Gibrat's Law and Quantile Regressions: an Application to Firm Growth

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Abstract

The nexus between firm growth, size and age in U.S. manufacturing is examined through the lens of quantile regression models. This methodology allows us to overcome serious shortcomings entailed by linear regression models employed by much of the existing literature, unveiling a number of important properties. Size pushes both low and high performing firms towards the median rate of growth, while age is never advantageous, and more so as firms are relatively small and grow faster. These findings support theoretical generalizations of Gibrat's law that allow size to affect the variance of the growth process, but not its mean [Cordoba, J. C., 2008, A Generalized Gibrat's Law, *International Economic Review*, 49(4), 1463–1468].

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1 Introduction

The literature on industrial dynamics has devoted much attention to unveiling the nexus between firm size and growth. In this respect, the theoretical proposition known as Gibrat's law (Gibrat, 1931)—which predicts randomness of firm growth rates—has been widely tested. Linear econometric frameworks employed to validate this hypothesis have delivered mixed evidence (see Sutton, 1997 for a comprehensive review of the literature). While some studies have found a tendency for large firms to grow faster than small ones (Samuels, 1965, Singh and Whittington, 1975), others have appreciated a tendency of small firms to grow faster (Hall, 1987, Evans, 1987a,b, Dunne *et al.*, 1989).[1] More recently, Cordoba (2008) has introduced a generalization of Gibrat's law that allows size to affect the variance of the growth process, but not necessarily its mean. This property is relevant to both models of economic growth featuring balanced-growth conditions, as well as short-run frameworks attempting to explain business cycles as phenomena emerging from idiosyncratic shocks to different production units (see, e.g., Carvalho and Grassi, 2015). This note shows how a consensus among these views may be reached, once firm heterogeneity is properly accounted for and firm growth is tracked over a long time span. It does so by re-examining the size-growth conundrum through the lens of conditional quantile regressions.

Empirical tests of Gibrat's law have typically relied on cross-section regressions or short-panel econometric techniques that impose homogeneity in the parameters across units and over time (Urga *et al.*, 2003). On one hand, the first approach ignores the information contained in firm-specific time variation of growth rates. On the other hand, while considering information available for different periods of time, a major drawback of the second approach is to pool potentially heterogeneous firms as if their data were generated according to the same process. To overcome these problems, we examine firm growth by means of conditional quantile regressions (see Koenker and Bassett, 1978 and Koenker, 2005), so as to allow factors such as size and age to exert different effects depending on the speed at which firms expand/contract. In fact, there is no reason to anticipate that the marginal effects of the covariates on the shape of the density are invariant over the domain of firm growth.

Quantile regressions have been implemented in the analysis of the determinants of firm size (e.g., Machado and Mata, 2000 and Cabral and Mata, 2003) and growth (e.g., Coad, 2007, Coad and Rao, 2008, Reichstein *et al.*, 2010). However, these studies have typically focused on short time windows, while a vast body of empirical evidence has shown how the density of firm growth displays marked variation over time, both at business cycle and lower frequencies (Higson *et al.*, 2002, 2004 and Holly *et al.*, 2013). In light of this,

[1] Consistent with the assumption of decreasing returns to scale, these works show that small firms tend to grow faster than large ones. This implies a mean reversion effect on firm size, which introduces an overall limit to the variance of the size distribution, as firm size converges in the long run towards an optimal level.

we employ a long panel of COMPUSTAT data on manufacturing firms, accounting for the presence of time effects in the set of determinants of firm growth. In addition, we condition the quantiles of firm growth to firms' age. In this respect, Haltiwanger, Jarmin, and Miranda (2013) have recently stressed the importance of controlling for age when examining the relationship between growth and size.[2]

We detect marked heterogeneity in the impact of firm characteristics on the growth process. Age is never advantageous to firm growth, and more so as firms are relatively small and grow faster. By contrast, size exerts a negative (positive) effect on firms that grow above (below) the median rate, with the marginal impact increasing in absolute terms as firms grow/decline faster. This implies a tendency to mean reversion, so that size differences between firms are transitory. This is an important finding, as it lends support to the generalization of Gibrat's law that allows size to affect the variance of the growth process, but not its mean (see Cordoba, 2008). Our results are robust to controlling for firm turnover—as implied by the analysis of different balanced panels—as well as to splitting the sample over a number of dimensions (e.g., distinguishing between durable and non-durable producing firms). Notably, we report a marked tendency for relatively large firms to display a faster pace of convergence to the mean size in the overall sample. This signals a weak degree of adaptation to an evolving competitive environment as firms grow larger, but not necessarily older.

The remainder of the paper is laid out as follows: Section 2 introduces the quantile regression framework; Section 3 presents the data and reports quantile-based evidence on the relationship between firm growth, size and age; Section 4 concludes.

2 Quantile Regression Analysis

Quantile regressions are especially useful when dealing with non-identically distributed data. In these situations one should expect to observe significant discrepancies in the estimated 'slopes' at different quantiles with respect a given set of covariates (Machado and Mata, 2000). Such discrepancies may reflect not just into location shifts, but also into scale shifts (i.e., changes in the degree of dispersion) and/or asymmetry reversals (i.e., changes in the sign of the skewness). Define the τ^{th} quantile of the distribution of a generic variable y, given a vector of covariates $\mathbf{x}$, as:

$$Q_\tau \left(y|\mathbf{x}\right) = \inf \left\{ y|\, F\left(y|\, \mathbf{x}\right) \geq \tau \right\}, \quad \tau \in (0,1)\,, \tag{1}$$

where $F\left(y|\, \mathbf{x}\right)$ denotes the conditional distribution function. A least squares estimator of the mean regression model would be concerned with the dependence of the conditional mean of y on the covariates. The quantile regression estimator tackles this issue at

[2] In fact, their analysis shows that the systematic inverse relationship between firm size and net growth rates highlighted in prior analyses is entirely attributable to most new firms being classified in small size classes. By contrast, once firm age is controlled for, they report no systematic relationship between firm size and growth.

each quantile of the conditional distribution. In other words, instead of assuming that covariates shift only the location of the conditional distribution, quantile regression looks at the potential effects on the whole shape of the distribution. The statistical model we opt for specifies the τ^{th} conditional quantile of firm-level growth, g_{it}, as a linear function of the vector of covariates, $\mathbf{x}_{it}$:[3]

$$Q_\tau \left(g_{it}|\mathbf{x}_{it}\right) = \mathbf{x}_{it}'\boldsymbol{\beta}_\tau, \ \tau \in (0,1)\,. \tag{2}$$

3 Data and Model Specification

As it has been noted by Urga, Geroski, Lazarova, and Walters (2003), short panels are much more informative on high-frequency variations in corporate growth rates than they are on low-frequency variations. As a result, growth rates can appear more random than they really are and important long-run or secular variations in growth rates may be overlooked. Short panels may also erroneously lead one to reject the view that firms have natural life cycles or systematically evolve through number of stages (see Binder *et al.*, 2005). Finally, it is important to recall that reasonably long panels ($T > 30$) may alleviate problems of autocorrelated residuals. To account for these issues, we employ annual accounting COMPUSTAT annual data on manufacturing firms over six decades (1950-2010).[4]

Real sales are taken as a proxy for firm size, which is denoted by s_{it}.[5] We then compute firm i's growth rate as $g_{it} \equiv (s_{it} - s_{it-1}) / [(s_{it} + s_{it-1})/2]$.[6] This definition is widely employed in the literature on industrial dynamics, as it shares some useful properties of log-differences and has the advantage of accommodating entry and exit (see Haltiwanger *et al.*, 2013).[7] In line with the industrial dynamics tradition, the econometric framework includes firm-level $(t-1)$ size and age in the vector of covariates. In addition, we include industry dummies at the 3-digit SIC code level—which account for the fact that firm growth, size and age distributions vary by industry—as well time dummies, which aim

[3]Ideally, one would prefer to implement quantile panel regressions, allowing for both firm-specific and time effects (see, e.g., Powell, 2010). However, this is computationally demanding, even in the presence of a limited number of covariates. In Distante, Petrella, and Santoro (2015) we show that quantile estimates are robust to the exclusion of firm-specific effects.

[4]Our data selection has priviledged the time-dimension of the COMPUSTAT panel, along with its availability. On the downside, it might be argued that, in light of including only quoted companies, these data are biased towards relatively large firms. However, on a priori grounds there is no reason to believe that this property should be crucial in explaining our facts.

[5]Various measures – including the value of assets of a firm, employment and sales – have been traditionally used to proxy firm size. Where data have been available for the various measures the results have generally been invariant to the measure of size (see Evans, 1987 and Hall, 1987).

[6]We remove firms growing (declining) beyond a 100% rate. Replicating the analysis with growth rates defined as log-differences or under alternative cut-off intervals does not qualitatively affect the analysis.

[7]Along with being symmetric around zero and bounded between -2 (exit) and 2 (entry), this growth rate represents a second order approximation of the log-difference for growth rates.

at controlling for the behavior of the distribution over time. The resulting framework generalizes the first order Galton–Markov model $g_{it} = \beta s_{it-1} + u_{it}$, where u_{it} is an error term, assumed to be $i.i.d.$ across firms and over time. Note that $\beta < 0$ implies that small firms grow faster than bigger ones, while for $\beta > 0$ the opposite holds true. Gibrat's Law holds instead if the estimated parameter $\hat{\beta}$ is not significantly different from zero, so that growth turns out to be stochastic and independent of size.

Prior to looking the effects of firm size and age over the spectrum of firm growth, it is important to examine the behavior of the density over the time span we consider. Figure 1 graphs the quantiles of firm growth. A first observation to be made is that different parts of the distribution do not follow the same time path, neither at relatively high nor lower frequencies. As documented by Comin and Philippon (2006) and Comin and Mulani (2006), the density has slowly become more sparse over time. Our evidence points to increasing dispersion as a phenomenon that primarily hinges on the evolution of the tails of the distribution, while the interquantile range only displays a very moderate trending behavior. Heterogeneity is also pervasive at higher frequencies, with different quantiles displaying dissimilar degrees of co-movement with the business cycle. This tendency is at odds with the view that aggregate fluctuations must reflect spread-preserving shifts in the mean of the distribution, which would instead imply all quantiles denoting the same cyclical behavior. Altogether, these findings emphasize the importance of employing quantile regressions to deal with heterogeneity in firm-level growth.

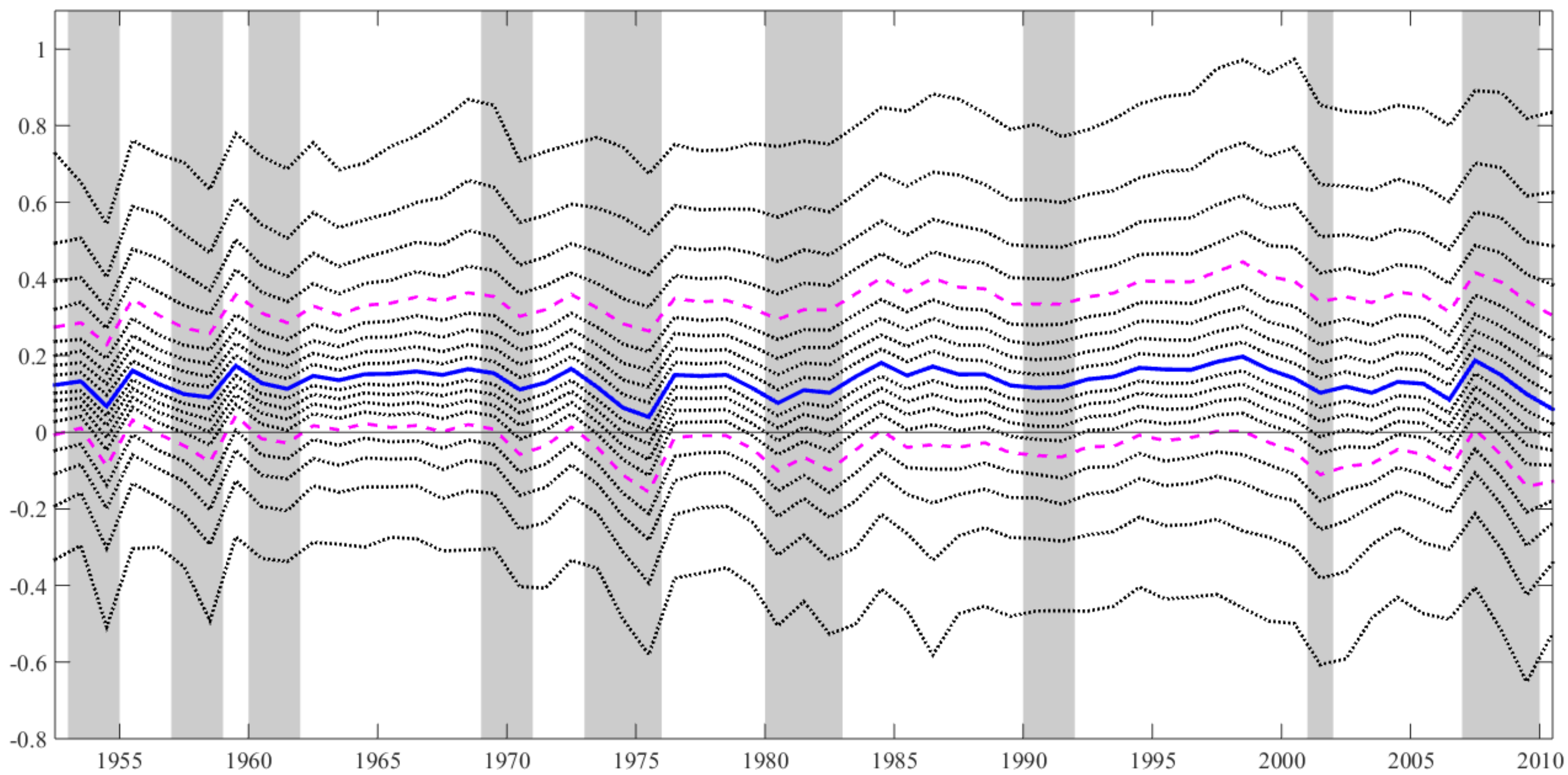
4 Firm Growth, Size and Age

Figure 2 plots the quantile treatment effects (QTE) associated with firm size and age, together with the OLS estimates from the pooled sample. The QTE of firm age is negative throughout the entire growth domain. Therefore, age is never advantageous, and more so for quantiles above the median rate of growth.[8] As for the QTE of firm size, this results as an affine transformation of the control distribution, crossing the zero axis at the median growth rate.[9] In other words, size acts as a scale shifter that exerts positive (negative) effects on LHS (RHS) of the median rate of growth. From an economic viewpoint, this property is consistent with a pattern of competitive convergence, as that reported by Fama and French (2000) with respect to firm profitability. In fact, this is a necessary condition for firms operating in a competitive environment to converge to the same size

[8] This finding is consistent with Evans (1987a), who finds that firm age is also important for the variability of firm growth and the probability of dissolution. Notably, these predictions are in line with Jovanovic (1982), whose theory of firm growth is based on entrepreneurs learning about their abilities over time.

[9] It is important to stress that these patterns are not driven by entry-exit dynamics. To test the robustness of our findings to this potential factor, as well as to changes in the sample composition, we have replicated our empirical exercise by taking six balanced panels of firms: 1950-2010, 1960-2010, 1970-2010, 1980-2010, 1990-2010, 2000-2010. The resulting QTE's are qualitatively in line with those reported in Figure 2.

FIGURE 1. QUANTILES OF FIRM GROWTH



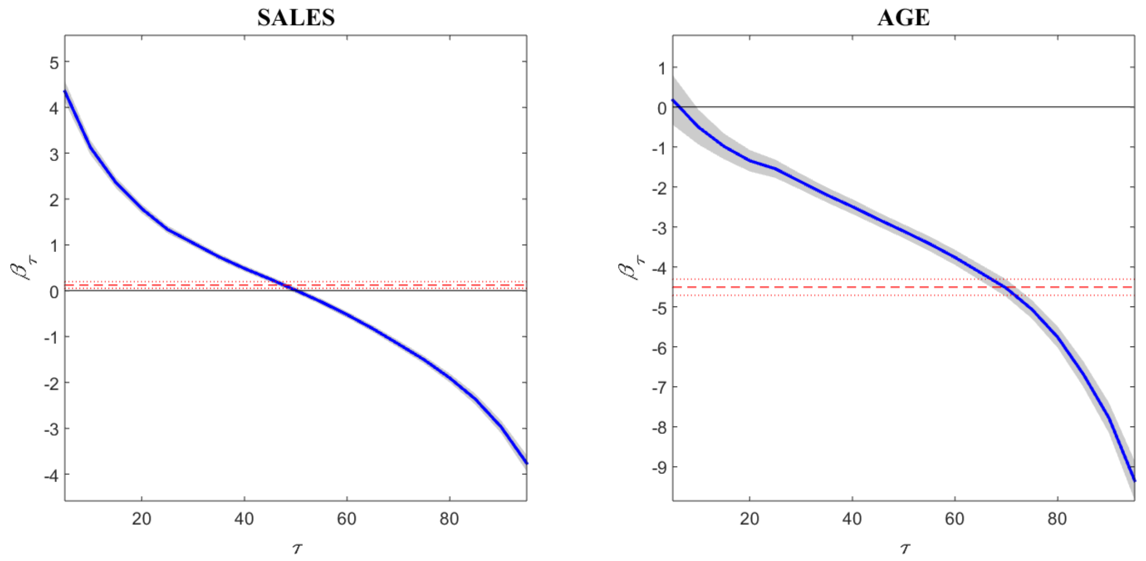
Notes: Figure 1 graphs the quantiles of firm-level growth of real sales over the 1950-2010 time window. The continuous line denotes the median, while the dashed lines denote the 25^{th} and 75^{th} quantiles. Recessionary episodes as identified by the NBER are denoted by the vertical bands.

in the long run, both within and between industries. Conditional on survival, the key implication of mean reversion is that small establishments grow faster than large ones. Other firms eventually mimic innovative products and technologies that allow them to expand, or simply allocate assets to more productive uses, so as to avoid the prospect of failure and take over. Eventually, this process erodes large firms' profitability.

These findings also need to be examined in light of the recent evidence by Haltiwanger, Jarmin, and Miranda (2013), who show the existence of no systematic relationship between firm size and growth, once they control for firm age. According to our evidence this remains a valid conclusion only if we focus on the median growth rate. In fact, the OLS estimate from the pooled sample is close to null in the case of firm size, whereas age exerts a negative effect. However, from a statistical viewpoint our evidence is not necessarily in contrast with that of Haltiwanger, Jarmin, and Miranda (2013). In fact, Cordoba (2008) has presented a generalization of Gibrat's law that allows size to affect the variance of the growth process, but not necessarily its mean. Models of economic growth typically impose that the expected growth rate of a variable is constant through time, and therefore independent of its size. From a modeling perspective the generalized Gibrat's law rationalizes a scale invariant process that embeds this property.[10] Supporting the existence of Gilbrat's law in its generalized form is important in that a growing literature that has sought to explain aggregate fluctuations based on firm-level idiosyncratic shocks. As pointed out by Gabaix (2011), the rate at which the variance of growth rates declines with firm size has implications for aggregate fluctuations (see also Luttmer, 2010). More recently Carvalho and Grassi (2015) and Grassi (2016) have introduced macro models where the generalized Gilbrat's law is the key mechanism generating a power law in firm size, which implies that idiosyncratic shocks do not wash out in the aggregate.

[10] As stressed by Cordoba (2008), many variables in economics grow over time and balanced growth conditions are often required on theoretical or empirical grounds. The generalized Gibrat's law provides a general process to account for these facts.

FIGURE 2. QUANTILE TREATMENT EFFECTS



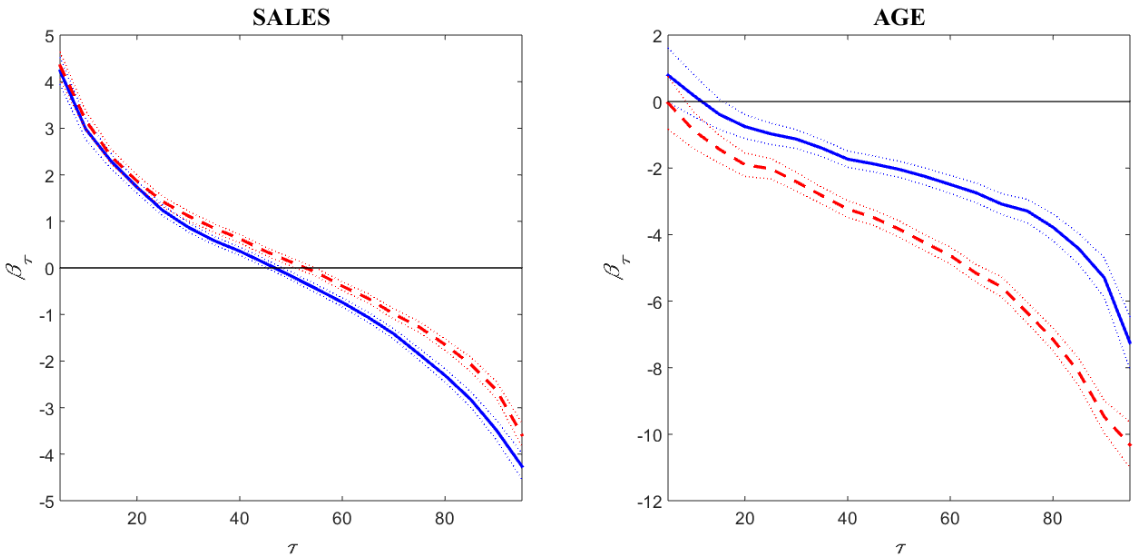
Notes: The continuos (blue) line is the estimated QTE associated with firm-specific lagged real sales (left panel) and age (right panel). The dashed (red) line indicates the OLS estimates from the pooled sample. All confidence intervals are obtained as the point estimate +/- twice its standard error.

4.1 Robustness

It is important to make sure that the tendencies we report do not result from pooling firms that operate in different sectors. To this end, Figure 3 graphs the QTE associated with firm size and age for firms operating in non-durable and durable manufacturing.[11] Interestingly, whereas the QTE associated with s_{it-1} does not display sizeable differences across different manufacturing industries, the negative effect exerted by firm age is more marked on non-durables producing firms. This is broadly consistent with Haltiwanger, Jarmin, and Miranda (2013), who show that firm entry rates are especially low in durable goods manufacturing, so that incumbents operating in this sector may be less vulnerable to competitive pressures brought by new entrants. Under these circumstances, age represents less of an obstacle to growth in the durable goods sector, as compared with firms operating in non-durable goods manufacturing.

[11] Opting for a further level of disaggregation leaves our results unaltered.

FIGURE 3. QUANTILE TREATMENT EFFECTS (Durables vs. Non-durables)

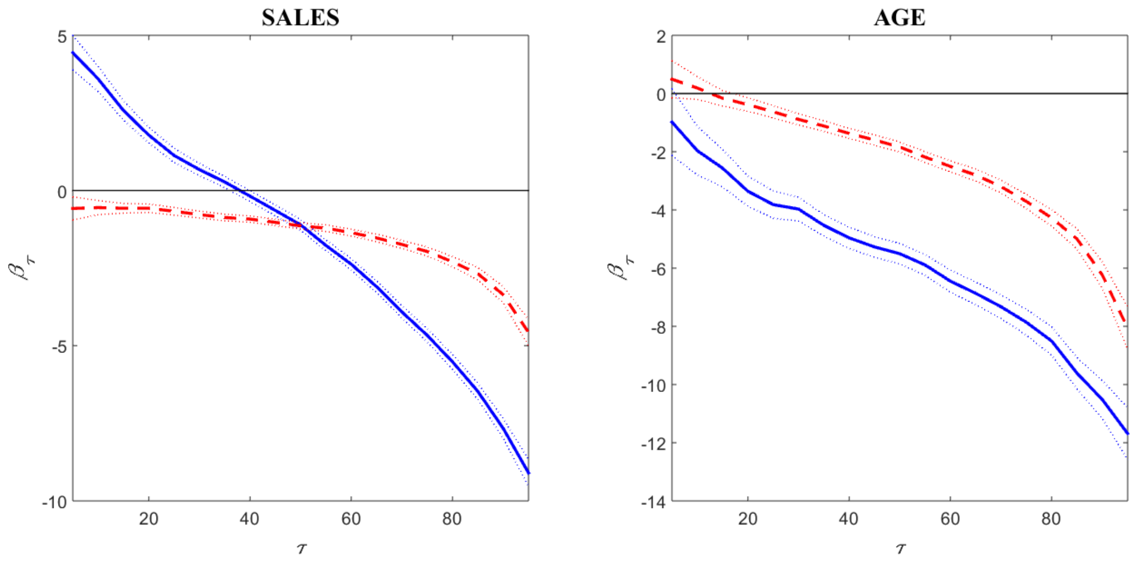


Notes: The continuos (blue) line denotes the estimated QTE associated with lagged real sales (left panel) and age (right panel) for firms operating in the manufacturing of durables, while the dashed (red) line denotes the analogous estimates for non-durables manufacturing firms. All confidence intervals are obtained as the point estimate +/- twice its standard error.

We finally examine how the growth quantiles of firms with different scales of production are affected by size and age.[12] To this end, we divide the sample according to firm size, regarding as large/small firms those who are located above/below the median level of real sales at any point time. Thus, we re-estimate our quantile regression model for each of the two sub-samples. Figure 4 reports the QTE resulting from this exercise. Notably, when accounting for the growth dynamics of large vs. small firms, size emerges as an element of moderation of firm growth, especially to large companies. In fact, the marginal impact of size on the growth of large firms is always negative, and more so as we move to the right over the spectrum of growth. This evidence signals that reversion to the mean may be faster for large companies. As suggested by strategic management theory, in fact, these companies are often focused on managing scale and efficiency, and their internal hierarchies and routines denote weak adaptation to an evolving competitive environment. Such management practices are hard to be eradicated, especially when they have historically been the basis for success (Reeves and Deimler, 2011).

[12] We also check the qualitative robustness of our key findings by splitting the original sample with respect to the median age, so as to compare young to relatively older firms. The results are virtually unchanged and have not been reported for reasons of space. However, they are available, upon request, from the authors.

FIGURE 4. QUANTILE TREATMENT EFFECTS (Large vs. Small Firms)



Notes: The continuos (blue) line denotes the estimated QTE associated with lagged real sales (left panel) and age (right panel) for small firms, while the dashed (red) line denotes analogous estimates obtained for large firms. All confidence intervals are obtained as the point estimate +/- twice its standard error.

5 Concluding Remarks

We examine firm growth in a long panel of manufacturing firms, through the lens of conditional quantile regressions. We control for the presence of time effects and allow firm-specific size and age to exert heterogeneous effects over the domain of firm growth. Size acts as a symmetric centripetal force—pushing both low and high performing firms towards the median rate of growth—while age is never advantageous to firm growth, and more so as firms grow faster. Along with implying a marked tendency to mean reversion, these findings emphasize the importance of employing regression methods that go 'beyond the mean' to cope with firm-level heterogeneity.

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