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# Communicating or Computing Over the MAC: Function-Centric Wireless Networks

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**Abstract**—Distributing data aggregation through multiple access channel (MAC) has been challenging in large wireless networks. In order to tackle the challenge, computing over the MAC (CP-MAC) scheme has been proposed as a promising communication-computation integrated way for function-centric networks. In this paper, we analyze the performance of the CP-MAC scheme, compared with the traditional communication-computation separated way, i.e., communicating over the MAC (CM-MAC) scheme. Function-centric wireless networks are considered, where the fusion center (FC) does not need the individual data of each node but only the target function. We begin with the ideal uniform-MAC scenarios, where the CP-MAC scheme is always better than the CM-MAC scheme. Then, practical non-uniform MAC scenarios are studied for both homogeneous networks with Rayleigh fading and heterogeneous networks with different path loss. Closed-form expressions of the achievable function rate are provided using the asymptotic theory of ordered statistics. It is found that the CP-MAC scheme is not always superior to the CM-MAC scheme. Simulation results are provided to verify and illustrate our derived results.

**Index Terms**—Achievable rate, data aggregation, data-centric, function-centric, multiple access channel, ordered statistics, wireless networks.

## I. INTRODUCTION

Future wireless networks are expected to connect an increasing number of nodes. For example, the 5G cellular system could provide Internet of Things (IoT) connections for up to 1 trillion devices, with a million connections per square kilometer [1]. In order to tackle this challenge, high-efficient data aggregation technologies are required in wireless multiple access channel (MAC) [2].

To improve the efficiency of data aggregation in wireless networks, most previous works have focused on two aspects.

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One is to reduce the data traffic through compression [3]–[6]. For example, sampling compression was studied in [3], [4] to reduce the number of sampling operations, where spatial and temporal sparsity properties were utilized to enhance the spectrum efficiency and energy efficiency of the networks. By converting the input data stream into another one with fewer bits, data compression was discussed in [5], [6] to obtain a more compact representation of the data. The other aspect is to improve the communication capacity through efficient resource allocation [7]–[9]. For example, non-orthogonal multiple access (NOMA) technique was explored in [7] to improve the throughput of the networks. Throughput-optimal problem of nodes scheduling for NOMA was tackled in [8] for a multi-carrier network. Considering the fairness between different nodes, the work in [9] investigated a fair opportunistic scheduler based on probabilistic scheduling.

The above mentioned works considered data-centric networks that require the individual data of each node. On the other hand, future networks become function-centric due to the development of various applications. The fusion center (FC) of these networks does not need the individual data of each node but only the target functions. For examples, in alarm detection of IoT networks [10], statistical data analysis for multi-sensor networks [11], and machine learning based on distributed data [12], only certain functions of the data are needed. The target function can be computed by using the individual data from each node at the FC. This is the traditional communication-computation separated way. In order to avoid inter-node interference, multi-access scheme will be used to incur excessive latency, especially for large networks. Thus, its performance is limited by the communicating over the MAC (CM-MAC) scheme.

A more intelligent scheme is computing over the MAC (CP-MAC), which utilizes the superposition property of wireless channel to compute the summation in the target function. It harnesses the inter-node interference rather than avoiding it. This leads to a communication-computation integrated way, where the target function with a summation structure can be directly received by the FC with all nodes' concurrently transmission. Some common functions such as arithmetic mean and geometric mean can be efficiently computed over the MAC.

So far CP-MAC has been studied from various perspectives. Analog CP-MAC was first investigated due to its low complexity. The FC recovered the underlying Gaussian source with respect to mean-squared error (MSE) through uncoded transmission in [13], where the channel input of the node

was merely a scaled version of its noisy observation. In order to combat synchronization errors, a robust analog function computation scheme was proposed by adopting random synchronization sequences in [14]. Considering that each node only has imperfect CSI, the worst-case MSE for analog CP-MAC was formulated and solved in [15]. Some experimental platforms were built to verify the idea of analog CP-MAC in [16], [17]. In order to obtain reliable functions, channel coding has been adopted to combat additive white gaussian noise (AWGN) for CP-MAC. The problem of reliably reconstructing a function of sources was first discussed in the seminal work [18], where linear source coding was adopted for the function computation over Gaussian MAC. Using nested lattice coding to compute the noisy modulo sum was investigated in [19], where each relay decoded linear combination of the sources in compute-and-forward relay. In [20], M. Goldenbaum *et al.* proposed a general form of functions, i.e., Nomographic functions, which could be computed over the MAC efficiently due to their summation structure. Considering the channel fading, transceiver design for CP-MAC has been further discussed. In [21], a uniform-forcing transceiver was designed to compensate the non-uniform fading of different nodes to a uniform level. Furthermore, transceiver designs for multiple functions computed over the MAC were discussed in [22], [23] utilizing antenna arrays at the FC and the nodes.

Both the CM-MAC scheme and the CP-MAC scheme have their own virtues and faults. For the CM-MAC scheme, multiple nodes can be scheduled in an opportunistic way to achieve multi-node diversity, but orthogonal radio resources are required to avoid inter-node interference. For the CP-MAC scheme, the inter-node interference is harnessed for computation, but the non-uniform MAC should be compensated to the uniform level, which is determined by the node with the worst channel gain. It incurs a vanishing computation rate when the number of nodes goes to infinity [24], [25]. Although it is important, the performance difference between the CM-MAC scheme and the CP-MAC scheme has never been studied for function-centric wireless networks.

Motivated by the above observations, in this paper we derive and compare the achievable function rates for both the CM-MAC scheme and the CP-MAC scheme in function-centric networks. The ideal scenarios with the uniform MAC are first discussed with/without additive white gaussian noise (AWGN). Furthermore, non-uniform MAC scenarios are investigated, where both homogeneous networks and heterogeneous networks are studied. A summary of main contributions are as follows.

- **Function-centric wireless network:** A function-centric wireless network is formulated where the FC does not need the individual reading of each node but the target function thereof. The CM-MAC scheme and the CP-MAC scheme are used to recover the target function at the FC.
- **Exact achievable function rate:** The exact achievable function rates are derived for both the CM-MAC scheme and the CP-MAC scheme. We begin with the discussion of the ideal uniform MAC, and then the non-uniform MAC for both homogeneous networks and heterogeneous networks are discussed.

Table I  
SOME COMMON NOMOGRAPHIC FUNCTIONS

Name	$\varphi_k$	$\psi$	$f$
Arithmetic Mean	$\varphi_k = s_k$	$\psi = \frac{1}{K}$	$f = \frac{1}{K} \sum_{k=1}^K s_k$
Weighted Sum	$\varphi_k = \omega_k s_k$	$\psi = 1$	$f = \sum_{k=1}^K \omega_k s_k$
Geometric Mean	$\varphi_k = \log(s_k)$	$\psi = \exp(\cdot)$	$f = \left( \prod_{k=1}^K s_k \right)^{\frac{1}{K}}$
Polynomial	$\varphi_k = \omega_k s_k^{\beta_k}$	$\psi = 1$	$f = \sum_{k=1}^K \omega_k s_k^{\beta_k}$
Euclidean Norm	$\varphi_k = s_k^2$	$\psi = (\cdot)^{\frac{1}{2}}$	$f = \sqrt{\sum_{k=1}^K s_k^2}$

- **Asymptotic closed-form expression:** The exact achievable function rate is complicated without closed-form expression, which lacks insights. Adopting the asymptotic theory of ordered statistics, the asymptotic closed-form expressions for achievable function rate are given to provide more insights.

The remainder of the paper is organized as follows. Section II presents the system model of function-centric networks. Section III derives and compares the achievable function rate for uniform MAC scenarios. Practical non-uniform MAC scenarios are studied in Section IV for both homogeneous networks and heterogeneous networks. Simulation results are provided in Section VI, followed by concluding remarks in Section VII.

## II. FUNCTION-CENTRIC WIRELESS NETWORKS

We consider a network with an FC and  $K$  nodes indexed by  $k \in \{1, 2, \dots, K\}$ . The reading of the node  $k$  is denoted as  $s_k$ .

The network is assumed to be function-centric, i.e., the FC does not need the individual reading of each node  $\{s_k\}$  but the target function  $f(s_1, s_2, \dots, s_K)$ . Furthermore, the target function of the FC is assumed to be nomographic which is defined as follows.

**Definition 1.** The Nomographic function is given by [20]

$$f = \psi \left[ \sum_{k=1}^K \varphi_k(s_k) \right], \quad (1)$$

where  $\psi(\cdot)$  is the post-processing function of FC, and  $\varphi_k(\cdot)$  is the pre-processing function of node  $k$ . Some common Nomographic functions are shown in Table I with different  $\psi(\cdot)$  and  $\varphi_k(\cdot)$ .

At node  $k$ , the data processing flow can be given as

$$s_k \rightarrow \varphi_k(s_k) \rightarrow \mathbf{w}_k \rightarrow \mathbf{x}_k, \quad (2)$$

where the reading  $s_k$  is first processed by the pre-processing function  $\varphi_k(\cdot)$ , the pre-processed reading  $\varphi_k(s_k)$  is then quantized into a length  $N$  binary vector  $\mathbf{w}_k \in \mathcal{B}^N$ , and the quantized vector  $\mathbf{w}_k$  is encoded into a length  $M$  codeword  $\mathbf{x}_k \in \mathcal{R}^n$  finally.

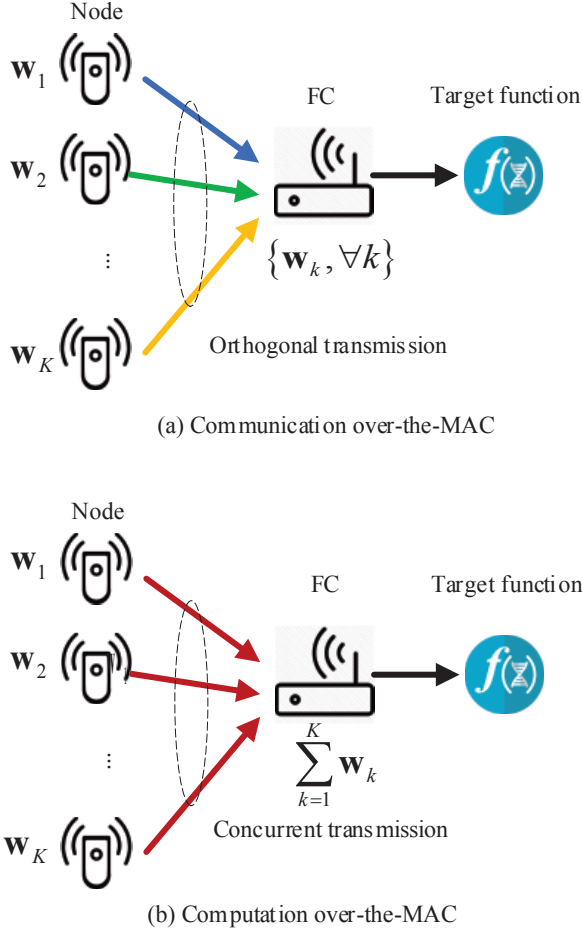


Figure 1. the CM-MAC scheme versus the CP-MAC scheme

At the FC, there are two multiple access schemes to recover the target function, which are illustrated as follows.

**1. CM-MAC:** As illustrated in Fig. 1(a), the CM-MAC scheme is a communicating and computing separated scheme, where the FC first recovers the individual messages  $\{\mathbf{w}_k\}$  of all nodes through orthogonal MAC, and then computes the target function  $f$  thereof.

The corresponding data processing flow at the FC is

$$\{\mathbf{y}_k\} \rightarrow \{\mathbf{w}_k\} \rightarrow \sum_{k=1}^K \mathbf{w}_k \rightarrow \sum_{k=1}^K \varphi_k(s_k) \rightarrow f, \quad (3)$$

where  $\{\mathbf{y}_k\}$  is the received signal with each node's orthogonal transmission, and  $\{\mathbf{w}_k\}$  is the decoded quantized vector of each node.

When all nodes' vectors are decoded, the FC computes  $\sum_{k=1}^K \mathbf{w}_k$ , and  $\sum_{k=1}^K \varphi_k(s_k)$  is then obtained by quantization recovery. Finally, the target function  $f$  is recovered after the post-processing function  $\psi(\cdot)$ . In order to avoid inter-node interference, orthogonal multiple access, e.g., TDMA, should be adopted, which incurs a high latency especially for a large number of nodes.

**2. CP-MAC:** As illustrated in Fig. 1(b), the CP-MAC scheme is a communicating and computing integrated scheme,

where the FC utilizes the superposition property of wireless channel to compute the summation part of the target function with all nodes' concurrent transmission.

The corresponding data processing flow of the FC is

$$\mathbf{y} \rightarrow \sum_{k=1}^K \mathbf{w}_k \rightarrow \sum_{k=1}^K \varphi_k(s_k) \rightarrow f, \quad (4)$$

where  $\mathbf{y}$  is the received signal when all nodes transmit simultaneously. The sum of the quantized vector  $\sum_{k=1}^K \mathbf{w}_k$  is decoded from  $\mathbf{y}$  directly without recovering the message of each node  $\mathbf{w}_k$ .

Then,  $\sum_{k=1}^K \varphi_k(s_k)$  is obtained by quantization recovery, and the target function  $f$  is recovered after the post-processing function  $\psi(\cdot)$  finally. Although CP-MAC can avoid individual message collection, the challenge is the computation error caused by the non-uniform fading of MAC and the AWGN.

In this work, we adopt the achievable function rate as the performance metric of function-centric networks, which can be defined as follows.

**Definition 2.** (Achievable function rate) Let  $f$  be the target function, and  $\hat{f}$  be the corresponding recovered function at the FC. The corresponding function error probability is  $\varepsilon = \Pr(\hat{f} \neq f)$ . Without considering the quantization error, the function rate  $R = 1/M$  (functions per channel use) is said to be achievable, if an arbitrary  $\varepsilon > 0$  can be satisfied with sufficiently large channel uses  $M$ .

In the following, we will derive and compare the achievable function rate of the CM-MAC scheme and the CP-MAC scheme for different scenarios including uniform MAC without AWGN, uniform MAC with AWGN, non-uniform MAC in homogeneous networks and non-uniform MAC in heterogeneous networks as shown in Fig. 2.

### III. ACHIEVABLE FUNCTION RATE FOR UNIFORM MAC

In this section, we discuss the uniform MAC scenario. First, the MAC without AWGN will be discussed followed by the MAC with AWGN.

#### A. Uniform MAC without AWGN

As shown in Fig. 2(a), the uniform MAC without AWGN is the most ideal scenario, where a reliable communication can be realized through each node's orthogonal transmission or a reliable summation can be realized through concurrent transmission from all nodes.

For the CM-MAC scheme, each node transmits messages in turn, and the received signal at the FC is

$$\mathbf{y}_k = \sqrt{\mu} \mathbf{w}_k, \quad (5)$$

where  $\mu$  is the uniform channel power gain.  $K$  channel uses are required to avoid inter-node interference and aggregate  $K$  nodes' messages. The corresponding achievable function rate is  $R_{CM} = 1/K$  (functions per channel use).

For the CP-MAC scheme, all nodes concurrently transmit messages, and the received signal of the FC is



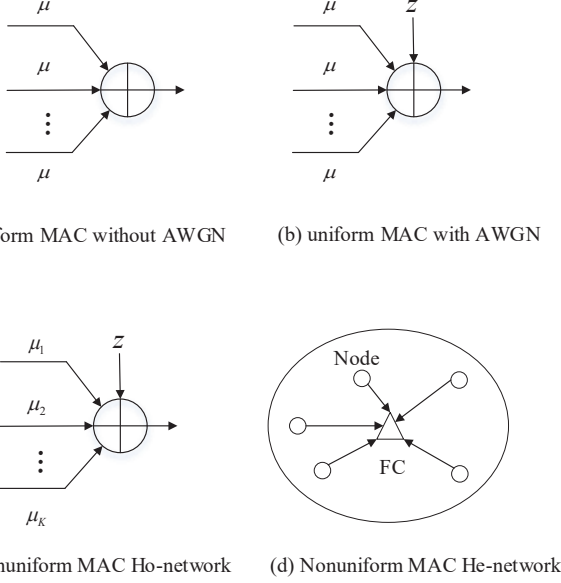


Figure 2. The illustration of different scenarios

$$\mathbf{y} = \sqrt{\mu} \sum_{k=1}^K \mathbf{w}_k, \quad (6)$$

where only one channel use is required to compute  $\sum_{k=1}^K \mathbf{w}_k$ , and the target function can be further recovered using the post-processing function at the FC. The corresponding achievable function rate is  $R_{CP} = 1$  (functions per channel use).

The achievable function rate gain of the CP-MAC scheme over the CM-MAC scheme can be expressed as

$$G_1 = \frac{R_{CP}}{R_{CM}} = K, \quad (7)$$

which linearly increases with the number of nodes.

### B. Uniform MAC with AWGN

As shown in Fig. 2(b), when AWGN is also considered, channel coding should be adopted to combat the AWGN and achieve reliable communicating or computing.

For the CM-MAC scheme, the received signal at the FC with orthogonal transmission is

$$\mathbf{y}_k = \sqrt{P_0 \mu} \mathbf{x}_k + \mathbf{z}, \quad (8)$$

where  $\mathbf{z}$  is the AWGN vector with each element distributed as  $\mathcal{N}(0, \sigma_z^2)$ . The total number of bits of  $K$  nodes is  $NK$ . Consider  $M$  channel uses, and the message rate in each channel use should not exceed the channel capacity, which can be expressed as

$$\frac{NK}{M} \leq \log_2 \left( 1 + \frac{P_0 \mu}{\sigma_z^2} \right), \quad (9)$$

where  $P_0$  is the transmit power constraint of the node, and  $\mu$  is the uniform MAC power gain. Thus, the achievable function rate of the CM-MAC scheme for uniform MAC with AWGN is

$$R_{CM} = \frac{1}{NK} \log_2 \left( 1 + \frac{P_0 \mu}{\sigma_z^2} \right). \quad (10)$$

For the CP-MAC scheme, nested lattice coding is adopted to combat the AWGN. By encoding the message vector  $\mathbf{w}_k$  into a length  $M$  nested lattice coding vector  $\mathbf{x}_k \in \mathcal{L}^M$ , the received signal with all nodes concurrently transmitting is

$$\mathbf{y} = \sqrt{P_0 \mu} \sum_{k=1}^K \mathbf{x}_k + \mathbf{z}. \quad (11)$$

The most attractive property of nested lattice coding is its linear property [26] as

$$\sum_{k=1}^K \mathbf{x}_k \bmod \Lambda_c \in \mathcal{L}^M, \quad (12)$$

where  $\mathcal{L}^M$  is a  $M$ -dimensions nested lattice codebook, and a summation of nested lattice codewords  $\mathbf{x}_k \in \mathcal{L}^M$  modulo the coarse lattice  $\Lambda_c$  only takes values on the codebook  $\mathcal{L}^M$ . Thus, the nested lattice coding can be adopted to protect a modulo- $q$  sum of the length- $N$  vectors, i.e.,  $\mathbf{v} = \bigoplus_{k=1}^K \mathbf{w}_k$ .

**Lemma 1.** (The achievable rate to decode modulo- $q$  sum) Given the received signal of the uniform MAC with AWGN in (11), the modulo- $q$  sum is decoded as  $\hat{\mathbf{v}} = \mathcal{D}(\mathbf{y})$ . The corresponding decoding error is  $\varepsilon = \Pr(\hat{\mathbf{v}} \neq \mathbf{v})$ . With sufficiently large number of channel use  $M$ , the following rate is achievable for an arbitrary  $\varepsilon > 0$  with nested lattice coding

$$R_{mod} = \frac{1}{M} \leq \log_2^+ \left( \frac{P_0 \mu}{\sigma_z^2} \right), \quad (13)$$

where  $\log_2^+(\cdot) = \max\{\log_2(\cdot), 0\}$

*Proof.* The proof follows from Theorem 3 in [19].  $\square$

If  $q$  is large enough, modulo- $q$  sum will not wrap around, which is equivalent to sum. That is  $\bigoplus_{k=1}^K \mathbf{w}_k = \sum_{k=1}^K \mathbf{w}_k$ . Thus, we can achieve reliable sum with sufficiently large  $q$ , and the achievable function rate of CP-MAC can be provided as follows.

**Proposition 1.** (The achievable function rate of CP-MAC) The achievable function rate of the CP-MAC scheme for uniform MAC with AWGN is

$$R_{CP} = \frac{1}{N + \log_2 K} \log_2^+ \left( \frac{P_0 \mu}{\sigma_z^2} \right). \quad (14)$$

*Proof.*  $\sum_{k=1}^K \mathbf{w}_k$  is a sum of  $K$  length- $N$  binary vectors, and its maximum value is  $K2^N$ . In order to avoid wrapping around for modulo- $q$  sum,  $q$  should satisfies  $q \geq K2^N$ . Assuming the number of channel uses is  $M$ , the message rate of modulo- $q$  sum in each channel use satisfies

$$\frac{\log_2 q}{M} \geq \frac{\log_2(K2^N)}{M}. \quad (15)$$

Furthermore, the message rate of modulo- $q$  sum in each channel use should not exceed the achievable rate to decode the modulo- $q$  sum with nested lattice coding in Lemma 1. Thus, we have

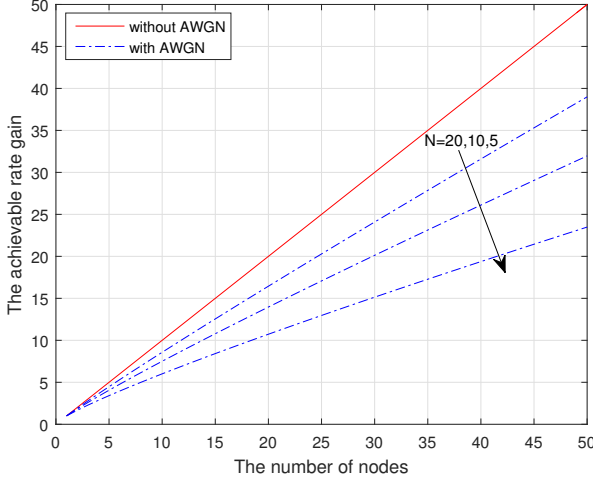


Figure 3. The achievable function rate gain versus the number of nodes and different length of message vector.

$$\log_2 + \left( \frac{P_0 \mu}{\sigma_z^2} \right) \geq \frac{\log_2 q}{M} \geq \frac{\log_2(K 2^N)}{M}. \quad (16)$$

The achievable function rate of CP-MAC is  $R_{CP} = 1/M$ , which completes the proof.  $\square$

According to (10) and (14), the achievable function rate gain of the CP-MAC scheme over the CM-MAC scheme can be approximated as

$$G_2 = \frac{R_{CP}}{R_{CM}} \approx \frac{NK}{N + \log_2 K}, \quad (17)$$

which is compared to the achievable function rate gain for uniform MAC without AWGN  $G_1$  in Fig. 3 for different numbers of nodes  $K$  and different lengths of message vector  $N$ .

**Remark 1.** When  $K > 1$ , one has  $G_1 > 1$  and  $G_2 > 1$ . In this case, the CP-MAC scheme is always superior to the CM-MAC scheme. Both  $G_1$  and  $G_2$  increase with the number of nodes  $K$ . The increasing rate of  $G_2$  is smaller than that of  $G_1$  due to channel coding. The increasing rate of  $G_2$  increases with the length of the message vector  $N$ . This means the increase of the length of the message vector will increase the benefit of using the CP-MAC scheme.

#### IV. ACHIEVABLE FUNCTION RATE FOR NON-UNIFORM MAC

In this section, we discuss the non-uniform MAC scenarios. We begin with the homogeneous networks, where all nodes experience i.i.d. fading. Then heterogeneous networks will be studied, where the nodes have different path loss.

##### A. Homogeneous Networks

We first focus on homogeneous networks where all nodes experience i.i.d. fading. The channel fading  $h_k$  between the node  $k$  and the FC is assumed to be Rayleigh fading. The

corresponding channel power gain  $\mu_k = |h_k|^2$  is exponentially distributed with PDF and CDF given by

$$f_{\mu_k}(x) = \frac{1}{\bar{\mu}} \exp\left(-\frac{x}{\bar{\mu}}\right), x \geq 0, \quad (18)$$

$$F_{\mu_k}(x) = 1 - \exp\left(-\frac{x}{\bar{\mu}}\right), x \geq 0, \quad (19)$$

where  $\bar{\mu}$  is the average channel power gain.

For the CM-MAC scheme, the received signal at the FC with orthogonal transmission is

$$\mathbf{y}_k = \sqrt{P_0 \mu_k} \mathbf{x}_k + \mathbf{z}, \quad (20)$$

and the achievable data transmission rate of the node  $k$  is

$$C_k(\mu_k) = \log_2 \left( 1 + \frac{P_0 \mu_k}{\sigma_z^2} \right). \quad (21)$$

In order to obtain multi-node diversity, we select the node  $m$  as  $m = \arg \max_k \mu_k$  in each channel use. The channel power gain between the selected node  $m$  and the FC is  $\mu_m = \max_k \mu_k$ , and the channel capacity of the selected node  $m$  is

$$C_m(\mu_m) = \max_k C_k(\mu_k) = \log_2 \left( 1 + \frac{P_0 \mu_m}{\sigma_z^2} \right). \quad (22)$$

The messages of  $K$  nodes are composed of  $NK$  bits. Assuming  $M$  channel uses, the average message rate in each channel use should not exceed the ergodic channel capacity as

$$\frac{NK}{M} \leq \mathbb{E} \left[ \log_2 \left( 1 + \frac{P_0 \mu_m}{\sigma_z^2} \right) \right]. \quad (23)$$

The PDF of  $\mu_m$  can be derived as

$$f_{\mu_m}(x) = \frac{K}{\bar{\mu}} \exp\left(-\frac{x}{\bar{\mu}}\right) \left[ 1 - \exp\left(-\frac{x}{\bar{\mu}}\right) \right]^{K-1}. \quad (24)$$

Thus, the achievable function rate for the CM-MAC scheme in homogeneous networks can be denoted as

$$\begin{aligned} R_{CM} &= \frac{1}{NK} \mathbb{E} \left[ \log_2 \left( 1 + \frac{P_0 \mu_m}{\sigma_z^2} \right) \right] \\ &= \frac{1}{NK} \int_0^\infty \log_2 \left( 1 + \frac{P_0 x}{\sigma_z^2} \right) f_{\mu_m}(x) dx \\ &\stackrel{(a)}{=} \frac{1}{\bar{\mu} N \ln 2} \sum_{i=0}^{K-1} C_{K-1}^i (-1)^i \int_0^\infty \ln \left( 1 + \frac{P_0 x}{\sigma_z^2} \right) \exp \left[ -\frac{(i+1)x}{\bar{\mu}} \right] dx \\ &\stackrel{(b)}{=} \frac{1}{\bar{\mu} N \ln 2} \sum_{i=0}^{K-1} C_{K-1}^i (-1)^i \left( \frac{\bar{\mu}}{i+1} \right) \exp \left[ \frac{(i+1)\sigma_z^2}{\bar{\mu} P_0} \right] \mathbb{E}_1 \left[ \frac{(i+1)\sigma_z^2}{\bar{\mu} P_0} \right]. \end{aligned} \quad (25)$$

where  $f_{\mu_m}(x)$  is given in (24), the procedure (a) is due to binomial expansion, the integral in procedure (b) is calculated according to [27, 4.337.2], and  $\mathbb{E}_1(\cdot)$  is the exponential integral function defined as  $\mathbb{E}_1(x) = \int_x^\infty (e^{-u}/u) du$ .

The exact achievable function rate for the CM-MAC scheme in homogeneous networks is complicated without closed-form expression. Therefore, we will provide the following results through asymptotic analysis.

**Lemma 2.** (The domain of attraction for maxima) Let  $X_{\max} = \max_k X_k$  where  $X_k, k \in \{1, \dots, K\}$  is a set of i.i.d. random variables with CDF  $F_X(x)$ . If the following condition holds

$$\lim_{K \rightarrow \infty} K \left\{ 1 - F_X \left[ X_{1-1/K} + x \left( X_{1-1/(Ke)} - X_{1-1/K} \right) \right] \right\} = \exp(-x), \quad (26)$$

$F_X(x)$  lies in the domain of attraction of Gumbel distribution for maxima. That is there exists constants  $a_K$  and  $b_K$  such that

$$\lim_{K \rightarrow \infty} \frac{X_{\max} - a_K}{b_K} \xrightarrow{d} \text{Gumbel distribution}, \quad (27)$$

where  $\xrightarrow{d}$  means converging in distribution. And  $a_K$  and  $b_K$  can be given by

$$a_K = F_X^{-1} \left( 1 - \frac{1}{K} \right), \quad (28)$$

and

$$b_K = F_X^{-1} \left( 1 - \frac{1}{Ke} \right) - F_X^{-1} \left( 1 - \frac{1}{K} \right), \quad (29)$$

respectively.

*Proof.* The proof follows from Theorem 3.3 in [28]  $\square$

According to Lemma 1, an approximated closed-form expression of  $R_{CM}$  can be provided for a large number of nodes  $K$ .

**Proposition 2.** (Asymptotic achievable function rate for CM-MAC) The CDF of  $C_k$  lies in the domain of attraction of the Gumbel distribution for maxima. That is

$$\frac{C_m(\mu_m) - \log_2 \left( 1 + \frac{P_0 \bar{\mu} \ln K}{\sigma_z^2} \right)}{\log_2 \left( \frac{P_0 \bar{\mu} \ln K}{\sigma_z^2} + \frac{P_0 \bar{\mu}}{\sigma_z^2} + 1 \right)} \xrightarrow{d} \text{Gumbel distribution}. \quad (30)$$

Thus, the achievable function rate of the CM-MAC scheme in homogeneous networks can be approximated as

$$R_{CM} = \frac{1}{NK} \log_2 \left( 1 + \frac{P_0 \bar{\mu} \ln K}{\sigma_z^2} \right), \quad (31)$$

for a large number of nodes  $K$ .

*Proof.* Refer to Appendix A.  $\square$

For the CP-MAC scheme, a uniform-forcing transmitter  $b_k$  should be adopted to compensate the non-uniform MAC into a uniform level. Also, the message vector  $\mathbf{w}_k$  should be encoded into a length  $M$  nested lattice coding vector  $\mathbf{x}_k \in \mathcal{L}^M$  to combat the AWGN. Then, the received signal at the FC with all nodes' concurrent transmission is

$$\mathbf{y} = \sum_{k=1}^K \sqrt{P_0 \mu_k} b_k \mathbf{x}_k + \mathbf{z} = \sqrt{P_0 \mu} \sum_{k=1}^K \mathbf{x}_k + \mathbf{z} \quad (32)$$

where  $b_k \in \mathcal{C}$  is the uniform-forcing transmitter designed as  $b_k = \sqrt{\mu}/\sqrt{\mu_k}$ . Considering the transmit power constant  $|b_k| \leq 1, \forall k$ , the uniform power level  $\mu$  depends on the minimum channel power gain, i.e.,

$$\mu = \min_k \mu_k, \quad (33)$$

whose PDF can be calculated as

$$f_\mu(x) = \frac{K}{\bar{\mu}} \left[ \exp \left( -\frac{x}{\bar{\mu}} \right) \right]^K. \quad (34)$$

According to the achievable function rate of the CP-MAC scheme for uniform MAC with AWGN in Proposition 1, the achievable function rate of the CP-MAC scheme in homogeneous networks can be calculated as

$$\begin{aligned} R_{CP} &= \mathbb{E} \left[ \frac{1}{N + \log_2 K} \log_2 \left( \frac{P_0 \mu}{\sigma_z^2} \right) \right] \\ &= \frac{1}{N + \log_2 K} \int_{\sigma_z^2/P_0}^{\infty} \log_2 \left( \frac{P_0 x}{\sigma_z^2} \right) \frac{K}{\bar{\mu}} \left[ \exp \left( -\frac{x}{\bar{\mu}} \right) \right]^K dx \\ &\stackrel{(a)}{=} \frac{1}{\ln 2 (N + \log_2 K)} \mathbb{E}_1 \left( \frac{\sigma_z^2 K}{P_0 \bar{\mu}} \right) \end{aligned} \quad (35)$$

where the integral in procedure (a) is calculated according to [27, 4.331.2].

According to (31) and (35), the achievable function rate gain of the CP-MAC scheme over the CM-MAC scheme can be approximated as

$$G_3 \approx \frac{NK}{N + \log_2 K} \frac{\mathbb{E}_1 \left( \frac{K \sigma_z^2}{P_0 \bar{\mu}} \right)}{\ln \left( 1 + \frac{P_0 \bar{\mu} \ln K}{\sigma_z^2} \right)}. \quad (36)$$

When  $x$  is large,  $\mathbb{E}_1(x) \approx \exp(-x)/x$ . Thus, for a large number of nodes  $K$ ,  $G_3$  can be approximated as

$$G_3 \approx G_2 \frac{\gamma}{\ln(1 + \gamma \ln K)} \exp \left( -\frac{K}{\gamma} \right), \quad (37)$$

where  $G_2$  is the achievable function rate gain for uniform MAC with AWGN in (17), and  $\gamma = P_0 \bar{\mu} / \sigma_z^2$  is the average received SNR at the FC.

**Remark 2.** Compared with the achievable function rate gain for uniform MAC with AWGN  $G_2$ , the non-uniform MAC will incur an exponential decrease with the increase of the number of nodes  $K$ . This means that the CP-MAC scheme is not always superior to the CM-MAC scheme, especially when the number of nodes  $K$  is large.  $G_3$  increases with the average received SNR  $\gamma$ , which means the increase of the average received SNR  $\gamma$  will increase the benefit of using the CP-MAC scheme.

### B. Heterogeneous Networks

We further extend the results to heterogeneous networks where the nodes are assumed to be randomly distributed in a circular network, and the distance between node  $k$  and the FC is  $r_k$ , whose PDF is given by

$$f_{r_k}(x) = \frac{2x}{r_c^2}, 0 \leq x \leq r_c, \quad (38)$$

where  $r_c$  is the radius of the network. The channel power gain between the node  $k$  and the FC can be modeled as

$$\beta_k = \frac{\Phi \mu_k}{r_k^n}, \quad (39)$$

where  $\Phi$  is a constant that captures the effects of carrier frequency, antenna gain, antenna height and other power factors,  $n$  denotes the path loss exponent, and  $\mu_k$  is the Rayleigh fading component. Thus, the PDF and CDF of  $\beta_k$  can be calculated as

$$f_{\beta_k}(x_k) = \frac{2\Phi^{\frac{2}{n}}}{nr_c^2} x_k^{-\frac{2}{n}-1} \gamma\left(\frac{2}{n} + 1, \frac{r_c^n x_k}{\Phi}\right), \quad (40)$$

$$F_{\beta_k}(\beta_k) = 1 - \frac{2\Phi^{\frac{2}{n}}}{nr_c^2} \beta_k^{-\frac{2}{n}} \gamma\left(\frac{2}{n}, \frac{r_c^n \beta_k}{\Phi}\right). \quad (41)$$

For the CM-MAC scheme, we adopt normalized channel power gain scheduling considering both multi-node diversity and multi-node fairness. In each channel use, we select the node as

$$m = \arg \max_k \mu_k. \quad (42)$$

The fast fading component of the channel power gain of the selected node is  $\mu_m = \arg \max_k \mu_k$ . Thus, the PDF of  $\mu_m$  can be also calculated according to the distribution of the maxima of i.i.d. variables as (24). The PDF of  $r_m$  is the same as that of the node  $k$  in (38). Given the fast fading component  $\mu_m$  and the distance  $r_m$ , the channel capacity of the node  $m$  can be expressed as

$$C_m(\mu_m, r_m) = \log_2 \left( 1 + \frac{P_0 \Phi \mu_m}{\sigma_z^2 r_m^n} \right). \quad (43)$$

The messages of  $K$  nodes are composed of  $NK$  bits, and the message rate in each channel use should not exceed the ergodic channel capacity. Thus, the achievable function rate for CM-MAC in heterogeneous networks can be obtained as

$$\begin{aligned} R_{CM} &= \frac{1}{NK} \mathbb{E} \left[ \log_2 \left( 1 + \frac{P_0 \Phi \mu_m}{\sigma_z^2 r_m^n} \right) \right] \\ &= \frac{1}{NK} \int_0^{r_c} \int_0^\infty \log_2 \left( 1 + \frac{P_0 \Phi x}{\sigma_z^2 y^n} \right) f_{\mu_m}(x) f_{r_m}(y) dx dy, \end{aligned} \quad (44)$$

where  $f_{\mu_m}(x)$  and  $f_{r_m}(y)$  are given in (24) and (38), respectively.

Then, we will provide the following asymptotic analysis. For a large number of nodes  $K$ , the achievable function rate

of the CM-MAC scheme in heterogeneous networks can be approximated as

$$\begin{aligned} R_{CM} &\stackrel{(a)}{\approx} \frac{1}{NK} \int_0^{r_c} \log_2 \left( 1 + \frac{P_0 \Phi \ln K}{\sigma_z^2 y^n} \right) \frac{2y}{r_c^2} dy \\ &= \frac{1}{NK} \log_2 \left( 1 + \gamma \frac{P_0 \Phi \ln K}{\sigma_z^2 y^n} \right) \frac{y^2}{r_c^2} \Big|_0^{r_c} \\ &\quad - \frac{1}{NK \ln 2} \int_0^{r_c} \frac{2y}{r_c^2} d \ln \left( 1 + \frac{P_0 \Phi \ln K}{\sigma_z^2 y^n} \right) \\ &= \frac{1}{NK} \log_2 \left( 1 + \frac{P_0 \Phi \ln K}{\sigma_z^2 r_c^n} \right) \\ &\quad + \frac{1}{NK \ln 2} \int_0^{r_c} \frac{2}{r_c^2} \frac{P_0 \Phi \ln K}{\sigma_z^2 r_c^n + P_0 \Phi \ln K} dy \\ &\stackrel{(b)}{\approx} \frac{1}{NK} \log_2 \left( 1 + \frac{P_0 \Phi \ln K}{\sigma_z^2 r_c^n} \right) + \frac{2n}{\ln 2 NK r_c^2}, \end{aligned} \quad (45)$$

where the procedure (a) is due to the approximation in Proposition 1, and the procedure (b) is obtained by using  $P_0 \Phi \ln K \gg \sigma_z^2 y^n$  for a large number of nodes  $K$ .

For the CP-MAC scheme, the uniform-forcing transmitter  $b_k$  in (32) should also be adopted to compensate the MAC to the uniform level, which is designed as  $b_k = \sqrt{\beta/\beta_k}$ . Considering the transmit power constant  $|b_k| \leq 1$ , the uniform power level  $\beta$  depends on the minimum channel power gain as

$$\beta = \min_k \{\beta_k\}. \quad (46)$$

Then, according to the distribution of the minima of i.i.d. variables, the PDF of  $\beta$  can be calculated as

$$f_\beta(x) = K f_{\beta_k}(x) [1 - F_{\beta_k}(x)]^{K-1}, \quad (47)$$

where  $f_{\beta_k}(x)$  and  $F_{\beta_k}(x)$  are given in (40) and (41), respectively.

Based on the achievable function rate of the CP-MAC scheme for uniform MAC with AWGN in Proposition 1, the achievable function rate of the CP-MAC scheme in heterogeneous networks can be calculated as

$$\begin{aligned} R_{CP} &= \mathbb{E} \left[ \frac{1}{N + \log_2 K} \log_2 \left( \frac{P_0 \beta}{\sigma_z^2} \right) \right] \\ &= \frac{1}{N + \log_2 K} \int_{\sigma_z^2/P_0}^\infty \log_2 \left( \frac{P_0 x}{\sigma_z^2} \right) f_\beta(x) dx, \end{aligned} \quad (48)$$

where  $f_\mu(x)$  is given in (47). It is also complicated without closed-form expression. We provide the asymptotic analysis as follows.

**Lemma 3.** (The domain of attraction for minima) Let  $X_{\min} = \min_k X_k$  where  $X_k, k \in \{1, \dots, K\}$  is a set of i.i.d. random variables with CDF  $F_X(x)$ . If, and only if,

$$\Gamma(F_{\beta_k}) = \inf \{x : F_{\beta_k}(x) > 0\} \quad (49)$$

and the function

$$F_X^*(x) = F_X \left( \Gamma - \frac{1}{x} \right), x < 0 \quad (50)$$



Table II  
ACHIEVABLE FUNCTION RATE AND RATE GAIN

Scenarios	Rate of CM-MAC	Rate of CP-MAC	Rate gain
Uniform MAC without AWGN	$R_{CM} = 1$	$R_{CP} = 1/K$	$G_1 = K$
Uniform MAC with AWGN	$R_{CM}$ in (10)	$R_{CP}$ in (14)	$G_2$ in (17)
Homogeneous non-uniform MAC	$R_{CM}$ in (31)	$R_{CP}$ in (35)	$G_3$ in (36)
Heterogeneous non-uniform MAC	$R_{CM}$ in (45)	$R_{CP}$ in (56)	$G_4$ in (57)

satisfies

$$\lim_{t \rightarrow -\infty} \frac{F_X^*(tx)}{F_X^*(t)} = x^{-\gamma}, \gamma > 0. \quad (51)$$

$F_X(x)$  lies in the domain of attraction of Weibull distribution for minima. That is there exists constants  $c_K$  and  $d_K$  such that

$$\lim_{K \rightarrow \infty} \frac{X_{\max} - c_K}{d_K} \xrightarrow{d} \text{Weibull distribution}, \quad (52)$$

where  $\xrightarrow{d}$  means converges in distribution. The  $c_K$  and  $d_K$  can be given by

$$c_K = \Gamma, \quad (53)$$

and

$$d_K = F_X^{-1}\left(\frac{1}{K}\right) - \Gamma, \quad (54)$$

respectively.

*Proof.* The proof follows from Theorem 3.4 in [28].  $\square$

**Proposition 3.** (Asymptotic achievable function rate for CM-MAC) The CDF of  $\beta$  lies in the domain of attraction of Weibull distribution for minima with the shape parameter 1 as follows

$$\frac{2r_c^n K}{(n+2)\Phi} \beta \xrightarrow{d} \text{Weibull distribution}. \quad (55)$$

The ergodic achievable function rate of the CP-MAC scheme in heterogeneous networks can be expressed as

$$R_{CP} = \frac{1}{\ln 2 [N + \log_2(K)]} E_1\left(\frac{2}{n+2} \frac{r_c^n K \sigma_z^2}{\Phi P_0}\right) \quad (56)$$

*Proof.* Refer to Appendix B.  $\square$

According to (45) and (56), the achievable function rate gain of the CP-MAC scheme over the CM-MAC scheme can be approximated as

$$G_4 \approx \frac{NK}{\ln 2 [N + \log_2(K)]} \frac{E_1\left[\frac{2Kr_c^n \sigma_z^2}{(n+2)P_0\Phi}\right]}{\log_2\left(1 + \frac{P_0\Phi \ln K}{r_c^n \sigma_z^2}\right)}, \quad (57)$$

When  $x$  is large,  $E_1(x) \approx \exp(-x)/x$ . Thus, for a large number of nodes  $K$ ,  $G_4$  can be approximated as

$$G_4 \approx G_2 \frac{(n+2)}{2} \frac{\gamma'}{\ln(1 + \gamma' \ln K)} \exp\left[-\frac{2K}{(n+2)\gamma'}\right], \quad (58)$$

Table III  
SIMULATION PARAMETERS

Parameter	Value
The number of nodes	$K = 100$
The length of the message vector	$N = 50$
The average channel power gain	$\bar{\mu} = 0$ dB
The transmit power to noise ratio	$P_0/\sigma_z^2 = 30$ dB
The scale of the network	$r_c = 3$ m
The path loss constant	$\Phi = 0.023568$
The path loss exponent	$n = 3$

where  $G_2$  is the achievable function rate gain for uniform MAC with AWGN in (17), and  $\gamma' = P_0\Phi/r_c^n \sigma_z^2$  is the average received SNR of the node at the edge of the network.

**Remark 3.** Similar to homogeneous networks, the non-uniform MAC will incur an exponential decrease of the achievable function rate gain with the increase of the number of nodes  $K$ . This also verifies that the CP-MAC scheme is not always superior to the CM-MAC scheme, especially when the number of nodes  $K$  is large. And  $G_4$  increases with the average received SNR for the node at the edge of the network  $\gamma'$ . This means the increase of the scale of the network  $r_c$  will decrease the benefit of using the CP-MAC scheme.

In conclusion, the achievable function rate and the rate gain of the CM-MAC scheme and the CP-MAC scheme for different scenarios are summarized in Table II. For uniform MAC scenarios, the CP-MAC is always superior to the CM-MAC scheme. The achievable rate gain of the CP-MAC scheme over the CM-MAC scheme of the uniform MAC with AWGN is smaller than that of the uniform MAC without AWGN. For non-uniform scenarios, the CP-MAC is not always better than the CM-MAC scheme. The achievable rate gain of the CP-MAC scheme over the CM-MAC scheme exponentially decreases with the number of nodes.

## V. NUMERICAL RESULTS AND DISCUSSION

In this section, we provide some simulation results to illustrate the performance of the CM-MAC scheme and the CP-MAC scheme. Both homogeneous networks and heterogeneous networks are discussed. The parameters are set as Table III unless specified otherwise.

### A. Homogeneous Networks

The achievable function rate versus the number of nodes  $K$  is shown in Fig. 4(a). For the CP-MAC scheme, the simulated results is exactly the same as theoretical ones. For the CM-MAC scheme, the simulated results is almost identical with the theoretical ones. The difference is due to the asymptotic

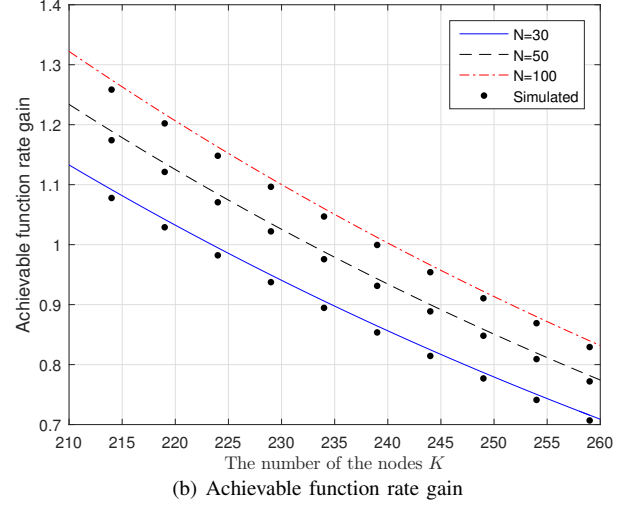
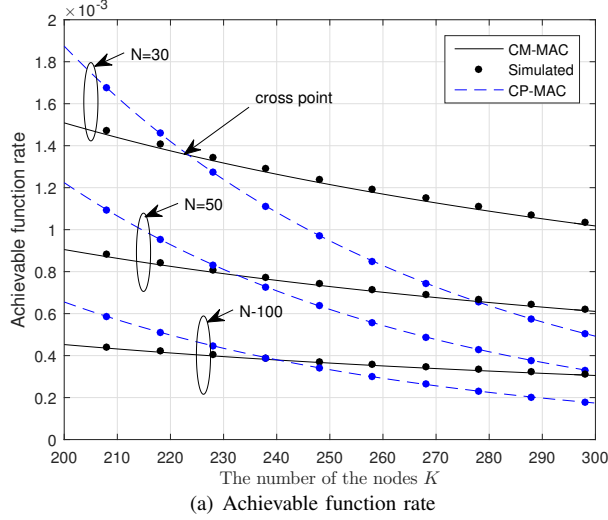


Figure 4. The achievable function rate and the rate gain versus the number of nodes

analysis used in the derivation. The achievable function rate decreases with the number of nodes for both the CP-MAC scheme and the CM-MAC scheme. The decreasing rate of the CP-MAC scheme is larger than that of the CM-MAC scheme. There is a threshold on the achievable function rate between the CP-MAC scheme and the CM-MAC scheme. When the number of nodes is larger than the threshold, the CM-MAC scheme is better than the CP-MAC scheme. Otherwise, the CP-MAC scheme is better. The achievable function rate decreases with the length of the message vector  $N$  for both the CP-MAC scheme and the CM-MAC scheme. Also, it can be seen the threshold of the number of nodes increases with  $N$ , which means that adopting the CP-MAC scheme is more beneficial when the length of the message vectors is large.

The achievable function rate gain versus the number of nodes  $K$  is shown in Fig. 4(b). The achievable function rate gain of the CP-MAC scheme over the CM-MAC scheme decreases with the number of nodes. When the number of nodes increases and the rate gain is smaller than 1, the CM-MAC scheme is superior to the CP-MAC scheme. While, the achievable function rate gain increases with the length of the message vector  $N$ , which also verifies that adopting the CP-MAC scheme is more beneficial when the length of the message vectors is large.

The achievable function rate versus the transmit power to noise ratio  $\gamma$  is illustrated in Fig. 5(a). When the transmit power to noise ratio increases, the achievable function rate linearly increases for the CM-MAC scheme and exponentially increases for the CP-MAC scheme. There is also a threshold on the achievable function rate between the CP-MAC scheme and the CM-MAC scheme. When transmit power to noise ratio is larger than the threshold, the CP-MAC scheme is better than the CM-MAC scheme. Otherwise, the CM-MAC scheme is better. The achievable function rate decreases with the number of nodes  $K$  for both the CP-MAC scheme and the CM-MAC scheme. Also, it can be seen the threshold of the transmit power to noise ratio increases with  $K$ , which means

that adopting the CM-MAC scheme is more beneficial when the number of nodes is large.

The achievable function rate gain versus the transmit power to noise ratio  $\gamma$  is illustrated in Fig. 5(b). The achievable function rate gain of the CP-MAC scheme over the CM-MAC scheme increases with the transmit power to noise ratio. When the transmit power to noise ratio increases and the rate gain is larger than 1, the CP-MAC scheme is superior to the CM-MAC scheme. While, the achievable function rate gain decreases with the number of nodes  $K$ , which also verifies that adopting the CM-MAC scheme is more beneficial when the number of nodes is large.

### B. Heterogeneous Networks

In Fig. 6(a), the achievable function rate versus the scale of the network  $r_c$  is illustrated. When the scale of the network increases, the achievable function rate linearly decreases for the CM-MAC scheme and exponentially decreases for the CP-MAC scheme. There is also a threshold on the achievable function rate between the CP-MAC scheme and the CM-MAC scheme. When the scale of the network is larger than the threshold, the CM-MAC scheme is better than the CP-MAC scheme. Otherwise, the CP-MAC scheme is better. We also consider different path loss exponent of the network  $n$ , whose value is normally in the range of 2 to 4<sup>1</sup>. The achievable function rate decreases with the path loss exponent  $n$  for both the CP-MAC scheme and the CM-MAC scheme. Also, it can be seen the threshold of the scale of the network decreases with the path loss exponent  $n$ , which means that adopting the CP-MAC scheme is more beneficial when the path loss exponent is small.

In Fig. 6(b), the achievable function rate gain versus the scale of the network  $r_c$  is illustrated. It decreases with the scale of the network. When the scale of the network increases and the rate gain is smaller than 1, the CM-MAC scheme

<sup>1</sup>2 is for propagation in free space, 4 is for relatively lossy environments.

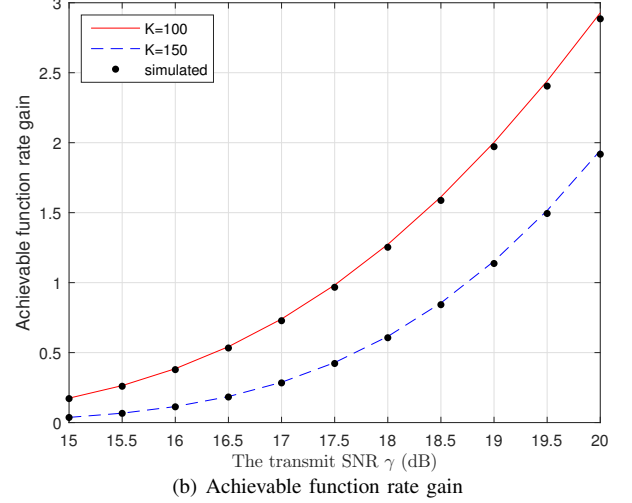
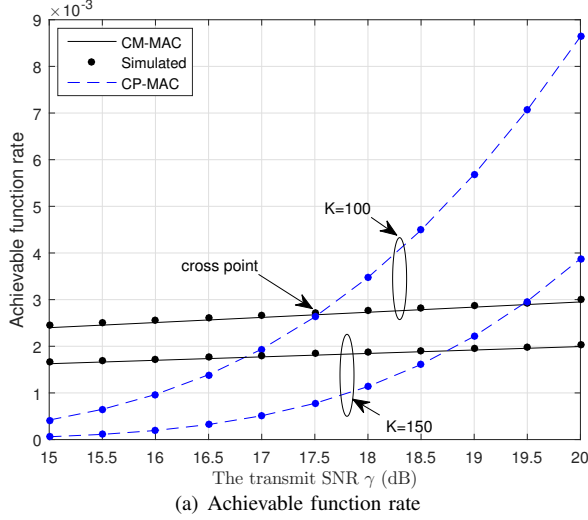


Figure 5. The achievable function rate and rate gain versus the transmit power to noise ratio

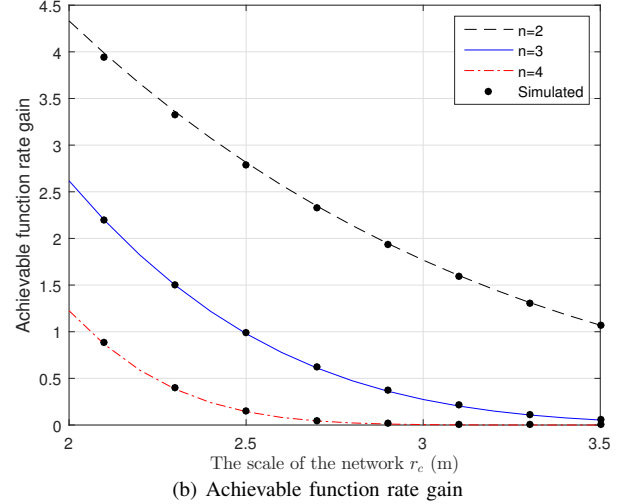
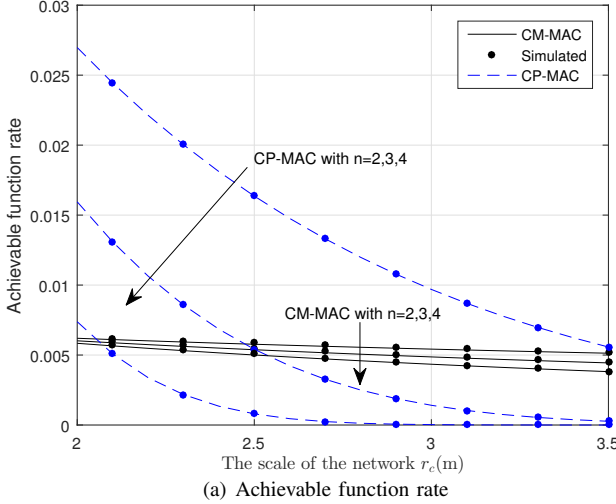


Figure 6. The achievable function rate and the rate gain versus the scale of the network

is superior to the CP-MAC scheme. While, the achievable function rate gain decreases with the path loss exponent  $n$ , which also verifies that adopting the CP-MAC scheme is more beneficial when the path loss exponent is small.

## VI. CONCLUSION

In this work, we have studied the achieve function rate of function-centric wireless networks for two schemes. One is the CM-MAC scheme where data is aggregated first and then the target function is computed. The other is the CP-MAC scheme where the summation part of the target function is computed utilizing the superposition property of wireless channel. The exact achieve function rate and the asymptotic closed-form expression have been provided for both schemes. We have first analyzed ideal uniform MAC scenarios, where the CP-MAC scheme is always superior to the CM-MAC scheme. Then practical non-uniform MAC scenarios have been further

studied for both homogeneous networks and heterogeneous networks. It has been found that the CP-MAC scheme is not always superior to the CM-MAC scheme with different network parameters.

## APPENDIX A PROOF OF PROPOSITION 2

The CDF of  $C_k(\mu_k)$  is

$$\begin{aligned} F_{C_k}(x) &= F_{\mu_k} \left( \frac{(2^x - 1) \sigma_z^2}{P_0} \right) \\ &= 1 - \exp \left( - \frac{(2^x - 1) \sigma_z^2}{P_0 \bar{\mu}} \right). \end{aligned} \quad (59)$$

And  $X_{1-1/K}$  and  $X_{1-1/(Ke)}$  in (26) can be given by

$$F_{C_k}^{-1} \left( 1 - \frac{1}{K} \right) = \log_2 \left( 1 + \frac{P_0 \bar{\mu} \ln K}{\sigma_z^2} \right), \quad (60)$$

and

$$F_{C_k}^{-1} \left( 1 - \frac{1}{Ke} \right) = \log_2 \left( 1 + \frac{P_0 \bar{\mu}}{\sigma_z^2} + \frac{P_0 \bar{\mu} \ln K}{\sigma_z^2} \right). \quad (61)$$

Then, we have that

$$\begin{aligned} & 1 - F_{C_K} \left[ F_{C_k}^{-1} \left( 1 - \frac{1}{K} \right) + x \left( F_{C_k}^{-1} \left( 1 - \frac{1}{Ke} \right) - F_{C_k}^{-1} \left( 1 - \frac{1}{K} \right) \right) \right] \\ &= 1 - F_{C_K} \left[ \log_2 \left( 1 + \frac{P_0 \bar{\mu} \ln K}{\sigma_z^2} \right) + \log_2 \left( \frac{P_0 \bar{\mu} \ln K + P_0 \bar{\mu} + \sigma_z^2}{P_0 \bar{\mu} \ln K + \sigma_z^2} \right)^x \right] \\ &= \exp \left( - \frac{(P_0 \bar{\mu} \ln K + \sigma_z^2)}{P_0 \bar{\mu}} \left( \frac{P_0 \bar{\mu} \ln K + P_0 \bar{\mu} + \sigma_z^2}{P_0 \bar{\mu} \ln K + \sigma_z^2} \right)^x - \frac{\sigma_z^2}{P_0 \bar{\mu}} \right). \end{aligned} \quad (62)$$

For a large  $K$ , the above equation can be further approximated as

$$\begin{aligned} & \exp \left( - \frac{(P_0 \bar{\mu} \ln K + \sigma_z^2)}{P_0 \bar{\mu}} \left( 1 + \frac{P_0 \bar{\mu}}{P_0 \bar{\mu} \ln K + \sigma_z^2} \right)^x - \frac{\sigma_z^2}{P_0 \bar{\mu}} \right) \\ & \approx \exp \left( - \ln K \left( 1 + x \frac{P_0 \bar{\mu}}{P_0 \bar{\mu} \ln K + \sigma_z^2} \right) \right) \\ & \approx \frac{1}{K} \exp(-x). \end{aligned} \quad (63)$$

Thus, the condition (26) in Lemma 1 is satisfied, and the CDF of  $C_k$  lies in the domain of attraction of the Gumbel distribution for maxima. That is

$$\frac{C_m(\mu_m) - \log_2 \left( 1 + \frac{P_0 \bar{\mu} \ln K}{\sigma_z^2} \right)}{\log_2 \left( \frac{P_0 \bar{\mu} \ln K}{\sigma_z^2} + \frac{P_0 \bar{\mu}}{\sigma_z^2} + 1 \right)} \xrightarrow{d} \text{Gumbel distribution.} \quad (64)$$

And we have that

$$\begin{aligned} \mathbb{E}[C_m(\mu_m)] & \stackrel{(a)}{=} \log_2 \left( 1 + \frac{P_0 \bar{\mu} \ln K}{\sigma_z^2} \right) \\ & + \xi \log_2 \left( 1 + \frac{P_0 \bar{\mu}}{\sigma_z^2} + \frac{P_0 \bar{\mu} \ln K}{\sigma_z^2} \right) \\ & \approx \log_2 \left( 1 + \frac{P_0 \bar{\mu} \ln K}{\sigma_z^2} \right), \end{aligned} \quad (65)$$

where the procedure (a) is due to the expectation of Gumbel distribution, and  $\xi$  is EulerMascheroni constant. The achievable function rate for CM-MAC is

$$R_{CM} = \frac{\mathbb{E}[C_m(\mu_m)]}{NK}, \quad (66)$$

which completes the proof.

#### APPENDIX B PROOF OF PROPOSITION 3

According to (41), we have

$$\Gamma(F_{\beta_k}) = \inf \{x : F_{\beta_k}(x) > 0\} = 0 > -\infty \quad (67)$$

When  $x \rightarrow 0$  we have

$$\begin{aligned} F_{\beta_k}^*(x) &= F_{\beta_k} \left( -\frac{1}{x} \right) \\ &= 1 - \frac{2\Phi^{\frac{2}{n}}}{nr_c^2} \left( -\frac{1}{x} \right)^{-\frac{2}{n}} \gamma \left( \frac{2}{n}, -\frac{r_c^n}{\Phi x} \right) \\ &\stackrel{(a)}{\approx} \frac{2}{2+n} \frac{r_c^n}{\Phi} x, \end{aligned} \quad (68)$$

where the procedure (a) is due to the Taylor expansion of incomplete gamma function. Then

$$\lim_{t \rightarrow -\infty} \frac{F_{\beta_k}^*(tx)}{F_{\beta_k}^*(t)} = x^{-1}. \quad (69)$$

$$c_n = 0, d_n = F_{\beta_k}^{-1} \left( \frac{1}{K} \right) \approx \frac{n+2}{2} \frac{\Phi}{r_c^n K} \quad (70)$$

Thus, we have

$$\frac{2r_c^n K}{(n+2)\Phi} \beta \xrightarrow{d} \text{Weibull distribution}, \quad (71)$$

with the shape parameter 1. The PDF of  $\beta$  can be approximated as

$$f_\beta(x) = \frac{2}{n+2} \frac{r_c^n K}{\Phi} \exp \left( -\frac{2}{n+2} \frac{r_c^n K x}{\Phi} \right). \quad (72)$$

Thus, the achievable function rate can be derived as

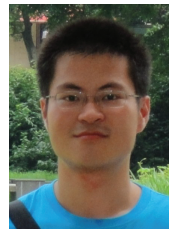
$$\begin{aligned} R_{CP} &= \frac{1}{N + \log_2(K)} \mathbb{E} \left[ \log_2^+ \left( \frac{P_0 \beta}{\sigma_z^2} \right) \right] \\ &= \frac{1}{N + \log_2(K)} \frac{2}{n+2} \frac{r_c^n K}{\Phi} \\ & \int_{\sigma_z^2/P_0}^{\infty} \log_2 \left( \frac{P_0 x}{\sigma_z^2} \right) \exp \left( -\frac{2}{n+2} \frac{r_c^n K x}{\Phi} \right) dx \\ &\stackrel{(b)}{=} \frac{1}{\ln 2 [N + \log_2(K)]} \text{E1} \left( \frac{2}{n+2} \frac{r_c^n K \sigma_z^2}{\Phi P_0} \right) \end{aligned} \quad (73)$$

where the procedure (b) can be calculated according to [27, 4.331.2]. It completes the proof.

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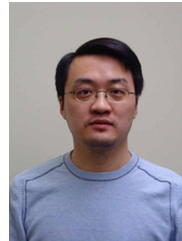


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