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1 **New radial turbine dynamic modelling in a low-temperature adiabatic Compressed Air Energy**  
2 **Storage system discharging process**

3 Wei He <sup>a</sup>, Jihong Wang <sup>a, b, 1</sup>, Yulong Ding <sup>c</sup>

4 <sup>a</sup> School of Engineering, University of Warwick, Coventry, United Kingdom, CV4 7AL, United Kingdom

5 <sup>b</sup> School of Electrical & Electronic Engineering, Huazhong University of Science & Technology, China

6 <sup>c</sup> Birmingham Centre for Energy Storage, School of Chemical Engineering, University of Birmingham,  
7 Birmingham B15 2TT, United Kingdom

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9 **Abstract:**

10 It is challenging to gain insight of a Compressed Air Energy Storage (CAES) system dynamic behaviour  
11 under various operation conditions due to its complexity with mixed mechanical, thermal, chemical  
12 and electrical processes in one. Although a number of studies are reported on CAES steady state and  
13 dynamic modelling to reveal its characteristics, few studies have been reported in whole CAES system  
14 dynamic modelling involving a radial turbine. This paper explores a new method to analyse the  
15 transient performance of radial turbines while it is integrated with whole low temperature Adiabatic-  
16 CAES system. The proposed modelling method approximates the average air flow within single  
17 stator/rotor stage. By applying the principle of energy and torque balance on the transmission shaft,  
18 the dynamic speed-torque characteristics of the turbine is obtained with a “quasi dynamic iterative  
19 searching” process. The model is then integrated to a simulation platform, which is created to  
20 synchronise the wide range of time scale dynamic responses including heat transfer, mechanical and  
21 electrical energy conversion processes. As every component of the low temperature adiabatic CAES  
22 system is built on its fundamental physical and engineering principles, the model is capable of  
23 revealing the system transient characteristics. Based on the model, various simulation studies are  
24 conducted and the results are compared with the operation data. It provides a valuable tool for  
25 preliminary design of a radial turbine to test its suitability in full and partial load operation conditions  
26 and its transient behaviours.

27 **Keywords:**

28 Adiabatic Compressed Air Energy Storage, Radial turbine, Dynamic Modelling, Fluctuated demand

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<sup>1</sup> Corresponding author: Tel: +44 (0)24 765 23780 (J Wang)  
E-mail address: [jihong.wang@warwick.ac.uk](mailto:jihong.wang@warwick.ac.uk)

## 39 1. Introduction

40 Energy storage recently attracts great attentions in addressing the issues associated with rapid  
41 growth of power generation from intermittent renewable energy sources. Among various  
42 technologies for electrical energy storage, Compressed Air Energy Storage (CAES) is one of two proven  
43 technologies suitable for building large-scale plants (over 100 MW). There are two successfully  
44 operated CAES plants in the world. The first utility-scale CAES is the 290 MW Huntorf plant in Germany  
45 using salt dome for air storage, which was built in 1978. The other is an 110 MW plant with a capacity  
46 of 26 hours in McIntosh, Alabama, USA. The common feature of the two CAES plants is that they both  
47 require fossil fuels to achieve its rated power. To avoid consuming the fossil fuels, adiabatic CAES (A-  
48 CAES) with Thermal Energy Storage (TES) become a new technology development direction.

49 Wolf and Budt proposed a low-temperature A-CAES (LTA-CAES) using multi-stage radial  
50 compressors and expanders, in which operational temperature of heat storage was between 95-200  
51 °C [1]. According to their analysis, advantages of the LTA-CAES include the fast start-up  
52 characteristics, wide-ranging part load, highly available thermal working fluid, low pressure drop of  
53 compressed air and potential plant profitability [1]. Luo et al also studied a LTA-CAES system and  
54 indicated the cycle efficiency and heat energy recycle efficiency can reach around 68% and 60%,  
55 respectively [2]. With the acceptable cycle efficiency, potential operational flexibility of the LTA-CAES  
56 plant makes it a promising and feasible solution. But recently, Wang et al presented the first public  
57 experiment on a LTA-CAES system [3]. Although the feasibility of the LTA-CAES was demonstrated in  
58 practice, only an averaged cycle efficiency of 22.6% was achieved in the pilot "TICC-500" plant [3]. The  
59 low efficiency was mainly caused by nature of the transient system operation of the A-CAES system in  
60 realistic operations, which reduced the efficiency of components when they were operated from their  
61 design conditions.

62 From the published literature, previous investigations developed dynamic analysis of CAES  
63 systems with volume-based expanders. Sun et al. developed a complete dynamic mathematical model  
64 of a hybrid CAES-wind turbine system, in which scroll expander is selected for storing the compressed  
65 air [4]. Krupke et al further demonstrated the benefits of improved efficiency and flexibility brought  
66 to the wind turbine by the hybridization with the scroll expander based CAES system to smooth the  
67 time variant fluctuations [5]. Compared to the volume-based expanders, prior studies about the  
68 turbine-based CAES system mainly focused on steady-state thermodynamic analysis. Liu and Wang  
69 thermodynamically analysed a modified A-CAES system using steady-state mathematical models, and  
70 found 3% exergy efficiency increase [6]. Yao et al searched for the Pareto front of a small-scale CAES  
71 system between thermodynamic and economic performance [7]. Guo et al proposed a CAES system  
72 integrated with ejector and indicated a 3.41% efficiency increase by thermodynamic analysis [8].  
73 Particularly, the radial turbine is recognised for high efficiency and light weight in high power capacity  
74 compared to volume based expanders. The power rating of a radial turbine is usually up to several  
75 megawatts [9], and thus, it has great potential in small-scale and medium-scale CAES systems. But the  
76 radial turbines usually have low efficiency in off-design operations [10]. As a consequence, evaluating  
77 the off-design operations' performance and the transitions of the CAES system with the radial turbine  
78 become particularly important due to the varied performance of the radial turbine and complex  
79 interactions between the component and system.

80 However, there are very limited studies that addressed the dynamic characteristics of turbines.  
81 Arabkoohsar et al. considered the off-design performance of compressors/turbines using an empirical  
82 relation in the steady-state thermodynamic modelling [11]. Wolf dynamically simulated a high-  
83 temperature A-CAES system in which main system response was dominated by the heat storage and  
84 turbines were modelled using empirical polynomial models [12]. Similarly, Sciacovelli et al.

85 investigated how packed bed heat storage dynamics inducing off-design conditions of turbines [13],  
86 but mechanical responses of turbines were not resolved. Li et al. only corrected the mass flow rate of  
87 the air turbine under off-design conditions in an integrated system of a diesel genset and a CAES unit  
88 [14]. Zhao et al. selected two operational modes of the CAES system and carried out the off-design  
89 operational analysis in different power levels and speed levels [15], in which each operation (100%  
90 load or partial load) was considered as the static operation. Briola et al. used the experimental  
91 characteristic curves as the basis and predicted the operative behaviour of turbomachines over a  
92 wider range of conditions by the affine transformation method [16].

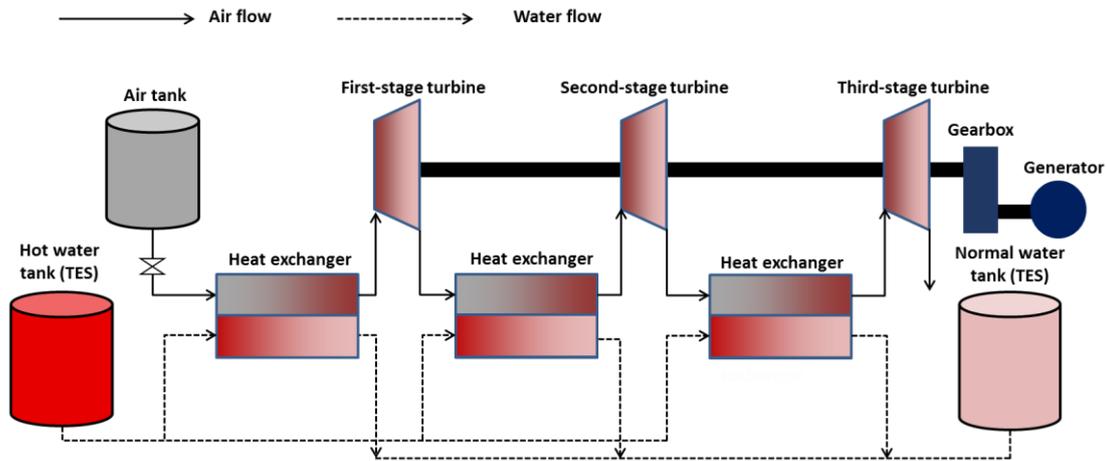
93 The complexity of turbine-generator rig dynamics is resulted from the multiple conversions  
94 between air momentum/potentials, mechanical and electrical energy. Therefore, a new “quasi  
95 dynamic iterative searching” method for modelling turbine dynamic behaviours is proposed in this  
96 study, aiming to understand the dynamic characteristics of turbines in a LTA-CAES operation cycle.  
97 The turbine dynamic changes is derived in response to the mechanical shaft dynamics which the  
98 turbine is connected to and the mechanical-electrical energy conversion happens through, by  
99 decoupling the air mass and momentum inside the turbine flow path as purely flow resistive [17]. This  
100 shaft dynamic response repeats every numerical step iteratively. The mean-line model is adopted to  
101 generate the characteristic equations of the turbine which are used for iterative searching of the  
102 balance states to match the shaft dynamic variations at every time-step. This model is built on the  
103 detailed knowledge of the turbines’ geometry, e.g. size of stator/rotor, to generate the characteristic  
104 curves of turbomachinery in various operational loads.

105 Furthermore, the LTA-CAES system has highly dynamic coupling in between components and wide  
106 variations of component dynamic response time scales. It is crucial to have simulation models accurate  
107 enough for main components. With these models, the assembled whole system model can reveal the  
108 system dynamic responses under various operation conditions and parameters. Therefore, this study  
109 starts from the key component (radial turbines) modelling and verification to the whole LTA-CAES  
110 system with integration of the turbines. A new “quasi dynamic iterative searching” method is  
111 proposed to reflect both the off-design operative behaviours and dynamic characteristics of the radial  
112 turbine in the LTA-CAES system. In addition to the radial turbines, the associated component models  
113 are also introduced which include heat exchanger (HEX), air storage, sensible thermal storage, and  
114 generator. The feasibility of the dynamic modelling methodology is investigated by comparing the  
115 simulation results with the published experimental data. Then, with the validated modelling method,  
116 effects of HEX’s design are evaluated, and a case study is presented in the paper to reveal the start-  
117 up and operation transitions of the LTA-CAES system and radial turbines between design and off-  
118 design operations.

## 119 120 **2. System configuration: discharge process of the low temperature Adiabatic Compressed Air** 121 **Energy Storage**

122 The system configuration considered in this study utilises the latest pilot A-CAES plant named  
123 “TICC-500”, which is built by the Key Laboratory of Cryogenics, Chinese Academy of Science [3]. The  
124 maximum output power of the generator is designed to be 500 kW. The pilot plant has five  
125 compression stages with inter-cooling and three expansion stages with inter-heating. The TES system  
126 employs pressurised water as the thermal fluid. Particularly in the discharge period, as shown in Fig.  
127 1, pressurised water from the hot water tank heats up compressed air from the air storage tank in the  
128 HEX before the air flowing into the turbine stages. Then, energy from the reheated compressed air is  
129 converted to the mechanical energy of the rotating turbines. The cooled water flows to the cold water  
130 tank which stores the cold for cooling down the compressed air during the charge period. Additionally,  
131 a throttle valve is placed before the HEX in the first stage to regulate the inlet pressure according to

132 the design operation. A single shaft connects all the three stage turbines, which drives the generator  
 133 through the gear box.  
 134



135  
 136 **Figure 1** Schematic diagram of the studied LTA-CAES discharge system

137  
 138

### 139 3. Mathematical models of the associated system components

140 This section introduces the mathematical models of the major components associated with the  
 141 LTA-CAES system. These models predict the performance of the components considering their basic  
 142 geometric parameters.

#### 143 3.1. Radial turbine

144 In a radial turbine, energy conversion can be approximately described by the Euler Equation of  
 145 Turbomachines [18]. Different geometries and fluid conditions significantly affect the performance of  
 146 the turbine. In an inflow radial (IFR) turbine used in CAES, compressed air enters the turbine  
 147 stator/rotor at the outer radius with one velocity and leaves at the inner radius with another velocity.  
 148 Fig. 2 illustrates the velocity triangles of air at inlet and outlet of stator and rotor. Fig. 2(a) shows the  
 149 velocity triangles in the design operation and those in the off-design operations are illustrated in Fig.  
 150 2(b). In the design operation, air flows into the rotor according to the designed inlet angle. In contrast,  
 151 in the off-design operations, the mismatch occurs between the flow angle and the designed inlet angle,  
 152 leading to energy losses. The turbine is regarded as a purely resistive fluid flow component in which  
 153 the accumulations of mass, momentum and energy inside the turbine flow path are negligible [17].  
 154 Therefore, average air flow in the stator/rotor without the unsteadiness of internal flow is considered  
 155 in the IFR modelling. The “quasi dynamic” speed-torque characteristics of the radial turbine is  
 156 modelled through the mechanical transmission of turbine-generator rig.

157 In the modelling of the radial turbine, the considered losses of air flow energy include: 1) frictional  
 158 losses in both the stator and the rotor. The fractional losses are incurred by the velocity gradient layers  
 159 within the air flow due to the viscosity. Air flows in both the design and off-design operations have  
 160 fractional losses; and 2) incidence losses due to the non-zero effective angle of incidence in the off-  
 161 design operations. A non-zero incidence angle can arise from variations in the rotating speed, the flow  
 162 rate, and the flow angle due to the changed inlet guide vane [19]. If the relative flow inlet angle of the  
 163 rotor is less than the designed relative flow angle, a separation zone could be formed on the front of  
 164 the suction surface and gradually move to the outlet of the rotor. In contrast, if the relative flow inlet  
 165 angle is larger than the designed relative flow angle, the separation zone occurs on the front of the  
 166 pressure surface.

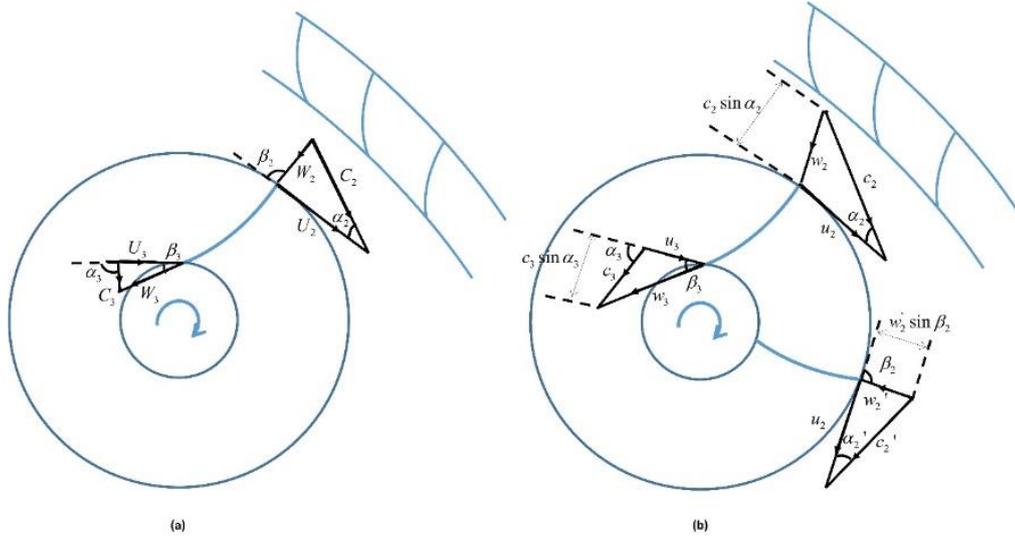


Figure 2 Schematic velocity triangles at inlet and outlet of the rotor.

The changes in the momentum of the air result in the work released by the turbine, which drives the externally applied torque, namely a generator in the A-CAES. The specific work can be described as follows,

$$w_t = \Delta h = u_2 c_{w2} - u_3 c_{w3} \quad (1)$$

where  $u$  and  $c_w$  are the tangential velocity of the rotor and air, respectively. When a 90 degree IFR turbine is considered at its nominal design condition, it leads to  $c_{w3} = 0$  and  $c_{w2} = u_2$ . Furthermore, the tangential velocity of the rotor is dependent on the rotor size and the rotation speed, which is

$$u_2 = \frac{\pi N D_2}{60} \quad (2)$$

where  $D$  is diameter of the rotor,  $N$  is rotational speed in rpm. The velocity of air at the inlet of the rotor is converted from the pressure drop of the compressed air in the stator. Velocity of the air at the outlet of the stator can be approximated by assuming adiabatic air flow in an equivalent nozzle with energy loss,

$$c_2 = \sqrt{2(\Delta h_{1-2s} - \Delta h_{2/loss})} = \sqrt{2\left(\frac{\kappa}{\kappa-1} R T_1 \left[1 - \left(\frac{p_2}{p_1}\right)^{\frac{\kappa-1}{\kappa}}\right] - \Delta h_{2/loss}\right)} \quad (3)$$

where  $c$  is absolute velocity,  $p$  is pressure,  $R$  is gas constant,  $T$  is temperature,  $\kappa$  is isentropic index, and  $\Delta h_{2/loss}$  is enthalpy loss due to passage friction in the stator. Subscript 1 and 2 denote the inlet and outlet of the stator. Energy loss in the stator can be approximated by [19],

$$\Delta h_{2/loss} = \frac{1}{2} \xi_2 c_2^2 \quad (4)$$

in which  $\xi_2$  is enthalpy loss coefficient of the stator.

189 As shown in Fig. 2(b), part-load operation results in the incidence losses due to the sudden  
 190 change of the flow direction which leads to flow separation. Wallace proposed a simplified method to  
 191 estimate this incidence loss in off-design operations based on energy conservation [20].

192 In the off-design operation, a sudden change of the flow direction forms shock, which is the  
 193 dissipation of the kinetic energy. Based on the assumed constant pressure from exit of the stator and  
 194 inlet of the rotor, equations of mass and energy conservation can be derived. The energy balance  
 195 becomes [20]

$$196 \quad c_p T_2 + \frac{c_2^2}{2} = c_p T_2' + \frac{c_2'^2}{2} + [c_2 \cos \alpha_2 - c_2 \sin \alpha_2 \frac{T_2'}{T_2} \cot \beta_2 - u_2] u_2 \quad (5)$$

197 where  $T_2'$  and  $c_2'$  are the varied temperature and velocity due to the sudden deflection of the flow,  $c_p$   
 198 is specific heat capacity.  $\alpha_2$  is rotor absolute inlet angle.  $\beta_2$  is rotor relative inlet angle. The equation  
 199 of continuity in radial direction is [20]

$$200 \quad \frac{w_2' \sin \beta_2}{c_2 \sin \alpha_2} = \frac{T_2'}{T_2} \quad (6)$$

201 where  $w_2'$  is the varied relative velocity. Based on Equations (5) and (6), the changed temperature  $T_2'$   
 202 due to the incidence loss can be obtained. Furthermore, the relative velocity of the air at the outlet of  
 203 the rotor is,

$$204 \quad w_3^2 = w_2^2 + 2(\Delta h_{2-3s} - \Delta h_{3loss}) + u_3^2 - u_2^2 \quad (7)$$

205 where  $\Delta h_{3loss} = \xi_3 w_3^2 / 2$  is enthalpy loss due to passage friction in the rotor [19]. Subscript 3 denotes  
 206 the outlet of the rotor. Therefore, considering all energy losses, the torque produced in the rotor can  
 207 be obtained. The torque is composed of two parts, namely, one is caused by the suddenly deflected  
 208 flow when entering the rotor, and the other one is produced within the rotor passage. Therefore, the  
 209 two torques can be expressed as [20]

$$210 \quad \tau_s = \frac{[c_2 \cos \alpha_2 - c_2 \sin \alpha_2 \frac{T_2'}{T_2} \cot \beta_2 - u_2] D_2}{2}; \tau_r = \frac{[c_2' \cos \alpha_2' + \frac{D_3}{D_2} c_3 \cos \alpha_3] D_2}{2} \quad (8)$$

211 where  $\tau_s$  is the torque due to flow deflection and  $\tau_r$  is the mechanical torque of the rotor. Accordingly,  
 212 the isentropic efficiency of the turbine is

$$213 \quad \eta_s = \frac{(\tau_s + \tau_r) \omega}{\Delta h_{13s}} = \frac{(\tau_s + \tau_r)}{\frac{\kappa}{\kappa-1} R T_1 \left[ \left( \frac{p_3}{p_1} \right)^{\frac{\kappa-1}{\kappa}} - 1 \right]} \frac{2\pi N}{60} \quad (9)$$

214 where  $\omega$  is the rotation speed of turbine in rad/s.

215 Simulation needs to find an appropriate pressure  $p_2$  satisfying the conservation of energy as  
 216 shown in equation (5) and the conservation of mass  $\dot{m}_N = \dot{m}_R$ , which are shown below [20],

$$217 \quad \dot{m}_N = c_2 \sin \alpha_2 \pi D_2 b_2 \rho_1 \left( \frac{p_2}{p_1} \right)^{1/k}; \dot{m}_R = c_3 \sin \alpha_3 \pi D_3 b_3 \rho_2 \frac{T_2}{T_2'} \left( \frac{p_3}{p_2} \right)^{1/k} \quad (10)$$

218 Therefore, when the continuity of mass flow is satisfied, an average air flow within the stator/rotor  
 219 stage is simulated and the performance of particular operation is estimated. Otherwise, if the two

220 mass flow rates are not equivalent, an iterative calculation has to be made to find the appropriate  
 221 pressure ratios through the stator and rotor.

222

### 223 3.2. Heat exchanger

224 A two stream HEX is considered in this work, which can be elements of any design, i.e. brazed-  
 225 plate, shell-and-tube or plate-and-plate frame. The HEX is modelled in one dimension which is parallel  
 226 to the flow direction of the fluid. It is assumed that negligible conduction occurs in the direction of the  
 227 flows compared to the convective heat transfer. A lumped overall heat transfer coefficient is used to  
 228 estimate the heat transfer between the two streams. There is no heat source or sink inside the HEXs.  
 229 Thermodynamic properties of the fluids across the discrete volumes, namely density and heat capacity  
 230 of both compressed air and pressurised water in the LTA-CAES, are assumed to be constant at any  
 231 time instant and the air properties are updated before the next time step based on the current  
 232 pressure and temperature. In addition, with assumed sufficient volume and heat transfer area of the  
 233 HEXs in the studied small-scale A-CAES system, velocities of the flows in the HEXs are relatively small  
 234 compared to the speed of sound, resulting in the small Mach numbers of both flows. Thus, the  
 235 simulation of HEXs assumes the incompressibility in both flows. Pressure drops of both fluids are not  
 236 considered at the early stage. There is no phase-change of the fluids in the HEXs. In the LTA-CAES  
 237 system, the water is pressurised to ensure its liquid state within the range of operating temperature.

238 Therefore, governing equations of the heat transfer within the HEXs are shown below,

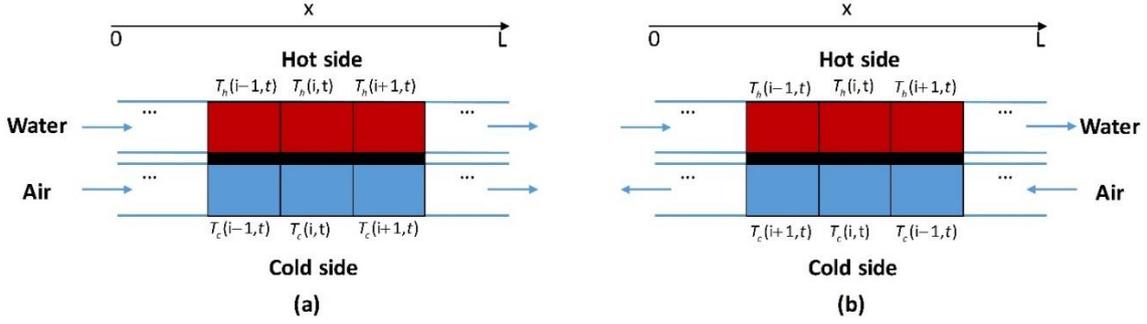
$$\begin{aligned}
 239 \quad & \rho_h c_{p,h} \left( \frac{\partial T_h(x,t)}{\partial t} + v_h \frac{\partial T_h(x,t)}{\partial x} \right) = q_{w,h} \\
 & \rho_c c_{p,c} \left( \frac{\partial T_c(x,t)}{\partial t} + v_c \frac{\partial T_c(x,t)}{\partial x} \right) = q_{w,c}
 \end{aligned} \tag{11}$$

240 where  $T$  is temperature of the fluid,  $c_p$  is heat capacity,  $\rho$  is density of the fluid.  $v$  is velocity of the  
 241 fluid. Subscript  $h$  and  $c$  denote the hot fluid and cold fluid respectively. In this study, during the  
 242 discharge period, compressed air (the cold fluid) is heated up through the HEX by the hot pressurised  
 243 water flowing on the other side of the HEX.  $q_w$  is the source term representing heat flux across the  
 244 HEX wall. The heat source term can be represented by

$$245 \quad q_{w,h} = \frac{h_{HEX} A (T_c(x) - T_h(x))}{V_h}; q_{w,c} = \frac{h_{HEX} A (T_h(x) - T_c(x))}{V_c}; \tag{12}$$

246 where  $V$  is volume of the HEX,  $h_{HEX}$  is overall heat transfer coefficient.

247 As it is difficult to solve the partial differential equations (PDEs) analytically, this study uses finite  
 248 difference method to solve the PDEs. Fig. 3(a) and Fig. 3(b) illustrate the discretisation of the flow  
 249 channels of co-current HEX and counter-current HEX respectively. Each of the flow channels is  
 250 discretised into a finite number of elements from the inlet to outlet.



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**Figure 3** Illustrated diagram of HEX simulation. The co-current HEX is shown in (a) and the counter-current HEX is shown in (b).

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Therefore, the finite difference method converts the PDEs to a system of algebraic equations, in which the convective term in the governing equation of both air and water is discretised using the first order upwind scheme. For simplicity, initial temperature distribution in both channels of the HEXs uses the constant value. With sufficient heat transfer area and volume, zero gradient boundary condition is set at the exits of both the flow channels. Consequently, boundary conditions and initial conditions of the co-current/counter-current HEXs are listed below,

$$T_h(x,0) = T_{W,0}; T_c(x,0) = T_{A,0}$$

260

$$T_h|_{x=0} = T_{W,0}; T_c|_{x=0} = T_{A,0}; \frac{\partial T_h}{\partial x}|_{x=L} = 0; \frac{\partial T_c}{\partial x}|_{x=L} = 0 \quad (\text{co-current HEX}) \quad (13)$$

$$T_h|_{x=0} = T_{W,0}; \frac{\partial T_c}{\partial x}|_{x=0} = 0; \frac{\partial T_h}{\partial x}|_{x=L} = 0; T_c|_{x=L} = T_{A,0} \quad (\text{counter-current HEX})$$

261

where  $x$  is the coordinate of position and  $L$  is the length of the flow channel.

262

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### 3.3. Electricity generator

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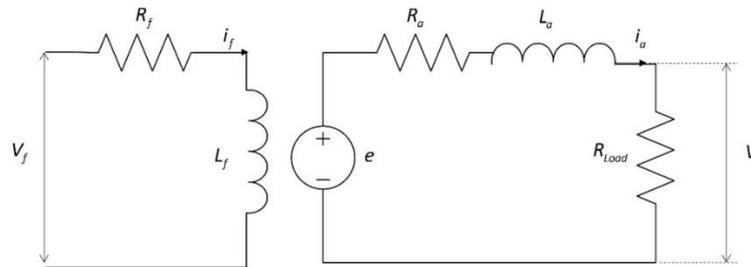
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The electricity generator can be either direct current (DC) or alternating current (AC). The investigated LTA-CAES system uses a DC electric generator to connect with the IFR turbine and generate electricity. In a separated excited DC generator, an external DC source supplies the field magnet winding. The equivalent circuit of a separated excited DC generator is illustrated in Fig. 4.



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270

**Figure 4** Illustrated resistance model of external excited DC generator.

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Therefore, the DC generator's torque,  $\tau_e$ , and back EMF,  $e$ , can be obtained, which are,

273

$$\tau_e = K_T i_a \quad (14)$$

274

$$e = K_E \omega_e \quad (15)$$

275 where  $K_T$  and  $K_E$  are torque constant and voltage constant respectively.  $\omega_e$  is the rotational speed  
 276 of the electric generator. In a separated excited DC machine, the voltage constant is proportional to  
 277 the field current  $i_f$ , which is

$$278 \quad K_E = L_{af} i_f \quad (16)$$

279 where  $L_{af}$  is the field-armature mutual inductance. According to Kirchhoff's circuit law, the circuit  
 280 shown in Fig. 3 can be presented as

$$281 \quad V_f = i_f R_f + L_f \frac{di_f}{dt} \quad (17)$$

$$282 \quad e = i_a R_a + L_a \frac{di_a}{dt} + i_a R_{load} \quad (18)$$

283 Substituting equation (15) into equation (18), the equation can be rewritten as

$$284 \quad \frac{di_a}{dt} = \frac{K_E \omega_e - i_a (R_a + R_{load})}{L_a} \quad (19)$$

285 where  $R_a$  is armature resistance,  $L_a$  is armature inductance,  $R_{load}$  is load resistance,  $i_a$  is armature  
 286 current,  $R_f$  is field resistance.

287 Moreover, the summation of the moments of inertias determines the dynamics of turbine-  
 288 generator rig. Inertias include the generator and the other components, because they are connected  
 289 to the same shaft. On the basis of the angular momentum conservation, the transfer of the mechanical  
 290 momentum can be described as,

$$291 \quad \frac{d\omega_e}{dt} = \frac{\tau_t - \tau_e - \tau_{fri}}{J_t + J_e} \quad (20)$$

292 where  $\tau_{fri}$  is mechanical losses due to friction, which is approximated by  $\tau_{fri} = B_m \omega$ , and  $B_m$  is the  
 293 viscous friction coefficient.  $J_t$  and  $J_e$  are the momentum of inertia of turbine and electrical generator  
 294 respectively. Substituting equation (14) into equation (20), the angular momentum balance can be  
 295 further presented as

$$296 \quad \frac{d\omega_e}{dt} = \frac{\tau_t - K_T i_a - B_m \omega_e}{J_t + J_e} \quad (21)$$

297

#### 298 3.4. Water tank (heat/cold storage)

299 As sensible heat/cold storage in the LTA-CAES using the pressurised water, TES modelling is  
 300 considered as water tank with an insulation layer. The model of heat/cold storage presents both hot  
 301 and cold water storage in the A-CAES system. The following assumptions are taken into account for  
 302 modelling the sensible TES: i) there is neither heat source nor sink in the water tank; ii) cylindrical  
 303 shape water tanks are considered; iii) only the temperature and the height of the water in the tank  
 304 are considered according to the incompressibility of water; iv) heat transfer between the water and  
 305 air in the tank is not considered; v) pressure drops of flow are not considered. Therefore, model of the  
 306 water tank is presented as,

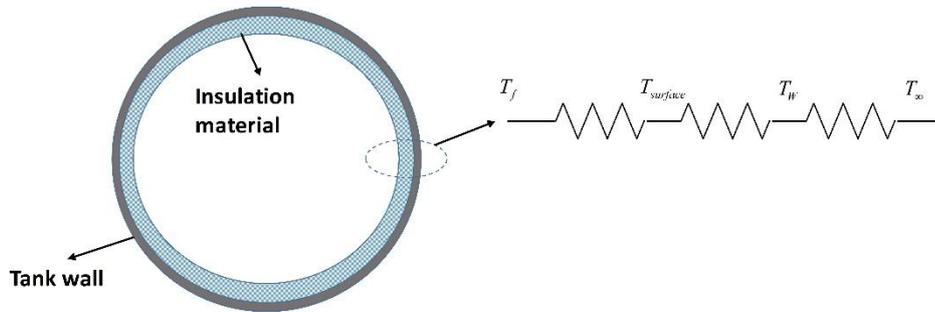
307

$$\frac{dT_{wt}}{dt} = \frac{(m_{in,w} h_{in,w} - m_{out,w} h_{out,w})}{c_{p,w} \rho_w A_{wt} H_{wt}} - \frac{h_{wt} A (T_{wt} - T_s)}{c_{p,w} \rho_w A_{wt} H_{wt}} \quad (22)$$

$$\frac{dH_{wt}}{dt} = \frac{m_{in,w} - m_{out,w}}{\rho_w A_{wt}}$$

308 where  $m_{in,w}$  and  $m_{out,w}$  are the mass flow rate of the water at inlet and outlet of the water tank.  $h_{in,w}$   
 309 and  $h_{out,w}$  are enthalpy of water at the inlet and the outlet, which are  $h_{in,w} = c_{p,w}(T_{out,w} - T_{wt})$  and  
 310  $h_{out,w} = c_{p,w}(T_{in,w} - T_{wt})$ .  $H_{wt}$  is the height of the water level in the tank.  $A_{wt}$  and  $A_{ws}$  are the bottom area  
 311 and surface area of the water tank.  $h_{wt}$  is the overall heat transfer coefficient between the TES tank  
 312 and ambience.

313 To consider the overall heat transfer coefficient between the internal water and ambience, Fig. 5  
 314 illustrates the heat transfer model using thermal resistances. Thus, the thermal resistance model  
 315 utilises an overall heat transfer coefficient to estimate the heat transfer through the insulation  
 316 material and the tank wall to ambience in radial direction. From the fluid inside the tank to ambience,  
 317 there are three thermal resistance layers. For simplicity, it is assumed that the temperature of the  
 318 tank wall is equivalent to that of the ambient air, namely  $T_w = T_\infty$ .  
 319



320

321 **Figure 5** Illustrated resistance model of the overall heat transfer coefficient of heat dissipation to the ambience.

322

323 Then, energy balance can be obtained based on the heat flux in the radial direction,

$$\frac{2\pi H k_{insulation}}{\ln(D_{out}/D_{in})} (T_f - T_{surface}) = h_{wall} \pi D_{out} H (T_{surface} - T_\infty) \quad (23)$$

325 where  $k_{insulation}$  is heat conductivity of the insulation material, and  $h_{wall}$  is natural heat transfer  
 326 coefficient between the tank wall and ambience.  $h_{wall}$  can be obtained using [21]

$$Nu(x) = \left[ \frac{7Gr(x)Pr^2}{100+105Pr} \right]^{1/4} + \frac{4(272+315Pr)}{35(64+63Pr)} \frac{x}{D_0} = \frac{h_{wall}}{k_f}; Gr(x) = \frac{g(T_{surface} - T_w)x^3}{\nu^2(T_{surface} + T_w)} \quad (24)$$

328 where  $x$  denotes the axial position of the water tank. Therefore, the overall heat transfer is [21]

$$\frac{1}{h_{wt}} = \frac{D_r \ln(D_{out}/D_{in})}{2k_{insulation}} + \frac{1}{h_{wall}} \frac{D_r}{D_{out}} \quad (25)$$

330 where  $D_r$  is the reference diameter paired with the overall heat transfer coefficient.

331

332 3.5. Air storage tank

333 An air tank is used as the compressed air storage. It is assumed that neither source nor sink is in  
 334 the tank. Besides air temperature variation, density and pressure of the air in the tank change during  
 335 the A-CAES operations. Therefore, based on the mass and energy conservations, the governing  
 336 equations describing compressed air in the air tank are [22],

$$337 \quad \frac{d\rho_{at}}{dt} = \frac{m_{in,a} - m_{out,a}}{V_{at}} \quad (26)$$

$$\frac{d(mU)}{dt} = (m_{in,a} h_{in,a} - m_{out,a} h_{out,a}) - h_{at} A_{as} (T_{at} - T_s)$$

338 where subscript *at* denotes air tank.  $m_{in,a}$  and  $m_{out,a}$  are the mass flow rate of the air at inlet and  
 339 outlet of the air tank.  $h_{in,a}$  and  $h_{out,a}$  are enthalpy of air at the inlet and the outlet, which are  
 340  $h_{in,a} = c_p(T_{out,a} - T_{at})$  and  $h_{out,a} = c_p(T_{in,a} - T_{at})$ .  $A_{as}$  is the surface area of the air tank.  $h_{at}$  is the overall  
 341 heat transfer coefficient between the air tank and ambience.

342 Using the ideal gas theory, internal energy of the compressed air in the storage can be presented  
 343 as,

$$344 \quad U = h - P / \rho; dh = c_p dT \quad (27)$$

345 Therefore, substituting equation (27) into energy equation shown in (26) leads to [22]

$$346 \quad \rho_{at} c_p \frac{dT}{dt} + \frac{m_{in,a}}{V_{at}} c_p (T_{at} - T_{in}) - \frac{dP_{at}}{dt} + \frac{h_{at} A_{at}}{V_{at}} (T_{at} - T_s) = 0 \quad (28)$$

347 Substituting temperature derivative expressed by equation (28) to the state equation of ideal air,  
 348 variation of air pressure in the storage tank can be obtained

$$349 \quad \frac{dP}{dt} = \frac{1}{V_{at}} (\kappa R T_{in,a} \dot{m}_{in,a} - \kappa R T \dot{m}_{out,a} + (\kappa - 1) h_{at} A_{at} (T_{at} - T_s)) \quad (29)$$

350 Using the proposed lumped volumetric heat transfer coefficient in [22], an experimental  
 351 correlation is fitted,

$$352 \quad h_{eff} = \frac{h_{at} A_{at}}{V_{at}} = a + b |m_{in,a} - m_{out,a}|^{0.8} \quad (30)$$

353 where *a* and *b* represent the heat transfer coefficient caused by natural convection and forced  
 354 convection in the storage.

### 355 3.6. Throttle valve

356 Air flow through the throttle valve is assumed to be isenthalpy processes, which satisfies

$$357 \quad h_u = h_d \quad (31)$$

358 where the subscript *u* and *d* present the upstream and downstream of the valve.

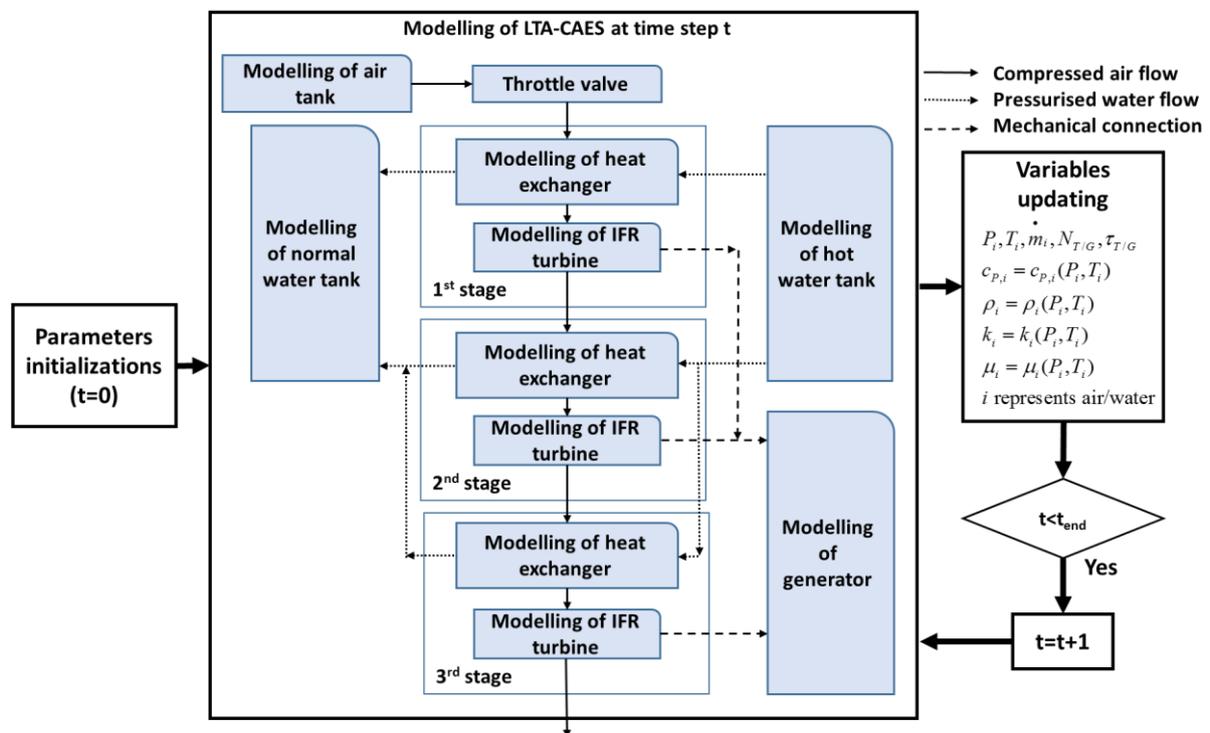
359

## 360 4. Dynamic modelling of the low temperature Adiabatic Compressed Air Energy Storage system 361 from the component level mathematical models

362 The whole system dynamic modelling is conducted by assembling all the models of the associated  
 363 components. By tracking the working fluids' thermodynamic variables, namely the compressed air and  
 364 the pressurised water, through inlets and outlets of the components, and interactions in between, the

365 component models are interconnected together to simulate the transitions and performance of the  
 366 whole system. The process irreversibility is the accumulation of the exergy losses in every components  
 367 that results in the whole system losses. The exergy decreases of both the fluids considered in the LTA-  
 368 CAES system discharging include air pressure drop in the air tank, water temperature reduction from  
 369 the hot water tank to the normal water tank, heat dissipations to ambience of the both air and water  
 370 tanks, throttling losses, flow losses (frictional losses and incidence losses) in the turbines, and exhaust  
 371 air loss. All of these exergy variations of air and water are mathematically described in the system  
 372 model presented in Section 3. Among them, the air pressure drop and the water temperature  
 373 decrease in the water tank can be recovered by combining the charging to fulfil the full energy storage  
 374 cycle. This study does not consider thermal inertias/resistances caused by the casting of components,  
 375 pressure drops of flow in pipes/channels, and thermal losses of generators.

376 Fig. 6 shows the dynamic modelling framework of the LTA-CAES discharge system structure. At the  
 377 beginning of the simulation, parameter initialisation is needed for all the component-level models,  
 378 such as design parameters of the air tank and the water tanks, geometric parameters of the radial  
 379 turbines, and parameters of the generator. At each time step, variables including pressure,  
 380 temperature and mass flow rate are simulated when the compressed air and the pressurised water  
 381 flow through different components in all the three stages. The speed-torque characteristics of the  
 382 shaft are calculated through the mechanical power transmission between the turbines and the  
 383 generator. The derivatives of these variables with respect to time are calculated based on the variables  
 384 at the current time step. These variable variations update the variables for the next time step. This  
 385 study builds up the component mathematical models using MATLAB programming/scripting and  
 386 assembles the whole system in Simulink environment to study its dynamic behaviour. Therefore, the  
 387 built-up ordinary differential equations are solved by ODE solvers in MATLAB/SIMULINK.  
 388



389

390

Figure 6 Schematic flowchart of dynamic modelling framework.

391

392

393

As stated in [23], ideal gas theory has limitations in applications of A-CAES and real gas properties of air need to be considered. Deviations of air properties between the ideal and real gas theories

394 increase when air has high pressure, and decrease when air has high temperature [24, 25].  
 395 Consequently, in a LTA-CAES system, the ideal gas theory may have large deviations due to the  
 396 relatively high pressure and low temperature in the operations. Therefore, the real gas effect is  
 397 considered in the modelling using *CoolProp*, which is an open-source thermophysical property library  
 398 which implements a series of thermophysical property correlations [26]. Thermodynamic properties,  
 399 such as heat capacity, density, viscosity and heat conductivity, of both the compressed air and the  
 400 pressurised water are updated using current pressure and temperature at each time step.

401

## 402 5. Results and discussion

403 With the model developed above, this Section presents the simulations of the dynamic LTA-CAES  
 404 system behaviours.

### 405 5.1. Validation of mathematical models of the components models and the system dynamic model

406 Before carrying out the analysis of the LTA-CAES by simulations, validations of these mathematical  
 407 models are first presented. In all the components, performance prediction of the turbine is the most  
 408 complicated due to the complex flow and design of the machine. Thus, validation of the one dimension  
 409 model is carried out by comparing the performance curves between the simulation and experimental  
 410 data. The test data of a high pressure radial turbine is used in the validation and the parameters of  
 411 the turbine are listed in Table 1. Fig. 7 shows the comparisons of the experimental and simulation  
 412 results. The detailed experimental parameters and the experimental performance curves can be found  
 413 in [27]. As shown in Fig. 7, the performance curve shows the acceptable agreement between the  
 414 experimental data and the simulation data. So the component-level mathematical model of the IFR  
 415 turbine is capable of predicting the performance of the turbine over a wide range of operation status  
 416 with different pressure ratios and rotational speeds.

417

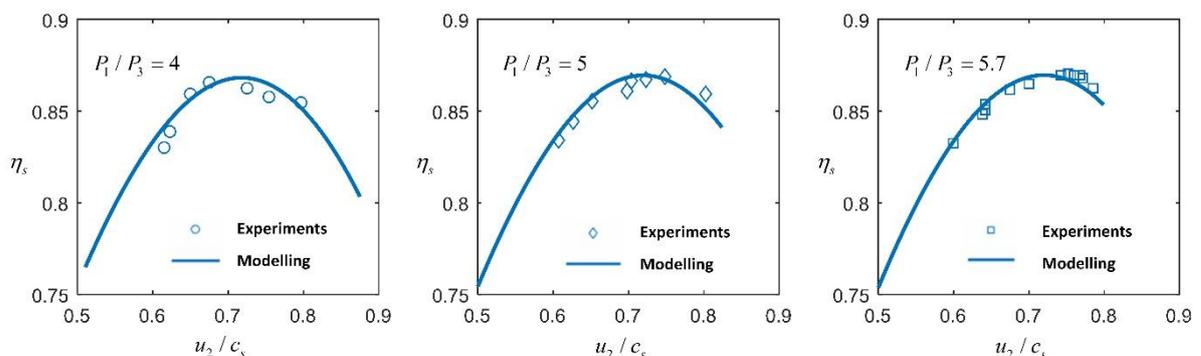
418

419 **Table 1** Parameters used in the validation of the turbine modelling. The parameters are read from [27].

Design parameter	Value	Geometric parameter	Value
Pressure ratio	5.73	Rotor inlet diameter, m	0.058
Inlet pressure, MPa	0.58	Rotor outlet mean diameter, m	0.026
Rotational speed, rpm	106,588	Rotor inlet width, m	0.006
Inlet temperature, °C	784	Rotor outlet width, m	0.021
Efficiency	87%	Rotor outlet angle, degree	57.4
Shaft power, kW	38	Nozzle outlet angle, degree	77.6

420

421



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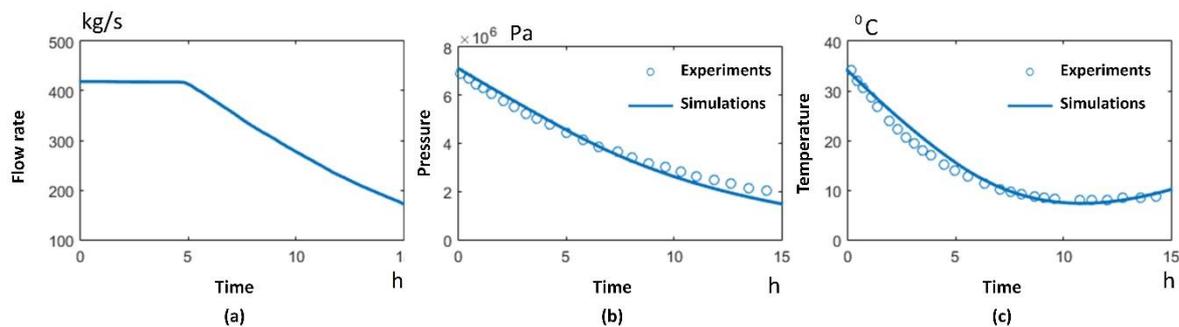
**Figure 7** validation of the mathematical model of IFR turbine.

424 Additionally, for the validation of air storage tank modelling, the published data and cavern  
 425 parameters from Huntorf plant are used in validation [22, 28]. Compared to the air storage tank in the  
 426 LTA-CAES system in this work, Huntorf CAES plant also uses the isochoric storage, namely constant-  
 427 volume air storage. Thus, both the air storage can be described by the same mathematical models  
 428 with different specifications of geometry and materials. The parameters of the cavern for air storage  
 429 in Huntorf CAES plant is listed in Table 2. A variable air mass flow rates during the discharge from the  
 430 experimental data [22, 28] is used as input, which is shown in Fig. 8(a). The pressure and temperature  
 431 variations during the discharge process are presented in Fig. 8(b) and 8(c), respectively. It indicates  
 432 that the modelling results agree with experimental data reasonably well.  
 433

434 **Table 2** Parameters of the cavern for air storage in Huntorf CAES plant. The parameters are from [22]

Parameter	Value
Air storage volume, m <sup>3</sup>	300,000
Air storage pressure range, MPa	4.6-6.6
Maximum cavern, MPa	7.2
Cavern wall temperature, °C	~50
Maximum discharge rate, MPa/h	0.1

441  
442



443

444

**Figure 8** validation of mathematical model of air storage tank.

445

446 At the system-level, the dynamic modelling is validated using the experimental data of the “TICC-  
 447 500” pilot plant [3]. Parameters of the system are listed in Table 3. In order to carry out component-  
 448 level modelling of the radial turbine, basic geometric parameters are needed for the simulation.  
 449 Therefore, based on the design parameters as shown in Table 3, this work estimates the geometric  
 450 parameters of the IFR turbines in three stages. The estimated parameters are shown in Table 4. For  
 451 operating the turbines in a range close to the design points, the simulation regulates the inlet pressure  
 452 to the design inlet pressure in every turbine-stage, if the inlet pressure is higher than the designed  
 453 value. As shown in Table 3, the highest water temperature of hot water tank is 108.2 °C and the  
 454 hydraulic pressure of water is 4 bar. The phase-changing temperature of water from the liquid state  
 455 to the gas state is approximately 143 °C, when the pressure of water is 4 bar. Thus, in the studied  
 456 operations of the “TICC-500” pilot plant, there is no two-phase flow of water in the hot water tank,  
 457 the HEXs, the normal water tank and pipes.

458

459 Through the comparisons, the performance of all the turbines are close to the performance of  
 460 those of designed operations. Because the optimum designs of these turbines are beyond the scope  
 461 of this study, these estimated geometric parameters are used to simulate the performance curve of  
 turbines at the system-level. In addition, a set of the gearbox and DC generators are also paired with

462 turbines. Energy loss is not considered in the mechanical transmission by the gearbox. Other  
 463 parameters used in the simulations are listed in Table 5. As the linear relationship between power and  
 464 torque of DC generators, at the early stage, a set of three 225HP (about 168 kW) DC generators are  
 465 bounded to present the 500 kW generator.

466 **Table 3** Parameters of “TICC-500” pilot A-CAES plant from [3].

467

Parameter	Value
Air storage volume, m <sup>3</sup>	100
Air storage pressure range, MPa	2.5-9.5
Pressure of water, MPa	0.4
Water tank volume, m <sup>3</sup>	12
Initial temperature of hot water tank, °C	108.2
Rated air mass flow rate, kg/h	8600
Rated speed, rpm	30,000
<i>First stage turbine</i>	
Inlet pressure, MPa	2.50
Rated outlet pressure, MPa	1.13
Inlet temperature, °C	100
Rated outlet temperature, °C	35
Rated adiabatic efficiency	82.6%
Rated shaft power, kW	150.5
<i>Second stage turbine</i>	
Inlet pressure, MPa	1.12
Rated outlet pressure, MPa	0.4
Inlet temperature, °C	100
Rated outlet temperature, °C	25
Rated adiabatic efficiency	81.0%
Rated shaft power, kW	185.5
<i>Third stage turbine</i>	
Inlet pressure, MPa	0.39
Rated outlet pressure, MPa	0.1
Inlet temperature, °C	100
Rated outlet temperature, °C	0.5
Rated adiabatic efficiency	81.6%
Rated shaft power, kW	236.3

468

469 **Table 4** Estimated geometric parameters of the three stage turbines and simulated performance.

Operational parameter	Value	Geometric parameter	Value
<i>First stage turbine</i>			
Inlet pressure, MPa	2.50	Rotor inlet diameter, m	0.1728
Outlet pressure, MPa	1.13	Rotor outlet mean diameter, m	0.0773
Inlet temperature, °C	100	Rotor inlet width, m	0.0044
Outlet temperature, °C	34.36	Rotor outlet width, m	0.0148
Adiabatic efficiency	82.34%	Nozzle outlet angle, degree	77
Shaft power, kW	149.4	Rotor outlet angle, degree	56.1
<i>Second stage turbine</i>			
Inlet pressure, MPa	1.12	Rotor inlet diameter, m	0.2226

Outlet pressure, MPa	0.4	Rotor outlet mean diameter, m	0.1050
Inlet temperature, °C	100	Rotor inlet width, m	0.0074
Outlet temperature, °C	18.23	Rotor outlet width, m	0.0253
Adiabatic efficiency	80.3%	Nozzle outlet angle, degree	77
Shaft power, kW	181.4	Rotor outlet angle, degree	56.1
<i>Third stage turbine</i>			
Inlet pressure, MPa	0.39	Rotor inlet diameter, m	0.2515
Outlet pressure, MPa	0.1	Rotor outlet mean diameter, m	0.1144
Inlet temperature, °C	100	Rotor inlet width, m	0.0192
Outlet temperature, °C	0.03	Rotor outlet width, m	0.0642
Adiabatic efficiency	78.72%	Nozzle outlet angle, degree	77
Shaft power, kW	225.2	Rotor outlet angle, degree	56.1

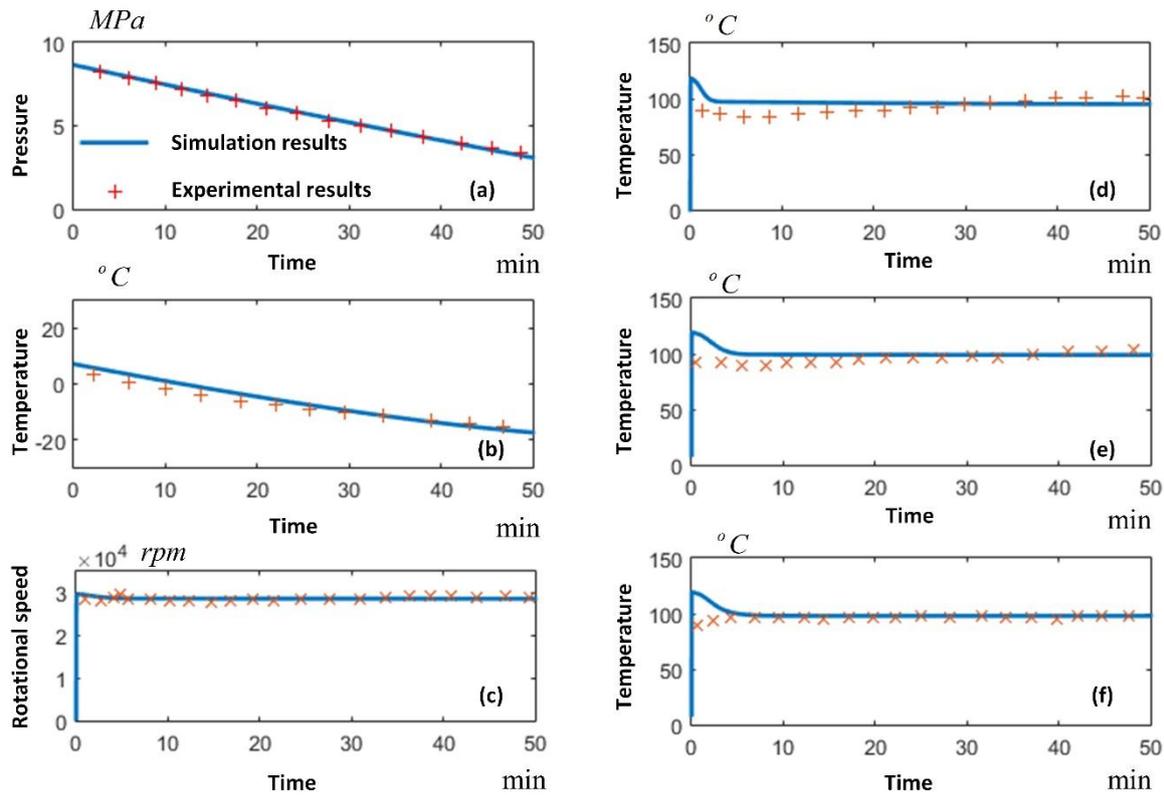
470

471 **Table 5** Parameters used in the simulations.

Parameter	Value
Armature resistance, $\Omega$ [29]	0.07571
Armature inductance, H [29]	0.001986
Field resistance, $\Omega$ [29]	33.46
Field inductance, H [29]	3.499
Field armature mutual inductance, H [29]	0.2874
Inertia, $\text{kg} \cdot \text{m}^2$ [29]	0.9177
Viscous friction torque, $\text{N} \cdot \text{m} \cdot \text{s}$ [29]	0.02289
Transmission ratio of gearbox	18
Transmission efficiency of gearbox	100%
HEX overall heat transfer coefficient, $\text{W} \cdot \text{m}^{-2} \cdot \text{K}^{-1}$	100
Diameter of TES tank, m	2
Height of TES tank, m	3
Averaged mass flow rate, kg/h [3]	7760

472

473 With the parameters listed in Tables 3, 4 and 5, dynamic modelling of the discharge process of the  
474 LTA-CAES system as shown in Fig. 1 is developed. Several results of the system during the discharge  
475 period are plotted in Fig. 9, including both the experimental data from [3] and the simulation results  
476 of temperature and pressure in storage tank in (a) and (b), rotational speed of the turbine in (c) and  
477 the inlet temperature of turbines in the three stages in (d), (e) and (f), respectively. As shown in these  
478 figures, although the experimental data may fluctuates during the discharge period, the simulated  
479 results closely track the dynamic responses of components in the system. In Fig. (d), (e) and (f), the  
480 simulation results agree with the experimental data during the quasi-steady-state operation (after  
481 about 5 mins) with a slight deviation between the experimental data and simulation at the beginning.  
482 This deviation is mainly caused by the inaccuracy of the initial condition's setting in simulating the  
483 HEXs, but the different initial temperature profiles' setting in the HEXs also gives an opportunity to  
484 analyse the thermal initial condition of the HEXs in the system dynamic modelling. With the insulation,  
485 heat losses of the water is negligible in the hot water tank to ambience during the 50 min discharging  
486 period. The water temperature in the hot water tank becomes 108.1 °C . Because of the normal water  
487 tank is positioned on the downstream of the HEXs and the turbines, the water temperature in the tank  
488 does not affect the performance of the turbines in the current discharging period. Without the  
489 parameters and initial condition of the normal water tank from [28], the temperature is not analysed  
490 in this work.



492

493

Figure 9 Validation of system level dynamic modelling framework.

## 494 5.2. Start-up of the low temperature Adiabatic Compressed Air Energy Storage discharge

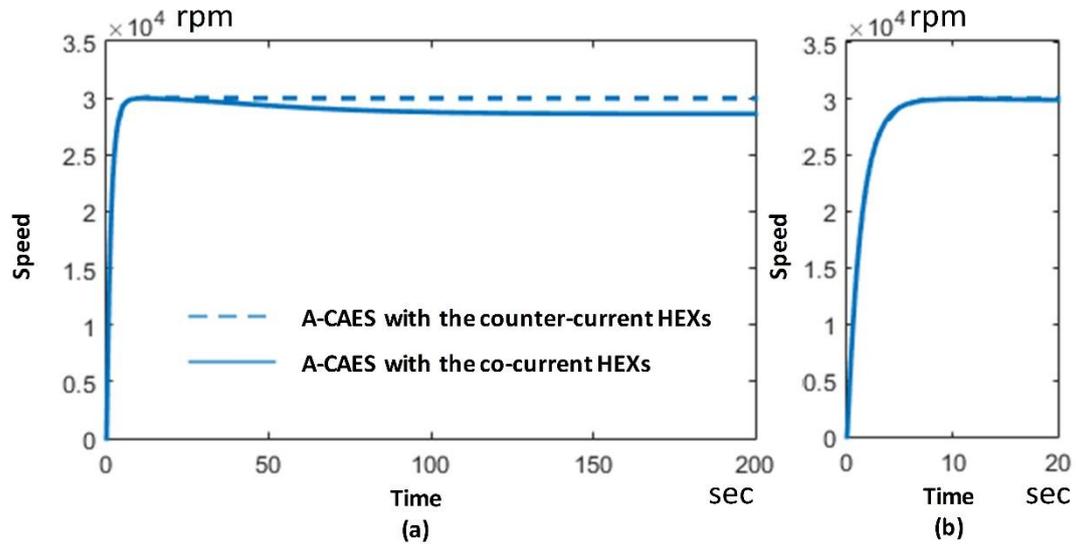
495 In the simulation, there is a slight decrease of the rotational speed of the turbine after the rapid  
 496 increase at the beginning during the start-up. In fact, the start-up can be divided into two periods. In  
 497 the first period, speed of the shaft increases rapidly due to the mechanical energy conversion between  
 498 the compressed air and the rotating shaft. It is governed by Equation (21) in which the changing rate  
 499 of the speed is influenced by the mechanical inertias connected to the shaft. Furthermore, the speed  
 500 gradually reduces to a certain level, which is a result of the delayed effect due to the thermal inertia  
 501 of heat transfer in the HEXs. As shown in Fig. 9(d), 9(e) and 9(f), because of the slowly established  
 502 steady temperature of the compressed air at the outlet of the HEXs, torques produced by the turbines  
 503 are affected. Consequently, these responses of heat transfer slow down building up the whole  
 504 system's steady-state operation.

505 To overcome this delayed thermal effect as simulated, in practice, at the start-up, the turbine is  
 506 usually operated first without connection with a generator. In order to gradually increase the  
 507 temperature of flowing air or casting, the turbine may be held in a lower speed initially and gradually  
 508 increasing to the rated value. When it rotates at the rated speed, if temperature distribution inside  
 509 the turbine is acceptable and inlet condition of air becomes steady, the external torque connects with  
 510 the turbine. This is how the steam turbine in a power plant operates in a "cold-start".

511 Furthermore, based on the mathematical model of the HEXs and dynamic modelling of the LTA-  
 512 CAES system, other possible configurations, such as the counter-current HEX, can be considered using  
 513 the basic models as presented by equation (11), (12) and (13). As shown in Fig. 1, the "TICC-500" pilot  
 514 plant utilised the co-current flow scheme of HEXs. It is well-known that the counter-current HEX  
 515 configuration has higher effectiveness than the co-current configuration. The outlet temperature of  
 516 the cold stream is possibly higher than that of the hot stream at the inlet of a counter-current HEX. In  
 517 contrast, the temperature of the cold stream cannot exceed the hot stream through the inlet to outlet

518 in a co-current configuration. Therefore, both the co-current and counter-current HEXs are simulated  
519 and the influenced dynamic system performances are compared.

520 In the simulations of the counter-current HEX using equation (11)-(13), since flow directions of  
521 two fluids are different, pairing the control volumes in the two flow channels to calculate the source  
522 term of heat flux across the wall is essential. The results of the LTA-CAES discharge are shown in Fig.  
523 10 and 11 in which the rotational speed is plotted in Fig. 10, and inlet temperature of the air flowing  
524 to the turbine in the three stages are plotted in Figure. 11. To compare the two flow schemes of the  
525 HEXs, all the simulations use the same parameters listed in Table 3, 4 and 5.



526

527 **Figure 10** The rotation speed of the shaft of A-CAES plants with co-current and counter-current HEXs in Fig. 10(a). The  
528 rising speed of the shaft at the beginning is particularly plotted in Fig. 10(b).

529

530 According to the results in Fig 10(a), it clearly indicates the faster steady-state response and higher  
531 effectiveness of the counter-current HEXs than the co-current HEXs. Although both the two A-CAES  
532 systems have almost identical rapid speed increase at the first several seconds as shown in Fig. 10(b),  
533 the system using the counter-current HEXs reaches a higher steady-state speed. There is a negligible  
534 decrease of the speed caused by the reduced thermal effect in the system using the counter-current  
535 HEXs in comparison to that using the co-current HEXs.

536 In fact, the fast response of the counter-current HEXs is a result of the enhanced mean net  
537 temperature difference and heat flux across the wall. With the same hot water stream (both the same  
538 temperature and flow rate), the improved heat transfer leads to higher temperature of the  
539 compressed air leaving the counter-current HEXs than that in the co-current HEXs, which is shown in  
540 Fig. 11. Because the initial very large rate of the air temperature increase in both systems is  
541 significantly caused by the initial condition for air temperature in the modelling, the focus of the  
542 simulation is on the physical dynamics of the heat transfer regardless of the initial temperature  
543 profiles, which is the varying temperature after the rapid increasing. As shown in Fig. 11(a), (c) and (e),  
544 compared to the co-current HEXs, there is no significant temperature decrease in the counter-current  
545 HEXs after the rapidly rising temperature at the beginning. Actually, air temperature in the counter-  
546 current HEXs also decreases but in a much modest way, which are shown in Fig. 11(b), (d) and (f).  
547 These achieved high steady-state air temperature in all the three stages improves the system response.  
548 Also, more input exergy of the compressed air flows to the turbine in every stage. These extra exergies  
549 produce more torques to drive the generator to a fast speed with the given electric load.

550

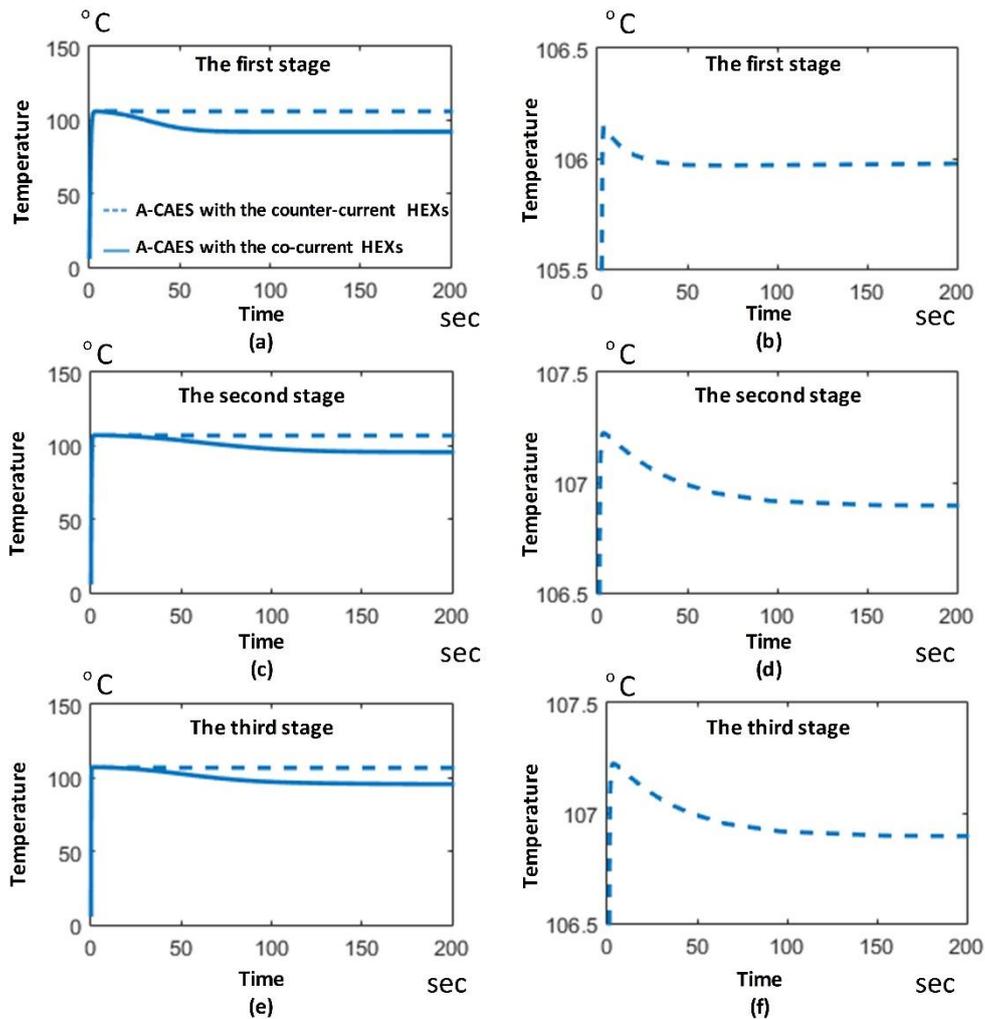


Figure 11 Temperature of compressed air at the inlet of the turbine in each stage.

551

552

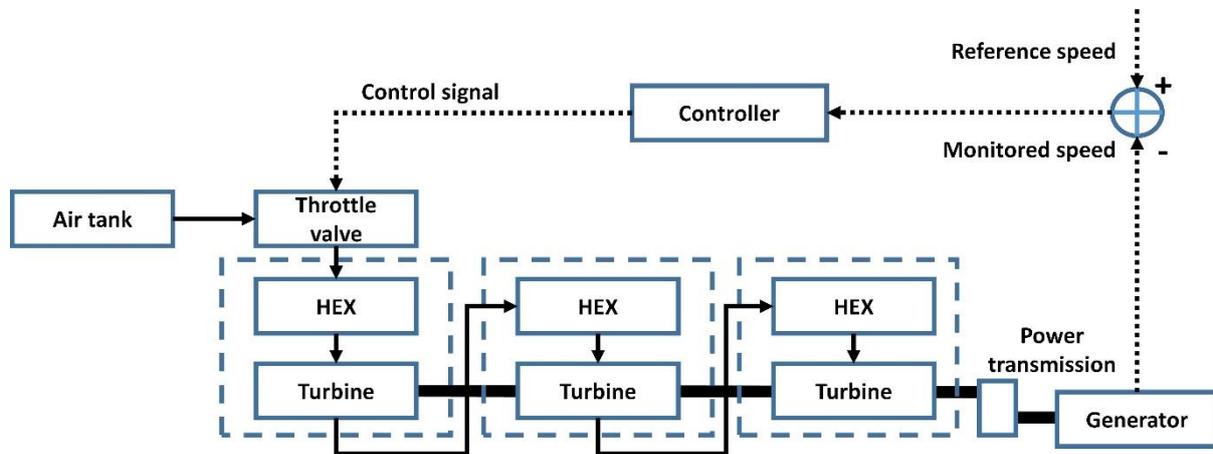
553

### 554 5.3. Effect of part-load operation of turbine in the low temperature Adiabatic Compressed Air Energy 555 Storage discharge

556 As an energy storage technology, flexible operation subject to a changing load or demand is a  
557 common application. Thus, this section presents a study of the operational transition due to a variable  
558 electric load. To follow the variable load, the A-CAES system is operated and controlled over a range  
559 of operations. In the discharge period, because of the difficulty to change the rotational speed of  
560 turbine-generator set, the rotational speed is maintained constant after the start-up [30]. To match  
561 the changed electric load without change of shaft speed, mass flow rate of the air flowing to the  
562 turbine is altered and controlled. Mass flow rate is controlled through changing the valve or inlet guide  
563 vane of the turbine at the first stage. In this study, the changing mass flow rate is achieved by a  
564 controller for tracking a reference rotational speed. The mass flow rate is assumed to be achieved by  
565 changing the displacement of valves in the first stage. Accordingly, performance of both the  
566 components and system will be changed due to the varied mass flow rate. This section aims to study  
567 how these variations occur dynamically and track the changed system performance due to the  
568 operation transitions.

569 Fig. 12 illustrates the diagram of the LTA-CAES with a PD controller. It needs to be noted that other  
570 control strategies are potentially feasible subject to applications. The reference speed uses the steady-  
571 state rotational speed with the electric load during the start-up. A signal pulse of the PD controller is

572 selected as 1 second in the simulation, which can be changed according to the particular applications  
 573 and experimental facilities. The tuned coefficients of the PD controller are proportional coefficient  
 574  $0.5 \times 10^{-4} \text{ kg}/(\text{s} \cdot \text{rpm})$  and derivative coefficient  $0.1 \times 10^{-5} \text{ kg}/\text{rpm}$ . The varied electric load represents the  
 575 variable power demand in practice. In power network, during the peak time, power load increases  
 576 and equivalent electrical resistance load decreases. In contrast, power load decreases and equivalent  
 577 electrical resistance load increases during the off-peak time. The followed profile of the variable  
 578 equivalent resistance load to the electric generator is shown in Fig. 13, which starts at  $1.45 \Omega$  for the  
 579 first 200 seconds, then gradually increases to  $1.85 \Omega$ , and finally decreases back to  $1.45 \Omega$  by a step  
 580 size of  $0.1 \Omega$  in sample period of 50 s. It needs to be noted that the selected variable load fitted to  
 581 operation between 100% and approximated 80% partial load. During the start-up, the controller is  
 582 deactivated and let the system change to the steady-state operation. Then, the controller is activated  
 583 at the time of 150 second. The operation exemplifies the capability of the dynamic modelling  
 584 framework to study the transient behaviour of the LTA-CAES discharge. The modelling library and  
 585 framework can also simulate other possible variable loads.  
 586

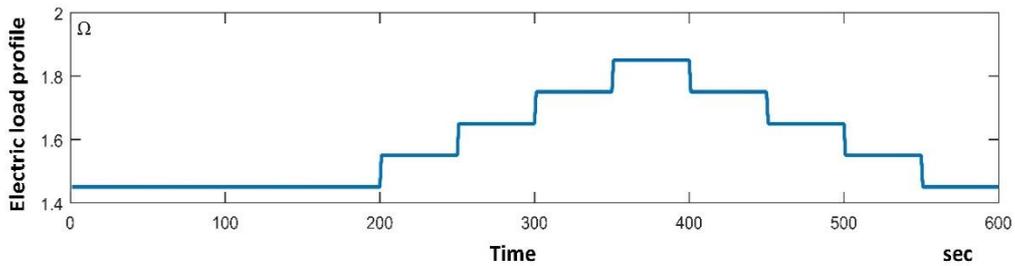


587  
 588 **Figure 12** Illustrated diagram of PD controller in rotational speed tracking.

589  
 590 Fig. (14)-(17) show the results of the dynamic responses of the LTA-CAES by following the variable  
 591 load profile, in which the rotational speed is shown in Fig. (14), the mass flow rate is in Fig. (15), total  
 592 output power of all three stages is presented in Fig. (16), and the isentropic efficiency of the first-stage  
 593 turbine is illustrated in Fig. (17). As show in Fig. 14, when the electrical load varies, the rotational speed  
 594 tends to establish a new balance between power generation and consumption. The sudden decrease  
 595 of electrical power load causes a decrease of current and external torque connected with the turbines.  
 596 The controller detects this sudden variation, automatically changes the mass flow rate to meet the  
 597 variable load and tracks the reference rotational speed. As shown in Fig. 14, although the rotational  
 598 speed slightly fluctuates around the reference speed, the operation transitions only take several  
 599 seconds back to the reference value based on the selected control signal frequency in the simulations.  
 600 The performance can be further improved by optimally selecting the controller strategy, which can be  
 601 investigated further using the proposed dynamic modelling method.

602 In addition, with the increase of electrical load to the generator, mass flow rate reduces in order  
 603 to meet the decreased external torque. The power generated from the turbines is also decreased to  
 604 accommodate to the decreased power consumption. As shown in Fig. 15 and 16, the mass flow rate  
 605 and power output slightly fluctuate at the beginning of each load changing period and gradually  
 606 change to the new steady-state values.

607 Because the geometric parameters of turbines are optimised for the design operations, the  
608 isentropic efficiency is lower when the machines are operated away from the design operation status.  
609 In these off-design operations, as the mass flow rate of the air decreases, the relative flow angle at  
610 the inlet of the rotor would be less than the design angle. Energy losses due to the progressive flow  
611 separation that is initially formed at the front of the suction surface are detrimental to the machine's  
612 efficiency. As plotted in Fig 17, using the first-stage turbine as an example, the reduced isentropic  
613 efficiencies are found in its partial load operations with low mass flow rates when the connecting  
614 electrical load is changing.  
615

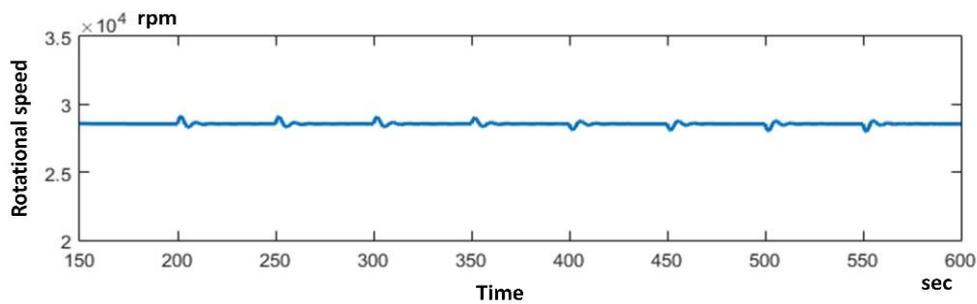


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Figure 13 Studied variable electrical load profile.

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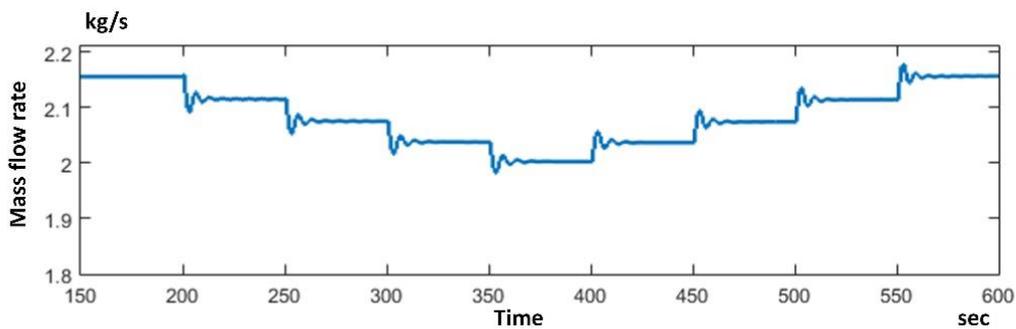


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Figure 14 Rotational speed during the transient operations.

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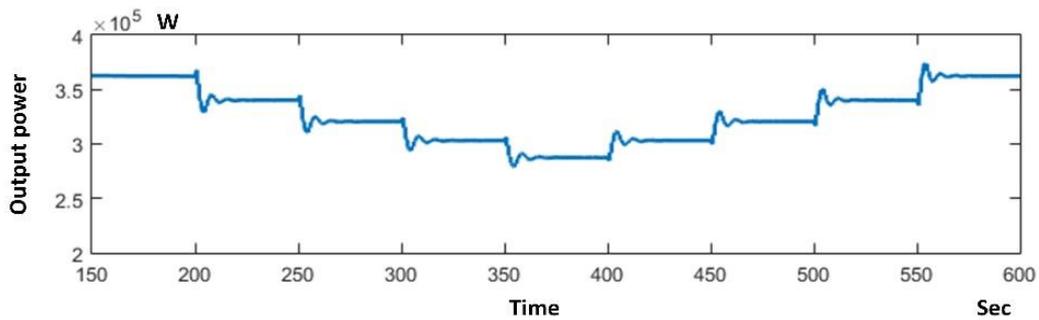


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Figure 15 Mass flow rate during the transient operations.

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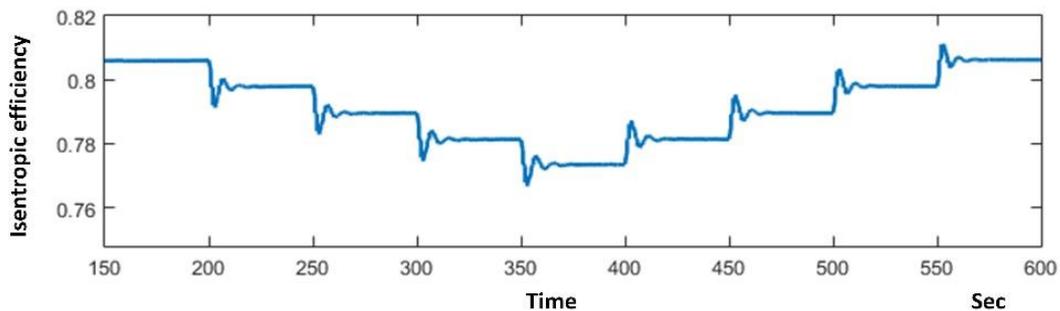


625

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**Figure 16** Output power generated of the three stages turbines during the transient operations.

627



628

629

**Figure 17** Isentropic efficiency of the first stage turbine during the transient operations.

630

## 631 6. Conclusion

632 With the proposed “quasi dynamic iterative searching” method, a simulation platform of the LTA-  
 633 CAES with radial turbines is built for analysing the system responses arisen by operation status change,  
 634 heat transfer, mechanical transmission and energy conversions. In order to verify the proposed  
 635 simulation platform and the associated components’ models, simulation results are validated using  
 636 experimental data from the published works. Then, a particular case study of the LTA-CAES system  
 637 discharging LTA-CAES is developed to analyse the system transience during start-up and operational  
 638 transitions. Based on the results, several conclusions can be drawn: 1) The developed simulation  
 639 platform of the LTA-CAES with radial turbines using the “quasi-dynamic iterative searching” shows  
 640 satisfactory agreement with the experimental data. 2) Based on the proposed simulation platform,  
 641 multiple time scales of the system responses are analysed in the LTA-CAES system discharging during  
 642 the start-up and operation transitions. The LTA-CAES system shows fast mechanical responses of the  
 643 shaft and slow thermal responses due to heat transfer in the HEXs during the start-up of discharging.  
 644 3) The LTA-CAES system with the counter-current HEXs has higher rotating speed in the steady-state  
 645 because of the enhanced heat transfer between the two fluids, compared to the LTA-CAES system  
 646 with the co-current HEXs. 4) The work demonstrates the compatibility of dynamic modelling  
 647 framework to combine with the controller for tracking a constant rotational speed subject to a variable  
 648 electric load. 5) In the operational transitions, when the electrical load increases, the air mass flow  
 649 rate decreases, matching the power generation to the reduced power load, and maintaining the  
 650 reference rotating speed. However, the incidence loss arisen by the decreased flow rate from the  
 651 design operation results in the turbine efficiency drop.

652

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