Two layer Markov model for prediction of future load and end of discharge time of batteries

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***Abstract*-To predict the remaining discharge energy of a battery, it is significant to have an accurate prediction of its end of discharge time (EoDT). In recent studies, the EoDT is predicted by assuming that the battery load profile (current or power) is *a priori* known. However, in real-world applications future load on a battery is typically unknown with high dynamics and transients. Therefore, predicting battery EoDT in an online manner can be very challenging. The purpose of this paper is to derive a load prediction method for capturing historical charge/discharge behaviour of a battery to generate the most probable future usage of it, enabling an accurate EoDT prediction. This method is based on a two layer Markov model for the load extrapolation and iterative model-based estimation. To develop the proposed concept, lithium-ion batteries are selected and the numerical simulation results show an improvement in terms of the accuracy of the EoDT prediction compared to methods presented in the literature.**

Keywords-load prediction, end of discharge time, prognosis, Markov models, lithium-ion battery

I. INTRODUCTION

In the context of batteries, end of discharge time of a battery is defined as the time threshold when the battery hits its practical limits (typically a cut-off voltage of circa 2.8-2.5V or a SOC <10%). It is not normally safe to use the battery after hitting these limits. Respectively, the remaining discharge time (RDT) of a battery is defined as the difference between the EoDT and the present time point. RDT defines the time that the battery can keep working safely with guaranteed performance. It is important to accurately estimate the battery EoDT/RDT in many practical applications. For example, in electric vehicles (EVs) the EoDT/RDT is requisite to estimate the remaining driving range (RDR). Accurate RDR estimation is vital to ease range anxiety or avoid breakdowns due to over-discharging [1]. Unlike battery state of charge (SoC) and state of health (SoH) estimations that only rely on battery historical use, EoDT is predicted based on both the battery past and future usage conditions. The EoDT calculation is also different with the state of power (SoP) prediction. While SoP employs a short term prediction algorithm to predict an incoming peak power point based on a time span of seconds, EoDT needs a long horizon prediction from the present time t to the time when the battery reaches its practical limits based on a time span of minutes or hours.

There are two different definitions for the EoDT in the literature. The first one is the time when the SoC hits a pre-determined limit [2] while the second one is when the terminal voltage reaches the lower cut-off voltage [3].

Based on these definitions, researchers have addressed the critical issues for the EoDT prognosis and the corresponding RDT prediction by using advanced filtering techniques, including particle filters [2], [4], and Kalman filters [5], [6] and data driven methods [3] for battery cells and packs. Although these studies show acceptable accuracy for the EoDT/RDT prediction, the proposed prediction approaches are based on the critical assumption that the battery charge/discharge profile is *a priori* known. In practical applications (i.e. EVs), the future load is known to be stochastic as it depends on many factors, such as: road traffic, driving style of driver, and road inclination [7].

Recent researches have been conducted to overcome this challenge. In [8], battery RDT prediction is performed by assuming that future load is mean value of the historical load. Future load characterization based on Gaussian distribution with the mean and variance of the historical load is presented in [9]. A Markov process with two states of the minimum and maximum load obtained from the historical data within a fixed length window is used for representing the future scenarios in [2]. A mission map which provides information of terrain as well as driving style in an EV is introduced in [10] to predict the battery load demand. The above methods can face critical challenges for the practical EoDT/RDT prognosis. The mission map method [10] requires specifying start and end points of the journey by the driver and the online identification of driving schedule. The mean-based prediction [8], although simple, cannot effectively extrapolate the loads with high transitions. This is because the mean calculation is an integration-based calculation and subsequently, damps the transitions. On the other hand, the Markov model [2] can capture dynamic behaviour of the load, but when the states of the Markov model are maximum and minimum of the historical load, the predicted load will be either over estimated or under estimated, especially when the battery data includes both charge and discharge values (positive and negative currents). Therefore, it is necessary to utilize a method that is capable of predicting transient loads with enough accuracy for EoDT prognosis purposes.

Due to the advantages of the Markov models in representing highly transient systems, this study proposes a two layer Markov model for battery EoDT/RDT prediction purposes. This is performed by a combination of a higher-level Markov model for characterization of future trend of battery charge and discharge, and a lower level Markov model for predicting actual load values. This two level configuration provides the opportunity to separately deal with the charge-discharge and states and the dynamic variation of the load value in each state. The higher level Markov model is homogeneous, while the lower level model is non-homogeneous with probabilities dependent on the first level Markov model states.

To evaluate the load and EoDT prediction method, a 18650 Lithium-ion battery is firstly selected. Next, the experimental data are used as an input to the load prediction mechanism and obtaining the future load profiles. Then the predicted load is applied to a validated equivalent circuit model (ECM) and finally the EoDT is calculated based on

both definitions mentioned before. The superiority of the proposed approach is clearly demonstrated through a comparison with both the mean-based predictor [8], and the single level Markov based predictor [2].

The remainder of this paper is organized as follows: in section 2 the preliminaries of Markov processes are given. In section 3 the two layer Markov model is addressed. Section 4 provides the simulation results on a lithium ion battery ECM and comparisons between the proposed and existing methods. Finally, section 5 gives conclusions.

II. Markov Process

The Markov model is a strong model both to analyse a time series and predict its future values. A time series is said to have the Markov property if the conditional probability distribution of the future states only depends on the present state and not the whole history of the occurred states. A Markov process $\{S_k, k \geq 0\}$ takes values in the finite set $\underline{\mathrm{N}} = \{1, 2, ..., N\}$ with conditional probabilities of:

$$\Pr\{S_{k+1} = j \mid S_k = i\} = \lambda_{ij} \;\; i, j \in \underline{\mathrm{N}} \tag{1}$$

Here, $\lambda_{ij} \geq 0$ is the transition probability (TP) from mode i at time k to mode j at time $k+1$. The transition probabilities are never negative and the total probability is equal to one, $\sum_{j=1}^{N} \lambda_{ij} = 1$. Each Markov process has a unique TP matrix (TPM) in the form of (2).

$$\mathbf{\Lambda} = \left[\lambda_{ij}\right], \;\; i, j \in \underline{\mathrm{N}} \tag{2}$$

Here, it is assumed that the Markov process used for load description is irreducible, i.e. it is possible to move from one mode to any modes in a countable number of jumps.

III. Two Layer Markov Model for Load Prediction

To address the concerns about the future load profiles for the purpose of EoDT/RDT prediction, here a stochastic load characterization by a two layer Markov model is proposed. The first level Markov model predicts the next charge/discharge state while the second level model predicts the load value state itself.

Generally, the probability of the next state of the battery load depends on all of its previous states which means that the knowledge of all existing states is required. However, by the causality coming from the practical conditions, the load follows the Markov property [2]. This means that the conditional probability of the battery load state can be written as (3) where S_k can represent either charge/discharge state or the load value state.

$$\begin{aligned} &\Pr(S_{k+1} = s_{k+1} \mid S_k = s_k,\, S_{k-1} = s_{k-1}, ..., S_1 = s_1) \\ &= \Pr(S_{k+1} = s_{k+1} \mid S_k = s_k) \end{aligned} \tag{3}$$

The conditional probabilities (or the transition probabilities) of (3) are initially unknown and should be calculated from the historical data of load time series.

A. Transition Probability Calculation

The probability of observing S_n as the state of the time series at the time point n, is obtained by (4) which results from the Markov property.

$$\Pr(S_n = s_n) = \Pr(S_1 = s_1) \prod_{k=2}^{n} \Pr(S_k = s_k \mid S_{k-1} = s_{k-1}) \tag{4}$$

Based on the above equation the likelihood, $L(.)$, of a given transition matrix is (5) where m_{ij} is the number of transitions from state i to state j [11].

$$L(\Lambda) = \Pr(S_1 = s_1) \prod_{i=1}^{N} \prod_{j=1}^{N} \lambda_{ij}^{m_{ij}} \tag{5}$$

The TPs are found such that the likelihood function is maximized. Taking the logarithm of the likelihood function and considering the restriction of $\sum_{j=1}^{N} \lambda_{ij} = 1$, leads to (6). Taking the derivative of (6) and setting it to zero will give the TPs as (7) which will maximize the function (5).

$$\begin{aligned} &\log(L(\Lambda)) = \\ &\log \Pr(S_1 = s_1) + \sum_{i,j} m_{ij} \log \lambda_{ij} + \sum_{i=1}^{j} (1 - \sum_{j} \lambda_{ij}) \end{aligned} \tag{6}$$

$$\lambda_{ij} = \frac{m_{ij}}{\sum_{j} m_{ij}} \tag{7}$$

B. Prediction of the Future Markov States

After calculating the TPM, the probability of the next state can be computed by the difference equation of (8).

$$\Pr(S_{k+1}) = \Pr(S_k)^T \Lambda \tag{8}$$

Consequently, the next state is the index of the $P(S_{k+1})$ vector with the maximum probability.

The accuracy of the TPs has a significant role in the accuracy of the prediction. Consequently, the quantity and quality of the data used for transition probability calculation is important. Here, it is assumed that the measurements from sensors are adequate for this purpose. The sensitivity of the predicted load to the TP accuracy is left for future studies.

C. High Level Markov Model: Predicting the Charge/ Discharge Trend

In vehicle applications acceleration results a discharge current for a battery while deceleration and braking provides charge for it because of recovery effect and regenerative braking. Input signals with both charge and discharge values are also observable at renewable energy-based systems, with charge due to the energy sources supplements and discharge due to the consumptions. The first level Markov model with states denoted by S_k, learns the charge or discharge pattern and forecasts the trend. This Markov model has two states defined at the set of $\underline{\mathrm{N}} = \{Ch, DCh\}$. It is assumed that the I_k is the battery input current and classified as Ch, if $I_k > 0$, and DCh if $I_k < 0$. Remarkably, the first level Markov process is homogeneous [12] which means that the TPs are time independent in each training window.

D. Low Level Markov Model: Prediction of the Load Value

Here it is assumed that the battery load in any of the of charge or discharge states has two internal states of SS_k defined as $\underline{M} = \{Min, Max\}$. The Max state of charge/discharge represents the high energy supply/consumption, and the Min state of charge/discharge represents the low-energy supply/consumption.

Remarkably, the second level Markov process is a non-homogeneous Markov process [12] with time-dependent TPs. This is because the second Markov process has two different TPMs, dependent on the states of the first level Markov process output.

$$\Gamma^{(S_k)} = \begin{bmatrix} \gamma_{11}^{(S_k)} & \gamma_{12}^{(S_k)} \\ \gamma_{21}^{(S_k)} & \gamma_{22}^{(S_k)} \end{bmatrix}, \quad \sum_{i,j} \gamma_{ij}^{(S_k)} = 1,\ S_k = Ch, DCh \tag{9}$$

For both Markov processes it is assumed that the initial state and its probability comes from the state of the last data used for training.

By two layer Markov model the next value of the load is predicted and applied to the battery model to obtain its state variables. The prediction is iteratively repeated until the predicted battery states hit the limits specifying the EoDT. The flowchart of the proposed method as well as its block diagram is given in fig. 1 and 2.

IV. Battery Model Description

To demonstrate the applicability of the proposed method, it is applied on the Panasonic 3.03Ah 18650 battery cell. The battery is assumed to be represented by a discrete-time first order ECM of fig. 3 described by (10).

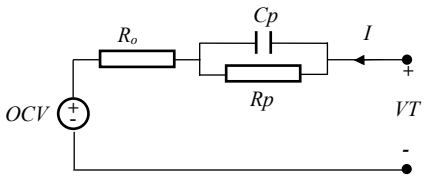

Fig. 3. The first order ECM of the battery

$$\begin{cases} SoC_{k+1} = SoC_k + I_k / Q\Delta t \\ Vp_{k+1} = e^{-\frac{\Delta t}{Rp_k Cp_k}} Vp_{k+1} + \left(1 - e^{-\frac{\Delta t}{Rp_k Cp_k}}\right) Rp_k I_k \\ V_{tk} = OCV_k + Vp_k + R_{ok} I_k \end{cases} \tag{10}$$

Q is the standard capacity of the cell, Vp is the battery polarization voltage, V_t is the terminal voltage, Rp and Cp are the polarization resistance and capacitance respectively. R_o is the ohmic internal resistance of the cell. The battery ECM parameters are obtained via a pulse power test at 1C and an OCV characterization experiment at 4 SoC values at 10 °C using a commercial Li-ion cell cycler. The model parameters are given at fig. 4(a)-(c). The SoC-OCV curve is also given at fig. 4(d). The sampling time is Δt = 1sec. This model is validated under different conditions showing a root mean square error (RMSE) of <50 mV.

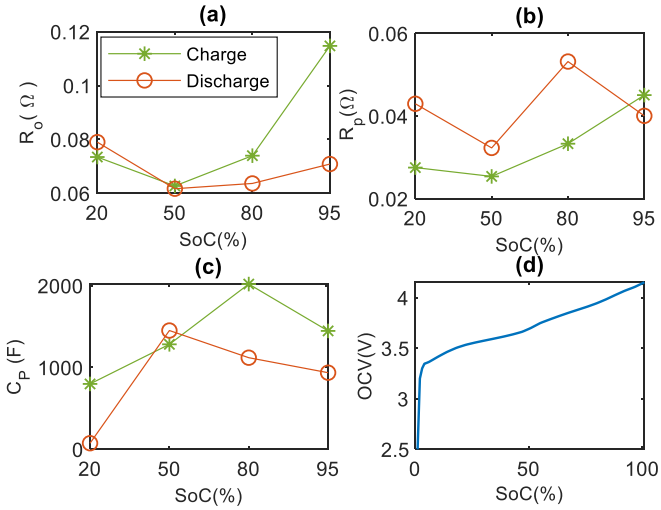

Fig. 4. The SoC dependent parameters and OCV curve of the battery modelled by a first order ECM

V. Simulation Results

The input current applied to the battery is shown in fig. 5, it follows the Artemis motorway charge/discharge driving style which is widely used by researchers to evaluate battery performance.

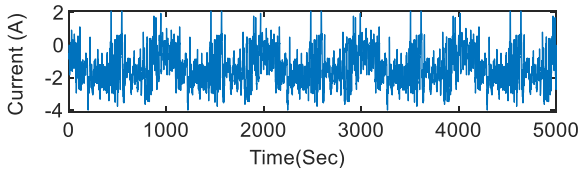

Fig. 5. The current profile for the Artemis motorway driving style

The prediction is updated each 2 minutes with the forgetting factor of 1. Since the Markov model is a stochastic model the future load profiles are not unique, therefore, the load profile generation is repeated for 5 different runs of Markov process for more reliability. The SoC and terminal voltage prediction results based on the load data between time [720, 780] are given by fig. 6, the initial SoC is assumed 65.5% and different realizations are shown in different colours. The final EoDT is obtained by calculating the mean of the distinctive runs.

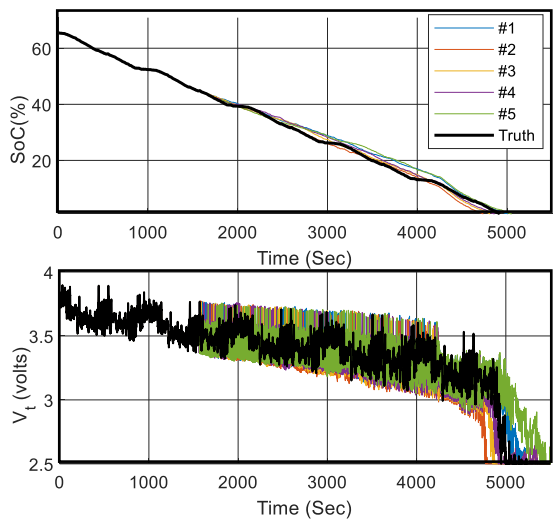

Fig. 6. SoC and terminal voltage predictions under Artemis motorway style

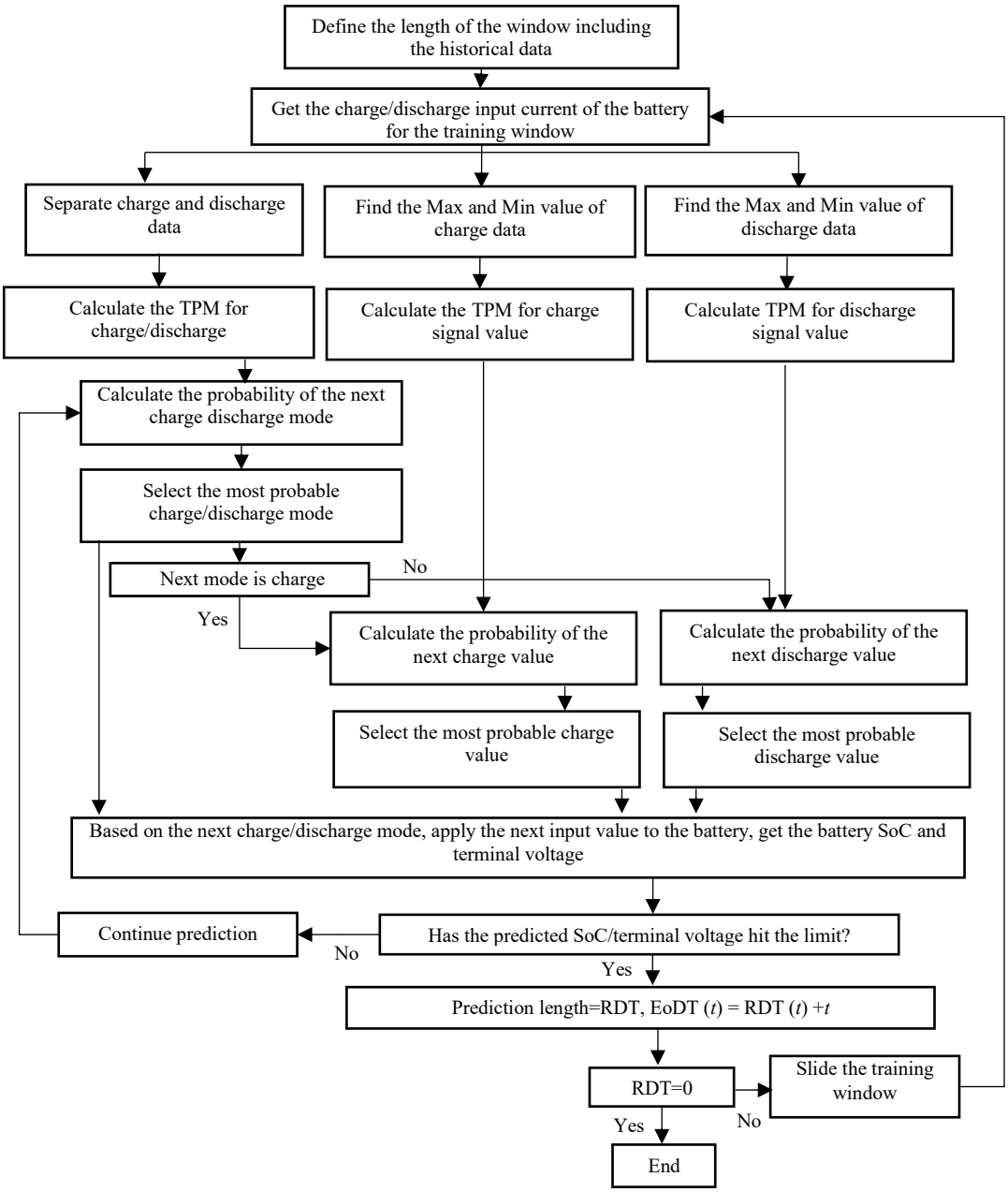

Fig. 1. Flowchart of the prediction method

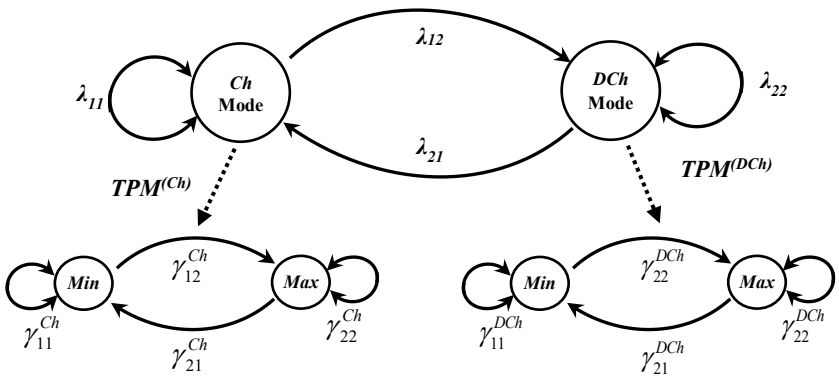

Fig. 2. Diagram of the two layer Markov model

The method is both compared with the EoDT prognosis based on the mean of the historical data as well as the single level Markov model for load prediction on fig. 7 and fig.8 respectively. Both definitions based on SoC and terminal voltage for the EoDT are simulated. The cut-off limits are SoC = 10% and V_T = 2.8 (volts) based on the battery supplier recommendations. The initial SoC is set to 65.5%. The true EoDT is obtained by applying the whole profile to the battery in advance. It is 72.5 minutes by the first definition, while 82.3 minutes by the second definition.

The results show that the load prediction based on the two layer Markov model shows more accurate results than the historical mean-based method. In fact, the mean-based method shows multiple overestimation peaks. This difference is due to the fact that predictions based on the historical mean cannot model the transitions in the load, while the two layer Markov model captures the dynamic variation through the transition between its finite numbers of states.

The root mean squared error of the EoDT prediction is given at table I both in minutes and the percentage of the error.

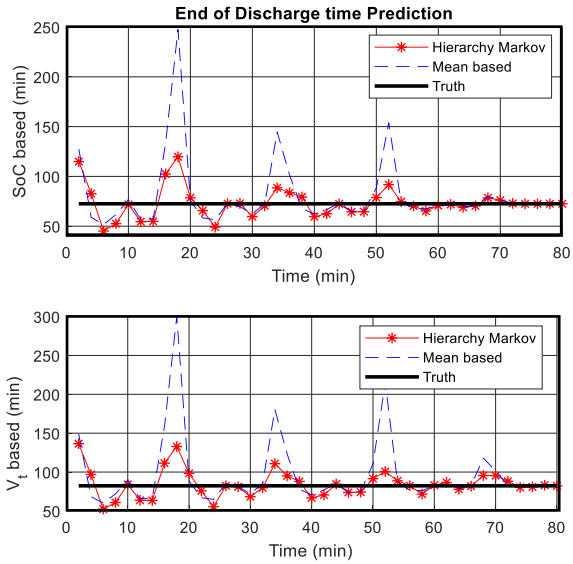

Fig. 7. Comparison of the proposed and mean based methods for the EoDT prognosis under Artemis motorway style

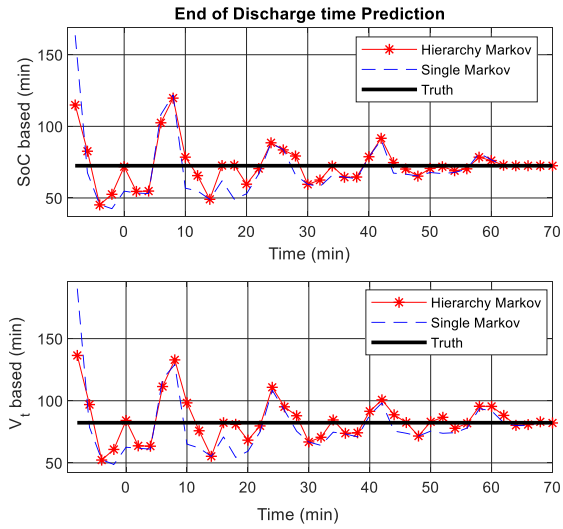

Fig. 8. Comparison between the proposed and the single Markov based method for the EoDT prognosis under Artemis motorway style

TABLE I. COMPARISON BETWEEN THE EODT PROGNOSIS RESULTS BY DIFFERENT METHODS

	SoC based RMSE (min)	Error Error (%)	V_T based RMSE (min)	Error Error (%)
archy Markov	14.9750	20.64%	17.5977	21.37%
Single Markov	21.8291	30.09%	24.8481	30.71%
Mean	36.2393	50.01%	47.8057	58.10%

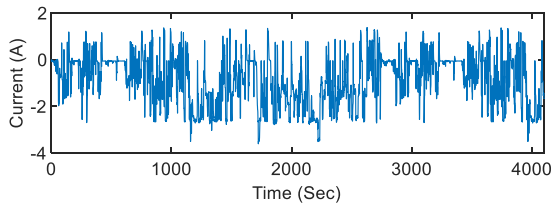

Fig. 9. The current profile for a random driving style

To further investigate the specifications of the method, it is also applied on a battery subject to an input current from a driver at some random driving conditions given at fig. 9. The EoDT prognosis based on SoC and V_T are respectively 41.85 and 67.01 minutes. The initial SoC is set 35%. As the results (fig. 10) show that the proposed method provides more accurate results in this case as well.

The reason that the two layer Markov model method shows better accuracy compared to the others is that by separating the values relevant to the charge and discharge states, the predicted profile shows more similar specifications (Min, Max, Mean and variance) to the segment of profile used for training. This is shown by fig. 11 for a sample time window of historical data and the predicted load. Accordingly, the histogram and the distribution of the predicted data by the proposed method captures the statistics of the historical data (the most probable load values) better than the single level Markov model. Also, this figure shows why the prediction based on the Mean value of the historical data does not properly represent the data.

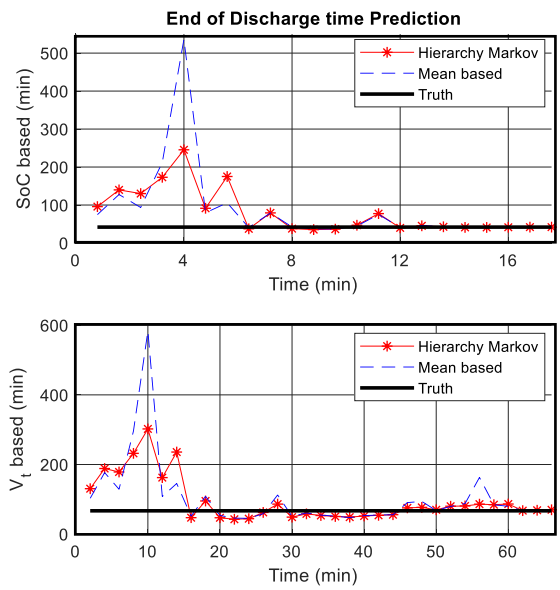

Fig. 10. Comparison between the proposed and the mean based method for the EoDT prognosis under the random load style

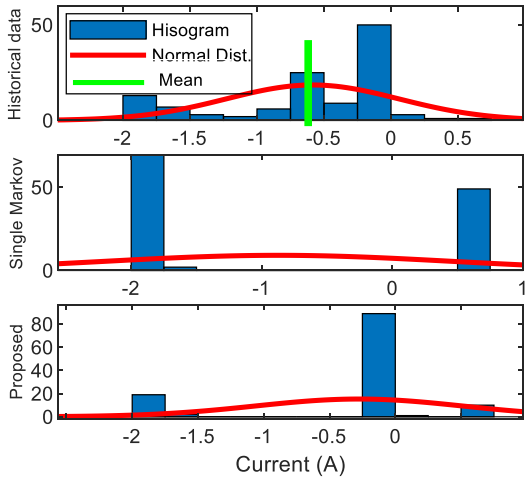

Fig. 11. Comparison between the histogram and distribution of the historical and predicted load by the proposed, the single Markov based and the mean based methods

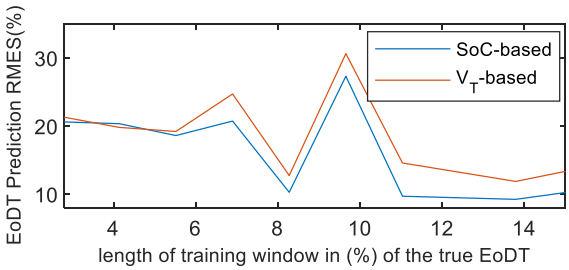

Fig. 12. The effect of the historical data length on the accuracy of the EoDT prediction error

The effect of the historical data length on the accuracy of the EoDT prediction error is given at fig. 12. Accordingly, the error of the prediction reduces with more data being used for training. But on the other hand, increasing the length of the required data for training makes the prediction rate slower and increases the possibility of overtraining at the same time. For high demanding application such as electric air-vehicles and automobile which are subject to very transient conditions the slow rate of the prediction may not be enough to predict the accurate value of the EoDT and protect the battery from being over-discharged.

VI. Conclusion

This paper suggests a method for the EoDT and RDT prediction based on the historical data of the charge and discharge of a battery. The method uses two Markov models for predicting the charge/discharge trend as well as the load value. It provides a better accuracy for prediction of the EoDT and RDT compared to the single level and mean-based prediction. This is due to the potential of the Markov models for capturing the transient behaviour of the load while the mean-based method filters all transitions and single Markov model mixes charge and discharge data. Future works will focus on addressing the effect of the historical data length on the accuracy of the predicted load and the selection of an optimum length either by optimization methods or adaptive techniques. Furthermore, the temperature effect on the battery model will be considered into account to improve the robustness and reliability of the prediction.

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