Essays on Economics of Information, Contract and Experimentation

by

Wentao Fu

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Declaration

I declare that the material contained in this thesis has not been used or published before. This thesis is my own work and it has not been submitted for another degree or at another university.

Abstract

This thesis consists of three chapters. In the first chapter, I explore a two-period economy with a three-tier hierarchy in which the principal without full commitment decides how and when to motivate a productive intermediary (agent one) to privately sub-contract and collaborate with another agent (agent two) on a project with uncertain quality. The dynamic moral hazard problem arises due to the agents' hidden effort choice and the opportunity for future work. Besides free riding, the agent one's exclusion and over-investment incentives need to be considered due to his private sub-contract option. Both the dominant incentive constraint in the optimal short term contract and the principal's investment decision depend on the project's value-cost ratio, the level of synergy in the partnership and the amount of patience. In general, the principal under-invests and stops earlier compared to the first-best outcome. However, there exist scenarios in which agent one always over-invests when the individual work is motivated, and the principal might compromise to motivate a higher effort level by over-investing relative to the static game, especially if the synergy is positive but small and the project's value-cost ratio is medium. In a two-tier hierarchy, the principal can be weakly better off, but the inefficiency caused by agent one's private link to the other agent still exists.

In the second chapter, I study how a principal motivates an uninformed agent to learn about, and reveal, his quality through private experiments. The principal commits to a reward scheme and she aims to assign the rewards to correspond as closely as possible to the quality of the agent. To get a high reward, the agent experiments privately and discloses the results selectively. I show that the optimal reward scheme features an increasing step function: the initial steps encourage a potential good type agent to continue experiments

after early successes; the later steps are designed to deter a bad type agent from over-experimentation after a failure, and the scheme becomes flat when enough successes are reported. If the agent's incentives to deviate from the intended path of experimentation are weak, a one-step function is optimal: the agent receives a bonus if he reports enough successes; otherwise, he only gets a non-negative compensation. I characterise the conditions where the principal achieves the same efficiency level relative to a public information environment.

The third chapter is an extension of the second chapter. I consider a situation in which an uninformed agent persuades a principal for a high reward through costly private experimentation. I show the existence of three types of equilibria as well as their conditions: no-experiment equilibrium, separating equilibria with learning and pooling equilibria with learning. The participation threshold determines the upper bound of the entire set of equilibria, and the over-experimentation determines the boundary between the separating and pooling equilibria with learning. As the agent's value-cost ratio or prior belief increases, the set of separating equilibria with learning shrinks but the set of pooling equilibria with learning expands. Moreover, when the agent can pre-commit to report a specific number of successes to prove his quality, he tends to commit to a number that is as small as possible.

Chapter 1

Motivating Partnership in R&D Projects

1.1 Introduction

Partnering and outsourcing are prevalent when firms and experts undertake innovative projects. According to Hagedoorn (1996, 2002) and Narula and Hagedoorn (1999), there is significant growth in inter-firm R&D partnering since 1960: non-equity, contractual forms, such as joint R&D pacts and joint development agreements, have become very important, and their share in the total of partnerships has far exceeded that of joint ventures especially in high-tech industry. The collaboration among researchers and experts is even more common. Collaborators can use the synergy to boost the probability of a project's success, and improve profitability by sharing the risk and cost burdens.

An investor is willing to encourage such collaboration if the benefit from the synergistic effect is high. However, the effort level of the firm or expert is normally private, as is his network, so the investor cannot directly contract with all of his partners. Thus, the investor doesn't know whether the project is conducted by the individual work or via collaboration, in which case she might over-pay for a low level of effort. As a result, the following questions arise: How does the investor motivate the private partnership via the grand contract? when is motivating the partnership optimal? Does the private partnership distort her investment decision and if so, how?

This chapter studies a simple three-tier hierarchy in a two-period economy, in which a principal without full commitment motivates agent one (intermediary) to sub-contract with the other agent (agent two) and collaborate on a risky project. The project's quality is either good or bad which is initially unknown, and only a good project can generate a single success with positive probability after the agent(s) exert effort. The agents make a binary effort decision, and, due to the synergy, the collaboration has a higher probability of success relative to the individual attempt, given the project is good. There is no direct link between the principal and the second agent, so she can only affect agent one's private decision about partnering through the grand contract that she offers.

The dynamic moral hazard problem arises due to the unobservable choice of effort and the lack of full commitment, in which the agents could potentially manipulate the principal's belief and distort efficiency through their private learning. Given the presence of the future opportunity to work, procrastination also increases the agency cost, in which the agent(s) hold an optimistic belief relative to the principal. Moreover, since agent one's partnering decision is also private, he has more channels and a higher incentive to deviate from the principal's goal. In the first place when the principal motivates the partnership, agent one (intermediary) has a free-riding incentive and an exclusion incentive. In the free-riding incentive, he can shirk and free-ride on agent two's effort and save the cost, and agent two has the same incentive. Meanwhile, in the exclusion incentive, he can deviate to exclude the other agent and work alone, and he can enjoy the large gain alone following success, such that he can manipulate the principal's and agent two's beliefs at different level even after a failure occurs. In the second case when the principal motivates the individual work, agent one has an over-investment incentive besides shirking. By doing so, he can boost the probability of success and his expected gain in the current period. If a failure occurs, agent one might reject the second opportunity due to pessimistic belief, even if the principal is still willing to invest.

In this chapter, I characterise the principal's optimal short term contract in each period when motivating the partnership and the individual work respectively. The second period is equivalent to a static game as there is no future opportunity to work, in which the level of synergy plays a crucial role for determining the agent(s)' dominant incentive. When the partnership is motivated, agent one's free-riding incentive dominates others for the positive synergy case, and his exclusion incentive dominates for the small negative synergy case[1]. In the first period, when the partnership is motivated, the conclusion is similar to the case above if the positive synergy is very large or the synergy is negative. However, if the synergy is positive but small, in a high enough quality project with a high value-cost ratio, the free-riding incentive dominates only for a very impatient agent; for a sufficient patient agent, the exclusion incentive dominates. Moreover, there exist scenarios in which agent one's over-investment incentive is always violated when motivating the individual work, and it would further distort the principal's optimal investment decision.

The principal's optimal investment decision in the first period depends on both amount of synergy and her patience level. The presence of the dynamic moral hazard problem leads the optimal stopping threshold to be earlier and the principal to under-invest relative to the first-best scenario. To reduce to cost of deterring potential deviations, the principal would optimally under-invest by motivating a weakly lower effort level, or even shut down the window of investing in the first period. When the synergy is positive and very large, she would motivate the partnership if the quality of the project is very high or very low. If the quality is medium, only an impatient principal would motivate the partnership; otherwise, she will withhold the investment in the first period. When the negative synergy is small, the principal would motivate the partnership if the quality is very high or the quality is medium and she is very impatient; otherwise, she would only motivate the individual work if she is sufficiently impatient, and not invest if she is very patient. When the synergy is positive but small, a similar conclusion can be achieved in the most scenarios, however, there also exist scenarios in which the principal compromises to a higher effort level relative to the decision in the static game. This happens when agent one's over-investment incentive is violated in a project with a

[1]In the small negative synergy case, the partnership can still achieve a higher probability of success relative to individual work.

medium value-cost ratio. In this case, the principal compromises to motivate partnership if she is impatient, thus she over-invests relative to the static game, which is where the inefficiency arises.

In a two-tier hierarchy, the principal is better off, as she can directly contract with both of the agents and the exclusion incentive can be discarded. Due to the presence of agent one's private option of partnering, the over-investment relative to the static game still exists, as well as the inefficiency when the positive synergy is small.

The rest of the chapter is organised as follows: section 1.2 summarises related literature; section 1.3 shows the model's setup and first-best results; the main results are in section 1.4, and Section 1.5 concludes. All the proofs are in Appendix A.

1.2 Related Literature

This chapter is closely related to the literature on strategic experimentation and financing innovation. Bergemann and Hege (2005) consider optimal investing and stopping when financing an innovation project with unknown quality, when the bargaining power is in the hands of an agent (entrepreneur) with no full commitment, and Hörner and Samuelson (2013) then consider the optimal sharing rule of profit when a principal obtains the bargaining power with limited liability, given binary (fixed) investment choice from principal and effort level from the agent in each short-term contract. Compared to their work, my work also focuses on the principal's short-term contract, but in a scenario with multiple agents, in which the principal cannot directly contract with both agents and one of the agents behave as an intermediary contractor. Bonatti and Höner (2011) consider collaboration among multiple agents who receive constant and equal payment after success, where the effort level path across time at equilibrium path in the presence of free-riding, the opportunity to work tomorrow, and the assistance of a deadline are discussed. In the contrast, in this chapter as one of the agent can privately sub-contract with the other, the exclusion incentive and over-investment incentive srise. Halac, Kartik and Liu (2016) consider the single agent's dynamic moral hazard problem with a long term contract. By contrast, I consider the similar dynamic moral hazard problem in multiple agents without full commitment. Buisseret (2015) points out the existence of over-investment behaviour compared to the first-best result in a single agent setting with continuous effort choice. Compared to his model, my work shows that in the multiple agents setting, the agent's private sub-contract option could lead to a result in which the principal might motivate a higher effort level compared to the static game, but still under-invests compared to the first-best.

Since agent one plays the role of intermediary, this work is related to the literature discussing hierarchic and decentralised contracts, where efficiency is mainly considered. Melumad, Mookherjee and Riechelstein (1995) confirm the conditions for providing additional incentive in delegated contracts, Severinov (2008) compares centralized, decentralized and hierarchic structure of contract when agents obtain private information, and concludes that the optimal form depends on the degree of complementarity/substitutability between two agentss input. Faure-Grimaud and Martimort (2001) model the one period optimal incentive scheme with an intermediary and an agent with private information, where there is no direct communication between principal and agent, but they only consider the agent working on the task alone. When a supervisor is introduced to monitor the private information of the agent, Faure-Grimaud, Laffont and Martimort (2003) show that acentralized and delegated contract achieves the same outcome. In the context where intermediary work jointly with other agents, Macho-Stadler and Pérez-Castrillo (1998), Sanchez and Hortala-Vallve (2005) compare the efficiency of the two-tier and three-tier hierarchies in a moral hazard environment. These works mainly focus on static settings, whereas my work focuses on the hidden action in dynamic settings with belief manipulation, which is missing in their work.

There are still other relevant papers. Gomes (2005) considers a multilateral contracting dynamic game with externalities where a randomly chosen agent, at every period, offers contracts to an endogenously selected group of agents. Compared to his model, my work discusses the ratchet effect due to the short-term contract and the over-investment compared to a static setting due to the private sub-contract option. Watson (2013) discusses the general property of contracting institutions, but he only focuses on the static setting.

1.3 Model

1.3.1 Setup

In a two-period economy with dates 0 and 1, there is a risky project, a risk neutral principal and two risk neutral agents. The quality of the project is initially unknown and is either good or bad, and the common prior belief of being good is $p_0 \in (0,1)$. If the project is bad, it can never be successful and yields a return zero. In contrast, if the project is good, a single success with positive return $R > 0$ can be achieved with a positive probability, and this probability depends on two agents' effort. In each period t, if the project is good, the probability of success is

$$Pr(\text{Success}|\text{Good}) = \lambda(e_{1,t} + e_{2,t} + \theta e_{1,t}e_{2,t}) \tag{1.3.1}$$

Where $e_{i,t} \in \{0,1\}$, $\lambda \in (0,1)$ and $\theta \in (-1, \frac{1-2\lambda}{\lambda})$. $e_{i,t}$ is the agent i's binary effort level in period t, where $i = 1,2$ and $t = 0,1$. λ measures the individual contribution to the probability of success, and θ is the multiplier of synergy in the partnership. A positive synergy exists when $\theta \geq 0$, and the teamwork can generate a higher probability of success compared to the sum of two agents' individual contribution[2]. On the other hand, the synergy is negative when $-1 < \theta < 0$, but the probability is still higher in that partnership compared to the case when only one agent works.

There is no direct link between the principal and agent two, thus she cannot offer a contract to agent two directly. But agent one is linked to agent two in his network. I assume the principal has no full commitment. In period t, the principal publicly proposes a share of the project's return to agent one, $\omega_{1,t} \in [0,1]$. As a prime contractor, agent one decides whether to offer a share of his gain to agent two, $\omega_{2,t} \in [0,1]$, and this partnership and sub-contract are not observable by the principal. If agent two accepts it, the two agents simultaneously choose private effort level $e_{i,t}$; otherwise, agent one works alone. The cost of effort is $C(e_{i,t}) = ce_{i,t}$, and the common discount factor is δ, where $c > 0$ and $\delta \in [0,1]$. The timeline in period t is shown in Figure 1.1.

[2] $\theta < \frac{1-2\lambda}{\lambda}$ guarantees $Pr(\text{Success}|\text{Good}) < 1$.

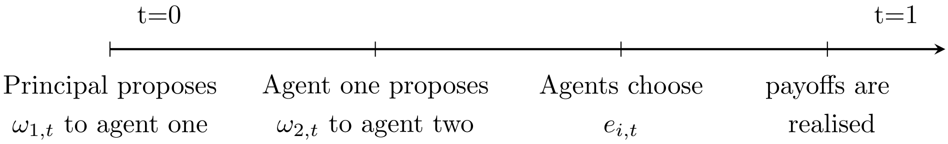

Figure 1.1: Timing in period t

The history in the first period is the null history. In the second period, $t = 1$, the public history is $h_1 = (O_0, \omega_{1,0})$, and the agent i's private history is $h_1^i = (O_0, \omega_{1,0}, \omega_{2,0}, e_{i,0})$, where O_0 is the output of the project in $t = 0$ and $O_0 \in \{\text{Success}, \text{No Sucess}\}$. The game ends if the success is achieved in the first period, and the posterior belief about the project's quality now is $Pr(\text{Good}|\text{Success}) = 1$. On the contrary, when success is not achieved, the principal is more pessimistic about the project's quality, and her posterior belief p_1^P is updated according to Bayes' rule:

$$p_1^P = \begin{cases} Pr(\text{Good}|\text{Teamwork}, \text{No Success}) = \frac{p_0[1-\lambda(2+\theta)]}{1-p_0\lambda(2+\theta)} = p_1 \\ Pr(\text{Good}|\text{One Works}, \text{No Success}) = \frac{p_0(1-\lambda)}{1-p_0\lambda} = \hat{p}_1 \end{cases} \tag{1.3.2}$$

I also denote agent i's private posterior belief by p_1^i and $\hat{p}_1^i$ respectively in the scenarios when agent two accepts the sub-contract and when he does not. Due to the linearity, this setting is equivalent to the one in which the principal and agent one offer a wage in the contracts without loss of generality. Agent i's payoff $u_t^i(p_t^i)$ and the principal's profit $\pi_t(p_t^P)$ in period t can be written as:

$$\begin{aligned} u_t^1(p_t^1) &= p_t^1(1-\omega_{2,t})\omega_{1,t}(e_{1,t} + e_{2,t} + \theta e_{1,t}e_{2,t})\lambda R - ce_{1,t} \\ u_t^2(p_t^2) &= p_t^2\omega_{2,t}\omega_{1,t}(e_{1,t} + e_{2,t} + \theta e_{1,t}e_{2,t})\lambda R - ce_{2,t} \\ \pi_t(p_t^P) &= p_t^P(1-\omega_{1,t})(e_{1,t} + e_{2,t} + \theta e_{1,t}e_{2,t})\lambda R \end{aligned} \tag{1.3.3}$$

1.3.2 First-Best Policy

In the first-best case, all actions are observable and contractable. Thus neither of the agents have a moral hazard problem, and the posteriors of the principal and the agents are consistent, $p_t^P = p_t^i$. In period t, the agent i should accept the contract and work if his continuation value $U^i(p_t^i)$ is non-negative:

$$\text{IR}_{i,t}: \quad U_t^i(p_t^i) = u_t^i(p_t^i) + \delta \left[1 - p_t^i \lambda(e_{1,t} + e_{2,t} + \theta e_{1,t} e_{2,t})\right] U_{t+1}^i(p_{t+1}^i) \geq 0 \tag{1.3.4}$$

Similarly, the principal would propose the grand contract if her continuation value $V_t(p_t^P)$ is non-negative:

$$V_t(p_t^P) = \pi_t(p_t^P) + \delta \left[1 - p_t^P \lambda(e_{1,t} + e_{2,t} + \theta e_{1,t} e_{2,t})\right] V_{t+1}(p_{t+1}^P) \geq 0 \tag{1.3.5}$$

Since the posterior belief p_t^P shrinks after the failure, the principal's continuation value shrinks as well. This implies that she would only invest in a period when her static profit from operating the project via a "partnership" or agent one's "individual work" is positive:

$$p_1^P \lambda(2+\theta)R - 2c \geq 0 \quad \text{or} \quad p_t^P \lambda R - c \geq 0 \tag{1.3.6}$$

The contacts depend on the project's the value-cost ratio and the belief about its quality, which implies that the principal would not motivate the agent(s) to operate the project as long as she is sufficiently pessimistic and the relation in (1.3.6) breaks down. Thus the socially efficient time to stop operating the project is reached when the posterior belief makes (1.3.6) binding, which is denoted by P^E. The properties of the first-best's optimal contracts $\left(\omega_{1,t}^P(p_t^P), \omega_{2,t}^P(p_t^P)\right)$ are characterised in Lemma 1.3.1.

Lemma 1.3.1. *In the first-best,* $p^E = \min\left\{\frac{2c}{(2+\theta)\lambda R}, \frac{c}{\lambda R}\right\}$ *such that, for* $p_t^P \geq p^E$*,*
1) when a partnership is motivated, $\left(\omega_{1,t}^P(p_t^P), \omega_{2,t}^P(p_t^P)\right) = \left(\frac{2c}{(2+\theta)\lambda R p_t^P}, \frac{1}{2}\right)$*; when the individual work is motivated,* $\left(\omega_{1,t}^P(p_t^P), \omega_{2,t}^P(p_t^P)\right) = \left(\frac{c}{\lambda R p_t^P}, 0\right)$.
2) in the static game at t, the partnership always dominates the individual work when the synergy is positive; when the synergy is negative, the partnership dominates the individual work if $\frac{R}{c} \geq \frac{1}{(1+\theta)\lambda R p_t^P}$*, and the individual work dominates the partnership if* $\frac{R}{c} \in \left[\frac{1}{\lambda p_t^P}, \frac{1}{(1+\theta)\lambda R p_t^P}\right)$.

In a partnership, the probability of success is higher when two agents work, and the principal gains the extra benefit by $p_t^P \lambda(1+\theta)R$. However, the cost is also higher, and she needs to pay more to the agents with c. The principal would prefer the partnership only if the extra benefit can cover the extra cost.

The static game can be considered as the period in which the principal is not going to invest anymore if the project fails, or when the principals is myopic with $\delta = 0$. In the static game at t, when the synergy is positive, $\theta \in \left[0, \frac{1-2\lambda}{\lambda}\right)$, the principal always prefers to motivate the partnership in period t if she is not sufficiently pessimistic, $p_t^P \geq p^E$. This also implies that the project's value-cost ratio is high enough, $\frac{R}{c} \geq \frac{2}{(2+\theta)\lambda p_t^P}$. Now the principal proposes a grand contract such that the prime contractor's participation constraint binds. The prime contractor, agent one, offers half of his own gain to agent two to form the partnership, and both of them work. On the other hand, when the synergy is negative, $\theta \in \left(-1, \min\left\{0, \frac{1-2\lambda}{\lambda}\right\}\right)$, the principal's choice varies at different beliefs. In a project with a high value-cost ratio, $\frac{R}{c} \geq \frac{1}{(1+\theta)\lambda p_t^P}$, for the principal, the collaboration is still more profitable compared to agent one working alone; and she prefers to have agent one working alone if the belief shrinks such that $\frac{R}{c} < \frac{1}{(1+\theta)\lambda R p_t^P}$. The principal then proposes a share of the project's return such that agent one is indifferent between working alone and rejection. The prime contractor would not propose a positive share to agent two. In a project with a medium level of the value-cost ratio with $\frac{R}{c} \in \left(\frac{1}{\lambda p_t^P}, \frac{1}{(1+\theta)\lambda p_t^P}\right]$, the partnership is dominated by to agent one working alone, as the extra benefit from the partnership is fairly low. Thus the principal would motivate agent one to work alone if she is still optimistic with $p_t^P \geq p^E$.

In the two-period economy, the principal's optimal plan at $t = 0$ is affected by her patience, and its properties in the first-best are characterised in Lemma 1.3.2, in which agent i's optimal effort level at $t = 0$ in the first-best scenario is denoted by $e_{i,0}^P$. Thus $\left(e_{1,0}^P(p_0), e_{1,0}^P(p_0)\right) = (1,1)$ and $\left(e_{1,0}^P(p_0), e_{1,0}^P(p_0)\right) = (1,0)$ represents the partnership, and the individual work is motivated at $t = 0$. In the first best scenario, since the principal would only invest in the project if her prior belief is higher than the efficient stopping threshold, this Lemma only focuses on the case when $p_0 \geq p^E$. This Lemma also implies that the principal never delays the investment by withholding the offer and only having the project be conducted at $t = 1$.

Lemma 1.3.2. *In the first-best, at t=0, for* $p_0 \geq p^E$*,*
1) when the synergy is postive, $\left(e_{1,0}^P(p_0), e_{2,0}^P(p_0)\right) = (1,1)$*;*

2) when the synergy is negative, $\left(e^{P}_{1,0}(p_0), e^{P}_{2,0}(p_0)\right) = (1,1)$ *if* $\frac{R}{c} \geq \frac{1-2p_0\lambda(1+\theta)}{p_0\lambda(1+\theta)[1-\lambda(2+\theta)]}$*;* $\left(e^{P}_{1,0}(p_0), e^{P}_{2,0}(p_0)\right) = (1,0)$ *if* $\frac{R}{c} \in \left[\frac{1}{\lambda p_0}, \frac{1}{(1+\theta)\lambda p_0}\right)$*; if* $\frac{R}{c} \in \left[\frac{1}{(1+\theta)\lambda p_0}, \frac{1-2p_0\lambda(1+\theta)}{p_0\lambda(1+\theta)[1-\lambda(2+\theta)]}\right)$, $\exists \delta^E \in (0,1)$ *such that*

$$\left(e^{P}_{1,0}(p_0), e^{P}_{2,0}(p_0)\right) = \begin{cases} (1,1) & \delta \in \left[0, \delta^E\right] \\ (1,0) & \delta \in \left(\delta^E, 1\right] \end{cases} \tag{1.3.7}$$

3) in the first-best, three-tier and two-tier contracting structures achieve the same efficiency level.

When the partnership is formed at $t = 0$, the probability of success is much higher than for the individual work, and the expected gain from the current period is boosted. At the same time, however, once no success occurs, the principal becomes more pessimistic, which lowers the expected gain in the second period. Alternatively, she can withhold or invest less at beginning. By doing so, her current expected gain now is reduced as the probability of success is lower. However, she would not be too pessimistic after the first failure, which leads the expected gain in the second period to be higher. Thus, the principal would optimally make the trade-off at $t = 0$.

Lemma 1.3.2.1) characterises the scenarios in which the principal's decision consists with her optimal choice in a static game with the same belief. When the synergy is positive, the principal would always prefer to motivate the collaboration at $t = 0$. This implies that the gain from boosting a higher probability of success is large enough to cover the loss in the future due to a pessimistic belief.

Lemma 1.3.2.2) shows that, if the synergy is negative and the project's quality is very high with $\frac{R}{c} \geq \frac{1-2p_0\lambda(1+\theta)}{p_0\lambda(1+\theta)[1-\lambda(2+\theta)]} > \frac{1}{(1+\theta)\lambda p_1}$, the principal still prefers to motivate the partnership at $t = 0$, which is consistent with her choice in the static game. This is because the project's return is very high and the principal doesn't want to delay or withhold the investment. When the synergy is negative and the project's value-cost ratio is low, $\frac{R}{c} \in \left[\frac{1}{\lambda p_0}, \frac{1}{(1+\theta)\lambda p_0}\right)$, the principal would prefer to motivate the individual work at $t = 0$, in which case the principal's extra gain from the partnership is not large enough to cover the cost of supporting a higher level of effort. As a result, she compromises on the individual work. When the project is quality is medium,

$\frac{R}{c} \in \left[\frac{1}{(1+\theta)\lambda p_0}, \frac{1-2p_0\lambda(1+\theta)}{p_0\lambda(1+\theta)[1-\lambda(2+\theta)]}\right)$, the principal would only motivate the partnership at $t = 0$ when she is sufficiently patient. If the principal insists on her best choice of a static game, the probability of success and her current static expected profit would both be higher at $t = 0$. However, if a failure arrives, her posterior belief also drops substantially, which reduces her expected profit at $t = 1$. Alternatively, she can motivate the individual work at $t = 0$, and would not be too pessimistic at $t = 1$ if a failure occurs. Her expected profit at $t = 1$ now would be larger, even though she suffers a lower current expected profit at $t = 0$. The result shows that, when the principal is patient enough, the extra gain at $t = 1$ can cover her loss at $t = 0$ by choosing the individual work at the beginning.

If the principal can directly propose contracts to both of the agents, in the first-best case, two agents would accept the contract and work if their participation constraints are satisfied. Since the moral hazard problem doesn't exist, the optimal shares that the principal proposes to the agents would also be the same as those in Lemma 1.3.2, and the socially efficient stopping time should be the same as well. As a result, the principal's profit level would be the same as that in the presence of a sub-contract.

1.4 Second-Best Analysis

Now I discuss the principal's second-best policy, in which the sub-contracting behaviour and the agents' effort level are neither observable nor contractable. In this situation, the principal can only use the grand contract to motivate agent one to form a partnership with agent two. In this section, I firstly characterise the properties of the static game, which can be treated as period $t = 1$, an then move to period $t = 0$.

1.4.1 Static Game

The static game can be considered as the last period of this two-period economy, or a scenario in which both the principal and agents are myopic and the discount factor is 0. Since the beliefs must be correct on the equilibrium path, each party now has the same posterior belief p_t^P. Given the grand contract

$\omega_{1,t}$ and the subcontract $\omega_{2,t}$, the agents' payoffs in the static game at t are shown in the Table 1.1.

		i=2 Work	i=2 Shirk
i=1	Work	$(1-\omega_{2,t})\omega_{1,t}p_t^P\lambda(2+\theta)R-c$, $\omega_{2,t}\omega_{1,t}p_t^P\lambda(2+\theta)R-c$	$(1-\omega_{2,t})\omega_{1,t}p_t^P\lambda R-c$, $\omega_{2,t}\omega_{1,t}p_t^P\lambda R$
	Shirk	$(1-\omega_{2,t})\omega_{1,t}p_t^P\lambda R$, $\omega_{2,t}\omega_{1,t}p_t^P\lambda R-c$	0, 0

Table 1.1: Agent i's payoff in the static game at t

When the partnership is desired, the free-riding incentive now arises due to private effort. To form the partnership, agent one needs to propose a share such that agent two's additional gain from working is weakly greater than that from shirking and free-riding. Given that agent two works, agent one also has the incentive to free ride on agent two's work, thus the share in the sub-contract also needs to satisfy that agent one is weakly better off by working. These conditions can be simplified as follows:

$$\begin{aligned} \overbrace{\omega_{2,t}\omega_{1,t}p_t^P\lambda(1+\theta)R}^{\text{agent two's gain from working}} &\geq \overbrace{c}^{\text{gain from free-riding}} \\ \underbrace{(1-\omega_{2,t})\omega_{1,t}p_t^P\lambda(1+\theta)R}_{\text{agent one's gain from working}} &\geq \underbrace{c}_{\text{gain from free-riding}} \end{aligned} \tag{1.4.1}$$

Thus, to support the partnership, the principal can simply add up the two constraints in (1.4.1):

$$\omega_{1,1}p_t^P(1+\theta)\lambda R \geq 2c \tag{1.4.2}$$

Since the principal cannot observe the sub-contract, agent one has extra incentives and more options to deviate from the principal's desire. Firstly, agent one could simply propose zero share to exclude agent two and work alone. In this exclusion incentive, agent one can pocket the entire gain from the grand contract, even if the probability of success is lower. To mitigate this issue, the principal needs to propose a grand contract such that the agent is weakly better off by sub-contracting to agent two rather than excluding him. It implies the following constraint:

$$\underbrace{\omega_{1,t}p_t^P(1+\theta)\lambda R}_{\text{agent one's gain from the partnership}} \geq \underbrace{\frac{2+\theta}{1+\theta}c}_{\text{gain from excluding agent two}} \tag{1.4.3}$$

Secondly, agent one has the incentive of complete outsourcing, in which he can motivate agent two to work alone by offering a share such that agent two's participation constraint of working alone is binding. By doing so, in Table 1.1, it's clear that agent one's static payoff would be the same as that when he works alone and excludes agent two. Thus agent one is indifferent between these two incentives in the static game, due to the identical individual contribution λ and operation cost c. In the following analysis, I assume agent one would work alone if he were indifferent between working alone and complete outsourcing.

On the other hand, when the principal motivates the individual work, agent one may still have the incentive to subcontract to agent two. In a static game, the principal can propose a share such that the expected gain from working alone can just cover the effort cost c. Since the sub-contracting behaviour is not observable, he can simply privately form a partnership with agent two and work together, which potentially yields a higher surplus.To prevent such deviation, the principal needs to make sure that the share guarantees that (1.4.2) doesn't hold and agent two rejects the offer, or (1.4.3) doesn't hold.

Compared to the first-best, it's now clear that the principal has to leave a positive surplus for agent one to sustain the partnership between the two agents, and her own surplus is reduced. Alternatively, the principal can just propose a share such that agent one's participation constraint from working alone is satisfied. By doing so, the principal sustains a larger share of surplus although the probability of success is lower. As the value-cost ratio remains unchanged, the principal prefers the agents' teamwork only if she is sufficiently optimistic. The lower bound of this belief is denoted by p^T, in which the principal is indifferent between the partnership and the individual work. Also, if the cost of motivating operating the project is too high, the principal stops proposing any grand contracts. This happens when the principal is so pessimistic that the share offered to agent one needs to be larger than the entire potential profit from having the agent(s) working. This belief at the stopping threshold is denoted by p^*.

Lemma 1.4.1. *In the second-best,* $p^* = \min\left\{\frac{2c}{(1+\theta)\lambda R}, \frac{c}{\lambda R}\right\} \geq p^E$*, and* $p^T = \max\left\{\frac{(3+\theta)c}{(1+\theta)^2\lambda R}, \frac{((3+2\theta)c}{(1+\theta)^3\lambda R}\right\}$ *such that, in the static game, for* $p_t^P \geq p^*$*,*
1) if $\theta \in \left[0, \frac{1-2\lambda}{\lambda}\right)$*, for* $\frac{R}{c} \geq \max\left\{\frac{2}{(1+\theta)\lambda p_t^P}, \frac{3+\theta}{(1+\theta)^2\lambda p_t^P}\right\}$*,* $\left(e_{1,t}^S(p_t^P), e_{2,t}^S(p_t^P)\right) = (1,1)$ *and* $\left(\omega_{1,t}^S(p_t^P), \omega_{2,t}^S(p_t^P)\right) = \left(\frac{2c}{(1+\theta)\lambda Rp_t^P}, \frac{1}{2}\right)$*;* $\left(e_{1,t}^S(p_t^P), e_{2,t}^S(p_t^P)\right) = (1,0)$ *and* $\left(\omega_{1,t}^S(p_t^P), \omega_{2,t}^S(p_t^P)\right) = \left(\frac{c}{\lambda Rp_t^P}, 0\right)$ *for* $\frac{R}{c} \in \left[\frac{1}{\lambda p_t^P}, \frac{3+\theta}{(1+\theta)^2\lambda p_t^P}\right)$*. Specially,* $p^* \geq p^T$ *if* $\theta \geq 1$*.*
2) if $\theta \in \left(-1, \min\left\{0, \frac{1-2\lambda}{\lambda}\right\}\right)$*, for* $\frac{R}{c} \geq \frac{3+2\theta}{(1+\theta)^3\lambda p_t^P}$*,* $\left(e_{1,t}^S(p_t^P), e_{2,t}^S(p_t^P)\right) = (1,1)$ *and* $\left(\omega_{1,t}^S(p_t^P), \omega_{2,t}^S(p_t^P)\right) = \left(\frac{(2+\theta)c}{(1+\theta)^2\lambda Rp_t^P}, \frac{1+\theta}{2+\theta}\right)$*;* $\left(e_{1,t}^S(p_t^P), e_{2,t}^S(p_t^P)\right) = (1,0)$ *and* $\left(\omega_{1,t}^S(p_t^P), \omega_{2,t}^S(p_t^P)\right) = \left(\frac{c}{\lambda Rp_t^P}, 0\right)$ *for* $\frac{R}{c} \in \left[\frac{1}{\lambda p_t^P}, \frac{3+2\theta}{(1+\theta)^3\lambda p_t^P}\right)$*.*

The optimal contracts in the static game at t, $\left(\omega_{1,t}^S(p_t^P), \omega_{2,t}^S(p_t^P)\right)$, are then summarised in Lemma 1.4.1, in which $e_{i,t}^S(p_t^P)$ is the agent i's optimal effort level at t. It shows that the level of synergy plays a crucial role for both free-riding and exclusion incentives. When the synergy is positive, the free-riding incentive dominates, and the principal can simply have (1.4.2) binding. The optimal grand contract is then offered such that agent one is indifferent between working and shirking in the partnership, and agent one would offer half of his gain to agent two. Moreover, when the positive synergy is very large, $\theta \in \left[1, \frac{1-2\lambda}{\lambda}\right)$, the principal always prefers the partnership to agent one working alone as long as operating the project is still profitable for her, $p_t^P \in \left[\frac{2c}{(1+\theta)\lambda R}, 1\right)$, which is equivalent to the case when the project has a high value-cost ratio, $\frac{R}{c} \geq \frac{2c}{(1+\theta)\lambda Rp_t^P}$. In this case, the principal only needs to transfer a small share to agent one which is lower than the cost of individual work, and then agent one would form the partnership with agent two and work hard. If the principal motivates the individual work, agent one would find it's always profitable to deviate to collaboration as the synergy is very strong.

On the other hand, when the synergy is positive but small, $\theta \in [0, 1)$, the principal would prefer the partnership only if she is sufficiently optimistic, $p_t^P \in \left[\frac{(3+\theta)c}{(1+\theta)^2\lambda R}, 1\right)$, in which case the benefit from the partnership can still cover the cost of dealing with the free-riding problem. This also implies the value-cost ratio is sufficiently high, $\frac{R}{c} \geq \frac{3+\theta}{(1+\theta)^2\lambda Rp_t^P}$. However, if the principal is pessimistic, $p_t^P \in \left[\frac{c}{\lambda R}, \frac{(3+\theta)c}{(1+\theta)^2\lambda R}\right)$, she would only motivate agent one to work alone as the cost of forming the partnership is too high. In this case, the value-cost ratio stays at the medium level, $\frac{R}{c} \in \left[\frac{c}{\lambda R p_t^P}, \frac{(3+\theta)}{(1+\theta)^2\lambda R p_t^P}\right)$.

When the synergy is negative, the exclusion incentive dominates. Agent one gains more in this deviation even if he still has the option of free-riding on agent two's hard work. Thus the principal needs to have (1.4.3) binding. To form the partnership, the principal proposes a larger share in the grand contract, and agent one would propose less than half portion in the optimal sub-contract. This is beneficial for the principal only if she is sufficiently optimistic, $p_t^P \in \left[\frac{(3+2\theta)c}{(1+\theta)^3\lambda R}, 1\right)$. If the principal is pessimistic, $p_t^P \in \left[\frac{c}{\lambda R}, \frac{(3+2\theta)c}{(1+\theta)^3\lambda R}\right)$, it's too costly to deal with the agent one's exclusion incentive. Thus the principal would only offer a smaller share to agent one to have him working alone.

The principal stops proposing grand contracts to support the project when she is very pessimistic and her belief drops below the threshold p^*. This is because the cost of motivating the agent(s) to work is too high and the principal cannot gain any positive profit. Compared to the socially efficient stopping threshold p^E in the first-best, the principal stops earlier in the second-best as $p^* \geq p^E$. This is because the principal has to leave positive surplus to agent one to mitigate potential incentives of deviation, and her own net expected profit is smaller.

1.4.2 Belief Manipulation

Now move backward to the first period, $t = 0$. All parties now have the same prior belief p_0. In contrast with the static game, now a potential future opportunity exists to conduct the project in period $t = 1$. In the environment without full commitment, due to the unobservable sub-contracts and effort level, the agents could potentially misbehave at $t = 0$ to manipulate other parties' beliefs about the project's quality, which affects the optimal contracts offered at $t = 1$. Such deviation doesn't exist once there is full commitment.

Consider agent two first. Given the principal's belief p_1^P and agent one's belief p_1^1, the principal would offer optimally offer $\omega_{1,1}^*(p_1^P)$ and agent one would optimally offer $\omega_{1,1}^*(p_1^1)$ at $t = 1$. Thus, given agent two's private belief p_1^2, he would optimally choose between rejection and acceptance at $t = 1$, and his optimal expected payoff would be:

$$\hat{u}_1^2(p_1^2; p_1^P, p_1^1) = \underset{e_{2,1}\in\{0,1\}}{Max} \left\{0, p_1^2 \omega_{2,1}^*(p_1^1)\omega_{1,1}^*(p_1^P)\left[1+(1+\theta)e_{1,1}\right] - e_{2,1}c\right\} \tag{1.4.4}$$

When agent one works alone at $t = 0$, there is no room for deviation by agent two. When the partnership is motivated at $t = 0$, on the equilibrium path, a common posterior belief is obtained by all parties, $p_1^i = p_1^P = p_1$, and agent two's expected payoff at $t = 1$ is $\hat{u}_1^2(p_1; p_1, p_1)$, which occurs with probability $1 - p_0\lambda(2 + \theta)$. However, he may reap more benefit by shirking at $k = 0$. Similarly to the static game, agent two can save the cost c and free-ride on agent one's hard work to achieve a positive gain at $t = 0$. Moreover, by doing so, he also has a more optimistic belief compared to other parties at $t = 1$ as $\hat{p}_1 > p_1$, and his optimal expected payoff would be $\hat{u}_1^2(\hat{p}_1; p_1, p_1)$ with a higher probability $1-p_0\lambda$. This implies that the free-riding incentive can additionally give agent two an additional gain at $t = 1$. As a result, the sub-contract should satisfy the following incentive constraint to have agent two working at $t = 0$:

$$\begin{aligned} IC_{2,0}^{FR}: \quad & \omega_{2,0}\omega_{1,0}p_0\lambda(1+\theta)R \\ & \geq c + \delta \underbrace{\left\{(1-p_0\lambda)\hat{u}_1^2(\hat{p}; p_1, p_1) - \left[1 - p_0\lambda(2+\theta)\right]\hat{u}_1^2(p_1; p_1, p_1)\right\}}_{\text{agent two's gain from belief manipulation by free-riding: } B_1^2(\hat{p}_1; p_1, p_1)} \end{aligned} \tag{1.4.5}$$

Compared to agent two's free-riding incentive in the static game, the extra terms are present on the side of shirking. It's clear that the gain from the belief manipulation, $B_1^2(\hat{p}_1; p_1, p_1)$, is always positive[3], so agent one needs to offers a larger share in the sub-contract to mitigate this free-riding problem, compared to the case in the static game.

Agent one has similar incentive of belief manipulation at $t = 0$, which can be achieved through more channels, as long as the principal continues investing at $t = 1$. Given the principal's belief p_1^P and the agent 2's belief p_1^2, the principal would offer optimally offer $\omega_{1,1}^*(p_1^P)$ and agent two would only accept the sub-contract if $\omega_{2,1} \geq \omega_{2,1}^*(p_1^2)$ at $t = 1$. Thus, given agent one's

[3]Since $\hat{p}_1 > p_1$, $\hat{u}_1^2(\hat{p}_1; p_1) \geq \hat{u}_1^2(p_1; p_1)$.

belief p_1^1, his expected payoff at $t = 1$ can be represented as:

$$\hat{u}_1^1(p_1^1; p_1^P, p_1^2) =$$
$$\underset{e_{1,1}\in\{0,1\}}{Max} \left\{0, e_{1,1}(p_1^1\omega_{1,1}^*(p_1^P)\lambda R - c), p_1^1(1-\omega_{2,1}^*(p_1^P))\omega_{2,1}^*(p_1^2)[1+(1+\theta)e_{1,1}]\lambda R - ce_{1,1}\right\} \tag{1.4.6}$$

It shows that agent one would optimally choose between rejection, individual work and the partnership, given his private belief.

Suppose the principal motivates the partnership at $t = 0$. At $t = 1$, given the equilibrium belief p_1, the principal would offer $\omega_{1,1}^*(p_1)$ and agent two would exert effort if $\omega_{2,1} \geq \omega_{2,1}^*(p_1)$ is offered, and agent one obtains $\hat{u}_1^1(p_1; p_1, p_1)$ at $t = 1$, which occurs with probability $1-p_0\lambda(2+\theta)$. Similarly to the static game, agent one can free-ride on agent two's hard work and save the cost. Also, if a failure occurs, agent one is more optimistic than the principal and agent two with $\hat{p}_1 > p_1$, and this information is private. From (1.4.6), it's easily to see that agent one can achieve a weakly higher expected payoff $\hat{u}_1^1(\hat{p}_1; p_1, p_1)$, and it occurs with a even higher probability $1 - p_0\lambda$. Therefore, even though agent two's current expected payoff is lower, he can receive an even higher payoff at $t = 1$ to compensate for the loss. To mitigate agent one's free-riding incentive at $t = 0$, the following incentive constraint needs to be satisfied:

$$\begin{aligned}(1-\omega_{2,0})&\omega_{1,0}p_0\lambda(1+\theta)R \\ &\geq c + \delta \underbrace{\left\{(1-p_0\lambda)\hat{u}_1^1(\hat{p}; p_1, p_1) - [1-p_0\lambda(2+\theta)]\,\hat{u}_1^1(p_1; p_1, p_1)\right\}}_{\text{agent one's gain from belief manipulation by free-riding: } B_1^1(\hat{p}_1; p_1, p_1)}\end{aligned} \tag{1.4.7}$$

This is the same to agent two's free-riding incentive constraint at $t = 0$ in (1.4.5). the last two terms on the right hand side of the inequality (1.4.7) represents the gain from belief manipulation by free-riding, $B_1^1(\hat{p}_1; p_1, p_1)$. Together with 1.4.5, the free-riding problem at $t = 0$ can be tackled when the following constraint is satisfied:

$$IC_{1,0}^{FR}: \qquad \omega_{1,0}p_0\lambda(1+\theta)R \geq 2c + \delta\left[B_1^2(\hat{p}_1; p_1, p_1) + B_1^1(\hat{p}_1; p_1, p_1)\right] \tag{1.4.8}$$

To resolve the free-riding problem at $t = 0$, the principal needs to leave a larger share to agent one, compared to that in the static game.

Secondly, agent one can manipulate other parties' beliefs by excluding agent two at $t = 0$. After no success occurs at $t = 0$, the principal holds a posterior belief $p_1^P = p_1$, and agent two believes that $p_1^2 = \hat{p}_1$. Then the principal would offer $\omega_{1,1}^*(p_1)$ and the agent 2 would accept the sub-contract if $\omega_{2,1} \geq \omega_{2,1}^*(\hat{p}_1)$ at $t = 1$. However, agent one's private belief now is $p_1^1 = \{p_0, \hat{p}_1\}$, and he is still more optimistic than both of the other parties, where his expected payoff at $t = 1$ would be $\hat{u}_1^1(p_1^1; p_1, \hat{p}_1)$ with probability $1 - p_0\lambda\tilde{e}_{1,0}$, where $\tilde{e}_{1,0}$ is agent one's potential effort choice in deviation at $t = 0$, and $\tilde{e}_{1,0} \in \{0, 1\}$. To mitigate the agent 1's exclusion incentive at $t = 0$, the principal needs to offer a contract $\omega_{1,0}$ such that the benefit from the partnership is weakly higher than the benefit from working alone or not working at all at $t = 0$. This incentive concern is summarised in (1.4.9), in which I let $B_1^1(p_1^1; p_1^P, \hat{p}_1)$ be the agent i's gain from belief manipulation by exclusion incentive, where $B_1^1(p_1^1; p_1^P, \hat{p}_1) = (1 - p_0\lambda\tilde{e}_{1,0})\hat{u}_1^1(p_1^1; p_1, \hat{p}_1) - [1 - p_0\lambda(2 + \theta)]\,\hat{u}_1^1(p_1; p_1, p_1)$:

$$
\begin{aligned}
IC_{1,0}^E : \quad & \overbrace{\omega_{1,0} p_0 \lambda (2 + \theta - \tilde{e}_{1,0}) R}^{\text{gain from the partnership}} \\
& \geq \underbrace{c(1 - \tilde{e}_{1,0}) + \frac{2+\theta}{1+\theta}\left(c + \delta B_1^2(\hat{p}_1; p_1, p_1)\right)}_{\text{gain from excluding agent two}} + \delta B_1^1(p_1^1; p_1, \hat{p}_1)
\end{aligned} \tag{1.4.9}
$$

On the right hand side of agent one's exclusion incentive constraint at $t = 0$, the first term represents the saved own cost, and the second term represents the saved cost which is paid for mitigating agent two's free-riding and belief manipulation incentive. Together with his own gain from belief manipulation by exclusion incentive, $B_1^1(p_1^1; p_1^P, \hat{p}_1)$, these imply that the principal needs to offer a larger share and leave more surplus to agent one, compared to the exclusion incentive in the static game.

Thirdly, agent one also has the option of delegating the entire work to agent two at $t = 0$. However, with Lemma 1.4.2, I show that this complete outsourcing incentive is always weakly dominated by agent one's exclusion incentive.

Lemma 1.4.2. *Exclusion incentive weakly dominates complete outsourcing.*

If agent one motivates agent two to work alone at $t = 1$, agent two's participation constraint needs to be satisfied when he is paid to cover the entire cost. Due to the identical individual contribution and cost, agent one makes the same gains as in the case when he works alone. If agent two is asked to work alone at $t = 0$, due to future opportunity to conduct the project again, he has the incentive to shirk and manipulate the agent 1's belief to gain a higher surplus at $t = 0$. This implies that agent one has to pay more than the effort cost c at $t = 0$ to have agent two working. In contrast, he can exclude agent two and work alone, in which case he just pays the effort cost c. Therefore, the following analysis can just be focused on the free-riding incentive and the exclusion incentive.

Now consider the incentives in the case where the principal motivates the individual work at $t = 0$. On the equilibrium path, all parties hold a common posterior belief $\hat{p}_1$ after no success occurs at $t = 0$. Thus the principal would optimally offer $\omega^*_{1,1}(\hat{p}_1)$ and agent two would only accept the sub-contract if $\omega_{2,1} \geq \omega^*_{2,1}(\hat{p}_1)$ at $t = 1$. This gives agent one the optimal expected payoff $\hat{u}^1_1(\hat{p}_1; \hat{p}_1, \hat{p}_1)$ with probability $1 - p_0\lambda$. From the perspective of agent one, he has the incentive to shirk at $t = 0$: by doing so, his belief remains the same and he is more optimistic than the other two parties. Thus, his optimal expected payoff at $t = 1$ would be $\hat{u}^1_1(p_0; \hat{p}_1, \hat{p}_1)$ with probability 1. This suggests that the principal needs to offer a share such that agent one is weakly better off by working alone compared to shirking at $t = 0$:

$$p_0\omega_{1,0}\lambda R \geq c + \delta \overbrace{\underbrace{\left[\hat{u}^1_1(p_0; \hat{p}_1, \hat{p}_1) - (1 - p_0\lambda)\hat{u}^1_1(\hat{p}_1; \hat{p}_1, \hat{p}_1)\right]}_{\text{gain from belief manipulation by shirking}}}^{B^1_1(p_0;\hat{p}_1,\hat{p}_1)} \tag{1.4.10}$$

Similarly to the static game, due to the unobservable sub-contract behaviour, agent one can deviate to collaborate with agent two at $t = 0$ to extract the future value. This is agent one's over-investment incentive. On the one hand, agent one has to leave a share of the return to agent two to form the partnership; on the other hand, however, he can boost the probability of success, which leads the current expected payoff to be higher. By doing so, if no success occurs, the principal still has the posterior belief $\hat{p}_1$ and offers $\omega^*_{1,1}(\hat{p}_1)$ at $t = 1$. However, agent two would have a pessimistic belief p_1 if he works at $t = 0$.

Lemma 1.4.3. *When the individual work is motivated at $t = 0$ but agent one deviates to collaborate, he would either work alone or reject the offer at $t = 1$.*

Lemma 1.4.3 suggests that agent two would not participate in the project after such deviation at $t = 0$, even if the partnership is desired by the principal. This is because both agents are more pessimistic than the principal, and the share of the return offered by the principal is not large enough to support the cost of the partnership, and it's possibly too small to support an individual work. To mitigate agent one's over-investment incentive, the following constraint must be satisfied:

$$
\begin{aligned}
&p_0\omega_{1,0}(1+\theta)\lambda R < \max\{M_1, M_2\}\\
M_1 =&2c + \delta\left\{(1-p_0\lambda)\hat{u}_1^1(\hat{p}_1;\hat{p}_1,p_1) - [1-p_0\lambda(2+\theta)]\,\hat{u}_1^1(p_1;\hat{p}_1,p_1)\right\}\\
M_2 =&\frac{2+\theta}{1+\theta}c + \delta\left\{(1-p_0\lambda)\hat{u}_1^1(\hat{p}_1;\hat{p}_1,\hat{p}_1) - [1-p_0\lambda(2+\theta)]\,\hat{u}_1^1(p_1;\hat{p}_1,p_1)\right\}
\end{aligned}
\tag{1.4.11}
$$

The first inequality, $p_0\omega_{1,0}(1+\theta)\lambda R < M_1$, suggests that the principal's grand contract cannot be too large such that agent two accepts the sub-contract and works in deviation. As agent one still can free-ride in the collaboration by deviation and holds the same belief as the principal, but is more optimistic than agent two, he gains $\hat{u}_1^1(\hat{p}_1;\hat{p}_1,p_1)$ with probability $1-p_0\lambda$. This is crucial when the synergy is positive, in which case agent two would not work given agent one shirks in the collaboration. The second inequality, $p_0\omega_{1,0}(1+\theta)\lambda R < M_2$, shows that agent one cannot achieve a better outcome by deviating from his equilibrium strategy, even if agent two accepts the sub-contract and works.

The optimal grand and sub-contracts $\left(\omega_{1,t}^*(p_t^P), \omega_{2,t}^*(p_t^P)\right)$ when motivating the partnership and the individual work respectively are summarised in Proposition 1.4.1. It shows that the last period of the two-period economy is equivalent to a static game. This is obvious since there is no future opportunity to conduct the project, and all parties' beliefs must be the same on the equilibrium path. Also, there is no room for the agent(s) to manipulate the belief. Therefore, they would behave in the same way as for the static game.

Proposition 1.4.1. *In the second-best,* $\left(\omega_{1,1}^*(p_1^P), \omega_{2,1}^*(p_1^P)\right) = \left(\omega_{1,1}^S(p_1^P), \omega_{2,1}^S(p_1^P)\right)$. $\left(\omega_{1,0}^*(p_0^P), \omega_{2,0}^*(p_0^P)\right) = \left(\omega_{1,0}^S(p_0^P), \omega_{2,0}^S(p_0^P)\right)$ *for* $\hat{p}_1 < p^*$. *For* $\hat{p}_1 \geq p^*$, *given* *the individual work is motivated at* $t = 0$*, (1.4.11) must be satisfied and* $\left(\omega^*_{1,0}(p_0), \omega^*_{2,0}(p_0)\right) = \left(\frac{c+\delta B^1_1(p_0;\hat{p}_1,\hat{p}_1)}{\lambda R p_0}, 0\right)$*; given the partnership is motivated at* $t = 0$*,* $\omega^*_{2,0}(p_0) = \frac{c+\delta B^2_1(\hat{p}_1;p_1,p_1)}{\omega^*_{1,0}(p_0)(1+\theta)\lambda R p_0}$*, and*

1) if $\theta \in \left(-1, \min\left\{0, \frac{1-2\lambda}{\lambda}\right\}\right)$*,* $\omega^*_{1,0}(p_0) = \frac{(2+\theta)c+\delta\left[(2+\theta)B^2_1(\hat{p}_1;p_1,p_1)+(1+\theta)B^1_1(\hat{p}_1;p_1,\hat{p}_1)\right]}{(1+\theta)^2\lambda R p_0}$*;*

2) if $\theta \in \left[1, \frac{1-2\lambda}{\lambda}\right)$*,* $\omega^*_{1,0}(p_0) = \frac{2c+\delta\left(B^2_1(\hat{p}_1;p_1,p_1)+B^1_1(\hat{p}_1;p_1,p_1)\right)}{(1+\theta)\lambda R p_0}$*;*

3) if $\theta \in \left[0, \min\left\{1, \frac{1-2\lambda}{\lambda}\right\}\right)$*, for* $p^T > p_1 \geq p^*$ *with* $\frac{\hat{p}_1}{p_1} \in \left(1, \frac{2}{1+\theta}\right)$*,* $\omega^*_{1,0}(p_0) = \frac{2c+\delta\left(B^2_1(\hat{p}_1;p_1,p_1)+B^1_1(\hat{p}_1;p_1,p_1)\right)}{(1+\theta)\lambda R p_0}$*; for* $p_1 \geq p^T$*, or* $p^T > p_1 \geq p^*$ *with* $\frac{\hat{p}_1}{p_1} \geq \frac{2}{1+\theta}$*,* $\exists \tilde{\delta} \in (0,1)$*, such that:*

$$\omega^*_{1,0}(p_0) = \begin{cases} \frac{2c+\delta\left(B^2_1(\hat{p}_1;p_1,p_1)+B^1_1(\hat{p}_1;p_1,p_1)\right)}{(1+\theta)\lambda R p_0} & \delta \in [0, \tilde{\delta}) \\ \frac{(2+\theta)c+\delta\left[(2+\theta)\hat{B}^2_1(\hat{p}_1;p_1,p_1)+(1+\theta)\hat{B}^1_1(\hat{p}_1;p_1,\hat{p}_1)\right]}{(1+\theta)^2\lambda R p_0} & \delta \in (\tilde{\delta}, 1] \end{cases} \tag{1.4.12}$$

At $t = 0$, for $\hat{p}_1 < p^*$, the principal's belief drops below the stopping threshold after a low effort level is motivated, thus she would not continue investing at $t = 1$ anymore. As a result, there is also no room for the agent(s) to manipulate the belief, and the concern of the incentives should be the same as those in the static game. This leads that the optimal grand and sub-contract should be the same as those in the static game.

For $\hat{p}_1 \geq p^*$, the incentives for belief manipulation need to be taken into account, due to the presence of the future opportunity. Proposition 1.4.1 suggests that, when motivating agent one's individual work, the principal needs to offer a share such that agent one is indifferent between working alone and shirking and belief manipulation, which makes (1.4.10) binding. Compared to the static game, the extra part $\frac{\delta B^1_1(p_0;\hat{p}_1,\hat{p}_1)}{\lambda R p_0}$ reflects the share which is transferred to prevent agent one from manipulating other parties' beliefs. Then agent one would not offer any sub-contracts to agent two on the equilibrium path, $\omega^*_{2,0}(p_0) = 0$. However, unlike the static game, the principal also needs to prevent agent one from seeking a higher effort level to boost the current expected gain in this case, in which (1.4.11) needs to be satisfied. This implies that there exist some scenarios in which the principal would fail to motivate the individual work at $t = 0$, and the detail is discussed in Corollary 1.4.1. It suggests that, if the individual work is desired by the principal, agent one would always deviate to a higher effort level by the partnership when the individual contribution is large, the multiplier of the synergy is sufficiently large, and when he is very optimistic and not patient. In such a scenario, agent one is very impatient such that he prefers to explore the project as much as possible and achieve success as early as possible, even if the principal is willing to wait. As a result, the principal has to compromise on a sub-optimal choice by motivating a higher effort level or withholding the investment in such cases.

For $\hat{p}_1 \geq p^*$, given the partnership is motivated at $t = 0$, the optimal sub-contract $\omega^*_{2,0}(p_0)$ must satisfy that agent two is indifferent between working and shirking, in which (1.4.5) binds. Moreover, compared to the sub-contract in the static game, the extra share $\frac{\delta B_1^2(\hat{p}_1;p_1,p_1)}{\omega^*_{1,0}(p_0)(1+\theta)\lambda R p_0}$ represents the surplus that agent one sacrifices to prevent agent two from belief manipulation. Similarly, in the grand contract $\omega^*_{1,0}(p_0)$, the principal needs to take care of agent one's free-riding and exclusion incentives, in which (1.4.8) and (1.4.9) must be satisfied. When the synergy is negative, it shows that agent one's exclusion incentive dominates, and he finds that the most profitable deviation is to exclude agent two and work alone. Thus, the extra term $\frac{\delta\left[(2+\theta)\hat{B}_1^2(\hat{p}_1;p_1,p_1)+(1+\theta)\hat{B}_1^1(\hat{p}_1;p_1,\hat{p}_1)\right]}{(1+\theta)^2\lambda R p_0}$ represents the share necessary to prevent agent one from belief manipulation by exclusion incentive. On the other hand, when the synergy is positive and very large, $\theta \in \left[1, \frac{1-2\lambda}{\lambda}\right)$, agent one's free-riding incentive dominates, in which he prefers to free ride on agent two's hard work among different channels of deviation. By doing so, the cost of the sub-contract would be less than the cost of working alone. Therefore, (1.4.8) must bind in this case, and the extra term $\frac{\delta\left(B_1^2(\hat{p}_1;p_1,p_1)+B_1^1(\hat{p}_1;p_1,p_1)\right)}{(1+\theta)\lambda R p_0}$ is paid to deter the belief manipulation.

The analysis is more complex when the synergy is positive but small, in which case $\theta \in \left[0, \min\left\{1, \frac{1-2\lambda}{\lambda}\right\}\right)$. For $p^T > p_1 \geq p^*$ with $\frac{\hat{p}_1}{p_1} \in \left(1, \frac{2}{1+\theta}\right)$, the incentive for free-riding at $t = 0$ dominates others. In this case, the principal optimally motivates the individual work at $t = 1$ if no success occurs at $t = 0$, and the difference in the posteriors at different effort levels doesn't vary too much. If agent one deviates, he would still stick to the individual work at $t = 1$ after his deviation at $t = 0$. Also, in this case, there is no room for agent two to manipulate the belief as he would not be hired at $t = 1$.

In all other cases at the small positive synergy, agent one still finds that the incentive of excluding agent two and shirking is dominated, and he is made better off by ensuring that at least one unit of effort is exerted. However, his patience level now plays a crucial role, and the share varies when deterring agent one's belief manipulation. When agent one is not patient, in which case his discount factor is very small, $\delta \in [0, \tilde{\delta})$, agent one's free-riding incentive dominates and (1.4.8) must bind. when agent one is very patient, $\delta \in [\tilde{\delta}, 1]$, the exclusion incentive dominates, and the offer should be the same as that in which the synergy is negative. These results also hold when the positive synergy is very large. In this case, the loss from shirking after excluding agent two is too large, and agent one doesn't want to withhold effort.

Corollary 1.4.1. *In the second-best, at $t = 0$,*
1) if $\theta \in \left[1, \frac{1-2\lambda}{\lambda}\right)$, the individual work cannot be motivated;
2) if $\theta \in \left(-1, \min\{\frac{1-2\lambda}{\lambda}, 1\}\right)$ with $\hat{p}_1 \geq p^T$ and $\frac{\hat{p}_1}{p_1} \geq \max\left\{\frac{2}{1+\theta}, \frac{2+\theta}{(1+\theta)^2}\right\}$, or $\theta \in \left[0, \min\{\frac{1-2\lambda}{\lambda}, 1\}\right)$ with $p^T > \hat{p}_1 \geq p^*$, $\exists \lambda_v \in (0,1)$, $\theta_v \in \left(-1, \min\left\{1, \frac{1-\lambda}{\lambda}\right\}\right)$ and $p^v \in (0,1)$ such that the individual work cannot be motivated when $\lambda \in [\lambda_v, 1)$, $\theta \in \left[\theta_v, \min\left\{\frac{1-2\lambda}{\lambda}, 1\right\}\right)$, $p_0 \in (0, p^v)$ and $\delta \in [\delta_v, 1]$, in which $c + \delta_v B_1^1(p_0; \hat{p}_1, \hat{p}_1) = \max\{M_1, M_2\}$.

Given the structure of the optimal grand and sub-contracts have been characterised, the focus shifts to the timing of motivating the partnership and the individual work. At $t = 0$, the principal's continuation value for the partnership and individual work can be written as:

$$\begin{aligned} V_0^{\mathrm{CO}}(p_0) &= p_0\left(1 - \omega_{1,0}^{\mathrm{CO}}(p_0)\right)\lambda(2+\theta)R + \delta[1 - p_0\lambda(2+\theta)]\pi_1^*(p_1) \\ V_0^{\mathrm{WA}}(p_0) &= p_0\left(1 - \omega_{1,0}^{\mathrm{WA}}(p_0)\right)\lambda R + \delta(1 - p_0\lambda)\pi_1^*(\hat{p}_1) \end{aligned} \tag{1.4.13}$$

Where $\pi_1^*(_1^P)$ is her optimal profit at $t = 1$ given belief p_1^P, and $\omega_{1,0}^{\mathrm{CO}}(p_0)$ and $\omega_{1,0}^{\mathrm{WA}}(p_0)$ represents the optimal grand contract when motivating the partnership and the individual work respectively, which are given in Proposition 1.4.1. Besides, the principal has the option to delay the investment, in which case she would only offer the grand contract at $t = 1$ and the associated continuation value would be $V_0^{\mathrm{D}}(p_0) = \delta\pi_1^*(p_0)$. By doing so, the principal can completely deter the possibility of belief manipulation, and offer a smaller share in the grand contract with a optimistic belief p_0. On the other hand, she still suffers a loss from her impatience due to the presence of the discount factor $\delta \in [0, 1]$, and also wastes the chance of making a second attempt after the first failure. The difference between the principal's continuation value for the partnership and the individual work at $t = 0$ can be represented as:

$$\begin{aligned}\triangle V_0^{\text{CW}}(p_0) =& \overbrace{p_0 \lambda R(1+\theta)}^{\text{extra gain}} - \overbrace{p_0 \lambda R \left[(2+\theta)\omega_{1,0}^{\text{CO}} - \omega_{1,0}^{\text{WA}}\right]}^{\text{extra cost for higher effort level}} \\ & - \underbrace{\delta \left\{(1-p_0\lambda)\pi_1^*(\hat{p}_1) - [1-p_0\lambda(2+\theta)]\pi_1^*(p_1)\right\}}_{\text{loss from the future profit}}\end{aligned} \tag{1.4.14}$$

Compared to the individual work, if the principal motivates the partnership at $t = 0$, she can boost the probability of success and gain the extra $p_0 \lambda R(1+\theta)$; on the other hand, she has to pay the extra cost $p_0 \lambda R[(2+\theta)\omega_{1,0}^{\text{CO}} - \omega_{1,0}^{\text{WA}}]$ to compensate the higher effort level. Also, she still suffers a loss in terms of future profit. This is because she would be more pessimistic if no success occurs after the first attempt, and has to offer a larger share to motivate the agents to work at $t = 1$, together with a lower probability of success which would lower the expected value. Similarly, compared to no investment at $t = 0$, if the principal motivates the partnership, she can have a positive static profit at $t = 0$; however, she also suffers a loss in the next period if the project fails, in which she is more pessimistic and the expected profit would be even lower. This is summarised in the expression of $\triangle V_0^{\text{CN}}(p_0)$, where

$$\triangle V_0^{\text{CN}}(p_0) = \underbrace{p_0 \left(1-\omega_{1,0}^{\text{CO}}(p_0)\right) \lambda(2+\theta)R}_{\text{net profit from the partnership}} - \underbrace{\delta \left\{\pi_1^*(p_0) - [1-p_0\lambda(2+\theta)]\pi_1^*(p_1)\right\}}_{\text{loss in the future profit}} \tag{1.4.15}$$

When the individual work is compared to no investment at $t = 0$, the logic is the same as that for the partnership, and the detail is shown in $\triangle V_0^{\text{WN}}(p_0)$, where

$$\triangle V_0^{\text{WN}}(p_0) = \underbrace{p_0 \left(1-\omega_{1,0}^{\text{WA}}(p_0)\right) \lambda R}_{\text{net profit from the individual work}} - \underbrace{\delta \left\{\pi_1^*(p_0) - [1-p_0\lambda(2+\theta)]\pi_1^*(p_1)\right\}}_{\text{loss in the future profit}} \tag{1.4.16}$$

Therefore, the principal would motivate the partnership only if its continuation value from doing so is larger than that from motivating a lower effort and by delaying the investment, in which $\triangle V_0^{\text{CW}}(p_0)$ and $\triangle V_0^{\text{CN}}(p_0)$ are both positive. A similar argument would be applied if other options are optimal. Then the principal's optimal choice of the effort level at $t = 0$ is demonstrated in Proposition 1.4.2, in which $e^*_{i,t}(p^P_t)$ is agent i's optimal effort level at t and would be motivated in the optimal grand and sub-contracts given the belief p^P_t.

Proposition 1.4.2. *In the second-best, at* $t = 0$,
1) if $\theta \in \left[1, \frac{1-2\lambda}{\lambda}\right)$, $\exists \delta^* \in (0,1)$, *such that,*

1.a) for $\frac{R}{c} \in \left[\frac{2}{p_0\lambda(1+\theta)}, \frac{2}{\hat{p}_1\lambda(1+\theta)}\right) \bigcup \left[\frac{2[1-p_1\lambda(1+\theta)]}{p_1\lambda(1+\theta)[1-\lambda(2+\theta)]}, +\infty\right)$, $\left(e^*_{1,0}, e^*_{2,0}\right) = (1,1)$;

1.b) for $\frac{R}{c} \in \left[\frac{2}{\hat{p}_1\lambda(1+\theta)}, \frac{2[1-p_1\lambda(1+\theta)]}{p_1\lambda(1+\theta)[1-\lambda(2+\theta)]}\right)$, $\left(e^*_{1,0}, e^*_{2,0}\right) = (1,1)$ *when* $\delta \in [0, \delta^*]$, *and* $\left(e^*_{1,0}, e^*_{2,0}\right) = (0,0)$ *when* $\delta \in (\delta^*, 1)$, *in which* $V^{CO}_0(p_0)|_{\delta^*} = V^{NO}_0(p_0)|_{\delta^*}$.

2) if $\theta \in \left(-1, \min\left\{0, \frac{1-2\lambda}{\lambda}\right\}\right)$, $\exists \overline{\frac{R}{c}} \geq \frac{3+2\theta}{p_0\lambda(1+\theta)^3}$ *and* $0 \leq \overline{\delta}^*_c \leq \overline{\delta}^*_w \leq 1$ *such that*

2.a) for $\frac{R}{c} \geq \overline{\frac{R}{c}}$, $\left(e^*_{1,0}, e^*_{2,0}\right) = (1,1)$;

2.b) for $\frac{R}{c} \in \left[\frac{3+2\theta}{p_0\lambda(1+\theta)^3}, \overline{\frac{R}{c}}\right)$, $\left(e^*_{1,0}, e^*_{2,0}\right) = (1,1)$ *at* $\delta \in [0, \overline{\delta}_v]$, $\left(e^*_{1,0}, e^*_{2,0}\right) = (1,0)$ *at* $\delta \in (\overline{\delta}_c, \overline{\delta}_w]$, *and* $\left(e^*_{1,0}, e^*_{2,0}\right) = (0,0)$ *at* $\delta \in (\overline{\delta}_w, 1]$; *specially, for* $\frac{R}{c} \in \left[\frac{3+2\theta}{p_0\lambda(1+\theta)^3}, \frac{1}{\hat{p}_1\lambda}\right)$, $\left(e^*_{1,0}, e^*_{2,0}\right) = (1,1)$ *and* $\overline{\delta}^*_c = \overline{\delta}^*_w = 1$;

2.c) for $\frac{R}{c} \in \left[\frac{1}{p_0\lambda}, \frac{3+2\theta}{p_0\lambda(1+\theta)^3}\right)$, $\left(e^*_{1,0}, e^*_{2,0}\right) = (1,0)$ *at* $\delta \in [0, \overline{\delta}^*_w]$, *and* $\left(e^*_{1,0}, e^*_{2,0}\right) = (0,0)$ *at* $\delta \in (\overline{\delta}^*_w, 1]$; *specially, for* $\frac{R}{c} \in \left[\frac{1}{p_0\lambda}, \min\left\{\frac{1}{\hat{p}_1\lambda}, \frac{3+2\theta}{p_0\lambda(1+\theta)^3}\right\}\right)$, $\left(e^*_{1,0}, e^*_{2,0}\right) = (1,0)$ *and* $\overline{\delta}^*_w = 1$.

3) if $\theta \in \left[0, \min\left\{1, \frac{1-2\lambda}{\lambda}\right\}\right)$, $\exists \tilde{\frac{R}{c}} \geq \frac{3+\theta}{(1+\theta)^2 p_1}$, $0 \leq \tilde{\delta}^*_c \leq \tilde{\delta}^*_w \leq 1$ *such that*

3.a) for $\frac{R}{c} \geq \tilde{\frac{R}{c}}$, $\left(e^*_{1,0}, e^*_{2,0}\right) = (1,1)$;

3.b) for $\frac{R}{c} \in \left[\frac{3+\theta}{p_0\lambda(1+\theta)^2}, \tilde{\frac{R}{c}}\right)$, $\left(e^*_{1,0}, e^*_{2,0}\right) = (1,1)$ *at* $\delta \in (0, \tilde{\delta}_c]$, $\left(e^*_{1,0}, e^*_{2,0}\right) = (1,0)$ *at* $\delta \in (\tilde{\delta}_c, \tilde{\delta}_w]$, *and* $\left(e^*_{1,0}, e^*_{2,0}\right) = (0,0)$ *at* $\delta \in (\tilde{\delta}_w, 1]$;

3.c) for $\frac{R}{c} \in \tilde{S} = \left[\max\left\{\frac{2}{p_0\lambda(1+\theta)}, \frac{1}{\hat{p}_1\lambda}\right\}, \frac{3+\theta}{p_0\lambda(1+\theta)^2}\right)$ *with (1.4.11) being violated,* $\left(e^*_{1,0}, e^*_{2,0}\right) = (1,0)$ *at* $\delta \in [0, \delta_v)$, $\left(e^*_{1,0}, e^*_{2,0}\right) = (1,1)$ *at* $\delta \in [\delta_v, \tilde{\delta}^*_c]$, $\left(e^*_{1,0}, e^*_{2,0}\right) = (0,0)$ *at* $\delta \in (\tilde{\delta}^*_c, 1]$;

3.d) for $\frac{R}{c} \in \tilde{S}$ *with (1.4.11) being satisfied, or* $\frac{R}{c} \in \left[\frac{1}{p_0\lambda}, \frac{3+\theta}{p_0\lambda(1+\theta)^2}\right) \setminus \tilde{S}$, $\left(e^*_{1,0}, e^*_{2,0}\right) = (1,0)$ *at* $\delta \in [0, \overline{\delta}^*_w]$, *and* $\left(e^*_{1,0}, e^*_{2,0}\right) = (1,0)$ *at* $\delta \in (\tilde{\delta}^*_w, 1]$; *specially, for* $\frac{R}{c} \in \left[\frac{1}{p_1\lambda}, \min\left\{\frac{1}{\hat{p}_1\lambda}, \frac{3+\theta}{p_0\lambda(1+\theta)^2}\right\}\right)$, $\left(e^*_{1,0}, e^*_{2,0}\right) = (1,0)$ *and* $\tilde{\delta}^*_w = 1$.

In general, Proposition 1.4.2 shows that the principal would optimally reduce the investment level and motivate a lower effort level at $t = 0$ due to the high cost of deterring the dynamic moral hazard problem. It follows that the optimal effort level at $t = 0$ would be lower than that in the static game. However, due to the presence of the private sub-contract, when the positive synergy is small and the principal fails to deter agent one's over-investment behaviour, she would also over-invest in these scenarios, in which the optimal effort level is higher than that in the static game but still weakly lower than that in the first-best. Compared to the first-best, the principal's optimal choice at t is distorted and she weakly under-invests at all levels of the value-cost ratio, leaving positive surplus to the agent(s) due to the hidden effort and private sub-contract.

When the project's value-cost ratio is not high enough to support the cost of a second trial after the first failure, Proposition 1.4.2 shows that the principal's optimal decision at $t = 0$ in this two-period economy is the same as that in the static game. This is obvious and the principal never delays the investment, and all parties behave as myopic players. However, when the value-cost ratio is high enough to cover the cost of the second investment after the failure, the principal's decision is distorted due to impatience and the fear of belief manipulation.

Proposition 1.4.2.1) shows that, if the synergy is very large, and the project's value-cost ratio is very high, the principal would optimally motivate the high effort level to boost the probability of success, and the benefit is much higher than the cost of deterring the agents' belief manipulation by all different channels of deviation. On the other hand, when the value-cost ratio is medium with $\frac{R}{c} \in \left[\frac{2}{(1+\theta)\lambda p_1}, \frac{2[1-p_1\lambda(1+\theta)]}{p_1\lambda(1+\theta)[1-\lambda(2+\theta)]}\right)$, her patience plays a crucial role. When she is very impatient, the cost of deterring the agents' deviation is not high from her point of view, so she would stick to motivating the high effort level through the partnership. However, when she is very patient, she weights the future value a lot, in which case the cost of deterring the agents' deviation is too high to be covered, and she would delay the investment at $t = 0$, and only invest the partnership at $t = 1$. The option of the individual work totally is excluded, in which agent one would always deviate to privately collaborate with agent two and boost the current expected gain, even if the principal is willing to sustain a lower effort level.

Proposition 1.4.2.2) and 1.4.2.3) suggest that the principal behaves similarly when the synergy is negative or positive but small. In both cases, when the project's quality is high, the principal still finds it beneficial to motivate the higher effort level since the cost of doing that is now comparatively low and could be covered by the extra gain. When the project's quality is medium and the synergy is positive but small, there exists a scenario where the principal would over-invest at $t = 0$, specifically, $p^T > p_0 > \hat{p}_1 \geq p^*$ and $p_0\lambda(2+\theta)R \geq \max\left\{\frac{2}{1+\theta}c, \frac{2+\theta}{(1+\theta)^2}\right\}$ and (1.4.11) is violated. In this scenario, the partnership is dominated by the individual work but is still better than no investment in a static game. When the principal tries to motivate the individual work at $t = 0$, which is her best choice in a static game, agent one always find that he's better off by deviating to collaborate with agent two. Thus, he cannot be deterred from deviating to privately over-invest. As a result, the principal has to compromise and distort to a higher effort level to exhaust the future possibility of investment and shut down the window of belief manipulation by the agent(s). This inefficiency arises due to the presence of the private sub-contract, and one example of this scenario is shown below. The observation ceases to exist when the synergy is negative for the same quality of the project. In the rest of the scenarios, when the project's quality is low or medium with (1.4.11) being satisfied, such distortion doesn't exist, and the principal would stick to motivating the individual work at $t = 0$ if she is very impatient, which is her optimal choice in the static game; if she is sufficiently patient, in which case she weights the cost of the deterring the dynamic moral hazard problem very high, she would withhold the investment and only invest at $t = 1$.

Example 1.4.1. $\frac{R}{c} = 15.5$, $p_0 = 0.3$, $\theta = 0.6$, $\lambda = 0.3$, $\delta = 0.9$. Thus $\hat{p}_1 = 0.2308$, $p_1 = 0.0862$, $p^T = 0.3024$ and $p^* = 0.2151$. In the first-best, $\left(e^P_{1,0}, e^P_{2,0}\right) = (1,1)$. In the static game, $\left(e^S_{1,0}, e^S_{2,0}\right) = (1,0)$. When the partnership is motivated, $\left(\omega^{\text{CO}}_{1,0}, \omega^{\text{CO}}_{2,0}\right) = (0.8961, 0.5)$ and $V^{\text{CO}}_0(p_0) = 0.377$; when she choose not to invest at $t = 0$, $V^{\text{NO}}_0(p_0) = 0.3555$. Therefore, $\left(e^*_{1,0}, e^*_{2,0}\right) = (1,1) = \left(e^P_{1,0}, e^P_{2,0}\right) \geq \left(e^S_{1,0}, e^S_{2,0}\right)$.

1.4.3 Comparisons to Two-Tier Hierarchy

Now I compare the three-tier hierarchic structure to the two-tier one, in which the principal can directly contract with agent two. Both structures are shown in Figure 1.2. The dynamic moral hazard problem can be partially mitigated but still exists. For simplicity, it assumes that the principal would only hire agent one if she motivates the individual work[4]. Agent one can be considered as the insider of a small network, and agent two is the outsider.

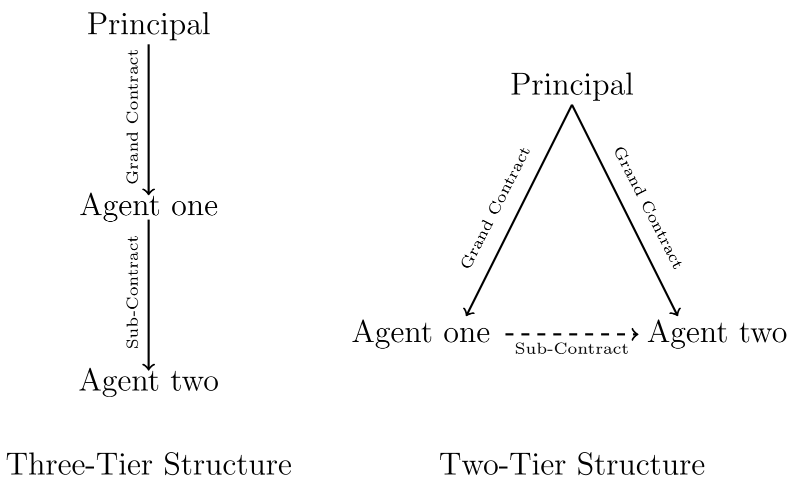

Figure 1.2: Links among players

In this environment, agent one's exclusion incentive can be dropped when motivating the collaboration. Now $\omega^*_{1,t}$ would be the total share offered to both of the agents. Since the two agents are identical, they would equally share the gain in the contract. If the synergy is positive and very large this change doesn't play a role, since the exclusion incentive is dominated by the free-riding incentive, which is the same as that in the three-tier structure. On the other hand, if the synergy is positive but small or negative, taking the exclusion incentive away can improve the principal's profitability: the exclusion-work incentive dominates and the associated constraint needs to be binding in the three-tier structure environment. Now the principal can offer a relatively smaller total share to the agents, and the stopping threshold would be the same as that in the three-tier structure with positive synergy, as well as the threshold in which the principal is indifferent between the collaboration and the individual work.

When motivating the individual work, the private sub-contract still matters. In the first place, if the link between two agents is dropped, the

[4]If this assumption is removed, the principal can randomly contract with one of the agents when motivating the individual work. It can reduce the agent's gain from the belief manipulation further since he faces an uncertainty of being offered a contract, which makes the principal better off compared to the situation with this assumption.

principal is much better off since she doesn't need to worry about the agent's over-investment incentive and whether (1.4.11) is violated or not, so the principal would not compromise to over-invest at $t = 0$. In the second place, if the link still exists, the principal still faces exactly the same consideration as she does in the three-tier structure with positive synergy. At $t = 0$, the analysis of the principal's optimal choice is the same as those in the three-tier structure environment with positive synergy and the free-riding incentive dominating.

As a result, from the perspective of the principal, the two-tier structure is weakly better than the three-tier one, and they can reach the same efficiency level when the synergy is positive and free-riding incentive dominates. From the perspective of the agents, they are weakly worse off since they have less options to deviate and the surplus they can keep is also less. The conclusion is summarised in Proposition

Proposition 1.4.3. *The principal is weakly better off and the agents are weakly worse off in a two-tier structure compared to a three-tier one.*

1.5 Conclusion

This paper analyses the principal's optimal contract and timing of motivating two agents collaborating on a risky project, in which she has no direct link to one of the agents nor full commitment. In a two-period economy with the presence of a dynamic moral hazard problem, the effect of agent one's private sub-contract on the principal's optimal decision is highlighted, as well as the comparisons between the three-tier and two-tier structure.

The principal's optimal contract for motivating the collaboration and the individual work in each period are fully characterised. Due to the presence of agent one's private sub-contract and the principal's inability to contract with agent two, if the principal motivates the collaboration, agent one's exclusion incentive dominates when the synergy is negative, or when the positive synergy is small and he is sufficiently patient. It follows that the principal has to leave more surplus to agent one to deter the deviation and belief manipulation. When the positive synergy is very large, or when it's small but agent one is impatient, the free-riding incentive dominates and the offer should be the same as that when the principal can contract directly with both agents. On the other hand, if motivating the individual work, besides the incentive of shirking, the principal also needs to deter agent one from over-investing by privately collaborating with agent two.

The principal's choice of motivating the optimal effort level in each period is also fully characterised. Without the full commitment, to deter belief manipulation by the agent(s), the principal tends to reduce the investment and motivate a lower effort level in this first period. Meanwhile, due to agent one's private sub-contract, there exist scenarios in which agent one's over-investment incentive cannot be deterred. It follows that the principal compromises to over-invest by motivating an effort level that exceeds her optimal choice in a static game, and inefficiency arises. Moreover, this result suggests that the two-tier structure is weakly better than the three-tier one.

Even though this paper only considers a two-period economy, the key properties of many periods have been covered. In future work, it would be interesting to explore the properties of a general network with more agents, in which the discussion of efficiency would help to determine the optimal network structure.

Chapter 2

Optimal Contract to Reward Private Experimentation

2.1 Introduction

Seeking and blundering are good, for it is only by seeking and blundering we learn.
— Johann Wolfgang von Goethe, *Faustus*

I study an environment in which a principal motivates an uninformed agent to learn and reveal his quality through costly and private experiments. The principal aims to assign the rewards to correspond as closely as possible to the quality of the agent, and she commits to a reward scheme before evidence is acquired. However, The agent, whose quality is initially unknown, only wants to get a high reward, and he experiments privately and discloses the results selectively. Thus the optimality of the principal's commitment arises as a key question.

For instance, consider a professor who wants to deliver a fair reference letter for an undergraduate student. Knowing that the student can privately take many exams and internships and selectively report the results, the professor can commit to only write a good letter if the student reports enough successes. Is this commitment optimal?

The reward scheme that is committed to by the principal is designed to reflect the agent's true quality, but it also affects the agent's incentives to acquire and reveal the evidence. A harsh standard in the commitment might deter the acquisition of the evidence due to a high cost of experiments, but a relaxed one may contaminate the informativeness of the evidence.

In designing the optimal reward scheme, the principal takes into account two options that the agent has to deviate from the intended path of experimentation: *early-stop* and *over-experimentation*. The early-stop incentive occurs when the total experimental cost is larger than the benefit from continuing experiments. In this case, a potential good type stops experimenting too early, to save experimental cost, and he doesn't learn enough and compromises on a lower reward level. The over-experimentation incentive emerges when he fails after he has acquired many successes. Now the agent has learned that he is a bad type. However, since the principal doesn't observe any experiments or results, the bad type agent can still continue to acquire successes and pretend to be a good type by hiding the unfavourable results in his later report. Thus, the bad type can still achieve a high reward level. Both incentives would cause a mismatch between types and rewards.

Building upon these two incentives, my first main result suggests that the optimal reward scheme is an increasing step function. When the motivated number of successes is small and the agent's incentives to deviate from the intended path of experimentation are weak, a one-step function is optimal: the agent receives the conditional expected value as a bonus if he reports enough successes; otherwise, he only gets a non-negative compensation. As this number increases, in which case the conditional expected value becomes higher than the expected cost of acquiring one more success by a bad type agent, extra steps are added after many successes have been acquired, to deter the bad type's over-experimentation behaviour, who only needs a few more successes to pretend to be a good type. These steps make a bad type agent indifferent between over-experimenting and stopping immediately. Furthermore, if the failures are not verifiable, and the agent's prior expected value cannot cover the expected total cost of the experiments, the early-stop incentive distorts the reward scheme at early stage, since the agent cannot prove that an experiment has been carried out and failed. Thus the additional steps are required to encourage a potential good type to continue the remaining experiments, which make a potential good type agent indifferent between aborting experiments and continuing. As a result, the distortion results in that a bad type agent is always over-paid relative to his true value. I also show that it's always optimal to screen the bad type agent from the potential good agent when motivating a strictly positive number of successes. Therefore, the number of motivated successes is the same as the number of experiments which are conducted without failures.

My second main result shows that the optimal motivated number of successes reported by the potential good type agent is always weakly greater than the largest number whose expected total cost can be covered by the agent's prior expected value. On the one hand, the more successes that a potential good type agent discloses, the more accurate the principal's reward could be. This force leads to a higher motivated number of experiments. However, on the other hand, when the motivated number is too large such that the conditional expected value is higher than the cost of over-experimentation, or the agent's prior expected value cannot cover the experiments' expected total cost, the principal has to sacrifice and pay more to a bad type to motivate the agent to conduct so many experiments and deter the bad type agents from over-experimenting and contaminating the informativeness of the reported successes. This force discourages the principal from motivating a large number of experiments, and the optimal number is determined by the trade-off between these two forces. Moreover, when the good type agent's value is too low or the cost of an experiment is too high, the principal would optimally motivate no experiments and assign the *ex ante* expected value to the agent.

In a public information environment where the experiments and results are publicly observed, I show that the one step function is still optimal. Moreover, when the failures are verifiable and the agent's value-cost ratio is low, the private experimentation achieves the same efficiency level as the public environment. In this case, the agent is able to prove that he has indeed conducted the experiment and failed, and the incentive for deviation from over-experimenting is weak.

I explore two extensions of the model, which show the robustness of my findings. Firstly, I introduce a small probability of bad luck in each experiment, which is privately observed and causes a failure for both types. The results suggest that this environment is equivalent to the scenario with unverifiable failures, since the failures caused by the bad luck provide no information about the agent's type but the incentive to stop at an earlier stage. Secondly, I consider a scenario with finite opportunity for experimenting, and show that the principal's optimal reward scheme is consistent with that with infinite opportunity.

The rest of the chapter is organised as follows. Section 2.2 summarises related literature. Section 2.3 shows the model's setup as well as some preliminary results and the benchmark. The main analysis of the private experimentation is demonstrated in section 2.4. Extensions can be found in section 2.5. Section 2.6 finally concludes. All proofs not shown in the main text are given in Appendix B.

2.2 Literature Review

This work relates to literature about private experimentation. Henry (2009) considers a scenario where the number of experiments is pre-determined, and the agent is not able to stop until all experiments are conducted regardless of their results. My work differs in allowing the agent to decide whether to continue after each experiment. Ispano (2015) shows the conditions such that the sender optimally reveals the unverifiable bad news. My work compares the difference when failures are verifiable and when they are not. Moreover, their work doesn't consider the receiver's optimal commitment from the perspective of mechanism design, which is the key result in my work.

The closest work is by Felgenhauer and Schulte (2014). They characterise the parameter range in which the persuasion equilibria with cut-off rule exists in costly private experimentation with symmetric information structure. In their work, the receiver makes a binary decision, and the sender applies a sanitisation strategy in which all unfavourable results are concealed. In contrast, my model considers an asymmetric information environment, and the principal offers a reward scheme according to different reported results, rather than a binary approval decision. My work follows the mechanism design approach and Delgenhauer and Schulte's does not. The cut-off function in my model is similar to their cut-off approval rule, but the interpretations are different. Also, the cut-off function is not always optimal, and the alternative step function is introduced, which is absent in their work. Other differences are that I use, and I show that, the sanitisation strategy is not optimal for the agent when failures are verifiable.

There is a large literature in strategic experimentation, such as Bergmann and Hege (2005), Halac, Kartik and Liu (2016), and Henry and Ottaviani (2014), etc. Bergman and Hege (2005) show the optimal way to finance an innovative project without full commitment, and Halac, Kartik and Liu (2016) focus on the scenario with full commitment. Henry and Ottaviani (2014) show that the principal free rides on the agent's experiments when results are public information. In most cases where results are private information, the principal or the receiver can use the timing of when they observe success to determine monetary transfer: this is a key difference from the current model, which does not include such timing.

This work also relates to literature on information disclosure and persuasion. Rayo and Segal (2010) and Kolotilin (2015) focus on the sender's optimal mechanism; Kamenica and Gentzkow (2011) find the optimal way for the sender to design the structure of the experiment, and Bergemann, Bonatti and Smolin (2015) consider a monopolist who can design the experiment and set the selling price. They all focus on public experimentation, where experimental results can be publicly observed. In contrast, this work mainly focuses on the private case, and compares the results to public case. Glazer and Rubinstein (2004, 2006) and Hart, Kremer and Perry (2016) analyse how commitment can help the principal to improve outcomes in evidence games where the agent's set of hard evidence is exogenously given and he cannot generate any other evidence. By comparison, this work considers how the optimal commitment changes if the agent can acquire hard evidence at a cost.

DeMarzo, Kremer and Skrzypacz (2017) also consider an uninformed agent who chooses one test among many different tests and strategically reveals the result to the market. In their paper, the market is competitive, and the agent has only one chance to take a test, in which the null result with positive probability is introduced and is not verifiable. By contrast, my model focuses on the softest test in which the good type always passes and the bad type passes with some probability, and the agent has infinite opportunity for experimenting. In my work, both the scenarios, when the failure is verifiable and not verifiable, are discussed. Also, my model focuses on the principal's optimal full commitment, which is absent in their paper.

This work can be compared to literature on signalling and screening, for example Spence (1973), and Rothchild and Stiglitz (1976). In their models, there is no learning process for the agent, and every type of agent can mimic the behaviour of others for a price. Here, the agent would learn his own type through experiments, and a potential good type cannot perfectly masquerade as bad type. Additionally, in this work, the agent can only be more optimistic that his type is good if no failure occurs, and full screening cannot be achieved.

Other literature that this work relates to includes Hörner and Skrzypacz (2014) and Celik (2015) who focus on gradually revealing information, and Hörner and Skrzypacz (2014) who show that sequential tests can help to mitigate the hold-up problem. Kruse and Strack (2015) focuses on mechanism design for an optimal stopping time, and a cut-off rule is proposed as optimal.

2.3 Model

2.3.1 Setup

An agent (he) wants to get reward (or evaluation) from a principal (she). Initially the agent's type[1] (value), $M_i \in \{M, 0\}$, is unknown, but a common prior is shared: type is good (G) with probability p_0, and its value is $M_G = M$, where $p_0 \in (0,1)$ and $M \in \mathbb{R}^+$; it's bad (B) with probability $1-p_0$, and value is $M_B = 0$.

The agent can learn his own type through the private experiments[2]. The constant cost of each experiment is c, where $c \in \mathbb{R}^+$. In each experiment, a good type agent can always succeed. However, a bad type can only succeed with probability $1-\theta$, where $\theta \in (0,1)$. θ can be considered as the threshold to pass a test. The experiment has the property of the softest test, where a good type always passes the test and a bad type fails it with some positive probability. Since experiments are privately observed, the agent can selectively report a subset of acquired results. Give he conducts n experiments, denote

[1]This setting is equivalent to that in which the agent sells a project with unkown quality.

[2]It can also be interpreted that there are many different tests with the same level of cost and threshold for passing.

the reported number of successes and failures by k^g and k^b respectively, where $n, k^g, k^b \in \mathbb{N}$.

Before experimentation, the principal can offer a reward scheme $a(\cdot)$ to the agent which specifies the different reward levels corresponding to different combinations of reported results by the agent, where $a : \mathbb{N} \times \mathbb{N} \rightarrow \mathbb{R}^+$. Specifically, $a(\cdot) = a(k^g, k^b)$. The principal cares about the precision and fairness of the reward, and her payoff function is: $-(a(k^g, k^b) - M_i)^2$.

The risk neutral agent only cares about the reward level, and his payoff function is $a(k^g, k^b) - nc$, given he has conducted n experiments and reported k^g successes and k^b failures. All parameters and payoff functions are common knowledge, and the timeline is shown below:

1. Principal offers the reward scheme, and agent chooses whether accept or not.
2. Agent begins to run experiments after acceptance.
3. Agent stops experiments and selectively reports to principal.
4. Payoffs are realised.

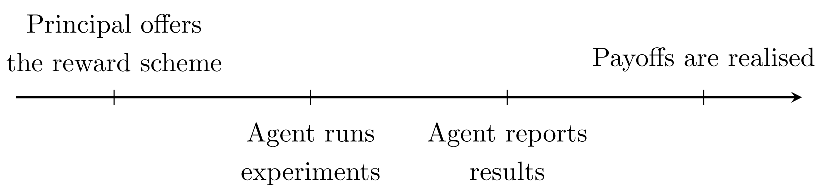

Figure 2.1: Timeline

2.3.2 Preliminaries

This section shows some preliminary results, which can help to simplify the future analysis. Now consider how the posterior belief is updated when different results are acquired. If the agent has acquired $k \in \mathbb{N}$ successes without failures, he becomes more optimistic on that his type is good, and the posterior belief denoted by p_k is updated according to Bayes' rule:

$$Pr(\text{Good}|k, 0) = \frac{p_0}{p_0 + (1 - p_0)(1 - \theta)^k} = p_k \tag{2.3.1}$$

In this case, the agent has no reason to hide the successes. This is because, on the equilibrium path, the more successes are reported the higher the posterior belief the principal has, which leads a higher expected value. Instead, if the agent has acquired at least one failure, he learns that he is a bad type since only the bad type can fail with positive probability in an experiment. Thus, no matter how many successes he has achieved before, his posterior belief now is $Pr(G|k^b \geq 1) = 0$.

Since one failure is enough for the principal to learn the agent is bad with value $M_B = 0$, the principal needs not to provide any incentives for the bad type agent to reveal more than one failures. On the one hand, more failures don't affect the belief if the principal has observed one failure, and the precision of the reward is not affected. On the other hand, If the agent continues to experiment after the first failure, he either gets more failures or more successes. Thus he can hide failures and report more successes to achieve a higher reward level. Since the success is verifiable, the only way for a bad type to pretend to be good is to over-experiment—continuing experiments to acquire successes by luck after his first failure arrives. On expectation, a bad type agent can achieve one more success with cost $\frac{c}{1-\theta}$. To mimic the good type's behaviour, the bad type agent's selective report satisfies: $k^g + k^b \leq k$. This is the agent's over-experimentation incentive, which would be discussed later in details, as well as the verifiability of failures.

Notice that the principal's commitment always motivates the potential good type agent, who hasn't failed yet, to reveal a certain number of successes, therefore, to determine the optimal commitment, the question can be decomposed into the following two parts: firstly, given the principal motivates the potential good agent to reveal a arbitrary k successes, the properties of the associated reward scheme, $a^k(k^g, k^b)$, needs to be characterised; secondly, the optimal number k^* then can be found, which maximised the principal's *ex ante* expected payoff. Thus, the optimal commitment is the optimal reward scheme $a^{k^*}(k^g, k^b)$, and k^* successes are motivated to be reported by the potential good agent.

Claim 2.3.1. *$a^0(\cdot) = p_0 M$.*

When $k = 0$, no successes are motivated to be reported, and claim 2.3.1 suggests that the principal would optimally assign a single reward level to the agent regardless of the agent's report, which equals the agent's prior expected value. The agent would not conduct any experiments on the equilibrium path, and no further information about the agent's quality is provided.

When $k \in \mathbb{N}^+$, a single reward level cannot be optimal, otherwise the agent can easily deviate to conduct no experiments and gain the same reward as those who report some successes. Some features associated with the optimal reward scheme can be summarised in Lemma 2.3.1.

Lemma 2.3.1. *Given the commitment motivates the potential good agent to report $k \in \mathbb{N}^+$ successes, the optimal associated reward scheme is equivalent to the reward scheme with: for $\forall k^g, k^b \in \mathbb{N}$,*
1) $a^k(k^g, 1) \geq a^k(k^g, k^b > 1)$;
2) $a^k(k^g \geq k, k^b \geq 1) = 0$;
3) $a^k(k^g \geq k, 0) = a^k(k, 0)$.

Lemma 2.3.1.1) states that it is optimal to assign at most the same level of reward to those reporting more failures, since one failure is enough for the principal to learn that the agent is a bad type and more failures don't affect the principal's posterior belief on the agent's type. Given the reported number of successes, the principal punishes the bad type agent who has more failures by assigning a lower reward level. Lemma 2.3.1.2) pushes the punishment to the maximum when an agent reports more than enough successes together with some failure(s). The principal learns the agent is bad when observing failures, and she also learns that the bad type has over-experimented to acquire so many successes after his first failure. This punishment is just the bad type's true value, which prevents such behaviour from occurring when a contract is accepted. Thus, the bad type agent would conceal failures when pretending to be a potential good one.

Lemma 2.3.1.3) shows that, to guarantee that an agent's ex ante optimal plan is to reveal enough successes k given the reward scheme from principal, the marginal benefit from running one more trial is strictly less than the marginal cost after continuously obtaining k successes, and it must also be true for all numbers greater than k. If not, the agent would continue experimenting as long as no failure occurs. For any other reward scheme satisfying such criteria, it would lead to the same end as the one with assigning the same level of reward to those reporting more than enough successes without failure. Therefore, the remaining analysis can focus on $a^k(k, 0)$, $a^k(k^g < k, 0)$ and $a^k(k^g < k, 1)$ only.

2.3.3 Benchmark: Public Experimentation

Before solving the main model, I discuss a benchmark to provide useful background and intuition, in which the experiments are conducted publicly and the results are observable by both parties. If a failure occurs, the principal learns immediately that the agent is a bad type, and any further experiments don't affect the principal's belief or improve the precision of the reward. The agent cannot hide any failures, or over-experiment to pretend that failure never occurs.

When the principal's commitment motivates the potential good agent to acquire k successes, the agent's expected payoff is:

$$U_P^A(k, p_0) = -c + [p_0 + (1-p_0)(1-\theta)]\, U_P^A(k-1, p_1) + (1-p_0)\theta a^k(0,1) \quad (2.3.2)$$

Denote by $U_P^A(k-j, p_j)$ the agent's continuation value of acquiring the remaining $k-j$ successes with current belief p_j, where $j \leq k$. After paying the cost c in the first experiment, with probability $p_0 + (1-p_0)(1-\theta)$, the agent succeeds and becomes more optimistic with a posterior belief p_1, and his continuation value of acquiring the remaining $k-1$ successes is $U_P^A(k-1, p_1)$; with probability $(1-p_0)\theta$, the agent fails, and then he learns that his type is bad, and receives a reward level $a^k(0,1)$. The agent's *ex ante* expected payoff can also be simplified as:

$$U(k, p_0) = \mathbb{E}\left(a^k(k^g, k^b) \middle| k, p_0\right) - \tilde{k}c, \quad \text{where } \tilde{k} = \sum_{i=1}^{k} \frac{p_0}{p_{i-1}} \quad (2.3.3)$$

where $\tilde{k}$ is the expected number of experiments that the agent would run after accepting the contract, and it equals to the summation of the likelihood ratio of prior to posterior beliefs. Thus $\tilde{k}c$ is the *ex ante* expected total cost[3].

[3] In this plan, the agent only needs to continue experimenting when no failure occurs, thus it's easily to see the probability that no failure occurs in k experiment is $p_0 + (1-p_0)(1-\theta)^k$, and the probability that first failure occurs in j_{th} experiments is $(1-p_0)(1-\theta)^{j-1}\theta$. Therefore, the expected total cost is: $\left[p_0 + (1-p_0)(1-\theta)^k\right] kc + \sum_{i=1}^{k}(1-p_0)(1-\theta)^{i-1}\theta(i-$

Following this plan, the agent's *ex ante* expected gain[4] is $\mathbb{E}\left(a^k(k^g, k^b)\middle|k, p_0\right)$. The agent would accept the contract and run experiments as long as Individual Rationality (IR) is satisfied, and his *ex-ante* expected payoff is non-negative. Thus the principal maximises her expected payoff conditional on the prior belief and the number of experiments provided incentives to run:

$$\begin{aligned} &\underset{a(\cdot)\in R^+}{Max} && V(k,p_0) = \mathbb{E}\left(-\left(a^k(\cdot) - M_i\right)^2 \middle| k, p_0\right) \\ &\text{s.t}: && \text{IR}: \ U_P^A(k,p_0) \geq 0 \end{aligned} \tag{2.3.4}$$

Proposition 2.3.1. *In public experimentation:*
1) Given the commitment motivates the potential good agent to report $k \in \mathbb{N}^+$ successes, the optimal associated reward scheme is a one-step function at k (OF), specifically:

$$OF = \begin{cases} a^k(j<k,1) = \max\left\{0, \tilde{k}c - p_0 M\right\} \\ a^k(k,0) = p_k M + \max\left\{0, \tilde{k}c - p_0 M\right\} \end{cases} \tag{2.3.5}$$

2) The optimal number k^P satisfies $\overline{k} \leq k^P < \infty$, where

$$\overline{k} = \begin{cases} \max\{k \in \mathbb{N} : p_0 M \geq \tilde{k}c\} & p_0 M \geq c \\ 0 & p_0 M < c \end{cases}$$

Given the principal motivates the potential good agent to report $k \in \mathbb{N}^+$ successes, the optimal reward scheme is a one-step function (OF), and it can be interpreted as follows: the principal commits that the agent gets a high reward level if reporting weakly more than k successes without failures, and he is treated as a bad type if reporting strictly less successes with one failure. The agent then runs experiments after accepting the contract, and reports all results he acquires. Thus when any less than k successes are reported, there is a failure associated.

The participation threshold $\overline{k}$ is now introduced, which is the largest number of experiments whose expected total cost can be covered by the agent's

1)$c = \sum_{i=1}^{k}\left[p_0 + (1-p_0)(1-\theta)^{i-1}\right] c = \sum_{i=1}^{k} \frac{p_0}{p_{i-1}} c$.

[4] $\mathbb{E}\left(a^k(k^g, k^b)\middle|k, p_0\right) = [p_0 + (1-p_0)(1-\theta)^k]a^k(k,0) + \sum_{i=0}^{k-1}(1-p_0)(1-\theta)^i\theta a^k(i,1)$.

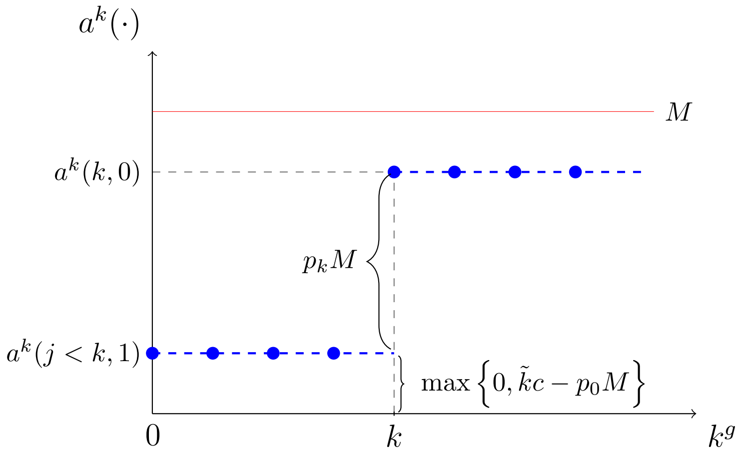

Figure 2.2: OF in public experimentation

prior expected value. When the prior expected value is very low, $p_0M < c$, it can cover the cost for at least one experiment, in which case $\overline{k} = 0$. When the prior expected value is high, $p_0M \geq c$, the threshold equals to the round down of the number of experiments such that the prior expectation equals to the expected total cost[5], $p_0M = \tilde{k}c$. When the incentivised number in the contract is smaller than this threshold, $k < \overline{k}$, the agent is willing to carry out the principal's desired plan since the prior expectation of agent's value is higher than the *ex ante* expected total cost, $p_0M \geq \tilde{k}c$. Then the principal can simply assign the bad type's true value to those facing early failures, and the posterior expectation, p_kM, to those who have achieved the desired number of successes without failure. These reward levels are the same as the agent's self evaluation after learning through experiments.

If the principal incentivises more than the participation threshold, $k > \overline{k}$, she needs to take care of the agent's individual rationality, IR, to guarantee the agent's acceptance of her contract in the first place. *Ex ante*, it's too costly to achieve the desired number of successes without failures, $\tilde{k}c > p_0M$. Thus it becomes optimal to assign a positive reward level to a bad type reporting one failure, which is just equal to the excess cost—the difference between the expected total cost and the prior expectation. This can be treated as a reward

[5]If k is continuous, the threshold number would be the one such that the prior expectation just equals to the total expected cost. Due to the discreteness of experiment, the threshold needs to be rounded down.

for audacity, or compensation for fear of failure, even if the agent's type turns out to be bad. If the exact k successes are acquired without failure, the agent gains a bonus which is equal to the posterior expectation, $p_k M$. It captures the feature that the principal is risk averse, and she optimally reduces the risk of making a mistake—assigning weakly a higher level of reward to every *ex post* scenario. This is equivalent to the principal paying the excess cost up front when agent accepts the contract, and incentivise him to collect the bonus if the desired number of successes are obtained without failure.

In the public experimentation, the principal optimally motivates the potential good agent to report k^P successes, which is always weakly greater than the first threshold $\overline{k}$. On the one hand, a higher reward is closer to the true valuation of good type, which is the gain in the *ex post* scenario where the agent's type is good; on the other hand, the loss also increases in *ex post* scenarios where the agent's type is bad. Therefore, the principal is only willing to push k^P as large as possible if the gain can cover the loss. This k^P must be finite, since the good type's value is finite and increasing required successes without failure would raise losses in every *ex post* scenario beyond the number whose associated reward is closest to the good type's value. A numeric example is shown below.

Furthermore, Proposition 2.3.1.2) also implicitly shows that it could be optimal for the principal to motivate no experiments and only assign the single reward to the agent. It happens when the good agent's value-cost ratio $\frac{M}{c}$ is too low, in which case it's too costly to motivate the any positive amount of experiments: the gain from a more accurate reward for the good agent cannot cover the loss from over-paying the bad type agent.

Example 2.3.1. When $\theta = 0.6$, $p = 0.4$, $M = 3$ and $c = 1$, then the participation threshold number is $\overline{k} = 1$. In public experimentation, the optimal number is $k^P = 2$ and its associated reward scheme is CF $= \begin{cases} a^2(2,0) = 2.86 \\ a^2(j < 2, 1) = 0.44 \end{cases}$ and the principal's expected payoff is $V^P(2, p_0) = -0.89$.

2.4 Analysis in Private Experimentation

In private experimentation, experiments and results are privately observed by the agent, thus extra incentives must be provided in the principal's commitment.

Given $j < k$ successes are acquired without failures in the first j experiments, the agent now is more optimistic that his type is good with posterior p_j, and his continuation payoff of continuing experiments would be:

$$U(k-j, p_j) = -c + \frac{p_0}{p_j} U(k-j-1, p_{j+1}) + (1-p_j)\theta \max\left\{a^k(j,0), a^k(j,1)\right\} \tag{2.4.1}$$

If continuing one experiment, by paying the cost c, he gets one more success with probability $\frac{p_0}{p_j}$ and his continuation payoff would be $U(k-j-1, p_{j+1})$; with probability $(1-p_j)\theta$, he fails and his posterior belief of being good type drops to 0. In this case, the bad type agent gets $U(k-j, 0)$, where

$$U(k-j, 0) = \max_{n \in \mathbb{N}^+,\, j \leq n \leq k} \max\left\{a^k(n,0), a^k(n,1)\right\} - (n-j)\frac{c}{1-\theta} \tag{2.4.2}$$

Now the bad type agent privately learns his type is bad, which the principal doesn't observe. He only cares about whether the future experiments can generate a net benefit since the cost of previous experiments is sunk. On expectation, when paying the cost $\frac{c}{1-\theta}$, he can still acquire one more success. Thus he can over-experiment to get more successes or even continue doing so until k successes are acquired. Depending on the reward scheme, he decides to reveal or not reveal the failures. This is called over-experimentation incentive. If the bad type is doing so, the report is less informative.

After j successes without failures, if the potential good type decides to stop, he would reveal all the successes and get $a^k(j,0)$ when the failure is verifiable, or make a fake failure and get $\max\left\{a^k(j,0), a^k(j,1)\right\}$ when the failure is not verifiable. In this deviation, even though the reward is lower, the potential good agent can save the cost of experiments. Thus the agent's incentive of early-stop rises. To motivate the potential good type agent to continue and fulfil the initial plan, the reward scheme must guarantee that the continuation payoff $U(k-j, p_j)$ is higher than that of early-stop, which demonstrates the agent's first type of incentives constraints IC_j^S:

$$IC_j^S: \quad U(k-j, p_j) \geq \begin{cases} a^k(k,0) & \text{verifiable failure} \\ \max\left\{a^k(j,0), a^k(j,1)\right\} & \text{unverifiable failure} \end{cases} \tag{2.4.3}$$

Lemma 2.4.1. *When motivating the potential good agent to report $k \in \mathbb{N}^+$ successes, the optimal associated reward scheme satisfies: for $0 \leq j < k$,*

$$\max\left\{a^k(j+1,1), a^k(j+1,0)\right\} - \max\left\{a^k(j,1), a^k(j,0)\right\} \leq \frac{c}{1-\theta} \tag{2.4.4}$$

When a bad type agent's over-experimentation incentive is taken into account, Lemma 2.4.1 suggests that, when motivating a positive number of successes to be reported, the associated optimal scheme always separates the bad type agent from the potential good type one. The principal prefers to deliver the bad type agent a reward that is as low as possible, but it also creates a high incentive of over-experimenting for those who just need a few successes to pretend to be the potential good ones. Compared to a reward scheme which makes bad type agents who fail late pooling with the potential good type ones, the principal can always construct a profitable deviation, in which the potential good agent receives the same as before but the bad type agents are now at most indifferent between over-experimenting and not. As a result, the principal makes strictly less loss in this deviation. The constraints in Lemma 2.4.1 are now the second type of incentive constraints that are designed to deter the bad type agent's over-experimentation incentive, IC_j^F, in which the extra gain from over-experimentation is less than the extra expected cost of doing so. As a result, the number of motivated successes, which are reported by the potential good agent, equals to the number of motivated experiments that the agent needs to conduct without failures.

Given k successes are motivated to be reported by the potential good agent, there are $k-1$ early-stop incentives constraints IC^S, and $k-1$ over-experimentation incentive constraints IC^F. Together with IR, the principal faces $2k-1$ constraints.

2.4.1 Verifiable failures

The verifiability of failures plays a crucial role in determining the optimal contract offered by the principal. When it's verifiable, it's the hard evidence to prove that the agent indeed has paid the cost, conducted the experiment and failed. When it's not verifiable, the such conducted experiment cannot be proved, and the principal cannot distinguish the agent who fails from those haven't undertaken it. This section focuses on the scenario in which failures are verifiable. The scenario with unverifiable failures is discussed later in Section 2.4.2.

Lemma 2.4.2. *In private experimentation with verifiable failures, for $k^g < k$,*

$$a^k(k^g, 1) \geq a^k(k^g, 0) = 0 \tag{2.4.5}$$

When the failures are verifiable, the agent's must be a bad type if less than the required number of successes are reported without failures since the principal provides enough incentives to the potential good type to continue experimenting after early success. A reward for honesty can be created by assigning a weakly higher reward level to those who report a failure. As the largest punishment to those who pretend to face a failure, the principal then can simply assign the bad type's true value, $a^k(j < k, 0) = 0$, if "no failure presented" when fewer successes are reported. Thus IC^S and IC^F can be simplified as:

$$\begin{aligned}
&\mathrm{IC}^{S,V}_{0\leq j\leq k-1}: && U(k-j, p_j) \geq 0 \\
&\mathrm{IC}^{F,V}_{0\leq j<k-1}: && a^k(j+1,1) - a^k(j,1) \leq \frac{c}{1-\theta} \\
&\mathrm{IC}^{F,V}_{k-1}: && a^k(k,0) - a^k(k-1,1) \leq \frac{c}{1-\theta}
\end{aligned} \tag{2.4.6}$$

Thus, this simplification shows that IR is the same as $\mathrm{IC}^{S,V}_0$. After the success in the first trial, the agent is more optimistic, and he is willing to carry out the remaining experiments. Thus, as long as IR is satisfied, $\mathrm{IC}^{S,V}_{1\leq j\leq k-1}$ must be slack, in which the early-stop incentive can be discarded.

These constraints demonstrate that the associated optimal reward scheme must share the property of screening: a bad type would not blend into a potential good type, and the potential good type would not pretend to be bad. However, full screening cannot be achieved here. This is because the agent can only be more optimistic after more successes are achieved without failure and treat himself as a potential good type, but he cannot be sure whether he is a good type or a lucky enough bad type who hasn't failed yet.

Proposition 2.4.1. *In private experimentation with verifiable failures,*
1) Given the commitment motivates the potential good type agent to report $k \in \mathbb{N}$ successes, the associated optimal reward scheme $a^k(\cdot)$ is:

a) CF when $k \leq \hat{k}$, where $\hat{k} = \begin{cases} \max\{k \in \mathbb{N} : p_k M \leq \frac{c}{1-\theta}\} & p_1 M \leq \frac{c}{1-\theta} \\ 0 & p_1 M > \frac{c}{1-\theta} \end{cases}$*;*

b) Type I multi-step function (MF-I) when $k > \hat{k}$;

2) The optimal number k_V^* satisfies $\overline{k} \leq k_V^* < \infty$. Especially, $k_V^* = k^P$ if $\hat{k} \geq k^P$.

Here the over-experimentation threshold $\hat{k}$ is introduced, which measures the largest number of reported successes where the agent's conditional expected value is weakly smaller than the expected cost of a bad type achieving a success. Proposition 2.4.1.1.a) shows CF, the optimal cut-off reward scheme in public experimentation, is still optimal when the motivated number of experiments is low, and the over-experimentation threshold $\hat{k}$ determines the scope of CF in private experimentation with verifiable failures.

The demonstration of the optimal reward scheme at different value-cost ratio can be summarised in Figure 2.4. When the agent's value-cost ratio is low, $\frac{M}{c} \in \left[0, \frac{1}{1-\theta}\right]$, $\hat{k} \to \infty$, the over-experimentation threshold $\hat{k}$ doesn't affect the optimality of CF. Now optimal commitment is exactly the same as that in public experimentation. Intuitively, this happens when the good type is not superior enough or the experimental cost is relatively high. Also, since the failures are verifiable, once the commitment is made and experiments are carried out, all reports which have less than the required level of successes must contain a failure, and the agent would disclose all of the information he acquires.

When the agent's value-cost ratio is medium, $\frac{M}{c} \in \left(\frac{1}{1-\theta}, \frac{1}{(1-\theta)p_1}\right]$, $0 < \hat{k} < \infty$, in which case the difference between the good and bad type's values are not too large, or the cost of a single experiment is relatively low. Now the over-experimentation threshold equals to the rounded down number of reported successes where the agent's conditional expected value equals to the expected cost of a bad type achieving a success, $p_k M = \frac{c}{1-\theta}$. When the motivated number of experiments is lower than the over-experimentation threshold, $k \leq \hat{k}$, and the optimal commitment in public experimentation could still be optimal in private with verifiable failures, when the over-experimentation threshold is sufficiently high, $\hat{k} \geq k^P$. However, if the motivated number of experiments is too large, $k > \hat{k}$, CF cannot be applied. Consider the following situation when the first failure occurs at the k_{th} experiment. If the principal still sticks to CF, the agent would definitely over-experiment since such behaviour would lead to an extra gain $p_k M$ which can cover the expected cost of acquiring one more success for the bad type. As a result, the incentive constraint $\text{IC}^{F,V}_{k-1}$ is violated and report becomes less informative. Therefore, alternative reward schemes need to be considered, and the optimal one among them is a type-I multi-step function (MF-I), which is proposed in Proposition 2.4.1.1.b).

Definition 1. Type I multi-step function (MF-I) is a reward scheme such that $\text{IC}^{F,V}_{l \leq j \leq k-1}$ are all binding, where $l = \max\{l \in \mathbb{N} : p_0 M \geq \sum_{i=l+1}^{k} \frac{p_0}{p_i} \frac{c}{1-\theta}\ \&\ 0 \leq l \leq k-1\}$:

$$\text{MF-I} = \begin{cases} a^k(j < l, 1) = \max\left\{0, \tilde{k}c - p_0 M\right\} \\ a^k(l, 1) = p_l M - \sum_{i=l+1}^{k} \dfrac{p_l}{p_i} \dfrac{c}{1-\theta} + \max\left\{0, \tilde{k}c - p_0 M\right\} \\ a^k(l < j < k, 1) = (j-l)\frac{c}{1-\theta} + a^k(l, 1) \\ a^k(k, 0) = (k-l)\frac{c}{1-\theta} + a^k(l, 1) \end{cases} \tag{2.4.7}$$

The structure of MF-I demonstrates the feature of "setting blocks at the end": the difference of rewards between rewards among a neighboured number of reported successes cannot exceed $\frac{c}{1-\theta}$, which is the cost level of acquiring a success for a bad type. For those failures which occurs early enough, that is before l_{th} experiments, it's still too costly to over-experiment: the agent is then happy to stop and report what he acquires, and the reward level could be same as those in CF at the same level since failures are verifiable. Notice that, the later the first failure occurs, the stronger the incentive for the agent to over-experiment. Thus it's optimal to have $\text{IC}^{F,V}_{l \leq j \leq k-1}$ be all binding. As a result, not only are bad types overpaid, but also they are treated differently when the first failure occurs in different experiments: the later the failure, the higher is the reward associated. Meanwhile, it's easy to see that when the required number of successes k are reported, the associated reward level for a potential good type in MF-I is strictly lower than that in CF[6]. One example of MF-I's structure can be found in Figure 2.3.

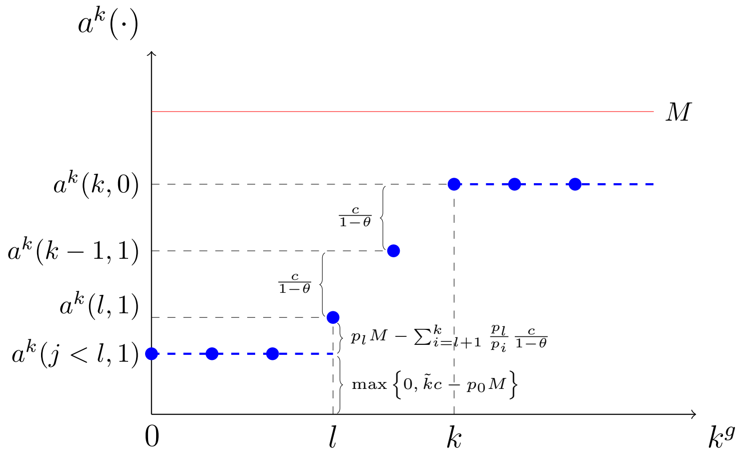

Figure 2.3: MF-I in private experimentation (Example: $k > 2$ and $l = k - 2$)

If the agent's value-cost ratio is sufficiently high, $\frac{M}{c} \in \left(\frac{1}{(1-\theta)p_1}, \infty\right)$, in which case the good type is much better than the bad type or the cost of a single experiment is too low, the agent would over-experiment at any positive number of reported successes, where $\hat{k} = 0$. This leads to all IC^{F,V} are binding. As a result, the potential good type is still underpaid relative to the agent's self-evaluation, but the bad type is overpaid.

Under the optimal reward scheme, the principal still optimally motivate the potential good type agent to report a number of successes that is higher than the participation threshold $\overline{k}$, even if it exceeds the over-experimentation threshold $\hat{k}$. This is because the benefit of making the reward level for a potential good type closer to the good type's true valuation M is sufficiently large to cover the expected increased loss of overpayment in other *ex post* scenarios where the type is bad. The lower bound of optimal amount k_V^* is the same as that in public experimentation, and the exact number depends

[6]When comparing the reward level to those reporting required k successes without failure in CF and MF-I, it has $p_k M - (k-l)\frac{c}{1-\theta} - \left(p_l M - \sum_{i=l+1}^{k} \frac{p_l}{p_i} \frac{c}{1-\theta}\right) > 0$.

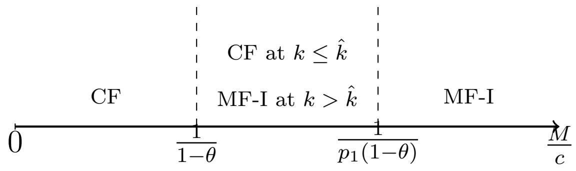

Figure 2.4: Optimal reward scheme with verifiable failures at different value-cost ratio

on the parameter range, and CF and MF-I are the only possible candidates of reward scheme. A numeric example is shown below.

Example 2.4.1. When $\theta = 0.6$, $p = 0.4$, $M = 3$ and $c = 1$, then the participation threshold is $\overline{k} = 1$, and the over-experimentation threshold is $\hat{k} = 2$. In private experimentation with verifiable failures, at $k = 2$, the associated optimal reward scheme is CF $= \begin{cases} a^2(2,0) = 2.86 \\ a^2(j < 2, 1) = 0.44 \end{cases}$, and the principal's expected payoff is $V_V(2, p_0) = -0.89$; at $k = 3$, the associated optimal reward scheme is MF-I $= \begin{cases} a^3(3,0) = 3.65 \\ a^3(2,1) = 1.15 \\ a^3(j < 2, 0) = 0.94 \end{cases}$, and the principal's expected payoff is $V_V(3, p_0) = -1.19$. In the optimal contract, $k_V^* = 2$ and its associated reward scheme is CF.

2.4.2 Unverifiable failures

The situation becomes more complicated when failures are not verifiable. This implies that the agent can easily or cheaply lie when reporting failures, which occurs when the hard evidence of failure is hard to find or stored or cost of fake evidence is cheap. Thus the principal cannot tell whether the experiment associated with the failure has been indeed carried out. If it is easy for the agent to conceal a failure, the idea of "rewarding honesty" in public and private experimentation with verifiable failures cannot be applied, and Lemma 2.4.2 doesn't hold. If the principal assigns a strictly higher reward to those reporting failures, the potential good type agent would pretend to be a bad type if it's beneficial to do so. On the one hand, when pretending to be those who face an early failure, the gain is smaller than continuing to undertake the remaining experiments; but, on the other hand, the agent can save experimental costs. Therefore, the best that the principal can do is to assign the same level to those reporting less than required number of successes with and without failures, and $\mathrm{IC}^{F,NV}$ and $\mathrm{IC}^{S,NV}$ can now be written as:

$$\begin{aligned} &\mathrm{IC}^{S,NV}_{0\leq j\leq k-1}: && U(k-j,p_j) \geq a^k(j,0) = a^k(j,1) \\ &\mathrm{IC}^{F,NV}_{0\leq j< k-1}: && a^k(j+1,1) - a^k(j,1) \leq \frac{c}{1-\theta} \\ &\mathrm{IC}^{F,NV}_{k-1}: && a^k(k,0) - a^k(k-1,1) \leq \frac{c}{1-\theta} \end{aligned} \tag{2.4.8}$$

With the help of incentive constraints above, the agent would disclose all of the information acquired on the equilibrium path, and "masquerading" behaviour is deterred: $\mathrm{IC}^{F,NV}$ prevent from "pretending to be good" and $\mathrm{IC}^{S,NV}$ deter "pretending to be bad". Together with IR, there are still $2k-1$ constraints, whose number is the same as under private experimentation with verifiable failures, but they are more strict. Thus it's natural to check whether CF and MF-I proposed before are feasible, and whether they would still be optimal if all constraints were satisfied; otherwise, other reward schemes need to be considered.

Proposition 2.4.2. *In private experimentation with unverifiable failures,*
1) Given the commitment motivates the potential good type agent to report $k\mathbb{N}^+$ successes, the associated optimal reward scheme is:

a) CF when $k \leq \min\left\{\hat{k}, \overline{k}\right\}$;

b) MF-I when $\hat{k} < k \leq \overline{k}$;

c) Type II multi-step function (MF-II) when $\overline{k} < k \leq \hat{k}$;

d) Type III multi-step function (MF-III) when $k > \max\left\{\hat{k}, \overline{k}\right\}$.

2) The optimal number k^*_{NV} satisfies $\overline{k} \leq k^*_{NV} < \infty$.

Proposition 2.4.2.1) shows that the structure of the optimal reward scheme is determined mutually by the participation and over-experimentation threshold. When the motivated number of experiments is smaller than both, $k < \min\left\{\hat{k}, \overline{k}\right\}$, the agent finds that, in CF, the benefit from early-stop is too small, and the cost of over-experimentation after failure is too high. This happens when the value-cost ratio stays at a medium level, $\frac{M}{c} \in \left[\frac{1}{p_0}, \frac{1}{(1-\theta)p_1}\right]$, where $0 < \overline{k} < \infty$ and $\hat{k} > 0$. Instead, when the ratio is sufficiently high or too low, $\frac{M}{c} \in \left[0, \frac{1}{p_0}\right) \cup \left(\frac{1}{(1-\theta)p_1}, \infty\right)$ where now $\hat{k} = 0$ or $\overline{k} = 0$, if the principal still sticks to a CF scheme, at least one incentive constraint is violated, and the agent then would either stop earlier without failure or over-experiment after early failure. As a result, CF is no longer optimal in this case. The discussion of the optimal reward scheme at different value-cost ratio when failures are not verifiable is summarised in Figure 2.6.

Since there is no straightforward way of knowing which threshold number is larger, different scenarios need to be discussed. If the participation threshold is relatively larger, $\hat{k} < \overline{k}$, MF-I is optimal when the number of desired experiments is between the two threshold numbers, $\hat{k} < k \leq \overline{k}$. Since the desired number of experiments is still smaller than the participation threshold, the agent doesn't gain from pretending to be bad by stopping early without failure. On the other hand, however, now $k > \hat{k}$ is large enough to generate a gain from over-experimentation, specially if a failure occurs when the required number of successes is almost achieved. Then only $\text{IC}^{F,NV}$ need to be attended to, and MF-I scheme is optimal.

If the over-experimentation threshold is relatively larger, $\overline{k} < \hat{k}$, when the incentivised number k is between two thresholds, the conclusions are different. When $k < \hat{k}$, the agent is not willing to pretend to be a good type by over-experimentation since the benefit from such behaviour cannot cover the cost. However, when $k > \overline{k}$, in which the prior expectation cannot cover the *ex-ante* expected total cost, the agent find that it's better to pretend to be a bad type and receive a relatively smaller reward after accepting the principal's contract, among which the worst case is that the agent reports failure immediately without any experiments. To deal with this, the type II step function (MF-II) is introduced.

Definition 2. The type II multi-step function (MF-II) is a reward scheme such that $\text{IC}^{S,NV}_{0\leq j\leq m}$ are all binding, where $m = \max\{m \in \mathbb{N} : p_0 M \leq \sum_{i=m+1}^{k} \frac{p_0}{p_{i-1}} c \;\&\; 0 \leq m \leq k-1\}$:

1) When $0 \leq m < k-1$,

$$\text{MF-II} = \begin{cases} a^k(0,0) = 0 \\ a^k(0 < j \leq m, 1) = \sum_{i=1}^{j} \frac{p_i}{p_{i-1}} c \\ a^k(m < j < k, 1) = \displaystyle\sum_{i=m+1}^{k} \frac{p_{m+1}}{p_{i-1}} c - p_{m+1} M + a^k(m,1) \\ a^k(k,0) = p_k M + a^k(m+1,1) \end{cases} \tag{2.4.9}$$

2) When $m = k-1$,

$$\text{MF-II} = \begin{cases} a^k(0,0) = 0 \\ a^k(0 < j < k, 1) = \displaystyle\sum_{i=1}^{j} \frac{p_i}{p_{i-1}} c \\ a^k(k,0) = \displaystyle\sum_{i=1}^{k} \frac{p_i}{p_{i-1}} c \end{cases} \tag{2.4.10}$$

Definition 3. The type III multi-step function (MF-III) is a reward scheme such that $IC^{S,NV}_{0 \leq j \leq m}$ and $\mathrm{IC}^{F,NV}_{l \leq j \leq k-1}$ are all binding, $0 \leq m < l \leq k-1$:

$$\text{MF-III} = \begin{cases} a^k(0,1) = 0 \\ a^k(0 < j \leq m, 1) = \sum_{i=1}^{j} \frac{p_i}{p_{i-1}} c \\ a^k(m < j < l, 1) = \displaystyle\sum_{i=m+1}^{k} \frac{p_{m+1}}{p_{i-1}} c - p_{m+1} M + a^k(m,1) \\ a^k(l,1) = p_l M - \displaystyle\sum_{i=l+1}^{k} \frac{p_l}{p_i} \frac{c}{1-\theta} + \sum_{i=m+1}^{k} \frac{p_{m+1}}{p_{i-1}} c - p_{m+1} M + a^k(m,1) \\ a^k(l < j < k, 1) = (j-l)\frac{c}{1-\theta} + a^k(l,1) \\ a^k(k,0) = (k-l)\frac{c}{1-\theta} + a^k(l,1) \end{cases} \tag{2.4.11}$$

MF-II captures the feature of "building stairs at beginning": the principal raises the reward level for those whose first failure occurs at some early stage to compensate for the high expected total cost, until the agent is optimistic enough to carry out the remaining experiments if no failure occurs. This implies that this reward scheme makes only the first $\mathrm{IC}^{S,NV}_{0\leq j\leq m}$ constraints binding. Then the remaining reward levels in MF-II share the same feature as those in CF: if the motivated number of successes k is reported, the agent can claim a bonus $p_k M$. If the value-cost ratio is too small, all $\mathrm{IC}^{S,NV}$ are binding, and steps are built until the very end. These stairs in MF-II have a different effect on an agent's behaviour compared to those blocks at the end of MF-I where the bad type is deterred from over-experimentation.

If the motivated number k is larger than both of the thresholds, $k > \max\left\{\hat{k}, \overline{k}\right\}$, the agent has an incentive to stop earlier without failures at beginning and to over-experiment at the end. On the one hand, due to $k > \overline{k}$, the agent finds that the *ex ante* expected total cost is too high to follow the planned commitment, and he would be better off by stopping earlier without failure. On the other hand, since $k > \hat{k}$, even if previous incentives are solved, the agent would over-experiment since pretending to be good type is more attractive than ceasing to experiment. Therefore, both types of incentives need to be addressed, and the type III step function (MF-III) is optimal among all feasible alternatives.

MF-III then is the mixture of MF-I and MF-II: stairs at beginning and blocks at the end. In this reward scheme, the first $\mathrm{IC}^{S,NV}_{0\leq j\leq m}$ and the last $\mathrm{IC}^{F,NV}_{l\leq j\leq k-1}$ are binding. At the beginning, the principal raises the reward level for those facing early failure to guarantee experiments are conducted; when enough experiments have been carried out, she sets blocks by fixing the neighboured number of reported good successes at $\frac{c}{1-\theta}$ to deter over-experimentation. Examples of MF-II's and MF-III's structure are shown in Figure 2.5.

Proposition 2.4.2.2) shows that the lowest possible optimal number of experiments motivated by the principal equals to the participation threshold, $k^*_{NV} \geq \overline{k}$, which is consistent with public and private experimentation with verifiable failures. One numeric example is shown below.

Example 2.4.2. When $\theta = 0.6$, $p = 0.4$, $M = 3$ and $c = 1$, then the participation threshold is $\overline{k} = 1$, and the over-experimentation threshold is $\hat{k} = 2$. In private experimentation with unverifiable failures, at $k = 1$, the associ-

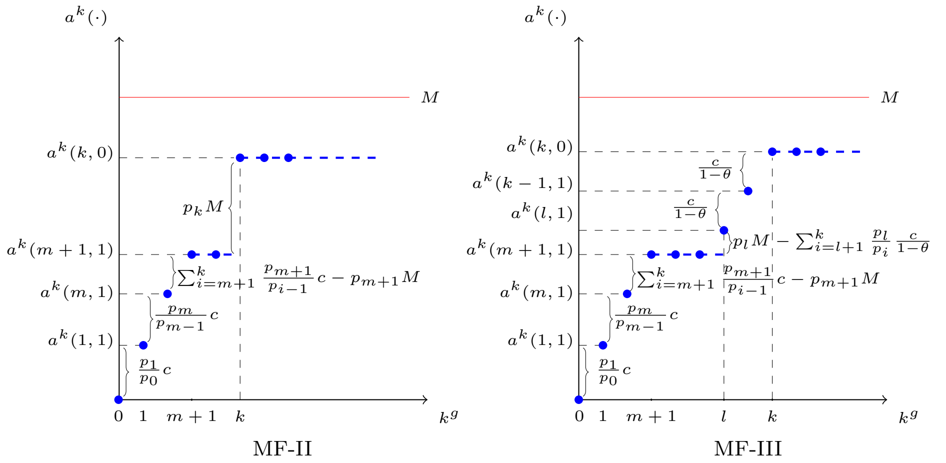

Figure 2.5: MF-II and MF-III in private experimentation (Example: $m = 2$, $l = k - 2$)

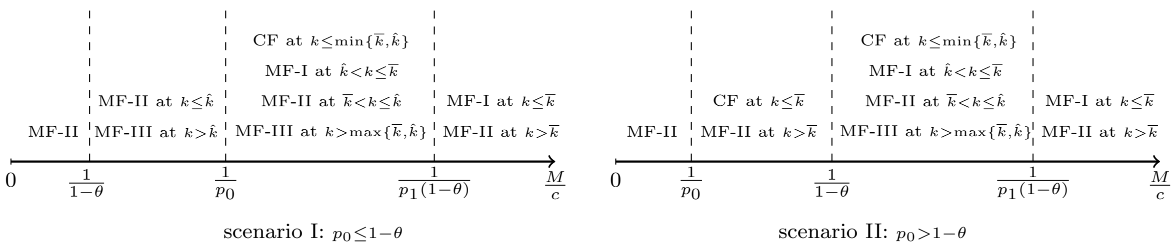

Figure 2.6: Optimal reward scheme with unverifiable failures at different value-cost ratio

ated optimal reward scheme is , CF $= \begin{cases} a^1(1,0) = 1.88 \\ a^2(0,1) = 0 \end{cases}$, and the principal's expected payoff is $V_{NV}(1, p_0) = -1.35$; at $k = 2$, the associated optimal reward scheme is MF-II $= \begin{cases} a^2(2,0) = 3.11 \\ a^2(1,1) = 0.69 \\ a^2(0,1) = 0 \end{cases}$, and the principal's expected payoff is $V_{NV}(2, p_0) = -0.9993$; at $k = 3$, the associated optimal reward scheme is MF-III $= \begin{cases} a^3(3,0) = 4.17 \\ a^3(2,1) = 1.67 \\ a^3(1,1) = 1.46 \\ a^3(0,1) = 0 \end{cases}$, and the principal's expected payoff is $U^P_{NV}(3, p_0) = -1.68$. In the ptimal contract, $k^*_{NV} = 2$ and its associated reward scheme is MF-II.

2.4.3 Comparisons between Public and Private Experimentation

Previous results have characterised the properties of the optimal reward scheme given different motivated numbers of experiments, and showed that the optimal number must be weakly larger than the participation threshold $\overline{k}$, which is the largest number of experiments whose expected total cost can be covered by the agent's prior expected value. However, these results don't suggest that it's always optimal to motivate a positive number of experiments.

Corollary 2.4.1. *There exist parameter ranges such that the principal is better off conducting no experiments and assigning a single reward level p_0M to both types.*

Imagine the case when the cost of an experiment is very high, which implies the excess cost is also prohibitively high. Thus, the principal finds it's too costly to incentivise a single experiment, even if a success can help improve posterior beliefs. Following similar reasoning, when the value-cost ratio, $\frac{M}{c}$, is too low, the principal would not motivate the agent to run any experiments, since the benefit from improving the precision of reward is not enough to cover the cost of doing so.

Corollary 2.4.2. *When $\frac{M}{c} \leq \frac{1}{1-\theta}$, private experimentation with verifiable failures is equivalent to public experimentation.*

Corollary 2.4.2 suggests that, when the value-cost level is low, the principal's optimal contract and the efficiency level in private experimentation with verifiable failures are the same as the public one. When $\frac{M}{c} < \frac{1}{1-\theta}$, $\hat{k} \to \infty$, and the over-experimentation threshold doesn't play a role since the extra gain from over-experimentation is too low. As a result, in private experimentation with verifiable failures, the principal can optimal motivate the same number of experiments by CF as that in public information environment.

When the value-cost ratio is increasing, it shows that a good type agent becomes relatively more valuable, or the experimental cost is relatively cheaper. Thus the prior expectation now can cover larger numbers of experiments, meaning the participation threshold $\overline{k}$ becomes larger. In the public case, the excess cost is also getting smaller, implying the reward levels for both types become more precise and closer to the posterior beliefs. As a result, the principal is willing to weakly increase the optimal number of experiments, which is the positive effect.

Proposition 2.4.3. *1) as $\frac{M}{c}$ increases, $\overline{k}$ and k^P are increasing and $\hat{k}$ is decreasing;*
2) k^*_V and k^*_{NV} are increasing as M increases.

When the value-cost ratio is getting larger, the over-experimentation threshold $\hat{k}$ becomes smaller, which leads that it becomes more attractive to deviate for a bad type who fails when only one more success is needed to prove he is a potential good type. Thus the bad type in such situation has the strongest incentive to over-experiment and it's easier to violate the original incentive constraint. As a result, the principal needs to distort the reward scheme at an earlier point of failure to prevent such behaviour, so that the imprecision occurs at an earlier stage and the principal is willing to reduce the number of experiments. This is the negative effect, but it is not the only effect. Since the participation threshold also increases in the private case, it's not clear which effect dominates. If the increasing of the value-cost is solely from the increasing of the good type's value M, then the positive effect dominates, and the principal is willing to raise the optimal number of experiments; if the value-cost ratio increases only due to increase in the cost of experiments, the result is ambiguous.

Now consider the reward levels that different types received under the optimal reward scheme. For a bad type whose value is zero, he can achieve a (weakly) positive reward level as long as he accepts the contract offered by the principal. In the public case, the principal pays the excess cost to ensure that the agent does not deviate from conducting experiments. In the private case, bad types who fail early would receive different reward levels, which are increasing as the early failure occurs later. As a result, even if the agent learns his type is bad after observing an early failure, he can still receive something from the principal, which is weakly higher than his true value. Now both the principal and the agent have the posterior belief p_k, and the agent's a posterior expectation is $p_k M$. The agent can now receive the highest reward level in the associated optimal reward scheme. In the public case, the potential good type can receive the amount of the posterior expectation as a bonus, on top of the excess cost. The reward level is now higher than the posterior expectation. However, in the private case, due to the distortion from the over-experimentation threshold, the highest reward level should cooperate with other reward levels for the bad type to guarantee the informativeness of reported successes, and result is ambiguous when comparing to the posterior expectation level.

Proposition 2.4.4. *1) A bad type is always overpaid in both the public and private cases;*
2) A potential good type is overpaid in the public case, but this doesn't always hold in the private case.

2.5 Extensions

2.5.1 Bad Luck

Now a small probability of bad luck is introduced. In each experiment, the bad luck occurs with a small but strictly positive probability $\sigma \in (0, 1)$, in which both types fail. It's privately observed by the agent when bad luck occurs. This can be considered as an exogenous negative shock which causes the failure of both types, for instance, a bad health condition causes a capable candidate to fail a CFA test.

When the bad luck occurs in an experiment, a failure occurs. This negative shock provides no information about the agent's quality, and the principal may still want the agent to continue if not enough successes are acquired. Thus, the principal can tolerate more failures which are caused only by bad luck. Also, since the bad luck may happen in every experiment, the number of failures are uncertain before experiments are carried out. Thus the principal optimally assigns them the same reward level,

$$a_B^k(k^g, k^b > 1) = a_B^k(k^g, 1) \quad \text{and} \quad a_B^k(k, 0) = a_B^k(k^g, 1) \tag{2.5.1}$$

Now the term "k experiments incentivised to run" in $a_B^k(k^g, k^b)$ means k experiments which are not affected by bad luck. Due to the presence of the failure caused by the bad luck, a potential good type now can easily masquerade as a bad type to save future cost. Thus, when bad luck arrives, the principal still needs the agent's expected benefit from continuing initial plan to be higher than stopping immediately and reporting results,

$$\mathrm{IC}^{S,B}_{0\le j\le k-1}: \quad U_B(k-j, p_j) \ge a_B^k(j,1) \ge a_B^k(j,0) \tag{2.5.2}$$

If failures are verifiable, Lemma 2.4.2 holds then $a_B^k(j<k, 0) = 0$; if they are not, then $a_B^k(j,1) = a_B^k(j,0)$. Meanwhile, those who have already learned that they are a bad type should be deterred from further experimentation, and the incentives are given such that the benefit from over-experimentation to acquire one more success is lower than its cost, where

$$\begin{aligned} &\mathrm{IC}^{F,B}_{0\le j<k-1}: \quad a_B^k(j+1,1) - a_B^k(j,1) \le \frac{c}{(1-\theta)(1-\sigma)} \\ &\mathrm{IC}^{F,B}_{k-1}: \qquad a_B^k(k,0) - a_B^k(k-1,1) \le \frac{c}{(1-\theta)(1-\sigma)} \end{aligned} \tag{2.5.3}$$

Then it's easily seen that the incentives provided are the same as conditions (2.4.8) in private experimentation with unverifiable failures and no bad luck, in which the cost level is $\frac{c}{1-\sigma}$. Therefore, the associated reward scheme would be the same as that in proposition 2.4.2 at a different cost level. Also, since the cost is higher, the *ex ante* expected total cost is higher, leading the threshold number $\overline{k}$, the number for which the prior expectation can cover the *ex ante* expected total cost, to be lower. Under the optimal commitment, the agent continues to experiment until either the first failure, which is not caused by bad luck, occurs, or the required number of successes are achieved. Also, the agent is indifferent between reporting one failure and not. Results are summarised in the following proposition, where all types may obtain more than one failure.

Proposition 2.5.1. *When a bad luck exists with probability* $\sigma \in (0,1)$ *in each experiment and is privately observed by the agent, the associated optimal reward scheme is the same as that in private experimentation with unverifiable failures and no bad luck at cost level* $\frac{c}{1-\sigma}$.

2.5.2 Finite Opportunities for Experimentation

Up to this point, it has been assumed that the agent can conduct an infinite number of experiments. Imagine now a scenario where opportunity T is finite, $T < \infty$. Firstly, consider the situation in which the number of experiments provided incentives to run is $k < T$. The participation incentive constraints should be the same as IC^S shown previously, since these motivate the agent to run a sufficiently number of experiments. Suppose the agent fails at the k_{th} experiment. To prevent the agent from pretending to be a potential good types by over-experimentation, the following condition must be satisfied:

$$\begin{aligned} & \frac{(1-\theta^{T-k})}{1-\theta}\left(-c+(1-\theta)a_F^k(k,0)\right)+\theta^{T-k}a_F^k(k-1,1) \leq a_F^k(k-1,1) \\ \Longrightarrow \quad & -\frac{c}{1-\theta}+a_F^k(k,0) \leq a_F^k(k-1,1) \end{aligned} \tag{2.5.4}$$

Similar constraints can be obtained for $j < k-1$. Notice that these are exactly the same as IC^F shown previously, so the optimal reward scheme should be the same as before.

If the incentive is to run T experiments, the constraints are slightly different. Imagine that the agent fails for the first time in T_{th} experiments. Now he has no further opportunity to continue experimenting, and the only remaining option is to disclose results selectively. Thus the principal only needs to motivate the agent to disclose information as her desire. For an early first failure in j_{th} experiments where $j < T$, the agent may still continue experimenting to get a higher reward level, thus $\text{IC}^F_{0\leq j<T-1}$ must be satisfied. On the side of IC^S constraints, stopping early without failure can be interpreted as revealing that the agent is bad type when failures are verifiable, and $a_F^k(k^g < k, 0) = 0$ would be optimal and CF scheme is optimal; when failures are not verifiable, the principal still needs to provide incentives to prevent the agent from pretending to be a bad type, which would be the same as $IC^{S,NV}_{0\leq j\leq T-1}$ in (2.4.8).

Proposition 2.5.2. *When the number of private experiments T is finite, given the commitment motivates the potential good agent to report $0 < k \leq T$ experiments,*
1) If $k < T$, the associated optimal reward scheme is the same as that proposed in $T \to \infty$;
2) If $k = T$, the associated optimal reward scheme is CF when failures are verifiable; when failures are not verifiable, CF is optimal if $T \leq \overline{k}$, and MF-II is optimal otherwise;
3) The optimal number is the same as that in the public case with $T \to \infty$ when T is sufficiently large.

Proposition 2.5.2 shows that the reward schemes proposed in section 2.4 are still optimal at every positive number of experiments, even if the agent cannot conduct experiments infinitely. Pushing motivated number of experiments to the boundary T can mitigate incentives for misbehaving, especially when failures are verifiable the same reward scheme in public experimentation, CF, can be applied. However, the proposition above also shows it is not always optimal to do so. When the boundary is sufficiently loosen, in which case T is sufficiently large, it doesn't help to improve the principal's benefit, and the optimal amount stays the same as that in the scenario with infinite opportunities, $T \to \infty$.

2.6 Conclusion

This chapter characterises the properties of the principal's optimal commitment when the agent can privately run costly experiments and selectively report favourable results.

The single cut-off function, or one-step function, is the optimal reward scheme in public experimentation, and it is still optimal in the private scenario if the motivated number of successes reported by the potential good agent is small and the agent's incentives to deviate from the intended path of experimentation are weak. When this number rises, a multi-step function is introduced, where a bad type agent receives a different level of rewards when reporting different numbers of successes. These different levels feature two potential types of agent's incentives of deviation in a private environment, which encourage a potential good type agent to continue experimenting after early successes of block a bad type agent from over-experimentation after an early failure. Moreover, the principal faces a trade-off when determining the optimal motivated number of successes, which is at least the largest number whose expected total cost can just be covered by the prior expectation of the good type agent's value. These results are robust when introducing finite opportunities for experiments or privately observed bad luck.

There is still room to improve upon the current work, which only considers the scenario of learning from bad news. In future research, a more general setting on information structure could be considered, and it would also be interesting to introduce the strategic third party that designs experiments.

Chapter 3

Private Experimentation and Persuasion

3.1 Introduction

A candidate on the job market needs to show sufficient quality in his CV to persuade an employer to employ him and to negotiate a good salary. The candidate undertakes various activities to signal skills and learning. For instance, he studies hard to achieve a high GPA, undertakes an internship, or sits professional certification exams such as the CFA. The employer is usually skeptical about the candidate's own reporting since most of these activities are private. The agent will disclose success to send a positive signal to the employer but when a failure occurs, the candidate may conceal the outcome, or the participation in the activity, or the number of attempts at the activity. These considerations potentially undermine the informativeness of the candidate's CV. The question then arises: how many achievements are sufficient for the candidate? How does the employer interpret the candidate's CV? when making a salary offer?

When selling a new software, a technology firm needs to show that the product has been thoroughly tested. The client doesn't usually observe which tests are undertaken before the firm disclose them and he is skeptical about the results, since the firm can disclose fewer results or even retake the same tests multiple times until the product passes. Therefore, how many test attempts and how many good results does the firm need to show?

This chapter considers a situation in which an uninformed agent persuades a principal for a high reward (evaluation) through private experimentation. The agent's type is initially unknown, and is either good or bad. The type can be learned through experiments. The information structure of an experiment is asymmetric: a good type agent always succeeds and a bad type agent can fail with positive probability. The result generated in an experiment is hard evidence, which can be forged. Since the experiments and results are privately observed, the agent has infinite opportunity for experimenting and he can selectively report the favourable results. The principal without full commitment delivers a reward to the agent based on the disclosed results, who cares about the precision of the evaluation.

I characterise three possible types of equilibria given the restriction on the principal's off-equilibrium path belief: no-experiment equilibrium, separating equilibria with learning and pooling equilibria with learning. In the first type, the agent doesn't conduct any experiments and receives his prior expected value as the reward. This equilibrium always exists as long as the principal's off-equilibrium path belief makes the agent worse off by conducting any positive amount of experiments. In a separating equilibrium with learning, the agent would stop experimenting either when he has acquired enough successes without failures or when he fails before that. In this case, the bad type agent will over-experiment since the extra benefit is less than his expected cost of doing so. Thus the disclosed successes are informative, where the bad type agent has been separated from the potential good ones. This type of equilibrium exists when the agent's value-cost ratio is medium and the number of successes reported by the potential good type is small. In a pooling equilibrium with learning, some bad type agents would over-experiment and report the same number of successes as the potential good ones' when their first failures arrive late enough. The agent's report becomes less informative, and the reward for the potential good type agent falls. This type of equilibrium exists when the agent's value-cost ratio is not too low and the number of successes reported by the potential good type is sufficiently high.

Moreover, I show that both the participation threshold and the over-experimentation threshold affect the set of equilibria. The participation threshold is the largest number of experiments whose expected total cost can be covered by the agent's prior expected value. It determines the upper bound of the entire set of equilibria, which is the largest possible number of successes reported by the agent on the equilibrium path. For any number of experiments over this, the agent be better off by deviating and not conducting any experiments. The over-experimentation threshold measures the largest number of successes where the agent's conditional expected value is smaller than the expected total of acquiring one more success by a bad type agent. It is also the boundary between the separating and pooling equilibria with learning. When the number of successes reported by the potential good type is less than the over-experimentation threshold, a bad type agent will not continue experimenting after a failure.

When the agent's value-cost ratio or the prior belief increases, the agent is willing to conduct more experiments. On the one hand, it leads the participation threshold rises and the set of equilibria expands as well.On the other hand, a bad type agent has stronger incentive to over-experiment after he has failed since the extra benefit also rises. Thus the over-experimentation threshold falls, which also causes that the set of separating equilibria with learning shrinks. As a result, the principal believes that it's easier for a bad type agent to over-experiment, and the set of pooling equilibria with learning expands. When the probability of succeeding for a bad type agent increases, a positive compound effect on the participation threshold also suggests that the set of equilibria expands. However, its effect on the over-experimentation threshold is ambiguous.

When the agent can pre-commit to the number of successes he plans to acquire to prove that he is a potential good type, I show that the agent tends to commit to the number that is as small as possible. This is because the fear of failure deters his willingness to experiment, and his optimal decision would as close as possible to that in the public experimentation scenario. His optimal commitment is also constrained by the restriction on the principal's off equilibria path belief.

The rest of the chapter is organised as follows. Section 3.2 summarises the related literature. The setup of the model and the benchmark are demonstrated in section 3.3. Section 3.4 discusses the analysis of the equilibria, and section 3.5 considers the agent's commitment. Section 3.6 concludes. All proofs not shown in the main text are given in Appendix C.

3.2 Literature Review

This paper is related to the literature about private experimentation. The closest work are by Felgenhauer and Schulte (2014) and Fu (2017). Felgenhauer and Schulte (2014) characterise the parameter range in which the persuasion equilibria with cut-off rule exist in costly private experimentation with symmetric information structure. In their work, the receiver makes a binary decision, and the sender applies a sanitisation strategy in which all unfavourable results are concealed. In contrast, my model considers an asymmetric information environment, and the principal assigns the reward level according to the disclosed information rather than a binary approval decision. The principal's strategy on the equilibrium path shares the property of the cut-off rule, but the discussion of her off-equilibrium path belief is absent in their work. Fu (2017) discusses the principal's optimal contract for evaluating the agent based on the reported experimental results, in which the principal can offer a reward scheme before experiments are conducted. In contrast, the principal doesn't have full commitment in the current work, and she can only assign a reward based on her posterior belief. Also, this work also discusses the agent's commitment, which is absent in the other papers.

Henry (2009) also considers a scenario in which the agent can pre-commit to the number of experiments, and he cannot stop until all experiments have been conducted regardless of their results. My work differs in allowing the agent to decide whether to continue after each experiment. Felgenhauer and Loerke (2013) compares public experimentation and private experimentation with symmetric information structure. Both works conclude that the agent tends to the public experimentation as less experiments are conducted, and Felgenhauer and Loerke (2013) also show that there is a deterrence effect which makes the principal and the agent better off in private experimentation. My work, by comparison, shows that the agent is weakly better off in public experimentation, but the deterrence effect doesn't exist due to the presence of asymmetric information structure.

This work relates to literature in strategic experimentation. Bergmann and Hege (2005) shows the optimal way to finance an innovative project without full commitment, Henry and Ottaviani (2014) show that the principal free rides on the agent's experiments when results are public information, and Halac and Kremer (2017) show that inefficiency is increased due to the agent's career concern in a bad news setting. In most cases where results are private information, the principal or the receiver can use the timing of when they observe success to determine the monetary transfer: this is a key difference from the current model, which does not include such timing.

This work also relates to literature on information disclosure and persuasion. Rayo and Segal (2010) and Kolotilin (2015) focus on the sender's optimal mechanism; Kamenica and Gentzkow (2011) finds the optimal way for the sender to design the structure of the experiment, and Bergemann, Bonatti and Smolin (2015) consider a monopolist who can design the experiment and set the selling price. They all focus on public experimentation, where results can be publicly observed. In contrast, My work mainly focuses on the private case, and also compared the difference between public and private case. Glazer and Rubinstein (2004, 2006) and Hart, Kremer and Perry (2017) analyse the commitment in evidence games where the agent's set of hard evidence is exogenously given. Compared to them, the agent can privately generate hard evidence given his type in my work. DeMarzo, Kremer and Skrzypacz (2017) also consider an uninformed agent who chooses one test among many different tests and strategically reveals the result to the market. In their paper the market is competitive, and the agent has only one chance to take a test, in which the null result with positive probability is introduced and is not verifiable. The decision of the principal in my work shares the same property as that of the competitive market. Compared to DeMarzo, Kremer and Skrzypacz (2017), the information structure of the experiment is exogenously given in my work, and it has the property of the softest test in which the good type always succeeds but the bad type fails with positive probability. Also, in my work, the agent has infinite opportunity for experimenting even though the information structure of the test is fixed. My work also discusses the agent's optimal commitment, which is absent in theirs.

This work can be compared to literature on signalling, for example

Spence (1973). In their models, there is no learning process for the agent, and every type of agent can costly mimic the behaviour of others for a price. In contrary, in my work, the agent has to learn his type first through the experiments. Also, when the reported number of successes increases, it's harder for a potential good type agent to separate himself from the bad types as the incentive for pooling is also increasing, and this is different from that in the literature.

3.3 Model

3.3.1 Description of the model

A risk-neutral agent (he) wants to get a reward (or evaluation) from a principal (she) who has no full commitment. The agent is either good (G) or bad (B), whose type M_i is initially unknown, where $M_i \in \{M, 0\}$ and $i \in \{G, B\}$. A common prior is shared. With probability p_0, his type is good and the value is $M_G = M$; with probability $1 - p_0$, his type is bad and the value is $M_B = 0$. $p_0 \in (0, 1)$ and $M \in \mathbb{R}^+$.

The agent can learn and prove his type through the private experiments. The cost of each experiment is constant c, where $c \in \mathbb{R}^+$, and the number of opportunities is infinite. In each experiment, a good type agent can always succeed; however, a bad type can only succeed with probability $1-\theta$, where $\theta \in (0, 1)$. The result in an experiment is hard evidence, which cannot be forged. After the k_{th} experiment in which the agent has acquired n^g successes and n^b failures, he decides whether to continue experimenting, $S\left(n^g, n^b\right) = 0$, or to stop and disclose the experimental results, $S\left(n^g, n^b\right) = 1$, where $k, n^g, n^b \in \mathbb{N}$, $k = n^g + n^b$ and $S : \mathbb{N} \times \mathbb{N} \to \{0, 1\}$. Once the agent reveals the results, he cannot run any further experiments. Moreover, since the experiments and results are privately observed by the agent, he can selectively report a subset of the acquired results which consists of k^g successes and k^b failures, where $k^g, k^b \in \mathbb{N}$, $k^g \leq n^g$ and $k^b \leq n^b$.

Based on the agent's report, the principal assigns a reward to the agent, and the reward level is $a : \mathbb{N} \times \mathbb{N} \to \mathbb{R}^+$. The principal has no full commitment, and only cares about the precision of the reward. Specifically, given the reward

a and the agent's type M_i, her payoff function is: $v(a, M_i) = -(a - M_i)^2$. Meanwhile, the agent only cares about the reward level, and his payoff function is: $u(k, k^g, k^b) = a - kc$. The timing of the game is shown as follows:

1. The agent runs experiments.
2. The agents selectively report the experimental results.
3. The principal gives the reward to the agent according to the report.
4. Payoffs are realised.

Histories and Equilibrium. After the k_{th} experiment, the agent's private history consists of the number of experiments he has run and the number of successes and failures which he has acquired, $h_k^A = \left(k, n^g, n^b\right)$, and his posterior belief is $p^A_{\left(n^g,n^b\right)} = Pr(M_i = M|n^g, n^b)$. If he stops and reports k^g success and k^b successes, his expect payoff is $\mathbb{E}\left[u(k, k^g, k^b)|n^b, n^g, k^g, k^b\right]$; instead, if he continues experimenting, he pays the experimental cost c, and gains $U\left(p^A_{\left(n^g+1,n^b\right)}\right)$ when he succeeds with probability $p^A_{\left(n^g,n^b\right)}$; however, if he fails with probability $1 - p^A_{\left(n^g+1,n^b\right)}$, he can only receive $U\left(p^A_{\left(n^g,n^b+1\right)}\right)$. Thus his expected payoff $U(p^A_{\left(n^g,n^b\right)})$ after the k_{th} experiment would be:

$$\begin{aligned} &U\left(p^A_{\left(n^g,n^b\right)}\right) \\ =&\left(1 - S_{\left(n^g,n^b\right)}\right)\left[-c + p^A_{\left(n^g,n^b\right)}U\left(p^A_{\left(n^g+1,n^b\right)}\right) + (1 - p^A_{\left(n^g,n^b\right)})U\left(p^A_{\left(n^g,n^b+1\right)}\right)\right] \\ &+ S_{\left(n^g,n^b\right)}\mathbb{E}\left[u(k, k^g, k^b)|n^b, n^g, k^g, k^b\right] \end{aligned} \tag{3.3.1}$$

After experimental results are disclosed, the public history consists of the number of successes and failures which the agent reports, $h^P = \left(k^g, k^b\right)$. The principal's posterior belief now is $p^P_{\left(k^g,k^b\right)} = Pr(M_i = M|k^g, k^b)$, and her expected payoff is $\mathbb{E}\left[v(a, M_i)|k^g, k^b\right]$.

I restrict attention to the set of Perfect Bayesian Equilibria (PBE) in the pure strategy, and a candidate equilibrium can be described as a collection $\left\{\left\{S^E_{n^g,n^b}\right\}_{n^g\geq 0, n^b\geq 0}, (k^g_E, k^b_E), a^E, p^E\right\}$, which satisfies the following conditions:

- *Sequential Rationality*: the agent's strategy $\left(\left\{S^E_{n^g,n^b}\right\}_{n^g\geq 0, n^b\geq 0}, (k^g_E, k^b_E)\right)$

maximises $U\left(p^A_{\left(n^g,n^b\right)}\right)$; given the agent's report, the principal's strategy a^E maximises $\mathbb{E}\left[v(a,M_i)|k^g,k^b\right]$;

- *Belief Consistency*: when $\left(k,(k^g,k^b)\right) = \left(k^E,(k^g_E,k^b_E)\right)$, $\left(k,(k^g,k^b)\right)$ is "on the equilibrium" and $p^E_{\left(k^g_E,k^b_E\right)} = p^P_{\left(k^g_E,k^b_E\right)}$, which is determined by Bayes' rule; otherwise, $(k,(k^g,k^b)$ is "off the equilibrium", and it requires $p^P_{\left(k^g,k^b\right)} \in [0,1]$.

In this paper, the off-equilibrium path belief is refined by the *belief monotonicity*: given the equilibrium level of the reported failure(s) k^b_E, the principal's belief weakly increases if more successes are reported with zero probability; given the equilibrium level of the reported success(es) k^g_E, the principal's belief weakly decreases if more failures are reported with zero probability. This refinement can simplify the conditions for the existence of equilibria, and the conclusion would still be robust without it.

3.3.2 Benchmark: public experimentation

Consider public experimentation as a benchmark for studying the model. The experiments and results are publicly observable in this situation, and the agent cannot hide any unfavourable results, nor claim that he hasn't conducted any experiments. Thus the principal's belief coincides with the agent's, $p^P_{\left(k^g,k^b\right)} = p^A_{\left(n^g,n^b\right)}$. Claim 3.3.1 suggests that the principal optimally has the reward level $a^*(p^P)$ equal to the agent's conditional expected value given her belief p^P. This is a standard result in the evidence game by Hart, Kremer and Perry (2017), due to the single peakedness of the principal's payoff function.

Claim 3.3.1. *Given the principal's belief* $p^P_{\left(k^g,k^b\right)}$,

$$a^*\left(k^g,k^b\right) = \mathbb{E}\left[v(a,M_i)|k^g,k^b\right] = p^P_{\left(k^g,k^b\right)}M$$

Proof. Given the principal's belief p^P, she solves the following maximisation problem:

$$\max_{a\in\mathbb{R}^+} \mathbb{E}\left[-\left(a-M_i\right)^2|k^g,k^b\right] \quad \Longrightarrow \quad \max_{a\in\mathbb{R}^+} -p^P(a-M)^2-(1-p^P)a^2$$

Thus the optimal solution is $a^*\left(k^g, k^b\right) = a^*(p^P_{\left(k^g,k^b\right)}) = p^P_{\left(k^g,k^b\right)}M$. $\square$

Given the agent has run k experiments and no failure occurs, both parties are more optimistic on that the agent is a good type, and the posterior belief is updated according to Bayes' rule:

$$Pr(M_i = M|k, 0) = \frac{p_0}{p_0 + (1 - p_0)(1 - \theta)^k} = p_k \tag{3.3.2}$$

In this case, the agent receives $p_k M$ as reward, and the total cost is kc. However, if the first failure occurs in $j + 1_{th}$ experiment, where $j \in \mathbb{N}$ and $j \leq k$, both parties learn that the agent's type is bad as only a bad type fails in an experiment. The agent receives zero even if he has j successes and has paid cost jc, and he has no incentive to conduct any further experiments since the principal has observed the failure. This implies that, if the agent plans to stop experimenting after acquiring k successes, he would stop experimenting when either k successes have been achieved without failure, or when a failure occurs before that. Thus, the agent's continuation payoff at beginning $CU^P(p_0)$ can be simplified as:

$$CU^P(p_0) = S_{(0,0)}p_0M + \left(1 - S_{(0,0)}\right)\left(p_0M - \tilde{k}c\right), \text{ where } \tilde{k} = \sum_{i=1}^{k} \frac{p_0}{p_{i-1}} \tag{3.3.3}$$

$\tilde{k}$ can be interpreted as the expected number of experiments that the agent runs, which is simply equal to the sum of the ratio of the prior and posterior beliefs.

Lemma 3.3.1. *In public experimentation, the agent doesn't run any experiments and $a^P(p_0) = p_0M$.*

Proof. Notice that $p_0M > p_0M - \tilde{k}c$ for $\forall k \geq 1$ since $c > 0$, thus $S^P_{(0,0)} = 1$ and $a^P(p_0) = p_0M$. $\square$

Lemma 3.3.1 shows that the public results of the experiments deter the learning process: the agent would never run experiments in public as long as the cost of the experiment is positive, regardless of the prior belief. The principal has no full commitment, thus she cannot commit to only assign a positive reward level to those who achieves a certain amount of successes. For the agent, the fear of failure stops him at the outset, and he doesn't learn anything in this case.

3.4 Equilibrium analysis

Now consider the case where experiments and results are private. Since the success is positive evidence of a potential good type, the agent would disclose all of his successes, where $k^g = n^g$. Recall that only a bad type fails in an experiment, thus the agent is not willing to reveal failure(s) if any. If he does reveal the failure, the principal immediately learns that the agent is a bad type, and would give him zero. He also cannot forge a success since it's hard evidence. Alternatively, the agent can claim that he hasn't run many experiments and achieves only successes. By doing so, he receives the at least the same as that when disclosing the failure(s). At the stage of disclosing evidence, these are the standard results in information disclosure literature.

Moreover, since the experiments and results are private information before disclosure, a bad type agent can still continue conducting experiments and collecting more successes with positive probability as many times as he wants. This is called over-experimentation behaviour.

Lemma 3.4.1. *In any equilibrium, a potential good type agent conducts weakly more experiments than a bad type does.*

Lemma 3.4.1 suggests that a potential good type always has weakly more successes relative to a bad type agent. This is because a bad type agent has a higher expected cost of acquiring one more success, and it's easy for a potential good type to separate himself from the bad type by conducting more experiments with less cost. As a result, in general, only three possible scenarios regarding the behaviour "on the equilibrium path" need to be considered:

- *No experiment equilibrium*: the agent doesn't run any experiments.
- *No over-experimentation scenario (separating equilibrium with learning)*: the agent plans to run $k > 0$ experiments at the beginning, and stops and discloses all the successes when he acquires k successes without failure(s) or when he fails before that.

- *Over-experimentation scenario (pooling equilibrium with learning)* : when the first failure occurs after some early successes, he continues to experiment until k successes are acquired; otherwise, he stops immediately.

In the scenarios without over-experimentation behaviour, the agent would stop experimenting once he has learned that his type is bad on the equilibrium path. Thus the potential good type is separated, and the bad type would receive zero even if he might still have some successes. The experimental results are informative, where $p^P_{(k,0)} = p^A_{(k,0)} = p_k$. This is similar to the separating equilibrium in a signalling game, but it's not the same as the agent needs to learn his type through the costly experiments.

In the scenario with over-experimentation behaviour, the experimental results become less informative. Suppose again the first failure occurs in $j+1_{th}$ experiments when the agent plans to acquire k successes at the very beginning, where $0 \leq j < k$. If he stops and discloses the j successes that he has achieved, the principal's posterior belief is $p^P_{(j,0)}$ and the reward level would be $p^P_{(j,0)}M$. Instead, if the agent continues experimenting, the expected experimental cost to guarantee another $k-j$ successes would be $\frac{k-j}{1-\theta}c$, and he can receive $p^P_{(k,0)}M$. Therefore, the agent is always willing to do so if the extra benefit is larger than the expected cost, in which case the following condition is satisfied:

$$\left(p^P_{(k,0)} - p^P_{(j,0)}\right) M > \frac{k-j}{1-\theta}c \tag{3.4.1}$$

Assume now condition (3.4.1) is violated at $j-1$. Since $p^P_{(j,0)}$ is weakly increasing and the right hand of condition (3.4.1) is decreasing as j increases, the bad type agent who fails after $j+1_{th}$ experiment would also have the over-experimentation incentive as condition (3.4.1) is also satisfied for him. With a similar argument, the bad type agent who fails before the $j+1_{th}$ experiment would stop immediately as the expected net benefit from over-experimenting is negative. As a result, the principal's belief on the equilibrium path, $p^{P,O}_{(k,0)}$, is updated according to the Bayes' rule:

$$\begin{aligned} p^{P,O}_{(k,0)} =& \frac{p_0}{\underbrace{p_0 + (1-p_0)(1-\theta)^k}_{\text{agent hasn't failed}} + \underbrace{\sum_{i=j}^{k-1}(1-p_0)(1-\theta)^i\theta}_{\text{agent fails after } j+1_{th} \text{ experiment}}} \\ =& \frac{p_0}{p_0 + (1-p_0)(1-\theta)^j} = p_j < p_k \end{aligned} \tag{3.4.2}$$

This result implies that the principal now has a lower posterior belief on that the agent is good when k successes are reported. Also, the informativeness of the reported successes is less relative to the separating equilibrium with learning. As a result, the potential good type is also worse off.

Lemma 3.4.2. *In any equilibrium where the potential good type agent reports $k > 0$ successes, $a^E(k^g < k, 0) = 0$.*

Proof. In both of the scenarios above, only the bad type agent reports less successes on the equilibrium path, thus $p^P_{(k^g<k,0)} = 0$ and $a^E(k^g < k, 0) = p^P_{(k^g<k,0)}M = 0$. □

Lemma 3.4.2 suggests that the number of successes reported by the potential good type agent on the equilibrium path plays a role in the acceptance threshold, in which the principal only recognises the agent as a bad type when fewer successes are reported. Therefore, in condition (3.4.1), $p^P_{(j,0)} = 0$ on the equilibrium path. It's easily to see that a bad agent whose first failure occurs in the k_{th} experiment has the strongest incentive to over-experiment, because he is so close to showing that his type is good, and he will do so if the extra benefit $p^P_{(k,0)}M$ is large enough to cover the cost $\frac{c}{1-\theta}$. Thus, the agent's expected payoff by stopping experiments and disclosing successes after $k > 0$ successes are acquired, $U^k(p_0)$, can be simplified as:

$$U^k(p_0) = \frac{p^P_{(k,0)}}{p_k}p_0M - \tilde{k}c + \underbrace{\sum_{i=0}^{k-1}(1-p_0)(1-\theta)^i\theta \max\left\{0, p^P_{k,0}M - \frac{(k-i)c}{1-\theta}\right\}}_{\text{over-experimentation incentives}} \tag{3.4.3}$$

Compared to the choice of no-experiment, the agent would report k successes if his expected payoff from doing so is non-negative, $U^k(p_0) \geq 0$.

Lemma 3.4.2 also concludes that experimenting forever cannot be an equilibrium. Suppose it is. Then the principal would consider the agent as a bad type on the equilibrium path if the agent reports any finite number of successes. As the costly experiments and results are private information which needs to be disclosed after the agent stops experimenting, the agent would deviate not to run any experiments, which is a contradiction.

Before the characterisation of the equilibria, it is worth discussing the off-equilibrium path belief, in which the agent reports more successes relative to the amount on the equilibrium path. Suppose that the agent has already acquired $k \geq 0$ successes. If he sticks to the strategy on the equilibrium path, the reward level would be $p^P_{(k,0)}M$. Now consider the deviations. If more successes are reported together with some failures, as a reasonable off-equilibrium belief, the principal would learn this agent is a bad type, $p^P_{\left(k^g>k, k^b>0\right)} = 0$, since only a bad type agent can failure in an experiment. Alternatively, when more successes are reported without failures, the *belief monotonicity* suggests that the principal would have a weakly higher posterior belief. If so, the potential good type agent may have the incentive to continue experimenting to acquire more successes as his posterior belief is higher and the expected cost to guarantee a successes is lower. The bad type agent may also have the incentive to deviate to continue experimenting due to the weakly higher reward off the equilibrium path.

Lemma 3.4.3. *In any equilibrium where the potential good type agent reports $k \geq 0$ successes, $p^P_{(k^g>k,0)}$ satisfies that, for $\forall n \in \mathbb{N}^+$,*

$$\begin{aligned}\sum_{j=0}^{n-1}(1-p^P_{(k,0)})(1-\theta)^j\theta\left[\max_{i\in\{0,\dots,n-j\}} p^P_{(k+i,0)}M - \frac{ic}{1-\theta}\right]&\\ \leq \frac{p_k\left(p_{k+n}-p^P_{(k+n,0)}\right)}{p_{k+n}}M + \sum_{i=1}^{n}\frac{p_k}{p_{k+i-1}}c&\end{aligned} \tag{3.4.4}$$

Lemma 3.4.3 demonstrates the further restriction on the off-equilibrium path belief, which guarantees that neither the bad nor the potential good type would deviate to continue experimenting after the equilibrium number of successes has been achieved. The left hand side in (3.4.4) is the simplified expression due to the *belief monotonicity* refinement, and the complete expression is shown in the proof in the Appendix.

Proposition 3.4.1. *In private experimentation, when (3.4.4) is satisfied,*
1) no-experiment equilibrium exists in which the agent doesn't conduct any experiments and $a^E(p_0) = p_0 M$*;*
2) a participation threshold $\overline{k} = \begin{cases} \max\left\{k \in \mathbb{N} : p_0 M \geq \tilde{k}c\right\} & p_0 M \geq c \\ 0 & p_0 M < c \end{cases}$ *and an over-experimentation threshold* $\hat{k} = \begin{cases} \max\left\{k \in \mathbb{N} : p_k M \leq \frac{c}{1-\theta}\right\} & p_1 M \leq \frac{c}{1-\theta} \\ 0 & p_1 M > \frac{c}{1-\theta} \end{cases}$
exist, such that:

2.a) when $\frac{M}{c} \in \left[0, \frac{1}{p_0}\right)$*, no-experiment equilibrium is unique;*

2.b) when $\frac{M}{c} \in \left[\frac{1}{p_0}, \frac{1}{p_1(1-\theta)}\right]$*, separating equilibria with learning exist at* $0 < k \leq \min\left\{\overline{k}, \hat{k}\right\}$*, in which* $a^E(k,0) = p_k M$*,* $a^E\left(k^g < k, 0\right) = 0$*, and the agent would stop and disclose all the successes when* k *successes are achieved without failures or when he fails before that;*

2.c) when $\frac{M}{c} \in \left(\max\left\{\frac{1}{p_0}, \frac{1}{1-\theta}\right\}, +\infty\right)$*, pooling equilibria with learning exist at* $\hat{k}+1 < k \leq \overline{k}$*, in which* $a^E(k,0) = p_{\hat{k}+l} M$*,* $a^E\left(k^g < k, 0\right) = 0$*, and the potential good type agent and the bad type agent whose first failure occurs after* $\hat{k}+l+1_{th}$ *experiment would report* k *successes, where* $0 < l \leq k - \hat{k}$ *such that* $\frac{M}{c} \in \left(\max\left\{\frac{k-\hat{k}-l}{p_{k+l}(1-\theta)}, \frac{k-\hat{k}-l}{p_{\hat{k}+l-1}} + \sum_{i=1}^{\hat{k}+l} \frac{1}{p_{i-1}}\right\}, \frac{k-\hat{k}-l+1}{p_{k+l}(1-\theta)}\right] \neq \varnothing$*.*

The existence for each type of equilibrium and it's conditions can be characterised, which are listed in Proposition 3.4.1. It shows that, given condition (3.4.4) is satisfied, the existence of different types of equilibria is determined by the value-cost ratio $\frac{M}{c}$ and two thresholds: the participation threshold $\overline{k}$ and the over-experimentation threshold $\hat{k}$. In general, the results suggest that the no-experiment equilibrium is unique when the value-cost ratio is too low, and only pooling equilibria with learning survive when the value-cost ratio is too high. The separating equilibria with learning only possibly exist at the medium level of the value-cost ratio. These are discussed in detail in the following paragraphs. Also, the existence of equilibria in different ranges of the value-cost ratio is listed in Figure 3.1, in which yellow, cyan and grey represents the region of existence of no-experiment equilibrium, separating equilibria with learning and pooling equilibria with learning respectively.

Proposition 3.4.1.1) shows the existence of the no-experiment equilibrium, in which neither the agent nor the principal learns. This is obvious when value-cost ratio is too low, $\frac{M}{c} < \frac{1}{p_0}$, in which it's too costly for the agent to run one experiment relative to the gain, and the principal doesn't have full commitment to cover the excess. Thus it's unique in this case. The no-experiment equilibrium also exists when the value-cost ratio is high, $\frac{M}{c} \geq \frac{1}{p_0}$, but it requires a restriction on the off-equilibrium path belief, which are given in (3.4.4). This restriction prevents the agent from deviating to conduct more experiments, in which the expected extra gain is smaller than the expected cost. It could still hold even if the principal holds "the more the better" belief under some parameter range.

Proposition 3.4.1.2) demonstrates the conditions for the existence of separating and pooling equilibria with learning, as well as their properties. The results suggest that the separating or pooling equilibria with learning exist when the reported number of successes by the potential good type is smaller than the participation threshold $\overline{k}$. The participation threshold is the largest number of experiments whose expected total cost $\tilde{k}c$ can be covered by the agent's prior expected value $p_0 M$. When $k > \overline{k}$, the agent would never conduct any positive number of experiments since the expected cost is too large and the principal cannot commit to compensating for the excess. Thus, the agent is better off by deviating to no-experiment choice. When $p_0 M < c$, this threshold is zero, thus the agent would not run any experiments on the equilibrium path, so that only the no-experiment equilibrium survives. This is shown in Proposition 3.4.1.2.a). On the other hand, when $p_0 M \geq c$, the participation threshold is the rounded down number which makes $U^k(p_0) = 0$, since the expected payoff is weakly decreasing as the number of reported successes is increasing.

In Proposition 3.4.1.2.b), to support a separating equilibrium with learning, the bad type agent must not have an incentive to over-experiment. Therefore, the over-experimentation threshold $\hat{k}$ is introduced, which is the largest number of reported successes where conditional expected value $p_k M$ is smaller than the bad type agent's expected cost of acquiring one more success $\frac{c}{1-\theta}$. When $p_1 M > \frac{c}{1-\theta}$, there always exists at least one bad type agent who would over-experiment since the extra benefit is always higher than the cost. The separating equilibria with learning collapse in this case as $\overline{k} = 0$. When $p_1 M \leq \frac{c}{1-\theta}$, the extra benefit then can be smaller than the cost of over-experimentation if the reported number of successes is low, $k \leq \hat{k}$. Therefore, the possible separating equilibria with learning only exist when the number of successes reported by the potential good type is constrained by both of the thresholds, $0 < k \leq \min\left\{\overline{k}, \hat{k}\right\}$. In this case, the reported successes are informative: only the potential good type would report enough successes, and the principal learns that the agent must be a bad type when observing fewer successes are reported. Also, (3.4.4) needs to be satisfied, otherwise the potential good type agent would always deviate to continue experimenting after k successes are achieved. When the value-cost ratio is at the medium level where $\frac{1}{p_0} \leq \frac{M}{c} \leq \frac{1}{1-\theta}$, the concern about the over-experimentation vanishes as $\hat{k} \to \infty$, in which case the bad type agent would never over-experiment as the extra benefit is so small. This also implies that a pooling equilibrium with learning would not exist in this case.

Proposition 3.4.1.2.c) suggests that the pooling equilibria with learning would only survive when the value-cost ratio is high enough, $\frac{M}{c} > \max\left\{\frac{1}{1-\theta}, \frac{1}{p_0}\right\}$. Among these equilibria, the lowest possible number of successes reported by the potential good type agent is $\hat{k} + 2$. Intuitively, on the one hand, the lowest number must be higher than the participation threshold, otherwise the extra benefit is not enough to support a bad type to over-experiment on the equilibrium path; on the other hand, the reported successes now are less informative since the principal knows at least one bad type agent over-experiments on the equilibrium path— the one who has the strongest incentive to do so as he only need one more successes to pretend to be a good type. The less informative evidence would make the principal assign a lower reward level to the agent, which mitigates the bad type's over-experimentation incentive. Specifically, suppose the equilibrium exists at $k = \hat{k} + 1$, the principal believes that $p^P_{\left(\hat{k}+1,0\right)} = p_{\hat{k}}$ on the equilibrium path, and would assign $a\left(\hat{k}+1, 0\right) = p_{\hat{k}} M$. But the conditional expected value now is lower than the cost of over-experimenting and acquiring one more success, in which case no bad type agents want to over-experiment. Therefore, the pooling equilibria can be found at $\hat{k} + 1 < k \leq \hat{k}$. In such an equilibrium, all the bad type agents whose first failures occur after the $\hat{k} + l + 1_{th}$ experiment would continue experimenting until k successes are acquired. This is because the extra benefit $p_{\hat{k}+l}M$ can maximumly cover the total expected cost of acquiring $k-\hat{k}-l$ more successes for the bad type agent, $\frac{k-\hat{k}-l+1}{1-\theta}c < p_{\hat{k}+l}M \leq \frac{k-\hat{k}-l}{1-\theta}c$. Meanwhile, it also requires that the agent's expected payoff in his initial plan is positive, $U_O^k(p_0) = p_0 M - \sum_{i=1}^{\hat{k}+l} \frac{p_0}{p_{i-1}}c - \frac{p_0(k-\hat{k}-l)}{p_{\hat{k}+l-1}}c$.

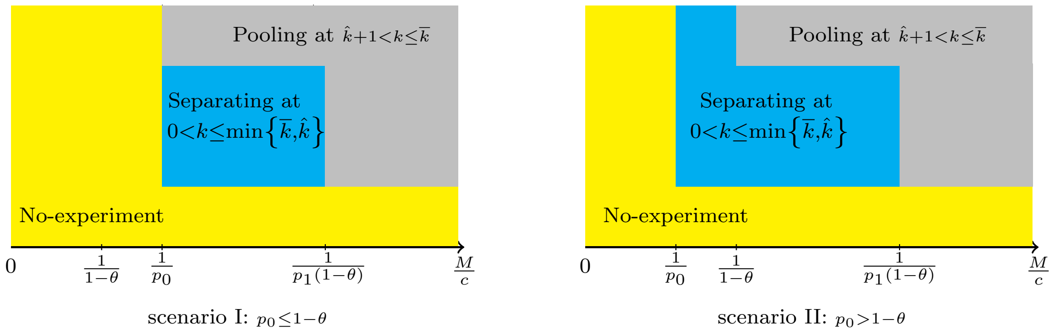

Figure 3.1: Existence of three types of equilibria in different value-cost ratios

Proposition 3.4.2. *The agent is weakly better off with public experimentation, but the principal is weakly better off with private experimentation.*

Compared to public experimentation, Proposition 3.4.2 suggests that private experimentation makes the agent weakly worse off but the principal weakly better off. This is mainly driven by the principal's skeptical thinking. When the experiments and results are public, the principal doesn't need to worry about the possibility of over-experimentation, as the beliefs of the principal and the agent are aligned. However, in private experimentation, the principal has different concerns when a certain number of successes reported. On the one hand, the principal knows that the agent has the opportunity for additional experiments if the cost is not too large, so she is skeptical about the agent's type when the number of reported successes is very small as she might think that the agent has failed; on the other hand, when a large number of successes is reported, she might also think that a bad type may have achieve it by over-experimentation. This result is similar to that in Henry (2009) and Felgenhauer and Loerke (2013). Henry (2009) discusses the scenario in which the agent pre-commits to a number of experiments and he cannot stop until all of the experiments have been conducted. He argued that the agent runs strictly less experiments in the public case. In contrast, my work analyses the scenario where the agent chooses whether to continue or stop experimenting after each result is realised, and the agent's commitment would be discussed in the next section. Felgenhauer and Loerke (2013) find that there exists a deterrence effect which causes both the principal and the agent to be better off in private experimentation with exogenous precision of the experiment, but this result doesn't hold in my work. An example is also given below.

Example 3.4.1. Suppose $p = \theta = 0.7$, $M = 3.5$ and $c = 1$. Thus $\overline{k} = 2$ and $\hat{k} = 1$. In public experimentation, the agent doesn't run experiments, thus $U_P(p_0) = a^P(0,0) = p_0 M = 2.45$, $V_P(p_0) = -p_0(1-p_0)M^2 = -2.5725$. In private experimentation, in the no-experiment equilibrium given restrictions on the principal's off-equilibrium path belief (3.4.4) is satisfied, $U_N(p_0) = U_P(p_0) = p_0 M = 2.45$ and $V_N(p_0) = V_P(p_0) = -2.5725$. In the separating equilibrium with learning, $p^P_{(1,0)} = p_1 \approx 0.886076$, $a^E(1,0) = p_1 M \approx 3.10127$, $a^E(0,0) = 0$, $U^1_S(p_0) = p_0 M - c = 1.45$ and $V^1_S(p_0) = -p_0(1-p_1)M^2 \approx 0.976899$. The pooling equilibrium with learning doesn't exist since $\hat{k}+1 = \overline{k}$.

Proposition 3.4.3 demonstrates how the participation threshold $\overline{k}$ and over-experimentation threshold $\hat{k}$ vary when the value-cost ratio $\frac{M}{c}$, prior belief p_0 and the experiment's "pass threshold" θ for the bad type agent.

Proposition 3.4.3. *$\overline{k}$ is weakly increasing in $\frac{M}{c}$, p_0 and θ, and $\hat{k}$ is weakly decreasing in $\frac{M}{c}$ and p_0.*

When the value-cost ratio increases, on the one hand, the agent becomes relatively more valuable, and his prior expected value can cover more experiments' total expected cost. Thus, he is able to conduct more experiments in his initial plan at the beginning, which leads the participation threshold $\overline{k}$ to increase. On the other hand, the larger value-cost ratio also gives the bad type agent stronger incentive to over-experiment, since the extra benefit is larger than before. With the presence of skeptical thinking, the principal believes that more bad type agents would now over-experiment. Thus the informativeness of the reported successes is lower, which follows that the over-experimentation threshold $\hat{k}$ decreases.

When the agent's prior belief increases, the agent's prior expected value is higher, which tends to raise the participation threshold. However, a higher prior belief also means that the expected total cost forthe same amount of experiments is also higher, and this negative effect tends to reduce the participation threshold. As a compounded effect, the analysis shows that the positive effect dominates, which leads the participation threshold $\overline{k}$ to be increasing. Meanwhile, when the agent is very optimistic on that he is a good type, he has a stronger incentive to over-experiment after he fails. This is because his loss from the failure is larger than that when he is pessimistic with a lower prior belief.

When θ increases, it implies that it's harder for a bad type agent to achieve a success in an experiment. In the initial plan, the agent remains uninformed. A higher θ implies that the failure arrives faster if the agent is a bad type. Thus, if he plans to stop once a failure occurs, the expected total cost of the given number of experiments falls. As a result, he can plan to more experiments, which leads a higher participation threshold. However, the compounded effect of the "pass threshold" θ on the over-experimentation threshold is ambiguous. On the one hand, a higher θ implies that the agent learns faster when a success arrives when he hasn't failed yet, in which the posterior value is higher after reporting the successes. Therefore the extra benefit of over-experimenting is larger for a bad type. On the other hand, the agent's expected cost of acquiring one more success, $\frac{c}{1-\theta}$, is also higher, which mitigates the agent's incentive to over-experiment. As a result, the two forces acting opposite directions mean that the compounded effect is ambiguous.

Notice that the upper bound of the set of equilibria is determined by the participation threshold, thus an increasing in $\overline{k}$ also implies that the entire set is expanding, given that the restrictions on the off-equilibrium path belief are still satisfied. This result is achieved when the value-cost ratio or the prior belief increases. However, at the same time, the over-experimentation threshold is decreasing as it's harder to mitigate the agent's incentive for over-experimenting. Thus, given that the same amount of successes are reported, the principal tends to discount their informativeness due to skeptical thinking. Therefore, the set of separating equilibria with learning shrinks, which exists when the reported number of successes are less than both the participation and over-experimentation thresholds. With the increasing $\overline{k}$, the set of pooling equilibria with learning expands. These results are summarised in Corollary

3.4.1.

Corollary 3.4.1. *As $\frac{M}{c}$ or p_0 increases, the set of separating equilibria with learning shrinks but the set of pooling equilibria with learning tends to expand.*

Proof. Given (3.4.4) holds, the set of separating equilibria with learning exist at $0 < k \leq \min\left\{\overline{k}, \hat{k}\right\}$, and the set of pooling equilibria with learning exist at $\hat{k} + 1 < k \leq \overline{k}$. From Proposition 3.4.3, $\overline{k}$ increases and $\hat{k}$ decreases as $\frac{M}{c}$ or p_0 increases. Thus, $\min\left\{\overline{k}, \hat{k}\right\}$ is decreasing as well as $\hat{k} + 1$. As a result, $\left\{k \in \mathbb{N} : 0 < k \leq \min\left\{\overline{k}, \hat{k}\right\}\right\}$ shrinks, but $\left\{k \in \mathbb{N} : \hat{k} + 1 < k \leq \overline{k}\right\}$ gets larger. □

3.5 The agent's commitment

In this section, the agent now can commit to report a certain number of successes to prove that he is a potential good type before he conducts experiment(s). The timing of the game is changed as follows:

1. The agent commits to report $k \geq 0$ successes;
2. The agent runs experiments;
3. The agent selectively reports the experimental results;
4. The principal gives the reward to the agent according to the report;
5. Payoffs are realised.

With the help of the commitment, the agent can pick a preferred equilibrium that maximises his own expected payoff, but the credibility of the commitment needs to be considered. The agent can possibly commit not to run any experiments or report any successes. In this case, the agent refuses to learn through experiments, and he receives his prior expected value $p_0 M$ as a reward if it's credible. However, the principal worries that the agent might deviate and continue experimenting, in which case the commitment is not credible. Such deviation occurs when the restriction on the off-equilibrium path belief (3.4.4) is violated at $k = 0$. If so, the agent compromises to commit to report a strictly positive number of successes. When committing to report $k > 0$ successes, the agent also implicitly proves that he must be a bad type when less than k successes are reported. Thus the principal's posterior belief

$p^P_{(k,0)}$ is updated in the same way as that in the scenario where the agent cannot commit. In the first place, the agent would only commit to report a number of successes which is smaller than the participation threshold $k \leq \overline{k}$, otherwise he would achieve negative expected payoff. In the second place, the agent's over-experimentation incentive still affects the informativeness of the evidence. The arguments are summarised in Proposition 3.5.1, which characterises the agent's optimal commitment and the existence condition.

Proposition 3.5.1. *In private experiment, the agent optimally commits to report k^* successes to prove that he is a potential good type, where*
1) $k^* = \min\left\{k \in \mathbb{N} : 0 \leq k \leq \min\left\{\overline{k}, \hat{k}\right\}\right\}$ *if* $\exists k$ *such that (3.4.4) is satisfied at* $0 \leq k \leq \min\left\{\overline{k}, \hat{k}\right\}$*;*
2) $k^* = k_p \in \underset{k \in \mathbb{N},\ \hat{k}+1<k\leq\overline{k}}{\arg\max} U^k_O(p_0)$ *if (3.4.4) is only satisfied at* $\hat{k}+1 < k \leq \overline{k}$*;*
3) otherwise, k^* *doesn't exist.*

Proposition 3.5.1.1) suggests that the agent tends to commit to report the smallest number of successes which satisfy the restrictions on the principal's off-equilibrium path belief (3.4.4) in the region where both the participation and over-experimentation incentives are satisfied, $0 < k \leq \min\left\{\overline{k}, \hat{k}\right\}$. In this case, the bad type agent doesn't over-experiment, and the reported successes are very informative. Therefore, his expected payoff would be the same as that when the agent cannot commit, $U^k_S(p_0) = p_0 M - \sum_{i=1}^{k} \frac{p_0}{p_{i-1}} c$. Since the agent's expected payoff is weakly decreasing as k increases in this case, to maximise his own expected payoff, the agent would optimally choose the lowest number of successes in which (3.4.4) is still satisfied. Ideally, the agent would only commit to report one success. However, if (3.4.4) at $k = 1$ is violated, the agent would always deviate to report more, in which case such a commitment is not credible. Therefore, the agent compromises to commit to report two successes. Similar argument can continue until the agent find the lowest number of successes in this region which satisfied the restriction (3.4.4). In this region, the agent's interest contrasts with the principal's since the principal is strictly better off when the reported number of successes increases.

If (3.4.4) fails in the previous region, the agent seeks to commit to report $\hat{k} + 1 < k \leq \overline{k}$ successes. Proposition 3.5.1.2) shows that the agent would commit to report the number of successes which maximise his expected payoff in this region, $U_O^k(p_0)$. The principal learns that some bad type agents have an incentive to over-experiment, which is the same as in the scenario when the agent cannot commit, and the agent's expected payoff would be $U_O^k(p_0) = p_0 M - \sum_{i=1}^{\hat{k}+l} \frac{p_0}{p_{i-1}} c - \frac{p_0(k-\hat{k}-l)}{p_{\hat{k}+l-1}} c$. Notice that, in this case, the expected total cost is higher than that when the agent commits to report $0 < k \leq \min\left\{\overline{k}, \hat{k}\right\}$ successes, so this region is strictly dominated, and the agent would only commit to report a number of successes in this region when the restriction on the off-equilibrium path belief (3.4.4) is violated in other regions.

Meanwhile, the expected total cost is not monotonic with respect to the number of successes committed to report, so the agent might not always prefer to commit to report a lower number in this region. This is because, on the one hand, reporting a larger number of successes implies that the agent needs to run more experiments, and the total expected cost tends to increase; on the other hand, such a larger number also brings a weakly higher reward, and the agent's incentive of over-experimenting is much stronger, where more cost from over-experimentation can be covered and the total expected cost tends to decrease. Moreover, if the restriction (3.4.4) is not satisfied at this optimal choice, the agent compromises to another sub-optimal choice in this region, which maximise his expected payoff except the first choice. This argument would stop until (3.4.4) is satisfied. As a result, it also implies that the agent might commit to report the highest number $\overline{k}$, which is most preferred by the principal among all equilibria.

If (3.4.4) fails in the region $0 \leq k \leq \overline{k}$, Proposition 3.5.1.3) suggests that the agent cannot make a credible commitment. Also, there doesn't exist any equilibrium in private experimentation when the agent cannot commit, which has been discussed in previous section.

3.6 Conclusion

This chapter characterises the properties of three different types of equilibria in private experimentation as well as the conditions for their existence, in which the experiment has an asymmetric information structure. When the restriction on the principal's off-equilibrium path belief is satisfied, a no-experiment equilibrium can possibly exist regardless of the agent's value-cost ratio, the separating equilibria with learning only exist at the medium level of value-cost ratio, and pooling equilibria with learning exist when the value-cost ratio is not too low. The participation threshold determines the upper bound of the equilibria, and the existence of an over-experimentation threshold determines the boundary between separating and pooling equilibria with learning. Also, the results suggest that the agent is worse off but the principal is better off relative to public experimentation. When the agent can commit before experimenting, he tends to optimally commit to report a small number of successes to prove that he is a potential good type. But, constrained by the principal's off-equilibrium path belief, the agent might commit to report a lager number.

Since this project is still work in progress, there is room to improve it. In future study, it would be interesting to extend this model to a finite multiple stage game, in which the agent can still experiment and disclose results after his first report.

Appendix A

Proofs for Chapter 1

Proof of Lemma 1.3.1

The efficient stopping threshold p^E just makes (1.3.6) binding. This implies that: when the synergy is positive, $p^E = \frac{2c}{(2+\theta)\lambda R} \leq \frac{c}{\lambda R}$; when the synergy is negative, $p^E = \frac{c}{\lambda R} < \frac{2c}{(2+\theta)\lambda R}$.

In the first-best, agent(s) would accept the contract as long as the participation constraint(s) bind(s). Thus, it can be only focused on $u_t^i(p_t^P)$. When the partnership is motivated, agent two's participation constraint must bind in the first-best. From (1.3.4), the optimal sub-contract satisfies: $\omega_{2,t}^P(p_t^P) = \frac{c}{\omega_{21,t}^P(p_t^P)(2+\theta)\lambda R p_t^P}$. Thus, agent one's participation constraint can be simplified as:

$$p_t^P \omega_{1,t}^P(p_t^P)(2+\theta)\lambda R - 2c \geq 0 \tag{A.0.1}$$

Having (A.0.1) binding, the optimal grand contract in the first-best would be $\omega_{1,t}^P(p_t^P) = \frac{2c}{(2+\theta)\lambda R p_t^P}$. Substituting it back to (A.0.1), the optimal sub-contract in the first-best would be $\omega_{2,t}^P(p_t^P) = \frac{1}{2}$. In this case, the principal's profit at t would be: $\pi_t(p_t^P, \omega_{i,t}^P) = p_t^P(2+\theta)\lambda R - 2c$.

When the individual work is motivated, agent one's participation constraint of working alone needs to be binding, thus $\omega_{1,t}^P(P_t^P) = \frac{c}{\lambda R p_t^P}$. Since agent one now works alone, he offers nothing to agent two, $\omega_{2,t}^P(p_t^P) = 0$. Therefore, the principal's profit at t in this case would be: $\pi_t(p_t^P, \omega_{i,t}^P) = p_t^P \lambda R - c$.

In the static game at t, the principal prefers the partnership to the individual work if the difference between the principal's profit at t from the partnership and the individual work is positive:

$$\triangle\pi_t^F(p_t^P,\omega_{i,t}^P) = p_t^P(1+\theta)\lambda R - c \geq 0 \quad \Leftrightarrow \frac{R}{c}\frac{1}{(1+\theta)\lambda p_t^P} \Leftrightarrow p_t^P \geq \frac{c}{(1+\theta)\lambda R} \tag{A.0.2}$$

When the synergy is positive, $\triangle\pi_t^P(p_t^F,\omega_{i,t}^P) \geq 0$ since $p_t^P \geq p^E \geq \frac{c}{(1+\theta)\lambda R}$, which implies that the partnership is always preferred in this case. On the other hand, when the synergy is negative, $\frac{c}{(1+\theta)\lambda R} > \frac{c}{\lambda R} = p^E$. Thus, the partnership still dominates the individual work if $\frac{R}{c} \geq \frac{1}{(1+\theta)\lambda p_t^P}$ as (A.0.2) is still satisfied; if $\frac{R}{c} \in \left[\frac{1}{\lambda p_t^P}, \frac{1}{(1+\theta)\lambda p_t^P}\right)$, (A.0.2) doesn't hold and the individual work dominates in this case.

Proof of Lemma 1.3.2

Firstly, I prove that the principal always prefers to invest in the project at $t = 0$ as long as $p_0 \geq p^E$. Consider when the principal sticks to the choice in the static game at $t = 0$. Depending on the individual work and the partnership is optimally motivated at $t = 0$ respectively, the difference between the principal's continuation value from investing and not investing would be:

$$\begin{aligned}\triangle V_t^I(p_0) =& \pi_0(p_0^P,\omega_{i,0}^P) - \delta\pi_0(p_0^P,\omega_{i,0}^P) \\ &+ \delta\left\{[1-p_0\lambda]\pi_1(\hat{p}_1,\omega_{i,1}^P),[1-p_0(2+\theta)\lambda]\pi_1(p_1,\omega_{i,1}^P)\right\}\end{aligned} \tag{A.0.3}$$

When the principal doesn't invest at $t = 0$, the posterior belief equals to the prior belief, thus her gain at $t = 1$ would be the same as that in the static game, and it's discounted due to the presence of a discount factor. This is represented in the second term in (A.0.3). Since $\pi_t(p_t^P,\omega_{i,t}^P) \geq 0$ and $\delta \in (0,1)$, $\triangle V_t^I(p_0)$ is always positive. As a result, no-investment at $t = 0$ is dominated for $p_0 \geq p^E$.

Now the focus shifts to the choice between the partnership and the individual work. At $t = 0$, in the first best, the principal would motivate the partnership if the difference between the principal's continuation value from the partnership and the individual work is positive:

$$\triangle V_t^F(p_0) = \triangle\pi_0^F(p_0^P,\omega_{i,0}^P) + \delta[1-p_0(2+\theta)\lambda]\pi_1(p_1,\omega_{i,1}^P) - \delta(1-p_0\lambda)\pi_1(\hat{p}_1,\omega_{i,1}^P) \geq 0 \tag{A.0.4}$$

$$\implies \quad \triangle\pi_0^F(p_0^P, \omega_{i,0}^P) \geq \delta(1-p_0\lambda)\pi_1(\hat{p}_1, \omega_{i,1}^P) - \delta[1-p_0(2+\theta)\lambda]\pi_1(p_1, \omega_{i,1}^P) \tag{A.0.5}$$

Since $\hat{p}_1 > p_1$, $\pi_1(\hat{p}_1, \omega_{i,1}^P) \geq \pi_1(p_1, \omega_{i,1}^P)$. Together with $\theta > -1$, the right hand side of (A.0.5) is always positive, $\delta(1-p_0\lambda)\pi_1(\hat{p}_1, \omega_{i,1}^P) - \delta[1-p_0(2+\theta)\lambda]\pi_1(p_1, \omega_{i,1}^P) \geq 0$. Thus the right hand side reaches the maximum when $\delta = 1$.

Consider the situation where the synergy is positive. When $\frac{R}{c} \geq \frac{2}{(2+\theta)\lambda p_1}$, from Lemma 1.3.1.2), the principal would motivate the partnership at $t = 1$ no matter whether the partnership or the individual work is motivated at $t = 0$, and

$$\begin{aligned}
\triangle V_t^F(p_0) &\geq \triangle\pi_0^F(p_0^P, \omega_{i,0}^P) - (1-p_0\lambda)\pi_1(\hat{p}_1, \omega_{i,1}^P) + [1-p_0(2+\theta)\lambda]\pi_1(p_1, \omega_{i,1}^P) \\
&= [1-p_0\lambda(2+\theta)]\left[p_1\lambda(1+\theta)R - \frac{1-2p_0\lambda(1+\theta)}{1-p_0\lambda(2+\theta)}c\right] \\
&\propto p_1\lambda(1+\theta)R - \frac{1-2p_0\lambda(1+\theta)}{1-p_0\lambda(2+\theta)}c \geq \frac{2+2\theta}{2+\theta}c - \frac{1-2p_0\lambda(1+\theta)}{1-p_0\lambda(2+\theta)}c \\
&\propto (2+2\theta)\left[1-p_0\lambda(2+\theta)\right] - (2+\theta)\left[1-2p_0\lambda(1+\theta)\right] = \theta \geq 0
\end{aligned} \tag{A.0.6}$$

This implies that (A.0.4) holds for $\forall\delta \in [0,1]$, thus partnership is preferred at $t = 0$. When $\frac{R}{c} \in \left[\frac{2}{(2+\theta)\lambda\hat{p}_1}, \frac{2}{(2+\theta)\lambda p_1}\right)$, according to Lemma 1.3.1.2), the principal would not invest at $t = 1$ if the partnership is motivated at $t = 0$, and

$$\begin{aligned}
\triangle V_t^F(p_0) &\geq \triangle\pi_0^F(p_0^P, \omega_{i,0}^P) - (1-p_0\lambda)\pi_1(\hat{p}_1, \omega_{i,1}^P) + [1-p_0(2+\theta)\lambda]\pi_1(p_1, \omega_{i,1}^P) \\
&= [1-p_0\lambda(2+\theta)]\left[\frac{1-2p_0\lambda}{1-p_0\lambda(2+\theta)}c - p_1\lambda R\right] \\
&\propto \frac{1-2p_0\lambda}{1-p_0\lambda(2+\theta)}c - p_1\lambda R \geq \frac{1-2p_0\lambda}{1-p_0\lambda(2+\theta)}c - \frac{2}{2+\theta}c \\
&\propto (2+\theta)(1-2p_0\lambda) - 2\left[1-p_0\lambda(2+\theta)\right] = \theta \geq 0
\end{aligned} \tag{A.0.7}$$

This implies that (A.0.4) holds for $\forall\delta \in [0,1]$, and then the partnership is preferred at $t = 0$. When $\frac{R}{c} \in \left[\frac{2}{(2+\theta)\lambda p_0}, \frac{2}{(2+\theta)\lambda\hat{p}_1}\right)$, the principal never invests at $t = 1$ since the posterior belief falls below the efficient stopping threshold, $p_1 < \hat{p}_1 < p^E$. Thus at $t = 0$, the principal's choice is the same as that in the static game. Since $\theta \geq 0$, the partnership is preferred.

Consider the situation in which the synergy is negative. With Lemma 1.3.1.2), when $\frac{R}{c} \geq \frac{1-2p_0\lambda(1+\theta)}{p_0\lambda(1+\theta)[1-\lambda(2+\theta)]} > \frac{1}{(1+\theta)\lambda p_1}$, the principal would motivate the partnership at $t=1$ no matter whether the partnership or the individual work is motivated at $t=0$, then

$$\begin{aligned} \triangle V_t^F(p_0) &\geq \pi_0^F(p_0^P, \omega_{i,0}^P) - (1-p_0\lambda)\pi_1(\hat{p}_1, \omega_{i,1}^P) + [1-p_0(2+\theta)\lambda]\pi_1(p_1, \omega_{i,1}^P) \\ &= p_0\lambda(1+\theta)[1-\lambda(2+\theta)]R - [1-2p_0\lambda(1+\theta)]\,c \\ &\geq [1-2p_0\lambda(1+\theta)]\,c - [1-2p_0\lambda(1+\theta)]\,c = 0 \end{aligned} \tag{A.0.8}$$

This implies that (A.0.8) holds for $\forall\delta[0,1]$, and then the partnership is preferred at $t=0$. When $\frac{R}{c} \in \left[\frac{1}{\lambda p_0}, \frac{1}{(1+\theta)\lambda p_0}\right)$, $\triangle\pi_0^F(p_0^P, \omega_{i,0}^P) < 0$ then (A.0.5) doesn't hold as its right hand side is positive. Thus the principal would motivate the individual work at $t=0$. When $\frac{R}{c} \in \left[\frac{1}{(1+\theta)\lambda p_1}, \frac{1-2p_0\lambda(1+\theta)}{p_0\lambda(1+\theta)[1-\lambda(2+\theta)]}\right)$, the principal would still motivate the partnership at $t=1$. Thus (A.0.4) and (A.0.5) imply that

$$\begin{aligned} \delta &\leq \frac{p_0\lambda(1+\theta)R - c}{p_0\lambda^2(1+\theta)(2+\theta)R - 2p_0\lambda(1+\theta)c} \\ &= 1 + \frac{p_0\lambda(1+\theta)[1-\lambda(2+\theta)]R - [1-2p_0\lambda(1+\theta)]c}{p_0\lambda^2(1+\theta)(2+\theta)R - 2p_0\lambda(1+\theta)c} \\ &\leq 1 + \frac{[1-2p_0\lambda(1+\theta)]c - [1-2p_0\lambda(1+\theta)]c}{p_0\lambda^2(1+\theta)(2+\theta)R - 2p_0\lambda(1+\theta)c} = 1 \end{aligned} \tag{A.0.9}$$

Therefore, (A.0.4) holds if $\delta \in [0, \delta^E)$, where $\delta^E = \frac{p_0\lambda(1+\theta)R-c}{p_0\lambda^2(1+\theta)(2+\theta)R-2p_0\lambda(1+\theta)c}$. When $\frac{R}{c} \in \left[\frac{1}{(1+\theta)\lambda\hat{p}_1}, \frac{1}{(1+\theta)\lambda p_1}\right)$, according to Lemma 1.3.1.2), the principal would motivate the partnership only if the individual work is motivated at $t=0$, thus (A.0.4) and (A.0.5) imply that

$$\begin{aligned} \delta &\leq \frac{\triangle\pi_0^F(p_0^P, \omega_{i,0}^P)}{(1-p_0\lambda)\left[p_1\lambda(2+\hat{\theta})R - 2c\right] - [1-p_0\lambda(2+\theta)]\max\{p_1\lambda R - c, 0\}} \\ &\leq \frac{p_0\lambda(1+\theta)R - c}{p_0\lambda(1+\theta)R - p_0\lambda(1+\theta)c} = 1 - \frac{[1-p_0\lambda(1+\theta)]\,c}{p_0\lambda(1+\theta)R - p_0\lambda(1+\theta)c} < 1 \end{aligned} \tag{A.0.10}$$

As a result, (A.0.4) holds if $\delta \in [0, \delta^E)$, where $\delta^E = \frac{p_0\lambda(1+\theta)R-c}{p_0\lambda(1+\theta)(\lambda R-c)}$ in this case. When $\frac{R}{c} \in \left[\frac{1}{(1+\theta)\lambda p_0}, \frac{1}{(1+\theta)\lambda \hat{p}_1}\right)$, according to Lemma 1.3.1.2), the principal would not motivate the partnership at $t=0$, then (A.0.4) and (A.0.5) imply that

$$\begin{aligned}\delta &\leq \frac{\triangle\pi_0^F(p_0^P, \omega_{i,0}^P)}{(1-p_0\lambda)(p_1\lambda\hat{R}-c) - [1-p_0\lambda(2+\theta)]\max\{p_1\lambda R - c, 0\}} \\ &\leq \frac{p_0\lambda(1+\theta)R-c}{p_0\lambda(1+\theta)(\lambda R-c)} = 1 + \frac{p_0\lambda(1-\lambda)(1+\theta)R - [1-p_0\lambda(1+\theta)]c}{p_0\lambda(1+\theta)(\lambda R-c)} \\ &= 1 + \frac{(1-p_0\lambda)c - [1-p_0\lambda(1+\theta)]c}{p_0\lambda(1+\theta)(\lambda R-c)} = 1 + \frac{p_0\lambda\theta c}{p_0\lambda(1+\theta)(\lambda R-c)} < 1\end{aligned} \tag{A.0.11}$$

As a result, (A.0.4) holds if $\delta \in [0, \delta^E)$, where $\delta^E = \frac{p_0\lambda(1+\theta)R-c}{p_0\lambda(1+\theta)(\lambda R-c)}$ in this case.

Now check the efficiency level between the two-tier and three-tier structure in the first-best. In the two-tier structure, in which the principal can directly offer contracts to both of the agents, Denote the contract offered to the agent i at t by $s_{i,t}$, where $s_{i,t} \in [0,1]$, $i = 1,2$ and $t = 0,1$. Thus, $\omega_{1,t} = s_{1,t} + s_{2,t}$ and $\omega_{1,t}\omega_{2,t} = s_{2,t}$. In the first best, to motivate the agent(s) to work, the participation constraint(s) need(s) to be binding:

$$\begin{aligned} & U_t^i(p_t^i) = p_t^i s_{i,t}\lambda(2+\theta)R - c + \delta[1-p_t^i\lambda(2+\theta)]U_{t+1}^i(p_{t+1}^i) \geq 0 \\ \text{or} \quad & U_t^i(p_t^i) = p_t^i s_{i,t}\lambda R - c + \delta(1-p_t^i\lambda)U_{t+1}^i(p_{t+1}^i) \geq 0 \end{aligned} \tag{A.0.12}$$

This is the same as that in (1.3.6). Therefore, the analysis and the conclusion should be the same as that in the three-tier structure.

Proof of Lemma 1.4.1

When the synergy is positive, to motivate the partnership, (1.4.1), (1.4.2) and (1.4.3) and imply that

$$p_t^P\omega_{1,t}^S(p_t^P)(1+\theta)\lambda R = \max\left\{2c, \frac{2+\theta}{1+\theta}c\right\} \text{ and } \omega_{2,t}(p_t^P) = \frac{c}{\omega_{1,t}(p_t^P)\lambda(1+\theta)Rp_t^P} \tag{A.0.13}$$

Notice that $2 \geq \frac{2+\theta}{1+\theta}$ in this case, so $\left(\omega_{1,t}^S(p_t^P), \omega_{1,t}^S(p_t^P)\right) = \left(\frac{2c}{(1+\theta)\lambda R p_t^P}, \frac{1}{2}\right)$, and the principal's profit at t now is $\pi_(p_t^P, \omega_{1,t}^S(p^*)) = p_t^P\lambda(2+\theta)R - \frac{2(2+\theta)}{1+\theta}c$. On the other hand, to motivate the individual work, agent one's participation constraint of working alone needs to be binding as no return would be generated if he shirks. This implies that $\left(\omega_{1,t}^S(p_t^P), \omega_{1,t}^S(p_t^P)\right) = (c, 0)$ and the principal's profit now is $\pi(p_t^P, \omega_{1,t}^S(p^*)) = p_t^P \lambda R - c$. If agent one deviates to collaborate with agent two, from Table 1.1, it shows that agent two would accept the offer if $\theta \geq 1$. If so, the principal would always motivate the partnership as long as the profit is positive. As as result, the optimal stopping threshold in this case would be $p^* = \frac{2c}{(1+\theta)\lambda R}$. If $0 \leq \theta < 1$, the individual work can still be motivated as agent two would reject if agent one deviates to collaborate. In this case, the principal would only motivate the partnership if the difference between the principal's profit from the collaboration and the individual work is positive:

$$\triangle\pi(p_t^P, \omega_{1,t}^S(p^*)) = p_t^P \lambda(1+\theta)R - \frac{3+\theta}{1+\theta}c \geq 0 \quad \Longrightarrow \quad \frac{R}{c} \geq \frac{3+\theta}{(1+\theta)^2\lambda p_t^P} \tag{A.0.14}$$

Then the principal is indifferent when $p_t^P = p^T = \frac{(3+\theta)c}{(1+\theta)^2\lambda R}$. Therefore, in this case, the principal would motivate the partnership if $\frac{R}{c} \geq \frac{3+\theta}{(1+\theta)^2\lambda p_t^P} > \frac{2}{(1+\theta)\lambda p_t^P}$; and she would motivate the individual work if $\frac{1}{\lambda p_t^P} \leq \frac{R}{c} < \frac{3+\theta}{(1+\theta)^2\lambda p_t^P}$. As a result, the stopping threshold is $p^* = \frac{c}{\lambda R}$.

When the synergy is negative, $\frac{2+\theta}{1+\theta} > 2$ in (A.0.13). To motivate the partnership, $\left(\omega_{1,t}^S(p_t^P), \omega_{1,t}^S(p_t^P)\right) = \left(\frac{(2+\theta)c}{(1+\theta)^2\lambda R p_t^P}, \frac{1+\theta}{2+\theta}\right)$, and the principal's profit is $\pi(p_t^P, \omega_{1,t}^S(p^*)) = p_t^P\lambda(2+\theta)R - \left(\frac{2+\theta}{1+\theta}\right)^2 c$. On the other hand, to motivate the individual work, the optimal contracts should be the same as those in which the synergy is positive. This is because the participation constraint is the same. It implies that $\left(\omega_{1,t}^S(p_t^P), \omega_{1,t}^S(p_t^P)\right) = (c, 0)$, and the principal's profit in this case is $\pi(p_t^P, \omega_{1,t}^S(p^*)) = p_t^P\lambda R - c$. Therefore, the principal would only motivate the partnership if the difference between the principal's profit from the collaboration and the individual work is positive:

$$\triangle\pi(p_t^P, \omega_{1,t}^S(p^*)) = p_t^P \lambda(1+\theta)R - \frac{3+2\theta}{(1+\theta)^2}c \geq 0 \quad \Longrightarrow \quad \frac{R}{c} \geq \frac{3+2\theta}{(1+\theta)^3\lambda p_t^P} \tag{A.0.15}$$

Then the principal is indifferent when $p_t^P = p^T = \frac{(3+2\theta)c}{(1+\theta)^3\lambda R}$. Notice that $\frac{(3+2\theta)c}{(1+\theta)^3\lambda R} > \frac{2c}{(1+\theta)\lambda R}$, then the principal would motivate the partnership if $\frac{R}{c} \geq \frac{3+2\theta}{(1+\theta)^3\lambda p_t^P}$. Moreover, since $\frac{(3+2\theta)c}{(1+\theta)^3\lambda R} > \frac{c}{\lambda R}$, the principal would motivate the individual work if $\frac{R}{c} \in \left[\frac{1}{\lambda p_t^P}, \frac{3+2\theta}{(1+\theta)^3 \lambda p_t^P}\right)$. At last, she stops investing when $p_t^P \lambda R - c < 0$. As a result, $p^* = \frac{c}{\lambda R}$.

Proof of Lemma 1.4.2

Consider the situation in which the individual work is motivated at $t = 0$. If agent one sticks to the equilibrium strategy, his continuation value would be:

$$U_0^{1,\mathrm{WA}}(p_0) = p_0 \omega_{1,0} \lambda R - c + \delta(1 - p_0\lambda)\hat{u}_1^1(\hat{p}_1; \hat{p}_1, \hat{p}_1) \tag{A.0.16}$$

Alternatively, if agent one deviates to delegate the entire work to agent two, he must ensure that agent two is weakly better off by working:

$$\underbrace{p_0 \omega_{2,0} \omega_{1,1} \lambda R}_{\text{gain from working}} \geq \underbrace{c}_{\text{gain from shirking}} + \delta \underbrace{\left[\hat{u}_1^2(p_0; \hat{p}_1, \hat{p}_1) - (1 - p_0\lambda)\hat{u}_1^2(\hat{p}_1; \hat{p}_1, \hat{p}_1)\right]}_{\text{gain from belief manipulation: } B_1^2(p_0; \hat{p}_1; \hat{p}_1)} \tag{A.0.17}$$

Thus agent one offers a sub-contract such that (A.0.17) binds, and his continuation value would be:

$$\begin{aligned} U_0^{1,\mathrm{CR}}(p_0) &= p_0 \omega_{1,0} \lambda R - c - B_1^2(p_0; \hat{p}_1; \hat{p}_1) + \delta(1 - p_0\lambda)\hat{u}_1^1(\hat{p}_1; \hat{p}_1, \hat{p}_1) \\ &\leq p_0 \omega_{1,0} \lambda R - c + \delta(1 - p_0\lambda)\hat{u}_1^1(\hat{p}_1; \hat{p}_1, \hat{p}_1) = U_0^{1,\mathrm{WA}}(p_0) \end{aligned} \tag{A.0.18}$$

This implies that agent one is weakly better off by working alone rather than complete resourcing. Now consider the other situation in which the partnership is motivated at $t = 0$. If agent one deviates to work alone, his continuation value would be:

$$U_0^{1,\mathrm{E}}(p_0) = p_0 \omega_{1,0} \lambda R - c + \delta(1 - p_0\lambda)\hat{u}_1^1(\hat{p}_1; p_1, \hat{p}_1) \tag{A.0.19}$$

Alternatively, if agent one deviates to complete resourcing and delegate the entire work to agent two, he must ensure that agent two is weakly better off by working rather than shirking:

$$\underbrace{p_0 \omega_{2,0} \omega_{1,1} \lambda R}_{\text{gain from working}} \geq \underbrace{c}_{\text{gain from shirking}} + \delta \underbrace{\left[\hat{u}_1^2(p_0; p_1, \hat{p}_1) - (1 - p_0\lambda)\hat{u}_1^2(\hat{p}_1; p_1, \hat{p}_1)\right]}_{\text{gain from belief manipulation: } B_1^2(p_0; p_1; \hat{p}_1)} \tag{A.0.20}$$

Thus agent one offers a sub-contract such that (A.0.20) binds, and his continuation value would be:

$$
\begin{aligned}
U_0^{1,\mathrm{EC}}(p_0) &= p_0\omega_{1,0}\lambda R - c - B_1^2(p_0;p_1;\hat{p}_1) + \delta(1-p_0\lambda)\hat{u}_1^1(\hat{p}_1;p_1,\hat{p}_1) \\
&\leq p_0\omega_{1,0}\lambda R - c + \delta(1-p_0\lambda)\hat{u}_1^1(\hat{p}_1;p_1,\hat{p}_1) = U_0^{1,\mathrm{E}}(p_0)
\end{aligned}
\tag{A.0.21}
$$

This implies that agent one is weakly better off by excluding agent two relative to complete resourcing. To sum up, the exclusion incentive dominates complete resourcing incentive.

Proof of Lemma 1.4.3

When the individual work is motivated at $t=0$ but agent one deviates to form the partnership, after no success occurs, the principal's posterior belief is $\hat{p}_1$ and the agents have the belief p_1. At $t=1$, the principal offers $\omega_{1,1}^*(p_1^P)$, and agent two would accept the sub-contract if $\omega_{2,1} \geq \omega_{2,1}^*(p_1)$. Since period $t=1$ is the last period in this economy, it's analysis should be the same as that in the static game. Thus, $\omega_{1,1}^*(\hat{p}_1) = \omega_{1,1}^S(p_1)$ and $\omega_{2,1}^*(p_1) = \omega_{2,1}^S(p_1)$. To mitigate agent two's free-riding problem, agent two's free-riding incentive constraint binds, $\omega_{2,1}^*(p_1) = \frac{c}{p_1\omega_{1,1}^*(\hat{p}_1)(1+\theta)\lambda R}$. The following discussion checks if agent one's free-riding incentive constraint (A.0.22) is satisfied:

$$
p_1\omega_{1,1}^*(\hat{p}_1)(1+\theta)\lambda R \geq 2c \tag{A.0.22}
$$

If it's satisfied, it needs to be proven that agent one's payoff would be higher by working alone or rejecting the grand contract at $t=1$; If the free-riding incentive is violated, it needs to be shown that either agent two would reject the sub-contract or agent one's expected payoff is lower by free-riding.

If $\hat{p}_1 < p^*$, the principal would not invest at $t=1$, and agent one would not accept a zero paid contract. Thus it can be focused on the case where $\hat{p}_1 \geq p^*$. If $\hat{p}_1 \geq \max\{p^*, p^T\}$, the principal would motivate the partnership at $t=1$, and offers $\omega_{1,1}^*(\hat{p}_1) = \max\left\{\frac{2c}{(1+\theta)\lambda R\hat{p}_1}, \frac{(2+\theta)c}{(1+\theta)^2\lambda R\hat{p}_1}\right\}$. In the scenario with positive synergy, (A.0.22) is not satisfied. Given agent one shirks, agent two's gain would be negative: $\frac{1}{1+\theta}c - c \leq 0$. Thus agent two would reject the sub-contract, leaving agent one working alone. Agent one would then reject the grand contract since his expected payoff from the individual work is negative,

$\frac{p_1}{\hat{p}_1}c - c < 0$. On the other hand, in the scenario with negative synergy, it shows that agent one's payoff from the individual work is higher than that from the collaboration and the free-riding. As a result, agent one's optimal payoff at $t = 1$ would be $\hat{u}_1^1(p_1; \hat{p}_1, p_1) = \max\left\{\frac{p_1}{\hat{p}_1}\frac{1}{1+\theta}c - c, \frac{p_1}{\hat{p}_1}\frac{2+\theta}{(1+\theta)^2}c - c, 0\right\}$

If $\hat{p}_1 \in \left[p^*, p^T\right]$, the principal would motivate the individual work at $t = 1$, and offers $\omega_{1,1}^*(\hat{p}_1) = \frac{c}{\lambda R \hat{p}_1}$. Then the (A.0.22) is not satisfied. When the synergy is positive, agent two would reject the sub-contract. Agent one would also reject the grand contract since his expected payoff from the individual work is negative. When the synergy is negative, agent one still gets negative payoff from free-riding, and he would still reject the grand contract.

Proof of Proposition 1.4.1

Period $t = 1$ is the last period in this economy, so there is no chance for the agent(s) to manipulate the other parties' beliefs. Therefore, the incentives of the agents are the same as those in the static game, which implies that the analysis should be the same. Notice that, on the equilibrium path, each party would have the common belief p_1^P at $t = 1$, therefore, $\omega_{i,1}^*(p_1^P) = \omega_{i,1}^S(p_1^P)$. For $\hat{p}_1 < p^*$, the principal would not invest at $t = 1$ after no success occurs at $t = 0$, since the net profit would be negative if she invests. Therefore, the incentives of the agent(s) are the same as those in the static game, which make the optimal grand and sub-contracts are the same, $\omega_{i,0}^*(p_0) = \omega_{i,0}^S(p_0)$

Now the focus shifts to the scenarios with $\hat{p}_1 \geq p^*$. Consider the situation in which agent one's individual work is motivates at $t = 0$. This requires that (1.4.10) binds, and $\omega_{1,0}^* = \frac{c+\delta B_1^1(p_0;\hat{p}_1,\hat{p}_1)}{\lambda R p_0}$. Meanwhile, (1.4.11) needs to be satisfied, otherwise agent one would deviate.

Consider the other situation in which the partnership is motivated at $t = 0$. From (1.4.5), agent two's free-riding incentive constraint must bind. Thus the optimal sub-contract $\omega_{2,0}^*(p_0)$ must satisfy:

$$\omega_{2,0}^*(p_0) = \frac{c + \delta B_1^2(\hat{p}_1; p_1, p_1)}{\omega_{1,0}^*(p_0)(1+\theta)\lambda R p_0} \tag{A.0.23}$$

Also, when (1.4.5) binds, agent one's continuation value from the collaboration would be:

$$\begin{aligned} U_0^{1,\mathrm{CO}}(p_0) = & p_0\omega_{1,0}(2+\theta)\lambda R - \frac{3+2\theta}{1+\theta}c - \frac{2+\theta}{1+\theta}\delta B_1^2(\hat{p}_1; p_1, p_1) \\ & + \delta[1-p_0\lambda(2+\theta)]\hat{u}_1(p_1; p_1, p_1) \end{aligned} \tag{A.0.24}$$

If agent one deviates to free-ride on the agent 2's work, his continuation value from such deviation is:

$$U_0^{1,\mathrm{FR}}(p_0) = p_0\omega_{1,0}\lambda R - \frac{1}{1+\theta}c - \frac{1}{1+\theta}\delta B_1^2(\hat{p}_1; p_1, p_1) + \delta(1-p_0\lambda)\hat{u}_1(\hat{p}_1; p_1, p_1) \tag{A.0.25}$$

If agent one deviates to exclude the agent 2 and indeed exerts effort, his continuation value is shown in (A.0.19). If he deviates to exclude agent two and shirks, his continuation value would be:

$$U_0^{1,\mathrm{ES}}(p_0) = \delta\hat{u}_1(p_0; p_1, \hat{p}_1) \tag{A.0.26}$$

Therefore, to support the partnership, the principal needs to offer a grand contract such that:

$$U_0^{1,\mathrm{CO}}(p_0) \geq \max\left\{U_0^{1,\mathrm{FR}}(p_0), U_0^{1,\mathrm{E}}(p_0), U_0^{1,\mathrm{ES}}(p_0)\right\} \tag{A.0.27}$$

This is equivalent to that (1.4.8) and (1.4.9) are satisfied.

When the synergy is negative, $\theta \in \left(-1, \min\left\{0, \frac{1-2\lambda}{\lambda}\right\}\right)$, the difference between agent one's continuation value from the free-riding and exclusion-work is:

$$\triangle U_0^{1,\mathrm{FE}}(p_0) = \frac{\theta}{1+\theta}c - \frac{1}{1+\theta}\delta B_1^2(\hat{p}_1; p_1, p_1) + \delta(1-p_0\lambda)\underbrace{[\hat{u}_1(\hat{p}_1; p_1, p_1) - \hat{u}_1(\hat{p}_1; p_1, \hat{p}_1)]}_{\text{"}<0\text{''}} \tag{A.0.28}$$

It's clear that (A.0.28) is negative when $\theta < 0$, in which case agent two's free-riding incentive is dominated by the exclusion-work incentive. Notice that (1.4.9) represents agent one's incentives constraints of working alone and shirking after he has excluded agent two, thus (1.4.8) and (1.4.9) can be simplified as:

$$
\begin{aligned}
IC_{1,0}^{\mathrm{FR}} &: \ \omega_{1,0}p_0\lambda R \geq \frac{2}{1+\theta}c + \frac{\delta\left(B_1^2(\hat{p}_1, p_1, p_1)(1+\theta) + B_1^1(\hat{p}_1; p_1, p_1)\right)}{1+\theta} \\
IC_{1,0}^{\mathrm{EW}} &: \ \omega_{1,0}p_0\lambda R \geq \frac{2+\theta}{(1+\theta)^2}c + \frac{\delta\left[(2+\theta)B_1^2(\hat{p}_1, p_1, p_1) + (1+\theta)B_1^1(\hat{p}_1; p_1, \hat{p}_1)\right]}{(1+\theta)^2} \\
IC_{1,0}^{\mathrm{ES}} &: \ \omega_{1,0}p_0\lambda R \geq \frac{3+2\theta}{(1+\theta)(2+\theta)}c + \frac{\delta\left[(2+\theta)B_1^2(\hat{p}_1, p_1, p_1) + (1+\theta)B_1^1(p_0; p_1, \hat{p}_1)\right]}{(1+\theta)(2+\theta)}
\end{aligned}
\tag{A.0.29}
$$

Notice that the principal would make the $\omega_{1,0}^*$ as low as possible, then the dominant incentive should have the largest right hand side in (A.0.29). The dominant incentive constraint would be binding and the constraints of others would be slack. From (A.0.29), the difference between the right hand side of $IC_{1,0}^{\mathrm{EW}}$ and $IC_{1,0}^{\mathrm{ES}}$:

$$
\triangle\mathrm{RHS}^{\mathrm{WS}} = \frac{1-\theta-\theta^2}{(1+\theta)^2(2+\theta)}c + \delta\left[\frac{B_1^2(\hat{p}; p_1, p_1)}{(1+\theta)^2} + \frac{B_1^1(\hat{p}; p_1, \hat{p}_1)}{1+\theta} - \frac{B_1^1(p_0; p_1, \hat{p}_1)}{2+\theta}\right]
\tag{A.0.30}
$$

It's clear that the first term on the right hand side of (A.0.30) and $B_1^2(\hat{p}_1; p_1, p_1)$ are positive when $\theta \in \left(-1, \min\left\{0, \frac{1-2\lambda}{\lambda}\right\}\right)$. For $p^1 \geq p^T$, this right hand side can be simplified as:

$$
\begin{aligned}
\triangle\mathrm{RHS}^{\mathrm{WS}} &> \delta\left[\frac{B_1^2(\hat{p}; p_1, p_1)}{(1+\theta)^2} + \frac{B_1^1(\hat{p}; p_1, \hat{p}_1)}{1+\theta} - \frac{B_1^1(p_0; p_1, \hat{p}_1)}{2+\theta}\right] \\
&\geq \delta\left[\frac{B_1^1(\hat{p}; p_1, \hat{p}_1)}{1+\theta} - \frac{B_1^1(p_0; p_1, \hat{p}_1)}{2+\theta}\right] \\
&= \delta c\left[\frac{[1-p_0\lambda(2+\theta)](2\theta+3)}{(1+\theta)^2(2+\theta)} - \frac{(1-p_0\lambda)(3+2\theta)}{(1+\theta)^2} + \frac{1}{2+\theta} + \frac{1}{1+\theta}\frac{p_0}{\hat{p}_1}\right] \\
&\propto \frac{1}{1+\theta}\frac{p_0}{\hat{p}_1} - \frac{2+2\theta}{2+\theta} > \frac{1}{\theta} - \frac{2+2\theta}{2+\theta} = \frac{-\theta}{(1+\theta)(2+\theta)} > 0
\end{aligned}
\tag{A.0.31}
$$

Thus agent one's incentive constraint of working dominates the incentive of shirking after excluding agent two, and it needs to be binding. For $p^T > p_1 \geq$

p^*, $\hat{u}_1(p_1;p_1,p_1)=0$, and (A.0.30) can be simplified as:

$$\begin{aligned}\triangle\text{RHS}^{\text{WS}} &> \delta\left[\frac{B_1^2(\hat{p};p_1,p_1)}{(1+\theta)^2}+\frac{B_1^1(\hat{p};p_1,\hat{p}_1)}{1+\theta}-\frac{B_1^1(p_0;p_1,\hat{p}_1)}{2+\theta}\right]\\ &\geq \delta\left[\frac{B_1^1(\hat{p};p_1,\hat{p}_1)}{1+\theta}-\frac{B_1^1(p_0;p_1,\hat{p}_1)}{2+\theta}\right]\\ &= \delta\left[\frac{1-p_0\lambda}{1+\theta}\hat{u}_1(\hat{p}_1;p_1,\hat{p}_1)-\frac{1}{2+\theta}\hat{u}_1(p_0;p_1,\hat{p}_1)\right]\end{aligned} \tag{A.0.32}$$

Notice that, $\hat{u}_1(\hat{p}_1;p_1,\hat{p}_1)=\frac{\hat{p}_1(2+\theta)}{p_1}c-\frac{3+2\theta}{1+\theta}c$ and $\hat{u}_1(p_0;p_1,\hat{p}_1)=\frac{p_0(2+\theta)}{p_1}c-c-\frac{2+\theta}{1+\theta}\frac{p_0}{\hat{p}_1}c$ if $\frac{\hat{p}_1}{p_1}\geq\frac{2+\theta}{(1+\theta)^2}$, then in (A.0.32):

$$\begin{aligned}&\triangle\text{RHS}^{\text{WS}}\\ &>\delta\left[\frac{1-p_0\lambda}{1+\theta}\hat{u}_1(\hat{p}_1;p_1,\hat{p}_1)-\frac{1}{2+\theta}\hat{u}_1(p_0;p_1,\hat{p}_1)\right]\\ &\propto\frac{1-p_0\lambda}{1+\theta}\left[\frac{\hat{p}_1}{p_1}(2+\theta)c-\frac{3+2\theta}{1+\theta}c\right]-\frac{1}{2+\theta}\left[\frac{p_0}{p_1}(2+\theta)c-c-\frac{2+\theta}{1+\theta}\frac{p_0}{\hat{p}_1}c\right]\\ &\propto\frac{1-p_0\lambda(2+\theta)}{2+\theta}-\frac{(1-p_0\lambda)(2+\theta)}{(1+\theta)^2}+\frac{p_0}{\hat{p}_1(1+\theta)}\\ &\geq\frac{p_0}{\hat{p}_1}\frac{(2+\theta)(1-\lambda)}{(1+\theta)^2}-\frac{(1-p_0\lambda)(2+\theta)}{(1+\theta)^2}=\frac{(1-p_0\lambda)(2+\theta)}{(1+\theta)^2}-\frac{(1-p_0\lambda)(2+\theta)}{(1+\theta)^2}\\ &=0\end{aligned} \tag{A.0.33}$$

Agent one's incentive constraint of working still dominates others in this case. If $\frac{\hat{p}_1}{p_1}\in\left(1,\frac{2+\theta}{(1+\theta)^2}\right)$, $\hat{u}_1(\hat{p}_1;p_1,\hat{p}_1)=\frac{\hat{p}_1}{p_1}c-c$ and $\hat{u}_1(p_0;p_1,\hat{p}_1)=\frac{p_0}{p_1}c-c$, then in (A.0.32):

$$\begin{aligned}\triangle\text{RHS}^{\text{WS}} &> \delta\left[\frac{1-p_0\lambda}{1+\theta}\hat{u}_1(\hat{p}_1;p_1,\hat{p}_1)-\frac{1}{2+\theta}\hat{u}_1(p_0;p_1,\hat{p}_1)\right]\\ &\propto\frac{1-p_0\lambda}{1+\theta}\left(\frac{\hat{p}_1}{p_1}c-c\right)-\frac{1}{2+\theta}\left(\frac{p_0}{p_1}c-c\right)\\ &\propto\frac{1-p_0\lambda(2+\theta)}{(1+\theta)(2+\theta)}-\frac{1-p_0\lambda(2+\theta)}{(1+\theta)(2+\theta)}=0\end{aligned} \tag{A.0.34}$$

Again, agent one's incentive constraint of working dominates others in this case. To sum up, when the synergy is negative, agent one's incentive of excluding agent two and working alone dominates, in which case

$$\omega^*_{1,0}(p_0) = \frac{(2+\theta)c + \delta\left[(2+\theta)\hat{B}^2_1(\hat{p}_1; p_1, p_1) + (1+\theta)\hat{B}^1_1(\hat{p}_1; p_1, \hat{p}_1)\right]}{(1+\theta)^2\lambda R p_0}.$$

Consider the small positive synergy scenario, $\theta \in \left[0, \min\left\{\frac{\sqrt{5}-1}{2}, \frac{1-2\lambda}{\lambda}\right\}\right)$. In this scenario, the first term on the right hand side of (A.0.30) is still positive. For $p_1 \geq p^T$, in (A.0.30):

$$\begin{aligned}
&\triangle \mathrm{RHS}^{\mathrm{WS}}\\
>&\frac{1-p_0\lambda}{1+\theta}\hat{u}_1(\hat{p}_1; p_1, \hat{p}_1) - \frac{\delta}{2+\theta}\hat{u}_1(p_0; p_1, \hat{p}_1) - \delta\frac{1-p_0\lambda(2+\theta)}{(1+\theta)(2+\theta)}\hat{u}_1(p_1; p_1, p_1)\\
=&\delta c\left[\frac{1-p_0\lambda(2+\theta)}{(1+\theta)^2} - \frac{(1-p_0\lambda)(2+\theta)}{(1+\theta)^2} + \frac{1}{1+\theta}\frac{p_0}{\hat{p}_1}\right] \propto \frac{1}{1+\theta}\frac{p_0}{\hat{p}_1} - \frac{1}{1+\theta}\\
>&0
\end{aligned} \tag{A.0.35}$$

Then agent one's incentive of the shirking after excluding agent two is dominated in this case. For $p^T > p_1 \geq p^*$, $\hat{u}_1(p_1; p_1, p_1) = 0$. Notice that, If $\frac{\hat{p}_1}{p_1} \geq \frac{2}{1+\theta} > \frac{2+\theta}{(1+\theta)^2}$, $\hat{u}_1(\hat{p}_1; p_1, \hat{p}_1) = \frac{\hat{p}_1(2+\theta)}{p_1}c - \frac{3+2\theta}{1+\theta}c$ and $\hat{u}_1(p_0; p_1, \hat{p}_1) = \frac{p_0(2+\theta)}{p_1}c - c - \frac{2+\theta}{1+\theta}\frac{p_0}{\hat{p}_1}c$, so the analysis would be the same as that in (A.0.33), in which case agent one's incentive of the shirking after excluding the agent 2 is dominated. If $\frac{\hat{p}_1}{p_1} \in \left(1, \frac{2+\theta}{(1+\theta)^2}\right)$, $\hat{u}_1(\hat{p}_1; p_1, \hat{p}_1) = \frac{\hat{p}_1}{p_1}c - c$ and $\hat{u}_1(p_0; p_1, \hat{p}_1) = \frac{p_0}{p_1}c - c$, so the analysis must be the same as that in (A.0.34). Therefore, agent one's incentive of the shirking after excluding agent two is dominated in all different cases with small positive synergy. As a result, it can be only focused on the free-riding incentive and exclusion-work incentive.

Notice that agent one's free-riding incentive dominates if $\triangle U^{1,\mathrm{FE}}_0(p_0)$ is positive,

$$\begin{aligned}
\triangle U^{1,\mathrm{FE}}_0(p_0) =& \frac{\theta c}{1+\theta}c - \frac{1}{1+\theta}\delta B^2_1(\hat{p}_1; p_1, p_1)\\
&+ \delta(1-p_0\lambda)\left[\hat{u}_1(\hat{p}_1; p_1, p_1) - \hat{u}_1(\hat{p}_1; p_1, \hat{p}_1)\right] \geq 0\\
\Longrightarrow \quad \delta \leq& \frac{\theta c}{B^2_1(\hat{p}_1; p_1, p_1) + (1+\theta)(1-p_0\lambda)\left[\hat{u}_1(\hat{p}_1; p_1, \hat{p}_1) - \hat{u}_1(\hat{p}_1; p_1, p_1)\right]}
\end{aligned} \tag{A.0.36}$$

If $p_1 \geq p^T$, (A.0.36) can be simplified as

$$\delta \leq \frac{\theta}{\frac{\lambda(1-p_0)(2+\theta)^2}{1-\lambda(2+\theta)} + p_0\lambda} \implies \delta \leq \frac{\theta[1-\lambda(2+\theta)]}{\lambda(2+\theta)^2 - p_0\lambda[(3+\theta)(1+\theta)+\lambda(2+\theta)]} \tag{A.0.37}$$

The denominator is always positive as $1-\lambda(2+\theta) > 0$. When the positive synergy is less than 1, to prove that the right hand side of (A.0.37) is less than 1, I take the difference between the numerator and the denominator as follows:

$$\begin{aligned}\theta[1-\lambda(2+\theta)] &- \lambda(2+\theta)^2 + p_0\lambda[(3+\theta)(1+\theta)+\lambda(2+\theta)] \\ &< \theta[1-\lambda(2+\theta)] - \lambda(2+\theta)^2 + \lambda[(3+\theta)(1+\theta)+\lambda(2+\theta)] \\ &= (\theta-1)[1-\lambda(2+\theta)] < 0\end{aligned} \tag{A.0.38}$$

Therefore, the right hand side of (A.0.37) is less than 1, and it holds for $\forall\theta \in \left[0, \min\left\{\frac{1-2\lambda}{\lambda}, 1\right\}\right)$. Agent one's free-riding incentive dominates when $\delta \in [0, \tilde{\delta}]$, where $\tilde{\delta} = \frac{\theta[1-\lambda(2+\theta)]}{\lambda(2+\theta)^2 - p_0\lambda[(3+\theta)(1+\theta)+\lambda(2+\theta)]}$. This result holds for $\forall\theta \in \left[0, \min\left\{\frac{1-2\lambda}{\lambda}, 1\right\}\right)$.

If $p^T > p_1 \geq p^*$, $B_1^2(\hat{p}_1; p_1, p_1) = 0$. For $\frac{\hat{p}_1}{p_1} \geq \frac{2}{1+\theta}$, $\hat{u}_1(\hat{p}_1; p_1, \hat{p}_1) = \frac{(2+\theta)\hat{p}_1}{p_1}c - \frac{3+2\theta}{1+\theta}c$ and $\hat{u}_1(\hat{p}_1; p_1, p_1) = \frac{\hat{p}_1}{p_1}c - c$, then, from (A.0.36),

$$\delta \leq \frac{\theta}{\frac{(1+\theta)^2(1-\lambda)p_0}{p_1} - (2+\theta)(1-p_0\lambda)} \tag{A.0.39}$$

To prove that its right hand side is less than 1, the difference between the numerator and the denominator would be rewritten as:

$$\begin{aligned}&\theta - \frac{(1+\theta)^2(1-\lambda)p_0}{p_1} - (2+\theta)(1-p_0\lambda) \\ &\underbrace{< \theta - \frac{(1+\theta)^2 p_0}{p_1} + (2+\theta)(1-p_0\lambda) < \theta - 2(1+\theta) + (2+\theta)(1-p_0\lambda)}_{\text{since } \frac{p_0}{p_1} > \frac{\hat{p}_1}{p_1} \geq \frac{2}{1+\theta}} \\ &= -(2+\theta)p_0\lambda < 0\end{aligned} \tag{A.0.40}$$

Therefore, agent one's free-riding incentive dominates when $\delta \in [0, \tilde{\delta}]$, where

$\tilde{\delta} = \frac{\theta}{\frac{(1+\theta)^2(1-\lambda)p_0}{p_1} - (2+\theta)(1-p_0\lambda)}$. For $\frac{\hat{p}_1}{p_1} \in \left(1, \frac{2}{1+\theta}\right)$, $\hat{u}_1(\hat{p}_1; p_1, \hat{p}_1) = \hat{u}_1(\hat{p}_1; p_1, p_1) = \frac{\hat{p}_1}{p_1}c - c$, and then $\triangle U_0^{1,\mathrm{FE}}(p_0)$ in (A.0.36) is always positive, which implies that the free-riding incentive is always dominates in this case, and it associated incentive constraint needs to be binding. As a result, $\omega^*_{1,0}(p_0) = \frac{2c+\delta\left(B_1^2(\hat{p}_1;p_1,p_1)+B_1^1(\hat{p}_1;p_1,p_1)\right)}{(1+\theta)\lambda R p_0}$. When $\delta \in [\tilde{\delta}, 1]$, the exclusion-work incentive dominates, and the optimal grand contract should be the same as that in the negative synergy case.

Now consider the large positive synergy, where $\theta \in \left[1, \frac{1-2\lambda}{\lambda}\right)$. Now $p^T \geq p^*$. For $p_1 \geq p^T$, To show the situation in which case the free-riding incentive also dominates the exclusion-shirk incentive, the difference of the right hands between $IC^{\mathrm{FR}}_{1,0}$ and $IC^{\mathrm{ES}}_{1,0}$ in (A.0.29) also needs to be positive, where

$$
\begin{aligned}
&\triangle \mathrm{RHS}^{\mathrm{FS}} \\
=&\frac{1}{(1+\theta)(2+\theta)}c + \delta\left[\frac{\theta B_1^2(\hat{p}_1;p_1,p_1)}{1+\theta} + \frac{B_1^1(\hat{p}_1;p_1,p_1)}{1+\theta} - \frac{B_1^2(p_0;p_1,\hat{p}_1)}{2+\theta}\right] \\
\propto&\frac{1}{2+\theta} + \delta\left[\frac{\theta(2+\theta)(1-p_0)\lambda}{1-\lambda(2+\theta)} + \theta p_0\lambda - \frac{\lambda(1+\theta)(1-p_0)}{(1-\lambda)[1-\lambda(2+\theta)]}\right] \\
>&\delta\left[\frac{\theta(2+\theta)(1-p_0)\lambda}{1-\lambda(2+\theta)} - \frac{\lambda(1+\theta)(1-p_0)}{(1-\lambda)[1-\lambda(2+\theta)]}\right] \\
\propto&\theta^2 - 1 + \theta[1-\lambda(2+\theta)] > 0
\end{aligned}
\tag{A.0.41}
$$

Then the free-riding incentive dominates the exclusion-shirk incentive in this case. To determine the relationship between the free-riding and exclusion-work incentives, the analysis is similar to that in which case $\theta \in \left[0, \min\left\{1, \frac{1-2\lambda}{\lambda}\right\}\right)$ and $p_1 \geq p^T$. The difference between the numerator and the denominator in

(A.0.37) would be:

$$\begin{aligned}\theta[1-\lambda(2+\theta)]-\lambda(2+\theta)^2+p_0\lambda[(3+\theta)(1+\theta)+\lambda(2+\theta)]\\ =&\theta[(2+\theta)^2-(3+\theta)(1+\theta)-\lambda(2+\theta)]\\ &-\lambda\left\{(2+\theta)^2-p_0[(3+\theta)(1+\theta)+\lambda(2+\theta)]\right\}\\ >&\theta\left\{(2+\theta)^2-p_0[(3+\theta)(1+\theta)+\lambda(2+\theta)]\right\}\\ &-\lambda\left\{(2+\theta)^2-p_0[(3+\theta)(1+\theta)+\lambda(2+\theta)]\right\}\\ =&(\theta-\lambda)\left\{(2+\theta)^2-p_0[(3+\theta)(1+\theta)+\lambda(2+\theta)]\right\}>0\end{aligned} \tag{A.0.42}$$

Therefore, the free-riding incentive also dominates the exclusion-work incentives in this case.

Finally, consider the positive synergy is medium, in which case $\theta \in \left[\frac{\sqrt{5}-1}{2}, \min\left\{1, \frac{1-2\lambda}{\lambda}\right\}\right)$. For $p_1 \geq p^T$, notice that the right hand side of (A.0.41) is a linear function of the discount factor δ, and $\triangle\text{RHS}^{\text{FS}}$ is positive when $\delta = 0$. Now let $\delta = 1$, then (A.0.41) becomes

$$\begin{aligned}\triangle\text{RHS}^{\text{FS}}|_{\delta=1} \propto& \frac{1}{2+\theta}-\frac{(1-\theta^2)\lambda(1-p_0)}{[1-\lambda(2+\theta)](1-\lambda)}+\frac{\lambda\theta(1-p_0\lambda)}{1-\lambda}\\ >&\frac{1-\lambda-\lambda(2+\theta)[\theta^2-\lambda+(1-\theta^2)p_0]}{(2+\theta)[1-\lambda(2+\theta)](1-\lambda)}\\ \propto& 1-\lambda-\lambda(2+\theta)[\theta^2-\lambda+(1-\theta^2)p_0]\\ >& 1-\lambda-\lambda(2+\theta)(\theta^2-\lambda+1-\theta^2)\\ =& (1-\lambda)[1-\lambda(2+\theta)]>0\end{aligned} \tag{A.0.43}$$

As a result, the $\triangle\text{RHS}^{\text{FS}}$ is always positive for $\forall\delta[0,1]$ in this case.

For $p_1 \geq p^T$, agent one's incentive of free-riding dominates the incentive of exclusion-work if $(A.0.37)$ is satisfied, which is the same as that with small positive synergy. For $p^T > p_1 \geq p^*$, $B_1^2(\hat{p}_1; p_1, p_1) = 0$ and $\hat{u}_1(p_1; p_1, p_1) = 0$. If $\frac{\hat{p}_1}{p_1} \in \left(1, \frac{2}{1+\theta}\right)$, $\hat{u}_1(\hat{p}_1; p_1, p_1) = \frac{p_0}{p_1}c - c$ and $\hat{u}_1(p_0; p_1, \hat{p}_1) = \frac{p_0}{p_1}c - c$. Then, in

(A.0.41),

$$\triangle\mathrm{RHS}^{\mathrm{FS}} = \frac{1}{(1+\theta)(2+\theta)}c + \delta c\left[\frac{1-\lambda(2+\theta)}{(1+\theta)(2+\theta)}\frac{p_0}{p_1} - \frac{1-p_0\lambda(2+\theta)}{(1+\theta)(2+\theta)}\right]$$
$$= \frac{1}{(1+\theta)(2+\theta)}c + 0 > 0 \tag{A.0.44}$$

This implies that the exclusion-shirk incentive is dominated by the free-riding incentive in this case. If $\frac{\hat{p}_1}{p_1} \geq \frac{2}{1+\theta}$, $\hat{u}_1(\hat{p}_1; p_1, p_1) = \frac{p_0}{p_1}c - c$ and $\hat{u}_1(p_0; p_1, \hat{p}_1) = \frac{(2+\theta)p_0}{p_1}c - c - \frac{(1+\theta)p_0}{(2+\theta)\hat{p}_1}$. Then, in (A.0.41),

$$\begin{aligned}
&\triangle\mathrm{RHS}^{\mathrm{FS}}\\
=&\frac{1}{(1+\theta)(2+\theta)}c + \delta\frac{1-p_0\lambda}{1+\theta}\left(\frac{\hat{p}_1}{p_1}c - c\right) - \frac{\delta}{2+\theta}\left(\frac{(2+\theta)p_0}{p_1}c - c - \frac{(2+\theta)p_0}{(1+\theta)\hat{p}_1}c\right)\\
>&\frac{\delta}{(1+\theta)(2+\theta)}c + \delta\frac{1-p_0\lambda}{1+\theta}\left(\frac{\hat{p}_1}{p_1}c - c\right) - \frac{\delta}{2+\theta}\left(\frac{(2+\theta)p_0}{p_1}c - c - \frac{(2+\theta)p_0}{(1+\theta)\hat{p}_1}c\right)\\
\propto&\frac{1}{(1+\theta)(2+\theta)} + \frac{1-p_0\lambda}{1+\theta}\left(\frac{\hat{p}_1}{p_1} - 1\right) - \frac{1}{2+\theta}\left(\frac{(2+\theta)p_0}{p_1} - 1 - \frac{(2+\theta)p_0}{(1+\theta)\hat{p}_1}\right)\\
\propto&(2-\theta) + \lambda^2(3+\theta) + \lambda^2 p_0(1-\theta-\theta^2) + \lambda p_0(\theta^2+2\theta) + \lambda(2\theta^2+3\theta-4)
\end{aligned} \tag{A.0.45}$$

Notice that, in this case, $\lambda \in (\frac{1}{3}, \frac{1}{2})$ and $\theta \in \left(\frac{\sqrt{5}-1}{2}, 1\right)$, then, from the results[1] in Software "Mathematica", (A.0.45) is always positive in this case. Therefore, the exclusion-shirk incentive is dominated in this case. Moreover, to check whether the free-riding incentive or the exclusion-work incentive dominates, the analysis is the same as before and results in (A.0.40) still hold.

Proof of Corollary 1.4.1

Notice that the left hand side of the two inequalities in (1.4.11) are the same, then it only needs to be checked wether the grand contract for motivating the individual work satisfies the inequality with the larger right hand side, where $\omega_{1,0}^{\mathrm{IW}}(p_0) = \frac{c+\delta B_1^1(p_0;\hat{p}_1,\hat{p}_1)}{\lambda R p_0}$. The difference of the right hand

[1] The code is: Manipulate[Plot[$(2-\theta)+\lambda^2(3+\theta)+\lambda^2 p_0(1-\theta-\theta^2)+\lambda p_0(\theta^2+2\theta)+\lambda(2\theta^2+3\theta-4)$,{$\theta$,$\frac{\sqrt{5}-1}{2}$,1}],{$\lambda$,$\frac{1}{3}$,$\frac{1}{2}$}{$p_0$,0,1}]

sides are:

$$\begin{aligned} \triangle \mathrm{RHS}^{\mathrm{IW}} &= \frac{\theta}{1+\theta}c + \delta(1-p_0\lambda)\left(\hat{u}_1^1(\hat{p}_1;\hat{p}_1,p_1) - \hat{u}_1^1(\hat{p}_1;\hat{p}_1,\hat{p}_1)\right) \\ &= \frac{\theta}{1+\theta}c + \delta(1-p_0\lambda)\max\left\{-\frac{\theta}{1+\theta}c, 0\right\} \end{aligned} \tag{A.0.46}$$

It's clear that the right hand side of the first inequality is larger in (1.4.11) when the synergy is positive, thus it can only be focused on whether the first inequality is satisfied. On the other hand, when the synergy is negative, the focus shifts to the second inequality in (1.4.11).

When $\theta \geq \left[1, \frac{1-2\lambda}{\lambda}\right)$, (1.4.11) requires:

$$(1+\theta)c + (1+\theta)\delta B_1^1(p_0;\hat{p}_1,\hat{p}_1) + \delta\frac{1-p_0\lambda(2+\theta)}{1+\theta}c < 2c \tag{A.0.47}$$

Its left hand side is an increasing function of δ and reaches the minimum at $\delta = 0$. However, $(1+\theta)c \geq 2c$. This implies for $\forall\delta \in [0,1]$, (1.4.11) is always violated.

Now consider the small positive synergy, $\theta \in \left[0, \min\left\{\frac{1-2\lambda}{\lambda}, 1\right\}\right)$. Firstly, for $\hat{p}_1 \geq p^T$ and $\frac{\hat{p}_1}{p_1} \geq \frac{2}{1+\theta}$, (1.4.11) becomes:

$$\delta\left[B_1^1(p_0;\hat{p}_1,\hat{p}_1) - \frac{(1-p_0\lambda)(1-\theta)}{(1+\theta)^2}c\right] < \frac{1-\theta}{1+\theta}c \tag{A.0.48}$$

The left hand side of the inequality above is a continuous linear function of δ, and it's always satisfied at $\delta = 0$. Now check whether it's still satisfied at $\delta = 1$. If it is, then it's satisfied for $\forall\delta \in [0,1]$; if it's violated, according to the intermediate value theorem, there must exist $\delta_v \in (0,1)$, such that (1.4.11) is violated for $\delta \in [\delta_v, 1]$, and satisfied for $\delta \in [0, \delta_v)$. Similar argument would be applied for the all rest of proof of Corollary 1.4.1. At $\delta = 1$, (A.0.48) requires:

$$\begin{aligned} & B_1^1(p_0;\hat{p}_1,\hat{p}_1) - \frac{(1-p_0\lambda)(1-\theta)}{(1+\theta)^2}c - \frac{1-\theta}{1+\theta}c < 0 \\ \Longrightarrow\quad & \propto \theta^2 + \theta - 2 + 2\lambda(2+\theta) - p_0\lambda(\theta^2 + 3\theta + 2\lambda) < 0 \\ \Longrightarrow\quad & p_0\lambda > \frac{\theta^2 + \theta - 2 + 2\lambda(2+\theta)}{\theta^2 + 3\theta + 2\lambda} \end{aligned} \tag{A.0.49}$$

It's clear that (A.0.49) always holds if $\lambda \in \left(0, \frac{1-\theta}{2}\right]$ since the numerator $\theta^2 + \theta - 2 + 2\lambda(2+\theta)$ is negative in this case. Thus I focus on the scenario with $\lambda \in \left(\frac{1-\theta}{2}, \frac{1}{2}\right)$, in which case $\theta \in \left(\frac{\sqrt{5}-1}{2}, 1\right)$. Notice that $\frac{\hat{p}_1}{p_1} \geq \frac{2}{1+\theta}$, and it requires:

$$p_0\lambda \leq \frac{\theta - 1 + \lambda(3+\theta)}{3\theta + \theta^2 + (2+\theta)(1-\theta)\lambda} \quad \text{and} \quad \lambda \in \left(\frac{1-\theta}{1+\theta}, \frac{1}{2}\right) \tag{A.0.50}$$

Since $2\lambda > (2+\theta)(1-\theta)\lambda$ and $\theta^2 - 1 + \lambda < 0$ when $\lambda \in \left(\frac{1-\theta}{2}, \frac{1}{2}\right)$ and $\theta \in \left(\frac{\sqrt{5}-1}{2}, 1\right)$, it must be true that $\frac{\theta^2+\theta-2+2\lambda(2+\theta)}{\theta^2+3\theta+2\lambda} < \frac{\theta-1+\lambda(3+\theta)}{3\theta+\theta^2+(2+\theta)(1-\theta)\lambda}$ and $\lambda \in \left(\frac{1-\theta}{1+\theta}, \frac{1}{2}\right)$. Therefore, (A.0.48) doesn't hold for $p_0 \left(0, \frac{\theta^2+\theta-2+2\lambda(2+\theta)}{\lambda(\theta^2+3\theta+2\lambda)}\right]$ in this case. As a result, according to intermediate value theorem, $\exists \lambda_v \in \left(\frac{1-\theta}{2}, \frac{1}{2}\right)$ and $\exists \theta_v \in \left(\frac{\sqrt{5}-1}{2}, 1\right)$, such that (1.4.11) holds for $\forall p_0 \in (0,1)$, in which case $\lambda \in \left[\lambda_v, \frac{1}{2}\right)$ and $\theta \in \left[\theta_v, \min\left\{1, \frac{1-2\lambda}{\lambda}\right\}\right)$. Thus, (A.0.48) doesn't hold at $\delta = 1$. Therefore, (1.4.11) is violated when $\delta \in [\delta_v, 1]$ in this case, where $\delta_v = \frac{(1-\theta^2)c}{(1+\theta)^2 B_1^1(p_0;\hat{p}_1,\hat{p}_1) - (1-p_0\lambda)(1-\theta)}$. Aslo, it's easily to see that (A.0.49) is violated if $p_0 \in (0, p^v]$, in which case $p^v = \min\left\{1, \frac{\theta^2+\theta-2+2\lambda(2+\theta)}{\lambda(\theta^2+3\theta+2\lambda)}\right\}$.

Secondly, for $\hat{p}_1 \geq p^T$ and $1 < \frac{\hat{p}_1}{p_1} < \frac{2}{1+\theta}$, in (1.4.11):

$$\begin{aligned}&\delta\left\{B_1^1(p_0;\hat{p}_1,\hat{p}_1) - \frac{(1-p_0\lambda)(1-\theta)}{(1+\theta)^2}c + \frac{[1-p_0\lambda(2+\theta)][2p_1 - (1+\theta)\hat{p}_1]}{\hat{p}_1(1+\theta)^2}c\right\} \\ &< \frac{1-\theta}{1+\theta}c\end{aligned} \tag{A.0.51}$$

By applying the same argument as the previous case, the inequality holds at $\delta = 0$. At $\delta = 1$, it requires:

$$\frac{\lambda + \theta - 1 - p_0\lambda\theta}{(1+\theta)(1-\lambda)} < 0 \quad \Longrightarrow \quad p_0\lambda > \frac{\lambda+\theta-1}{\theta} \tag{A.0.52}$$

Since $1 < \frac{\hat{p}_1}{p_1} < \frac{2}{1+\theta}$, in which $p_0\lambda > \frac{\theta-1+\lambda(3+\theta)}{3\theta+\theta^2+(2+\theta)(1-\theta)\lambda}$, it implies that

$$\frac{\lambda+\theta-1}{\theta} - \frac{\theta - 1 + \lambda(3+\theta)}{3\theta+\theta^2+(2+\theta)(1-\theta)\lambda} \propto \lambda^2 - \lambda - \theta < 0 \tag{A.0.53}$$

Therefore, (A.0.52) holds at $\delta = 1$. As a result, (1.4.11) is always satisfied for $\forall\delta \in [0,1]$ in this case.

Thirdly, for $p^T > \hat{p}_1 \geq p^*$, (1.4.11) requires

$$\delta B_1^1(p_0; \hat{p}_1, \hat{p}_1) < \frac{1-\theta}{1+\theta} c \tag{A.0.54}$$

Again, it's satisfied at $\delta = 0$. At $\delta = 1$, it becomes:

$$\frac{1-p_0\lambda}{1-\lambda} c - \frac{2}{1+\theta} c < 0 \quad \Longrightarrow \quad p_0\lambda > \frac{\theta - 1 + 2\lambda}{1+\theta} \tag{A.0.55}$$

If $0 < \theta \leq 1 - 2\lambda$, (A.0.55) is always satisfied as the numerator is negative. However, if $\theta \in \left(1-2\lambda, \min\{\frac{1-2\lambda}{\lambda}, 1\}\right)$, it implies that $0 < \frac{\theta-1+2\lambda}{(1+\theta)\lambda} < 1$. Thus, together with $p_0 \in \left(0, \frac{\theta-1+2\lambda}{(1+\theta)\lambda}\right]$, (A.0.55) is violated, and it follows that (A.0.54) is violated at $\delta = 1$. As a result, for $\delta \in \left[\frac{1-\theta}{(1+\theta)B_1^1(p_0;\hat{p}_1,\hat{p}_1)}, 1\right]$, $\theta \in \left(1-2\lambda, \min\{\frac{1-2\lambda}{\lambda}, 1\}\right)$ and $p_0 \in \left(0, \frac{\theta-1+2\lambda}{(1+\theta)\lambda}\right]$, (1.4.11) is violated. In this case, $p^v = \frac{\theta-1+2\lambda}{(1+\theta)\lambda}$, $\delta_v = \frac{(1-\theta)c}{(1+\theta)B_1^1(p_0;\hat{p}_1,\hat{p}_1)}$, and $\theta_v = 1-2\lambda$, which are different from previous levels of threshold.

Now consider the negative synergy, $\theta \in \left(-1, \min\left\{\frac{1-2\lambda}{\lambda}, 0\right\}\right)$, in which it only needs to be checked whether the second inequality in (1.4.11) is satisfied. Firstly, $\hat{p}_1 \geq p^T$ and $\frac{\hat{p}_1}{p_1} \geq \frac{2+\theta}{(1+\theta)^2}$, (1.4.11) becomes:

$$\delta \left[B_1^1(p_0; \hat{p}_1, \hat{p}_1) - \frac{(1-p_0\lambda)(1-\theta-\theta^2)}{(1+\theta)^3} c \right] < \frac{1-\theta-\theta^2}{(1+\theta)^2} c \tag{A.0.56}$$

The inequality is satisfied at $\delta = 0$. At $\delta = 1$, it can be simplified as:

$$\theta^2 + \theta - 1 + (2-\theta^2)\lambda < p_0\lambda \left[\lambda(1-\theta-\theta^2) + \theta^2 + 2\theta\right] \tag{A.0.57}$$

Since $\frac{\hat{p}_1}{p_1} \geq \frac{2+\theta}{(1+\theta)^2}$, it requires that $p_0\lambda\left[\lambda(1-\theta-\theta^2) + \theta^2 + 2\theta\right] \leq \frac{\theta^2+\theta-1+\lambda(3+2\theta)}{2+\theta}$, which also implies that:

$$\begin{aligned} &\theta^2 + \theta - 1 + (2-\theta^2)\lambda < \frac{\theta^2+\theta-1+\lambda(3+2\theta)}{2+\theta} \\ \Longrightarrow \quad &(1-2\theta^2-\theta^3)\lambda < (1-\theta-\theta^2)(1+\theta) \end{aligned} \tag{A.0.58}$$

This inequality is always satisfied for $\forall\lambda \in (0,1)$ and $\forall\theta \in \left(-1, \min\{0, \frac{1-\lambda}{\lambda}\}\right)$ in this case. Now it can focus on the value of p_0. If it's coefficient is positive, $[\theta^2 + \theta - 1 + \lambda(3+\theta)]\lambda > 0$, it must be true that $\theta^2 + \theta - 1 +$

$\lambda(3+\theta) > 0$. This scenario exists at $\lambda \in \left(\frac{-\theta(2+\theta)}{1-\theta-\theta^2}, 1\right)$ and $\theta \in (-1, \theta_{v'}]$, where $\frac{-\theta_{v'}(2+\theta_{v'})}{1-\theta_{v'}-\theta_{v'}^2} = \frac{1-\theta_{v'}-\theta_{v'}^2}{3+\theta_{v'}}$. In contrast, it's implies that (A.0.56) is violated in this case if $p_0 \in \left(0, \frac{\theta^2+\theta-1+(2-\theta^2)\lambda}{\lambda[\lambda(1-\theta-\theta^2)+\theta^2+2\theta]}\right)$ with $\lambda \in \left(\frac{1-\theta-\theta^2}{2-\theta^2}, 1\right)$ and $\theta \in (-1, \theta_{v'}]$. Thus (1.4.11) is violated at $\delta = 1$. As a result, (1.4.11) would be violated at $\delta \in \left[\frac{\frac{1-\theta-\theta^2}{(1+\theta)^2}c}{B_1^1(p_0;\hat{p}_1,\hat{p}_1)-\frac{(1-p_0\lambda)(1-\theta-\theta^2)}{(1+\theta)^3}c}, 1\right]$, $p_0 \in \left(0, \frac{\theta^2+\theta-1+(2-\theta^2)\lambda}{\lambda[\lambda(1-\theta-\theta^2)+\theta^2+2\theta]}\right)$ with$\lambda \in \left(\frac{1-\theta-\theta^2}{2-\theta^2}, 1\right)$ and $\theta \in (-1, \theta_{v'}]$. On the other hand, if $[\theta^2+\theta-1+\lambda(3+\theta)]\lambda < 0$, it requires that $\theta^2+\theta-1+(2-\theta^2)\lambda < 0$. Thus, (A.0.56) is violated at $p_0 \in \left(\frac{\theta^2+\theta-1+(2-\theta^2)\lambda}{\lambda[\lambda(1-\theta-\theta^2)+\theta^2+2\theta]}, 1\right)$ with $\lambda \in \left(0, \frac{1-\theta-\theta^2}{3+\theta}\right)$. However, since $\frac{\theta^2+\theta-1+(2-\theta^2)\lambda}{\lambda[\lambda(1-\theta-\theta^2)+\theta^2+2\theta]} > 1$ in this scenario, it can be discarded.

Secondly, for $\hat{p}_1 \geq p^T$ and $1 < \frac{\hat{p}_1}{p_1} < \frac{2+\theta}{(1+\theta)^2}$, from (1.4.11),

$$\begin{aligned}\delta \Bigg\{ B_1^1(p_0;\hat{p}_1,\hat{p}_1) + \frac{[1-p_0\lambda(2+\theta)][(2+\theta)p_1-(1+\theta)^2\hat{p}_1]}{\hat{p}_1(1+\theta)^3}c \\ -\frac{(1-p_0\lambda)(1-\theta-\theta^2)}{(1+\theta)^3}c\Bigg\} < \frac{1-\theta-\theta^2}{(1+\theta)^2}c\end{aligned} \tag{A.0.59}$$

The inequality is satisfied at $\delta = 0$. At $\delta = 1$, it can be simplified as:

$$\frac{\theta^2+\theta-1}{(1+\theta)^2} < 0 \tag{A.0.60}$$

This implies that (A.0.59) is also satisfied at $\delta = 1$. Therefore, (1.4.11) is always satisfied in this case.

Thirdly, for $p^T > \hat{p}_1 \geq p^*$, from (1.4.11),

$$\delta B_1^1(p_0;\hat{p}_1,\hat{p}_1) < \frac{1-\theta-\theta^2}{(1+\theta)^2}c \tag{A.0.61}$$

The inequality is satisfied at $\delta = 0$. At $\delta = 1$, this condition can be simplified as:

$$\frac{p_0}{\hat{p}_1}c - \frac{2+\theta}{(1+\theta)^2}c < 0 \implies p_0 > \frac{\theta^2+\theta-1+(2+\theta)\lambda}{\lambda(1+\theta)^2} \tag{A.0.62}$$

Notice that the numerator is always negative as $\theta \in (-1, 0)$ and $1-\lambda(2+\theta) > 0$, thus (A.0.62) is always satisfied for $\forall\delta \in [0, 1]$.

Proof of Proposition 1.4.2

If $p_0 \geq p^* > \hat{p}_1$, $\pi_1^*(p_1) = \pi_1^*(\hat{p}_1) = 0$. Notice that $\delta \in [0,1]$, then the principal would never delay the investment. When comparing the profits from the partnership and the individual work, the analysis should be the same as that in the static game with belief p_0, so Lemma 1.4.1 can be applied. When $\theta \in \left(1, \frac{1-2\lambda}{\lambda}\right)$, it is equivalent to the case in which $\frac{R}{c} \in \left[\frac{2}{(1+\theta)\lambda p_0}, \frac{2}{(1+\theta)\lambda \hat{p}_1}\right)$; when $\theta \in \left(-1, \frac{1-2\lambda}{\lambda}\right)$, it equivalent to the case in which $\frac{R}{c} \in \left[\frac{1}{\lambda p_0}, \frac{1}{\lambda \hat{p}_1}\right)$. The rest of proof of Proposition 1.4.2 would focus on the scenarios in which $\hat{p}_1 \geq p^*$. Moreover, notice that $B_1^i(\cdot)$ is a linear function of the cost c, it can be represented as $B_1^i(p_1^i; p_1^P, p_1^j) = \beta_1^i(p_1^i; p_1^P, p_1^j)c$, where $i, j = 1, 2$ and $i \neq j$.

Firstly, consider the large positive synergy in which case $\theta \in \left[1, \frac{1-2\lambda}{\lambda}\right)$. According to Corollary 1.4.1, the principal would only choose between the partnership and delaying the investment, and she would prefer the partnership at $t = 0$ if the difference between the benefit from these two choices, $\triangle V_0^{\text{CN}}(p_0)$, is positive. When $\hat{p}_1 \geq p^* \geq p_1$, the principal would not invest at $t = 1$ after the failure from the collaboration, and $\pi_1^*(p_1) = 0$. As a result, (1.4.15) is always positive as $\delta \in [0, 1]$, and $e_{i,0}^*(p_0) = 1 = e_{i,0}^P(p_0)$. This scenario is equivalent to $\frac{R}{c} \in \left[\frac{2}{(1+\theta)\lambda \hat{p}_1}, \frac{2}{(1+\theta)\lambda p_1}\right)$. When $p_1 \geq p^*$, $\pi_1^*(p_1) > 0$, and $V_0^{\text{CN}}(p_0)$ is positive at $\delta = 0$. This scenario is equivalent to $\frac{R}{c} \geq \frac{2}{(1+\theta)\lambda p_1}$. Then I check if it's still positive at $\delta = 1$. If it is, the partnership at $t = 0$ is preferred; if it's not, then $\exists \delta^* \in (0, 1)$, such that the partnership is preferred for $\delta \in [0, \delta^*)$. The rest of the proof would also follow the similar argument. At $\delta = 1$, $V_0^{\text{CN}}(p_0)$ can be simplified as:

$$\begin{aligned}
\triangle V_0^{\text{CN}}(p_0)|_{\delta=1} =& [1 - p_0\lambda(2+\theta)]p_1\lambda(2+\theta)R \\
& - \frac{2(2+\theta)}{1+\theta}\left[1 - p_0\lambda(1+\theta) + \frac{(2+\theta)\lambda(1-p_0)}{1-\lambda(2+\theta)}\right]c \qquad \text{(A.0.63)} \\
\propto & p_1\lambda R - \frac{2 - 2p_1\lambda(1+\theta)}{(1+\theta)[1-\lambda(2+\theta)]}c
\end{aligned}$$

This implies that $\triangle V_0^{\text{CN}}(p_0)|_{\delta=1} \geq 0$ if $\frac{R}{c} \geq \frac{2[1-p_1\lambda(1+\theta)]}{p_1\lambda(1+\theta)[1-\lambda(2+\theta)]} > \frac{2}{(1+\theta)p_1\lambda}$. Therefore, $e_{i,0}^*(p_0) = 1$ when $\frac{R}{c} \geq \frac{2[1-p_1\lambda(1+\theta)]}{p_1\lambda(1+\theta)[1-\lambda(2+\theta)]}$. When $\frac{R}{c} \in \left[\frac{2}{(1+\theta)p_1\lambda}, \frac{2[1-p_1\lambda(1+\theta)]}{p_1\lambda(1+\theta)[1-\lambda(2+\theta)]}\right)$, $\triangle V_0^{\text{CN}}(p_0)|_{\delta=1} < 0$, thus $e_{i,0}^*(p_0) = 1$ for $\delta \in [0, \delta^*]$, and $e_{i,0}^*(p_0) = 0$ for $\delta \in (\delta^*, 1]$, where $\triangle V_0^{\text{CN}}(p_0)|_{\delta^*} = 0$.

Now consider the scenarios with the negative synergy, in which case $\theta \in \left(-1, \min\left\{0, \frac{1-2\lambda}{\lambda}\right\}\right)$. When all the parameters make (1.4.11) to be satisfied, it must be true that both $\triangle V_0^{\mathrm{CN}}(p_0)$ and $\triangle V_0^{\mathrm{CW}}(p_0)$ are positive if the partnership is motivated, which can be simplified as:

$$\begin{cases} \frac{R}{c} \geq & \frac{[1-\delta p_0\lambda(2+\theta)](2+\theta)+\delta\left[(2+\theta)\beta_1^2(\hat{p}_1;p_1,p_1)+(1+\theta)\beta_1^1(\hat{p}_1;p_1,\hat{p}_1)\right]}{p_0\lambda(1+\theta)^2[1-\delta\lambda(2+\theta)]} = \left(\frac{\overline{R}}{c}\right)^{\mathrm{cn}} \\ \frac{R}{c} \geq & \frac{2+\theta}{1+\theta}\left(\frac{\overline{R}}{c}\right)^{\mathrm{cn}} - \frac{(1+\theta)^2-\delta\left[p_0\lambda(2+\theta)^2(1+\theta)-(1+\theta)^2\beta_1^1(p_0;\hat{p}_1,\hat{p}_1)\right]}{p_0\lambda(1+\theta)^3[1-\delta\lambda(2+\theta)]} = \left(\frac{\overline{R}}{c}\right)^{\mathrm{cw}} \end{cases} \tag{A.0.64}$$

For $p_0 \geq p^T = \frac{(3+2\theta)c}{(1+\theta)^3\lambda R}$, the partnership is always preferred at $\delta = 0$ since $\frac{R}{c} \geq \frac{3+2\theta}{p_0\lambda(1+\theta)^3} > \frac{2+\theta}{p_0\lambda(1+\theta)^2}$. Thus, for $\forall\delta \in [0,1]$, the partnership is preferred as long as $\frac{R}{c} \geq \max\left\{\left(\frac{\overline{R}}{c}\right)^{\mathrm{cn}}|_{\delta=1}, \left(\frac{\overline{R}}{c}\right)^{\mathrm{cw}}|_{\delta=1}, \frac{3+2\theta}{p_0\lambda(1+\theta)^3}\right\} = \frac{\overline{R}}{c}$. Such value-cost ratio always exists as $\frac{R}{c} \in [0, +\infty)$. On the other hand, if (1.4.11) is not satisfied, $\triangle V_0^{\mathrm{CW}}(p_0)$ can be discarded for $\delta \in [\delta_v, 1]$ as the individual work cannot be motivated in this case. For $\delta \in [0, \delta_v]$, the partnership is preferred only if $\frac{R}{c} \geq \max\left\{\left(\frac{\overline{R}}{c}\right)^{\mathrm{cn}}|_{\delta=\delta_v}, \left(\frac{\overline{R}}{c}\right)^{\mathrm{cw}}|_{\delta=\delta_v}, \frac{3+2\theta}{p_0\lambda(1+\theta)^3}\right\}$. Therefore, for $\delta \in [0,1]$, it has to be true that $\frac{R}{c} \geq \max\left\{\left(\frac{\overline{R}}{c}\right)^{\mathrm{cn}}|_{\delta=1}, \left(\frac{\overline{R}}{c}\right)^{\mathrm{cw}}|_{\delta=\delta_v}, \frac{3+2\theta}{p_0\lambda(1+\theta)^3}\right\}$. It's clear that such value-cost ratio still exists, which is denoted by $\frac{\overline{R}}{c}$.

For $\frac{R}{c} \in \left[\frac{3+2\theta}{p_0\lambda(1+\theta)^3}, \frac{\overline{R}}{c}\right)$, the difference of the static profits are still positive, $p_0\lambda(2+\theta)R - \frac{3+2\theta}{(1+\theta)^2}c \geq 0$ and $p_0\lambda(2+\theta)R - \left(\frac{2+\theta}{1+\theta}\right)^2 c \geq 0$. From (A.0.64), it must be true that $\exists\overline{\delta}_{\mathrm{cw}} \in [0,1]$ and $\exists\overline{\delta}_{\mathrm{cn}} \in [0,1]$ such that the partnership is preferred at $\delta \in \left[0, \min\left\{\overline{\delta}_{\mathrm{cw}}, \overline{\delta}_{\mathrm{cn}}\right\}\right]$ when (1.4.11) is satisfied, then $\overline{\delta}_c^* = \min\left\{\overline{\delta}_{\mathrm{cw}}, \overline{\delta}_{\mathrm{cn}}\right\}$ in this case. On the other hand, when (1.4.11) is violated and the individual work cannot be motivated at $\delta \in [\delta_v, 1]$, the principal would preferred to the partnership at $\delta \in \left[0, \max\left\{\min\left\{\overline{\delta}_{\mathrm{cw}}, \overline{\delta}_{\mathrm{cn}}\right\}, \delta_v\right\}\right]$, and $\overline{\delta}_c^* = \max\left\{\min\left\{\overline{\delta}_{\mathrm{cw}}, \overline{\delta}_{\mathrm{cn}}\right\}, \delta_v\right\}$ in this case. The individual work is preferred to no investment only if $\triangle V_0^{\mathrm{WN}}(p_0)$ is positive, which can be simplified as:

$$\triangle V_0^{\mathrm{WN}}(p_0) = p_0\lambda R - c \geq \delta\left[B_1^2(p_0;\hat{p}_1,\hat{p}_1) - (1-p_0\lambda)\pi_1^*(\hat{p}_1) + \pi_1^*(p_0)\right] \tag{A.0.65}$$

Notice the left hand side is always positive, thus this inequality must hold at $\delta = 0$. But it might be violated at $\delta = 1$. For instance, for $\frac{R}{c} \in \left[\frac{3+2\theta}{p_0\lambda(1+\theta)^3}, \frac{1}{p_1\lambda}\right)$, (A.0.65) is violated as $B_1^2(p_0;\hat{p}_1,\hat{p}_1) = \pi_1^*(\hat{p}_1) = 0$. Therefore, $\exists\overline{\delta}_{\mathrm{wn}} \in [0,1]$ such that the individual work is preferred to no investment at $\delta \in [0, \overline{\delta}_{\mathrm{wn}}]$.

Together with $\triangle V_0^{\mathrm{CN}}(p_0) < 0$, no investment at $t = 0$ is preferred at $\delta \in \left(\max\left\{\overline{\delta}_{\mathrm{cn}}, \overline{\delta}_{\mathrm{wn}}\right\}, 1\right]$ when (1.4.11) is satisfied, and then $\overline{\delta}_{\mathrm{w}}^* = \max\left\{\overline{\delta}_{\mathrm{cn}}, \overline{\delta}_{\mathrm{wn}}\right\} \leq \overline{\delta}_{\mathrm{c}}^*$ in this case. When (1.4.11) is violated, $e_{i,1}^* = 0$ at $\delta \in \left(\overline{\delta}_{\mathrm{c}}^*, 1\right]$, in which case $\overline{\delta}_{\mathrm{w}}^* = \overline{\delta}_{\mathrm{c}}^*$.

Consider $\frac{R}{c} \in \left[\frac{1}{p_0\lambda}, \frac{3+2\theta}{p_0\lambda(1+\theta)^3}\right)$. For $\frac{R}{c} \in \left[\frac{1}{p_0\lambda}, \min\left\{\frac{1}{\hat{p}_1\lambda}, \frac{3+2\theta}{p_0\lambda(1+\theta)^3}\right\}\right)$, $\left(e_{1,0}^*, e_{2,0}^*\right) = (1,0)$ as it's equivalent to a static game. For $\frac{R}{c} \in \left[\frac{1}{\hat{p}_1\lambda}, \frac{3+2\theta}{p_0\lambda(1+\theta)^3}\right)$, $V_0^{\mathrm{CW}}(p_0)$ can be simplified as:

$$\begin{aligned}
V_0^{\mathrm{CW}}(p_0) =& p_0\lambda(1+\theta) - \frac{3+2\theta}{(1+\theta)^2}c - \delta[p_0\lambda^2(1+\theta)R - p_0\lambda(1+\theta)c - (\frac{p_0}{\hat{p}_1} - 1)c \\
&+ \frac{(2+\theta)(1-p_0\lambda)c}{1+\theta}\max\left\{\frac{\hat{p}_1(2+\theta)}{p_1(1+\theta)} - \frac{3+\theta}{1+2\theta}, \frac{p_0}{\hat{p}_1} - 1\right\}] \\
\leq& -\delta\left[p_0\lambda^2(1+\theta)R - p_0\lambda(1+\theta)c - (\frac{p_0}{\hat{p}_1} - 1)c - \frac{p_0(2+\theta)(1-p_0\lambda)}{\hat{p}_1(1+\theta)}\right] \\
\propto& -\left[\frac{\lambda(2+\theta)}{1-\lambda(2+\theta)} + \frac{1-p_0}{(1-\lambda)[1-\lambda(2+\theta)]}\right] < 0
\end{aligned} \tag{A.0.66}$$

This implies that the collaboration is dominated by the individual work in this case. The analysis between the individual work and no investment at $t = 0$ follows the same argument as that in (A.0.65). As a result, now $\left(e_{1,0}^*, e_{2,0}^*\right) = (0,0)$ at $\delta \in (\overline{\delta}_{\mathrm{wn}}, 1]$ and $\left(e_{1,0}^*, e_{2,0}^*\right) = (1,0)$ at $\delta \in [0, \overline{\delta}_{\mathrm{wn}}]$, in which case $\overline{\delta}_w^* = \overline{\delta}_{\mathrm{wn}}$.

Finally, consider the small positive synergy, where $\theta \in \left[0, \min\{\frac{1-2\lambda}{\lambda}, 1\}\right)$. For $p_0 \geq p^T = \frac{(3+\theta)c}{(1+\theta)^2\lambda R}$, the argument would be the same as that in the negative synergy scenario with $p_0 \geq p^T = \frac{(3+2\theta)c}{(1+\theta)^3\lambda R}$, thus the similar threshold $\frac{\tilde{R}}{c}$ must exist in which case $\frac{\tilde{R}}{c} \geq \frac{3+\theta}{p_0\lambda(1+\theta)^2\lambda}$. For $\frac{R}{c} \in \left[\frac{3+\theta}{p_0\lambda(1+\theta)^2}, \frac{\tilde{R}}{c}\right)$, the argument would be the same as that in the negative synergy scenario with $\frac{R}{c} \in \left[\frac{3+2\theta}{p_0\lambda(1+\theta)^3}, \frac{\overline{R}}{c}\right)$, and then similar $\tilde{\delta}_c^*$ and $\tilde{\delta}_w^*$ can be achieved.

For $\frac{R}{c} \in \left[\frac{1}{p_0\lambda}, \frac{3+\theta}{p_0\lambda(1+\theta)^2}\right)$, $\triangle V_0^{\mathrm{CW}}(p_0)$ is always negative. Since the game is equivalent to a static one for $\frac{R}{c} \in \left[\frac{1}{p_0\lambda}, \min\left\{\frac{1}{\hat{p}_1\lambda}, \frac{3+\theta}{p_0\lambda(1+\theta)^2}\right\}\right)$, $\left(e_{1,0}^*, e_{2,0}^*\right) =$

$(1,0)$. For $\frac{R}{c} \in \left[\frac{1}{\hat{p}_1\lambda}, \min\left\{\frac{3+\theta}{p_0\lambda(1+\theta)^2}, \frac{1}{p_1\lambda}\right\}\right)$, $\triangle V_0^{\mathrm{CN}}(p_0)$ can be simplified as:

$$\triangle V_0^{\mathrm{CN}}(p_0) = p_0\lambda(2+\theta)R - \frac{2(2+\theta)}{1+\theta}c - \delta(p_0\lambda R - c) \tag{A.0.67}$$

If $\frac{R}{c} \in \left[\max\left\{\frac{2}{p_0\lambda(1+\theta)}, \frac{1}{\hat{p}_1\lambda}\right\}, \min\left\{\frac{3+\theta}{p_0\lambda(1+\theta)^2}, \frac{1}{p_1\lambda}\right\}\right) \subseteq \tilde{S}$, $\triangle V_0^{\mathrm{CN}}(p_0) \geq 0$ at $\delta = 0$ and $\triangle V_0^{\mathrm{CN}}(p_0) < 0$ at $\delta = 1$.Thus exists $\tilde{\delta}_{\mathrm{cn}} \in (0,1)$ such that $V_0^{\mathrm{CN}}(p_0) \geq 0$ at $\delta \in [0, \tilde{\delta}_{\mathrm{cn}}]$. Therefore, when (1.4.11) is violated, $\left(e^*_{1,0}, e^*_{2,0}\right) = (1,1)$ at $\delta \in [\delta_v, \tilde{\delta}_{\mathrm{cn}}]$, $\left(e^*_{1,0}, e^*_{2,0}\right) = (0,0)$ at $\delta \in (\tilde{\delta}_{\mathrm{cn}}, 1]$ and $\left(e^*_{1,0}, e^*_{2,0}\right) = (1,0)$ at $\delta \in [0, \delta_v)$. In this case, $\tilde{\delta}^*_c = \tilde{\delta}_{\mathrm{cn}}$, and it's clear that the interval $[\delta_v, \tilde{\delta}_{\mathrm{cn}}]$ is non-empty when (1.4.11) is violated. When (1.4.11) is satisfied, the partnership is dominated by the individual work as $\triangle V_0^{\mathrm{CW}}(p_0)$ is negative. The principal now only compares the individual work and no investment at $t = 0$, and the analysis follows the same argument as that in (A.0.65). As a result, in this case, $\left(e^*_{1,0}, e^*_{2,0}\right) = (0,0)$ at $\delta \in (\tilde{\delta}_{\mathrm{wn}}, 1]$, and $\left(e^*_{1,0}, e^*_{2,0}\right) = (1,0)$ at $\delta \in [0, \tilde{\delta}_{\mathrm{wn}}]$ and $\tilde{\delta}^*_w = \tilde{\delta}_{\mathrm{wn}}$.

For $\frac{R}{c} \in \left[\frac{1}{p_1\lambda}, \frac{3+\theta}{p_0\lambda(1+\theta)^2}\right)$, $\triangle V_0^{\mathrm{CN}}(p_0)$ would be:

$$\triangle V_0^{\mathrm{CN}}(p_0) = p_0\lambda(2+\theta)R - \frac{(2+\theta)^2}{(1+\theta)^2}c - \delta\left[p_0\lambda R(2+\theta) + B_1^1(p_0; \hat{p}_1, \hat{p}_1)\right] \tag{A.0.68}$$

For $\frac{R}{c} \in \left[\max\left\{\frac{2+\theta}{p_0\lambda(1+\theta)^2}, \frac{1}{p_1\lambda}\right\}, \frac{3+\theta}{p_0\lambda(1+\theta)^2}\right) \subseteq \tilde{S}$, $\triangle V_0^{\mathrm{CN}}(p_0) \geq 0$ at $\delta = 0$, thus exists $\tilde{\delta}_{\mathrm{cn}} \in (0,1]$ such that $V_0^{\mathrm{CN}}(p_0) \geq 0$ at $\delta \in [0, \tilde{\delta}_{\mathrm{cn}}]$. Therefore, when (1.4.11) is violated, $\left(e^*_{1,0}, e^*_{2,0}\right) = (1,1)$ at $\delta \in [\delta_v, \tilde{\delta}_{\mathrm{cn}}]$, $\left(e^*_{1,0}, e^*_{2,0}\right) = (0,0)$ at $\delta \in (\tilde{\delta}_{\mathrm{cn}}, 1]$, and $\left(e^*_{1,0}, e^*_{2,0}\right) = (1,0)$ at $\delta \in [0, \delta_v)$. In this case, $\tilde{\delta}^*_c = \tilde{\delta}_{\mathrm{cn}}$, and it's clear that the interval $[\delta_v, \tilde{\delta}_{\mathrm{cn}}]$ is non-empty when (1.4.11) is violated. When

(1.4.11) is satisfied, at $\delta \in [0, \tilde{\delta})$, $V_0^{\text{CW}}(p_0)$ can be simplified as:

$$
\begin{aligned}
V_0^{\text{CW}}(p_0) =& p_0\lambda(1+\theta)R - \frac{3+\theta}{1+\theta}c \\
&- \delta\left[p_0\lambda^2(1+\theta)R - p_0\lambda(1+\theta)c - (\frac{p_0}{\hat{p}_1} - 1)c - \frac{p_0(2+\theta)(1-p_0\lambda)}{\hat{p}_1(1+\theta)}\right] \\
\leq& -\delta\left[p_0\lambda^2(1+\theta)R - p_0\lambda(1+\theta)c - (\frac{p_0}{\hat{p}_1} - 1)c - \frac{p_0(2+\theta)(1-p_0\lambda)}{\hat{p}_1(1+\theta)}\right] \\
\propto& -\left[\frac{\lambda(2+\theta)}{1-\lambda(2+\theta)} + \frac{1-p_0}{(1-\lambda)[1-\lambda(2+\theta)]}\right] < 0
\end{aligned}
\tag{A.0.69}
$$

Similarly, at $\delta \in [0, \tilde{\delta})$, $V_0^{\text{CW}}(p_0)$ can be simplified as:

$$
\begin{aligned}
V_0^{\text{CW}}(p_0) =& p_0\lambda(1+\theta) - \frac{3+2\theta}{(1+\theta)^2}c - \delta[p_0\lambda^2(1+\theta)R - p_0\lambda(1+\theta)c - (\frac{p_0}{\hat{p}_1} - 1)c \\
&+ \frac{(2+\theta)(1-p_0\lambda)}{1+\theta}c\max\left\{\frac{\hat{p}_1(2+\theta)}{p_1(1+\theta)} - \frac{3+\theta}{1+2\theta}, \frac{p_0}{\hat{p}_1} - 1\right\}] \\
\leq& -\delta\left[p_0\lambda^2(1+\theta)R - p_0\lambda(1+\theta)c - (\frac{p_0}{\hat{p}_1} - 1)c - \frac{p_0(2+\theta)(1-p_0\lambda)}{\hat{p}_1(1+\theta)}\right] \\
\propto& -\left[\frac{\lambda(2+\theta)}{1-\lambda(2+\theta)} + \frac{1-p_0}{(1-\lambda)[1-\lambda(2+\theta)]}\right] < 0
\end{aligned}
\tag{A.0.70}
$$

These imply that the collaboration is dominated by the individual work in this case. The analysis between the individual work and no investment at $t = 0$ follows the same argument as that in (A.0.65). As a result, in this case, $\left(e_{1,0}^*, e_{2,0}^*\right) = (0,0)$ at $\delta \in (\tilde{\delta}_{\text{wn}}, 1]$, and $\left(e_{1,0}^*, e_{2,0}^*\right) = (1,0)$ at $\delta \in [0, \tilde{\delta}_{\text{wn}}]$ and $\tilde{\delta}_w^* = \tilde{\delta}_{\text{wn}}$.

For $\frac{R}{c} \in \left[\frac{1}{p_0\lambda}, \frac{3+\theta}{p_0\lambda(1+\theta)^2}\right) \setminus \tilde{S}$, the argument would be the same as that for For $\frac{R}{c} \in \left[\frac{1}{p_0\lambda}, \frac{3+2\theta}{p_0\lambda(1+\theta)^3}\right)$ with the negative synergy, in which (1.4.11) is always satisfied. As a result, in this case, $\left(e_{1,0}^*, e_{2,0}^*\right) = (0,0)$ at $\delta \in (\tilde{\delta}_w^*, 1]$, and $\left(e_{1,0}^*, e_{2,0}^*\right) = (1,0)$ at $\delta \in [0, \tilde{\delta}_w^*]$.Now $\tilde{\delta}_w^* = \tilde{\delta}_{\text{wn}}$ if (1.4.11) is satisfied; otherwise, $\tilde{\delta}_w^* = \max\left\{\tilde{\delta}_{\text{wn}}, \delta_v\right\}$.

Proof of Proposition 1.4.3

The notation follows that in the proof of Lemma 1.3.2.3). When motivating the collaboration, since two agents are identical, $s_{i,t} = \frac{1}{2}\omega_{1,t}$, and the optimal sub-contract at $t = 1$ in positive synergy case is $\omega^*_{2,t} = \frac{1}{2}$. Therefore, the free-riding incentive at t would be exactly the same as that in positive synergy case. As a result, when the synergy is positive, both the principal's profit maximisation problem and the agents' surplus are the same as those in three-tier structure since the free-riding incentive constraint binds in both environment; when the synergy is negative, the principal is strictly better off in the two-tier one since exclusion incentive is discarded which is binding in the three-tier one. Also, $s^*_{1,t} + s^*_{2,t}$ would be exactly the same as those in Lemma 1.4.1 and Proposition 1.4.1 with $\theta \geq 1$. This implies that agent one's surplus is strictly less in the negative synergy case, and agent two's stays the same.

When motivating the individual work, both the incentive of shirking and the over-investment incentive need to be satisfied, which are the same as those in the three-tier structure if the link between two agents still exists. In this case, the same contract as the three-tier one would be offered and (1.4.11) needs to be satisfied, thus the principal achieves the same profit level as the three-tier structure. If the link doesn't exist anymore, the principal only needs to consider the agent's incentive of shirking. In this case, the contract would still be the same as that in three-tier structure. As a result, the principal's profit is weakly higher with less distortion. This also implies that agent one's surplus is less. Agent two still gets zero.

When considering the optimal choice at $t = 0$, the analysis should be exactly the same as that in Proposition 1.4.2 with positive synergy if the link between two agents still exits. The principal reaches the same profit level as that in the three-tier structure. If the link doesn't exist, constraint (1.4.11) can be discarded, and the principal would never over-invest, in which case her expected profit is strictly higher.

Appendix B

Proofs for Chapter 2

B.0.1 Proofs for Preliminaries and Public Experimentation

Proof of Claim 2.3.1

When $a^0(\cdot)$ is a single reward, the agent would not conduct any experiments since the reported successes would not increase the reward. Thus, the principal would not observe any results of experiments being reported, and she solves the following maximisation problem:

$$\max_{a^0(0,0)} - p_0 \left(a^0(0,0) - M\right)^2 - (1 - p_0) \left(a^0(0,0)\right)^2$$

The optimal solution then is $a^0(\cdot) = a^0(0,0) = p_0 M$.

Proof of Lemma 2.3.1

Lemma 2.3.1.1): When the agent reports more than one failure, the principal learns that the agent's type is bad and he must have over-experimented after his first failure. Notice that the principal wants to deter such behaviour on the equilibrium path, thus $a^k(k^g, 1) \geq a^k(k^g, k^b > 1)$ is a plausible candidate to achieve such goal. If there exists a contract with $a^k(k^g, 1) < a^k(k^g, k^b > 1)$ which can achieve the same goal, the bad type's equilibrium path behaviour in such contract would be the same as that with $a^k(k^g, 1) \geq a^k(k^g, k^b > 1)$.

Lemma 2.3.1.2): Suppose the principal motivates a potential good type agent to report k successes without failures. When the agent reports $k^g \geq k$ successes with one failure, the principal learns that the agent is a bad type and he has over-experimented. To deter such deviation from the bad type agent, the principal would assign $a^k(k^g \geq k, 0) \geq a^k(k^g \geq k, 1) \geq 0$. Thus $a^k(k^g \geq k, 1) = 0$ is a candidate which can achieve such goal, and it's equal to bad type's true value. For any other contracts with $a^k(k^g \geq k, 1) > 0$ which can achieve the same goal, the bad type's equilibrium path behaviour would be the same as the contract with $a^k(k^g \geq k, 1) = 0$. Moreover, together with Lemma 2.3.1.1), $a^k(k^g \geq k, k^b \geq 1) = 0$.

Lemma 2.3.1.3): After achieving k successes without failures, the agent would stop experimenting if:

$$[1 - \theta(1 - p_j)] \triangle a^k(j + 1, 0) < c, \quad j > k \tag{B.0.1}$$

where $\triangle a^k(j + 1, 0) = a^k(j + 1, 0) - a^k(j, 0)$. Thus $\triangle a^k(j + 1, 0)$ must be bounded, $\triangle a^k(j + 1, 0) \in \left[0, \frac{c}{1-\theta(1-p_j)}\right)$. For $\forall k, j \in \mathbb{N}$, $\triangle a^k(j + 1, 0) \in [0, c)$ since $1 - \theta(1 - p_j)$ is increasing as j increases, the same incentive can be achieved by setting $\triangle a^k(j + 1, 0) = 0$.

Proof of Proposition 2.3.1

Given motivating the potential good type agent to report k successes, the principal's *ex ante* expected payoff can be represented as following:

$$\begin{aligned} V_P(k, p_0) = &- p_0 \left(a^k(k, 0) - M\right)^2 - (1 - p_0)(1 - \theta)^k \left(a^k(k, 0)\right)^2 \\ &- \sum_{i=0}^{k-1} (1 - p_0)(1 - \theta)^i \theta \left(a^k(i, 1)\right)^2 \end{aligned} \tag{B.0.2}$$

Given $a^k(\cdot)$ in proposition 2.3.1.1) is committed, the expected payoff above can be simplified as:

$$V_P(a^k(\cdot)) = -p_0(1 - p_k)M^2 - \left(\max\left\{0, \tilde{k}c - p_0 M\right\}\right)^2 \tag{B.0.3}$$

Proof by contradiction then can be applied in order to achieve the conclusion that $a^k(\cdot)$ in Proposition 2.3.1.1) is optimal and unique on the equilibrium path. Suppose not, then, given Properties in Lemma 2.3.1 are satisfied, there must exist another reward scheme $\tilde{a}^k(\cdot)$:

$$\begin{cases} \tilde{a}^k(j<k,1) = \max\left\{0, \tilde{k}c - p_0 M\right\} + \epsilon_j \\ \tilde{a}^k(k,0) = p_k M + \max\left\{0, \tilde{k}c - p_0 M\right\} + \epsilon_k \end{cases} \tag{B.0.4}$$

where $(\epsilon_k, ..., \epsilon_0) \in \mathbb{R}^k$ and $(\epsilon_k, \epsilon_{k-1}, ..., \epsilon_0) \neq \mathbf{0}$, such that $V_P(\tilde{a}^k(\cdot)) \geq V_P(a^k(\cdot))$ and IR constraint is still satisfied. Now the principal's expected payoff can be represented as:

$$\begin{aligned} V_P(\tilde{a}^k(\cdot)) = & - p_0 \left(p_k M + \max\left\{0, \tilde{k}c - p_0 M\right\} + \epsilon_k - M\right)^2 \\ & - (1-p_0)(1-\theta)^k \left(p_k M + \max\left\{0, \tilde{k}c - p_0 M\right\} + \epsilon_k\right)^2 \\ & - \sum_{i=0}^{k-1} (1-p_0)(1-\theta)^i \theta \left(\max\left\{0, \tilde{k}c - p_0 M\right\} + \epsilon_i\right)^2 \\ = & - p_0(1-p_k)M^2 - \left(\max\left\{0, \tilde{k}c - p_0 M\right\}\right)^2 - 2\max\left\{0, \tilde{k}c - p_0 M\right\}\bar{\epsilon} \\ & - \left\{\left[p_0 + (1-p_0)(1-\theta)^k\right]\epsilon_k^2 + \sum_{i=0}^{k-1}(1-p_0)(1-\theta)^i \theta \epsilon_i^2\right\} \end{aligned} \tag{B.0.5}$$

Where $\bar{\epsilon} = \left[p_0 + (1-p_0)(1-\theta)^k\right]\epsilon_k + \sum_{i=0}^{k-1}(1-p_0)(1-\theta)^i \theta \epsilon_i$. Since $\tilde{a}^k(\cdot)$ must satisfy IR constraint:

$$\begin{aligned} & \mathbb{E}(\tilde{a}^k(k^g, k^b)|k, p_0) - \tilde{k}c \geq 0 \\ \Longrightarrow \quad & \bar{\epsilon} \geq -\max\left\{0, \tilde{k}c - p_0 M\right\} + \tilde{k}c - p_0 M = \min\left\{\tilde{k}c - p_0 M, 0\right\} \\ \Longrightarrow \quad & \max\left\{0, \tilde{k}c - p_0 M\right\}\bar{\epsilon} \geq \max\left\{0, \tilde{k}c - p_0 M\right\}\min\left\{\tilde{k}c - p_0 M, 0\right\} \geq 0 \end{aligned} \tag{B.0.6}$$

Notice $(\epsilon_k, \epsilon_{k-1}, ..., \epsilon_0) \neq \mathbf{0}$:

$$\left[p_0 + (1-p_0)(1-\theta)^k\right]\epsilon_k^2 + \sum_{i=0}^{k-1}(1-p_0)(1-\theta)^i \theta \epsilon_i^2 > 0 \tag{B.0.7}$$

Combining from (B.0.5) to (B.0.7), the following result can be achieved:

$$
\begin{aligned}
V_P(\tilde{a}^k(\cdot)) - V_P(a^k(\cdot)) = - &\left\{ \left[p_0 + (1-p_0)(1-\theta)^k\right] \epsilon_k^2 + \sum_{i=0}^{k-1} (1-p_0)(1-\theta)^i \theta \epsilon_i^2 \right\} \\
&- 2\max\left\{0, \tilde{k}c - p_0 M\right\} \overline{\epsilon} \quad < 0
\end{aligned} \tag{B.0.8}
$$

which contradicts to $V_P(\tilde{a}^k(\cdot)) \geq V_P(a^k(\cdot))$. Therefore, it can concluded that $a^k(\cdot)$ is uniquely optimal on the equilibrium path.

The following part focuses on the optimal motivated number of experiments, k^P in public experimentation. When $k \leq \overline{k}$, $\max\left\{0, \tilde{k}c - p_0 M\right\} = 0$, from (B.0.3):

$$
V_P(a^k(\cdot)) = -p_0(1-p_k)M^2 \tag{B.0.9}
$$

and the principal's expected payoff is increasing as k increases, and this implies $k^P = \overline{k}$ in this scenario.

When $k > \overline{k}$, $\max\left\{0, \tilde{k}c - p_0 M\right\} = \tilde{k}c - p_0 M$, (B.0.3) becomes:

$$
V_P(a^k(\cdot)) = -p_0(1-p_k)M^2 - \left(\tilde{k}c - p_0 M\right)^2 \tag{B.0.10}
$$

the second term $\left(\tilde{k}c - p_0 M\right)^2$ is increasing as k increases, and it would undermine the benefit of exploration through experiment. Notice that when $k \to \infty$, $V_P(a^k(\cdot)) \to -\infty$, the optimal number k^P must be finite, $k^P < \infty$.

B.0.2 Proofs for Private Experimentation

Proof of Lemma 2.4.1

Suppose, when a potential good agent is motivated to report $k \in \mathbb{N}^+$ successes, in the optimal reward scheme $a^k(\cdot)$, there exit(s) some $j \in \mathbb{N}$ and $0 \leq j < k$, such that

$$
\max\left\{a^k(j+1,1), a^k(j+1,0)\right\} - \max\left\{a^k(j,1), a^k(j,0)\right\} > \frac{c}{1-\theta}
$$

To simplify the notation, I let $\rho_i = \max\left\{a^k(i,1), a^k(i,0)\right\}$ for $i = 0, ..., k$. Given the scheme $a^k(\cdot)$, I use α_k to denote the reward that a potential good type agent receives and use $\alpha_{0\leq i<k}$ to denote the reward that is actually re-

ceived by the bad type agent whose first failure occurs in $i+1_{th}$ experiment. In each of the following steps, I construct a profitable and feasible deviation for the principal to show the contradiction without violating IC^S in (2.4.3).

Step 1. Suppose $a^k(k,0)-\rho_{k-1} > \frac{c}{1-\theta}$. A bad type agent with $k-1$ successes would over-experiment since the expected extra gain is higher than the extra cost of doing so. Thus $\alpha_k = \alpha_{k-1} = a^k(k,0)$ and $\alpha_{j<k-1} = \rho_{\hat{n}}$, where $\hat{n} \in \max\limits_{n\in\mathbb{N}, j\le n\le k} \rho_n - (n-j)\frac{c}{1-\theta}$. The principal's expected payoff now is

$$V(a^k(\cdot)) = -p_0\left(a^k(k,0)-M\right)^2 - (1-p_0)(1-\theta)^{k-1}\left[a^k(k,0)\right]^2 - \sum_{j=1}^{k-2}(1-p_0)(1-\theta)^j\theta\alpha_j^2 \tag{B.0.11}$$

and the continuation payoff of a potential good agent with posterior belief p_j can be simplified as

$$\begin{aligned} U(k-j,p_j) =& \frac{p_j}{p_k}a^k(k,0) + (1-p_j)(1-\theta)^{k-j-1}\theta\left[a^k(k,0)-\frac{c}{1-\theta}\right] \\ &+\sum_{i=j}^{k-2}(1-p_j)(1-\theta)^i\theta U(i-j,0) - \sum_{i=j}^{k}\frac{p_j}{p_i}c \end{aligned} \tag{B.0.12}$$

which satisfies IC^S in (2.4.3).

Now consider a different reward scheme $\tilde{a}^k(\cdot)$, in which $\tilde{a}^k(k-1,0) = \tilde{a}^k(k-1,1) = a^k(k,0) - \frac{c}{1-\theta}$, and $\tilde{a}^k(i,n) = a^k(i,n)$ for $i = 1,...,k-2,k$ and $n\in\mathbb{N}$. The bad type agent with $k-1$ successes would not over-experiment since the extra gain of doing so equals to the expected cost. Thus $\tilde{\alpha}_k = \alpha_k$, $\tilde{\alpha}_{k-1} = \alpha_k - \frac{c}{1-\theta}$ and $\tilde{\rho}_j = \rho_j$ for $j = 0,...,k-2$. Notice that, under $\tilde{a}^k(\cdot)$, the continuation payoff of the bad type agent with $j<k-1$ successes is

$$\begin{aligned} &\tilde{U}(k-j,0) = \\ &\max\left\{\underbrace{\tilde{\alpha}_k}_{=\alpha_k} - (k-j)\frac{c}{1-\theta}, \underbrace{\tilde{\alpha}_{k-1}-(k-j-1)\frac{c}{1-\theta}}_{>\rho_{k-1}-(k-j-1)\frac{c}{1-\theta}}, \max_{n\in\mathbb{N}^+, i<k-1}\rho_n - (n-j)\frac{c}{1-\theta}\right\} \\ &\ge U(k-j,0) \end{aligned} \tag{B.0.13}$$

Thus, the continuation payoff of the potential good agent with posterior belief

p_j would be

$$
\begin{aligned}
\tilde{U}(k-j,p_j) =& \frac{p_j}{p_k}a^k(k,0) + (1-p_j)(1-\theta)^{k-j-1}\theta\left[a^k(k,0) - \frac{c}{1-\theta}\right] \\
&+ \sum_{i=j}^{k-2}(1-p_j)(1-\theta)^i\theta\tilde{U}(i-j,0) - \sum_{i=j}^{k}\frac{p_j}{p_i}c \\
\geq& U(k-j,p_j)
\end{aligned}
\tag{B.0.14}
$$

This inequality implies that $\mathrm{IC}^S_{0\leq j<k-1}$ in (2.4.3) are still satisfied under the new reward scheme $\tilde{a}^k(\cdot)$. Notice that now the potential good agent with posterior belief p_{k-1} can get $a^k(k,0) - \frac{c}{1-\theta}$ if he stops immediately, then

$$
\begin{aligned}
&\tilde{U}(1,p_{k-1}) - \left[a^k(k,0) - \frac{c}{1-\theta}\right] \\
&= \frac{p_{k-1}}{p_k}a^k(k,0) + (1-p_{k-1})\theta\left[a^k(k,0) - \frac{c}{1-\theta}\right] - c - \left[a^k(k,0) - \frac{c}{1-\theta}\right] \\
&= \frac{p_{k-1}}{p_k}\frac{c}{1-\theta} - c \propto 1-(1-\theta) = \theta > 0
\end{aligned}
\tag{B.0.15}
$$

Therefore, IC^S_{k-1} is also satisfied and (B.0.13) and (B.0.15) imply that the new scheme $\tilde{a}^k(\cdot)$ can also motivates a potential good type agent to report k successes on equilibrium path.

If there exits $0 \leq j' < k-1$ such that $\rho_{k-1} - \rho_{j'} \leq (k-1-j')\frac{c}{1-\theta}$ and $a^k(k,0) - \rho_{j'} \leq (k-j')\frac{c}{1-\theta}$ under $a^k(\cdot)$, they also hold under $\tilde{a}^k(\cdot)$ since $\tilde{a}^k(k,0) = a^k(k,0)$, $\tilde{\rho}_{j'} = \rho_{j'}$ and

$$
\tilde{\rho}_{k-1} - \tilde{\rho}_{j'} = a^k(k,0) - \frac{c}{1-\theta} - \rho_{j'} \leq (k-j')\frac{c}{1-\theta} - \frac{c}{1-\theta} = (k-1-j')\frac{c}{1-\theta}
\tag{B.0.16}
$$

which implies that $\tilde{\alpha}_{j'} = \alpha_{j'}$. If there exits $0 \leq j'' < k-1$ such that $\rho_{k-1} - \rho_{j''} > (k-1-j'')\frac{c}{1-\theta}$, this bad type agent would stop over-experimenting once he achieves $k-1$ successes under $\tilde{a}^k(\cdot)$, which makes $\tilde{\alpha}_{j''} < \alpha_{j''}$. As a result, $\tilde{\alpha}_j \leq \alpha_j$ for $0 \leq j < k-1$.

Under $\tilde{a}^k(\cdot)$, the principal's expected payoff can be written as

$$\begin{aligned}
&V(\tilde{a}^k(\cdot)) \\
=& - p_0 \left(a^k(k,0) - M\right)^2 - (1-p_0)(1-\theta)^k \left[a^k(k,0)\right]^2 \\
&- (1-p_0)(1-\theta)^{k-1}\theta \left[a^k(k,0) - \frac{c}{1-\theta}\right]^2 - \sum_{j=1}^{k-2}(1-p_0)(1-\theta)^j\theta\tilde{\alpha}_j^2 \\
>& - p_0 \left(a^k(k,0) - M\right)^2 - (1-p_0)(1-\theta)^{k-1} \left[a^k(k,0)\right]^2 - \sum_{j=1}^{k-2}(1-p_0)(1-\theta)^j\theta\alpha_j^2 \\
=& V(a^k(\cdot))
\end{aligned} \tag{B.0.17}$$

This shows that the principal is strictly better off by offering $\tilde{a}^k(\cdot)$ instead of $a^k(\cdot)$. This contradicts to that $a^k(\cdot)$ is optimal. As a result, $a^k(k,0) - \rho_{k-1} \leq \frac{c}{1-\theta}$ must hold.

Step 2. Suppose $\rho_{k-1} - \rho_{k-2} > \frac{c}{1-\theta}$. Together with the result in step 1, a bad type agent with $k-2$ successes would over-experiment and stop when he achieves $k-1$ successes. Thus $\alpha_k = a^k(k,0)$, $\alpha_{k-1} = \alpha_{k-2} = \rho_{k-1}$ and $\alpha_{j<k-2} = \rho_{\hat{n}}$. Then principal's expected payoff now is

$$\begin{aligned}
V(a^k(\cdot)) =& - p_0 \left(a^k(k,0) - M\right)^2 - (1-p_0)(1-\theta)^k \left[a^k(k,0)\right]^2 \\
&- (1-p_0)(1-\theta)^{k-1} \left(\rho_{k-1}\right)^2 - (1-p_0)(1-\theta)^{k-1} \left(\rho_{k-1}\right)^2 \\
&- \sum_{j=1}^{k-3}(1-p_0)(1-\theta)^j\theta\alpha_j^2
\end{aligned} \tag{B.0.18}$$

and the continuation payoff of a potential good agent with posterior belief p_j is

$$\begin{aligned}
&U(k-j, p_j) \\
=& - \sum_{i=j}^{k} \frac{p_j}{p_i}c + \frac{p_j}{p_k}a^k(k,0) + (1-p_j)(1-\theta)^{k-j-1}\theta\rho_{k-1} \\
&+ (1-p_j)(1-\theta)^{k-j-2}\theta\left(\rho_{k-1} - \frac{c}{1-\theta}\right) + \sum_{i=j}^{k-2}(1-p_j)(1-\theta)^i\theta U(i-j,0)
\end{aligned} \tag{B.0.19}$$

which satisfies IC^S in (2.4.3).

Consider another reward scheme $\check{a}^k(\cdot)$, in which $\check{a}^k(k-2,0) = \check{a}^k(k-2,1) = \rho_{k-1} - \frac{c}{1-\theta}$, and $\check{a}^k(i,n) = a^k(i,n)$ for $i = 1,...,k$ and $i \neq k-2$. Now the bad type agent with $k-2$ success would not over-experiment since the extra gain is the same as the expected cost of doing so. Thus $\check{\alpha}_k = \alpha_k$, $\check{\alpha}_{k-1} = \alpha_{k-1} = \rho_{k-1}$, $\check{\alpha}_{k-2} = \rho_{k-1} - \frac{c}{1-\theta}$ and $\check{\rho}_j = \rho_j$ for $j = 0,...,k-3$. Thus, under $\check{a}^k(\cdot)$, the continuation payoff of the bad type agent with $j < k-2$ successes is

$$\begin{aligned}
&\check{U}(k-j,0) \\
&= \max\left\{\underbrace{\check{\alpha}_{k-1}}_{=\rho_{k-1}} - (k-1-j)\frac{c}{1-\theta}, \underbrace{\check{\alpha}_{k-2} - (k-j-2)\frac{c}{1-\theta}}_{>\rho_{k-2}-(k-j-2)\frac{c}{1-\theta}}\right. \\
&\qquad \left., \max_{n\in\mathbb{N}^+, i<k-2} \rho_n - (n-j)\frac{c}{1-\theta}\right\} \\
&\geq U(k-j,0)
\end{aligned} \tag{B.0.20}$$

which implies that $\text{IC}^S_{0\leq j<k-2}$ in (2.4.3) are still satisfied under $\check{a}^k(\cdot)$. Since $\check{a}^k(k,0) = a^k(k,0)$ and $\check{\alpha}_{k-1} = \alpha_{k-1}$, IC^S_{k-1} is also satisfied. Notice that now the potential good type agent with posterior belief p_{k-2} can get $\alpha_{k-1} - \frac{c}{1-\theta}$ if he stops immediately, then

$$\begin{aligned}
&\check{U}(2,p_{k-2}) - \left(\rho_{k-1} - \frac{c}{1-\theta}\right) \\
&= \frac{p_{k-2}}{p_{k-1}}\left[\overbrace{-c + \frac{p_{k-1}}{p_{p_k}}a^k(k,0) + (1-p_{k-1})\theta\alpha_{k-1}}^{\geq\rho_{k-1} \text{ from } \text{IC}^S_{k-1}}\right] \\
&\quad + (1-p_{k-2})\theta\left(\rho_{k-1} - \frac{c}{1-\theta}\right) - c - \left(\rho_{k-1} - \frac{c}{1-\theta}\right) \\
&\geq \frac{p_{k-2}}{p_{k-1}}\rho_{k-1} + (1-p_{k-2})\theta\left(\rho_{k-1} - \frac{c}{1-\theta}\right) - c - \left(\rho_{k-1} - \frac{c}{1-\theta}\right) \\
&= \frac{p_{k-2}}{p_{k-1}}\frac{c}{1-\theta} - c \propto 1 - (1-\theta) = \theta > 0
\end{aligned} \tag{B.0.21}$$

This means that IC^S_{k-2} is also satisfied. Also, (B.0.20) and (B.0.21) show that the $\check{a}^k(\cdot)$ can also motivates a potential good type agent to report k successes on equilibrium path.

If there exists $0 \leq i' < k-2$ such that $\rho_{k-2} - \rho_{i'} \leq (k-2-i')\frac{c}{1-\theta}$ and $\rho_{k-1} - \rho_{i'} \leq (k-1-i')\frac{c}{1-\theta}$ under $a^k(\cdot)$, they also hold under $\check{a}^k(\cdot)$ since $\check{\alpha}_{k-1} = \alpha_{k-1} = \rho_{k-1}$, $\check{\rho}_{i'} = \rho_{i'}$ and

$$\check{\alpha}_{k-2} - \check{\rho}_{i'} = \rho_{k-1} - \frac{c}{1-\theta} - \rho_{i'} \leq (k-1-i')\frac{c}{1-\theta} - \frac{c}{1-\theta} = (k-2-i')\frac{c}{1-\theta} \tag{B.0.22}$$

This implies that $\check{\rho}_{i'} = \rho_{i'}$. If there exits $0 \leq i'' < k-2$ such that $\rho_{k-2} - \rho_{i''} > (k-2-i'')\frac{c}{1-\theta}$, this bad type agent would stop over-experimenting once he achieves $k-2$ successes under $\check{a}^k(\cdot)$, which makes $\check{\alpha}_{i''} < \alpha_{i''}$. Therefore, $\check{\alpha}_i \leq \alpha_i$ for $0 \leq i < k-2$.

Under $\check{a}^k(\cdot)$, the principal's expected payoff is

$$\begin{aligned}
&V(\check{a}^k(\cdot)) \\
&\quad = -p_0\left(a^k(k,0) - M\right)^2 - (1-p_0)(1-\theta)^k\left[a^k(k,0)\right]^2 \\
&\qquad - (1-p_0)(k-1)^{k-1}\theta(\rho_{k-1})^2 - (1-p_0)(1-\theta)^{k-2}\theta\left(\rho_{k-1} - \frac{c}{1-\theta}\right)^2 \\
&\qquad - \sum_{j=1}^{k-3}(1-p_0)(1-\theta)^j\theta\check{\alpha}_j^2 \\
&\quad > -p_0\left(a^k(k,0) - M\right)^2 - (1-p_0)(1-\theta)^{k-1}\left[a^k(k,0)\right]^2 \\
&\qquad - (1-p_0)(k-1)^{k-1}\theta(\rho_{k-1})^2 \\
&\qquad - (1-p_0)(k-1)^{k-2}\theta(\rho_{k-1})^2 \\
&\qquad - \sum_{j=1}^{k-2}(1-p_0)(1-\theta)^j\theta\alpha_j^2 = V(a^k(\cdot))
\end{aligned} \tag{B.0.23}$$

This shows that, compared to $a^k(\cdot)$, the principal can find a profitable deviation by offering $\check{a}^k(\cdot)$. This contradicts to that $a^k(\cdot)$ is optimal. As a result, $\rho_{k-1} - \rho_{k-2} \leq \frac{c}{1-\theta}$ must hold.

Step 3. Repeat the similar argument sequentially with descending order from $j = k-3$ to $j = 0$ and suppose $\rho_{j+1} - \rho_j > \frac{c}{1-\theta}$. I can always construct another feasible deviation $\dot{a}^k(\cdot)$, in which $\dot{a}^k(j,1) = \dot{a}^k(j,0) = \rho_{j+1} - \frac{c}{1-\theta}$, and $\dot{a}^k(i,n) = a^k(i,n)$ for $i = 1,..,k$ and $i \neq j$. With the similar argument in step 2, IC_i^S in (2.4.3) are still satisfied for $i = 0,...,k$ and $i \neq j$. Similarly, for the good agent with posterior belief p_j,

$$\begin{aligned}\dot{U}(k-j,p_j)-&\left(\rho_{j+1}-\frac{c}{1-\theta}\right)\\&=\frac{p_j}{p_{j+1}}U(k-j-1,p_{j+1})+(1-p_j)\theta\left(\rho_{j+1}-\frac{c}{1-\theta}\right)\\&\quad-c-\left(\rho_{j+1}-\frac{c}{1-\theta}\right)\\&\geq\frac{p_j}{p_{j+1}}\rho_{j+1}+(1-p_j)\theta\left(\rho_{j+1}-\frac{c}{1-\theta}\right)-c-\left(\rho_{j+1}-\frac{c}{1-\theta}\right)\\&=\frac{p_j}{p_{j+1}}\frac{c}{1-\theta}-c\propto\theta>0\end{aligned}\tag{B.0.24}$$

Then IC_j^S is also satisfied. Furthermore, by applying the same argument as those in step 2, it must be true that $\dot{\alpha}_n\leq\alpha_n$ for $n=0,...,j$. Therefore, the principal now can receive

$$\begin{aligned}&V(\dot{a}^k(\cdot))\\=&-p_0\left(a^k(k,0)-M\right)^2-(1-p_0)(1-\theta)^k\left[a^k(k,0)\right]^2-\sum_{i=j+1}^{k}(1-p_0)(1-\theta)^i\theta\alpha_i^2\\&-(1-p_0)(1-\theta)^j\theta\left(\rho_{j+1}-\frac{c}{1-\theta}\right)^2-\sum_{i=1}^{j-1}(1-p_0)(1-\theta)^i\theta\dot{\alpha}_i^2\\>&-p_0\left(a^k(k,0)-M\right)^2-(1-p_0)(1-\theta)^k\left[a^k(k,0)\right]^2-\sum_{i=j+1}^{k}(1-p_0)(1-\theta)^i\theta\alpha_i^2\\&-(1-p_0)(1-\theta)^j\theta\rho_{j+1}^2-\sum_{i=1}^{j-1}(1-p_0)(1-\theta)^i\theta\alpha_i^2=V(a^k(\cdot))\end{aligned}\tag{B.0.25}$$

This result contradicts to $\rho_{j+1}-\rho_j>\frac{c}{1-\theta}$. As a result, $\rho_{j+1}-\rho_j\leq\frac{c}{1-\theta}$ must hold. To sum up, $\rho_{j+1}-\rho_j$ must hold for $0\leq j<k$.

Given motivating the potential good agent to report k successes, the principal solves the following utility maximisation problem in private experimentation:

$$
\begin{aligned}
\underset{a^k(\cdot)\geq 0}{Max} \quad & V(k,p_0) = \mathbb{E}\left(-\left(a^k(\cdot)-M_i\right)^2 \middle| k,p_0\right) \\
\text{s.t}: \quad & \text{IR}: \ U(k,p_0)\geq 0 \\
& \text{IC}^S: \ U(k-j,p_j)\geq a^k(j,0), \ 0\leq j<k \\
& \text{IC}^F: \ \max\left\{a^k(j+1,1),a^k(j+1,0)\right\} \\
& \qquad\qquad -\max\left\{a^k(j,1),a^k(j,0)\right\}\leq \frac{c}{1-\theta}
\end{aligned} \tag{B.0.26}
$$

The Karush-Kuhn-Tucker condtion (KKT) can be applied to solve the constrained maximisation problem above, since the feasible set under the constraints is convex and utility function is continuous and quasi-concave. However, it's too tedious and not convenient to follow the logic if the details of KKT are shown. Thus an alternative way could be adopted—proof by contradiction, which is similar as the proof of Proposition 2.3.1.1).

Proof of Lemma 2.4.2

Suppose $0 \leq a^k(j<k,1) < a^k(j,0)$, the agent would never disclose failures if any. Thus, on the equilibrium path, the principal can only observe successes are reported, and constraints become:

$$
\begin{aligned}
\text{IR}': \quad & \sum_{i=1}^{k}\frac{p_0}{p_i}\left[a^k(i,0)-a^k(i-1,0)\right]+a^k(0,0)\geq \sum_{i=1}^{k}\frac{p_0}{p_{i-1}}c \\
\text{IC}^{S'}: \quad & \sum_{i=j+1}^{k}\frac{p_0}{p_i}\left[a^k(i,0)-a^k(i-1,0)\right]\geq \sum_{i=j+1}^{k}\frac{p_0}{p_{i-1}}c \\
\text{IC}^{F'}: \quad & a^k(j+1,0)-a^k(j,0)\leq \frac{c}{1-\theta}, \ 0\leq j<k
\end{aligned} \tag{B.0.27}
$$

The structure of constraints is the same as that in private experimentation with unverifiable failures. Instead, if assigning $a^k(j<k,1)\geq a^k(j,0)$, the principal gives the incentive to the agent to disclose all acquired realisations. She can do so because failures are verifiable and the agent can prove himself

that he indeed ran experiments but failed. Constrains now are:

$$
\begin{aligned}
&\text{IR}: \quad \sum_{i=1}^{k} \frac{p_0}{p_i}\left[a^k(i,1) - a^k(i-1,1)\right] + a^k(0,1) \geq \sum_{i=1}^{k} \frac{p_0}{p_{i-1}} c \\
&\text{IC}^S: \quad \sum_{i=j+1}^{k} \frac{p_0}{p_i}\left[a^k(i,1) - a^k(i-1,1)\right] + \frac{p_0}{p_j}\left[a^k(j,1) - a^k(j,0)\right] \geq \sum_{i=j+1}^{k} \frac{p_0}{p_{i-1}} c \\
&\text{IC}^F: \quad a^k(j+1,1) - a^k(j,1) \leq \frac{c}{1-\theta} \quad \& \quad a^k(k,0) - a^k(k-1,1) \leq \frac{c}{1-\theta}
\end{aligned}
\tag{B.0.28}
$$

Notice that the feasible set in (B.0.27) is weakly smaller than that in (B.0.28), and the latter reaches the largest when $a^k(j < k, 0) = 0$. Without solving original maximisation problem, it can be concluded that the solution in scenario "$a^k(j < k, 1) \geq a^k(j, 0) = 0$" is weakly better than that in scenario "$0 \leq a^k(j < k, 1) < a^k(j, 0)$". This conclusion can also be confirmed later when the optimal reward schemes with verifiable and unverifiable failures are compared, and this is because that the structure of solution to maximisation problem when "$0 \leq a^k(j < k, 1) < a^k(j, 0)$" is the same as that with unverifiable failures.

Proof of Proposition 2.4.1

Consider the optimal reward scheme CF in public experimentation. When $k \leq \hat{k}$, $p^k M \leq \frac{c}{1-\theta}$ and this implies that IC^F and IC^S are always satisfied in CF. Moreover, since CF is the optimal reward scheme in public experimentation which contains least constraints, CF must be the optimal reward scheme in this current scenario as well. However, when $k > \hat{k}$, $p_k M > \frac{c}{1-\theta}$ and this leads the last IC^F, $a^k(k,0) - a^k(k-1,1) \leq \frac{c}{1-\theta}$, to be violated.

Now proof by contradiction can be applied to check if the Type-I step function (MF-I) proposed in Proposition 2.4.1.1.b) is optimal when $k > \hat{k}$. Suppose not, then, there must exist another reward scheme $b^k(\cdot)$:

$$
\begin{cases}
b^k(j < k, 1) = a^k(j, 1) + \eta_j \\
b^k(k, 0) = a^k(k, 0) + \eta_k
\end{cases}
\tag{B.0.29}
$$

where $(\eta_k, ..., \eta_0) \in \mathbb{R}^k$ and $(\eta_k, ..., \eta_0) \neq \mathbf{0}$, such that $V_V\left(b^k(\cdot)\right) \geq V_V\left(a^k(\cdot)\right)$ and all constraints are satisfied, where $V_V(\cdot)$ is the principal's expected payoff in private experimentation with verifiable failures. The principal's expected payoff with $b^k(\cdot)$ then can be represented as:

$$\begin{aligned}
&V_V(b^k(\cdot))\\
=&-p_0\left(a^k(k,0)+\eta_k-M\right)^2-(1-p_0)(1-\theta)^k\left(a^k(k,0)+\eta_k\right)^2\\
&-\sum_{i=0}^{k-1}(1-p_0)(1-\theta)^i\theta\left(a^k(i,1)+\eta_i\right)^2\\
=&V_V(a^k(\cdot))-\left\{\frac{p_0}{p_k}\eta_k^2+\sum_{i=0}^{k-1}(1-p_0)(1-\theta)^i\theta\eta_i^2\right\}-2\max\left\{0,\tilde{k}c-p_0M\right\}\overline{\eta}\\
&+2p_0M\eta_k-2(k-l)\frac{p_0}{p_k}\frac{c}{1-\theta}\eta_k-2\frac{c}{1-\theta}\sum_{i=l+1}^{k-1}(1-p_0)(1-\theta)^i\theta(i-l)\eta_i\\
&-2\left(p_lM-\sum_{i=l+1}^{k}\frac{p_l}{p_i}\frac{c}{1-\theta}\right)\left[\frac{p_0}{p_k}\eta_k+\sum_{i=l}^{k-1}(1-p_0)(1-\theta)^i\theta\eta_i\right]
\end{aligned} \tag{B.0.30}$$

Where $\overline{\eta}=\frac{p_0}{p_k}\eta_k+\sum_{i=0}^{k-1}(1-p_0)(1-\theta)^i\theta\eta_i$. Notice $b^k(\cdot)$ must satisfy IR, and it's similar to (B.0.6):

$$\begin{aligned}
&\mathbb{E}(b^k(\cdot)|k,p_0)-\tilde{k}c\geq 0\\
\Longrightarrow\quad&\overline{\eta}\geq-\max\left\{0,\tilde{k}c-p_0M\right\}+\tilde{k}c-p_0M=\min\left\{\tilde{k}c-p_0M,0\right\}\\
\Longrightarrow\quad&\max\left\{0,\tilde{k}c-p_0M\right\}\overline{\eta}\geq\max\left\{0,\tilde{k}c-p_0M\right\}\min\left\{\tilde{k}c-p_0M,0\right\}\geq 0
\end{aligned} \tag{B.0.31}$$

Meanwhile, $\text{IC}^{F,V}$ must be satisfied:

$$\begin{cases}
b^k(k,0)-b^k(k-1,1)\leq\frac{c}{1-\theta}\\
...\\
b^k(l+1,1)-b^k(l,1)\leq\frac{c}{1-\theta}\\
...\\
b^k(1,1)-b^k(0,1)\leq\frac{c}{1-\theta}
\end{cases}\Longrightarrow\begin{cases}
\eta_k-\eta_{k-1}\leq 0\\
...\\
\eta_{l+1}-\eta_l\leq 0
\end{cases}\Longrightarrow\eta_{l\leq j<k}\leq-\eta_k \tag{B.0.32}$$

Now apply this result to Equation (B.0.30):

$$
\begin{aligned}
&V_V(b^k(\cdot)) \\
\leq &V_V(a^k(\cdot)) - \left\{ \frac{p_0}{p_k}\eta_k^2 + \sum_{i=0}^{k-1}(1-p_0)(1-\theta)^i\theta\eta_i^2 \right\} - 2\max\left\{0, \tilde{k}c - p_0M\right\}\overline{\eta} \\
&+ 2p_0M\eta_k - 2\frac{c}{1-\theta}\eta_k\left[(k-l)\frac{p_0}{p_k} + \sum_{i=l+1}^{k-1}(1-p_0)(1-\theta)^i\theta(i-l)\right] \\
&- 2\left(p_lM - \sum_{i=l+1}^{k}\frac{p_l}{p_i}\frac{c}{1-\theta}\right)\left[\frac{p_0}{p_k} + \sum_{i=l}^{k-1}(1-p_0)(1-\theta)^i\theta\right]\eta_k \\
= &V_V(a^k(\cdot)) - \left\{ \frac{p_0}{p_k}\eta_k^2 + \sum_{i=0}^{k-1}(1-p_0)(1-\theta)^i\theta\eta_i^2 \right\} - 2\max\left\{0, \tilde{k}c - p_0M\right\}\overline{\eta} \\
&+ 2\left(p_0M - \sum_{i=l+1}^{k}\frac{p_0}{p_i}\frac{c}{1-\theta}\right)\eta_k - 2\left(p_0M - \sum_{i=l+1}^{k}\frac{p_0}{p_i}\frac{c}{1-\theta}\right)\eta_k \\
= &V_V(a^k(\cdot)) - \left\{ \frac{p_0}{p_k}\eta_k^2 + \sum_{i=0}^{k-1}(1-p_0)(1-\theta)^i\theta\eta_i^2 \right\} - 2\max\left\{0, \tilde{k}c - p_0M\right\}\overline{\eta}
\end{aligned} \tag{B.0.33}
$$

Since $(\eta_k, ..., \eta_0) \neq \mathbf{0}$:

$$
\frac{p_0}{p_k}\eta_k^2 + \sum_{i=0}^{k-1}(1-p_0)(1-\theta)^i\theta\eta_i^2 > 0 \tag{B.0.34}
$$

Combing (B.0.31), (B.0.33) and (B.0.34), it can be concluded that:

$$
\begin{aligned}
V_V(b^k(\cdot)) - V_V(a^k(\cdot)) \leq &- \left\{ \frac{p_0}{p_k}\eta_k^2 + \sum_{i=0}^{k-1}(1-p_0)(1-\theta)^i\theta\eta_i^2 \right\} \\
&- 2\max\left\{0, \tilde{k}c - p_0M\right\}\overline{\eta} < 0
\end{aligned} \tag{B.0.35}
$$

This result contradicts to $V_V(b^k(\cdot)) \geq V_V(a^k(\cdot))$. Therefore, the proposed reward scheme in Proposition 2.4.1.1) is optimal.

When $\hat{k} \geq k^P$, CF is still feasible at level k^P, thus k^P must be the number of successes which gives the principal least expected loss for $k \leq \hat{k}$. On the other hand, for $k > \hat{k}$, compared to CF, MF-I is the optimal reward scheme with more constraints and it implies that the principal's expected loss is higher under MF-I scheme than that under CF scheme give the same number of experiments k. Therefore, k is dominated by k^P for $\forall k > \hat{k}$ in this case.

When $\hat{k} < k^P$, CF is no longer feasible at level k^P, and MF-I is optimal and the proof is the same as above. To show that the optimal number of experiments in private environment is still higher than the first threshold number, $k_V^* \geq \overline{k}$, it needs to be proved that principal is better off as k increases when $k \leq \overline{k}$. In the case where $k \leq \overline{k} \leq \hat{k}$ or $k < \hat{k} < \overline{k}$, the optimal reward scheme is CF and rest of proof would be the same as that in in proof of proposition 2.4.1.2).

Notice that $V_V(k)$ is the principal's expected payoff in the optimal reward scheme given the incentive to run k experiments if no failure occurs, In the case where $k = \hat{k} < \overline{k}$, it needs to be shown that $V_V(\hat{k}+1) \geq V_V(\hat{k})$. The difference between $V_V(\hat{k}+1)$ and $V_V(\hat{k})$ is:

$$\begin{aligned}
&V_V(\hat{k}+1) - V_V(\hat{k}) \\
&\qquad = - p_0 \left[p_{\hat{k}} M + \left(1 - \frac{p_{\hat{k}}}{p_{\hat{k}+1}}\right) \frac{c}{1-\theta} \right]^2 \\
&\qquad\quad - (1-p_0)(1-\theta)^{\hat{k}+1} \left[p_{\hat{k}} M + \left(1 - \frac{p_{\hat{k}}}{p_{\hat{k}+1}}\right) \frac{c}{1-\theta} \right]^2 \\
&\qquad\quad - (1-p_0)(1-\theta)^{\hat{k}}\theta \left(p_{\hat{k}} M - \frac{p_{\hat{k}}}{p_{\hat{k}+1}} \frac{c}{1-\theta} \right)^2 + p_0 \left(1 - p_{\hat{k}}\right) M^2 \\
&\qquad = \underbrace{\left(2p_0 M - \frac{p_0}{p_{\hat{k}+1}} \frac{c}{1-\theta} + \frac{p_0 p_{\hat{k}}}{p_{\hat{k}+1}^2} \frac{c}{1-\theta} \right)}_{\text{“}>0\text{” as } p_0 M > \frac{p_0}{p_{\hat{k}+1}} \frac{c}{1-\theta}} \left(1 - \frac{p_{\hat{k}}}{p_{\hat{k}+1}}\right) \frac{c}{1-\theta} \\
&\qquad\quad + (1-p_0)(1-\theta)^{\hat{k}}\theta \left(\frac{p_{\hat{k}}}{p_{\hat{k}+1}} \frac{c}{1-\theta} \right)^2 > 0
\end{aligned} \tag{B.0.36}$$

This result suggests that the principal would strictly prefer $\hat{k}+1$ experiments are conduct rather than $\hat{k}$.

Consider the other case where $\hat{k} < k \leq \overline{k}$. It needs to be proved that the principal's expected payoff is an increasing function of k in this region. Thus the problem is equivalent to show that $V_V(k+1) - V_V(k) > 0$ in this region. At $k+1$, the "l" in Definition 1 would be $l(k+1) = l$ or $l(k+1) = l+1$, depending on the parameters. If $l(k+1) = l$, then the extra expected payoff that the principal can gain from increasing one more experiment is:

$$\begin{aligned}
&V_V(k+1) - V_V(k) = \\
&\qquad 2p_0 M\frac{c}{1-\theta} - \frac{p_0}{p_{k+1}}(2k-2l+1)\left(\frac{c}{1-\theta}\right)^2 \\
&\qquad + \frac{p_0}{p_l}\left[\left(p_l M - \sum_{i=l+1}^{k+1}\frac{p_l}{p_i}\frac{c}{1-\theta}\right)^2 - \left((p_l M - \sum_{i=l+1}^{k}\frac{p_l}{p_i}\frac{c}{1-\theta}\right)^2\right] \\
&\quad = \frac{p_0}{p_{k+1}}\frac{c}{1-\theta}\left[2p_{k+1}M - (2k-2l+1)\frac{c}{1-\theta}\right] \\
&\qquad - \frac{p_l}{p_{k+1}}\frac{c}{1-\theta}\left(2p_0 M - \sum_{i=l+1}^{k+1}\frac{p_0}{p_i}\frac{c}{1-\theta} - \sum_{i=l+1}^{k}\frac{p_0}{p_i}\frac{c}{1-\theta}\right)
\end{aligned} \tag{B.0.37}$$

Thus,

$$\begin{aligned}
Sign\left(V_V(k+1) - V_V(k)\right) = &\frac{p_{k+1}}{p_0}\left[2p_0 M - \frac{p_0}{p_{k+1}}(2k-2l+1)\frac{c}{1-\theta}\right] \\
&- \frac{p_l}{p_0}\left(2p_0 M - \sum_{i=l+1}^{k+1}\frac{p_0}{p_i}\frac{c}{1-\theta} - \sum_{i=l+1}^{k}\frac{p_0}{p_i}\frac{c}{1-\theta}\right)
\end{aligned} \tag{B.0.38}$$

Notice that

$$(k-l)\frac{p_0}{p_{k+1}} < \sum_{i=l+1}^{k}\frac{p_0}{p_i} \quad \text{and} \quad p_{k+1} > p_l$$

Together with(B.0.38), it can be achieved that (B.0.37) is strictly positive.

If $l(k+1) = l$, the principal's gain from one more experiment is:

$$\begin{aligned}
&V_V(k+1) - V_V(k) \\
&\quad = \frac{p_{l+1}}{p_0}\left(p_0 M - \sum_{i=l+2}^{k+1}\frac{p_0}{p_i}\frac{c}{1-\theta}\right)^2 - \frac{p_l}{p_0}\left(p_0 M - \sum_{i=l+1}^{k}\frac{p_0}{p_i}\frac{c}{1-\theta}\right)^2 \\
&\qquad + \sum_{i=l+2}^{k+1}\frac{p_0}{p_i}(\frac{c}{1-\theta})^2 - \sum_{i=l+1}^{k}\frac{p_0}{p_i}(\frac{c}{1-\theta})^2 - 2\frac{p_0}{p_{k+1}}(k-l)(\frac{c}{1-\theta})^2
\end{aligned} \tag{B.0.39}$$

Notice that

$$(k-l)\frac{p_0}{p_{k+1}} < \sum_{i=l+2}^{k+1}\frac{p_0}{p_i} < \sum_{i=l+1}^{k}\frac{p_0}{p_i} \quad \text{and} \quad p_{l+1} > p_l$$

thus (B.0.39) is strictly positive. Therefore, it's always true that $V_V(k+1) > V_V(k)$ when $\hat{k} < k \leq \overline{k}$. As a result, if $k \leq \overline{k}$, the principal is always better off by increasing the motivated number of successes in the commitment, and it implies that $k_V^* \geq \overline{k}$. To prove $k_V^* < \infty$, the argument is the same as that in the proof of proposition 2.3.1.2).

Proof of Proposition 2.4.2

The arguments and steps to prove the optimality of reward scheme proposed in Propostion 2.4.2.1) are similar to those in proof of Proposition 2.3.1.1) and 2.4.1.1), and proof by contradiction is applied.
Proposition 2.4.2.1.a): When $k \leq \min\left\{\hat{k}, \overline{k}\right\}$, it can be seen that $\tilde{k}c \leq p_0 M \leq \frac{p_0}{p_k}\frac{c}{1-\theta}$, which implies that $\text{IC}^{S,NV}$ and $\text{IC}^{F,NV}$ are satisfied under CF scheme. Thus CF must be the optimal reward scheme in this scenario.

Proposition 2.4.2.1.b): When $\hat{k} < k \leq \overline{k}$, it becomes that $p_k M > \frac{c}{1-\theta}$ and $p_0 M \geq \tilde{k}c$. Now CF scheme leads the last $\text{IC}^{F,NV}$, $p_k M < \frac{c}{1-\theta}$, to be violated. MF-I scheme satisfies all $\text{IC}^{F,NV}$ and $\text{IC}^{S,NV}$ in this scenario, so it must be optimal.

Proposition 2.4.2.1.c): When $\overline{k} < k \leq \hat{k}$, $p_0 M < \tilde{k}c$ and $p_k M \leq \frac{c}{1-\theta}$, it's easy to check that both CF scheme and MF-I scheme violate at least one $\text{IC}^{S,NV}$. Then proof by contradiction can be applied to check the optimality of MF-II in this scenario. Suppose MF-II is not optimal, then there exists another feasible reward scheme $d^k(\cdot)$:

$$\begin{cases} d^k(j<k,1) = a^k(j,1) + \tau_j \\ d^k(k,0) = a^k(k,0) + \tau_k \end{cases} \tag{B.0.40}$$

where $(\tau_k, ..., \tau_0) \in \mathbb{R}^k$ and $(\tau_k, ..., \tau_0) \neq \mathbf{0}$, such that $V_{NV}\left(d^k(\cdot)\right) \geq V_{NV}\left(a^k(\cdot)\right)$ and all constraints are satisfied. When $0 \leq m < k-1$, the principal's expected payoff with $d^k(\cdot)$ is:

$$\begin{aligned}
V_{NV}(d^k(\cdot)) =& -p_0\left(a^k(k,0)+\tau_k-M\right)^2-(1-p_0)(1-\theta)^k\left(a^k(k,0)+\tau_k\right)^2\\
&-\sum_{i=0}^{k-1}(1-p_0)(1-\theta)^i\theta\left(a^k(i,1)+\omega_i\right)^2\\
=&V_{NV}(a^k(\cdot))-\left\{\frac{p_0}{p_k}\eta_k^2+\sum_{i=0}^{k-1}(1-p_0)(1-\theta)^i\theta\eta_i^2\right\}\\
&-2\sum_{n=1}^{m}\frac{p_n}{p_{n-1}}c\left[\frac{p_0}{p_k}\tau_k+\sum_{i=n}^{k-1}(1-p_0)(1-\theta)^i\theta\tau_i\right]\\
&-2\left(\sum_{i=m+1}^{k}\frac{p_{m+1}}{p_{i-1}}c-p_{m+1}M\right)\left[\frac{p_0}{p_k}\tau_k+\sum_{i=m+1}^{k-1}(1-p_0)(1-\theta)^i\theta\tau_i\right]
\end{aligned} \tag{B.0.41}$$

Since IR and all IC^{S,NV} are satisfied, the following inequalities must be true:

$$\begin{cases}
\frac{p_0}{p_k}\tau_k+\sum_{i=0}^{k-1}(1-p_0)(1-\theta)\theta\tau_i\geq 0\\
\frac{p_0}{p_k}\tau_k+\sum_{i=1}^{k-1}(1-p_0)(1-\theta)\theta\tau_i\geq (1-p_0)(1-\theta)\theta\tau_0\\
\frac{p_0}{p_k}\tau_k+\sum_{i=2}^{k-1}(1-p_0)(1-\theta)\theta\tau_i\geq (1-p_0)(1-\theta)^2\theta\tau_1\\
\ldots\\
\frac{p_0}{p_k}\tau_k+\sum_{i=m+1}^{k-1}(1-p_0)(1-\theta)\theta\tau_i\geq (1-p_0)(1-\theta)^{m+1}\theta\tau_m
\end{cases} \tag{B.0.42}$$

If $\tau_0\geq 0$, it's true that $\frac{p_0}{p_k}\tau_k+\sum_{i=1}^{k-1}(1-p_0)(1-\theta)\theta\tau_i\geq 0$ from the second inequality in (B.0.42); if $\tau_0<0$, it also states that $\frac{p_0}{p_k}\tau_k+\sum_{i=1}^{k-1}(1-p_0)(1-\theta)\theta\tau_i\geq 0$ from the first inequality in (B.0.42). Thus it always holds that $\frac{p_0}{p_k}\tau_k+\sum_{i=1}^{k-1}(1-p_0)(1-\theta)\theta\tau_i\geq 0$. Similarly, if $\tau_1\geq 0$, $\frac{p_0}{p_k}\tau_k+\sum_{i=2}^{k-1}(1-p_0)(1-\theta)\theta\tau_i\geq 0$ and this is achieved from the third equality in (B.0.42); if $\tau_1\geq 0$, it's still obtained that $\frac{p_0}{p_k}\tau_k+\sum_{i=2}^{k-1}(1-p_0)(1-\theta)\theta\tau_i\geq 0$ from $\frac{p_0}{p_k}\tau_k+\sum_{i=1}^{k-1}(1-p_0)(1-\theta)\theta\tau_i\geq 0$. Thus it can always hold that $\frac{p_0}{p_k}\tau_k+\sum_{i=2}^{k-1}(1-p_0)(1-\theta)\theta\tau_i\geq 0$. Together with the same logic and (B.0.42), it's concluded that:

$$\frac{p_0}{p_k}\tau_k+\sum_{i=j}^{k-1}(1-p_0)(1-\theta)\theta\tau_i\geq 0,\text{ where }1\leq j\leq m+1 \tag{B.0.43}$$

Meanwhile, notice that $(\tau_k, ..., \tau_0) \neq \mathbf{0}$ and $\sum_{i=m+1}^{k} \frac{p_{m+1}}{p_{i-1}} c - p_{m+1}M \geq 0$, in (B.0.41):

$$\begin{aligned} V_{NV}(d^k(\cdot)) \leq & V_{NV}(a^k(\cdot)) - \left\{ \frac{p_0}{p_k}\eta_k^2 + \sum_{i=0}^{k-1}(1-p_0)(1-\theta)^i\theta\eta_i^2 \right\} \\ < & V_{NV}(a^k(\cdot)) \end{aligned} \tag{B.0.44}$$

This result contradicts to $V_{NV}(d^k(\cdot)) \geq V_{NV}(a^k(\cdot))$, therefore, MF-II scheme is optimal in this scenario when $0 \leq m < k-1$. When $m = k-1$, similarly, the principal's expected payoff now is:

$$\begin{aligned} V_{NV}(d^k(\cdot)) = & V_{NV}(a^k(\cdot)) - \left\{ \frac{p_0}{p_k}\eta_k^2 + \sum_{i=0}^{k-1}(1-p_0)(1-\theta)^i\theta\eta_i^2 \right\} \\ & - 2\sum_{n=1}^{k-1}\frac{p_n}{p_{n-1}}c\left[\frac{p_0}{p_k}\tau_k + \sum_{i=n}^{k-1}(1-p_0)(1-\theta)^i\theta\tau_i\right] - 2\frac{p_0}{p_{k-1}}c\tau_k \end{aligned} \tag{B.0.45}$$

Together with (B.0.42), it shows that the last two terms in (B.0.44) are both negative. Notice that the second term in (B.0.44) is strictly negative, therefore MF-II scheme is optimal when $m = k-1$. To sum up, it can be concluded that MF-II is optimal when $\overline{k} < k \leq \hat{k}$.

Proposition 2.4.2.1.d): When $k > \max\left\{\hat{k}, \overline{k}\right\}$, it states that $\frac{p_0}{p_k}\frac{c}{1-\theta} < p_0M < \tilde{k}c$. Now CF and MF-I violate at least one $\mathrm{IC}^{S,NV}$, and MF-II violates at least one $\mathrm{IC}^{F,NV}$. Thus consider the optimality of MF-III. Suppose MF-III is not optimal in this scenario, then there exists another feasible reward scheme $e^k(\cdot)$:

$$\begin{cases} e^k(j<k,1) = a^k(j,1) + \omega_j \\ e^k(k,0) = a^k(k,0) + \omega_k \end{cases} \tag{B.0.46}$$

where $(\omega_k, ..., \omega_0) \in \mathbb{R}^k$ and $(\omega_k, ..., \omega_0) \neq \mathbf{0}$, such that $V_{NV}\left(e^k(\cdot)\right) \geq V_{NV}\left(a^k(\cdot)\right)$ and all constraints are satisfied. The principal's expected payoff with $e^k(\cdot)$ then can be represented as:

$$
\begin{aligned}
V_{NV}(e^k(\cdot)) =& -p_0\left(a^k(k,0)+\omega_k-M\right)^2-(1-p_0)(1-\theta)^k\left(a^k(k,0)+\omega_k\right)^2\\
&-\sum_{i=0}^{k-1}(1-p_0)(1-\theta)^i\theta\left(a^k(i,1)+\omega_i\right)^2\\
=&V_{NV}(a^k(\cdot))-\left\{\frac{p_0}{p_k}\omega_k^2+\sum_{i=0}^{k-1}(1-p_0)(1-\theta)^i\theta\omega_i^2\right\}+2p_0M\omega_k\\
&-2(k-l)\frac{p_0}{p_k}\frac{c}{1-\theta}\omega_k-2\frac{c}{1-\theta}\sum_{i=l+1}^{k-1}(1-p_0)(1-\theta)^i\theta(i-l)\omega_i\\
&-2\left(p_lM-\sum_{i=l+1}^{k}\frac{p_l}{p_i}\frac{c}{1-\theta}\right)\left[\frac{p_0}{p_k}\omega_k+\sum_{i=l}^{k-1}(1-p_0)(1-\theta)^i\theta\omega_i\right]\\
&-2\sum_{n=1}^{m}\frac{p_n}{p_{n-1}}c\left[\frac{p_0}{p_k}\omega_k+\sum_{i=n}^{k-1}(1-p_0)(1-\theta)^i\theta\omega_i\right]\\
&-2\left(\sum_{i=m+1}^{k}\frac{p_{m+1}}{p_{i-1}}c-p_{m+1}M\right)\left[\frac{p_0}{p_k}\omega_k+\sum_{i=m+1}^{k-1}(1-p_0)(1-\theta)^i\theta\omega_i\right]
\end{aligned}
\tag{B.0.47}
$$

The rest of proof is similar to the proofs of MF-I and MF-II. Since IR and IC^{S,NV} are satisfied, the following inequalities can be achieved, which are similar to (B.0.42):

$$
\begin{cases}
\frac{p_0}{p_k}\omega_k+\sum_{i=0}^{k-1}(1-p_0)(1-\theta)\theta\omega_i\geq 0\\
\frac{p_0}{p_k}\omega_k+\sum_{i=1}^{k-1}(1-p_0)(1-\theta)\theta\omega_i\geq (1-p_0)(1-\theta)\theta\omega_0\\
\frac{p_0}{p_k}\omega_k+\sum_{i=2}^{k-1}(1-p_0)(1-\theta)\theta\omega_i\geq (1-p_0)(1-\theta)^2\theta\omega_1\\
\cdots\\
\frac{p_0}{p_k}\omega_k+\sum_{i=m+1}^{k-1}(1-p_0)(1-\theta)\theta\omega_i\geq (1-p_0)(1-\theta)^{m+1}\theta\omega_m
\end{cases}
\tag{B.0.48}
$$

Similar to (B.0.43), the same the logic in proof of MF-II can be applied and the following inequality can be achieved:

$$
\frac{p_0}{p_k}\tau_k+\sum_{i=j}^{k-1}(1-p_0)(1-\theta)\theta\omega_i\geq 0,\ \text{where } 1\leq j\leq m+1 \tag{B.0.49}
$$

This implies that the last two terms in (B.0.47) are negative, noticing that

$\frac{p_0}{p_k}\omega_k + \sum_{i=2}^{k-1}(1-p_0)(1-\theta)\theta\omega_i \geq 0$.

Also, since $\text{IC}^{F,NV}$ are satisfied, similar to (B.0.32), the following inequalities are true:

$$\begin{cases} \omega_k - \omega_{k-1} \leq 0 \\ ... \\ \omega_{l+1} - \omega_l \leq 0 \end{cases} \implies \begin{cases} -\omega_{k-1} \leq -\omega_k \\ ... \\ -\omega_l \leq -\omega_{l+1} \leq ... \leq -\omega_k \end{cases} \tag{B.0.50}$$

Then (B.0.47) becomes:

$$\begin{aligned} V_{NV}(e^k(\cdot)) \leq & V_{NV}(a^k(\cdot)) - \left\{ \frac{p_0}{p_k}\omega_k^2 + \sum_{i=0}^{k-1}(1-p_0)(1-\theta)^i\theta\omega_i^2 \right\} \\ & + 2\left(p_0M - \sum_{i=l+1}^{k}\frac{p_0}{p_i}\frac{c}{1-\theta}\right)\omega_k - 2\left(p_0M - \sum_{i=l+1}^{k}\frac{p_0}{p_i}\frac{c}{1-\theta}\right)\omega_k \\ = & V_{NV}(a^k(\cdot)) - \left\{ \frac{p_0}{p_k}\omega_k^2 + \sum_{i=0}^{k-1}(1-p_0)(1-\theta)^i\theta\omega_i^2 \right\} \end{aligned} \tag{B.0.51}$$

Again, notice $(\omega_k, ..., \omega_0) \neq \mathbf{0}$:

$$\begin{aligned} V_{NV}(e^k(\cdot)) \leq & V_{NV}(a^k(\cdot)) - \left\{ \frac{p_0}{p_k}\omega_k^2 + \sum_{i=0}^{k-1}(1-p_0)(1-\theta)^i\theta\omega_i^2 \right\} \\ < & V_{NV}(a^k(\cdot)) \end{aligned} \tag{B.0.52}$$

This result contradicts to $V_{NV}(e^k(\cdot)) \geq V_{NV}(a^k(\cdot))$, therefore, MF-III scheme must be optimal in this scenario.

Proposition 2.4.2.2): if $\overline{k} \leq \hat{k}$, for $\forall k \leq \overline{k}$, CF is optimal and then principal is strictly better off as k increasing, which can be obtained from the proof of proposition 2.3.1.2); if $\hat{k} < \overline{k}$, for $\forall k \leq \overline{k}$, optimal reward scheme is either CF or MF-I, and the optimal amount in this region is $\overline{k}$, which is the same proof as that in proposition 2.4.1.2). Therefore, $k^*_{NV} \geq \overline{k}$. To prove $k^*_{NV} < \infty$, it's the same argument as that in proposition 2.3.1.2).

Proof of Corollary 2.4.1

If the principal motivates the agent not to run any experiments, a single reward scheme should be determined by the prior belief p_0, regardless of public and private experimentation. Thus the principal's expected payoff is $V_{NO}(p_0) = -p_0(1-p_0)M^2$. In stead, given the incentive to run any positive number of experiments k, the principal can achieve $V_P(a^k(\cdot))$, $V_V(a^k(\cdot))$ and $V_{NV}(a^k(\cdot))$ in public and private cases respectively, which are shown in previous proofs. It's clear that the set of parameter ranges is not empty, which satisfies:

$$V_{NO}(p_0) \geq \left\{V_P(a^k(\cdot)), V_V(a^k(\cdot)), V_{NV}(a^k(\cdot))\right\}$$

Proof of Corollary 2.4.2

When $\frac{M}{c} \leq \frac{1}{1-\theta}$, $\hat{k} \to \infty$. From Proposition 2.4.1, CF is always optimal for $\forall k \in \mathbb{N}^+$ in private experimentation with verifiable failures. Thus the optimal reward scheme is always the same as that in public experimentation. As a result, $k_V^* = k^P$ and $V_p^{CF}\left(k^P, p_0\right) = V_V^{CF}\left(k_V^*, p_0\right)$.

Proof of Proposition 2.4.3

For $p_0 M \geq c$, the participation threshold can be rewritten into $\overline{k} = \max\{k \in \mathbb{N} : p_0\frac{M}{c} \geq \tilde{k}\}$. When $\frac{M}{c}$ increases, the left hand side of the inequality constraint is increasing, and it implies that this condition can hold for a larger number of experiments. As a result, the first threshold $\overline{k}$ becomes larger. For $p_0 M < c$, when $\frac{M}{c}$ increases, this inequality is easier to be violated, thus $\overline{k}$ tends to become larger. To sum up, $\overline{k}$ is increasing as $\frac{M}{c}$ increases.

For $p_1 M \leq \frac{c}{1-\theta}$, the over-experimentation threshold can be rewritten into $\hat{k} = \max\{k \in \mathbb{N} : \frac{M}{c} \leq \frac{1}{(1-\theta)p_k}\}$. When $\frac{M}{c}$ increases, the left hand side of the inequality constraint is increasing, and it implies that this condition would be violated at a lower level of experiment. As a result, the second threshold $\hat{k}$ shrinks. For $p_1 M > \frac{c}{1-\theta}$, $\hat{k}$ stays at zero when $\frac{M}{c}$ increases. To sum up, the participation threshold $\hat{k}$ is decreasing as $\frac{M}{c}$ increases.

In the public case, $k^P \geq \overline{k}$, which implies that the lower bound of the potential optimal number of experiments is increasing. Now it can focus on the number which satisfied $k > \overline{k}$. Firstly, take the first different between $k+1$ and k:

$$V_P(k+1) - V_P(k) = \left[p_0(p_{k+1} - p_k)(\frac{M}{c})^2 - \left(\frac{p_0}{p_k} + 2\sum_{i=1}^{k}\frac{p_0}{p_{i-1}} - 2p_0\frac{M}{c}\right)\frac{p_0}{p_k}\right]c^2 \tag{B.0.53}$$

Then the first derivate with respect to $\frac{M}{c}$ can be achieved:

$$\frac{\partial(V_P(k+1)-V_P(k))}{\partial\frac{M}{c}} = 2\left(\sum_{i=1}^{k+1}\frac{p_0}{p_{i-1}}c+\sum_{i=1}^{k}\frac{p_0}{p_{i-1}}c-p_0M\right)\frac{p_0}{p_k}\frac{c^2}{M}>0 \tag{B.0.54}$$

This strictly positive first difference for $\forall k>\overline{k}$ suggests that the (local and global) maximum point is getting larger as $\frac{M}{c}$ increases. Together with that $\overline{k}$ is increasing as $\frac{M}{c}$ increases, it can be concluded that k^P is increasing as $\frac{M}{c}$ is increases.

Similar arguments can be applied in the private experimentation scenario. In private with verifiable failures, if $\overline{k}<k_V^*\leq\hat{k}$, the conclusion is the same as (B.0.53). If $k_V^*>\max\left\{\overline{k},\hat{k}\right\}$, it can be focus on the difference of the principal's expected payoff at $k+1$ and k, for $\forall k>\max\left\{\overline{k},\hat{k}\right\}$. Similar to (B.0.37) and (B.0.39), if $l(k+1)=l$, it's first derivative with respect to M would be

$$\frac{\partial(V_V(k+1)-V_V(k))}{\partial M}=p_0\frac{c}{1-\theta}\left(2-\frac{p_l}{p_{k+1}}\right)+2\frac{p_0^2}{p_k}c>0 \tag{B.0.55}$$

Instead, if $l(k+1)=l+1$, the first derivative becomes

$$\begin{aligned}\frac{\partial(V_V(k+1)-V_V(k))}{\partial M}=&2p_{l+1}\left(p_0M-\sum_{i=l+2}^{k+1}\frac{p_i}{p_0}\frac{c}{1-\theta}\right)\\&-2p_l\left(p_0M-\sum_{i=l+1}^{k}\frac{p_i}{p_0}\frac{c}{1-\theta}\right)+2\frac{p_0^2}{p_k}c>0\end{aligned} \tag{B.0.56}$$

The positive signs in (B.0.55) and (B.0.56) imply the first difference is increasing as agent's value M increases, and it leads the maximum point k_V^* to increase together with $\overline{k}$ increasing.

In private with unverifiable failures, if $\hat{k}\leq\overline{k}$, for $\forall k>\overline{k}$, then associated optimal reward scheme is MF-III, then the first difference of principal's expected could be obtained. From Definition 3, if $m(k+1)=m+1$, the first derivative would be

$$\begin{aligned}
&\frac{\partial(V_{NV}(k+1)-V_{NV}(k))}{\partial M} \\
&= \frac{\partial(V_V(k+1)-V_V(k))}{\partial M} + 2(p_{m+2}-p_{m+1})\left(p_0 M + \sum_{i=m+2}^{k} \frac{p_0}{p_{i-1}} c\right) \\
&\quad + 2\frac{p_0}{p_k}(p_{m+2}-p_0)c > 0
\end{aligned} \tag{B.0.57}$$

Instead, if $m(k+1) = m = m(k)$, the first derivative becomes:

$$\frac{\partial(V_{NV}(k+1)-V_{NV}(k))}{\partial M} = \frac{\partial(V_V(k+1)-V_V(k))}{\partial M} + 2\frac{p_0}{p_k}(p_{m+2}-p_0)c > 0 \tag{B.0.58}$$

The positive signs in (B.0.57) and (B.0.58) suggest that the first difference is increasing as M increases for $\forall k > \overline{k} \geq \hat{k}$. As a result, k^*_{NV} increases due to the same reason in private with verifiable failures.

If $\hat{k} > \overline{k}$, for $\forall k > \hat{k}$, MF-III is still optimal and the conclusions are the same as (B.0.57) and (B.0.58). For $\overline{k} < k < \hat{k}$, MF-II is optimal, and it can focus on the first derivative of the first difference of the principal's expected payoff with respect to M. From Definition 2, if $m(k) = m < k-1$ and $m(k+1) = m+1$

$$\begin{aligned}
&\frac{\partial(V_{NV}(k+1)-V_{NV}(k))}{\partial M} \\
&= 2p_0(p_{k+1}-p_k)M + 2(p_{m+2}-p_{m+1})\left(p_0 M + \sum_{i=m+2}^{k} \frac{p_0}{p_{i-1}} c\right) \\
&\quad + 2\frac{p_{m+2}}{p_k} p_0 c > 0
\end{aligned} \tag{B.0.59}$$

if $m(k) = m < k-1$ and $m(k+1) = m$, the first derivative is

$$\frac{\partial(V_{NV}(k+1)-V_{NV}(k))}{\partial M} = 2p_0(p_{k+1}-p_k)M + 2\frac{p_{m+2}}{p_k} p_0 c > 0 \tag{B.0.60}$$

if $m(k) = k-1$, the first derivative becomes

$$\frac{\partial(V_{NV}(k+1)-V_{NV}(k))}{\partial M} = 2\frac{p_{k+1}}{p_k} p_0 c > 0 \tag{B.0.61}$$

From the positive signs in (B.0.59), (B.0.60) and (B.0.61), it shows that the first difference is increasing as M increases in this case. For $\forall k > \hat{k} > \overline{k}$, the conclusions would be the same as (B.0.57) and (B.0.58). To sum up, it concludes that k^*_{NV} is increasing as M increases.

Proof of Proposition 2.4.4

The bad type agent's value is zerio if early failure occurs and he would learn it. For the potential good type, his posterior value is $p_k M$ given he successfully collected k successes in k experiments without failure, and he has the posterior belief p_k that his type is good. The proofs below are comparing these values to the rewards that different types of agents can received in the optimal contracts of public and private experimentation respectively.

In the public case, the optimal contract would deliver the bad type agent a reward level $a^{k^P}(k < k^P) = \max\left\{0, (\sum_{i=1}^{k^P} \frac{p_0}{p_{i-1}} c - p_0 M)^2\right\} \geq 0$, and it implies that the bad type is overpaid. For the potential good type, he would receive the reward level $a^{k^P}(k = k^P) = p_{k^P} M + a^{k^P}(k < k^P) \geq p_{k^P} M$, and it's clear to see that he is also overpaid.

In the private case with verifiable failures, the bad types who face a later failure would receive a weakly higher reward. Comparing the lowest reward among them, the bad type would receive $\max\left\{0, (\sum_{i=1}^{k^*_V} \frac{p_0}{p_{i-1}} c - p_0 M)^2\right\} \geq 0$, so it shows that all bad types are overpaid at differently level of early failure. However, for the potential good type, he would receive

$$(k^V - l(k^V))\frac{c}{1-\theta} + p_{l(k^*_V)} M - \sum_{i=l(k^*_V)+1}^{k^*_V} \frac{p_0}{p_i} \frac{c}{1-\theta} + \max\left\{0, (\sum_{i=1}^{k^*_V} \frac{p_0}{p_{i-1}} c - p_0 M)^2\right\}$$

and it's not clear whether it's higher than $p_{k^*_V} M$, and it concludes that the potential good type is not necessarily overpaid.

In the private case with unverifiable failures, the lowest possible reward that a bad type agent receives under the optimal contract is 0, which is the same as his true valuation. Therefore the bad type is weakly overpaid. For the potential good type, with similar argument in private with verifiable bad ones, the conclusion is still not clear whether he is overpaid or not, when comparing the reward that the potential good type receives to $p_{k^*_{NV}} M$.

Proof of Proposition 2.5.1

Given the agent has acquired j successes without failures, the benefit from fulfilling the remaining experiments is:

$$\begin{cases} U_B(k-j,p_j) = -c + (1-\sigma)U_B^A(k-j-1,p_{j+1}) + \sigma U_B^A(k-j-1,p_j) \\ U_B(1,p_{k-1}) = -c + (1-\sigma)a_B^k(k,0) + \sigma U_B^A(1,p_{k-1}) \end{cases} \tag{B.0.62}$$

Where $0 \leq j < k-1$. It can be simplified as:

$$U_B(k-j,p_j) = \frac{p_0}{p_k}a^k(k,0) + \sum_{i=j}^{k}(1-p_0)(1-\theta)^j\theta a^k(j,1) - \sum_{i=j}^{k}\frac{p_0}{p_{i-1}}\frac{c}{1-\sigma} \tag{B.0.63}$$

Then condtions (2.5.2) now become

$$\mathrm{IC}_{0\leq j\leq k-1}^{S,B}: \quad \frac{p_0}{p_k}a^k(k,0)+\sum_{i=j}^{k}(1-p_0)(1-\theta)^j\theta a^k(j,1) \geq a^k(j,1)+\sum_{i=j}^{k}\frac{p_0}{p_{i-1}}\frac{c}{1-\sigma} \tag{B.0.64}$$

Conditions (B.0.64) then are the same as those in private experimentation with unverifiable failures as well as IR constraint, and the cost level of a single experiment is $\frac{c}{1-\sigma}$. Also, $\mathrm{IC}^{F,B}$ in (2.5.3) are the same as those in conditions (2.4.8). Additionally, when failures are verifiable, Lemma 2.4.2 can be applied. Therefore, the principal is maximising the expected payoff under the same constraints in the scenario with unverifiable failures, and the optimal solution should be the same.

Proof of Proposition 2.5.2

1) When $k < T$, the agent still has further opportunity for over-experimenting even if the first failure occurs in the k_{th} experiment. Thus the condition (2.5.4) must be satisfied. Similarly, when the first failure occurs in the $j+1_{th}$ experiment, where $j < k$, to prevent agent from pretending to be those whose have more successes, the following incentive constraints need

to be satisfied:

$$\begin{aligned}
&\frac{(1-\theta^{T-j-1})}{1-\theta}\left[-c+(1-\theta)a_F^k(j+1,1)\right]+\theta^{T-j-1}a_F^k(j,1)\leq a_F^k(j,1)\\
\implies \quad &-\frac{c}{1-\theta}+a_F^k(j+1,0)\leq a_F^k(j,1)
\end{aligned} \tag{B.0.65}$$

These constraints together with (2.5.4) are the same as IC^F in (2.4.6) and (2.4.8). Meanwhile, to prevent agent from stopping experimenting earlier without a failure, the following $\text{IC}^{S,F}$ constraints need to be satisfied:

$$\text{IC}^{S,F}_{0\leq j\leq k-1}: \quad U_F(k-j,p_j)\geq a_F^k(j,0) \tag{B.0.66}$$

These conditions are exactly the same as those IC^S when failures are verifiable and not verifiable respectively. Therefore, the principal is solving the same maximisation problem as that in $T\to\infty$, and the optimal solution should be the same.

2) When $k=T$, the constraint (2.5.4) can be removed since the first failure occurs in the last experiment and the agent has no chance to over-experiment. But other constraints in (B.0.65) and (B.0.66) are still the same as those in section 2.4.1 and 2.4.2 when failures are verifiable and unverifiable respectively. When failures are verifiable, it can be easily show that CF scheme satisfies all these constraints, so it has be optimal. In the other scenario where failures are not verifiable, it can be shown that all constraints are satisfied under CF and MF-II when $T\leq\overline{k}$ and $T>\overline{k}$ respectively. This is still true even if $T>\hat{k}$.

3) Denote by $V_F(k)$ the principal's expected payoff given the incentive to run k experiments in finite opportunity case. In public experimentation, CF is optimal from Proposition 2.5.2.1), thus $V_{F,P}(k)=V_P(k)$ for $\forall k>0$. Therefore, if $T\geq k^P$, the principal can just provide the incentive to run k^P experiments if no failure occurs, and achieve the same expected payoff as that in the public case with infinite opportunities.

If experiments are private and failures are verifiable, since more constraints are binding and the feasible set shrinks, the principal is worse off relative to the private case, $V_V(k)\leq V_P(k)$ for $\forall k>0$. Notice that $V_V(k)$ and $V_P(k)$ are decreasing functions when $k>k^P$ and $k>k_V^*$ respectively, it's must be true that $\exists k_V=\max\{k\in\mathbb{N}:V_P(k)\leq V_V(k_V^*)\}$ and $V_P(k)\leq V_P(k_V)$

for $\forall k \geq k_V$. From Proposition 2.5.2.2), CF is still optimal at $k = T$ in the case with finite opportunities, which implies that it's still true that $V_{F,P}(T) = V_P(T)$. Notice that $k_V^* \leq k_V$, the principal would optimally motivate to agent to run $k_V^* < T$ experiments When $T > k_V$.

If experiments are private and failures are not verifiable, the expected payoff under MF-II scheme is weakly higher than that under MF-III given the same number of experiments is motivated, $V_{NV}(k) \leq V_V(k)$ for $\forall k > \overline{k}$, according to Definition 3 and Proposition 2.4.2.1). Notice that $V_{NV}(k)$ and $V_V(k)$ are decreasing function when $k > k_V^*$ and $k > k_{NV}^*$ respectively, thus it's must be true that $\exists k_{NV} = \max\{k \in \mathbb{N} : V_V(k) \leq V_{NV}(k_{NV}^*)\}$ and $V_V(k) \leq V_V(k_{NV})$ for $\forall k \geq k_{NV}$. From Proposition 2.5.2.2), MF-II is optimal at $k = T > \overline{k}$ in the case with finite opportunities, and $V_{F,NV}(T) = V_V(T)$. Notice that$k_{NV}^* \leq k_{NV}$, as a result, the principal would optimally motivate the agent to run $k_{NV}^* < T$ experiments in the optimal contract when $T > k_{NV}$.

Appendix C

Proofs for Chapter 3

Proof of Lemma 3.4.1

I prove this Lemma by using the following claim.

Claim C.0.1. *The expected cost of acquiring a success for a potential good type agent is lower than that for a bad type, and it's decreasing as his posterior belief increases.*

Proof. Suppose a potential good type agent has n successes without failures. Now his posterior belief is $p^A_{(n,0)} = p_n$. If he conducts one more experiment, he can acquire a success with probability $p_n + (1 - p_n)(1 - \theta)$; if he fails with probability $(1 - p_n)\theta$, he knows that he is actually a bad type and the expected cost of acquiring a success becomes to $\frac{c}{1-\theta}$. Thus, the expected cost of acquiring a success for the potential good type would be $\frac{(1-p_n\theta)c}{1-\theta} < \frac{c}{1-\theta}$. Moreover, when p_n increases, the numerator in the expected cost is lower as the coefficient of p_n is negative. $\square$

This claim suggests that only a bad type might have the incentive to stop before k successes are acquired, and the potential good type agent has a stronger incentive to conduct more experiments.

Proof of Lemma 3.4.3

I prove this lemma by using the following clam.

Claim C.0.2. *Given $k \geq 0$ successes have been acquired, a potential good type agent has a stronger incentive to continue experimenting relative to a bad type agent.*

Proof. Suppose now the agent has $k \geq 0$ successes already, which is required on the equilibrium path. If the agent is a potential good type who hasn't failed yet, his posterior belief is $p^A_{(k,0)} = p_k$. To acquire another $N > 0$ successes, his expected payoff $U^n(p_k)$ would be:

$$U^n_G(p_k) = \frac{p_k}{p_{k+n}} p^P_{(k+n,0)} M - \sum_{i=1}^{n} \frac{p_k}{p_{k+i-1}} c$$
$$+ \sum_{j=0}^{n-1} (1-p_k)(1-\theta)^j \theta \max\left\{ \max_{i \in \{0,\dots,n-j\}} p^P_{(k+i,0)} M - \frac{ic}{1-\theta}, p^P_{(k,0)} M, \dots, p^P_{(k+j,0)} M \right\} \tag{C.0.1}$$

With *belief monotonicity*, where $p^P_{(k+n+1,0)} \geq p^P_{(k+n,0)}$, $U^n(p_k)$ can be simplified as:

$$\begin{aligned} U^n_G(p_k) =& \frac{p_k}{p_{k+n}} p^P_{(k+n,0)} M - \sum_{i=1}^{n} \frac{p_k}{p_{k+i-1}} c \\ &+ \sum_{j=0}^{n-1} (1-p_k)(1-\theta)^j \theta \left[\max_{i \in \{0,\dots,n-j\}} p^P_{(k+i,0)} M - \frac{ic}{1-\theta} \right] \end{aligned} \tag{C.0.2}$$

Similarly, if a bad type agent deviates to acquire n more successes, his expected payoff would be:

$$U^n_B(p^A = 0) = p^P_{(k+n,0)} M - \frac{nc}{1-\theta} \tag{C.0.3}$$

Thus, the difference of their expected payoff would be:

$$\begin{aligned}
U_G^n(p_k)-&U_B^n(p^A=0)\\
\geq&\frac{p_k}{p_{k+n}}p^P_{(k+n,0)}M-p^P_{(k+n,0)}M+\frac{nc}{1-\theta}\\
&-\sum_{i=1}^{n}\frac{p_k}{p_{k+i-1}}c+\sum_{j=0}^{n-1}(1-p_k)(1-\theta)^j\theta\left[p^P_{(k+n,0)}M-\frac{(n-j)c}{1-\theta}\right]\\
=&\frac{nc}{1-\theta}-\sum_{i=1}^{n}\frac{p_k}{p_{k+i-1}}c-\sum_{j=0}^{n-1}(1-p_k)(1-\theta)^j\theta\frac{(n-j)c}{1-\theta}\\
=&\sum_{i=1}^{n}\frac{p_k}{p_{k+i-1}}\frac{c}{1-\theta}-\sum_{i=1}^{n}\frac{p_k}{p_{k+i-1}}c=\sum_{i=1}^{n}\frac{p_k}{p_{k+i-1}}\frac{\theta c}{1-\theta}>0
\end{aligned}\tag{C.0.4}$$

Therefore, if $U_B^n(p^A=0)\geq p^P_{(k,0)}M$, it must be true that $U_G^n(p_k)>p^P_{(k,0)}M$. □

Claim C.0.2 suggests that if the principal's posterior belief makes the potential good type has no incentive to continue experimenting, the bad type would also not to do so. Thus, the potential good agent would not to continue experimenting if the current payoff is larger than the expected payoff of continuing experimenting. This implies that $U_G^n(p_k)\leq p^P_{(k,0)}M$ is satisfied for $\forall n\in\mathbb{N}^+$:

$$\begin{aligned}
&\sum_{j=0}^{n-1}(1-p^P_{(k,0)})(1-\theta)^j\theta\left[\max_{i\in\{0,\dots,n-j\}}p^P_{(k+i,0)}M-\frac{ic}{1-\theta}\right]\\
&\qquad\qquad\leq\frac{p_k\left(p_{k+n}-p^P_{(k+n,0)}\right)}{p_{k+n}}M+\sum_{i=1}^{n}\frac{p_k}{p_{k+i-1}}c
\end{aligned}\tag{C.0.5}$$

If this condition is not satisfied, the potential good type would always continue experimenting, which contradicts to an equilibrium where the potential good type stops after k successes are acquired.

Proof of Proposition 3.4.1

Proof of 3.4.1.1). Given the agent doesn't run any experiments, the principal would optimally assign $p^P_{(0,0)}=p_0$, $a(p_0)=p_0M$. Consider the deviation of the agent. Since (3.4.3) is satisfied, the agent has no incentive to run more experiments.

The following claim is useful when proving 3.4.1.2).

Claim C.0.3. *There doesn't exist an equilibrium with learning at $k > \overline{k}$.*

Proof. Suppose a separating equilibrium with learning exists, in which the agent's strategy is to run $k > 0$ experiments without failures, and stop once k successes have been acquired or he faces an early failure. Thus, $a^E(p^P_{(k,0)}) = p_k M$. From (3.4.3), $U^k_S(p_0) = p_0 M - \tilde{k}c$. Notice that the agent's expected payoff is decreasing as k increases, $k > \overline{k}$ implies that $U^k_S(p_0) < U^{\overline{k}}_S(p_0) < 0$. This implies that the agent would be better off by deviating to stop at the beginning. Thus the separating equilibrium with learning doesn't exists in this case. Suppose a pooling equilibrium with learning exists, in which the bad type agent whose first failure occurs after $j + 1_{th}$ experiments would over-experiment till k successes are acquired. In this case, $a^E(p^P_{(k,0)}) = p_j M$, and $U^k_O(p_0) = p_0 M - \tilde{k}c - \sum_{i=j}^{k-1}(1-p_0)(1-\theta)^{i-1}\theta(k-i)c = p_0 M - \sum_{i=1}^{j} \frac{p_0}{p_{i-1}}c - \frac{p_0(k-j)}{p_{j-1}}c < p_0 M - \tilde{k}c$. Thus, when $k > \overline{k}$, $U^k_O(p_0) < 0$ and the agent would always deviate. Thus the pooling equilibrium with learning also doesn't exists in this case. □

Proof of 3.4.1.2.a). Claim C.0.3 suggests that $\overline{k}$ plays the role of participation threshold, and all the equilibrium must satisfy that $k \leq \overline{k}$. Therefore, when $p_0 M < c$, $\overline{k} = 0$, which implies the only equilibrium left is no-experiment equilibrium. The following proofs would focus on the scenario when $p_0 M \geq c$.

Proof of 3.4.1.2.b). Consider a separating equilibrium with learning, in which the agent's strategy is to conduct $k > 0$ experiments without failures, and stop once k successes have been acquired or he faces an early failure. In this case, on the equilibrium path, only the potential good type agent would report k successes, and the bad type agent would report less. This implies that the principal's posterior belief is the same as that of the potential good type agent when observing k successes, $p^P_{(k,0)} = p^A_{(k,0)} = p_k$. Thus, $a^E\left(p^P_{(k,0)}\right) = p_k M > 0$ and $a^E(p^P_{(k^g<k,0)}) = 0$.

The agent has no incentive to conduct more experiments since (3.4.3) is satisfied. Consider the agent's incentive of conducting less experiments. If he deviates to conduct fewer experiments, the agent would be worse off since

$a^E(p^P_{(k^g<k,0)}) = 0$. Now check the bad type's over-experimentation incentive. The bad type agent whose first failure occurs occurs in k_{th} experiment has the strongest incentive to over-experiment, since he only needs one more success to pretend to be a good type. Thus, such bad type agent will not over-experiment if the extra benefit $p_k M$ is less than it's cost $\frac{c}{1-\theta}$ by dosing so,

$$p_k M \leq \frac{c}{1-\theta} \quad \implies \quad k \leq \hat{k} = \begin{cases} \max\left\{k \in \mathbb{N} : p_k M \leq \frac{c}{1-\theta}\right\} & p_1 M \leq \frac{c}{1-\theta} \\ 0 & p_1 M > \frac{c}{1-\theta} \end{cases} \tag{C.0.6}$$

$\hat{k}$ is the over-experimentation threshold, and $k \leq \hat{k}$ suggests that all the bad types would not over-experiment. Therefore, the set of separating equilibria with learning would must satisfy $\left\{k \in \mathbb{N} : 0 < k \leq \min\{\hat{k}, \overline{k}\}\right\}$. Moreover, when $\frac{M}{c} > \frac{1}{p_1(1-\theta)}$, $\hat{k} = 0$ and $\left\{k \in \mathbb{N} : 0 < k \leq \min\{\hat{k}, \overline{k}\}\right\} = \varnothing$. As a result, the separating equilibria exist only when $\frac{M}{c} \in \left[\frac{1}{p_0}, \frac{1}{p_1(1-\theta)}\right]$.

Now check the over-experimentation incentive. Suppose now the agent fails in k_{th} experiment, where the agent has the strongest incentive to over-experiment. He receives zero If he sticks to the strategy on the equilibrium path. Alternatively, if he continues experimenting and collects one more success by chance, he would be treated as a potential good type agent and receive $p_k M$. Thus, to prevent such behaviour on the equilibrium path, the number of experiments k must satisfy:

$$p_k M \leq \frac{c}{1-\theta} \tag{C.0.7}$$

Where $\frac{c}{1-\theta}$ is the expected cost of acquiring a success for a bad type agent. Notice that the left hand side of (C.0.7) is increasing as k increases, the over-experimentation threshold $\hat{k}$ can be found, which is the largest number of experiments such that the bad type agent has no incentive to over-experiment no matter when he fails,

$$\hat{k} = \begin{cases} \max\left\{k \in \mathbb{N} : p_k M \leq \frac{c}{1-\theta}\right\} & p_1 M \leq \frac{c}{1-\theta} \\ 0 & p_1 M > \frac{c}{1-\theta} \end{cases} \tag{C.0.8}$$

if $\frac{M}{c} \in \left[\frac{1}{p_0}, \frac{1}{p_1(1-\theta)}\right]$, the over-experimentation threshold is strictly positive, therefore, in a candidate of separating equilibrium with learning, it must be true that $k \leq \hat{k}$. As a result, the number of successes reported by the potential good type on the equilibrium path must satisfied $0 < k \leq \max\left\{\overline{k}, \hat{k}\right\}$. Moreover, once k successes have been acquired by the potential good type, he has no incentive to continue experimenting as (3.4.4) is satisfied.

Proof of 3.4.1.2.c). In a pooling equilibrium with learning, exists bad type agent(s) who must over-experiment on the equilibrium path, thus it must be true that $\hat{k} < k \leq \overline{k}$. When $\frac{M}{c} \in \left[\frac{1}{p_0}, \frac{1}{\theta}\right]$, $p_k M < M \leq \frac{c}{1-\theta}$, and it implies that $\hat{k} \to \infty$, in which case the set of pooling equilibria is empty set. Therefore, to guarantee that $\hat{k} < \infty$, the only possible value-cost ratio would be $\frac{M}{c}\left(\max\left\{\frac{1}{p_0}, \frac{1}{1-\theta}\right\}, +\infty\right)$. The following Claim C.0.4 suggests that the possible pooling equilibria must satisfy $\hat{k} + 1 < k \leq \overline{k}$.

Claim C.0.4. *There doesn't exist an equilibrium with learning at* $k = \hat{k} + 1$.

Proof. Previous arguments have shown that the claim holds for $\overline{k} \leq \hat{k}$. Consider $\hat{k} < \overline{k}$. Suppose a separating equilibrium with learning exists, the contradiction is obvious since $p_{\hat{k}+1} M > \frac{c}{1-\theta}$ and the bad type agent whose first failure occurs in $\hat{k} + 1_{th}$ experiment would deviate to over-experiment. Consider a pooling equilibrium with learning. Suppose only the potential good type and the bad type whose first failure occurs in $\hat{k} + 1_{th}$ experiment would report $\hat{k} + 1$ successes on the equilibrium path, then $p^P_{\left(\hat{k}+1,0\right)} = p_{\hat{k}}$. However, since $p_{\hat{k}} M \leq \frac{c}{1-\theta}$, the bad type agent would not over-experiment, which is a contradiction. $\square$

With Claim C.0.4, there doesn't exists pooling equilibrium if $\overline{k} = \hat{k} + 1$. Therefore, the existence of the pooling equilibrium with learning can be restricted in $\hat{k} + 1 < k \leq \overline{k}$ with $\overline{k} > \hat{k} + 1$.

Suppose the only a bad type agent whose first failure occurs after $\hat{k} + l + 1_{th}$ experiment would over-experiment on the equilibrium path, the principal's posterior belief would be $p^P_{(k,0)} = p_{\hat{k}+l}$ and $p^P_{(k^g<k,0)} = 0$, where $0 < l \leq \overline{k} - \hat{k}$. Thus, $a^E(k,0) = p_{\hat{k}+l} M$ and $a^E(k^g k, 0) = 0$. To support this as an equilibrium, in the first place, $k \leq \overline{k}$ guarantees that the agent would not deviate to no-experiment choice. In the second place, For the agent whose first failure occurs in $\hat{k}+l+1_{th}$, he would indeed over-experiment if the extra benefit is larger than the expected total cost by doing so:

$$p_{k+l}M > \frac{k-\hat{k}-l}{1-\theta}c \quad \Longrightarrow \quad \frac{M}{c} > \frac{k-\hat{k}-l}{p_{\hat{k}+l}(1-\theta)} \tag{C.0.9}$$

Meanwhile, for the bad type agent whose first failure occurs in $\hat{k}+l_{th}$ experiment, he needs to have no incentive to over-experiment, which requires that the total expected cost to do so is larger than the extra benefit:

$$p_{k+l}M \leq \frac{k-\hat{k}-l+}{1-\theta}c \quad \Longrightarrow \quad \frac{M}{c} \leq \frac{k-\hat{k}-l+1}{p_{\hat{k}+l}(1-\theta)} \tag{C.0.10}$$

Therefore, to support such pooling equilibrium, it requires the value-cost ratio belongs to the following non-empty set:

$$\frac{M}{c} \in \left(\frac{k-\hat{k}-l}{p_{\hat{k}+l}(1-\theta)}, \frac{k-\hat{k}-l+1}{p_{\hat{k}+l}(1-\theta)}\right] \subset \left(\max\left\{\frac{1}{p_0}, \frac{1}{1-\theta}\right\}, +\infty\right)$$

Proof of Proposition 3.4.2

In public experimentation, since only the no-experiment equilibrium exists, the agent's expected payoff would be $U_P\left(p_0\right) = p_0 M$ and the principal's expected payoff is $V_P\left(p_0\right) = -p_0(1-p_0)M^2$.

In private experimentation, given (3.4.3) is satisfied, if the no-experiment equilibrium survives, both the agent's and the principal's expected payoff, $U_N\left(p_0\right)$ and $V_N\left(p_0\right)$, would be the same as that in public experimentation, where $U_N\left(p_0\right) = U_P\left(p_0\right) = p_0 M$ and $V_N\left(p_0\right) = V_P\left(p_0\right) = -p_0(1-p_0)M^2$.

When the separating equilibria with learning survive, consider the one in which the potential good type agent reports $0 < k \leq \min\left\{\overline{k}, \hat{k}\right\}$ successes. The agent's expected payoff would be strictly worse off relative to public experimentation: $U_S^k\left(p_0\right) = p_0 M - \sum_{i=1}^{k} \frac{p_0}{p_{i-1}} c < p_0 M = U_P\left(p_0\right)$. From the perspective of the principal, she would be strictly better off since $V_S^k\left(p_0\right) = -p_0(1-p_k)M^2 > -p_0(1-p_0)M^2 = V_P\left(p_0\right)$.

When the pooling equilibria with learning survive, consider the one in which the potential good type reports $\hat{k}+1 < k \leq \overline{k}$ successes and the bad type agent whose first failure occurs after $\hat{k}+l+1_{th}$ experiment would over-experiment, where $0 < l \leq k - \hat{k}$. In this case, the agent's expected payoff would be $U_O^k(p_0) = p_0 M - \sum_{i=1}^{\hat{k}+l} \frac{p_0}{p_{i-1}} c - \frac{p_0(k-\hat{k}-l)}{p_{\hat{k}+l-1}} c < p_0 M = U_P(p_0)$, thus the agent is worse off. For the principal, her expected payoff would be $V_O^k(p_0) = -p_0(1-p_{\hat{k}+l})M^2 > -p_0(1-p_0)M^2 = V_P(p_0)$, so she is strictly better off relative to public experimentation.

Proof of Proposition 3.4.3

Consider $\overline{k}$ first. When $p_0 M < c$, $\overline{k} = 0$. This condition can be rewritten as $p_0 \frac{M}{c} < 1$ In this case, the change of θ doesn't affect the participation threshold. As p_0 or $\frac{M}{c}$ increases, $p_0 \frac{M}{c}$ is getting larger, which makes the inequality is harder to be satisfied. If it's violated, $\overline{k}$ becomes to 1. When $p_0 M \geq c$, $\overline{k} = \max \left\{ k \in \mathbb{N} : p_0 \frac{M}{c} \geq \tilde{k} \right\}$. It's easy to see that $p_0 \frac{M}{c}$ is increasing as $\frac{M}{c}$ increases, which implies a larger number of experiments whose expected total cost can be cover. Thus $\overline{k}$ rises in this case. Now consider the marginal effect of θ and p_0 on this condition:

$$
\begin{aligned}
\frac{\partial U^k(p_0,\theta)}{\partial \theta} &= (1-p_0) \sum_{i=1}^{k} (i-1)(1-\theta)^{i-2} c > 0 \\
\frac{\partial U^k(p_0,\theta)}{\partial p_0} &= M - \sum_{i=1}^{k} [1-(1-\theta)^{i-1}] c \\
&> \underbrace{p_0 \left[M - \sum_{i=1}^{k} [1-(1-\theta)^{i-1}] c \right] > \sum_{i=1}^{k} (1-\theta)^{i-1} c}_{\text{since } p_0 M - \sum_{i=1}^{k} [p_0 + (1-p_0)(1-\theta)^{i-1}] c \geq 0} \geq 0
\end{aligned}
\tag{C.0.11}
$$

These two first derivatives suggest that $U^k(p_0,\theta)$ is increasing as p_0 or θ increases, holding k constant, which implies more experiments' expected total cost can be covered by the prior expected value. Therefore, $\overline{k}$ is weakly increasing in this case.

Now consider $\hat{k}$. When $\frac{M}{c} > \frac{1}{p_1(1-\theta)}$, $\hat{k} = 0$. When $\frac{M}{c}$ increases, the participation threshold stays the same as the left hand side of the condition increases. Now the first derivatives of $\frac{1}{p_k(1-\theta)}$ with respect to p_0 and θ can be calculated, where $k \in \mathbb{N}^{+}$:

$$\begin{aligned}\frac{\partial \frac{1}{p_k(1-\theta)}}{\partial \theta} &= \frac{p_0 + (1-p_0)(1-\theta)^k(1-k)}{p_0(1-\theta)^2} \\ \frac{\partial \frac{1}{p_k(1-\theta)}}{\partial p_0} &= -\frac{(1-\theta)^{k-1}}{p_0^2} < 0\end{aligned} \tag{C.0.12}$$

Therefore, $\frac{1}{p_k(1-\theta)}$ decreases when p_0 increases at $\forall k \geq 1$. When $\frac{M}{c} > \frac{1}{p_1(1-\theta)}$, $\frac{1}{p_1(1-\theta)}$ decreases as p_0 increases. Therefore, it's easier to have the over-experimentation threshold staying at 0. When θ increases, the marginal changes at $k = 1$ is positive, which implies $\frac{1}{p_1(1-\theta)}$ gets larger, and it's harder to have the over-experimentation threshold staying at 0. When $\frac{M}{c} \geq \frac{1}{\theta}$, $\hat{k} \to \infty$. This condition is easier to be satisfied when $\frac{M}{c}$ or θ increases, but is independent of p_0. When $\frac{1}{1-\theta} \leq \frac{M}{c} \leq \frac{c}{p_1(1-\theta)}$, $\hat{k}$ is finite. Since $\frac{1}{p_k(1-\theta)}$ is decreasing as p_0 increases, the condition $\frac{M}{c} \leq \frac{1}{p_k(1-\theta)}$ is harder to have the $\hat{k}$ staying at current level. As a result, $\hat{k}$ tends to fall. When $\frac{M}{c}$ increases, it's also hard to have the over-experimentation threshold staying a the current level since the left hand side of the condition increases, which leads that $\hat{k}$ also tends to fall in this case. Notice that the sign of $\frac{\partial \frac{1}{p_k(1-\theta)}}{\partial \theta}$ varies at different k, thus it cannot be achieved a monotonic effect of θ on $\hat{k}$.

Proof of Proposition 3.5.1

Firstly, I prove that it's not credible when the agent commits to report $k > \overline{k}$ successes or $k = \hat{k} + 1$ successes with $\hat{k} < \overline{k}$. Suppose he commits to report $k > \overline{k}$ successes, then the principal learns that he must be a bad type when less than k successes are reported. If the agent doesn't over-experiment, his expected payoff would be $p_0 M - \sum_{i=1}^{k} \frac{p_0}{p_{i-1}} c < 0$ as $k > \overline{k}$. Thus, the agent will always deviate and not to run any experiments, which implies such commitment is not credible. Therefore, the agent would only commit to $k \leq \overline{k}$. Suppose the agent commits to report $k = \hat{k}+1$ successes with $\hat{k} < \overline{k}$, he receives $p_{\hat{k}} M$ as a reward when reporting $\hat{k}+1$ successes since the principal learns that the bad type agent who fails in $\hat{k} + 1_{th}$ experiment would over-experiment. However, since $p_{\hat{k}} M \leq \frac{c}{1-\theta}$, such bad type agent would deviate and not to over-experiment. Therefore, this commitment is also not credible.

Secondly, I prove that the agent is better off by committing to report a number of successes at $0 \leq k \leq \min\left\{\overline{k}, \hat{k}\right\}$ rather than that at $\hat{k}+1 < k \leq k$. In the region $0 \leq k \leq \min\left\{\overline{k}, \hat{k}\right\}$, when committing to report k' successes, the agent receives $p_{k'}M$ when reporting k' successes since no bad type agents would over-experiment, thus his expected payoff would be $U_S^{k'}(p_0) = p_0 M - \sum_{i=1}^{k'} \frac{p_0}{p_{i-1}} c$, which is the same as that when the agent cannot commit. In the region $\hat{k}+1 < k \leq \overline{k}$, the agent's reward level would be $p_{\hat{k}+l}M$ when reporting k successes, since the principal knows that the bad type agent whose first failure occurs after $\hat{k}+l+1_{th}$ experiment has incentive to over-experiment, where $0 < l \leq k-\hat{k}-l$, thus his expected payoff would be the same as that when the agent cannot commit, $U_O^k(p_0) = p_0 M - \sum_{i=1}^{\hat{k}+l} \frac{p_0}{p_{i-1}} c - \frac{p_0(k-\hat{k}-l)}{p_{\hat{k}+l-1}} c < p_0 M - \sum_{i=1}^{k} \frac{p_0}{p_{i-1}} c$. Since $k' < k$, $U_O^k(p_0) < p_0 M - \sum_{i=1}^{k} \frac{p_0}{p_{i-1}} c < p_0 M - \sum_{i=1}^{k'} \frac{p_0}{p_{i-1}} c = U_S^{k'}(p_0)$. Therefore, the agent strictly prefers committing to report a number of successes at $0 \leq k \leq \min\left\{\overline{k}, \hat{k}\right\}$.

Thirdly, I show that the agent prefers committing to report a smaller number at $0 \leq k \leq \min\left\{\overline{k}, \hat{k}\right\}$. In this region, the agent's expected total cost $\sum_{i=1}^{k} \frac{p_0}{p_{i-1}} c$ is weakly increasing as k decreases. Thus, the agent's optimal choice k^* would be the smallest k in this region. Ideally, $k^* = 0$.

Fourthly, I show that the agent comprises to commit to report a larger number to makes (3.4.4) being satisfied. Suppose the agent choose $k = 0$. To support it's credibility, the restrictions on the principal's off-equilibrium path belief (3.4.4) needs to be satisfied. If not, the agent would always deviate to run at least one experiment and report the successes if any, which leads the initial commitment to be non-credible. In this case, the agent has to choose the second lowest number, $k = 1$. Now it needs to check if (3.4.4) is violated in this case. If it's not, $k^* = 1$. If it is, the agent seeks to the next lowest number except the previous ones. This process would hold in the region $0 \leq k \leq \min\left\{\overline{k}, \hat{k}\right\}$. Therefore, Proposition 3.5.1.1) summarises the process above in this region.

Fifthly, I show that the expected total cost in the region $\hat{k}+1 < k \leq k$ is not monotonic with respect to k. Suppose when committing to report k successes, the bad type agent whose first failure occurs after $\hat{k}+l+1_{th}$ experiment has incentive to over-experiment. This requires $\frac{k-\hat{k}-l}{1-\theta} c < p_{\hat{k}+l} M \leq \frac{k-\hat{k}-l+1}{1-\theta} c$. If the agent commits to report $k+1$ successes, it implies that, at most, the bad type agent whose first failure occurs after $\hat{k}+l+2_{th}$ experiment would over-experiment, as $p_{\hat{k}+l+1}M > \frac{k-\hat{k}-l}{1-\theta}c$. In this case, his expected payoff would be $U_O^{k+1}(p_0) = p_0 M - \sum_{i=1}^{\hat{k}+l+1} \frac{p_0}{p_{i-1}}c - \frac{p_0(k-\hat{k}-l)}{p_{\hat{k}+l}}c$. Thus,

$$U_O^{k+1}(p_0) - U_O^{k}(p_0) \propto -[p_0+(1-p_0)(1-\theta)^{\hat{k}+l}\theta] + (k-\hat{k}-l)(1-p_0)(1-\theta)^{\hat{k}+l-1}\theta \tag{C.0.13}$$

It can easily see that this difference is not always negative or positive under different parameter range. Alternatively, the agent's optimal choice in the region at $\hat{k}+1 < k \leq k$ can be rewritten as $k_p \in \underset{k\in\mathbb{N},\hat{k}+1<k\leq k}{\arg\max}\ U_O^k(p_0)$. As a result, if (3.4.4) is violated in the region $0 \leq k \leq \min\left\{\overline{k}, \hat{k}\right\}$, the agent would consider $k^* = k_p$. If (3.4.4) is still violated at $k^* = k_p$, the agent would choice the sub-optimal choice in this region, which maximises $U_O^k(p_0)$ except k_p. The process is then summarised Proposition 3.5.1.2).

Finally, if (3.4.4) is violated at $0 \leq k \leq \overline{k}$, the agent's commitment would not be credible at any level in this region.

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