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Endogenous games with goals: side-payments among goal-directed agents

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Abstract Boolean games have been developed as a paradigm for modelling societies of goal-directed agents. In boolean games agents exercise control over propositional variables and strive to achieve a goal formula whose realization might require the opponents' cooperation. The presence of agents that are goal-directed makes it difficult for an external authority to be able to remove undesirable properties that are inconsistent with agents' goals, as shown by recent contributions in the multi-agent literature. What this paper does is to analyse the problem of regulation of goal-direct agents from within the system, i.e., what happens when agents themselves are given the chance to negotiate the strategies to be played with one another. Concretely, we introduce *endogenous games with goals*, obtained coupling a general model of goal-directed agents (*strategic games with goals*) with a general model of pre-play negotiations (*endogenous games*) coming from game theory. Strategic games with goals are shown to have a direct correspondence with strategic games (Proposition 1) but, when side-payments are allowed in the pre-play phase, display a striking imbalance (Proposition 4). The effect of side-payments can be fully simulated by taxation mechanisms studied in the literature (Proposition 7), yet we show sufficient conditions under which outcomes can be rationally sustained without external intervention (Proposition 5). Also, integrating taxation mechanisms and side-payments, we are able to transform our starting models in such a way that outcomes that are theoretically sustainable thanks to a pre-play phase can be actually sustained even with limited resources (Proposition 8). Finally, we show how an external authority incentivising a group of agents can be studied as a special agent of an appropriately extended endogenous game with goals (Proposition 11).

1 Introduction

One of the key objectives of Artificial Intelligence is that of devising interaction platforms for distributed computational entities pursuing different, when not con-

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flicting, design objectives (Castelfranchi, 1998). As a consequence of this, the effective handling of conflict in agents' societies is of tremendous importance to achieve desirable systemic properties. More often than not, however, a centralised *off-line design* type of solution (Shoham and Tennenholtz, 1995), where useful social laws are determined before the interaction starts and computational entities can be guided towards their pursuit, is not always an option (Walker and Wooldridge, 1995), and the field of Distributed Artificial Intelligence has come up with a number of decentralised solutions, elsewhere called *from within the system* (Walker and Wooldridge, 1995), involving computational entities that are able to negotiate and agree on *locally* desirable properties (Rosenschein and Genesereth, 1985; Davis and Smith, 1983). Disciplines such as Distributed Problem Solving (Davis and Smith, 1983), Computational Social Choice (Chevaleyre et al, 2007), Automated Negotiation (Jennings et al, 2001; Endriss et al, 2006) — to mention a few — are all attempts to develop endogenous solutions for regulating potential conflicts among artificial agents, by for instance allowing them to exchange information, cast a ballot, engage in a discussion and so forth.

A well-known model of a society of goal-directed artificial agents investigated in AI is boolean games (Harrenstein et al, 2001), a compact and computationally well-behaved representation of strategic interaction by means of logical formulas. In boolean games agents exercise control over propositional variables and strive to achieve a goal formula whose realization might require the opponents' cooperation, disregarding the cost associated to each action, if need be.

Recently a theory of *incentive engineering* has been devised (Wooldridge et al, 2013), where an external authority, i.e., the principal, steers the outcome of the game towards certain *desirable* properties, by imposing a taxation mechanism on the agents that makes the outcomes that do not comply with those properties less appealing to them. However, the task of the principal turns out to be a non-trivial one as, due to the structure of the so-called quasi-dichotomous preferences, there is no monetary compensation that can convince agents to give up their goal. In all the other cases, though, agents behave as cost-minimisers and desirable systemic properties satisfying agents' goals have been shown to be implementable by the appropriate system of incentives.

The present contribution[1] stems from a complementary perspective and studies, instead, how games with quasi-dichotomous preferences can be transformed from within the system, by endowing agents with the possibility of sacrificing a part of their payoff received at a certain outcome in order to convince other agents to play a certain strategy. Concretely, we introduce *endogenous games with goals*,

[1] This paper is based on (Turrini, 2013), but it significantly extends it, overcoming a number of its limitations, generalising all its results and adding new ones. In details, while (Turrini, 2013) studied the framework of boolean games, this paper studies a generalisation thereof, strategic games with goals, which avoids the somewhat unnatural assumptions on which the original models were based - as we make clear later on when comparing boolean games with strategic games with goals; while Turrini (2013) studied unbounded side-payments only, this paper studies the possibility of bounded side-payments, i.e., players have budget constraints, but still allowing for the unbounded case; while Turrini (2013) studied updates by side-payments using an ad hoc adjustment factor, the endogenous games with goals studied here avoid that complication. This paper, unlike Turrini (2013), extensively motivates the use of quasi-dichotomous preference relations and presents a significantly larger number of formal results - with full proofs - on both the static (without side-payments) and the dynamic (with side-payments) models.

obtained coupling strategic games with goals, a generalisation of boolean games to a full-blown model of goal-directed agency, with the machinery of *endogenous games* coming from game theory (Jackson and Wilkie, 2005). We illustrate our idea and, informally, our setting in the following example.

Energy sharing in block world Consider two robots, Ann and Bob, and a heavy block in front of them. Ann and Bob can, together, lift the block and, once lifted, move it to the left. The block is such that only the coordinated action of both robots would lift it and, possibly, move it. Let us assume that each robot can independently decide whether to try and lift the block and, once lifted, try and move it. Robots are programmed to achieve a desired state of affairs and, let us assume for the moment, endowed with enough battery power to allow them to perform all sort of tasks. In particular, let us assume Ann wants to have the block lifted and moved to the left, while Bob simply wants to try and lift the block, so he does not really care whether the block ends up being actually lifted or it will be moved once lifted. Try and lift the block costs Ann 5 units of energy, while it costs Bob 10. Try and move the block left, once lifted, costs Ann 3 and Bob 10. As a general rule, when confronted with the choice of achieving a state satisfying their design objective or achieving a state that does not, Ann and Bob will always go for its satisfaction, disregarding energy consumption. When instead goal satisfaction is not an issue and the choice is between two outcomes satisfying the goal or between two outcomes not satisfying it, then our robots will always opt for the least expensive course of action.

It goes without saying that in a scenario of this kind Bob will try and lift the block. But, being indifferent between trying and move the block left and staying put once the block is lifted, he will then simply look at the resulting energy costs and decide to stay put. Ann, instead, would like to see the block lifted and moved left, but she knows that, once the block is lifted, Bob will not make an effort to move it. Therefore, confronted with the choice between making an effort to try and lift the block and not to move at all, she will opt for the latter. In the end, Bob will have his goal satisfied, consuming 10 energy units, while Ann will not, consuming 0 energy units.

Suppose though that robots were connected to each other by an *energy channel*, which would allow them, before any action has yet to take place, to commit to bear a part of the effort the opponent is undertaking at a certain outcome, should that outcome be reached. This is simply implemented by first allowing the robots to promise each other a certain amount of battery power as a function of the outcome of the game, and then by having a mechanism to enforce these promises once their actions are taken.

While in the previous scenario Ann, although *depending* on Bob for the realization of her goal (cfr. (Bonzon et al, 2009a,b; Ben-Naim and Lorini, 2014) for a qualitative account of dependencies in boolean games and (Grossi and Turrini, 2012) in strategic games), could not have any say on Bob's decision-making, now she can adopt a richer strategy and, before Bob takes any decision, offer him, say, 20, for trying and move the block left. In the resulting situation both Ann and Bob will have their goal satisfied, Ann consuming 28 units of energy and Bob consuming nothing, which is a more satisfactory solution for both. Notice, however, that the solution is not stable, as Ann has an incentive to deviate to more parsimonious offers in the pre-play phase without compromising the realization of her own goal.

This paper analyses scenarios of this kind, where goal-directed agents can overcome undesirable properties of an interaction by undergoing a pre-play negotiation phase, offering to bear a part of their opponents' costs, should some particular course of actions been taken.

The added value of the analysis presented here, intuitively introduced in the example above, is two-fold:

- it complements the framework of incentive engineering for boolean games (Wooldridge et al, 2013), studying those situations in which agents can reach desirable properties *without* external intervention;
- it provides a quantitative resolution to dependence relations (Castelfranchi et al, 1992; Grossi and Turrini, 2012), broadly studied for the case of boolean games (Bonzon et al, 2009a,b; Ben-Naim and Lorini, 2014), allowing agents to influence each other's decision-making by the offer of monetary incentive.

We carry out the analysis studying the general setting of endogenous games with goals, in relation with both boolean games and endogenous games, focussing on the properties of the resulting equilibria.

Paper Structure In Section 2 we introduce strategic games with goals, studying their formal connection with strategic (normal form) games coming from game-theory. In Section 3 we study endogenous games with goals, adding to strategic games with goals the dynamics brought into play by the possibility of exchanging side-payments in the pre-play phase. In Section 4 we carry out an equilibrium analysis of these structures, showing results of pure strategy equilibrium survival and discussing the connection with what known from game theory. In Section 5 we integrate side-payments with taxation mechanisms, devising a procedure that ensures desirable properties to be reached, even when the possibility of making transfers is limited. Section 6 discusses related literature, paying particular attention to the formal relation with boolean games, while Section 7 presents possible extensions of the framework. Finally, in Section 8 we wrap up the work pointing to possible future research directions.

2 Strategic games and quasi-dichotomous preferences

In this section we describe a general approach to characterizing goal-directed artificial agents acting in a common shared world. We do so by explicitly enriching a strategic game with a distinguished set of goals, one for each agent.

As well-known, a strategic (normal form) game $\mathcal{S}$ is a tuple $(N, \{\Sigma_i\}_{i\in N}, \pi)$, where N is a set of agents, each Σ_i a set of strategies for agent $i \in N$ and $\pi : \prod_{i\in N} \Sigma_i \times N \to \mathbb{R}$ a payoff function, assigning to each agent his payoff at each strategy profile. Henceforth we abbreviate $\pi(\sigma, i)$ as $\pi_i(\sigma)$, $\prod_{i\in N} \Sigma_i$ as Σ, — extending the conventions to similar cases — and denote $NE(\mathcal{S})$ the set of pure strategy Nash equilibria of strategic game $\mathcal{S}$, with $NE^{\Delta}(\mathcal{S})$ being its mixed extension.

Strategic games with goals are defined as follows.

Definition 1 (Strategic games with goals)
A *strategic game with goals* $\mathcal{G}$ is a tuple $(\mathcal{S}, \{G_i\}_{i\in N})$ where $\mathcal{S} = (N, \{\Sigma_i\}_{i\in N}, \pi)$ is a strategic game and each $G_i \subseteq \Sigma$ is a set of goal states for agent i.

Intuitively a *goal* is a state of the game the agent is directed to and, when given the possibility, would not want to trade for a non-goal state, no matter what the payoff assigned by the function π is.

If we think of each agent as associated to a colour, we obtain a particularly intuitive representation of strategic games with goals, where goal states for each agent are assigned the agent's colour. The algebra of goal states is defined by operation on colours, e.g., a goal state shared by a blue agent and red agent is represented as a purple state. Figure 1 is an example of how strategic games with goals can be displayed.

	L	R
U	$3, 3$	$0, 5$
D	$5, 0$	$1, 1$

Fig. 1 Agents' goals. Column wants the game to end up in the set $\{(L, D), (R, D)\}$, Row in the set $\{(R, D)\}$. The coalition $\{Column, Row\}$ wants the shared outcome (R, D) to be realised. As a convention, Row obtains the first component in the displayed payoff vectors, Column the second.

Example 1 Our starting example can be modelled as a strategic game with goals[2] The set of agents is $\{Ann, Bob\}$, each having the same set of strategies, representing the subsequent combination of their atomic actions: (lL), where she tries and lift the block and once lifted to move it left, (lN) where she stays put, and finally (nL) and (nN) representing strategies starting with her not trying and lift the block. We will regard the robots' strategies that do not lift the block as utility-equivalent for an agent, i.e. identical for decision-making purposes, as moving the block to the left can only be brought about on the condition that the block itself is lifted. Combination of strategies by each agent represent our strategy profiles, upon which both goals and payoffs are formulated. The strategic game with goals depicted next, where Ann is the red row agent and Bob the blue column one, models our scenario.

We have not yet provided the tools to analyse Figure 2 at a formal level. However some intuitive considerations can already be made. For instance, if we disregarded goals, the outcomes at which agents do nothing would be the only stable and rational ones. If we instead consider goals, we can see that Ann is not, alone, able to satisfy her own goal, while Bob is, by simply trying and lift the

[2] As the reader will have noticed, the example might, possibly more realistically, be modelled making its temporal dimension explicit. Robots are, in fact, involved in an *extensive* form of interaction, being able to lift a block and *once lifted* moving it somewhere. We can use strategic games without loss of generality, though. As well known (Osborne and Rubinstein, 1994), each extensive form game can be canonically represented as a strategic normal form game, where the history-based strategies in the former can be encoded as one-shot strategies of the latter. A more subtle point concerns the choice of the solution concept, as subgame perfection, rather than Nash-equilibrium, is usually thought to be a more adequate tool for the analysis of rational play in extensive games (Osborne and Rubinstein, 1994). In normal form games this boils down to requiring a Nash equilibrium outcome to survive iterated elimination of weakly dominated strategies. This requirement can be incorporated in our framework by substituting the set of Nash equilibria $NE^{\Delta}(\mathcal{S})$ with the set of Nash equilibria surviving iterated elimination of weakly dominated strategies $IEWDS(NE^{\Delta}(\mathcal{S}))$. None of the point we are making here would be affected by this modification.

	lL	*lN*	*nL*	*nN*
lL	−8, −20	−8, −10	−8, 0	−8, 0
lN	−5, −20	−5, −10	−5, 0	−5, 0
nL	0, −20	0, −10	0, 0	0, 0
nN	0, −20	0, −10	0, 0	0, 0

Fig. 2 Ann, Bob and the block. The costs agents incur in as a result of interaction are modelled as negative payoffs.

block. Also, Bob would rather not perform any action once the block is lifted, as it would mean paying 10 more for no gain in terms of goal satisfaction. All these considerations show the presence of an asymmetric structure of interdependence among the agents, where Bob enjoys a stronger position.

To understand which outcomes are game-theoretically *rational* and which ones are *stable* in strategic games with goals, in other words, to do equilibrium analysis, we will need to construct a utility function which faithfully incorporates the intuitive difference between goals agents have and resources spent or gained by them at each given outcome.

2.1 Quasi-dichotomous preferences

Consider a drone that is programmed to destroy enemy headquarters. If several courses of actions ensure the target to be destroyed, the drone is required to choose the least expensive one in terms of, e.g., casualties. The drone strategy is constructed in such a way that an outcome where the target is destroyed will always be preferred to an outcome where the target is not destroyed, no matter how many casualties will be involved.

Being able to construct entities that prioritise goals such as the drone we have just described, notice, does not force us to commit to a world in which all agents have priorities on goals. However, we maintain that the very existence of such a machine forces other entities, e.g., humans, to form beliefs about agents having that type of preferences. [3] In any case, using priorities we can technically allow other entities, e.g., humans or other drones [4], to be purely utility maximisers or cost minimisers, by simply setting the goal-states to be the empty set, or the set of all outcomes.

In this paper we certainly do not intend to provide a descriptive model of human behaviour, which should be backed by appropriate experimental findings, but we point out that quite a few accounts in cognitive psychology show how human behaviour can be more easily explained in terms of attainability of objectives rather than sole utility maximisation. Examples are the Maslow's theory of hierarchical needs (Maslow, 1943), according to which human decision-making prioritises the attainment of basic drives (e.g., hunger drive) before addressing the others

[3] I am thankful to Marcus Pivato for pointing to this fact.

[4] We could reprogramme our drone in such a way that it aims at destroying the target only if it does not involve the killing of a thousand casualties. What we are doing now is to construct a machine that assigns a negative utility to the killing of casualties that can actually *compensate* failed target destruction.

(e.g., differences in styles of hair-dress) [5]; Herbert Simon's model of *satisfacing* decision-making (Simon, 1957); and most modern accounts of human risk-taking in lotteries, such as Kahneman and Tversky's Prospect Theory (Kahneman and Tversky, 1979).

We stress it once more, in our account goal states represent those outcomes that allow for *no compensation* in terms of secondary rewards. A goal state is better than a non-goal state, and will remain so, no matter what the payoff associated to the secondary reward is. When goal realization is however not an issue, secondary aspects play a role and the agent will always try to maximise the resulting payoff.

This fact induces what is technically called *quasi-dichotomy* of a preference relation (Wooldridge et al, 2013). For decision-making purposes we need however to make a choice on how to actually implement it in a fully formal way. The first that comes to mind is that of working with truly lexicographic preferences (Rubinstein, 2006), i.e., states being identified with a tuple (x, n) where $x \in \{0, 1\}$ and $n \in \mathbb{R}$, the first entry encoding whether the state is a goal state or not and the second entry encoding secondary materialistic aspects. Therefore, a state z is to be preferred to a state z' whenever $z \geq^{LEX} z'$, where $\geq^{LEX}$ is the lexicographic order between the two. Under this interpretation, goal states are *de facto* assigned an infinite payoff, which makes them better than a non-goal state no matter what the payoff of the latter is.

Lexicographic orders on outcomes do not however have a corresponding von Neumann-Morgenstern utility representation (Wilson, 1996; Knoblauch, 2003; Blume et al, 1991; Rubinstein, 2006) and the resulting games do not in general display fundamental game-theoretical properties, such as existence of Nash equilibria [6].

To overcome this difficulty we introduce a purely technical device that, intuitively, allows us to treat infinite utility as extremely big utility. This is enough to make the structures we work with game-representable, without however modifying the goal-directed character of agents' decision-making. Concretely, we define a family of *boost factors* Ω_i, one for each agent i, each of which encodes how much more a agent values a certain goal state with respect to a non-goal state. In other words, a boost factor is a measure of the relative distance that, at each game, a certain goal state for an agent finds itself with respect to all other non-goal states. Notice that boost factors are mechanisms to ensure that goal states *remain* better than non-goal states, but they implicitly also give a measure of the *risk* that a agent is willing to undertake to achieve a goal state, i.e., they encode a preference relations over mixed profiles containing both goal states and non-goal states.

For a given strategic game with goals $\mathcal{G} = ((N, \{\Sigma_i\}_{i\in N}, \pi), \{G_i\}_{i\in N})$, let $\mathbb{R}_{\pi_i} = \{x \in \mathbb{R} \mid \pi_i(\sigma) = x, \text{ for some } \sigma \in \Sigma\}$. Technically, a **boost factor** is a function $\omega_i^{\mathcal{G}} : \mathbb{R}_{\pi_i} \to \mathbb{R}$ associating to each payoff how much this payoff is *boosted* if it is the payoff of a goal state. Each $\omega_i^{\mathcal{G}}$ is required to be such that:

[5] The examples of drives come directly from Maslow. In (Maslow, 1943) he emphasises the fact, which is fully taken up in our account, that humans have *superficial needs*, attempting the satisfaction of which is conditional to the attainment of the *basic needs*. Basic and superficial needs can, in turn, have inner hierarchy, i.e., the famous "Maslow Pyramid".

[6] Technically, a preference order representing lotteries over outcomes which are lexicographically ordered does not satisfy the axiom of continuity, it does however satisfy the axioms of monotonicity, simplification of compound lotteries and independence Osborne and Rubinstein (1994).

$$\omega_i^{\mathcal{G}}(x) \geq \omega_i^{\mathcal{G}}(y) \text{ if and only if } x \geq y \tag{1}$$

Intuitively, a goal state with associated payoff x is at least as good to i as a goal state with associated payoff y if and only if x is at least as high as y.

$$\omega_i^{\mathcal{G}}(x) > \pi_i(\sigma) \text{ for all } \sigma \notin G_i \tag{2}$$

In other words, a state satisfying a goal is always better than a state not satisfying it.

A strategic game with goals $\mathcal{G}$ associated to a profile of boost factors $\omega \in \Omega = \prod_{i\in N} \Omega_i$ is said to be **instantiated by** ω, and this is denoted $\mathcal{G}(\omega)$. Intuitively, when playing $\mathcal{G}(\omega)$ each agent i judges the betterness of goal states according to the boost factor ω_i. Here is the definition of utility taking them into account.

Definition 2 (Utility) Let $\mathcal{G}(\omega)$ be a strategic game with goals instantiated by a profile of boost factors ω with $\mathcal{G} = ((N, \Sigma, \pi), \{G_i\}_{i\in N})$. The **utility function** $u^{\mathcal{G}(\omega)} : N \times \Sigma \to \mathbb{R}$ assigning to each agent the payoff $u_i^{\mathcal{G}(\omega)}(\sigma)$ he receives at outcome σ is defined as follows.

$$u_i^{\mathcal{G}(\omega)}(\sigma) = \begin{cases} \omega(\pi_i(\sigma)) & \text{IF } \sigma \in G_i \\ \pi_i(\sigma) & \text{otherwise} \end{cases}$$

So, the function u is constructed by the combination of π and ω, i.e., the payoff function and the profile of boost factor, respectively. The latter takes care of the fact that goal states are always better than non-goal states, no matter what the payoff is associated to the latter by the function π.

We would like at this point to clarify the possible conceptual ambiguity that might arise from having two different functions, π and u, which associate a vector of numerical values to each outcome. The function π, which we will always refer to as a *payoff function*, encodes the secondary, intuitively purely monetary, aspects of a certain state. The function u instead, which we will always refer to as a *utility function*, incorporates the primary aspects, i.e., goal realization, possibly associated to a state. Thereby, a state might have (relatively) high utility and (relatively) low payoff, if for instance the state satisfies a goal, but it might also have (relatively) low utility and (relatively) high payoff, if for instance it is the only state not satisfying a goal. In the end, what the agents look at when taking a decision is the utility function which, in our case, is built upon the payoff function and the goal states.

It is also worth noticing that boost factors allow us to reason about hypothetical utility distributions, e.g., by comparing expressions such as $\omega^{((N,\Sigma,\pi),\{G_i\}_{i\in N})}$ and $\omega^{((N,\Sigma,\pi'),\{G_i\}_{i\in N})}$. This extremely important feature will be fully exploited later on, as the role of boost factors is not only to declare that goal states are better, but, again, to keep them so independently of any monetary compensation.

For a given strategic game with goals $(\mathcal{S}, \{G_i\}_{i\in N})$, with $\mathcal{S} = (N, \Sigma, \pi)$, and instantiated with boost factor profile ω, the **induced strategic game** is the game $\mathcal{S}' = (N, \Sigma, u)$, where u is calculated according to Definition 2.

Figure 3 shows how.

	L	R
U	$-3, -3$	$0, -5$
D	$-5, 0$	$-1, -1$

	L	R
U	$-3, -3$	$0, -5$
D	$-5, 1$	$3, 0$

Fig. 3 From a strategic game with goals to its induced strategic game: each agent's boost factor assigns $+3$ to all his goal states with respect to his best non-goal state, maintaining the relative distance among goal states. For example, the reason why Column is getting 1 at outcome (D, L) of the induced strategic game is because he is getting 0 at outcome (D, R) — which is in turn because -3 is the payoff of his best non-goal state and $-3 + 3 = 0$ — and the original relative distance between (D, R) and (D, L) is of 1.

2.1.1 Expected Utility

We introduce an extra novel feature with respect to the standard treatment of boolean games: we allow agents to randomise over possible strategies. This will make it possible to draw a comparison with the equilibrium existence results known for endogenous games, which rely on Nash's theorem, the well-known result on the existence of Nash equilibria with mixed strategies in normal form games (Nash, 1950). As usual, we first denote $\Delta(\Sigma_i)$ the set of probability distributions over the strategies of agent i and, for $\delta \in \Delta(\Sigma_i)$, we denote $\delta(\sigma_i)$ the probability that **mixed strategy profile** δ assigns to $\sigma_i \in \Sigma$. We call the set $s(\delta) = \{\sigma_i \in \Sigma_i \mid \delta(\sigma_i) > 0\}$ the **support** of δ.

Definition 3 (Expected Utility)
Let δ be a mixed strategy profile available at strategic game $\mathcal{S}$ with a payoff function π, $\sigma \in s(\delta)$ a pure strategy profile in the support of δ, and $\delta(\sigma)$ the probability of σ to occur according to δ. The **expected utility** of δ for agent i is defined as follows.

$$E_i(\delta) = \sum_{\sigma \in s(\delta)} \pi_i(\sigma)\delta(\sigma)$$

To compute the expected utility on a strategic game with goals, instantiated with a boost factor, we compute the expected utility in its induced strategic game.

Example 2 Consider again our starting example and assume now that each agent values a goal state 10 units more than each non-goal state, keeping the relative distance among them. More precisely, the game is instantiated by a family of boost factors $\{\omega_{Ann}, \omega_{Bob}\}$ such that for each $\sigma \in G_{Ann}$ — the case of Bob is symmetric — $\omega_{Ann}(\pi_{Ann}(\sigma)) = \pi_{Ann}(\rho) + \varphi + 10$, where $\rho = max_{\sigma' \notin G_{Ann}} \pi_i(\sigma')$, and $\varphi = max_{\sigma' \in G_{Ann}}(\pi_{Ann}(\sigma) - \pi_{Ann}(\sigma'))$. The matrix next shows how the game is transformed, with payoffs and goals replaced by a standard utility function induced by the family of boost factors.

	lL	lN	nL	nN
lL	$10, 10$	$-8, 20$	$-8, 0$	$-8, 0$
lN	$-5, 10$	$-5, 20$	$-5, 0$	$-5, 0$
nL	$0, 10$	$0, 20$	$0, 0$	$0, 0$
nN	$0, 10$	$0, 20$	$0, 0$	$0, 0$

Fig. 4 Ann, Bob and the block. The utility function implements a family of boost factors where each agent values goals 10 units of payoff more than each non goal state.

Two features are worth observing. The first is that the utility function respects the preference quasi-dichotomy of the agents: all outcomes that satisfy Ann's (respectively, Bob's) goals are better in terms of utility than the outcomes that do not. In the table, Ann's highest utility is achieved at the state where the block is lifted and the moved left, while for Bob, at all states where the block is lifted. The second feature worth observing is that the shape this function takes is determined by the family of boost factors instantiating the game. For each agent, a boost factor works as a sort of psychological predisposition that weighs how important a goal is with respect to all other states. Under this specific instance Ann and Bob happen to construct the utility in a similar way, literally keeping a fixed distance of 10 between the two, but many more variations are possible.

In the newly obtained game we can see, by dominance argument, there are two payoff-equivalent pure strategy Nash equilibria, where Bob plays lN and Ann either nL or nN — as Nash equilibria are all strategies where Bob plays lN and Ann mixes between nL and nN — but in none of them the block is lifted, in spite of Bob's effort.

Representation result The following results establish a correspondence between strategic games and strategic games with goals.

Proposition 1 (Strategic games with goals and strategic games)

1. *Let* $\mathcal{G} = ((N, \{\Sigma_i\}_{i\in N}, \pi), \{G_i\}_{i\in N})$ *be a strategic game with goals and* ω *a profile of boost factors. There exists a strategic game* $\mathcal{S}' = (N, \{\Sigma_i\}_{i\in N}, \pi')$ *such that, for each* $\sigma \in \prod_{i\in N} \Sigma_i$ *and each* $i \in N$*, we have that* $u_i^{\mathcal{G}(\omega)}(\sigma) = \pi'(\sigma)$.
2. *Let* $\mathcal{S} = (N, \{\Sigma_i\}_{i\in N}, \pi)$ *be a strategic game. Then there exists a strategic game with goals* $\mathcal{G}' = (\mathcal{S}', \{G\}_i)$ *with* $\mathcal{S}' = (N, \{\Sigma_i\}_{i\in N}, \pi')$ *such that, for all profiles of boost factors* ω*, for each* $\sigma \in \prod_{i\in N} \Sigma_i$ *and each* $i \in N$*,* $u_i^{\mathcal{G}'(\omega)}(\sigma) = \pi'(\sigma)$.

Proof For the first item, start out with a strategic game with goals $\mathcal{G} = ((N, \{\Sigma_i\}_{i\in N}, \pi), \{G_i\}_{i\in N})$, and a boost factor profile ω. Construct a strategic $\mathcal{S} = (N, \{\Sigma_i\}_{i\in N}, \pi')$ for which the payoff function is such that $u_i^{\mathcal{G}(\omega)}(\sigma) = \pi'(\sigma)$ for each $\sigma \in \prod_{i\in N} \Sigma_i$ and each $i \in N$.

For the second item, consider the strategic game $\mathcal{S}$, given by the tuple $(N, \{\Sigma_i\}_{i\in N}, \pi)$ and construct the strategic game with goals $\mathcal{G}' = (\mathcal{S}', \{G\}_i)$ with $\mathcal{S}' = \mathcal{S}$ and for each $i \in N$, set $G_i = \emptyset$. It follows that for all profiles of boost factors ω we have that for each $\sigma \in \prod_{i\in N} \Sigma_i$ and each $i \in N$, $u_i^{\mathcal{G}(\omega)}(\sigma) = \pi_i(\sigma)$.

Thus, strategic games with goals and strategic games display a straightforward correspondence and the proof of Proposition 1 provides an automatic procedure to convert them into one another.

The reader might at this stage understandably be puzzled. We have introduced strategic games with goals, and have called them a *generalisation* of strategic games, showing that, in the end, the two structures are substantially equivalent. The rest of the paper is devoted to showing that the introduction of dynamic operations, such as the possibility of side-payments in a pre-play phase, brings to light surprising differences between these structures.

2.2 The case of Boolean Games

Boolean games, first introduced in (Harrenstein et al, 2001), are a simple model of strategic interaction and technically a very special case of strategic games with goals, where agents are endowed with goal formulas and can set the propositions they control either to true or false, paying a cost for each action performed. We describe them next and we motivate the need for our generalisation.

Definition 4 (Boolean Games) A **boolean game** is a tuple

$$(N, \Phi, c, \{\gamma_i\}_{i\in N}, \{\Phi_i\}_{i\in N})$$

where:

- N is a finite set of agents;
- Φ a finite set of propositional atoms; $c : N \times V \to \mathbb{R}_+$ is a **cost function**, associating to each agent the cost he incurs in when some valuation $v \in V$ of the atoms Φ obtains;
- γ_i is a boolean formula, constructed on the set Φ, denoting the **goal** of agent i;
- $\Phi_i \subseteq \Phi$ is the nonempty set of atoms controlled by agent i. As standard (van der Hoek and Wooldridge, 2005), we assume that for $j \neq i$, $\Phi_i \cap \Phi_j = \emptyset$ and that $\bigcup\{\Phi_i \mid i \in N\} = \Phi$, i.e., controlled atoms partition the whole space.

A **choice** of agent i is a function $v_i : \Phi_i \to \{\mathbf{tt}, \mathbf{ff}\}$, representing agent i's decision to set the atoms he controls to either true or false . We denote V_i as the set of possible choices of agent i. An **outcome** $v \in \prod_{i\in N} V_i$ of the boolean game $\mathcal{B}$ is a collection of choices, one per agent. An outcome induces a valuation function, assigning a boolean value to each propositional atom (van der Hoek and Wooldridge, 2005). Therefore, we denote $V = \prod_{i\in N} V_i$ the set of all possible valuation functions and, for $v \in V$ and φ being a boolean formula constructed on Φ, we write $v \models \varphi$ (resp. v $\not\models \varphi$) to say that φ holds (resp. does not hold) under the valuation v.

The utility in boolean games is calculated using a boost factor μ_i, with $\mu_i = max_{v\in V}(c_i(v))$, selecting the payoff of the worst outcome that can happen to agent i, i.e., the updated valuation that is most costly to him, and adding it to the goal states, together with a sufficiently small real ϵ.

$$u_i^{\beta(\mathcal{B})}(v) = \begin{cases} \epsilon + \mu_i - c_i(v) & \text{IF } v \models \gamma_i \\ -c_i(v) & \text{otherwise} \end{cases}$$

Figure 5 shows an example of this translation.

	s_C	$\neg s_C$
s_R	3, 3	0, 5
$\neg s_R$	5, 0	1, 1

	s_C	$\neg s_C$
s_R	−3, −3	0, −5
$\neg s_R$	−5, 6	5, 5

Fig. 5 From a boolean game to its induced strategic game. Each agent i is endowed with boost factor μ_i.

When constructing a strategic game starting from a given boolean game we soon realise the downside of the two main restrictions — otherwise extremely desirable from a computational point of view — differentiating boolean games from the larger class of strategic game with goals:

- The number of strategies agents can play are always, 2^n for some natural number n, which can vary for each agent;
- The relative distance between goal states and non-goal states is given by the $\mu + 1$ boost factor, which is fixed for each agent.

The first constraint is not particularly restrictive, but it shows an intuitive difference between boolean games and strategic games with goals: while in boolean games agents fully control propositional variables — i.e., they can always decide whether to set them to true or false — strategic games with goals can be thought of as a sort of boolean games with *admissible* valuation functions, and thereby more general structures. The second constraint is somewhat more restrictive, as the choice of the $\mu + 1$ boost factor is extremely committal and bears a number of consequences especially if we take utility to be transferable, as we do in our framework. $\mu + 1$ is a *regret-based* (or even *pride-based*) boost factor: the worse non-goal states are for an agent, the higher the payoff at his goal states; the factor is independent of the actual numerical value assigned by the cost function, i.e., it does not vary along with the absolute values of its domain; what is more, it is fixed for all agents, i.e., all agents apply exactly the same distance to separate goal states and non-goal states.

These are among the reasons why we think that the more general approach allowed by strategic games with goals is in order, which incorporates the important features of the $\mu + 1$ class but leaves also space for more variety.

We would like however to point out that when dynamic operations, such as side-payments or taxation mechanisms, are not allowed, boolean games are equivalent to strategic games, and therefore to strategic games with goals, modulo positive affine transformation.

Proposition 2 (Strategic games and boolean games)

1. *Let $\mathcal{B} = (N, \Phi, c, \{\gamma_i\}_{i \in N}, \{\Phi_i\}_{i \in N})$ be a boolean game. Then there exist a strategic game $\mathcal{S} = (N, \{\Sigma_i\}_{i \in N}, \pi)$ and a bijective function $f : V \to \prod_{i \in N} \Sigma_i$ such that for all $i, u_i^{\mathcal{B}}(v) = \pi_i(f(v))$.*
2. *Let $\mathcal{S} = (N, \{\Sigma_i\}_{i \in N}, \pi)$ be a strategic game such that, for each $i \in N$ there is $n \in \mathbb{N} \backslash \{0\}$ with $|\Sigma_i| = 2^n$. Then there exist a boolean game $\mathcal{B} = (N, \Phi, c, \{\gamma_i\}_{i \in N}, \{\Phi_i\}_{i \in N})$, a $k \in \mathbb{N}$, and a bijective function $f : V \to \prod_{i \in N} \Sigma_i$ such that for all $i, u_i^{\mathcal{B}}(v) = \pi_i(f(v)) - k$.*

Proof For the first item, simply construct a strategic game such that f is a bijection and then set each $\pi_i(f(v))$ to return the value given by $u_i^{\mathcal{B}}(v)$. For the second one, pick again f to be bijection, and, for a sufficiently large $k \in \mathbb{N}$, set each γ_i to $\bot$ — i.e., $p \wedge \neg p$ for some $p \in \Phi_i$ — and each $u_i^{\mathcal{B}}(v)$ to $\pi_i(f(v)) - k$.

Proposition 3 (Strategic games with goals and boolean games)

1. *Let $\mathcal{B} = (N, \Phi, c, \{\gamma_i\}_{i \in N}, \{\Phi_i\}_{i \in N})$ be a boolean game. Then there exist a strategic game with goals $(\mathcal{S} = (N, \{\Sigma_i\}_{i \in N}, \pi), G_i)$ and a bijective function $f : V \to \prod_{i \in N} \Sigma_i$ such that for all $i, u_i^{\mathcal{B}}(v) = u_i(f(v))$.*

2. *Let* $(\mathcal{S} = (N, \{\Sigma_i\}_{i \in N}, \pi), G_i)$ *be a strategic game such that, for each* $i \in N$ *there is* $n \in \mathbb{N} \setminus \{0\}$ *with* $|\Sigma_i| = 2^n$ *. Then there exist a boolean game* $\mathcal{B} = (N, \Phi, c, \{\gamma_i\}_{i \in N}, \{\Phi_i\}_{i \in N})$*, a* $k \in \mathbb{N}$*, and a bijective function* $f : V \to \prod_{i \in N} \Sigma_i$ *such that for all* $i, u_i^{\mathcal{B}}(v) = u_i(f(v)) - k$*.*

Proof Direct consequence of the definitions and the previous two results.

3 Endogenous games and quasi-dichotomous preferences

Endogenous games (Jackson and Wilkie, 2005) have been introduced as an extension of normal form games with a pre-play negotiation phase, where agents have the possibility, before the game starts, to spend the amount of utility received at certain outcomes to influence their opponents' decision-making.

This section imports the ideas and the techniques of endogenous games to games with quasi-dichotomous preferences and it shows that, in spite of the static representation results of the previous section, the presence of goal states does make a difference when side-payments are allowed.

Following (Jackson and Wilkie, 2005)[7] we enrich a strategic game $(N, \{\Sigma_i\}_{i \in N}, \pi)$ with a family $\{T_i\}_{i \in N}$ where, for each agent i, T_i is a set of functions of the form $\tau_i : \Sigma \times N \to \mathbb{R}_+$. Each such function specifies how much payoff agent i secures the other agents in case some particular outcome obtains. We call each $\tau \in \prod_{i \in N} \tau_i$ a **transfer function** and a tuple $(\mathcal{S}, \{T_i\}_{i \in N})$ an **endogenous game**. We moreover denote τ^0 the transfer function such that for all $i, j \in N$ and $\sigma \in \Sigma$ we have $\tau_i(\sigma, j) = 0$, which we call the *void* transfer function.

We require each set of transfer functions T_i to satisfy the following minimal conditions:

$$\text{If } \tau'_i \text{ is such that } \tau'_i(\rho, j) = 0 \text{ for some } j \in N, \rho \in \Sigma, \text{ then } \tau'_i \in T_i \tag{3}$$

The property says that each agent is always able to offer nothing to any agent at any outcome.

$$\forall \tau'_i \in T_i, \forall \tau_{-i} \in \prod_{j \in N \setminus i} T_j, \exists \tau_i \in T_i, \forall \sigma \in \Sigma, \forall j \in N : \tau_i(\sigma, j) = \tau_j(\sigma, i) + \tau'_i(\sigma, j) \tag{4}$$

The property says that each agent is always able to give the money back once received, possibly adding an available counteroffer.

[7] While the approach to game transformation by side-payments adopted here (Jackson and Wilkie, 2005) provides an elegant technical framework that well suits strategic games with goals, we remind the reader of the existence of earlier related work in the game theory literature (Guttman, 1978, 1987; Rosenthal, 1975; Kalai, 1981; Varian, 1994; Farrell, 1998). Pre-play negotiations were originally conceived as a tool to overcome inefficiency introduced by the non-cooperative structure of the interaction, without forcing the agents to form coalitions. In particular, to regulate the effect of individuals' actions on other individuals' welfare, what in economic theory is called *externalities*. There is abundant literature on these, e.g., Coase (1960), (Meade, 1952), (Maskin, 1994), and a comprehensive account of models of pre-play negotiations is to be found in (Goranko and Turrini, 2012), where a multi-step model of pre-play negotiations in non-cooperative games, addressing some limitations of (Jackson and Wilkie, 2005), is developed and which will briefly be described in Section 7.

We argue that the conditions we impose on transfer functions are satisfied in many cases, and represent a significant departure from the literature on pre-play negotiations, where agents are endowed with the family $\{\mathcal{T}_i\}_{i\in N}$, incorporating *all* transfer functions (Jackson and Wilkie, 2005; Guttman, 1978; Kalai, 1981; Farrell, 1998; Goranko and Turrini, 2012).

An **endogenous game with goals** is defined in the expected way, i.e., as a tuple $((\mathcal{S}, \{G_i\}_{i\in N}), \{T_i\}_{i\in N})$ where $(\mathcal{S}, \{G_i\}_{i\in N})$ is a strategic game with goals and $(\mathcal{S}, \{T_i\}_{i\in N})$ is an endogenous game.

It is useful to think of an endogenous game (with goals) as a game consisting of two phases:

– A pre-play phase, where agents simultaneously play a transfer function;
– An actual game play, where agents play a strategy and the payoff of the starting game is updated taking the selected transfers into account.

Definition 5 (Update by side-payments)
Let $\mathcal{S} = (N, \{\Sigma_i\}_{i\in N}, \pi)$ be a strategic game and let τ be a transfer function. The **play of** τ **in** $\mathcal{S}$ is the strategic game $\tau(\mathcal{S}) = (N, \{\Sigma_i\}_{i\in N}, \pi')$ where, for $\sigma \in \Sigma, i \in N$,

$$\pi'_i(\sigma) = \pi_i(\sigma) + \sum_{j\in N} \tau_j(\sigma, i) - \sum_{j\in N} \tau_i(\sigma, j)$$

In a nutshell, when a game is updated by a transfer function each agent:

– adds to his payoff at each outcome all the transfers that he receives from the other agents at that outcome;
– subtracts from his payoff at each outcome all the transfers he makes to the other agents at that outcome.

Example 3 We have observed how our robots would act if they were to take their decisions on what to do with the block independently and simultaneously with no form of commitment, coordination or communication being allowed. In the resulting situation, i.e., in all equilibria of the game, Bob would have his goal satisfied, against a relatively high cost, while Ann would not, although not paying any cost.

Let us now introduce a mechanism, which we will refer to as the *energy channel*, connecting all agents involved in the interaction. The task of the energy channel is to allow robots to share effort in the performance of their action. In particular, a robot could end paying part of the cost of another robot as a function of the outcome of the game. We can think of the energy channel as a mechanism acting after the game has taken place: each robot will execute a task, employing a certain amount of energy, which might get compensated through the energy channel. We assume, for the sake of simplicity, that the energy channel has no dispersion. The way the channel can be used is as follows: before the interaction begins, each robot involved can commit to bear an amount of effort (dually, offer an amount of payoff) to some other robot, should a certain outcome be reached. For instance, Figure 6 shows how the starting strategic game with goals is updated by an offer by Ann to Bob of 8 units of payoff at all outcomes where Bob tries and lift the block and, once lifted, tries and move it left.

	lL	lN
lL	$-8(-20), -20(+20)$	$-8, -10$
lN	$-5(-20), -20(+20)$	$-5, -10$
nL	$0(-20), -20(+20)$	$0, -10$
nN	$0(-20), -20(+20)$	$0, -10$

Fig. 6 An offer of 20 units of payoff by Ann to Bob for moving the block left. Strictly dominated strategies nL and nN by Bob are disregarded.

It is worth noticing that the offer depicted is only a partial transfer profile, to which Bob can respond with his own simultaneous counteroffer. But if we were to read the offer as representing the only true updates of the matrix, considering the rest void transfers, then we could notice that such a pre-play exchange would fundamentally change the set of rational outcomes.[8] In the novel game, there is only one strictly dominant strategy equilibrium, moreover independent of the way the game is instantiated: Bob plays lL and so does Ann. In the only equilibrium outcome, both robots are satisfying their goal, Bob paying no cost, Ann paying 28. This solution is clearly better for both than any Nash equilibrium of the original game.

Let $\mathcal{E} = ((\mathcal{S}, \{G_i\}_{i \in N}), \{T_i\}_{i \in N})$ be an endogenous game with goals and let $\mathcal{E}(\omega)$ be its instantiation with the boost factor profile ω[9]. A pair (τ, σ), for τ being a transfer function and σ a strategy profile available at $\mathcal{S}$, is a **sustainable solution** of $\mathcal{E}$ if there is a strategy in the two-phase game that is a subgame perfect equilibrium and where (τ, σ) is played on the equilibrium path. In other words in order for a pair (τ, σ) to be a sustainable solution of an endogenous game with goals $\mathcal{E} = (\mathcal{S}, \{G_i\}_{i \in N}), \{T_i\}_{i \in N})(\omega)$ we require that there exists a strategy for $\mathcal{E}(\omega)$, i.e., a specification of a strategy profile σ' for the actual game play after every transfer function τ', such that:

1. (τ, σ) is a Nash Equilibrium in the two-phase game;
2. for every (τ', σ') in the strategy specification, σ' is a Nash-equilibrium in every subgame $\tau'(\mathcal{S}, \{G_i\}_{i \in N})(\omega)$ of the two-phase game.

Subgame perfect equilibria rule out *incredible threats* (Osborne and Rubinstein, 1994), in our case the fact that some transfers might be discouraged by the play of strategies that are dominated after the transfers in question are made.

For a sustainable solution (τ, σ) of $\mathcal{E} = ((\mathcal{S}, \{G_i\}_{i \in N}), \{T_i\}_{i \in N})$ instantiated with ω we say that σ is a **surviving equilibrium** if it is a Nash equilibrium of $(\mathcal{S}, \{G_i\}_{i \in N})(\omega)$.

3.1 Boost factors and game update

When a pre-play phase precedes a strategic game with goals instantiated with a family of boost factors, we need to understand how the boost factors react to

[8] As a result of the pre-play offer, some payoffs originally representing costs have now become positive. We can think of this phenomenon as the robots being promised extra reserve battery power in case a certain outcome is reached, which can be stored without actually being used.

[9] The calculation of the utility after a transfer function has occurred is carried out as expected, i.e., in the updated strategic games with goals instantiated by ω.

the game update. In particular, if boost factors are to encode a relative distance between goal states and non-goal states, we do want these changes to be reflected in the dynamics introduced by the possibility of side-payments.

To do so, we indicate $\omega^{(\mathcal{S},\{G_i\}_{i\in N})}$ the boost factor applied to game $(\mathcal{S},\{G_i\}_{i\in N})$ and omit the superscript when obvious. We indicate with $\Omega\uparrow$ the set of all boost factors satisfying the following property, under the assumption that π is the payoff function of $\mathcal{S}$ and π^{τ} of $\tau(\mathcal{S})$:

$$\omega_i^{(\mathcal{S},\{G_i\}_{i\in N})}(x) \geq \omega_i^{(\tau(\mathcal{S}),\{G_i\}_{i\in N})}(x) \Leftrightarrow \exists\sigma \notin G_i \text{ such that } \forall\sigma' \notin G_i, \pi_i(\sigma) \geq \pi_i^{\tau}(\sigma') \quad (5)$$

What the definition says is that, fixing a boost factor, the utility of goal states is pushed upwards the higher the payoff non-goal states yield. It is pushed downwards otherwise.

We will also consider a different type of boost factor, modelling agents that value more goal states the further away they are from the worst possible outcome they could end up in. It is the *regret-based* boost factor that generalises the μ factor typical of boolean games (Wooldridge et al, 2013). We indicate with $\Omega\downarrow$ the set of all boost factor profiles satisfying the following property:

$$\omega_i^{(\mathcal{S},\{G_i\}_{i\in N})}(x) \geq \omega_i^{(\tau(\mathcal{S}),\{G_i\}_{i\in N})}(x) \Leftrightarrow \exists\sigma \notin G_i \text{ such that } \forall\sigma' \notin G_i, \pi_i(\sigma) \leq \pi_i^{\tau}(\sigma') \quad (6)$$

What the definition says is that, fixing a boost factor, the utility of goal states is pushed upwards the lower the payoff non-goal states yield. It is pushed downwards otherwise.

As we will see later, having a boost factor profile in the set $\Omega\downarrow$ makes a huge difference in agents' strategic behaviour, as agents can deliberately strive to increase their cost at non-goal states for the sole reason of increasing their payoff at goal states.

Unless otherwise specified we consider boost factor profiles in the set $\Omega\uparrow$.

Example 4 Let us go back again to our working example and assume that the game is instantiated by a boost factor profile ω in the set $\Omega\uparrow$, working as specified by the constraints we have imposed. Also assume that the space of transfers is $\mathcal{T}$. The original Nash equilibria of the game (lN, nL) and (lN, nN), and any mixing thereof, are not surviving, as no matter what transfer we use to sustain them, Ann will always be in a position to make (lL, lL) a unique dominant strategy equilibrium, by means of a transfer compensating Bob while still making her better off. It is also not difficult to see that the profile of offers illustrated in the previous example, where Ann is paying 20 to Bob to make him try and move the block left is not sustainable. Ann could have obtained the same result by offering Bob just enough, i.e., 10 plus an arbitrarily small amount ϵ, to make his strategy lL uniquely dominant. But even that would not be enough! In fact, by the same reasoning, Ann could have offered $10+\frac{\epsilon}{2}$ and get the job done. As we could go on forever, there is no least amount Ann could offer on top of 10, so, in the end won't be able to turn lL into a unique dominant strategy equilibrium, *guaranteeing its stability.* However the profile (lL, lL) of the original game is sustainable, as Figure 7 shows.

No matter how the game is instantiated — as long as we fix a boost factor profile ω in the set $\Omega\uparrow$ — Ann will have no incentive to deviate for this profile of offers. If she did it, and they ended up not playing lL, she would certainly not

	lL	lN
lL	$-8(-10), -20(+10)$	$-8, -10$
lN	$-5(-10), -20(+10)$	$-5, -10$
nL	$0(-10), -20(+10)$	$0, -10$
nN	$0(-10), -20(+10)$	$0, -10$

Fig. 7 A sustainable solution. Ann offers Bob 10 for moving the block left and both set of play lL in the continuation.

be better off. But neither would she, if they ended up playing lL anyway. For Bob the reasoning is similar.

At this point the reader may have wonder what the difference actually is between strategic games with goals and strategic games, and what the need therefore would be to study endogenous games with goals as separate structures. After all, we have shown a rather straightforward correspondence between the two (Proposition 1).

The following proposition is a hint of the fact that the similarity between them is not as obvious as the previous results might suggest. Concretely, when introduce dynamic operations that update the game (such as side-payments), the correspondence ceases to exist.

Proposition 4 (Dynamic asymmetry)
Let $(\mathcal{S}, \{G_i\}_{i\in N})$ with $\mathcal{S} = (N, \{\Sigma_i\}_{i\in N}, \pi)$ be a strategic game with goals, ω a boost factor profile, τ^0, τ' be two transfer functions, $\mathcal{S}^0, \mathcal{S}'$ the strategic games corresponding to $\tau^0(\mathcal{S}, \{G_i\}_{i\in N})(\omega)$ and $\tau'(\mathcal{S}, \{G_i\}_{i\in N})(\omega)$, respectively. It is not necessarily the case that $\tau'(\mathcal{S}^0) = \mathcal{S}'$.

Proof Consider the strategic game with goals $(\mathcal{S}, \{G_i\}_{i\in N})$, such that $\mathcal{S} = (\{A, B\}, \{\Sigma_i\}_{i\in N}, \pi)$, $G_B = \emptyset$, $G_A = \{\sigma^*_A, \sigma^*_B\}$, $\pi_i(\sigma) = 0$ for all $i \in N, \sigma \in \Sigma$ and $|\Sigma_A| = |\Sigma_B| = 2$. Consider the transfer functions τ^0 and τ' such that $\tau'_A(\sigma, B) = 2$, for all $\sigma \notin G_A$ while $\tau'_A(\sigma^*, B) = 0$ and $\tau'_B(\sigma, A) = 0$ for all $\sigma \in \Sigma$. Notice now that $u_A^{(\tau^0(\mathcal{S}),\{G_i\}_{i\in N})(\omega)}(\sigma^*) = \omega_A^{(\tau^0(\mathcal{S}),\{G_i\}_{i\in N})}(0) > 0$, that $u_A^{(\tau'(\mathcal{S}),\{G_i\}_{i\in N})(\omega)}(\sigma^*) = \omega_A^{(\tau'(\mathcal{S}),\{G\}_i)(\omega)}(-2) > -2$ and that $\omega_A^{(\tau'(\mathcal{S}),\{G\}_i)(\omega)}(-2) < 0$. σ^* yields agent A a payoff $x \geq 0$ in $\tau'(\mathcal{S}^0)$ but not in $\mathcal{S}'$.

The proposition shows that updating a strategic game with goals and its corresponding strategic game with the same transfer profile does not necessarily maintain the correspondence between the two structures.

The reason lies on the fact that the utility function is calculated in such a way that a agent gets a boosted payoff in outcomes satisfying his goal, but always relative to the value yielded by the non-goal states. The results in the next section will show that this imbalance bears heavy consequences in terms of equilibrium analysis.

4 Equilibria in the two-phase game

This section is devoted to the exploration of the formal properties of rational outcomes in endogenous games with goals. In particular, of how the properties of equilibria in those structures are related to the ones already known in the literature.

The classical results on equilibrium survival for the case of endogenous games (Jackson and Wilkie, 2005) are centred on the notion of solo payoff, i.e., the payoff that a single agent i can guarantee if the opponents $-i$ do not make any transfer.

Definition 6 (Solo payoff)
Let $\mathcal{E}(\omega) = ((\mathcal{S}, \{G_i\}_{i\in N}), \{T_i\}_{i\in N})(\omega)$ be an endogenous game with goals instantiated by a boost factor profile ω. The **solo payoff** $\hat{s}_i$ for agent i, where sup is the least upper bound operator, is

$$\hat{s}_i^{\mathcal{E}(\omega)} = \sup_{\tau_i}(min_{\rho\in NE^{\Delta}((\tau_i,\tau^0{}_{-i})(\mathcal{E}(\omega))}E^{(\tau_i,\tau^0{}_{-i})(\mathcal{E}(\omega))}(\rho))$$

In words, the solo payoff of agent i is given by the best transfer i can make, under the expectation that:

- his opponents will not make any transfer
- will play the worst for i Nash equilibrium in each subgame.

When the underlying game is fixed and no confusion can arise, we use the notation $\hat{s}_i$. Outcomes at which agent i is receiving at least $\hat{s}_i$ can be considered *desirable* from i's perspective: he could not improve his final payoff even if he was the only one allowed to make monetary offers.

Two important facts are known for endogenous games (Jackson and Wilkie, 2005):

1. when $N \leq 2$, a Nash equilibrium survives if and only if each agent gets at least his solo payoff;
2. when $N > 2$, every pure strategy Nash equilibrium survives.

As common with models of pre-play negotiations, and bargaining in general, there is a fundamental distinction between situations in which two or more agents are involved. Intuitively, this is due to the fact that, in the two agent case, each agent that receives an offer is always able to counter it with an offer that leads the game to the status quo, i.e., each agent can always give the money back. With three agents it is not possible for a agent to unilaterally restore the money transfers.

The first result carries over to endogenous games with goals. The argument we use is similar to the one by (Jackson and Wilkie, 2005), but our result holds for a broader class of transfer functions.

Proposition 5 $\mathcal{E}(\omega) = ((\mathcal{S}, \{G_i\}_{i\in N}), \{T_i\}_{i\in N})(\omega)$ *be an endogenous game with goals instantiated by a boost factor profile* ω *and such that the each* T_i *satisfies conditions 3 and 4. A Nash equilibrium strategy profile* ρ *is surviving if and only if* $u_i(\rho) \geq \hat{s}_i^{\mathcal{E}(\omega)}$

Proof From right to left, consider a Nash equilibrium strategy profile ρ such that $u_i(\rho) \geq \hat{s}_i^{\mathcal{E}(\omega)}$. To see that it is surviving, construct a subgame perfect equilibrium of the two phase game such that (τ^0, ρ) is played on the equilibrium path. This is possible by the fact that each T_i satisfies condition 3. Off the equilibrium path instead, if some agent offers transfers in the first period, then identify the worst equilibrium for that agent in the resulting second-stage game and have that be played in the continuation. Play any Nash equilibrium instead if more than one agent deviates from τ^0. This is an equilibrium, as no agent can improve upon (τ^0, ρ). From left to right, instead, suppose that $u_i(\rho) < \hat{s}_i^{\mathcal{E}(\omega)}$ for some surviving Nash equilibrium ρ. But this means that there exists a transfer function τ such that all Nash equilibria x of $(\tau_i, \tau^0_- i)((\mathcal{S}, \{G_i\}_{i\in N}))$ are such that $u_i(x) > u_i(\rho)$. Consider now any transfer function τ^* such that (τ^*, ρ) is a surviving equilibrium. By condition 4, agent i can switch to a profile of transfers τ_i' such that $\forall \sigma \in \Sigma : \sum_{j\in N\setminus i} \tau_i'(\sigma, j) = \sum_{j\in N\setminus i} \tau_j^*(\sigma, i) + \sum_{j\in N\setminus i} \tau_i(\sigma, j)$. We can observe that $(\tau_i' \tau^*_{-i})(\mathcal{S}, \{G_i\}_{i\in N}) = \tau(\mathcal{S}, \{G_i\}_{i\in N})$, which shows that τ_i is a profitable deviation.

One might expect that, due to the correspondence between strategic games and strategic games with goals, also the result for $N > 2$ would carry over. It might come as a surprise that, in fact, it does not.

Proposition 6 *There exists an endogenous game with goals $\mathcal{E}(\omega)$ with more than 2 agents and Nash equilibrium outcome σ of $\mathcal{E}(\omega)$ that is not a surviving equilibrium.*

Proof Consider the strategic game with goals $(\mathcal{S}, \{G_i\}_{i\in N})(\omega)$ where $\mathcal{S}$ is given by the tuple $(\{A, B, C\}, \{\Sigma_i\}_{i\in\{A.B,C\}}, \pi)$, and where $G_B = G_C = \emptyset$, $G_A = \{\sigma_A^*, \sigma_B^*, \sigma_C^*\}$, $\pi_i(\sigma) = 0$ for all $i \in N, \sigma \in \Sigma$. Suppose moreover that, for $M = max_{i\in N,\sigma,\sigma'\in\Sigma}\{\pi_i(\sigma) - \pi_i(\sigma')\}$ be the highest payoff difference a agent can incur in the game, agents' A's transfers functions are such $\forall \tau_{-A}, \exists \tau_A \in T_A, \forall \sigma \in \Sigma, \forall j, k \in -A : \tau_A(\sigma, j) = \sum_{k\in -A} \tau_k(\sigma, j) + M$. In other words, agent A has enough resources to be able to compensate all other agents for the result of every negotiation. Clearly each $(\sigma_A^*, \sigma_B^*, \sigma_C')$ for $\sigma_C' \neq \sigma_C^*$ is a Nash equilibrium. However it is not a surviving one. For suppose it was and take $(\tau, (\sigma_A^*, \sigma_B^*, \sigma_C'))$ to be the strategy played on the equilibrium path. Now for each payoff that agent C is getting at $(\tau, (\sigma_A^*, \sigma_B^*, \sigma_C'))$ agent A is better off deviating to a transfer τ_A' such that $\tau_A'(C, \sigma^*) > \pi_C^\tau(\sigma_A^*, \sigma_B^*, \sigma_C') - \pi_C^\tau(\sigma_A^*, \sigma_B^*, \sigma_C^*)$, making it worthwhile for C to satisfy A's goal. Such deviation exists by the properties of T_A.

The idea of the proof is quite simple: when goal realization is at stake and a agent has enough resources, she could go to any length to have it satisfied. So Nash equilibria of the initial game will not survive if a joint deviation of a group of agents satisfies the goal of some other resourceful agent without compromising their own. Figure 8 illustrates one more such scenario.

	L	R
U	1, 1, 1	1, 1, 1
D	1, 1, 1	1, 1, 1

1

	L	R
U	1, 1, 1	1, 1, 1
D	1, 1, 1	1, 1, 1

2

Fig. 8 A three agent game, with the third agent choosing the matrix to be played. Row wants $(U, L, 1)$ to be realised and, no matter what the distribution of payoffs after the pre-play phase looks like, he is willing to compensate the other agents to go along that strategy. Notice that Row's deviation in the pre-play phase is effective, as it does not compromise the opponents' goals. The Nash equilibrium outcome $(D, R, 2)$ is not surviving.

Proposition 6 is of fundamental importance, not only because it shows that there is a strategic game with goals with $|N| > 2$ where a pure strategy Nash equilibrium is not surviving, at odds with well-known results for the strategic games case, but because Proposition 1 was suggesting a straightforward correspondence between strategic games and strategic games with goals. The reason of this imbalance was already hinted at by Proposition 4, which showed how the application of the same transfer function to a strategic game and to its corresponding strategic game with goals was not guaranteed to yield corresponding structures. Proposition 6 uses the same idea to show that dynamic factors such as transfer functions are enough to falsify fundamental results for the otherwise statically correspondent strategic games [10].

Example 5 Consider again the Nash equilibria of our two-agent starting scenario: (lN, nL) and (lN, nN), and any mixing thereof. We showed previously how none of them was a surviving equilibrium, constructing a deviation for every possible transfer profile imposed on the starting game. Proposition 5 shows that we could simply check that the payoff-equivalent (lN, nL) and (lN, nN) are not yielding Ann her solo payoff. This is readily done, as fixing Bob's transfer to be void, there exist a transfer function, i.e., the one offering Bob $10 + \epsilon$ for playing lL, *guaranteeing* Ann a positive payoff, even if Bob played the worst for Ann's Nash equilibrium. This, once again, is independent of how the game is instantiated.

Suppose we added now a third agent, Charles, with no goals and two actions, A and B, choosing between two copies of the original game, and constant payoff at every state (see Figure 9). The presence of Charles would not affect the sustainability of the solution in either of the two games. No matter what the transfer profile is that they play, Ann would always be able to turn the tables and offer the agents enough compensation for making them worth while to play the desired profile. The original Nash equilibria are therefore not surviving, even with three agents. Notice, once again, that this relies on the space of offers being $\mathcal{T}_i$[11].

	lL	lN
lL	$-8, -20, 0$	$-8, -10, 0$
lN	$-5, -20, 0$	$-5, -10, 0$
nL	$0, -20, 0$	$0, -10, 0$
nN	$0, -20, 0$	$0, -10, 0$

A

	lL	lN
lL	$-8, -20, 0$	$-8, -10, 0$
lN	$-5, -20, 0$	$-5, -10, 0$
nL	$0, -20, 0$	$0, -10, 0$
nN	$0, -20, 0$	$0, -10, 0$

B

Fig. 9 Ann, Bob and Charles.

In spite of the negative result discussed above, we are still able to show a sufficient condition for all pure strategy Nash equilibria to survive in certain strategic games with goals, independently of the number of agents involved.

Proposition 7 (Survival)
Let $(\mathcal{S}, \{G_i\}_{i \in N})$ be strategic game with goals and let it be instantiated by a boost factor profile ω. A pure strategy Nash equilibrium σ of $\mathcal{S}$ survives whenever $u_i(\sigma) \geq \hat{s}_i$ for each $i \in N$.

[10] In the terminology of (Harrenstein et al, 2014) such outcomes would be called *soft* equilibria, meaning that they can be rationally removed by the appropriate use of allowed game transformations.

[11] In fact, a weaker requirement is sufficient, i.e., that for every τ sustaining an original Nash equilibrium of the game, Ann had enough resources to construct a profitable deviation.

Proof We need to construct a subgame perfect equilibrium of the two-phase game where σ is played. On the equilibrium path let agents play the profile (τ^0, σ). Off the equilibrium path, if a single agent deviates from τ^0 pick the worst Nash equilibrium for that agent and have that played in the continuation. If more than a agent deviates from τ^0, play any Nash equilibrium. We can observe that no agent i can get more than $\hat{s}_i$ by deviating from τ^0, given the choices played in each subgame. Moreover σ is a pure strategy Nash equilibrium of $\mathcal{S}$.

What we have shown is that when a Nash equilibrium σ gives the agents at least their solo payoff then that Nash equilibrium survives. In fact the proof of the proposition shows even more, i.e., that the agents will actually obtain at least their solo payoff in that equilibrium.

The case of regret-based boost factors For families of strategic games with goals instantiated by boost factors in $\Omega\downarrow$ the results are even more striking.

Proposition 8 (No survival)
There exist $(\mathcal{S}, \{G_i\}_{i\in N})(\omega)$ *with* $|N| = 3$, $\omega \in \Omega\downarrow$ *and* $\{\sigma^*\} = NE(\tau^0((\mathcal{S}, \{G_i\}_{i\in N})(\omega)))$ *such that:*

- $\sigma' \in \bigcap_{i\in N} G_i$ *if and only if* $\sigma'_i = \sigma^*_i$
- σ^* *is not a surviving equilibrium.*

Proof Let $\mathcal{G}(\omega)$ with $\mathcal{G} = ((N, \Sigma, \pi), \{G_i\}_{i\in N})$ be such that $N = \{1, 2, 3\}$ and for each $i \in N$ we have that $\Sigma_i = \{\sigma_i, \rho_i\}$ and $G_i = \{\sigma' \mid \sigma'_i = \sigma_i\}$. The payoff function is such that $\pi_1(\rho) = -10$, for and $\pi_1(\sigma') = 0$ for $\sigma' \neq \rho$, with $\pi_1(\sigma') = \pi_2(\sigma') = \pi_3(\sigma')$ for each $\sigma' \in \Sigma$. The outcome σ is clearly a dominant strategy equilibrium. Suppose now that it is also a surviving equilibrium and that the transfer function τ is part of the solution. But then there exists some agent i for which $u_i^{\tau(\mathcal{G})(\omega)}(\sigma) \leq u_j^{\tau(\mathcal{G})(\omega)}(\sigma)$ for some $j \neq i$. This means, by the properties of boost factors in $\Omega_\downarrow$ that i is better off deviating to a transfer function (τ'_i, τ_{-i}) such that $\sum_{j\in N} \tau'_i(\rho), j) > \sum_{j\in N} \tau_i(\rho), j)$ while $\tau'_i(\sigma', j) = \tau_i(\sigma', j)$ for each $\sigma \neq \rho$. No matter what equilibrium will be played in the resulting subgame, i will be increasing is payoff in σ. Contradiction.

So not only have we constructed a non-surviving pure strategy Nash equilibrium in the case of more than two agents, but also an outcome that is shared joint goal among all agents and even dominant strategy profile in every continuation, which is a straightforward truth for boost factor profiles $\omega \in \Omega\uparrow$. As anticipated, with boost factor profiles in $\Omega\downarrow$ agents can increase their cost at non-goal states for the sole reason of increasing their payoff at goal states, which is exactly what happens here. Figure 10 displays once more this fact.

What we have shown tells us, once again, how having boost factors where the payoff at goal states is increased by decreasing the payoff at non goal states, as done for instance with boolean games in (Wooldridge et al, 2013; Turrini, 2013), has unwanted consequences. In (Turrini, 2013) for instance, sufficient conditions for survival of pure strategy Nash equilibria in boolean games with costs (Wooldridge et al, 2013) are shown to be extremely demanding and require, besides the standard condition on solo-payoffs, also an upper bound on the payoff agents can receive as a result of transfers and the presence of a $|N|$ outcomes with maximal aggregated sum of payoffs.

	L	R
U	$0,0,0$	$0,0,0$
D	$0,0,0$	$0,0,0$

1

	L	R
U	$0,0,0$	$0,0,0$
D	$0,0,0$	$-10,-10,-10$

2

Fig. 10 A three agent game, with the third agent (Table) choosing the matrix to be played. Row wants any outcome consistent with U to be realised, Column any outcome consistent with L and Table any outcome consistent with 1. Notice that all agents are in control of their own goal satisfaction. To ease readability we avoid displaying all coalitional colours which are, as expected, $\{Row, Column\}$, $\{Table, Column\}$, $\{Row, Column, Table\}$, $\{Row, Table\}$. Rather, we only label the strategies corresponding to individual agents' goals. The maroon outcome $(U, L, 1)$ is Row's, Column's and Table's joint goal and happens to be a dominant strategy equilibrium of the game. However it is not a surviving equilibrium as, no matter what the distribution of payoffs after the pre-play phase looks like, there will always be a agent that can increase his cost at outcome $(D, R, 2)$ for the sole reason to increase his payoff at outcome $(U, L, 1)$.

5 An integrated framework for resource-bounded agents

In this section we integrate the possibility for agents to exchange side-payments with the possibility for an external authority to impose taxes. Taxes, we will see, can effectively be used to overcome the limitations in resources agents have during the pre-play phase. We also explore further the relation between centralised and decentralised mechanisms.

Consider a strategic game with goals $(\mathcal{S}), \{G_i\}_{i\in N})(\omega)$. We say that an outcome σ is **potentially-dominant** if it is a dominant strategy equilibrium of some $(\tau(\mathcal{S}), \{G_i\}_{i\in N})(\omega)$ for $\tau \in \prod_{i\in N} \mathcal{T}_i$. The space of potentially dominant outcomes is quite large. In a strategic game, for instance, every outcome is potentially dominant. Also, the desirable solution of our starting example, where Ann can convince Bob to move the block left, is potentially dominant. Intuitively, potentially dominant outcomes are conflict-free, i.e., ideally agents could ensure their achievement in equilibrium. As such, an external authority might want some of these outcomes to be attainable rational outcomes of a negotiation. However, it might be the case that agents do not have enough resources to sustain them, i.e., their family of transfer functions is much more restricted than $\mathcal{T}$, or agents might have no way of constructing a surviving equilibrium given the initial payoffs. The purpose of this section is to study taxation mechanisms that *guarantee* these type of outcomes to be turned into surviving equilibria.

Wooldridge et al. (Wooldridge et al, 2013) define taxation mechanisms on a boolean game as functions associating to every agent a monetary *sanction* at each particular outcome. We employ this notion for the more general framework of strategic games with goals, defining them as functions $\alpha_i : \Sigma \to \mathbb{R}_+$, subtracting the taxes received from a agent at a certain outcome to his payoff.

To ease the connection with our previous definitions we introduce the strategic game $\alpha(\mathcal{S})$, i.e., the game $\mathcal{S}$ to which the taxation mechanism $\alpha = \prod_{i\in N} \alpha_i$ is applied, as the strategic game $(N, \{\Sigma_i\}_{i\in N}, \pi')$ where for each $i \in N, \sigma \in \Sigma$ we have that $\pi'_i(\sigma) = \pi_i(v) - \alpha_i(\sigma)$. The way taxes modify goals' value is similar to what happens for the case of transfer functions:

$$\omega_i^{(\mathcal{S}, \{G_i\}_{i\in N})}(x) \geq \omega_i^{(\alpha(\mathcal{S}), \{G_i\}_{i\in N})}(x) \Leftrightarrow \exists \sigma \notin G_i \text{ such that } \forall \sigma' \notin G_i, \pi_i(\sigma) \geq \pi'_i(\sigma') \tag{7}$$

Transfer functions and taxation mechanisms are of a rather different kind. While the former ones consist of payoff redistributions among agents at certain outcomes, without adding or subtracting to the agents' total payoff, the latter ones explicitly inject new sanctions into the system to modify agents' decision-making. However, in a technical sense, we can always find a taxation mechanism having the same effect of a transfer function on an underlying game.

Proposition 9 (From side-payments to taxes)
Let $(N, \{\Sigma_i\}_{i \in N}, \pi')$ be a strategic game and τ a transfer function. There exists a taxation mechanism α such that $NE(\tau(\mathcal{S})) = NE(\alpha(\mathcal{S}))$.

Proof Straightforward.

Proposition 9 shows that, in some respect, taxation mechanisms can *simulate* transfer functions. It must however be said that simulations of this kind only make sense when both taxation mechanisms and transfers functions are fixed. But unlike taxation mechanisms, that are decided externally, transfer functions bear further strategic considerations. As made clear in the motivating example, it is not enough to establish that a transfer function induces equilibria in the resulting game, but we also need to establish whether the transfer function itself is part of a larger equilibrium, i.e., whether agents are not better off by switching to different transfers.

The procedure we present shows how to use taxation mechanisms to make potentially dominant outcomes survive. It takes an outcome of the original game that is potentially dominant and yields a taxation mechanism that turns it into a surviving Nash equilibrium.

Algorithm 1

Input An outcome σ of $(\mathcal{S}, \{G_i\}_{i \in N})(\omega)$ with $\mathcal{S} = (N, \{\Sigma_i\}_{i \in N}, \pi)$ that is potentially dominant.
Output A taxation mechanism α on $\mathcal{S}$.
Steps
1. **Let** $\alpha_i(\sigma') = 0$, *for each* $i \in N, \sigma' \in \Sigma$;
2. **While** $u_i^{(\alpha(\mathcal{S}), \{G_i\}_{i \in N})(\omega)}(\sigma) < \hat{s}_i^{((\alpha(\mathcal{S}), \{G_i\}_{i \in N}), \{T_i\}_{i \in N})(\omega)}$, **do** $\alpha_i(\sigma') := \alpha_i(\sigma') + 1$, *for each* $\sigma' \neq \sigma \in \Sigma$;
3. **Return** α.

Proposition 10 (Survival by taxation)
Let σ be a potentially dominant outcome of an instantiated strategic game with goals $(\mathcal{S}, \{G_i\}_{i \in N})(\omega)$ with $\mathcal{S} = (N, \{\Sigma_i\}_{i \in N}, \pi)$. There exists a taxation mechanism α such that σ is a surviving equilibrium of $((\alpha(\mathcal{S}), \{G_i\}_{i \in N}), \{T_i\}_{i \in N})(\omega)$.

Proof To see that Algorithm 1 guarantees this fact we only need to observe that the construction of α at step 1 and step 2 ensures that the payoff $u_i^{(\alpha(\mathcal{S}), \{G_i\}_{i \in N})(\omega)}(\sigma)$ will eventually reach $\hat{s}_i^{((\alpha(\mathcal{S}), \{G_i\}_{i \in N}), \{T_i\}_{i \in N})(\omega)}$ as σ is potentially dominant.

External authorities as agents One of the reasons why it is important to study decentralised solutions is the fact that centralised solutions are generally more costly and not always available. Our previous results suggest that this can be a fundamental problem in terms of attainment of desirable properties: we might want a potentially dominant outcome to be surviving but we find out that it will not be surviving unless some taxation mechanism is introduced. The question we ask here is whether endogenous mechanisms are inherently weak, i.e., whether there is no way of making some outcome that can survive thanks to an external authority also survive in *some* minimal modification of the original endogenous game, minimal in the sense that agents' transfer powers and objectives remain the same. What we show is that decentralised mechanisms are powerful enough to encode centralised mechanisms. The idea is simple: we expand an endogenous games with goals $((\mathcal{S}, \{G_i\}_{i\in N}), \{T_i\}_{i\in N})(\omega)$ with one *distinguished* agent i^* such as:

1. $\Sigma_{i^*} = \{\sigma_{i^*}\}$, i.e., the distinguished agent only has a single action at its disposal;
2. $T_{i^*} = \mathcal{T}_{i^*}$, i.e., the distinguished agent can offer unbounded incentives to all agents at all outcomes [12];
3. For each agent $i \neq i^*$, transfer function $\tau_i \in T_i$ and outcome σ we have that $\tau_i(\sigma, i^*) = 0$, i.e., the distinguished agent cannot receive incentives.
4. the rest remains as in $((\mathcal{S}, \{G_i\}_{i\in N}), \{T_i\}_{i\in N})(\omega)$. [13]

For an endogenous game with goals $((\mathcal{S}, \{G_i\}_{i\in N}), \{T_i\}_{i\in N})(\omega)$ we call $((\mathcal{S}, \{G_i\}_{i\in N})), \{T_i\}_{i\in N})^*(\omega)$ an **internalised expansion** if it is obtained by augmenting $((\mathcal{S}, \{G_i\}_{i\in N}), \{T_i\}_{i\in N})(\omega)$ with a agent i^* satisfying the properties above.

We can now show the following:

Proposition 11 *Let $((\mathcal{S}, \{G_i\}_{i\in N}), \{T_i\}_{i\in N})(\omega)$ be an endogenous game with goals and $((\mathcal{S}, \{G_i\}_{i\in N})), \{T_i\}_{i\in N})^*(\omega)$ be an internalised expansion. Then, for each potentially dominant profile σ with $\{(\sigma, \sigma_{i^*})\} = G_{i^*}$ and for some taxation mechanism α:*

σ is surviving equilibrium of $((\alpha(\mathcal{S}), \{G_i\}_{i\in N}), \{T_i\}_{i\in N})(\omega)$ if and only if (σ, σ_{i^*}) is a surviving equilibrium of $((\mathcal{S}, \{G_i\}_{i\in N})), \{T_i\}_{i\in N})^*(\omega)$.

Proof From right to left. Pick a transfer profile τ^* making (σ, σ_{i^*}) a surviving equilibrium in $((\mathcal{S}, \{G_i\}_{i\in N})), \{T_i\}_{i\in N})^*(\omega)$, which exists by the assumptions. Now replicate the transfers of all agents $j \neq i^*$ and construct α such that $\alpha_i(\sigma) = -\tau^*_{i^*}(\sigma, \sigma_{i^*}, i)$. From left to right, engineer the minimal τ^*_i that makes (σ, σ_{i^*}) unique dominant strategy equilibrium. This is possible because $\{(\sigma, \sigma_{i^*})\} = G_{i^*}$ and because σ is potentially dominant. Then add incentives at each outcome and player, until it the equilibrium becomes surviving, iterating as in Algorithm 1.

Notice that the taxation mechanism α in the proposition need not be minimal, in particular it need not be the one returned by Algorithm 1 on input $((\mathcal{S}, \{G_i\}_{i\in N}), \{T_i\}_{i\in N})(\omega)$.

[12] This assumption is just for technical convenience. In fact, as Algorithm 1 shows, we need a finite amount of monetary injection on a system to make a potentially dominant outcome be surviving.

[13] As σ_{i^*} is the only action available to player i^*, there is a natural bijection with the starting set of strategy profiles, remaining players, goals and payoff functions.

6 Related Work

The multi-agent systems community has produced a number of models and results that are related to our contribution and the topic of distributed solution of multi-agent decision problems is one of its core concerns. As an example, the recently concluded COST action *Agreement Technologies* [14] has emphasised the need of having "computer systems in which autonomous software agents negotiate with one another, typically on behalf of humans, in order to come to mutually acceptable agreements".

Our conviction is that the endogenous games approach, combined with a proper goal-oriented modelling of agent interaction, [15] is able to provide a platform for analysis for these computer systems. In particular we stress how it provides a *decentralised, non-cooperative and goal-oriented approach to multi-agent interaction.*

Especially in the past decade, there has been a tremendous focus on computational analysis of distributed processes, in the context of the related research lines of automated negotiation, incentive engineering and coalition formation, thanks to the growth of algorithmic game theory as unifying tool for analysing distributed processes. We discuss these related reseach lines, emphasising the added value of our framework with respect to the focus and the assumptions they build upon.

6.1 Negotiation

Several contributions exist in the field of artificial intelligence that have studied negotiation among rational agents. This has mostly boiled down to the analysis of the algorithmic properties of allocation procedures studied in game theory and social choice (comprehensive surveys of negotiation in multi-agent systems are (Kraus, 2001) and (Rosenschein, 1994)). Very few articles have dealt with the problem of escaping inefficiency of strategic interaction by agents striking deals. Among them the seminal contribution by Rosenschein and Genesereth (Rosenschein and Genesereth, 1985) in which, possibly for the first time, game-theoretic models of contracts are studied to resolve conflict in agent societies. In their framework each agent is basically a agent in a normal form game but he is given the possibility to specify a set of joint moves, i.e., strategy profiles, that he is willing to accept as outcomes. If the intersection of all agents' suggestions is nonempty, then a profile inside that set is chosen by a fair arbiter and executed. Moreover, a payoff function, conditional on each agent move and suggested outcomes, is used to study equilibrium outcomes. While the notion of deal in (Rosenschein and Genesereth, 1985) does have some intuitive connections with sustainable outcomes, there are a number of substantial differences:

– We have added *goal-states* to a starting normal form game, capturing agents goal-directedness (Castelfranchi, 1998) as different from maximal payoff, working on a more general, quasi-dichotomous, preference relation.

[14] `http://www.agreement-technologies.eu`

[15] We note on the fly that the tradition of modelling goal states in the multi-agent systems community is divided between a purely qualitative logic-based account of goals (e.g., the BDI logic-based tradition) and the game-theoretic approach, that treats goals as synonym of maximal payoff states under a classical utility function. To the best of our knowledge, the first approach combining the two is Boolean Games with costs.

- We have kept the interaction inherently strategic. Agents are not allowed to coordinate on joint offers, nor to propose their play, but to provide monetary incentives to their fellow agents, in order to modify their decision-making. In the tradition of pre-play negotiations, the starting normal form game, even when updated by transfers, remains non-cooperative.
- We have explicitly studied a pre-play negotiation phase, where offers affect the game to be played, *before* the game starts. Rational outcomes of the pre-play negotiations have been studied as subgame perfect equilibria of the two phase game.

In general, the models of negotiations studied in the multi-agent systems community emphasise the distributed and non-cooperative nature of the decision-making process, which are also building blocks of our approach. There are also approaches to the regulation of a multi-agent system that are somewhat related to ours, involving either centralised solutions - an external authority that can effectively transform the interaction of the decision-makers - or cooperative solutions - where individuals are allowed to join forces and coordinate towards the realisation of collective improvement, e.g., utility maximisation. Both approaches have roots in game theory and have been taken up in the multi-agent systems literature which has further analysed their computational properties.

6.2 Centralised solutions

While our approach to the use side-payments as a decentralised negotiation mechanism has ancient roots in the game-theoretic tradition (e.g., (Coase, 1960; Meade, 1952; Maskin, 1994)), there has been work in the social choice literature that has used side payments to model incentive engineering, in particular to model bribery. In this area, at least from the paper on lobbying and legislative bargaining by Helpman and Persson (Helpman and Perrson, 1998), bribery has been modelled as a centralised single-agent decision problem, where an external authority attempts to influence the behaviour of a group of decision-makers, e.g., a committee, by offering monetary incentive. This approach is taken up in a variety of computational studies of bribery and lobbying in the multi-agent systems community, e.g., (Faliszewski et al, 2009) in which an external agent tries to ensure a victory of one of the candidates via bribing some voters to change their votes. Other, more distant, forms of centralised solutions studied in game theory are the principal-agent problem (Grossman and Hart, 1983) and the theory of coalitional incentives in (Winter, 2010), which aim at answering questions of the kind "how much should a single agent or a group of agents be paid to sustain a certain effort?"

Unlike those approaches, ours is inherently decentralised and multi-agent: every participant in the interaction can be allowed to offer incentive to every other participant, in order to reach a more satisfactory outcome. Our belief is that modelling bribery as merely centralised decision problem, in particular without allowing counteroffers from the bribed parties, is often an unwanted oversimplification which our proposal allows to overcome. We point moreover out that this is independent on whether offers happen in normal form games or voting games, where the outcome is determined by a voting rule. Clearly the bribing strategies will be different, e.g., to pass a legislation there might be need to bribe the majority of agents, but this does not change the need of a decentralised multi-agent pre-play (respectively, pre-vote) phase.

Other centralised approaches have been studied in the multi-agent systems community, involving utility transfer, possibly the most notable one being the idea of mediator by Monderer and Tennenholtz (Monderer and Tennenholtz, 2006), which study models of external agents (the mediators) that can modify the payoff redistribution from outside - rather than endogenously - and can achieve more efficient outcomes in non-cooperative game.

6.3 Cooperative solutions

Our approach is inherently non-cooperative. Even though agents are allowed to offer incentives to modify the decision-making of their fellow agents, they always do this at their cost and they always rationally choose to transfer monetary payoff if this is increasing their individual expected utility in the resulting game. In all equilibria they also transfer the minimal amount of money needed to convince other agents to change their decision. In this sense, with a pre-play phase agents are allowed to "bargain" over the outcome to be played (a relation between extensive form pre-play negotiations and Rubinstein's bargaining games (Rubinstein, 1982) is formally shown in (Goranko and Turrini, 2012)) but they are *not* allowed to play as a coalition. If agents were allowed to play as a coalition and pick the best possible outcomes for themselves, then the game would clearly trivialise. The fact that agents are keeping their individual perspective and looking at their own expected utility, allows us to treat transfers as strategies of a larger game.

An important game-theoretic approach to the study of coalitions acting on a starting non-cooperative game is Aumann's model of strong equilibria, (Aumann, 1959), which can be seen as the cooperative solution concept of the core applied to a non-cooperative game (Osborne and Rubinstein, 1994). The theory of cooperative game has also analysed redistributions of payoff in coalitional games, i.e., the so-called TU games (Osborne and Rubinstein, 1994). We point out once more how these approaches do not preserve the bargaining nature typical of pre-play negotiations.

An interesting yet only remotely related example of use of cooperative approaches for manipulating elections is (Zuckerman et al, 2011). As the authors speficy, the term (coalitional) manipulation is referred to "a situation where a voter (a group of voters) casts votes not according to his (their) true preferences, but rather to ob- tain some goal". Despite the differences this suggests an interesting application for models of goal-directed agency.

7 Discussion

The framework we have developed here could be extended in many ways, to cover more structured types of interaction. In what follows we elaborate on several abstractions that our framework implicitly contains and on ways to overcome them.

On the structure of goal states In a strategic game with goals, a goal state is modelled as a strategy profile, a combination of actions for each agent that results in an outcome of the game. When choosing between two outcomes, an agent will first check whether the outcomes satisfy her goal, and only after she will consider the associated payoff. The cognitive theory of goals is rather complex (see for instance the number of differences in goal modes made in (Conte and Castelfranchi, 1995)) and our modelling choices abstract away from many of its features. The first that comes to mind is the difference between *terminal goals*, i.e., states that are desired fixed objectives, and *non-terminal goals*, i.e, states that are instrumental to the realization of the other goals. It seems to us that working on extensive form structures could be a first step to make the difference explicit between these two types of goals. Our models already *contain* these type of structures — as well known extensive game can be canonically translated into normal form games (Osborne and Rubinstein, 1994) — however the extensive form representation can facilitate means-end reasoning. If terminal goals can be seen as static objectives an agent wants to achieve, certainly non-terminal goals have a clear dynamic. If an agent realises to have a device at his disposal to realise his objective to lift a block, we would certainly want the agent to go on and *want* to use the device to lift the block. This type of goal dynamics can effectively be captured in extensive games with imperfect knowledge, where the means-end relations can be learned.

On the structure of the negotiation phase The model of pre-play negotiation adopted here is a more general version of the one in (Jackson and Wilkie, 2005), where agents simultaneously exchange binding offers in case a certain outcome is achieved. One could employ a more refined binary structure to model negotiations, as the one developed (Goranko and Turrini, 2013, 2012), where agents alternate offers and counteroffers on a starting normal form game and only stop if agents are happy with the updated game. More in details, starting from a normal form game G, there is a turn function t determining how agents take turns. Each agent to move can either pass or make a binding offer of payoff to the opponents, on top of the ones already made thus far. [16]. If all agents pass, the game terminates. This has been shown to yield a proper bargaining game, of the type analysed in (Osborne and Rubinstein, 1994), where agents bargaining on what game they will end up playing. It is known that, when in bargaining games with valuable time, i.e., when they agents' payoff is discounted taking into account the length of the negotiation, agent quickly settle for fair and efficient divisions of payoff.

Each of these features can be desirable when modelling a particular type of application, and for each of them the effect can be observed on the properties of surviving equilibria.

Pre-play negotiations and agent dependencies Pre-play negotiations are a mechanism to *solve* non-cooperative interaction, giving agents the possibility to sacrifice a part of the payoff received at a certain outcome to convince their fellow agents to change their strategy. Intuitively this is possible only if agents making the offer:

– are interested in having some other agents changing their decision-making;

[16] In Goranko and Turrini's model (Goranko and Turrini, 2012), the offers can be made on the strategies played by opponents (unconditional offers) or conditional on matching counteroffers (conditional offers). Without entering technical details, it is sufficient to know that both type of offers can be (at least statically) simulated by the transfer functions studied here.

– they can sacrifice enough payoff to persuade their fellow agents and still make the resulting outcome worthwhile for everyone involved in the exchange.

Intuitively, only when agents are not indifferent to the way their opponents play, offers make sense. This is the sort of *dependence relation* first described in (Castelfranchi et al, 1992), incorporated by (Bonzon et al, 2009a) in the analysis of boolean games with binary preference[17] and by (Grossi and Turrini, 2012) in the analysis of strategic games. The solutions to the games given both in (Bonzon et al, 2009a) and (Grossi and Turrini, 2012) allow agents to literally exchange favours, modelling no quantitative bargain and seeing agents as potentially resource-unbounded. The framework we present here allows for a more fine-grained resolution of a strategic games with goals, but what the properties are of the dependence relations that actually determine the outcome a negotiation is still not understood.

Voting games The framework developed here can have interesting applications in domains where actors are engaged in a collective decision. If we think of each actor wanting to achieve a certain set of outcomes of a collective decision that is determined by a given voting rule, then it is interesting to study how *pre-vote* negotiations can effectively affect the outcome of a collective decision, providing as well a novel model to study bribing in elections (Faliszewski et al, 2009). Voting rules can easily be implemented in a boolean game by making a certain variable true at an outcome if and only if it is selected by that rule. For instance, under the majority rule, a proposition p will hold at outcome σ if and only if there is a group C that is at least as large as the majority such that every member i of C has set a proposition p_i true at σ. Resources (payoff) can be assigned to agents as a function of the outcome of the collective decision. Directions in this sense are already being taken (Grandi et al, 2014).

8 Conclusion

We have studied strategic games where agents are endowed with designated goal states and with the possibility of offering side-payments to their fellow agents in order to influence their decision-making. The perspective we have taken integrates the framework of strategic games with goals, a generalisation of the boolean games studied in artificial intelligence, with that of endogenous games with side-payments, studied in game theory. We have seen that the resulting games display specific properties that make them worth studying in their own sake (Propositions 1, 2, 3 and 4) and the classical results available on Nash equilibria survival do not generalise (Proposition 6). We have however provided sufficient conditions that Nash equilibria need to have in order to survive (Proposition 7), independently of the number of agents involved. We have also shown that, with an appropriate use of taxation mechanisms, every outcome consistent with agents' goals can be turned into a surviving Nash equilibrium (Algorithm 1). Future research efforts

[17] The models adopted in (Bonzon et al, 2009a) are very close to the original account of boolean games given in (Harrenstein et al, 2001), where boolean games can be seen as a simple version of the ones defined here, but with no cost function. Under these restrictions boolean games become a special case of strategic games.

will be devoted to studying the interaction between side-payments in strategic games with goals and more realistic taxation mechanism that carry out imperfect redistribution of wealth, i.e., extract payoff units to some agents at certain outcomes redistributing a part of it to possibly different agents at possibly different outcomes. Attention will also be paid to the relation with mechanism design and the algorithmic properties of the procedures under study.

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