Mandatory savings, information and welfare

Theory and empirical evidence

Conrado Cuevas López

A thesis presented for the degree of
Doctor of Philosophy in Economics

Department of Economics
University of Warwick
September 2017

Contents

List of Figures

List of Tables

Acknowledgements

I want to thank my supervisors, Pablo Beker, Dan Bernhardt and Herakles Polemarchakis for all their help, for the many hours I spent working with them, for their encouragement, patience and honesty. I also thank CONICYT and the Department of Economics of the University of Warwick for their financial support.

To Clemente, thanks for making Thursdays the best day of the week. To Stephanie, thanks for keeping me *en garde*. I love you both.

Declaration

This thesis is submitted to the University of Warwick in support of my application for the degree of Doctor of Philosophy. It has not been submitted in any previous application for any degree. Chapter one has been composed in collaboration with Dan Bernhardt, and Mario Sanclemente, who helped me in conducting the survey and analyzing its data. The survey had the approval of the Humanities and Social Sciences Research Ethics Committee, Ethical Application Reference: 135/14-15. Chapter two has been composed in collaboration with Pablo Beker.

Abstract

In Chapter 1 we document how pension investments by individuals in the Chilean social security system are influenced by portfolio recommendations of Happy and Loaded, a pension advice firm. Following H&L's recommendations about which of five portfolios to invest in, investors shift amounts that often exceed 20% of portfolios value and 1.3% of Chilean annual GDP, in a week. We uncover what drives investment recommendations, the resulting return consequences for the Chilean stock market and social security portfolios, and the characteristics of followers and their investment outcomes. Paradoxically, investors who followed H&L's advice would have earned more by sticking with their original portfolio over time, regardless of the portfolio selected. These findings provide a cautionary tale for the design of privatized social security systems.

In chapter 2 we study the value of public information in a stochastic pure exchange economy where agents trade assets in financial markets to reallocate risk, and a subset of those agents face a mandatory savings constraint. As the mandatory savings constraint depends on equilibrium prices, changes in information may allow a Planner that faces the same constraints as the agents in terms of the available assets, information, and savings constraints, to obtain Pareto improvements relative to the equilibrium without information. Changes in information cause the posteriors to change, thus affecting equilibrium prices and shifting the constraints that the Planner has to satisfy. We provide conditions for the arrival of new information before trading to obtain ex-post and ex-ante welfare improvements relative to the initial equilibrium without information. The reaction of prices to the arrival of new information is key in our analysis. We relate the value of information in exchange economies with the literature on Bayesian persuasion.

Chapter 1

The Pied Piper of Pensioners

1.1 Introduction

On March 6, 2012, the Chilean government took the unprecedented step of ordering *Felices y Forrados* (Happy and Loaded, henceforth H&L) to cease providing pensioners guidance on portfolio choice for their retirement savings. The government's action stemmed from concerns that massive swings in investment flows induced by investors following H&L's advice were destabilizing the Chilean economy. The order was revoked on April 24, 2012. Still, the government's worries did not cease, as indicated by an April 26, 2013 report from the Financial Stability Board stating "...movements between different pension funds have increased markedly...movements of this quantity, in such a short term, affect the system as a whole by affecting the prices of some financial assets, creating stress on market infrastructures..."

Chile's private social security system is perhaps the most widely-emulated social security design in the world. More than 20 countries have adopted variants of the Chilean design (Berstein, Larraín, Pino, and Morón (2006)), including countries in Latin America,[1] central and eastern Europe,[2] Africa (Nigeria), Asia (Kazakhstan, Singapore), and developed countries (Sweden, Denmark). Aspects of the Chilean design are central to a recent Blackstone proposal for a privatized social security plan in the United States.[3] It is clearly important for policy makers to understand the consequences of its design for Chile, and to understand how consequences might differ in other countries, in order to gain insights into how the design could be improved.

The Chilean social security system is a fully funded, defined contribution, multi-fund, personal account system. On December 31, 2014, total savings were $165 billion USD, or roughly 60% of the Chilean GDP in 2014. Average pension savings were $38,600 USD, or about 54% of total net wealth (Behrman, Mitchell, Soo, and

[1] Peru, Colombia, Argentina, Uruguay, Bolivia, Mexico, El Salvador, Costa Rica, Dominican Republic, Nicaragua and Ecuador.

[2] Bulgaria, Croatia, Estonia, Hungary, Latvia, Poland, Russia, and Slovakia.

[3] See Ghilarducci, James, et al. (2016).

Bravo (2012)). Formal workers must save 10% of monthly earnings. Workers choose a Pension Fund Administrator (AFP) to manage their investments. Government regulation sharply limits asset allocations by AFPs to reduce the possibility of poor pension performance that could endanger retirement savings. Each AFP only offers five types of funds ordered from A to E by their riskiness. Fund A, which is largely invested in foreign mutual funds, ETFs and domestic stocks, is the riskiest; while fund E, which is invested mainly in government and Central Bank bonds, and bank deposits, is the safest.

This chapter shows how the design of Chile's social security system led to massive, coordinated portfolio reallocations following the recommendations of a single financial advisor, H&L. These transfers often exceeded one percent of Chile's annual GDP, sometimes exceeding the *total monthly trading volume* in domestic stocks. We uncover what drives H&L's recommendations, and show how they altered investment strategies of pensioners. We derive the impacts on asset prices in domestic financial markets, including portfolio and stock market returns. Using administrative and survey data, we document that followers of H&L are far wealthier than the typical pension investor, with over twice the savings. Followers are also more educated, quite financially sophisticated and informed about how their pension returns compare to alternative buy-and-hold strategies. Nonetheless, we find that *most* investors who flocked to follow H&L's advice would have done better to stick with their original portfolio over time, *no matter which portfolio they originally held.* We show that most of this underperformance emerges because not only have pension investors come to believe that H&L's recommendations have value, but so has the market. As a result, stock prices adjust to reflect H&L's recommendations before pension investors can transfer their funds, with the outcome that pension investors end up buying high and selling low. These findings provide a cautionary tale for the design of privatized social security systems.

H&L, founded in July 2011, charges a small annual fee of about $24 USD for advice.[4] Advice takes the form of emails, issued after the close of a trading day, typically instructing clients to switch 50% or 100% of savings from portfolio A to portfolio E, or vice versa. Between July 2011 and September 2016, H&L issued instructions to switch savings from one portfolio to another on 35 occasions. Using daily flow data from October 2011 to September 2016, we first document that investors have come to *believe* that H&L's advice is sound. H&L started with 54 paid followers, and did not have new customers until after its 4th recommendation.[5] Remarkably,

[4] H&L's founder claims to have developed a statistical model, based on Harry Markowitz' 1952 model, that allows him to forecast the performance of social security portfolios.

[5] We have H&L's administrative data through September 30, 2016. These data include the payment history of all clients plus basic demographics like gender and age. H&L has two types of clients: premium and basic. Premium followers pay an annual fee of $24USD, while basic followers have a free three month trial period, in which they receive announcements with a three day lag. The number of premium followers reached 66,000 in August 2016, and there are many more second-hand followers.

beginning with the sixth recommendation, accompanied by surges in Google trends and Google searches for H&L, each new recommendation led to net shifts of more than 25,000 investors (a mix of paid subscribers and second-hand followers) to the newly endorsed portfolio, and away from the portfolio that had previously been endorsed. Strikingly, the five recommendations issued between April 2013 and January 2014 on a date $t-1$,[6] on average, led over 100,000 (net) investors to switch to the recommended portfolio over the first six trading days that their requests could be executed. The average funds shifted following these five recommendations exceeded 20% of portfolio E's value at the time of the new recommendation, equivalent to *more than 1.3% of the Chilean GDP in 2013*. Cumulative net flows to the risky portfolios A and B were of comparable magnitudes. H&L-recommended changes directly precede *every* large shift in pension investments.

Having established the remarkable impact of H&L's advice on pension investments, we investigate what drives H&L's advice, whether investors benefit, and whether and how H&L's advice affects portfolio returns, and domestic stock markets.

1. We find that H&L's advice primarily reflects the immediate past performance of the Chilean stock market: very high past returns on the Chilean market directly precede recommendations to transfer funds into risky portfolio A and out of safe portfolio E; while very bad returns are associated with the opposite recommendation pattern.

2. We find positive announcement effects on the day following a recommendation of portfolio A (negative announcement effects after a recommendation of portfolio E) for portfolios A through D and Chilean stock market indexes, followed by positive cumulative excess returns on days $t+3$ to $t+7$ where portfolio transfers are high.

These latter results lead naturally to a conjecture that the induced transfers of funds in and out of portfolios must have had short-run price impacts on domestic equity prices. This leads us to search for impacts of these mass transfers on stock market volume. There are *none*. Domestic stock market volumes are *not* unusually high on days where portfolio transfers are high: AFPs seem to accommodate mass transfers by adjusting positions in liquid foreign equity markets, and not illiquid domestic markets.

We then ask: do investors benefit from following H&L's advice? We get at this in two ways. We first use H&L's payment records to compute returns for each follower starting with the first announcement he could follow until the last announcement for which his account with H&L was active, i.e., until his subscription expired.

[6] The timing convention is that date $t-1$ is the date on which the recommendation is made (after the close of trading), so that the recommendation is known at the open of trading day t.

We then determine whether a follower's return exceeded that from a buy-and-hold strategy for each portfolio. We find that most followers are hurt by following H&L's advice: fewer than 10% of followers beat portfolio E and fewer than 30% beat their riskiest option. We then compare the performance of H&L's strategy with that of holding the other portfolios, starting at *any* of the first twenty announcements through September 30, 2016. Without adjusting for risk, an investor would have done better following H&L from the first announcement than investing in any of the other five portfolios. However, only 54 investors did this; and at any other starting point H&L is not the option with the highest return. Indeed, at almost all other starting points, investors would have done better to stick with whichever portfolio they held, *no matter what it was*.

These results beg the question: why do so many investors come to believe that H&L's recommendations have value? We establish a sense in which H&L's recommendations look very good. Someone who hypothetically could transfer funds at the *exact* moment a recommendation was made—rather than one day later—would have earned returns that exceed those from buying and holding *any* of the other portfolios for nine of the first fourteen recommendations, and would outperformed portfolio A for fifteen of the first twenty recommendations. Thus, this hypothetical possibility reverses the ordering of returns. Paradoxically, what harms followers is not that they come to believe H&L's recommendations have value, but rather that the *entire market* does. As a result, the Chilean stock market experiences a large positive announcement effect following a recommendation by H&L to switch to the risky portfolio A, and a large negative announcement effect after a recommendation to switch away from the risky portfolio A. Followers cannot switch portfolios in time to benefit from the announcement effect—they end up buying high and selling low, reducing their cumulative portfolio returns by 20-25%. The difference in results reflects the time it takes AFPs to shift funds between pension portfolios and the complex pricing rules used by AFPs to value transferred funds; pricing rules that may conceal from investors the fact that they do not benefit from the announcement effect.

These findings lead us to investigate further *why* individuals follow H&L, despite the under-performance of their investments. To identify whether followers are less financially sophisticated or less informed, we surveyed a *large* sample of over 8,700 current followers of H&L. We then contrast our survey results with findings regarding the population characteristics of pension investors derived from the broad Social Protection Survey (EPS).

Surprising results obtain. H&L followers are *far* more sophisticated than the average investor. In particular, they are more educated, with higher incomes and over twice the savings of the average investor. They are also much more likely to make additional voluntary savings, highlighting their patience and understand-

ing of the tax benefits. Answers to questions related to risk diversification and compound interest underscore that H&L followers are extremely financially sophisticated, especially relative to non-followers. One striking illustration of this is that 64% of followers correctly calculate a compound interest problem vs. only 3-5% of non-followers! One might then conjecture that followers are just uninformed about portfolio returns or that they systematically over-estimate returns from following H&L vis à vis buy-and-hold strategies. This conjecture is also false. Followers are well informed about portfolio returns. Among followers providing full rankings, 57% correctly rank the 12 month returns on portfolios A, C and E, and slightly more than half correctly rank portfolio E's return above H&L's. Paradoxically, the key reasons that investors give for why they follow H&L are: higher returns, minimize losses, and they trust H&L more than their AFP.

Our findings have implications for the design of privatized social security systems in which individuals have some choice over pension investments that may serve as a primary source of retirement funds. These lessons are relevant for the many countries that use variants of the Chilean design. One goal of such systems is to align investments better with individual attitudes toward risk. A second goal is to minimize risks of inadequate retirement savings due to bad investment choices or moral hazard by investment advisors. The Chilean design addresses this by sharply limiting the portfolio choices available to investors. We show that despite these constraints—or perhaps because of them—investors may be harmed due to responses by the market. Our analysis highlights the risk that with few portfolio alternatives, information arrival, here taking the form of recommendations by H&L, can lead to massive coordinated reallocations of funds. In Chile, liquidity provision is not overwhelmed due to the limited portfolio exposure to the domestic market, which allows AFPs to accommodate large portfolio transfers by adjusting holdings of liquid foreign assets, mitigating price impacts. However, greater exposure to domestic markets—as might occur in the United States or England—could magnify the ramifications of transfers induced by such information arrival.

Ghilarducci, James, et al. (2016) lay out a personal savings plan to confront what they term "the US retirement savings crisis." This Blackstone plan contains many features of the Chilean system: savings are mandatory, private accounts are managed by professionals, and it is built on personal choice. The authors do not detail the set of portfolio alternatives nor specify how individuals can change portfolio selection. We highlight mechanisms and considerations that should enter the design of these details.

1.1.1 Related literature

Our work contributes to a broad literature that studies how the design of a social security system affects the economy. Edwards (1998) argues that the massive amount

of assets held by AFPs helped Chile by contributing to a more dynamic and modern capital market, allowing private firms to rely on long-term financing. Joubert (2015) builds a dynamic model analyzing how the Chilean pension system affects a household's labor supply, formal/informal sector choice, and saving decisions. He shows that the presence of a large informal sector, not covered by the mandatory savings system, offers workers competitive earnings opportunities, thus mandatory pensions contributions can create a significant reduction in formality and tax revenue. Mesa, Bravo, Behrman, Mitchell, and Todd (2008) analyze savings, participation patterns and the financial literacy of investors in the Chilean social security system. They find that only a small fraction knows basic key details such a the payroll tax or commission rates.

Our analysis also pertains to a literature on the informativeness of analyst recommendations. Just as investors in Chile come to believe H&L's recommendations, as reflected by the announcement returns, researchers find that recommendation upgrades by financial analysts are associated with positive announcement returns (Stickel (1995), Womack (1996), Barber, Lehavy, McNichols, and Trueman (2006), Ivković and Jegadeesh (2004), Loh and Stulz (2010)). More generally, analyst recommendation changes contain relevant information, and investment strategies based on portfolio constructions that use recommendation information have positive value (Barber, Lehavy, McNichols, and Trueman (2001), Jegadeesh, J. Kim, Krische, and Lee (2004), Jegadeesh and W. Kim (2009), Boni and Womack (2006)). Consistent with this, Dahlquist, Martinez, and Söderlind (2016) find that active investors in Sweden's Premium Pension System tend to follow recommendations of financial advisors, and that, gross of advisor fees, active investors seem to outperform passive ones. Jegadeesh, J. Kim, Krische, and Lee (2004) show that analysts often recommend stocks with positive momentum; we show that H&L adopts an even shorter-horizon momentum strategy.[7]

Our analysis also relates to a literature showing that better-performing mutual funds draw greater cash inflows (Chevalier and Ellison (1997), Zheng (1999); see Bernhardt and R. J. Davies (2009) for a theoretical model). Of note, we show that the advent of H&L changed how individuals make pension investments. Prior to H&L, shifts in investments reflected long-term portfolio performances: investments flowed to portfolios that had higher returns over the previous three months. Once H&L began to have influence, only its recommendations (which we show are based on the *immediate* past performance of the market) affected portfolio flows.

Carlin and S. W. Davies (2016) theoretically analyze the implementation of state sponsored retirement plans, showing how the optimal menu of options and default option depends on the financial sophistication of participants and their behavioral biases. They assume that only unsophisticated investors make bad active trading

[7] Inderst and Ottaviani (2012) and Gennaioli, Shleifer, and Vishny (2015) provide theoretical analyses of strategic financial advisor behavior.

decisions, making it optimal to limit access to risky portfolios. Here we provide evidence that agents adopting active strategies tend to do worse; but in Chile these investors are far more sophisticated than the average investor.

The fact that so many followers pay for advice that seems to have harmed them has suggestive links to Powdthavee and Riyanto (2015). Their experimental analysis shows that people can be induced to believe in an "expert" who provides "transparently useless" advice. People pay for the advice, perceiving a hot hand after a string of successes; while long streaks of incorrect predictions generate beliefs that the "unlucky" agent's luck was likely to revert.

1.1.2 The Chilean social security system

The Chilean Social Security System has three key pillars: a welfare pillar, a mandatory contribution pillar and a voluntary savings pillar. For a full description see Superintendence of Pensions (2010). The mandatory contribution pillar is a defined contribution, multi-fund, personal account system. Each worker accumulates savings in a personal account until retirement. Formal workers must save at least 10% of monthly wages up to a cap. Participation by self-employed workers was voluntary prior to 2018.

The savings accounts are privately managed by the AFPs. There are currently six AFPs. AFPs are highly regulated and face constraints on their investments. Individuals choose among five portfolios, portfolios A to E, which differ in their exposure to stocks and other variable yield instruments. The portfolios are ordered according to riskiness: Portfolio A is the riskiest, while portfolio E is the safest. Table 1.1 presents the distribution of assets in each portfolio on the last day of 2014. Of note, less than 20% of the risk in portfolios A and B reflects exposure to domestic securities; most of the risk exposure is to foreign assets such as ETFs or mutual funds. Further (unreported) investigation reveals that the *composition* of domestic stock holdings is *very* similar across portfolios; the portfolios differ largely only according to the *scale factor* weighting domestic stock holdings. This composition can vary within a month: AFPs alter their holdings in response to market conditions. In particular, we will provide evidence that following large portfolio shifts, AFPs do not alter investments in domestic equities. Instead, they accommodate these shifts in their holdings of more liquid (foreign) assets. In practice, the returns of portfolios A through E are similar across AFPs; see Table 1.2.[8]

Individuals make two important choices—the choice of administrator and the allocations of savings across portfolios. Men under the age of 55 and women under 50 face no constraints on portfolio choices; and no restrictions apply to voluntary

[8] AFPs differ more in terms of the fees that they charge. H&L's recommendation that its followers select one of two specific AFPs—one due to higher returns, and the other due to lower fees—has not varied over time. However, H&L's motto is "if you are happy with your AFP, stick with it."

Table 1.1: Total assets of each portfolio (millions $US) and asset distribution (in %) on 12/31/2014.

	A	B	C	D	E
Total assets	26,348	27,169	61,277	26,385	24,253
Asset distribution					
Domestic stocks	11.8	12.8	10.0	3.8	0.7
Domestic mutual and investment funds	2.5	2.6	2.4	0.9	0.1
Foreign ETFs	16.6	12.2	9.4	6.5	3.7
Foreign mutual and investment funds	61.9	45.3	32.6	21.5	1.9
Other foreign assets	1.2	1.0	1.0	1.5	1.7
Central bank bonds	0.4	3.7	5.9	9.4	15.0
Government bonds	0.7	7.6	14.4	22.7	28.3
Bank bonds	1.6	4.6	8.5	11.6	11.7
Domestic firm bonds	1.8	4.5	8.6	9.4	10.2
Deposits	0.3	3.5	3.3	8.9	22.9
Others	1.2	2.4	3.8	3.9	3.8

Table 1.2: Real annual returns across AFPs: 27 Sept 2002-Aug 2015

Portfolio	CAPITAL	CUPRUM	HABITAT	PLANVITAL	PROVIDA*
A	6.78%	6.83%	6.88%	6.43%	6.78%
B	5.75%	5.90%	5.85%	5.67%	5.50%
C	5.13%	5.68%	5.65%	5.29%	5.08%
D	4.86%	5.18%	5.19%	4.58%	4.67%
E	4.23%	4.20%	4.33%	3.47%	3.78%

*MODELO, the 6th AFP, entered 2010, and had similar returns over that subperiod.

savings. Older workers cannot select portfolio A, and pensioners cannot select portfolio B (for mandated savings). A worker who does not choose a fund is assigned a default option that places weights on portfolios B, C and D that depend on the worker's age. Workers are otherwise free to shift savings from one portfolio or AFP to another, and there are no fees associated with such transfers. Transfers among portfolios within an AFP are made four working days after a request,[9] unless the total transfer request from a particular fund exceeds 5% of its value, in which case the excess is delayed to the next working day, on a first come first serve basis. Transfers between AFPs are delayed until the first working day of the following month if made during the first fifteen days of a month; otherwise they are delayed until the fifteenth day of the following month. Using the convention that date t is the first trading day after a recommendation to switch savings from one portfolio to another, AFPs first transfer savings based on a recommendation on trading day $t+3$. Importantly, when a transfer is made on $t+3$ due to an investor acting on a recommendation made after close on day $t-1$, an AFP uses asset prices on day t (e.g., weighted average for domestic stocks and closing prices for foreign stocks) to value the funds being transferred.

[9] This delayed response by AFPs is administrative in nature.

1.2 H&L's impact

1.2.1 Portfolio transfers

H&L makes recommendations after the close of a trading day. Figure 1.1 presents overwhelming evidence that social security investors come to believe in H&L recommendations: their new recommendations explain the bulk of transfers in and out of the different portfolios.[10] Beginning with the fifth recommendation by H&L, advice to switch from A to E causes funds to flow from portfolio A to E, and advice to switch from E to A leads to flows from E to A. Not only is *every* large shift in pension investments directly preceded by a recommendation from H&L to redirect investments in that way, but so is *every moderate* shift, save one.

H&L's administrative records provide us the number of official (paid) followers. To estimate the number of second-hand followers, given the age restrictions on who can hold portfolios A and B, we use recommendations to move all money in or out of portfolio E that all investors can follow. A conservative estimate of the number of second-hand followers is given by the *net* number of accounts shifted on days $t+3$ through $t+8$ in the direction suggested by a recommendation minus the number of official followers. The measure is conservative because some paying followers may not follow a given recommendation.[11] Table 1.3 shows that by the 11th recommendation, H&L had more than 100,000 followers. The table also reveals that followers have more than double the average savings of typical pension investors.

Table 1.3: Number of followers, estimated number of second hand followers and their average savings switched (CLP), and total number of members of the system and their average savings in the month of the announcement.

Recommendation		Followers			System	
Number	Type	Official	Second hand	Mean savings switched	Members	Mean savings
5	buy E	1,113	8,201	16,124,583	9,169,709	7,061,000
6	sell E	3,168	19,430	19,020,284	9,169,709	7,061,000
7	buy E	3,587	30,688	17,687,873	9,220,325	7,165,000
8	sell E	4,456	21,627	17,643,342	9,220,325	7,165,000
9	buy E	7,486	46,309	17,106,715	9,220,325	7,165,000
10	sell E	17,130	45,696	18,414,688	9,373,955	7,521,000
11	buy E	18,010	86,994	17,434,146	9,461,060	7,453,000
12	sell E	27,132	76,054	17,426,833	9,509,439	7,637,000
13	buy E	42,304	79,830	16,582,820	9,509,439	7,637,000
14	sell E	44,935	45,818	16,565,331	9,509,439	7,637,000
15	buy E	45,736	66,489	17,093,872	9,634,711	8,149,000
20	buy E	51,431	55,226	19,634,988	9,746,467	9,079,000

Investors who followed H&L's first few recommendations earned higher returns than those who did not change portfolio allocations. Figure 1.2 shows that—likely as a result—H&L experienced a massive upsurge in media coverage and investor

[10] Daily data on portfolio flows around the initial recommendation on July 27, 2011 do not exist.

[11] Mean savings of followers are even more conservatively estimated as they may only shift some savings.

Figure 1.1: H&L recommendations and daily net flows to portfolios A and E (billions of CLP).

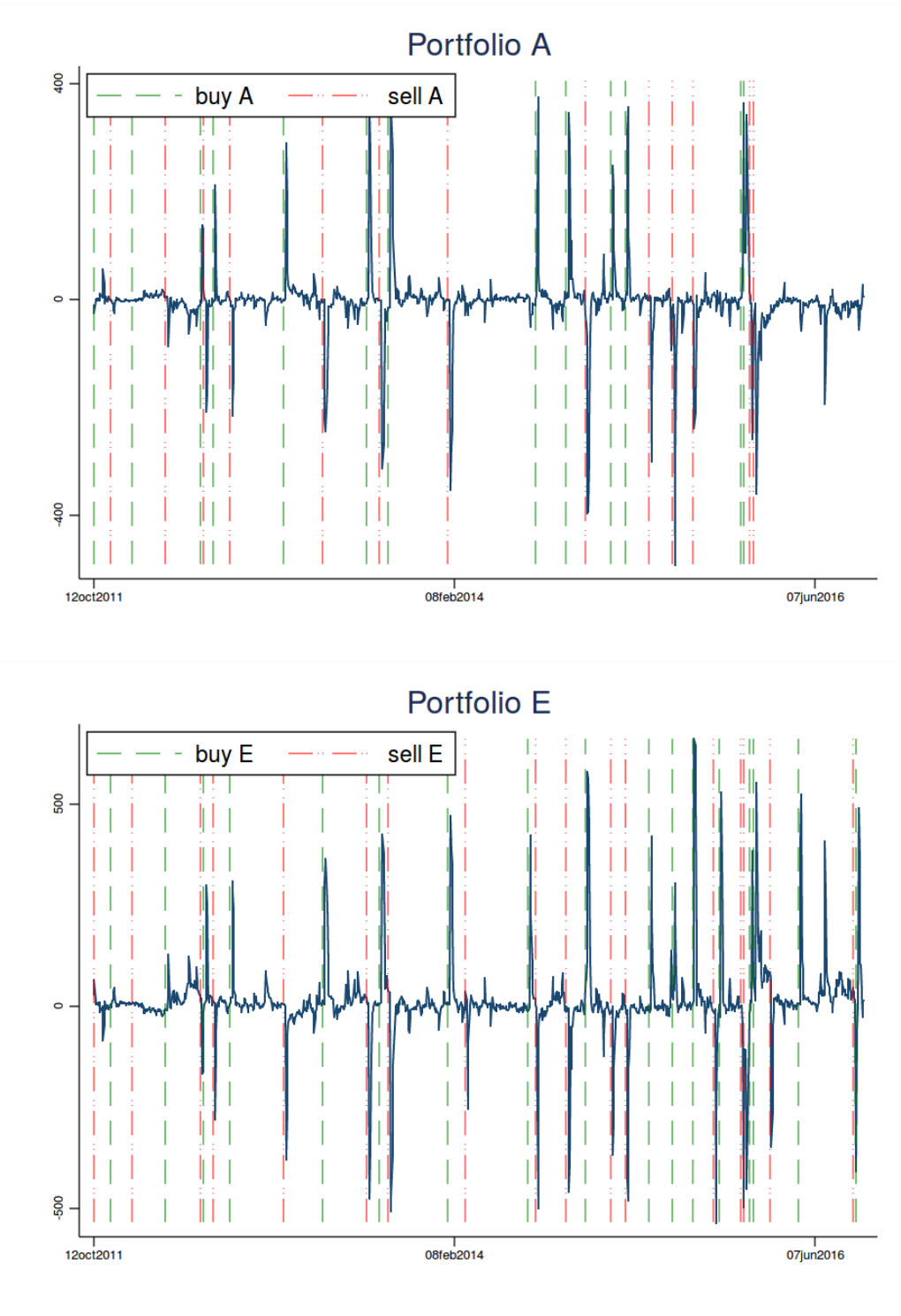
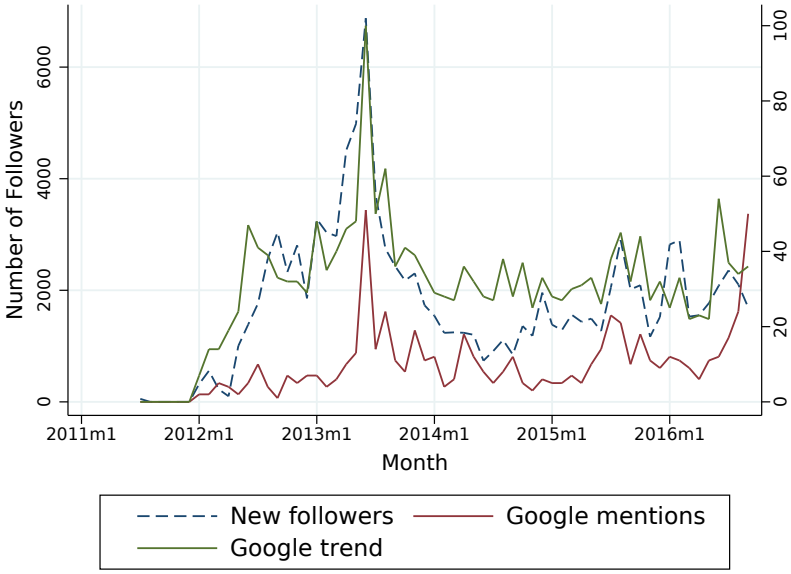


Figure 1.2: Investors interest, media coverage, and new subscriptions.



attention followed by massive numbers of investors beginning to follow H&L's advice. We document this surge using data from Google trends, searching for the phrase "Felices y Forrados." Figure 1.2 shows the Google trend index, taking on the value of 100 in the month where the most users "Googled" H&L, and a "Google search" series, constructed using a monthly search on Google of the same phrase. We only count results from media sites, news sites, and opinion blogs. Both series show similar trends. H&L was almost unknown before 2012. Media coverage and interest from Internet users increase and then explode, peaking in July 2013; after this point, interest in H&L remains steady. We see that the numbers of new clients closely track these Google indexes.

Table 1.4 shows the daily net flows to portfolios after recommendations. For the last six recommendations to shift 100% of funds from portfolio A to portfolio E or vice versa,[12] the net inflows/outflows for each of these portfolios exceed 1,000 billion CLP. These flows represent as much as 25% of the total value of portfolio E as of day $t+2$, or *more than* 1.5% of the annual Chilean GDP. These flows reveal that pension investors *believe* that H&L's recommendations contain material information about the future performances of portfolios. The data show heavy flows of savings into the recommended portfolio following a recommendation on date t on days $t+3$ to $t+8$, but *not* at earlier dates.[13] Underscoring the high fraction of active investors influenced by H&L's advice, Superintendence of Pensions (2013) using data through May 2013, found that 60% of pension investors used the default investment strategy.

[12]All other recommendations, the bulk of which were made later, were to shift 50% of funds from one of portfolios A, C and E to another.

[13]The patterns in Figure 1.1 and Table 1.4 also show up in net numbers of accounts shifted. Portfolios B and C follow similar patterns to those documented in Figure 1.1 and Table 1.4 for portfolio A, but they are of smaller magnitudes. This reflects that some investors are over the age limit for investing in portfolio A, making portfolios B or C their closest feasible alternative.

Table 1.4: Net flows to portfolios (in billions of CLP) around new recommendation days. Day t represents the first trading day after a recommendation is made. Net flow to portfolio X on day $t+s$ is defined as the inflow minus the outflow to portfolio X on day $t+s$.

						Portfolio A				
Date	From	To	$t+3$	$t+4$	$t+5$	$t+6$	$t+7$	$t+8$	sum	$\frac{\text{sum}}{\text{value on } t+2}$
13-Oct-2011	E	A	0	-3	1	-1	-2	1	-4	0.00
23-Nov-2011	A	E	-8	-21	-26	-16	-3	-3	-76	-0.01
12-Jan-2012	E	A	-1	0	-3	-3	-1	-2	-12	0.00
30-Mar-2012	A	E	-18	-87	-9	1	-1	3	-110	-0.01
20-Jun-2012	E	A	137	130	99	20	2	-4	385	0.03
29-Jun-2012	A	E	-209	-177	-10	-6	-2	-7	-412	-0.03
20-Jul-2012	E	A	211	109	61	4	-3	3	384	0.03
30-Aug-2012	A	E	-217	-193	-153	-22	-12	-8	-605	-0.05
3-Jan-2013	E	A	289	273	191	50	26	21	850	0.06
4-Apr-2013	A	E	-223	-245	-244	-208	-160	-142	-1223	-0.08
18-Jul-2013	E	A	378	357	281	249	95	22	1382	0.11
19-Aug-2013	A	E	-304	-313	-269	-248	-207	-63	-1404	-0.10
9-Sep-2013	E	A	405	356	272	116	10	2	1162	0.09
27-Jan-2014	A	E	-353	-348	-273	-242	-134	-22	-1372	-0.09
7-Mar-2014	E	E+C	-13	-29	-16	-6	-9	-17	-89	-0.01
4-Aug-2014	E+C	E	-52	-30	-27	-24	-7	-9	-148	-0.01
20-Aug-2014	E	E+A	375	133	48	16	18	13	603	0.04
3-Nov-2014	E+A	A	345	291	61	37	108	39	881	0.05
16-Dec-2014	A	E	-397	-394	-305	-232	-67	-6	-1400	-0.08
13-Feb-2015	E	E+A	247	206	94	31	24	11	613	0.04
19-Mar-2015	E+A	A	311	355	117	85	14	6	888	0.05
14-May-2015	A	E+A	-301	-170	-61	-50	-4	2	-584	-0.03
9-Jul-2015	E+A	C+E	-492	-292	-108	-46	4	-2	-936	-0.05
25-Aug-2015	C+E	E	-239	-210	-180	-45	7	-14	-681	-0.04
14-Oct-2015	E	E+C	8	-22	-9	16	1	-3	-9	0.00
27-Oct-2015	E+C	E	-29	-30	-15	-12	-4	-2	-93	-0.01
17-Dec-2015	E	E+A	293	363	87	87	201	342	1372	0.08
23-Dec-2015	E+A	A	201	342	140	31	14	5	733	0.04
7-Jan-2016	A	E+A	-259	-235	-52	-44	-9	-7	-605	-0.03
18-Jan-2016	E+A	E	-282	-360	-100	-115	-109	-60	-1026	-0.06
23-Feb-2016	E	E+C	-14	-24	-19	-20	-11	-19	-107	-0.01
2-May-2016	E+C	E	-34	-58	-29	-23	-9	-6	-158	-0.01
7-Sep-2016	E	E+C	-5	-11	-12	-14	-3	-57	-101	-0.01
14-Sep-2016	E+C	E	-57	-32	-11	-19	-8	-7	-135	-0.01

						Portfolio E				
Date	From	To	$t+3$	$t+4$	$t+5$	$t+6$	$t+7$	$t+8$	sum	$\frac{\text{sum}}{\text{value on } t+2}$
13-Oct-2011	E	A	3	8	5	7	10	2	35	0.01
23-Nov-2011	A	E	18	36	45	30	9	8	146	0.03
12-Jan-2012	E	A	7	3	8	9	7	7	41	0.01
30-Mar-2012	A	E	22	128	10	-3	-1	-5	150	0.03
20-Jun-2012	E	A	-166	-161	-106	-17	5	14	-430	-0.06
29-Jun-2012	A	E	299	253	19	13	9	13	606	0.09
20-Jul-2012	E	A	-281	-133	-64	3	10	5	-460	-0.06
30-Aug-2012	A	E	310	285	240	40	27	18	920	0.13
3-Jan-2013	E	A	-379	-366	-270	-71	-38	-32	-1157	-0.13
4-Apr-2013	A	E	303	353	364	313	250	248	1831	0.25
18-Jul-2013	E	A	-474	-475	-360	-335	-127	-27	-1799	-0.17
19-Aug-2013	A	E	402	424	376	365	330	128	2025	0.23
9-Sep-2013	E	A	-508	-474	-367	-147	-8	0	-1504	-0.13
27-Jan-2014	A	E	469	466	385	352	209	38	1918	0.19
7-Mar-2014	E	E+C	-253	-252	-90	-7	3	21	-579	-0.05
4-Aug-2014	E+C	E	422	221	107	70	33	19	871	0.07
20-Aug-2014	E	E+A	-500	-181	-57	-20	-30	-19	-807	-0.06
3-Nov-2014	E+A	A	-458	-395	-80	-51	-155	-53	-1191	-0.09
16-Dec-2014	A	E	578	561	452	383	111	9	2094	0.17
13-Feb-2015	E	E+A	-366	-295	-152	-52	-34	-20	-919	-0.06
19-Mar-2015	E+A	A	-429	-479	-154	-119	-19	-8	-1208	-0.09
14-May-2015	A	E+A	419	234	86	66	5	0	809	0.07
9-Jul-2015	E+A	C+E	304	212	83	10	-14	-1	594	0.05
25-Aug-2015	C+E	E	661	642	510	168	1	42	2025	0.14
14-Oct-2015	E	E+C	-535	-278	-88	-90	-22	-4	-1016	-0.06
27-Oct-2015	E+C	E	529	359	103	60	12	3	1065	0.07
17-Dec-2015	E	E+A	-422	-496	-108	-108	-272	-450	-1857	-0.11
23-Dec-2015	E+A	A	-272	-450	-185	-38	-17	-8	-970	-0.06
7-Jan-2016	A	E+A	384	342	75	70	22	15	907	0.06
18-Jan-2016	E+A	E	436	553	157	194	186	110	1635	0.11
23-Feb-2016	E	E+C	-348	-264	-50	-16	19	14	-644	-0.03
2-May-2016	E+C	E	404	523	114	91	23	17	1173	0.06
7-Sep-2016	E	E+C	-392	-408	-95	-61	-2	489	-470	-0.02
14-Sep-2016	E+C	E	489	368	105	100	42	23	1127	0.05

Table 1.5 provides other metrics for the amounts of funds transferred. For each recommendation, we compare the cumulative net transfers to portfolio E on days $t+3$ to $t+8$ with measures of the "size" of the Chilean economy. These net portfolio reallocations can exceed 5% of GDP for the associated quarter; and they sometimes exceed the total value of *all* trade of domestic stocks on the Santiago Stock Exchange in the month of the recommendation.

Table 1.5: Net flow to portfolio E ($t+3$ to $t+8$) as percentage of: the value of portfolio E in $t+2$, the domestic GDP in the quarter of the recommendation, and the total value of transactions in domestic stocks in the Santiago stock exchange in the month of the announcement.

Recom. #	Direction of Recom.	Value at $t+2$ of Portfolio E	of Quarterly GDP shifted	monthly total dollar trading volume in domestic stocks
5	A to E	2.894	0.48	7.52
6	E to A	-6.020	-1.36	-15.13
7	A to E	8.950	1.91	21.35
8	E to A	-6.133	-1.47	-22.75
9	A to E	12.578	2.93	57.34
10	E to A	-12.789	-3.48	-46.11
11	A to E	25.438	5.48	99.24
12	E to A	-17.081	-5.33	-105.40
13	A to E	22.760	6.00	137.09
14	E to A	-13.257	-4.45	-83.67
15	A to E	18.671	5.35	83.14
16	E to E+C	-4.647	-1.62	-40.21
17	E+C to E	6.942	2.41	43.93
18	E to E+A	-5.856	-2.23	-40.68
19	E+A to A	-8.882	-3.01	-38.71
20	A to E	16.784	5.29	169.24
21	E to E+A	-6.290	-2.38	-75.35
22	E+A to A	-8.849	-3.12	-78.12
23	A to E+A	6.734	2.10	56.24
24	E+A to E+C	4.568	1.54	61.22
25	C+E to E	14.438	5.24	198.62
26	E to E+C	-6.231	-2.44	-99.69
27	E+C to E	6.971	2.56	104.55
28	E to E+A	-11.424	-4.46	-205.00
29	E+A to A	-6.413	-2.33	-107.08
30	A to E+A	6.455	2.20	71.39
31	E+A to E	10.875	3.97	128.68
32	E to E+C	-3.463	-1.56	-75.26
33	E+C to E	6.298	2.88	92.27
34	E to E+C	-1.955	-1.15	-37.72
35	E+C to E	4.859	2.76	90.41

We next estimate flow regressions of the form:

$$y_\tau = \alpha + \sum_{s=-3}^{10} \beta_s \delta_{E\tau}(s) + \epsilon_\tau,$$

where y_τ are daily flows on date τ (measured in percentage terms) as a function of

the indicator function:

$$\delta_{E\tau}(s) = \begin{cases} \rho & \text{if } \tau - s \text{ is the first trading day after a recommendation to sell fraction } \rho \\ & \text{of portfolio E,} \\ -\rho & \text{if } \tau - s \text{ is the first trading day after a recommendation to buy fraction } \rho \\ & \text{of portfolio E,} \\ 0 & \text{otherwise.} \end{cases}$$

Thus, $\delta_{E\tau}(0) = 1$ if on the previous day $\tau - 1$, H&L recommended shifting all holdings from portfolio E to portfolio A.

The left panel of Table 1.6 presents results. The coefficients on the indicator functions are very large and highly statistically significant on dates $t + 3$ to $t + 9$. The astonishing fit (adjusted R^2 of 0.64 for portfolio E) reveals that H&L's advice is *the* primary driver of fluctuations in transfers of funds between portfolios.[14] The findings indicate that (a) investors respond quickly and massively to recommendations; (b) there is no leakage of information about a recommendation prior to its announcement—investors do not systematically shift in or out of a portfolio before a new recommendation; and (c) investors finish responding to a recommendation within a week.

To see whether investor responses differ according to the direction of a recommendation (i.e., to or from A) we estimate the following regression:

$$y_\tau = \alpha + \sum_{s=2}^{10} \beta_s \delta_{E\tau}(s) + \sum_{s=2}^{10} \phi_s \delta_{A\tau}(s) + \epsilon_\tau,$$

where y_τ are daily flows on date τ (measured in percentage terms) as a function of:

$$\delta_{E\tau}(s) = \begin{cases} \rho & \text{if } \tau - s \text{ is the first trading day after a recommendation to sell fraction } \rho \\ & \text{of portfolio E,} \\ 0 & \text{otherwise.} \end{cases}$$

and

$$\delta_{A\tau}(s) = \begin{cases} \rho & \text{if } \tau - s \text{ is the first trading day after a recommendation to sell fraction } \rho \\ & \text{of portfolio A,} \\ 0 & \text{otherwise.} \end{cases}$$

[14]Empirical identification of the causal impact of H&L's recommendations is clean: (a) Figures 1 and 2 show that reallocations and recommendations coincide only after investors started showing interest in H&L as measured by the Google indicators; (b) the fact that only tiny numbers of investors switched portfolios in the direction of H&L's first few recommendations reveals that a common force does not drive both recommendations and reallocations. Indeed, H&L only had 54 paid followers for its first four recommendations.

Table 1.6: Recommendations and portfolio transfers. OLS regression of $y_\tau = \alpha + \sum_{s=-3}^{10} \beta_s \delta_{E\tau}(s) + \epsilon_\tau$ and $y_\tau = \alpha + \sum_{s=2}^{10} \beta_s \delta_{E\tau}(s) + \sum_{s=2}^{10} \phi_s \delta_{A\tau}(s) + \epsilon_\tau$ using daily data for the period 12Oct2011–30Sep2016, where y_τ is the percentage net flow to portfolio X on day τ: y_τ is the value of the inflow minus the value of the outflow to portfolio X on τ divided by the value of the portfolio on day $\tau - 1$.

	A	B	C	D	E	A	B	C	D	E
$\delta_E(-3)$	0.007	-0.002	-0.002	0.007	0.036					
	(0.036)	(0.013)	(0.014)	(0.004)	(0.074)					
$\delta_E(-2)$	0.010	0.002	-0.027	0.006*	0.049					
	(0.029)	(0.012)	(0.025)	(0.003)	(0.047)					
$\delta_E(-1)$	0.061	0.018	-0.031	0.001	-0.018					
	(0.044)	(0.018)	(0.026)	(0.003)	(0.065)					
$\delta_E(0)$	0.062	0.016	-0.015*	-0.002	-0.034					
	(0.046)	(0.017)	(0.008)	(0.002)	(0.070)					
$\delta_E(1)$	0.030	-0.000	-0.011*	-0.002	0.013					
	(0.022)	(0.010)	(0.007)	(0.003)	(0.073)					
$\delta_E(2)$	0.062***	0.020**	-0.003	-0.002	-0.096*	0.061**	0.017*	-0.003	-0.003	-0.039
	(0.022)	(0.009)	(0.003)	(0.002)	(0.053)	(0.024)	(0.010)	(0.005)	(0.002)	(0.048)
$\delta_E(3)$	1.629***	0.469***	0.278***	-0.014**	-3.683***	1.749***	0.482***	0.260**	-0.031***	-3.464***
	(0.224)	(0.065)	(0.082)	(0.007)	(0.410)	(0.370)	(0.101)	(0.117)	(0.010)	(0.596)
$\delta_E(4)$	1.520***	0.472***	0.240***	-0.018**	-3.458***	1.501***	0.470***	0.219**	-0.038***	-3.143***
	(0.206)	(0.064)	(0.068)	(0.008)	(0.360)	(0.342)	(0.107)	(0.097)	(0.011)	(0.588)
$\delta_E(5)$	0.951***	0.303***	0.095***	-0.007	-2.051***	0.882***	0.272***	0.078**	-0.030***	-1.835***
	(0.167)	(0.055)	(0.025)	(0.006)	(0.330)	(0.234)	(0.076)	(0.032)	(0.007)	(0.443)
$\delta_E(6)$	0.602***	0.197***	0.063***	0.000	-1.263***	0.439**	0.136**	0.048**	-0.017***	-0.931***
	(0.147)	(0.049)	(0.015)	(0.005)	(0.294)	(0.173)	(0.055)	(0.019)	(0.004)	(0.358)
$\delta_E(7)$	0.325***	0.113***	0.028***	-0.004	-0.648***	0.198***	0.069***	0.012	-0.011***	-0.374**
	(0.099)	(0.034)	(0.010)	(0.004)	(0.208)	(0.074)	(0.027)	(0.010)	(0.003)	(0.167)
$\delta_E(8)$	0.175***	0.064**	-0.005	0.002	-0.338**	0.165**	0.055**	-0.039	-0.009**	-0.227**
	(0.062)	(0.026)	(0.023)	(0.004)	(0.145)	(0.074)	(0.025)	(0.048)	(0.004)	(0.107)
$\delta_E(9)$	0.097***	0.040***	-0.006	-0.001	-0.191***	0.122***	0.043***	-0.030	-0.005	-0.181***
	(0.032)	(0.014)	(0.017)	(0.003)	(0.069)	(0.042)	(0.015)	(0.037)	(0.003)	(0.068)
$\delta_E(10)$	0.104***	0.038***	0.015	-0.004	-0.162**	0.093**	0.034**	0.004	-0.007*	-0.089
	(0.037)	(0.015)	(0.011)	(0.003)	(0.073)	(0.043)	(0.015)	(0.014)	(0.004)	(0.117)
$\delta_A(2)$						-0.076*	-0.028*	0.004	0.001	0.173*
						(0.040)	(0.016)	(0.007)	(0.004)	(0.095)
$\delta_A(3)$						-1.643***	-0.503***	-0.108	0.005	3.617***
						(0.291)	(0.095)	(0.081)	(0.010)	(0.549)
$\delta_A(4)$						-1.624***	-0.499***	-0.142**	0.007	3.614***
						(0.249)	(0.077)	(0.071)	(0.013)	(0.395)
$\delta_A(5)$						-1.032***	-0.330***	-0.063***	-0.006	2.166***
						(0.247)	(0.081)	(0.020)	(0.009)	(0.494)
$\delta_A(6)$						-0.781***	-0.260***	-0.058***	-0.012	1.577***
						(0.236)	(0.081)	(0.019)	(0.008)	(0.465)
$\delta_A(7)$						-0.461***	-0.159***	-0.042***	-0.003	0.948***
						(0.177)	(0.061)	(0.016)	(0.007)	(0.368)
$\delta_A(8)$						-0.185*	-0.075	-0.024	-0.012**	0.459*
						(0.103)	(0.047)	(0.015)	(0.006)	(0.274)
$\delta_A(9)$						-0.069	-0.037	-0.014*	-0.005**	0.207
						(0.053)	(0.026)	(0.009)	(0.003)	(0.126)
$\delta_A(10)$						-0.118*	-0.046*	-0.014*	0.000	0.246**
						(0.066)	(0.026)	(0.008)	(0.004)	(0.107)
Obs.	1,239	1,239	1,239	1,239	1,239	1,239	1,239	1,239	1,239	1,239
R^2	0.545	0.481	0.161	0.030	0.649	0.566	0.500	0.087	0.080	0.603
Adj. R^2	0.539	0.475	0.151	0.018	0.645	0.560	0.493	0.073	0.067	0.597

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1
Intercept not reported

The right panel of Table 1.6 shows that investor responses do not differ materially according to the direction of a recommendation: a recommendation of portfolio A leads to transfers into A and out of E that roughly equal transfers from portfolio A and into E following a recommendation of E. Table 1.6 also reveals clear but muted responses of investors in portfolios B and C to recommendations to switch to/from E: the funds shifted in and out of portfolios B and C are less than 25% of those for portfolio A. These "echo" shifts reflect that some investors are too old to invest in portfolio A, making portfolios B or C their closest feasible alternative.

Did the advent of H&L change in other ways how investors allocated savings? Chevalier and Ellison (1997) and Sirri and Tufano (1998) have established that higher past returns of a mutual fund increase cash flows into the fund. We now modify Sirri and Tufano's approach to study the relationship between measures of

past performance of portfolio A and net flows to portfolios A and E to determine whether this relationship changed once investors began following H&L's advice. Because there are no daily data on portfolio flows prior to October 2011, we use monthly data for the period October 2002–September 2016. Table 1.7 presents OLS estimates of the following equation:

$$\text{Flow}_t = \alpha + \left(\sum_{s=1}^{3} \beta_{1,s} \text{Return A}_{t-s} + \sum_{s=1}^{3} \beta_{2,s} \text{Risk A}_{t-s} + \beta_3 \log \text{TA}_{t-1} \right) \times (1 - d_t)$$
$$+ \left(\sum_{s=1}^{3} \beta_{4,s} \text{Return A}_{t-s} + \sum_{s=1}^{3} \beta_{5,s} \text{Risk A}_{t-s} + \beta_6 \log \text{TA}_{t-1} \right) \times d_t + \beta_7 d_t + \epsilon_t.$$

Flow_t is the net flow to portfolio X in month t as a percent of the total assets in X on the last day of month $t-1$; Return A_{t-s} is the monthly return on portfolio A computed as the log difference using the price on the last day of month $t-s$ and the price on the last day of month $t-(s+1)$; Risk A_{t-s} is the standard deviation of the daily returns of portfolio A in month $t-s$; and TA_{t-1} is the total assets of portfolio A on the last day of month $t-1$. We consider two formulations for the dummy variable d_t indicating H&L's presence in the market: one where $d_t = 1$ once H&L enters the market in October 2011, so $d_t = 0$ before October 2011; and one where $d_t = 1$ starting in April 2012, when substantial numbers of investors began shifting investments in line with H&L's recommendation.

Table 1.7 shows how H&L's entry changed investor behavior. Prior to H&L, and consistent with Chevalier and Ellison (1997) and Sirri and Tufano (1998), higher lagged monthly returns of portfolio A led investors to shift funds into A and out of E. After H&L's entry, investments no longer vary with the long-term performance of portfolio A—investors only rely on H&L.

1.2.2 Returns

Having established how H&L's recommendations affected the flow of investors and money in and out of portfolios, we now investigate asset returns around the recommendation announcements. This lets us (a) uncover what drives H&L's recommendations to switch portfolios; (b) derive the informational consequences (announcement effects) of recommendations for different assets; and (c) probe the impacts of the portfolio transfers on asset returns.

To do this, we estimate regressions of the form:

$$y_\tau = \alpha + \sum_{s=-2}^{0} \beta_s \delta_{E\tau}(s) + \beta_1 \delta_{E\tau}(1:2) + \beta_2 \delta_{E\tau}(3:7) + \beta_3 \delta_{E\tau}(8:10) + \epsilon_\tau,$$

Table 1.7: The effect of past performance on monthly cash flow to portfolios A and E, before and after the emergence of H&L.

	$d_t = 1$ starting October 2011		$d_t = 1$ starting April 2012	
	Port. A	Port. E	Port. A	Port. E
		Before H&L		
Return A_{t-1}	0.212*	-1.456***	0.223**	-1.437***
	(0.110)	(0.342)	(0.099)	(0.322)
Return A_{t-2}	0.210***	-0.882***	0.204***	-0.876***
	(0.077)	(0.277)	(0.070)	(0.261)
Return A_{t-3}	0.176*	-0.288	0.163*	-0.282
	(0.098)	(0.192)	(0.089)	(0.178)
Risk A_{t-1}	1.057	-9.436**	1.117	-8.940**
	(1.417)	(4.608)	(1.309)	(4.393)
Risk A_{t-2}	1.311	-3.457	1.133	-3.744
	(1.243)	(4.665)	(1.040)	(4.280)
Risk A_{t-3}	-0.697	2.237	-0.695	2.373
	(0.851)	(2.702)	(0.782)	(2.469)
$\log TA_{t-1}$	-0.022***	0.016**	-0.022***	0.015**
	(0.007)	(0.007)	(0.006)	(0.007)
		After H&L		
Return A_{t-1}	-0.097	0.022	-0.063	0.020
	(0.219)	(0.406)	(0.236)	(0.468)
Return A_{t-2}	0.131	-0.264	0.150	-0.253
	(0.180)	(0.365)	(0.236)	(0.502)
Return A_{t-3}	-0.178	0.319	-0.190	0.410
	(0.163)	(0.331)	(0.217)	(0.460)
Risk A_{t-1}	-2.180	1.132	-2.534	1.176
	(4.286)	(9.147)	(5.067)	(11.381)
Risk A_{t-2}	-3.968	10.778	-6.495	15.470
	(3.551)	(6.997)	(5.182)	(11.411)
Risk A_{t-3}	4.346*	-9.525*	6.813**	-11.427
	(2.350)	(4.962)	(3.220)	(7.191)
$\log TA_{t-1}$	-0.047	0.066	-0.040	0.040
	(0.049)	(0.097)	(0.059)	(0.118)
d_t	0.770	-1.612	0.573	-0.864
	(1.488)	(2.935)	(1.782)	(3.546)
Constant	0.641***	-0.376*	0.638***	-0.354*
	(0.192)	(0.191)	(0.189)	(0.182)
Obs.	164	164	164	164
R^2	0.399	0.359	0.408	0.363
Adj. R^2	0.338	0.294	0.348	0.298

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

where y_τ is the daily return[15] on day τ of a selected asset (social security portfolios A and E,[16] and IPSA stock market index).[17] $\delta_{E\tau}(0)$ is an indicator function that takes on the value 1 if a recommendation was made on the previous day $(\tau - 1)$ to sell portfolio E, it takes on the value -1 if a recommendation was made on the previous day to buy portfolio E, and it is zero otherwise. $\delta_{E\tau}(1:2)$ is the analogous indicator function for recommendations made either one or two days earlier (i.e., on days $\tau - 2$ or $\tau - 3$). The indicator function $\delta_{E\tau}(3:7)$ captures days where portfolio transfers are high following recommendations; and the indicator function $\delta_{E\tau}(8:10)$ captures days where portfolio transfers have largely returned to normal.

To control for market returns, we also augment regressions by adding the return of a foreign stock market index as a regressor. We use the return on the MSCI ACWI and MSCI World indexes as controls when using portfolio returns as a de-

[15]Returns are defined as: $r_\tau = 100\,(\log(p_\tau) - \log(p_{\tau-1}))$.

[16]The website of the superintendency of pensions provides daily data on official prices for all portfolios of all AFPs. These official prices reflect regulations that specify the prices used to value each asset (e.g. closing price for foreign equity, or weighted average price for domestic stocks). We use the weighted average price, lagged one day because the price of a portfolio on day τ reflects the day $\tau - 1$ prices of the underlying assets.

[17] IPSA is an index of the 40 stocks with the highest annual volume from the set of stocks with a market capitalization that exceed USD 200 MM and a free-float of at least 5%. The index is market capitalization weighted and free float adjusted, and includes dividends.

pendent variable.[18] When using the return of a domestic stock market index as a dependent variable, we use returns on the MSCI Emerging Markets and MSCI Emerging Markets Latin America indexes.[19][20]

Table 1.8: Recommendations and asset returns. OLS regression of $y_\tau = \alpha + \sum_{s=-2}^{0} \beta_s \delta_{E\tau}(s) + \beta_1 \delta_{E\tau}(1:2) + \beta_2 \delta_{E\tau}(3:7) + \beta_3 \delta_{E\tau}(8:10) + \epsilon_\tau$ using daily data for the period 3Jan2011–30Sep2016, where y_τ is the daily return (in %) of a social security portfolio, or stock market index.

	Social Security Portfolios				Stock Market		
	A	A	A	E	IPSA	IPSA	IPSA
$\delta_E(-2)$	0.479***	0.225***	0.250***	-0.00379	0.750***	0.308*	0.352**
	(0.0809)	(0.0741)	(0.0736)	(0.0184)	(0.177)	(0.174)	(0.162)
$\delta_E(-1)$	0.642***	0.184**	0.204***	0.000443	1.024***	0.467***	0.432**
	(0.106)	(0.0758)	(0.0756)	(0.0249)	(0.186)	(0.179)	(0.172)
$\delta_E(0)$	0.251*	0.176**	0.199**	-0.0110	0.541**	0.264*	0.351**
	(0.143)	(0.0810)	(0.0847)	(0.0254)	(0.224)	(0.146)	(0.152)
$\delta_E(1:2)$	-0.0635	-0.00894	-0.0147	-0.00210	-0.0220	0.0674	0.103
	(0.0767)	(0.0508)	(0.0525)	(0.0175)	(0.137)	(0.119)	(0.116)
$\delta_E(3:7)$	0.233***	0.112***	0.116***	0.00642	0.280**	0.142	0.146*
	(0.0699)	(0.0370)	(0.0382)	(0.00863)	(0.120)	(0.0869)	(0.0782)
$\delta_E(8:10)$	-0.0519	-0.0388	-0.0353	-0.0384*	-0.252	-0.267*	-0.251**
	(0.0716)	(0.0393)	(0.0391)	(0.0210)	(0.159)	(0.150)	(0.128)
ACWI		0.419***					
		(0.0206)					
World			0.403***				
			(0.0209)				
Emerging Markets						0.488***	
						(0.0312)	
Latin America							0.355***
							(0.0207)
Observations	1,436	1,436	1,436	1,436	1,437	1,437	1,437
R^2	0.048	0.501	0.484	0.006	0.045	0.356	0.385
Adjusted R^2	0.044	0.499	0.482	0.002	0.041	0.353	0.382

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1
Intercept not reported

What drives H&L's advice? Table 1.8 reveals that H&L's advice overwhelmingly reflects the *immediate* past performance of the Chilean stock market. On the two trading days *prior* to a recommendation to switch into portfolio A and out of E, on average, the Chilean stock market rose by 1.8%. Roughly the opposite occurs prior to recommendations to switch out of portfolio A and into E. That is, following good returns on the Chilean stock market, H&L recommends that investors hold the risky portfolio A, and following bad returns on the market, H&L recommends that they hold the safe portfolio E. This suggests that H&L employs a simple, very short-term, momentum strategy.

[18] The ACWI index captures large and mid-cap representation in 23 developed markets and 24 emerging markets countries. The World index captures large and mid cap representation in 23 developed markets countries.

[19] The Emerging Markets index captures large and mid-cap representation in 24 emerging markets countries. The Emerging Markets Latin America index captures large and mid-cap representation for five emerging markets countries in Latin America.

[20] Results similar to those for portfolio A obtain when using returns of portfolios B and C as dependent variables. The magnitude of the effects fall due to the lesser risk exposure of these portfolios. Findings similar to those for the IPSA index obtain for the IGPA index of all stocks on the Santiago Exchange with an annual volume above UF 10,000 (US$400,000-450,000), free float of at least 5% and a market presence of at least 5%, and for the INTER-10 index, which consists of 10 stocks selected from the IPSA, listed in foreign markets through ADRs, with the highest annual volume or if we use indexes without dividends. We also considered the US$ over CLP exchange rate and indexed Central Bank bonds.

Table 1.9: Cumulative returns of the ACWI and VIX indexes.

Type of recommendation	ACWI	VIX
Increase risk exposure		
2	0.07	-0.19
4	0.00	-0.05
6	0.03	-0.18
8	0.03	-0.17
10	0.02	-0.19
12	0.02	-0.03
14	0.02	-0.06
16	0.01	0.01
18	0.02	-0.15
19	0.02	-0.13
21	0.01	-0.09
22	0.03	-0.19
26	0.01	-0.09
28	0.00	-0.09
29	0.00	-0.23
32	0.04	-0.27
34	0.01	-0.07
Reducing risk exposure		
1	0.00	0.19
3	-0.06	0.02
5	0.00	-0.01
7	-0.01	-0.02
9	0.00	0.12
11	0.00	0.11
13	-0.01	0.07
15	-0.02	0.37
17	-0.02	0.29
20	-0.04	0.36
23	0.01	-0.10
24	-0.02	0.08
25	-0.10	1.14
27	0.01	0.02
30	-0.04	0.25
31	-0.03	0.00
33	-0.01	0.17
35	-0.03	0.40

To reinforce this conclusion, we show that the cumulative return (the sum of daily returns) of the ACWI and VIX indexes in the week before an announcement predict the nature of the recommendation.[21] Table 1.9 divides recommendations according to whether the direction of the recommendation represents an increase or a reduction in risk exposure. A momentum strategy suggests that preceding a recommendation to increase risk exposure, we should see positive cumulative returns on the ACWI index, and negative returns on the VIX index; and prior to a recommendation to reduce risk exposure, the opposite should obtain. This is precisely what we find. Indeed, prior to *every* recommendation to increase risk, the five day cumulative return on the ACWI index is positive, and the analogous return on the VIX index is negative. The opposite happens for the vast bulk of recommendations to reduce risk exposure.[22]

Putting these results together with those in Table 1.7 proves revealing: it is not

[21] The CBOE Volatility Index (VIX) is a measure of market expectations of near-term volatility conveyed by S&P 500 stock index option prices.

[22] These results are robust to choosing different short term horizons (e.g. four, six or seven days).

that investors no longer base investments on past performance; rather, via the model that H&L uses to formulate recommendations, investors have switched from basing investments on intermediate-term historical performance to basing investments on very recent market performance. Moreover, many more investors now implicitly follow this momentum strategy.

Consequences of H&L's advice. Table 1.8 also reveals positive announcement effects on the day after a recommendation of portfolio A. Most notably, there are abnormal returns of half a percent on the Chilean stock market, and smaller announcement effects for portfolio A that remain significant when we control for foreign market returns. In addition, after a recommendation of risky portfolio A on day $t-1$ there are positive abnormal returns on both the stock market and portfolio A on days $t+3$ to $t+7$ where pension transfers are high, followed by slight reversals. Similar return patterns emerge for portfolios B, C and D, where the magnitudes decline as the riskiness of the portfolio declines, and there are no systematic return patterns for portfolio E, which is comprised of very liquid, information-insensitive securities.

At first blush, these results suggest that reinvestments by AFPs in response to the portfolio reallocations on days $t+3$ to $t+7$ had price impacts. It *looks* as if AFPs trades in the stock market induced by the re-allocations of pension savings moved the market—it *looks* as if the market was not sufficiently liquid to absorb the re-allocations. To investigate, we explore the impact of portfolio reallocations on domestic stock market trading volume, regressing log of total daily trading volume in millions of CLP on indicator functions that take on the value of one s trading days after a new recommendation, for $s = -2, -1, 0, 1...10$. Table 1.10 presents those results. Surprisingly, Figure 1.3 and Table 1.10 reveal that there is *no pass through from the massive portfolio re-allocations to stock market trading volume.* Trades by AFPs induced by portfolio re-allocations do *not* drive the excess returns on dates $t+3$ through $t+7$.

Figure 1.3: Trading volume of IPSA and H&L's announcements

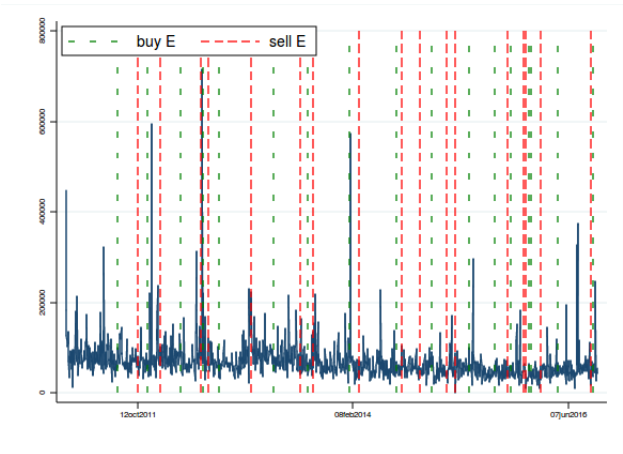


Table 1.10: Trading volume around new recommendations. OLS regression of $y_\tau = \alpha + \sum_{s=-2}^{10} \beta_s \delta_{E\tau}(s) + \epsilon_\tau$ using daily data from 3Jan2011–30sep2016, where y_τ is daily volume of a stock market index, and $\delta_{E\tau}(s)$ is an indicator function that equals 1 s days after a new recommendation.

	IPSA	IGPA	INTER-10
$\delta_E(-2)$	-0.040	-0.061	0.038
	(0.079)	(0.084)	(0.096)
$\delta_E(-1)$	0.057	0.029	0.127
	(0.065)	(0.071)	(0.079)
$\delta_E(0)$	-0.005	-0.023	0.029
	(0.077)	(0.079)	(0.077)
$\delta_E(1)$	-0.267	-0.080	-0.047
	(0.202)	(0.093)	(0.111)
$\delta_E(2)$	-0.035	-0.051	-0.000
	(0.115)	(0.113)	(0.141)
$\delta_E(3)$	-0.062	-0.063	-0.099
	(0.063)	(0.068)	(0.084)
$\delta_E(4)$	-0.040	0.007	-0.037
	(0.071)	(0.084)	(0.100)
$\delta_E(5)$	-0.100	-0.115	-0.030
	(0.091)	(0.097)	(0.114)
$\delta_E(6)$	-0.079	-0.103	-0.065
	(0.079)	(0.078)	(0.100)
$\delta_E(7)$	-0.037	-0.030	-0.014
	(0.076)	(0.083)	(0.083)
$\delta_E(8)$	0.009	-0.034	0.085
	(0.068)	(0.069)	(0.070)
$\delta_E(9)$	-0.024	-0.043	-0.035
	(0.088)	(0.089)	(0.110)
$\delta_E(10)$	0.135	0.088	0.147
	(0.114)	(0.117)	(0.132)
Obs.	1,436	1,436	1,436
R^2	0.012	0.005	0.006
Adj. R^2	0.003	-0.004	-0.003

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1
Intercept not reported

The question becomes: what does underlie these return patterns? Suggestive insights come from decomposing the sample into early recommendations (recommendations one to six) and later recommendations (recommendations seven onwards)—as individual investors only began switching between pension portfolios in large numbers beginning with the seventh recommendation (see Figure 1.1). We estimate regressions of the form:

$$\begin{aligned} y_\tau &= \alpha + \sum_{s=-2}^{0} \beta_s \delta_{E\tau}(s) + \beta_1 \delta_{E\tau}(1:2) + \beta_2 \delta_{E\tau}(3:7) + \beta_3 \delta_{E\tau}(8:10) \\ &\quad + \sum_{s=-2}^{0} \phi_s \gamma_{E\tau}(s) + \phi_1 \gamma_{E\tau}(1:2) + \phi_2 \gamma_{E\tau}(3:7) + \phi_3 \gamma_{E\tau}(8:10) + \epsilon_\tau. \end{aligned}$$

The δ_E indicators are active (taking on values of $\pm\rho$) only for the first six recommendations, and the γ_E indicators become active starting with the seventh recommendation, and the notation is as before. Table 1.11 underscores that portfolio reallocations do not underlie these excess returns. For early recommendations that almost no investors acted on, there are large abnormal returns on days $t+3$ to $t+7$ of about 0.7-0.8% on stock market indexes and 0.5% on portfolio A. Thus, initially investors would have benefited from following H&L's advice. However, once large numbers of investors began to follow H&L's recommendations, the excess returns in the stock market vanish: portfolio reallocations and market illiquidity do *not* drive

the observed return patterns.[23]

Table 1.11: Recommendations and asset returns. OLS regression of $y_\tau = \alpha + \sum_{s=-2}^{0} \beta_s \delta_{E\tau}(s) + \beta_1 \delta_{E\tau}(1:2) + \beta_2 \delta_{E\tau}(3:7) + \beta_3 \delta_{E\tau}(8:10) + \sum_{s=-2}^{0} \phi_s \gamma_{E\tau}(s) + \phi_1 \gamma_{E\tau}(1:2) + \phi_2 \gamma_{E\tau}(3:7) + \phi_3 \gamma_{E\tau}(8:10) + \epsilon_\tau$ using daily data for the period 3Jan2011–30Sep2016, where y_τ is the daily return (in %) of a social security portfolio, stock market index, exchange rate (foreign currency over CLP), or government bond.

	Social Security Portfolios				Stock Market			Placebo
	A	A	A	E	IPSA	IPSA	IPSA	S&P500
$\delta_E(-2)$	0.678***	0.154	0.185	-0.027	1.160***	0.453	0.561	0.645**
	(0.115)	(0.139)	(0.134)	(0.035)	(0.333)	(0.414)	(0.364)	(0.258)
$\delta_E(-1)$	0.398**	0.056	0.062	-0.009	1.035***	0.688*	0.587*	0.768***
	(0.166)	(0.105)	(0.101)	(0.050)	(0.364)	(0.358)	(0.337)	(0.280)
$\delta_E(0)$	0.312	0.151	0.174	0.013	0.531	0.210	0.259	0.326
	(0.200)	(0.104)	(0.113)	(0.055)	(0.443)	(0.268)	(0.268)	(0.371)
$\delta_E(1:2)$	-0.059	-0.019	-0.030	0.006	-0.016	0.107	0.052	-0.318
	(0.113)	(0.073)	(0.079)	(0.049)	(0.203)	(0.171)	(0.194)	(0.384)
$\delta_E(3:7)$	0.475**	0.198***	0.208**	-0.012	0.755**	0.441**	0.459***	0.702**
	(0.193)	(0.076)	(0.081)	(0.020)	(0.317)	(0.215)	(0.177)	(0.354)
$\delta_E(8:10)$	0.099	0.067	0.089	-0.120*	-0.434	-0.641	-0.455	-0.020
	(0.193)	(0.096)	(0.094)	(0.063)	(0.437)	(0.424)	(0.351)	(0.526)
$\gamma_E(-2)$	0.410***	0.242***	0.266***	0.021	0.621***	0.255	0.291*	0.603**
	(0.100)	(0.091)	(0.092)	(0.019)	(0.204)	(0.186)	(0.176)	(0.252)
$\gamma_E(-1)$	0.758***	0.244***	0.269***	0.004	1.055***	0.402**	0.397**	1.130***
	(0.127)	(0.095)	(0.095)	(0.029)	(0.211)	(0.197)	(0.193)	(0.252)
$\gamma_E(0)$	0.242	0.192*	0.215**	-0.021	0.583**	0.314*	0.417**	0.080
	(0.174)	(0.104)	(0.108)	(0.026)	(0.246)	(0.174)	(0.185)	(0.235)
$\gamma_E(1:2)$	-0.055	0.002	-0.000	-0.011	-0.036	0.027	0.110	-0.275
	(0.099)	(0.066)	(0.067)	(0.014)	(0.179)	(0.157)	(0.145)	(0.175)
$\gamma_E(3:7)$	0.130**	0.076*	0.079*	0.013	0.072	0.009	0.011	0.148
	(0.055)	(0.042)	(0.042)	(0.009)	(0.100)	(0.083)	(0.081)	(0.101)
$\gamma_E(8:10)$	-0.115*	-0.082**	-0.086**	-0.005	-0.181	-0.117	-0.169	-0.162
	(0.061)	(0.038)	(0.038)	(0.011)	(0.134)	(0.113)	(0.107)	(0.134)
ACWI		0.417***						
		(0.021)						
World			0.401***					
			(0.021)					
Emerging Markets						0.487***		
						(0.030)		
Latin America							0.353***	
							(0.020)	
Observations	1,436	1,436	1,436	1,436	1,437	1,437	1,437	1,446
R^2	0.058	0.503	0.486	0.019	0.056	0.363	0.391	0.037
Adjusted R^2	0.050	0.498	0.481	0.011	0.048	0.357	0.385	0.029
	Testing equality of before and after coefficients (p-value)							
$\delta_E(-2) = \gamma_E(-2)$	0.079	0.594	0.614	0.225	0.167	0.660	0.502	0.906
$\delta_E(-1) = \gamma_E(-1)$	0.085	0.176	0.128	0.816	0.962	0.482	0.623	0.340
$\delta_E(0) = \gamma_E(0)$	0.790	0.783	0.790	0.570	0.918	0.744	0.628	0.577
$\delta_E(1:2) = \gamma_E(1:2)$	0.976	0.832	0.774	0.744	0.941	0.727	0.812	0.919
$\delta_E(3:7) = \gamma_E(3:7)$	0.086	0.160	0.160	0.250	0.040	0.062	0.022	0.135
$\delta_E(8:10) = \gamma_E(8:10)$	0.289	0.150	0.083	0.071	0.581	0.231	0.435	0.795

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1
Intercept not reported

Instead, following later recommendations, the market immediately responds with announcement effects on stock market indexes of 0.5–0.6%. Thus, it appears that at the outset H&L correctly anticipated the near-term performance of the risky portfolio A. In turn, this seems to have led to a shared belief among individual investors *and* the market that H&L's recommendations contain valuable information. As a result large numbers of individual investors started following H&L's advice. Paradoxically, though, *because* the market also comes to believe that H&L's advice has value, this information is immediately incorporated into portfolio valuations and stock prices. Unfortunately for followers of H&L, it takes time to transfer

[23] A recent working paper (Da, Larrain, Sialm, and Tessada (2016)) also looks at H&L's returns, positing that trades by AFPs might be driving the return patterns. The absence of increased trading volume in domestic stock markets in the days following recommendations indicates that this premise is incorrect.

investments between portfolios. Consequently, investors following H&L's advice fail to benefit from these announcement effects.

As a placebo test, the last column of Table 1.11 reports results using daily returns of the S&P 500 index as the dependent variable, as returns on this index are highly correlated with those on the domestic stock market and portfolio A. For early recommendations, we expect results similar to those for the domestic stock market indexes; but for later recommendations, we expect positive lagged returns and no announcement effects. This is exactly what we find.

1.2.3 Do followers gain?

We next investigate whether investors benefit from following H&L's advice. We employ two approaches:

1. We use H&L's payment records to compute returns for each follower starting with the first announcement he could follow through the last announcement before his subscription expired. We then determine whether the follower's return exceeded that from a buy-and-hold strategy for each feasible portfolio over that period of time.

2. We compare the cumulative returns starting at each of the first twenty announcements through September 30, 2016 from (a) following H&L's strategy vs. (b) investing and then holding any given portfolio.

Because some investors must select from less risky portfolios due to their age or gender, we divide followers into risk categories according to the riskiest portfolio that they can choose. If age or gender is missing, or if the age when the account was open is below eighteen, we drop the observation.[24]

Table 1.12 compares returns for each follower starting with the first announcement he could follow through the last announcement before his subscription expired with the analogous return from a buy-and-hold strategy for each (age and gender) feasible portfolio over that period of time. The top panel reveals the percentage of followers that obtained a higher return than they would have obtained from holding an alternative (age and gender) feasible portfolio presuming that followers request that investments be shifted as soon as a recommendation is received. Over 70% obtained lower returns that those they would have received from investing in the riskiest portfolio that they could choose, and over 90% earned worse returns than those from holding the safe portfolio E. The table also plots the distribution of net returns: the median is negative and a non-trivial fraction of followers earn a net

[24] Of the 111,351 observations in H&L's administrative records, about 22% do not have information for gender or age, or has a starting age below eighteen. Results are robust to assigning risk type A to the dropped observations.

return of -10% or worse; in contrast, those who outperformed the social security portfolios, barely did so.

The bottom panel of Table 1.12 compares returns of followers if the pricing convention were changed so that returns reflect prices at the moment a request is made, rather than with a one day lag. Consistent with our regression results, followers would have done far better if they could have benefitted from the stock market announcement effect. Plausibly, this finding can reconcile why some individuals continued to follow H&L despite their poor actual performance—they may not have recognized this poor performance.[25] The complex pricing rules used by AFPs to value portfolio transfers may make difficult for followers to realize that the announcement effect is hurting them rather than helping them. We recall: following a request to switch portfolios on day $t-1$, when the AFP moves the money on day $t+3$ (as is typical), the AFP uses the portfolio price on day $t+1$ to value the funds being shifted, which is computed based on the day t prices of the underlying assets.

Table 1.13 provides even starker evidence. The table compares the return from following H&L strategy with those from buying and holding each of the five portfolios. The comparison considers an investor who starts following H&L at recommendation k for $k = 1, ..., 20$ and continues to do so until September 30, 2016. That is, we compare the return from beginning to follow H&L at different points in time with all possible buy and hold strategies. Column $t-1$ shows the return assuming a follower acts the same day the recommendation is made. We see that with the exception of the first recommendation, any other starting point is outperformed by at least one the five portfolios, and, in almost all cases, H&L's strategy ranks last. Column $t-2$ shows the analogous returns were investments transferred at the exact moment when H&L made its recommendation at prices based on close of that trading day—returns that H&L can highlight, and returns that an investor would obtain if he could benefit from the announcement effect. For 9 of the first 14 recommendations, this return exceeds that from buying and holding *any* portfolio, and for 15 of the first 20 recommendations, this return exceeds that from buying and holding portfolio A, essentially reversing the true return pattern. Comparing columns $t-1$ and $t-2$ reveals the impact of the announcement effect, which results in investors buying portfolio A high, and selling it low, and the consequences for cumulative returns.

Columns $t-3$ and $t-4$ show that a hypothetical investor who could have acted on H&L's advice two or three days before the recommendation was made would have vastly outperformed any buy and hold strategy. This, of course, simply reflects the market/portfolio returns that entered into H&L's decision to change recommendations. But, such returns raise the possibility that some investors may

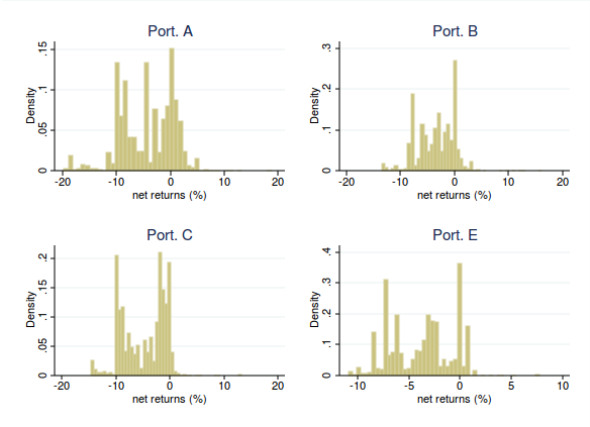
[25] In the next subsection we will show that this fact can only partially explain why so many workers follow H&L. In particular, many followers *know* that H&L underperformed buy and hold in the previous 12 months.

Table 1.12: Percentage of followers by risk type that beat a *Buy & Hold* strategy for different feasible portfolios. Followers are assumed to act immediately on all recommendations while their subscription is active. The top panel reflects the existing pricing convention. The bottom panel supposes that the pricing convention is changed so that returns reflect prices at the moment a request is made, rather than with a one day lag.

Acting immediately

Risk type	A					B				C		
Benchmark	A	B	C	D	E	B	C	D	E	C	D	E
	0.27	0.21	0.12	0.11	0.08	0.24	0.11	0.08	0.06	0.25	0.23	0.05

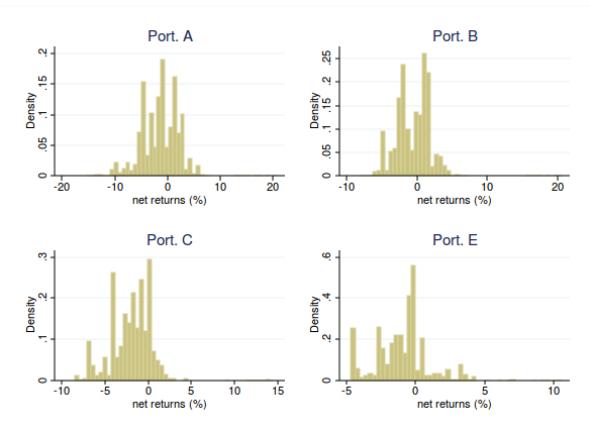
Distribution of net returns



Incorporating announcement effect

Risk type	A					B				C		
Benchmark	A	B	C	D	E	B	C	D	E	C	D	E
	0.36	0.49	0.21	0.07	0.16	0.46	0.25	0.14	0.23	0.32	0.32	0.47

Distribution of net returns



naively lump those returns in with the returns that they actually realize and conclude that the performance of H&L is better than it actually was. The qualitative results in Table 1.13 obviously hold if returns are adjusted by risk. Appendix C presents the analogous table computed using the modified Sharpe ratio (Israelsen (2005)).

Table 1.13: Nominal cumulative return for H&L's strategy and the five social security portfolios starting with any of the first 20 recommendations until September 30, 2016. Column $t-1$: cumulative return for follower acting the day a recommendation is made. Column $t-j$, $j = 2, 3, 4$: hypothetical cumulative return for a follower who acts j days before a recommendation is made. Column $t-2$ captures the return that would obtain were investments transferred as soon as a recommendation is made.

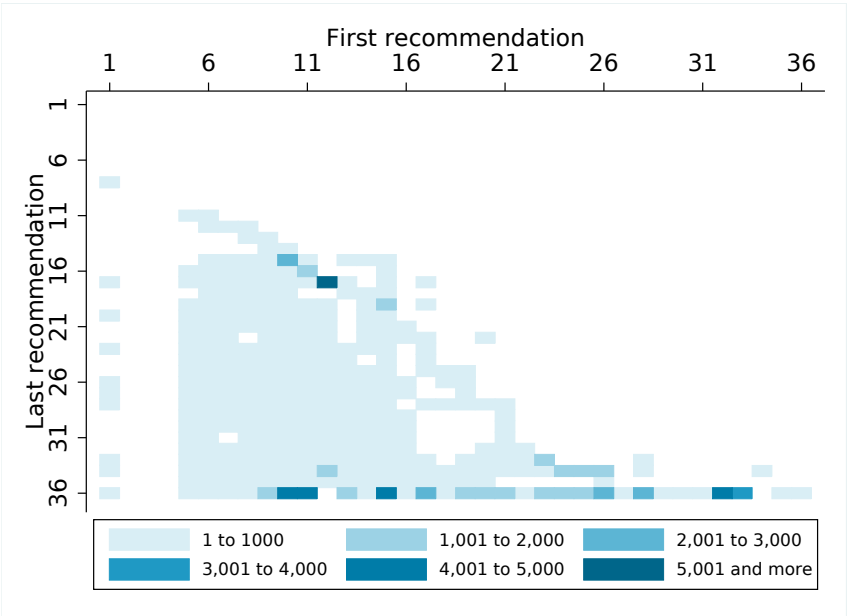
Starting	Benchmark					H&L			
recomm.	A	B	C	D	E	$t-1$	$t-2$	$t-3$	$t-4$
1	0.399	0.393	0.461	0.484	0.514	0.525	0.632	0.805	0.932
2	0.535	0.480	0.504	0.485	0.460	0.470	0.573	0.737	0.858
3	0.541	0.483	0.509	0.492	0.472	0.476	0.553	0.706	0.790
4	0.483	0.438	0.474	0.462	0.440	0.444	0.519	0.669	0.749
5	0.390	0.367	0.420	0.430	0.433	0.354	0.429	0.565	0.620
6	0.458	0.410	0.438	0.428	0.409	0.331	0.407	0.541	0.596
7	0.443	0.399	0.426	0.419	0.402	0.318	0.413	0.545	0.595
8	0.466	0.414	0.435	0.424	0.403	0.318	0.415	0.546	0.592
9	0.472	0.422	0.439	0.418	0.388	0.324	0.419	0.545	0.584
10	0.360	0.337	0.377	0.380	0.376	0.312	0.403	0.523	0.560
11	0.349	0.327	0.361	0.362	0.355	0.302	0.389	0.484	0.512
12	0.342	0.326	0.348	0.337	0.329	0.277	0.359	0.451	0.478
13	0.345	0.329	0.346	0.330	0.314	0.279	0.351	0.435	0.448
14	0.339	0.322	0.341	0.326	0.312	0.277	0.347	0.432	0.446
15	0.277	0.271	0.286	0.269	0.260	0.217	0.252	0.310	0.312
16	0.216	0.221	0.245	0.241	0.247	0.205	0.241	0.298	0.301
17	0.130	0.142	0.168	0.174	0.182	0.142	0.175	0.229	0.230
18	0.097	0.113	0.140	0.149	0.160	0.120	0.153	0.211	0.214
19	0.123	0.129	0.146	0.147	0.150	0.111	0.142	0.196	0.199
20	0.123	0.125	0.135	0.136	0.135	0.110	0.128	0.156	0.154

This evidence is striking given that H&L only had 54 followers at the time of the first announcement, and did not attract new clients until announcement five (see the top panel of Figure 1.4, which plots the matrix of clients according to the first and last announcement they could follow according to their payment records).[26] This figure shows that an important part of H&L's subscribers were still active in the last announcement in our sample, and many have been following for over a year. The bottom panel of Figure 1.4 reveals that over halpf of H&L customers renew their membership, and so presumably choose to continue following recommendations despite the poor realized returns from doing so. This leads us to ask: Why do followers act on H&L's recommendations? Are they aware of their performance? We investigate these questions next, presenting results from a survey of H&L clients.

[26] Our sample features 35 announcements, but the matrix in Figure 1.4 has 36 rows and columns because some followers joined after the last announcement (or their membership has not expired).

Figure 1.4: Membership length of H&L followers according to their payment records. The top figure shows the heat map of the matrix with the first and last announcement while the account was active for each follower. The bottom table shows the distribution of followers according to the length of their membership in years, and we decompose each category in whether they are active or inactive (expired) as of September 30, 2016.

Membership length



Distribution of membership length

	Number of followers	Inactive	Active
One year or less	53,037	29,594	23,433
	47%	56%	44%
Between 1 and 2 years	7,862	3,686	4,176
	7%	47%	53%
Between 2 and 3 years	17,022	8,267	8,755
	15%	49%	51%
Between 3 and 4 years	18,494	5,469	13,025
	16%	30%	70%
More than 4 years	15,833	716	15,117
	14%	5%	95%
Total	112,248	47,732	64,516
	100%	43%	57%

1.2.4 Characterizing followers

Table 1.3 reveals that paid and second-hand followers of H&L have over twice the savings of the average investor, consistent with Fuentes, Searle, and Villatoro (2013) who find that active investors in the system have higher savings than those who do not change portfolio choices. To glean more insights, we conducted a survey during October and November 2016 of followers of H&L and other advisers providing a similar service. We invited them to distribute the survey to followers by email. Appendix A lists the questions. We also launched our survey in Facebook. The Facebook post was randomly displayed to stratified segments of adults in Chile according to age and gender. We now compare the results of our survey with the

EPS.

The vast majority of respondents—almost 87%—claim to be current followers of H&L (see Table 1.18 in appendix B), so our analysis focuses on this sub-population. We contrast these respondents with the 68% of respondents from the 2015 EPS who claim to be members of the AFP system. We divide AFP members into two groups according to whether they claim to know which portfolio(s) they are holding. About 77% of AFP members do *not* know their portfolio (see Table 1.19 in appendix B). We label these groups AFP DNK if do not know the portfolio, and AFP otherwise. Table 1.14 provides some demographics of these groups. Striking differences appear when comparing education and income: H&L followers are far more educated (almost 75% have university degrees vs. 36% for AFP members who know their portfolios, and only 14% for those who do not), they have incomes that are several multiples of the other two groups, and they are more than twice as likely to hold savings other than mandatory savings. These three findings are consistent with Fuentes, Searle, and Villatoro (2013).[27] The three groups do not differ substantially in risk aversion. Interestingly, a *smaller* proportion of followers self-describe themselves as having a good or very good financial knowledge.

Greater savings and income, and higher education suggest that followers are more sophisticated than typical members of the other two groups. Our survey features two questions pertaining to financial sophistication. These questions belong to a group of "sophisticated" questions designed by the United States Health and Retirement Study (HRS) to identify knowledge of key financial concepts (Behrman, Mitchell, Soo, and Bravo (2012)). These two questions were also asked in the 2009 and 2012 EPS. One question is a TRUE/FALSE question about risk diversification; and the other question asked respondents to compute a two-period compound interest problem. We coded answers as correct or incorrect. Table 1.15 reveals that 85% of followers answered the diversification question correctly, but non-followers did no better than would be expected by chance. The question related to compound interest is even more telling: 64% of followers answer the question correctly versus only 3-5% of non-followers. In sum, followers are *very* financially sophisticated, whereas the typical investor is quite unsophisticated.

The fact that followers are sophisticated does not ensure that they are aware of the poor performance of H&L's strategy. To test their awareness we had respondents rank portfolios A, C, E and their own savings in terms of returns over the past twelve months. The survey was "open" from October to November 2016; Figure 1.5 plots these twelve month returns starting on the 30th September until the end of November. The figure shows that the ranks of portfolios A, C and E do not change over this period, but the rank of H&L's strategy varies depending on the specific day agents used to compute the ranking. Therefore, we only compare the relative

[27] Fuentes, Searle, and Villatoro (2013) provide evidence about the characteristics of active members of the system who request transfers among the different portfolios between 2008–2013.

Table 1.14: Demographics from our survey and the 2015 EPS. Table entries represent percent of observations in that category.

	Gender	
	Male	Female
H&L	79.3	20.7
AFP	61.4	38.6
AFP DNK	51.8	48.2

	Hold other savings	
	Yes	No
H&L	69.4	30.6
AFP	32.4	67.6
AFP DNK	19	81

	Age			
	18-34	35-54	55-64	65+
H&L	25.6	57.2	15	2.3
AFP	29.2	58.1	10.5	2.3
AFP DNK	35.9	43.9	12	8.1

	Income (USD)			
	500	500-1,000	1,000-2,000	2,001+
H&L	2.5	11.6	37.3	48.5
AFP	31.8	32.2	27.6	8.5
AFP DNK	65.9	25.5	7.3	1.3

	Education				
	Primary	Secondary	Tech. Degree	Uni. Degree	Postgraduate
H&L	0	4	21.7	56.5	17.8
AFP	3.7	40.1	20.9	30.1	5.2
AFP DNK	19.6	53.6	13.2	12.7	1

	Financial knowledge – self perception (2012 EPS)				
	Very bad	Bad	Intermediate	Good	Very good
H&L	6.7	28.2	53.5	10.7	0.9
AFP	6.8	16.1	47.7	24.9	4.6
AFP DNK	16.9	29.2	41.8	11.1	1

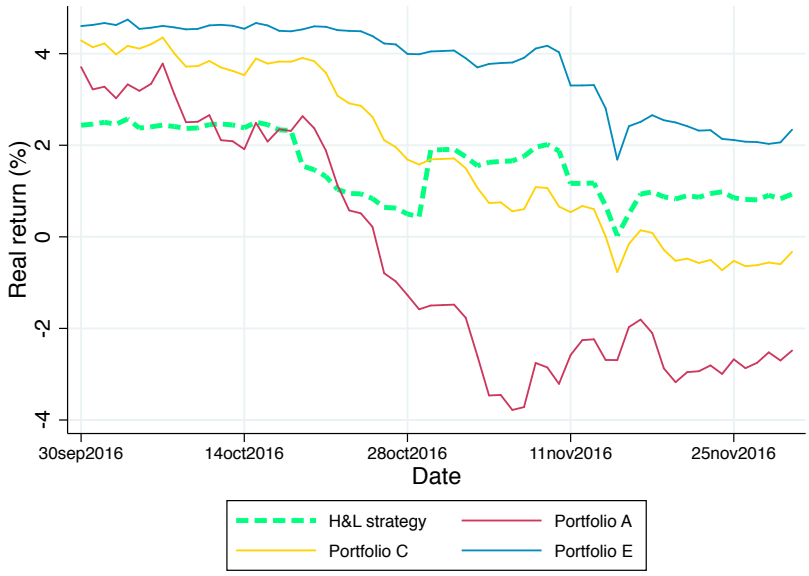
	Risk aversion (2012 EPS)										
	0	1	2	3	4	5	6	7	8	9	10
H&L	2.45	1.76	3.57	5.93	6.81	23.44	15.39	20.78	14.12	2.39	3.38
AFP	5.01	1.86	2.77	5.46	6.08	17.9	12.62	13.6	14.24	8.75	11.71
AFP DNK	7.81	4.39	6.13	6.98	7.12	17.39	11.18	11.65	10.71	4.59	12.05

Table 1.15: Percent of correct answers to questions related to financial sophistication.

	Risk diversification	Compound interest
H&L followers	85%	64%
AFP	55%	5%
AFP DNK	48%	3%

rank of their own savings with respect to portfolio E.

Figure 1.5: Twelve month return during survey period for each portfolio and H&L's strategy.



Using only observations of individuals who have been following H&L for over a year, we divide respondents who ranked portfolios into groups. The "full-ranking" group consists of those who ranked all three portfolios and their own savings. We divide those who only give a partial ranking into three groups: partial ranking group 1 consists of those who ranked portfolio E, their own portfolio, and either A or C. Partial ranking group 2 consists of those who provided rankings for at least two portfolios but failed to provide a ranking for their own savings. Partial ranking group 3 consists of those who ranked portfolio E and their own savings only. About 40% of followers do not provide rankings, and about 9% only give his own ranking and a portfolio other than E, and are omitted.

Table 1.16 shows that those who rank portfolio returns tend to be well informed—over 60% rank the returns on portfolios A, C and E correctly, and little over half correctly recognize that the return on the safe portfolio E exceeded the return from following H&L's strategy[28] and over 75% of those giving a response correctly assessed that the real return on their savings from following H&L's strategy in the past twelve months was between 0 and 4%. In sum, most investors have good ideas of the actual return that they obtained, and a non-trivial majority of followers pay extensive attention to the relative performance of different pension portfolios. Some investors overestimate the performance of H&L's strategy relative to the safe portfolio E, but even then a majority get this ordering right.

Thus, we have documented that (1) followers are highly financially sophisticated—far more so than the average investor—and they have a lot more at stake, (2) most

[28] This question is demanding—a reader should reflect on how the 12-month return on their own portfolio ranked vis à vis the S&P 500 or MSCI World Index.

Table 1.16: Portfolios ranking and H&L relative performance.

Group	Sample size Num. respondents	(A <C <E) Correct answers	(H&L <E) Correct answers
Full ranking	2,964	1,693 57%	1,489 50%
Partial ranking 1	300	197 66%	191 64%
Partial ranking 2	1,083	702 65%	
Partial ranking 3	89		50 56%

Respondents are classified into four groups: (i) Full ranking, those who ranked all portfolios including their own savings; (ii) Partial ranking 1 have rankings for portfolio E, their own portfolio and either A or C; (iii) Partial ranking 2 have rankings for at least two portfolios but not for their own savings; (iv) Partial ranking 3 have rankings for portfolio E and their own savings only.

Table 1.17: Real returns on own savings over the previous 12 months. Actual return on H&L's strategy fluctuated between 0 and 4%.

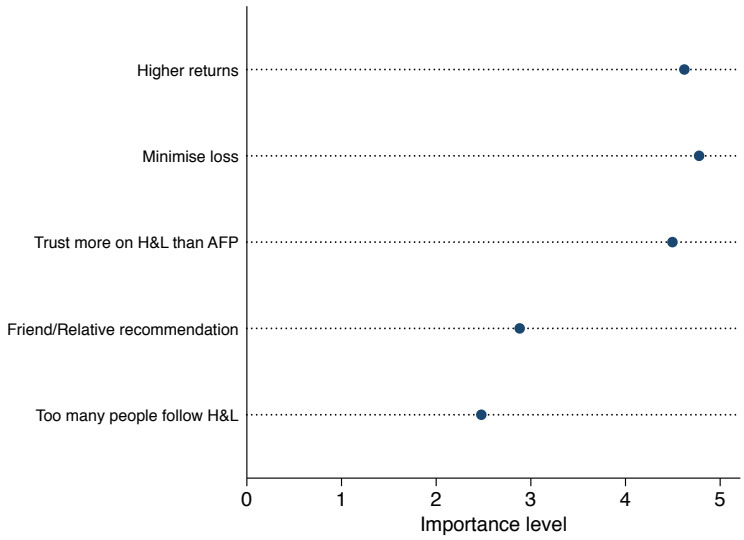
Group	Num. of respondents	Less than 0%	Between 0 and 4%	Higher% than 4%	Don't know / No Response
Full ranking	1,859	37 1.9%	1,202 64.7%	361 19.4%	259 13.9%
Partial ranking 1	178	1 1%	122 69%	33 19%	22 12%
Partial ranking 2	676	13 2%	435 64%	129 19%	99 15%
Partial ranking 3	60	0 0%	45 75%	8 13%	7 12%
No ranking	1,852	39 2%	953 52%	267 14%	593 32%

Respondents are classified into four groups: (i) Full ranking, those who ranked all portfolios including their own savings; (ii) Partial ranking 1 have rankings for portfolio E, their own portfolio and either A or C; (iii) Partial ranking 2 have rankings for at least two portfolios but not for their own savings; (iv) Partial ranking 3 have rankings for portfolio E and their own savings only.

followers were harmed by following H&L, and (3) a majority of followers is aware of the bad performance. Why then do investors follow H&L? Our survey asks respondents to rank five possible reasons for why they follow their current advisor.[29] Figure 1.6 presents a puzzle: it reveals that current H&L followers indicate that the most important reasons for following H&L are: higher returns, loss minimization and trust...

Of course, inverting the standard investment caveat, past bad performance is no guarantee of future bad performance. The fact that most respondents cite trust in H&L as a key reason for following him, may indicate that they believe that H&L offers them a higher *expected* return, or that they value having someone "looking after" their savings for them. Moreover, some followers appear to over-estimate relative returns of H&L, possibly due to the complex pricing rules used by AFPs to value transferred funds, and some may also confuse the good performance of a portfolio just prior to H&L's recommendation. The bottom line remains that understanding how and why individuals allocate pension investments as they do is germane not only to the Chilean Social Security system, but to retirement savings everywhere.

Figure 1.6: Why do current followers follow H&L?



1.3 Conclusion

We document the massive effects that portfolio recommendations by the pension advisory service H&L have had on pension investments by individuals, and the consequences for the Chilean stock market. Investors have come to believe that H&L's recommendations contain information with predictive value—leading to pension transfers that amount to as much as 25% of total portfolio holdings and 1.5% of

[29] Only a few respondents selected an "other" option, and its average importance level is low.

GDP. Prior to investors following this advice in large numbers, investors could have benefited from it; but once investors started following H&L's advice, stock prices responded before investors could reallocate pension investments in time to benefit. As a result, pension investors are harmed by following H&L's advice; most investors would have done better to stick with *whatever* pension position they had at the outset.

We establish that H&L bases recommendations on the immediate past performances of the Chilean stock market—recommending riskier portfolios after the market rises, and recommending safer portfolios after it falls. The extreme short-run nature of the recommendation model means that others cannot forecast and frontrun H&L's advice; consistent with this, we do not observe abnormal portfolio flows or stock volume before new recommendations.

Individuals following H&L have high incomes and are highly educated and financially sophisticated, especially relative to non-followers. Moreover, followers have a good understanding of the relative performances of different portfolios, albeit modestly over-estimating the returns from following H&L. Remarkably, despite evidence of poor past performance, the key reasons that individuals give for following H&L are: high returns, minimize risk and trust.

Our findings suggest care in the design of "privatized" social security. "Privatized" systems seek to align individual investments with risk attitudes, while avoiding shortfalls in savings due to bad investment choices or moral hazard by investment advisors. One way to do this is to limit the set of investment alternatives. We show that even with sharp limitations, sophisticated pension investors can still be harmed. Our analysis also highlights a potentially adverse consequence of limited choice sets. With few investment alternatives, common information arrival—here taking the form of portfolio recommendations by H&L—can result in massive portfolio reallocations. In Chile, these reallocations did not overwhelm liquidity provision because AFPs can allocate them to liquid foreign equity markets, avoiding volume surges in the Chilean stock market. This might not be possible for savings plans with greater exposure to domestic equity markets.

References

Barber, Brad, Reuven Lehavy, Maureen McNichols, and Brett Trueman (2001). "Can investors profit from the prophets? Security analyst recommendations and stock returns". In: *The Journal of Finance* 56.2, pp. 531–563.

— (2006). "Buys, holds, and sells: The distribution of investment banks' stock ratings and the implications for the profitability of analysts' recommendations". In: *Journal of accounting and Economics* 41.1, pp. 87–117.

Behrman, Jere R, Olivia S Mitchell, Cindy K Soo, and David Bravo (2012). "How financial literacy affects household wealth accumulation". In: *The American economic review* 102.3, pp. 300–304.

Bernhardt, Dan and Ryan J Davies (2009). "Smart fund managers? Stupid money?" In: *Canadian Journal of Economics/Revue canadienne d'économique* 42.2, pp. 719–748.

Berstein, Solange, Guillermo Larraín, Francisco Pino, and Eduardo Morón (2006). "Chilean Pension Reform: Coverage Facts and Policy Alternatives [with Comments]". In: *Economía* 6.2, pp. 227–279.

Boni, Leslie and Kent L Womack (2006). "Analysts, industries, and price momentum". In: *Journal of Financial and Quantitative analysis* 41.1, pp. 85–109.

Carlin, Bruce I and Shaun William Davies (2016). "The Implementation of State Sponsored Retirement Plans". In:

Chevalier, Judith and Glenn Ellison (1997). "Risk taking by mutual funds as a response to incentives". In: *Journal of Political Economy* 105.6, pp. 1167–1200.

Da, Zhi, Borja Larrain, Clemens Sialm, and José Tessada (2016). *Coordinated Noise Trading: Evidence from Pension Fund Reallocations*. Tech. rep. National Bureau of Economic Research.

Dahlquist, Magnus, José Vicente Martinez, and Paul Söderlind (2016). "Individual investor activity and performance". In: *The Review of Financial Studies* 30.3, pp. 866–899.

Edwards, Sebastian (1998). "The Chilean pension reform: A pioneering program". In: *Privatizing social security*. University of Chicago Press, pp. 33–62.

Financial Stability Board (2013). "Minutes of meeting held on July 26, 2013". In:

Fuentes, Olga, Pamela Searle, and Félix Villatoro (2013). "Active Investment Decisions of Members in the Chilean DC Pension System: Performance and Learning over time". In:

Gennaioli, Nicola, Andrei Shleifer, and Robert Vishny (2015). "Money doctors". In: *The Journal of Finance* 70.1, pp. 91–114.

Ghilarducci, Teresa, Hamilton James, et al. (2016). *A Comprehensive Plan to Confront the Retirement Savings Crisis*. Tech. rep. Schwartz Center for Economic Policy Analysis (SCEPA), The New School.

Inderst, Roman and Marco Ottaviani (2012). "Financial advice". In: *Journal of Economic Literature* 50.2, pp. 494–512.

Israelsen, Craig (2005). "A refinement to the Sharpe ratio and information ratio". In: *Journal of Asset Management* 5.6, pp. 423–427.

Ivković, Zoran and Narasimhan Jegadeesh (2004). "The timing and value of forecast and recommendation revisions". In: *Journal of Financial Economics* 73.3, pp. 433–463.

Jegadeesh, Narasimhan, Joonghyuk Kim, Susan D Krische, and Charles Lee (2004). "Analyzing the analysts: When do recommendations add value?" In: *The journal of finance* 59.3, pp. 1083–1124.

Jegadeesh, Narasimhan and Woojin Kim (2009). "Do analysts herd? An analysis of recommendations and market reactions". In: *The Review of Financial Studies* 23.2, pp. 901–937.

Joubert, Clement (2015). "Pension design with a large informal labor market: Evidence from Chile". In: *International Economic Review* 56.2, pp. 673–694.

Loh, Roger K and René M Stulz (2010). "When are analyst recommendation changes influential?" In: *The review of financial studies* 24.2, pp. 593–627.

Mesa, Alberto Arenas de, David Bravo, Jere R Behrman, Olivia S Mitchell, and Petra E Todd (2008). "The Chilean pension reform turns 25: lessons from the social protection survey". In: *Lessons from pension reform in the Americas*. Oxford University Press, pp. 23–58.

Powdthavee, Nattavudh and Yohanes E Riyanto (2015). "Would you pay for transparently useless advice? A test of boundaries of beliefs in the folly of predictions". In: *Review of Economics and Statistics* 97.2, pp. 257–272.

Sirri, Erik R and Peter Tufano (1998). "Costly search and mutual fund flows". In: *The journal of finance* 53.5, pp. 1589–1622.

Stickel, Scott E (1995). "The anatomy of the performance of buy and sell recommendations". In: *Financial Analysts Journal* 51.5, pp. 25–39.

Superintendence of Pensions (2010). *The Chilean Pension System*.

— (2013). *Objetivo de los Fondos de Pensiones y los Traspasos Frecuentes de Fondo*. Tech. rep.

Womack, Kent L (1996). "Do brokerage analysts' recommendations have investment value?" In: *The journal of finance* 51.1, pp. 137–167.

Zheng, Lu (1999). "Is money smart? A study of mutual fund investors' fund selection ability". In: *The Journal of Finance* 54.3, pp. 901–933.

Appendix

1.A Survey questions

1. Do you currently follow the recommendation of an adviser to manage your social security savings?
2. Currently do you follow any of the following advisers in social media? If Yes in 1:
3. When did you start following the recommendations of your current adviser?
4. Why do you follow the announcements of your current adviser?
5. In the last twelve months, what is the return on your savings?
6. Rank portfolios A, C, E and your own savings in terms of returns in the last twelve months
7. How much time did it pass since you first heard about your current adviser and when you started following the recommendations?
8. Usually, how fast do you act upon recommendations?
9. Did you follow the recommendations of another adviser before? If No in 1:
10. What is the return on your savings in the last twelve months?
11. Rank portfolios A, C, E in terms of returns in the last twelve months
12. Are mandatory savings your main source of savings for retirement?
13. Did you follow the recommendations of another adviser before? For everyone:
14. Gender
15. Education
16. Age
17. Income (monthly individual income)

18. Where do you live?

19. Generally, how is your knowledge on financial issues?

20. In a scale from 0 to 10, how risk averse are you? (where 0 is not willing to take any risk)

21. True or false: buying share of a single firm is less risky compared to buying, with the same money, shares of different companies.

22. Let's say you have 200 in a savings account. The account pays 10% interest rate per year. How much do you have after two years?

23. Do you have voluntary savings?

1.B EPS and survey samples

This appendix reports sample statistics for followers and members of the AFP system. Table 1.18 reports the number of observations in our survey, and the number of observations in the different EPS. Table 1.19 decomposes the observations used in our analysis.

Table 1.18: Survey observation numbers by method of collection

Own survey		**EPS† survey**		
Source	Obs.	Source	Obs.	Obs.(iw)
H&L email list	9,373	EPS 2015	16,906	13,560,981
Other advisors	118	EPS 2012	15,998	12,718,525
Facebook	547	EPS 2009	14,463	12,765,015

† The Obs. (iw) column refers to the expanded sample by "importance weights" provided in the EPS.

Table 1.19: Sample selection

Source		Sample size	Per cent of total N
Our survey	current H&L followers	8,703	86.7
EPS 2015	in AFP system	9,253,512	68.2
	Know portfolio	2,090,012	22.5
	Don't know portfolio	7,163,500	77.5
EPS 2012	in AFP system	8,431,177	66.3
	Know portfolio	2,577,376	30.5
	Don't know portfolio	5,853,801	69.5
EPS 2009	in AFP system	8,288,982	64.9
	Know portfolio	2,905,235	35.0
	Don't know portfolio	5,383,747	65.0

1.C Modified Sharpe ratio

In this appendix we show the modified Sharpe ratio (Israelsen (2005)) for H&L's strategy and for each portfolios besides portfolio E. We use Portfolio E as the risk free portfolio when calculating the modified Sharpe ratio. The original Sharpe ratio is not valid when the excess return is negative. The modified ratio corrects this. As Table 1.13 reports, portfolio E yields the highest return for the period analyzed. Interpretations of the modified ratio are similar to those of the original Sharpe ratio: a higher ratio reveals a higher return after controlling for risk.

Let ER be the average of the daily excess return of portfolio X relative to portfolio E, and SD the standard deviation of the daily excess return. The modified Sharpe ratio is defined as:

$$\text{Modified Sharpe ratio } = \frac{\text{ER}}{\text{SD}^{\text{ER/abs ER}}}, \tag{1.1}$$

where abs ER is the absolute value of ER. From equation (1.1) we see that if ER is positive then the Sharpe ratio and the modified version coincide. If ER is negative then the exponent in the denominator is -1, and so the modified Sharpe ratio becomes the product of ER and SD, instead of their ratio.

The results presented in Table 1.20 are obtained adjusting equation (1.1) in two ways: first we multiply the ratio by 252, the (average) number of trading days in a year; and second, because both ER and SD are close to zero, when ER is negative the ratio becomes very small, therefore we multiply negative entries by 100,000 so that scales are similar.

Table 1.20: Modified Sharpe Ratios for H&L's strategy and social security portfolios A, B, C & D starting with any of the first 20 recommendations until September 30, 2016. Column $t-1$: modified Sharpe ratio if a follower acts the day a recommendation is made. Column $t-j$ for $j = 2, 3, 4$: modified Sharpe ratio if follower acts j days before a recommendation is made.

Starting	Benchmark				H&L			
recomm.	A	B	C	D	$t-1$	$t-2$	$t-3$	$t-4$
1	-6.46	-5.65	-1.59	-0.48	0.83	4.64	9.76	12.75
2	2.70	1.24	2.97	3.21	0.85	4.71	9.88	12.90
3	2.63	0.88	2.63	2.73	0.58	3.89	9.40	11.97
4	1.97	0.33	2.62	3.22	0.58	3.92	9.59	12.09
5	-1.86	-3.02	-0.30	-0.03	-3.45	0.21	6.27	8.26
6	2.38	0.49	2.54	3.07	-3.72	0.33	6.52	8.61
7	2.13	0.25	2.18	2.76	-4.05	0.99	7.29	9.14
8	2.95	1.01	2.86	3.57	-4.13	1.05	7.34	9.04
9	3.81	2.24	4.52	5.06	-3.13	2.19	8.36	9.80
10	-0.14	-2.06	0.38	0.93	-3.55	2.08	8.33	9.77
11	0.39	-1.52	0.87	1.50	-3.14	2.67	8.15	9.36
12	1.30	0.26	2.40	2.04	-3.57	2.56	8.30	9.54
13	2.21	1.50	3.87	3.63	-2.35	3.13	8.57	9.10
14	2.08	1.21	3.76	3.54	-2.42	3.11	8.65	9.23
15	1.67	1.40	3.82	2.72	-3.60	-0.36	4.91	5.01
16	-3.25	-2.30	0.01	-0.20	-3.85	-0.18	5.14	5.33
17	-8.74	-5.08	-1.11	-0.34	-4.95	-0.40	5.52	5.53
18	-11.53	-6.29	-1.67	-0.46	-5.09	-0.28	6.09	6.25
19	-4.43	-2.81	-0.26	-0.12	-5.65	-0.69	6.01	6.32
20	-1.43	-1.22	0.23	0.57	-3.77	-0.73	3.29	3.02

Chapter 2

The social value of information in economies with mandatory savings

2.1 Introduction

We study the value of information in a stochastic pure exchange economy where all agents trade assets in financial markets to reallocate risk and a subset of those agents face a mandatory savings constraint. In their 2008 book, Barr and P. Diamond write about what they call the "aging crisis". The authors discuss how reductions in mortality, fertility and lower labor force participation of older men in recent decades have increased pension costs to unsustainable levels in many countries. Accordingly, countries have responded by introducing changes to their pension systems. One such change has been the development of mandatory fully funded individual savings accounts, as in the famous case in Chile in 1981.

In economies with mandatory individual savings the government may give consumers the possibility to decide where to invest their savings. In the literature on the design of such systems, information is considered as a key input in the consumer's decision making process. This is because not only these systems involve complicated rules but also due to the widespread financial illiteracy among consumers in many countries. Thus "more" information is normally viewed as desirable.

However there is another channel through which information can have an effect on welfare: information release may affect prices. Surprisingly, to the best of our knowledge, this channel has been largely ignored in the literature on pensions even though the theoretical literature suggests that the welfare effects on prices might be ambiguous and dependent on the asset structure. Chapter 1 shows how the release of information has led to massive coordinated movements between the available social security portfolios in Chile, affecting the prices of the social security portfolio, the domestic stock market and other assets. Thus the release of new information appears to be affecting also those who did not act upon the new information and those outside the pension system. So, it is key to understand the welfare effect of information-

driven price changes when some agents face mandatory savings constraints.

We consider a two-period exchange economy with two agents, a single consumption good and uncertainty about the state of nature in period 1. There are two states of nature that realise in period 1 but no aggregate uncertainty. In period zero, agents trade two Arrow securities and a risk-free bond.[1] One of the agents faces a mandatory savings constraint modelled as lower bound on the value of the holdings of the risk-free bond, and short-sale constraints on the other assets. We call her the constrained agent. The other agent is unconstrained.[2] We call her the unconstrained agent. We model an information structure as signals that lead agents to update their common prior belief over the states of nature via Bayes' rule.

We study the welfare effects caused by information that arrives before agents trade in financial markets. In particular, we look for an information structure that allows a benevolent planner to obtain a Pareto improvement relative to the equilibrium in which agents do not have access to information before trading, the *uninformative equilibrium* hereafter. Throughout our analysis we assume the Planner faces the same constraints as the agents in terms of the available assets, information, and savings constraints. We consider two welfare notions depending on whether one evaluates allocations ex-ante or ex-post, i.e. before or after observing the signal.

In the absence of the savings and short-sell constraints, the existing assets would allow the agents to generate any desirable future consumption vector. However, the savings constraints and short-sale constraints prevent the equilibrium from being fully Pareto efficient when they are binding. Since the savings constraint is modeled as a lower bound on the value of the holdings of the risk-free bond, it depends on the equilibrium (gross) interest rate. Different information structures lead to different posteriors and thus result in different equilibrium prices and, possibly, different interest rate. Therefore changes in information can shift the lower bound on the holdings of the risk-free bond. If the savings constraint is relaxed, the Planner can access allocations that are Pareto superior, but that were not feasible at the uninformative equilibrium interest rate.

When studying the existence of ex-post improvements, i.e. Pareto improvements under each signal, we show that an ex-post improvement exists if and only if the savings constraint is relaxed under *every* signal. We begin our analysis by asking if there exists an alternative allocation and a posterior different to the common prior such that the alternative allocation satisfies the savings constraint at the initial equilibrium prices, and that Pareto dominates the uninformative equilibrium allo-

[1] We consider mandatory savings in a redundant assets to mimic how these type of social security systems work in practice.

[2] We work with an unconstrained agent because that is what we observe in the Chilean case. Only formal workers are force to save. If we were to assume that both agents have mandatory savings constraints, but the unconstrained agent were completely free to choose the Arrow-Debreu securities, then we would need to have the bond in positive net supply, but we expect all the result to hold in such a case.

cation when utilities are computed using the new posterior. The answer is no. For every alternative posterior, the allocations that Pareto dominate the uninformative equilibrium allocation are outside the constrained feasible set of the uninformative equilibrium. *All* the Pareto superior allocations violate the savings constraint at initial equilibrium prices. Thus ex-post constrained Pareto improvement exist if and only if the constraint set of the Planner is enlarged for *every* signal.

To study ex-ante improvements, i.e. Pareto improvements in expected value, we define the Pareto frontier as the maximum utility the constrained agent can attain in the constrained feasible set as a function of her posterior belief and the utility of the unconstrained agent. We relate the existence of ex-ante improvements to the concavity of the Pareto frontier. We show that there exist ex-ante improvements if and only if the concavification of the Pareto frontier evaluated at the prior belief and the uninformative equilibrium utility level for the unconstrained agent lies above the uninformative equilibrium utility for constrained agent, i.e. the Pareto frontier evaluated at that point.

The Pareto frontier fails to be concave in beliefs and the utility of the unconstrained agent under general conditions. In the simple case where the savings constraint does not prevent full consumption smoothing across states, the objective function defining the Pareto frontier is independent of the posterior. Therefore the posterior only affects the Pareto frontier through its effect on the equilibrium interest rate and, therefore, on the constrained feasible set. Thus if prices do not change with changes in information it is not possible to obtain ex-ante improvements. Consequently, changes in prices are a necessary condition for the existence of ex-ante improvements.

We show there exist a threshold on the posteriors such that equilibria are first best if and only if the posterior is weakly above that threshold. Since there is no aggregate uncertainty, then the gross interest rate is equal to one in *every* first best equilibria. Fixing the utility of the unconstrained agent, this means that the constrained feasible set, and thus the Pareto frontier, is constant for posteriors above the threshold. For posteriors below the threshold, the savings constraint is binding in equilibrium and the equilibrium interest rate is strictly less than one, as equilibrium prices must induce the unconstrained agent to increase her consumption in period zero. If the common prior is below the threshold, then the constrained feasible set in the uninformative equilibrium contains the constrained feasible set for posteriors above the threshold. This implies that the Pareto frontier at the uninformative equilibrium attains a higher value relative to the constant value to the right of the threshold.

Following Kamenica and Gentzkow (2011), we say a distribution of posteriors is Bayes' plausible if the expected posterior is equal to the prior. If the common prior is below the threshold, then for any Bayes' plausible distribution of posteriors

with support equal to the common prior and a posterior to right of the threshold, the expected value of the Pareto frontier is greater than the Pareto frontier at the threshold. Consequently, the frontier fails to be concave in the posterior belief.

Finally, we show that there exist information structures that allow the Planner to obtain ex-ante improvements, i.e. we show that the concavification of the Pareto frontier lies above the Pareto frontier when evaluated at the prior and the initial utility for the unconstrained agent. To show existence of ex-ante improvements we keep the utility of the constrained agent fixed at the uninformative equilibrium level for every posterior. We fix a support for the posteriors and we look for the probabilities of the posteriors that satisfy Bayes' plausibility. Then we ask what are the probabilities (of the posteriors) that keep the constrained agent indifferent with respect to the initial equilibrium. We show that if the prior is close to the aforementioned threshold, the Bayes' plausible probabilities differ from the probabilities that keep the constrained agent indifferent. In particular, the constrained agent is better off relative to the uninformative equilibrium and as the unconstrained agent is indifferent by construction, we obtain an ex-ante improvement.

Our work is related to a large literature that analyze the effect of public information in two-periods competitive economies with homogeneous beliefs and complete markets to share risk.[3] In a seminal paper Hirshleifer (1971) considers a situation where initially uninformed agents are revealed the true state of the world before trading and, therefore, no risk sharing trade that benefits all agents is possible. Initially uninformed traders cannot all be made better off even if the new information that is revealed before trading is only partially revealing. Marshall (1974) showed that the contract curve is independent of the posteriors when beliefs are homogeneous (and markets are complete), therefore if the equilibrium without information lies in the contract curve, then it also belongs to the contract curve for every vector of posteriors. Thus there are no ex-post improvements. In the special case where initial endowments are an equilibrium in the economy without information, he showed that changes in information cannot reduce the ex-ante utility of any agent, as they can always stand pat and do not act upon the new information, nor increase agents' ex-ante utility. When endowments are not an equilibrium without information, he argued that public information cannot obtain Pareto improvements. This result, closely related to the Sunspot theorem by Cass and Shell (1983), was formally proved by Hakansson, Kunkel, and Ohlson (1982) who gave a set of sufficient and necessary conditions for public information to have social value in pure exchange economies under uncertainty. Full Pareto efficiency of the initial equilibrium is a sufficient condition for public information to be of no social value.

When prior beliefs are heterogeneous, Marshall (1974) gave an early example that public information can have social value. Ng (1977) showed that when initial

[3]With the exception of Maurer and Tran (2016), this literature do not consider markets that are open before signals arrive.

endowments are an equilibrium in the economy without information, then the arrival of information makes some individual better off and no individual worse off. This follows directly from the fact that agents can always decide not to trade. If beliefs are heterogeneous, new information will make individuals willing to trade, hence they cannot be worse off. He also showed that when initial endowments are not an equilibrium in the economy without information, if prices are the same in the economies with and without information, then the equilibrium with information represents an ex-ante constrained Pareto improvement when posteriors coincide after the release of information. The results follows from a standard revealed preference argument.[4]

Recently, Maurer and Tran (2016) study the value of public information in an economy with multiple consumption and trading dates. They show that when beliefs are heterogeneous, the Hirshleifer effect is reversed if information arrives before the first round of trading. This result holds when agents anticipate small benefits from risk sharing and large benefits from intertemporal consumption smoothing.

Gottardi and Rahi (2014) consider the case of a pure exchange economy with one good, two periods and incomplete markets. They show that ex-post Pareto improvements can be attained for any change in information, by adjusting agents' asset holdings to the new information.[5] However when comparing different equilibria they show that the overall effect on welfare can go in any direction. The difference lies in an additional welfare effect that arises due to the adjustment in equilibrium prices. They conclude that competitive markets typically do not deal with changes in information in a way that is welfare-improving even though it is feasible to do so.[6] In contrast, in our model ex-post improvements are only possible under special conditions. The uninformative equilibrium is not ex-post constrained Pareto efficient if and only if the uninformative equilibrium interest rate is greater than the equilibrium interest rate of *all* the signals of the informative information structure.

From a methodological point of view, this paper is also related to the literature on Bayesian persuasion. Kamenica and Gentzkow (2011) study a symmetric information model where two players, a sender and a receiver, interact. The sender can

[4]In his analysis, Ng (1977) uses the value of optimization to measure changes in welfare. That is, he evaluate different allocations at the new posteriors induced by the new information, but he fails to integrate across all the signals.

[5]Since the condition for constrained Pareto optimally of equilibria can be characterized in terms of the equality of the marginal rate of substitution between assets and present consumption for all agents (P. A. Diamond, 1967), it typically depends on the posterior of the agents. If one keeps the allocation fixed and change the posteriors, nothing ensures that the condition for constrained Pareto optimality holds for the new posteriors.

[6]They relate their analysis to the *Hishleifer effect* and the *Blackwell effect*. The former effect follows from the example by Hirshleifer (1971) and it is related to how information affects welfare through the change in equilibrium prices. The latter effect makes reference to Blackwell (1951), who showed that agents can hedge risk more efficiently by adjusting their portfolios on the new information.

send signals to the receiver, whom upon observing the signal takes an action that affects the payoff of both agents. The authors show that the sender can send signals that result in the receiver taking an action that gives the sender higher expected utility, relative to the action taken based on the prior, if and only if the concavification of the sender's expected utility lies above the expected utility function when evaluated at the action the receiver takes under his prior. Thus we can relate our analysis for a competitive economy to the literature on Bayesian persuasion. We can think of the Planner as the sender, whose payoff function is given by the Pareto frontier. The agents play the role of the receiver. They observe the signal sent by the Planner and take an action, their excess demand, to maximize their payoff, taking prices as given. These actions affect the Planner's payoff as they determine equilibrium prices and the constrained feasible set.

In the next section we formally introduce our model. In section three we define an equilibrium in our economy. In section four we relate changes in formation and welfare, and in the next two sections we study ex-post and ex-ante improvements. We finish with conclusions in section seven. All proofs are relegated to the appendix.

2.2 The model

There are two periods, 0 and 1, and a single consumption good. Uncertainty, which is resolved at period 1, is described by $S = 2$ states of the world.

The economy is populated by $H = 2$ agents indexed by h. A consumption plan for agent h is given by $x^h = \left(x_0^h, x_1^h, x_2^h\right)$. Agent h has endowments $w^h = (w_0^h, w_1^h, w_2^h)$ where $w_0^h > 0$ is agent h's endowment in period 0 and $w_s^h > 0$ is agent h's endowment in state $s = 1, 2$. Endowments are such that $\sum_h w_s^h = w$ for $s = 0, 1, 2$.

Agents' common belief about the probability of state one is denoted by $\pi \in [0, 1]$. Both agents have utility functions $V(x^h, \pi) : \mathbb{R}_+^3 \times [0, 1] \mapsto \mathbb{R}$ that can be represented as a time-separable expected utility function with Bernoulli function $v : \mathbb{R}_+ \mapsto \mathbb{R}$. That is, for every consumption plan x^h,

$$V(x^h, \pi) = v(x_0^h) + \pi v(x_1^h) + (1 - \pi)v(x_2^h).$$

We assume that v is twice continuously differentiable, strictly increasing, strictly concave, and that $\lim_{x_s^h \to 0} v'(x_s^h) = \infty$, where v' denotes the first derivative of v with respect to its argument.

2.2.1 Financial markets

There are $L = 3$ securities that are traded in a competitive market at period 0. The payoffs of these assets are in units of the consumption good in period 1. Assets 1 and 2 are Arrow-Debreu securities while asset 3 is a risk free bond paying one unit

of the consumption good in all states. Let

$$\Phi = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix},$$

be the payoff matrix, where the element (s, l) of Φ is the payoff of asset l in state s.

Let $q = (q_0, \ldots, q_3) \in \mathbb{R}^4_{++}$ be the vector of period 0 prices, where q_0 is the price of the consumption good and q_l is the price of asset $l = 1, 2, 3$, and let $z^h = (z^h_1, z^h_2, z^h_3) \in \mathbb{R}^3$ be agent h's vector of asset holdings. We say a consumption plan x^h can by financed at (q, w^h) by a portfolio z^h if:

$$q_0 x^h_0 + \sum_s q_s z^h_s = q_0 w^h_0,$$
$$x^h_s = w^h_s + z^h_s + z^h_3, \forall s = 1, 2.$$

One of the two agents living in this economy faces a mandatory savings constraint. Let's call her agent c (for *constrained*). Agent c's mandatory savings constraint is such that the value of her holdings of asset three have to satisfy:

$$q_3 z^c_3 \geq q_0 \theta_3, \tag{2.1}$$

where $\theta_3 > 0$ is an exogenous parameter. The savings constraint on asset three implies that she has a lower bound on the value of her holdings of the risk free bond, and this lower bound depends on equilibrium prices.

In addition to the mandatory savings constraint, agent c faces short-sale constraint on assets one and two. That is, her holdings of assets one and two have to satisfy:

$$z^c_l \geq \theta_l, \tag{2.2}$$

with $\theta_l \leq 0$ an exogenous constant for $l = 1, 2$. Short-sale constraints are needed in order to have equilibria where the savings constraint is binding. If there were no short-sale constraints, then agent c could undo the savings constraint (2.1) as she could generate any date 1 consumption plan in $\mathbb{R}^2_+$ with the existing assets. In that case any equilibrium would be fully Pareto optimal.[7] The other agent, agent u (for *unconstrained*), faces no constraints besides the usual budget constraint.

[7] An alternative approach, used in a previous version of this chapter, is to assume that the government can target agents' savings directly, i.e. we could assume the government imposes a lower bound on total savings. If this lower bound depends on equilibrium prices all the result in the chapter hold.

2.2.2 Information

Prior to trading agents observe a public signal possibly correlated with the state of the world s. We fix a finite set of signal realizations $\mathcal{Y} = (y_1, y_2, y_3)$.[8] Once signal y_k is observed agents update their common prior belief about the state of nature, π^0, to the posterior belief $\pi(y_k)$ via Bayes' rule. Let $\text{pr}(y_k|s)$ denote the conditional probability of signal k given state s. The $S \times K$ matrix of conditional probabilities:

$$Y = \begin{pmatrix} \text{pr}(y_1|1) & \text{pr}(y_2|1) & \text{pr}(y_3|1) \\ \text{pr}(y_1|2) & \text{pr}(y_2|2) & \text{pr}(y_3|2) \end{pmatrix},$$

is called an information structure. Let $\text{pr}(y_k) = \sum_s \text{pr}(y_k|s)\pi_s^0$. Then posteriors are given by:

$$\pi_s(y_k) = \frac{\text{pr}(y_k|s)\pi_s^0}{\text{pr}(y_k)}.$$

If Y is such that $\text{pr}(y_k|1) = \text{pr}(y_k|2)$ for all k then the posteriors coincide with the prior for all signals. We call such an information structure *uninformative*. We call the information structure *informative* otherwise.

2.3 Equilibrium

Both agents take the price vector q as given. Agent c's budget set, denoted $\mathbb{B}^c(q, w^c)$, is the set of consumption plans that can be financed at (q, w^c) by a portfolio z^c that satisfies constraints (2.1) - (2.2). That is,

$$\mathbb{B}^c(q, w^c) \equiv \Bigg\{ x^c \in \mathbb{R}^3_+ \;\Bigg|\; \exists z^c \in \mathbb{R}^3 \text{ s.t. } q_0 x_0^c + \sum_s q_s z_s^c = q_0 w_0^c,\ x_s^c = w_s^c + z_s^c + z_3^c\ \forall s, \\ z_s^c \geq \theta_s\ \forall s,\ q_3 z_3^c \geq q_0 \theta_3 \Bigg\}.$$

Since agent u neither faces a savings nor short-sale constraints, her budget set, denoted $\mathbb{B}^u(q, w^u)$, is simply defined as the set of consumption plans that can be financed at (q, w^u) by a portfolio z^u. That is,

$$\mathbb{B}^u(q, w^u) \equiv \left\{ x^u \in \mathbb{R}^3_+ \;\middle|\; \exists z^u \in \mathbb{R}^3 \text{ s.t. } q_0 x_0^u + \sum_s q_s z_s^u = q_0 w_0^u,\ x_s^u = w_s^u + z_s^u + z_3^u\ \forall s \right\}.$$

A financial market equilibrium is defined as follows:

Definition 1. Given π, a financial market equilibrium is a collection of prices $\overline{q} \in \mathbb{R}^4_{++}$, consumption plans $\overline{x} = (\overline{x}^u, \overline{x}^c) \in \mathbb{R}^6_+$ and portfolios $\overline{z} = (\overline{z}^u, \overline{z}^c) \in \mathbb{R}^6$, where

[8] The number of signal realization is set equal to the number of agents plus the number of states of the world minus one. We need at least three signals to use Carathéodory's theorem in the proof of Corollary 4.

$\overline{z}^h$ finances the consumption plans $\overline{x}^h$ at (q, w^h), such that:

1. $\overline{x}^h \in \text{argmax } \left\{V^h(x^h, \pi) \,\middle|\, x^h \in \mathbb{B}^h(\overline{q}, w^h)\right\} \ \forall h \in \{c, u\}$,
2. $\sum_h \overline{z}^h = 0$.

Hereafter we refer to any consumption plan satisfying Definition 1 as an *equilibrium allocation.*

To prove the existence of a financial market equilibrium for all posteriors $\pi \in [0, 1]$, we need the following assumptions on $c's$ endowment vector:

$$\begin{aligned} &w_0^c \geq \theta_3, \\ &w_s^c \geq -\theta_s \text{ for } s = 1, 2. \end{aligned} \tag{A0}$$

Proposition 1. *If (A0) is satisfied, then a financial market equilibrium exist for all* $\pi \in [0, 1]$.

From here onward we normalize $q_0 = 1$, i.e. the equilibrium price of the consumption good at period 0 is set equal to one. Proposition 1 is proved in several steps. First we define the concept of a non-arbitrage equilibrium following Magill and Quinzii (2002). Then we show the equivalence between both types of equilibria in the absence of arbitrage. Working with a non-arbitrage equilibrium is useful because it allows us to use the standard existence proof in models with contingent consumption.

Assumption (A0) is needed to ensure that c's optimization problem has a solution for every $\pi \in [0, 1]$. The savings and short-sale constraints imply that there is a lower bound on x_s^c for $s = 1, 2$ given by:[9]

$$x_s^c \geq w_s^c + \theta_s + \frac{\theta_3}{q_1 + q_2} \text{ for } s = 1, 2. \tag{2.3}$$

The second condition in assumption (A0) implies that the lower bound on x_s^c is positive when prices are strictly positive. When both lower bounds are binding, c's period 0 consumption is given by: $x_0^c = w_0^c - \theta_3 - \sum q_s \theta_s$. If prices are positive, then the first condition in (A0) ensures $x_0^c \geq 0$.

Given a vector of prices q, we say there is no arbitrage if there does not exist $z \in \mathbb{R}^3$ such that:

$$\begin{bmatrix} -q \\ \Phi \end{bmatrix} z > 0.$$

It is not difficult to show that with our payoff matrix there is no arbitrage if and only if $q_1 + q_2 = q_3$.[10] It is direct to see that u's optimization problem has no solution if there are arbitrage opportunities. Thus every equilibrium satisfies absence

[9] To see this just replace the lower bounds on z_l^c for all l in the expressions for x_s^c for $s = 1, 2$.

[10] If $q_1 + q_2 > q_3$ use the vector $z = (-1, -1, 1)$. If $q_1 + q_2 < q_3$ use the vector $z = (1, 1, -1)$.

of arbitrage. Since the gross interest rate is given by $R = (q_1 + q_2)^{-1}$, then in equilibrium $R = q_3^{-1}$.

Before discussing the effect of changes in information on welfare, we show that, given a posterior, if the Planner is constrained to satisfy c's mandatory savings and short-sale constrains he cannot obtain a Pareto improvement. We follow the literature on constrained Pareto optimality and assume that the Planner can freely allocate period 0 consumption, but period 1 consumption can only be allocate using the existing assets. The allocation assigned by the Planner to agent c is required to satisfy the short-sale constraints and the savings constraint, that is $q_3 z_3^c \geq \theta_3$ where q_3 is the period zero equilibrium price of the bond *before* the Planner redistribute assets and period 0 consumption.

Definition 2. The consumption allocation $x \in \mathbb{R}^6_+$ is constrained feasible if there exists an asset allocation $z \in \mathbb{R}^6$ such that:

1. $x_s^h = w_s^h + z_s^h + z_3^h$, for $s = 1, 2$. $\forall h$,
2. $\sum_h z^h = 0$,
3. $z_s^c \geq \theta_s$, for $s = 1, 2$,
4. $z_3^c \geq \theta_3 R$.

Proposition 2. *There is no constrained feasible allocation that Pareto dominates the financial market equilibrium allocation.*

To prove Proposition 2 we follow the standard proof of constrained Pareto optimality in economies with two period and a single consumption good, with the difference that we request allocations to be constrained feasible.

We define the constrained feasible set under posterior π, CFS(R), as the set of all constrained feasible allocations. Formally:

Definition 3. The constrained feasible set, CFS(R), is the set of all constrained feasible allocations:

$$\begin{aligned}\text{CFS}(R) = \Big\{ & x \in \mathbb{R}^6_+ \mid \exists z \in \mathbb{R}^6 \text{ s.t. } x_s^h = w_s^h + z_s^h + z_3^h \ \forall (h, s), \\ & z_s^c \geq \theta_s \ \forall s, \ z_3^c \geq \theta_3 R \text{ and } \sum_h z^h = 0 \Big\}.\end{aligned}$$

It is straightforward to redefine the constrained feasible set independently of the portfolio of agent c by replacing the lower bounds on the components of this vector straight into the expressions for x_s^c:

Definition 4. Given a posterior π, the constrained feasible set, $\text{CFS}'(R)$, is the set of all constrained feasible allocations:

$$\text{CFS}'(R) = \left\{ x \in \mathbb{R}_+^{2\times 3} \mid x_s^c \geq w_s^c + \theta_s + \theta_3 R \ \forall s, \text{ and } x^u = w - x^c \right\}.$$

It is trivial to show that $\text{CFS}(R) \subset \text{CFS}'(R)$. It is also true that $\text{CFS}'(R) \subset \text{CFS}(R)$.[11] Notice that if we were to write Definition 2 in terms of contingent consumption instead, we would have to rewrite the conditions for feasibility, and replace conditions 3. and 4. by equation (2.3).

2.4 Changes in information and welfare

Proposition 2 tells us that changing the posterior, and hence the information structure, is a necessary condition for obtaining Pareto improvements if the Planner is force to satisfy c's constraints at equilibrium prices.

The mandatory savings constraint for agent c implies the Planner has to satisfy a constraint that depends on equilibrium prices and, therefore, on the agent's posteriors. By changing the information structure, the Planner can change the posterior of the agents and, as R is a function of π, shift the mandatory savings constraints to reach allocations that were not feasible under the original information structure. Thus it may be the case that changing the information structure allows the Planner to reach allocations that are Pareto superior with respect to the starting information structure, but that weren't feasible before the change in information. For example if R is increasing in π, by inducing a posterior below the prior the Planner can enlarge the constrained feasible set. For this reason from now on we make explicit the dependence of the constrained feasible set on the posterior, i.e. we wright $\text{CFS}(R(\pi))$.

We wish to study the welfare consequences of changes in information. Our reference point will always be what we call the *uninformative equilibrium*. Under an *uninformative* information structure, by definition $\pi(y_k) = \pi^0$ for $k = 1, 2, 3$. We define an uninformative equilibrium, as an equilibrium of the economy where agents make decisions on the basis of the prior π_0.

By Proposition 1 there exist at least one vector of prices that clears the market. If the equilibrium is unique, it is clear that this unique equilibrium is the uninformative equilibrium. If there are multiple equilibria, we pick one of them, and set the uninformative equilibrium equal to it for every signal realisation. Therefore, by definition, the uninformative equilibrium is signal invariant.

Definition 5. Consider an information structure Y. Let $x(y_k)$ be an (equilibrium) allocation when signal y_k is observed in the economy with information structure Y.

[11] To prove that $\text{CFS}'(R) \subset \text{CFS}(R)$, fix $z_3^c = \theta_3 R$ and notice that with the existing asset structure, and ignoring c's constraints, the agents can generate any period 1 consumption in $\mathbb{R}_+^2$. Hence there exist a z_s^c such that $w_s^c + z_s^c + z_3^c = x_s^c \geq w_s^c + \theta_s + \theta_3 R$ for $s = 1, 2$. Our choice of z_3^c implies: $z_s^c \geq \theta_s$ for $s = 1, 2$. Finally $x^u = w - x^c$ implies $\sum_h z^h = 0$.

Then, $x(Y) = (x(y_1), x(y_2), x(y_3))$ is an (equilibrium) allocation under information structure Y. If Y is the uninformative information structure, then we define an uninformative equilibrium allocation as the equilibrium allocation under information structure Y, say $x(Y)$, that satisfies $x(y_1) = x(y_2) = x(y_3)$.

Starting from the uninformative equilibrium, we will look for an *informative* information structures that allows the Planner to obtain welfare improvements in a sense that will be made precise below. We consider two welfare notions depending on whether one evaluate allocations ex-ante or ex-post, i.e. before or after observing the signal. Formally:

Definition 6. Let $\overline{Y}$ and $\widehat{Y}$ be an uninformative and an informative information structure respectively. Let $\pi(y_k)$ and $\text{pr}(y_k)$ for $k = 1, 2, 3$, be the posteriors and the probabilities of the posteriors under $\widehat{Y}$, respectively. We say $x(\overline{Y})$ is ex-ante constrained Pareto efficient under $\widehat{Y}$ if there is no allocation under information structure $\widehat{Y}$, call it $x(\widehat{Y}) = (\widehat{x}(y_1), \widehat{x}(y_2), \widehat{x}(y_3))$, such that $\widehat{x}(y_k) \in \text{CFS}(R(\pi(y_k)))$ for all k and:

$$\sum_k \text{pr}(y_k) V(\widehat{x}^h(y_k), \pi(y_k)) \geq \sum_k \text{pr}(y_k) V(\overline{x}^h(y_k), \pi(y_k)),$$

for all $h \in H$ and with strict inequality for some h.

Definition 7. Let $\overline{Y}$ and $\widehat{Y}$ be an uninformative and an informative information structure respectively. Let $\pi(y_k)$ and $\text{pr}(y_k)$ for $k = 1, 2, 3$, be the posteriors and the probabilities of the posteriors under $\widehat{Y}$, respectively. We say $x(\overline{Y})$ is ex-post constrained Pareto efficient under information structure $\widehat{Y}$ if there is no allocation under information structure $\widehat{Y}$, call it $x(\widehat{Y}) = (\widehat{x}(y_1), \widehat{x}(y_2), \widehat{x}(y_3))$, such that $\widehat{x}(y_k) \in \text{CFS}(R(\pi(y_k)))$ for all k and:

$$V(\widehat{x}^h(y_k), \pi(y_k)) \geq V(\overline{x}^h(y_k), \pi(y_k)),$$

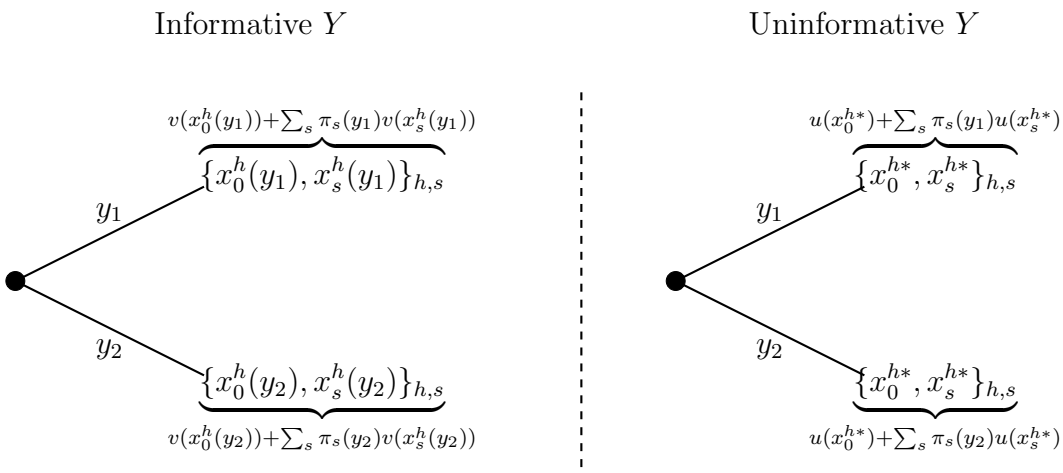
for all $h \in H$ and with strict inequality for some h, for all k.

From Definition 6 we see that the concept of an ex-ante constrained Pareto improvement is related to an (constrained) improvement in expected value. Definition 7 provides a stronger concept of improvement: a (constrained) Pareto improvement for every signal at the new posteriors. Clearly an ex-post improvement is a sufficient condition for an ex-ante improvement.

Consider the right diagram in Figure 2.1. There we show, for a two dimensional information structure, the uninformative equilibrium under each signal, denoted by x^*. Remember that this equilibrium is constant across signals by definition. In the left diagram we show some constrained feasible allocation under every signal of an informative information structure. Notice that $x(y_1)$ and $x(y_2)$ may differ. When

looking for ex-ante improvements, we take each utility level under the informative information structure, and compute the expected value for every agent using the probability of the signals as weights. Then we compare this expected value with the utility that the agents obtain in the uninformative equilibrium. We look if they are better off before receiving information. When looking for ex-post improvements, we compare the $x(y_1)$ allocation with x^*, and $x(y_2)$ with x^* separately. If there is a constrained Pareto improvement for both cases, then we have an ex-post constrained Pareto improvement. We look if they are better off after receiving each signal.

Figure 2.1: Ex-ante and Ex-post constrained Pareto improvements.



To prove the existence of ex-ante improvements one could work with the ex-ante utility function used in Definition 6. Noticing that the existence of ex-ante improvement is closely related to the shape of the boundary of the utility possibility set, the Pareto frontier, helps us to simplify the problem. Formally, we define the Pareto frontier as a function $F : [0,1] \times \mathbb{R} \mapsto \mathbb{R}$ given by:

$$\begin{aligned} F(\pi, V) = \{ \underset{x^c}{\text{Max}} \quad & v(x_0^c) + \sum \pi_s v(x_s^c) \\ \text{s. t.} \quad & x_s^c \geq w_s^c + \theta_s + \theta_3 R(\pi) \text{ for } s = 1, 2, \\ & v(w - x_0^c) + \sum \pi_s v(w - x_s^c) \geq V \ \} . \end{aligned} \tag{2.4}$$

Since we assume v is strictly increasing, the last constraint in (2.4) will always bind at a solution. Furthermore, Bayes' rule implies that posteriors have to satisfy: $\sum_k \text{pr}(y_k)\pi(y_k) = \pi^0$ for any information structure. This condition is what Kamenica and Gentzkow (2011) call *Bayes-plausibility* of posteriors. Since by definition $V(x^c, \pi) = v(x_0^c) + \sum \pi_s v(x_s^c)$, we can relate the existence of an ex-ante constrained Pareto improvement to the properties of function F.

Proposition 3. *Let π^0 and V_0^h be the common prior and the equilibrium utility level of agent h in the uninformative equilibrium , respectively. The uninformative equilibrium allocation is not ex-ante constrained Pareto efficient if and only if there exist vectors $(\tau_1, \tau_2, \tau_3) \in \Delta^2$, $(\pi_1, \pi_2, \pi_3) \in [0,1]^3$ and $(V_1, V_2, V_3) \in \mathbb{R}^3$ such that:*

1. $\sum_k \tau_k \pi_k = \pi^0$,

2. $\sum_k \tau_k V_k = V_0^u$,

3. $\sum_k \tau_k F(\pi_k, V_k) > F(\pi^0, V_0^u)$.

The proof of Proposition 3 is direct from the definitions of an ex-ante improvement and the Pareto frontier, using constrained Pareto optimality of the uninformative equilibrium, and the fact that the uninformative equilibrium is signal invariant. Proposition 3 shows that we can work with the Pareto frontier to study the existence of an ex-ante improvement. In Corollary 4 we relate this to the concavification of F.

Definition 8. Let $g : X \mapsto \mathbb{R}$. The concavification of g is given by: cav $g(x) \equiv \sup\{y|(x,y) \in co(g)\}$, where $co(g)$ denotes the convex hull of the graph of g.

From Definition 8 it is direct to notice that cav g is concave, and everywhere weakly greater than g. While there are alternative definitions for the concavification of a function, the one we use is useful when applying Carathéodory's theorem when proving the "if" part in Corollary 4.

Corollary 4. *There exist an ex-ante constrained Pareto improvement over the uninformative equilibrium if and only if the concavification of F at (π^0, V_0^u) is greater than $F(\pi^0, V_0^u)$. There cannot be an ex-ante constrained Pareto improvement if F is concave.*

In Kamenica and Gentzkow (2011) the authors study a symmetric information model where a sender can choose a signal she reveals to a receiver, who takes a (contractable) action that affects the payoff of both agents. They ask whether there exist a signal that leads the receiver to take an action that benefits the sender, relative to the equilibrium where actions are taken based on the prior beliefs. They show that there exist such a signal if and only if the concavification of the expected utility of the Sender evaluated at the action based on the prior beliefs lies above the expected utility of the Sender evaluated at the same point.

Similar to their analysis we can relate the existence of an information structure that makes the Planner better off with the concavification of the Pareto frontier. We can think of the Planner as the sender, who can choose the information structure. Agents c and u play the role of the receiver. They observe the signal and their actions (excess demands) determine the equilibrium interest rate, which affect the Planner's payoff through the constrained feasible set.

A difference between our model and those considered in Kamenica and Gentzkow (2011), is that in their paper Bayes plausibility is the only constraint that the sender has to satisfy. Our sender also has to make sure that the levels of utility he assigns to agent u in every signal are such that she is indifferent with respect to the

uninformative equilibrium in expected value. In a sense this makes the problem of the sender more flexible as his expected payoff function can fail to be concave in beliefs, utility levels, or both. However is makes the analysis more complicated.

The *concavification* of a function has also been used to analyze the value of knowledge in a game theoretical context. Aumann and Maschler (1995) use it to analyze whether a player benefits from using his knowledge of chance's choice in a infinite 2-person game. To our knowledge there are no papers taking this approach when studying the value of public information in a market economy. The "usual" approach is to work with information structure defined as we did in section 2 (or by joint probability distributions as in Gottardi and Rahi (2014)). From now on we think of information structures as a vector of posteriors and a vector of probabilities of signals such that *Bayes-plausibility* is satisfied.

2.5 Ex-post improvements

In a similar model, but without c's savings and short-sale constraints, Hakansson, Kunkel, and Ohlson (1982) showed that if prior beliefs and information structures are homogeneous, utility functions are time-additive, and the uninformative equilibrium is fully Pareto efficient, then there cannot be an ex-ante improvement over the uninformative equilibrium. Thus no ex-post improvements exist either.

Consider an uninformative equilibrium where c's savings constraint is not binding. Since with the existent assets agent h can generate any consumption vector in $\mathbb{R}^3_+$, this implies that such an equilibrium belongs to the Pareto set and hence is fully Pareto efficient. Therefore a necessary condition for the existence of an ex-post improvement is for c's savings constraint to be binding in the uninformative equilibrium. To ensure this is the case, we need to introduce some assumptions in the parameters of the model.

First, endowments in state one and state two have to differ, otherwise there is no uncertainty. We assume that c is relatively rich in state one:

$$w_1^c > w_2^c. \tag{A1}$$

Second, a priori we do not know which of the two lower bounds on c's consumption in period one is higher. However it simplifies the analysis if knew their relative size. We assume that the lower bound on x_1^c is the biggest of the two:

$$w_1^c + \theta_1 \geq w_2^c + \theta_2. \tag{A2}$$

Define the priors $\underline{\pi}_1$, $\underline{\pi}_2$ and $\underline{\pi}$ as:

$$\underline{\pi}_1 \equiv \frac{2\theta_3 + 2(\theta_1 + w_1^c) - w_0^c - w_2^c}{w_1^c - w_2^c},$$
$$\underline{\pi}_2 \equiv \frac{2\theta_3 + 2(\theta_2 + w_2^c) - w_0^c - w_2^c}{w_1^c - w_2^c},$$
$$\underline{\pi} \equiv \max\{\underline{\pi}_1, \underline{\pi}_2\} = \underline{\pi}_1.$$

Proposition 5 show that $\underline{\pi}$ is a threshold such that if the prior is below $\underline{\pi}$, then the uninformative equilibrium is not fully Pareto efficient. For $\underline{\pi}$ to be in the interval $(0, 1)$ we have to assume:

$$0.5(w_0^c + w_1^c) > \theta_1 + \theta_3 + w_1^c > 0.5(w_0^c + w_2^c). \tag{A3}$$

Proposition 5. *Assume (A1) and (A2) hold. Equilibria are full Pareto efficient if and only if $\pi \geq \underline{\pi}$. If an equilibrium is not fully Pareto efficient, then the lower bound on x_1^c is binding at that equilibrium, i.e. $x_1^c = w_1^c + \theta_1 + \theta_3 R(\pi)$.*

Assumptions (A1) and (A2) are without loss of generality, we just specify the relative magnitudes to simplify our analysis. If we write (A1) with the reverse inequality, then the condition in Proposition 5 would also hold with the reverse inequality. If (A2) were to hold with the reverse inequality then $\underline{\pi} = \underline{\pi}_2$, but we would have to adjust assumption (A3) to make sure that $\underline{\pi}_2 \in (0, 1)$. Finally, assumption (A3) is important. This condition ensures that we can partition $[0, 1]$ in two non-empty sets such that equilibria are first best if and only if the posterior is in one of the partitions, or equilibria are not first best and at least one of the lower bounds on c's period one consumption is binding if the posterior is in the other partition. We exploit this difference in the section on ex-ante improvements.

From now on assume $\pi^0 \leq \underline{\pi}$. We showed at the end of section 3 that equilibria are constrained Pareto efficient. The following proposition shows that if the constrained feasible set does not change with changes in the information structure, the Planner cannot obtain an ex-post constrained Pareto improvement.

Proposition 6. *Let x be the equilibrium allocation under π^0. There is no $\widehat{x} \in CFS(R(\pi^0))$ and $\pi^1 \in [0, 1]$ such that $V(\widehat{x}^h, \pi^1) \geq V(x^h, \pi^1)$ for all h and with strict inequality for some h.*

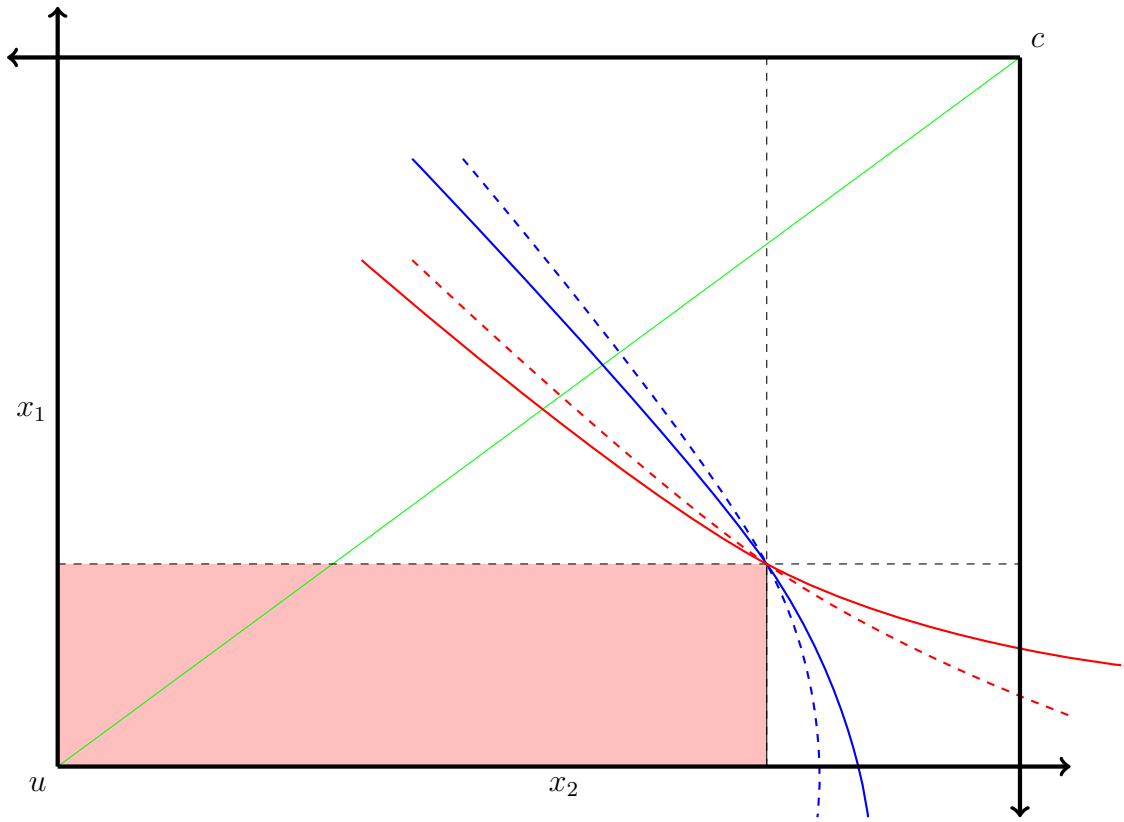
By constrained Pareto optimality, it follows that the allocations that make both agents better off at the prior are not constrained feasible. Proposition 6 shows that this remains true even if we use a different posterior to compute the utility that both agents get from consumption.

To understand the idea behind Proposition 6 see Figure 2.2. For simplicity. assume there is no consumption in period 0, and that c faces some lower bound on period 1 consumption, depicted by the dashed straight lines in the figure. The

shaded area represents the constrained feasible set. Assume both constraints are binding at equilibrium and that this equilibrium is not in the Pareto set, which is depicted by the diagonal of the box. The solid indifference curves represent this equilibrium. We need to understand how the indifference curves that pass trough the initial equilibrium allocation change when we increase the posterior of state one. By definition the new indifference curves, depicted by the dashed lines, pass through the initial equilibrium. To keep u indifferent we need to increase consumption in one state and reduce consumption in the other state, but this implies reducing one of c's consumptions. By doing that we are choosing an allocation outside of the constrained feasible set.

If only one constraint is binding at equilibrium,[12] then the argument we just gave is not enough. If only the constraint on x_1^c is binding, then $x_1^c > x_2^c$ and $x_2^u > x_1^u$.[13] Thus we could reduce x_1^u and increase x_2^u to make u indifferent without leaving the constrained feasible set. But as $\text{MRS}_{1,2}^c < \text{MRS}_{1,2}^u$ this change cannot make c better off, where $MRS_{s,s'}^h = \frac{\partial v(x_1^h)}{\partial x_1^h} / \frac{\partial v(x_2^h)}{x_2^h}$.

Figure 2.2: Indifference curves rotate at the equilibrium allocation when the posterior of state one is increased.



Thus the only way in which we can obtain ex-post improvements is if we can find a vector of posteriors such that the equilibrium interest rate under each signal is

[12] By Proposition 5 there are no equilibria that are not fully Pareto efficient and the lower bound on x_1^c is not binding.

[13] In this case the lower bound on x_2^c is somewhere to the right of the vertical dashed line depicted in the figure.

below the interest rate of the uninformative equilibrium. In that case the constrained feasible set in the uninformative equilibrium is contained in the constrained feasible set of each signal of the *informative* information structure. The next Theorem is a direct consequence of Proposition 6.

Theorem 7. *Let π^k for $k = 1, 2, 3$ be the posteriors under some informative information structure. The uninformative equilibrium is ex-post constrained Pareto efficient if and only if $R(\pi^k) \geq R(\pi^0)$ for some k.*

The analysis above shows that ex-post improvements are possible if and only if the constrained feasible set under π^0 is contained in the constrained feasible set of the posteriors under *all* signals of the *informative* information structure. This is the case when $R(\pi^0) > R(\pi(y_k))$ for all k. When considering marginal changes in the posterior, then an ex-post improvement exists if and only if R attains a strict local maximum at π^0. This differs with Gottardi and Rahi (2014) who show that when markets are incomplete ex-post improvements are always feasible, and in no way this depends on how prices react to the new information.

The next corollary is implied directly by Theorem 7:

Corollary 8. *If R is monotonic, then the uninformative equilibrium is ex-post constrained Pareto efficient for every informative information structure.*

The shape of the equilibrium interest rate as a function of the posterior is key for the existence of ex-post improvements. In appendix 2.H we show that Corollary 8 is not an empty statement. In Lemma 3 we use the implicit function theorem to prove that when (A2) holds with equality R is a monotone increasing function below $\underline{\pi}$ for *any* v strictly increasing and strictly concave.

2.6 Ex-ante improvements

In this section we study the existence of ex-ante constrained Pareto improvements. In the first subsection we study the shape of the function F, and in particular we investigate if it is concave. In the second subsection we look for sufficient conditions for ex-ante improvements to exist.

For the discussion in the main text we assume that assumption (A2) is satisfied with equality, i.e $w_1^c + \theta_1 = w_2^c + \theta_2$. The proofs in the general case are relegated to the appendix.

2.6.1 Non-concavity of the Pareto frontier

In Corollary 4 we have shown that a necessary condition for the existence of ex-ante improvements is for F to be non-concave. To prove the non-concavity of $F(\pi, V)$, we show it is not concave in π when V is given by u's utility level at the uninformative

equilibrium, $V^u(\pi^0)$. We define a new function $f(\pi, \pi^0) \equiv F(\pi, V^u(\pi^0))$ and show that $f(\pi, \pi^0)$ is not concave in π. Formally,

$$\begin{aligned} f(\pi, \pi^0) = F(\pi, V^u(\pi^0)) = \{ \operatorname*{Max}_{x^c} \quad & v(x_0^c) + \sum \pi_s v(x_s^c) \\ \text{s. t.} \quad & x_s^c \geq w_s^c + \theta_s + \theta_3 R(\pi) \text{ for } s = 1, 2, \\ & v(w - x_0^c) + \sum \pi_s v(w - x_s^c) \geq V^u(\pi^0) \ \} . \end{aligned} \tag{2.5}$$

When $w_1^c + \theta_1 = w_2^c + \theta_2$, there is *always* full smoothing in period 1 consumption in equilibrium, as both lower bounds on period 1 consumption coincide. The solution to the maximization problem in (2.5) is characterized by $x_1^c = x_2^c$ for all π.[14] Therefore the function f coincides with the function $\widehat{f}$ defined below:

$$\begin{aligned} \widehat{f}(\pi, \pi^0) = \{ \operatorname*{Max}_{\{x_0^c, x_1^c\}} \quad & v(x_0^c) + v(x_1^c) \\ \text{s. t.} \quad & x_1^c \geq w_1^c + \theta_1 + \theta_3 R(\pi) \ , \\ & v(w - x_0^c) + v(w - x_1^c) \geq V^u(\pi^0) \ \} . \end{aligned}$$

Notice that the function $\widehat{f}$ depends on π only through $R(\cdot)$ as the objective function is independent of the posterior. If $\widehat{f}$ is defined at $\underline{\pi}$, then $\widehat{f}(\pi, \pi^0)$ is equal to some constant $\underline{f}(\pi^0)$ for all $\pi \geq \underline{\pi}$, as $R(\pi) = 1$ for all $\pi \geq \underline{\pi}$, i.e. the constrained feasible set is constant to the right of $\underline{\pi}$.[15] Also notice that $\widehat{f}(\pi^0, \pi^0) \geq \widehat{f}(\pi, \pi^0)$ for all $\pi \geq \underline{\pi}$, this follows from the fact that the constrained feasible set under $\pi \geq \underline{\pi}$ is a proper subset of the constrained feasible set under π^0, as $R(\pi^0) < 1 = R(\pi)$.[16] Therefore $\widehat{f}(\pi^0, \pi^0)$ cannot be lower than $\widehat{f}(\underline{\pi}, \pi^0)$. In fact, as the constrained feasible set under every posterior, and in particular π^0, is a convex set and the maximiser of the problem defining $\widehat{f}(\pi, \pi^0)$ belongs to CFS$(R(\pi^0))$ for all $\pi \geq \underline{\pi}$, strict concavity of v implies that $\widehat{f}(\pi^0, \pi^0) > \widehat{f}(\pi, \pi^0)$ for all $\pi \geq \underline{\pi}$.

Consider the posteriors $\pi^1 = \pi^0$ and $\pi^2 = 1$. There exist a $\tau \in (0, 1)$ such that $\tau\pi^1 + (1-\tau)\pi^2 = \underline{\pi}$. But $\tau\widehat{f}(\pi^1, \pi^0) + (1-\tau)\widehat{f}(\pi^2, \pi^0) > \widehat{f}(\underline{\pi}, \pi^0)$. Hence the function $\widehat{f}$, and therefore f, is not concave in π.

Proposition 9. *Assume f is defined at $(\underline{\pi}, \pi^0)$. If (A2) is satisfied with equality, or if (A2) is satisfied with strict inequality and f is not differentiable at $\underline{\pi}$, then f is not concave.*

In the general case, when (A2) is satisfied with strict inequality, it is not longer true that the objective function in the problem defining f is independent of the

[14] If the constraints on x_1^c and x_2^c are both binding at the solution, then $x_1^c = x_2^c$ at the solution. If both constraints are not binding, then $x_0^c = x_1^c = x_2^c$ at the solution. If only the constraint on x_1^c is binding, then at the solution $x_0^c = x_2^c > x_1^c$, but then if we assign to c the constrained feasible allocation $(x_0^c, \widetilde{x}^c, \widetilde{x}^c)$, where $\widetilde{x}^c = \pi x_1^c + (1-\pi)x_2^c$, we make both agents better off as v is strictly concave. Therefore we cannot have $x_0^c = x_2^c > x_1^c$ at a solution. To discard the case when $x_0^c = x_1^c > x_2^c$ at equilibrium, we use the same logic.

[15] The same argument implies that if $\widehat{f}$ is defined at $\underline{\pi}$ for a given π^0, then it is defined for all $\pi \in [\underline{\pi}, 1]$.

[16] See appendix 2.H.

posterior, as the solution may not display full smoothing in period 1. This implies that we cannot use the approach explained above to prove non-concavity. Lemma 4 in appendix 2.H shows that $R(\pi) < 1$ for all $\pi < \underline{\pi}$, so while it is still true that CFS($R(\pi)$)$\subset$CFS($R(\pi^0)$) for all $\pi \geq \underline{\pi}$, now the objective function is (weakly) increasing in π, as it is always true that $x_1^c \geq x_2^c$. Therefore when comparing π^0 with a posterior to the right of $\underline{\pi}$ we have two forces going in opposite directions.

To prove non-concavity in the general case we do an analysis around $\underline{\pi}$. We show that when f is non-differentiable at $\underline{\pi}$ its slope marginally to the right of $\underline{\pi}$ is bigger than its slope marginally to the left of $\underline{\pi}$. This condition allows us to prove non-concavity independently of the actual sign of these slopes. A sufficient condition for non-differentiability of f is:

$$w_0^u - w_2^u \neq -2(\theta_1 + \theta_3). \tag{A4}$$

Numerical results confirm that the intersection between the subset of parameter values that satisfy assumptions (A0)-(A3) and (A2) with strict inequalities, and the subset of parameter values that satisfy (A4) is not empty.

So far we have assumed that f is defined at $(\underline{\pi}, \pi^0)$. In appendix 2.I, Lemma 8 provides a sufficient condition for f to be defined at $(\underline{\pi}, \pi^0)$. Using the implicit function theorem we show that if π^0 is sufficiently close to $\underline{\pi}$, then f is indeed defined at $(\pi^0, \underline{\pi})$. When (A2) holds with equality, looking at Figure 2.3 it is not difficult to imagine why this is the case. If π^0 is arbitrarily close to $\underline{\pi}$, then points A and C are arbitrarily close, as the allocation in A is always in the interior of the box, we obtain feasibility of point C.

Notice that the analysis above helps us explain why there cannot be an improvement if R is independent of π, or in the standard pure exchange economy with complete markets and no aggregate uncertainty. In both cases the objective function and the constraints are independent of π. Strict concavity of the objective function and convexity of the constrained set imply concavity of F.

2.6.2 Sufficient conditions for an ex-ante improvement

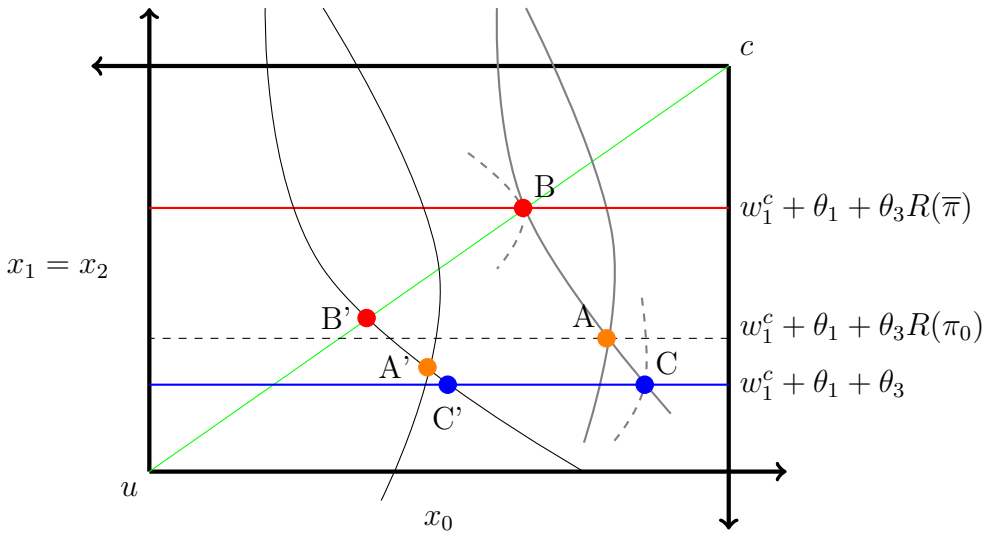
In the previous subsection we argued that f is not concave. In this subsection we first determine the actual shape of f and that of its concavification. We complete the analysis with a sufficient condition for the existence of ex-ante constrained Pareto improvements. In what follows we assume $\pi^0 < \underline{\pi}$ to ensure the uninformative equilibrium is not fully Pareto efficient.

By Corollary 4 and the definition of f, to see if ex-ante improvements are possible we need to compare cav f and f at the point (π^0, π^0). For simplicity, in the main text we assume that $f(\pi, \pi^0)$ is defined for all $\pi \in [0, 1]$ and that (A2) holds with equality. In the appendix we provide general proofs when (A2) holds with weak

inequality.

Remember that when (A2) is satisfied with equality we always have $x_1^c = x_2^c$ in equilibrium. Therefore we can depict the equilibrium in an Edgeworth box with period 0 and period 1 consumption in the horizontal and vertical axes, respectively. In Figure 2.3, the dashed horizontal line represents the lower bound on period one consumption and the uninformative equilibrium is given by a point like A for any $\pi_0 < \underline{\pi}$. The Pareto set, characterized by full consumption smoothing across periods (and states), is depicted by the diagonal line connecting the bottom left and upper right corners of the box.

Figure 2.3: Uninformative equilibrium and allocations $\overline{x}^h(\pi^0)$ and $\underline{x}^h(\pi^0)$.



Let $\mathbb{1} \in \mathbb{R}^3$ be the vector of ones. Consider the allocation

$$\overline{x}^c(\pi^0) = \mathbb{1}\overline{x}(\pi^0) \quad \text{and} \quad \overline{x}^u(\pi^0) = \mathbb{1}w - \overline{x}^c(\pi^0),$$

where $\overline{x}^c(\pi^0)$ is a constant consumption plan for c that gives u her uninformative equilibrium utility, $V^u(\pi^0)$. That is, $\overline{x}(\pi^0)$ is the solution to

$$2v(w - x) = V^u(\pi^0).$$

Allocation $(\overline{x}^c(\pi^0), \overline{x}^u(\pi^0))$ is depicted as point B in Figure 2.3, the point where u's indifference curve that passes through the uninformative equilibrium allocation (point A) intersects the Pareto set. Notice that this allocation gives c the highest possible utility conditional on u being indifferent with respect to the uninformative equilibrium. Thus, if $(\overline{x}^c(\pi^0), \overline{x}^u(\pi^0))$ is constrained feasible for π, $f(\pi, \pi^0) = V(\overline{x}^c(\pi^0), \pi)$.

Consider also the allocation

$$\underline{x}^c(\pi^0) = \left(\underline{x}(\pi^0), w_1^c + \theta_1 + \theta_3, w_1^c + \theta_1 + \theta_3\right) \quad \text{and} \quad \underline{x}^u(\pi^0) = \mathbb{1}w - \underline{x}^c(\pi^0),$$

where $\underline{x}(\pi^0)$ is the period 0 consumption level for c such that if she consumes $w_1^c+\theta_1+\theta_3$ in both states in period 1, then u is indifferent with respect to the uninformative equilibrium. That is, $\underline{x}(\pi^0)$ is the solution to:

$$v(w-x)+v(w_1^u-\theta_1-\theta_3)=V^u(\pi^0).$$

Allocation $(\underline{x}^c(\pi^0),\underline{x}^u(\pi^0))$ is depicted as point C in Figure 2.3. Notice that $\underline{x}^c(\pi^0)$ is constrained feasible for every posterior as $R(\pi)\leq 1$ for every $\pi\in[0,1]$.

By construction u is indifferent between the uninformative equilibrium and allocations $\underline{x}^u(\pi^0)$ and $\overline{x}^u(\pi^0)$. We have drawn Figure 2.3 in such a way that these allocations are well defined, however it may be the case that one or both of them are not feasible. Lemma 9 in appendix 2.J gives us a sufficient condition for both allocations to be feasible. Assume for now that both $\overline{x}(\pi^0)$ and $\underline{x}(\pi^0)$ are in $[0,w]$.

Define $\overline{\pi}(\pi^0)$ as a belief such that $\overline{x}(\pi^0)$ is c's equilibrium consumption in state one. That is, $\overline{\pi}(\pi^0)$ is the value of π that solves:

$$\overline{x}(\pi^0)=w_1^c+\theta_1+\theta_3R(\pi).$$

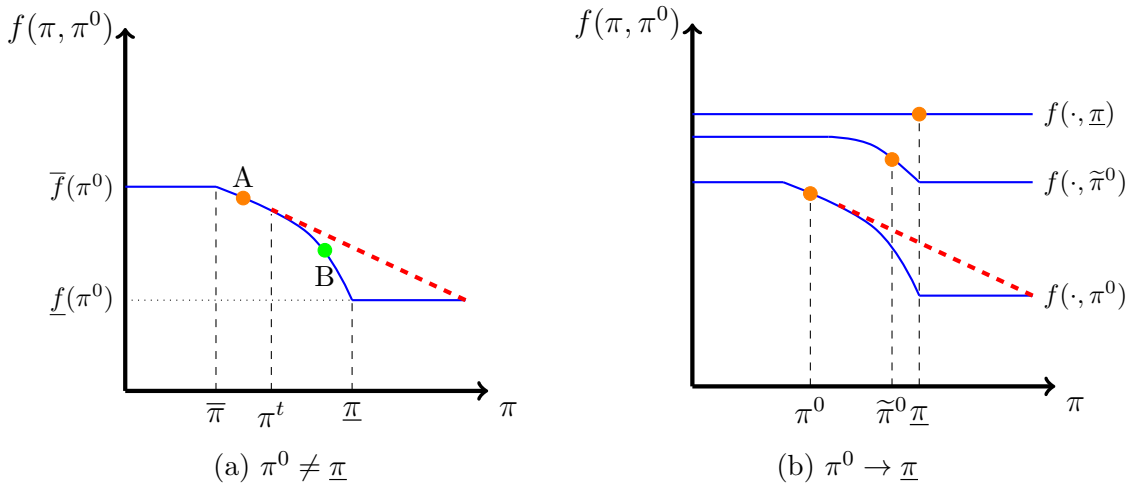
Graphically, $\overline{\pi}(\pi^0)$ is the belief that makes the lower bound on state one consumption to pass through point B in Figure 2.3. Assume for now that $\overline{\pi}(\pi^0)\in[0,1]$. Note that $\overline{\pi}(\pi^0)<\pi^0$ as R is monotone increasing below $\underline{\pi}$ whenever (A2) holds with equality.

As $R(\pi)$ is increasing in π for all $\pi<\underline{\pi}$, then $\overline{x}^c(\pi^0)$ is constrained feasible for all $\pi\leq\overline{\pi}(\pi^0)$. Consequently, f is constant, say equal to $\overline{f}(\pi^0)$, to the left of $\overline{\pi}(\pi^0)$. Note that $f(\pi^0,\pi^0)$ is c's utility associated with the indifference curve through point A, $f(\overline{\pi}(\pi^0),\pi^0)$ is c's utility associated with the indifference curve through point B. Since point A is not in the Pareto set by Proposition 5, then $f(\pi,\pi^0)>f(\pi^0,\pi^0)$ for all $\pi\leq\overline{\pi}(\pi^0)$.

The function f is constant to the left of $\overline{\pi}(\pi^0)$ and in the previous subsection we argue it is also constant to the right of $\underline{\pi}$, i.e. $f(\pi,\pi^0)=\underline{f}(\pi^0)$ for all $\pi\geq\underline{\pi}$. A priori we do not know the shape that f takes for $\pi\in[\overline{\pi}(\pi^0),\underline{\pi}]$. If f is convex in this interval, then we have ex-ante improvements, as the concavification of f in this case is equal to f to the left of $\overline{\pi}(\pi^0)$ and to the right of $\underline{\pi}$, and the straight line joining $f(\underline{\pi},\pi^0)$ and $f(\overline{\pi}(\pi^0),\pi^0)$ elsewhere.

In Figure 2.4a we have drawn f assuming it is strictly concave for $\pi\in[\overline{\pi},\underline{\pi}]$. In this case the concavification of f is given by the straight line starting from $f(1,\pi^0)$ that is tangent to f, and cav f coincides with f for posteriors below the tangency point. Denote the tangency point by π^t. Therefore, we have ex-ante improvements if and only if $\pi^0>\pi^t$.

Figure 2.4: f and its concavification when (A2) holds with equality.



(a) $\pi^0 \neq \underline{\pi}$ (b) $\pi^0 \to \underline{\pi}$

Suppose we have a common prior π^0 below π^t so that cav $f(\pi^0, \pi^0)$ coincides with $f(\pi^0, \pi^0)$ as in point A in Figure 2.4a. A hasty conjecture would be that for a common prior π^0 to the right of π^t, cav $f(\pi^0, \pi^0)$ would lie strictly above $f(\pi^0, \pi^0)$, like in point B in the figure. However, notice that $\overline{\pi}$, π^t, $\overline{f}$ and $\underline{f}$ are all functions of π^0. So by increasing π^0 we change $\overline{\pi}$ and π^t and also the value that f attains at those points and at the point $\underline{\pi}$, as by changing the common prior we are changing the expected wealth of the agents in the uninformative equilibrium and the position of u's indifference curve at this equilibrium.

To understand how f changes as we increase the prior it is useful to go back to Figure 2.3. In Figure 2.3 we show the effect of an increase in the prior on allocations $\overline{x}^h$ and $\underline{x}^h$. If we change π^0 to $\widetilde{\pi}^0 > \pi^0$, as u is relatively poor in period 1 (assumption (A1)), and as utilities are independent of the probability of state one (since (A2) holds with equality), u's indifference curve at the uninformative equilibrium is shifted down. The same argument gives us that c's indifference curve is shifted upwards and the uninformative equilibrium moves from point A to point A', i.e. c attains a higher utility at the uninformative equilibrium. Thus $f(\widetilde{\pi}^0, \widetilde{\pi}^0) > f(\pi^0, \pi^0)$. By definition, $\overline{f}(\widetilde{\pi}^0)$ is the utility of agent c at the point where u's uninformative equilibrium cuts the Pareto set. As u's indifference is shifted down it cuts the Pareto set in a point that gives c more consumption in every period, thus $\overline{f}(\widetilde{\pi}^0) > \overline{f}(\pi^0)$ (see points B' and B). As R is monotone increasing below $\underline{\pi}$ and $w_1^c + \theta_1 + R(\overline{\pi}(\widetilde{\pi}^0)) > w_1^c + \theta_1 + R(\overline{\pi}(\pi^0))$, then $\overline{\pi}(\widetilde{\pi}^0) > \overline{\pi}(\pi^0)$. Finally, as u's indifference curve is shifted down and $\underline{x}_1^c = w_1 + \theta_1 + \theta_3$ is unchanged the point where it cuts the horizontal line at $\underline{x}_1^c = w_1 + \theta_1 + \theta_3$ gives more period zero consumption to c, i.e. $\underline{x}_0^c$ is increased. Thus $\underline{f}(\widetilde{\pi}^0) > \underline{f}(\pi^0)$ (see point C' and C).

The analysis above explains why f is shifted upwards when we increase agents' prior, as shown in Figure 2.4b. The extreme case when $\pi^0 = \underline{\pi}$ is also shown in Figure 2.4b. In that case the uninformative equilibrium is in the Pareto set, and it is constrained feasible for every posterior, hence f is flat, and it lies above $f(\pi, \pi^0)$

for all $\pi \in [0,1]$ and all $\pi^0 < \underline{\pi}$. See point A in Figure 2.5.

Thus we see that even though f fails to be concave, it is not direct to see if cav f is above f at the point (π^0, π^0). If for a given prior we have $\pi^0 < \pi^t(\pi^0)$, it is also not clear if by changing the prior we can move the economy to an uninformative equilibrium that is not ex-ante constrained Pareto efficient.

We will show that ex-ante improvements exist if π^0 is close to $\underline{\pi}$. Let's redefine the prior on state one as $\pi^0(\epsilon) \equiv \underline{\pi} - \epsilon$, with $\epsilon \in [0, \underline{\pi}]$. Therefore now all $\overline{x}$, $\underline{x}$ and $\overline{\pi}$ are functions of ϵ.

The uninformative equilibrium displays full smoothing if and only if $\pi^0 \geq \underline{\pi}$, hence $\overline{\pi}(\epsilon) = \pi^0(\epsilon)$ if and only if $\epsilon = 0$. Above we argue that $\overline{\pi}(\epsilon) < \pi^0(\epsilon)$ for all $\epsilon > 0$.[17] Consider the posteriors $\pi^1 = \overline{\pi}(\epsilon)$, $\pi^2 = 1$ and $\pi^3 = \pi^0(\epsilon)$.

Notice that $\overline{x}^c(\epsilon)$ is constrained feasible under π^1, and $\underline{x}^c(\epsilon)$ is constrained feasible under π^2. For an ex-ante improvement to exist, it is *sufficient* to find $\tau^1 \in (0,1)$ and $\tau^3 \in [0,1)$ such that $\tau^1 + \tau^3 < 1$ and:

$$\begin{aligned}
\pi^0(\epsilon) &= \tau^1\pi^1 + (1-\tau^1-\tau^3)\pi^2 + \tau^3\pi^3, \\
V_0^c &< \tau^1 V(\overline{x}^c(\pi^0), \pi^1) + (1-\tau^1-\tau^3)V(\underline{x}^c(\pi^0), \pi^2) + \tau^3 V_0^c,
\end{aligned}$$

since u is indifferent with respect to the uninformative equilibrium by construction.

Bayes plausibility implies that τ^1 has to satisfy:

$$\tau^1 = (1-\tau^3)\frac{\pi^0(\epsilon) - \pi^2}{\pi^1 - \pi^2} = (1-\tau^3)\frac{\pi^0(\epsilon) - 1}{\overline{\pi}(\epsilon) - 1} \equiv \widetilde{\tau}(\epsilon).$$

Let

$$\overline{V}(\epsilon) \equiv V(\overline{x}^c(\pi^0), \overline{\pi}(\epsilon)),$$

$$\underline{V}(\epsilon) \equiv V(\underline{x}^c(\pi^0), 1),$$

$$V_0(\epsilon) \equiv V_0^c.$$

Agent c is ex-ante indifferent between mixing $\overline{V}(\epsilon)$, $\underline{V}(\epsilon)$ and $V_0(\epsilon)$, and the uninformative equilibrium if she gets $\overline{V}(\epsilon)$ with probability:

$$\tau^1 = (1-\tau^3)\frac{V_0(\epsilon) - \underline{V}(\epsilon)}{\overline{V}(\epsilon) - \underline{V}(\epsilon)} \equiv \widehat{\tau}(\epsilon).$$

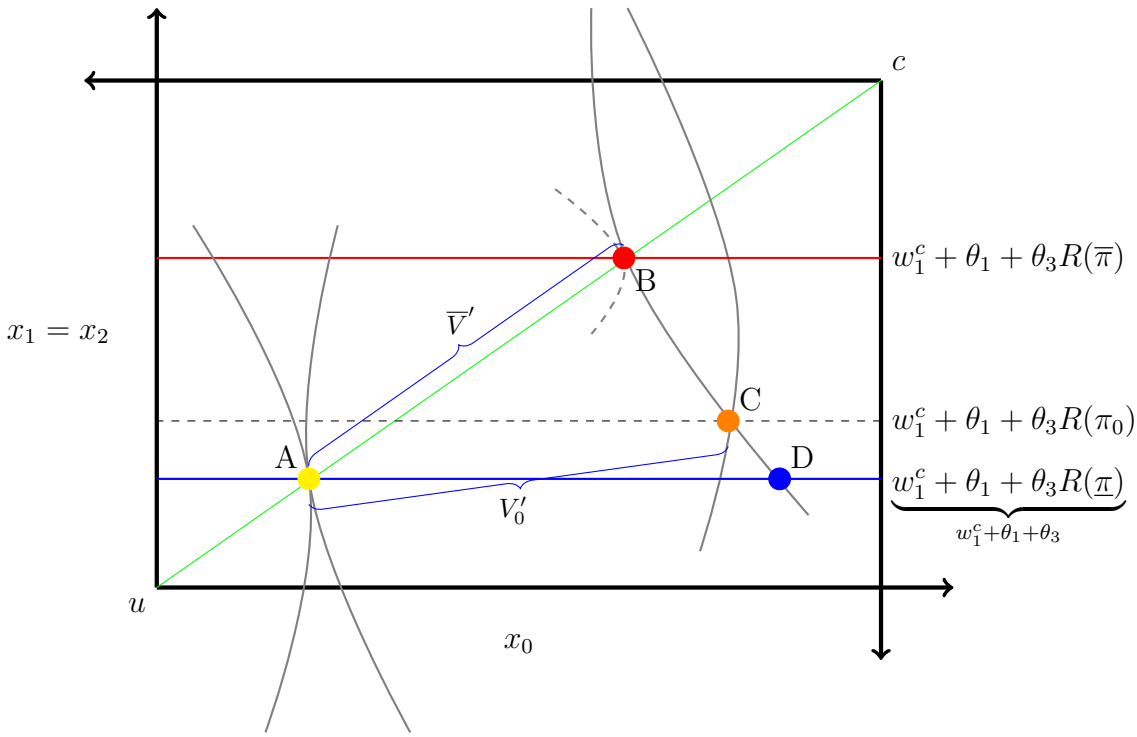
If $\widehat{\tau}(\epsilon) \leq \widetilde{\tau}(\epsilon)$, then there exist an ex-ante constrained Pareto improvement as $\overline{V}(\epsilon) > V_0(\epsilon) > \underline{V}(\epsilon)$. Equality between $\widehat{\tau}$ and $\widetilde{\tau}$ is sufficient as the Planner could still smooth period 0 consumption between signals.

The limit as ϵ goes to 0 of $\widetilde{\tau}(\epsilon)$ is equal to $1 - \tau_3$, as $\overline{\pi}(\epsilon)$ converges to $\pi^0(\epsilon)$ as

[17] At a first glance there is no reason to think that $\overline{\pi}(\epsilon) \in [0,1]$. For example if $\overline{x}(\pi^0) < \min\{w_1^c + \theta_1 + \theta_3 R(\pi)\}$ then $\overline{x}(\pi^0)$ is never constrained feasible, or if R is strictly increasing below $\underline{\pi}$, we may have $\overline{x}(\pi^0) < w_1^c + \theta_1 + \theta_3 R(0)$ in which case the same conclusion applies. In appendix 2.J we formally show that $\overline{\pi}$ is well defined in an interval around $\epsilon = 0$.

ϵ tends to zero. If we can show that the limit as ϵ tends to 0 of $\widehat{\tau}(\epsilon)$ is equal to $\overline{a}(1-\tau_3)$ for some constant $\overline{a} < 1$, then we would have proved the existence of an ex-ante improvement.

Figure 2.5: Indifference curves at $\epsilon = 0$.



In Figure 2.5 we show the picture that led us to think that the limit of $\widehat{\tau}$ is strictly lower than $1-\tau_3$. For this, we need the limit of $\frac{V_0(\epsilon)-\underline{V}(\epsilon)}{\overline{V}(\epsilon)-\underline{V}(\epsilon)}$ to be strictly lower than one. From the figure we see that the ratio is strictly lower than one for every $\epsilon > 0$. Agent c's indifference curve at point C represents $V_0(\epsilon)$, her indifference curve at point B represents $\overline{V}(\epsilon)$, and her indifference curve at point D represents $\underline{V}(\epsilon)$. Thus it is direct to see that $\frac{V_0(\epsilon)-\underline{V}(\epsilon)}{\overline{V}(\epsilon)-\underline{V}(\epsilon)} < 1$ for all $\epsilon > 0$. However this is not sufficient to prove that the limit of the ratio is strictly below one, as the ratio is not defined at $\epsilon = 0$. Point A depicts the equilibrium when $\epsilon = 0$. In that point $\overline{V}$, $\underline{V}$ and V_0 take the same value.

If we increase ϵ marginally starting from $\epsilon = 0$, the indifference curves of the agents are shifted from point A to those in the right of the figure. From c's perspective, point B is associated with higher utility relative to point C, i.e. $\overline{V}(\epsilon) > V_0(\epsilon)$. As we can think of the (right) derivatives of $\overline{V}$ and V_0 at $\epsilon = 0$ as the distance between c's indifference curves at points A and and B, or points A and C respectively, this suggest that if we apply L'Hôpital's rule to compute the limit of $\frac{V_0(\epsilon)-\underline{V}(\epsilon)}{\overline{V}(\epsilon)-\underline{V}(\epsilon)}$, we obtain a limit strictly lower than one.

Theorem 10. *Suppose either (A2) is satisfied with equality or (A4) holds. Then there exist a $\delta > 0$ such that if $\epsilon \in (0, \delta)$ then the uninformative equilibrium is not ex-ante constrained Pareto efficient.*

However when using L'Hôpital's rule to compute the limit of $\widehat{\tau}(\epsilon)$, we need to apply it twice as in the limit when the increment of ϵ goes to zero, V_0', $\overline{V}'$, and $\underline{V}'$ coincide. Therefore we use an alternative approach to compute the limit of $\widehat{\tau}(\epsilon)$. First we define the function $g(a,\epsilon) = a\overline{V}(\epsilon) + (1-a)\underline{V}(\epsilon) - V_0(\epsilon)$, mapping $\mathbb{R} \times [0, \underline{\pi}]$ into $\mathbb{R}$. Then we show that there exist $\overline{a} < 1$ such that $g(\overline{a}, \epsilon)$, as a function of ϵ, attains a *strict* local minimum at $(\overline{a}, 0)$, and that $g(\overline{a}, 0) = 0$. Thus $g(\overline{a}, \epsilon) > 0$ for all ϵ in a neighborhood of zero. Using the definition of g, $g(\overline{a}, \epsilon) > 0$ is equivalent to $\overline{a}\overline{V}(\epsilon) + (1-\overline{a})\underline{V}(\epsilon) - V_0(\epsilon) > 0$, or $\frac{V_0(\epsilon)-\underline{V}(\epsilon)}{\overline{V}(\epsilon)-\underline{V}(\epsilon)} < \overline{a} < 1$ for all ϵ in the neighborhood. This last expression tells us that the limit of $\widehat{\tau}$ is strictly below $1 - \tau_3$.

When assumption (A2) holds with strict inequality, the difference is that we do not know a priori if $\overline{\pi}(\epsilon)$ is above or below $\pi^0(\epsilon)$. In appendix 2.J we show that $\overline{\pi}$ is a continuous function in an interval around $\epsilon = 0$. As $\overline{\pi} = \pi^0$ if and only if $\epsilon = 0$, continuity implies that $\overline{\pi}(\epsilon)$ is always above or below $\pi^0(\epsilon)$. If $\overline{\pi}(\epsilon) < \pi^0(\epsilon)$, then the analysis explained above is still valid. If $\overline{\pi}(\epsilon) > \pi^0(\epsilon)$, then we need to set $\pi^2 = \pi^0(\epsilon) - \gamma$ for some fixed and small $\gamma > 0$.

2.7 Conclusion

We have shown that in economies where savings and short-sale constraints may prevent equilibrium from being fully Pareto efficient, public information may have positive social value. Information-driven price changes may allow a benevolent social planner facing the same information and asset constraints, to obtain *ex-ante* constrained Pareto improvements under quite general conditions. Unlike Gottardi and Rahi (2014), ex-post improvements are attainable only under special conditions for the equilibrium interest rate. The reaction of prices due to the arrival of new information is a necessary condition for information to have social value. Thus we need to be careful when judging the welfare implications of such price changes. Chapter 1 documents that the Chilean authorities viewed the arrival of new information as bad for the economy because it affected asset prices. Our analysis showed that a Planner could take advantage of such a situation and improve welfare. However, our result on ex-post improvements suggest that the authorities are right to be worried if they are concerned with welfare from an ex-post point of view.

Our results provide new insights on the value of public information in exchange economies where equilibria are not necessarily fully Pareto efficient. Gottardi and Rahi (2014) show that with incomplete markets a Planner can obtain ex-post improvements for any initial information structure, by locally changing the information agents receive before trading. In our setting, constrained ex-post improvements are possible only under special circumstances: the uninformative equilibrium interest rate needs to be above the equilibrium interest rate for every signal of the informative information structure. In the main result of the chapter we show that ex-ante

improvements exist if the common prior is sufficiently close to the threshold dividing first best equilibria from equilibria where the savings constraint and short-sale constraint are binding. These ex-ante improvements are not marginal in nature. We consider a situation where posteriors can be far away from the prior, in fact one of the posteriors is equal to one.

Finally, we have shown that the study of the value of information in exchange economies can be simplified by adopting the techniques used in the literature on Bayesian persuasion. The simplification lies in realizing that information structures can be defined as a vector of posteriors and a vector of probabilities of the posteriors such that Bayes plausibility is satisfied. By taking the Planner as the sender and the agents as the receivers, whose actions affect the Planner's payoff by changing equilibrium prices and the constrained feasible set, we can relate the value of information to the concavity of the Planner's utility function, the Pareto frontier. The difference between our model and the standard problem in the Bayesian persuasion literature is that on top of Bayes plausibility, the sender has to make sure that the utility levels he assigns to the unconstrained agent, leave her indifferent with respect to the uninformative equilibrium.

Our analysis used the condition that the total endowment is constant in every period and state of the world. Possible extension of this work may be to relax this assumption and see if our results extend to this more general setting. Also, in our current model there is no reason why agents should face mandatory constraints, therefore the best thing the Planner could do is to remove the mandatory savings altogether. We plan to extend our model to the case of preferences involving hyperbolic discounting or temptation, where having mandatory savings constraints can be optimal. Finally, we have been silent about the existence of ex-ante improvement through marginal changes in posteriors, in future work we plan to relate this to Radner and Stiglitz (1984).

References

Aumann, Robert J and Michael Maschler (1995). *Repeated games with incomplete information*. MIT press.

Barr, Nicholas and Peter Diamond (2008). *Reforming pensions: Principles and policy choices*. Oxford University Press.

Blackwell, David (1951). "Comparison of experiments". In: *Proceedings of the second Berkeley symposium on mathematical statistics and probability*.

Cass, David and Karl Shell (1983). "Do sunspots matter?" In: *Journal of political economy* 91.2, pp. 193–227.

Diamond, Peter A (1967). "The role of a stock market in a general equilibrium model with technological uncertainty". In: *The American Economic Review* 57.4, pp. 759–776.

Gottardi, Piero and Rohit Rahi (2014). "Value of information in competitive economies with incomplete markets". In: *International Economic Review* 55.1, pp. 57–81.

Hakansson, Nils H, J Gregory Kunkel, and James A Ohlson (1982). "Sufficient and necessary conditions for information to have social value in pure exchange". In: *The Journal of Finance* 37.5, pp. 1169–1181.

Hirshleifer, Jack (1971). "The private and social value of information and the reward to inventive activity". In: *The American economic review* 61.4, pp. 561–574.

Kamenica, Emir and Matthew Gentzkow (2011). "Bayesian persuasion". In: *The American Economic Review* 101.6, pp. 2590–2615.

Magill, Michael and Martine Quinzii (2002). *Theory of incomplete markets*. Vol. 1. Mit press.

Marshall, John M (1974). "Private incentives and public information". In: *The American Economic Review*, pp. 373–390.

Maurer, Thomas Andreas and Ngoc-Khanh Tran (2016). "The Hirshleifer Effect in a Dynamic Setting".

Ng, David S (1977). "Pareto-optimality of authentic information". In: *The Journal of Finance* 32.5, pp. 1717–1728.

Radner, Roy and Joseph Stiglitz (1984). "A Nonconcavity in the Value of Information". In: *Bayesian models in economic theory* 5, pp. 33–52.

Appendix

2.A Proof of Proposition 1

To prove the existence of an equilibrium, instead of working with the financial market equilibrium defined in Definition 1, it is easier to write the model in terms of contingent consumption. For this, we define a non-arbitrage equilibrium following Magill and Quinzii (2002). We then prove the equivalence between both types of equilibria. First, let's define alternative budget sets for both agents:

$$\begin{aligned}
\mathcal{B}^c(p, w^c) &= \left\{ x^c \in \mathbb{R}^3_+ \,\middle|\, \sum_{s=0}^{2} p_s x^c_s = \sum_{s=0}^{2} p_s w^c_s,\ x^c_s \geq w^c_s + \theta_s + \frac{p_0 \theta_3}{p_1 + p_2}\ \forall s \in \{1, 2\} \right\}, \\
\mathcal{B}^u(p, w^u) &= \left\{ x^u \in \mathbb{R}^3_+ \,\middle|\, \sum_{s=0}^{2} p_s x^u_s = \sum_{s=0}^{2} p_s w^u_s \right\}.
\end{aligned} \tag{2.6}$$

where $p = (p_0, p_1, p_2)$ is the vector of contingent consumption prices. Using the budget sets in (2.6) we can now define a non-arbitrage equilibrium:

Definition 9. Given π, a non-arbitrage equilibrium is a collection of prices $\overline{p} \in \mathbb{R}^3_{++}$ and consumption plans $\overline{x} = (\overline{x}^u, \overline{x}^c) \in \mathbb{R}^6_+$, such that:

1. $\overline{x}^h \in \text{ argmax } \left\{ V^h(x^h, \pi) \,\middle|\, x^h \in \mathcal{B}^h(\overline{p}, w^h) \right\}\ \forall h \in \{c, u\}$,
2. $\sum_h \left(\overline{x}^h - w^h \right) = 0$.

Having defined both type of equilibria, we can now prove their equivalence. First we show the equivalence of both budget sets in the absence of arbitrage.

Lemma 1. *Let $q = (1, q^1) \in \mathbb{R}^4_{++}$, and $p = (1, p^1) \in \mathbb{R}^3_{++}$. If $q^1 = p^1 \Phi$, then $\mathbb{B}^h(q, w^h) = \mathcal{B}^h(p, w^h)$ for all h.*

Proof. Notice that the non-arbitrage condition $q^1 = p^1 \Phi$ implies: $q^1_1 = p^1_1$, $q^1_2 = p^1_2$, and $q^1_3 = p^1_1 + p^1_2$. Assume $x^h \in \mathbb{B}^h(q, w^h)$ for all h, then:

$$x^h_0 - w^h_0 = -q^1 z^h = -p^1 \Phi z^h = -\sum_s p^1_s \Phi_s z^h, \tag{2.7}$$

where Φ_s represents row s of matrix Φ. The budget constraint in period 1 implies:

$$x^h_s - w^h_s = \Phi_s z^h,$$

thus we can rewrite (2.7) as:

$$x_0^h - w_0^h = -\sum_s p_s^1 (x_s^h - w_s^h). \tag{2.8}$$

Furthermore as $z_s^c \geq \theta_s$ for $s = 1, 2$, and $q_3^1 z_3^c \geq \theta_3$:

$$x_s^c = w_s^c + z_s^c + z_3^c \geq w_s^c + \theta_s^c + \frac{\theta_3}{q_3^1}. \tag{2.9}$$

Equations (2.8) and (2.9) imply that $x^h \in \mathcal{B}^h(p, w^h)$ for all h.

Assume now $x^h \in \mathcal{B}^h(p, w^h)$ for all h. Then $x_s^c - w_s^c \geq \theta_s + \theta_3/(p_1^1 + p_2^1)$. Fix $z_3^c = \theta_3/q_3^1$. As $q_3^1 = p_1^1 + p_2^1$, there exist $z_s^c \geq \theta_s$ for $s = 1, 2$, such that:

$$x_s^c - w_s^c = z_s^c + z_3^c \geq \theta_s + \theta_3/q_3^1.$$

Replacing $x_s^c = w_s^c + z_s^c + z_3^c$ into $\sum_{s=0}^2 p_s x_s^c = \sum_{s=0}^2 p_s w_s^c$ we obtain $x_0^c + \sum_k q_k^1 z_k^c = w_0^c$. These two results imply that $x^c \in \mathbb{B}^h(q, w^h)$. For u the result follows from noticing that she can freely choose the $z^u \in \mathbb{R}^3$. Then for any $x_s^u - w_s^u$ there exist $z^u \in \mathbb{R}^3$ such that $x_s^u - w_s^u = z_s^u + z_3^u$. □

From the equivalence between the two budget sets under the no arbitrage condition, when can prove the equivalence between the two type of equilibria.

Lemma 2.

1. *If $(\overline{x}, \overline{z}, \overline{q})$ is a financial market equilibrium with $\overline{q} = (1, \overline{q}^1)$, then $(\overline{x}, \overline{p})$, with $\overline{p} = (1, \overline{p}^1)$, and $\overline{p}^1$ satisfying $\overline{q}^1 = \overline{p}^1 \Phi$, is a non-arbitrage equilibrium.*

2. *If $(\overline{x}, \overline{p})$ is a non-arbitrage equilibrium with $\overline{p} = (1, \overline{p}^1)$, then there exist portfolios $\overline{z}^u$ and $\overline{z}^c$ and asset prices $\overline{q} = \overline{p}^1 \Phi$ such that $(\overline{x}, \overline{z}, (1, \overline{q}))$ is a financial market equilibrium.*

Proof.

1. By Lemma 1 $\overline{x}^h \in \operatorname{argmax}\{V(x^h, \pi) \,|\, x^h \in \mathbb{B}^h(\overline{q}, w^h)\}$ imply $\overline{x}^h \in \operatorname{argmax}\{V(x^h, \pi) \,|\, x^h \in \mathcal{B}^h(\overline{p}, w^h)\}$ for all h. As $\sum_h z_l^h = 0$ for $l = 1, 2, 3$, then $\sum_h (x_s^h - w_s^h) = 0$ for $s = 0, 1, 2$.

2. By Lemma 1 $\overline{x}^h \in \operatorname{argmax}\{V(x^h, \pi) \,|\, x^h \in \mathcal{B}^h(\overline{p}, w^h)\}$ imply $\overline{x}^h \in \operatorname{argmax}\{V(x^h, \pi) \,|\, x^h \in \mathbb{B}^h(\overline{q}, w^h)\}$ for all h. As $\sum_h (x_s^h - w_s^h) = 0$ for $s = 0, 1, 2$, then $\sum_h z_l^h = 0$ for $l = 1, 2, 3$.

□

Using the equivalence between financial market equilibria and non-arbitrage equilibria, we can now prove the existence of a financial market equilibrium following the standard proof involving contingent consumption.

Proof of Proposition 1:

We will prove the existence of a non-arbitrage equilibrium, and then invoke Lemma 2.

It is well-known that agent u's optimal demand function for contingent consumption is continuous, homogeneous of degree zero, satisfies Walras' law, satisfies non-negativity, and has the appropriate boundary behavior. The proof can be found, for example, in Hildenbrand and Kirman (1988). Below we will argue that c's demand function has the same properties. Let's start with continuity: As $V(\cdot)$ is continuous in consumption, if we can show that c's budget correspondence, defined in (2.6), is compact-valued and continuous, continuity of c's demand function follows from the maximum theorem. That $\mathcal{B}^c(p,w)$ is compact-valued is direct when prices are stictly positive. Let's study its continuity.

Upper hemi continuity: Take a sequence $(p_n, w_n) \in \mathbb{R}^4_{++} \times \mathbb{R}^3_+$ converging to $(p,w) \in \mathbb{R}^4_{++} \times \mathbb{R}^3_+$. Let $(x_n) \in \mathbb{R}^3_+$ be a sequence such that $(x_n) \in \mathcal{B}^c(p_n, w_n)\ \forall n$. Our assumptions on θ_l for $l = 1,2,3$, guarantee that the budget correspondence is never the empty set. Clearly the sequence (x_n) is bounded below by the zero vector. Let $\overline{p} = \max_s (\sup_n p_{s,n}) > 0$, where $s \in \{0,1,2\}$; $w^* = \max_s (\sup_n w_{s,n}) > 0$, and $\underline{p} = \min_s (\inf_n p_{s,n}) > 0$.[18] Then $x_{s,n} \leq \frac{\overline{p} w^*}{\underline{p}}$ for all s and n. Hence the sequence (x_n) is bounded, and by the Bolzano-Weierstrass Theorem it has a convergent subsequence: $x_{n_k} \to x$. Since $x_{n_k} \in \mathcal{B}^c(p_{n_k}, w_{n_k})$: $\sum_{s=0}^2 p_{s,n_k} x_{s,n_k} \leq \sum_{s=0}^2 p_{s,n_k} w_{s,n_k}$, and $x_{s,n_k} \geq w_{s,n_k} + \theta_s + \frac{p_{0,n_k}\theta_3}{p_{1,n_k}+p_{2,n_k}}$ for $s = 1,2$. Taking limits it's direct to see that $x \in \mathcal{B}^c(p,w)$, as weak inequalities hold at the limit.

Lower hemi continuity: Fix $(p,w) \in \mathbb{R}^4_{++} \times \mathbb{R}^3_+$. Let O be an open subset of $\mathbb{R}^3_+$ such that $\mathcal{B}^c(p,w) \cap O \neq \emptyset$. Suppose $\mathcal{B}^c(p,w)$ is not lower hemi continuous at (p,w), then for every $n \in \mathbb{N}$ there exist a (p_n, w_n) within a $\frac{1}{n}$-neighborhood of (p,w) such that $\mathcal{B}^c(p_n, w_n) \cap O = \emptyset$. Take any $x \in \mathcal{B}^c(p,w) \cap O$ such that x is in the interior of $\mathcal{B}^c(p,w)$. Then $\lambda x \in \mathcal{B}^c(p,w) \cap O$ for $\lambda \in (0,1)$ sufficiently close to 1. But as (p_n, w_n) converges to (p,w) and $px - pw < 0$ and $x_s > w_s^x + \theta_s + p_0\theta_3/(p_1+p_2)$ for $s = 1,2$, continuity of $\lambda p_n x - p_n w_n$ and $x_s - w_s^x - \theta_s - p_0\theta_3/(p_1+p_2)$ implies that $\lambda p_n x - p_n w_n < 0$, and $x_s > w_{s,n}^c + \theta_s + p_{0,n}\theta_3/(p_{1,n}+p_{2,n})$, for $s = 1,2$; for n large enough. But then $x \in \mathcal{B}^c(p_n, w_n)$ for such n. This contradicts O and $\mathcal{B}^c(p_n, w_n)$ being disjoint. Thus the budget correspondence is continuous, and we obtain continuity of demand using the maximum theorem.

Walras' law follows from strong monotonicity of preferences, and homogeneity of

[18] Let ϵ be such that $p_s - \epsilon > 0$, where p_s is the limit of the convergent sequence $(p_{s,n})$. Then there exist N such that for all $n \geq N$: $p_s + \epsilon > p_{s,n} > p_s - \epsilon > 0$. Therefore $p_{s,n} > \min\{p_s - \epsilon, \min\{p_{s,1}, ..., p_{s,N-1}\}\} > 0$ for all n, as the sequence only takes strictly positive values. This implies $\inf_n p_{s,n} > 0$. A similar argument gives us that w^* and $\overline{p}$ are strictly positive and do not diverge to infinity.

degree zero follows from the fact that the budget set does not change if we multiply all prices by the same constant.

Define the excess demand function of agent h as:

$$\phi^h(p) = x^h(p, w^h) - w^h,$$

where $x^h(p, w^h)$ is h's Walrasian demand function. The aggregate excess demand function of the economy is:

$$\phi(p) = \sum_h \phi^h(p).$$

As $x^h(\cdot)$ is continuous, homogeneous of degree zero and satisfy Walras' law, these properties are directly inherited by $\phi(p)$. As $x^h(\cdot) \geq 0$, this implies there exist an $m > 0$ such that $\phi_s(p) > -m$ for every s and all p. Finally we have to prove that if $p^n \to p$, where $p \neq 0$ and $p_s = 0$ for some s, then $\max\{\phi_0(p^n), \phi_1(p^n), \phi_2(p^n)\} \to \infty$. Suppose this is not true. Then the sequences $\max\left\{\phi_0^h(p^n), \phi_1^h(p^n), \phi_2^h(p^n)\right\}$ does not diverge to infinity for any h, and so each of the $\phi_s^h(p^n)$ for $s = 0, 1, 2$ does not diverge to infinity for any h. Assume the value of c's endowment is different from zero at the limit. Then, there is a bounded set $B \subset \mathbb{R}^3_+$ such that $\phi^c(p^n) \cap B \neq \emptyset$ for infinitely many n. Then the sequence $(\phi^c(p^n)) \in B$ has a convergent subsequence. Let ϕ^{c*} be the limit of this subsequence, and define $x^{c*} = \phi^{c*} + w^c$. Then $x^{c*} \in \mathbb{R}^3_+$ and $px^{c*} = pw^c$. Take any other $x^c \in \mathbb{R}^3_+$ such that $px^c \leq pw^c$. If $px^c < pw^c$, then for n large enough $p^n x^c < p^n w^c$. Let $x_n^c = \phi^c(p^n) + w^c$, then $x_n^c \succsim x^c$. By continuity of preferences: $x^{*c} \succsim x^c$. If $px^c = pw^c$ we can find a sequence $(\widehat{x}_n^c)$ converging to x^c with $p\widehat{x}_n^c < pw^c$, but then $x^{*c} \succsim \widehat{x}_n^c$, and by continuity of preferences: $x^{*c} \succsim x^c$. But this is a contradiction since by strong monotonicity the demand of c at p is not well defined , because by consuming more of the good with price equal to zero, she can increase her utility at no cost. If the value of c's endowment at the limit is equal to zero, then the result follows from doing the sames analysis for u as total endowment w is assumed to be strictly positive.

As the excess demand function is defined for all strictly positive price vectors, and satisfy all the properties explained above, existence of equilibrium follows from Proposition 17.C.1 in Mas-Colell, Whinston, and Green (1995). □

2.B Proof of Proposition 2

Let $x = (x^u, x^c)$ be the competitive equilibrium allocation and normalize $q_0 = 1$. Assume x is not constrained Pareto optimal, then there exist a constrained feasible allocation $\overline{x}$ and a supporting portfolio $\overline{z}$ such that $V(\overline{x}^h, \pi) \geq V(x^h, \pi)$ for all h and, say, $V(\overline{x}^c, \pi) > V(x^c, \pi)$. As $\overline{x}$ is feasible, $\sum_h \overline{x}_s^h - w_s^h \leq 0$ for $s = 1, 2$. This implies $\Phi \sum_h \overline{z}^h \leq 0$. As equilibrium prices q satisfy no arbitrage:[19] $q \sum_h \overline{z}^h \leq 0$. Local non

[19]Otherwise the optimization problem of agent u has no solution.

satiation of preferences imply that: $\overline{x}_0^u + \sum_{s=1}^3 q_s \overline{z}_s^u \geq w_0^u$. Similarly, as $V(\overline{x}^c, \pi) > V(x^c, \pi)$: $\overline{x}_0^c + \sum_{s=1}^3 q_s \overline{z}_s^c > w_0^c$. Adding across consumers: $\sum_h \overline{x}_0^h + q \sum_h \overline{z}^h > w$. Thus $\sum_h \overline{x}_0^h > w$ which contradicts constrained feasibility of $\overline{x}$. □

2.C Proof of Proposition 3

If there exist an ex-ante Pareto improvement, then there exist an informative information structure $\widehat{Y}$, and a constrained feasible allocation under $\widehat{Y}$, $(\widehat{x}(y_1), \widehat{x}(y_2), \widehat{x}(y_3))$ with:

$$\begin{aligned} \sum_k \mathrm{pr}(y_k) V^u(\widehat{x}^u(y_k), \pi(y_k)) &= \sum_k \mathrm{pr}(y_k) V^u(\overline{x}^u(y_k), \pi(y_k)), \\ \sum_k \mathrm{pr}(y_k) V^c(\widehat{x}^c(y_k), \pi(y_k)) &> \sum_k \mathrm{pr}(y_k) V^c(\overline{x}^c(y_k), \pi(y_k)). \end{aligned} \tag{2.10}$$

where $\overline{x}$ is the uninformative equilibrium allocation. Set $\pi_k = \pi(y_k)$ and $\tau_k = \mathrm{pr}(y_k)$ for $k = 1, 2, 3$, where $\pi(y_k)$ and $\mathrm{pr}(y_k)$ are the posteriors and signal probabilities implied by $\widehat{Y}$. Bayes' rule imply that the posteriors under $\widehat{Y}$, $\pi(y_k)$ for all k have to satisfy condition 1 in the proposition. Let $V_k = V^u(\widehat{x}(y_k), \pi(y_k))$ for $k = 1, 2, 3$, then condition 2 is also satisfied. The allocation $(\widehat{x}^c(y_1), \widehat{x}^c(y_2), \widehat{x}^c(y_3))$ satisfy all the constraint in (2.4) when F is evaluated at $(\pi(y_1), V_1)$, $(\pi(y_2), V_2)$, and $(\pi(y_3), V_3)$ respectively. Hence $F(\pi(y_k), V_k) \geq V^c(\widehat{x}^c(y_k), \pi(y_k))$, and using (2.10), condition 3 is satisfied, as the uninformative equilibrium is constrained Pareto efficient by Proposition 2 and $\sum_k \mathrm{pr}(y_k) V^c(\overline{x}^c(y_k), \pi(y_k)) = V_0^c = F(\pi^0, V_0^u)$.

Assume there exist vectors $(\tau_1, \tau_2, \tau_3) \in \Delta^2$, $(\pi_1, \pi_2, \pi_3) \in [0, 1]^3$ and a vector of utility levels $(V_1, V_2, V_3) \in \mathbb{R}^3$ such that conditions 1, 2 and 3 are satisfied. Let $\widehat{x}^c(y_k)$ be the argmax of the optimization problem defining $F(\pi_k, V_k)$, and $\widehat{x}^u(y_k) = w - \widehat{x}^c(y_k)$ for $k = 1, 2, 3$. Then $(\widehat{x}(y_1), \widehat{x}(y_2), \widehat{x}(y_3))$ is constrained feasible. Furthermore $V^u(\widehat{x}^u(y_k), \pi_k) = V_k$, and $V^c(\widehat{x}^c(y_k), \pi_k) = F(\pi_k, V_k)$ for $k = 1, 2, 3$. Then conditions 2, and 3 imply that the allocation $(\widehat{x}(y_1), \widehat{x}(y_2), \widehat{x}(y_3))$ obtains an ex-ante Pareto improvement over the uninformative equilibrium, as $F(\pi^0, V_0^u) = V_0^c = \sum_k \tau_k V^c(\overline{x}^c(y_k), \pi_k)$. □

2.D Proof of Corollary 4

We will start from the last part of the corollary. If F is concave, then $\sum_k \mathrm{pr}(y_k)\pi(y_k) = \pi^0$ and $\sum_k \mathrm{pr}(y_k) V_k = V_0^u$ implies $\sum_k \mathrm{pr}(y_k) F(\pi(y_k), V_k) \leq F(\pi^0, V_0^u)$. Therefore we cannot have an ex-ante improvement by Proposition 3. Next we need to show that if there exist an ex-ante Pareto improvement, then cav $F(\pi^0, V_0^u)$ is greater than $F(\pi^0, V_0^u)$. If F coincide with cav F at the point (π^0, V_0^u), then when $\sum_k \mathrm{pr}(y_k)\pi(y_k) = \pi^0$ and $\sum_k \mathrm{pr}(y_k) V_k = V_0^u$ we have $\sum_k \mathrm{pr}(y_k) \mathrm{cav}\ F(\pi^k, V_k) \leq \mathrm{cav}\ F(\pi^0, V_0^u) = F(\pi^0, V_0^u)$, as the concavification is concave. But by definition cav $F(\pi^k, V_k) \geq F(\pi^k, V_k)$ for all k, therefore $\sum_k \mathrm{pr}(y_k) F(\pi^k, V_k) \leq F(\pi^0, V_0^u)$. Finally, assume the

concavification of F is greater than F at (π^0, V_0^u). Carathéodory's theorem states that if a point m of $\mathbb{R}^3$ lies in the convex hull of the graph of F, then m lies in a 2-simplex with vertices in the graph of F. As the concavification of F is the boundary of the closure of the convex hull of F, cav $F(\pi^0, V_0^u)$ belongs to the boundary of a 2-simplex with vertices in the graph of F. Therefore $(\pi^0, V_0^u, \text{cav}F(\pi^0, V_0^u)) = \sum_{k=1}^{3} \alpha_k m_k$, with $\sum_k \alpha_k = 1$, $\alpha_k \geq 0$ for all k, and where $m_k = (\pi_k, V_k, F(\pi_k, V_k))$ is a point in the graph of F for $k = 1, 2, 3$. As cav $F(\pi^0, V_0^u) > F(\pi^0, V_0^u)$, setting $\text{pr}(y_k) = \alpha_k$ we obtain that $\sum_k \text{pr}(y_k) F(\pi^k, V_k) > F(\pi^0, V_0^u)$. □

2.E Proof of Proposition 5

We start with the first part of the proposition. Let's work with the model in terms of contingent consumption. Assume the equilibrium under π is full Pareto efficient. Full Pareto optimality in our economy is characterized by full consumption smoothing, i.e. $x_0^h = x_s^h$ for $s = 1, 2$ for all h. From agents' optimality conditions it is easy to see that this allocation implies that equilibrium prices are $(p_0, p_1, p_2) = (1, \pi, 1-\pi)$, and $x_1^c = \frac{1}{2}(w_0^c + \pi w_1^c + (1-\pi) w_2^c)$. In equilibrium, as $R(\pi) = 1$, we must have $x_1^c \geq w_1^c + \theta_1 + \theta_3$, hence:

$$\frac{1}{2}(w_0^c + \pi w_1^c + (1-\pi) w_2^c) \geq w_1^c + \theta_1 + \theta_3. \tag{2.11}$$

Solving the inequality in (2.11) for π we obtain:

$$\pi \geq \frac{2\theta_3 + 2(\theta_1 + w_1^c) - w_0^c - w_2^c}{w_1^c - w_2^c} = \underline{\pi}_1. \tag{2.12}$$

Assume now that $\pi \geq \underline{\pi}_1$. From inequality (2.12) we can go back to (2.11) and therefore the allocation $x_s^h = \frac{1}{2}\left(w_0^h + \pi w_1^h + (1-\pi) w_2^h\right)$ for $s = 0, 1, 2$ and for all h satisfy both lower bounds on c's period 1 consumption, and jointly with prices $(p_0, p_1, p_2) = (1, \pi, 1-\pi)$ is the equilibrium when the posterior is π. As there is full smoothing across time and states of the world, the equilibrium is fully Pareto efficient.

Now we prove the inverse of the second part of the proposition. First, notice that in any equilibrium where c's lower bound on state one consumption is not binding, then the lower bound on state 2 consumption cannot be binding. As the lower bound on x_2^c is the smallest of the two lower bound, if it were binding, then c could increase her utility by marginally reducing x_1^c and increasing x_2^c to bring both consumptions closer together. Assume now that in equilibrium the lower bounds on c's state one consumption is not binding (and therefore the lower bound on x_2^c is

also not binding), then from the optimality conditions of both agents we have:

$$\frac{u'(x_s^h)}{u'(x_0^h)} = \frac{p_s}{\pi_s} \text{ for } s = 1, 2, \ \forall h. \tag{2.13}$$

Equation (2.13) implies that the equilibrium allocation satisfies $x_0^h = x_1^h = x_2^h$ for all h. We obtain full Pareto efficiency of the equilibrium. □

2.F Proof of Proposition 6

Assume, as a way of contradiction, that there exist $\widehat{x}$ and π^1 such that $V(\widehat{x}^h, \pi^1) \geq V(x^h, \pi^1)$ for all h and with strict inequality for some h. By convexity of CFS(R), if there exist such Pareto improvement, then there exist a marginal improvement. Let's compute the directional derivative of $V(x^u, \pi^1)$:

$$D_\alpha V_0^u = v'(x_0^u)\alpha_0 + \pi^1 v'(x_1^u)\alpha_1 + (1-\pi^1)v'(x_2^u)\alpha_2, \tag{2.14}$$

where x_s^h is h's equilibrium consumption in state s in the uninformative equilibrium, with period 0 denoted by $s = 0$. For u to be indifferent with respect to the uninformative equilibrium we need $D_\alpha V_0^u = 0$. When u is indifferent we can solve equation (2.14) for α_0:

$$\alpha_0 = \frac{-\left(\pi^1 v'(x_1^u)\alpha_1 + (1-\pi^1)v'(x_2^u)\alpha_2\right)}{v'(x_0^u)}. \tag{2.15}$$

Computing the directional derivative for c, forcing changes in consumption to be feasible, we obtain:

$$D_\alpha V_0^c = -v'(x_0^c)\alpha_0 - \pi^1 v'(x_1^c)\alpha_1 - (1-\pi^1)v'(x_2^c)\alpha_2. \tag{2.16}$$

Feasible changes that leave u indifferent have to satisfy (2.15). Replacing (2.15) into (2.16):

$$D_\alpha V_0^c = -\alpha_1 \pi^1 v'(x_1^u)\left(\frac{v'(x_1^c)}{v'(x_1^u)} - \frac{v'(x_0^c)}{v'(x_0^u)}\right) - \alpha_2(1-\pi^1)v'(x_2^u)\left(\frac{v'(x_2^c)}{v'(x_2^u)} - \frac{v'(x_0^c)}{v'(x_0^u)}\right).$$

If in the uninformative equilibrium all constrains are binding, then $x_0^h \neq x_1^h \neq x_2^h$. In particular we have $x_1^c > x_2^c > x_0^c$ and $x_0^u > x_2^u > x_1^u$. As v' is decreasing:

$$\frac{v'(x_s^c)}{v'(x_s^u)} - \frac{v'(x_0^c)}{v'(x_0^u)} < 0 \text{ for } s = 1, 2. \tag{2.17}$$

Therefore, for c to be better off we need at least one $\alpha_s > 0$. This means increasing u's consumption in one of the states in period 1, or reducing c's consumption. But as both lower bounds are assumed to be binding, those changes are not constrained feasible. There is no constrained feasible allocations that attains a Pareto

improvement.

If only the constraint on x_1^c is binding in the uninformative equilibrium, then $x_0^h = x_2^h$ for all h, and $x_1^c > x_0^c$. In this case equation (2.17) still holds for $s = 1$. For $s = 2$ the expression is equal to zero. This implies that c is made better off if and only if $\alpha_1 > 0$, i.e. if and only if u consumption in $s = 1$ is increased. This change is not constrained feasible and we reach the same conclusion: there is no constrained feasible allocations that attains a Pareto improvement. □

2.G Proof of Theorem 7

Assume without loss of generality that $R(\pi^1) \geq R(\pi^0)$. Then $\text{CFS}(R(\pi^1)) \subseteq \text{CFS}(R(\pi^0))$. By Proposition 6 there is no feasible allocation in $\text{CFS}(R(\pi^0))$, and so in $\text{CFS}(R(\pi^1))$, that Pareto dominates the uninformative equilibrium when utility is computed using π^1.

Assume that $R(\pi^k) < R(\pi^0)$ for $k = 1, 2, 3$. Let x be the uninformative equilibrium allocation. As $x_1^h \neq x_0^h$ for all h, x is not in the Pareto set under any posterior. This means that for π^k there exist a different allocation, call it $\widehat{x}$, in the Pareto set, i.e. such that $V(\widehat{x}^h, \pi^k) \geq V(x^h, \pi^k)$ for all h with strict inequality for some h. If $\widehat{x} \in \text{CFS}(R(\pi^k))$ for all k then the uninformative equilibrium is not ex-post constrained Pareto efficient. If $\widehat{x} \notin \text{CFS}(R(\pi^k))$ for some k, consider the allocation $\widetilde{x} = \lambda x + (1 - \lambda)\widehat{x}$ with $\lambda \in [0, 1]$. For λ sufficiently close to one, we can make $\widetilde{x} \in \text{CFS}(R(\pi^k))$ and $\widetilde{x} \neq x$. Strict convexity of preferences tells us that $V(\widetilde{x}^h, \pi^k) \geq V(x^h, \pi^k)$ for all h with strict inequality for some h. Thus the uninformative equilibrium is not ex-post constrained Pareto efficient. □

2.H Properties of the equilibrium gross interest rate

Lemma 3. *If (A2) is satisfied with equality, i.e.* $w_1^c + \theta_1 = w_2^c + \theta_2$, *then* R *is increasing in* $(0, \underline{\pi})$.

Proof. Assume $w_1^c + \theta_1 = w_2^c + \theta_2$ and $\pi \in (0, \underline{\pi})$.[20] Then when both constraints are binding there is full consumption smoothing in period 1. This implies $q_1/q_2 = \pi/(1 - \pi)$, or $R = \pi/q_1$. From u's FOC $Rv_1' - v_0' = 0$, where $v_s' \equiv v'(x_s^u)$. As both constraints are binding $x_s^u = w_s^u - \theta_s - \theta_3 R$ for $s = 1, 2$; and $x_0^u = w_0^u + \theta_3 + q_1\theta_1 + q_2\theta_2 = w_0^u + \theta_3 + (\pi\theta_1 + (1 - \pi)\theta_2)R^{-1}$. Let $\phi(R, \pi) \equiv Rv_1' - v_0'$, then using the implicit function theorem:

$$R' = -\frac{\phi_\pi}{\phi_R} = \frac{v_0''(\theta_1 - \theta_2)R^{-1}}{v_1' + Rv_1''(-\theta_3) - v_0''\left(\frac{-(\pi\theta_1 + (1-\pi)\theta_2)}{R^2}\right)} > 0, \tag{2.18}$$

[20] it is easy to check that when $w_1^c + \theta_1 = w_2^c + \theta_2$, then $\underline{\pi}_2 = \underline{\pi}_1$.

where the inequality follows from both the numerator and denominator in (2.18) being positive. Assumption $w_1^c + \theta_1 = w_2^c + \theta_2$ implies $\theta_1 - \theta_2 = w_2^c - w_1^c < 0$. As $R > 0$ and $v'' < 0$, the numerator is positive. As $\theta_3 > 0$ and $\theta_s \leq 0$ for $s = 1, 2$; the denominator is also positive. □

It is straightforward to show that for all $\pi \geq \underline{\pi}$: $R(\pi) = 1$. The following lemma shows that for all other posteriors the gross interest rate is strictly less than one.

Lemma 4. *If $\pi < \underline{\pi}$, then $q_1 > \pi$, $q_2 \geq 1 - \pi$, and $R(\pi) < 1$.*

Proof. Under the assumptions of the lemma and given a vector of prices, u's optimization problem has a unique interior solution. Equilibrium prices follow from u's first order conditions, which are sufficient conditions for utility maximization under the assumptions of the lemma. These can be written as:

$$\begin{aligned} q_1 &= \pi \frac{v'(x_1^u)}{v'(x_0^u)}, \\ q_2 &= (1-\pi) \frac{v'(x_2^u)}{v'(x_0^u)}. \end{aligned} \tag{2.19}$$

When $\pi < \underline{\pi}$, the first best allocation is not constrained feasible and so we need to have: $x_1^c > x_0^c$ and $x_2^c \geq x_0^c$. This is equivalent to: $x_1^u < x_0^u$ and $x_2^u \leq x_0^u$. As $v'' < 0$, from (2.19) we see that $q_1 > \pi$ and $q_2 \geq 1 - \pi$, and so $q_1 + q_2 > 1$. This implies $R = (q_1 + q_2)^{-1} < 1$. □

When (A2) is satisfied with equality Lemma 3 gives us continuity of R. If (A2) is satisfied with strict inequality the following lemma gives us continuity of R in an interval around $\underline{\pi}$.

Lemma 5. *There exist a neighborhood around $\underline{\pi}$ such that the equilibrium gross interest rate is a continuous function of π.*

Proof. If (A2) holds with equality the result follows from Lemma 3. Assume (A2) holds with strict inequality and let's work with the model in terms of contingent consumption.

When $\pi = \underline{\pi}$ equilibrium prices are the solution to the system:

$$\begin{aligned} \pi v'\left(x_1^u\right) - p_1 v'\left(x_0^u\right) &= 0, \\ (1-\pi) v'\left(x_2^u\right) - p_2 v'\left(x_0^u\right) &= 0. \end{aligned}$$

When (A2) holds with strict inequality, the lower bound on x_1^c is strictly above the lower bound on x_2^c, thus: $x_1^u = w_1^u - \theta_1 - \theta_3/(p_1+p_2)$, $x_2^u = x_0^u = (w_0^u + p_2 w_2^u + p_1(\theta_1 + \theta_3/(p_1 + p_2$ $p_2) > w_2^u - \theta_2 - \theta_3/(p_1 + p_2)$, where the last inequality follows from the fact that at $\pi = \underline{\pi}$, the equilibrium features full consumption smoothing.

The prices p_1 and p_2 are the endogenous variables, and π is a parameter. This system has a solution at $\pi = \underline{\pi}$, given by $p_1 = \pi$ and $p_2 = 1 - \pi$. If we can show

that the determinant of the Jacobian of endogenous variables is not zero at $\pi = \underline{\pi}$, then the implicit function theorem will give us continuity of equilibrium prices, and therefore continuity of the equilibrium gross interest rate, in a neighborhood of $\underline{\pi}$. The Jacobian of endogenous variables is given by:

$$\begin{pmatrix} \pi v_1'' \frac{\partial x_1^u}{\partial p_1} - v_o' - p_1 v_0'' \frac{\partial x_0^u}{\partial p_1} & \pi v_1'' \frac{\partial x_1^u}{\partial p_2} - p_1 v_0'' \frac{\partial x_0^u}{\partial p_2} \\ (1-\pi) v_2'' \frac{\partial x_2^u}{\partial p_1} - p_2 v_0'' \frac{\partial x_0^u}{\partial p_1} & (1-\pi) v_2'' \frac{\partial x_2^u}{\partial p_2} - v_o' - p_2 v_0'' \frac{\partial x_0^u}{\partial p_2} \end{pmatrix}, \tag{2.20}$$

where $v_s' \equiv \frac{\partial u(x_s^u)}{\partial x_s^u}$. Evaluating (2.20) at $\pi = \underline{\pi}$, where $x_0^u = x_1^u = x_2^u$, $q_1 = \pi$ and $q_2 = (1-\pi)$, it simplifies to:

$$\begin{pmatrix} \underline{\pi} v_1'' \left(\frac{\partial x_1^u}{\partial p_1} - \frac{\partial x_0^u}{\partial p_1} \right) - v_o' & \underline{\pi} v_1'' \left(\frac{\partial x_1^u}{\partial p_2} - \frac{\partial x_0^u}{\partial p_2} \right) \\ 0 & -v_o' \end{pmatrix}, \tag{2.21}$$

The determinant of (2.21) is given by:

$$-\underline{\pi} v_1'' v_0' \left(\frac{\partial x_1^u}{\partial p_1} - \frac{\partial x_0^u}{\partial p_1} \right) + (v_0')^2 > 0, \tag{2.22}$$

where the sign of the expression in (2.22) follows from strict concavity of v, and $\frac{\partial x_1^u}{\partial p_1} - \frac{\partial x_0^u}{\partial p_1} = \frac{\theta_3 - \theta_1}{2-\underline{\pi}} > 0$, as $\theta_3 > 0$ and $\theta_1 \leq 0$. □

Finally we show that (A4) implies that R is not differentiable at $\underline{\pi}$.

Lemma 6. *If $w_0^u - w_2^u \neq -2(\theta_1 + \theta_3)$, then R is not differentiable at $\underline{\pi}$.*

Proof. The right derivative of R at $\underline{\pi}$ is equal to zero as $R(\pi) = 1$ for all $\pi \geq \underline{\pi}$. If (A2) is satisfied with equality, the result follows from Lemma 3. Otherwise, in a interval around $\underline{\pi}$ the equilibrium gross interest rate can be define as:

$$R(\pi) = \begin{cases} R^{FB}(\pi), & \text{if } \pi > \underline{\pi}, \\ R^{B}(\pi), & \text{if } \pi \leq \underline{\pi}. \end{cases}$$

where R^{FB} is the interest rate under first best equilibrium prices, i.e. $R^{FB}(\pi) = 1$ for all π. On the other hand, $R^B(\pi) = (p_1 + 1 - \pi)^{-1}$, where p_1 given by the solution of u's first order conditions when $p_2 = 1 - \pi$ and $x_0^u = x_2^u$.

This definition helps us see that $R_-'(\underline{\pi}) = R'^B(\underline{\pi})$. Using the fact that $R^B(\underline{\pi}) = 1$, $R'^B(\underline{\pi}) = p_1'(\underline{\pi}) - 1$. Therefore we need to show that the first derivative of p_1 with respect to π is different from one. At $\pi = \underline{\pi}$ equilibrium prices follow from the system shown in the proof of Lemma 5. These prices feature $p_2 = 1 - \underline{\pi}$. Using the implicit function theorem:

$$\frac{\partial p_1}{\partial \pi} = \frac{-\left(v_1' + \pi v_1'' \frac{\partial x_1^u}{\partial \pi} - p_1 v_0'' \frac{\partial x_0^u}{\partial \pi} \right)}{\pi v_1'' \frac{\partial x_1^u}{\partial p_1} - v_0' - p_1 v_0'' \frac{\partial x_0^u}{\partial p_1}}.$$

In the limit $v_0' = v_1'$, and it is easy to check that $\frac{\partial x_1^u}{\partial \pi} = -\frac{\partial x_1^u}{\partial p_1}$. Therefore $\partial p_1/\partial \pi = 1$ if and only if $\frac{\partial x_0^u}{\partial \pi} = -\frac{\partial x_0^u}{\partial p_1}$. In equilibria where $p_2 = 1 - \pi$ and the constraint on x_1^c is binding we have:

$$x_0^u = \frac{1}{2-\pi}\left(w_0^u + (1-\pi)w_2^u + p_1\left(\theta_1 + \frac{\theta_3}{p_1 + 1 - \pi}\right)\right).$$

Taking derivatives with respect to p_1 and π:

$$\begin{aligned}
\frac{\partial x_0^u}{\partial p_1} &= \frac{1}{2-\pi}\left(\theta_1 + \frac{\theta_3}{p_1+1-\pi}\right) - \frac{1}{2-\pi}\left(\frac{p_1\theta_3}{(p_1+1-\pi)^2}\right), \\
\frac{\partial x_0^u}{\partial \pi} &= \frac{x_0^u}{2-\pi} + \frac{1}{2-\pi}\left(-w_2^u + \frac{p_1\theta_3}{(p_1+1-\pi)^2}\right).
\end{aligned}$$

So, $\frac{\partial x_0^u}{\partial \pi} = -\frac{\partial x_0^u}{\partial p_1}$ if and only if:

$$\begin{aligned}
&x_0^u = w_2^u - \theta_1 - \theta_3, \\
&w_0^u + (1-\underline{\pi})w_2^u + \underline{\pi}\theta_1 + \underline{\pi}\theta_3 = (2-\underline{\pi})(w_2^u - \theta_1 - \theta_3), \\
&w_0^u - w_2^u = -2(\theta_1 + \theta_3).
\end{aligned}$$

□

2.I Proof of Proposition 9

Here we provide a proof for the case when (A2) is satisfied with strict inequality. First we will argue that f and its argmax are continuous functions at $\underline{\pi}$.

Lemma 7. *Fix π_0. Let π be in the interval around $\underline{\pi}$ where R is continuous and assume f is defined at $f(\pi, \pi^0)$. The function f is continuous in π. Let x^* be the argmax of $f(\pi, \pi^0)$, then x^* is a continuous function of π.*

Proof. The function v is strictly increasing, this implies that the last constraint in the definition of f in equation (2.5) will always bind at a solution. When that equation holds with equality we can solve it for x_0^c obtaining x_0^c as a function of π, π^0 and x_s^c for $s = 1, 2$:

$$x_0^c = g(x_1^c, x_2^c, \pi, \pi^0) \equiv w - v^{-1}\left(V^u(\pi^0) - \sum_s \pi_s v(w - x_s^c)\right).$$

The function f coincides with the function f^1 defined as:

$$\begin{aligned}
f^1(\pi, \pi^0) = \Big\{ \underset{\{x_1^c, x_2^c\}}{\text{Max}} \quad & v(g(x_1^c, x_2^c, \pi, \pi^0)) + \sum \pi_s v(x_s^c) \\
\text{s. t.} \quad & x_s^c \geq w_s^c + \theta_s + \theta_3 R(\pi) \text{ for } s = 1, 2 \Big\}.
\end{aligned}$$

We will argue that f^1 is continuous. Consider the correspondence:

$$\Gamma(\pi) = \left\{(x_1^c, x_2^c) \in [0, w]^2 \,|x_s^c - w_1^c - \theta_1 - \theta_3 R(\pi) \geq 0 \;\forall s = 1, 2\right\},$$

Theorem 2.2. in chapter 7 in De la Fuente (2000) tells us that if the functions defining $\Gamma(\pi)$ are continuous, concave in (x_1^c, x_2^c) for a given π, if $\Gamma(\pi)$ is compact and if there exist $\widehat{x} = (\widehat{x}_1, \widehat{x}_2) \in \Gamma(\pi)$ such that $\widehat{x}_s - w_1^c - \theta_1 - \theta_3 R(\pi) > 0 \;\forall s = 1, 2$, then $\Gamma(\pi)$ is continuous at π. Clearly the functions defining Γ are continuous and concave in consumption, and given a posterior the set $\Gamma(\pi)$ is compact. Assumption (A3) implies $w_1^c + \theta_1 + \theta_3 R(\pi) < w$, therefore the point $\widehat{x} = (w, w)$ satisfy both conditions with strict inequality. Hence $\Gamma(\pi)$ is continuous in π.

As the function v is continuous and strictly increasing, its inverse is continuous. This gives us continuity of the objective function defining f^1. By Berge's theorem of the maximum the function $f^1(\pi, \pi^0)$ is continuous in π and its argmax is nonempty and upper hemicontinuous. As f^1 and f (and their argmax) are equivalent, we obtain continuity of f and upper hemicontinuity of x^*.

Now we will argue that x^* is single valued for a given π, therefore obtaining continuity of x^*. For this we work directly with f. The maximization problem in f is characterized by a constraint set that is convex and a objective function that is strictly concave in consumption for a given π, therefore x^* is the unique optimal solution. □

Proof of Proposition 9:

The derivative of f with respect to π (where it exists) is given by:

$$\frac{\partial f(\pi, \pi^0)}{\partial \pi} = v(x_1) - v(x_2) - \theta_3 R'(\pi)(\lambda_1 + \lambda_2) + \frac{v'(x_0)}{v'(w - x_0)} \left(v(w - x_1) - v(w - x_2)\right),$$

where λ_1 and λ_2 are the Lagrange multipliers associated with the constraints on x_1^c and x_2^c respectively. For all posteriors strictly above $\underline{\pi}$, $R'(\pi) = 0$. At $\underline{\pi}$ the right derivative of R is equal to zero and the left derivative is non-negative. Solving the maximization problem defining f we obtain:

$$\lambda_1 = \frac{v'(x_0^c)}{v'(w - x_0^c)} \pi v'(w - x_1^c) - \pi v'(x_1^c),$$

a continuous function of π. If both lower bounds on consumption are not binding, then $\lambda_1 = \lambda_2 = 0$, and it is straightforward to show that the Planner assigns an allocation featuring full smoothing across time and states. At $\underline{\pi}$ the CFS is a proper subset of CFS$(R(\pi^0))$ $(R(\pi^0) < 1)$. Therefore at $\underline{\pi}$ we must have $\lambda_1 > 0$, otherwise the first best allocation is constrained feasible in the uninformative equilibrium, violating constrained Pareto optimality of the uninformative equilibrium. By continuity, for π slightly below $\underline{\pi}$ it is still true that $\lambda_1 > 0$. If $R'_-(\underline{\pi}) \neq 0$, where

R'_- denotes the left derivative of R, then we have:

$$v'_-(\underline{\pi}, \pi^0) - v'_+(\underline{\pi}, \pi^0) = -\theta_3 R'_-(\underline{\pi})(\lambda_1 + \lambda_2) < 0.$$

$R'_-(\underline{\pi})$ is different from zero by Lemma 6. We want to show that there exist $\epsilon > 0$ such that:

$$f(\underline{\pi}, \pi^0) < 0.5 f(\underline{\pi} + \epsilon, \pi^0) + 0.5 f(\underline{\pi} - \epsilon, \pi^0) \tag{2.23}$$

Taking a first order Taylor approximation around $\underline{\pi}$ we can write $f(\underline{\pi}+\epsilon)$ and $f(\underline{\pi}-\epsilon)$ as:

$$f(\underline{\pi} + \epsilon, \pi^0) = f(\underline{\pi}, \pi^0) + f'_+(\underline{\pi}, \pi^0)\epsilon + o(\epsilon)$$
$$f(\underline{\pi} - \epsilon, \pi^0) = f(\underline{\pi}, \pi^0) - f'_-(\underline{\pi}, \pi^0)\epsilon + o(\epsilon)$$

where $o(\epsilon)$ represents the remainder. If (2.23) is not true then:

$$\begin{aligned} 0 &\geq \epsilon(f'_+(\underline{\pi}, \pi^0) - f'_-(\underline{\pi}, \pi^0)) + o(\epsilon) \\ 0 &\geq (f'_+(\underline{\pi}, \pi^0) - f'_-(\underline{\pi}, \pi^0)) + \frac{o(\epsilon)}{\epsilon} \end{aligned} \tag{2.24}$$

When $R'_- \neq 0$, $f'_+(\underline{\pi}, \pi^0) - f'_-(\underline{\pi}, \pi^0) > 0$. If the approximation error in the Taylor approximation is non-negative, then (2.24) is a contradiction. If the approximation error is negative, then as $\frac{o(\epsilon)}{\epsilon}$ tends to zero as ϵ goes to zero, for ϵ small enough we can make the modulus of $\frac{o(\epsilon)}{\epsilon}$ smaller than $f'_+(\underline{\pi}, \pi^0) - f'_-(\underline{\pi}, \pi^0)$ and again we obtain a contradiction. □

The next lemma provides a sufficient condition for f to be defined at $(\underline{\pi}, \pi^0)$.

Lemma 8. *There exist $\delta > 0$ such that if $\pi^0 \in (\underline{\pi} - \delta, \underline{\pi})$, then f is defined at $(\underline{\pi}, \pi^0)$*

Proof. Let $x^u_s(\pi)$ be u's equilibrium allocation when the posterior is π for $s = 0, 1, 2$. When the posterior is equal to $\underline{\pi}$ we have: $x^u_s(\underline{\pi}) = w^u_1 - \theta_1 - \theta_3$ for $s = 0, 1, 2$, by definition of $\underline{\pi}$. This allocation is constrained feasible by Proposition 1. Define $\underline{x}^u$ as:

$$\underline{x}^u(\pi) = v^{-1}\left(V(x^u(\pi), \pi) - v(x^u_1(\underline{\pi}))\right),$$

i.e. $\underline{x}^u(\pi)$ is the period 0 consumption that leaves u indifferent between the equilibrium under posterior π, and consuming $(\underline{x}^u(\pi), x^u_1(\underline{\pi}), x^u_1(\underline{\pi}))$. Clearly $\underline{x}^u(\underline{\pi}) = x^u_0(\underline{\pi})$, hence $\underline{x}^u(\underline{\pi}) \in (0, w)$. If the equilibrium interest rate is continuous in π, so it is each component of the vector of equilibrium allocations. Therefore $\underline{x}^u(\pi)$ is continuous in π. This implies that there exist a δ such that for all $\pi \in (\underline{\pi} - \delta, \underline{\pi})$: $\underline{x}^u(\pi) \in (0, w)$. Hence the set defined by the constraints in the definition of f is not empty for all $\pi^0 \in (\underline{\pi} - \delta, \underline{\pi})$. Continuity of R in an interval around $\underline{\pi}$ is proved in Lemma 5 in appendix 2.H. □

2.J Proof of Theorem 10

First we argue that if π^0 is close to $\underline{\pi}$ then $\underline{x}(\pi^0)$ and $\overline{x}(\pi^0)$ are well defined.

Lemma 9. *There exist a $\delta > 0$ such that if $\pi^0 \in (\underline{\pi} - \delta, \underline{\pi})$, then $\underline{x}(\pi^0)$ and $\overline{x}(\pi^0)$ are in $[0, w]$.*

Proof. $\underline{x}(\pi^0) \in [0, w]$ follows from Lemma 8, as $\underline{x}(\pi^0) = w - \underline{x}^u(\pi^0)$. Define $\overline{x}^u$ as:

$$\overline{x}^u(\pi) = v^{-1}\left(\frac{V(x^u(\pi), \pi)}{2}\right).$$

clearly $\overline{x}^u(\underline{\pi}) = x_s^u(\underline{\pi})$ for $s = 0, 1, 2$, hence $\underline{x}(\underline{\pi}) \in (0, w)$. If the equilibrium interest rate is continuous in π, so it is each component of the vector of equilibrium allocations. Therefore $\overline{x}^u(\pi)$ is continuous in π. This implies that there exist a δ such that for all $\pi \in (\underline{\pi}, \underline{\pi} - \delta)$: $\overline{x}^u(\pi) \in (0, w)$. But then $\underline{x}(\pi^0) = w - \underline{x}^u(\pi^0) \in [0, w]$. □

Now we will show that $\overline{\pi}(\epsilon)$ is continuous and well defined around $\epsilon = 0$. For simplicity let's assume that $\pi^0(\epsilon) = \underline{\pi} - \epsilon$ with $\epsilon = [-\gamma, \underline{\pi}]$ and γ a small positive number such that $\underline{\pi} + \gamma < 1$. We do this to simplify notation and make π^0 differentiable at $\epsilon = 0$.

Lemma 10. *Assume that (A2) holds with equality, or that (A2) holds with strict inequality and (A4) holds ($R'_-(\underline{\pi}) \neq 0$), then there exist a $\widehat{\epsilon} > 0$ such that $\overline{\pi}(\epsilon)$ is continuous for all $\epsilon \in [0, \widehat{\epsilon})$.*

Proof. We start with the case when (A2) holds with strict inequality. Let v^{-1} be the inverse of v. By the inverse function theorem v^{-1} is continuously differentiable. The posterior $\overline{\pi}(\epsilon)$ is the π that solves the following equation:

$$w - v^{-1}\left(\frac{V^u(\pi^0(\epsilon))}{2}\right) = w_1^c + \theta_1 + \theta_3 R(\pi). \tag{2.25}$$

At $\epsilon = 0$ the solution to (2.25) is $\pi = \underline{\pi} \in (0, 1)$. In appendix 2.H we argued that for all $\pi \leq \pi^0$ the equilibrium gross interest rate is given by R^B. We also showed that R^B is differentiable at $\underline{\pi}$, and its derivative is, by definition, equal to $R'_-(\underline{\pi})$. Therefore we can rewrite (2.25) as:

$$w - v^{-1}\left(\frac{V^u(\pi^0(\epsilon))}{2}\right) = w_1^c + \theta_1 + \theta_3 R^B(\pi). \tag{2.26}$$

As all the functions in (2.26) are continuously differentiable and $R^{B'}$ is different from zero at $\epsilon = 0$ by (A4), the implicit function theorem gives us continuity of $\overline{\pi}(\epsilon)$ in an interval around $\epsilon = 0$. Continuity tells us that $\overline{\pi}(\epsilon) \in (0, 1)$ for all ϵ in an interval around $\epsilon = 0$. When (A2) is satisfied with equality, monotonicity of R^B allows us to use the implicit function theorem and arrive at the same conclusions. □

Let's define the function $g(a,\epsilon) = a\overline{V}(\epsilon) + (1-a)\underline{V}(\epsilon) - V_0(\epsilon)$, mapping $\mathbb{R}\times[0,\underline{\pi}]$ into $\mathbb{R}$. We next study the properties of g. **Properties of g:**
Notice that $g(a,0) = 0$ for all a. When $\epsilon = 0$, then $\pi^0 = \overline{\pi} = \underline{\pi}$, therefore $\overline{V}(0) = V_0(0) = \underline{V}(0)$. Next, we will show that the right derivative of g with respect to ϵ at $(a,0)$ is zero for all a.

Claim 1. $\lim_{\epsilon\to 0^+} g_\epsilon(a,\epsilon) = 0$ *for all* a.

Proof. Taking the first derivative of g with respect to ϵ:

$$\frac{\partial g(a,\epsilon)}{\partial \epsilon} = a\overline{V}'(\epsilon) + (1-a)\underline{V}'(\epsilon) - V_0'(\epsilon).$$

Remember that the functions $\overline{V}(\epsilon)$, $\underline{V}(\epsilon)$ and $V_0(\epsilon)$ are given by:

$$\begin{aligned}
\overline{V}(\epsilon) &= 2v(\overline{x}(\epsilon)),\\
\underline{V}(\epsilon) &= v(\underline{x}(\epsilon)) + v(w_1^c + \theta_1 + \theta_3),\\
V_0(\epsilon) &= v(x_0(\epsilon)) + \sum_s \pi_s^0 v(x_s(\epsilon)).
\end{aligned} \tag{2.27}$$

where $x_0(\epsilon)$ is c's uninformative equilibrium allocation in period 0 when prior is $\pi^0 = \underline{\pi} - \epsilon$, similarly $x_s(\epsilon)$ is c's uninformative equilibrium allocation in state s. Consumption levels $\overline{x}(\epsilon)$ and $\underline{x}(\epsilon)$ are defined by:

$$\begin{aligned}
2v(w-\overline{x}(\epsilon)) &= v(w-x_0(\epsilon)) + \sum_s \pi_s^0(\epsilon) v(w-x_s(\epsilon)),\\
v(w-\underline{x}(\epsilon)) + v(w_1^u - \theta_1 - \theta_3) &= v(w-x_0(\epsilon)) + \sum_s \pi_s^0(\epsilon).v(w-x_s(\epsilon))
\end{aligned} \tag{2.28}$$

Differentiating the functions defined in (2.27) with respect to ϵ:

$$\begin{aligned}
\overline{V}'(\epsilon) &= 2v'(\overline{x}(\epsilon))\overline{x}'(\epsilon),\\
\underline{V}'(\epsilon) &= v'(\underline{x}(\epsilon))\underline{x}'(\epsilon),\\
V_0'(\epsilon) &= v'(x_0(\epsilon))x_0'(\epsilon) + \sum_s \pi_s^0 v'(x_s(\epsilon))x_s'(\epsilon) - v(x_1(\epsilon)) + v(x_2(\epsilon)).
\end{aligned} \tag{2.29}$$

Using (2.28) we can get an expressions for $\overline{x}'(\epsilon)$ and $\underline{x}'(\epsilon)$:

$$\begin{aligned}
-2v'(w-\overline{x}(\epsilon))\overline{x}'(\epsilon) &= -\,v'(w-x_0(\epsilon))x_0'(\epsilon) - \sum_s \pi_s^0 v'(w-x_s(\epsilon))x_s'(\epsilon) +\\
&\quad v(w-x_1(\epsilon)) - v(w-x_2(\epsilon)),\\
-v'(w-\underline{x}(\epsilon))\underline{x}'(\epsilon) &= -\,v'(w-x_0(\epsilon))x_0'(\epsilon) - \sum_s \pi_s^0 v'(w-x_s(\epsilon))x_s'(\epsilon) +\\
&\quad v(w-x_1(\epsilon)) - v(w-x_2(\epsilon)).
\end{aligned} \tag{2.30}$$

When ϵ goes to 0^+: $x_0 = x_1 = x_2 = \overline{x} = \underline{x}$. Let $\phi(0^+) \equiv \lim_{\epsilon\to 0^+}\phi(\epsilon)$. In the limit

we can rewrite (2.30) as:

$$\begin{aligned} 2\overline{x}'(0^+) &= x_0'(0^+) + \sum_s \underline{\pi}_s x_s'(0^+), \\ \underline{x}'(0^+) &= x_0'(0^+) + \sum_s \underline{\pi}_s x_s'(0^+). \end{aligned} \tag{2.31}$$

Evaluating (2.29) at $x_0 = x_1 = x_2 = \overline{x} = \underline{x}$ and using (2.31) we have:

$$\overline{V}'(0^+) = V_0'(0^+) = \underline{V}'(0^+).$$

□

Next, we show that $g(1,0)$ is a strict local minimum.

Claim 2. *If* $\lim_{\epsilon\to 0^+}(x_0'(\epsilon) - x_1'(\epsilon)) \neq 0$ *holds, then* $\lim_{\epsilon\to 0^+} g_{\epsilon\epsilon}(1,\epsilon) > 0$.

Proof. Notice that $g_{\epsilon\epsilon}(1,\epsilon) = \overline{V}''(\epsilon) - V_0''(\epsilon)$. Let us now compute the second derivatives of V_0, and $\overline{V}$:

$$\begin{aligned} V_0''(\epsilon) =& v''(x_0(\epsilon))x_0'(\epsilon)^2 + v'(x_0(\epsilon))x_0''(\epsilon) + \sum_s \pi_s^0 \left(v''(x_s(\epsilon))x_s'(\epsilon)^2 + v'(x_s(\epsilon))x_s''(\epsilon)\right) + \\ & 2\sum_s (-1)^s v'(x_s(\epsilon))x_s'(\epsilon), \\ \overline{V}''(\epsilon) =& 2v''(\overline{x}(\epsilon))\overline{x}'(\epsilon)^2 + 2v'(\overline{x}(\epsilon))\overline{x}''(\epsilon), \end{aligned}$$

where we have used the fact that the second derivative of π^0 with respect to ϵ is zero. The second derivative of $\overline{x}(\epsilon)$ has to satisfy:

$$\begin{aligned} & 2v''(w - \overline{x}(\epsilon))\overline{x}'(\epsilon)^2 - 2v'(w - \overline{x}(\epsilon))\overline{x}''(\epsilon) = v''(w - x_0(\epsilon))x_0'(\epsilon)^2 - \\ & v'(w - x_0(\epsilon))x_0''(\epsilon) + \sum_s \pi_s^0 \left(v''(w - x_s(\epsilon))x_s'(\epsilon)^2 - v'(w - x_s(\epsilon))x_s''(\epsilon)\right) + \\ & 2\sum_s (-1)^s v'(w - x_s(\epsilon))x_s'(\epsilon). \end{aligned} \tag{2.32}$$

Now we can compute $\overline{V}''(0^+) - V_0''(0^+)$:

$$\begin{aligned} \overline{V}''(0^+) - V_0''(0^+) =& v''(x_0(0^+))\left(2\overline{x}'(0^+)^2 - x_0'(0^+)^2 - \sum_s \pi_s^0 x_s'(0^+)^2\right) + \\ & v'(x_0(0^+))\left(2\overline{x}''(0^+) - x_0''(0^+) - \sum_s \pi_s^0 x_s''(0^+)\right) - \\ & 2v'(x_0(0^+))\sum_s (-1)^s x_s'(0^+). \end{aligned} \tag{2.33}$$

Using (2.32):

$$2\overline{x}''(0^+) - x_0''(0^+) - \sum_s \pi_s^0 x_s''(0^+) = \frac{v''(w - x_0(0^+))}{v'(w - x_0(0^+))}\left(2\overline{x}'(0^+)^2 - x_0'(0^+)^2 - \sum_s \pi_s^0 x_s'(0^+)^2\right) + 2\sum_s (-1)^s x_s'(0^+). \tag{2.34}$$

Equations (2.33) and (2.34) imply:

$$\overline{V}'' - V_0'' = \left(v''(x_0(0^+) + v'(x_0(0^+))\frac{v''(w - x_0(0^+)}{v'(w - x_0(0^+)}\right)\left(2\overline{x}'(0^+)^2 - x_0'(0^+)^2 - \sum_s \pi_s^0 x_s'(0^+)^2\right).$$

As $v'' < 0$, $g_{\epsilon\epsilon}(1, 0^+) > 0$ if and only if $2\overline{x}'(0^+)^2 - x_0'(0^+)^2 - \sum_s \pi_s^0 x_s'(0^+)^2 < 0$. Assume (A2) is satisfied with strict inequality, then around $\underline{\pi}$ we have $x_0 = x_2$, and $x_0' = x_2'$ (See appendix 2.H). Using (2.31) to replace $\overline{x}'(0^+)$:

$$\begin{aligned} x_0'(0^+)^2 + \sum_s \pi_s^0 x_s'(0^+)^2 - \frac{1}{2}\left(x_0'(0^+) + \sum_s \pi_s^0 x_s'(0^+)\right)^2 &> 0, \\ (2-\pi)x_0'(0^+)^2 + \pi x_1'(0^+)^2 - \frac{1}{2}\left((2-\pi)x_0'(0^+) + \pi x_1'(0^+)\right)^2 &> 0, \\ (2-\pi)\pi x_0'(0^+)^2 + (2-\pi)\pi x_1'(0^+)^2 - 2(2-\pi)\pi x_0'(0^+)x_1'(0^+) &> 0, \\ x_0'(0^+)^2 + x_1'(0^+)^2 - 2x_0'(0^+)x_1'(0^+) &> 0, \\ \left(x_0'(0^+) - x_1'(0^+)\right)^2 &> 0. \end{aligned} \tag{2.35}$$

If (A2) is satisfied with equality instead, then we have $x_1 = x_2$ and $x_1' = x_2'$, and the same conclusion follows. □

Now we characterize when is it that condition $\lim_{\epsilon \to 0^+} (x_0'(\epsilon) - x_1'(\epsilon)) \neq 0$ holds.

Claim 3. $\lim_{\epsilon \to 0^+} (x_0'(\epsilon) - x_1'(\epsilon)) = 0$ *if and only if* $R_-'(\underline{\pi}) = 0$.

Proof. In an interval around $\underline{\pi}$ equilibrium prices are such that $p_2 = 1 - \pi$ and R follows from u's F.O.C:

$$Rv_1' - v_0' = 0,$$

where $v_s' = \partial u(x_s^u)/\partial x_s^u$. Differentiating this F.O.C. with respect to ϵ and taking the limit as ϵ approaches 0^+:

$$\begin{aligned} -R_-'(\underline{\pi})v_1' + v_1''x_1'(0^+) - v_0''x_0'(0^+) &= 0, \\ -v_0'R_-'(\underline{\pi}) + v_0''(x_1'(0^+) - x_0'(0^+)) &= 0. \end{aligned}$$

Therefore $x_1'(0^+) = x_0'(0^+)$ if and only if $R_-'(\underline{\pi}) = 0$. □

From Claim 3 we see that (A2) being satisfied with equality, or assumption (A4) are sufficient conditions for g to attain a strict local minimum at $(1, 0)$. This gives us the following lemma:

Lemma 11. *If (A2) is satisfied with equality, or if (A4) holds, then* $\lim_{\epsilon\to 0^+}\widehat{\tau} = (1-\tau_3)\overline{a} < 1-\tau_3$

Proof. The second derivative of g with respect to ϵ: $g_{\epsilon\epsilon} = a\overline{V}''(\epsilon) + (1-a)\underline{V}''(\epsilon) - V_0''(\epsilon)$ is continuous in a. As $g_{\epsilon\epsilon}(1,0) > 0$, continuity in a implies that there exist $\overline{a} < 1$ such that $g_{\epsilon\epsilon}(\overline{a},0) > 0$. Using claims 1 and 2, $g(\overline{a},\epsilon)$, as a function of ϵ, attains a strict local minimum at $(\overline{a},0)$ with $g(\overline{a},0) = 0$. Therefore on a neighborhood of $\epsilon = 0$, $g(\overline{a},\epsilon) > 0$ for all ϵ. Thus $\overline{a}\overline{V}(\epsilon) + (1-\overline{a})\underline{V}(\epsilon) - V_0(\epsilon) > 0$ for all ϵ in the interval. Solving for $\overline{a}$:

$$1 > \overline{a} > \frac{V_0(\epsilon) - \underline{V}(\epsilon)}{\overline{V}(\epsilon) - \underline{V}(\epsilon)} \ \forall \epsilon \in [0,\overline{\epsilon}). \tag{2.36}$$

But (2.36) implies that the limit of $\widehat{\tau}/(1-\tau_3)$ as ϵ tends to 0 is strictly lower than one, giving us the result in the lemma. □

<u>Proof of Theorem 10</u>:
Set $\mu < (1-\tau_3)\frac{1-\overline{a}}{2}$, where $\overline{a} = \lim_{\epsilon\to 0}\widehat{\tau}/(1-\tau_3) < 1$. For this μ there exist a $\widehat{\delta}$ such that for all $\epsilon \in [0,\widehat{\delta})$, $|\widehat{\tau} - \overline{a}(1-\tau_3)| < \mu$. Similarly, for this μ there exist a $\widetilde{\delta}$ such that for all $\epsilon \in [0,\widetilde{\delta})$, $|\widetilde{\tau} - (1-\tau_3)| < \mu$. Set $\delta = \min\{\widehat{\delta},\widetilde{\delta}\}$, then for all $\epsilon \in [0,\delta)$ we have:

$$\begin{aligned} \overline{a}(1-\tau_3) - \mu < \widehat{\tau} < \overline{a}(1-\tau_3) + \mu, \\ (1-\tau_3) - \mu < \widetilde{\tau} < (1-\tau_3) + \mu. \end{aligned} \tag{2.37}$$

Equation (2.37) imply that for all such ϵ, $\widehat{\tau} < \widetilde{\tau}$ as $\overline{a}(1-\tau_3) + \mu < (1-\tau_3) - \mu$. □

Bibliography

Aumann, Robert J and Michael Maschler (1995). *Repeated games with incomplete information*. MIT press.

Barber, Brad, Reuven Lehavy, Maureen McNichols, and Brett Trueman (2001). "Can investors profit from the prophets? Security analyst recommendations and stock returns". In: *The Journal of Finance* 56.2, pp. 531–563.

— (2006). "Buys, holds, and sells: The distribution of investment banks' stock ratings and the implications for the profitability of analysts' recommendations". In: *Journal of accounting and Economics* 41.1, pp. 87–117.

Barr, Nicholas and Peter Diamond (2008). *Reforming pensions: Principles and policy choices*. Oxford University Press.

Behrman, Jere R, Olivia S Mitchell, Cindy K Soo, and David Bravo (2012). "How financial literacy affects household wealth accumulation". In: *The American economic review* 102.3, pp. 300–304.

Bernhardt, Dan and Ryan J Davies (2009). "Smart fund managers? Stupid money?" In: *Canadian Journal of Economics/Revue canadienne d'économique* 42.2, pp. 719–748.

Berstein, Solange, Guillermo Larraín, Francisco Pino, and Eduardo Morón (2006). "Chilean Pension Reform: Coverage Facts and Policy Alternatives [with Comments]". In: *Economía* 6.2, pp. 227–279.

Blackwell, David (1951). "Comparison of experiments". In: *Proceedings of the second Berkeley symposium on mathematical statistics and probability*.

Boni, Leslie and Kent L Womack (2006). "Analysts, industries, and price momentum". In: *Journal of Financial and Quantitative analysis* 41.1, pp. 85–109.

Carlin, Bruce I and Shaun William Davies (2016). "The Implementation of State Sponsored Retirement Plans". In:

Cass, David and Karl Shell (1983). "Do sunspots matter?" In: *Journal of political economy* 91.2, pp. 193–227.

Chevalier, Judith and Glenn Ellison (1997). "Risk taking by mutual funds as a response to incentives". In: *Journal of Political Economy* 105.6, pp. 1167–1200.

Da, Zhi, Borja Larrain, Clemens Sialm, and José Tessada (2016). *Coordinated Noise Trading: Evidence from Pension Fund Reallocations*. Tech. rep. National Bureau of Economic Research.

Dahlquist, Magnus, José Vicente Martinez, and Paul Söderlind (2016). "Individual investor activity and performance". In: *The Review of Financial Studies* 30.3, pp. 866–899.

De la Fuente, Angel (2000). *Mathematical methods and models for economists*. Cambridge University Press.

Diamond, Peter A (1967). "The role of a stock market in a general equilibrium model with technological uncertainty". In: *The American Economic Review* 57.4, pp. 759–776.

Edwards, Sebastian (1998). "The Chilean pension reform: A pioneering program". In: *Privatizing social security*. University of Chicago Press, pp. 33–62.

Financial Stability Board (2013). "Minutes of meeting held on July 26, 2013". In:

Fuentes, Olga, Pamela Searle, and Félix Villatoro (2013). "Active Investment Decisions of Members in the Chilean DC Pension System: Performance and Learning over time". In:

Gennaioli, Nicola, Andrei Shleifer, and Robert Vishny (2015). "Money doctors". In: *The Journal of Finance* 70.1, pp. 91–114.

Ghilarducci, Teresa, Hamilton James, et al. (2016). *A Comprehensive Plan to Confront the Retirement Savings Crisis*. Tech. rep. Schwartz Center for Economic Policy Analysis (SCEPA), The New School.

Gottardi, Piero and Rohit Rahi (2014). "Value of information in competitive economies with incomplete markets". In: *International Economic Review* 55.1, pp. 57–81.

Hakansson, Nils H, J Gregory Kunkel, and James A Ohlson (1982). "Sufficient and necessary conditions for information to have social value in pure exchange". In: *The Journal of Finance* 37.5, pp. 1169–1181.

Hildenbrand, Werner and Alan P Kirman (1988). *Equilibrium analysis: variations on themes by Edgeworth and Walras*. Vol. 28. North Holland.

Hirshleifer, Jack (1971). "The private and social value of information and the reward to inventive activity". In: *The American economic review* 61.4, pp. 561–574.

Inderst, Roman and Marco Ottaviani (2012). "Financial advice". In: *Journal of Economic Literature* 50.2, pp. 494–512.

Israelsen, Craig (2005). "A refinement to the Sharpe ratio and information ratio". In: *Journal of Asset Management* 5.6, pp. 423–427.

Ivković, Zoran and Narasimhan Jegadeesh (2004). "The timing and value of forecast and recommendation revisions". In: *Journal of Financial Economics* 73.3, pp. 433–463.

Jegadeesh, Narasimhan, Joonghyuk Kim, Susan D Krische, and Charles Lee (2004). "Analyzing the analysts: When do recommendations add value?" In: *The journal of finance* 59.3, pp. 1083–1124.

Jegadeesh, Narasimhan and Woojin Kim (2009). "Do analysts herd? An analysis of recommendations and market reactions". In: *The Review of Financial Studies* 23.2, pp. 901–937.

Joubert, Clement (2015). "Pension design with a large informal labor market: Evidence from Chile". In: *International Economic Review* 56.2, pp. 673–694.

Kamenica, Emir and Matthew Gentzkow (2011). "Bayesian persuasion". In: *The American Economic Review* 101.6, pp. 2590–2615.

Loh, Roger K and René M Stulz (2010). "When are analyst recommendation changes influential?" In: *The review of financial studies* 24.2, pp. 593–627.

Magill, Michael and Martine Quinzii (2002). *Theory of incomplete markets*. Vol. 1. Mit press.

Marshall, John M (1974). "Private incentives and public information". In: *The American Economic Review*, pp. 373–390.

Mas-Colell, Andreu, Michael Dennis Whinston, and Jerry R Green (1995). *Microeconomic theory*. Vol. 1. Oxford university press New York.

Maurer, Thomas Andreas and Ngoc-Khanh Tran (2016). "The Hirshleifer Effect in a Dynamic Setting".

Mesa, Alberto Arenas de, David Bravo, Jere R Behrman, Olivia S Mitchell, and Petra E Todd (2008). "The Chilean pension reform turns 25: lessons from the social protection survey". In: *Lessons from pension reform in the Americas*. Oxford University Press, pp. 23–58.

Ng, David S (1977). "Pareto-optimality of authentic information". In: *The Journal of Finance* 32.5, pp. 1717–1728.

Powdthavee, Nattavudh and Yohanes E Riyanto (2015). "Would you pay for transparently useless advice? A test of boundaries of beliefs in the folly of predictions". In: *Review of Economics and Statistics* 97.2, pp. 257–272.

Radner, Roy and Joseph Stiglitz (1984). "A Nonconcavity in the Value of Information". In: *Bayesian models in economic theory* 5, pp. 33–52.

Sirri, Erik R and Peter Tufano (1998). "Costly search and mutual fund flows". In: *The journal of finance* 53.5, pp. 1589–1622.

Stickel, Scott E (1995). "The anatomy of the performance of buy and sell recommendations". In: *Financial Analysts Journal* 51.5, pp. 25–39.

Superintendence of Pensions (2010). *The Chilean Pension System*.

— (2013). *Objetivo de los Fondos de Pensiones y los Traspasos Frecuentes de Fondo*. Tech. rep.

Womack, Kent L (1996). "Do brokerage analysts' recommendations have investment value?" In: *The journal of finance* 51.1, pp. 137–167.

Zheng, Lu (1999). "Is money smart? A study of mutual fund investors' fund selection ability". In: *The Journal of Finance* 54.3, pp. 901–933.