Growth Options and Credit Risk*

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Abstract

We calibrate a dynamic model of credit risk and analyze the relation between growth options and credit spreads. Our model features real and financing frictions, a technology with decreasing returns to scale, and endogenous investment options driven by both systematic and idiosyncratic shocks. We find a negative relation between credit spreads and growth options, after controlling for determinants of credit risk. This negative relation is due to the current decision to invest and the associated change in leverage which, in the presence of external financing needs and financing frictions, increase credit spreads while reducing the value of future investments. We do not find evidence that growth options accrue value in response to systematic risk, thus increasing credit risk premia.

JEL Classifications: G12, G32

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The empirical evidence of the relation between credit risk and different proxies for the value of growth options is inconclusive. For example, controlling for book leverage, the market to book ratio is negatively related to credit spreads, while the relationship is positive when controlling for market leverage. Research and development expenses are negatively related to credit risk, when scaled by sales and controlling for book leverage, but positively correlated when scaled by investments in physical assets and controlling for market leverage. Intangible assets are positively related to credit spreads, while non-physical assets have a negative correlation. Innovative technology, measured by citations per patent applications, is instead negatively related to credit spreads. Taken individually all these measures can be interpreted as being related to the firm's growth potential. However, they do not provide consistent and easily rationalizable economic evidence.

The ambiguity is due to the fact that a few challenges arise when taking the matter directly to the data. First and most important, the value of growth options is not directly observable. Empirical proxies, such as those mentioned above, can be constructed but they might contain information that is not unequivocally related to the value of the option to invest in the future.

Second, a firm's asset composition (the relative importance of the value of growth options to the value of assets in place) is not exogenous. Rather, it is endogenously determined by past and current investments. Hence, in a world in which growth options are in limited supply because of decreasing returns to scale, a firm may have more growth options because it has invested less in the past. For the same reason, it might have less debt and therefore lower credit risk.

Third, the asset composition of a firm might reveal not only valuable information about future prospects of the firm, but also about its current risk profile. The relationship between exposure to systematic risk and growth options has been widely studied. While most papers arrive to the conclusions that such relationship is positive, recently Babenko et al. (2016) show that if the option exercise increases exposure to firm-specific risk, then it reduces the firm's exposure to systematic risk and thus also credit risk premia. Conversely, if future investment increase exposure to systematic risk, they will also make the debt contract riskier. Hence, even this systematic channel that drives the relation between credit risk and growth options is ambivalent.

We address these challenges by calibrating a dynamic model of credit risk, in which debt dynamics are driven by investments, and the firm faces idiosyncratic and systematic shocks, countercyclical risk premia, corporate taxes, real adjustment costs and external financing frictions. A key feature of the model is that it allows us to precisely measure quantities that would otherwise be unobservable: the present value of growth options (i.e., PVGO), and their systematic risk exposure. We show that an asset composition that is richer in growth options (i.e., high PVGO relative to assets or equity) is related to lower credit spreads when we control for other determinants of credit risk (such as, leverage and cash flow volatility).

The negative relation between growth options and credit spreads is motivated by two tensions between the main economic forces in the model. First, the assumption of decreasing returns to scale creates a trade off between current investment and future expansion: the more the firm convert growth options by investing in productive capital, the lower the incentive to grow in the future. Second, the tension between debt financing and growth option creates opposite incentives: in order to invest more, the firm wants to issue more debt to mitigate financing constraints; on the other hand, too much debt increases the risk that future net worth will be negative, thus causing the loss of growth options.

Further, we use the simulated economy to shed some light on the role of the firm's exposure to aggregate shocks. We find that the beta of growth options is negatively related to the firm's asset composition and the value of its growth options, which suggests, at least in our calibration, a non-positive relation between growth options and credit risk premia.

Finally, another important feature of the model is that it allows us to consider the relation between growth options and the market to book ratio. To this end, we calibrate the model to match the empirical relation between credit spread and market to book ratio, which is negative controlling for book leverage and positive controlling for market leverage, while preserving the fundamental negative relation between credit risk and PVGO. In this regard, our work is complementary to other studies that construct and calibrate models that feature exogenous (e.g., Arnold et al. 2013) and endogenous (i.e., Kuehn and Schmid 2014) investment opportunities.

We show that in certain instances the market to book ratio is hard to interpret as measure of growth opportunities because of its close relationship with leverage and the value of the asset in place (PVAP). First, an increase in the value of growth options increases the market to book ratio and contemporaneously decreases market leverage, thus creating a confounding effect that produces a positive coefficient in a regression of credit spreads on market to book ratios (when controlling for market leverage). Controlling for book leverage or removing the effect of PVGO from market leverage, one instead obtains a negative coefficient. Second, the sensitivity of credit spreads to market to book is affected by the fact that the latter contains information not only regarding growth options but also to PVAP. In fact, because of decreasing returns to scale, as the size of the firm becomes larger, the market to book ratio becomes less sensitive to PVGO and more sensitive to PVAP. As a result, credit spreads are less sensitive to growth options, when the firm has proportionally fewer opportunities to grow (i.e., the firm is already big).

1. Related Literature

By way of analyzing the relation between growth options and credit risk, our paper contributes to the recent credit risk literature in several respects. Huang and Huang (2012) show that traditional structural models of credit risk, similar to Merton (1974) and Leland (1994), when matching empirically observed leverage ratios and default probabilities, cannot generate realistic credit spreads. Such limitation is often referred to as the *credit spread puzzle*.

Chen et al. (2009) propose an extension of the traditional Merton (1974) framework by introducing habit formation into a pricing kernel that provides countercyclical risk premia. This innovation allows the standard Merton model, in which the capital structure is static and there is no investment, to produce an average credit spread on corporate debt similar to the one empirically observed in the BBB credit class, while matching the average leverage and the average default probability of BBB firms. Bhamra et al. (2010) and Chen (2010) extend the above framework of state dependent risk premia to the case in which firms can dynamically adjust their capital structure through issuance of new debt, while the asset follows an exogenous stochastic process. Chen (2010) shows the importance of considering pro-cyclical recovery rates. Bhamra et al. (2010) solve the credit spread puzzle by imposing an initial cross-sectional distribution of leverage. Relative to these papers, we endogenously obtain a realistic cross-sectional distribution of firms, and focus on the cash flow generating process and the effect of realized idiosyncratic risk on the firm's policy.

Recent studies in finance integrate corporate decision into asset pricing. Berk et al. (1999) and Zhang (2005) show that asset pricing contributions show that the dynamic of a firm's assets plays a central role in determining the risk profile of a firm. Leary and Roberts (2005) and Hennessy and Whited (2005), among many others, show how investment policies in the presence of financing frictions determine capital structure decisions. Among others, Davydenko and Strebulaev (2007) show that, controlling for determinants of credit risk, proxies of growth options (market to book asset, R&D expenses to total investment, and intangibles to total assets) are positively related to credit spreads. We confirm some of these empirical regularities in our sample. Differently from these studies, recognizing that the value of growth option is unobservable, we calibrate a model that allows us to produce an exact measure of growth options, while endogenously replicating the empirical negative relation between growth options and leverage (see, e.g. Fama and French 2002). By regressing the credit spread on asset composition (growth options versus asset in place), while controlling for leverage and other determinants of credit risk, we show that the value of growth options is negatively related to credit spreads.

In a model of a firm endowed with an exogenous option to expand and a static but initially optimal capital structure, Arnold et al. (2013) produce positive association between credit spreads and growth options. Differently from Arnold et al. (2013), our simple model shows that when growth options are in limited supply because of decreasing returns to scale and the firm is financially constrained, the investment and the ensuing financing decision lead to a contemporaneous reduction of the value of growth opportunities and increase of the credit spread.

Differently from Kuehn and Schmid (2014), who examine a model very similar to ours, we produce an exact measure of growth options (i.e., PVGO) in our simulated economy. We complement their findings by showing that the impact of systematic risk on growth options can be dominated by the influence of idiosyncratic risk, and that a positive relationship between credit spreads and market to book ratio is possible (when controlling for market leverage) even when the underlying economic correlation between PVGO and credit spreads is negative.

2. Data and empirical evidence

We assemble data from various sources. Firm-level accounting and financial information is obtained from the merged CRSP-COMPUSTAT files. In order to align the data to the model, when applicable variables are scaled by the sum of the book values of equity and total financial debt (i.e., short term liabilities plus long term debt), as opposed to total assets (i.e., AT), thus eliminating the impact of trade credit.

To avoid various problems related to the optionality included in corporate bonds, different taxation regimes, and liquidity issues, we rely on the spreads of senior unsecured CDS contracts with a five year tenor as our main measure of credit spreads. We obtain the data from Markit for the period from 2001 throughout 2013. In order to align market and accounting data, we compute the average of the daily mid-point CDS quotes over the last two months of the fiscal cycle.

In a further effort to eliminate concerns about liquidity of market CDS prices, we focus on firms that belong to the S&P 500 index and for which we can observe CDS prices.[1] Additionally, we eliminate from the sample utilities and firms in the financial sector. Ultimately, our sample contains 311 unique firms and a total of 2548 firm-year observations.

The construction of a few variables deserves some consideration. Asset volatility is not directly observable. Similar to other papers in the literature, we follow Bharath and Shumway (2008) and compute the asset volatility as the weighted average of the equity (i.e., implied volatility from ATM one-month to maturity options) and debt volatility.

We also construct a few proxies of the value of growth opportunities. Following the numerical procedure proposed by Trigeorgis and Lambertides (2014), we compute the GO score. For each firm j and for each time period t we construct a measure of PVGO as

$$\frac{PVGO_{jt}}{V_{jt}} = \frac{V_{jt} - \frac{CF_{jt}}{i_{jt}}}{V_{jt}},$$

where V_{jt} is the value of the firm, CF_{jt} is the firm cash flow and i_{jt} is the weighted average cost of capital. Next, we run cross-sectional regressions of $PVGO_{jt}/V_{jt}$ on a number of firm controls. Each year, a firm GO score is obtained as the regression model fit, using the average of the regressions coefficients over the previous three years and current right-hand side variables. As proxies for the value of intangibles we adopt the amount of intangible

[1]Examining the measures of CDS liquidity suggested by Qiu and Yu (2012) and Feldhutter et al. (2016) confirms that the CDS market for S&P 500 firms is more liquid. The average (median) number of contributors per quote for firms in the SP500 is 7.2 (6.1), while the average number of contributors per quote for firms not in the SP500 is 4.4 (3.2). The average (median) number of quotes per month is 22 (22) and 20 (21), for firms in and outside of the SP500 respectively.

assets recorded in the balance sheet or the amount of non-fix assets (e.g., Davydenko and Strebulaev 2007). Finally, as growth is often linked to innovation, alternative proxies for growth opportunities can be constructed from research and development expenses (e.g., Chan et al. 2001) or from patent application for new technology. We follow Dass et al. (2017) and compute the number of citations per patent.[2]

Default events are collected by merging several sources: Moody's KMV, Bloomberg, Standard and Poor's, and FISD Mergent. These events include Chapter 7 and Chapter 11 filings, missed payments of interest and principal, and are related to both bank and publicly held debt. There are nine default events in our sample from 2001 to 2013. We only consider defaults that happen in the last year a firm is listed in one of the three exchanges, and for which we can observe CDS prices in the immediate past. Details about the list of defaults are reported in the Appendix E. The sample default frequency is computed as the time-series average of the cross-sectional frequency of defaults in each year in the sample.

When applied to the model calibration, we homogenize the sample by removing industry fixed effects from the data (Hennessy and Whited 2007). We report descriptive statistics in Table 1.

2.1 CDS spreads and growth options

In Table 2 we present results of Fama-MacBeth regressions of credit spreads on several empirical measures of growth options. We include the market to book ratio, the GO score of Trigeorgis and Lambertides (2014), R&D intensity (defined as R&D expenses divided by sale), the ratio of R&D expenses to capital investments, the ratio of intangible capital to total assets, the ratio of non-fix assets to total assets, and, limited to the years before 2010, the ratio of citations to number of patent applications based on Dass et al. (2017). We control for asset volatility, the capital stock of the firm, and two measures of leverage (i.e., book and market). Each regression also includes industry fixed effects based on Ken French 12 industries classifications.

The table provides mixed evidence about the empirical relationship between growth options and credit risk. Using different proxies for the value of growth opportunities one can reach diametrically opposite conclusions. The most commonly used measure of growth op-

[2]We thank Steven Xiao for graciously sharing his data with us.

tions is the market to book ratio. In Columns 1 and 2 we see that the ratio enters with a negative sign when controlling for book leverage and with a positive sign when controlling for market leverage (see for example Davydenko and Strebulaev 2007; Kuehn and Schmid 2014).

In columns 3 and 4, we include the GO score of Trigeorgis and Lambertides (2014) in the regression specification in place of the market to book ratio. We find a negative regression coefficient regardless of which measure of leverage is used.

Intangibles, Columns 5 and 6, enters the regression with a positive coefficients (when controlling for market leverage). Non-fix assets load instead negatively and significantly on credit spreads when controlling for book leverage, and positively, but insignificantly, when controlling for market leverage. Research and developments expenses also offer contradictory evidence. When scaled by sales, Columns 9 and 10, they are negatively related to credit spreads; when scaled by capital expenses, Columns 11 and 12, they are instead positively related to credit spreads. Finally, Columns 13 and 14, we find that the coefficients for citation counts are also negative and statistically significant.

In summary, even abstracting from possible endogeneity concerns, the relationship between credit spreads and measures of growth options is unclear. In the remaining part of the paper we develop a theoretical framework that allows to not only provide a precise measure of growth options but also to study the equilibrium relation of such measure to credit spreads.

3. Credit Risk, Investment and Growth Options in a Simple Model

In this section, we construct a simplified one-period version of our model that allows us to present the basic relation between options to grow and credit risk using closed-form expressions. Our approach differs from Arnold et al. (2013) because in our model growth options emerge *endogenously*.[3]

[3] To simplify the exposition, we exclude for the moment corporate taxes, real and financial frictions, and discounting. A proper treatment of discounting is not essential to the argument presented in this section. A version of the simple model featuring a stochastic discount factor with countercyclical risk premia is available upon request.

We consider a cross section of ongoing firms that make simultaneous investment and financing decisions at the beginning of the period given the current net worth (cash flow from operations, plus residual value of asset, net of debt). The investment is in a decreasing returns to scale technology. Because capital depreciates, there is a finite unconstrained optimal investment level. Investment is financed with a blend of internal equity and debt financing. We consider two frictions: first, we assume that *no external equity* can be raised, so there is an incentive to use debt financing. By issuing debt, the firm becomes exposed to default risk. The second friction, *bankruptcy costs*, gives an incentive to keep the debt issuance at a minimum. With these two frictions, the efficient investment may not be achieved.

At the end of the period, each firm has the option to expand the production capacity. We assume that the debt must be repaid before the investment is made and that no external equity or new debt can be used to finance the expansion. Hence, the option is not available if the net worth is negative. If the end-of-period net worth is higher than the unconstrained optimal capital stock, the excess is paid as dividend. To simplify the analysis, we exclude bankruptcy costs after the first period, so that debt financing becomes irrelevant in the follow-on investment decisions.

Two features make this setup apt to analyze the interaction between growth options and credit risk. The first is that the assumption of decreasing returns to scale creates a trade off between current investment and future expansion: the more the firm invests now, the lower the incentive to grow later. The second is the tension between debt financing and growth option: on the one hand, the firm wants to issue more debt at the beginning of the period to mitigate the financing constraint and invest more; on the other hand, too much debt increases the risk that the end-of-period net worth is negative and the growth option is lost. These features create a realistic inverse relation between growth options and leverage.

More formally, at $t = 0$ a firm in the cross section has some capital stock, k, some debt that is immediately due, b, and observes a productivity shock, z, with domain $[0, \infty[$. Given the state (z, k, b), the current net worth is $w = zk^{\alpha} + (1-\delta)k - b$, where $\alpha < 1$ is a return to scale parameter, and depreciation at a rate $\delta > 0$ is necessary for a finite investment decision. We denote $\mathcal{A}(z, w)$ the set of firm's choices (k', b') that satisfy the (no equity issuance) budget constraint at (z, w):

$$k' \leq w + D(z, k', b'), \tag{1}$$

where $D(z, k', b')$ is the fair price of single-period debt when the total face value is b' and capital stock is k'. In Appendix A, we derive the following expression for the value of equity at $t = 0$:

$$S(z, w) = \max_{(k',b') \in \mathcal{A}(z,w)} \left\{ w - k' + (1-\eta) \int_0^{\underline{z}(k',b')} (w' + b') \Gamma(dz'|z) + b' \left[1 - \Gamma(\underline{z}(k',b')|z)\right] \right.$$
$$\left. + \int_{\underline{z}(k',b')}^{\overline{z}(k',b')} \left[\phi(z')(w')^{\alpha} + (1-\delta)w'\right] \Gamma(dz'|z) + \int_{\overline{z}(k',b')}^{+\infty} \left[w' + V(z')\right] \Gamma(dz'|z) \right\}. \quad (2)$$

In this equation, $w' = w(z', k', b')$ is the net worth at $t = 1$, in correspondence of (z', k', b'), $\eta < 1$ is the bankruptcy cost parameter, $\Gamma(z'|z)$ is the conditional cumulative probability of end-of-period shock z', and $V(z) = (1-\alpha)(\alpha/\delta)^{\frac{\alpha}{1-\alpha}} \phi(z)^{\frac{1}{1-\alpha}}$. In equation (2), at $t = 1$ the firm invests all the net worth up to unconstrained optimal level of capital, $\hat{k}(z)$, and pays out any excess cash $w - \hat{k}(z)$.

In equation (2) there are two important thresholds for z' at $t = 1$: $\underline{z}$ and $\overline{z}$. The default threshold, $\underline{z}(k', b') = (k')^{1-\alpha} \max\{b'/k' - (1-\delta), 0\}$, is found by solving condition $w(z', k', b') = 0$. The threshold $\overline{z}(k', b')$ defines the lowest z' such that the net worth is sufficient to invest at the unconstrained level, $w(z', k', b') = \hat{k}(z')$ (if there is no such z', we assume $\overline{z}(k', b') = \infty$).

In equation (2), the first line is the dividend at $t = 0$ (which is affected by bankruptcy costs), and the second line is the continuation value. The latter comprises the present value of the growth option (PVGO),

$$G(z, w) = \int_{\underline{z}(k',b')}^{\overline{z}(k',b')} \left[\phi(z')(w')^{\alpha} + (1-\delta)w'\right] \Gamma(dz'|z), \quad (3)$$

and the value of future dividend payments, $\int_{\overline{z}(k',b')}^{+\infty} [w' + V(z')] \Gamma(dz'|z)$, where $V(z')$ is the value of dividends after $t = 1$. Notably, from (3) the growth options is more valuable the higher $\overline{z}(k', b')$ and the lower $\underline{z}(k', b')$, which happens when book leverage, b'/k', is low.[4] We

[4] As for $\overline{z}(k', b')$, the statement can be proved by observing that $\overline{z}$ is implicitly defined by equation $m(z, k, b) = 0$, where $m(z, k, b) = zk^{\alpha} + (1-\delta)k - b - [\alpha/\delta\varphi(z)]^{\frac{1}{1-\alpha}}$. If a finite $\overline{z}$ exists, then $\partial m/\partial z \neq 0$. In particular, if $\varphi(z)$ is exponential, as we will assume later, then $\partial m/\partial z < 0$. Then,

$$\frac{\partial \overline{z}}{\partial k} = -\frac{\partial m/\partial k}{\partial m/\partial z} > 0 \qquad \text{and} \qquad \frac{\partial \overline{z}}{\partial b} = -\frac{\partial m/\partial b}{\partial m/\partial z} < 0,$$

because $\partial m/\partial k = z\alpha k^{\alpha-1} + (1-\delta)$, and $\partial m/\partial b = -1$.

define the present value of asset in place (PVAP) as the sum of current and future dividends, $S(z,w) - G(z,w)$, giving the usual decomposition of equity value in PVGO plus PVAP.

Optimal capital, k^*, and debt, b^*, are found by solving the program in (2). Because of decreasing returns to scale and depreciation, the optimal capital stock is finite. The optimal financing choice at $t = 0$ can lead to either risk-free or risky debt issuance when $b^*/k^* \leq (1-\delta)$ and $b^*/k^* > (1-\delta)$, respectively. Bankruptcy costs create an incentive to minimize the issuance of risky debt. Because the debt price is strictly increasing in b', the constraint in (1) is binding: $k^* = w + D(z, k^*, b^*)$. The firm will issue risky debt only when it needs to finance investments in excess of the net worth (i.e., $k^* > w$) and at the same time $b^*/k^* > (1-\delta)$. We focus on situations in which the firm's net worth is low enough, and the firm issues risky debt, which occurs only when $w < \delta k^*$ (the debt being risk-free in the alternative case).[5]

The optimal capital, k^*, is monotonically increasing in z and so the extent to which the firm can grow further is reduced. The relation of growth options with credit risk depends on the effect that the shock has on the related financing decision, and ultimately on b^*/k^*. However, the interaction between credit risk and growth option is made non-trivial by the fact that the shock that triggers investment and debt issuance also increases the net worth, thus reducing the need of debt financing.

We investigate the relation among investment, growth options and credit risk using numerical simulations of the model under the assumption that $\log z' = \rho_z \log z + \sigma_z \varepsilon$, with $\varepsilon \sim \mathcal{N}(0,1)$, and using the parameters $\rho_z = 0.7$, $\sigma_z = 0.3$, $\alpha = 0.4$, $\eta = 0.1$, and $\delta = 0.12$.[6] In all these simulations, we set the level of current capital and debt so that firms are constrained and issue risky debt.

In Figure 1 we plot the investment and financing policies, Panel (a), and the credit spreads and PVGO over asset, Panel (b), against the productivity shock z. Given two firms at the same (k,b), for which they are currently solvent, the firm that is exposed to the larger z installs a higher k^*, as expected. Because the greater shock also increases the net worth, even if the debt increases, the firm ends up with lower book leverage and credit spread. Despite the initial investment being larger, the growth option in correspondence of

[5]We obtain the threshold δk^* as the w such that even the largest issuance of risk-free debt $b^* = (1-\delta)k^*$ would be insufficient to satisfy the financing constraint (i.e., $k^* > w + b^*$). In the dynamic model presented later on, the requirement $w < \delta k^*$ is satisfied because the tax incentive to debt financing makes b sufficiently high so that w is low enough to have credit risk.

[6]Derivations of the closed-form expressions of debt and equity value are in Appendix A.

the larger shock is more valuable. This is because of two effects. First, a persistent positive shock is informative of future positive shocks and increases the optimal investment $\hat{k}(z')$, thus increasing $\overline{z}(k', b')$. Second, the lower book leverage decreases $\underline{z}(k', b')$. Both these effects lead to a higher PVGO.

In Panels (c) and (d) of Figure 1 we examine the response to the same productivity shock ($z = 1$) of firms with different initial production capacities (in the neighborhood of $\hat{k}(z)$). The smaller firm has more incentives to invest, but because of the lower net worth it also faces more severe financing constraints. Hence, the effect of size on investment and leverage is not straightforward. In Panel (c), we show that the smaller firm invests less and issues relatively more debt because the effect of increased financing constraints is dominant. Overall, Panel (d), the smaller firm ends up with more leverage, which increases the cost of debt, and reduces PVGO because the default threshold, $\underline{z}(k', b')$, is higher.

Our setup allows us to analyze the effect of different proportions of PVGO on credit spread, for firms of equal value. In Figure 2, a different asset composition is achieved by choosing, among firms with same $S(z, w)$ for a given b, different couples (z, k): a higher z is necessarily paired with a lower k. As seen in Panels (a) and (b), a firm with higher z (and lower k) invests more, which reduces the value of the residual growth options. The effect of investment is to increase leverage, and therefore the credit spread. On the other hand, Panels (c) and (d), a firm with relatively higher k (and therefore lower z) invests less, which increases the value of the residual growth option. At the same time, the firm with higher capital chooses a relatively lower debt, and therefore a lower leverage and credit spread.

Arnold et al. (2013) show that there is a *positive* relation between credit spread and PVGO for firms with same leverage and different growth options. We study a similar cross-sectional comparison in the context of our model, where growth options result endogenously from optimal investments and financing decisions using risky debt. In Figure 3, we compare firms with different levels of growth options, when book leverage (b^*/k^*) is the same. While the credit spread appears almost constant for the different levels of z and k, because b^*/k^* is the first order determinant of credit spreads, the value of the options to grow is decreasing in the current investment, which is higher for higher z and lower k. Differently from Arnold et al. (2013), in our model growth options *are not* positively related to credit spreads, for firms with same book leverage.

Overall, in a simplified setup we can create a cross-section in which we either hold $S(z, w)$

constant (Figure 2), or b^*/k^* constant (Figure 3), but not both (i.e., the cross-section would become a singleton). The analysis shows that the relation between growth options and credit spread is negative when we control for value, and it is (virtually) absent when we control for book leverage.

In the next section we present a dynamic model with realistic frictions and endogenous states. Such features are necessary to conduct cross-sectional comparison of firms with *same value and same (book) leverage*.

4. Dynamic Model

We propose a dynamic model of corporate decisions that is characterized by firm heterogeneity and endogenous default. Differently from the simple model, we include corporate taxes, real adjustment costs, external equity financing frictions, debt adjustment costs, operating leverage, financial distress costs, and consider countercyclical risk premia. The model is therefore similar, in spirit, to that of Hennessy and Whited (2007) in the description of the firm's decisions, and to those of Berk et al. (1999), Zhang (2005) and Gomes and Schmid (2010) in the choice of a reasonably simple (exogenously specified) pricing kernel.

4.1 The economy

Information is revealed and decisions are made at a set of discrete dates $\{0, 1, \ldots, t, \ldots\}$. The time horizon is infinite. The economy is composed by a utility maximizing representative agent and a fixed number of heterogenous firms ($j = 1, \ldots, J$) that produce the same good. Firms make dynamic investment and financing decisions and are allowed to default on their obligations. Defaulted firms are restructured and then continue operations, so as to guarantee a constant number of firms in the economy. The agent consumes the dividends paid by the firms and saves by investing in the financial market. We do not close the economy and derive the equilibrium, but instead choose an exogenously specified stochastic discount factor.

There are two sources of risk that capture variation in the firm's productivity. The first, z_j, captures variations in productivity caused by firms' specific events. Idiosyncratic shocks are independent across firms, and have a common transition function $Q_z(z_j, z'_j)$. z_j denotes the current (or time–t) value of the variable, and z'_j denotes the next period (or time–$(t+1)$)

value.

The second source of risk, x, captures variations in productivity caused by macroeconomics events. The aggregate risk is independent of the idiosyncratic shocks and has transition function $Q_x(x, x')$. Q_z and Q_x are stationary and monotonic Markov transition functions that satisfy the Feller property. z and x have compact support. For convenience of exposition, we define the state variable $s = (x, z)$, whose transition function, $Q(s, s')$, is the product of Q_x and Q_z. As there is no risk of confusion, we drop the index j in the rest of the section.

4.2 Firm policies

We assume that firm's decisions are made to maximize shareholders' value. An intuitive description of the chronology of the firm's decision problem is presented in Figure 4. At t, the two shocks $s = (x, z)$ are realized and the firm cash flow is determined based on current capital stock, k, and debt, b. Immediately after that, the firm simultaneously chooses the new set of capital, k', and debt, b' for the period $]t, t+1]$. This decision determines d, the residual cash flow to shareholders, which can be positive (a dividend) or negative (an injection of new equity).

At t, the cash flow from operations (EBITDA) depends on the idiosyncratic and aggregate shocks, and on the current level of asset in place, $\pi = \pi(x, z, k) = e^{x+z}k^{\alpha} - f$, where $\alpha < 1$ models decreasing returns to scale and $f \geq 0$ is a operating cost parameter that summarizes all operating expenses excluding interest on debt.

The capital stock of the firm might change over time. The asset depreciates both economically and for accounting purposes at a constant rate $\delta > 0$. After observing the realization of the shocks at time t, the firm chooses the new production capacity k', which will be in operation during the period $]t, t+1]$. The firm can either increase or decrease the production capacity, and the net investment equals to $I = k' - k(1-\delta)$. Similar to Abel and Eberly (1994) and many others after them, we assume that the change in capital entails an asymmetric and quadratic adjustment cost $h(I, k) = \left(\lambda_1 \mathbf{1}_{\{I>0\}} + \lambda_2 \mathbf{1}_{\{I<0\}}\right) I^2/\delta k$, where $0 < \lambda_1 < \lambda_2$ model costly reversibility, and $\mathbf{1}_{\{.\}}$ is the indicator function. The economic interpretation of λ_i, $i = 1, 2$, is straightforward: it is the per cent cost of a (dis)investment $I = \delta k$.

The debt level might also change over time. At any date, the firm can issue a one–period zero–coupon unsecured bond. As is shown in Figure 4, at time t the firm chooses the nominal value of the debt contract b' that will be repaid at $t+1$. If the firm is solvent, the market value of such bond, $D(s, p')$, depends on the current state s and on the choices of the debt and the capital stock, $p' = (k', b')$, that are made after observing the shocks.

Changing the debt level entails a proportional adjustment cost, $\theta|b' - b|$, with $\theta \geq 0$. Since the issuance decision is contemporaneous to repayment of the nominal value of old debt b, the debt decision generates a net cash flow equal to $D(s, p') - b - \theta|b' - b|$.

We assume a linear corporate tax function with rate τ. The tax code allows deduction from the taxable income of the depreciation of assets in place, δk, and of interest expenses.[7] Modeling deduction of the interest at maturity of the bond would entail keeping track of the value of the debt at issuance, therefore increasing the number of state variables. For the sake of numerical tractability, we assume that the expected present value of the end-of-period interest payment $b' - D(s, p')$, which we denote $PI(s, p')$, can be expensed when the new debt is issued at time t. In case of linear corporate tax, and assuming knowledge of the equilibrium conditional default probability, this is equivalent to the standard case of deduction at $t+1$. The after–tax cash flow from operations plus the net proceeds from the debt decision is

$$v = v(s, p, p') = (1-\tau)\pi + \tau\delta k + \tau PI(s, p') + D(s, p') - b - \theta|b' - b|. \tag{4}$$

We incorporate insolvency on a cash flow basis as an additional element to standard trade-off costs that are already present in our model. The firm is insolvent, $v < 0$, if the after–tax cash flow from operations plus the net proceeds from debt policy is lower than the value of the debt that is due. In this case, if the default option is not exercised by shareholders, the company must raise enough new equity capital to cover the cash shortfall and pay a proportional transaction cost $\xi \geq 0$. In other words, to raise capital for $v < 0$, the firm pays a cost $v\xi$. The rationale for modeling insolvency stems from the fact that in the real world there are some state contingent financial penalties associated with high leverage (for example, the loss of intangible assets and the disruption of operations), which are paid by shareholders and are hard to measure. These state contingent costs are included in our

[7] The firm is allowed to deduct interest when solvent. In case of insolvency, both the principal and the interest are forgiven by the debt holders in exchange of the ownership of the firm and the interest payment cannot be deducted.

framework in a reduced form by assuming that, in case of financial distress, the firm receives only a fraction of the capital that is injected by the shareholders. As shown later, these costs are crucial for the success of the calibration of the model.

The equity payout is therefore equal to $y = y(s, p, p') = v\left(1 + \xi \mathbf{1}_{\{v<0\}}\right) - I - h(I, k)$ where, on the right-hand side, the first term is the after–tax cash flow from operations, inclusive of the distress cost, and the other terms are the net proceeds from (dis)investment. Finally, the distribution to shareholders is

$$d = d(s, p, p') = y \cdot (1 + \zeta \mathbf{1}_{\{y<0\}}), \tag{5}$$

whereby, if the payout is positive, the firm pays a dividend to the current shareholders, and if the payout is negative (i.e., the firm issues new shares), d is the equity injection. In the latter case, the company incurs a proportional issuance cost $\zeta \geq 0$, as only y is the actual inflow of the corporation.

4.3 The value of corporate securities

Following Berk et al. (1999), Zhang (2005), and Gomes and Schmid (2010), we exogenously define a pricing kernel that depends on the aggregate source of risk, x. The associated one–period stochastic discount factor $M(s, s')$ defines the risk-adjustment corresponding to a transition from the current state x to state x'. We assume that M is a continuous function of both arguments.

The firm can issue two types of securities, debt and equity, whose equilibrium prices are determined under rational expectations in a competitive market. The cum–dividend price of equity, $S(s, p)$, is the sum of current distribution, d, and the present value of the expected future optimal distributions, which is equal to the next period price $S(s', p')$. Since this sum can be negative, a limited liability provision is also included (i.e., default on a value basis), in which case the firm's equity is worthless:

$$S(s, p) = \max\left\{0, \max_{p'} \left\{d(s, p, p') + \mathbb{E}_s\left[M(s, s')S(s', p')\right]\right\}\right\}. \tag{6}$$

The value function, S, is the solution of functional equation (6). We define $\omega = \omega(s, p)$ as in indicator function that captures the event of default. Note that, if $\omega = 0$, the optimal investment and financing decision is $F(s,p) = p^*$, where $p^* = (k^*, b^*)$ is the optimal choice of the second argument in the max in (6). The optimal policy is therefore summarized by (ω, F).

A growth opportunity at a given date is the option to increase the production capacity at a later date. The present value of growth opportunities (PVGO) at (s,p) is

$$G(s,p) = \mathbb{E}_s \left[M(s,s') S(s',p^*) \chi_{\{k^{**}>k^*\}} \right] \tag{7}$$

where the optimal policy at (s,p) is $p^* = (k^*, b^*)$, and $\chi_{\{k^{**}>k^*\}}$ is the indicator function that the optimal investment policy at (s', p^*), denoted k^{**}, is strictly higher than k^*.

As for the debt contract, the end-of-period payoff to debt holders, $u(s',p')$, depends on the current policy, $p' = (k', b')$, the new realization of the shocks s', and on whether the firm is in default:

$$u(s',p') = b'(1-\omega(s',p')) + [\pi' + \tau\delta k' + k'(1-\delta)]\,(1-\eta)\omega(s',p'). \tag{8}$$

In case of default, similarly to Hennessy and Whited (2007), the bondholders receive the sum of the cash flow from operations, the depreciated book value of the asset, and the tax shield from depreciation, all net of a proportional bankruptcy cost, η. Hence, at issuance the debt value is

$$D(s,p') = \mathbb{E}_s \left[M(s,s') u(s',p') \right]. \tag{9}$$

One final item that needs to be evaluated is the expected present value of the interest payment, $PI(s,p')$, which enters the determination of the after tax cash flow in (4):

$$PI(s,p') = [b' - D(s,p')]\, \mathbb{E}_s \left[M(s,s')(1-\omega(s',p')) \right]. \tag{10}$$

Because the interest is deductible only if the firm is not in default, the expectation term is the conditional price of a default contingent claim.

In Appendix B, we prove the existence of the solution of the program given by the Bellman operator in equation (6), subject to the constraints in equations (9) and (10). The model is solved numerically by simultaneously finding the optimal value of S, D and PI. We describe the numerical approach in Appendix C.

4.4 Credit default swap spread

In the observed sample, we use firm-level data on credit default swaps (CDS) as a measure of credit spreads. In the model, we consider a T-period CDS contract that insures a debt obligation with face value equal to one against the first default. [8] The CDS price (spread) is

$$cds(s,p) = (1-R)\frac{\sum_{n=1}^{T} \overline{\mathcal{P}}_n(s,p)}{\sum_{n=1}^{T} \left(\overline{\mathcal{P}}_n(s,p) + \overline{\mathcal{S}}_n(s,p)\right)}, \tag{11}$$

where R is the recovery rate on the face value of a unit bond, $\overline{\mathcal{P}}_n$ and $\overline{\mathcal{P}}_n$ are the prices of contingent claim that pay \$1 if the firm does and does not default for the first time in n, respectively. We provide a complete derivation of equation (11) in Appendix D.

4.5 Stochastic discount factor

We assume that the idiosyncratic shock z and the aggregate shock, x, follow autoregressive processes of first order, $z' = (1-\rho_z)\overline{z} + \rho_z z + \sigma_z \varepsilon'_z$ and $x' = (1-\rho_x)\overline{x} + \rho_x x + \sigma_x \varepsilon'_x$, respectively. In the above equations, for $i = x, z$, $|\rho_i| < 1$ and ε_i are i.i.d. and obtained from a truncated standard normal distribution, so that the actual support is compact around the unconditional average. We assume that ε_z are uncorrelated across firms and time and are also uncorrelated with the aggregate shock, ε_x. The parameters ρ_z, σ_z, and $\overline{z}$ are the same for all the firms in the economy, $\overline{z}$ and $\overline{x}$ denote the long term mean of idiosyncratic risk and of macroeconomic risk, respectively, $(1-\rho_i)$ is the speed of mean reversion, and σ_i is the conditional standard deviation. With this specification, the transition function Q satisfies all the assumptions required for the existence of the value function (see Appendix B).

Finally, following Zhang (2005), we specify the stochastic discount factor (SDF) as

$$M(s,s') = \beta e^{g(x)(x-x')}, \tag{12}$$

where the state–dependent coefficient of risk–aversion is defined as $g(x) = \gamma_1 + \gamma_2(x - \overline{x})$, with $0 < \beta < 1$, $\gamma_1 > 0$ and $\gamma_2 < 1$.

Following the literature, the aggregate risk parameters are taken from Cooley and Prescott

[8] We exclude from our analysis the managerial and agency implications that the presence of a CDS market has on firm policies (see for example, Bolton and Oehmke 2011; Saretto and Tookes 2013; Subrahmanyam et al. 2014; Danis and Gamba 2018).

(1995) and converted to annual frequency. We obtain a value for the persistence of the systematic risk (ρ_x) and the aggregate volatility (σ_x) of 0.815 and 0.013, respectively. We adopt the parameters of the stochastic discount factor calibrated by Liu and Zhang (2014). The personal discount factor (β) is set to 0.988, and the SDF parameters (γ_1 and γ_2) to 17 and -1000, respectively. These parameters produce an annualized average real interest rate of 2.72%, annualized volatility of risk free rate of 2.02%, and an average Sharpe ratio of 0.37.

5. Calibration

After fixing the five parameters that describe the aggregate source of risk and the SDF, the model has 13 more parameters. Because the depreciation rate and the operating cost parameters are very closely related, we fix the depreciation rate δ at 7.5% to approximate the average depreciation rate in our sample. Following Warusawitharana and Whited (2016) we also fix the equity adjustment costs to 5%. We calibrate all the remaining 11 parameters by minimizing the sum of square deviations of a set of quantities that are observable in the data and in the simulated economy.

Important objectives of the calibration exercise are that the model captures the basic relationship between determinants of credit risk and credit spreads. The model should therefore match the average credit spread, the (book and market) leverage ratio, the frequency of debt issuances and defaults that are observed in the real economy. The model should also create an acceptable amount of total risk, and thus simulated firms should have the same average, standard deviation and autocorrelation of profitability as real firms. Because the driving force of the model is the ability to modify the production capacity, we also require the simulated economy to have the same average and standard deviation of investments as the real economy. As the relevant sources of total risk match up with the economy, firms should exhibit similar market to book ratios.

We also incorporate into the calibration the conditional dependence of credit spreads from the most popularly used proxy of growth options, the market to book ratio. We do so by asking the model to match the regression coefficients of the market to book ratio in Fama-MacBeth regressions equal to those reported in Columns 1 and 2 of Table 2. Thus, we include both coefficients from regressions that control for book and market leverage.

Finally, as most of the economic mechanisms that we want to investigate are portrayed in the literature as cross-sectional relations, we also ask the model to generate a representative cross-section of firms. We do so by matching credit spreads of ten portfolios of firms sorted on book leverage. Similar calibration results would be obtained by sorting firms on market leverage.

5.1 Calibrated parameters

We report the calibrated parameters in Panel B of Table 3. The idiosyncratic shocks is about as persistent as the aggregate shock, 0.822 versus 0.815, but more volatile, 0.352 versus 0.013. Both parameter are close to values commonly used in the literature.

The estimated marginal corporate tax rate, τ, is 0.154, close to the estimates produced by Graham (1996a) and Graham (1996b) (i.e., approximately 16% for our sample). In contrast to Hennessy and Whited (2005) and Altinkilic and Hansen (2000), debt adjustment costs, θ, are small at 0.2% possibly due to the nature of the firms in our sample that are all large and with access to the corporate bond market, and thus face lower cost at issuing new debt (relative to small firms that rely primarily on bank loans).

The estimate for the production function parameter α is 0.387. There are large bounds around figures reported in the literature: from 0.30 in Zhang (2005) to 0.75 in Riddick and Whited (2009). We obtain an estimate for the operating cost parameter, f, equal to 0.671 (unit of capital). As the average size of the firms, in unit of capital, is around 6.5, the figure translates into a cost of about 10% per year.

The calibrated value of the bankruptcy cost parameter, η, is 0.421. There is not a very strong consensus on what this parameter should be (Gomes and Schmid 2010; Hennessy and Whited 2007). Glover (2016) estimates default cost parameters at the firm level (using a simpler model) and finds an average value of 0.432, and values that range from 0.189 for lower rated firms to 0.568 for AAA rated companies. Our number matches quite well with those reported by Glover (2016).

The value of the expected recovery rate parameter, R, is 0.400 and it is close to the empirical evidence: Glover (2016) presents an average recovery rate equal to 0.423 based on Moody's data (although on a longer sample); Elkamhi et al. (2012) reports implied estimates based on 5-year CDS contracts of 0.338 and 0.143, for senior and subordinated reference obligations, respectively. A brief analysis of the Moody's Recovery data set in our sample period gives us an average market recovery (i.e., price at which the claim trades one month after the default event) of 36% for the entire U.S. sample of industrial firms. By comparison, the sample of nine defaults used in the calibration had a market recovery rate of 47%.

A small set of parameters does not have any direct benchmark for comparison: the investment cost parameter, λ_1, is close to zero. The disinvestment cost parameter, λ_2, is calibrated at 0.964, meaning that a disinvestment of δk incurs an adjustment cost equal to 96% of the disinvestment. The figure makes the model almost similar to that of Kuehn and Schmid (2014) where capital is irreversible (i.e., 100% disinvestment cost). Finally, the financial distress costs has an economically relevant magnitude of 5.3%.

5.2 Model fit

In Panel C of Table 3, we compare the values of the moment conditions computed from the observed empirical sample (*Data*) to the moment conditions computed from the simulated sample (*Model*).

The model produces a distribution of five-year credit spreads (0.979% average and 1.655% standard deviation) that approximates well the corresponding empirical distribution (1.132% average and 1.941% standard deviation). All decile portfolios based on book leverage are also very close to their empirical counterparts. The one-year default frequency of 0.668% is close to the respective empirically observed quantity, 0.694%. Similarly, the model generates a leverage distribution that is very close to the one observed in the data: the average market and book leverage ratios being slightly larger in the model than in the data. The model also produces relatively similar dynamic debt policies: the debt issuance frequency is 21% relative to 17% in the data.

As for the real side of the firm, both averages and standard deviations of profitability (0.257 and 0.218) and investments (0.083 and 0.124) are close to their data counterparts (0.216 and 0.153 for profitability, and 0.073 and 0.120 for investment), indicating that the model generates the proper amount of fundamental risk.

A proper calibration of the risk profiles should also translate into relatively accurate equity prices. For example, firms in the simulated economy exhibits similar average market to book ratios (2.53 versus 2.44).[9] Similarly, to Nikolov and Whited (2014) the model

[9]As described in the Data section, all variables are scaled by the sum of debt and book value of eq-

however has difficulty at matching the standard deviation of the market to book ratio.

Finally, the model is able to match the quantitative relation between credit spreads and market to book ratio. The market to book sensitivity of credit spreads is -0.588 when controlling for book leverage and 0.175 when controlling for market leverage, which compare well to -0.425 and 0.161, respectively, in the data. Since it is not possible to produce an equivalent measure of variation in credit spreads due to an exact measure of growth options, this is as close as we can get in terms of establishing quantitative bounds to the impact of future investment prospects to current credit risk.

6. Growth Options and Credit Risk

First we study the relation between credit spreads and the exact measure of the present value of growth opportunities computed based on equation (7) (i.e., PVGO). Second, we study how PVGO is related to systematic risk. Finally, we connect the findings from the simulated economy to the existing empirical literature.

6.1 Growth options and credit spreads

In Table 4, we report results of a Fama-MacBeth regression of credit spreads on growth options, leverage, asset volatility and either the value of the equity or the size of the productive assets. As the simulation involves generating N firms for T periods over S simulated economies, we report the estimate obtained by averaging (cross-sectional estimated coefficients) first across time and then across simulated economies. Standard errors are computed as the standard deviation across simulated economies of the time series averages.

We construct two measures of growth options: the value of the investment option (i.e., PVGO divided by physical assets) and the asset composition (i.e., the proportion of the value of the firm represented by growth options — PVGO divided by equity). Each measure allows us to investigate the relation between credit risk and growth options from a different perspective and replicate the results presented in Figures 2 and 3, where we kept fixed equity and leverage to specific value.

uity, instead of total assets. As a consequence the average market to book ratio in the data is equal to approximately 2.4 when scaled by the sum of debt and equity. It is instead 1.8 when scaled by total assets.

After controlling for leverage, asset volatility and the size of the invested capital, the relation between the credit spread of the average firm and growth options is negative. In particular, as described in Section 3, controlling for the value of the equity, more valuable growth options are the result of a lower current investment, and such choice reduces external financing needs and leverage. Table 4 confirms this theory when we control for leverage and total cash flow volatility. That is true whether we consider the total value of growth options scaled by physical assets and control for the value of the equity, or whether we consider the proportion of PVGO relative to equity. The negative coefficients in the regressions indicate that better future prospects and/or an asset composition that is titled towards growth options, as opposed to asset in place, signal a greater ability of the firm to repay current obligations. At the same time, because growth options and leverage are chosen simultaneously, the negative coefficient shows that a firm with lower credit risk has higher ability to exercise future growth opportunities, for this reason making them more valuable.

From a quantitative perspective, we can relate the coefficient on PVGO over Asset of Column 1, to the coefficient on the market to book ratio (controlling for book leverage) reported in Panel C of Table 3. A one standard deviation change in the market to book ratio, which contains economic information about both growth options and assets in place, reduces credit spreads by 59 basis points. A one standard deviation change in PVGO over asset, instead, reduces credit spreads by approximately 45 basis points, thus confirming that growth options have a larger impact on credit spreads than current productive capital. The result seems at least comforting that on average most of the information contained in the market to book ratio is related to growth options. We will return to this comparison later in Section 6.3

Since the presence of growth options is typically associated with small innovative firms, it is worth wondering how the relationship between PVGO and credit spreads varies across firms with different size. In our model, this effect is captured by the curvature of the production function: firms with small productive capital has higher marginal productivity of capital, relative to larger firms. We repeat the analysis described above, but for each time and simulated sample, we sort data into ten groups based on the initial capital stock and thus run ten separate regressions. We then average the ten coefficients across time and across simulated economy and plot them against the ten equity groupings in Figure 5. While the coefficients (of PVGO over Asset and of PVGO over Equity) are negative for all ten size groups, we note that they vary (increase) across groups. In other words, the negative association between growth options and credit risk is more important for small firms than they are for large firms, thus confirming the economic intuition described in previous sections.

6.2 Growth options and systematic risk exposure

So far, we have discussed reasons why the relationship between growth options and credit risk should be negative. However, an established assumption in the asset pricing literature (since Berk et al. 1999) is that growth options represent a source of risk for shareholders because they are levered claims on the firm's assets exposure to systematic risk. If growth options add risk to the equity because they increase the exposure of total assets (i.e., asset in place plus options to grow) to aggregate uncertainty, by extension they also increase the riskiness of debt claims and therefore predict a positive relationship between growth options and credit spreads, as argued for example by Arnold et al. (2013) and Kuehn and Schmid (2014).

In a more recent contribution, Babenko et al. (2016) argue that the nature of growth options might have counter-intuitive implication for the systematic risk exposure of a firm: in particular, growth options that draw value from idiosyncratic risk decrease the systematic exposure of the firm. Therefore as the nature of growth options (the extent to which they derive value from firm-specific shocks) affects the systematic risk exposure of the assets, it also affects the relation between the value of the options to grow and credit spreads.

In particular, if growth options draw their value from firm-specific shocks, their beta is decreasing with such shocks. In that case, the relation between growth options and credit risk may not be positive: according to Babenko et al. (2016), in fact, if growth options derive their value from idiosyncratic risk, their beta is negatively related to the same shocks that increase their value. Hence, as growth options become more valuable, they decrease the systematic risk exposure of the firm, and therefore can be associated with lower (rather than higher) credit spreads.

We examine the relation between asset composition and/or the value of growth options and their betas in the context of our calibration. We report such relationships in Table 5 in the form of Fama-MacBeth regression coefficients, where we control for leverage, productive asset (because options are limited by decreasing returns to scale) and total value of the firm. The calculation of PVGO beta follows the state-contingent formula presented by Zhang

(2005), but adapted to the definition of PVGO presented in our equation (7). The four columns report different specifications that confirm the same idea: controlling for leverage and the value of the equity, firms with a higher proportion of growth options have lower PVGO beta. Notably, the signs of the control variables confirm the economic intuition: a higher leverage increases the systematic risk of growth options. On the other hand, a bigger size has the effect of making growth options more systematic.

The result shows that the dependence of credit risk on systematic risk may not give a positive relation between growth options and credit risk, with the sign (and intensity) of the relation potentially depending on the model specification and parameterization (see also Carlson et al. 2004; Zhang 2005; Cooper 2006; Obreja 2013). For example, Kuehn and Schmid (2014) with a different set of parameters and a different formulation for the stochastic discount factor reach very different conclusions.

6.3 Growth options and market to book ratio

We have required that the model calibration matches the quantitative relation between credit spreads and market to book ratio. As we noticed at the outset, the sign of this relationship changes with the measure of leverage included in the regression. We show here how this apparent contradiction is unrelated to the baseline economic relationship between credit risk and growth options, which in our model is negative.

In Table 6 we present results of Fama-MacBeth regressions of credit spreads in the simulated economy. The first two columns report the specifications that give origin to the coefficients used in the calibration exercise, reported in Table 3 and confirm the signs that we find in the data (see also Table 2).

The fact that market leverage contains information about debt capacity as well as growth options is not new to the literature (see for example a non technical discussion by Nash et al. 2003). The advantage of our model is that we can decompose the various effects, as we are able to compute an exact measure of PVGO:

$$\text{Market Leverage} = \frac{b^*}{b^* + S} = \frac{b^*}{b^* + A + G}.$$

where G is the value of growth options and A is the asset in place. Therefore, an increase in G has the effect of simultaneously decreasing market leverage and increasing whatever measure of growth options is included in the regression. In Column 3, we substitute the traditional version of market leverage with a specification that excludes G from the denominator:

$$\text{Market Leverage without PVGO} = \frac{b^*}{b^* + S - G} = \frac{b^*}{b^* + A} \tag{13}$$

The substitution of the market leverage with this "alternative" definition leads the sign market to book to *turn back* to negative.

We observe the exact same sequence in the sign of the coefficients (i.e., negative, positive, negative) when we replace market to book with the two direct measures on growth options (i.e., PVGO over Asset and PVGO over Equity). Controlling for book leverage leads to a negative sign, controlling for market leverage to a positive one, and back to a negative one when controlling for the version of market leverage that does not include PVGO.

Overall, the results presented in Table 6 suggest that the positive relation between credit spreads and market to book is due to a confounding effect caused by market leverage being negatively related to PVGO.

6.4 PVGO and market to book ratio

The economic magnitude of impact of PVGO on credit spreads reported in Table 6 differs from the equivalent impact of the market to book ratio. Comparing Column 4 to Column 1 we see that, while one standard deviation change in PVGO over assets is associated with an average decrease in the firm's credit spread of 45 basis points, an increase of one standard deviation in market to book is associated with a decrease of 58 basis points. Please remember that right hand side variables are standardized before estimating the regression coefficients. We argue that such difference in sensitivity is related to the fact that market to book is, in certain states, an inaccurate proxy of the value of growth options, because it contains information that is also related to the assets in place.

To understand why the credit spread sensitivity to market to book is larger than the sensitivity to PVGO, we use equation (7) to decompose the market to book:

$$\text{Market to book} = \frac{S + b^*}{k^*} = \frac{G}{k^*} + \frac{A}{k^*} + \frac{b^*}{k^*}.$$

As the right hand side variables in Table 6 are standardized, the regression coefficients are equal to covariances. Given the above decomposition, the covariance of credit spreads and market to book can be decomposed into the covariance of credit spreads with PVGO over asset, PVAP over asset, and book leverage, respectively.

The latter is already accounted for in Table 6, because in these regressions we control for book leverage. Hence, the sensitivity on market to book is larger than the one to PVGO over asset because the former includes also a sensitivity of credit spreads to PVAP over asset, which is negative (i.e., more assets in place reduce credit risk).

When is market to book a good proxy for growth options? From the decomposition above, this is the case when the wedge between market to book and PVGO over asset is small. Such wedge grows larger when the asset in place and book leverage become also large. Because of decreasing returns to scale of the production technology, this happens when the production capital is high.

Indeed, we find that the correlation between market to book and PVGO over asset becomes weaker for larger firms. In Figure 6 we present conditional correlations of market to book to PVGO over assets and PVAP over assets for different groups of firms with similar size (i.e., we sort each cross-section of simulated firms in ten groups based on their initial capital). The figure shows that market to book is more weakly correlated with PVGO over asset and more strongly correlated with PVAP over asset for larger firms. An implication of this relation is that in the model, the correlation between PVGO and PVAP is negative (i.e., approximately –20%)

7. Conclusion

We develop a structural model of credit risk that allows for dynamic investment and financing policies in response to both idiosyncratic and systematic shocks, and that features financing and real frictions. The model allows us to compute a precise measure of the value of growth options.

We show that, in a world with decreasing returns to scale, the ability to make profitable investments by a financially constrained firm induces an increase in leverage and leads to a reduction of valuable opportunities to grow in the future, thus delivering a negative relation between growth options and credit spreads

However, this is not what the empirical finance literature finds: using market leverage as a control variable, the regression coefficients of growth options (proxied by market to book ratio) on credit spreads is positive. The positive sign is often ascribed to fact that both growth options and debt are claims mostly exposed to systematic risk. We show that this is not the case. We also show that the reason why credit spreads and market to book are positively related when controlling for market leverage is primarily because of a negative relation between market leverage and growth options. Finally, we describe circumstances in which market to book is a good proxy for the value of growth options.

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Figure 1: Growth options and credit risk

This figure shows optimal investment, k^*, debt financing, b^*, credit spreads, $b^*/D(z,k^*,b^*) - 1$, growth opportunities over asset, $G(z,w)/k^*$, and market to book, $(S(z,w)+b^*)/k^*$, for the simple model. We plot these quantities as function of z for given (k,b) in Panels (a)-(b), and against k for given (z,b) in Panels (c)-(d). Parameters are: $\rho_z = 0.7$, $\sigma_z = 0.3$, $\alpha = 0.4$, $\eta = 0.1$, and $\delta = 0.12$. To have credit risk, we create a positive financing gap by setting in panels (a)-(b) $k = \hat{k} - 1.75$ and $b = \hat{k} - 0.1$, and in panels (c)-(d) $z = 1$ and $b = \hat{k}$.

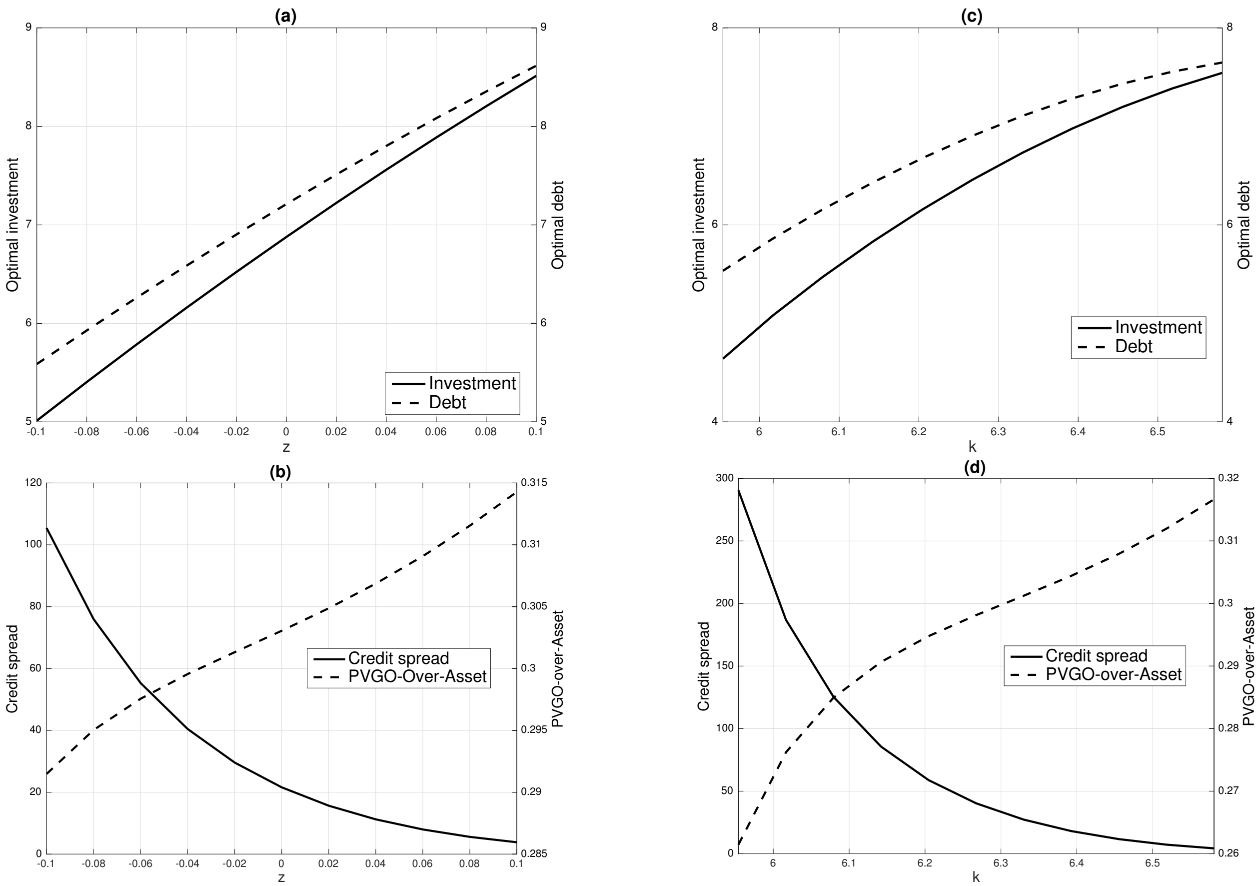

Figure 2: Growth options and credit risk for constant equity value

For firms with same equity value $S(z,w)$ and b, this figure shows optimal investment, k^*, debt financing, b^*, credit spreads, $b^*/D(z,k^*,b^*)-1$, growth opportunities over asset, $G(z,w)/k^*$, and market to book, $(S(z,w)+b^*)/k^*$, for the simple model. We plot these quantities as function of z in Panels (a)-(b), and against k in Panels (c)-(d). Parameters are: $\rho_z = 0.7$, $\sigma_z = 0.3$, $\alpha = 0.4$, $\eta = 0.1$, and $\delta = 0.12$. The plots are constructed by solving the program in (2) at different z and k for a given b, while holding equity value $S(z,w)$ constant, and then plotting the relevant quantities against z and k.

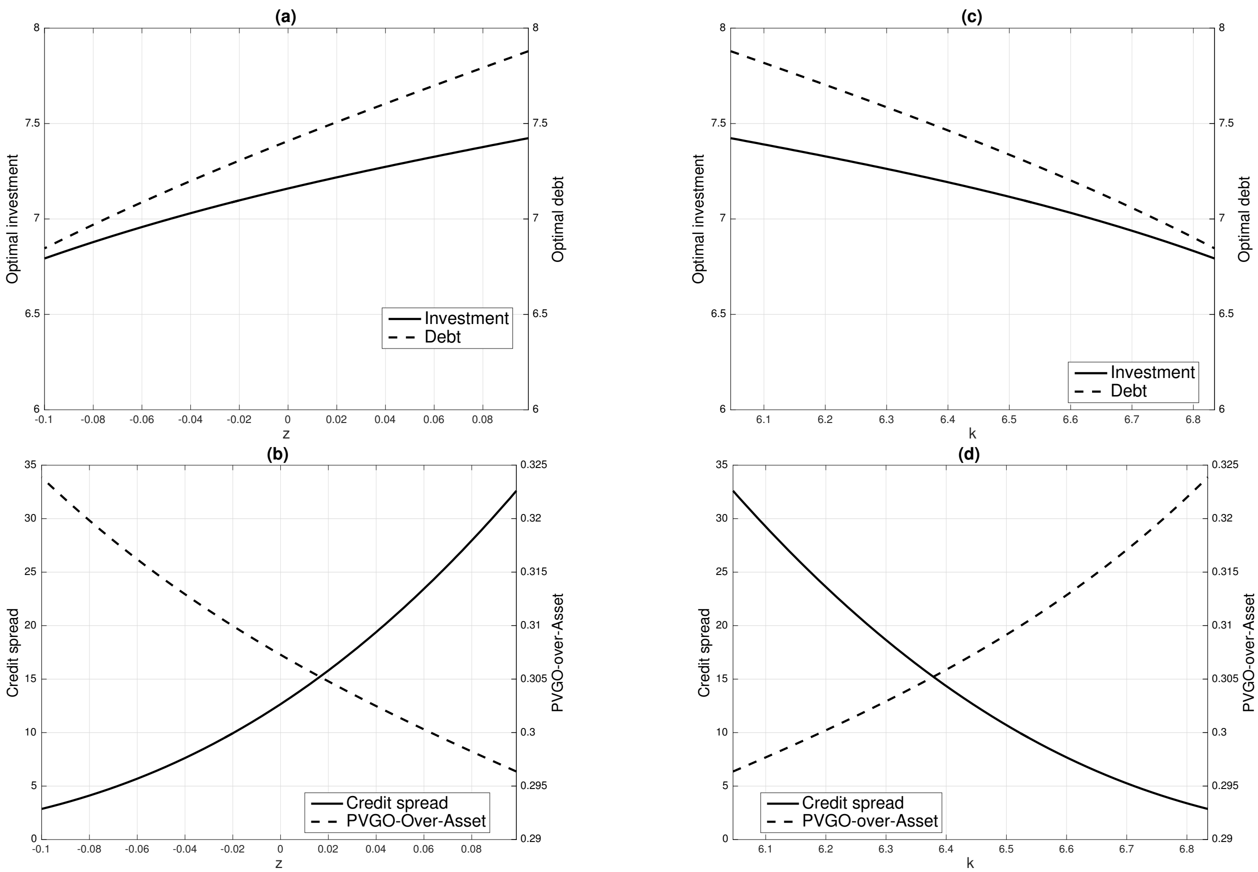

Figure 3: Growth options and credit risk for constant book leverage

For firms with same book leverage resulting from their decisions, b^*/k^*, this figure shows optimal investment, k^*, debt financing, b^*, credit spreads, $b^*/D(z, k^*, b^*) - 1$, growth opportunities over asset, $G(z, w)/k^*$, and market to book, $(S(z, w) + b^*)/k^*$, for the simple model. We plot these quantities as function of z in Panels (a)-(b), and against k in Panels (c)-(d). Parameters are: $\rho_z = 0.7$, $\sigma_z = 0.3$, $\alpha = 0.4$, $\eta = 0.1$, and $\delta = 0.12$. The plots are constructed by solving the program in (2) at different z and k for a given b, while holding book leverage, b^*/k^*, constant at 1.1, and then plotting the relevant quantities against z and k.

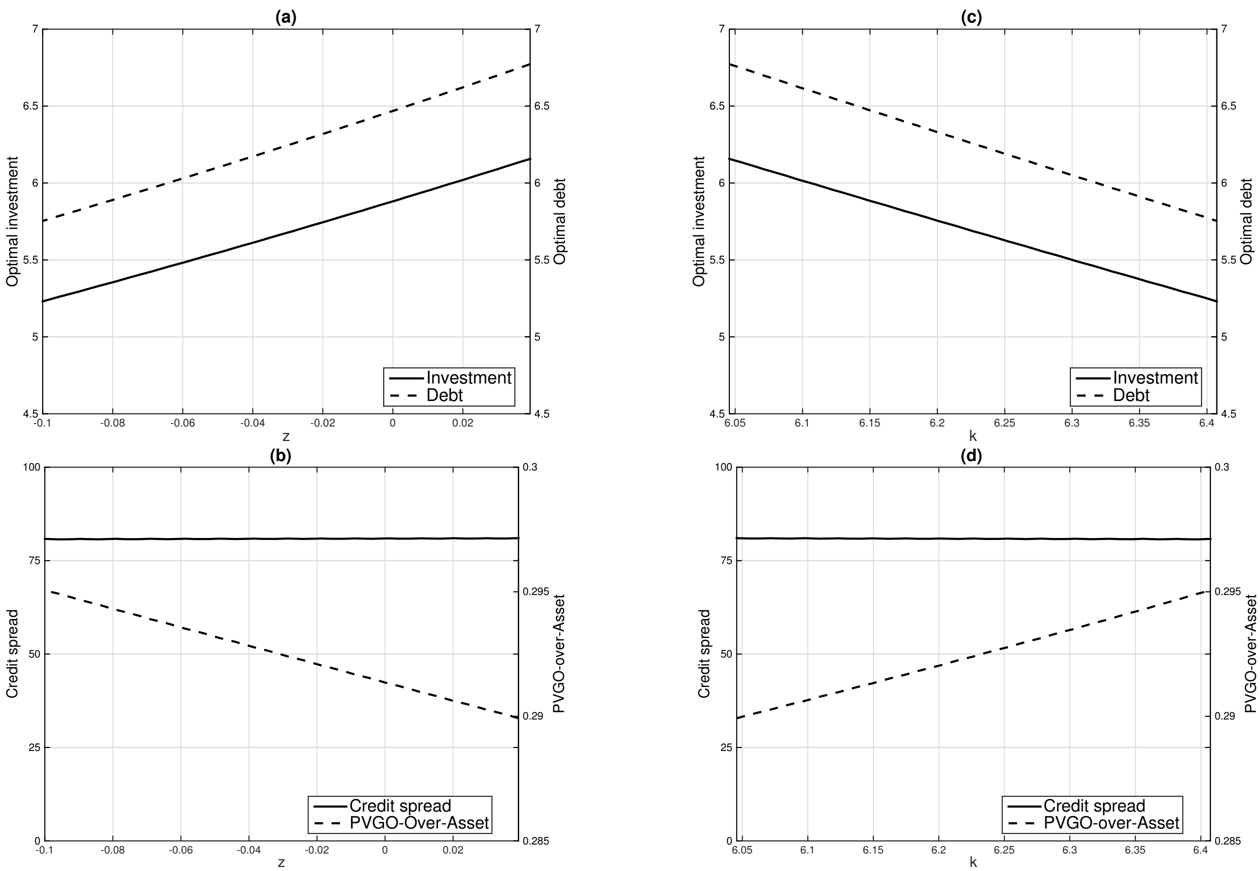

Figure 4: Model time line

This figure offers a description of the chronology of the firm's recursive decision problem. At t, the shocks $s = (x, z)$ are realized, and the firm's cash flow is determined based on the capital stock k and the debt b, or $p = (k, b)$. Immediately after t, the firm chooses the new set of capital and debt, as the combination $p' = (k', b')$ that maximizes the value of the equity, given by the sum of the current cash flow, d, plus the continuation value.

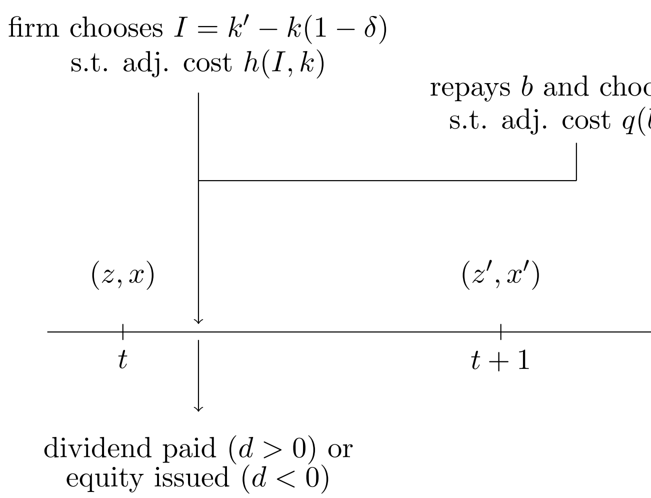

Figure 5: Credit spreads and PVGO

This figure plots the sensitivity of credit spreads to two measure of growth options: PVGO over Asset and PVGO over Equity. The sensitivities are computed from the simulated economy by estimating the coefficients of regression specifications similar to those reported in Columns (1) and (3) of Table 4, respectively, on different subsamples constructed by sorting the initial asset (i.e., the capital stock, k, at the beginning of the period) into ten deciles. The sensitivities are then plotted against the relative asset deciles, where 1 represents the smallest group and 10 the largest.

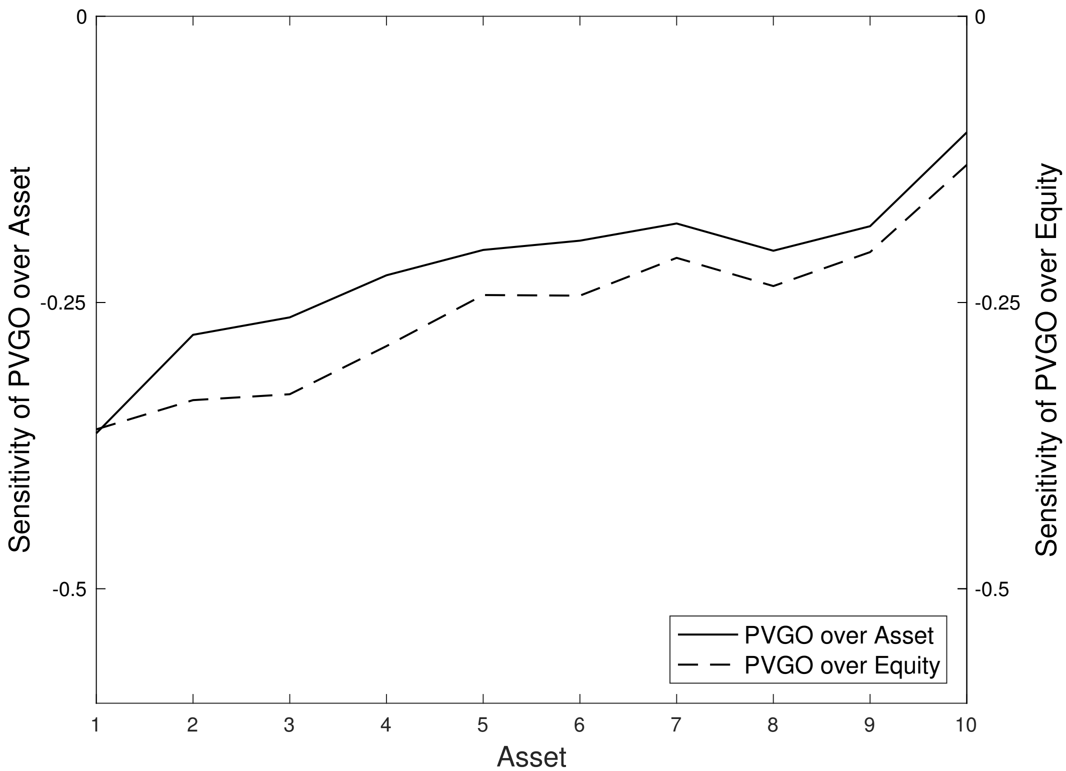

Figure 6: Market to book and PVGO

This figure plots the conditional correlations of market to book with PVGO over Asset and PVAP over Asset. The correlations are computed from the simulated economy by sorting the initial asset (i.e., the capital stock, k, at the beginning of the period) into ten deciles. The correlations are then plotted against the the relative asset deciles, where 1 represents the smallest group and 10 the largest.

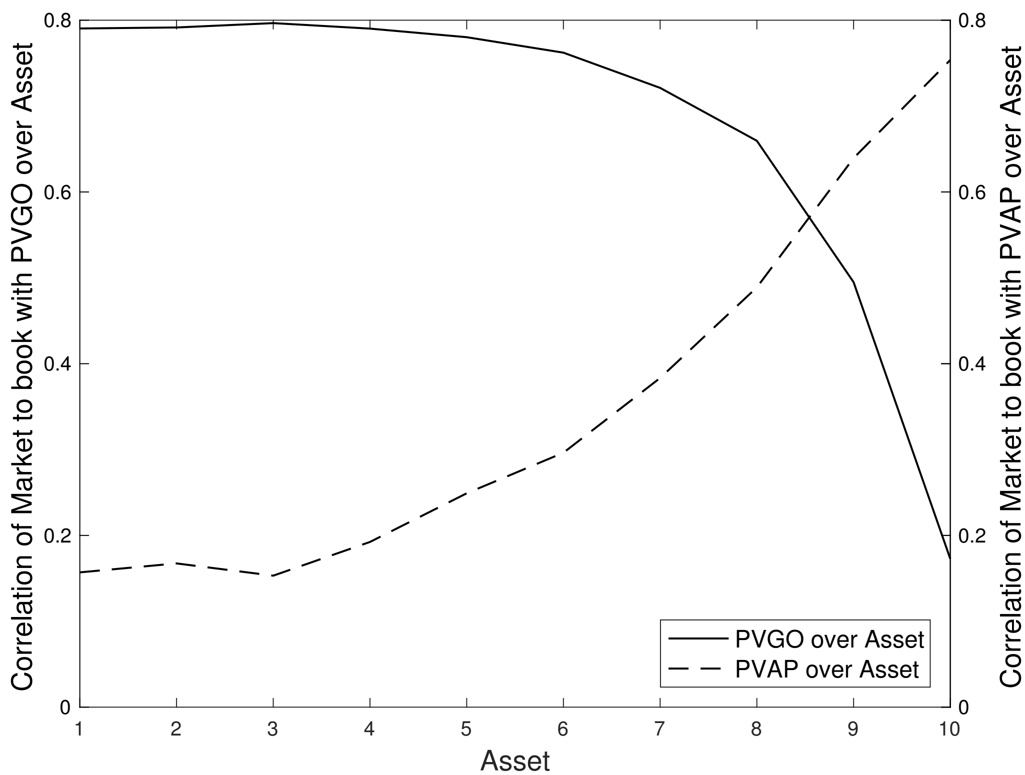

Table 1: Summary statistics

This table presents summary statistics for the sample of non-financial firms in the S&P 500 index during the 2001–2013 period. After cleaning data and requiring that firms have a CDS price in the last month of their fiscal year, we obtain a sample composed by 311 firms and 2548 firm-year observations. We report below summary statistics for the variables used in the model calibration and in the regressions reported below.

Before removing industry fix effects					
			Percentiles		
	Mean	St.Dev	1^{st}	50^{th}	99^{th}
5-year CDS Spread	1.412	2.214	0.101	0.615	9.993
1-year CDS Spread	1.164	3.882	0.027	0.244	11.855
Book Leverage	0.460	0.308	0.024	0.425	1.157
Market Leverage	0.245	0.172	0.007	0.202	0.798
Asset Volatility	0.261	0.165	0.087	0.219	0.724
Size	9.332	1.037	7.500	9.512	12.539
Profitability	0.234	0.167	0.007	0.224	0.714
Investments	0.093	0.125	-0.258	0.085	0.446
Depreciation	0.075	0.043	0.025	0.063	0.194
Market-to-Book	2.412	1.595	0.805	1.962	8.365
GO Score	-0.052	0.258	-0.814	-0.050	0.511
R&D to Sales	0.055	0.077	0.000	0.024	0.318
R&D to Capex	1.444	2.074	0.000	0.683	8.578
Intangibles	0.209	0.184	0.000	0.160	0.714
Non-Fix Assets	0.686	0.227	0.121	0.748	0.975
Citations per Patent	1.658	2.327	0.000	0.000	8.784
After removing industry fix effects — Calibration only					
			Percentiles		
	Mean	St.Dev	1^{st}	50^{th}	99^{th}
5-year CDS Spread	1.132	1.941	-1.863	0.599	8.689
1-year CDS Spread	0.903	2.611	-1.544	0.235	10.592
Book Leverage	0.420	0.284	-0.024	0.389	1.100
Market Leverage	0.212	0.152	-0.044	0.184	0.717
Profitability	0.216	0.153	-0.052	0.210	0.639
Investments	0.073	0.120	-0.255	0.067	0.409
Market-to-Book	2.443	1.434	0.209	2.232	7.567
Defaults					
Number of Defaults	9.000				
Annual Default Frequency	0.694				

Table 2: Credit spreads and proxies of growth options

The table presents the results of Fama-MacBeth regressions of 5-year CDS spreads on asset volatility, size, book leverage, market leverage, market to book ratio, the measure of growth opportunities computed following Trigeorgis and Lambertides (2014), two measures of R&D intensity (R&D scaled by sales and by capital expenditures), the ratio of intangibles total assets, the ratio of non-fix assets to total assets, and the ratio citations to patent applications based on Dass et al. (2017). All independent variables are winsorized at the 99% thresholds, to remove outliers, and subsequently standardized, for ease of interpretation. *t*-statistics are reported in parentheses. A constant is estimated but not reported. The regressions also include industry fixed effects based on Ken French 12 industry groupings. The sample includes non-financial firms in the S&P 500 index during the 2001–2013 period. After cleaning data and requiring that firms have a CDS price in the last month of their fiscal year, we obtain a sample composed by 311 firms and 2548 firm-year observations.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
Asset Volatility	1.049	0.965	1.209	1.141	1.073	0.967	1.066	0.955	1.094	0.964	1.237	1.097	1.258	1.196
	(4.53)	(4.88)	(4.19)	(4.30)	(4.45)	(4.99)	(4.42)	(4.88)	(4.50)	(4.94)	(4.46)	(4.91)	(4.59)	(4.83)
Asset	0.050	-0.016	0.046	-0.011	0.100	-0.017	0.086	-0.020	0.110	-0.015	0.111	0.003	0.135	0.043
	(1.15)	(-0.37)	(1.08)	(-0.31)	(2.38)	(-0.41)	(2.21)	(-0.51)	(2.77)	(-0.40)	(2.44)	(0.06)	(2.97)	(0.98)
Book Leverage	0.712		0.387		0.690		0.676		0.662		0.772		0.776	
	(6.56)		(5.03)		(5.82)		(5.81)		(5.78)		(5.99)		(6.07)	
Market Leverage		1.049		0.612		0.972		0.962		0.952		1.021		0.897
		(6.45)		(5.39)		(7.19)		(7.28)		(7.14)		(6.57)		(6.57)
Market-to-book	-0.425	0.161												
	(-7.72)	(3.12)												
GO score			-0.634	-0.309										
			(-5.72)	(-4.82)										
Intangibles					0.017	0.042								
					(0.64)	(2.36)								
Non-Fix Assets							-0.078	0.023						
							(-3.62)	(1.11)						
R&D Intensity									-0.130	-0.006				
									(-2.69)	(-0.13)				
R&D Investments											0.064	0.083		
											(2.28)	(2.20)		
Citations per Patent													-0.144	-0.038
													(-5.06)	(-2.19)
Average R^2	0.567	0.660	0.630	0.650	0.488	0.661	0.485	0.655	0.491	0.656	0.555	0.673	0.560	0.615

Table 3: Model calibration

This table presents the calibration results of the firm model. In Panel A, we report the list of model parameters that are kept fix at values taken from the literature. In Panel B, we present the the 11 parameters that are being calibrated. Finally in Panel C we compare the 27 quantities that are weighted to calibrate the model. In the left column (*Data*) we report the value of the moment conditions computed from the observed empirical sample, while in the right column (*Model*) we report the moment conditions computed from the simulated sample. Data is from various sources and spans the period between January 2003 throughout December 2013, as described in Section 2.

Panel A: Fixed Parameters		
Systematic Productivity Autocorrelation	ρ_x	0.815
Systematic Productivity Volatility	σ_x	0.013
Annual Discount Factor	β	0.988
Constant Price of Risk Parameter	γ_1	17
Time-varying Price of Risk Parameter	γ_2	-1000
Annual Depreciation Rate	δ	0.075
Equity Adjustment Cost	ζ	0.050
Panel B: Calibrated Parameters		
Idiosyncratic Productivity Autocorrelation	ρ_z	0.822
Idiosyncratic Productivity Volatility	σ_z	0.352
Corporate Taxes	τ	0.154
Debt Adjustment Cost	θ	0.002
Production Function	α	0.387
Fix Cost	f	0.671
Distress Cost	ξ	0.053
Cost of Expansion	λ_1	0.016
Cost of Contraction	λ_2	0.964
Bankruptcy Cost	η	0.421
Recovery on Debt	R	0.400

Panel C: Moments		
	Data	Model
1-year Default Frequency	0.694	0.668
E[5-year CDS Spread]	1.132	0.979
E[Book Leverage]	0.420	0.584
E[Market Leverage]	0.212	0.257
E[Market to Book]	2.443	2.528
E[Investment]	0.073	0.083
E[Profitability]	0.216	0.257
Std[5-year CDS Spread]	1.941	1.655
Std[Book Leverage]	0.284	0.266
Std[Market Leverage]	0.152	0.163
Std[Market to Book]	1.434	0.674
Std[Investment]	0.120	0.124
Std[Profitability]	0.153	0.218
AC(Profitability)	0.686	0.738
Debt Issuance Frequency	0.175	0.210
β_Q| Book Leverage	-0.425	-0.588
β_Q| Market Leverage	0.161	0.175
E[Leverage Portfolio CDS Spread]:		
1 Low Leverage	0.392	0.403
2	0.630	0.535
3	0.703	0.627
4	0.763	0.705
5	0.803	0.795
6	0.928	0.918
7	0.986	1.018
8	1.191	1.231
9	1.604	1.504
10 High Leverage	2.178	1.951

Table 4: Credit spreads and growth options

This table presents the results of Fama-MacBeth regressions of credit spreads on leverage, asset volatility, two measures of the size of the firm, Equity or Asset, and two measures of growth options: PVGO over Assets and PVGO over Equity. All results are obtained by estimating the regression parameters for each of the 100 simulated economies and subsequently averaging across the 100 estimations. The standard errors are obtained as standard deviations of the 100 estimates of the parameters. A constant is estimated but not reported.

	(1)	(2)	(3)	(4)
Volatility	0.046	0.025	0.019	-0.007
	(5.38)	(4.91)	(3.73)	(-1.53)
Book Leverage	0.566	0.586	0.488	0.543
	(19.94)	(21.28)	(17.18)	(24.08)
Asset	-0.133		-0.146	
	(-5.60)		(-6.74)	
Equity		-0.223		-0.311
		(-5.19)		(-5.82)
PVGO over Asset	-0.449	-0.309		
	(-8.73)	(-12.67)		
PVGO over Equity			-0.147	-0.129
			(-5.91)	(-7.78)
Average R^2	0.716	0.736	0.679	0.732

Table 5: Growth options and systematic risk

This table presents the results of Fama-MacBeth regressions of systematic exposure of growth options (i.e., the beta of PVGO) on leverage, equity, physical assets and asset compositions. Asset composition is measured by PVGO over Equity and PVGO over Asset. All results are obtained by estimating the regression parameters for each of the 100 simulated economies and subsequently averaging across the 100 estimations. The standard errors are obtained as standard deviations of the 100 estimates of the parameters. A constant is estimated but not reported.

	(1)	(2)	(3)	(4)
Volatility	0.197	-0.125	0.265	-0.104
	(2.54)	(-1.41)	(3.35)	(-0.98)
Book Leverage	4.285	2.626	4.439	4.125
	(5.35)	(4.10)	(4.72)	(4.22)
Asset	3.018		1.976	
	(5.04)		(3.59)	
Equity		-0.126		-0.080
		(-0.87)		(-0.47)
PVGO over Asset	-0.907	-2.481		
	(-5.30)	(-7.46)		
PVGO over Equity			-3.109	-3.617
			(-8.18)	(-7.53)
Adjusted-R^2	0.343	0.322	0.433	0.431

Table 6: Credit spreads and market to book ratio

The table presents the results of Fama-MacBeth regressions of 5-year CDS spreads on asset volatility, size, book leverage, market leverage, and market to book ratio (t-stats in parentheses). We report results from the simulated sample, which are obtained by estimating the regression parameters for each of the 100 simulated economies and subsequently averaging across the 100 estimations. The standard errors are calculated as standard deviations of the 100 estimates of the parameters. A constant is estimated but not reported.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Volatility	0.022	0.005	0.002	0.046	0.013	0.013	0.019	0.018	0.006
	(4.66)	(2.51)	(1.02)	(5.38)	(7.64)	(5.64)	(3.73)	(8.17)	(3.06)
Asset	0.115	0.066	0.018	-0.133	0.044	-0.003	-0.146	0.032	-0.023
	(17.01)	(9.61)	(1.74)	(-5.60)	(9.06)	(-0.45)	(-6.74)	(4.38)	(-2.90)
Book Leverage	0.748			0.566			0.488		
	(16.18)			(19.94)			(17.18)		
Market Leverage		0.986			0.881			0.857	
		(12.69)			(12.57)			(12.92)	
Market to Book	-0.588	0.175	-0.093						
	(-7.67)	(10.17)	(-6.44)						
PVGO over Asset				-0.449	0.063	-0.218			
				(-8.73)	(8.28)	(-24.99)			
PVGO over Equity							-0.147	0.017	-0.265
							(-5.91)	(4.14)	(-23.61)
Market Leverage without PVGO			0.733			0.760			0.830
			(10.62)			(12.47)			(12.70)
Average Adjusted-R^2	0.785	0.885	0.668	0.716	0.866	0.739	0.679	0.867	0.747

Appendix

A. Analytic solution of the simple model

Derivation of the security valuation equations

We derive here equation (2). The value of equity at $t = 0$, assuming the firm is currently solvent, is

$$S(z,w) = \max_{(k',b')\in\mathcal{A}(z,w)} \left\{ w - k' + D(z,k',b') + \int_0^{+\infty} \Psi(z',w')\Gamma(dz'|z) \right\}, \tag{14}$$

where

$$\Psi(z',w') = \begin{cases} 0 & \text{if } w' < 0 \\ \widetilde{S}(z',w') & \text{if } 0 \leq w' \end{cases} \tag{15}$$

is the value of equity at the end of the period. In (15), if $w' < 0$ the firm defaults and there is no option to grow. Because bankruptcy costs are excluded in the follow-on period and debt becomes irrelevant, the value of equity absent debt financing is

$$\widetilde{S}(z,w) = \max_{k' \leq w} \left\{ w - k' + \phi(z)(k')^{\alpha} + (1-\delta)k' \right\}, \tag{16}$$

where $\phi(z) = \int_0^{+\infty} z'\Gamma(dz'|z)$. We denote $\hat{k}(z) = [\alpha\phi(z)/\delta]^{1/(1-\alpha)}$ the unconstrained (i.e., ignoring the budget constraint $k' \leq w$) optimal level of capital stock in program (16): Because the optimal *constrained* (by $k' \leq w$) new asset level is never higher than $\hat{k}(z)$, we have

$$\widetilde{S}(z,w) = \begin{cases} \phi(z)w^{\alpha} + (1-\delta)w & \text{if } 0 \leq w < \hat{k}(z) \\ w - \hat{k}(z) + \phi(z)\left[\hat{k}(z)\right]^{\alpha} + (1-\delta)\hat{k}(z) & \text{if } w \geq \hat{k}(z). \end{cases} \tag{17}$$

The value of debt is

$$D(z,k',b') = (1-\eta)\int_0^{\underline{z}(k',b')} (w'+b')\Gamma(dz'|z) + b'\left[1 - \Gamma(\underline{z}(k',b')|z)\right]. \tag{18}$$

Substituting the expression (18) in (14) and $\hat{k}(z)$ in (17), we obtain equation (2).

The simple model in the presence of cash holdings

If the firm was allowed to save cash in the first period, there would be a third decision variable, c'. Equation (2) would remain unchanged, except for investment in savings, $-c'$, at $t = 0$, and the fact that net worth w', and the thresholds $\underline{z}$ and $\overline{z}$ at $t = 1$ would be based on net debt, $b' - c'$, as opposed to only b'. However, the ability to save would not change the optimal decision to invest at $t = 0$. The intuition is simple: both the investment in cash holdings and in productive capital have the effect of increasing w', $\underline{z}$, and $\overline{z}$ at $t = 1$, ultimately increasing the value of growth options.[10] However, cash holdings earn the risk-free rate (zero in our case), which is lower than the expected return on capital stock for positive NPV investments. So, the firm would rather invest the marginal dollar in productive capital than in cash holdings.

Analytic solution in a special case

We now derive the value of debt and equity in closed-form expressions. By assuming that $\log z' = \rho_z \log z + \sigma_z \varepsilon$, with $\varepsilon \sim \mathcal{N}(0,1)$, we can write

$$D(z,k',b') = (1-\eta)(k')^{\alpha}\Phi(z)F(\ell_2) + (1-\eta)(1-\delta)k'F(\ell_1) + b'\left[1 - F(\ell_1)\right], \tag{19}$$

and

$$S(z,w) = \max_{(k',b') \in \mathcal{A}(z,w)} \Big\{ w - k' + b'\left[1 - F(\ell_1)\right] + (1-\eta)(1-\delta)kF(\ell_1) + (1-\eta)k^{\alpha}\Phi(z)F(\ell_2) + \int_{\log \underline{z}(k',b')}^{\infty} \widetilde{S}(z',w')f(\varepsilon)d\varepsilon \Big\}, \tag{20}$$

where f and F are respectively the density and the cumulative normal standard distribution, $w = e^z k^{\alpha} + (1-\delta)k - b$, and

$$\ell_1 = \frac{1}{\sigma_z}\left[\log \underline{z}(k',b') - z\rho_z\right], \qquad \ell_2 = \ell_1 - \sigma_z, \qquad \Phi(z) = e^{z\rho_z + \frac{1}{2}\sigma_z^2}.$$

[10] Acharya et al. (2012) show that a shock that increases the value of growth options increases also the incentive to save and therefore reduces the default threshold and the risk of default. However, in their case real investment is excluded.

In (19), we have derived

$$\int_{-\infty}^{\log \underline{z}(k',b')} e^{z'} d\Gamma(z'|z) = \Phi(z) \int_{-\infty}^{\ell_1} \frac{e^{-\frac{1}{2}(\varepsilon-\sigma_z)^2}}{\sqrt{2\pi}} d\varepsilon = \Phi(z) F(\ell_2).$$

It should be noted that when $b'/k' \leq (1-\delta)$, then $\ell_1, \ell_2 \to -\infty$, $F(\ell_1) = F(\ell_2) = 0$, debt is risk-free, $D(z,k',b') = b'$, and therefore $S(z,w) = \widetilde{S}(z,w)$.

B. Proof of existence of the solution of the model

The proof of the existence of the solution S of the functional equation (6) (together with the optimal policy F), subject to the constraints for D in equations (9) and for PI in (10) mostly follows standard arguments by Stokey and Lucas (1989).

It is easy to prove that the choice set of the program in (6) is compact. As for k', given the current capital stock k, the firm cannot sell more than the current capital, or $k' \geq 0$, and it makes no economic sense to invest more than $\overline{k}$, such that $\pi(\overline{x}, \overline{z}, \overline{k}) - \delta\overline{k} = 0$, given the strict concavity of the production function, and $\overline{x}$ and $\overline{z}$ are the best possible outcomes for the systematic and the idiosyncratic shock, respectively.

Given the incentive to issue debt, quite naturally, $b' \geq 0$. On the other hand, there is an upper bound for b', say $\overline{b}$, because there is a tradeoff between the positive effect that increasing the debt has on current year dividend (v, in equation (4), is increasing in both D and PI, and they are increasing in b', although at a decreasing rate), and the negative effect that it has on next year expected dividend (v is decreasing in b), and this effect is magnified by a factor $(1+\xi)$ if the after–tax cash flow is negative (which is the case if the debt is high), and by a factor $1+\varphi$ if the net proceed from selling capital is insufficient. So, we may conclude that the choice sets for k' and b' are compact. Then, we have the following

Lemma 1 *Under the stated conditions that the decision set is compact, Q has the Feller property, M is continuous, and ω is almost everywhere continuous, the functions D and PI are continuous in (s,p').*

To prove this result, we focus on D, as the same argument carries over to PI. In particular, we show that $D(s,p') = \int u(s',p')M(s,s')Q(s,ds')$ is continuous. By separating the two components of the payoff function u, this boils down to proving that the function $\int \omega(s',p')M(s,s')Q(s,ds')$ is continuous in (s,p'), where $\omega(s',p')$ is the default indicator function, which is bounded and is almost everywhere continuous (i.e., the set of the discontinuity points has zero measure). If this is true, then D is continuous because it is a linear combination of continuous functions.

For notational convenience, let define $H(s,s') = M(s,s')Q(s,s')$. Because M is continuous and the support of s and s' is compact, M is bounded. We need to prove that $f(s,p') = \int \omega(s',p')H(s,ds')$ is continuous. Indeed, M does not create any concerns in this respect, given Q has the Feller property (see Lemma 9.5 in Stokey and Lucas (1989)). Because the state space is compact, there is a sequence $\{(s_n,p'_n)\}$ converging to (s,p'). We just need to prove that $|f(s_n,p'_n) - f(s,p')|$ converges to zero for $n \to \infty$. We observe that

$$|f(s_n,p'_n) - f(s,p')| \leq |f(s_n,p'_n) - f(s,p'_n)| + |f(s,p'_n) - f(s,p')|. \tag{21}$$

As for the second component in the right–hand–side of (21), we have

$$\begin{aligned} |f(s,p'_n) - f(s,p')| &= \left| \int \omega(s',p'_n)H(s,ds') - \int \omega(s',p')H(s,ds') \right| \\ &\leq \int \left|\omega(s',p'_n) - \omega(s',p')\right| H(s,ds'). \end{aligned}$$

By separating the support of s' into the subset in which ω is continuous from the subset in which it is discontinuous, $|\omega(s',p'_n) - \omega(s',p')|$ converges to zero for $n \to \infty$ when in the first subset. In the second subset, the behavior of ω is not important because it has zero measure.

As for the first component in (21),

$$|f(s_n,p'_n) - f(s,p'_n)| = \left| \int \omega(s',p'_n)H(s_n,ds') - \int \omega(s',p'_n)H(s,ds') \right|$$

$$\leq \int \omega(s',p'_n) \left|M(s_n,s')Q(s_n,ds') - M(s,s')Q(s,ds')\right| \leq \overline{M} \int \omega(s',p'_n) \left|Q(s_n,ds') - Q(s,ds')\right|,$$

where $\overline{M}$ is the maximum value of M.

Because Q has the Feller property, then it is continuous in the first argument. This can be proved by contradiction: Assume that Q is discontinuous in $\bar{s}$. Then, for an arbitrary continuous function g, the function $G(s) = \int g(s')Q(s,ds')$ would be discontinuous in $\bar{s}$.

Therefore, $|Q(s_n,ds') - Q(s,ds')|$ converges to zero for $n \to \infty$, and this concludes the proof of the lemma.

Proposition 1 *There is a unique solution S to the program in equations* (6), (9), *and* (10). *S is increasing in $s = (x, z)$ and k, and is decreasing in b.*

To prove this proposition, define the operator T as follows:

$$(TS)(s,p) = \max\left\{0, \max_{p'}\left\{d(s,p,p') + \beta\mathbb{E}_s\left[m(s,s')S(s',p')\right]\right\}\right\},$$

where we have specified $M(s,s') = \beta m(s,s')$, with $\beta < 1$, so that the risk-free component of the stochastic discount factor is highlighted.[11]

First, we will prove that if S is a continuous function on the compact set for (s,p), also TS is continuous on the same compact set. Second, we will show that $S_1 \leq S_2$ implies $TS_1 \leq TS_2$. Third, we will show that for any $a \geq 0$, $T(S+a) \leq TS + \beta a$. Then, because the last two are the Blackwell sufficient conditions, if the operator T satisfies them it is a contraction and the fixed point exists and is unique because the set of continuous functions equipped with the sup-norm is a complete metric space.

We first show that $(TS)(s,p)$ is continuous for all (s,p). From Lemma 1, both D and PI are continuous. Therefore, v, y and d in (5) are continuous. Because the feasible set for p' is compact, using the Maximum Theorem (see Theorem 3.6 in Stokey and Lucas (1989)) the function

$$L(s,p) = \max_{p'}\left\{d(s,p,p') + \mathbb{E}_s\left[M(s,s')S(s',p')\right]\right\}, \tag{22}$$

is continuous. Because $(TS)(s,p) = \max\{0, L(s,p)\}$, also TS is continuous.

As for monotonicity of the operator T, let S_1 and S_2 such that $S_1(s,p) \leq S_2(s,p)$ for all (s,p). Then the following is straightforward:

$$\begin{aligned}(TS_1)(s,p) &= \max\left\{0, \max_{p'}\left\{d(s,p,p') + \mathbb{E}_s\left[M(s,s')S_1(s',p')\right]\right\}\right\} \\ &\leq \max\left\{0, \max_{p'}\left\{d(s,p,p') + \mathbb{E}_s\left[M(s,s')S_1(s',p')\right]\right\}\right\} = (TS_2)(s,p).\end{aligned}$$

[11] The fact that M is bounded make this assumption harmless.

Lastly, let

$$
\begin{aligned}
[T(S+a)](s,p) &= \max\left\{0, \max_{p'}\left\{d(s,p,p') + \beta\mathbb{E}_s\left[m(s,s')\left(S(s',p')+a\right)\right]\right\}\right\} \\
&= \max\left\{0, \max_{p'}\left\{d(s,p,p') + \beta\mathbb{E}_s\left[m(s,s')S(s',p')\right]\right\} + \beta a\right\} \\
&\leq \max\left\{0, \max_{p'}\left\{d(s,p,p') + \beta\mathbb{E}_s\left[m(s,s')S(s',p')\right]\right\}\right\} + \beta a \\
&= (TS)(s,p) + \beta a.
\end{aligned}
$$

Monotonicity of S with respect to $s = (x, z)$ is due to strict monotonicity of both π and Q with respect to x and z. Monotonicity of S with respect to k and b is based on monotonicity of d in equation (5) with respect to these variables. This concludes the proof of the proposition.

Finally, the fact that the default indicator function, which has values in $\{0, 1\}$, is discontinuous only in a set with zero measure can be proved by observing that the change of value from 0 to 1 occurs where the continuous and strictly monotonic function L, defined in equation (22), is equal to zero. L is strictly monotonic because S is monotonic, Q is monotonic and π is strictly monotonic.

C. Numerical procedure

The solution to the Bellman equation (6) with the constraints in equations (9) and (10), and the related optimal policy is obtained by discretizing the state space of $s = (x, z)$ and the control variables $p = (k, b)$. Because the stochastic process of systematic risk is quite persistent we discretize the two exogenous processes using the numerical approach proposed by Rouwenhorst (1995). x is discretized with $N_x = 7$ points and z with $N_z = 11$. The discretized set of values for capital stock is $\{k_j = k_u(1-d)^j \mid j = 1, \ldots, N_k\}$, and the set of discrete debt levels is $\left\{b_j = j\frac{b_u}{N_b} \mid j = 1, \ldots, N_b\right\}$ with $N_k = 31$ and $N_b = 21$.

The fixed point of the Bellman equation, S, is found using a value function iteration algorithm, and the algorithm is halted when the maximum change of value on S between two iteration is below the tolerance 10^{-5}. The simulated economy is composed of 100 different samples, characterized by different histories of the aggregate state variable. Each sample is composed by a panel of 300 independent firms, which are characterized by different histories of the idiosyncratic variable. To make the simulated economy comparable to the observed data, each firm's history, after the first 20 periods are discarded, is composed by 10 periods. The desired quantities are obtained by applying the optimal policy at each step. If a company defaults at a given step, it is restarted at the steady state for k and b one step later.

D. CDS spread

Hull and White (2000) show that if default event, interest rate, and recovery rate are independent, the spread of a multi-period CDS is equivalent to the yield spread on a corporate bond with the same maturity. Because in our economy default event, interest rate and recovery rate are not independent, we provide details about how CDS spreads are computed in the model.

The credit default swap is a T-period contract that insures against the first default on a debt contract with face value equal to one. For the sake of clarity, we use time subscripts, and assume that current time is t and the firm is in a non-default state ($\omega(s_t, p_t) = 0$). Default along a given trajectory occurs the first time $t+n$ in which $\omega(s_{t+n}, p_{t+n}) = 1$. Until then, the law of motion of the policy is $p_{t+j+1} = F(s_{t+j}, p_{t+j})$.

While the CDS is based on a debt contract with unit face value, the default policy and recovery depends on the actual amount of debt b_{t+n} at the default date, besides the state and the actual stock of capital as of that date. Therefore, we must normalize the recovery by b_{t+n}. Let $\mathcal{H}(s_t, s_{t+1})$ be the price at the current state, s_t, of a contingent claim that pays \$1 if state s_{t+1} occurs: $\mathcal{H}(s_t, s_{t+1}) = Q(s_t, s_{t+1})M(s_t, s_{t+1})$. Define the price of a contingent claim that pays \$1 only if the firm defaults for the first time n periods from now as

$$\begin{aligned}\mathcal{P}_n(s,p) &= \mathcal{P}_n(s_t,p_t) \\ &= \mathbb{E}_{s_t}\left[\mathcal{H}(s_{t+n-1}, s_{t+n})\omega(s_{t+n}, p_{t+n}) \prod_{j=1}^{n-1} \mathcal{H}(s_{t+j-1}, s_{t+j}) \left(1 - \omega(s_{t+j}, p_{t+j})\right)\right].\end{aligned}$$

We also define the price of a contingent claim that pays \$1 if the firm does not default within the first n periods as

$$\mathcal{S}_n(s,p) = \mathcal{S}_n(s_t,p_t) = \mathbb{E}_{s_t}\left[\prod_{j=1}^{n} \mathcal{H}(s_{t+j-1}, s_{t+j}) \left(1 - \omega(s_{t+j}, p_{t+j})\right)\right],$$

and the price of a contingent claim that pays the loss-given-default, $\ell(s_{t+n}, p_{t+n}) = \eta(\pi_{t+n} +$

$(1-\delta)k_{t+n} + \tau\delta k_{t+n})/b_{t+n}$, only if the firm defaults for the first time n periods from now as

$$\mathcal{L}_n(s,p) = \mathcal{L}_n(s_t,p_t) \\ = \mathbb{E}_{s_t}\left[\mathcal{H}(s_{t+n-1},s_{t+n})\omega(s_{t+n},p_{t+n})\ell(s_{t+n},p_{t+n})\prod_{j=1}^{n-1}\mathcal{H}(s_{t+j-1},s_{t+j})\left(1-\omega(s_{t+j},p_{t+j})\right)\right].$$

Then, the CDS spread is equal to

$$cds(s,p) = \frac{\sum_{n=1}^{T}\mathcal{L}_n(s,p)}{\sum_{n=1}^{T}\left(\mathcal{P}_n(s,p)+\mathcal{S}_n(s,p)\right)}. \tag{23}$$

The above argument is consistent with the valuation of a contract that insures against default of a one-dollar face value multi-period debt contract issued by the firm, assuming that all debt contracts have the same priority at default, and the firm, which rolls over single-period debt, never decides to have $b_t = 0$ before default or maturity of CDS, whichever occurs first.

The CDS spread in (23) is calculated within our numerical procedure as follows. Using the discrete-state version of the transition probability function, the state of the firm is summarized by $X = (s,p)$, and given the optimal policy of the firm (ω, F), we can define the state price at X of X' as

$$H(X,X') = \begin{cases}\mathcal{H}(s,s') & \text{if } \omega(X)=0 \text{ and } p'=F(X)\\ 0 & \text{if } \omega(X)=1 \text{ or } p'\neq F(X).\end{cases}$$

The size of this square matrix is $N_xN_zN_kN_b$. By deleting from H all rows with $\omega(X)=1$ and all columns with $\omega(X')=0$, we define H_d, the matrix of prices of contingent claims that pay one unit if we have a transition from a non–default state to a default state in one period. Similarly, by deleting from H all rows with $\omega(X)=1$ and all columns with $\omega(X')=1$, we define H_{nd}, the matrix of prices of contingent claim that pay one unit if there is a transition from a non–default state to a non–default state.

We can calculate the prices of the contingent claims defined above using linear algebra: $\mathcal{P}_i = H_{nd}^{i-1}H_dI_d$, where I_d is a column vector of ones with as many components as the number of default states; $\mathcal{S}_i = H_{nd}^{i}I_{nd}$, where I_{nd} is a column vector of ones with as many components as the number of non–default states; $\mathcal{L}_i = H_{nd}^{i-1}H_dL_d$, where L_d is a column vector, whose components are the state contingent loss-given-default $\ell = \eta(\pi + (1-\delta)k + \tau\delta k)/b$. Then, the CDS spread is

$$cds = \frac{\sum_{i=1}^{T} \mathcal{L}_i}{\sum_{i=1}^{T} (\mathcal{Q}_i + \mathcal{P}_i)}.$$

However, this calculation is quite demanding from the numerical stand point and cannot be done within the simulated method of moments. Therefore, for the sake of the calibration, we reduce the dimensionality of the problem by resorting to two simplifications.

First, we assume that over the interval $[t, t+T]$, the firm's policy remains the same as the one in the first period: $p_{t+j} = p_{t+1} = F(s_t, p_t)$, for all $j = 1, \ldots, T-1$. This implies that also the default policy is $\omega(s_{t+j}, p_{t+j}) = \omega(s_{t+j}, p_{t+1})$, for all $j = 1, \ldots, T-1$. With this assumption, given a non-default state $(s, p) = (s_t, p_t)$, and $p' = F(p, s)$, we can restrict ourselves to the state price matrix $\overline{H}(s, s') = \mathcal{H}(s, s')$. Notably, the size of this square matrix is $N_x N_z$. From this, in line with what done in the general case, we define $\overline{H}_d$, by dropping from $\overline{H}$ all the columns corresponding to $\omega(s', p') = 0$, and $\overline{H}_{nd}$, by dropping all columns corresponding to $\omega(s', p') = 1$.

Second, because also the loss-given-default depends on the policy at the date right before the default, which cannot be determined given the above assumption of a constant firm policy, we assume that the recovery on par is constant at R. With these simplifications, the credit spread is

$$cds(s,p) = (1-R)\frac{\sum_{n=1}^{T} \overline{\mathcal{P}}_n(s,p)}{\sum_{n=1}^{T} \left(\overline{\mathcal{P}}_n(s,p) + \overline{\mathcal{S}}_n(s,p)\right)}.$$

R is the recovery rate on the face value of a unit bond, and $\overline{\mathcal{P}}(s,p)$ differs from $\mathcal{P}(s,p)$ in that the policy p does not change with the evolution of aggregate and idiosyncratic shocks. In our numerical procedure, the above pricing equation is implemented as follows:

$$cds = (1-R)\frac{\sum_{i=1}^{T} \overline{\mathcal{P}}_i}{\sum_{i=1}^{T} \left(\overline{\mathcal{P}}_i + \overline{\mathcal{S}}_i\right)},$$

where $\overline{\mathcal{P}}_i = \overline{H}_{nd}^{i-1} \overline{H}_d \overline{I}_d$, in which $\overline{I}_d$ is a column vector of ones with as many components as the number of default states, s, and $\overline{\mathcal{S}}_i = \overline{H}_{nd}^{i} \overline{I}_{nd}$, in which $\overline{I}_{nd}$ is a column vector of ones with as many components as the number of non–default states, s.

We validate the accuracy the formula, and of the simplifying assumption, by computing CDS spreads at different maturities and comparing them with the observed counterparts.

E. Composition of the default sample

There are nine default events in our sample of firms from 2001 to 2013. We only consider defaults that happen in the last year a firm is listed in one of the exchanges, and for which we can observe CDS prices in the immediate past. Summary details about the sample defaults were extracted from the Moody's Default and Recovery Dataset and are reported in the table below. All firms, except SPRINT that completed a distressed exchange, entered Chapter 11. None of the firms was liquidated as a consequence of the default. The average market recovery rate (i.e., the price at which the claim trades in the market one month after the default event) is equal to 47 cents on the dollar. The sample default frequency used in the calibration is computed as the average across years of the cross-sectional frequency of defaults in each year in the sample.

Gvkey	Company Name	Default Date	Amount	Recovery	
3734	DANA CORP	1-Mar-06	1582	55.11	12-Dec-07
3851	DELTA AIR LINES INC	14-Sep-05	3,554	17.81	25-Apr-07
5073	GENERAL MOTORS CO	1-Jun-09	34,046	13.48	10-Jul-09
6307	KMART	22-Jan-02	2,277	42.21	22-Apr-03
10984	SPRINT CORP	11-Nov-03	750	58.00	11-Nov-03
11535	WINN-DIXIE STORES INC	22-Feb-05	300	88.83	10-Nov-06
12589	HEALTHSOUTH CORP	28-Mar-03	3,391	48.09	12-Aug-03
63605	CALPINE CORP	20-Dec-05	7,827	62.46	19-Dec-07
140033	GENON ENERGY INC	15-Jul-03	2,800	45.96	8-Dec-05