Determination of the yield *loci* of four sheet materials (AA6111-T4, AC600, DX54D+Z, and H220BD+Z) by using uniaxial tensile and hydraulic bulge tests)

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Abstract

In sheet metal forming simulation, a flow curve and a yield criterion are vital requirements for obtaining reliable numerical results. It is more appropriate to determine a flow curve by using biaxial stress condition tests, such as the hydraulic bulge test, than a uniaxial test because hardening proceeds higher strains before necking occurs. In a uniaxial test, higher strains are extrapolated, which might lead to incorrect results. The bulge test, coupled with the digital image correlation (DIC) system, is used to obtain stress–strain data. In the absence of the DIC system, analytical methods are used to estimate hardening. Typically, such models incorporate a correction factor to achieve correlation to experimental data. An example is the Chakrabarty and Alexander method, which uses a correction factor based on the n-value. Here, the Chakrabarty and Alexander approach was modified using a correction factor based on normal anisotropy. When compared with DIC data, the modified model was found to be able to better predict the hardening curves for the materials examined in this study. Because a biaxial flow curve is required to compute the biaxial yield stress, which is an essential input to advanced yield functions, the effects of the various approaches used to determine the biaxial stress–strain data on the shape of the BBC2005 yield *loci* were also investigated. The proposed method can accurately predict the magnitude of the biaxial yield stress, when compared with DIC data, for all materials investigated in this study.

Keywords: Biaxial flow curve, Yield criterion, Bulge test, Normal anisotropy

1. Introduction

Numerical simulation of sheet metal forming processes, such as the stamping process, plays an important role in the design phase [1]. The reliability of simulation depends on the accuracy of the mechanical characterization of the materials [2]. One of the most important items in material characterization, which has a major impact on the quality of the forming simulations, is the plastic flow curve [3,4]. In the field of sheet metal forming, various mechanical tests can be used in order to obtain stress–strain curves, such as tensile, hydraulic bulge, through-thickness disk compression, plane strain and shear tests [5,6]. The uniaxial tensile test is the most commonly used such test. It provides stress–strain data up to the point of diffuse necking at very low levels of plastic strain, when compared with the ones attained in some forming processes [3,7,8]. However, in sheet forming processes, the level of plastic deformation can be higher. Therefore, it is more appropriate to use biaxial loading tests, such as the hydraulic bulge test, which can reach higher plastic strain levels before necking and fracture occur [9]. Another reason that the hydraulic bulge test is more appropriate is that the biaxial mode is the major deformation mode in many sheet forming processes [10,11]. Finally, the hydraulic bulge test has the advantage of providing the value of one of the key material parameters, namely, the biaxial yield stress, that is required to define most advanced yield functions [12].

The hydraulic bulge test, in combination with a digital image correlation (DIC) system, is the state of the art in the determination of biaxial stress–strain curves [8,13,14]. However, there are three occasions when it is more suitable to use analytical models: in the absence of a DIC system [13, 15], during high temperature testing when the view of the optical systems is obscured by vapor and smoke [11] and when investigating fundamental effects of material properties on ductility [18]. These models are becoming increasingly important with the greater adoption of elevated forming methods such as warm forming.

Analytical methods establish stress and strain indirectly by identifying the instantaneous bulge radius and the thickness at the dome apex during a test. One of the earliest models was developed by Hill [16] who created a method for calculating the bulge radius through geometric considerations alone. Panknin [17] later improved on the model by accounting for the curvature of the sheet material around the tooling fillet radius, as this significantly affects the bulge of the sheet.

Hill also introduced a model for the determination of the thickness at the pole of the bulge [16]. This was based on the geometry of a sheet material that is bulged under hydrostatic pressure. The shape of the material in the polar region is assumed to be approximately spherical and strain relationships are derived in the circumferential and thickness directions However, this relationship results in sheet thickness that is uniform at each stage of the test. However, in reality sheet thickness is thinnest at the polar region and thickest at the equatorial regions because of the strain concentrating effect of the spherical geometry. To introduce the more realistic case of varying thickness. Chakrabarty and Alexander [18] introduced the parameter, $\lambda = 1\text{-}n$, where n is the work-hardening index of the material, to Hill's equation. This modification was able to better explain the polar thickness strain of soft copper compared to Hill's theory. Similar to Hill [16], Kruglov *et al.* [19] developed method for determining the dome apex thickness by considering the instantaneous geometry of the dome shell alone. Kruglov's formulation was more general and accounted for the distribution in thickness from the equator to the pole. Their model was used to predict the thickness of titanium sheet in the superplastic deformation regime. They treated the material as isotropic and reported a 10% error in their calculation of polar thickness. Despite this, Koc *et al*. [11] concluded that the best approach was one that combined Kruglov's thickness determination approach with Panknin's polar radius method. Lăzărescu *et al.*. [13,20] subsequently made improvements to the overall accuracy of this method. The accuracy was improved by incorporating a correction factor related to the dome apex thickness to account for the non-uniformity of the strain distribution on the pole. It was observed that the accuracy of this could be improved further [7]. A systematic study showed that evolution of the sheet thickness is dependent of the anisotropy [21, 40]. This corroborates the findings of the current study.

The other key factor in the material characterization is the yield function, which plays an important role in the accurate prediction of forming defects, such as thinning and splitting [22-25]. Several researchers have proposed advanced criteria, such as the Banabic yield criteria [26-29] and the Barlat yield criteria [30,31], to describe the plastic behaviour of materials accurately because they incorporate a large number of parameters. These advanced criteria account for the biaxial stretching regime that is the dominant regime in sheet metal forming [10,11]. It has been proven that these advanced models can overcome the inaccuracies of classical models, such as Hill'48 [32], by improving the description of the plastic behaviour of the metallic sheets. Recently, Banabic *et al.*. [2] presented a review of the most recently proposed yield criteria for describing anisotropic plastic behaviour.

The primary objective of this paper is to explore the importance of sheet anisotropy to the evolution of sheet thickness during a bulge test. Although Kruglov *et. al* [19] demonstrated the importance of geometry to instantaneous sheet thickness, the relationship between anisotropy and thinning suggest that it will be a particularly relevant in calculating instantaneous thickness. Our starting point is therefore to couple the Panknin correction for die radius [17] with the Chakrabarty and Alexander relationship for instantaneous sheet thickness at the pole [18]. By modifying Chakrabarty and Aleaxander's correction factor to include the effects of anisotropy, we developed a relationship to test the importance of anisotropy over the work hardening ability of the material. Hydraulic bulge experiments were performed to test the hypothesis. Calculations for material hardening and BBC2005 yield locus were obtained in three ways: with the Panknin-Kuglov, Panknin-Chakrabarty and the proposed Panknin-Chakrabarty (modified) combinations. These were compared to a DIC-based method for calculating hardening and yield loci. The comparisons were repeated for 4 alloys: AA6111-T4 and AC600 aluminium alloys and DX54D, and H220BD steel alloys. The comparisons demonstrated the importance of anisotropy and produced a method for obtaining hardening and yield loci in the absence of digital image correlation equipment.

The paper is structured as follows. In section 2, the theories and methods for stress–strain determinations by using the hydraulic continuous bulging test are summarized. In section 2.1, the framework for the proposed modification is presented. The methods for determining the biaxial yield stress and a description of the BBC2005 are given in sections 2.3 and 2.4, respectively. In section 3, the experimental setup, materials, and analysis methods used in this study are described. In section 4, the analysis results are presented and discussed. Finally, conclusions are presented in section 5.

2. Biaxial flow curve determination

Membrane theory is a common approach used in the determination of biaxial flow curves [17,33]. Flow curves are determined based on the analysis of variables measured in the bulge test [8], in which a specimen is clamped with a blank holder, as illustrated in Fig. 1. The theory is only valid when the ratio of the sheet thickness to the bulge diameter is small [33]. This theory is based on the assumptions that the through thickness stress σ_3 is zero and that a relationship can be established using Laplace's formula:

$$\frac{\sigma_1}{\rho_1} + \frac{\sigma_2}{\rho_2} = \frac{p}{t} \tag{1}$$

where σ_1 and σ_2 are the principal stresses on the material surface, ρ_1 and ρ_2 are the polar curvature radii, p is the bulge pressure, and t is the specimen thickness at the dome apex. The bulge test is considered to represent an axisymmetric case; therefore, the principal stresses can be assumed to be equivalent and equal to the membrane stress, i.e., $\sigma_1 = \sigma_2 = \sigma_b$. The same conclusion can be drawn for the curvature radii, i.e., $\rho_1 = \rho_2 = \rho$. Given these simplifications, the current biaxial stress or membrane stress σ_b is defined as follows:

$$\sigma_b = \frac{p\rho}{2t} \tag{2}$$

For the purpose of strain calculation, the constant volume condition of Eq. (3) is used. This assumption is based on the fact that the plastic deformation in metals and alloys occurs without any appreciable change in volume [34].

$$\varepsilon_1 + \varepsilon_2 = -\varepsilon_3 \tag{3}$$

Therefore, the thickness strain or the biaxial strain ε_b can be determined as follows:

$$\varepsilon_b = -\varepsilon_3 = \ln \frac{t_o}{t} \tag{4}$$

The primary assumptions considered in this study are the following:

- The through-thickness stress is zero,
- The principal stresses are equal to the membrane stress,
- The principal curvature radii are equal,
- The incompressibility condition is valid

Assuming a balanced equi-biaxial stress state could results in a noticeable error for strongly anisotropic materials [7].

2.1 Experimental and analytical methods

The pressure p, polar radius ρ, and current thickness at the dome apex t must be calculated to determine the biaxial stress–strain relationship by using Eqs. (2) and (4). The pressure p is recorded using a sensor attached to the hydraulic device. However, the other quantities are not measured in a direct manner; rather, they are obtained from other experimental data, namely, the polar height h. Hill [16] developed a method for determining the bulge radius without considering the effect of the die fillet radius R. To improve the accuracy of this method, Panknin [17] developed the formula given in Eq. (5) for the polar radius as a function of the fillet radius. Panknin’s method yields a valid result for ratios of the bulge height to the radius of the die of up to 0.56 [4] or even higher [11].

$$\rho = \frac{1}{2h}(\frac{d}{2} + R)^2 + \frac{h}{2} - R \tag{5}$$

In Eq. (5), d is the diameter of the die cavity, R is the die fillet radius, and h is the dome height. The parameters d and R are constants related to the experimental device, and the parameter h is a variable that is measured using a displacement sensor, e.g., a linear voltage displacement transducer (LVDT) or a DIC system [11,13].

Dome apex thickness evolution data can be derived experimentally by using data measured by a DIC system or calculated using various approaches. Hill [16] proposed the following relationship to predict the polar thickness t at the pole:

$$t = t_o[\frac{1}{1 + (h/(d/2))^2}]^2 \tag{6}$$

Chakrabarty and Alexander [18] modified Hill's formula by taking into account the hardening effect to improve the model's accuracy. An unknown parameter λ, which is a function of the strain hardening exponent of the material, was introduced into Hill's formula [18]:

$$\lambda = 1 - n \tag{7}$$

For all practical cases, λ must lie between 0 and 1 to ensure that the rate of plastic work will be positive. Chakrabarty and Alexander proposed that the thickness strain rate at the pole be defined as follows:

$$\frac{d\varepsilon_b}{dh} = (1 + \lambda)\frac{2h}{(d/2)^2 + h^2} \tag{8}$$

Eq. (8) can be integrated to derive the following expression for the thickness strain ε_b at the dome apex:

$$\varepsilon_b = (2 - n)\ln(1 + \frac{h^2}{(d/2)^2}) \tag{9}$$

Because Eq. (9) is equal to Eq. (4),

$$\varepsilon_b = -\varepsilon_3 = \ln\frac{t_o}{t} = (2 - n)\ln(1 + \frac{h^2}{(d/2)^2}) \tag{10}$$

The thickness at the dome apex can then be expressed as follows:

$$t = t_o[\frac{1}{1 + (h/(d/2))^2}]^{2-n} \tag{11}$$

Eq. (11) reduces to Eq. (6) when $\lambda = 1$. Moreover, the Ross and Prager assumptions [35] are obtained if $\lambda = 0$ [18].

Kruglov *et al.*. [19] developed a simple method for determining the dome apex thickness. The method assumes that the meridian stresses are uniformly distributed along the surface thickness [19]. The expression proposed by Kruglov for the thickness at the pole is as follows:

$$t = t_o[\frac{(d/2)/\rho}{\sin^{-1}((d/2)/\rho)}]^2 \tag{12}$$

2.2 Proposed methodology

Based on the constant-volume assumption, the principal strains generated during a biaxial test can be related to compressive thickness strain. Biaxial strain deformation is known to be sensitive to plastic anisotropy [33]. It is proposed that the accuracy of the polar thickness prediction obtained using the method proposed by Chakrabarty and Alexander [18] could be improved by considering the effect of the plastic strain ratio rather than the hardening effect.

We assume that λ is a function of the normal plastic anisotropy. Because the R-values for the range of materials tested in this study range from 0.5 to 2, we made the following assumptions concerning the relationship between $\overline{R}$ and λ:

$$\lambda = \begin{cases} 2\overline{R} - 1 & \mathit{for} \quad 0.5 \le \overline{R} \le 1.0 \\ 2 - \overline{R} & \mathit{for} \quad 1.0 \le \overline{R} \le 2.0 \end{cases} \tag{13}$$

where $\overline{R}$ is the coefficient of normal anisotropy, which is computed as follows:

$$\overline{R} = \frac{R_0 + 2R_{45} + R_{90}}{4} \tag{14}$$

The term *normal* refers to the direction perpendicular to the sheet. The variation of the plastic properties along the thickness of the sheet is characterized by the $\overline{R}$ parameter [36]. Equation (15) ensures that the condition concerning the rate of plastic work is not violated.

$$0 \le \lambda(\overline{R}) \le 1 \tag{15}$$

For most aluminium alloys, the corresponding values of $\overline{R}$ lie within the range of $0.5 \le \overline{R} \le 1$. Thus**,** the through-thickness logarithmic strain and dome apex thickness at the pole can be calculated using the following relationships:

$$\varepsilon_b = (2\overline{R})\ln(1+\frac{h^2}{(d/2)^2}) \tag{16}$$

$$t = t_o[\frac{1}{1+(h/(d/2))^2}]^{2\overline{R}} \tag{17}$$

For most steel alloys, the assumption concerning λ is different from the one for aluminium grades. The value of the normal anisotropy parameter for steels lies in the range of 1 to 2. Therefore, the polar strain and thickness are calculated using the following expressions:

$$\varepsilon_b = (3-\overline{R})\ln(1+\frac{h^2}{(d/2)^2}) \tag{18}$$

$$t = t_o[\frac{1}{1+(h/(d/2))^2}]^{3-\overline{R}} \tag{19}$$

For $\lambda = 2\overline{R}-1$, the upper and lower bounds of the proposed method correspond to Hill's approach [16] and Ross and Prager's assumption [35], respectively. When $\lambda = 2-\overline{R}$, the opposite condition holds.

2.3 Determination of the biaxial yield stress

The method of the 0.2% offset for the initial biaxial yield stress is not reliable [37] because the biaxial flow curve is inaccurate at low levels of plastic strain [3]. Therefore, the principle of equivalent plastic work is used to calculate the average initial biaxial yield stress [3,14,37]. The principle of equivalent plastic work can be written for the uniaxial and biaxial stress states as follows:

$$W_u = \int \sigma_u d\varepsilon_u = W_b = \int \sigma_b d\varepsilon_b \tag{20}$$

where W_u and W_b are the plastic work per unit volume in the cases of uniaxial and biaxial stress states, respectively; σ_u and σ_b are the uniaxial and biaxial stresses, respectively; and $d\varepsilon_u$ and $d\varepsilon_b$ are the uniaxial and biaxial plastic strain increments, respectively. If equality prevails, then the yield stresses of the same material for the different stress states are identical [3,14]. In that case, the average ratio is calculated. One of the methods for determining this ratio is the approach proposed by Lee *et al.*.[37]. In this approach, for a certain strain range or plastic work range, the ratio between σ_b and σ_u is evaluated, and then the average ratio is calculated [3,37]. This average ratio is multiplied by the uniaxial yield stress to obtain the biaxial yield stress. Theoretically, the average ratio should be independent of the selected range of the plastic strain; however, such value affects the resulting average ratio [3]. Other methods exists and are discussed by Sigvant *et al.*.[3].

2.4 BBC2005

Anisotropic yield criteria or functions are used in academia and industry to describe the onset of plastic deformation at different stress states [1,2]. Different methods can be used to develop anisotropic yield functions [2]. These methods are employed to transform existing isotropic formulations into anisotropic ones [10].

One of the approaches to perform such transformation is to include new parameters or coefficients into an isotropic function as in the case of the BBC2005 that is used in our work. The BBC2005 yield function includes new plastic anisotropy parameters into Hershey's formulation [1,29].

Banabic *et al.*. [29] proposed a yield function referred to as BBC2005 that is implemented in the AUTOFORM 4.1 program. This yield function can be written in the following form:

$$F = \overline{\sigma} - Y = 0 \tag{21}$$

where F is the yield function, $\overline{\sigma}$ is the BBC2005 equivalent stress, and Y is the instantaneous reference yield stress of the material. The BBC2005 equivalent stress can be written as follows:

$$\overline{\sigma} = [a(\Lambda + \Gamma)^{2k} + a(\Lambda - \Gamma)^{2k} + b(\Lambda + \Psi)^{2k} + b(\Lambda - \Psi)^{2k}]^{\frac{1}{2k}} \tag{22}$$

The terms Γ, Ψ, and Λ are defined as follows:

$$\begin{aligned} \Gamma &= L\sigma_{11} + M\sigma_{22} \\ \Psi &= \sqrt{(N\sigma_{11} - P\sigma_{22})^2 + \sigma_{12}\sigma_{21}} \\ \Lambda &= \sqrt{(Q\sigma_{11} - R\sigma_{22})^2 + \sigma_{12}\sigma_{21}} \end{aligned} \tag{23}$$

where a and b > 0 are material coefficients and k is the exponent related to the crystal structure of the materials (3 for steel sheets and 4 for aluminium sheets). The parameters L, M, N, P, Q, and R involved in Eq. (23) are the remaining plastic anisotropy parameters. These eight plastic anisotropy parameters or coefficients have to be identified numerically by solving the nonlinear system associated with the BBC2005 yield function. The details of the system of the nonlinear equations associated with BBC2005 can be found in [1,29].

Each yield criterion has its own requirements. Particularly, each yield function has a certain number of mechanical parameters (Yield stresses, e.g. ; and *R*-values, e.g.) that must be obtained from experimental tests. These mechanical parameters work as inputs to the yield functions [41]. The BBC2005 yield function requires the determination of eight mechanical parameters. Three uniaxial yield stresses and three R-values are obtained from three different directions ($0^{\circ}, 45^{\circ}, 90^{\circ}$). The values of these six parameters are obtained using a uniaxial tensile test that must be complemented by other tests, such as a compression test, to compute the biaxial R-value [30], and the hydraulic bulge test, to determine the biaxial yield stress [14].

Generally, these eight mechanical parameters are fed to a system of nonlinear equations associated with the yield function [41]. The roots of the nonlinear system is the plastic anisotropy coefficients, that are used to calibrate the yield function shape to certain experimental points as in the case of advanced models such as BB2005. The roots of such systems are called plastic anisotropy parameters (or coefficients or constants). These parameters are fed to the yield function in order to define the anisotropic locus. The roots can be identified using different numerical procedures such as Newton Raphson, minimisation of an error function, and genetic algorithm [41].

3. Experiments and materials

As stated previously, the determination of biaxial stress–strain data requires the instantaneous measurement or calculation of certain variables. The experimental method and combinations of different analytical approaches were used in this study. The pressure was measured using a sensor. The polar radius was determined experimentally by using the DIC system and calculated analytically by using Panknin's approach. The thickness at the dome apex was determined using the DIC system and calculated analytically by using the method proposed by Chakrabarty and Alexander [18], its proposed modification, and the Kruglov *et al.*. [19] method. Continuous hydraulic bulging experiments were performed to validate the proposed approach. The continuous hydraulic bulging tests were performed on samples with diameters of 180 mm at an equivalent strain rate of 0.002 s-1. Stochastic pattern was applied to the samples' surfaces. The DIC measurement consists of two digital CCD cameras with a resolution of 1280x1024 pixels. These cameras were used to capture many pictures at different level of deformation. The frames were analysed in order to determine the 3D coordinate, principle logarithmic strains, radii of the principle curvatures. With the assumption of the validity of the volume consistency, the thickness strain is measured. The mean curvature radius, that is used in the calculation of the biaxial stress, is calculated following the equation recommended by the ISO procedure i.e. ($\rho= 2/(1/\rho_1+1/\rho_2)$). A spherically shaped surface near the pole was assumed for the calculation of the major and minor curvature radii. Biaxial flow curves were determined using four materials: AA6111-T4, AC600, DX54D+Z, and H220BD+Z. A summary of the approaches used in this study to determine the biaxial stress–strain curves is shown in Fig. 2. Furthermore, tensile tests equipped with two extensometers were performed in the three directions ($0^\circ, 45^\circ, 90^\circ$). The data obtained from the tensile tests are the uniaxial flow curves, yield stresses, and plastic strain ratios. The uniaxial data were used to compute the biaxial yield stresses and yield *loci*. To calculate the biaxial plastic anisotropy parameter, which is required to calculate the yield *loci*, compression tests were performed on samples with diameters of 10 mm at an equivalent strain rate of 0.001 s^{-1}. Sheep suet, provided by Erichsen GmbH & Co. KG, Germany, was used as a lubricant in the compression tests. The method for determining the biaxial R-value ($R_b=\varepsilon_{TD}/\varepsilon_{RD}$) has been presented elsewhere [30].

A summary of the material properties derived from the tensile and compression tests is presented in Table 1. The initial thicknesses were measured with a micrometre.

Table 1 Average mechanical properties of the materials

Material	t_0 [mm]	YS_0 [MPa]	YS_{45} [MPa]	YS_{90} [MPa]	R_0 [-]	R_{45} [-]	R_{90} [-]	R_b [-]	$\bar{R}$ [-]
AA6111-T4	0.92	138	128	126	0.699	0.539	0.509	1.299	0.572
AC600	0.90	144	141	142	0.615	0.399	0.658	0.962	0.518
DX54D+Z	0.74	162	168	165	2.007	1.699	2.370	0.902	1.944
H220BD+Z	0.69	248	260	259	1.666	1.544	2.107	0.871	1.715

4. Results and discussion

4.1 Polar thickness vs. dome height

The polar thickness was measured using the DIC software and compared with the predictions obtained using the Chakrabarty and Alexander, Kruglov, and modified Chakrabarty and Alexander equations, i.e. Eq. (11), Eq. (12), Eq. (17), and Eq. (19). Fig. 3 illustrates the polar thickness as a function of the dome height for different materials. The proposed method (P-C-M) accurately predicts the polar thickness as measured by the DIC system for all of the materials investigated in this study. Kruglov's approach also leads to the same conclusion, except for the H220BD+Z material. For this material, Kruglov's prediction gradually diverges with increasing dome height. The predicted value tends to be greater than the corresponding experimental value. In contrast, Chakrabarty and Alexander's prediction gradually diverges with increasing apex height, and the predicted value tends to be lower than the experimental value. It should be noted that height measurements up to 24 mm and 28 mm were used in the calculations of the polar thickness for the aluminium and steel alloys, respectively.

4.2 Pressure vs. polar strain

The relation between the polar strain and the pressure is illustrated in Fig. 4. The strain is calculated from Eq. (4), using thickness data measured with the ARAMIS software. Strains determined from the continuous bulging experiment were compared with strains predicted using the Chakrabarty and Alexander method (Eq. 9), the Kruglov method (Eq. 4, Eq. 12), and the modified Chakrabarty and Alexander method (Eq. 16, Eq. 18). The Panknin–Chakrabarty & Alexander (P-C) model under predicted the experimental values, whereas the Panknin–Kruglov (P-K) method overestimated the experimental values. Conversely, the modified method (P-C-M) tended to predict the magnitudes and trends of the experimental data well.

4.3 Biaxial flow curves

Finally, the biaxial stress–strain curves were calculated and compared with the measured flow curves provided by the DIC system. The results are plotted in Fig. 5a-d for AA6111-T4, AC600, DX54D+Z, and H220BD+Z, respectively. Overall, the predictions obtained with the proposed method (P-C-M) capture the trends and magnitudes of the experimental data well. In contrast, the P-C model approximates the magnitude of the data but does not predict the trend as well as the P-C-M model does. The predictions obtained with the P-K method tended to be accurate for the steel grades. The plotted data for the aluminium alloys and steel grades were approximated using Voce and power law-type equations, respectively. One bulge test for each material was used and fitted. The tensile test results for all of the materials are illustrated in Fig. 5. The presented uniaxial tensile tests are for sheets tested along the rolling direction.

The flow stress curves shown in Fig. 5 for the aluminium alloys were fitted to Voce's equation [38] form. The hardening constants (A, B, and C) obtained are listed in Table 2.

Table 2 Hardening parameters for Voce type hardening equation ($\sigma = A - B\ \exp(-C\varepsilon)$)

Approach	**AA6111-T4**			**AC600**		
	A	B	C	A	B	C
DIC	376.1	269.3	6.99	338.3	243.7	8.145
P-C	470.5	352.1	3.233	421.6	309.8	3.685
P-K	342	223	9.217	312.6	200.9	10.69
P-C-M	369	251.4	6.803	322.3	212.3	9.312
T	372	231.9	10.04	320.3	177.5	11.72

The flow stress curves shown in Fig. 5 for the steel alloys were fitted to Hollomon's [39] equation form. The strength coefficient (K) and strain hardening exponent (n) values obtained are listed in Table 3. The fitting of the aluminium and steel flow stress curves to these model forms was performed for potential use in future research. Tables 2 and 3 presented the biaxial flow curves obtained from hydraulic bulge tests. These were measured using the DIC data and calculated using the Panknin model coupled with the Chakrabarty and Alexander (P-C), Kruglov (P-K), and modified Chakrabarty and Alexander (P-C-M) methods. These tables also presented the uniaxial tensile test data (T) for sheets tested along the rolling direction. As expected, the tensile flow curve fails at a lower elongation than the biaxial flow curves. Should flow stress be required for longer elongations, the biaxial flow curve may be extrapolated using either the Hollomon or Voce equations.

Table 3 Hardening coefficients in Hollomon's equation ($\sigma = K\varepsilon^n$)

Approach	**DX54D+Z**		**H220BD+Z**	
	K	n	K	n
DIC	688.9	0.3066	713	0.2367
P-C	705.4	0.3504	770.5	0.3018
P-K	651.3	0.2707	688.8	0.219
P-C-M	663.7	0.2865	719.7	0.2524
T	546.21	0.2466	560	0.169

4.4 Stress ratios

For each material, the biaxial flow curves obtained from hydraulic bulge tests were measured using the DIC data (A) and calculated using the Chakrabarty and Alexander (P-C), Kruglov (P-K), and modified Chakrabarty and Alexander (P-C-M) methods. Therefore, for each material, four different biaxial flow curves were obtained. The numbers of samples used in the hydraulic bulge and uniaxial tests that were employed to derive the stress ratios are shown in Fig. 6. For instance, for the AA6111-T4 alloy, three uniaxial flow curves and five biaxial flow curves measured with the DIC were used to obtain 15 variations in the σ_b / σ_u ratio. As Fig. 6 shows, for certain strain ranges or plastic work ranges, the average of the variations between σ_b and σ_u was evaluated for each method, and the average ratio was then calculated (σ_b / σ_u Avg.). The average ratio multiplied by the uniaxial yield stress YS_0 is the biaxial yield stress YS_b as summarized in Tables 4 and 5. It must be stated that the utilised uniaxial tensile data are for sheets tested along the rolling direction. Moreover, the results shown in Fig. 6 correspond to the average of 15, 15, 20, and 20 calculated variations for AA6111-T4, AC600, DX54D+Z, and H220BD+Z respectively.

Table 4 The average ratios (σ_b/σ_u Avg.) and uniaxial yield stresses (YS_0) for different materials with various approaches

	Materials							
	AA6111-T4		AC600		DX54D+Z		H220BD+Z	
Approach	σ_b/σ_u Avg.	YS_0 [MPa]	σ_b/σ_u Avg.	YS_0 [MPa]	σ_b/σ_u Avg.	YS_0 [MPa]	σ_b/σ_u Avg.	YS_0 [MPa]
A	0.918	138	0.956	144	1.143	162	1.116	248
P-C	0.865		0.903		1.063		1.047	
P-K	0.9115		0.954		1.120		1.105	
P-C-M	0.906		0.951		1.117		1.093	

Table 5 The biaxial yield stresses (**YS_b**) for all the materials determined by various methods

	Materials			
Approach	YS_b −AA6111-T4 [MPa]	YS_b −AC600 [MPa]	YS_b −DX54D+Z [MPa]	YS_b −H220BD+Z [MPa]
A	126.68	137.68	184.97	278.15
P-C	119.41	130.10	172.33	259.71
P-K	125.79	137.47	182.52	273.60

P-C-M	125.06	137.03	182.04	271.11

4.5 Yield loci

To study the effect of the approach used for biaxial stress–strain curve determination on the shape of the BBC2005 yield locus, using the four different sheet alloys, the flow curves for the materials were first determined using the DIC technique in conjunction with three combinations of models, for the determination of the thickness and polar curvature at the dome apex. These experimentally measured and calculated flow curves were used to compute the biaxial yield stresses for the different approaches, for all of the materials considered in this study. This was done by adopting the approach proposed by Lee *et al.*.[34], which is based on the principle of equivalent plastic work. To compute the biaxial yield stress, the raw biaxial flow curves for any used approach, together with the uniaxial tests for the same material are employed (see section 4.4).

Fig. 7 illustrates the yield *loci* for the materials considered in this study. The results are plotted in Fig. 7 a-d for AA6111-T4, AC600, DX54D+Z, and H220BD+Z, respectively. Overall, the proposed method (P-C-M) predicts the biaxial yield stress well when compared to the one determined using the DIC technique. In contrast, the P-C model underestimates the biaxial yield stress. The P-K method predicts the biaxial yield stress with accuracy similar to that of the P-C-M method for all of the material grades. These conclusions can be taken from Table 5.

Tables 6-9 present the material coefficients a, b, L, M, N, P, Q, and R calculated using a Newton solver for the materials examined in this study.

Table 6 BBC2005 anisotropy coefficients for the aluminium alloy AA6111-T4

	Material constants							
Approach	a	b	L	M	N	P	Q	R
A	1.480789	0.606917	0.426028	0.525034	0.462843	0.466455	0.493022	0.549261
P-C	0.798241	0.685386	0.49554	0.594287	0.458535	0.465789	0.490753	0.553323
P-K	1.380706	0.616569	0.433598	0.5326	0.4624	0.466365	0.492567	0.549836
P-C-M	1.303193	0.62438	0.439905	0.538899	0.462025	0.466293	0.492235	0.550285

Table 7 BBC2005 anisotropy coefficients for the aluminium alloy AC600

	Material constants							
Approach	a	b	L	M	N	P	Q	R
A	1.043736	0.316884	0.469887	0.484009	0.465538	0.46074	0.560136	0.585803
P-C	0.577512	0.371103	0.535313	0.551638	0.463799	0.457495	0.553768	0.579774
P-K	1.028116	0.318392	0.471491	0.485663	0.465494	0.460662	0.559923	0.585590

P-C-M	0.996282	0.321516	0.474849	0.489125	0.465401	0.460499	0.55949	0.585157

Table 8 BBC2005 anisotropy coefficients for the aluminium alloy DX54D+Z

	Material constants							
Approach	a	b	L	M	N	P	Q	R
A	0.431829	0.48905	0.463888	0.433533	0.501546	0.498695	0.547105	0.53948
P-C	0.241193	0.514689	0.543878	0.517541	0.50318	0.49716	0.544893	0.537786
P-K	0.389268	0.49406	0.477531	0.447828	0.501794	0.498467	0.546748	0.539047
P-C-M	0.381273	0.495037	0.480292	0.450722	0.501845	0.498419	0.546675	0.538968

Table 9 BBC2005 anisotropy coefficients for the aluminium alloy H220BD+Z

	Material constants							
Approach	a	b	L	M	N	P	Q	R
A	0.368627	0.447045	0.494265	0.44379	0.508937	0.506312	0.548284	0.512384
P-C	0.20526	0.47114	0.57665	0.530911	0.511548	0.504868	0.546016	0.510827
P-K	0.322588	0.45302	0.512286	0.4628	0.509461	0.506052	0.547869	0.51186
P-C-M	0.299114	0.456269	0.522687	0.473779	0.509774	0.505888	0.547607	0.51161

4.6 Evaluation of the analytical methods

The analytical calculations were compared to the DIC-based calculations to evaluate their accuracy. Figures 3 to 7 show that the Panknin-Chakrabarty (P-C) method does not lead to accurate calculations of sheet thickness (Fig.3), pressure vs. strain curves (Fig.4), plastic work (Fig.6) and yield loci (Fig.7). It does however, provide reasonable agreement for flow curves (Fig.5). The Panknin-Kruglov (P-K) and the modified Panknin-Chakrabarty (P-C-M) matched the DIC-based method calculations more closely. To assess the accuracy of the P-K and the P-C-M calculations, values from Figs. 3 to 7 were extracted and compared in Table 10.

Table 10 Comparison of the P-K and the P-C-M methods against the DIC method

	a) A6111	**b) AC600**	**c) DX54**	**d) H220**
Fig. 3 (Sheet thickness) at 20mm height		**in mm**		
P-K	0.823	0.775	0.651	0.596
P-C-M	0.802	0.768	0.632	0.569
DIC	0.807	0.752	0.628	0.579
P-K error	2.0%	3.1%	3.7%	2.9%
P-C-M error	-0.6%	2.1%	0.6%	-1.7%
Fig.4 (Pressure vs. strain) at 0.2 strain		**in MPa**		
P-K	6.488	5.954	7.428	7.72
P-C-M	5.935	5.874	6.848	6.93
DIC	5.966	5.463	6.726	6.99
P-K error	8.7%	9.0%	10.4%	10.4%

P-C-M error	-0.5%	7.5%	1.8%	-0.9%

Fig.5 (Flow curve) at 0.5 strain	**in MPa**			
P-K	340	311	541	590
P-C-M	360	320	545	604
DIC	368	334	559	605
P-K error	-7.6%	-6.9%	-3.2%	-2.5%
P-C-M error	-2.2%	-4.2%	-2.5%	-0.2%

Table 10 shows that for the calculation of sheet thickness, the pressure vs. strain curves and the subsequent derivation of flow curves, the Panknin-Chakrabarty (modified) method was, in all cases, closer to the DIC-based method. No discernible error could be identified in the identification of normalised stress ratio with respect to plastic work (Fig.6) because of the noise in the data. Little difference was detected in the initial yielding of the materials (Fig.7) because negligible thinning occurs at this point. With increasing deformation, the flow curve comparison at 0.5 strain (Table 10) shows that the Panknin-Chakrabarty (modified) method becomes increasingly more accurate than the Panknin-Kruglov method. The Panknin-Chakrabarty (modified) method will, therefore, produce flow curves more suitable for modelling hot forming processes and some cold forming ones such as tyre tub components that can reach strains up to 0.5 strain. No consistent patterns were detected on the accuracy of the calculations based on material type.

5. Conclusions

This paper demonstrates the importance of anisotropy to the thinning of material during the bulge test. This was done by modifying the Chakrabarty and Alexander equation to incorporate the effects of anisotropy. When compared Kruglov's thinning solution, the modified Chakrabarty and Alexander equation showed that anisotropy becomes more important with increasing strain. As a result, thinning and flow curves were better predicted (Table 1) with the modified equation for two aluminium and 2 steel sheet materials. In the absence of a continuous and in-line thickness measurement system, the proposed method, coupled with the Panknin method, was found to be a reliable way to determine the biaxial flow curve and hence the biaxial yield stress compared to ones obtained using the DIC technique.

Compliance with ethical standards

Conflict of interest

The authors declare that they have no conflict of interest.

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Informed consent

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Figure captions

Fig. 1 Geometry of the bulge test (R = 6 mm, d = 100 mm)

Fig. 2 Methodologies used in the study for the biaxial flow curve determination

Fig. 3 Variation of polar thickness with dome height for (a) AA6111-T4; (b) AC600; (c) DX54D+Z; and (d) H220BD+Z. (A: ARAMIS, P-C: Panknin–Chakrabarty & Alexander, P-K: Panknin–Kruglov, and P-C-M: Proposed model)

Fig. 4 Variation of oil pressure with polar strain for (a) AA6111-T4, (b) AC600, (c) DX54D+Z, and (d) H220BD+Z. (A: ARAMIS, P-C: Panknin–Chakrabarty & Alexander, P-K: Panknin–Kruglov, and P-C-M: Proposed model)

Fig. 5 The uniaxial curve vs. biaxial flow curves obtained with different methods: (a) AA6111-T4, (b) AC600, (c) DX54D+Z, and (d) H220BD+Z. (A: ARAMIS, P-C: Panknin–Chakrabarty, P-K: Panknin–Kruglov, P-C-M: Proposed model, and T: Rolling direction flow curve)

Fig. 6 The $\sigma_{b/}\sigma_u$ ratio as a function of plastic strain for different methodologies: (a) AA6111-T4, (b) AC600, (c) DX54D+Z, and (d) H220BD+Z. (A: ARAMIS, P-C: Panknin–Chakrabarty and Alexander, P-K: Panknin–Kruglov, and P-C-M: Proposed model)

Fig. 7 Yield *loci* obtained with different methodologies: (a) AA6111-T4, (b) AC600, (c) DX54D+Z, and (d) H220BD+Z