Optimal Stabilisation Policies in Interdependent Economies. A Game Theoretic Approach

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Summary

This thesis examines the inefficiencies which arise from decentralised policy making in an interdependent world economy. Policy making is modelled as a non-cooperative, non-zero sum game between economies. Each economy is assumed to have a loss function defined on relevant macroeconomic variables. The constraints facing any particular economy are determined by the joint action of all the economies. Conventional Nash and Stackelberg solution concepts are examined for simple comparative static models. These solution concepts are then extended to models in which the constraint set has a dynamic structure. It is demonstrated that even when countries can agree on ultimate targets policy will still be inefficient because of sub-optimal choices of convergent paths to steady-state. A simple macro-economic model of an interdependent world economy is used to demonstrate this analytically. This is followed by numerical simulations of a more general model. Although the inefficiency discussed above is clearly present it appears to be numerically small suggesting that some ultimate conflict of interests is necessary for decentralised policy to have large welfare effects.

Contents

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Chapter One

Introduction

Chapter One

Introduction

This thesis has been motivated by an interest in two areas of economic theory. These are the theory of the transmission of economic disturbances between countries and the use of game theory to model the interaction of small groups of agents. A natural combination of these interests is the modelling of interacting stabilisation policies in an interdependent world economy. This is not just of abstract theoretical interest. As I hope to show in this introduction, a number of periods of unsatisfactory performance in recent economic history can, at least partly, be explained by a failure to coordinate policies.

In the post war period there have been two very different international monetary systems. The Bretton Woods system was conceived during the war,brought into operation shortly after and lasted until 1973. This system was characterised by fixed exchange rates alterable only in response to 'fundamental disequilibrium' in a country's payments position. In 1973 this system was abandoned in favour of one of flexible exchange rates though occasionally governments have intervened to manipulate the foreign exchange market. Both systems have proved to have several disadvantages.

A possible cause of the failure of the Bretton Woods system was the asymmetric way in which balance of payments disequilibria were treated. Deficit countries were forced to deflate or to devalue but there was effectively no sanction on surplus countries c.f. Williamson (1977) for a more detailed discussion. Thus fixed exchange rates were seen as a severe constraint on the ability of a country to pursue a full employment macroeconomic policy. The central role of the dollar as the international reserve currency also required the U.S. monetary authorities to design policy with regard to the demand for world liquidity rather than domestic considerations. The conflict between the two understandably created difficulties.

It was hoped that the shift to flexible exchange rates would relieve both problems by allowing countries to direct policy towards full employment targets and at the same time allow the United States to pursue an independent monetary policy. The main worry was that countries would indulge in the kind of competitive depreciation which was a feature of the inter-war floating period. The danger of this is that the resulting instability of exchange rates could act as a disincentive to trade. In actual fact, although exchange rates have proved quite volatile, competitive depreciation does not seem to have been a major problem. Indeed the reverse seems to have been the case as countries have appreciated their currencies to combat inflation as the policy emphasis has shifted from unemployment.

International factors have therefore been important under both systems of exchange rates. The reasons why floating the exchange rate have not led to the desired decoupling of policies are discussed in chapter two. A natural question is therefore to what extent countries could improve on decentralised outcomes by coordination of macroeconomic policy. This question will provide the central theme of this thesis.

The plan of the thesis is as follows. Chapter two provides a survey of the literature on the transmission of economic policy between countries. This begins with the small open-economy case and moves on to consider models of interdependence. It is shown that as models have become more general they have come to concentrate more on dynamic adjustment through time rather than comparative static equilibria. Chapter three discusses policy optimisation among a small group of agents. The solution concepts of game theory are set out in a general form and are then used to analyse a simple two-country game using the Mundell fixed price model. Chapter four generalises the problem to the case where the state variables have a dynamic structure by setting out solution concepts for differential games. This enables two sources of inefficiency from uncoordinated policy to be contrasted. The first of these results from incompatibility of targets adopted by agents. The second is due to an inefficient choice of adjustment path even when targets are mutually compatible. Chapter five considers the problem of bringing down inflation in a two country model using the techniques of chapter four. The emphasis in this chapter is on obtaining general analytical results. Unfortunately this requires the use of a very simplified economic model to keep the problem tractable. Chapter six examines a less restrictive model by use of numerical simulation. Both chapters five and six emphasise real wage resistance as a factor influencing the transmission of inflation from one country to another.

Chapter Two

The International Transmission of Macroeconomic Policy

Chapter Two

(2:1) Introduction

The purpose of this chapter is to provide a survey of the existing literature on macroeconomic policy in open economies. There are several ways of going about this. One is to provide a chronological account of the development of the theory emphasising the various controversies which have arisen. A second is to begin with a simple model with highly restrictive assumptions and demonstrate the effects of relaxing these assumptions. The approach adopted in this chapter is something of a mixture of the two. Simple models are specified which capture the spirit of various analyses and it is shown that for the most part these simple models can be seen as special cases of a more general model. For example, the Mundell-Fleming and monetary models are derived by imposing values on the price adjustment coefficient in a more general model.

The plan of the chapter is as follows. Section (2:2) analyses the impact of macroeconomic policy in a small open economy (SOE). The issues considered will be the effects of the exchange rate regime, the degree of capital mobility and the degree of price flexibility. In section (2:3) this analysis is extended to the case where the economy is large enough to affect world aggregates. A two country model is specified which can be thought of as a system of two large economies or as one large economy facing an amalgam of smaller economies. Section (2:4) moves away from the positive analysis of the previous sections to consider some normative issues, i.e. in what way can the authorities best determine policy to achieve their objectives. Section (2:5) contains conclusions.

(2:2) The effects of policy in an SOE

Modern open economy macroeconomics really begins with the extension of the Keynesian multiplier model to incorporate the external sector c.f. Machlup (1943). This is a special case of the more general fixed-price, Hicksian IS/LM system in which the interest elasticity of aggregate expenditure is assumed to be zero. Since the IS/LM system is better suited for the analysis of capital mobility this framework will be used for the development of the Keynesian model.

The closed economy IS/LM system is made up of three markets, i.e. those for goods, money and bonds. It is solved by finding the interest rate/output combination which simultaneously clears the goods and money markets and relying on Walras' law to ensure that the bond market clears. The generalisation of the model to incorporate an external sector adds a fourth market in the form of the market for foreign exchange. Mundell (1963) and Fleming (1962) extended the IS/LM model to this four market system. They assumed that equilibrium at any point in time corresponds to intersection of the IS and LM curves. By Walras' law this implicitly requires that any excess demand/supply in the foreign exchange market must have as its counterpart excess supply/demand in the bond market.

Analysis of the foreign exchange market usually disaggregates transactions into two categories, current and capital account. The analytical advantage of this is that the determinants of the balance on these categories are different. The current account balance depends on the level of output at home and in the rest of the world and the relative price of domestic and foreign goods while the capital account balance is a function of the relative yields of domestic and foreign financial assets. Assuming both domestic and world prices to be fixed enables the Mundell-Fleming model to be summarised by the following set of equations.

(2:1)	$Y = Y(r, g, E, A)$	- IS curve
(2:2)	$r = r(Y, M)$	- LM curve
(2:3)	$\dot{R} = B = C(Y, Y^*, E) + K(r, r^*, \dot{E}^e/E)$	- Reserve changes
(2:4)	$M = \phi(R + D)$	- Money Supply
(2:5)	$\dot{D} = -\alpha\dot{R}$	- Sterilisation

Notation is as follows:

Y = output

r = rate of interest

E = exchange rate (price of unit of foreign currency)

g = government expenditure

A = all other autonomous expenditures

M = money supply

B = balance of payments

C = current account

K = capital account

R = reserves of foreign currency

D = domestic credit base

* indicates a foreign variable

Consider first the fixed exchange rate case. Assuming that the authorities do not intervene to sterilise reserve changes, i.e.

$\dot{D} = 0$, implies a single source of dynamics in the form of the change in the money stock due to an external payments imbalance. This can be written:

(2:6) $\dot{M} = \phi(C(Y, Y^*) + K(r, r^*))$

where ϕ, the money multiplier, is by definition positive. Rewriting the IS/LM system as a reduced form, i.e.

(2:1') $Y = Y(r, g, E, A) = Y'(g, M, E, A)$

(2:2') $r = r(Y, M) = r'(g, M, E, A)$

enables stability conditions to be derived. Stability requires $\partial\dot{M}/\partial M < 0$ which in turn requires $\phi(C_Y Y'_M + K_r r'_M) < 0$. This is satisfied if all partial derivatives have their expected signs.

The properties of this model are well known. Both fiscal and monetary policy generate positive impact effects on output but the effects of a monetary expansion are completely eroded by the subsequent loss of reserves. There are two possible responses to a fiscal expansion.

(1) If $C_Y Y'_g + K_r r'_g > 0$ then the increase in the rate of interest generates a capital account surplus which more than offsets the deterioration in the current balance at the impact equilibrium. There is therefore a second round monetary expansion which reinforces the output increase from the initial fiscal expansion.

(2) If $C_Y Y'_g + K_r r'_g < 0$ then the impact equilibrium exhibits a payments deficit. This leads to a loss of reserves which erodes some, but not all, of the initial expansionary effects of fiscal policy (providing $K_r \neq 0$).

Consider now the flexible exchange rate case. Expected changes in the exchange rate are assumed to be exogenous, i.e. $\dot{E}^e$ is independent of current variables. Rather than the money stock adjusting in response to payments imbalance it is assumed that the exchange rate moves to correct the disequilibrium. Assuming a partial adjustment mechanism enables the dynamics of the exchange rate to be written

$$(2:7) \qquad \dot{E} = -\theta(C(Y, Y^*, E) + K(r, r^*))$$

where θ, the adjustment coefficient, can either reflect the speed of adjustment of the market under a freely floating system or a reaction function coefficient of the policy authorities under a managed float.

If the adjustment path is stable, then the conventional policy results can be derived. Both fiscal and monetary policy are effective in increasing output in the short and long run but the effectiveness of fiscal policy declines as capital mobility increases becoming zero with perfect mobility. Note that if the adjustment path involves a non-zero balance of payments it is necessary to assume full sterilisation to disentangle the effects of policy from subsequent monetary effects.

For stability $\partial\dot{E}/\partial E < 0$ is necessary which requires $C_E + C_Y Y'_E + K_r r'_E > 0$. This is clearly satisfied if $C_E > 0$, i.e. if the Marshall-Lerner condition is satisfied. The existence of adjustment lags makes $C_E < 0$ quite a plausible case in the short-run.

Several criticisms of the Mundell-Fleming model have been put forward as follows:

(1) The most obvious criticism is the assumption of fixed prices. This can be relaxed by including a Phillips' curve without changing the basic insights of the model providing the adjustment of prices to output is sluggish. Sachs (1980) replaces this assumption with that of a fixed real wage. This reverses the conclusion that monetary policy is more effective than fiscal policy under flexible exchange rates since changes in the nominal exchange rate no longer imply changes in the real exchange rate.

(2) Determination of the capital account is specified in flow terms. This implies that a net capital inflow can be maintained indefinitely by a positive interest differential. A more satisfactory approach is to model adjustment towards a desired stock of financial assets which is determined by relative yields and risk. This kind of portfolio model has been made popular by Tobin (1958 and 1968) and has been applied to markets in international financial assets by Kouri and Porter (1974) and Branson (1981).

(3) The static expectations assumption (i.e. $\dot{E}^e$ independent of current economic variables) is clearly unsatisfactory. Dornbusch (1976) replaces it with the assumption of model consistent expectations (discussed later in this section). Niehans (1975) argues that expectations can be endogenised in such a way as to remove the instability problems associated with short-run J curve effects. The basic argument is that when the Marshall-Lerner condition is not satisfied in the short-run a monetary expansion produces a deficit on both current and capital accounts. Depreciation is unable to correct the current account deficit but may be able to create an

offsetting capital inflow if it creates the expectation of a subsequent appreciation. This idea of exchange rate overshooting in response to monetary policy has since become familiar through papers by Dornbusch (1976) and Buiter and Miller (1981).

The Mundell-Fleming model is clearly a Keynesian model in that its central concern is the determination of the output level with prices assumed exogenous. For a number of years in the late 1960's/ early 1970's its chief rival was seen as the 'monetary approach' which assumed flexible prices and fixed output c.f. Frenkel and Johnson (1976). The stylised features of this model are as follows. Firstly nominal expenditure is determined by a fixed proportional relationship to the money stock. With fixed exchange rates and perfect commodity arbitrage prices are determined via the purchasing power parity relationship. This determines nominal income since output is assumed to be exogenous. The balance of payments is then determined as the difference between nominal income and expenditure. Changes in the money supply therefore affect the balance of payments by changing nominal expenditure but do not affect output or prices.

As in the Mundell-Fleming case the monetary approach to the balance of payments can be described in terms of Walras' law. In this case there are only two markets to be considered, that for money balances and that for foreign currency. An excess supply of money necessarily implies an excess demand for foreign currency and vice-versa. Thus the two models, under very different assumptions, come to the qualitatively similar conclusion that a small country in a fixed exchange rate regime cannot pursue an independent monetary policy.

In the flexible exchange rate case the monetary approach becomes a theory of the exchange rate rather than a theory of the balance of payments. The argument here is that monetary expansion results in a proportionate increase in the price level which by PPP implies a proportionate depreciation of the exchange rate. This implies that bilateral exchange rates are determined as the ratio of the money stocks in the two countries and the rate of change of the exchange rate is given by the difference in the rates of monetary growth. More sophisticated versions of the monetary approach to the exchange rate can be developed incorporating features such as non-traded goods, interest sensitive money demand and productivity changes (c.f. Dornbusch and Krugman (1976)) but the basic causal factor in exchange rate determination remains monetary policy.

Several detailed critiques of the monetary approach exist c.f. Currie (1976) and Hahn (1977). Currie points out that the incorporation of additional financial assets into the model eliminates the simple identity between monetary expansion and the balance of payments. For example, a balance of payments deficit may have as its counterpart the issue of bonds by the government to finance its budget deficit rather than an excess supply of money with the private sector. Both authors argue that the monetary approach relies on the strong assumption of fixed output which renders it incapable of generating interesting short-run dynamics while its long-run properties are consistent with a whole range of less restrictive models. In addition to its theoretical shortcomings the monetary approach has also suffered the empirical falsification of one of the basic hypotheses underlying it in the form of the purchasing power parity hypothesis. Frenkel (1980) has documented the poor performance of this hypothesis since the return to floating exchange rates in 1973.

By the mid seventies therefore two approaches to modelling macroeconomic policy in an open economy had emerged. It would be wrong to view them as rivals (though they were viewed as such at the time) since they were designed to do different things. The Mundell-Fleming model was designed to explain short-run dynamics while the monetary approach was designed to explain long-run trends and the assumptions underlying the models reflect these purposes. Clearly, however, some unification of the theories was desirable if nothing else to reduce pointless arguments. Dornbusch (1976) achieved this by providing a model which has the theoretically desirable property that in the long-run a given increase in the money stock will result in an equiproportionate rise in prices and exchange rate depreciation but is nevertheless capable of generating short-run output dynamics. The Dornbusch model also marked a shift from the view that the exchange rate is determined by the relative prices of goods as in the monetary model towards the view that the international market in financial securities is of prime importance. The model is usually expressed as a set of log-linear equations as in (2:8)-(2:11).

(2:8) $y = u + \delta(e-p) - \delta r$ - IS curve

(2:9) $m - p = ky - \lambda r$ - LM curve

(2:10) $\dot{p} = \pi(y - \bar{y})$ - Phillips curve

(2:11) $\dot{e}^e = r - r^*$ - Uncovered Interest Parity

Lower case letters denote natural logarithms. All variables are as defined previously with the addition of u the shift parameter in the IS curve, p the price level and $\bar{y}$ the natural level of output. Equilibrium in the foreign exchange market is determined by the requirement that the yields on domestic and foreign assets be equal (2:11).

The Dornbusch model provides a significant theoretical advance on the flexible exchange rate Mundell-Fleming model by remedying two of the criticisms noted earlier. Firstly it allows prices to adjust in response to divergences from the natural output level. The sluggish nature of the adjustment gives rise to short-run output dynamics. Secondly it replaces the assumption of exogenous or static expectations concerning exchange rate movements with the requirement that expectations must be model-consistent or rational. In a deterministic framework this is equivalent to assuming perfect foresight on the part of foreign exchange speculators. This feature gives rise to overshooting of the exchange rate similar to that considered by Niehans. Consider a monetary expansion. In the long-run this requires an equiproportionate rise in prices and depreciation of the exchange rate. However, in the short-run prices are not free to move instantaneously implying an excess supply of real money balances and a rate of interest lower than that in the rest of the world.* This requires an expected appreciation to maintain the Uncovered Interest Parity condition. It is therefore necessary for the exchange rate to depreciate to a value below its new equilibrium and then to appreciate along the adjustment path.

* This conclusion is qualified somewhat if output is flexible since it may rise sufficiently to increase the rate of interest if the effect of the short-run exchange rate depreciation is powerful enough.

Since the publication of Dornbusch's 1976 paper a literature has grown up around the idea of exchange rate overshooting. A thorough discussion of this is almost a thesis in itself but it is worthwhile mentioning some significant contributions. Frenkel and Rodriguez (1982) showed that the existence of less than perfect capital mobility weakened the argument for overshooting. Wilson (1979) demonstrated that the expectations formation mechanism described by Dornbusch in which model consistency is imposed on a regressive expectations coefficient is not equivalent to rational expectations when policy shocks are anticipated. Fully rational expectations can imply movements in exchange rates which precede the actual implementation of policy. Buiter and Miller (1981) extended the Dornbusch model to incorporate steady-state inflation and used it to analyse recent British macroeconomic performance.

Thus the asset market approach to the exchange rate provides the closest thing yet to a unified theory of the macroeconomics of the small open economy. Relaxation of the perfect capital mobility assumption gives the current account some role to play in exchange rate determination without changing the basic insights c.f. Branson (1981) for a portfolio balance approach. Of course important questions remain unanswered. In particular the dynamics of wages and prices are badly specified. Therefore an important question becomes the extent to which expectations of inflation by wage bargainers can be considered rational. However, this issue will not be considered here. Instead section (2:3) moves on to consider the repercussion effects of policy when the economy described is large enough to affect world aggregates.

(2:3) The transmission of policy in an interdependent world

This section makes use of some simple two-country models to analyse the transmission of policy when each country is large enough to have a significant effect on the other. The discussion will be as follows. First the comparative static results for fiscal and monetary policy in the two-country Mundell model will be compared with those for the SOE case. This will be followed by an extension of the monetary model to the two country case as in Dornbusch (1973). Finally some models incorporating sticky prices will be discussed. Hamada and Sakurai (1978) analyse the case where capital is immobile while Miller (1982) emphasises this feature.

Mundell (1968) provides a two country model under the assumption of fixed prices. In this development of his model it is also assumed that capital is perfectly mobile. This simplifies the analysis by ensuring that the rate of interest is equal in the two countries. The IS curves can be written in the general form

(2:12) $Y = Y(r, G, Y^*, E)$ – Home country IS curve

$Y^* = Y^*(r, G^*, Y, E)$ – Foreign country IS curve

The form of the monetary equilibrium condition depends on the exchange rate regime. If exchange rates are fixed then each country's money stock is endogenous. Imagine a world in which each country creates an initial stock of money but either loses from, or adds to, this stock according to whether it runs a balance of payments deficit or surplus. Thus the available stock of money is distributed between the countries according to their demands.

The condition for monetary equilibrium is therefore simply that the available world stock of money equals the sum of the demands of individual countries.

$$(2.13) \qquad M + M^* = M(Y, r) + M^*(Y^*, r)$$

This gives three equations in three endogenous variables, the output levels and the rate of interest. To analyse the effects of policy note that (2:13) can be rewritten as

$$(2.13') \qquad r = r(Y, Y^*, M + M^*)$$

Thus it can immediately be stated that a monetary expansion by either country will raise output in both by pushing down the world rate of interest. The effects of fiscal policy are not so clear. Consider a domestic fiscal expansion. This unambiguously raises domestic output but may lower foreign output if the effects of the rise in the rate of interest outweigh the effects of the increase in Y.

If exchange rates are flexible then payments imbalances are assumed to be corrected by appreciation or depreciation. This gives each country monetary independence thus replacing the single monetary equilibrium condition (2:13) with the pair of conditions (2:14)

$$(2{:}14) \qquad M = M(Y, r)$$

$$M^* = M^*(Y^*, r)$$

This yields a system of four equations in four unknowns, the output levels, the rate of interest and the exchange rate. The effects of policy are as follows. A fiscal expansion by either country will raise output in both. A monetary expansion by one country, e.g. the home country, will increase output there but lower it elsewhere. This can be seen from the monetary equilibrium condition. With a fixed foreign money stock a decline in the world rate of interest due to a domestic monetary expansion must imply a decline in foreign output.

The large country results therefore contrast with the SOE results in that policy options which the SOE model predicts to be ineffective regain some power. In the fixed exchange rate case monetary policy is effective because each country supplies a significant proportion of the world money stock. Under flexible exchange rates fiscal policy regains some power because each country is able to engineer significant multiplier effects on the world level of output. The somewhat counterintuitive result that a domestic monetary expansion lowers output abroad under flexible rates arises because monetary policy works by causing a depreciation of the exchange rate of the expanding country which reduces demand elsewhere.

As in the SOE case, the fixed price model is useful for explaining short-run movements in output when prices or inflation rates are sticky. However, it has nothing to say about the behaviour of prices in response to deviations of output from its natural level. Dornbusch (1973) presents a two-country monetary model in which the emphasis is on the transmission of inflation via monetary policy. The assumptions for this model are the same as for the SOE monetary approach, i.e. output is fixed and nominal expenditure is proportional to the money stock. However, in this case each country can have a significant impact on both the price level and the rate of inflation, even under fixed exchange rates, by virtue of their size. The rate of hoarding for each country is defined as the difference between nominal income and expenditure or the rate of accumulation of foreign assets. In the absence of a fractional reserve system this also measures the rate of increase of the money stock. It can be written for the home country as

$$(2{:}15) \qquad \bar{\dot{M}} = P\bar{Y} - VM$$

Short-run equilibrium is defined by the condition that the home country's rate of hoarding must equal the foreign country's rate of dishoarding. Assuming purchasing power parity this can be written

$$(2{:}16) \qquad P\bar{Y} - VM = EV^*M^* - EP\bar{Y}^*$$

In a fixed exchange rate regime the price level is assumed to adjust to meet this condition. In the long-run the redistribution of the world money stock between the two countries eliminates any non-zero hoarding rate. The difference in this case from the SOE model is that monetary policy does influence prices even when PPP holds. Under flexible exchange rates the analysis is unchanged from the SOE case with monetary policy determining domestic prices and movements in the exchange rate being determined by differences in monetary policy.

The above model has sensible long-run features and provides useful insights into the transmission of inflation between countries. However, again the assumption of fixed output renders it incapable of explaining the short-run output dynamics which are the strength of

Keynesian type models. The only way of achieving this is to build in short-run price stickiness and to explain persistent deviations from the equilibrium or natural level of output it is usually necessary to build in some form of stickiness of expected price changes also. A model of this type has been put forward by Hamada and Sakurai (1978). This will be discussed at some length in chapter six, but it is worthwhile noting some important features at this stage. For the fixed exchange rate case the model is quite similar to the Dornbusch (1973) model in that disturbances are transmitted via balance of payments disequilibrium. However, both output and prices respond so a balance of payments surplus leads to inflation and increases in output. An additional channel of transmission is analysed in that consumers are assumed to regard the products of the two countries as imperfect substitutes implying that the terms of trade is a function of relative output levels. Thus, under flexible exchange rates, a foreign recession has inflationary consequences for the domestic economy.

Hamada and Sakurai's model marks a significant extension of the theory of international economic interdependence. However, like all models which assume sticky prices plus PPP it has difficulty explaining the sharp movements in exchange rates during the period of floating exchange rates since 1973. Miller (1982) puts forward a two country model capable of explaining this feature by concentrating on an asset market theory of exchange rates of the kind discussed in section (2:2). Basically, either a fiscal expansion or monetary contraction can result in an over appreciation of the currency of the initiating country. This is due to the increase in the rate of interest relative to the other country and the requirement that the exchange rate subsequently depreciates to maintain portfolio equilibrium in the foreign exchange market. This is similar to the overshooting hypothesis discussed previously. The advantage of the two country framework is that it emphasises that it is policy differences which lead to wild swings in exchange rates. Thus coordinated fiscal and monetary policies can be implemented without leading to instability in the foreign exchange market.

(2:4) Policy design in open economies

Much of the early literature concerning the design of policy for open economies concentrates on the conflict between internal and external balance. By this is meant the difficulty of simultaneously achieving a target level of output and balance of payments equilibrium. This definition of the problem was first put forward by Meade (1951). Using the analysis of Tinbergen (1956) the problem can be seen as one of finding at least as many policy instruments as targets.

Policy design is obviously crucially dependent on the model accepted by the authorities. Consider the small country monetary model with fixed exchange rates. In this case output is assumed fixed and prices are tied to world prices by the PPP relationship. The only variable which the authorities are capable of affecting is the balance of payments (by use of monetary policy to control nominal expenditure). Therefore no policy dilemma exists and the authorities simply set the money stock at whatever level necessary to generate the target payments balance. Similarly in the monetary model under flexible exchange rates output is fixed and the balance of payments is zero leaving the price level to be determined by monetary policy.

Unlike the monetary model the flexible output model does generate policy dilemmas, at least under fixed exchange rates. Expansionary policies increase output at the expense of a worsening trade balance. The obvious way to deal with this is to allow the exchange rate to move to correct the imbalance. This is described by Johnson (1961) as the use of expenditure changing policies to control output and expenditure shifting policies to control the balance of payments. However, under a system of fixed exchange rates such as that operating before 1973 it may not be possible to make the frequent adjustments necessary to pursue an effective policy. Mundell (1962) suggested an alternative choice of instruments if capital flows are sensitive to interest rates. In addition, Mundell suggested a way of assigning particular instruments to particular targets. The model can be summarized as follows. Suppose both output and the balance of payments depend on a fiscal instrument, say government spending and a monetary instrument, the rate of interest. This is written:

$$(2{:}17) \qquad Y = Y(\overset{+}{g}, \overset{-}{r})$$

$$(2{:}18) \qquad B = B(\overset{-}{g}, \overset{+}{r})$$

The Mundell assignment procedure states that fiscal policy should be used to control output and the rate of interest used to control the balance of payments if

$$\left| \frac{dr}{dg} \right|_{dY=0} > \left| \frac{dr}{dg} \right|_{dB=0}$$

This can alternatively be stated as requiring instruments to be associated with the targets on which they have relatively the greater effect. If adjustment takes place sequentially with each instrument moving to meet its assigned target in turn then this leads to stable convergence to the desired pair of values. The alternative assignment would lead to instability.

Policy design of this kind must be seen as inefficient since by simultaneous choice of instruments the authorities could .

move immediately to the desired levels of the target variables rather than by the iterative procedure described above. However, in a world of uncertainty where the authorities wish to make their intentions clear to the private sector there may be offsetting advantages in being able to announce simple reaction functions in terms of one variable and one target. This is discussed by Vines, Maciejowski and Meade (1983) who refer to the alternative simultaneous choice of instruments as cross-linked Keynesianism.

An alternative way of expressing the above discussion can be obtained by anticipating the analysis of chapter three to note that the optimal decision rule when a controller has a quadratic objective function and linear constraint set can be written as

$$(2{:}19) \qquad u = G(x - x_a)$$

where u is vector of controls, x a vector of states and x_a a vector of targets. Assuming the number of controls to be equal to the number of targets the Mundell assignment method simply involves restricting the matrix G to be diagonal. Note also that the choice of policy by decentralised authorities in cases such as this can be modelled as a non-cooperative game c.f. Kydland (1975).

During the Bretton Woods period fixed exchange rates came to be seen as a severe constraint on macroeconomic policy making. From a Keynesian point of view this arose because of the balance of payments deficits arising from expansionary policies. Even if these could be financed by a capital inflow attracted by high rates of interest it seemed undesirable to have foreign claims against the economy build up indefinitely. From a monetarist point of view the constraint was that a small economy must always accept an exogenously determined rate of inflation. Therefore models with very different theoretical bases gave the similar normative policy conclusion that a move to flexible exchange rates was desirable. It was hoped that this would give greater autonomy in policy design by reducing the extent to which policy disturbances were transmitted abroad. Unfortunately the emergence of transmission mechanisms not incorporated in the popular theories of the time have led to the frustration of this hope. For example, the simple Mundell-Fleming model predicts that a small economy will be insulated from fluctuations in export demand by movements in the real exchange rate. However, this result does not hold if there is real wage resistance by wage bargainers.

Given that flexible exchange rates do not, except under very special assumptions, afford complete insulation against external shocks a reasonable strategy might be to design a combination of monetary and exchange rate rules to minimise the effects of such disturbances. This approach is adopted by Artis and Currie (1981) who conclude that the best 'regime' depends on what shocks are of most importance for the economy in question. Currie and Levine (1983) extend this idea to dynamic models and consider the design of simple policy feedback rules which have good operating characteristics under a variety of stochastic disturbances.

The extension of policy design methods to include dynamic problems has been made necessary by the increasing recognition of the importance of dynamics in macroeconomic problems. Work of this kind for open economies has been carried out by Driffill (1982). This uses a Buiter-Miller type model to consider the optimal design of monetary policy to control inflation. Due to the complex nature of the problem numerical simulations are used to examine the solution. More discussion of this method will follow in later chapters.

Most of the existing literature on policy design for open economies has concentrated on the SOE case. However, a small literature has developed concerning policy design in models of interdependent economies. This has made use of game theoretic methods to analyse cooperative and non-cooperative behaviour c.f. Hamada (1974, 1975 and 1976).

(2:5) Conclusions

The purpose of this chapter has been to survey the literature on the transmission of economic policy between countries. Rather than give a blow by blow chronological account of the various controversies which have arisen the approach has been to use simple stylised models to capture the essential features of particular theoretical standpoints. It has also been argued that some sort of synthesis has been achieved between the very different Keynesian and monetary models in that modern models incorporate long-run monetarist features while retaining the short-run Keynesian output dynamics.

It must be recognised that a number of important features of the macroeconomic models used have not been adequately discussed. In particular no real justification for the assumption of price stickiness has been given. Such a justification would involve appeal to such institutional features as long-term contracts and would require too much space in a thesis concentrating on open economy aspects of macroeconomics. Similarly a thorough discussion of alternative expectations formation mechanisms has not been provided.

Future chapters go on to consider policy optimisation problems using particular open-economy models. It is hoped that this chapter will help show how these models fit into what has become an enormous literature.

Chapter Three

A static framework for the analysis of policy optimisation with interdependent economies

Chapter Three

3:1 Introduction

The use of optimal policy design methods was a natural consequence of the development of econometric models of the macro-economy. Since the theoretical framework underlying these models was the comparative static 1S/LM model the policy dilemmas facing the authorities were thought of as trade-offs between the values taken by variables at a point in time. Standard optimisation methods were therefore sufficiently general to capture the problem and there was no need to employ dynamic optimisation techniques. Applications of this kind can be seen in Tinbergen (1956). Since this time macroeconomic theory and macroeconometric models have come to concentrate much more on dynamic adjustment processes rather than comparative static equilibria. Thus the policy choices facing the authorities are now seen in terms of alternative adjustment paths to a steady-state rather than alternative comparative static equilibria. This has required more sophisticated optimisation techniques such as dynamic programming or Pontryagin methods (cf chapter four for discussion of these).

As well as the increasing concentration on dynamics in macroeconomic analysis it has also been argued that macroeconomic model-builders should recognise that the structures they observe are not policy invariant but result from the interaction of optimising agents. This point is put forcefully by Lucas (1976). The Lucas critique has implications for economic theory as well as econometric model-building. In particular when modelling the optimal policy choices of, for example, the national monetary authorities account needs to be taken of the effects on the behaviour of other agents sensitive to these choices. For this example it is natural to think of the private sector and the monetary authorities of other countries as being affected. This kind of interdependence in decision making can be handled analytically by the application of game theoretic methods. Numerous examples of models in which optimal policy is modelled as a game between the government and the private sector can be found in the literature, for recent examples c.f. Barro and Gordon (1983) and Backus and Driffill (1984(a) and 1984(b)). In contrast this thesis concentrates mainly on the interdependency of decision making between separate national policy authorities taking as given the structure of decision making within the economies. Future chapters concentrate on dynamic problems using differential game methods. However, the technically demanding nature of these solutions often obscures the insights which the simpler static or 'one-shot' games have to offer. Indeed, models in which a static games are repeated through time are often capable of capturing more sophisticated strategic behaviour by players. This chapter therefore makes use of one-shot game methods to consider a number of international economic policy issues both in the hope that this will make the differential game models considered later more accessible and because they are of considerable interest in themselves.

In a series of pioneering papers Hamada (1974, 1975 and 1976) applied game theoretic methods to models of interacting economies in which monetary policy influenced the balance of payments and output. This chapter builds on foundations laid in these papers but gives a more rigorous derivation of policy reaction functions and considers some important new issues. These include the problem of cheating in Stackelberg leader games and the sustainability of cooperative equilibria in repeated games. To conform with the static structure of the game the economic models considered here are restricted to those capable of a comparative static representation. Thus frequent use is made of the Mundell (1968) two country model. Despite this restrictiveness a number of interesting results are derived.

The plan of the chapter is as follows. Section two discusses the solution concepts of interest. Section three applies these to the two country Mundell model assuming that the authorities' loss functions are quadratic in the state variables. Section four considers the alternative specification of parabolic loss functions and section five contains conclusions.

3:2 Solutions for one shot linear quadratic games

The class of games considered in this section are referred to as linear-quadratic because they assume minimisation of a quadratic loss function subject to a linear constraint set. This has proved a very popular specification of the policy optimisation problem because it has the property that Theil (1958) terms certainty equivalence. This means that if a Gaussian disturbance is added to the constraint set then the problem can be solved by setting this equal to its expected value of zero. Thus a stochastic problem can be treated as if it were deterministic. However, this does not imply that the quadratic objective function is always the most desirable on theoretical grounds and so section four of this chapter considers the parabolic loss function as an alternative.

Before going on to consider multi-player games it is worth looking at the single controller or 'game against nature' case. Here the controller is assumed to have a loss function of the form

$$(3:1) \qquad \frac{1}{2}\left[(x-x^a)^T Q(x-x^a) + u^T Ru\right]$$

where x is a vector of states on which the controller's preferences are defined, x^a is a vector of targets or desired values of these states, u is a vector of instruments available to the controller and Q and R are symmetric, positive definite weighting matrices. The states are related to the instruments by a set of reduced form equations such as (3:2)

$$(3:2) \qquad x = Bu$$

where B is an nxm matrix where n is the number of states and m the number of controls.

Tinbergen defines a system as controllable if all targets can be satisfied simultaneously. It is straightforward to show that this requires there to be at least as many instruments as targets, i.e. $m \geq n$. Unfortunately if the controller attaches costs to the instruments themselves then this condition can never be satisfied. However, this does not imply the non-existence of an optimal vector of controls, merely that the controller must trade off deviations of states from their desired values. Thus most systems of interest for a macroeconomic policy maker are likely to be uncontrollable by the Tinbergen definition but this simply implies that the authorities will be conducting the sort of balancing act between competing targets which is normally seen as their role.

Setting up the Lagrangean and minimising with respect to the controls gives

$$(3:3) \qquad u = -R^{-1}B^{T}p$$

where p is the vector of shadow prices on the states. This can alternatively be expressed in terms of the targets.

$$(3:4) \qquad u = [I + R^{-1}B^{T}QB]^{-1}[R^{-1}B^{T}Qx^{a}]$$

The certainty equivalence property states that this will also be the solution to the stochastic problem in which there is an additive disturbance term in the state equation (3:2). If control is costless then the R^{-1} matrix is undefined. However, a solution can be shown to exist by use of a limiting argument. (3:4) can be rewritten

$$(3{:}4') \qquad u = \left[R + B^T Q B\right]^{-1} \left[B^T Q x^a\right]$$

Taking limits as the elements of R tend to zero yields

$$(3{:}4'') \qquad u = \left[B^T Q B\right]^{-1} B^T Q x^a$$

It is possible for a system such as this to be controllable in the Tinbergen sense if $m \geq n$. Then there is no unique choice of optimal control vector since any subset of at least n controls could be used to achieve the targets. However this non-uniqueness problem will never arise if control is costly.

The case of multiple controllers changes the state vector to

$$(3{:}5) \qquad x = \sum_{i=1}^{N} B_i u_i$$

where N is the number of controllers or players. Each player is assumed to have a loss function of the form

$$(3{:}6) \qquad \frac{1}{2} \left[(x-x_i{}^a)^T Q_i (x-x_i{}^a) + u_i{}^T R_i u_i\right] \qquad i = 1,\ldots,N$$

Consider the two player game. By substituting the state equation into player one's loss function an indirect loss function in terms of the controls of the two players can be written

$$(3{:}7) \qquad \frac{1}{2} \left[u_1{}^T (B_1{}^T Q_1 B_1 + R_1)\, u_1 + u_2{}^T B_2{}^T Q_1 B_2 u_2 + 2u_1{}^T B_1{}^T Q_1 B_2 u_2 \right.$$

$$\left. + x_1{}^{aT} Q_1 x_1{}^a - 2u_1{}^T B_1{}^T Q_1 x_1{}^a - 2u_2{}^T B_2{}^T Q_1 x_1{}^a\right]$$

By symmetry a similar function can be derived for player two. If u_1 and u_2 are scalars then the loss contours can be drawn as concentric ellipses in u_1, u_2 space defined around a central point where all targets are satisfied simultaneously.

To determine his optimal strategy each player must form some expectation about the response of other players to his strategy. In the Industrial Economics literature this is referred to as the conjectural variations assumption. The simplest possible assumption is that of zero response or the Nash assumption. Under this assumption each player chooses a loss minimising strategy on the basis of other players controls being fixed. There is Nash equilibrium when all players choose strategies which satisfy the assumptions of others. Consider the simplest possible game in which there are two players with one instrument each. The optimal choice of strategy for each player can be shown diagrammatically by the tangency of one of his own loss contours to the assumed fixed strategy line of the other player. By varying the strategy of the other player the best response strategy locus or Nash reaction function can be traced out. Nash equilibrium occurs when each player is simultaneously on his Nash reaction function. Such an equilibrium is illustrated in figure one.

Figure One

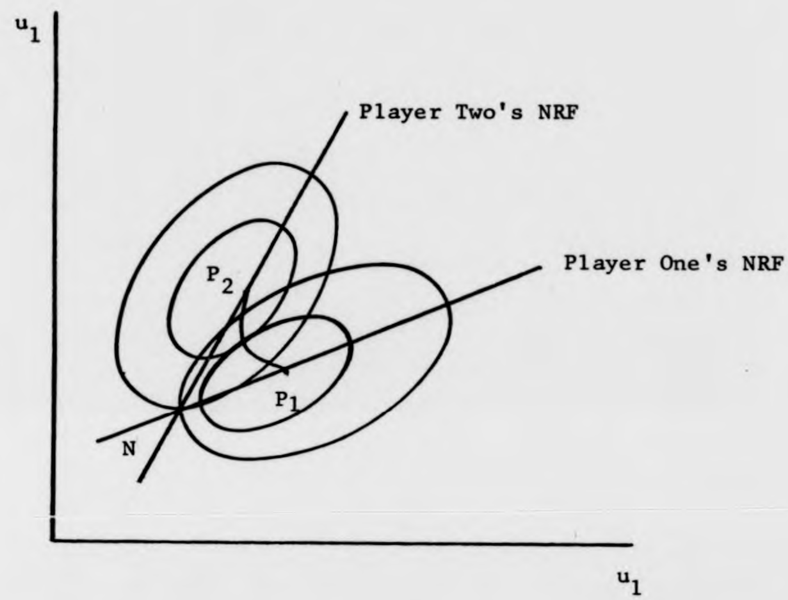
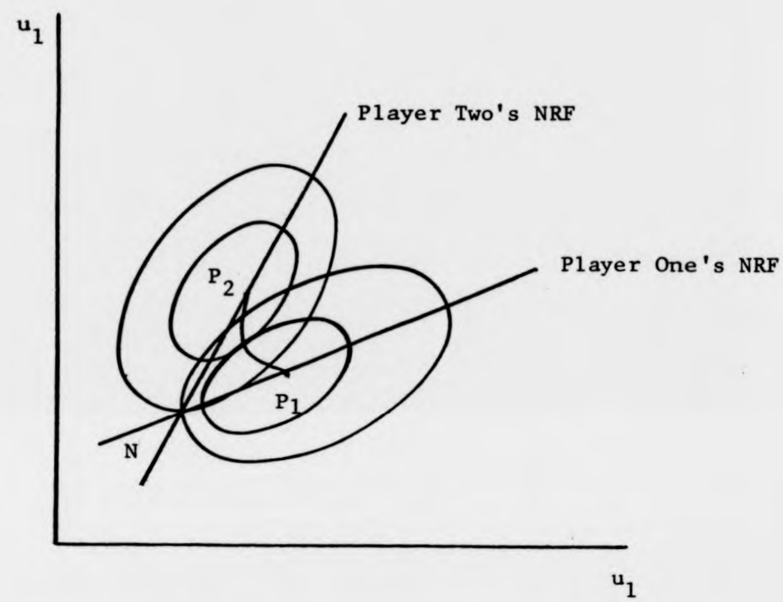


Figure One



P_1 and P_2 are the central or 'bliss' points for player one and two respectively. If their targets are mutually compatible then these points coincide. Both Nash reaction functions pass through this point and the Nash equilibrium is Pareto Efficient.

If the players' targets are not mutually compatible then the Nash equilibrium will not be Pareto Efficient. This is because each player imposes an externality on the other when choosing his optimal strategy. Under a system of decentralised control this leads to a sub-optimal outcome.

In algebraic terms the Nash reaction functions can be derived by differentiating the appropriate Lagrangean with respect to the control vector and substituting for the states to obtain.

$$(3{:}8) \qquad u_i = \left[I + R_i^{-1} B_i^T Q_i B_i\right]^{-1} \left[-R^{-1} B_i^T Q_i \left(\sum_{j \neq i} B_j u_j\right) + R_i^{-1} B_i^T Q_i x_i^a\right]$$

i.e. each player's optimal control vector is a function of the control vectors of the other players and his own targets. The Nash equilibrium corresponds to the intersection of all Nash reaction functions and can be derived as a function of players' preferences as defined by their weighting matrices and their targets.

$$(3{:}9) \qquad x = \left[I + \sum_i B_i R_i^{-1} B_i^T Q_i B_i\right]^{-1} \left[\sum_i B_i R_i^{-1} B_i^T Q_i x_i^a\right]$$

Pareto efficient solutions can be derived by assuming a single controller whose loss function is a weighted average of the individual players loss functions, i.e.

$$(3:10) \qquad u_i = -w_i^{-1}R_i^{-1}B_i^T\Sigma w_jQ_j(x-x_i^a)$$

This means that in choosing the controls previously allocated to individual players the Pareto Efficient controller takes into account the preferences of all the players in the game. By varying the weights w_i the controller can locate the solution anywhere along the locus of tangencies of loss contours shown as P_1P_2 in figure one.

For many years the most popular application of game theory in economics was to the oligopoly problem. Different Nash solutions were derived depending on whether strategies were defined in terms of output (Cournot) or price (Bertrand). The sub-optimality of Nash solutions was argued to be the reason for the formation of cartels or the emergence of tacit collusion. At the same time there was dissatisfaction with the apparent naivity of the Nash solution and attempts were made to build models incorporating more sophisticated conjectural variations assumptions. One of the most popular of these has been the Stackelberg leader model usually applied to an industry consisting of a dominant firm and a number of smaller firms. The smaller firms treat the dominant firm's strategy as parametric but it incorporates their responses into its calculations. The key feature of Stackelberg leader models is the division of the players into two groups, the leader(s) (dominant firm) and the followers (the fringe group).

In terms of the linear quadratic game the Stackelberg leader model for the two player game can be derived by first defining the follower's reaction function as

(3:12) $u_2 = G_2x_2^a - G_2B_1u_1 \quad G_2 = [I + R_2^{-1}B_2^TQ_2B_2]^{-1}R_2^{-1}B_2^TQ_2$

where player two is the follower and is assumed to behave according to Nash assumptions. The state equation facing the leader can now be redefined as

(3:13) $x = (B_1 - B_2G_2B_1)u_1 + B_2G_2x_2^a$

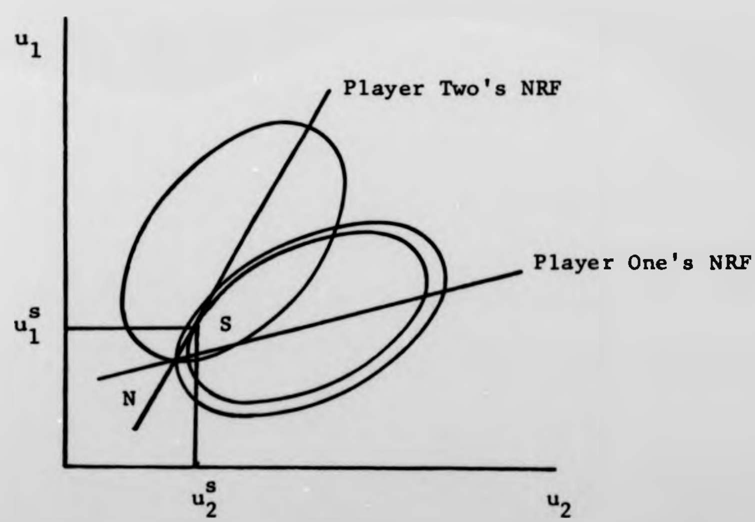
The leader's optimal control vector can now be derived by minimisation of his loss function subject to the redefined constraint set (3:13). This yields

(3:14) $u_1 = -R_1^{-1}[B_1^T - (B_2G_2B_1)^T]p_1$

i.e. the leader's optimal strategy takes into account the follower's reaction function G_2.

For the single instrument case the Stackelberg solution can be shown diagrammatically as the tangency of one of player one's loss contours to player two's Nash reaction function c.f. figure two.

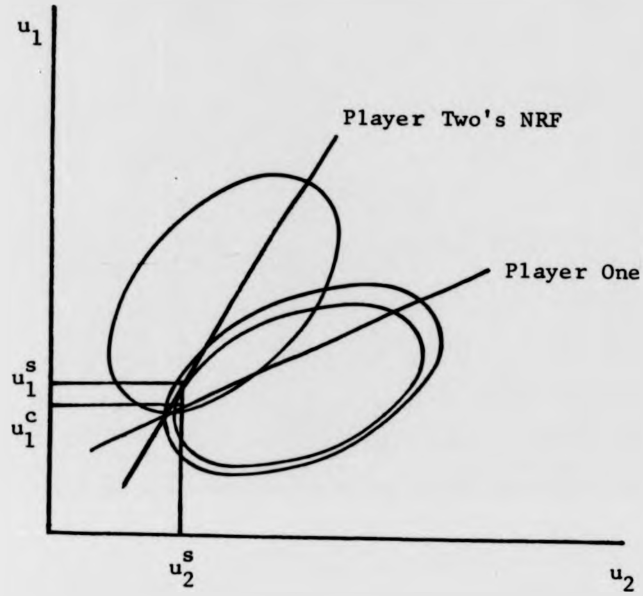
Figure Two



The Stackelberg solution requires one player to be dominant in some sense though it is often unclear what factors give it this dominant role. Because of this the most popular application of this solution concept in macroeconomics has been to the analysis of optimal government policy with government as leader and the private sector as follower. Another application suggested by Kydland (1975) is to the case where decisions are taken by separate policy making institutions in a fixed sequence. For example, the US Congress acquires a leadership role relative to the Federal Reserve simply because the sequence in which decisions are taken enables it to announce its strategy first.

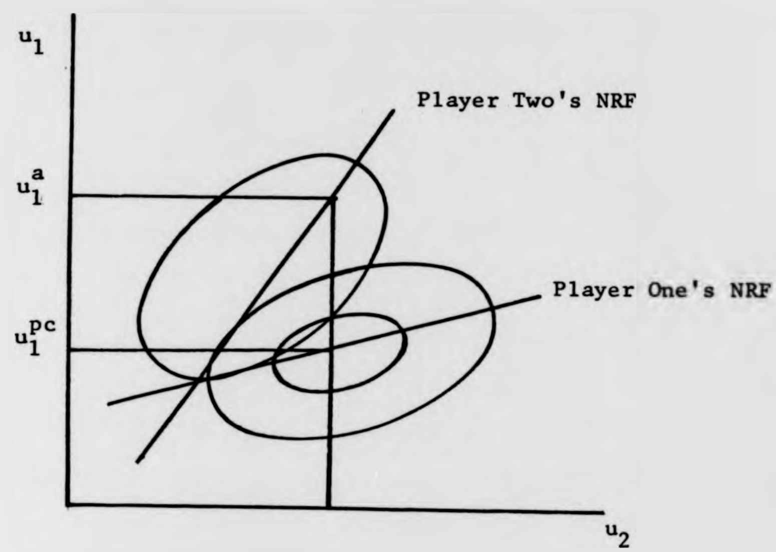
Stackelberg leader models, although superficially more sophisticated than Nash, run into logical problems when the actual sequence of decision and implementation is considered. In particular strong incentives usually arise for the leader to renege on pre-announced strategies. Hamaleinen (1981) describes this as the problem of cheating. Consider a game divided into two periods, a pre-play period during which the players decide and announce their optimal strategies and a period in which they actually implement their strategies. If the game has a leader-follower structure then the leader carries out the optimisation problem defined by his loss function and the constraint set (3:13) and announces the stragegy u_1^S. The follower then chooses an optimal strategy u_2^S based on this. Now in the implementation period the leader's optimal strategy is no longer u_1^S since his constraint set is not determined by the reaction function (3:12) but by the fixed control vector u_2^S. The leader always has an incentive to abandon his announced strategy and choose an alternative on his own Nash reaction function, i.e. u_1^C in figure three.

Figure Three



Hamaleinen calls this two-stage cheating. It involves a certain degree of myopia on the part of the leader since he does not anticipate and try to exploit the fact that the follower's strategy is based on his announcements rather than his actual choice. By making use of this the leader can improve his welfare still further. Indeed if the reaction functions are linear and the choice of control can lie anywhere along the real line, then by a suitable announcement the leader can attain his central or 'bliss' point. Hamaleinen calls this perfect cheating. The effect of it is to give the leader control of the follower's strategy as well as his own. Figure four illustrates such a solution where u_1^a is the leader's announced strategy and u_1^{PC} is the strategy he actually plays.

Figure Four



For a single one-shot game the Stackelberg solution therefore begins to look a little implausible. However, one argument is that it describes an essentially dynamic problem within a static context. If the game is repeated then the leader has an incentive to stick to his announced strategy despite the potential gains from cheating. This is because the erosion of his credibility over time due to cheating will lead to the collapse of the game to the Nash solution and the loss of the leadership benefits in future periods. The incentives to establish a reputation for sticking to announcements are discussed by Ulph (1984) in the context of a multi-period union-employer wage bargaining game.

The Stackelberg solution therefore makes more sense in a repeated game model. In this setting it can be viewed as a simple allocation of roles to players to achieve a Pareto improvement on the Nash outcome. However, the one-shot Stackelberg solution by no means guarantees such an improvement as the model considered in Section 3:3 demonstrates.

In a repeated game setting the Stackelberg solution which generates a Pareto improvement is one possible equilibrium for an infinitely repeated game. However, apart from games with a very special structure it is by no means the only one. Indeed, what has become known as the folk theorem of repeated games states that any choice of strategies which generates a Pareto improvement on the one-shot Nash solution can be sustained as an equilibrium. Since the Stackelberg solution is not in general Pareto efficient there exist sustainable equilibria which dominate it.

The multiplicity of equilibria in infinitely repeated games has naturally lead game theorists to look for plausible restrictions on the structure of the game which would reduce this number, preferably to some unique equilibrium. Lockwood (1983) introduces discounting into the players' welfare functions but while this shrinks the set of possible equilibria it is not sufficient to generate a unique outcome. Another approach has been to impose a finite time horizon on the game. This succeeds in generating a unique equilibrium but in a somewhat undesirable manner.

In order to discuss repeated games with a finite time horizon it is first necessary to define the idea of a perfect equilibrium. Basically this is an equilibrium sustainable by credible threats. A threat is a conditional strategy of the form "if you choose action A then 1 will respond with action B . Such a conditional strategy is credible if in the event of the other player choosing A the optimal response action is B. In a repeated game there is said to be sub-game perfection if the strategies adopted by the players satisfy the perfectness criterion at every stage or taking as given the previous history of the game. Perfectness results in the collapse of the game to a repetition of the one-shot Nash solution in the case of repeated games of full information with a finite time horizon. Selten (1975) refers to this result as the 'chain-store paradox'. Demonstration of this result is very simple. Consider the last period of a finite game. Past history is given and there is no future therefore rationality requires the one-shot Nash outcome to be the solution. Given this, then, in the last but one period there can be no incentive to play anything other than the one-shot Nash strategy in the hope that the other player will respond by maintaining a tacitly collusive strategy in the next period. Solving the game recursively leads to the conclusion that there are no credible threats which can sustain an equilibrium other than repeated one-shot Nash.

The imposition of a finite time horizon coupled with the assumption of full information therefore succeeds in generating a single equilibrium. However, this is at the cost of rejecting all the Pareto superior outcomes which intuition suggests agents will attempt to realise. Recent developments in repeated game theory have involved attempts to allow for equilibria other than one-shot Nash whilemaintaining the assumption of a finite time horizon. The basic way in which this is done is by relaxation of the full information assumption c.f. Kreps and Wilson (1982 a and b) and Kreps, Milgrom, Roberts and Wilson (1982). Further discussion of these developments will be given in relation to the specific economic model considered in section 3:3.

To conclude this section it is worth summarising the main results. In the one-shot game without cooperation the Nash equilibrium is the natural solution since it is the only equilibrium which has the property that all players are simultaneously playing their best response strategies. Stackelberg leader solutions are sustainable in repeated games if they generate a Pareto improvement on Nash but there is an infinity of potential solutions. This infinity of sustainable equilibria can be reduced to a single possible equilibrium in a repeated game with a finite time horizon by the chain store paradox argument.

(3:3) A simple two-country model

This section uses the two-country Mundell model to analyse optimal policy in an interdependent world. The exchange rate and prices are assumed to be fixed throughout. Alternative specifications of the model are used to consider fiscal and monetary policy. For the fiscal policy case the interest rate plays no role in the model. This can be rationalised by either assuming that aggregate demand is interest insensitive and there is no capital mobility or that these effects are present but are prevented from having any effect because of a passive government monetary policy which stabilises interest rates. The two countries are referred to as the home and foreign countries with foreign variables being indicated by an asterisk. Under the above assumptions aggregate demand in the two economies can be written

$$(3{:}15) \qquad \begin{aligned} Y &= cY + g + mY^* - mY \\ Y^* &= cY^* + g^* + mY - mY^* \end{aligned}$$

For simplicity the economies are assumed to be structurally similar.

The preferences of the policy authorities are assumed to be captured by a quadratic loss function defined in terms of deviations of output and the balance of payments from target values and the level of government spending.

$$(3{:}16) \qquad L = \frac{1}{2}(Y-Y^a)^2 + \frac{\beta}{2}(B-B^a)^2 + \frac{\theta}{2}g^2$$

with a similar loss function being defined for the foreign country.

The determination of output levels and the balance of payments can be represented as a reduced form consisting of a set of linear simultaneous equations

$$(3:17) \qquad \begin{bmatrix} Y \\ Y^* \\ B \end{bmatrix} = \begin{bmatrix} \alpha_1 & \alpha_2 \\ \alpha_2 & \alpha_1 \\ m(\alpha_2-\alpha_1) & -m(\alpha_2-\alpha_1) \end{bmatrix} \begin{bmatrix} g \\ g^* \end{bmatrix}$$

$$\alpha_1 = \frac{s + m}{\Delta}, \quad \alpha_2 = \frac{m}{\Delta}, \quad \Delta = s(s + 2m), \quad B = -B^* = m(Y^*-Y)$$

The problem is one of minimising (3:16) subject to the constraint set (3:17). Making the Nash assumption of zero conjectural variations enables the Nash reaction function for the home country to be derived by differentiating (3:16) with respect to g subject to (3:17). This yields

$$(3:18) \qquad g = \frac{-(\alpha_1\alpha_2 - \beta m^2(\alpha_2-\alpha_1)^2)}{\theta+\alpha_1^2+\beta m^2(\alpha_2-\alpha_1)^2} g^* + \frac{\alpha_1}{\theta+\alpha_1^2+\beta m^2(\alpha_2-\alpha_1)^2} Y^a$$

$$\frac{-\beta m(\alpha_2-\alpha_1)}{\theta+\alpha_1^2+\beta m^2(\alpha_2-\alpha_1)^2} B^a$$

The reaction function is therefore linear in the other country's strategy. Government spending increases with the size of the output target and decreases with the balance of payments target.

The slope of the reaction function can be either positive or negative according to

$$\beta \gtrless \frac{\alpha_1\alpha_2}{m^2(\alpha_2-\alpha_1)^2} = \frac{s+m}{ms^2}$$

Therefore it is more likely to be upward sloping.

(1) The greater the weight put on the balance of payments target.

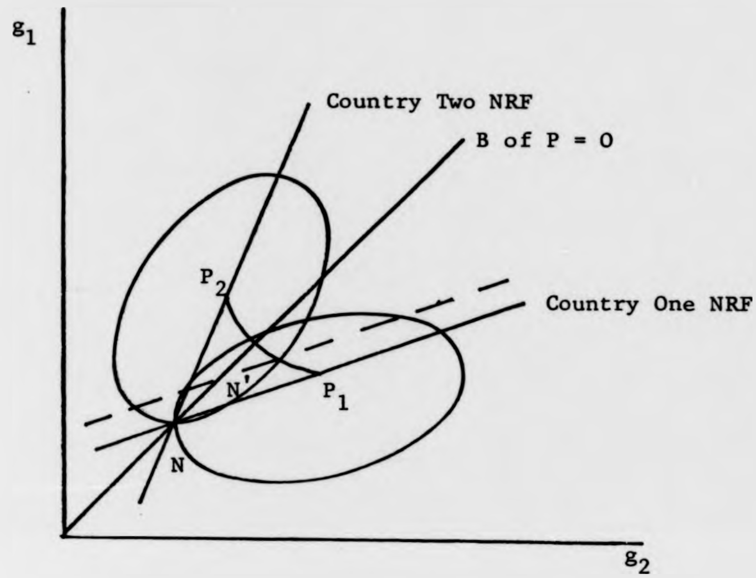
(2) The more open the economy is, as measured by the marginal propensity to import

(3) The larger the marginal propensity to save.

These conditions can be given some intuitive justification. Consider a cut in foreign government expenditure, this reduces home output and worsens the home balance of payments. If there is a large weight on the balance of payments it is more likely that the policy authorities in the home country will respond by cutting expenditure to defend the balance of payments rather than increasing it to defend the output level. Similarly if the marginal propensity to save is low and the marginal propensity to import is high this implies a small government expenditure multiplier. A rise in spending therefore has a small effect on output and a large effect on the balance of payments making it likely that the domestic economy will respond to a foreign contraction by contracting itself.

The Nash equilibrium can be depicted diagrammatically with government expenditure levels on the axes.

Figure Five



Under the assumption of symmetry the balance of payments equilibrium locus is simply the 45° line. If both countries desire a balance of payments surplus then the home country's ideal point lies below the 45° line and the foreign country's lies above it. This implies that the reaction functions intersect below the Pareto Efficient locus, i.e. decentralised policy optimisation gives a deflationary bias to the world economy. If each country has symmetric targets, i.e. same balance of payments surplus and output targets, then the Nash equilibrium will correspond to balance of payments equilibrium but at a lower than optimal output level (point N in the diagram). If the home country has either a larger output or smaller payments surplus target than the foreign country then this results in a parallel shift upwards of its reaction function as shown by the broken line and an equilibrium such as N' with a balance of payments deficit.

If both countries desired a balance of payments deficit then the home country's ideal point would be above the 45° line and the foreign country's below it.* This means that the reaction functions intersect above the Pareto Efficient locus therefore giving an over-expansionary bias to the world economy.

It can be shown that similar equilibria can be derived with monetary policy as the choice variable of the authorities. Aggregate demand is redefined to be a function of the rate of interest.

(3 19) $$Y = cY - br + g + m(Y^{*}-Y)$$

$$Y^{*} = cY^{*} - br^{*} + g^{*} + m(Y-Y^{*})$$

Financial arbitrage ensures that the rate of interest is equal in the two countries i.e. $r = r^{*}$. The balance of payments is defined as the change in foreign exchange reserves which is equal to the difference between the demand for money and the level of the domestic credit base made available by the authorities, i.e.

(3:20) $$B = M_d - C = KY - \lambda r - C$$

assuming a demand for money function which is linear in output and the rate of interest. Since the authorities are assumed not to pursue an active fiscal policy setting $g = g^{*} = 0$ enables the reduced form of the system to be written

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* This may seem an implausible case but Appendix A outlines a simple model in which a balance of payments deficit target can be justified as the result of the desire of government to maximise the welfare of the representative individual in the economy.

$$(3:21)\qquad \begin{bmatrix} Y \\ Y^* \\ B \end{bmatrix} \begin{bmatrix} \alpha & \alpha \\ \alpha & \alpha \\ -1/2 & 1/2 \end{bmatrix} \begin{bmatrix} C \\ C^* \end{bmatrix}$$

The loss function is that given in (3:16) with C replacing g as the policy instrument. Differentiating with respect to C subject to the constraint set (3:21) enables the reaction function for the home country to be derived as

$$(3:22)\qquad C = \frac{\beta/4-\alpha_1^2}{\theta+\alpha_1^2+\beta/4} C^* + \frac{\alpha_1}{\theta+\alpha_1^2+\beta/4} Y^a - \frac{\beta/2}{\theta+\alpha_1^2+\beta/4} B^a$$

This has positive slope if

$$\beta > 4\alpha_1^2$$

This condition can be given a similar explanation to that for fiscal policy. With β large if the foreign country cuts its credit base then the home country is more likely to cut its own credit base to defend the Balance of Payments rather than increase it to defend the output level. Similarly the smaller α_1 is the less the increase in output relative to the decline in the balance of payments associated with an expansion of credit.

The comparative static results also carry through from the fiscal policy. If both countries have balance of payments surplus targets then the Nash equilibrium has a deflationary bias, if they have deficit targets then it has an inflationary bias. If one country has a higher output/lower payments surplus target then it runs a balance of payments deficit. It is only in the case where targets are mutually compatible that the Nash equilibrium is Pareto Efficient.

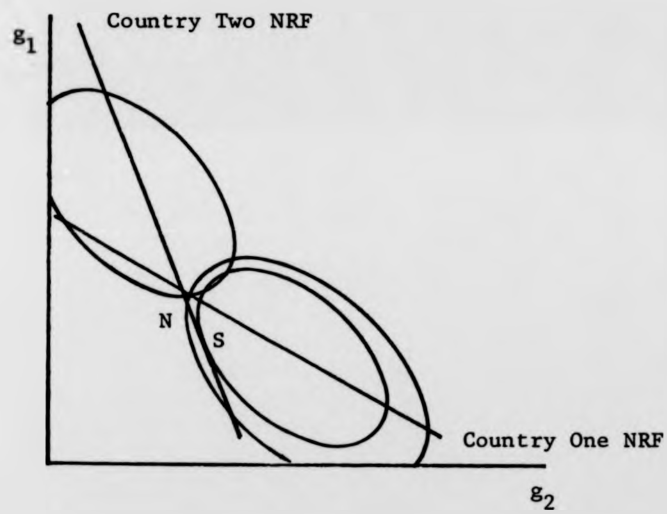
So far the discussion has been purely in terms of one-shot games. If the game is repeated then tacit agreement to limit the inefficiencies due to uncoordinated behaviour may emerge. However, the analysis of section (3:2) suggested that the scope for this is limited by existence of a finite time horizon. In the case considered here the policy makers are governments which have to seek periodic re-election. Thus cooperation may break down as individual governments adopt Nash behaviour in an attempt to maximise their chances of re-election via the short-run benefits available from succesful cheating.

Given the difficulties of maintaining a tacit cooperative solution the Stackelberg leader solution may well provide a means of improving on the Nash outcome by means of a simple allocation of roles to the players. It has the advantage that the follower or followers play Nash strategies which do not involve any incentive to cheat during the course of the game. The leader of course does have an incentive to cheat but if he can somehow be restrained from doing so then the Stackelberg solution becomes a possibility.

However, there is no guarantee that the Stackelberg solution will generate a Pareto improvement. Whether it does or not depends crucially on the slope of the reaction functions. Consider the case where both countries derive a balance of payments surplus and the reaction functions are upward sloping. The Stackelberg solution for such a game can be portrayed in a diagram similar to figure three. This solution corresponds to a loss contour for the leader which is interior to the Nash contour by definition. Since the follower's reaction function is upward sloping and there is a deflationary bias to the Nash equilibrium it follows that the follower's welfare must improve as he moves up his reaction function in response to the expansionary policy of the leader.

Consider now the case in which the reaction functions are downward sloping. The Stackelberg solution for such a case is shown in figure six.

Figure Six



Since the leader must choose a point on the follower's reaction function which is interior to his own Nash contour and the reaction function slopes downwards through the Nash point it follows that the Stackelberg equilibrium must correspond to a loss contour inferior to the Nash solution for the follower. Thus the leader-follower game structure improves the welfare of the leader but reduces that of the follower when the reaction functions are downward sloping.

The results for the Stackelberg leader model for the case where countries have balance of payments surplus targets can therefore be summarised as follows. Starting from a symmetric Nash position, with upward sloping reaction functions the leader pursues a more expansionary policy which induces the follower to do likewise. The equilibrium has higher output for both countries but a payments deficit for the leader, there is a Pareto improvement in welfare. If the reaction functions are downward sloping the leader pursues a contractionary policy relative to Nash. This induces an expansionary policy by the follower. The leader runs a balance of payments surplus but both output levels fall. Welfare improves for the leader but deteriorates for the follower. Therefore even if the leader-follower structure was to be imposed on the game there is no reason to believe that it would result in a Pareto improvement in welfare.

3:4 Parabolic loss functions

In section 3:3 the preferences of the policy authorities were assumed to be described by a quadratic loss function. Although this has been a common assumption in the literature on optimal goverment policy it may have certain undesirable features. In particular the implication that positive and negative deviations from target values are weighted equally is questionable. For instance, in the Mundell model considered in section 3:3 it is likely that a given positive deviation of the balance of payments from its target value will be given less weight than the equivalent negative deviation. Therefore this chapter considers an alternative form of the objective function which is quadratic in some variables and linear in others. This is referred to as a parabolic loss function.

Consider the case where deviations from the target value of income enter the loss function quadratically but the balance of payments term enters linearly. This implies a loss function of the form

$$(3:23) \qquad L = \frac{1}{2}(Y-Y^a)^2 - \beta(B-B_a) + \frac{\theta}{2}u^2$$

where u is either government expenditure or domestic credit. In the case of fiscal policy the constraint set is assumed to be (3:17). The first order condition for the minimisation of (3:23) with respect to g and subject to (3:17) yields the home country's reaction function

$$(3{:}24) \qquad g = \frac{-\alpha_1\alpha_2}{(\alpha_1^{\,2}+\theta)} g^* + \frac{\alpha_1}{(\alpha_1^{\,2}+\theta)} Y^a - \frac{\beta m(\alpha_1-\alpha_2)}{(\alpha_1^{\,2}+\theta)}$$

The slope is always negative and less than one in absolute value in contrast to the quadratic loss function case in which the sign of the slope is ambiguous. There is therefore no possibility of improving welfare by allocating the role of leader to one of the countries in this case. The intercept is increasing in the output target but decreasing in the weight put on the balance of payments.

To consider monetary policy the relevant constraint set is (3:21). Taking first order conditions gives the reaction function.

$$(3{:}25) \qquad C = \frac{\alpha_1^{\,2}}{(\alpha_1^{\,2}+\theta)} C^* + \frac{\alpha_1}{(\alpha_1^{\,2}+\theta)} Y^a - \frac{\beta}{2(\alpha_1^{\,2}+\theta)}$$

This has similar properties to the fiscal policy reaction function in that the slope is always negative. However, if there is no cost put on control, i.e. $\theta = 0$ then the slope has value minus one. If this is true for both countries then the possible Nash equilibria are not straightforward. This is due to the fact that in this case the reaction functions are parallel. Therefore if the targets are not compatible there is no interior solution since the reaction functions do not intersect. If the targets are compatible the reaction functions coincide and there are an infinity of possible equilibria. It is therefore necessary to assume at least some cost on use of the controls in this case.

The general conclusion to be drawn from the analysis is therefore that the results are qualitatively similar to the case of quadratic loss functions. Reaction functions can be derived which are linear in the other country's control and the inefficiency of Nash equilibrium depends on the incompatibility of targets. However, the parabolic loss functions results in a significant simplification in that it removes the ambiguity about the sign of the slope of the reaction function.

3:5 Conclusions

The purpose of this chapter has been to discuss the standard solution concepts of game theory both for games consisting of a single play and games consisting of repeated plays through time. This has led to the conclusion that, despite its apparently naive conjectural variations assumption, the Nash equilibrium is the natural way to model uncoordinated behaviour in a single play, or one-shot, game. For repeated games with perfect information and a finite time horizon the Nash solution remains the relevant solution concept though infinitely repeated games and games with imperfect information make alternative equilibria possible. The Stackelberg leader solution was shown to be of some interest as a simple 'rule of thumb' to reduce the loss from uncoordinated behaviour.

Having discussed the solution concepts in abstract they were then applied to policy decisions in the two country Mundell framework. This led to the conclusion that if countries pursued balance of payments surplus targets an excessively deflationary outcome would result. Improvement on this requires explicit coordination of policy or the adoption of an alternative policy regime which reduces the extent to which economic fluctuations are transmitted from one country to another. It could be argued that it was the failure of countries to coordinate policy which led to the break-up of the Bretton Woods system and the adoption of floating exchange rates as an attempt to find such a regime.

Two alternative specifications of the policy authorities' loss functions were considered, the quadratic and parabolic functions. These gave broadly similar qualitative results. The overall conclusion to be drawn from this chapter is that in cases where the game can be represented as a single play or a succession of identical single plays then the inefficiency of uncoordinated policy actions derives from the incompatibility of targets. This can be contrasted with the problem of dynamic inefficiency discussed in future chapters which occurs when the state variables which provide the constraint set subject to which players choose their optimal strategies have an explicit dynamic structure. Dynamic inefficiency implies that even when players agree on the steady-state towards which the state variables should converge uncoordinated behaviour results in the choice of an inefficient convergent path.

Appendix A

In the main body of the chapter use was made of a government loss function which set targets for output and the balance of payments. Although this is certainly descriptive of the way in which governments have behaved in the past it is open to the criticism that it is inconsistent with the maximisation of the utility of the representative individual in the economy. The purpose of this appendix is to show that this inconsistency can be resolved within a simple Keynesian disequilibrium model.

Consider an economy consisting of a number of identical individuals. This assumption enables their consumption decisions to be determined by a single aggregate utility function.

$$\text{(A1)} \qquad U = U(C_1, C_2)$$

where C_1 is current period consumption and C_2 is future consumption. The second period budget constraint facing the consumption sector is

$$\text{(A2)} \qquad C_2 = Y_2 + Y_2 + (1+r)(Y_1 - C_1)$$

Income is assumed to be treated as exogenous. Since the consumption function was assumed to be linear it is convenient to assume that (A1) is log-linear. This enables the maximisation problem of the consumption sector to be written

(A3) $$\max_{C_1, C_2} \quad \ln C_1 + \delta \ln C_2 + \lambda \left(C_1 + \frac{C_2}{1+r} - Y_1 - \frac{Y_2}{1+r} \right)$$

This enables the period one consumption to be derived as

(A4) $$C_1 = \frac{Y_1}{1+\delta} + - \frac{Y_2}{(1+r)(1+\delta)}$$

If consumers expect income to grow by a fraction t then this can be written as a standard Keynesian consumption function relating current consumption to current income.

(A5) $$C_1 = \frac{2+r+t}{(1+\delta)(1+t)} Y_1$$

The marginal propensity to consume is increasing in t and δ, and decreasing in r. If it is greater than one consumers wish to dissave and if it is less than one they wish to save.

Now suppose that investment is determined separately as a function of the rate of interest, income and a vector of exogenous influences i.e.

(A6) $$I = I(r, Y, \zeta)$$

ζ is assumed to measure such factors as the state of business confidence which may not vary in any systematic way with current economic variables. Since savings and investment are determined separately there is no guarantee that at any particular value of income the two will be equal. In a closed economy the multiplier would bring ex post savings and investment into equality but in an open economy the two can differ if there is a non-zero balance of payments.

Thus there are two channels through which the consumption sector can save or dissave, i.e. investment and the balance of payments. In determining its optimal policy the government therefore has to take into account the implications for both current output and the balance of payments since this affects the consumption possibilities in future time periods. Suppose the government has some estimate of full capacity output Y_f, however at Y_f the implied savings-investment gap is not equal to the implied balance of payments e.g. consumers wish to save more than the sum of firm's desired investment and the balance of payments surplus. The government will therefore be willing to trade-off a negative deviation of output from full capacity with an improvement in the balance of payments. A loss function which targets output and the balance of payments is therefore consistent with the maximisation of the utility of the consumption sector in the context of a Keynesian disequilibrium model.

Chapter Four

The use of differential games to model policy interdependence

Chapter Four

(4:1) Introduction

Chapter three introduced some important solution concepts of game theory and used them to analyse policy optimisation in a simple model of interacting economies. However, the linear-quadratic structure considered was applicable only to models in which the states could be expressed as a set of linear, simultaneous equations. In many macroeconomic models the focus of interest is not on the comparative static equilibrium of the model but the dynamic path of the model towards (or, if unstable, away from) the equilibrium. The analysis of policy optimisation in such models requires the techniques of differential game theory. The purpose of this chapter is to set out these techniques for the general linear quadratic differential game. Chapters five and six then make use of these techniques to analyse specific economic models.

In chapter three the source of the inefficiency of decentralised decision making was shown to be the incompatibility of the targets the players put on the state variables. If the states evolve through time then there is another potential source of inefficiency in that agents may agree on the steady-state to which the system should converge but choose an inefficient path towards the steady-state. The reason for this inefficient choice is failure to coordinate their policies. In a differential game it is possible for both these sources of inefficiency to be present simultaneously. However, since the problem of inconsistent targets was given a good deal of attention in chapter three, this chapter assumes that targets are consistent in order to focus on the second problem which is referred to as the problem of dynamic inefficiency.

The plan of the chapter is as follows. Section (4:2) considers the single controller problem emphasising the role of discounting in the loss function, and the effects of forward looking expectations variables. The implications of multi-controller games are discussed in section (4:3) which uses Nash assumptions to characterise uncoordinated behaviour and contrasts this with coordinated or Pareto Efficient control. The strategy space on which Nash strategies are defined becomes of considerable importance in differential games. Section (4:4) discusses a simple Stackelberg leader-follower game structure and section (4:5) contains conclusions.

(4:2) The single controller problem

This section considers the single player or 'game against nature' control problem. The Pontryagin minimum principle is used to minimise a quadratic loss function subject to a set of linear differential equations. The notation follows that of Miller and Salmon (1983) and many of the same issues are considered. However, some additional features are added to the model such as discounting.

The policy authorities are assumed to have a loss function which is an integral over time of a quadratic function of a vector of states x and a vector of controls u.

$$(4:1) \qquad L = \frac{1}{2}\int_0^{t_1} (x^T Q x + u^T R u)e^{-\delta t} + F(x(t_1))\,dt$$

x and u are implicitly time-indexed. The constraint set consists of a set of linear differential equations

$$(4:2) \qquad \dot{x} = Ax + Bu$$

A dot above a variable indicates its derivative with respect to time. The matrices A and B are assumed to be time invariant. A, the matrix relating the rate of change of the states to their levels at a point in time, is referred to as the state-transition matrix. The system of equations (4:2) is referred to as the state-space representation of the model.

Assuming that the policy authorities wish to minimise the loss function (4:1) subject to the constraints (4:2) then the problem can be solved by setting up the Hamiltonian function (c.f. Intriligator

(1971)). The Hamiltonian for this problem can be written.

$$(4:3) \qquad H = \frac{1}{2}\,(x^TQx + u^TRu)e^{-\delta t} + \mu^T(Ax + Bu)$$

where μ^T is a vector of costate variables analogous to the Lagrange multipliers of the static optimisation problem. The first order conditions for the minimisation of (4:3) yield the optimal control trajectory for the problem defined by (4:1) and (4:2). Differentiating with respect to the controls yields

$$(4:4) \qquad u = -R^{-1}B^T \frac{\mu}{e^{-\delta t}}$$

It can be shown that substituting (4:4) back into (4:3) and differentiating with respect to x and μ yields a set of differential equations which describes the behaviour of the system under control c.f. Dixit (1976). These equations can be written

$$(4:5) \qquad \dot{x} = \frac{\partial H^*}{\partial \mu} = Ax - BR^{-1}B^T \frac{\mu}{e^{-\delta t}}$$

$$\dot{\mu} = \frac{-\partial H^*}{\partial x} = -Qxe^{-\delta t} - A^T\mu$$

where H^* is the result of substituting (4:4) into (4:3). By substituting $p = \mu/e^{-\delta t}$ the equations can be written in matrix form as:

$$\begin{bmatrix} \dot{x} \\ \dot{p} \end{bmatrix} = \begin{bmatrix} A & -BR^{-1}B^T \\ -Q & -A^T \end{bmatrix} \begin{bmatrix} x \\ p \end{bmatrix}$$

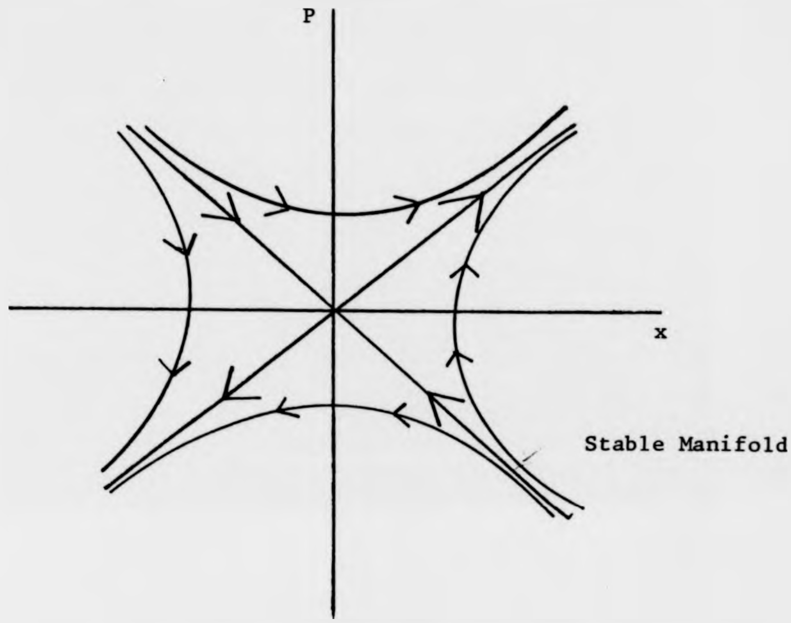
The effect of discounting is therefore to make the evolution of the costates an explicit function of time as well as the level of the states. These equations will be referred to as the state-space representation of the model under control. To determine the time path of the system it is also necessary to place boundary or terminal conditions on the states and costates. For many economic problems of interest the initial values of the states are predetermined but their terminal values are not. Therefore there is no problem about specifying the boundary conditions at time 0. At the terminal date the choice of optimal controls can be seen as a conventional minimisation problem, i.e. the optimal choice implies that the costate should equal the marginal change in the instantaneous loss function from a change in the state. These conditions can be written

$$(4{:}7) \qquad x(0) = \bar{x}(0)$$

$$p(t_1) = \frac{\partial F}{\partial x(t_1)}$$

Equations (4:6) and (4:7) are called the canonical equations. The case of a single state can be illustrated by means of a phase diagram. The state-transition matrix in (4:6) has a saddle-point structure, i.e. it has an equal number of positive and negative eigenvalues. Thus if the steady-state of the system is given by $\dot{x} = \dot{p} = 0$ then there is a unique stable path to the equilibrium as shown in figure one.

Figure One



If the control problem has a finite time horizon then the system will not follow the stable path. However, the more distant the horizon the closer the approximation to the stable path. This gives rise to the turnpike property in optimal growth models where the optimal growth path approximates the balanced growth path for much of the planning period.

The discussion of control problems with a finite horizon suggests that considerable simplification can be achieved by replacing this assumption with that of an infinite horizon. This replaces the second transversality condition in (4:7) with the requirement that the system must follow the stable path. This in turn implies that the solution path must be a function only of the negative eigenvalues of (4:6) rather than all of them as in the finite horizon case.

To show how this simplifies the analysis the vector of states and costates needs to be transformed into canonical form. The canonical variables z are defined by

(4:8) $$\begin{bmatrix} x \\ p \end{bmatrix} = C \begin{bmatrix} z \end{bmatrix}$$

where C is the matrix of column eigenvectors of the state-transition matrix. By a well known transformation (c.f. Glaister (1972) p.79) the differential equation system (4:6) can be rewritten in terms of the canonical variables as

(4:9) $$\begin{bmatrix} \dot{z}_s \\ \dot{z}_u \end{bmatrix} = \begin{bmatrix} \Omega_s & 0 \\ 0 & \Omega_u \end{bmatrix} \begin{bmatrix} z_s \\ z_u \end{bmatrix}$$

i.e. each of the canonical variables is associated with a single eigenvalue, and the eigenvalues have been divided into those less than zero (subscripted s) and those greater than zero (subscripted u).

For the infinite horizon case the transversality conditions merely require the system to converge on the steady-state. This means that only the stable roots should play a role in the adjustment process, which requires that the canonical variables corresponding to the unstable roots should be set equal to zero. Note that (4:8) can be rewritten

$$(4.8') \quad \begin{bmatrix} x \\ p \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix} \begin{bmatrix} z_s \\ z_u \end{bmatrix}$$

if $z_u = 0$ this implies $x = C_{11} z_s$ and $p = C_{21} z_s$

$$(4.10) \quad \dot{x} = C_{11} e^{\Lambda_s t} C_{11}^{-1} x$$

Using the boundary condition that $x(0)$ is fixed, the evolution of the states and costates can be derived as

$$(4.11) \quad x(t) = C_{11} e^{\Lambda_s t} C_{11}^{-1} x(0)$$

$$p(t) = C_{21} e^{\Lambda_s t} C_{11}^{-1} x(0)$$

where $e^{\Lambda_s t}$ is the matrix with $e^{\lambda i}$ on the diagonal (where $\lambda_i (i=1,n)$ are the stable eigenvalues of the state-transition matrix) and zeros elsewhere. Thus the time path of the system is open-loop in that it depends only on information available in the initial period.

Although the trajectory of the system is open-loop the optimal control vector can always be written in feedback form.

$$(4.12) \quad u(t) = -R^{-1} B^T C_{21} x(t)$$

This is still an open-loop control since the trajectory of the x vector is determined wholly by initial conditions. However, it will frequently be of use in future chapters to know that there exists a feedback control rule equivalent to the open-loop formulation of the problem.

All the analysis so far has assumed that the initial values of the states are fixed. However, for many macroeconomic models, particularly those which incorporate rational expectations and financial asset prices, this is not a valid assumption. For example, in the Dornbusch (1976) model of exchange rate determination the exchange rate jumps instantaneously due to anticipations of future events. Miller and Salmon suggest a way of dealing with this which effectively means that the costate corresponding to the jump variable should be regarded as predetermined rather than the variable itself. The logic behind this is that since the state is free to move the controller will always choose a policy such that its associated costate is initially zero. Thus the state-transition matrix should be partitioned into jump and non-jump variables as in (4:13)

$$(4{:}13)\qquad \begin{bmatrix} \dot{x}_1 \\ \dot{p}_2 \\ \dot{p}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} A_{11} & -J_{12} & -J_{11} & A_{12} \\ -Q_{21} & -A_{22}^T & -A_{12}^T & -Q_{22} \\ -Q_{11} & -A_{21}^T & -A_{11}^T & -Q_{12} \\ A_{21} & -J_{22} & -J_{21} & A_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ p_2 \\ p_1 \\ x_2 \end{bmatrix}$$

$$J = BR^{-1}B^T$$

The subscripts indicate that the matrices have been partitioned appropriately. The open-loop dynamics of this system can then be derived in the same way as in the case where all the states were predetermined (c.f. Miller and Salmon for futher details).

However, there is one important respect in which optimal policy differs when the state variables are free to jump. This is that the optimal policy derived by the minimum principle cannot be derived by the alternative method of dynamic programming. This has been termed the problem of time inconsistency c.f. Kydland and

Prescott (1977). The basic reason for this is that if the state variables jump in response to policy announcements then it is in the interests of the policy maker to continuously revise announced policy so that the states take on the values which make their associated costates zero. An important factor here is the reponse of the private sector to such behaviour. Further consideration of this issue will be given in section (4:4) on Stackelberg leader games.

The discussion of the single controller problem has therefore lead to the following conclusions. Firstly it has been shown that discounting is very easy to model via a simple transformation of the costate variables, therefore in the analysis which follows discounting is not considered explicitly. Secondly, the infinite horizon problem has been shown to offer considerable analytical simplification and to be a good approximation to problems with a lengthy time horizon, therefore an infinite horizon will be assumed in future problems. The final important conclusion is that although dynamic programming is an equivalent solution to the minimum principle for cases in which the states are predetermined this is not the case when the states are free to jump.

(4:3) Nash and Pareto Efficient Solutions

This section uses Nash and Pareto Efficient solution concepts to contrast uncoordinated and coordinated policy making. It is assumed that there are only two players or controllers but this can easily be generalised. Following the discussion of section (4:2) it is assumed that the loss functions of the players are undiscounted integrals of a quadratic function of the states and the players' own controls defined over the interval zero to infinity, i.e.

$$(4:14) \quad \frac{1}{2}\int_{o}^{\infty} (x^TQx + u_1^TRu_1)\ dt$$

With a similar function being defined for player two. The lack of discounting may cause problems if the loss function becomes unbounded, however this will not be the case if the players' targets are mutually compatible so that the states and controls converge to their desired values (x and u in (4:14) are defined in terms of deviations from targets). Even if this were not the case then methods exist by which a solution can be determined, e.g. the overtaking criterion.

The states are assumed to be determined jointly by the actions of the two players according to the set of differential equations (4:15)

$$(4:15) \quad \dot{x} = Ax + B_1u_1 + B_2u_2$$

As in the case of static games each player must make some strategic assumption about the behaviour of the other. The Nash assumption that each player regards the other's strategy as independent of his own is again particularly convenient. However, in the dynamic case it is not as obvious what constitutes a strategy. One possibility is simply to extend the static definition of a strategy as a vector of controls to incorporate the time element by indexing the vector on the interval zero to infinity. This is termed the Open-Loop Nash (OLN) assumption since the strategies constitute vectors of controls determined at the beginning of the game. An alternative is to specify the strategy space in terms of reaction functions. This implies that each player announces that he will respond to the states in a certain way subject to which the other player formulates his optimal policy. An equilibrium constitutes a pair of mutually compatible reaction functions and is termed the Closed-Loop Nash (CLN) solution.

The OLN solution can be derived by assuming that each player sets up a Hamiltonian for the problem. The optimal decision rules can then be derived in terms of the individual's costates. This is given for player one in (4:16).

$$u_1 = R_1^{-1} B_1^T p_1 \tag{4:16}$$

Substituting this into the Hamiltonian and differentiating with respect to the states and costates for both players enables the state-space representation of the problem under OLN control to be derived as:

$$\begin{bmatrix} \dot{x} \\ \dot{p}_1 \\ \dot{p}_2 \end{bmatrix} = \begin{bmatrix} A & -B_1 R_1^{-1} B_1^T & -B_2 R_2^{-1} B_2^T \\ -Q_1 & -A^T & 0 \\ -Q_2 & 0 & -A^T \end{bmatrix} \begin{bmatrix} x \\ p_1 \\ p_2 \end{bmatrix} \tag{4:17}$$

The time path of this system can once again be determined by making the canonical transformation.

$$\begin{bmatrix} x \\ p_1 \\ p_2 \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix} \begin{bmatrix} Z_s \\ Z_u^1 \\ Z_u^2 \end{bmatrix} \tag{4.18:}$$

Due to the infinite time horizon the transversality conditions simply require the system to be restricted to the stable manifold, i.e. $Z_u^1 = Z_u^2 = 0$. Therefore the costates can be written as functions of the states.

$$(4:19) \qquad p_1 = C_{21}C_{11}^{-1}x \quad \text{and} \quad p_2 = C_{31}C_{11}^{-1}x$$

Therefore the optimal decision rule for player one can be written as feedback form as:

$$(4:20) \qquad u_1 = -R_1^{-1} B_1^T C_{21}C_{11}^{-1}x$$

Now consider the steady-state of the system as defined by setting $\dot{x} = 0$ in (4:15). This means that the states can be written:

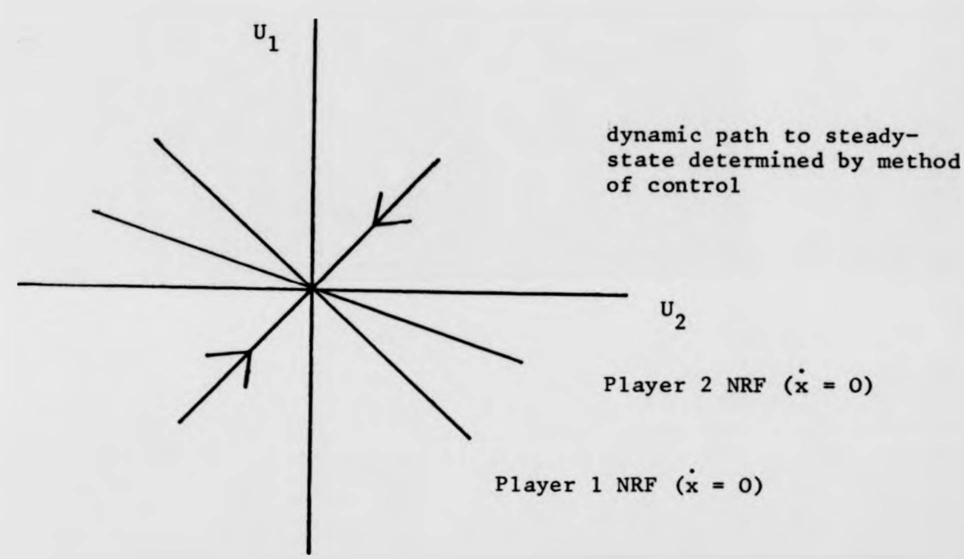
$$(4:21) \qquad x = A^{-1}B_1u_1 + A^{-1}B_2u_2$$

Substituting into (4:20) and rearranging yields an expression for the optimal control vector as a reaction function.

$$(4:22) \qquad u_1 = (I + R_1^{-1} B_1^T C_{21}C_{11}^{-1}A^{-1}B_1)^{-1}(-R_1^{-1}B_1^TC_{21}C_{11}^{-1}A^{-1}B_2u_2)$$

This is a static reaction function similar to (3:8) which has been simplified by the assumption that the target values of the states are zero (or alternatively that (4:21) is expressed in terms of deviations from target values). The steady-state can therefore be thought of as corresponding to the intersection of two static Nash reaction functions. This enables the decomposition of the inefficiency of Nash equilibrium discussed at the beginning of the chapter into the inefficiency of the steady-state and the inefficiency of the the dynamic path towards it. Figure two illustrates this.

Figure Two



Thus the OLN solution is derived by assuming that each player acts as a single controller facing an exogenous environment one feature of which is the control vector of the other player defined on the interval zero to infinity. The CLN solution can be derived in a similar manner except that, instead of adding an additional exogenous element to the state equation, the other player's control adds an additional source of dynamics.

Consider the problem for player one. Player two's feedback control rule is treated as exogenous, i.e.

(4:23) $\quad u_2 = G_2 x$

is treated as a constraint in player one's minimisation problem. This enables the differential equations in the states to be rewritten:

$$(4{:}24) \qquad \dot{x} = (A + B_2G_2)x + B_1u_1$$

The problem facing player one is to choose a feedback control rule G_1 (or alternatively a vector $U(t)$ which can be expressed in feedback form) to minimise his losses. The Hamiltonian for this problem can be written

$$(4{:}25) \qquad H_1 = \frac{1}{2}(x^TQ_1x + u_1^TR_1u_1 + u_2^TR_{12}u_2) + p_1^T((A + B_2G_2)x + B_1u_1)$$

where the loss function has been expanded to include the possibility that player one cares explicitly about player two's controls. The differential equations describing the system under control can therefore be derived as:

$$(4{:}26) \qquad \begin{bmatrix} \dot{x} \\ \dot{p}_1 \\ \dot{p}_2 \end{bmatrix} = \begin{bmatrix} A & -B_1R_1^{-1}B_1^T & -B_2R_2^{-1}B_2^T \\ -(Q_1+G_2^1R_{12}G_2) & -(A+B_2G_2)^T & 0 \\ -(Q_2+G_1^1R_{21}G_1) & 0 & -(A+B_1G_1)^T \end{bmatrix} \begin{bmatrix} x \\ p_1 \\ p_2 \end{bmatrix} \qquad (1)$$

In the applications of the CLN solution in chapter six it is assumed that $R_{12} = R_{21} = 0$ on the grounds that any effect of player two's controls on player one's welfare comes through their effect on the states rather than directly via their inclusion in the loss function.

If the state variables are capable of jumping then there are several possible CLN solutions. Firstly if players can implement optimal time-inconsistent closed loop strategies then an equilibrium could be defined as a pair of these. Alternatively if they were restricted to time consistent feedback rules an equilibrium could exist in these also. This topic is not pursued at length here since the model used in chapter six has all state variables predetermined.

OLN and CLN models are therefore different ways of modelling uncoordinated behaviour. In order to model coordinated behaviour it is necessary to introduce another solution concept, i.e. that of Pareto Efficient (PE) control. This transforms the two player model into a single player model by assuming a single controller whose objective function is a weighted average of the individual players' objective functions. Thus the weighted loss function can be written:

$$(4{:}27)\qquad L = \frac{1}{2}\int_{o}^{\infty} \sum_{i=1}^{2} w_i (x^T Q x + \sum_{j=1}^{2} u_j^T R_{ij} u_j)\, dt$$

Making the simplifying assumption $R_{ij} = 0,\ i \neq j$ enables the optimal control vectors to be derived as

$$(4{:}28)\qquad u_i = w_i^{-1} R_i^{-1} B_i^T p$$

where p is the vector of costates defined in the Hamiltonian. The state-space form of the model under PE control can therefore be written

$$(4{:}29)\qquad \begin{bmatrix} \dot{x} \\ \dot{p} \end{bmatrix} = \begin{bmatrix} A & -\sum_{i=1}^{2} w_i^{-1} B_i R_i^{-1} B_i^T \\ -\sum_{i=1}^{2} w_i Q_i & -A^T \end{bmatrix} \begin{bmatrix} x \\ p \end{bmatrix}$$

As in the single controller case of section (4:2) PE control is time-consistent providing the states do not jump in reponse to announced policy.

As in the single controller case of section (4:2) PE control is time-consistent providing the states do not jump in response to announced policy.

(4:4) Stackelberg leader models

In chapter three the Stackelberg leader model was defined as a game in which one player correctly assesses the effects of his policy choice on the choices of others but these others treat his policies as parametric. There is a straightforward generalisation of this model for dynamic game models c.f. Simaan and Cruz (1973), Kydland (1975), and Miller and Salmon (1983 and 1984). However, the definition of the strategy space, i.e. whether it is the open-loop time paths of the controls or a feedback control rule on the states, is even more important than under Nash assumptions.

Consider first the case where strategies are defined as the open-loop time paths of the controls. There are assumed to be two players, one leader and one follower. Since the follower acts according to Nash assumptions the derivation of its optimal control is exactly the same as that for the OLN case in section (4:3), i.e.

$$(4:30) \qquad u_2 = -R_2^{-1} B_2^T p_2$$

$$\text{and} \quad \dot{p}_2 = \frac{-\partial H_2}{\partial x} = -Q_2 x - A^T p_2$$

The follower's costate vector p_2 measures the values of changes in the state vector. Since these values are functions of the leader's controls the leader can be seen as affecting the follower's controls through the channel of his costates c.f. Miller and Salmon (1983). Thus the leader's minimisation problem can be set up as

$$(4:31) \qquad \text{Min } \frac{1}{2}\int_{o}^{\infty} (x^T Q_1 x + u_1^T R_1 u_1 + u_2^T R_{12} u_2)\, dt$$

$$\text{subject to} \quad \dot{x} = Ax + B_1 u_1 + B_2 u_2$$

$$u_2 = -R_2^{-1} B_2{}^T p_2$$

$$\dot{p}_2 = -Q_2 x - A^T p_2$$

Therefore the Hamiltonian for the leader incorporates costates on the follower's costates, i.e.

$$(4:32) \qquad H_1 = \frac{1}{2}(x^T Q_1 x + u_1{}^T R_1 u_1 + p_2{}^T J_{12} p_2) + p_1{}^T (Ax + B_1 u_1 - J_2 p_2)$$

$$+ p_2^{*T} (-Q_2 x - A^T p_2)$$

where $J_2 = B_2 R_2^{-1} B_2^T$ and $J_{12} = B_2 R_2^{-1} R_{12} R_2{}^{-1} B_2{}^T$

Differentiating with respect to u_1 yields the leader's optimal control rule

$$(4:33) \qquad u_1 = -R_1^{-1} B_1^T p_1$$

and differentiating with respect to x, p_1 and p_2^* enables the system as a whole to be written

$$\text{(4:34)} \quad \begin{bmatrix} \dot{x} \\ \dot{p}_2^* \\ \dot{p}_1 \\ \dot{p}_2 \end{bmatrix} = \begin{bmatrix} A & 0 & -J_1 & -J_2 \\ 0 & A & J_2 & -J_{12} \\ -Q_1 & Q_2 & -A^T & 0 \\ -Q_2 & 0 & 0 & -A^T \end{bmatrix} \begin{bmatrix} x \\ p_2^* \\ p_1 \\ p_2 \end{bmatrix}$$

The leader's control problem is effectively that of a single controller in a model where the state variables are capable of jumps. This is because the follower's costates are a function of the entire time path of the leader's controls. Therefore even though the states are initially predetermined the leader realises that his choice of control causes the follower's costates to jump. The leader will therefore choose a policy such that the marginal value of a change in the follower's costates to him is zero, i.e. $p_2^*(0) = 0$. Along the open-loop time path however $p_2^*(t) \neq 0$, this implies that the leader has a constant temptation to abandon the original policy and implement a new one which sets $p_2^*(t) = 0$, i.e. the open-loop Stackelberg solution is time inconsistent.

The similarity between this model and the single controller model with jump variables is not coincidental. In practice such models in macroeconomics should be treated as Stackelberg leader models. For example the policy optimisation problem in models with rational expectations should be thought of as a Stackelberg game with government as leader and the private sector as follower. To take a particular example in the Dornbusch exchange rate model it is the optimising behaviour of speculators reacting to announced government policy which leads to exchange rate over-shooting. This takes the argument back to the basic point made by Lucas (1976) that in designing policy the government should recognise that it faces a set of agents minimising their own loss functions.

This problem of an agent being tempted to revise his original plan even without any unexpected disturbances is familiar in a context other than the time inconsistency literature, i.e. the idea of cheating in Stackelberg games discussed by Hamaleinen (1981). As the above argument shows the problem of time inconsistency arises because of the role of government as leader in a Stackelberg game with the private sector. The problem of cheating arises when the private sector reacts to announced government policy thus giving government the opportunity to say one thing and do another.

Given that time or dynamic inconsistency is a feature of these models the question arises of how to deal with it. In the context of the government/private sector game a number of authors have suggested the adoption of fixed, easily monitored policy rules c.f. Kydland and Prescott. However, by precluding response to stochastic disturbances this merely replaces one source of inefficiency with another. Buiter (1981) suggests a solution method which involves dividing the controls into a deterministic time inconsistent element and an innovation contingent feedback element. However, this does nothing to answer the basic time inconsistency criticism.

Calvo (1978) has argued that even if the objective function of the government is to maximise the welfare of the representative individual in the economy the time inconsistency problem will still arise. However, intuition suggests that the problem is likely to be much more severe in cases where the government and public loss functions are very different. Indeed a great deal of the time inconsistency literature including the original Kydland and Prescott paper seems to be a restatement of the familiar monetarist proposition that fixed policy rules are a way of restraining malevolent governments from dysfunctional behaviour.

Although it has most frequently been applied to the government/ private sector game the Stackelberg leader model is in principle applicable to a much wider range of problems. For example in a multi-country model one country could assume a dominant role in policy making. The time inconsistency problem is likely to be more serious in such models both because the loss functions are more likely to conflict and because the players are of more even size. This second point implies that it is harder for the leader to maintain his position by virtue of relative power, therefore the game is much more likely to degenerate into a Nash game. Miller and Salmon show that if the follower accurately anticipates cheating by the leader then the game becomes one of closed-loop Nash. If the follower fails to anticipate cheating at all then the solution to the game is the dynamic equivalent of the perfect cheating solution discussed in chapter three.

(4:5) Conclusions

The purpose of this chapter has mainly been expository. Differential game solutions have been set out which describe uncoordinated and coordinated policies. The Stackelberg leader model has been discussed and related to the time inconsistency debate. Chapter five and six now go on to draw material from this chapter to analyse policy optimisation in a world of interacting economies.

Footnote

(1) A straightforward analytical solution of the problem defined by equations 4:25 and 4:26 is not possible since it requires knowledge of the feedback matrices G_1 and G_2. However, a numerical solution for any particular problem is fairly easy. The simulations in chapter six use the following method. An initial guess was made of the values of G_1 and G_2. This was in fact the feedback form of the OLN solution. Each player's optimal open-loop policy was then calculated given the feedback control matrix of the other and expressed in feedback form. This process was continued until the assumed matrices and the optimal feedback form matrices coincided. In practice it took four to five iterations to ensure this convergence.

Chapter Five

Optimal disinflation with interdependent economies

Chapter Five

(5:1) Introduction

The purpose of this chapter is to make use of the techniques of differential games introduced in the previous chapter to analyse some problems for policy makers in open economies. Again an important distinction is that of inefficiencies due to incompatible targets and dynamic inefficiency due to a sub-optimal choice of time path towards the steady-state. This chapter will consider a two-country model of the world economy in which each country wishes to guide the economy towards a steady-state with zero inflation at the natural level of output. Thus incompatibility of targets is ruled out by assumption though dynamic inefficiency remains a problem.

One problem associated with research of this kind is that the complex nature of macroeconomic models of the open economy means that application of optimal control methods usually leads to analytically intractable solutions which offer little insight. Consider a model in which the states are the output levels and rates of inflation of the two countries and in which control is exercised via some policy instrument such as the rate of monetary growth. The equations of motion of the world economy could be expressed as a system of differential equations.

$$\dot{x} = f(x, \theta) \tag{5:1}$$

where x is the vector of states and θ is the vector of policy instruments. Assuming that (5:1) can be linearised, either by appropriate choice of functional forms for the behavioural relationships in the model or by taking a linear approximation around the equilibrium, and that the policy authorities have quadratic loss functions, enables the problem to be analysed within the context of the differential game framework discussed in chapter four. However, the state vector consists of four variables and each player sets up costates on all these. This leads to a system of twelve linear differential equations to describe the system under control. In the general case the time path of the system will be a function of all twelve eigenvalues of the state-transition matrix. By assuming an infinite time horizon the solution path can be restricted to the stable manifold meaning that it will be a function of the four negative eigenvalues only. However, this still leaves a problem of considerable difficulty.

One solution to the above problem has been to carry out numerical simulations of control problems. Driffill (1982) and Currie and Levine (1985) both contain examples of this for single controller models of the open economy. This approach provides useful insights and chapter six uses it to analyse dual controller problems in which policies are transmitted between countries. However, numerical simulations can always be criticised as being specific to the parameter values chosen. Sachs (1983) employs an alternative method which involves drastically simplifying the underlying economic model in order to concentrate on the control aspects of the problem. In his formulation the state vector is reduced to a single variable, inflation, and a single control, output, which the policy authorities use to steer inflation towards its target value of zero. Such an approach is not without defects, by over-simplifying the economic model many interesting features may disappear from the analysis. However, it does appear to be a promising way of obtaining tractable results from control problems.

This chapter makes use of the Sachs approach to model policy interaction between countries in an interdependent world economy. The problem considered is that of bringing inflation down from an initially high level while minimising the output costs involved. A simplified model of interacting economies is used which is consistent with a variety of assumptions concerning the underlying economic structure. Section (5:2) considers the control problem for a closed economy. This is of use for comparison with the open economy models considered later. Section (5:3) goes on to consider the policy optimisation problem for an open economy treating the rest of the world as exogenous. This can be interpreted as the small open economy case or as the individual choice of a large country exhibiting Nash behaviour. Section (5:4) examines the behaviour of a two-country system under Open-Loop Nash and Pareto Efficient control. Section (5:5) contains conclusions.

(5:2) Optimal inflation control in a closed economy

The closed economy case can be seen as a benchmark model which throws the open economy aspects of the problem into sharp relief. The problem facing the country is assumed to be that of bringing inflation down from an initially high level subject to the constraint that core inflation adjusts sluggishly in response to persistent deviations of output from its natural level. Wage inflation is assumed to be determined according to a standard expectations augmented Phillips curve.

$$(5:2) \qquad \dot{w} = \psi x + \pi$$

w and x are the natural logarithms of the wage and output levels respectively. π is core or expected inflation which is assumed to adjust according to equation (5:3).

$$(5:3) \qquad \dot{\pi} = \gamma(\dot{p} - \pi)$$

p is the log of the price level which is assumed to be a constant mark-up on wage costs enabling (5:2) to be written in terms of prices rather than wages. The problem facing the policy authorities is to use the control variable, output, to direct the state variable, core inflation, towards its target value of zero in such a way as to minimise the loss function defined by (5:4).

$$(5:4) \qquad \frac{1}{2}\int_{o}^{\infty} \pi_t^2 + rx_t^2 \, dt$$

As discussed in the previous chapter, the quadratic loss function offers analytical simplicity and has the desirable feature of punishing large deviations from the targets severely. The infinite time horizon simplifies the transversality conditions by simply requiring the system under control to depend only on stable, i.e. negative, eigenvalues. This problem can be solved easily by use of the Pontryagin minimum principle. The Hamiltonian can be written:

$$(5{:}5) \qquad H = \frac{1}{2}\pi_t^2 + \frac{r}{2}x_t^2 + \lambda_t\,(\gamma\psi x_t)$$

where λ is the costate variable. Differentiating with respect to output yields the optimal choice of control in terms of the costate.

$$(5{:}6) \qquad x_t = \frac{-\gamma\psi}{r}\lambda_t$$

Substituting back into the Hamiltonian and differentiating with respect to the state and costate enables the state-space representation of the dynamics of core inflation and its associated costate to be written:

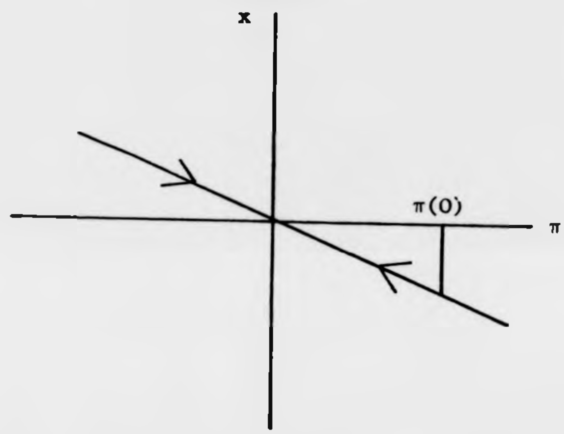
$$(5{:}7) \qquad \begin{bmatrix} \dot{\pi} \\ \dot{\lambda} \end{bmatrix} = \begin{bmatrix} 0 & \dfrac{-\gamma^2\psi^2}{r} \\ -1 & 0 \end{bmatrix} \begin{bmatrix} \pi \\ \lambda \end{bmatrix}$$

The above system is saddle-point stable, the state transition matrix exhibiting repeated roots with opposite signs.

$$w_{1,2} = \pm\sqrt{\frac{\gamma^2\psi^2}{r}}$$

The transversality conditions require the adjustment path to depend only on the negative root. Since the state is fixed instantaneously by history the costate must jump in to put the system on the stable adjustment path. Output is a function of the costate only so the phase diagram depicting the system can be drawn in terms of output and core inflation.

Figure 1



The optimal time path of output can be written:

$$(5{:}8) \qquad x_t = -\frac{\gamma\psi}{r}\,\frac{\pi(0)e^{w_1 t}}{|w_1|}$$

where w_1 is the negative eigenvalue of the state-transition matrix. The optimal anti-inflation strategy is therefore to push output below its natural level but allow it to rise gradually as inflation falls. The initial recession will be deeper

(1) The more effectively output controls inflation, i.e. the larger the parameters γ and ψ.

(2) The lower the weight put on deviations of output from its natural level in the loss function.

(3) The higher the initial rate of inflation.

The most unsatisfactory aspect of this analysis is the use of the output level as the control variable. A more realistic approach would be to model the rate of change of output as subject to control. However, this would require the examination of a system consisting of two predetermined state variables so it is left to chapter six in which numerical simulations are employed.

(5:3) Inflation control in an open economy

The main difference between the problem for the open economy and that for the closed economy is that, even if domestic prices are a fixed mark-up on wages, the real wage can vary due to changes in the terms of trade. It has been frequently argued that the level of the real wage is an important influence on the rate of growth of nominal wages, c.f. Sargan (1964), Henry, Sawyer and Smith (1976), Grubb, Jackman and Layard (1982) for empirical tests of this hypothesis. Sachs (1981) and Dornbusch (1983) argue that the real wage effect or real wage resistance can be an important channel for the transmission of macroeconomic disturbances under a system of flexible exchange rates. If domestic prices are a fixed mark-up on domestic wages then the real wage varies monotonically with the real exchange rate. A simple way of incorporating real wage stickiness is therefore to assume that inflation is negatively related to the terms of trade.

$$(5:9) \qquad \dot{p} = \psi x + \pi + \delta(p^{*} + e - p)$$

where p^{*} and e are the logs of the foreign price level and the nominal exchange rate.

The consumer price level has been implicitly assumed to be an exponential function of the price level of domestic goods and the domestic price level of foreign produced goods. This formulation can be justified by assuming that domestic and foreign produced goods are imperfect substitutes and that consumers' utility is Cobb-Douglas in them. This enables the rate of change of the consumer price index to be written:

$$(5{:}10) \qquad \dot{p}_c = \alpha\dot{p} + (1-\alpha)(\dot{p}^* + \dot{e})$$

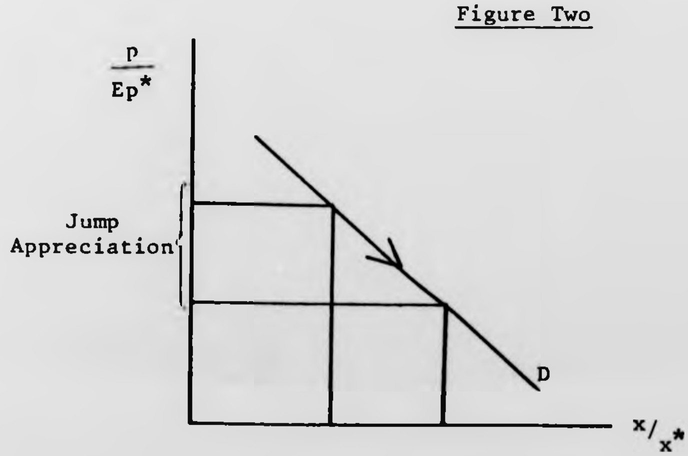
or rearranging terms

$$\dot{p}_c = \dot{p} + (1-\alpha)(\dot{p}^* + \dot{e} - \dot{p})$$

i.e. the rate of change of domestic prices plus $(1-\alpha)$ times the rate of change of the terms of trade. The determination of the terms of trade or the real exchange rate is therefore of central importance in the open economy inflation process.

How the real exchange rate is determined depends on whether or not capital is internationally mobile and on the intermediate instrument used to control output. Consider the case where capital is immobile. The exchange rate is assumed to equate demand and supply in the foreign exchange market which in this case depends only on goods market transactions. A country cutting its output to control inflation will expect an improvement in its terms of trade due to the reduced amount of its product on world markets. As output increases back towards the natural level this gain in the terms of trade is gradually eroded. Figure two illustrates this process

Figure Two



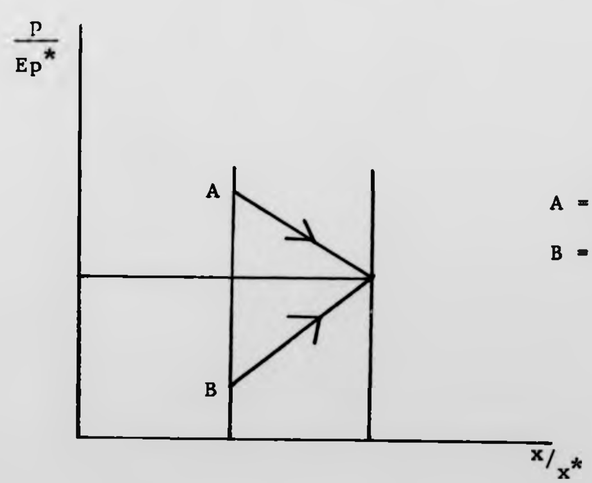
The terms of trade are determined by the world demand curve for domestic goods. It is assumed that the country has some market power implying a downward sloping demand curve. This is not unreasonable even if the country is small in terms of aggregate world output if its production is sufficiently specialised c.f. Branson (1983) for a discussion of this. The supply curve is vertical reflecting the assumption that output is the policy variable of government. There are two important effects on inflation. Firstly, when a restrictive policy is introduced there is an immediate appreciation of the real exchange rate, increasing real wages and therefore bringing immediate gains in inflation control. However, during the adjustment period the real exchange rate depreciates back towards its equilibrium value, making it harder to bring down inflation.

If capital is internationally mobile then the transmission mechanism is very different. For the case of perfectly mobile capital the exchange rate is determined solely by the requirements of portfolio equilibrium in asset markets rather than in the goods market. The impact of government policy on interest rates is of crucial importance in this case. If the demand for real money balances is determined by output and the rate of interest then the choice of intermediate instrument to control output determines the direction of change of the interest rate. Two alternatives are considered. (1) Fiscal policy is assumed to mean a move towards budget surplus, leaving the monetary growth rate constant initially. This will push domestic interest rates below foreign rates requiring an exchange rate appreciation to satisfy the uncovered interest parity condition. Since the real exchange rate must ultimately return to its original level this requires an immediate depreciation to make room for the subsequent gradual appreciation.

(2) Monetary policy is assumed to mean a cut in the monetary growth rate with an unchanged fiscal stance. This pushes the interest rate above the foreign rate requiring a depreciation to maintain portfolio equilibrium. In this case there must be an immediate appreciation to make room for the subsequent depreciation.

Of course in the long-run there is no such simple division of policies into fiscal and monetary policy. The requirements of the government budget identity require adjustment of both variables to achieve equilibrium. However, this division is of use in distinguishing cases in which one instrument is used more intensively than the other in the early stages of the planning period. Figure Three illustrates possible time paths for the real exchange rate under fiscal and monetary policy.

<u>Figure Three</u>



A = Monetary policy

B = Fiscal policy

To summarise: For the cases of zero capital mobility and monetary policy with mobile capital the exchange rate is overvalued, or alternatively the real wage is too high, along the adjustment path. For the case of fiscal policy with mobile capital the exchange rate is undervalued and real wages are too low. This is summarised in equation (5:11)

$$(5{:}11) \qquad (p^* + e - p) = \eta (x - x^*)$$

η is positive for the cases of zero capital mobility and monetary policy with mobile capital and negative for the case of a fiscal contraction with internationally mobile capital.

Along the adjustment path the real exchange rate depreciates if overvalued and appreciates if undervalued. This is summarised in equation (5:12)

$$(5{:}12) \qquad (\dot{p}^* + \dot{e} - \dot{p}) = \phi(p^* + e - p)$$

$$\text{or} \quad (\dot{p}^* + \dot{e} - \dot{p}) = \phi\eta (x - x^*)$$

By substituting (5:11) into (5:9) the rate of change of domestic prices can be written:

$$(5{:}13) \qquad \dot{p} = (\psi + \delta\eta)x - \delta x^* + \pi$$

If $\eta > 0$ then a cut in output has a more powerful instantaneous effect on inflation than in the closed economy case. This is because the output cut raises real wages as well as reducing the pressure of demand in the labour market. If $\eta < 0$ then a cut in output has a smaller instantaneous effect than in the closed economy case and could even have perverse effects if real wage resistance is strong enough i.e. it could produce stagflation with the rate of growth of nominal wages increasing as workers attempt to make good the fall in real wages even though output and employment have fallen. Such a case would be unstable so it is ruled out by assumption, i.e. it is assumed that $(\psi + \delta\eta) > 0$.

The rate of change of the consumer price index is a function of domestic price inflation and the rate of change of the real exchange rate. It can be written:

$$(5{:}14) \qquad \dot{p}_c = (\psi + \delta\eta - (1-\alpha)\phi\eta)x - (\delta\eta - (1-\alpha)\phi\eta)x^* + \pi$$

The model is now sufficiently developed to examine its behaviour under control. Foreign output is treated as exogenous, the loss function remains that specified in equation (5:4) but adjustment of core inflation now depends on the discrepancy between its current level and the rate of change of consumer prices, i.e.

$$(5{:}15) \qquad \dot{\pi} = \gamma(\dot{p}_c - \pi)$$

This creates a problem since the exchange rate jumps to put the economy on the adjustment path at the time the policy is implemented. This of course implies that the consumer price index jumps meaning that its rate of change is not defined. The starting value of $\dot{p}_c$ is therefore assumed to be equal to its value just after the introduction of the anti-inflation policy. What this implies is that agents can distinguish jumps in the price level from smooth changes. They incorporate jumps into wage-setting behaviour solely through the real wage term.

The constraint set facing the policy authorities can now be written:

(5:16) $$\dot{\pi} = \gamma\left[(\psi+\delta\eta - (1-\alpha)\phi\eta)x - (\delta\eta - (1-\alpha)\phi\eta)x^*\right]$$

$$= b_1x - b_2x^*$$

This enables the Hamiltonian for the problem to be written:

(5:17) $$H = \frac{\pi^2}{2} + \frac{rx^2}{2} + \lambda(b_1x - b_2x^*)$$

Differentiating with respect to output gives the optimal output level as a function of the costate.

(5:18) $$x_t = \frac{-b_1}{r}\lambda_t$$

Substituting (5:18) into (5:17) and differentiating with respect to the state and costate enables the state-space representation of the system under control to be derived as:

(5:19) $$\begin{bmatrix} \dot{\pi} \\ \dot{\lambda} \end{bmatrix} = \begin{bmatrix} 0 & -b_1^2/r \\ -1 & 0 \end{bmatrix} \begin{bmatrix} \pi \\ \lambda \end{bmatrix} + \begin{bmatrix} -b_2 \\ 0 \end{bmatrix} x^*$$

The eigenvalues of the state-transition matrix are again repeated with opposite signs.

$$w_{1,2} = \pm\sqrt{\frac{\gamma^2}{r}(\psi + \delta\eta - (1-\alpha)\phi\eta)^2}$$

and the time path of output can be written

$$(5{:}20) \qquad x_t = \frac{-b_1}{r}\,\frac{\pi(0)e^{w_1 t}}{|w_1|}$$

where w_1 is the negative eignevalue. The interesting question here is whether the initial recession chosen by the authorities will be deeper or shallower than for the closed economy case. This can be inferred by checking the conditions under which the coefficient b_1 is greater or less than $\psi\gamma$. An exactly equivalent method is to check whether the stable eigenvalue is larger or smaller in absolute magnitude. The possible outcomes are:

(1) In the case of immobile capital and that of monetary policy with mobile capital $\eta > 0$ so the policy authorities will choose a deeper initial recession if $\delta > (1-\alpha)\phi$. This means that the fall in inflation due to the initial increase in the real wage must outweigh the fact that the real exchange rate depreciates along the adjustment path thus making control of inflation more difficult. If $\delta < (1-\alpha)\phi$ then the authorities would choose a shallower recession than in the closed economy case. This is plausible if workers quickly come to see the higher real wage as the norm and resist its erosion by the subsequent deterioration of the terms of trade.

(2) In the case of fiscal policy with mobile capital $\eta < 0$ so the condition for the authorities to choose a deeper recession than for the closed economy case is $\delta > (1-\alpha)\phi$ i.e. real wage resistance must be sufficiently low that the benefits associated with an appreciating exchange rate more than compensate for the boost to inflation due to the fall in the real wage.

(5:4) Inflation control in an interdependent world

As well as considering the behaviour of a single country in isolation it is desirable to examine the operation of an inter-dependent system. To do this it is assumed that the world is divided into two countries, neither of which is small relative to the other. These are referred to as the domestic and foreign economies. This adds an extra dimension to the problem since each country must make some assumption about the strategic response of the other when formulating its optimal policy.

To characterise uncoordinated behaviour the natural solution concept is the Nash solution. This chapter uses the Open-Loop Nash (OLN) solution concept to model uncoordinated policy. Chapter six goes on to consider the alternative Closed-Loop Nash (CLN) solution. Coordinated behaviour is modelled by use of the Pareto Efficient (PE) solution concept.

Under OLN control each country treats the other's policy vector $u(t)$, defined on the interval 0 to ∞, as fixed. The Hamiltonian is therefore of the form (5:17). By use of the analysis in chapter four the state-space representation of the system under control can be derived as:

$$\begin{bmatrix} \dot{\pi} \\ \dot{\pi}^* \\ \dot{\lambda} \\ \dot{\lambda}^* \end{bmatrix} = \begin{bmatrix} 0 & 0 & \frac{-b_1^2}{r} & \frac{b_1 b_2}{r} \\ 0 & 0 & \frac{b_1 b_2}{r} & \frac{-b_1^2}{r} \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \pi \\ \pi^* \\ \lambda \\ \lambda^* \end{bmatrix} \tag{5:21}$$

The behaviour of this system through time is determined by the eigenvalues of the above state-transition matrix. Examination of these is made considerably easier by making use of the restriction of structural similarity to employ the Aoki (1981) technique of averaging and differencing. This enables the dynamics to be decoupled into those associated with averages and those with differences of variables. If both countries begin with the same rate of inflation then it is interesting to look at the system in averages since the symmetry of the problem means that this gives the solution for the individual countries. The system in averages can be derived as

$$(5{:}22) \qquad \begin{bmatrix} \dot{\pi}_a \\ \dot{\lambda}_a \end{bmatrix} \quad \begin{bmatrix} 0 & \frac{-1}{r}(b_1^2 - b_1 b_2) \\ -1 & 0 \end{bmatrix} \begin{bmatrix} \pi_a \\ \lambda_a \end{bmatrix}$$

The eigenvalues of the state-transition matrix determine the rate at which average inflation falls when each country implements an anti-inflation policy under OLN assumptions. These can be written

$$w_{1,2} = \pm\sqrt{\frac{\gamma^2}{r}(\psi^2 + \psi(\delta\eta - (1-\alpha)\phi\eta))}$$

The interesting question here is whether average inflation will fall more quickly or more slowly under OLN or PE control. To derive the equations of motion under PE control it is necessary to assume a single controller whose loss function is a weighted average of the individual countries' loss functions. Assuming that equal weights are put on each countries' losses enables the

Hamiltonian for the problem to be written

$$(5{:}23) \qquad H = \frac{\pi^2}{2} + \frac{\pi^{*2}}{2} + \frac{rx^2}{2} + \lambda(b_1x - b_2x^*) + \lambda^*(b_1x^* - b_2x)$$

Differentiating with respect to the output levels enables the optimal control rules to be written

$$(5{:}24) \qquad x_t = \frac{-b_1}{r}\lambda_t + \frac{b_2}{r}\lambda_t^*$$

$$x_t^* = \frac{-b_1}{r}\lambda_t^* + \frac{b_2}{r}\lambda_t$$

Substituting (5:24) into (5:23) and differentiating with respect to the state and costate enables the state-space representation of the model under coordinated control to be derived as

$$(5{:}25) \qquad \begin{bmatrix} \dot{\pi} \\ \dot{\pi}^* \\ \dot{\lambda} \\ \dot{\lambda} \end{bmatrix} = \begin{bmatrix} 0 & 0 & \frac{-1}{r}(b_1^2 + b_2^2) & \frac{2b_1b_2}{r} \\ 0 & 0 & \frac{2b_1b_2}{r} & \frac{-1}{r}(b_1^2 + b_2^2) \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \pi \\ \pi^* \\ \lambda \\ \lambda^* \end{bmatrix}$$

The relative speed of adjustment of the world economy under uncoordinated and coordinated control can be compared by taking averages of the system (5:25) and comparing the eigenvalues of the resulting system with those of (5:22).

Taking averages of (5:25) enables the PE system to be written as:

$$(5:26) \qquad \begin{bmatrix} \dot{\pi}_a \\ \\ \dot{\lambda}_a \end{bmatrix} = \begin{bmatrix} 0 & \frac{-1}{r}(b_1 - b_2)^2 \\ \\ -1 & 0 \end{bmatrix} \begin{bmatrix} \pi_a \\ \\ \lambda_a \end{bmatrix}$$

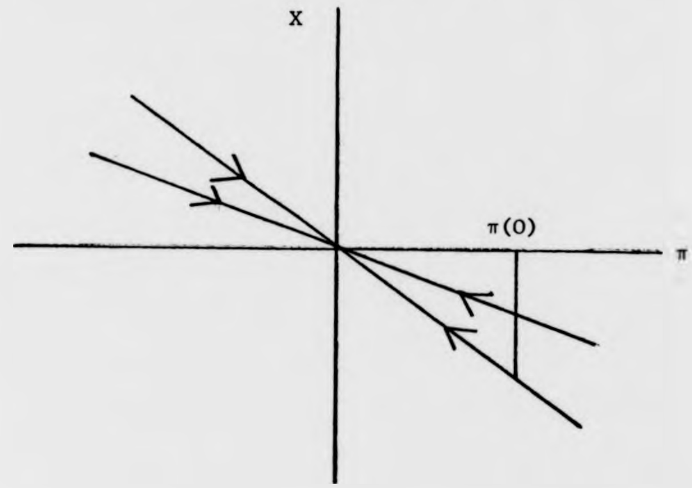
The eigenvalues of this system are:

$$w_{1,2} = \pm\sqrt{\frac{\gamma^2\psi^2}{r}}$$

Since the interaction terms are missing here the average of the two economies behaves in the same way as a single, closed economy. For inflation to decline faster under uncoordinated control it must be the case that the stable eigenvalue associated with (5:22) is larger in absolute value than that associated with (5:26). This requires $\delta\eta - (1-\alpha)\phi\eta > 0$ i.e. $b_2 > 0$.

The reason for the above result is intuitively obvious. If $b_2 > 0$ then each country perceives a gain to be made in controlling inflation by imposing an externality on the other country, e.g. by raising real wages at home and lowering them abroad it buys an immediate fall in domestic inflation while imposing inflationary costs on the other country. When policy is coordinated via a single controller this externality is internalised, since the controller realises that it merely shifts inflation from one country to the other. Figure four shows the phase paths for both OLN and PE control under the assumption that $b_2 > 0$.

Figure Four



In the cases where restrictive demand policies raise the real wage an uncoordinated policy will result in a deeper recession than a coordinated policy if the real wage effect is strong enough. If the real wage effect is not strong enough then the fact that the exchange rate depreciates along the adjustment path leads to slower than optimal adjustment under OLN control. If the restrictive demand policy produces a fall in the real wage (i.e. fiscal policy with mobile capital) then the OLN case will only result in faster than optimal adjustment if the real wage effect is relatively weak.

To analyse cases in which the countries begin with different rates of inflation it is necessary to look at the dynamics in differences. The system in differences under OLN control can be derived from (5:21) as

$$(5:27) \qquad \begin{bmatrix} \dot{\pi}_d \\ \dot{\lambda}_d \end{bmatrix} = \begin{bmatrix} 0 & \frac{-1}{r}(b_1^2 + b_1 b_2) \\ -1 & 0 \end{bmatrix} \begin{bmatrix} \pi_d \\ \lambda_d \end{bmatrix}$$

which has associated eigenvalues

$$w_{1,2} = \pm\sqrt{\frac{1}{r}\,(b_1^2 + b_1 b_2)}$$

Again the interesting comparison is between OLN and PE control or uncoordinated and coordinated policies. The dynamics in differences under PE control can be derived from (5:25) as

$$(5{:}28)\qquad \begin{bmatrix} \dot{\pi}_d \\ \dot{\lambda}_d \end{bmatrix} = \begin{bmatrix} 0 & \frac{-1}{r}\,(b_1 + b_2)^2 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} \pi_d \\ \lambda_d \end{bmatrix}$$

which has associated eigenvalues

$$w_{1,2} = \pm\sqrt{\frac{1}{r}\,(b_1 + b_2)^2}$$

The question of interest is whether OLN or PE control will result in faster convergence of initially different rates of inflation. This will be determined by which mode of control has the largest (in absolute magnitude) stable eigenvalue. The condition for PE control to give faster convergence is

$$(b_1 + b_2) > b_1^2 + b_1 b_2$$

This is obviously satisfied if $b_2 > 0$ and even if $b_2 < 0$ the condition will be satisfied if $|b_1| < |b_2|$.

At first sight this result may seem a little counter-intuitive. Even if coordinated policy results in slower adjustment of average inflation than uncoordinated policy, differences in initial rates of inflation will be eliminated more quickly. The explanation for this is that under PE control the controller makes efficient use of variations in the real exchange rate/real wage to enable the high inflation country to disinflate faster than the other. Under OLN control the high inflation country's attempts to exploit this effect are to some extent offset by the policy of the low inflation country.

This feature produces some interesting differences in the way the controls move initially. Under OLN control the difference in output in the initial time period can be written:

$$(5:29) \qquad x_d\,(0) = \frac{-b_1}{r}\,\frac{\pi_d(0)}{|w_1^{OLN}|}$$

i.e. the high inflation country always deflates more than the low inflation country. Under PE control the initial difference is

$$(5:30) \qquad x_d(o) = -\left(\frac{b_1 + b_2}{r}\right)\frac{\pi_d(0)}{|w_1^{PE}|}$$

If $b_2 > 0$ then it remains the case that the high inflation country has the deeper initial recession. However, if $b_2 < 0$ then the effect can be reversed if $\psi + 2b_2 < 0$. This means that the low inflation country will cut output by a greater amount. An example where this could be plausible is the case of fiscal policy with mobile capital. The controller chooses a smaller recession in the high inflation country because this means an increase in its real wage and therefore results in a faster decline in inflation.

(5:5) Conclusions

The approach of this chapter has been to set up a fairly simple model of interacting economies in order to emphasise the problems of policy optimisation. This has proved to have two main advantages. Firstly a number of different hypotheses about the underlying economic structure can be nested within this general framework, e.g. perfect or zero capital mobility. The second main advantage is that it enables tractable results to be obtained for policy optimisation problems.

These advantages are not without an associated cost. Obtaining the analytical results has required a number of strong assumptions. In particular allowing output to be perfectly flexible even in the short-run is somewhat unsatisfactory. Relaxation of this assumption would, however, necessitate numerical simulations. This approach is followed in chapter six where it is shown that relaxation of the flexible output assumption does not imply qualitative changes in the results.

There are several conclusions which can be drawn from the analysis. Adjustment to zero inflation from initially high rates has been shown to be inefficient. If the real wage can be raised by restrictive demand policies and this has a sufficiently large dampening effect on inflation then an excessive contraction of the world economy will result as each individual country attempts to speed the decline of its own inflation rate by shifting the burden of adjustment onto the rest of the world. This model therefore provides an explanation for competitive appreciation. Another important feature is that, although average inflation may fall faster than is optimal, differences in inflation will take longer to eliminate.

Chapter Six

Interdependent monetary policies in a two country model. Some numerical simulations

Chapter Six

Introduction

In the previous chapter a simple two-country model was used to analyse the inefficiency of uncoordinated control. To obtain tractable analytical results it was necessary to simplify the economic model so as to emphasise the control aspects of the problem. This has several associated problems. Firstly it may be the case that over-simplification of the model leads to results which are not robust when a more sophisticated model is employed. Secondly a simplified model inevitably neglects interesting economic phenomena.

For the reasons stated above it is desirable to examine the behaviour of a more fully specified macroeconomic model. Since the model then becomes too unwieldly to generate analytical results it becomes necessary to employ numerical simulation to assess its properties. This methodology has been used in several recent papers (c.f. Driffill (1983) and Currie and Levine (1984) to examine the behaviour of macroeconomic models with quite a high order of dynamics.

Numerical simulations can always be criticised on the ground that results derived from them are particular to the parameter values chosen. Examination of a grid of parameter values is tedious and usually only confirms that the qualitative nature of the model's solution is robust to quite wide variations of parameters around 'plausible' values. The approach taken in this chapter is therefore to examine only one set of parameter values for the economic model but to examine the effects of changes in the weights in the controllers' loss functions. This has the advantage of emphasising the way in which the controllers' preferences affect the time path of the system.

The particular economic model examined in this chapter is the Hamada and Sakurai (1978) model of interacting economies. Section two discusses the model and examines the influence of policy. This is followed by numerical simulation of the model under various types of uncoordinated and coordinated control. Section four extends the model to incorporate real wage resistance and numerical simulations of the extended model are presented in section five. Section six contains conclusions.

6:2 The Hamada and Sakurai model

Hamada and Sakurai (1978) set out two country models for both fixed and flexible exchange rate regimes. Since the transmission of monetary disturbances under fixed exchange rates via balance of payments disequilibrium is well known this chapter concentrates on the flexible exchange rate model. In this case the channel of interdependence between countries is via the terms of trade. The important issue for policy analysis is the link between monetary policy and the terms of trade.

The Hamada and Sakurai model consists of a pair of countries linked by trade in goods. All structural equations in the model occur in pairs and for expositional simplicity the countries are referred to as the home and foreign countries with foreign variables being distinguished by an asterisk. Structural similarity is imposed on the economies because this enables use of the Aoki (1981) technique of taking averages and differences of variables to reduce the order of the dynamics.

Wage inflation is determined by the countries' expectations augmented Phillips' curves.

$$(6{:}1) \qquad \begin{aligned} \dot{w} &= \psi x + \pi \\ \dot{w}^* &= \psi x^* + \pi^* \end{aligned}$$

These are assumed to be linear in the log of output with a unit coefficient on core inflation. Unlike H-S these relationsips are assumed to be linear. This is necessary for the system to be expressed as a set of linear differential equations for the purposes of the optimal control exercises which follow in the next section. There is assumed to be some sluggishness in the reaction of core inflation to observed inflation, i.e.

$$(6:2) \qquad \dot{\pi} = \gamma(\dot{p}_c - \pi)$$

$$\dot{\pi}^* = \gamma(\dot{p}_c^* - \pi^*)$$

The subscript c indicates that it is the rate of change of consumer prices which is important in determining core inflation.[1] In an open economy this will not be identical to producer price inflation since the consumer price index is a function of the price of domestic goods and the domestic price of foreign produced goods.

To analyse the determinants of consumer price inflation it is necessary to specify their utility function. The simplest possible case is to assume that utility is Cobb-Douglas in consumption of home and foreign goods. This implies that the relevant price index is also Cobb-Douglas in form. Taking logs and differentiating with respect to time yields the intuitively plausible result that consumer price inflation is equal to a weighted average of the rate of inflation of domestic prices and the domestic price of foreign goods with the expenditure shares as weights. The rate of change of consumer prices can therefore be written

$$(6:3) \qquad \dot{p}_c = \alpha\dot{p} + (1-\alpha)(\dot{p}^* + \dot{e})$$

$$\dot{p}_c^* = \alpha(\dot{p}-\dot{e}) + (1-\alpha)\dot{p}^*$$

To retain the assumption of symmetry it is necessary for the expenditure shares on the two products to be equal, i.e. $\alpha = 0.5$ and this is assumed from now on.

The production functions are assumed to be Cobb-Douglas in a single variable input, labour i.e.

$$(6:4) \qquad X = N^{\beta}$$

$$X^{*} = N^{*\beta}$$

It is assumed that the economies are on their demand for labour curves therefore implying that the real wage equals the marginal product of labour in each case. Imposing this condition, taking logs and differentiating with respect to time yields a pair of relationships in the rates of growth of nominal wages, producer prices and the logs of output.

$$(6:5) \qquad \dot{w} = \dot{p} - \frac{1-\beta}{\beta}\dot{x}$$

$$\dot{w}^{*} = \dot{p}^{*} - \frac{1-\beta}{\beta}\dot{x}^{*}$$

The demand for money is assumed to take a particularly simple form in that it is proportional to nominal expenditure. However, under flexible exchange rates absorption and income are identical so this can alternatively be stated as a proportional relationship between money demand and the nominal value of domestic production. Taking logs and differentiating with respect to time gives

$$(6{:}6)\qquad \theta = \dot{m} = \dot{p} + \dot{x}$$

$$\theta^* = \dot{m}^* = \dot{p}^* + \dot{x}^*$$

θ and θ^* the rates of monetary growth, are assumed to be set exogenously by the policy authorities.

Equations (6:1) to (6:6) constitute the structural equations of the model. In order to analyse the transmission of monetary policy the model can be reduced to a minimal state-space form in terms of the logs of output and core inflation levels. Combining equations (6:1), (6:5) and (6:6) yields the differential equations in output.

$$(6{:}7)\qquad \dot{x} = -\beta\psi x - \beta\pi + \beta\theta$$

$$\dot{x}^* = -\beta\psi x^* - \beta\pi^* + \beta\theta^*$$

To derive the dynamics of core inflation note that the rate of change of the real exchange rate can be written:

$$(6{:}8)\qquad \dot{p} - \dot{p}^* - \dot{e} = \dot{x}^* - \dot{x}$$

(c.f. Appendix for derivation of this result). This implies that the rate of change of the nominal exchange rate can be written

$$(6{:}9)\qquad \dot{e} = (\dot{p} + \dot{x}) - (\dot{p}^* + \dot{x}^*)$$

$$\text{or}\quad \dot{e} = \theta - \theta^*$$

This enables the rates of change of consumer prices (equation (6:3)) to be rewritten

$$(6:10) \qquad \dot{p}_c = \theta - \cdot 5\dot{x} - \cdot 5\dot{x}^*$$

$$\dot{p}_c^* = \theta^* - \cdot 5\dot{x} - \cdot 5\dot{x}^*$$

The rates of change of core inflation can therefore be derived by substituting (6:10) into (6:2) and using (6:7) to express $\dot{x}$ and $\dot{x}^*$ in terms of levels rather than rates of change. The minimal state-space representation of the model can then be written in matrix form as

$$(6:11) \qquad \begin{bmatrix} \dot{x} \\ \dot{x}^* \\ \dot{\pi} \\ \dot{\pi}^* \end{bmatrix} = \begin{bmatrix} -\beta\psi & 0 & -\beta & 0 \\ 0 & -\beta\psi & 0 & -\beta \\ \cdot 5\gamma\beta\psi & \cdot 5\gamma\beta\psi & -\gamma(1-\cdot 5\beta) & \cdot 5\gamma\beta \\ \cdot 5\gamma\beta\psi & \cdot 5\gamma\beta\psi & \cdot 5\gamma\beta & -\gamma(1-\cdot 5\beta) \end{bmatrix} \begin{bmatrix} x \\ x^* \\ \pi \\ \pi^* \end{bmatrix}$$

$$+ \begin{bmatrix} \beta & 0 \\ 0 & \beta \\ \gamma(1-\cdot 5\beta) & -\cdot 5\gamma\beta \\ -\cdot 5\gamma\beta & \gamma(1-\cdot 5\beta) \end{bmatrix} \begin{bmatrix} \theta \\ \theta^* \end{bmatrix}$$

As in the previous chapter considerable simplification in the expression of the model can be achieved by taking averages and differences of the state variables. This decouples the system into two separate 2 x 2 systems in averages and differences yielding obvious advantages for examination of cyclical and stability conditions. The system in averages can be derived as

$$(6{:}12)\qquad \begin{bmatrix} \dot{x}_a \\ \dot{\pi}_a \end{bmatrix} = \begin{bmatrix} -\beta\psi & -\beta \\ \gamma\beta\psi & -\gamma(1-\beta) \end{bmatrix} \begin{bmatrix} x_a \\ \pi \end{bmatrix} + \begin{bmatrix} \beta \\ \gamma(1-\beta) \end{bmatrix} \theta_a$$

The eigenvalues of the state-transition matrix in (6:12) can be derived as

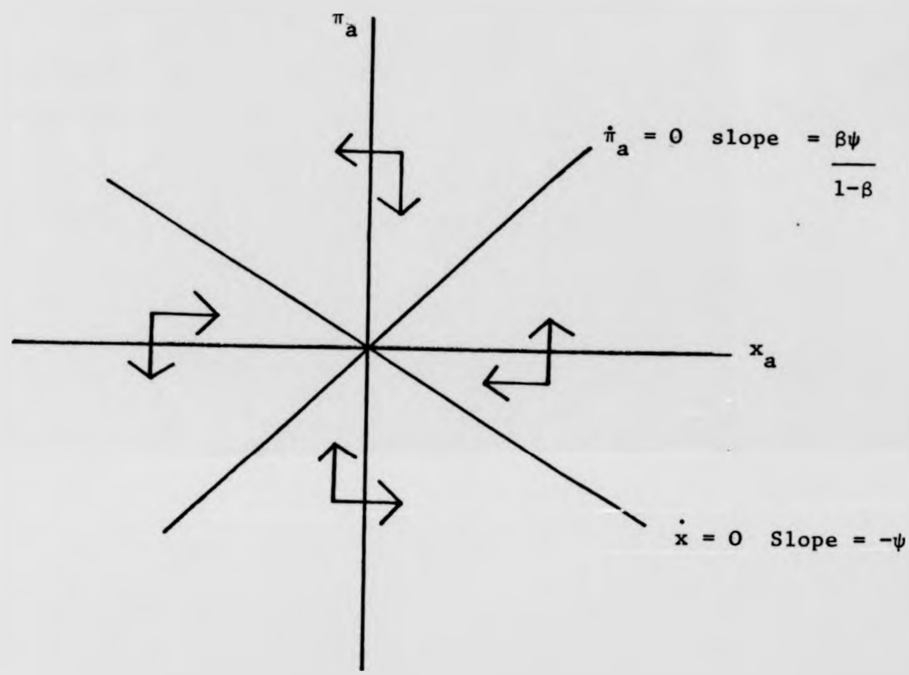
$$w_{1,2} = \frac{-(\beta\psi + \gamma(1-\beta) \pm \sqrt{(\beta\psi + \gamma(1-\beta))^2 - 4(\gamma\beta^2\psi+\beta\psi(1-\beta))}}{2}$$

Since, a priori, all coefficients are expected to be positive it can immediately be stated that the system is stable providing these expectations are not violated. The possibility of complex roots and therefore a cyclical approach to equilibrium exists if

$$4(\gamma\beta^2\psi+\beta\psi(1-\beta) > (\beta\psi+\gamma(1-\beta))^2$$

Since (6:12) is a (2x2) system it is possible to represent the dynamics of adjustment by means of a phase diagram. Figure one shows the equilibrium loci and the arrows of motion out of equilibrium for the case of zero money growth in both countries.

Figure One



The arrows of motion indicate that, as required, the system is globally stable.

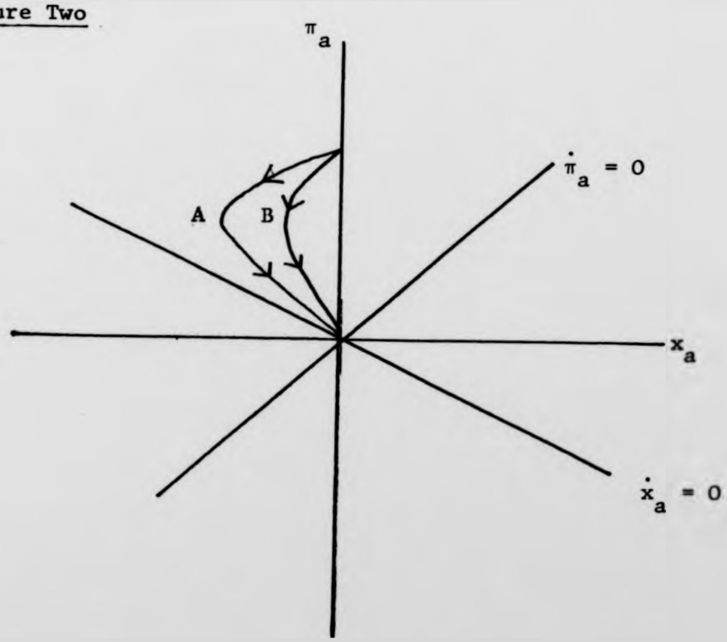
The nature of the choice facing the policy authorities can be analysed more clearly by looking at the equations of (6:12) separately. These can be rewritten

(6;12') $\quad \dot{x}_a = -\beta\psi x_a - \beta(\pi_a - \theta_a)$

(6:12") $\quad \dot{\pi}_a = \gamma\beta\psi x_a - \gamma(1-\beta)(\pi_a - \theta_a)$

In both cases the crucial variable determining the rate of change of the state variable is the gap between core inflation and the rate of monetary growth. Since core inflation is fixed instantaneously this can be viewed as the policy instrument of the authorities. Starting from an initial position where the world economy is at its natural level of output but there is positive inflation, the choice is whether to cut money growth quickly and achieve a quick fall in inflation at the expense of a sharp movement into recession or to adopt a more gradualist approach. The time paths which result from these strategies are illustrated in figure two.

Figure Two



Path A shows a 'cold turkey' strategy. The money growth rate is cut quickly and there is a short, but deep, recession. Path B shows a gradualist strategy the authorities keep the gap between core inflation and monetary growth small and the adjustment period is prolonged. However, along path B output doesn't fall too far below its natural level. The chosen path will tend to resemble B if the authorities put a large weight on output in their objective function. Another factor favouring a gradualist choice is an objective function which penalises large deviations from target level severely, such as the quadratic loss functions discussed in chapter four.

The above discussion of policy assumes a single controller with responsibility for managing the world economy as if it were a single, closed economy. And the discussion in chapter five showed, if there are several controllers and significant linkages between economies, the realised outcome is unlikely to approximate this. The important question then becomes the extent to which actual policy differs from that which would be chosen by a hypothetical single controller. Section two of this chapter examines this issue by means of numerical simulation of the model under various kinds of decentralised control.

Examination of the system in averages gives interesting results for the case in which both countries have the same initial rate of inflation. However, if the countries have different inflation rates then the relevant system becomes that in differences. This can be derived from (6:11) as:

$$(6:13) \qquad \begin{bmatrix} \dot{x}_d \\ \dot{\pi}_d \end{bmatrix} = \begin{bmatrix} -\beta\psi & -\beta \\ 0 & -\gamma \end{bmatrix} \begin{bmatrix} x_d \\ \pi_d \end{bmatrix} + \begin{bmatrix} \beta \\ \gamma \end{bmatrix} \theta_d$$

The eigenvalues of this state-transition matrix are derived by inspection as $-\beta\psi$ and $-\gamma$ implying stable adjustment to the steady-state. Again some additional insight can be gained by writing out the equations of (6:13) separately.

$$(6.13') \quad \dot{x}_d = -\beta\psi x_d - \beta((\pi-\theta) - (\pi^*-\theta^*))$$

$$(6.13'') \quad \dot{\pi}_d = -\gamma((\pi-\theta) - (\pi^*-\theta^*))$$

The rate of change of the variables in differences can be seen to depend on the relative severity of the deflationary strategies adopted. If the home country opens up a bigger gap between core inflation and the rate of monetary growth than the foreign country then it achieves a faster decline in inflation but at the cost of a sharper movement into recession.

Consider a situation in which both countries are initially at their natural levels of output but the home country has a zero rate of inflation while the foreign country has a positive rate. A cut in monetary growth by the foreign country will move it into recession and start a decline in inflation. However, the reduced amount of foreign output on the market moves the terms of trade against the home economy and produces inflation. This opens up a positive gap between core inflation and monetary growth and leads to recession in the home economy. This transmission mechanism is described by H-S as "a lower foreign rate of monetary expansion leads to a transitory period of stagflation at home".

Under decentralised control the home country will offset the deflationary strategy of the foreign country to some extent. It could cut its money growth rate so as to prevent some of adverse movement in the terms of trade which lead to an acceleration of inflation. Alternatively it could increase monetary growth in order to prevent a movement into recession. The strategy it adopts depends on the weights in the loss function. However, it is unlikely that the strategies adopted under decentralised control would be those chosen by a Pareto Efficient controller. Therefore the inefficiencies of uncoordinated control do not just result in inefficient movements of aggregate variables but also imply inefficient reactions to asymmetries in initial conditions. Without looking at specific numerical examples it is difficult to say in which direction the inefficiencies will occur, i.e. whether the resulting time path will be over contractionary or insufficiently restrictive.

6:3 Control simulations of the model

The purpose of this section is to examine the behaviour of the model under control and in particular the implications of decentralised control. Chapter four set out a number of non-cooperative differential game solution concepts. This chapter uses the Open-Loop Nash (OLN) and Closed-Loop Nash (CLN) regimes to describe decentralised control. These solutions are contrasted with the Pareto Efficient (PE) solution which is obtained by assuming a single controller whose loss function is a weighted average of the individual country's loss functions. Since the countries are treated symmetrically throughout these weights will be assumed equal.

The loss functions of the two countries are assumed to be quadratic in the log of output, core inflation and the rate of monetary growth. They can be written

$$(6{:}14) \qquad \int_0^\infty (q_x x^2 + q_\pi \pi^2 + r\theta^2)\, dt$$

$$\int_0^\infty (q_x x^{*2} + q_\pi \pi^{*2} + r\theta^{*2})\, dt$$

Despite the fact that the loss functions are undiscounted integrals they are defined since all variables approach a steady-state value of zero (for further discussion of this and the incorporation of a discount factor into the problem see chapter four). The choice of variables in the loss function deserves some comment. Output is an obvious variable to include but the choice of core inflation is not so clear. The reason for the inclusion of core inflation rather than any of the other possible indices (i.e. wage inflation, producer price inflation, consumer price inflation) is to keep the mathematical structure of the problem simple. This is because core inflation is not subject to discrete jumps at the beginning of the control period in the way that producer and consumer price inflation are, and does not complicate the calculation of the state transition matrix under control by requiring examination of cross-product terms in x and π in the way that wage inflation does. The final term θ is included to capture the costs of implementing the policy.

To determine the weights in the objective function the following procedure was implemented. First a single reference comparison was decided on, e.g. a 5% deviation of output from its natural level contributes as much to welfare loss as 10% inflation. This comparison was then used to find the implied relative weight on the output and inflation variables. Two alternatives were tried. Firstly a 5% shortfall in output was valued as highly as 10% inflation this resulted in a weight of 3.8 on the output variable when that on core inflation is 1.0. Secondly as 5% shortfall in output was valued as highly as 20% inflation this resulted in relative weights of 15.2 and 1.0. The weight on monetary growth was assumed to be unity throughout.

Open Loop Nash simulations

In chapter four the Open-Loop Nash solution concept was defined by the condition that agents announce control vectors over time which correspond to the minimum loss attainable given the control vectors of the other agents. Thus the strategy space is the value of monetary growth defined over the interval zero to infinity. The general format for the state-transition matrix under OLN control and the time path of the system can be found in chapter four.

<u>OLN Simulation 1</u>

The numerical values assumed for the parameters were as follows

$$\beta = \psi = \gamma = 0.5 \qquad q_x = 3.8 \qquad q_\pi = 1.0 \qquad r = 1.0$$

Since there are four predetermined state variables the state-transition matrix should have four negative or stable eigenvalues. These were derived as

-1.23143

-0.961277

-0.187475

-0.140512

None of the stable roots are complex implying that the approach to the steady-state is monotonic. The next stage of the numerical analysis was to examine the time path of the system. A starting value for the system was assumed in which each economy was at the natural level of output but inflation was 10%. Table one summarises the time path. Since the two countries are identical, the results for one country only are presented. Time periods are referred to as years for convenience.

Table one

OLN Control $q_x = 3.8$ $q_\pi = 1.0$ $r = 1.0$

T	x	π	θ	$\dot{w}$	$\dot{p}$	w/p	L_t/L_o
0	100.00	10.00	-4.00	10.00	3.00	.5000	100.00
1	95.92	7.29	1.00	5.21	3.11	.5212	85.59
2	95.08	5.66	2.92	3.14	3.14	.5258	67.21
3	95.32	4.54	3.14	2.15	2.65	.5245	49.30
4	95.88	3.72	2.88	1.61	2.25	.5214	34.93
5	96.49	3.06	2.49	1.27	1.88	.5182	24.35
6	97.01	2.53	2.11	1.03	1.55	.5152	16.85
7	97.53	2.09	1.76	0.84	1.31	.5126	11.61
8	97.95	1.73	1.47	0.69	1.09	.5105	7.99
9	98.29	1.44	1.22	0.57	0.90	.5087	5.50
10	98.58	1.19	1.01	0.48	0.74	.5072	3.78

Because the quadratic loss function punishes large deviations from targets heavily the optimal strategy is to cut monetary growth to negative rates during the first year of the planning period. As inflation falls and the economy moves into recession this restrictive stance is relaxed. Price inflation falls rapidly due to the cut in monetary growth and the movement into recession. Wage inflation and core inflation respond sluggishly to price inflation. Because both countries implement similar strategies there is no difference between consumer and producer price inflation and both the real and nominal exchange rates are constant. The real wage is higher than its natural level along the adjustment path due to the decline in output.

OLN Simulation 2

In order to assess the effects on the time path of changing the relative weights on output and inflation an alternative version was run with parameter values

$$\beta = \psi = \gamma = 0.5 \qquad q_x = 15.2 \qquad q_\pi = 1.0 \qquad r = 1.0$$

The stable eigenvalues of the state-transition matrix in this case are:

-2.117601
-1.923262
-0.092973
-0.082404

Again there are no complex roots so convergence to the steady-state is monotonic. A priori expectations suggest that a larger

Table 2

OLN Control $q_x = 15.2$ $q_\pi = 1.0$ $r = 1.0$

T	x	π	θ	$\dot{w}$	$\dot{p}$	w/p	L_t/L_o
0	100.00	10.00	-4.30	10.00	2.84	.5000	100.00
1	97.05	8.00	4.98	6.51	5.73	.5152	90.84
2	96.89	7.13	5.83	5.55	5.69	.5161	76.97
3	97.10	6.48	5.51	5.00	5.26	.5149	64.12
4	97.34	5.90	5.05	4.55	4.80	.5136	53.27
5	97.57	5.37	4.60	4.15	4.37	.5124	44.24
6	97.79	4.90	4.19	3.78	3.98	.5113	36.73
7	97.98	4.46	3.82	3.44	3.63	.5103	30.50
8	98.16	4.07	3.48	3.14	3.31	.5094	25.32
9	98.32	3.70	3.17	2.86	3.01	.5085	21.03
10	98.47	3.38	2.89	2.60	2.75	.5078	17.46

Figure 3a

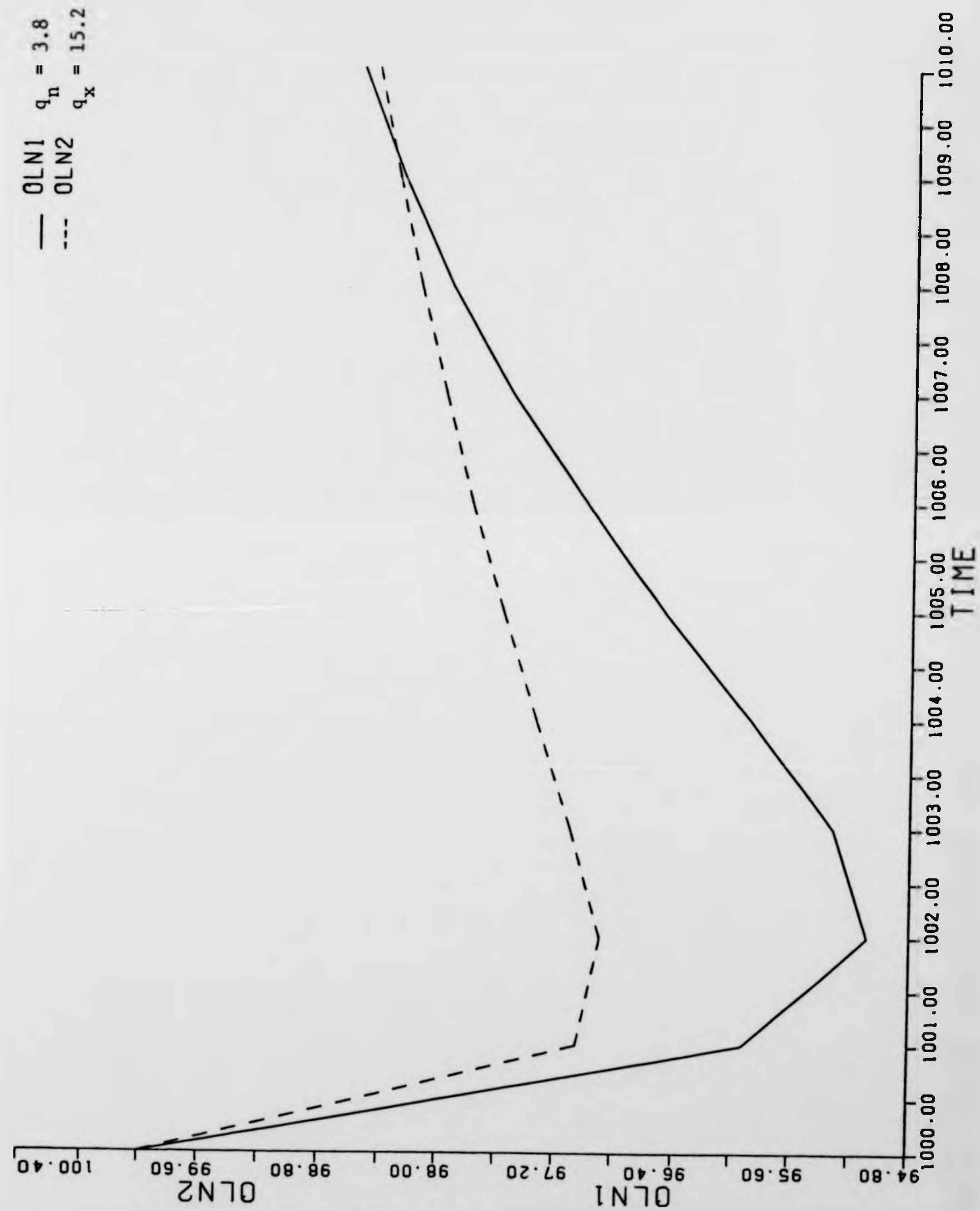

OLN1 q_n = 3.8
OLN2 q_x = 15.2
TIME
1000.00
1001.00
1002.00
1003.00
1004.00
1005.00
1006.00
1007.00
1008.00
1009.00
1010.00
OLN1
OLN2
94.80
95.60
96.40
97.20
98.00
98.80
99.60
100.40

Figure 3b

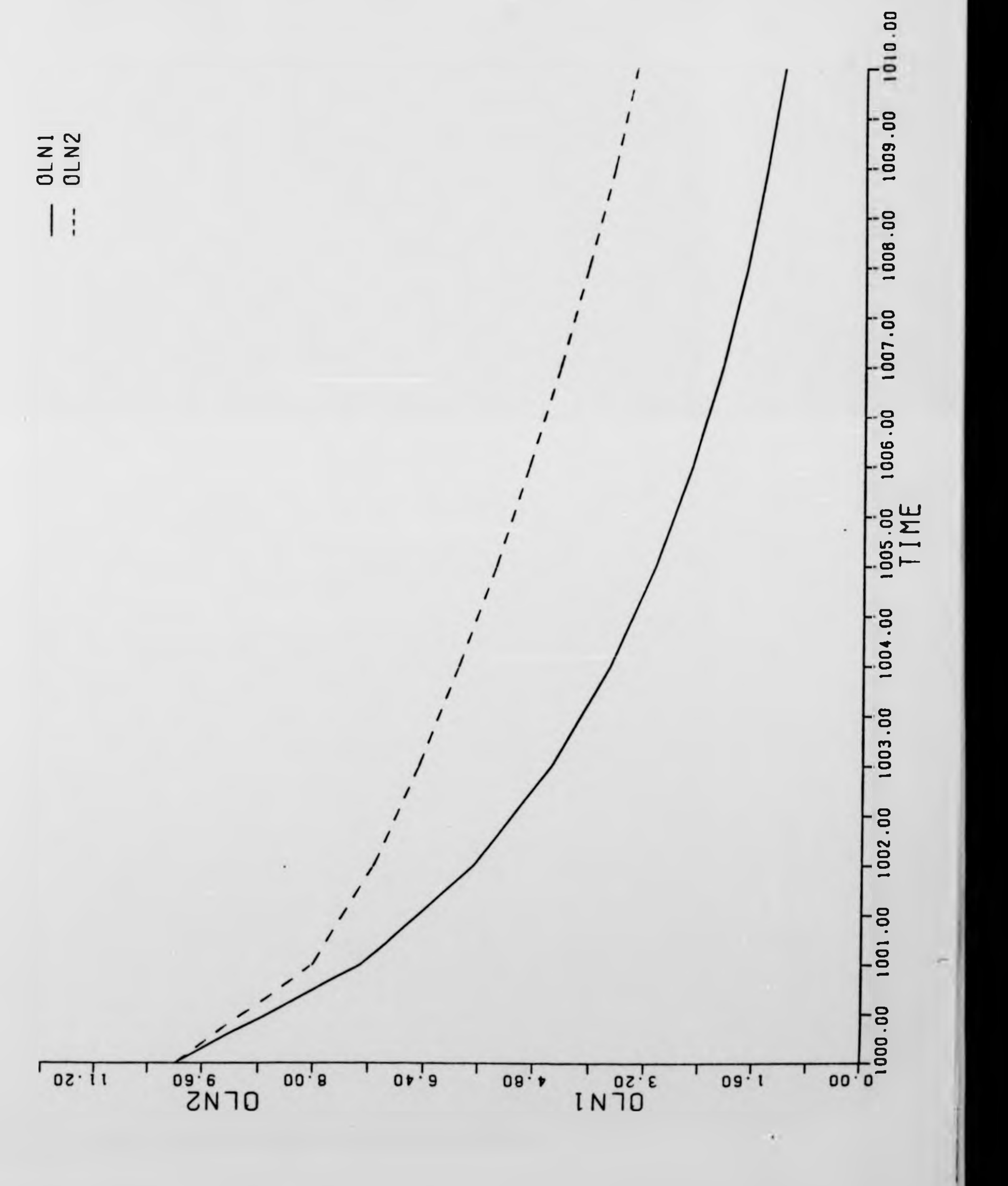
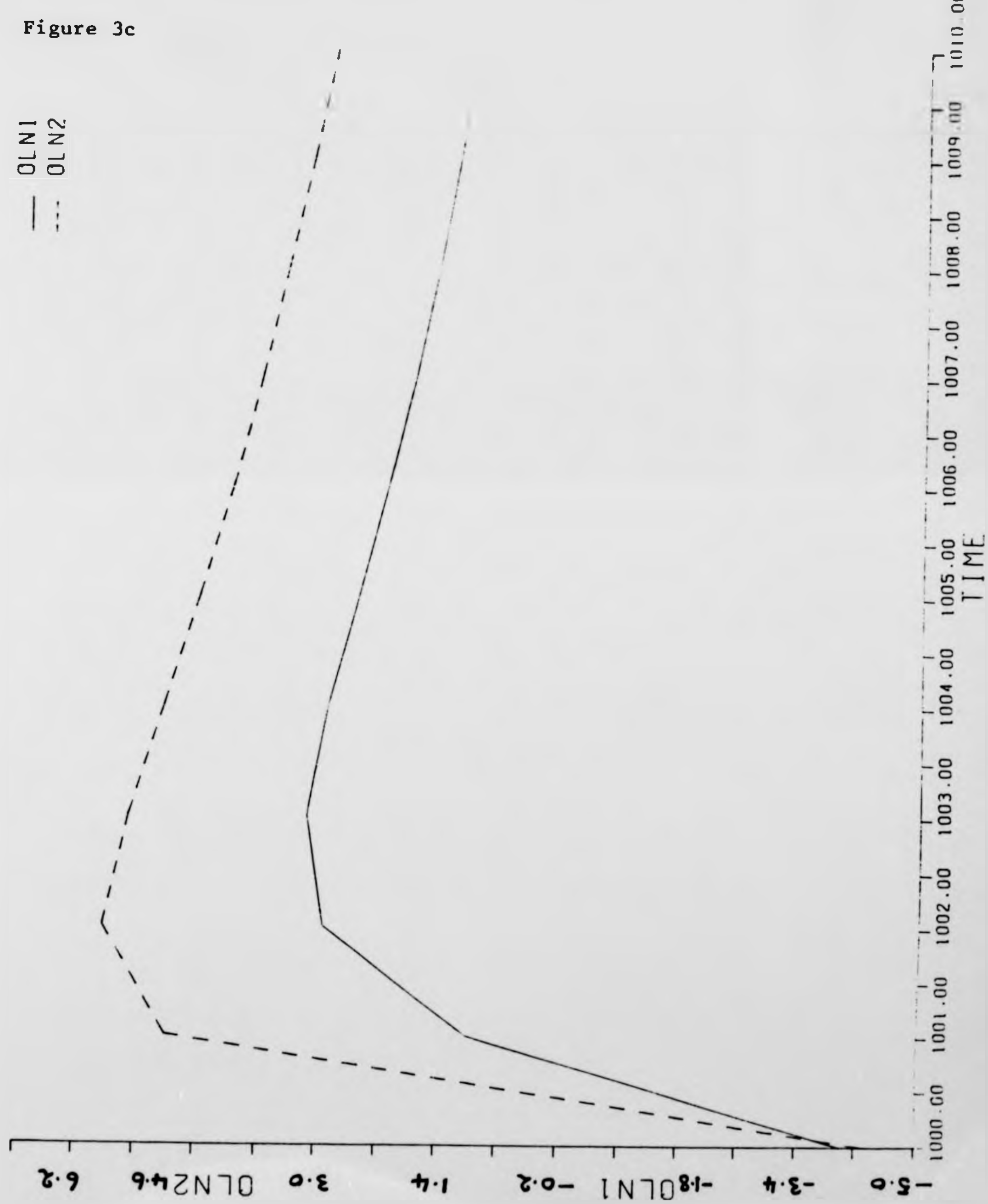

OLN1
OLN2
TIME
1000.00
1001.00
1002.00
1003.00
1004.00
1005.00
1006.00
1007.00
1008.00
1009.00
1010.00
OLN1
0.00
1.60
3.20
4.80
6.40
8.00
9.60
11.20
OLN2

Figure 3c


OLN1
OLN2
TIME
1000.00
1001.00
1002.00
1003.00
1004.00
1005.00
1006.00
1007.00
1008.00
1009.00
1010.00
OLN1
OLN2
-5.0
-3.4
-1.8
-0.2
1.4
3.0
4.6
6.2

weight on output will lead to slower adjustment with output remaining closer to its natural level. This is confirmed by examination of the time path of the system given in table two. The qualitative nature of the adjustment process does not change but the actual trajectory does. Figure three illustrates the time paths of output, core inflation and monetary growth and compares them with the case of $q_x = 3.8$.

CLN Simulations

The OLN solution corresponds to an equilibrium in strategies defined on the control vectors. An alternative is to define the strategy space in terms of reaction functions. This was discussed in chapter four and an algorithm was suggested for the derivation of the closed-loop solution. This algorithm was used to calculate the CLN solution for the same parameter values used in the OLN problem. The first set of parameters is

$$\beta = \psi = \gamma = 0.5 \quad q_x = 3.8 \quad q_\pi = 1.0 \quad r = 1.0$$

The state-transition matrix for this problem can be derived by numerical iteration. This yields four stable eigenvalues:

-1.231206
-0.960429
-0.187925
-0.140686

Table Three

CLN Control $q_x = 3.8$ $q_\pi = 1.0$ $r = 1.0$

T	x	π	θ	$\dot{w}$	$\dot{p}$	w/p	L_t/L_o
0	100.00	10.00	-4.44	10.00	2.78	.5000	100.00
1	95.91	7.50	1.24	5.20	3.22	.5213	85.30
2	95.08	5.66	2.91	3.13	3.02	.5259	66.89
3	95.32	4.54	3.14	2.14	2.65	.5245	49.02
4	95.88	3.71	2.87	1.60	2.24	.5215	34.71
5	96.49	3.05	2.49	1.26	1.88	.5182	24.17
6	97.05	2.52	2.10	1.02	1.56	.5152	16.71
7	97.53	2.08	1.76	0.84	1.30	.5126	11.51
8	97.95	1.73	1.46	0.69	1.08	.5105	7.91
9	98.29	1.43	1.21	0.57	0.89	.5087	5.44
10	98.58	1.18	1.01	0.47	0.74	.5072	3.73

These show little change from the OLN case suggesting that the CLN time path will also be similar. This is confirmed by examination of the time path in Table three.

For closed-loop solutions it becomes natural to express the model in feedback form in open-loop cases. The similarity between the two is another measure of the similarity of the OLN and CLN solutions. This can be seen in Table four.

Table Four

	Hypothetical OLN feedback form	CLN feedback control coefficients
Own output	-0.9681	-0.9664
Foreign output	-0.1085	-0.1083
Own inflation	-0.4129	-0.4148
Foreign inflation	-0.0287	-0.0291

The results are similar for the alternative set of parameter values, i.e.

$\beta = \psi = \gamma \quad q_x = 15.2 \quad q_\pi = 1.0 \quad q_r = 1.0$

This yields negative eigenvalues

-2.116896
-1.92218
-0.09311
-.082493

which are again very similar to the OLN values. Table five shows the time paths of the variables of interest which differ little from those under OLN control. Table six gives the feedback control coefficients.

<u>Table Five</u>

CLN Control $q_x = 15.2$ $q_\pi = 1.0$ $r = 1.0$

T	x	π	θ	$\dot{w}$	$\dot{p}$	w/p	L_t/L_o
0	100.00	10.00	-4.31	10.00	2.84	.5000	100.00
1	97.05	8.00	4.98	6.51	5.73	.5152	90.75
2	96.88	7.13	5.83	5.55	5.69	.5161	76.87
3	97.09	6.47	5.50	5.00	5.25	.5149	64.02
4	97.34	5.89	5.04	4.55	4.79	.5136	53.17
5	97.57	5.37	4.60	4.14	4.37	.5124	44.14
6	97.79	4.89	4.19	3.77	3.98	.5113	36.64
7	97.98	4.46	3.82	3.44	3.63	.5103	30.41
8	98.16	4.06	3.48	3.13	3.31	.5094	25.25
9	98.32	3.69	3.17	2.85	3.01	.5085	20.96
10	98.47	3.37	2.89	2.60	2.74	.5078	17.40

Table Six

	Hypothetical OLN feedback form	CLN feedback control coefficients
Own output	-2.6606	-2.6582
Foreign output	-0.1566	-0.1568
Own inflation	-0.4132	-0.4144
Foreign inflation	-0.0172	-0.0174

The conclusion to be drawn from these simulations is that the choice of solution concept to describe non-cooperative behaviour does not really make much difference. OLN and CLN solutions produce almost identical time paths.

Pareto Efficient Simulations

In chapter four a Pareto Efficient solution was derived by assuming a single controller with a loss function consisting of a weighted average of the individual countries' loss functions. In this chapter this solution is used to characterise cooperative behaviour among countries. The interesting question is whether the PE solution can improve welfare relative to the non-cooperative solutions, OLN and CLN.

Taking the first set of parameters, i.e. that with $q_x = 3.8$ yields a state-transition matrix with stable roots.

-1.221090

-0.990269

-0.144746

-0.178514

Table Seven

PE Control $q_x = 3.8$ $q_\pi = 1.0$ $r = 1.0$

T	x	π	θ	$\dot{w}$	$\dot{p}$	w/p	L_t/L_o
0	100.00	10.00	-4.14	10.00	2.93	.5000	100.00
1	96.03	7.36	1.53	5.34	3.43	.5207	85.48
2	95.22	5.79	3.13	3.34	3.24	.5251	67.41
3	95.45	4.70	3.31	2.37	2.84	.5238	49.91
4	95.98	3.88	3.02	1.83	2.43	.5209	35.29
5	96.55	3.23	2.63	1.47	2.05	.5178	25.36
6	97.08	2.69	2.23	1.21	1.72	.5150	17.84
7	97.54	2.25	1.88	1.00	1.44	.5126	12.51
8	97.94	1.88	1.58	0.84	1.21	.5105	8.76
9	98.27	1.57	1.32	0.70	1.01	.5088	6.13
10	98.55	1.31	1.11	0.59	0.85	.5073	4.29

Figure 4a

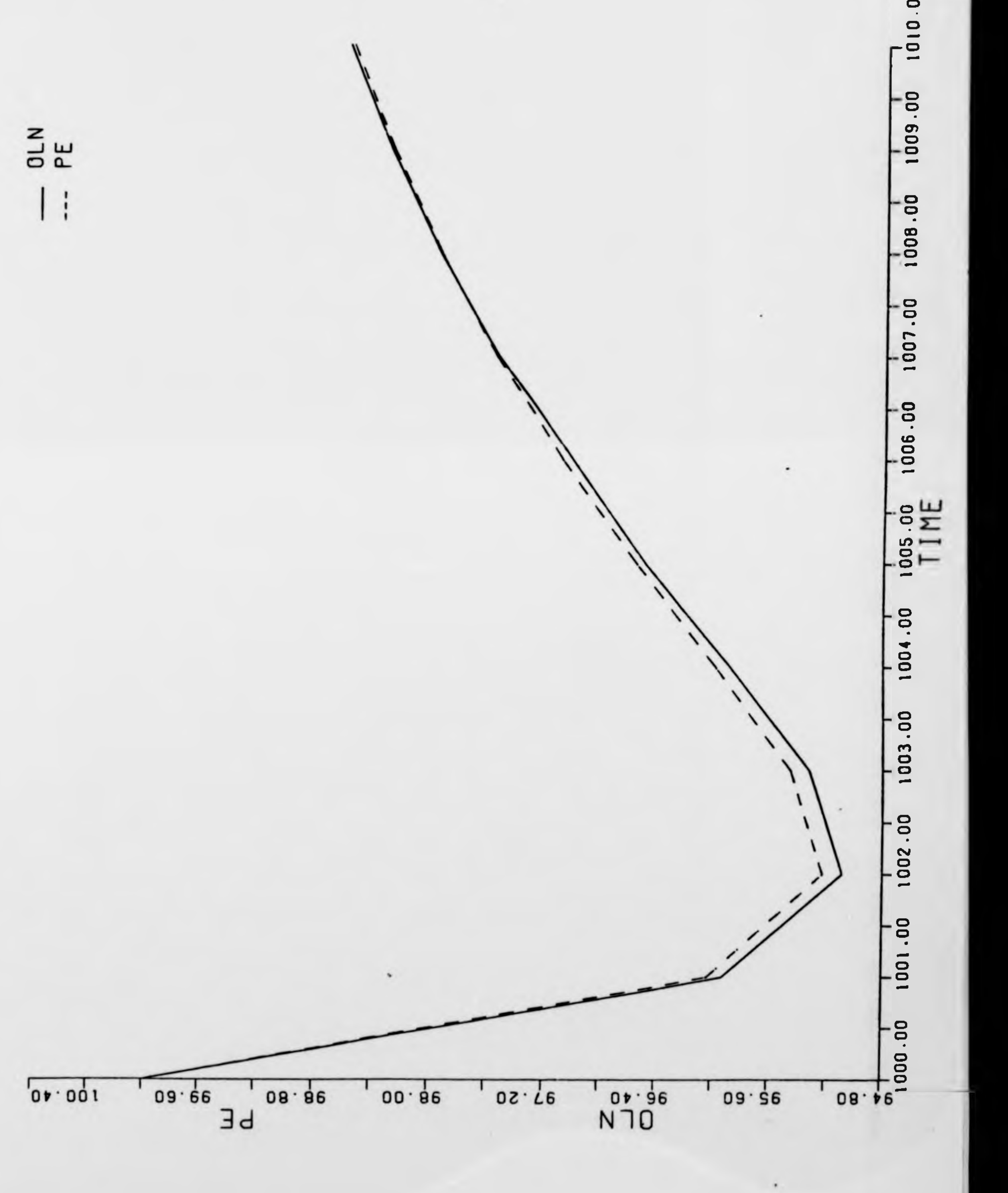
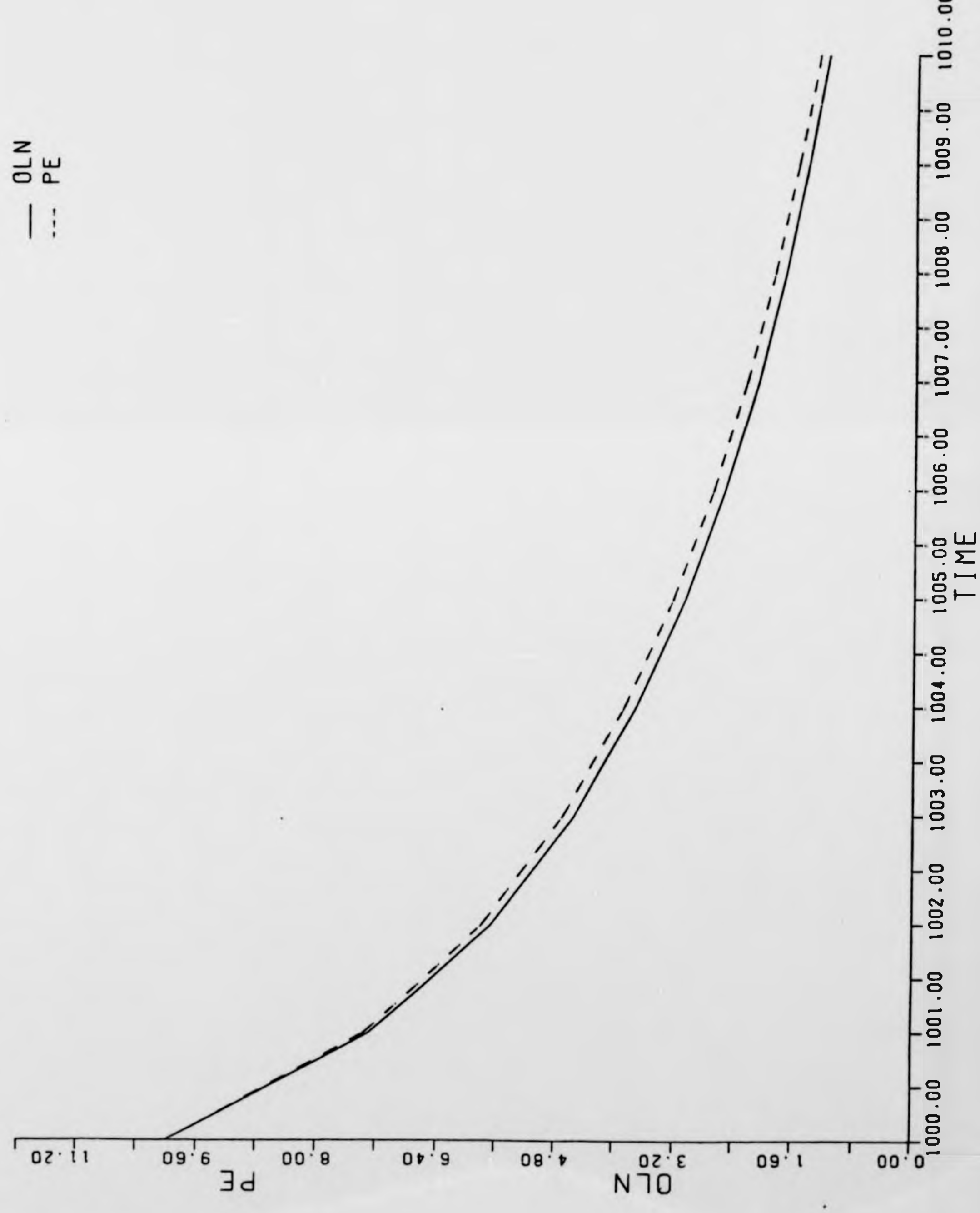

OLN
PE
TIME
1000.00
1001.00
1002.00
1003.00
1004.00
1005.00
1006.00
1007.00
1008.00
1009.00
1010.00
94.80
95.60
96.40
97.20
98.00
98.80
99.60
100.40

Figure 4b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PE
TIME
OLN
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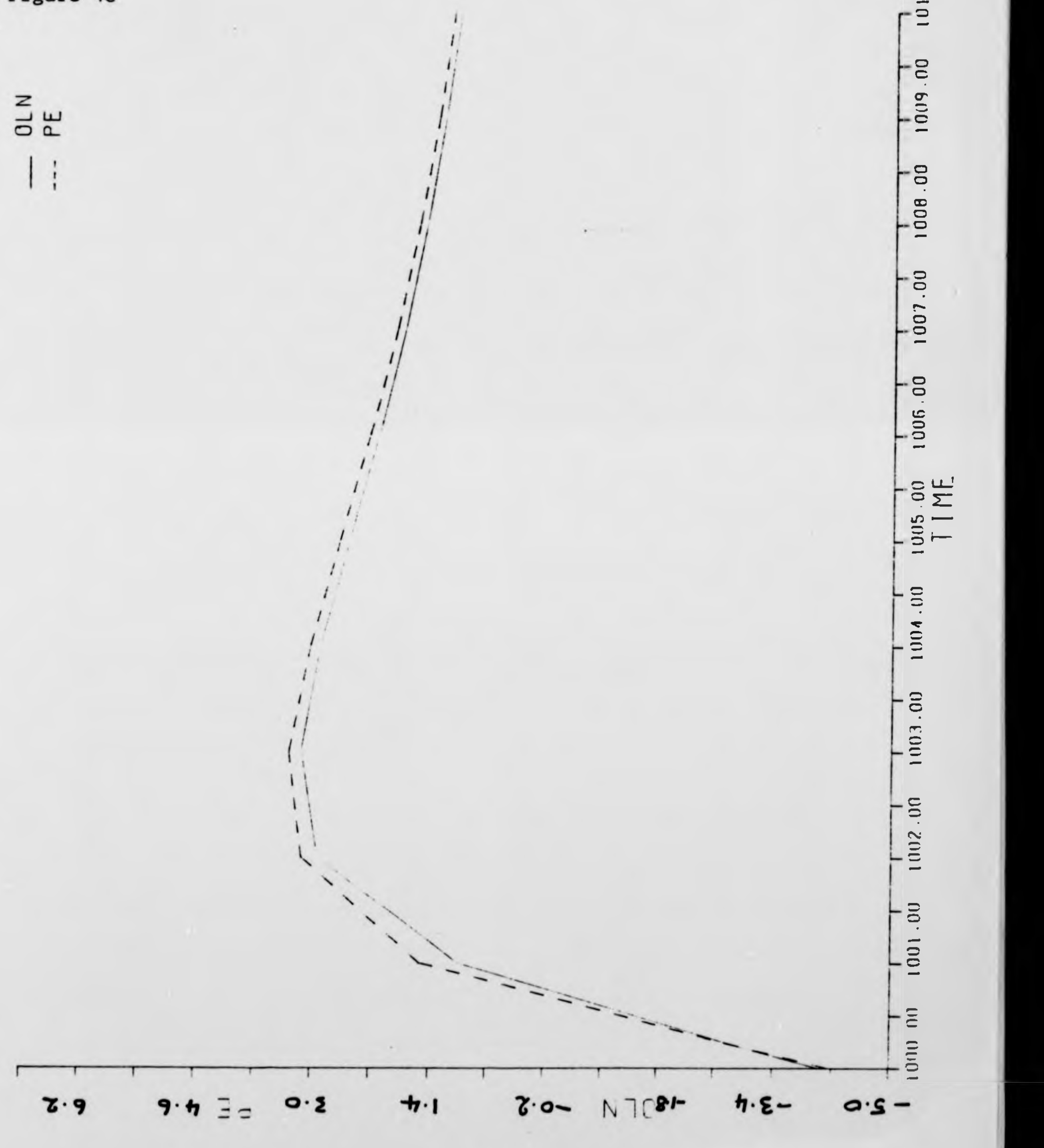
1010.00

Figure 4c


OLN
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TIME
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1010.00

The time path of the system is given in table seven. For the first eight periods of the policy the PE output level is above that for OLN control but the difference never exceeds 0.13 percentage points. Similarly for core inflation the maximum difference between the OLN and PE values is 0.17 percentage points. The similarity between the paths can be seen in Figure four.

The similarity of the uncoordinated and coordinated time paths is also the case for the model with a larger weight on inflation with $q_x = 15.2$ the stable eigenvalues of the state-transition matrix are:

-2.086942

-1.963261

-0.084706

-0.090042

These are similar to the OLN and CLN values suggesting that the time path will also be similar. This is confirmed by examination of table eight and figure five.

In both cases therefore the OLN and PE time paths are remarkably similar. This suggests that policy coordination will lead to relatively minor welfare gains, at least within the context of this model. Table nine gives the total welfare loss over the interval zero to infinity under the various types of control, taking OLN = 100.00.

Table Eight

PE Control $q_x = 15.2$ $q_\pi = 1.0$ $r = 1.0$

T	x	π	θ	$\dot{w}$	$\dot{p}$	w/p	L_t/L_o
0	100.00	10.00	-4.14	10.00	2.93	.5000	100.00
1	97.12	8.05	5.13	6.51	5.86	.5148	90.86
2	96.97	7.20	5.94	5.55	5.80	.5156	77.28
3	97.17	6.56	5.60	5.00	5.36	.5146	64.74
4	97.40	5.99	5.15	4.55	4.91	.5133	54.09
5	97.62	5.48	4.71	4.15	4.49	.5122	45.18
6	97.82	5.01	4.30	3.78	4.10	.5111	37.74
7	98.01	4.57	3.93	3.44	3.75	.5101	31.52
8	98.18	4.18	3.59	3.14	3.43	.5093	26.32
9	98.33	3.82	3.28	2.86	3.13	.5085	21.99
10	98.47	3.49	3.00	2.60	2.86	.5077	18.36

Figure 5a

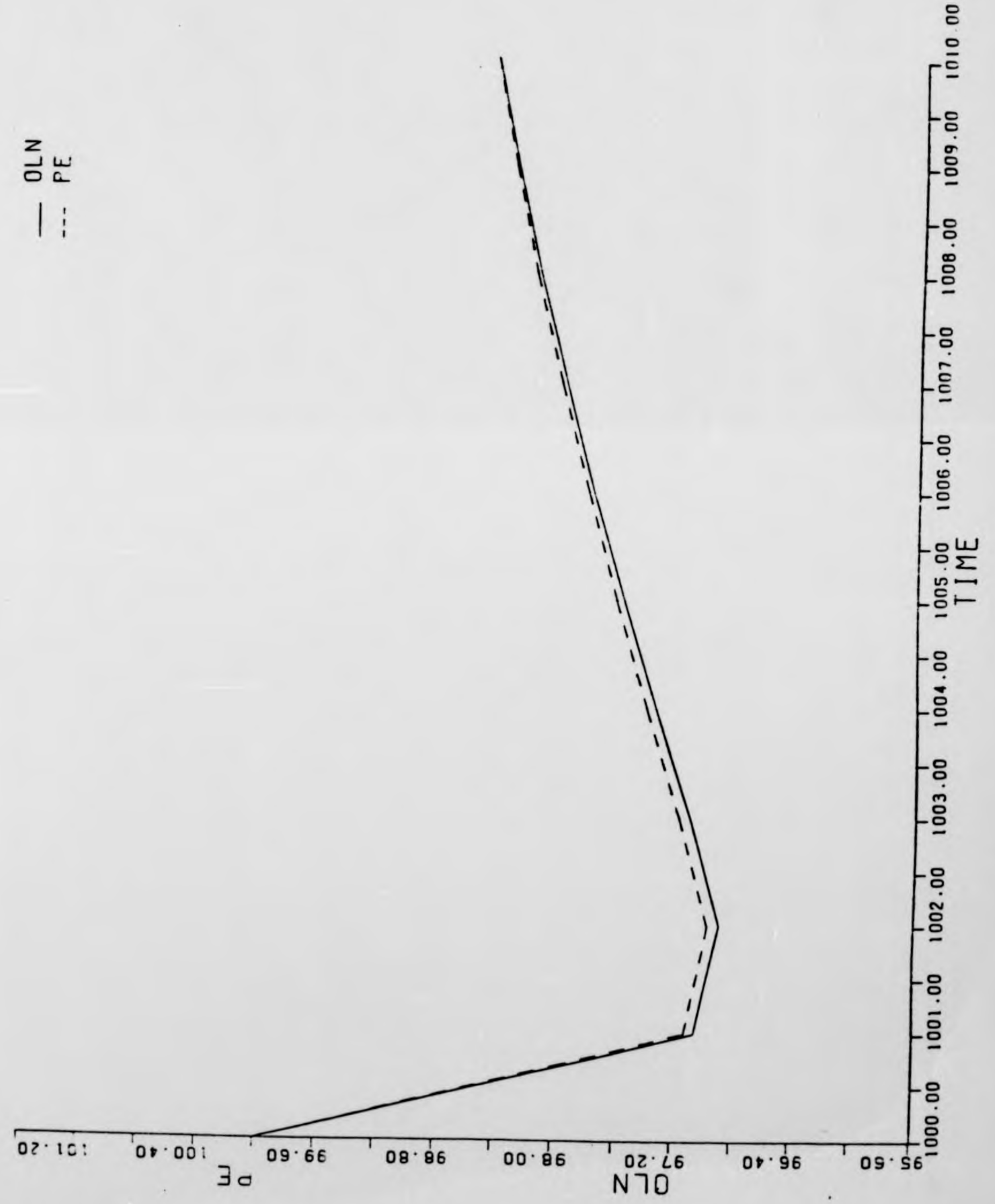
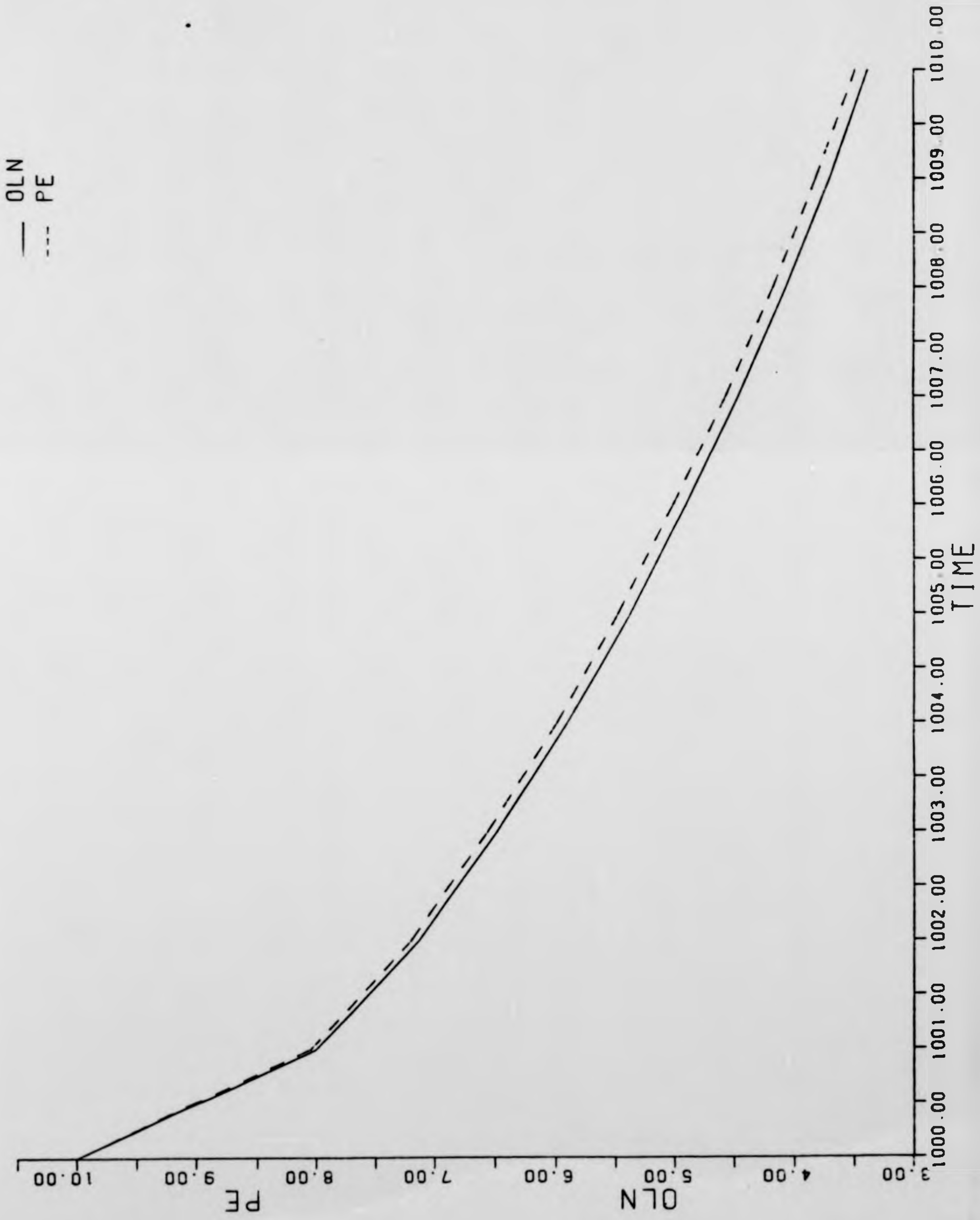


Figure 5b


OLN
PE
TIME
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1003.00
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1009.00
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PE
OLN
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Figure 5c

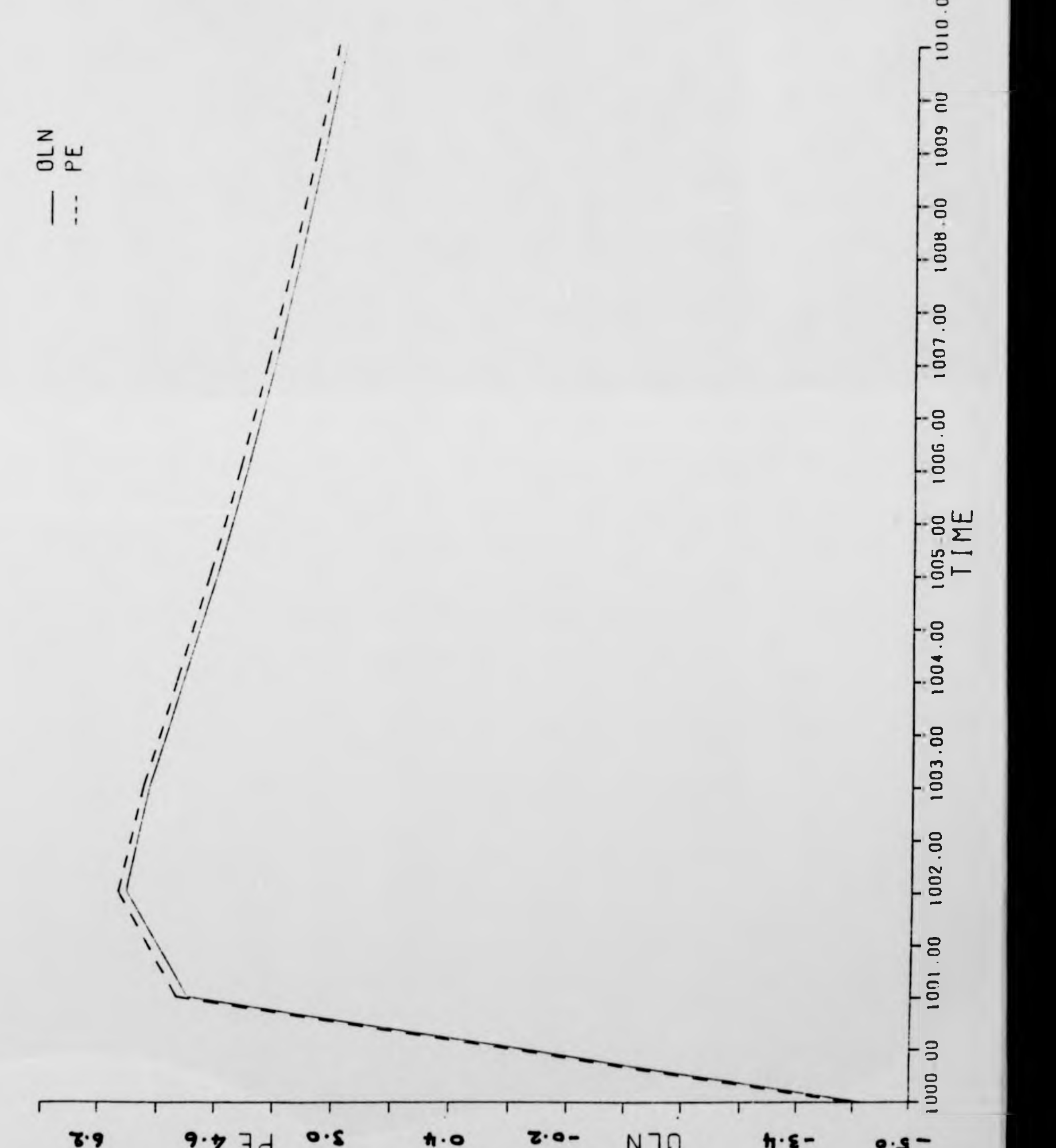


Table Nine

Welfare Loss from 0 to ∞

	$q_x = 3.8$	$q_x = 15.2$
OLN	100.00	100.00
CLN	98.69	98.18
PE	98.55	98.11

Table Nine confirms that there is relatively little improvement in welfare to be made by coordination of policy. A more promising way of reducing welfare losses would be to concentrate on improving the efficiency of policy for example, by speeding up the rate of adjustment of core inflation.

6:4 Real Wage Resistance

The H-S model discussed in section 6:2 gives no role to real wage resistance in the wage adjustment process. Since this was found to be of considerable importance in determining the solution path of the simple dynamic problem presented in chapter five the H-S model is now extended to incorporate this feature. This generates a model with considerably more complex dynamics than that of chapter five and therefore requires numerical methods to assess its control properties. Simulations for the extended model are presented in section 6:5.

As in chapter five the Phillips curve is assumed to be of the form

$$(6:15)\qquad \dot{w} = \psi x + \pi + \delta(w - p_c - Rw)$$

$$\dot{w}^* = \psi x^* + \pi^* + \delta(w^* - p_c{}^* - Rw^*)$$

i.e. wage inflation is sensitive to output, core inflation and the deviation of the log of the real wage as perceived by consumers from some target value, Rw. The log of the real wage can be written

$$(6:16)\qquad w - p_c = (w-p) - (1-\alpha)(p^* + e - p)$$

$$w^* - p_c = (w^* - p^*) - \alpha(p - p^* - e)$$

(6:16) decomposes the real consumer wage into two components. The first is the real product wage or the real wage as perceived by firms. The second is the terms of trade weighted by the expenditure share on foreign products. Since the home and foreign economies are symmetric the development of the model is continued in terms of the home economy only, also since it is desirable to retain the assumption of structural symmetry α is set equal to 0.5.

The log of the real product wage can be derived from the production function as

(6:17) $$w-p = \ln\beta - \frac{1-\beta}{\beta} x$$

The terms of trade can also be expressed in terms of output as

(6:18) $$p^* + e - p = x - x^*$$

Without significant loss of generality it is assumed that the target real wage is that which is consistent with the natural level of output, i.e. $Rw = \text{In}\beta$. This enables the wage adjustment equation to be rewritten

(6:19) $$\dot{w} = (\psi + \delta \left(\frac{1-\beta}{\beta}\right) + .5\delta)\ x - .5\delta x^* + \pi$$

An increase in output increases wage inflation for three reasons, firstly the straightforward Phillips curve effect via the parameter ψ, secondly because it lowers the real product wage as the marginal productivity of labour declines and thirdly because it causes an adverse movement in the terms of trade. Foreign output has a negative influence on domestic wage inflation because an increase in it will move the terms of trade in favour of the home country and thereby increase the real consumer wage. Note that

(6:19) differs from the wage adjustment equation in chapter five because it allows the real product wage to vary with output where previously it had been fixed due to the assumption of a constant mark-up of prices on wages.

The derivation of the minimal state-space representation of the model follows that in section 6:2. Combining (6:5), (6:6) and (6:19) leads to the differential equations in output.

$$\text{(6:20)} \qquad \begin{aligned} \dot{x} &= \beta\theta - (\beta\psi + \delta(1-\beta) + .5\delta\beta)x + .5\delta\beta x^* - \beta\pi \\ \dot{x}^* &= \beta\theta^* - (\beta\psi + \delta(1-\beta) + .5\delta\beta)x^* + .5\delta\beta x - \beta\pi^* \end{aligned}$$

The differential equations in core inflation can be written

$$\text{(6:21)} \qquad \begin{aligned} \dot{\pi} &= \gamma(\theta - .5x - .5x^* - \pi) \\ \dot{\pi}^* &= \gamma(\theta^* - .5x - .5x^* - \pi^*) \end{aligned}$$

Combining (6:20) and (6:21) enables the state-space representation of the model to be written in matrix form.

$$\begin{bmatrix} \dot{x} \\ \dot{x}^* \\ \dot{\pi} \\ \dot{\pi}^* \end{bmatrix} = \begin{bmatrix} -(\beta\psi+\delta(1-.5\beta)) & .5\delta\beta & -\beta & 0 \\ .5\delta\beta & -(\beta\psi+\delta(1-.5\beta)) & 0 & -\beta \\ .5\gamma(\beta\psi+\delta(1-\beta) & .5\gamma(\beta\psi+\delta(1-\beta)) & -\gamma(1-.5\beta) & .5\gamma\beta \\ .5\gamma(\beta\psi+\delta(1-\beta)) & .5\gamma(\beta\psi+\delta(1-\beta)) & .5\gamma\beta & -\gamma(1-.5\beta) \end{bmatrix} \begin{bmatrix} x \\ x^* \\ \pi \\ \pi^* \end{bmatrix}$$

$$+ \begin{bmatrix} \beta & 0 \\ 0 & \beta \\ \gamma(1-.5\beta) & -.5\gamma\beta \\ -.5\gamma\beta & \gamma(1-.5\beta) \end{bmatrix} \begin{bmatrix} \theta \\ \theta^* \end{bmatrix}$$

This can be reduced to a manageable order of dynamics by the process of taking averages and differences. The system in averages can be derived as

$$(6{:}23)\qquad \begin{bmatrix} \dot{x}_a \\ \dot{\pi}_a \end{bmatrix} = \begin{bmatrix} -(\beta\psi+\delta(1-\beta)) & -\beta \\ \gamma(\beta\psi+\delta(1-\beta)) & -\gamma(1-\beta) \end{bmatrix} \begin{bmatrix} x_a \\ \pi_a \end{bmatrix} + \begin{bmatrix} \beta \\ \gamma(1-\beta) \end{bmatrix} \theta_a$$

This system differs from that derived for the model without real wage resistance (6:12) in that output has a more powerful effect on both its own rate of change and that of core inflation. The eigenvalues of the state-transition matrix can be derived as:

$$w_{1,2} = \frac{-(a+\gamma(1-\beta) \pm\sqrt{(a+\gamma(1-\beta))^2 - 4(a\gamma(1-\beta) + \beta\psi)}}{2}$$

$$a \equiv \beta\psi + \delta(1-\beta)$$

which are both negative provided all coefficients have their expected positive sign, however the possibility of a cyclical adjustment path, due to complex roots, does exist.

The equations of (6:23) can be written separately as:

$$(6{:}23')\qquad \dot{x}_a = -(\beta\psi+\delta(1-\beta))x_a - \beta(\pi_a-\theta_a)$$

$$(6{:}23'')\qquad \dot{\pi}_a = \gamma(\beta\psi+\delta(1-\beta))x_a - \gamma(1-\beta)(\pi_a-\theta_a)$$

As in the previous model the restrictiveness of the monetary stance of the authorities can be assessed by looking at the gap between core inflation and the rate of monetary growth. The impact effect of this variable on the rates of change of output and core inflation remains the same as that in the earlier model but adjustment of the system will be faster due to the larger coefficients on average output.

The dynamics of the system in differences can be derived as

$$(6:24)\quad \begin{bmatrix} \dot{x}_d \\ \dot{\pi}_d \end{bmatrix} = \begin{bmatrix} -(\beta\psi+\delta) & -\beta \\ 0 & -\gamma \end{bmatrix} \begin{bmatrix} x_d \\ \pi_d \end{bmatrix} + \begin{bmatrix} \beta \\ \gamma \end{bmatrix} \theta_d$$

This system has two stable roots $-(\beta\psi+\delta)$ and $-\gamma$. Since $-(\beta\psi+\delta)$ is greater in absolute value than $-\beta\psi$ adjustment of the system in differences is faster in this case than in the case of no real wage resistance.

6:5 Control simulations of the model with the real wage effect

The control simulations of section 6:3 showed little welfare loss from decentralised control. Indeed if there were any significant costs attached to implementing a cooperative solution then decentralised control would be likely to be the more efficient system. It is possible that this result will not hold when real wage resistance is a feature of the model. This is because the real wage effect provides a much richer source of interaction between the economies. To examine this issue control simulations were run with parameters identical to those used previously except for the inclusion of a real wage term. Since OLN and CLN time paths have been found to be almost identical in the previous simulations, this section uses the OLN solution only to characterise uncoordinated behaviour. The loss functions of the policy authorities remain those specified in (6:14)

OLN and PE Simulations

For both simulations the parameter values are

$\beta = \psi = \gamma = 0.5 \quad q_x = 3.8 \quad q_\pi = 1.0 \quad r = 1.0$

The real wage resistance coefficient δ is assumed to have a value 0.5. The eigenvalues of the state-transition matrix under OLN control are:

-1.400903

-0.892846

-0.362065

-0.411880

There are no complex roots so the system should exhibit monotonic convergence. Table ten gives the time paths of the variables of interest. The model was also examined for the case $q_x = 15.2$ but since the results are qualitatively similar these results are not presented here.

Under PE control the stable roots are

-1.364333

-0.970749

-0.388710

-0.364207

Table eleven gives the time paths of the variables of interest.

Table Ten

OLN Control with Real Wage Resistance $q_x = 3.8$

T	x	π	θ	$\dot{w}$	$\dot{p}$	w/p	L_1/L_o
0	100.00	10.00	-4.71	10.00	2.64	.5000	100.00
1	96.21	6.86	-0.15	2.99	1.05	.5098	71.96
2	95.94	4.64	1.44	0.49	0.96	.5104	43.93
3	96.67	3.11	1.50	-0.28	0.61	.5086	23.42
4	97.52	2.08	1.22	-0.44	0.39	.5063	11.54
5	98.24	1.38	0.81	-0.39	0.21	.5044	5.42
6	98.79	0.92	0.63	-0.30	0.16	.5031	2.48
7	99.18	0.61	0.43	-0.22	0.11	.5021	1.12
8	99.45	0.40	0.29	-0.15	0.07	.5014	0.49
9	99.63	0.27	0.20	-0.10	0.05	.5009	0.22
10	99.75	0.18	0.13	-0.07	0.03	.5006	0.09

Table Eleven

PE Control with Real Wage Resistance $q_x = 3.8$

T	x	π	θ	$\dot{w}$	$\dot{p}$	w/p	L_t/L_o
0	100.00	10.00	-4.14	10.00	2.93	.5000	100.00
1	96.38	7.00	0.65	3.32	1.98	.5188	71.97
2	96.12	4.89	1.78	0.93	1.36	.5202	44.75
3	96.78	3.40	1.74	0.13	0.94	.5166	24.77
4	97.55	2.37	1.40	-0,11	0.65	.5125	12.87
5	98.22	1.65	1.05	-0.15	0.45	.5091	6.46
6	98.73	1.14	0.75	-0.13	0.31	.5064	3.19
7	99.11	0.79	0.54	-0.10	0.22	.5045	1.56
8	99.37	0.55	0.38	-0.07	0.15	.5031	0.75
9	99.56	0.38	0.26	-0.05	0.10	.5022	0.37
10	99.69	0.26	0.18	-0.04	0.07	.5015	0.18

Figure 6a

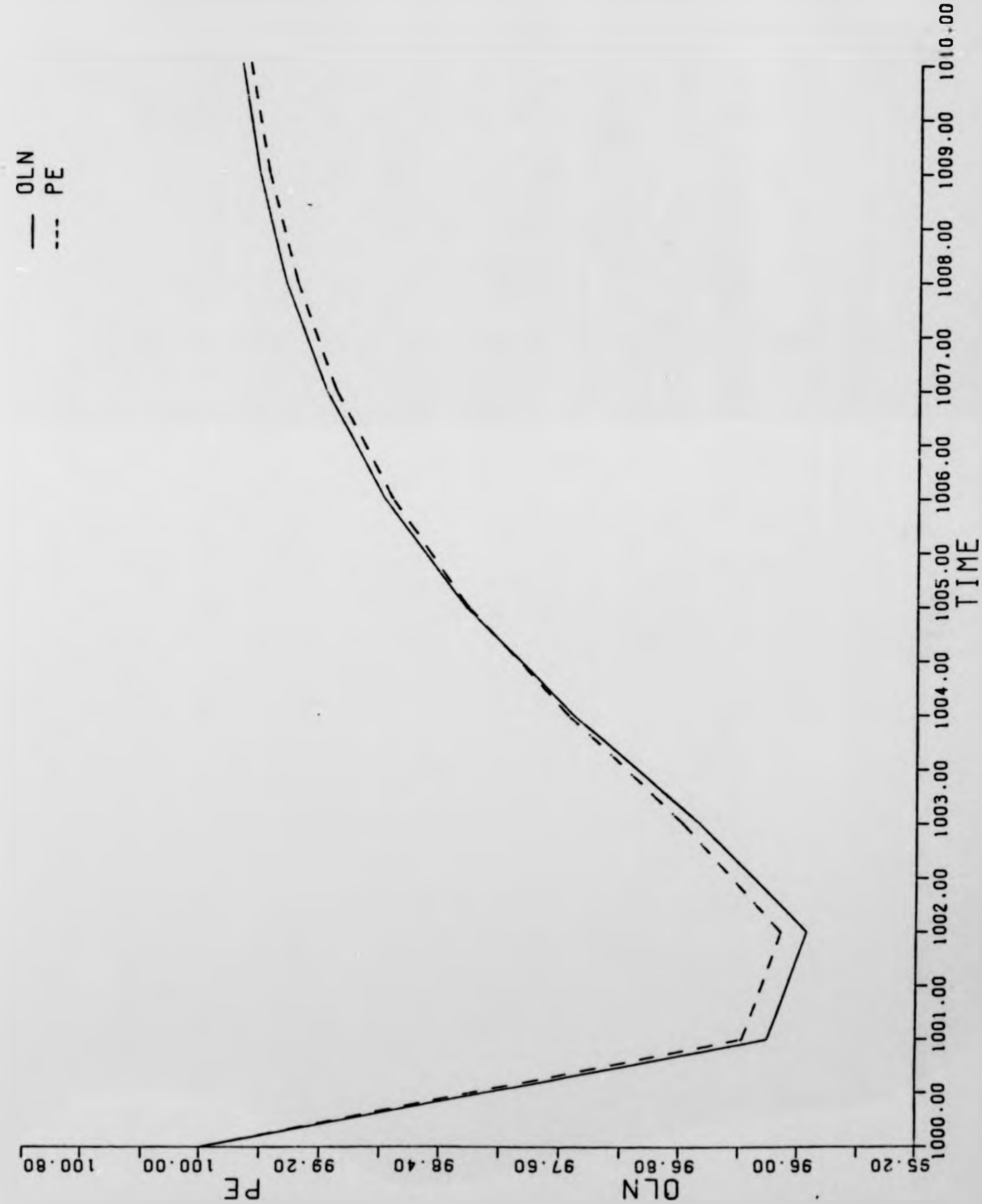


Figure 5b

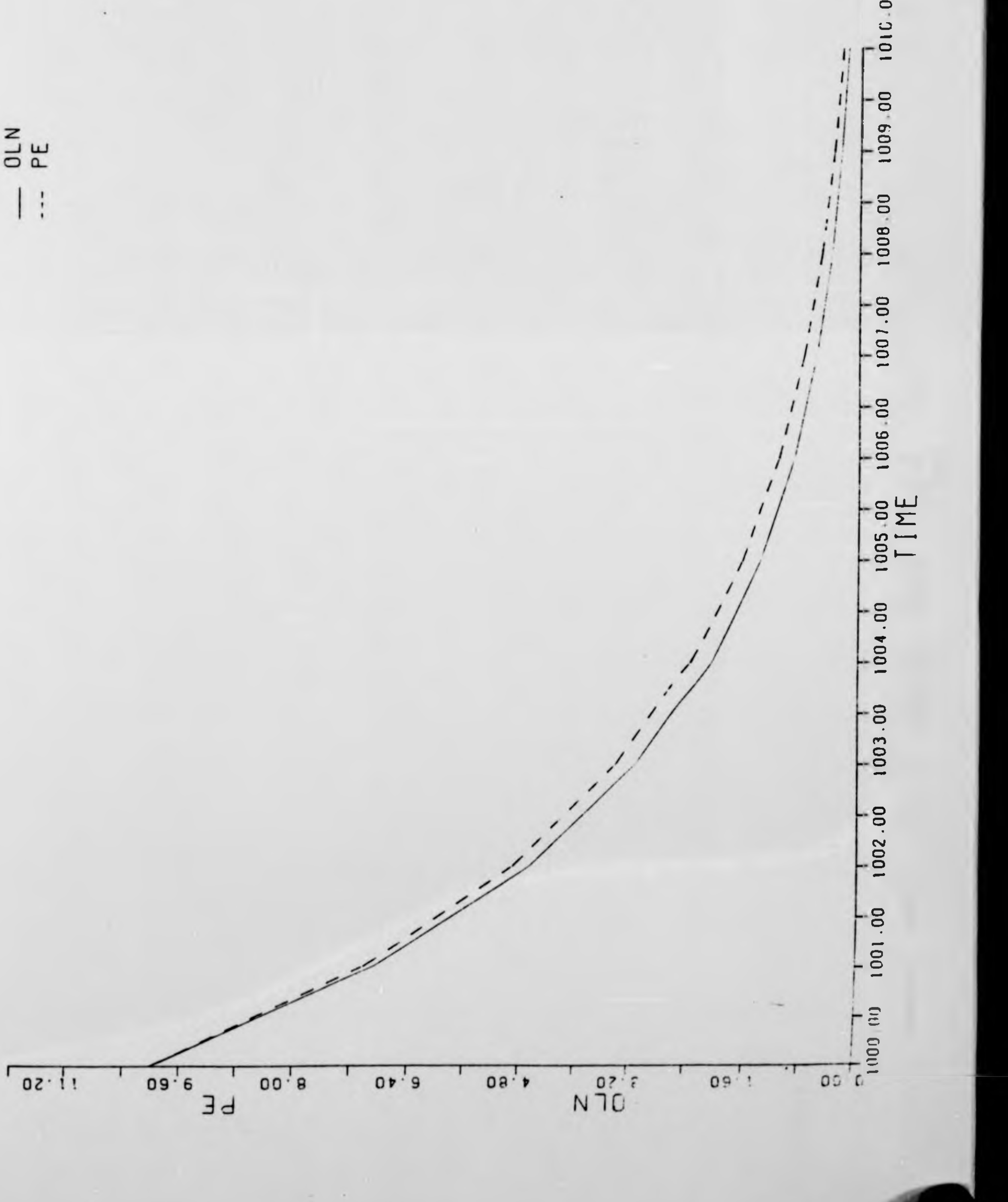
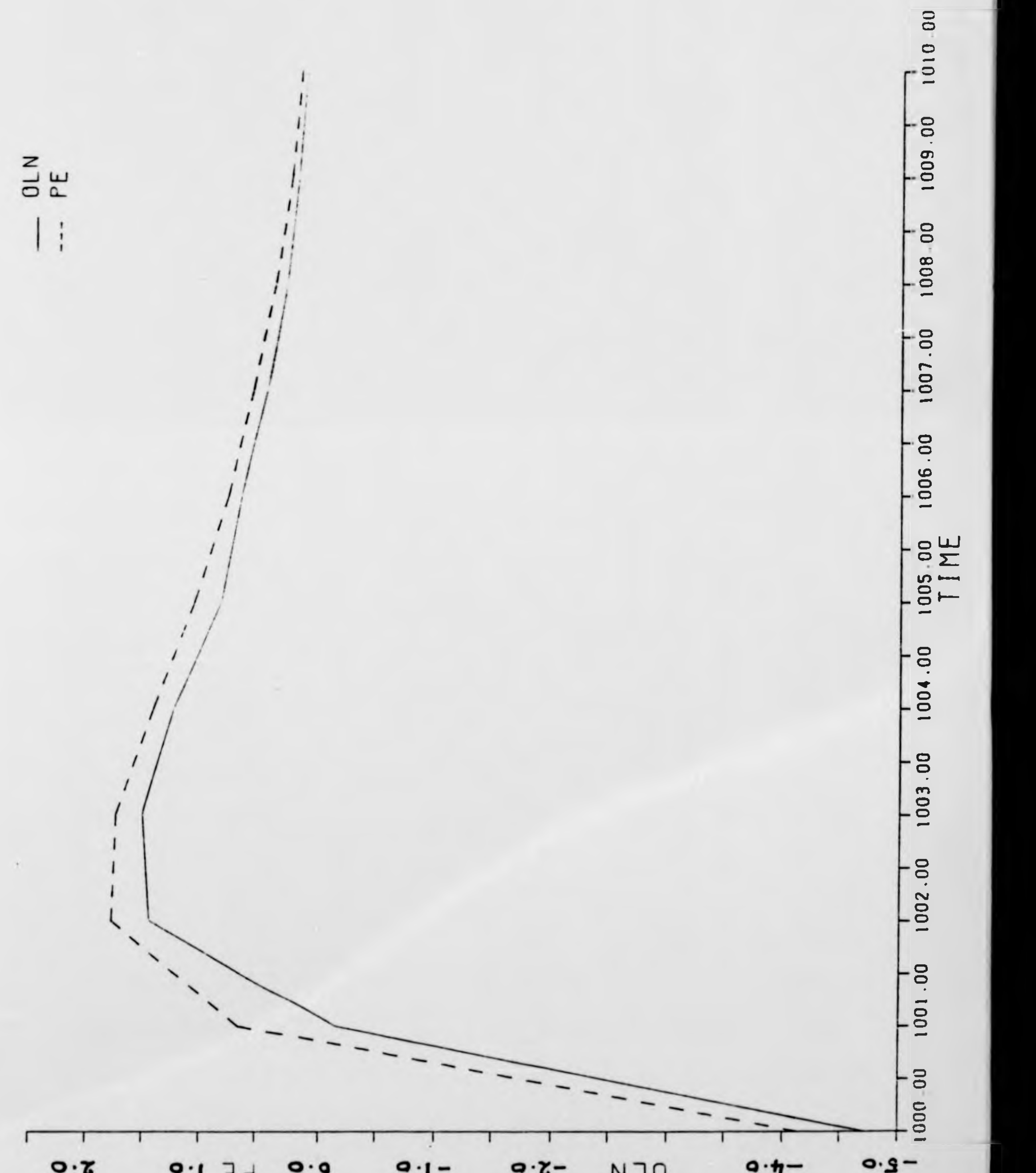


Figure 5c


OLN
PE
TIME
1000.00
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1003.00
1004.00
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1007.00
1008.00
1009.00
1010.00
-5.0
-4.0
-2.0
-1.0
0.0
1.0
2.0

Even with the more powerful interaction effects deriving from real wage resistance there is still little evidence of a substantial difference between the OLN and PE time paths. There does seem to be a tendency to excessive contraction during the early stages for the OLN case, but the magnitude of this effect is not large. Figure six compares the time paths under the two types of control.

The similarity of the OLN and PE solutions can be seen by comparison of the welfare losses over the interval zero to infinity. If the OLN loss is normalised as 100 then the PE loss is 98.99. Again this indicates that a very small fraction of the welfare loss involved in controlling inflation can be avoided by coordination of policies. The similarity of the solutions can also be seen by comparing the feedback forms of the control rules. c.f. Table 12.

Table 12

	OLN	PE
Own output	-.7737	-.7773
Foreign output	-.1003	-.1854
Own inflation	-.4118	-.4142
Foreign inflation	-.0592	0

For the Pareto Efficient Controller this indicates a feedback control matrix which is very close to being block diagonal. This indicates a fairly natural decentralisation of policy or association of particular instruments with particular targets.

Although there is no significant gain through coordination in the model without real wage resistance or that incorporating it there is a significant gain in welfare due to the speeding up of the disinflation process through the introduction of the real wage effect. Taking the losses from the model without the real wage effect as 100 then the losses from the model with the effect are 48.94 for OLN and 49.17 for PE control.

6:6 Conclusions

This chapter has examined coordinated and uncoordinated policy regimes within the context of a more fully specified model than that used in chapter five. Although this has reduced the conciseness and tractability of the results it provides a richer understanding of the policy interaction between economies.

The numerical simulations considered have shown the existence of dynamic inefficiency due to uncoordinated control, though the magnitude of the effect has been rather small. This should not be generalised to imply that such welfare loss will always be negligible. The results derived are particular to the parameter values chosen and, more importantly, to the models examined. Both models considered assume that the exchange rate is determined by trade-flows only. A model in which the capital account had influence would lead to much faster and bigger movements in exchange rates. This would lead to more severe policy coordination problems. Miller and Salmon (1984) derive some numerical results for such a model which show a more significant policy coordination problem than that found in this chapter.

The results of this chapter should therefore be seen as providing confirmation that the results of chapter five are robust to relaxation of the strong assumptions of the model used in that chapter. They should not lead to the conclusion that policy interdependence will be an insignificant problem whatever the economic model considered.

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1. Note that in contrast to the previous chapter the exchange rate is not subject to discrete jumps in this case. This is because it is determined by relative output levels c.f. eqn. (A:5) and these in turn are not capable of jumping.

Appendix A

In the main body of the chapter use was made of the equation $\dot{p} - \dot{e} - \dot{p}^* = \dot{x}^* - \dot{x}$ (c.f. equation 6:8)). This is now proved as follows.

Consumers' utility is assumed to be Cobb-Douglas in consumption levels of the commodities produced in the two countries, i.e.

(A1) $$U_d = C_d^{\alpha} C_d^{*1-\alpha}$$

$$U_f = C_f^{\alpha} C_f^{*1-\alpha}$$

Total production of the commodities is by definition equal to the sum of the consumption levels.

(A2) $$X = C_d + C_f$$

$$X^* = C_d^* + C_f^*$$

Suppose consumers in the domestic economy have income M and those in the foreign economy have income M^*. By a well known theorem Cobb-Douglas utility functions imply that the share of nominal expenditure on a product in total expenditure is given by the exponent on consumption of that product. This yields

(A3) $$PC_d = \alpha M \quad \text{and} \quad EP^*C_d^* = (1-\alpha)M$$

$$\frac{PC_f}{E} = \alpha M^* \quad \text{and} \quad EP^*\, C_f^* = (1-\alpha)EM^*$$

Rearranging this gives

$$\text{(A4)} \qquad P(C_d + C_f) = PX = \alpha(M + EM^*)$$

$$EP^* (C_d^* + C_f^*) = EP^*X^* = (1-\alpha)(M + EM^*)$$

Taking ratios of nominal expenditures on the products yields

$$\text{(A5)} \qquad \frac{PX}{EP^*X^*} = \frac{\alpha}{1-\alpha}$$

or $\dfrac{P}{EP^*} = \dfrac{\alpha}{1-\alpha} \dfrac{X^*}{X}$

or taking logs and differentiating with respect to time

$$\text{(A6)} \qquad \dot{p} - \dot{e} - \dot{p}^* = \dot{x}^* - \dot{x}$$

which is the required result.

Chapter Seven

Conclusion

Chapter Seven

Conclusion

The main theme of this thesis has been the inefficiency of uncoordinated national macroeconomic policies in an interdependent world economy. Two types of inefficiency have been considered. The first of these arises when countries adopt incompatible targets. Chapter three considers an obvious example of this in which both countries in a two-country model desire a balance of payments surplus, this extended earlier work by Hamada. The second type of inefficiency arises when targets are compatible but failure to coordinate policy leads to an inefficient dynamic adjustment path towards the steady-state. This is referred to as dynamic inefficiency. Chapters five and six consider examples of this in which both countries in a two country model wish to bring inflation down to zero from an initially high level.

Non-cooperative behaviour has been modelled throughout as the Nash solution to a two player, non-zero sum game. Cooperative behaviour has been modelled as the solution corresponding to that chosen by a single controller whose objective function is a weighted average of the individual countries' objective functions. In chapter three the question of whether or not an improvement on the Nash solution could be achieved by allocating the role of Stackelberg leader to one country. This would have the advantage of maintaining the non-cooperative structure of the game. However, the superiority of the Stackelberg over the Nash solution was found to be ambiguous.

Chapter five presented a simple model of an interdependent world economy. The policy instrument used by government to control inflation was assumed to be output (though an alternative interpretation is that output is perfectly controllable by use of some intermediate monetary or fiscal instrument). This rather strong assumption enabled tractable results to be derived for the model under control. These results indicated that the problem of coordination was more serious the greater the interdependence between the two countries. The main source of interdependence was the effect of variations in the ratio of outputs on the terms of trade.

Chapter six considered a more fully specified model of interacting economies which assumed that output adjusted sluggishly in response to changes in the monetary growth rate. Because of the increased complexity of the model numerical simulations were necessary to examine its behaviour. These confirmed that the analytical results of chapter five were robust to generalisation of the model. However, the welfare inefficiencies from decentralised control were surprisingly small. The models considered in chapter six focus on interaction through trade in goods only. If capital flows were to be introduced then it is possible that this would increase the welfare loss. Some indication of this can be obtained by consideration of the model in chapter five. There the welfare problems increased the more sensitive the real exchange rate was to changes in the output ratio. If capital flows result in the kind of exchange rate overshooting discussed in chapter two then they would increase the return to coordination.

To conclude. This thesis has considered some topics in a potentially large area of research. Future research might involve more detailed specification of the economic models used or more sophisticated methods of modelling strategic behaviour.

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