Perils of unconventional monetary policy[1]

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Abstract

Unconventional monetary policy, by relaxing restrictions on the composition of the balance sheet of the central bank, compromises control over the stochastic path of inflation; or, in open economies, over the stochastic path of exchange rates. If the composition of the balance sheet is unrestricted then the path of inflation is indeterminate. This is the case under pure quantitative easing, where the target is the size of real money balances. In contrast, credit easing policies restrict the composition of the portfolio by targeting a specific expansion in the maturity profile of bonds bought, and thus can implement a determinate path of inflation. The composition of the portfolios traded by monetary-fiscal authorities also determines premia in asset and currency markets.

Keywords: unconventional monetary policy; stochastic path of inflation.

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1 Introduction

Conventional monetary policy restricts assets on the balance sheet of the central bank to short-term Treasury Bills. Much analysis takes this as given and, as a result, the importance of restrictions on the central bank asset portfolio is typically overlooked. Unconventional monetary policy, specifically balance sheet policy in the classification of Borio and Zabai (2016), relaxes the restrictions on the central bank asset portfolio and allows for assets of varying maturity and risk profiles. In this paper, we show that a potentially less-restricted portfolio may eliminate the ability of the central bank to control the stochastic path of inflation.

We consider a stochastic cash-in-advance economy with flexible prices and a complete asset market, and we restrict attention to trades in securities of one-date maturity.[1] We explore the consequences of both Ricardian and non-Ricardian seigniorage policy; well-founded criticisms of the fiscal theory of the price level in Buiter (2002) and Drèze and Polemarchakis (2000) notwithstanding, the advantage of the non-Ricardian policy is that under both conventional and unconventional monetary policy it yields a determinate inital price level but nonetheless doesn't overturn our key finding. The conclusion in Nakajima and Polemarchakis (2005) that, surprisingly, had gone unnoticed, is that monetary policy that sets a path of short-term, nominal interest rates determines the path of expected or average inflation, but not the distribution of possible paths of inflation. The key result is that the stochastic path of inflation is determined by the adjustment of the portfolio of the monetary authority over time in response to market forces and expectations. As uncertainty unfolds, market forces and expectations determine the value of the assets on the balance sheet. As explained in Sims (2016), variations in the value of the balance sheet drive the stochastic path of monetary injections or withdrawals that, in turn, determines the path of inflation.

Once portfolio allocations are selected, the distribution of inflation outcomes is determinate. In fact, with full knowledge of the structure of the economic environment, the central bank could choose the portfolio weights to target a specific distribution of inflation. However, if the portfolio allocations are left unrestricted, the distribution of inflation is not pinned down. In this paper we show how different approaches to unconventional monetary policy affect this stochastic distribution of inflation through their effect on the portfolio of the central bank. Moreover, we extend the analysis to an open economy and show that the indeterminacy extends to the stochastic path of exchange rates. It is straightforward to show that this finding is robust to many changes in the environment including introducing price rigidity into the model.

Under a conventional (restricted) central bank asset portfolio, there is a determinate stochastic distribution of inflation outcomes even though central banks typically target only expected inflation. That is, restrictions on the portfolio of assets, even where they are driven by the central bank's conservative approach to risk, imply that the central bank can keep the distribution of inflation anchored as it targets expected inflation.

[1] Trades in long-lived assets can duplicate such trades, as in Kreps (1982), and allow for a role for the maturity structure of debt, as in Cochrane (2001) or Angeletos (2002).

Adoption of unconventional policies, such as in response to hitting the zero lower bound, may change the central bank's portfolio restrictions and lead to changes in the stochastic distribution of inflation. The composition of the portfolios traded by monetary-fiscal authorities also determines premia in asset and currency markets.

While a common feature of unconventional monetary policy is the expansion of their balance sheet, central banks have employed two distinct approaches to these policies which we discuss in detail below; namely, quantitative easing (QE) and credit easing (CE). QE focuses only on the expansion of the central bank liabilities and does not restrict the asset composition of the balance sheet. By contrast, CE targets a specific allocation of assets, much like conventional monetary policy that restricts open market operations to Treasury bills. Under CE, it is the explicit target for the composition of the balance sheet that allows the monetary authority to target the stochastic path of inflation: the target for the composition of the portfolio guarantees the necessary restrictions to obtain determinacy of the inflation distribution and limit the de-anchoring of the inflation distribution.

We also discuss a number of important extensions of the basic result. We show that policy rules that select the portfolio weights as a function of expected inflation do not overcome the indeterminacy problem. This may be surprising since policy rules may seem to imply restriction but we show they leave portfolio weights effectively unrestricted. Also, since unconventional policies are typically employed during times of crisis, when interest rates may be constrained by the effective zero lower bound. While our focus is on the implications of changes in the composition of risky assets in the portfolio of the central bank for the stochastic path of inflation, we show that this constraint does not alter our main finding.

The indeterminacy of QE in our benchmark model is nominal. While the central bank loses the control of inflation, the indeterminacy does not affect the attainable equilibrium allocations. If the central bank switched from interest rate to money supply policy, the indeterminacy would affect real allocations. Importantly, the indeterminacy becomes real if prices are sticky or the asset market is incomplete, as in Bai and Schwarz (2006).

Moving to an open economy environment, we highlight that QE transmits indeterminacy across central bank portfolios and, consequently the path of exchange rates. If markets are incomplete within each country, then fluctuations in central bank portfolios resonate abroad by affecting, not only the nominal exchange rate, but also asset prices and risk premia. Furthermore, if central banks set interest rates according to a Taylor-type rule that accounts for changes in the nominal exchange rate, as in Taylor (2001), and trading partners conducted QE, then they would not be able to guarantee the desired outcomes can be implemented. In other words, QE by trading partners manifests itself as indeterminacy of both nominal and real risk-premia, globally, and more importantly, even in countries that conduct traditional monetary policy or CE. These findings contribute to the growing view that variations in capital flows should be managed, and that these variations may stem from the monetary policy of trading partners.

The portfolio balance channel operates when bonds of different maturities are not perfect substitutes and traders have have maturity-specific bond demands. In this setting, the maturity structure of outstanding debt can affect term premia. Theoretical models

describing the portfolio balance channel such as Vayanos and Vila (2009) and Hamilton and Wu (2012) neglect the consequences of variations in the composition of the monetary authority portfolio on the stochastic path of inflation. We show that as the composition of the portfolios of monetary-fiscal authorities determine the stochastic path of prices, they also determine the nominal stochastic discount factor. Independent of changes in expectations about the path of short-term interest rates, the correlation between the discount factor and asset prices, and nominal exchange rates, then generates risk premia and biases whose size and sign corresponds to the chosen portfolio composition.

This paper contributes to a vast and important literature on indeterminacy of monetary equilibria: Sargent and Wallace (1975) pointed out the indeterminacy of the initial price level under interest rate policy; Lucas and Stokey (1987) derived the condition for the uniqueness of a recursive equilibrium with money supply policy; Woodford (1994) analysed the dynamic paths of equilibria associated with the indeterminacy of the initial price level under money supply policy. In this paper, we give the exact characterisation of recursive equilibria under quantitative easing with interest rate policy.

Here, we highlight the importance of the composition of the portfolio of the monetary authority for the determinacy of the path of prices, and even the determinacy of the price level.[2] Many papers take it for granted that a monetary authority trades exclusively in short-term, nominally risk-free bonds. For example, in the fiscal theory of the price level in Woodford (1994), and Dupor (2000) in an open economy. This is also the case in Dubey and Geanakoplos (2003), who argue that "outside money" suffices to eliminate the indeterminacy that prevails in economies with nominally denominated assets. The latter is an important claim, because, as noted by Cass (1984, 1985) and analysed in depth in Balasko and Cass (1989) and Geanakoplos and Mas-Colell (1989), when the asset market is incomplete, nominal indeterminacy has real effects.

The possible multiplicity of stochastic inflation paths at equilibrium was clear in Bloise, Drèze, and Polemarchakis (2005) and Nakajima and Polemarchakis (2005). In those papers the specification was Ricardian, equilibria were indeterminate, and the point was to demonstrate that the indeterminacy can be parametrised by the price level and a nominal martingale measure. Here, in contrast to Nakajima and Polemarchakis (2005), we show that under unconventional monetary policy, the path of inflation is indexed by the portfolio composition of the monetary-fiscal authority, independent of the Ricardian/non-Ricardian distinction. Magill and Quinzii (2014b) developed the argument that inflationary expectations can serve as an alternative parametrisation. Drèze and Polemarchakis (2000) pointed out the need for "comprehensive monetary policy" that sets the stochastic path of the term structure of interest rates (equivalently, all state-contingent short-term rates) in order to determine the path of inflation. Adao, Correia, and Teles (2014), and Magill and Quinzii (2014a) developed this theme. Importantly, in this argument, the way out of indeterminacy involved targets or restrictions on the returns of assets. Our point

[2]It is important to note that our results are driven by fluctuations in central bank wealth in an environment without "fiscal backing"; an issue highlighted in Del Negro and Sims (2015) and Hall and Reis (2015) among others. Our contribution to this literature is to emphasise and characterise the importance of the portfolio composition in this result.

here is that the composition of the balance sheet of the monetary authority matters as an instrument of immediate policy relevance. Although unconventional policy offers more *potential* instruments, if the policy maker does not choose them appropriately, then the policy objective of a determinate path of inflation cannot be implemented.

Our extension to show that feedback rules that set interest rates or the composition of the balance sheet as a function of future inflation are not be sufficient to obtain a determinate inflation path. "Simple" inflation processes may only be compatible with conventional monetary policy. Our argument does not derive from an open-ended horizon or the stability of a steady state as is standard in the literature on the determinacy of equilibrium paths under a Taylor rule, following Taylor (1993); important contributions are Woodford (1999) and Benhabib, Schmitt-Grohe, and Uribe (2001).

In Curdia and Woodford (2011), the portfolio under unconventional policies has the same risk-profile as (or is collinear with) the portfolio under conventional policy; yet, unconventional policies may have real effects in the presence of segmented markets. In practice central banks have accommodated trade to include non-collinear assets (bonds of longer maturities or private sector liabilities). It is the trade in these assets that we focus on and our formulation is in the spirit of Eggertsson and Woodford (2003) where the central bank chooses a portfolio among a set of state-contingent assets. Benigno and Nistico (2017) show that under unconventional monetary policy the central bank may suffer balance sheet losses, and policy aimed at preventing this will affect the path of inflation. Similarly, Bhattarai (2016), in the spirit of Sargent and Wallace (1981), shows that the path of real wealth of a central bank that is fiscally constrained, that is, one in which transfers to the Treasury are fixed, depends on the path of the relative prices of the assets on the balance sheet. We show that the nominal path of central bank wealth also depends on the composition of the portfolio chosen.[3]

1.1 Unconventional monetary policy in practice

We first explore the practical adoption of unconventional monetary policy in major economies as context before turning to the model. Unconventional policies are a mechanism to support credit and liquidity. Potential channels through which these unconventional policies have an effect include the portfolio rebalancing channel and the risk channel. However the implementation of policies vary as Bernanke (2009) explains:

> "*The Federal Reserve's approach to supporting credit markets is conceptually distinct from quantitative easing (QE), the policy approach used by the Bank of Japan from 2001 to 2006. Our approach–which could be described as 'credit easing' (CE)–resembles quantitative easing in one respect: It involves an expansion of the central bank's balance sheet. However, in a pure QE regime, the focus of policy is the quantity of bank reserves, which are liabili-*

[3]The central bank in Bhattarai (2016) fixes the portfolio composition indirectly by requiring that the transfers to the treasury from the different asset positions are always equal. The central bank in Benigno and Nistico (2017) chooses the sequence of one of the two assets directly.

ties of the central bank; the composition of loans and securities on the asset side of the central bank's balance sheet is incidental."

Heterogeneity in the implementation of unconventional policies makes our theoretical distinction between pure QE and pure CE particularly relevant. While no central bank has pursued pure QE or pure CE, some central banks have pursued policies much closer to CE and others policies much closer to QE. The key characteristic that makes the Fed's policies closer to CE is that it sought to expand it's balance sheet while committing to a specific asset composition in doing so.[4] The Federal Reserve Bank of New York published how (in terms of portfolio weights) the total Large Scale Asset Purchases (LSAP) program purchases would be distributed across maturity sectors and each round of asset purchases began with an announcement of the specific days on which it would be conducting the auctions and the maturity sectors in which it would be buying. In our model ex-ante restrictions on the composition of central bank assets resolve the indeterminacy.

The Fed's approach contrasts somewhat with the easing policies of other major central banks. During their first round of QE undertaken between 2001 and 2006, the Bank of Japan (BOJ) set new operational targets for monetary policy in terms of the central bank reserves held by financial intermediaries.[5] To achieve these targets, it made outright purchases of long-term Japanese government bonds, stocks held by commercial banks (October 2002 to September 2003) and ABS (July 2003 to March 2006), but the specific portfolio of these assets was not the target of the central bank. It cared only about the total level of the balances. More recent unconventional policies by the BOJ have targeted lending to banks rather than outright purchases of assets from secondary markets. Fawley and Christopher (2013) argue that the BOJ was still mainly concerned with generating reserves and provided limited restrictions on the range of assets. While the specific assets purchased by the BOJ may be a function of expectations of prices and premia, we show that even such a policy is insufficient to rule out indeterminacy.

The Bank of England's QE scheme was similar to the early BOJ scheme. The Monetary Policy Committee set an overall target for the amount of assets purchased through the Asset Purchase Facility (APF), the composition of assets was not the target. However, the Bank of England set some restrictions on the asset portfolio (medium- and long-term gilts) making the APF somewhat closer to the Fed, and CE, than the BOJ.[6]

Finally, early unconventional policies by the ECB als targeted lending to banks rather than outright purchases of assets from secondary markets. Nonetheless, Fawley and Christopher (2013) argue that these programs should be "considered pure QE in the sense that they targeted reserves and typically accepted a wide range of assets as collateral".

[4]The Federal Reserve reduced the target federal funds rate to effectively zero and also implemented a number of other programs and policies which led to significant changes to the Federal Reserve's balance sheet; for example, Bernanke (2009), Goodfriend (2011), Reis (2009) and Fawley and Christopher (2013).

[5]See Ugai (2007) and Maeda et al. (2005) for more details of the BOJ experience with QE.

[6]In early 2009, the APF initially bought high-grade corporate bonds and government gilts with maturity 5-25 years before switching to only government gilts over a slightly extended maturity range starting at three years. In any given purchase operation, a broad range of assets, such as gilts with maturities 10-25 years, were up for purchase. Prices and allocations were determined by market conditions.

2 The analytical argument

Monetary policy involves *quantitative easing* if open market operations extend to unrestricted portfolios of government bonds of different maturities or bonds issued by the private sector. It involves *credit easing* if open market operations extend beyond treasuries, but still target a specific composition for the balance sheet of the monetary-fiscal authority; as a limit case, monetary policy is *conventional* when open market operations are restricted to short term, nominally risk-free assets (Treasury Bills).

Fiscal policy is *Ricardian* if it is restricted to satisfy an intertemporal budget constraint or transversality condition; equivalently, if public debt vanishes for all possible, equilibrium or non-equilibrium, values of prices and interest rates. It is *non-Ricardian*, if it is not restricted to satisfy an intertemporal budget constraint; in particular, outside money or initial liabilities of the public towards the private sector are not taxed back.

Quantitative easing generates indeterminacy indexed by a *nominal pricing measure* over states of the world. This measure determines the distribution of rates of inflation, up to a moment that is determined by the risk-free rate and non-arbitrage. Ricardian policy leaves the initial price level indeterminate as well. Determinacy and, by extension, monetary and financial stability, obtain under credit easing or conventional monetary policy. The indeterminacy is nominal only as long as prices are flexible, monetary policy sets nominal rates of interest, and the asset market is effectively complete; otherwise, there are real effects. It is worth pointing as our analysis considers a process of continual re-balancing of the monetary-fiscal authority balance sheet, our argument applies equally to the unwinding of quantitative easing as well as to the initiation of it. What is essential is the type of policy that determines the stochastic evolution of the balance sheet.

In section 2.1 we present the main result in a three-period model. We show the indeterminacy inherent in a stochastic economy and link it to the mix of interest and non-interest bearing assets traded by the monetary-fiscal authority. We then show that the zero lower bound does not overturn the result (section 2.2). We consider the effects of unconventional monetary policy in an infinite horizon stochastic model in 2.3. In section 2.4 we show that this is not a consequence of non-stationary equilibria or of exogenous interest rate paths, and we make explicit the role of the composition of the portfolio of the monetary-fiscal authority portfolio in the determination of stochastic inflation rates. In the presence of pure quantitative easing, interest-rate feedback rules are insufficient to obtain determinacy. We then show that pure credit easing policies (which set portfolio weights exogenously) obtains determinacy while policies that allow for feedback rules determining the composition of assets is insufficient to rule out indeterminacy. Finally, restricting attention to "simple" inflation processes may only be compatible with conventional monetary policy.

2.1 A Three-Period Model

Activity extends over dates $t = 0, 1$, while a final third date, $t = 2$, serves for accounting purposes.[7] Uncertainty over states of the world, $s \in \{1 \ldots, s, \ldots\ S\}$, is realised at $t = 1$; each state occurs with probability $f(\cdot)$. These states could be purely extrinsic or they could refer to some other "fundamentals," such as monetary policy shocks across states. To maintain notational consistency with later sections, a date-event at date t is denoted s^t; with s^0 being one element, and $s^1|s^0$ being one of S elements, as is $s^2|s^1$.

At dates 0 and 1, a representative individual supplies $l(s^t)$ units of labour to produce perishable output $y(s^t) = l(s^t)$ in exchange for competitive nominal wages $w(s^t)$, while consumption is $c(s^t)$ and the price level is $p(s^t)$. As real wages are 1, equilibrium nominal wages equal the price level at each-date event.

Utility is derived from consumption of goods and leisure. The intertemporal utility is separable and that flow utility function is continuously differentiable, strictly increasing, strictly concave and satisfies boundary conditions. The lifetime utility of an individual is

$$u(c(s^0), 1 - l(s^0)) + \beta \textstyle\sum_{s^1} u(c(s^1), 1 - y(s^1)) f(s^1).$$

An individual cannot use labour income to purchase goods but must instead use cash obtained from the asset market. The asset market opens (and closes) before the goods market, and any cash proceeds from the sale of output must be carried over to the next date.[8]

An individual enters date 0 with nominal wealth $\tau(s^0)$; the asset market is then open and cash $\hat{m}(s^0)$ and a complete set of contingent claims are traded. He purchases an amount $\theta(s^1|s^0)$ of the elementary security that pays one unit of currency in state s^1 and zero otherwise; the price of this contingent claim is $q(s^1|s^0)$. While the asset market is open, the individual faces the budget constraint

$$\hat{m}(s^0) + \sum_{s^1|s^0} q(s^1|s^0)\theta(s^1|s^0) \leq \tau(s^0).$$

The nominally risk-free one-date interest rate at date 0 is $r(s^0)$; the no arbitrage condition yields

$$\frac{1}{1 + r(s^0)} = \sum_{s^1} q(s^1|s^0).$$

The cash-in-advance constraint implies that the household must have sufficient cash $(\hat{m}(s^0))$ to cover its purchases:

$$p(s^0)c(s^0) \leq \hat{m}(s^0).$$

The cash brought by the individual into the next date is

$$m(s^0) = \hat{m}(s^0) - p(s^0)z(s^0),$$

[7] In particular, debts are settled in this final date; there is no uncertainty after date 1.

[8] Our timing and monetary structure closely follows the cash-in-advance models in Lucas and Stokey (1987) and Nakajima and Polemarchakis (2005).

where we $z(s^0) = c(s^0) - l(s^0)$ is the net demand of the individual at date 0 (with a similar definition for $z(s^1|s^0)$.

It follows that the cash-in-advance constraint is

$$m(s^0) \geq p(s^0)y(s^0)$$

or[9]

$$\frac{r(s^0)}{1+r(s^0)}m(s^0) \geq \frac{r(s^0)}{1+r(s^0)}p(s^0)y(s^0).$$

At the start of the date 1, nature determines the state and household wealth is

$$\tau(s^1|s^0) = m(s^0) + \theta(s^1|s^0).$$

It follows that, at date 0, the household faces the constraint

$$p(s^0)z(s^0) + m(s^0) + \sum_s q(s^1|s^0)\theta(s^1|s^0) \leq \tau(s^0).$$

At date 1, uncertainty has been fully resolved. Similar reasoning yields a single constraint,

$$p(s^1|s^0)z(s^1|s^0) + m(s^1|s^0) + \frac{1}{1+r(s^1|s^0)}\theta(s^2|s^1) \leq \tau(s^1|s^0),$$

and the cash-in-advance constraint

$$m(s^1|s^0) \geq p(s^1|s^0)y(s^1|s^0).$$

At date 2, the final date, all debts are repaid; wealth at the end of the date cannot be negative:

$$\tau(s^2|s^1) = m(s^1|s^0) + \theta(s^2|s^1) \geq 0.$$

Since individuals strictly prefer more to less, there will be no slack.

The lifetime budget constraint is

$$p(s^0)z(s^0) + m(s^0)\frac{r(s^0)}{1+r(s^0)} + \sum_s q(s^1|s^0)z(s^1|s^0) + \sum_s q(s^1|s^0)m(s^1|s^0)\frac{r(s^1|s^0)}{1+r(s^1|s^0)} \leq \tau(s^0). \tag{1}$$

The optimization problem of an individual is to choose $c(s^0)$, $c(s^1|s^0)$, $y(s^0)$ and $y(s^1|s^0)$ subject to the intertemporal budget constraint and the cash-in-advance constraints; first order conditions are

[9] If $r(s^0) = 0$, then both sides are zero while if $r(s^0) > 0$, then utility-maximising households will not wish to hold excess cash between dates as doing so would forego a positive return.

$$\frac{\frac{\partial u[c(s^0),1-l(s^0)]}{\partial c(s^0)}}{\frac{\partial u[c(s^0),1-l(s^0)]}{\partial l(s^0)}} = 1 + r(s^0), \tag{2}$$

$$\frac{\frac{\partial u[c(s^1|s^0),1-l(s^1|s^0)]}{\partial c(s^1|s^0)}}{\frac{\partial u[c(s^1|s^0),1-l(s^1|s^0)]}{\partial l(s^1|s^0)}} = 1 + r(s^1|s^0), \tag{3}$$

$$\frac{\frac{f(s^1|s^0)\partial u[c(s^1|s^0),1-l(s^1|s^0)]}{\partial c(s^1|s^0)}}{\frac{\partial u[c(s^0),1-l(s^0)]}{\partial c(s^0)}} = \tilde{q}(s^1|s^0) = \frac{q(s^1|s^0)p(s^1|s^0)}{p(s^0)}. \tag{4}$$

The consolidated monetary-fiscal authority sets the nominal interest rate $r(s^0)$ and $r(s^1|s^0)$, manages its asset portfolio by choosing the securities which it trades, $\delta(s^1|s^0)$. The monetary fiscal authority enters with liabilities $T(s^0)$. The monetary fiscal authority prints money $M(s^0)$ which it introduces to the economy either via open market operations in the asset market. This yields

$$M(s^0) + \textstyle\sum_s q(s^1|s^0)\Theta(s^1|s^0) = T(s^0),$$

$$T(s^1|s^0) = M(s^0) + \Theta(s^1|s^0),$$

$$M(s^1|s^0) + \tfrac{1}{1+r(s^1|s^0)}\Theta(s^2|s^1) = T(s^1|s^0),$$

$$T(s^2|s^1) = M(s^1|s^0) + \Theta(s^2|s^1).$$

We write the composition of the monetary fiscal authority portfolio in each security, $\Theta(s^1|s^0)$, as a security specific share, $\delta(s^1|s^0)$, times a measure of the total size of the holdings, ϑ:

$$\Theta(s^1|s^0) = \delta(s^1|s^0)\vartheta(s^0).$$

The present-value budget constraint is

$$M(s^0)\frac{r(s^0)}{1+r(s^0)} + \sum_{s^1|s^0} q(s^1|s^0)M(s^1|s^0)\frac{r(s^1|s^0)}{1+r(s^1|s^0)} = T(s^0).$$

If initial liabilities, $T(s^0)$, vary with prices to satisfy the present-value budget constraint, then the fiscal-policy regime is Ricardian, otherwise it is non-Ricardian. Definition 3 gives a formal statement of the distinction in the infinite horizon section, and with minor modifications for notation, it holds here. The distinction between the two regimes results in either the initial price level being indeterminate or not. As our focus is on the distribution of second date prices, the fiscal-policy regime is not important for our results.

As described earlier, conventional monetary policy involves only trades of risk-free securities: $\delta(s^1|s^0) = 1/s$. QE leaves portfolio shares unrestricted, or, equivalently, they

depend on expected inflation rates. CE does not trade only risk-free securities but does restrict the portfolio shares, or, equivalently, they do not depend on expected inflation rates.

In addition to individual optimization, equilibrium requires market clearing and consistency:

$$c(s^0) = y(s^0), \qquad c(s^1|s^0) = y(s^1|s^0),$$

$$m(s^0) = M(s^0), \qquad m(s^1|s^0) = M(s^1|s^0),$$

$$\tau(s^0) = T(s^0), \qquad \tau(s^1|s^0) = T(s^1|s^0),$$

$$\tau(s^2|s^1) = T(s^2|s^1).$$

The no-arbitrage condition is used to express security prices $q(s^1|s^0)$ as depending on the nominal equivalent Martingale measure $\nu(s^1|s^0)$ in the form

$$q(s^1|s^0) = \frac{\nu(s^1|s^0)}{1 + r(s^0)}, \tag{5}$$

$$\nu(s^1|s^0) = \tilde{q}(s^1|s^0)\frac{1 + r(s^0)}{1 + \pi(s^1|s^0)}, where \tag{6}$$

$$\sum_{s^1|s^0} \nu(s^1|s^0) = 1. \tag{7}$$

where the tilde denotes the real value.

Definition 1. Given initial nominal wealth, $\tau(s^0) = T(s^0)$, and interest rate policy, $\{r(s^0), r(s^1|s^0)\}$, a competitive equilibrium consists of an allocation, $\{c(s^0), c(s^1|s^0), y(s^0), y(s^1|s^0)\}$, a portfolio of households, $\{m(s^0), m(s^1|s^0), \tau(s^1|s^0), \tau(s^2|s^1)\}$, a portfolio of the monetary-fiscal authority, $\{M(s^0), M(s^1|s^0), T(s^1|s^0), T(s^2|s^1)\}$, spot-market prices, $\{p(s^0), p(s^1|s^0)\}$, and asset prices, $\{q(s^1|s^0), q(s^2|s^1)\}$, such that

1. given $T(s^0)$ and $\{r(s^0), r(s^1|s^0), M(s^0), M(s^1|s^0)\}$, the debt portfolio $\{T(s^1|s^0), T(s^2|s^1)\}$ is determined;

2. the monetary authority accommodates the money demand, $M(s^0) = m(s^0)$ and $M(s^1|s^0) = m(s^1|s^0)$;

3. given interest rates, $\{r(s^0), r(s^1|s^0)\}$, spot-market prices, $\{p(s^0), p(s^1|s^0)\}$, and asset prices, $\{q(s^1|s^0), q(s^2|s^1)\}$, the household's problem is solved by $c(s^0)$, $c(s^1|s^0)$, $y(s^0)$, $y(s^1|s^0)$, $m(s^0)$, $m(s^1|s^0)$, $\tau(s^1|s^0)$, and $\tau(s^2|s^1)$;

4. all markets clear.

We now define the types of balance sheet policy under consideration

Definition 2. A balance sheet policy is

1. Unrestricted, and termed "Quantitative Easing", if the portfolio shares are not exogenously specified;

2. Restricted, and termed "Credit Easing", if the portfolio shares are exogenous;

3. Restricted, and termed "conventional", if the portfolio shares are exogenous and form a nominally riskless portfolio, i.e. $\delta(s^1|s^0) = \frac{1}{s}$.

We will show later that if policy selects the portfolio shares to depend on expected inflation, then equilibria will be indeterminate, as when they are unrestricted, while if they are chosen to depend on realised endogenous variables, equilibria will be determinate, as when they are "restricted". Our definition of conventional balance sheet policy conforms to the monetary-fiscal authority purchasing a complete portfolio of state-contingent bonds that form a nominally riskless 1 period bond.

The main result of this section is

Proposition 1. *Given interest rate policy,* $\{r(s^0), r(s^1|s^0)\}$ *where* $r(s^0) > 0$ *and* $r(s^1|s^0) > 0$*,*

1. *with Ricardian (non-Ricardian) Fiscal Policy, and unrestricted balance sheet policy, the initial price level* $p(s^0)$ *is indeterminate (determinate) and the stochastic inflation rate,* $(1 + \pi(s^1|s^0))$*, is indeterminate;*

2. *with Ricardian (non-Ricardian) Fiscal Policy, and restricted balance sheet policy, the initial price level* $p(s^0)$ *is indeterminate (determinate) and the stochastic inflation rate,* $(1 + \pi(s^1|s^0))$*, is determinate;*

Proof From market clearing and the first order conditions for optimization, equations (2) and (3) we obtain the allocation; and, given the allocation, the real price of the state-contingent bond, $\tilde{q}(s^1|s^0)$ from equation (4). From the present value budget constraint (1) and binding cash-in-advance constraints[10] we obtain an expression for the real value of initial nominal wealth. Under a non-Ricardian fiscal policy, this then determines the initial price level while under Ricardian fiscal policy initial liabilities of the monetary-fiscal authority vary with the initial price level, which is then indeterminate. Without further restrictions, the indexed bond price cannot be uniquely decomposed and the distribution of inflation rates is left undetermined.

Let a tilde over a variable denote the real value of the variable. The second date household budget constraint and market clearing gives us in equilibrium

[10] As interest rates are positive this will be true in equilibrium.

$$
\begin{aligned}
p(s^1|s^0)z(s^1|s^0) + m(s^1|s^0)\frac{r(s^1|s^0)}{1+r(s^1|s^0)} &= \theta(s^1|s^0) + m(s^0) \\
&= \delta(s^1|s^0)\vartheta(s^0) + m(s^0), \\
1 + \pi(s^1|s^0) &= \frac{\delta(s^1|s^0)\tilde{\vartheta}(s^0) + \tilde{m}(s^0)}{\tilde{m}(s^1|s^0)\frac{r(s^1|s^0)}{1+r(s^1|s^0)}}. \qquad (8)
\end{aligned}
$$

If the balance sheet is restricted we can take the ratio of equation (8), across states and obtain $S-1$ relationships between the expected inflation rates and the real scale of debt and the real value of money balances. The final equations to solve the system are obtained from the no-arbitrage requirement in equations (6) and (7). Real money balances are $\tilde{m}(s^0) = y(s^0)$ and $\tilde{m}(s^1|s^0) = y(s^1|s^0)$ and we obtain one equation to determine the scale of debt.

With the real scale of debt in hand, we obtain the distribution of inflation rates as a function of the portfolio weights. If policy is conventional or one of credit easing, then the portfolio weights are *restricted* and policy determines the path of inflation. On the other hand, under quantitative easing, the portfolio weights are *unrestricted* (either free or depends on expected inflation), and inflation is indeterminate. For example, suppose that the portfolio weights were were endogenous and a function of the expected inflation rate, of the form $\delta(s^1|s^0) = [1+\pi(s^1|s^0)]/[\sum_{s^1|s^0} 1+\pi(s^1|s^0)] \times constant_s$ such that $\sum_{s^1|s^0} \delta(s^1|s^0) = 1$. In this case, the state-contingent inflation rates cancel out in equation (8) and what remains is the sum of expected stochastic inflation rates. □

In Eggertsson and Woodford (2003), when real money balances are not saturated, the central bank portfolio uniquely determines the path of prices. This is because the composition of the portfolio depends on current state variables and is *restricted*. If QE focuses exclusively on expansion of real money balances, then the assumption made in their paper is a strong one. A relaxation of this assumption leaves the portfolio *unrestricted* even when the zero lower bound is not binding, the central bank loses control of the path of inflation.

2.2 The Effect of the Zero Lower Bound

Although our derivations above used positive interest rates, this was only for analytical convenience and our results remain robust to scenarios where the economy is either temporarily or permanently at the zero lower bound. One may also be concerned that our analysis precludes the expansion of real money balances through unconventional policy. When interest rates are positive, the path of real money balances are determined solely from the path of the real interest rate. In contrast, when interest rates are zero, the composition of assets held by the monetary-fiscal authority determines the path of real money balances.

2.2.1 Temporary Zero Lower Bound

Suppose that the economy is temporarily at the zero lower bound; at date 0 interest rates are 0, but in the second date the economy exits the zero bound and interest rates are positive. If date 0 interest rates were 0, then real money balances may be greater than real income at date 0; we lose one equation.

The real monetary fiscal-authority present-value budget constraint when date 0 interest rates are 0 is

$$\sum_s \tilde{q}(s^1|s^0)\tilde{M}(s^1|s^0)\frac{r(s^1|s^0)}{1+r(s^1|s^0)} = \frac{T(s^0)}{p(s^0)},$$

and, under a non-Ricardian policy, it gives us the initial price level.

The real date 0 budget constraint in equilibrium gives

$$\tilde{M}(s^0) + \tilde{\vartheta}(s^0)\sum_{s^1|s^0}\frac{\tilde{q}(s^1|s^0)}{1+\pi(s^1|s^0)}\delta(s^1|s^0) = \tilde{\tau}(s^0).$$

Note that

$$\frac{\tilde{q}(s^1|s^0)\delta(s^1|s^0)}{1+\pi(s^1|s^0)} = \tilde{q}(s^1|s^0)\delta(s^1|s^0)\frac{z(s^1|s^0) + \tilde{M}(s^1|s^0)\frac{r(s^1|s^0)}{1+r(s^1|s^0)}}{\delta(s^1|s^0)\tilde{\vartheta}(s^0) + \tilde{M}(s^0)},$$

and, if interest rates are positive, real money balances are $\tilde{m}(s^0) = y(s^0)$ and $\tilde{m}(s^1|s^0) = y(s^1|s^0)$. Furthermore $[T(s^0)/p(s^0)] = \tilde{\tau}(s^0)$ and $\tilde{M}(s^1|s^0) = \tilde{m}(s^1|s^0)$. In equilibrium,

$$\tilde{m}(s^0) + \tilde{\vartheta}(s^0)\textstyle\sum_{s^1|s^0}\tilde{q}(s^1|s^0)\delta(s^1|s^0)\frac{\tilde{m}(s^1|s^0)\frac{r(s^1|s^0)}{1+r(s^1|s^0)}}{\delta(s^1|s^0)\tilde{\vartheta}(s^0)+\tilde{m}(s^0)} =$$

$$\textstyle\sum_s \tilde{q}(s^1|s^0)\tilde{m}(s^1|s^0)\frac{r(s^1|s^0)}{1+r(s^1|s^0)},$$

and, together with the no-arbitrage condition, the real scale of debt can be solved for if monetary policy is restricted. Importantly, the real value of date 0 money balances depends on the portfolio shares; balance sheet policy determines both the size of real money balances and the path of inflation. Note that the Ricardian/non-Ricardian fiscal policy distinction is not driving the results; we only require that fiscal policy is chosen to be compatible with equilibrium.

2.2.2 Permanent Zero Lower Bound

If interest rates are zero in both dates, we require a more-fully-articulated fiscal policy though the Ricardian/non-Ricardian policy distinction remains unimportant. Let the monetary-fiscal authority levy indexed transfers at all date-events, in addition to the initial nominal liabilities.

The equilibrium monetary-fiscal authority real budget constraints now become, with all interest rates zero,

$$\tilde{m}(s^0) + \tilde{\vartheta}(s^0) \textstyle\sum_{s^1|s^0} \tilde{q}(s^1|s^0)\delta(s^1|s^0) + g(s^0) =$$

$$\tfrac{T(s^0)}{p(s^0)}(1 + \pi(s^1|s^0))g(s^1|s^0) = \tilde{\vartheta}(s^0)\delta(s^1|s^0) + \tilde{m}(s^0),$$

and the real present-value budget constraint is

$$g(s^0) + \sum_{s^1|s^0} \tilde{q}(s^1|s^0)g(s^1|s^0) = \frac{T(s^0)}{p(s^0)},$$

where g is the real value of the indexed nominal transfer. The household optimality conditions remain the same. The real scale of debt and the real value of date 0 money balances is solved from

$$\tilde{m}(s^0) + \tilde{\vartheta}(s^0) \textstyle\sum_{s^1|s^0} \tilde{q}(s^1|s^0)\delta(s^1|s^0) =$$

$$\textstyle\sum_s \tilde{q}(s^1|s^0)g(s^1|s^0) \sum_{s^1|s^0} \tilde{q}(s^1|s^0)g(s^1|s^0) \frac{1+r(s^0)}{\tilde{\vartheta}(s^0)\delta(s^1|s^0)+\tilde{m}(s^0)} = 1,$$

where the second equation is the no-arbitrage condition. Hence, in contrast to conventional policy or credit easing, if the portfolio weights are unrestricted under QE (either free or dependent on expected inflation rates) then the path of inflation cannot be determined from policy.

Date 0 real money balances are given by

$$\sum_{s^1|s^0} \frac{\tilde{q}(s^1|s^0)g(s^1|s^0)}{\frac{\sum_s \tilde{q}(s^1|s^0)g(s^1|s^0)-\tilde{m}(s^0)}{\sum_{s^1|s^0} \tilde{q}(s^1|s^0)\delta(s^1|s^0)}\delta(s^1|s^0) + \tilde{m}(s^0)} = \frac{1}{1 + r(s^0)}.$$

Loosely speaking, date 0 real money balances are increasing in the correlation between the portfolio shares and the present value of state-contingent real indexed transfers. To be explicit, our results remain robust to a world where unconventional policies simultaneously expand the size of the balance sheet and alter the composition of assets held in the portfolio.

Our results on the uniqueness of prices at the zero lower bound contrasts with Eggertsson and Woodford (2003) as fiscal transfers in their model are purely nominal and the presence of indexed transfers there would result in a link between the composition of the portfolio and inflation in their setting.

2.3 A stochastic dynamic economy

We now examine infinite-horizon equilibria to show that our results do not depend on a finite horizon or on not restricting attention to stationary equilibria. As our results

do not depend on interest rates, for analytical convenience we assume interest rates are positive throughout this section.

Time, t, is discrete, and it extends into the infinite future: t = 0,1 .... Events, s^t, at each date are finitely many. An immediate successor of a date-event is $s^{t+1}|s^t$, and, inductively, a successor is $s^{t+k}|s^t$. Conditional on s^t, probabilities of successors are $f(s^{t+1}|s^t)$ and, inductively, $f(s^{t+k}|s^t) = f(s^{t+k}|s^{t+k-1})f(s^{t+k-1}|s^t)$.

At a date-event, a perishable input, labor, $l(s^t)$, is employed to produce a perishable output, consumption, $y(s^t)$, according to a linear technology:

$$y(s^t) = a(s^t)l(s^t), \quad a(s^t) > 0.$$

A representative individual is endowed with 1 unit of leisure at every date-event. He supplies labor and demands the consumption good, and he derives utility according to the cardinal utility index $u(c(s^t), 1-l(s^t))$ that satisfies standard monotonicity, curvature and boundary conditions. The preferences of the individual over consumption-employment paths commencing at s^t are described by the separable, von Neumann-Morgenstern intertemporal utility function

$$u(c(s^t), 1 - l(s^t)) + \mathrm{E}_{s^t} \sum_{k>0} \beta^k u(c(s^{t+k}|s^t), 1 - l(s^{t+k}|s^t)),$$

where $0 < \beta < 1$. Balances, $m(s^t)$, provide liquidity services. Elementary securities, $\theta(s^{t+1}|s^t)$, serve to transfer wealth to and from immediate successor date-events. The price level is $p(s^t)$, and the wage rate is $w(s^t) = a(s_t)p(s^t)$, as profit maximization requires. The nominal, risk-free interest rate is $r(s^t)$.

At each date-event, the asset market opens after the uncertainty, s^t, has realized, and, as a consequence, purchases and sales in the markets for labor and the consumption good are subject to standard cash-in-advance constraints; the effective cash-in-advance constraint is[11]

$$a(s^t)p(s^t)l(s^t) \leq m(s^t).$$

Prices of elementary securities are

$$q(s^{t+1}|s^t) = \frac{\nu(s^{t+1}|s^t)}{1 + r(s^t)},$$

with $\nu(\cdot|s^t)$ a "nominal pricing measure" or transition probabilities, which guarantees the non-arbitrage relation

$$\sum_{s^{t+1}|s^t} q(s^{t+1}|s^t) = \frac{1}{1 + r(s^t)}.$$

Inductively,

$$\nu(s^{t+k}|s^t) = \nu(s^{t+k}|s^{t+k-1})\nu(s^{t+k-1}|s^t), \quad k > 1,$$

[11] Nakajima and Polemarchakis (2005) provide an explicit derivation.

and the implicit price of revenue at successor date-events is

$$q(s^{t+k}|s^t) = \frac{\nu(s^{t+k}|s^{t+k-1})}{1 + r(s^{t+k-1}|s^t)} q(s^{t+k-1}|s^t), \quad k > 1.$$

The individual has initial wealth $\tau(s^1) = \omega$. Initial wealth constitutes a claim against the monetary-fiscal authority; alternatively, it can be interpreted as outside money. It is exogenous in a non-Ricardian specification. In a Ricardian specification, it is endogenous satisfying the transversality condition imposed on monetary-fiscal policy.

The flow budget constraint is

$$p(s^t)z(s^t) + m(s^t) + \sum_{s^{t+1}|s^t} q(s^{t+1}|s^t)\theta(s^{t+1}|s^t) \leq \tau(s^t),$$

where $z(s^t) = c(s^t) - a(s^t)l(s^t)$ is the effective excess demand for consumption.

Wealth at successor date-events is

$$\tau(s^{t+1}|s^t) = \theta(s^{t+1}|s^t) + m(s^t),$$

and, after elimination of the trade in assets, the flow budget constraint reduces to

$$p(s^t)z(s^t) + \frac{r(s^t)}{1 + r(s^t)} a(s^t)p(s^t)l(s^t) + \sum_{s_{t+1}} q(s_{t+1}|s^t)\tau(s_{t+1}|s^t) \leq \tau(s^t).$$

Debt limit constraints are

$$-\tau(s^t) \leq \sum_{k>0} \sum_{s^{t+k}|s^t} q(s^{t+k}|s^t) \frac{1}{1 + r(s^t)} a(s^{t+k})p(s^{t+k}).$$

Alternatively, $\tilde{m}(s^t) = (1/p(s^t))m(s^t)$ are real balances, $\tilde{\tau}(s^t) = (1/p(s^t))\ \tau(s^t)$ is real wealth, $\pi(s^{t+1}|s^t) = (p(s^{t+1})/(p(s^t)) - 1$ is the rate of inflation, and

$$\tilde{q}(s^{t+1}|s^t) = q(s^{t+1}|s^t)(1 + \pi(s^{t+1}|s^t)) = \frac{\nu(s^{t+1}|s^t)(1 + \pi(s^{t+1}|s^t))}{1 + r(s^t)}$$

are prices of indexed elementary securities.

Real wealth at successor date-events is

$$\tilde{\tau}(s^{t+1}|s^t) = \left(\frac{\theta(s^{t+1}|s^t) + m(s^t)}{p(s^t)} \right) \frac{1}{1 + \pi(s^{t+1}|s^t)},$$

and the flow budget constraint reduces to

$$z(s^t) + \frac{r(s^t)}{1 + r(s^t)} a(s^t)l(s^t) + \sum_{s_{t+1}} \tilde{q}(s_{t+1}|s^t)\tilde{\tau}(s_{t+1}|s^t) \leq \tilde{\tau}(s^t).$$

First order conditions for an optimum are

$$\frac{\partial u(c(s^t),1-l(s^t))}{\partial c(s^t)} = \frac{\partial u(c(s^t),1-l(s^t))}{\partial l(s^t)} \left(\frac{a(s^t)}{1+r(s^t)}\right)^{-1},$$

$$\beta f(s^{t+1}|s^t)\frac{\partial u(c(s^{t+1}),1-l(s^{t+1}))}{\partial c(s^{t+1})}\tilde{q}(s^{t+1}|s^t)^{-1} = \frac{\partial u(c(s^t),1-l(s^t))}{\partial c(s^t)},$$

and the transversality condition is

$$\lim_{k\to\infty} \sum_{s^{t+k}|s^t} \tilde{q}(s^{t+k}|s^t)\tilde{\tau}(s^{t+k}|s^t) = 0.$$

The monetary-fiscal authority sets rates of interest and accommodates the demand for balances. It supplies balances, $M(s^t)$, and trades in elementary securities subject to a flow budget constraint is

$$M(s^t) + \sum_{s^{t+1}|s^t} q(s^{t+1}|s^t)\Theta(s^{t+1}|s^t) = T(s^t),$$

scaling the portfolio as $\vartheta(s^t)\delta(s^{t+1}|s^t) = \Theta(s^{t+1}|s^t)$ for some sequence of δ that satisfies $\sum_{s^{t+1}|s^t} \delta(s^{t+1}|s^t) = 1$ at each date event we obtain

$$M(s^t) + \vartheta(s^t) \sum_{s^{t+1}|s^t} q(s^{t+1}|s^t)\delta(s^{t+1}|s^t) = T(s^t),$$

that, after elimination of the trade in assets, reduces to

$$T(s^t) = \frac{r(s^t)}{1+r(s^t)}M(s^t) + \sum_{s^{t+1}|s^t} q(s^{t+1}|s^t)T(s^{t+1}|s^t),$$

where $T(s^t)$ and, similarly, $T(s^{t+1}|s^t)$ are obligations towards the private sector; initial obligations are $\Omega = T(s^0)$.

We will continue to use the definitions for balance sheet policy in Definition 2, with the minor change that the "conventional" policy will be defined as $\delta(s^{t+1}|s^t) = \frac{1}{s}$.

Definition 3. Fiscal policy is

1. Ricardian policy if it imposes on the monetary-fiscal authority the transversality condition

$$\lim_{k\to\infty} \sum_{s^{t+k}|s^0} q(s^{t+k}|s^0)T(s^{t+k}|s^0) = 0$$

or, equivalently, as prices vary, it sets the initial claims of the private sector as

$$\Omega = \frac{r(s^0)}{1+r(s^0)}M(s^0) + \sum_{t>0}\sum_{s^t|s^0} \frac{r(s^t|s^0)}{1+r(s^t|s^0)}q(s^t|s^0)M(s^t|s^0);$$

2. Non-Ricardian policy if it fixes initial liabilities Ω and equilibrium prices adjust to satisfy the transversality condition

$$\lim_{k\to\infty} \sum_{s^{t+k}|s^t} q(s^{t+k}|s^t)T(s^{t+k}|s^t) = 0.$$

Proposition 2. *Given interest rates, $\{r(s^t)\}$,*

1. *with Ricardian (non-Ricardian) fiscal policy, and unrestricted balance sheet policy, the initial price level $p(s^0)$ is indeterminate (determinate) and the stochastic path of inflation $(1+\pi(s^{t+1}|s^t)$ is indeterminate;*

2. *with Ricardian (non-Ricardian) fiscal policy, and restricted balance sheet policy, the initial price level $p(s^0)$ is indeterminate (determinate) and the stochastic path of inflation $(1+\pi(s^{t+1}|s^t)$ is determinate.*

Proof: For equilibrium, it is necessary and sufficient that the excess demand for output vanishes:

$$z(s^t) = c(s^t) - a(s^t)l(s^t) = 0.$$

From the first order conditions for an optimum, this determines the path of employment and consumption:

$$\frac{\partial u(c(s^t), 1-l(s^t))}{\partial c(s^t)} = \frac{\partial u(c(s^t), 1-l(s^t))}{\partial l(s^t)} \left(\frac{a(s^t)}{1+r(s^t)}\right)^{-1},$$

and, in turn, the prices of indexed elementary securities:

$$\beta f(s^{t+1}|s^t)\frac{\partial u(c(s^{t+1}), 1-l(s^{t+1}))}{\partial c(s^{t+1})}\tilde{q}(s^{t+1}|s^t)^{-1} = \frac{\partial u(c(s^t), 1-l(s^t))}{\partial c(s^t)}.$$

Substituting in the allocation just obtained, the present value budget constraint of the monetary-fiscal authority gives

$$\frac{\Omega}{p(s^0)} = \frac{r(s^0)}{1+r(s^0)}y(s^0) + \sum_{t>0}\sum_{s^t|s^0} \frac{r(s^t|s^0)}{1+r(s^t|s^0)}\tilde{q}(s^t|s^0)y(s^t|s^0).$$

The initial price level serves to guarantee that, at equilibrium, the transversality condition of the monetary-fiscal authority holds. If monetary-fiscal policy is Ricardian, the price level remains indeterminate. If it is non-Ricardian, in that initial claims are given, then the equilibrium path of nominal asset prices determines the present-discounted value of unindexed transfers and so the initial price level.

More importantly, without further restrictions, as is the case under QE, the decomposition of equilibrium asset prices into an inflation process, $\pi(\cdot|s^t)$, and a nominal pricing

measure, $\nu(\cdot|s^t)$, remains indeterminate: if the nominal pricing measure, $\nu(\cdot|s^t)$, is specified arbitrarily, the inflation process, $\pi(\cdot|s^t)$, adjusts to implement the equilibrium; that is, to satisfy

$$\tilde{q}(s^{t+1}|s^t) = \frac{\nu(s^{t+1}|s^t)(1+\pi(s^{t+1}|s^t))}{1+r(s^t))}.$$

To see the effect of a restricted balance sheet policy, consider the present value budget constraint of the monetary-fiscal authority at any state s^t that follows date-event s^{t-1}

$$\begin{aligned}\Theta(s^t|s^{t-1}) + M(s^{t-1}) &= \vartheta(s^{t-1})\delta(s^t|s^{t-1}) + M(s^{t-1}) \\ &= \frac{r(s^t)}{1+r(s^t)}M(s^t) + \sum_{i>1}\sum_{s^{t+i}|s^0}\frac{r(s^{t+i}|s^t)}{1+r(s^{t+i}|s^t)}q(s^{t+i}|s^t)M(s^{t+i}|s^t),\end{aligned}$$

In real terms, substituting in equilibrium values for real money balances and rearranging

$$\begin{aligned}&\tilde{\vartheta}(s^{t-1})\delta(s^t|s^{t-1}) + y(s^{t-1}) \\ &\quad = (1+\pi(s^t|s^{t-1}))\left\{\frac{r(s^t)}{1+r(s^t)}y(s^t) + \sum_{i>1}\sum_{s^{t+i}|s^0}\frac{r(s^{t+i}|s^t)}{1+r(s^{t+i}|s^t)}\tilde{q}(s^{t+i}|s^t)y(s^{t+i}|s^t)\right\}.\end{aligned}$$

For each state proceeding date-event s^{t-1} there are $S-1$ relationships of the above form from the definition of the portfolio weights. The final equation is the no-arbitrage equation for the martingale measure

$$\begin{aligned}q(s^t|s^{t-1}) &= \frac{\nu(s^t|s^{t-1})}{1+r(s^{t-1})}, \\ \nu(s^t|s^{t-1}) &= \tilde{q}(s^t|s^{t-1})\frac{1+r(s^{t-1})}{1+\pi(s^t|s^{t-1})}, where \\ \sum_{s^t|s^{t-1}}\nu(s^t|s^{t-1}) &= 1.\end{aligned}$$

If, as in the finite horizon section, the balance sheet policy weights were to depend on expected inflation in a state contingent way, then again the stochastic path of inflation is left undetermined.[12]

□

The determinacy in Woodford (1994) highlights the importance of the present value of the monetary-fiscal authority budget constraint in the determination of the price level. We examine here, and what is often overlooked, is that the stochastic evolution of government wealth is essential for the determination of the stochastic path of prices. That Woodford (1994) restricts attention to conventional policy, in which case the portfolio of the monetary-fiscal authority is composed solely of Treasury bills, obscures this second point. Our results are not dependent on the infinite horizon of the economy, or the open-endedness of the policies that we describe.

[12] As the argument is similar, for the sake of brevity we refer the reader to the previous section.

It may be confusing that we abstract from fiscal transfers after the initial date; we only do so because their implications are straightforward and do not affect the argument. It is worth pointing out, however, that the dichotomy between the nominal pricing measure and the initial price level that obtains when transfers are indexed, no longer holds when transfers are not indexed: the nominal pricing measure, indeterminate under quantitative easing, affects the aggregate volume of claims against the monetary-fiscal authority and, as a consequence, the initial price level as well.

Under QE, the nature of the interest-elasticity of money demand does not determine the stationary equilibrium path, though it determines the stability of the path. Following Sims (1994), the introduction of a portfolio of securities (rather than the single risk-free bond that was considered) under a policy of QE leaves the difference equations, that otherwise determine the path of money, to depend on the state-contingent return on the portfolio of the monetary-fiscal authority and the portfolio-to-money supply ratio. A given (stationary) distribution of portfolio returns, that satisfies the no-arbitrage condition given by the fixed short-term nominal interest rate, then corresponds to a stationary distribution of portfolios, even for the fiscal policy rules that were considered there. Put simply, adequate consideration of the interest-elasticity of money demand guarantees a stationary distribution, but not a determinate one.

2.4 A stationary economy

In this section we examine the nature of stationary equilibria, abstracting from consideration of the stability of stationary equilibria and, therefore, also from the interest-elasticity of money demand. We show that the argument extends to stationary economies and stationary equilibria or steady states.

The resolution of uncertainty follows a stationary stochastic process. Elementary states of the world are s, finitely many, and transition probabilities are $f(s'|s)$.

Rates of interest, $r(s)$, determine the path of consumption, $c(s'|s)$, and employment, $l(s'|s)$, at equilibrium, which, in turn, determine the prices of indexed elementary securities:

$$\beta f(s'|s)\frac{\partial u(c(s'),1-l(s'))}{\partial c(s')}\tilde{q}(s'|s)^{-1} = \frac{\partial u(c(s),1-l(s))}{\partial c(s)}$$

or

$$\tilde{Q} = \beta Du(s)^{-1}FDu(s').$$

Here,

$$Du(s) = diag\left(\ldots,\frac{\partial u(c(s),1-l(s))}{\partial c(s)},\ldots\right)$$

is the diagonal matrix of marginal utilities of consumption, and

$$F = (f(s'|s)) \quad \text{and} \quad \tilde{Q} = (\tilde{q}(s'|s))$$

are, respectively, the matrices of transition probabilities and of prices of indexed elementary securities.

With

$$\tilde{y} = (\ldots \frac{r(s)}{1+r(s)} a(s) l(s) \ldots)'$$

the vector of net, real expenditures on balances at equilibrium, real claims against the fiscal-monetary authority at the steady state,

$$\tilde{\tau} = (\ldots \tilde{\tau}(s), \ldots)',$$

are determined by the equation

$$\tilde{y} + \tilde{Q}\tilde{\tau} = \tilde{\tau} \quad \text{or} \quad \tilde{\tau} = (I - \tilde{Q})^{-1}\tilde{y};$$

since $0 < \beta < 1$,

$$(I - \tilde{Q})^{-1} = \sum_{k=0}^{\infty} \tilde{Q}^k = \sum_{k=0}^{\infty} \beta^k Du(s)^{-1} F^k Du(s'),$$

and, since F is a Markov transition matrix, while $\tilde{y} \gg 0$, the real claims against the monetary-fiscal authority at the steady state are strictly positive:

$$\tilde{\tau} \gg 0.$$

Non-Ricardian monetary-fiscal policy determines the initial price level by setting exogenously the level of initial nominal claims; otherwise, the price level remains indeterminate.

More importantly, the decomposition of equilibrium asset prices into an inflation process, $\pi(\cdot|s)$, and a nominal pricing measure, $\nu(\cdot|s)$, remain indeterminate:

$$\tilde{Q} = R^{-1} N \otimes \Pi,$$

where $\otimes$ denotes the Hadamard product. Here,

$$R = diag(\ldots, (1 + r(s)), \ldots)$$

is the diagonal matrix of interest factors, and

$$N = (\nu(s'|s)) \quad \text{and} \quad \Pi = ((1 + \pi(s'|s)))$$

are, respectively, the Markov transition matrix of "nominal pricing transition probabilities" and the matrix of inflation factors.

Alternative specifications of the stochastic process of inflation serve to characterize the set of equilibria and to highlight the role of the balance sheet policy of the monetary-fiscal authority. The role of the balance sheet policy is the focus of the analysis here while it was not dealt with in Drèze and Polemarchakis (2000) or Bloise, Drèze, and Polemarchakis (2005).

Proposition 3. *Given a stationary interest rate policy, $\{r(s)\}$,*

1. *Under Quantitative Easing or unrestricted balance sheet policy, the stationary inflation process, $\{\pi(s'|s)\}$ is indeterminate;*
2. *Under Credit Easing or restricted balance sheet policy, the stationary inflation process, $\{\pi(s'|s)\}$ is determinate;*
3. *Simple inflation processes obtain a determinate stationary path of inflation and are compatible with Conventional balance sheet policy.*

Proof:

QE: In the absence of restrictions on the balance sheet of the monetary fiscal authority, which is the case under QE, the set of steady state equilibria is indexed by the nominal pricing transition probabilities, $\nu(\cdot|s)$, that can be set arbitrarily, while the inflation factors, $\pi(\cdot|s)$, adjust to implement the equilibrium; alternatively, the inflation factors are set arbitrarily, up to a scale effect, and the nominal pricing transition probabilities adjust to implement the equilibrium.

The argument is as follows: with

$$(1 + \pi(s'|s)) = h(s)\gamma(s'|s),$$

an arbitrary (for the moment) decomposition of the inflation process into a term (the scale effect) that depends only on the current state and a term of (relative) inflation factors and is Markovian, the equilibrium condition takes the form

$$\tilde{Q} = R^{-1}N \otimes H\Gamma;$$

here, H be the diagonal matrix of the $h(s'|s)$ and Γ the matrix of the $\gamma(s'|\ s)$.

Given Γ, there are H, N that guarantee equilibrium; the argument is straightforward:

$$\tilde{Q} = R^{-1}N \otimes \Pi \Rightarrow \tilde{Q} = R^{-1}N \otimes H\Gamma \Rightarrow \tilde{Q} \oslash \Gamma = (R^{-1}H)N,$$

the last step, since H is a diagonal matrix.[13]

Since N is Markovian if and only if it is non-negative and $N\mathbf{1}_S = \mathbf{1}_S$,

$$(\tilde{Q} \oslash \Gamma)\mathbf{1}_S = (R^{-1}H)\mathbf{1}_S,$$

which allows us to solve for H.

With H, Γ in hand, we can solve for N, that shall indeed, be Markovian.

If N is given, there are H, Γ (or, equivalently, Π) that guarantee equilibrium.

[13] $\oslash$ denotes Hadamard division.

Taylor rules: We now show that the indeterminacy obtained is not ruled out by interest-feedback rules. Any process can be written uniquely as

$$(1+\pi(s'|s)) = h(s)\gamma(s'|s), \quad \gamma(s'|s) = \frac{\delta(s'|s)}{f(s'|s)}, \quad \sum_{s'} \delta(s'|s) = 1,$$

in which case,

$$h(s) = \mathrm{E}_{s'}(1+\pi(s'|s));$$

With $r(s)$ not set exogenously, but as a function of $h(s)$, this is a Taylor (1993) rule, and indeterminacy persists. In other words, policy that specifies the path of nominal interest rates as a function of expected inflation, does not pin down the stochastic path of inflation.[14]

Evidently, with $r(s)$ not set exogenously, but as a function of $h(s)$, equilibrium requires solution of the equation

$$\frac{h(s)}{1+r((h(s))} = \sum_{s' \in S} \frac{f(s'|s)}{\gamma(s'|s)} \beta \frac{\frac{\partial u(c(h(s')),1-l(h(s')))}{\partial c(h(s'))}}{\frac{\partial u(c(h(s)),1-l(h(s)))}{\partial c(s)}},$$

where the allocation, as a function of $h(s)$, is solved from the individual optimality conditions. If a solution to this system of equations exists and is unique, for example if the function/rule is linear, then the solution still depends on the (arbitrarily chosen) Γ.

CE: Alternatively,

$$(1+\pi(s'|s)) = h(s)\gamma(s'|s), \quad \gamma(s'|s) = \frac{\delta(s'|s)}{\tilde{\tau}(s')}, \quad \sum_{s'} \delta(s'|s) = 1,$$

in which case, $\delta(s'|s)$ are portfolio weights that determine the composition of assets in the balance sheet of the monetary-fiscal authority.

Monetary-fiscal policy conducted as CE sets the composition of the balance sheet; that is, it sets explicit positive portfolio weights, $\delta(s'|s) > 0$; claims against the monetary-fiscal authority in real terms, $\tilde{\tau}(s'|s)$, are determined, at the steady-state, by fundamentals, and, as a consequence, under CE, the matrix Γ is determined.

Since

$$N\mathbf{1}_S = \mathbf{1}_S \quad \Leftrightarrow \quad H\mathbf{1}_S = (R\tilde{Q} \oslash \Gamma)\mathbf{1}_S,$$

the Markov transition matrix, N, is well defined ($h \gg 0$) and determinate; it follows that the equilibrium is determinate as well.

Under conventional monetary-fiscal policy, the portfolio of the monetary-fiscal authority consists of Treasury bills, nominally risk-free bonds of short maturity. Here, this corresponds to one-date nominally risk-free bonds: $\delta(s'|s) = 1/S$.

[14] That the Taylor rule does not depend on realized rates of inflation is appropriate for (stochastic) steady-state equilibria.

Determinacy obtains for any arbitrary portfolio weights in the balance sheet of the monetary-fiscal authority that, importantly, only depend on fundamentals and/or realized variables at s. If the portfolio weights are chosen by policy to depend on endogenous nominal variables at s', such as the expected stochastic rate of inflation, then indeterminacy obtains. As an explicit example, consider that the portfolio weights depended on expectations of the future nominal value of wealth: $\delta(s'|s) = [\tilde{\tau}(s')(1+\pi(s'|s))]/[\sum_{s'} \tilde{\tau}(s')(1+\pi(s'|s))] \Rightarrow h(s'|s) = \sum_{s'} \tilde{\tau}(s')(1+\pi(s'|s))$. In a model where there are long-dated securities, setting portfolio weights as a function of expected nominal asset prices would have the same outcome. This contrasts with Magill and Quinzii (2014b) and Adao, Correia, and Teles (2014), where explicit targets for asset prices, independent of equilibrium, pin down portfolio weights. This would correspond to setting targets (or portfolio weights) to depend on fundamentals, or primitives, or according to exogenous rules (such as under conventional policy). Given the difficulties in obtaining information on the primitives or exogenous processes in the economy, policy makers in practice need to set targets based on either exogenous rules, or past, present or expected (future) endogenous variables. As unconventional policy aims to affect the future course of the economy, targets invariably will depend on forecasts of the future. Our point here is that constructing balance sheet portfolios based on such forecasts may result in the portfolios being compatible with other ex-post realisations (of inflation, asset prices etc) while rules based on past variables may not be compatible with stationary endogenous processes. In contrast, rules based on realised endogenous variables (such as current GDP, inflation etc) can implement a unique, if not desired, stationary path of inflation.

Simple inflation processes: It is instructive to consider whether restricting the inflation process to depend endogenously only on either the current or future state is compatible with an equilibrium policy choice. Suppose that the inflation process, which is endogenous, is restricted to take the form

$$(1+\pi(s'|s)) = h(s)\xi(s'),$$

where $\xi(s'|s) > 0$ is positive function of the fundamentals of the economy determined at the steady state and, as a consequence,

$$N \otimes \Pi = HNB.$$

Here,

$$b = (\ldots, \xi(s), \ldots), \quad \text{and} \quad h = (\ldots, h(s), \ldots),$$

and B and H are the associated diagonal matrices.

Then,

$$\tilde{Q} = R^{-1} N \otimes \Pi \quad \Leftrightarrow \quad R\tilde{Q}B^{-1} = HN,$$

which determines the inflation process as well as nominal pricing probabilities, since

$$N\mathbf{1}_S = \mathbf{1}_S \quad \Leftrightarrow \quad N = \left(diag(R\tilde{Q}B^{-1}\mathbf{1}_S)\right)^{-1} R\tilde{Q}B^{-1},$$

is a Markov transition matrix, as required.

This is indeed the case under conventional monetary policy. To see this, real wealth at successor date-events is

$$\tilde{\tau}(s') = \left(\frac{\theta(s'|s) + m(s)}{p(s)}\right) \frac{1}{1 + \pi(s'|s)},$$

conventional monetary policy purchases a portfolio that is nominally riskless and, in our framework, requires that

$$\theta(s'|s) = \frac{1}{s}$$

or

$$(1 + \pi(s'|s)) = \underbrace{\left(\frac{\theta(s) + m(s)}{p(s)}\right)}_{h(s)} \underbrace{\frac{1}{\tilde{\tau}(s')}}_{\xi(s')}$$

Suppose, instead, that inflation is restricted to depend endogenously only on the future state,

$$(1 + \pi(s'|s)) = h(s')\xi(s),$$

and, as a consequence,

$$N \otimes \Pi = BNH.$$

In this case,

$$\tilde{Q} = R^{-1} N \otimes \Pi \quad \Leftrightarrow \quad B^{-1} R\tilde{Q} = NH,$$

and

$$N\mathbf{1}_S = \mathbf{1}_S \quad \Leftrightarrow \quad N = B^{-1} R\tilde{Q} \left(diag((B^{-1} R\tilde{Q})^{-1} \mathbf{1}_S)\right)$$

that need not be positive. In other words equilibrium inflation may be restricted to depend endogenously only on the current state but not only on the future one. However such a restriction precludes analysis of the effects of unconventional monetary policy on changes in the composition of the balance sheet of the monetary-fiscal authority and their subsequent determination of the stochastic path of inflation.

□

3 A large open economy

We now relax the assumption that the economy is closed because although unconventional policy, especially in the US, was aimed at domestic markets, global consequences were evident. Predictably, the US dollar depreciated which led to calls of "Currency Wars" by political leaders. We show that the absence of adequate restrictions on the monetary-fiscal authority balance sheet causes the stochastic path of inflation and also exchange rates to be indeterminate. As earlier, the result is purely nominal, but with incomplete markets, the effects would be real and cause policy in trade partners to not be optimal.[15]

[15] The consequences of international spillovers of policy was argued by Taylor (2013) in a New-Keynesian framework, though our results show that optimal policy responses, in a Nash Equilibrium sense, are not unique if the equilibrium itself is not determinate.

There are two countries in the world, home and foreign, each inhabited by a representative agent. Foreign variables, both macro and those relating to foreign agents, will be denoted with an asterisk (*). The transactions of agents in the home and foreign country will be denoted with a subscript "h" and "f", respectively. It suffices to specify explicitly mostly only the constraints and variables relevant for the home agent and country; the foreign agent and country are equivalent.

At a date-event, a perishable non-tradable input, labor, $l(s^t)$, is employed to produce a perishable domestic tradable output, $y(s^t)$, according to a linear technology. The representative home individual is endowed with 1 unit of leisure at every date-event. He supplies non-tradable labor and demands the tradable consumption good, and he derives utility according to the cardinal utility index $u(c(s^t), 1 - l(s^t))$ that satisfies standard monotonicity, curvature and boundary conditions. The preferences of the individual over consumption-employment paths commencing at s^t are described by the separable, von Neumann-Morgenstern, intertemporal utility function. Balances, $m_h(s^t)$ and $m_f(s^t)$ provide liquidity services in the home and foreign country respectively. Elementary securities, $\theta(s^{t+1}|s^t)$, serve to transfer wealth to and from immediate successor date-events. The price level is $p(s^t)$, and the wage rate is $w(s^t) = a(s_t)p(s^t)$, as profit maximisation requires. The nominal, risk-free interest rate is $r(s^t)$. As the goods produced in each country are perfect substitutes, the law-of-one-price holds and determines the exchange rate $e^*(s^t) = p(s^t)/p^*(s^t)$.

At each date-event, the asset (and currency) market opens after the uncertainty, s^t, has realized, and, as a consequence, purchases and sales in the markets for labor and the consumption good are subject to standard cash-in-advance constraints; the effective cash-in-advance constraint is[16]

$$a(s^t)p(s^t)l(s^t) \leq m_h(s^t), \quad 0 \leq m_f^*(s^t).$$

Prices of elementary securities in the domestic country are

$$q(s^{t+1}|s^t) = \frac{\nu(s^{t+1}|s^t)}{1 + r(s^t)},$$

with $\nu(\cdot|s^t)$ the domestic "nominal pricing measure". Note that the nominal prices of elementary securities and the "nominal pricing measure" are unique to the currency in which they are denominated. As there are a complete set of state-contingent bonds in each currency, the prices of securities which deliver currency in the same state are related by the following no-arbitrage condition

$$\frac{\nu^*(s^{t+1}|s^t)}{\nu(s^{t+1}|s^t)} \left\{ \frac{1 + r(s^t)}{1 + r^*(s^t)} \right\} = \frac{e^*(s^{t+1}|s^t)}{e^*(s^t)}.$$

In other words, the path of nominal exchange rates depends on the ratio of the "nominal pricing measure" across countries and implies the uncovered interest parity condition

$$\frac{1 + r(s^t)}{1 + r^*(s^t)} e^*(s^t) = \sum_{s^{t+1}|s^t} \nu(s^{t+1}|s^t) e^*(s^{t+1}|s^t).$$

[16] Nakajima and Polemarchakis (2005) provide an explicit derivation.

This gives the risk-neutral expected exchange rate. As markets are complete, variations in the nominal equivalent martingale measure in each country only have nominal effects on the implicit premium in the exchange rate.[17] If there were nominal rigidities or frictions which prevented the law of one price from holding, then the covariance between the nominal equivalent martingale measure and nominal exchange rate, and hence the premium in expected exchange rates, would imply different allocations of (real) resources.

The individual has initial nominal wealth $\tau_h(s^t)$ and $\tau_f(s^t)$ in each country. Initial wealth constitutes a claim against the respective monetary-fiscal authority; alternatively, it can be interpreted as outside money. It is exogenous in a non-Ricardian specification. In a Ricardian specification, it is set endogenously so as to satisfy the transversality condition imposed on monetary-fiscal policy.

The monetary-fiscal authority in each country sets domestic one date rates of interest and accommodates the demand for domestic balances. It supplies domestic balances, $M(s^t)$, and trades in domestic elementary securities subject to a flow budget constraint as in the closed economy case.

A formal description of the model appears in the Appendix as part of the proof of Proposition 4 below.

We will use the balance sheet policy definitions in Definition 2 subject to the modification for conventional policy described in the previous section.

Proposition 4. *Given interest rates, $\{r(s^t)\}$,*

1. *with Ricardian (non-Ricardian) fiscal policy and unrestricted balance sheet policy in each country, the initial price level $p(s^0)$ and $p^*(s^0)$ and exchange rate $e^*(s^0)$ is indeterminate (determinate) and there is indeterminacy of the stochastic path of inflation and exchange rates;*

2. *in stationary equilibria, with unrestricted balance sheet policy, the path of inflation rates and exchange rates are indeterminate;*

3. *in stationary equilibria and restricted balance sheet policy, the path of inflation rates and exchange rates are determinate.*

The proof is in the Appendix.

Concerning the indeterminacy that obtains in the open economy, further remarks are in order:

1. Our results under unconventional QE policies remain valid if the law of one price failed to hold, as in Corsetti and Pesenti (2005), or if there were pricing rigidities. However, in these cases the indeterminacy may have real effects.

2. The indeterminacy under QE obtained is not a consequence of deviations from steady-state equilibria and will not be eliminated by an interest rate feed-back rule, such as a "Taylor rule". The non-stationary equilibria results presented above

[17] The difference between the risk neutral and objective expected exchange rate.

allow for extreme paths inflation and exchange rates. As the Fisher equation only guarantees an expected rate of inflation, it is entirely possible that there are paths of ever increasing inflation and a path of ever decreasing inflation (deflation), and consequently large stochastic changes in nominal exchange rates, and is reminiscent of the literature on speculative hyperinflation such as Obstfeld and Rogoff (1983).

3. The link between the path of the wealth of the central bank balance sheet and the path of inflation and exchange rates is also documented in Bhattarai (2016). Here we go on to show how this path also depends on the composition of the portfolio.

4. Our requirement that the present-value budget constraints of the monetary-fiscal authority in each country be satisfied individually is not innocuous. Equilibrium only requires that the individual household budget constraints are satisfied, and as a consequence, only the joint budget constraint of the two government budget constraints will be satisfied. In that case the non-Ricardian assumption only guarantees that the present value of the monetary liabilities of both central banks, weighted by the exchange rate, equals the initial nominal wealth, also weighted by the exchange rate. As a consequence, neither the price levels in each country nor the exchange rate is determinate. This is the point of Dupor (2000). Here, the non-Ricardian assumption in each country results in the price-level in each country to be uniquely determined. The subsequent indeterminacy is then restricted to the indeterminacy of the stochastic path of inflation, and is convenient to identify the role that QE plays in generating this indeterminacy.

5. A managed exchange rate, satisfying uncovered interest parity, will either transmit or eliminate the indeterminacy. If, for arguments sake, the home country conducts traditional monetary policy (and has a determinate path of inflation), then the foreign country may partake in quantitative easing and, provided that they also target a path of the exchange rate, then the law-of-one-price guarantees that foreign prices are also determinate. If, however, the home country also conducts quantitative easing, then management of the path of the exchange rate leaves the rates of inflation in each country indeterminate. This is because the law-of-one-price only determines the ratio of prices across countries to equal the nominal exchange rate, but the (stochastic) levels are left free.

6. Our argument allows monetary-fiscal authorities to arbitrarily select the composition of initial assets, and independently of the initial quantity of money and price level, which are determined by the initial fiscal liabilities. Our argument is valid when the monetary-fiscal authority attempts to affect the initial quantity of money by purchasing assets with newly printed money: this would be analagous to increasing the outstanding liabilities that need to be returned through seignorage profits

7. The argument holds for the policies of unwinding of quantitive easing that are dependent on realized rates of inflation. This will be made more explicit in the

following section, but intuitively, the monetary-fiscal authority here are faced with a new portfolio every date, due to the one-date contracts we focus on. This implies that the degrees of indeterminacy are $S-1$ in each country and state. Hence, even if the initial portfolio composition is fixed, the consequent evolution of the portfolio (ie the unwinding phase) is not and indeterminacy will result.

It is worth pointing out that the indeterminacy we obtain is not a consequence of the stochastic nature of our economy per se, but rather that, given the uncertainty, the non-colliniarity of assets traded by the monetary-fiscal authority. In a related note, McMahon et al. (2012), we examine the consequences of the recent European Central Bank (ECB) policy on purchasing the debt of member countries (Outright Monetary Transactions, or OMT). If the bonds purchased by the ECB are not expected to default, which such a policy is in fact designed to support, then the bonds of the member countries are collinear and there is no requirement to provide ex-ante restrictions on the composition of assets held by the ECB. If, however, such a policy cannot prevent default, then the bonds are no longer collinear and short-term interest rates may no longer be sufficient to determine the path of Eurozone inflation.

4 Unconventional Monetary Policy and Premia

We have shown the importance of the portfolio composition for determining equilibrium in closed (section 2) and open (section 3) economies. In this final analytical section, we consider the consequences of the portfolio rule for the nature of equilibrium.

In cash-in-advance specifications, liquidity costs generate a wedge between cash and credit goods, and consequently affect marginal utilities and equilibrium prices. This generates a positive correlation between the (real) stochastic discount factor and expected nominal interest rates, and, as a consequence, a real risk premium that causes the term structure of interest rates to lie above levels predicted by the pure expectation hypothesis. In a closed economy, Espinoza et al. (2009) show that the risk-premia generated by the non-neutrality of monetary policy exist in addition to the ones derived from the stochastic distribution of endowments as presented in Lucas (1978) and Breeden (1979). They provide a potential explanation for the Term Premium Puzzle.[18] In an open economy, the argument extends, whereby the path of nominal interest rates in each country can affect real risk-premia on the path of nominal exchange rates as in Peiris and Tsomocos (2015).[19] This is in contrast to equilibrium models where monetary policy is neutral, as in Lucas (1982), where, as risk premia are constant, interest rate differentials move

[18]There is a large literature on the difficulties of the uncovered interest parity holding empirically. The forward premium anomaly, as documented by Fama (1984), Hodrick (1987), and Backus et al. (1995) among others, states that when a currency's interest rate is high, that currency is expected to apprecash-in-advance constraintte. Roughly speaking, the expected change in the exchange rate is constant and interest differentials move approximately one-for-one with risk premia.

[19]In that paper, there are two countries each inhabited by a representative agent and who must use domestic money for domestic trades, such as in the present paper. A cash-in-advance structure means that nominal interest rates affect the wedge between the marginal utilities of income and expenditure.

one-for-one with the expected change in the exchange rate. We extend this literature by showing how the composition of the monetary-fiscal authority balance sheet, in addition to policy setting the path of interest rates or money supplies, affects premia in the bond and currency markets. These results extend to economies with incomplete markets and price rigidities, in which case the premia would also be real.

Proposition 5. *Given interest rates,* $\{r(s^t)\}$,

1. *the size and sign of premia of the term structure of interest rates in each country is indexed by the path of the composition of the monetary-fiscal authority portfolio in each country;*
2. *the size and sign of exchange rate premia is indexed by the path of the composition of the monetary-fiscal authority portfolio in each country;*

Proof:

Term Premia

Our analysis utilises the stationary equilibrium results obtained in the previous section. Consider the price, in the home country, of a two-date nominally risk-free bond, at state s:

$$q_2(s) = \sum_{s'} q(s'|s) \sum_{s''|s'} q(s''|s') = \frac{1}{1+r(s)} \sum_{s'|s} \frac{\nu(s'|s)}{1+r(s')}$$

In other words, the forward rate gives the risk-neutral expectation of the future one-date interest rates:

$$q_2(s)(1+r(s)) = \sum_{s'|s} \frac{\nu(s'|s)}{1+r(s')}.$$

The term premia are then described by

$$\sum_{s'|s} \frac{\nu(s'|s) - f(s'|s)}{1+r(s')}.$$

The stationary distribution of the term premia is[20]

$$N \otimes \hat{R}^{-1} - F \otimes \hat{R}^{-1} = [(R^{-1}H)^{-1}\tilde{Q} \oslash \Gamma - F] \otimes \hat{R}^{-1}.$$

Furthermore, markets are incomplete and agents may default upon their nominal obligtations. Monetary policy, by altering the wedge, affects the volume of real trade, and hence marginal utilities, default probabilities and implied risk neutral probabilities. Consequently, there is a covariance between nominal exchange rates, and real and nominal premia which affects the difference between the risk-neutral and objective expectation of future exchange rates. In the present paper, this difference is generated purely by altering the composition of assets traded by the monetary-fiscal authorities in each country.

[20] Recall that H is solved from

$$(\tilde{Q} \oslash \Gamma)\mathbf{1}_S = (R^{-1}H)\mathbf{1}_S.$$

$\hat{R}^{-1}$ is a matrix with each row being the elements of the diagonal of matrix R^{-1}. In other words, $\hat{R}^{-1}$ is a square matrix with identical rows where each column is the bond price for the realised state.

Recall that Γ is the matrix of portfolio weights relative to the payoff of the balance sheet of the monetary-fiscal authority. Hence, given the fundamentals of the economy, and a given path of one-date interest rates, the term premia depends on the composition of the monetary-fiscal authority balance sheet. More precisely, a correlation is generated between the nominal martingale measure and nominal interest rates which results in risk-neutral pricing being systematically biased (from subjective pricing alone).

Currency Premia

Recall the Uncovered Interest Parity equation in state s

$$e^*(s'|s)\frac{1+r(s'|s)}{1+r^*(s'|s)} = \sum_{s'|s} \nu(s'|s)e^*(s'|s),$$

where $\sum_{s'|s} \nu(s'|s)e^*(s'|s)$ is the risk neutral expectation of exchange rates. The realised distribution of exchange rates implies an (objective) expectation of $\sum_{s'|s} f(s'|s)e^*(s'|s)$. The difference between these two will be the currency premium. The stationary distribution of the premium is:

$$\begin{gathered} N \otimes E - F \otimes E = N \otimes \Pi \oslash \Pi^* - F \otimes \Pi \oslash \Pi^* = \\ R\tilde{Q} \oslash \Pi^* - F \otimes \Pi \oslash \Pi^* = (R\tilde{Q} - F \otimes \Pi) \oslash \Pi^* = \\ (R\tilde{Q} - F \otimes H\Gamma) \oslash (H^*\Gamma^*) \end{gathered}$$

Given that in each country H depends on R, $\tilde{Q}$ and Γ of that country, this is entirely in terms of real variables and nominal interest rates and portfolio weights set by policy. Furthermore there is a clear separation between home and foreign variables and policy parameters. It follows then that stationary portfolio weights chosen in each country correspond to varying premia in the currency markets. The sign and magnitude of the premium can be chosen arbitrarily by appropriate choices of nominal interest rates and portfolio weights. Note that varying the nominal interest rates results in the premium having a real (risk) component while varying the portfolio weights affects the stationary distribution of inflation and exchange rates which is purely nominal. From the equation, it is clear that the joint distribution of interest rates and inflation across countries matters in addition to the mean and variance of the inflation process in each country. □

Bhattarai et al. (2015) point out that a maturity mismatch on the central bank balance sheet occurs when the asset structure moves away from short term to long term under unconventional policy and, consequently, generates risk in the path of central bank net wealth. They highlight how such policy induce real effects through a "signalling" channel while here we highlight the effects on term premia which, in a framework with market incompleteness, would also have real effects. The term premia generated by altering the composition of the balance sheet is also reflected in the path of nominal central bank net wealth. To see this, note that the matrix of growth rates of nominal central bank net wealth is

$$(1+\pi(s'|s)) \otimes \left(\frac{\tilde{T}(s')}{\tilde{T}(s)}\right).$$

As well as the determining the matrix of stochastic inflation rates (shown earlier), in this section we showed how the portfolio shares affect the size and sign of term premia.

We have considered only interest rate targeting; the results do extend to policies that target the paths of money supplies. In that case, although the path of money is given by policy, fluctuations in demands for assets affect the path of interest rates and changes in the composition of monetary-fiscal authority portfolio has real effects. That is, in a money growth targeting regime, the path of real risk-premia depends on the composition of assets held by the monetary-fiscal authority.[21]

5 Concluding Remarks

We have shown that unconventional balance sheet policies may result in the central bank being unable to implement a determinate path of inflation in an environment in which it would otherwise be able to. We showed that the central bank can regain control of the path of inflation by selecting the composition of the assets on its balance sheet to be exogenous or depend on realised variables. We argued that this is the case under CE policies as well as conventional monetary policy.

In contrast, QE policies that focus on the size of the central bank balance sheet do not restrict the composition of assets and the path of inflation is indeterminate. This is the case under Taylor rules and when the portfolio composition policy depends on expected inflation. Our results extend to an open economy where the path of nominal exchange rates is left indeterminate under unrestricted quantiative easing policies. Finally we show that the portfolio composition determines the size and sign of term and currency premia.

An open question is whether other unconventional policies, such as forward guidance which is one of the other classifications of unconventional policy used by Borio and Zabai (2016), might overturn or mitigate our results. While we did not focus on other forms of unconventional policy, to the extent that we view forward guidance as a credible commitment to keep the path of interest rates fixed then it will not rule out the indeterminacy.

Finally, the results we have described are in an environment absent the financial frictions and liquidity constraints that characterised the financial crisis. Papers such as Gertler and Karadi (2011), Williamson (2012) and Schabert (2015) show that unconventional policies can have real effects by interacting with these constraints. The composition of the portfolio in our model has no welfare implications, but extending it to environments like these would allow careful analysis of what optimal portfolio choices would be, given the traditional tools of interest rate or money supply policies and the dependency of these with fiscal policy.

[21] Nakajima and Polemarchakis (2005) show this in a closed economy. Alvarez et al. (2009) consider an open economy similar to ours, but with segmented participation in the asset market; in the the absence of a credit good, monetary policy is otherwise neutral. If the monetary-fiscal authority portfolio is left unrestricted, introducing a credit good may mean that the correlation between interest rates and risk premia depend both on the path of money and the evolution of the portfolio composition.

6 Appendix

6.1 Proof of Proposition 4

Proof: The flow budget constraint is[22]

$$p(s^t)c_h(s^t) + e^*(s^t)p^*(s^t)c_f(s^t) + m(s^t)$$
$$+ \sum_{s^{t+1}|s^t} \left\{ q(s^{t+1}|s^t)\theta_h(s^{t+1}|s^t) + e^*(s^t)q^*(s^{t+1}|s^t)\theta_f(s^{t+1}|s^t) \right\}$$
$$\leq p(s^t)a(s^t)l(s^t) + \tau_h(s^t) + e^*(s^t)\tau_f(s^t).$$

Debt limit constraints are

$$\tau_h(s^t) + e^*(s^t)\tau_f(s^t) \geq -\sum_{k>0} \sum_{s^{t+k}|s^t} q(s^{t+k}|s^t) \frac{1}{1+r(s^t)} a(s^{t+k})$$

or, equivalently,

$$\lim_{k\to\infty} \sum_{s^{t+k}|s^t} q(s^{t+k}|s^t) \left\{ \tau_h(s^{t+k}|s^t) + e^*(s^{t+k}|s^t)\tau_f(s^{t+k}|s^t) \right\} \geq 0.$$

Wealth at successor date-events is

$$\tau_h(s^{t+1}|s^t) = \theta(s^{t+1}|s^t) + m(s^t) \quad \text{and} \quad \tau_f(s^{t+1}|s^t) = \theta_f(s^{t+1}|s^t),$$

and, after elimination of the trade in assets and using the law-of-one-price, the flow budget constraint reduces to

$$p(s^t)z(s^t) + \frac{r(s^t)}{1+r(s^t)} a(s^t)p(s^t)l(s^t) + \sum_{s_{t+1}} q(s_{t+1}|s^t)\tau(s_{t+1}|s^t) \leq \tau(s^t),$$

where $z(s^t) = c(s^t) - a(s^t)l(s^t)$ is the effective excess demand for consumption, $c(s^t)$ is the sum of consumption at home and abroad and $\tau(s^t) = \tau_h(s^t) + e^*(s^t)\tau_f(s^t)$.

$\tilde{m}(s^t) = (1/p(s^t))m(s^t)$ are real balances, $\tilde{\tau}(s^t) = (1/p(s^t))\ \tau(s^t)$ is real wealth, $\pi(s^{t+1}|s^t) = (p(s^{t+1})/(p(s^t)) - 1$ is the rate of inflation, and

$$\tilde{q}(s^{t+1}|s^t) = q(s^{t+1}|s^t)(1 + \pi(s^{t+1}|s^t)) = \frac{\nu(s^{t+1}|s^t)(1 + \pi(s^{t+1}|s^t))}{1 + r(s^t)}$$

are prices of indexed elementary securities[23].

Real wealth at successor date-events is

$$\tilde{\tau}(s^{t+1}|s^t) = \left(\frac{\theta(s^{t+1}|s^t) + m(s^t) + e^*(s^{t+1}|s^t)\theta(s^{t+1}|s^t)}{p(s^t)} \right) \frac{1}{1 + \pi(s^{t+1}|s^t)},$$

[22]Foreign money balances are dominated by foreign bonds and are zero in equilibrium, while the effective cash-in-advance constraint guarantees that domestic money balances are positive.

[23]From the no-arbitrage condition for assets, this is also the same in the foreign country.

and the flow budget constraint reduces to

$$z(s^t) + \frac{r(s^t)}{1+r(s^t)} a(s^t) l(s^t) + \sum_{s_{t+1}} \tilde{q}(s_{t+1}|s^t)\tilde{\tau}(s_{t+1}|s^t) \leq \tilde{\tau}(s^t).$$

First order conditions for an optimum are

$$\tfrac{\partial u(c(s^t),1-l(s^t))}{\partial c(s^t)} = \tfrac{\partial u(c(s^t),1-l(s^t))}{\partial l(s^t)} \left(\tfrac{a(s^t)}{1+r(s^t)}\right)^{-1},$$

$$\beta f(s^{t+1}|s^t) \tfrac{\partial u(c(s^{t+1}),1-l(s^{t+1}))}{\partial c(s^{t+1})} \tilde{q}(s^{t+1}|s^t)^{-1} = \tfrac{\partial u(c(s^t),1-l(s^t))}{\partial c(s^t)},$$

and the transversality condition is

$$\lim_{k\to\infty} \sum_{s^{t+k}|s^t} \tilde{q}(s^{t+k}|s^t)\tilde{\tau}(s^{t+k}|s^t) = 0.$$

For equilibrium, it is necessary and sufficient that the excess demand for output vanishes:

$$z(s^t) + z^*(s^t) = c(s^t) + c^*(s^t) - a(s^t)l(s^t) - a^*(s^t)l^*(s^t) = 0,$$

which determines the path of employment and consumption for each household:

$$\frac{\partial u(c(s^t), 1-l(s^t))}{\partial c(s^t)} = \frac{\partial u(c(s^t), 1-l(s^t))}{\partial l(s^t)} \left(\frac{a(s^t)}{1+r(s^t)}\right)^{-1};$$

in turn, this determines the prices of indexed elementary securities:

$$\beta f(s^{t+1}|s^t) \frac{\partial u(c(s^{t+1}), 1-l(s^{t+1}))}{\partial c(s^{t+1})} \tilde{q}(s^{t+1}|s^t)^{-1} = \frac{\partial u(c(s^t), 1-l(s^t))}{\partial c(s^t)}.$$

The initial price level serves to guarantee that, at equilibrium, the trans- versality condition of the monetary-fiscal authority holds. If monetary-fiscal policy is Ricardian, the price level remains indeterminate.

More importantly, without further restrictions, as is the case under QE, the decomposition of equilibrium asset prices into an inflation process, $\pi(\cdot|s^t)$, and a nominal pricing measure, $\nu(\cdot|s^t)$, remains indeterminate: if the nominal pricing measure, $\nu(\cdot|s^t)$, is specified arbitrarily, the inflation process, $\pi(\cdot|s^t)$, adjusts to implement the equilibrium; that is, to satisfy

$$\tilde{q}(s^{t+1}|s^t) = \frac{\nu(s^{t+1}|s^t)(1+\pi(s^{t+1}|s^t))}{1+r(s^t)}.$$

Furthermore, the path of the nominal exchange rate remains indeterminate. Arbitrary nominal pricing measures in each country determine the stochastic future exchange rate to satisfy

$$\frac{\nu^*(s^{t+1}|s^t)}{\nu(s^{t+1}|s^t)} \left\{\frac{1+r(s^t)}{1+r^*(s^t)}\right\} e^*(s^t) = e^*(s^{t+1}|s^t).$$

A stationary economy The argument extends to stationary economies and stationary equilibria or steady states.

The resolution of uncertainty follows a stationary stochastic process. Elementary states of the world are s, finitely many, and transition probabilities are $f(s'|s)$.

Rates of interest, $(r(s), r^*(s))$ determine the path of consumption, $(c(s), c^*(s))$ and employment, $(l(s), l^*(s))$ at equilibrium, which, in turn, determine the prices of indexed elementary securities:

$$\beta f(s'|s)\frac{\partial u(c(s'), 1-l(s'))}{\partial c(s')}\tilde{q}(s'|s)^{-1} = \frac{\partial u(c(s), 1-l(s))}{\partial c(s)}$$

or

$$\tilde{Q} = \beta Du(s)^{-1} F Du(s').$$

Note that the prices of indexed elementary securities is independent of the country. The nominal elementary securities, and hence martingale measures, across countries differ in their stochastic rates of inflation (and consequently the no-arbitrage condition).

Here,

$$Du(s) = diag(\ldots, \frac{\partial u(c(s), 1-l(s))}{\partial c(s)}, \ldots)$$

is the diagonal matrix of marginal utilities of consumption, and

$$F = (f(s'|s)) \quad \text{and} \quad \tilde{Q} = (\tilde{q}(s'|s))$$

are, respectively, the matrices of transition probabilities and of prices of indexed elementary securities.

For the home household,

$$\tilde{m} = (\ldots \frac{r(s)}{1+r(s)} a(s) l(s) \ldots)$$

is the vector of net, real balances at equilibrium,

$$\tilde{z} = (\ldots z(s) \ldots)$$

is the vector of excess demands and the real wealth at the steady state is given by

$$\tilde{\tau} = (\ldots \tau(s), \ldots).$$

$\tilde{\tau}$ is determined by by the equations

$$\tilde{z} + \tilde{m} + \tilde{Q}\tilde{\tau} = \tilde{\tau} \quad \text{or} \quad \tilde{\tau} = (I - \tilde{Q})^{-1} [\tilde{z} + \tilde{m}].$$

$$\tilde{z}^* + \tilde{m}^* + \tilde{Q}\tilde{\tau}^* = \tilde{\tau}^* \quad \text{or} \quad \tilde{\tau}^* = (I - \tilde{Q})^{-1} [\tilde{z}^* + \tilde{m}^*].$$

The real wealth of the monetary-fiscal authorities in the home country, $\tilde{T}$, is determined by

$$\tilde{M} + \tilde{Q}\tilde{T} = \tilde{T} \quad \text{or} \quad \tilde{T} = (I - \tilde{Q})^{-1}\tilde{M},$$

where $\tilde{M} = (\ldots \frac{r(s)}{1+r(s)} a(s) l(s) \ldots)$ and, since F is a Markov transition matrix, while $\tilde{M} \gg 0$, the real claims against the monetary-fiscal authority at the steady state are strictly positive:

$$\tilde{T} \gg 0.$$

Note that the real claims against the monetary-fiscal authorities can only be jointly determined, $\tilde{T} + \tilde{T}^* = \tilde{\tau} + \tilde{\tau}^*$.

As we have solved the entire real economy without nominal variables, the initial price level in each country remains indeterminate. More importantly, the decomposition of equilibrium asset prices into an inflation process, $\pi(\cdot|s)$, and a nominal pricing measure, $\nu(\cdot|s)$, remain indeterminate in each country. For the home country:

$$\tilde{Q} = R^{-1} N \otimes \Pi.$$

Here,

$$R = diag(\ldots, (1 + r(s)), \ldots)$$

is the diagonal matrix of interest factors, and

$$N = (\nu(s'|s)), \qquad \Pi = ((1 + \pi(s'|s))), \quad \text{and} \quad E = ((1 + \pi(s'|s))/(1 + \pi^*(s'|s)))$$

are, respectively, the matrices of "nominal pricing transition probabilities", inflation factors and exchange rate factors. The stochastic growth rates of nominal exchange rates are given by[24]

$$E = \Pi \oslash \Pi^*$$

In the absence of restrictions on the balance sheet of the monetary fiscal authority, which is the case under QE, the set of steady state equilibria is indexed by the nominal pricing transition probabilities, $\nu(\cdot|s)$, that can be set arbitrarily; the inflation factors, $\pi(\cdot|s)$, and exchange rate factors, $e(\cdot|s)$, then adjust to implement the equilibrium.

In each country, the composition of the balance sheet of the monetary-fiscal authority can be described by portfolio weights, $\delta(s'|s)$ (that is, $\sum_{s'} \delta(s'|s) = 1$), and scale factors $h(s)$, such that

$$h(s)\delta(s'|s) = \tilde{T}(s')(1 + \pi(s'|s));$$

this is the case since the inflation factor is the rate of exchange of output between a date-event and an immediate successor.

The equilibrium condition, then reduces to

$$\tilde{Q} = R^{-1} N \otimes H\Gamma = R^{-1} H N \otimes \Gamma.$$

Here,

$$H = diag(\ldots, h(s), \ldots)$$

[24]Entry-by-entry multiplication is $\otimes$, while $\oslash$ is entry-by-entry division.

is the diagonal matrix of scale factors, and

$$\Gamma = \left(\frac{\delta(s'|s)}{\tilde{T}(s')} \right)$$

is the matrix of portfolio weights relative to the payoff of the balance sheet.

Monetary-fiscal policy conducted as CE sets the composition of the balance sheet; that is, it sets positive portfolio weights, $\delta(s'|s) > 0$; claims against the monetary-fiscal authority in real terms, $\tilde{T}(s)$, are determined, at the steady-state, by fundamentals, and, as a consequence, under CE, the matrix Γ is determined.

Since

$$N\mathbf{1}_S = \mathbf{1}_S \quad \Leftrightarrow \quad H = (R\tilde{Q} \oslash \Gamma)\mathbf{1}_S,$$

the Markov transition matrix, N, is well defined ($h \gg 0$) and determinate; it follows that the equilibrium is determinate as well.

Under conventional monetary-fiscal policy, the portfolio of the monetary-fiscal authority consists of treasury bills, nominally risk-free bonds of short maturity. Here, this corresponds to one-date nominally risk-free bonds: $\delta(s'|s) = 1/S$. $\square$

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