in [r_\alpha, r_\beta]$ such that $G(r^*) = 0$.

To find the equilibrium conditions I set $\mathbb{E}[Prob(c \leq \hat{c})|\Omega] \equiv \Phi(\hat{c}(r)|\Omega)$, where $\hat{c} = R - (1+r)$, and I create a new function, $G(r) = (1+r)^{-1} - \Phi(\hat{c}(r)|\Omega)$. Note that $G(r)$ satisfies condition (i) as $\Phi(\hat{c}(r)|\Omega)$ and $(1+r)^{-1}$ are continuous functions. To show that condition (ii) is satisfied, I set $r_\alpha = 0$ where the function $(1+r)^{-1}$ reaches its maximum value, 1. Note that $G(0)$ is non-negative as $\Phi(\hat{c}(r)|\Omega, r = 0) < 1$. Hence, for an equilibrium to exist it is sufficient to show that there is at least one value of r, denoted as $\tilde{r}$, such that $G(\tilde{r}) < 0$. Figure B.1 illustrates the case where at least one equilibrium exists, and Figure B.2 the case where there is no equilibrium (market breakdown).

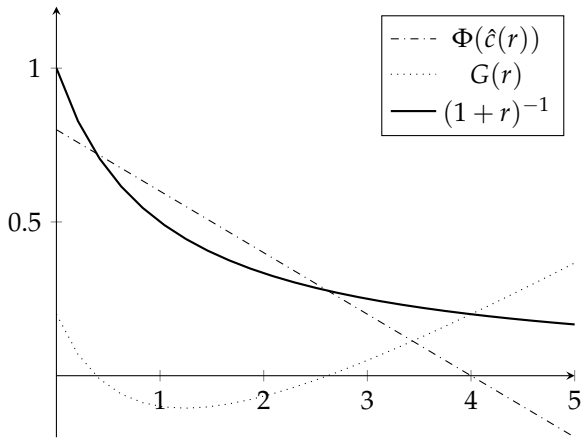


Figure B.1. Case where market collapses.

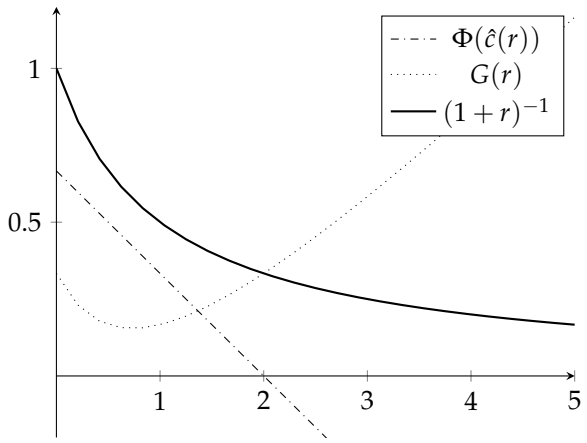


Figure B.2. Case where market survives.

B.3 Proof of Main Propositions

B.3.1 Proof of Proposition 1

Part one: The case where the introduction of a CRA alleviates under-financing of a project with positive probability refers to a state of the world where an LEC entrepreneur is financed only if he can differentiate himself from an HEC entrepreneur. This is true when the following conditions are

satisfied:

$$\lambda F_L(\hat{c}(r')) + (1-\lambda)F_H(\hat{c}(r')) < (1+r)^{-1} \tag{B.2}$$

$$F_L(r') \geq (1+r)^{-1} \tag{B.3}$$

Thus the proof of part one, pins down to showing that (B.2) and (B.3) are not mutually exclusive. By combining (B.2) and (B.3), we obtain:

$$F_L(\hat{c}(r')) > (1+r')^{-1} > \lambda F_L(\hat{c}(r')) + (1-\lambda)F_H(\hat{c}(r')) \tag{B.4}$$

Since, $F_L(r') \geq F_H(r')$ (due to the FOSD assumption), the is always an interest rate, denoted by r' which satisfies (B.4).

Part two: The case where the introduction of a CRA leads to under-financing of a project with positive probability refers to a state of the world where an HEC entrepreneur is financed only if he is pooled with an LEC entrepreneur. This case emerges when the following conditions are satisfied:

$$\lambda F_L(\hat{c}(r'')) + (1-\lambda)F_H(\hat{c}(r''))) \geq (1+r'')^{-1} \tag{B.5}$$

$$F_L(\hat{c}(r'')) < (1+r'')^{-1} \tag{B.6}$$

Thus, the proof of part two pins down to showing that (B.5) and (B.6) are not mutually exclusive. By combining (B.5) and (B.6), we obtain:

$$\lambda F_L(\hat{c}(r'')) + (1-\lambda)F_H(\hat{c}(r''))) > (1+r'')^{-1} > F_L(\hat{c}(r'')) \tag{B.7}$$

Since, $F_L(r'') \geq F_H(r'')$ (due to the FOSD assumption), the is always an

interest rate, denoted by r'' which satisfies (B.7).

B.3.2 Proof of Proposition 2

Part one: I present the case where the entrepreneur is of LEC type. Similar intuition applies for the case where the entrepreneur is of HEC type.

Recall that when the type of the entrepreneur is known, the probability of default is $1 - F_L(\hat{c}(r))$, if the entrepreneur is of LEC type, and $1 - F_H(\hat{c}(r))$, if the entrepreneur is of HEC type. Also $\hat{c} = R - (1 + r)$. Recall that an LEC entrepreneur promises $1 + r^*_{GR}$, when a CRA exists, and $1 + r^*_{II}$, otherwise, where $r^*_{II} > r^*_{GR}$. Recall also that $F_i(\hat{c}(r))$ is a non-decreasing function of $\hat{c}$. As a result, $\hat{c}_{GR} > \hat{c}_{II}$. Following the previous remarks, $F_L(\hat{c}(r_{GR})) > F_L(\hat{c}(r_{II}))$, which implies that the probability of default is higher in the regime without a CRA.

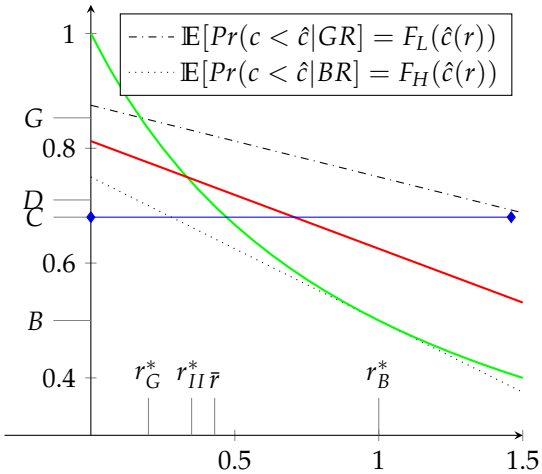


Figure B.3. r and $\tilde{s}$ with and without CRA

Part two: Suppose that we consider the ex-ante probability of success in a regime with a CRA. If the entrepreneur is of LEC type, he receives a good

rating, and the probability of success equals G, whereas the interest rate equals r_G^*. Similarly, if the entrepreneur is of HEC type, he receives a bad rating, and the probability of success equals to B, whereas the interest rate equals to r_B^*. Thus, in the regime with a CRA, the expected probability of effort equals $\lambda G + (1 - \lambda)B$, which is the linear combination of G and B. The blue line depicts the expected probability of success, which equals C when $\lambda = 0.5$.

Suppose now the regime without a CRA. The red line depicts the investor's beliefs about the probability of success as a function of r, for the case where $\lambda = 0.5$. Note that for any value of λ, there is a non-empty set $[r_G^*, \bar{r}]$ where the red line lies above the blue line. This is because for r_G^*, the red line captures the linear combination of G and D, whereas the blue line captures the linear combination of G with B, where B is always lower than D. Note also that for r_G^*, the green curve is above the red line, due to its continuity. In order to show the expected probability of effort when there is no CRA exceeds the expected probability of effort when there is a CRA, it is sufficient to show that the green curve crosses the red line for r_{II}^* smaller than $\bar{r}$. This is because at the crossing point, the value of $(1 + r_{II}^*)^{-1}$ equals the expected probability of success when there is no CRA. We prove that this is the case by the method of contradiction. The negative slope of the red line implies that the only case that the green curve crosses first the blue line and then the red line, i.e., crossing the red line for $r > \bar{r}$, is if the green curve is concave. The green curve (depicting $(1 + r)^{-1}$), however, is convex for each value of r. Thus, the green curve crosses the red line for r_{II}^* smaller than $\bar{r}$. Hence, the expected probability of success in a regime without a CRA exceeds the corresponding probability in a regime with a

CRA, i.e.,

$$\lambda F_L(\hat{c}(r^*_{II})) + (1-\lambda)F_H(\hat{c}(r^*_{II})) \geq \lambda F_L(\hat{c}(r^*_{GR})) + (1-\lambda)F_H(\hat{c}(r^*_{BR})) \quad \text{(B.8)}$$

which implies part two of the Proposition 2:

$$\begin{aligned} \lambda[1 - F_L(\hat{c}(r^*_{GR}))] + (1-\lambda)[1 - F_H(\hat{c}(r^*_{BR}))] \geq \\ \lambda[1 - F_L(\hat{c}(r^*_{II}))] + (1-\lambda)[1 - F_H(\hat{c}(r^*_{II}))] \end{aligned} \quad \text{(B.9)}$$

B.3.3 Proof of Proposition 3

Recall that the entrepreneur exerts effort as long as $c < R - (1 + r_k)$, and that the equilibrium condition for each regime is given by:

$$(1 + r^*_k)^{-1} = \begin{cases} F_L(\hat{c}(r^*_{GR})) & \text{if } k = \text{GR} \\ F_H(\hat{c}(r^*_{BR})) & \text{if } k = \text{BR} \\ \lambda F_L(\hat{c}(r^*_{II})) + (1-\lambda)F_H(\hat{c}(r^*_{II})) & \text{if } k = \text{II (no CRA)} \end{cases} \quad \text{(B.10)}$$

Part one: Part one is an implication of FOSD. The proof is rather intuitive and is based on the fact that, for a given interest rate r, the probability of default is weakly higher for an HEC project. As $F(\hat{c}(r))$ is an non-increasing function of $\hat{c}$ and $\hat{c}$ is a decreasing function of r, the curve of $F_L(\hat{c}(r))$ crosses the curve of $(1 + r)^{-1}$ before the curve of $F_H(\hat{c}(r))$ does, which implies that $(1 + r^*_{GR}) < (1 + r^*_{BR})$. Similarly, the curve $\lambda F_L(\hat{c}(r)) + (1-\lambda)F_H(\hat{c}(r))$ crosses the curve of $(1 + r)^{-1}$ on the right of the curve of $F_L(\hat{c}(r))$ and on the left of the curve of $F_H(\hat{c}(r))$ does, which implies that $r^*_{GR} < r^*_{II} < r^*_{BR}$.

Part two: In order to show part two I use Proposition 2. From part two of Proposition 2, I obtain:

$$\lambda F_L(\hat{c}(r^*_{II})) + (1-\lambda)F_H(\hat{c}(r^*_{II})) \geq \lambda F_L(\hat{c}(r^*_{GR})) + (1-\lambda)F_H(\hat{c}(r^*_{BR})) \quad \text{(B.11)}$$

After I substitute the equilibrium conditions into equation (B.11), I obtain:

$$(1+r^*_{II})^{-1} \geq \lambda(1+r^*_{GR})^{-1} + (1-\lambda)(1+r^*_{BR})^{-1} \quad \text{(B.12)}$$

which implies that:

$$(1+r^*_{II}) \leq \frac{(1+r^*_{GR})(1+r^*_{BR})}{\lambda(1+r^*_{BR}) + (1-\lambda)(1+r^*_{GR})} \quad \text{(B.13)}$$

The aim of this proof is to show that:

$$\lambda(1+r^*_{GR}) + (1-\lambda)(1+r^*_{BR}) \geq (1+r^*_{II}) \quad \text{(B.14)}$$

In order to show that (B.14) is satisfied, it is sufficient to show that:

$$\lambda(1+r^*_{GR}) + (1-\lambda)(1+r^*_{BR}) \geq \frac{(1+r^*_{GR})(1+r^*_{BR})}{\lambda(1+r^*_{BR}) + (1-\lambda)(1+r^*_{GR})} \quad \text{(B.15)}$$

Hence, the goal is to show that (B.15) holds. Relation (B.15) simplifies to:

$$2(\lambda^2-\lambda)(1+r^*_{GR})(1+r^*_{BR}) + (1-\lambda)\lambda(1+r^*_{GR})^2 + (1-\lambda)\lambda(1+r^*_{BR})^2 \geq 0$$

which simplifies further to:

$$(1-\lambda)\lambda[(1+r^*_{GR})^2 + (1+r^*_{BR})^2 - 2(1+r^*_{GR})(1+r^*_{BR})] \geq 0$$

which is satisfied as long as:

$$(1-\lambda)\lambda[(1+r^*_{GR})-(1+r^*_{BR})]^2 \geq 0 \tag{B.16}$$

where (B.16) holds, given that $\lambda \in (0,1)$. Thus, (B.14) is satisfied.

B.3.4 Proof of Proposition 4

Following the assumption that the CRA reveals its private signal truthfully and without cost, and given that the CRA's precision level is captured by α, the investors' beliefs about the probability of success are formed as follows:

$$\tilde{s} \equiv \begin{cases} \tilde{s}(\tilde{r}_{GS}) = F_L(\hat{c}(\tilde{r}_{GS}))\alpha + F_H(\hat{c}(\tilde{r}_{GS}))(1-\alpha) & \text{if Good Signal (GS)} \\ \tilde{s}(\tilde{r}_{BS}) = F_L(\hat{c}(\tilde{r}_{BS}))(1-\alpha) + F_H(\hat{c}(\tilde{r}_{BS}))\alpha & \text{if Bad Signal (BS)} \end{cases} \tag{B.17}$$

Given the level of precision α, the equilibrium condition for an entrepreneur with a good or a bad signal, respectively, are:

$$(1+\tilde{r}^*) = \begin{cases} (1+\tilde{r}^*_{GS}) = [F_L(\hat{c}(\tilde{r}_{GS}))\alpha + F_H(\hat{c}(\tilde{r}_{GS}))(1-\alpha)]^{-1} & \text{if GS} \\ (1+\tilde{r}^*_{BS}) = [F_L(\hat{c}(\tilde{r}_{BS}))(1-\alpha) + F_H(\hat{c}(\tilde{r}_{BS}))\alpha]^{-1} & \text{if BS} \end{cases} \tag{B.18}$$

Note that since $\alpha > 0.5$, if the market survives after a bad signal, this must be also true after a good signal. The proof of proposition 4 pins down to showing that for a given $\alpha = \tilde{\alpha} < 1$, there is at least an interest rate, denoted as $\tilde{r}$, where the following two conditions are not mutually

exclusive.

$$(1+\tilde{r}_{BR|\tilde{\alpha}})^{-1} \leq [F_L(\hat{c}(\tilde{r}_{BS}))(1-\tilde{\alpha}) + F_H(\hat{c}(\tilde{r}_{BS}))\tilde{\alpha}] \tag{B.19}$$

$$(1+\tilde{r}_{BR|\alpha=1})^{-1} > F_H(\tilde{r}_{BR|\alpha=1}) \tag{B.20}$$

where (B.19) implies that for $\alpha = \tilde{\alpha}$ an HEC entrepreneur raises funds (the market survives), and the (B.20) implies that for $\alpha = 1$ (benchmark case) an HEC entrepreneur can not raise funds (the market breaks down). Note that (B.19) and (B.20) are not mutually excursive because $F_H(\hat{c}(\tilde{r})) < F_L(\hat{c}(\tilde{r}))$, for any r.

B.3.5 Proof of Proposition 6

Note that the fee is strictly positive as long as there is at least one value of r such that:

$$F_L(\hat{c}(r))|P=0) \geq (1+r)^{-1} \tag{B.21}$$

Note also that a positive fee shifts the $F_L(r)$ curve downwards, thus, (B.21) is the sufficient condition for an equilibrium to exist. The fee is zero if for any r but r^*, $F_L(\hat{c}(r)|P=0) < (1+r)^{-1}$ and for $r = r^*$, $F_L(\hat{c}(r^*)|P = 0) \geq (1+r^*)^{-1}$. Observe that these two conditions reflect the minimum value of $F_L(.)$ for which the market does not collapse, which is unaffected by the profit-maximizing assumption due to the non-positive fee. Hence, allowing a CRA to charge a fee does not affect financing opportunities.

The proof of proposition 6 relies on the remarks that $\hat{c}_L$ is an decreasing function of P^{max}, and $F_L(\hat{c}(r))$ is a non-decreasing function of $\hat{c}_L$. Hence, $F_L(\hat{c}(r)|P = P^{max}) \leq F_L(\hat{c}(r)|P = 0)$, and the equilibrium interest

rate given by $(1+r^*)^{-1} = F_L(\hat{c}(r^*))$, will be higher if the CRA charges the profit maximizing fee.

B.4 Case where the entrepreneur observes a signal about the realized cost

The aim of this section is to show that the benchmark setting is qualitatively similar to a setup where the entrepreneur observes an informative signal about the cost, rather than its realized value. For mathematical convenience, and without loss of generality, I assume that this signal is unbiased:

$$\sigma_c = c + \epsilon$$

where $\epsilon \sim N(0, \sigma_\epsilon^2)$. This interpretation implies that if the realized signal is σ_c', then Bayesian updating results in $\mathbb{E}[c|\sigma_c = \sigma_c'] = \sigma_c'$.

I first deal with the implications on the entrepreneur's problem. Recall that exerting effort is profitable as long as $c \leq R - (1+r)$. Since the entrepreneur observes σ_c and not c, his critical condition for exerting effort will depend on σ_c (rather than c). Thus, the entrepreneur exerts effort as long as $\mathbb{E}[c|\sigma_c] \leq R - (1+r)$, which is true if $\sigma_c \leq R - (1+r)$. Hence the effort threshold $\hat{c} \equiv R - (1+r)$ switches to $\hat{\sigma}_c \equiv R - (1+r)$.

I now deal with the implications on the investors' problem. Recall that investors' beliefs about the probability of an entrepreneur exerting effort determine $(1+r)$. Following the analysis of the previous paragraph, the investors' beliefs about that probability equals $\mathbb{E}[Prob(\sigma_c \leq \hat{\sigma}_c)|\Omega]$, instead of $\mathbb{E}[Prob(c \leq \hat{c})|\Omega]$, where Ω is investor's information set. Ob-

serve that even though $\hat{c} = \hat{\sigma}_c$, in general $\mathbb{E}[Prob(\sigma_c \leq \hat{\sigma}_c)|\Omega]$ differs from $\mathbb{E}[Prob(c \leq \hat{c})|\Omega]$. In order to show that the two models are identical, it is sufficient to show that there exists a unique $\hat{\hat{\sigma}}_c$ such that $\mathbb{E}[Prob(\sigma_c \leq \hat{\hat{\sigma}}_c)|\Omega] = \mathbb{E}[Prob(c \leq \hat{c})|\Omega]$. If this is the case, then replacing $\hat{c}$ with $\hat{\hat{\sigma}}_c$ generates the same model. Hence, the whole exercise pins down to showing that there exists a $\hat{\hat{\sigma}}_c$ which satisfies the aforementioned property exists.

The proof of that is straightforward. Since the cumulative distribution function of σ_c and c is a continuous and strictly increasing function, then for each $(1 + r)$ there is a unique $\hat{c}$ associated to the value $\mathbb{E}[Prob(c \leq \hat{c})|\Omega]$. In addition, for each value of $\mathbb{E}[Prob(s \leq \hat{\hat{s}})|\Omega]$, there is a unique $\hat{\hat{\sigma}}_c$ associated with it. Hence, there exists a unique $\hat{\hat{\sigma}}_c$ for which $\mathbb{E}[Prob(\sigma_c \leq \hat{\hat{\sigma}}_c)|\Omega] = \mathbb{E}[Prob(c \leq \hat{c})\Omega]$.

B.5 Uniform Distribution Example

In this section, I assume that the cost of effort is uniformly distributed with $c_L \sim U[\alpha\bar{c}, \beta\bar{c}]$ and $c_H \sim U[\gamma\bar{c}, \delta\bar{c}]$. The analysis is meaningful only if the probability of default is strictly positive. The probability of default is zero if effort is exerted for any feasible value of c. Thus, the probability of default is always positive as long as $\bar{c} > R - 1$. Additionally, I keep the assumption that the implied expected rate of return of both types is non-negative, i.e., $\frac{R-1-E[c_i]}{1+E[c_i]} \geq 0$. In order to satisfy these two conditions and simplify calculations, I set $\alpha = 0$ and $\beta = \delta = 1$. The parameter γ captures the efficiency level of the HEC type compare to the efficiency level of the LEC type; as γ approaches 1, the efficiency level of the HEC type approaches the efficiency level of the LEC type.

B.5.1 Regime without a CRA

After I incorporate the entrepreneur's problem, investors' beliefs are given by:

$$\tilde{s}_{II} \equiv \lambda \tilde{s}_L + (1-\lambda)\tilde{s}_H = \lambda \frac{R-(1+r_{II})}{\bar{c}} + (1-\lambda)\frac{R-(1+r_{II})-\gamma\bar{c}}{\bar{c}(1-\gamma)} \quad \text{(B.22)}$$

Substituting $\tilde{s}_{II}$ into the investors's zero profit condition, the equilibrium interest rate, $\bar{r}^*_{II}$, is given by:

$$1 + r^*_{II} = \quad \text{(B.23)}$$
$$\frac{[(1-\lambda\gamma)(R)-\gamma\bar{c}(1-\lambda)] - \sqrt{[(1-\lambda\gamma)(R)-\gamma\bar{c}(1-\lambda)]^2 - 4(1-\lambda\gamma)(1-\gamma)\bar{c}}}{2(1-\lambda\gamma)}$$

It can be shown that the probability λ and the return (R) of the project is positively related to the probability of market survival. In contrast, the cost of effort is negatively related to the probability of market survival. For this set of distributions, the equilibrium condition is quadratic in r. In the case where the equation has two distinct roots, the smallest root is the one which maximizes entrepreneur's profits.

B.5.2 Regime with a non-profit maximizer CRA

The investor's beliefs depend on the rating as follows:

$$\tilde{s} = \begin{cases} \tilde{s}_{GR} = \frac{R-(1+r_{GR})}{\bar{c}} & \text{if Good Rating} \\ \tilde{s}_{BR} = \frac{R-(1+r_{BR})-\gamma\bar{c}}{\bar{c}(1-\gamma)} & \text{if Bad Rating} \end{cases} \quad \text{(B.24)}$$

The equilibrium interest rates are given by:

$$1+r^* = \begin{cases} 1+r^*_{GR} = \frac{R-\sqrt{R^2-4\bar{c}}}{2} & \text{if Good Rating} \\ 1+r^*_{BR} = \frac{(R-\gamma\bar{c})-\sqrt{(R-\gamma\bar{c})^2-4\bar{c}(1-\gamma)}}{2} & \text{if Bad Rating} \end{cases} \tag{B.25}$$

The market existence condition when an entrepreneur holds a bad rating is $(R-\gamma\bar{c})^2 \geq 4\bar{c}(1-\gamma)$. Similarly, when an entrepreneur holds a good rating, the market existence condition is given by $R^2 \geq 4\bar{c}$. When these conditions are satisfied, the introduction of CRA leads to inferior results, as it does not improve financing opportunities and it increases the expected default probability.

An interesting case is when an HEC entrepreneur is not efficient enough to prevent market breakdown, but he can raise capital if he is pooled with an LEC entrepreneur. This is because the emerging interest rate is smaller, and the condition which prevents market breakdown is less restrictive ($\frac{[(1-\lambda\gamma)R-\gamma\bar{c}(1-\lambda)]^2}{(1-\lambda\gamma)} \geq 4\bar{c}(1-\gamma) > (R-\gamma\bar{c})^2$). Without a CRA both types are financed. In contrast, in the regime with a CRA, this pooling is prevented, and only an LEC entrepreneur is financed. Hence, the introduction of a CRA leads to under-financing of an entrepreneur with positive expected net return.

Lastly, when an LEC entrepreneur can be financed only if they can be distinguished from an HEC entrepreneur ($\frac{[(1-\lambda\gamma)R-\gamma\bar{c}(1-\lambda)]^2}{(1-\lambda\gamma)(1-\gamma)} < 4\bar{c} \leq R^2$), then without a CRA, the market collapses and neither an LEC nor an HEC entrepreneur is financed. Thus, the introduction of a CRA prevents the market of an LEC entrepreneur from collapsing, alleviating the problem of under-financing.

B.5.3 Regime with a profit-maximizer CRA

Investors observe the rating and whether a rating is necessary for financing ($[(1-\lambda\gamma)R-\gamma\bar{c}(1-\lambda)]^2 \geq 4(1-\lambda\gamma)(1-\gamma)\bar{c}$), and they form their beliefs about the probability of default as follows:

$$\tilde{s} \equiv \begin{cases} \tilde{s}_{\underline{GR}} = \frac{R-(1+\underline{r}_R)(1+\underline{P}^{max})}{\bar{c}} & \text{If GR \& rating not necessary} \\ \tilde{s}_{\bar{GR}} = \frac{R-(1+\bar{r}_R)(1+\bar{P}^{max})}{\bar{c}} & \text{If GR \& rating necessary} \\ \tilde{s}_{NR} = \frac{R-(1+r_{NR})}{\bar{c}} & \text{If No Rating} \end{cases} \tag{B.26}$$

The combination of the CRA's, the entrepreneur's and the investors' problem, determine the equilibrium interest rates r_R^* and r_{NR}^* for an entrepreneur with or without rating respectively.

$$(1+r_R^*) \equiv \begin{cases} (1+\underline{r}_R^*) = \frac{\bar{c}}{R-(1+r_{II}^*)} & \text{if rating not necessary} \\ (1+\bar{r}_R^*) = 2 & \text{if rating necessary} \end{cases} \tag{B.27}$$

$$(1+r_{NR}^*) = \frac{(R-\gamma\bar{c}) - \sqrt{(R-\gamma\bar{c})^2 - 4\bar{c}(1-\gamma)}}{2} \tag{B.28}$$

As Proposition 6 indicates, the market existence conditions are unaffected by the profit maximizing assumption.

Appendix C

Security Design with Endogenous Implementation Choice

C.1 Proofs

C.1.1 Proof of Proposition 1

Conditional that the market survives, i.e., $\lambda_G > \lambda_G^{min} \equiv \frac{I-[p_B S+(1-p_B)F]}{(p_G-p_B)(S-F)}$, the optimal security solves the following maximization problem.

$$\underset{g(S)',g(F)'}{\text{Maximize}} \quad p_G(S-g(S)') + (1-p_G)(F-g(F)') \quad \text{s.t.}$$

$$\lambda_G[(p_G g(S)' + (1-p_G)g(F)'] + \lambda_B[p_B g(s)' + (1-p_B)g(F)'] = I \tag{C.1}$$

$$0 \leq g(S)' \leq S \tag{C.2}$$

$$0 \leq g(F)' \leq F \tag{C.3}$$

Optimality of $g^*(F)' = F$

The investors' participation constraint can be written as:

$$\overbrace{[\lambda_G p_G + \lambda_B p_B]}^{C_S} g(S)' + \overbrace{[\lambda_G(1-p_G) + \lambda_B(1-p_B)]}^{C_F} g(F)' = I \tag{C.4}$$

Suppose that the entrepreneur offers a security with corresponding payments $g^*(F)' = F$ and $g^*(S)'$ which solves (C.4) given that $g^*(F)' = F$. Suppose now that the entrepreneur considers switching from $g^*(F)' = F$ to $g(F)'' = F - \epsilon$. For the investor's participation constraint to be satisfied, the payment in case of success should be not lower than $g(S)'' = g^*(S)' + \frac{C_F}{C_S} \times \epsilon$. This deviation is profitable as long as its benefit, which is given by $(1-p_G) \times \epsilon$, exceeds its cost, which is given by $p_G \times \frac{C_F}{C_S} \times \epsilon$. By simple algebra, we obtain that:

$$\begin{aligned} (1-p_G) \times \epsilon > p_G \times \frac{C_F}{C_S} \times \epsilon \implies \\ \frac{\lambda_B(p_G - p_B)}{-\lambda_G p_G - \lambda_B p_B} > 0 \end{aligned} \tag{C.5}$$

where (C.5) is never satisfied because $p_G > p_B$. Hence, under the optimal security $g^*(F)$ reaches its maximum value, i.e. $g^*(F) = F$. By substituting $g^*(F)' = F$ into (C.1) and rearranging, we obtain:

$$g^*(S)' = \frac{I - F}{\lambda_G p_G + \lambda_B p_B} + F$$

Intuitive Criterion

Suppose the security which characterized in the previous analysis, g^*, i.e., $g^*(S) = \frac{I-F}{\lambda_G p_G + \lambda_B p_B} + F$, $g^*(F) = F$. Let us allow for an alternative security

g', where its corresponding payments are denoted as $g(S)'$ and $g(F)'$. For security g^* *not* to be an equilibrium in the contracting game, it must be that there is a deviation to security g', which is preferred by a good type but not by a bad type, i.e.,

$$\mathbb{E}\, U[g'|t = G, \mu] > EU[g^*|t = G, \mu = \lambda]$$

$$\mathbb{E}\, U[g'|t = B, \mu] < EU[g^*|t = B, \mu = \lambda]$$

which can be rearranged as:

$$p_G(g^*(S) - g'(S)) + (1 - p_G)(g^*(F) - g'(F)) > 0 \tag{C.6}$$

$$p_B(g^*(S) - g'(S)) + (1 - p_B)(g^*(F) - g'(F)) < 0 \tag{C.7}$$

Recall that $p_G > p_B$. Thus, for (C.6) and (C.7) not to be mutually exclusive, it must be the case that $g'(F) > g^*(F)$ and $g'(S) < g^*(S)$. However, $g'(F)$ cannot exceed $g^*(F)$, since $g^*(F)$ reaches its maximum feasible value, i.e., $g^*(F) = F$.

C.1.2 Proof of Corollary 1

Both types are better-off as long as $g^*(S) < g^*(S)'$, i.e.

$$\frac{I\lambda_G - F\lambda_G(1 - p_G) + \lambda_B p_B S}{\lambda_G p_G + \lambda_B p_B} < \frac{I - F}{\lambda_G p_G + \lambda_B p_B} + F \implies$$

$$I\lambda_G - F\lambda_G(1 - p_G) + \lambda_B p_B S < I - F\lambda_B - F\lambda_G + F\lambda_G p_G + F\lambda_B p_B \implies$$

$$I\lambda_G + \lambda_B p_B S < I - F\lambda_B + F\lambda_B p_B \implies$$

$$I\lambda_G + \lambda_B[p_B S + (1 - p_B)F] < I$$

which holds since, by assumption, $[p_B S + (1 - p_B)F] < I$.

C.1.3 Proof of Lemma 3 - Proposition 2

The optimal security solves the following maximization problem.

$$\underset{g(S),g(F),\bar{g}}{\text{Maximize}} \;\; p_G(S - g(S)) + (1 - p_G)(F - g(F)) \quad \text{s.t.}$$

$$p_G(S - g(S)) + (1 - p_G)(F - g(F)) \geq I - \bar{g} \qquad (ICC_G)$$

$$p_B(S - g(S)) + (1 - p_B)(F - g(F))) \leq I - \bar{g} \quad (ICC_B)$$

$$\lambda_G[p_G g(S) + (1 - p_G)g(F)] + \lambda_B \bar{g} \geq I \qquad (PC_I)$$

$$0 \leq g(S) \leq S \;\; , \;\; 0 \leq g(F) \leq F, \;\; 0 \leq \bar{g} \leq I \qquad (LL)$$

Binding ICC_B

First, we show that under the optimal security ICC_B binds. We do this by following the method of contradiction. Suppose that the optimal security is given by $g(S)$, $g(F)$ and $\bar{g}$, which satisfy PC_I, ICC_G and ICC_B is not binding, i.e.,

$$p_B(S - g(S)) + (1 - p_B)(F - g(F))) < I - \bar{g}$$

Now consider the following deviation. Suppose that we increase $\bar{g}$ to $\bar{g}'' = \bar{g} + \epsilon$. Then, the LHS of PC_I increases, thus, PC_I is still satisfied. Also, the RHS of ICC_G decreases, thus, ICC_G is still satisfied. This implies

that, as long as $\eta \leq \frac{\lambda_B \epsilon}{\lambda_g p_G}$, there is always a deviation where we decrease $g(S)$ to $g(S)'' = g(S) - \eta$, which does violate ICC_G or PC_I. Such deviation would be profitable, because it would increase the expected utility of the good type. Hence, the initial hypothesis that under the optimal security ICC_B is binding does not hold.

Optimality of $g^*(F) = F$

Suppose for now that ICC_G is slack (we will come back to this in the end of the proof). Given that ICC_B binds, by substituting ICC_B into PC_I and solving with respect to $g(S)$, we obtain:

$$g^*(S) = \frac{\lambda_B(-Fp_B + F + p_B S) + \lambda_G I}{\lambda_G p_G + \lambda_B p_B} - \frac{\overbrace{\lambda_G(1-p_G) + \lambda_B(1-p_B)}^{R}}{\lambda_G p_G + \lambda_B p_B} g^*(F) \tag{C.8}$$

Condition (C.8) implies that the entrepreneur could increase $g(F)$ by ϵ and decrease $g(S)$ by $R \times \epsilon$, without violating PC_I. Increasing $g(F)$ by ϵ decreases the entrepreneur's expected utility by $(1 - p_G) \times \epsilon$, whereas decreasing $g(S)$ by $R \times \epsilon$ increases entrepreneur's expected utility by $p_G \times R \times \epsilon$. By simple algebra, we obtain that:

$$\begin{aligned} p_G \times R \times \epsilon > (1 - p_G) \times \epsilon \implies \\ \frac{(\lambda_B)(p_G - p_B)}{\lambda_G p_G + \lambda_B p_B} \geq 0 \end{aligned} \tag{C.9}$$

where (C.9) is always satisfied, given that $p_G > p_B$. Hence, under the optimal security $g^*(F)$ reaches its maximum value, i.e. $g^*(F) = F$. Substituting

$g^*(F) = F$ into PC_I we obtain:

$$g^*(S) = \frac{\lambda_G(I - F(1 - p_G)) + \lambda_B p_B S}{\lambda_G p_G + \lambda_B p_B}$$

Also, by substituting $g^*(F) = F$ into ICC_B we obtain:

$$\bar{g}^* = I - p_B(S - g^*(S))$$

The last part of the proof is to explore whether ICC_G is slack under the optimal security. By substituting $g^*(F) = F$ into ICC_G, we obtain:

$$p_G(S - g^*(S)) \geq I - \bar{g}^*$$

Recall that under the optimal contract ICC_B binds, i.e.,

$$p_B(S - g^*(S)) = I - \bar{g}^*$$

Hence, given that $p_G > p_B$, under the optimal contract ICC_G is redundant by ICC_B.

Intuitive Criterion

Suppose the security which characterized in the previous analysis, g^*, i.e., $g^*(S) = \frac{\lambda_G(I-F(1-p_G))+\lambda_B p_B S}{\lambda_G p_G+\lambda_B p_B}$, $g^*(F) = F$, and $\bar{g}^* = I - p_B(S - g^*(S)$. Let us allow for an alternative security g', where its corresponding payments are denoted as $g(S)'$, $g(F)'$ and $\bar{g}'$. For the security g^* *not* to be an equilibrium in the contracting game, it must be that there is a deviation to security g'

which, is preferred by a good type but not by a bad type, i.e.,

$$\mathbb{E}\, U[g'|t = G, \mu] > EU[g^*|t = G, \mu = \lambda] \tag{C.10}$$

$$\mathbb{E}\, U[g'|t = B, \mu] < EU[g^*|t = B, \mu = \lambda] \tag{C.11}$$

which can be rearranged as:

$$p_G(g^*(S) - g'(S)) + (1 - p_G)(g^*(F) - g'(F)) > 0 \tag{C.12}$$

$$p_B(g^*(S) - g'(S)) + (1 - p_B)(g^*(F) - g'(F)) < 0 \tag{C.13}$$

Recall that $p_G > p_B$. Thus, for (C.12) and (C.13) *not* to be mutually exclusive, it must be the case that $g'(F) > g^*(F)$ and $g'(S) < g^*(S)$. However, $g'(F)$ cannot exceed $g^*(F)$ since $g^*(F)$ reaches its maximum feasible value, i.e., $g^*(F) = F$.

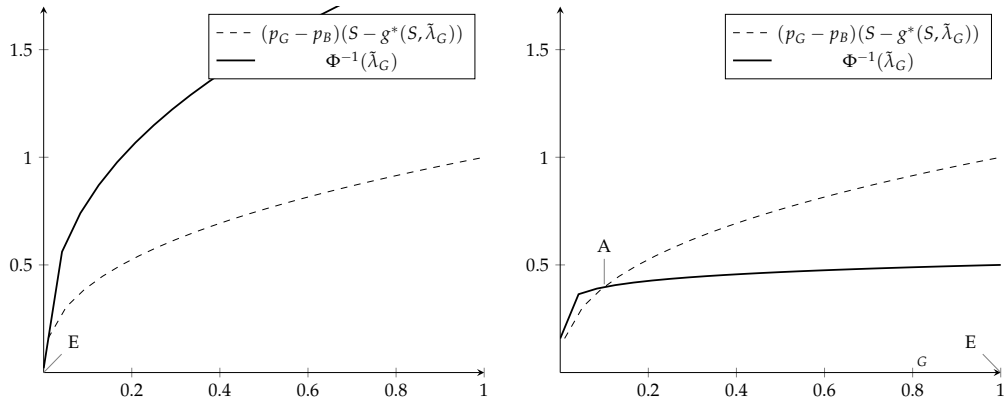
C.1.4 Proof of Lemma 5

Recall that if an interior equilibrium exists, it must satisfy the following condition:

$$\Phi^{-1}(\tilde{\lambda}_G) = (p_G - p_B)(S - g(S, \tilde{\lambda}_G)) \tag{C.14}$$

where $\tilde{\lambda}_G = \Phi(\hat{c})$.

If there is no value of $c' \in [\underline{c}, \bar{c}]$ for which the expected benefit, $((p_G - p_B)(S - g(S, \Phi(c')))$, exceeds c', then, there is no interior equilibrium. In this case, the entrepreneur never invests in her productivity level, and the market collapses. This case is captured in Panel A of Figure C.1.



Panel A Panel B

Figure C.1. Cases where there is no interior equilibrium.

Suppose now that there is $\hat{c} \in [\underline{c}, \bar{c})$ which satisfies (C.14). For $\hat{c}$ to be an interior equilibrium, investors' beliefs should be consistent, i.e., for cost $c' \in [\underline{c}, \bar{c})$ the expected benefit always exceeds c', whereas for cost $c'' \in (\hat{c}, \bar{c}]$, the expected benefit is always below c''. This necessary condition implies that if the expected benefit $((p_G - p_B)(S - g(S, \Phi(\bar{c})))$ exceeds $\bar{c}$, then the entrepreneur always invests in her productivity. Hence, the unique equilibrium in this case is for $\tilde{\lambda}_G = 1$. For instance, point A in Panel B of Figure C.1 can not be an interior equilibrium, because there is deviation to $\tilde{\lambda}_G = 1$, where the beliefs are consistent and the entrepreneur is better-off.

C.1.5 Proof of Proposition 4

Lemma 6 provides the necessary and sufficient conditions for an interior equilibrium to exist, for the regime where the entrepreneur is obliged to the project implementation. The payment $g^*(S, \tilde{\lambda}_G)'$ is determined in Lemma 7.

Lemma 6: Interior equilibrium - sufficient conditions

- *If there is no $c \in [\underline{c}, \bar{c}]$ which satisfies:*

$$c \leq (p_G - p_B)(S - g^*(S, \Phi(c))') \tag{C.15}$$

then, the unique equilibrium is for $\tilde{\lambda}_G = 0$, for which the market collapses.

- *If $\bar{c}$ satisfies:*

$$\bar{c} > (p_G - p_B)(S - g^*(S, \Phi(\bar{c})') \tag{C.16}$$

then, there is a unique equilibrium is for $\tilde{\lambda}_G = 1$.

- *If* (C.16) *is violated, and there exists $c \in [\underline{c}, \bar{c}]$ which satisfies* (C.17)*, then, the unique interior equilibrium is characterized by the maximum $\tilde{\lambda}_G \in (0, 1)$ which solves:*

$$\Phi^{-1}(\tilde{\lambda}_G) = (p_G - p_B)(S - g^*(S, \tilde{\lambda}_G)') \tag{C.17}$$

Given Proposition 1, the optimal security when the entrepreneur is obliged to project implementation, *given* $\tilde{\lambda}_G$, is captured in Lemma 7.

Lemma 7

$$g^*(F)' = F \ \ , \ \ g^*(S)' = \frac{I - F}{\tilde{\lambda}_G p_G + \tilde{\lambda}_B p_B} + F$$

In this regime, the entrepreneur invest in her productivity as long as:

$$\underbrace{c}_{\text{cost}} \leq \underbrace{(p_G - p_B)(S - g^*(S, \tilde{\lambda}_G)')}_{\text{benefit}} \equiv \hat{c} \tag{C.18}$$

The methodology that we follow in order to characterize the equilibrium conditions is identical with the methodology we followed in Lemma 5.

The proof of Lemma 6 is identical to the proof of Lemma 4. The main difference is that the payment in case of success is defined by Lemma 7 rather than Lemma 6.

C.1.6 Proof of Proposition 5

Recall that for a interior equilibrium to exist, the solid curve, which depicts $\Phi^{-1}(\tilde{\lambda}_G)$, should cross the expected benefit curve from below. Recall also that $\Phi^{-1}(\tilde{\lambda}_G)$ is continuous and increasing in $\tilde{\lambda}_G$. Also, by Corollary 1, and for any given value of λ_G, it holds:

$$g^*(S, \tilde{\lambda}_G)' \geq g^*(S, \tilde{\lambda}_G)$$

Hence, for any given value of $\tilde{\lambda}_G \in [\tilde{\lambda}_G^{min}, 1]$, for the expected benefit of investing in productivity, it holds:

$$(p_G - p_B)(S - g^*(S, \tilde{\lambda}_G)) \leq (p_G - p_B)(S - g^*(S, \tilde{\lambda}_G))$$

Thus, the dashed curve is always above the dotted curve. As a result, the intersection of the solid curve with the dashed curve always corresponds to a higher value of $\tilde{\lambda}_G$ than the one which corresponds to the intersection of the solid curve with the dotted curve.

Note that for $\tilde{\lambda}_G \in [0, \tilde{\lambda}_G^{min})$, the market collapses if the entrepreneur is obliged to project implementation. This corresponds to $\tilde{\lambda}_G^* = 0$.

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