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Digital Compensation of Chromatic Dispersion in 112-Gbit/s PDM-QPSK System

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ABSTRACT

High bit rates optical communication systems pose the challenge of their tolerance to linear and nonlinear fiber impairments. Coherent optical receivers using digital signal processing techniques can mitigate the fiber impairments in the optical transmission system, including the chromatic dispersion equalization with digital filters. In this paper, an adaptive finite impulse response filter employing normalized least mean square algorithm is developed for compensating the chromatic dispersion in a 112-Gbit/s polarization division multiplexed quadrature phase shift keying coherent communication system, which is established in the VPI simulation platform. The principle of the adaptive normalized least mean square algorithm for signal equalization is analyzed theoretically, and at the meanwhile, the taps number and the tap weights in the adaptive finite impulse response filter for compensating a certain fiber chromatic dispersion are also investigated by numerical simulation. The chromatic dispersion compensation performance of the adaptive filter is analyzed by evaluating the behavior of the bit-error-rate versus the optical signal-to-noise ratio, and the compensation results are also compared with other present digital filters.

Key Words: Coherent optical receivers, digital signal processing, chromatic dispersion equalization, polarization division multiplexed quadrature phase shift keying, normalized least mean square algorithm, adaptive finite impulse response filter

1. INTRODUCTION

Fiber impairments such as chromatic dispersion (CD) severely impact the performance of high speed optical fiber transmission systems\textsuperscript{1,2}. Although current systems use dispersion compensation fibers (DCFs) to compensate the chromatic dispersion distortion, this increases the cost of the transmission systems. Digital coherent receivers allow equalization for linear transmission impairments in the electrical domain\textsuperscript{3,4}, and have become a promising alternative approach to dispersion compensation fibers. While coherent detection was experimentally demonstrated as early as 1979, its use in commercial systems has been hindered by the additional complexity, due to the need to track the phase and polarization of the incoming signal\textsuperscript{5}. In a digital coherent receiver these functions are implemented in the electrical domain leading to a dramatic reduction in complexity. Furthermore since coherent detection maps the entire optical field within the receiver bandwidth into the electrical domain it maximizes the efficacy of the signal processing. This allows fiber impairments which have traditionally limited high bit rate systems to be overcome adaptively\textsuperscript{6-15}.

It is possible to completely compensate chromatic dispersion with zero penalty in coherent detection receivers by means of electronic equalization techniques\textsuperscript{13-15}. Several digital filters have been applied to compensate the chromatic dispersion in the time domain and the frequency domain\textsuperscript{14,15}. In this paper an adaptive finite impulse response (FIR) filter employing normalized least mean square (NLMS) algorithm is developed to compensate CD in a 112-Gbit/s polarization division multiplexed quadrature phase shift keying (PDM-QPSK) coherent optical transmission system. The principle of the NLMS algorithm and the structure of the adaptive filter are analyzed and investigated, and the influence of the step size on the convergence of the tap weights in the adaptive filter is also discussed and illustrated. The performance of the NLMS filter is characterized by evaluating the behavior of the bit-error-rate (BER) versus optical signal-to-noise ratio (OSNR) in the PDM-QPSK system using VPI simulation platform\textsuperscript{16}, and compared with a fiber dispersion FIR (FD-FIR) filter and a blind look-up adaptive filter\textsuperscript{14,15}. The tap weights distribution in the NLMS adaptive filter and the FD-FIR

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filter are investigated and compared in the CD equalization numerical simulation. The required taps number in the NLMS filter and FD-FIR filter and the fast Fourier transform (FFT) size in the blind look-up filter are also studied for compensating different fiber chromatic dispersion. The characteristics of the three digital filters are gradually analyzed and illustrated by comparing their CD compensation simulation results.

2. PRINCIPLE OF NORMALIZED LEAST MEAN SQUARE ADAPTIVE FILTER

The normalized least mean square algorithm is an iterative adaptive algorithm that can be used in the highly time varying signal environment. The NLMS algorithm uses the estimates of the gradient vector from the available data. The NLMS algorithm incorporates an iterative procedure that makes successive corrections to the weights vector in the direction of the negative of the gradient vector which eventually leads to the minimum mean square error.

The transfer function \( h(n) \) of the NLMS adaptive filter is given by

\[
\begin{align*}
\hat{h}(n+1) &= \hat{h}(n) + \frac{\mu e(n)\bar{x}(n)}{|\bar{x}(n)|^2} \\
e(n) &= d(n) - \hat{h}(n)\bar{x}(n)
\end{align*}
\]  

where \( \bar{x}(n) \) is the input signal vector, \( \bar{x}^H(n) \) is the Hermitian of \( \bar{x}(n) \), \( d(n) \) is the desired symbol, \( e(n) \) represents the estimation error between the output signal and the desired symbol, \( e^*(n) \) is the conjugation of estimation error \( e(n) \), and \( \mu \) is a coefficient called step size parameter. The transfer function vector \( \hat{h}(n) \) is updated in a symbol-by-symbol iterative manner, and achieves convergence when \( e(n) \) approaches to zero.

In order to guarantee the convergence of the transfer function \( \hat{h}(n) \), the step size \( \mu \) needs to satisfy the condition of \( 0 < \mu < 1/\lambda_{\text{max}} \), where \( \lambda_{\text{max}} \) is the largest eigenvalue of the correlation matrix \( R = \bar{x}(n)\bar{x}^H(n) \). The convergence speed of the algorithm is inversely proportional to the eigenvalue spread of the correlation matrix \( R \). When the eigenvalues of \( R \) are spread, convergence may be slow. The eigenvalue spread of the correlation matrix is estimated by computing the ratio of the largest eigenvalue to the smallest eigenvalue of the matrix. If the step size \( \mu \) is chosen to be very small, then algorithm converges very slowly. A large value of \( \mu \) may lead to a faster convergence, but may be less stable around the minimum value, an optimal value of step size is usually selected as 0.1.

![Figure 1. Schematic of the adaptive normalized least mean square filter.](image)

The schematic of the linear adaptive normalized least mean square equalizer with \( N \) weights is shown in Fig. 1, where \( T \) is the sampling period, and coefficient \( W_i \) is the tap weights corresponding to the NLMS transfer function vector.
The linear adaptive NLMS equalizer consists of a tapped delay line that stores data samples from the input signal. Once per symbol period, the equalizer outputs a weighted sum of the values in the delay line and updates the tap weights to prepare for the next symbol period. The tap weights value are updated according to the estimation error between the output signal and the desired signal.

3. SIMULATION INVESTIGATION OF PDM-QPSK TRANSMISSION SYSTEM

The installation of the 112-Gbit/s PDM-QPSK coherent optical transmission system established in the VPI simulation platform is illustrated in Fig. 2. The electrical data from four 28-Gbit/s pseudo random bit sequence (PRBS) generators are modulated into two orthogonally polarized QPSK optical signals by two Mach-Zehnder modulators, which are then integrated into one fiber transmission channel by a polarization beam combiner to form the 112-Gbit/s PDM-QPSK optical signal. Using an optical local oscillator (LO) in the coherent receiver, the received optical signals are mixed with the LO laser to be transformed into four electrical signals by the photodiodes and then digitalized by the analog-to-digital convertors (ADCs) at double sampling rates. Thus the impairments of chromatic dispersion in the transmission channel could be equalized with diverse digital filters.

![Figure 2. Schematic of 112-Gbit/s PDM-QPSK coherent optical transmission system.](image)

4. SIMULATION RESULTS OF CHROMATIC DISPERSION COMPENSATION

To illustrate the features of the NLMS filter, the compensation of CD from a standard single mode fiber with dispersion coefficient \( D = 16 \text{ ps}/(\text{nm} \cdot \text{km}) \) are investigated and analyzed by comparing with the FD-FIR filter. Compared with the iteratively updated tap weights in the NLMS adaptive filter, the tap weights in the FD-FIR filter have a relatively simple specification, the tap weights \( a_k \) in the FD-FIR filter is given by

\[
a_k = \left| \frac{j c T^2}{D z} \exp \left( -j \frac{\pi c T^2}{D z} k^2 \right) \right| - \frac{N}{2} \leq k \leq \frac{N}{2}
\]  

where \( D \) is the fiber chromatic dispersion coefficient, \( \lambda \) is the central wavelength of the transmission optical wave, \( z \) is the fiber length in the transmission channel, and \( N \) is the required minimum taps number for compensating the fiber dispersion.

The distribution of converged tap weights in the NLMS filter for 20 km fiber are shown in Fig. 3(a), (b) and (c), respectively, and the tap weights of the FD-FIR filter for 20 km fiber are shown in Fig. 3(d). We can see that in the NLMS filter, the central tap weights take more dominant roles in the chromatic dispersion equalization, while in the
FD-FIR filter, the tap weights magnitudes are constant for a certain length fiber, whereas the real parts and the imaginary parts of the tap weights vary periodically with the tap orders increasing. For a fixed fiber dispersion, the tap weights in NLMS adaptive filter approach to zero, when the corresponding taps order exceeds a certain value, and this value indicates the least required taps number for compensating the chromatic dispersion effectively. This also illustrates the optimization characteristic of the NLMS adaptive algorithm. While in the FD-FIR filter, the excessive tap weights do not approach to zero when the chromatic dispersion in the transmission fiber channel is equalized effectively.

The chromatic dispersion compensation results are shown in Fig. 4, where Fig. 4(a) indicates the CD equalization for 20 km and 6000 km fibers using the NLMS filter and the FD-FIR filter respectively, both digital filters using 9 taps for 20 km fiber and 2411 taps for 6000 km fiber, and Fig. 4(b) shows the influence of taps number increment on CD compensation effects for 20 km fiber with the two digital filters.

It could be seen in Fig. 4 that the NLMS adaptive filter could achieve the same CD compensation performance as the FD-FIR filter, and the NLMS filter have a better improvement with the increment of taps number. For an acceptable CD compensation performance (BER better than $10^{-3}$), the required minimum numbers of taps are 9 taps in the NLMS filter and 9 taps in the FD-FIR filter for 20 km fiber, and 2305 taps in the NLMS filter and 2411 taps in the FD-FIR filter for 6000 km fiber, and the NLMS filter needs fewer taps than the FD-FIR filter for an acceptable compensation effect with
the fiber length increasing.

Table 1. Necessary taps number in NLMS and FD-FIR filters and FFT-sizes in blind look-up filter for a certain length fiber with CD coefficient $D = 16\text{ps}/(\text{nm} \cdot \text{km})$

<table>
<thead>
<tr>
<th>Fiber length (km)</th>
<th>Taps number in NLMS filter</th>
<th>Taps number in FD-FIR filter</th>
<th>FFT-sizes in blind look-up filter</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>9</td>
<td>9</td>
<td>32</td>
</tr>
<tr>
<td>40</td>
<td>15</td>
<td>17</td>
<td>32</td>
</tr>
<tr>
<td>400</td>
<td>115</td>
<td>161</td>
<td>256</td>
</tr>
<tr>
<td>4000</td>
<td>1495</td>
<td>1607</td>
<td>2048</td>
</tr>
<tr>
<td>6000</td>
<td>2305</td>
<td>2411</td>
<td>2048</td>
</tr>
</tbody>
</table>

To investigate the required taps number in NLMS adaptive filter and FD-FIR filter for a fixed fiber dispersion, the required minimum taps number in the filters for different fiber length with the same CD coefficient are shown in Table 1. We could see that the NLMS adaptive could use fewer taps than FD-FIR filter to compensate the CD with the fiber length increasing.

The structure of the frequency domain blind look-up adaptive equalizer is shown in Fig. 5. In the blind look-up filter, the digitalized electrical signals are firstly transformed by the fast Fourier transform (FFT) operation and then multiplied by the inverse transfer function of the dispersive channel in frequency-domain, and then the processed signals are transformed into time domain signals by the inverse fast Fourier transform (IFFT) operation. Starting at the initial value 240 ps/nm, the dispersion applied to the adaptive filter is increased in steps of 2 ps/nm up to the negative of maximum possible dispersion in the 112-Gbit/s PDM-QPSK coherent optical transmission system. The simulation results of CD compensation for 20 km fiber using the blind look-up filter with various FFT-sizes N are illustrated in Fig. 4(c), the dotted line in Fig. 4(c) is the back-to-back measurement result.

![Figure 5. Schematic of blind look-up adaptive filter.](image)

In the above simulation results, we could see that the CD equalization of the NLMS adaptive filter could reach the same performance as the FD-FIR filter and the blind look-up adaptive filter with 32 symbols in the FFT operation.

### 5. CONCLUSIONS

In this paper an adaptive finite impulse response digital filter employing normalized least mean square algorithm is developed to compensate the chromatic dispersion in a 112-Gbit/s polarization division multiplexed quadrature phase shift keying coherent optical transmission system. The performance of the NLMS adaptive filter for CD equalization is compared with the FD-FIR filter and the blind look-up adaptive filter. The NLMS adaptive filter shows the best performance in CD compensation, whereas it requires slow iteration for guaranteed convergence. The FD-FIR filter affords simple analytical tap weights specification, whereas giving slightly poor performance with taps number increasing. The blind look-up filter shows the same performance as the NLMS filter, furthermore, it will decrease the computational complexity when the filter length is large.
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