Three Essays in International Financial Economics

by

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"Idle reader, you can believe me when I say that I'd like this book, as a child of my intellect, to be the most beautiful, the most gallant, and most ingenious one that could ever be imagined. But I haven't been able to violate the laws of nature, which state that each one begets his like." — Miguel de Cervantes

I cannot think of a better way to describe this thesis which is the product of my PhD than with Cervantes' words. The image of his ingenious gentleman has for the longest time been part of my home and symbolizes the past years better than any other. As for him, this battling of windmills by abstraction and determination would not have been possible or at least enjoyable without valued companions.

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Declarations

This thesis is submitted to the University of Warwick in support of the requirements for the degree of Doctor of Philosophy. I confirm that I have not submitted the thesis for a degree at another university. All work contained in this thesis is my own.

Abstract

This thesis consists of three papers on different topics on international financial markets. The first paper provides insights into the dynamics of decreasing asset prices. I show how preferences in line with cumulative prospect theory can explain the temporary reversal of the downward trend also know as *dead cat bounce* or *bear market rally*. The second paper in this thesis focuses on the dynamics of multi-venue trading. Departing from a new theoretical framework, I develop a novel measure of price discovery. I show the properties of the Toxic Arbitrage Information Share using simulations and a set of foreign exchange futures. The third paper analyzes the connection between foreign ownership and currency risk. I find that investors use stocks' FX exposure in order to implicitly hedge currency risk. Furthermore, there is no evidence that domestic investors and foreign investor should hold identical portfolios.

Chapter 1

Introduction

This thesis explores different aspects of international financial markets. The three main chapters, though very distinct in their focus, are aimed at analyzing how international financial markets work. Throughout the thesis, the methodology shifts from a purely theoretical contribution towards a mostly empirical analysis based on theoretical hypotheses. While Chapter 2 looks at the dynamics of asset prices during financial bubbles, Chapter 3 focuses on the interaction of different trading venues both theoretically and, in one example, empirically. In Chapter 4, I provide a new perspective on the link between exchange rates and portfolio diversification by international investors. The purpose of each chapter is to provide new insights into the dynamics and mechanisms which drive international financial markets. The chapters are presented in the form of papers. All three are single-authored.

Chapter 2

Financial bubbles are a common feature in asset prices and of special interest to investors and regulators alike. While the build-up of financial bubbles receives a lot of attention in the academic literature, their unraveling has been less studied. Chapter 2 provides the first paper to theoretically analyze the dynamics of decreasing asset prices. A common feature in unraveling financial bubbles is the temporary reversal of the downward trend, also known as *dead cat bounce* or *bear market rally.* I show that preferences according to cumulative prospect theory can lead an investor to take excessive risk and unprofitable positions in order to recover an initial loss in a declining market. The loss driven behavior results in premature re-entering into the market. Heterogeneous investors enter at the same time despite differences in their reference points, wealth, and initial loss. The resulting shift in aggregate demand can explain the sudden but temporary reversal common in declining asset prices.

Chapter 3

When an asset is traded across multiple venues, discrepancies between prices can lead to short lived arbitrage opportunities Foucault et al. (2016) differentiate between non-toxic arbitrage opportunities caused by liquidity trades and toxic arbitrage opportunities caused by information arrival. This paper shows that the direction of the latter provides valuable insights into price discovery and markets' information shares. Starting from a new theoretical framework of multi-venue trading, I derive a measure of information shares based on the relative frequency of toxic arbitrage opportunities. The resulting Toxic Arbitrage Information Share provides not only a point estimate but also approximate error bands. Additionally, this measure avoids common drawbacks such as arbitrary choice of observation frequency and does not rely on constant movements in asset prices as is the case for Vector Error Correction Model based measures. It therefore provides a valuable addition for the analysis of price dynamics, especially in low liquidity environments. I illustrate these advantages with a set of simulations and a unique data set of internationally traded foreign exchange futures.

Chapter 4

Currency risk is a central feature in international asset pricing and a major concern for international investors. A broad literature investigates why investors show a strong preference for domestic assets, but currency risk is found to be only a minor factor. The only source for currency risk considered in such studies is the risk resulting from exchanging foreign currency returns into domestic currency at the end of the holding period. Such risk can be fully hedged with currency futures. However, as asset returns in local currency are usually correlated with the exchange rate, the actual exchange rate risk investors face is more complex. This paper is the first to investigate the role of exchange rate risk for international investors by looking at stocks' exchange rate (FX) exposure and its connection with within country differences in foreign ownership. I show both theoretically and empirically that international investors use stocks' FX exposure to implicitly hedge currency risk. The results stand in contrast to the common assumption that foreign investors would find it optimal to invest into the same portfolio as domestic investors. The latter even holds if explicit hedging is free. While the results for developed markets are stable over time, there has been a change after the crisis in how currency risk is handled when holding emerging market stocks.

Chapter 2

Dead Cat Bounce

2.1 Motivation

During bear markets, in particular after the burst of a bubble, asset prices often experience a temporary reversal of the downward trend followed by a further decline. Investors commonly describe this phenomenon as *dead cat bounce* while authors such as Maheu et al. (2012) refer to it as *bear market rally.* Despite its importance for investors, the *dead cat bounce* has widely been ignored by the theoretical literature.

This paper provides a theoretical explanation for the *dead cat bounce* using insights from behavioral finance namely cumulative prospect theory. As a financial bubble bursts investors sell their holdings leading to large perceived losses. In the hope to recover these losses, such investors are willing to make excessively risky investments by re-entering the market after a further fall in prices. This re-entering leads to a temporary reversal in the sell-off. Thereby, the occurrence of a reversal does not rely on the arrival of fundamental information or coincided closing of short positions which are commonly given explanations.

Temporary reversals in falling prices are a common feature in capital, commodity, and currency markets as shown in Figure 2.1. In March 2000 the NASDAQ composite index saw the peak of a bubble which had been years in the making. Within a bit more than two months the index lost over a third of its value. In the subsequent 8 weeks the index rose by 33% narrowing the gap to its previous high to 15%. After this short lived recovery, the stock index plummeted once again, reaching its low point over two years later ending up with a total loss of nearly 80% compared to its peak.

The need for a better understanding of such events for investors is clearly given. Additionally, central banks and government bodies repeatedly intervene in currency crises and stock market bubbles. The coordinated intervention by China's "national team" of state financial institutions investing at least \$ 140 billion[1] in the equity market

[1] This is the estimate in the beginning of August 2015 according to the financial time: "Goldman estimates China's 'national team' stock rescue at \$144bn".

to curb the fall in stock prices has been a recent example. A failure to understand the downward dynamics of asset prices can easily lead to a misinterpretation of the success of such measures. If the intervention occurs towards the beginning of a *dead cat bounce*, the upward movement is likely to provide a false sense of success and an early end to supporting measures. The results even indicate that the timing of the temporary reversal may be triggered by intervention.

Similarly, there is a plethora of examples of central bank intervention in the FX market attempting to smooth or even reverse the unraveling of currency bubbles. These attempts are hazardous given the lack in understanding of downward dynamics. The temporary reversal of the downward trend is a natural starting point for a more in depth analysis.

This paper shows that (1) the demand can be split into a loss recovery component as well as an investment component. The loss recovery component is the amount the investor needs to invest in order to have a chance of recovering his losses. The investment component is what he is willing to invest beyond this point. (2) The demand is "objectively irrational" meaning that the investor will buy the asset despite negative expected return. Further, if he had not suffered an initial loss, he would not enter the market at this point. (3) There is a jump in demand from zero to the desired level once the price falls under a threshold, i.e. demand does not follow a continuous function. (4) This jump is likely to coincide among a large group of investors leading to a large and sudden shift in aggregate demand. The resulting jump in overall demand explains the strong, sudden reversal in the downward trend. Furthermore, the reversal can be triggered by trivial events and the end of the reversal does not need new fundamental information.

The remainder of the paper is structured as follows. The next section gives a brief overview of the different branches of literature related to this paper. Section 2.3 lays out the assumptions and set-up of the model. Sections 2.4 and 2.5 provide the investor's maximization problem as well as an analysis of his decision to re-enter. In Section 2.6, I analyze the timing when the investor reconsiders his demand and go into details of the reversal. Section 2.7 provides a brief summary of the results.

2.2 Literature

This paper combines several branches of literature on e.g. bear markets, currency crises, asset pricing, and behavioral finance. Yet, the largest contribution is to the literature on financial bubbles. Kaizoji and Sornette (2008) as well as Scherbina and Schlusche (2014) provide concise reviews of prominent approaches on the latter. From their summaries it becomes clear that the existing literature has focused on the build-up and burst of bubbles, ignoring its unraveling.

The wide interest in financial bubbles is not least found in the cost that bubbles can have for the economy as a whole. Jordà et al. (2015) analyze these costs, especially in connection with leverage. Similarly, Brunnermeier and Schnabel (2015) provide insights into the history of bubbles by collecting evidence from the main financial bubbles of the last 400 years. The costs also explain the large literature on central bank intervention. Roubini (2006), Posen (2006), and Conlon (2015) are only some examples of a long discussion on whether central banks should burst bubbles. This discussion shows the interest in central bank intervention and the need for a better understanding of its effects. Given the prominence of this question, it is surprising to see no contribution on the dynamics of bursting bubbles.

Over the last years, the research focus on bubbles has shifted from rational investors' behavior towards modeling the actions of noise traders using insights from behavioral research. Scherbina and Schlusche (2014) provide an overview of the recent literature on why financial bubbles occur highlighting different rational and behavioral approaches. The latter often emphasize the role of herd behavior as in Kaizoji (2010a,b).

Brunnermeier (2008) summarizes the different approaches by identifying four types of theoretical models for explaining bubbles. These four strands of models are: 1) all investors have rational expectations and identical information, 2) investors are asymmetrically informed and the existence of a bubble need not be common knowledge, 3) rational traders interact with behavioral traders and limits to arbitrage prevent rational investors from limiting the price impact of behavioral traders, 4) investors hold heterogeneous beliefs and agree to disagree about the fundamental value. This paper can be seen as the aftermath of his third category. In this class of models, limited arbitrage is caused by rational, well-informed and sophisticated investors' interaction with behavioral traders where the latter are subject to psychological biases. Abreu and Brunnermeier (2003) give an example of such a model. They show why rational arbitrageurs may fail to correct excessive price developments driven by noise traders. Building upon their findings, this paper shows that reversals can happen despite rational arbitrageurs and in absence of fundamental news. Further, I find that the interaction between the inhibited arbitrage in Abreu and Brunnermeier (2003) and the analyzed reversal here is likely to matter for a better understanding of unraveling bubbles.

The existing behavioral approaches for explaining financial bubbles are quite different from those used in behavioral asset pricing models. Instead of using psychological biases to explain noise trader behavior, the latter focus on optimizing behavioral traders using cumulative prospect theory (CPT) as highlighted by Giorgi and Hens (2006). Introduced by Kahneman and Tversky (1979) and further developed by Tversky and Kahneman (1992), the potential of CPT to explain puzzles in asset pricing has been widely recognized. Barberis (2013) emphasizes this by providing an overview of the contribution of CPT to the asset pricing literature. He also describes the four elements of prospect theory: 1) reference dependence, 2) loss aversion, 3) diminishing sensitivity and 4) probability weighting. Having analyzed the existing literature, he concludes that diminishing sensitivity, i.e. the curvature of the utility function for gains and losses, matters less in financial research. This is the case as the existing literature focuses on returns close to the reference value. This paper is the first attempt to apply prospect theory to bubbles. In contrast to Barberis (2013), the results in this paper are primarily driven by diminishing sensitivity, highlighting its importance in extreme market situations.

Most earlier work in behavioral asset pricing, such as by Barberis et al. (2001), assume loss averse investors to be homogeneous. Berkelaar and Kouwenberg (2009) find that the heterogeneity of investors in wealth and reference point matters. This is to some extent in contrast to the results here that a difference in wealth or reference value has no effect on the re-entering decision but only on the amount demanded.

Apart from the literature on financial bubbles, there is a small literature on bear market rallies. While this is the first theoretical paper on this topic, there has been some empirical work. A central problem in the empirical analysis of bear market rallies is the necessity for a clear definition and a clear separation between bull markets and bear market rallies. Maheu et al. (2012) analyze weekly S&P500 returns between 1885 and 2008 using a four state Markov switching model. The probability of a bear market being turned into a bear market rally is 96%, while the probability for it to turn into a bull market is 4%. This is partly due to the authors definition of a bear market rally which can turn into a bull or bear market. If we define a bear market rally as a period embedded in bear markets, the probability of a bear market to lead to a bear market rally is still 42%.[2] Maheu et al. (2012) hence emphasize the importance of the phenomenon. Furthermore, the average cumulative return in the bear market state is -12% while bear market rallies counteract this steep decline by yielding a cumulative return of 7% on average. The authors also conclude that bear market rallies are significantly larger than bull market corrections. Over the 123 years, the S&P500 spent around 16% of the time in

[2] In the authors' estimation, the probability of a bear market turning into a bull market is 4%, while the probability of turning into a bear market rally and subsequently into a bull market is 54%. The 42% given here form the remainder.

bear market rallies. These results quantify the importance of understanding downward dynamics and specifically bear market rallies. For future empirical work, my results indicate that it may be worth differentiating between bear market rallies close to the equilibrium level and those closer to the peak.

Overall, theoretical work on downward dynamics such as bear market rallies is scarce. An exception is the literature on fire sales such as by Miller and Stiglitz (2010). The idea that balance sheet effects matter seems to be largely accepted at least in times of crisis. However, fire sales merely exacerbate the downward pressure. It follows, that in presence of fire sales, the mechanism explaining temporary reversals will have to be even stronger. The next section introduces the model set-up and the investor's preferences.

2.3 Model set-up

This model focuses on an investor's portfolio reevaluation during the unraveling of a bubble after suffering an initial loss.

Consider an investor with preferences according to cumulative prospect theory (CPT) as introduced by Kahneman and Tversky (1979). The investor's utility function is split into a concave part for gains with respect to a reference value and a convex part for losses. This implies risk seeking behavior once losses have occurred and risk averse behavior after gains. Figure 2.2 illustrates the CPT utility function.

Using experimental data, Tversky and Kahneman (1992) find that the utility function is best described as

$$u(x) = \begin{cases} u^{+}(x) & \text{if} \quad x \geq 0 \\ u^{-}(-x) & \text{if} \quad x < 0 \end{cases} \tag{2.1}$$

$$u^{+}(x) = x^{\alpha} \quad \text{for} \quad x \geq 0 \quad \text{and} \quad u^{-}(-x) = -\lambda(-x)^{\beta} \quad \text{for} \quad x < 0.$$

In line with the experimental evidence by Tversky and Kahneman (1992), I assume $0.5 < \alpha = \beta < 1$. It is worth mentioning that while Bernard and Ghossoub (2010) depend on $\alpha < \beta$ for an interior solution in a model similar to this one, this assumption is not necessary here. Tversky and Kahneman (1992) find a loss aversion parameter of $\lambda \approx 2.25$. A higher loss aversion parameter λ leads to a stronger punishment for losses. For simplicity and without change in the results,[3] let λ be set to unity.

Assumption 1 *The investor's utility function is given by*

$$u(x) = \begin{cases} x^{\beta} & \textit{if} \quad x \geq 0 \\ -(-x)^{\beta} & \textit{if} \quad x < 0. \end{cases}$$

with $0.5 < \beta < 1$.

Following Kahneman and Tversky, x is given by the change of wealth relative to a reference value Y_0. In this model, I refrain from a fix definition of the reference value. Generally, it can be thought of as the expectation or aspiration the investor has for the asset. Given that the focus here is on financial bubbles, the reference value is likely to be inflated and overly optimistic. The change in wealth is dependent on the investor's demand for a single risky asset. Without loss of generality, assume that the risk-less alternative yields zero interest. The risky asset has two possible outcomes. With probability π the asset yields a high final value Y_g and with probability $(1-\pi)$ the asset realizes a final value $Y_b < Y_g$. Given a bubble setting, the good state can be considered less likely:

Assumption 2 *The probability of the positive outcome is given by*

$$\pi < 0.5.$$

[3]The implications of this simplification are analyzed in Appendix 2.8.10.

Due to their nature, bubble assets are likely to be perceived as having quasi binary outcomes. As an example, consider Internet stocks in the late 1990s. Investors hoped to "find the next Amazon" leading to large gains. At the same time, there was a large risk of bankruptcy for firms which did not become profitable. This makes the assumption of binary outcomes more sensible than in other settings.

In line with Kahneman and Tversky (1979), the probability π is subjective. Agents perceive probabilities in a distorted way where low probabilities are seen as larger, while larger probabilities are undervalued. In this setting, there is no need to specify what the objective probabilities are.

Figure 2.3 illustrates the chronological setting of the model. While the bubble grows, the investor enters the market buying one unit of the asset. For simplicity, assume that this is all his disposable wealth.[4]

Assumption 3 *The investor invests all his disposable wealth into the risky asset when first entering.*

The investor now forms his reference value Y_0. In contrast to the existing literature, I refrain from imposing a functional form on the reference value.[5] One can think of Y_0 as the price that the investor is convinced the asset will reach.

The burst of the bubble is assumed to be caused by a change in market expectation, leading to the subjective expected value

$$E(v_s) := \pi Y_g + (1 - \pi) Y_b. \tag{2.2}$$

After the bubble bursts, the investor sells all his assets to price $Y_r < Y_0$. Conse-

[4]Assumption 3 is not a necessary assumption for the results in this paper to hold. Section 2.8.11 provides more details on the effect of retained wealth. Overall, the behavior of the investor does not change, however additional savings relax the budget constraint and make the condition to observe loss driven behavior more restrictive.

[5]Most authors like Barberis et al. (2001), Gomes (2005), and Bernard and Ghossoub (2009) use the risk free rate as the growth rate for the reference value, which makes it easier to solve the models. One advantage of this paper is that this assumption is not necessary.

quently, he realizes a loss with respect to his reference value of $(Y_0 - Y_r)$. By assumption, the CPT investor is not able to engage in short selling. This is a reasonable assumption when seeing the CPT investor as less sophisticated.

Assumption 4 *No short selling by the CPT investor.*

From this point onward, the CPT investor is loss-driven. He is willing to take large risks hoping to recover his losses. After a further fall in the price to p, the investor decides on his new portfolio ζ. The timing of the decision is hence exogenous[6] while the amount invested is endogenous. A larger ζ implies a larger exposure to the risky asset. Due to the short selling constraint, $\zeta \geq 0$ must hold. Further, given that the investor sold his holdings at price Y_r the investor faces a budget constraint[7] of $\zeta p \leq Y_r$. Hence, the bounds of the new portfolio holdings are given by

$$0 \leq \zeta \leq \frac{Y_r}{p}. \tag{2.3}$$

$p < Y_r$ implies that the upper bound is larger or equal to one and increasing with a lower price p. As the price falls further, an investor is able to buy a larger amount of the asset. Finally, the investor realizes either Y_g or Y_b. Without loss of generality, I set $Y_b > 0$. This allows us to interpret Y_g, Y_0, Y_b, Y_r, and p as price levels.

Assumption 5 *The hierarchy of price levels as explained above is summarized as*

$$Y_g > p > Y_b \qquad \textit{and} \qquad Y_0 > Y_r > p.$$

[6]In this model, the timing when the investor reconsiders his portfolio is exogenous. This is a strong simplification as it implies the absence of intertemporal optimization. However, the setting is sufficient to show how the willingness of the investor depends on the price of the asset and to demonstrate the discontinuous reaction in demand resulting from CPT preferences.

[7]This budget constraint results from Assumption 3 that the investor is fully invested at the height of the bubble. Without this assumption and given the utility function, he is more likely to invest an amount smaller than unity. In that case the budget constraint is relaxed. Consequently, this budget constraint is the minimum budget constraint.

This paper can be limited to cases where the investment yields a negative subjective expected value. The limitation is justified as I am considering reversals at the height of the unraveling. If the asset yielded a positive return, the investment would be attractive for risk neutral investors as well. In this case, there would be no further downward pressure in the price and the reversal would not be temporary. Put differently, without selling pressure the asset would be close to its equilibrium price. This implies that the reconsideration would occur towards the end of the unrevealing rather than at its height.

Assumption 6 *The risky asset has a negative expected return*

$$\pi(Y_g - p) - (1 - \pi)(p - Y_b) < 0.$$

Assumption 6 provides a lower bound for the price when considering to re-enter, given by:

$$p > \pi Y_g + (1 - \pi)Y_b. \tag{2.4}$$

Given Assumptions 1 and 2 that $\beta < 1$ and $\pi < 0.5$, Assumption 6 also implies that an investor with CPT preferences who has not suffered an initial loss would not invest into the risky asset, as

$$\pi(Y_g - p)^\beta - (1 - \pi)(p - Y_b)^\beta < 0. \tag{2.5}$$

The implications of Assumption 6 for the reversal are discussed in more detail in Section 2.6. The next section provides the investor's maximization problem and the optimal portfolio holdings upon re-entering.

2.4 The investor's maximization

This section focuses on the investor's maximization problem when choosing the optimal portfolio ζ. There are two cases which need to be considered.

I) The investor is able to recover his initial losses with respect to his reference value in the good state.

II) Even with the good outcome, the investor cannot recover his initial loss with respect to his reference value.

In the latter case, this implies that the whole maximization takes place in the convex part of the utility function. It is crucial to keep in mind that the utility in the good state depends on the endogenous variable ζ. This implies that it depends on the size of ζ whether the agent can recover his losses and hence which case needs to be considered.

2.4.1 Case I: Complete recovery of losses possible

For now, let us assume that the investor makes an overall gain if the outcome is positive, i.e. Case I. It follows that given the utility function in Assumption 1, the investor's subjective expected utility when creating his optimal portfolio is given by

$$\begin{aligned} E(U) &= \pi u^{+}\left(x_g\right) + (1-\pi) u^{-}\left(x_b\right) \\ &= \pi \left(x_g\right)^{\beta} - (1-\pi)\left(-x_b\right)^{\beta}. \end{aligned} \tag{2.6}$$

Following the setting above, x_g and x_b are given by initial loss $Y_0 - Y_r$ and the gain and loss from the reinvestment, respectively:

$$x_g = \zeta(Y_g - p) - (Y_0 - Y_r) > 0 \quad \text{and} \quad x_b = \zeta(Y_b - p) - (Y_0 - Y_r) < 0.$$

It follows that

$$\begin{aligned} E(U) &= \pi[\zeta(Y_g - p) - (Y_0 - Y_r)]^\beta - (1-\pi)[-(\zeta(Y_b - p) - (Y_0 - Y_r))]^\beta \\ &= \pi[\zeta(Y_g - p) - (Y_0 - Y_r)]^\beta - (1-\pi)[\zeta(p - Y_b) + (Y_0 - Y_r)]^\beta. \end{aligned} \tag{2.7}$$

Loss recovery in the good state implies therefore that $\zeta(Y_g - p) - (Y_0 - Y_r) \geq 0$. For Case I to be true, the optimal investment needs to be at least

$$\zeta_{Min} = \frac{Y_0 - Y_r}{Y_g - p}. \tag{2.8}$$

The minimum investment is needed to have a chance to recover the initial loss. It represents the ratio of the initial loss with respect to the reference value and the potential gain by investing into the risky asset. Together with the upper bound, the minimum ζ for Case I leads to

$$\begin{aligned} \frac{Y_0 - Y_r}{Y_g - p} \leq \zeta \leq \frac{Y_r}{p} \\ p \leq Y_g \frac{Y_r}{Y_0}. \end{aligned} \tag{2.9}$$

This is the condition for Case I, i.e. for total loss recovery to be possible. It is important to note that this is independent of the bad outcome. Hence, the case differentiation is not connected to the expected value but only to the size of the positive outcome.

The investor's maximization problem is given by

$$\max_{\zeta} E(U), \tag{2.10}$$

leading to the following proposition:

Lemma 1 *Given Assumptions 1, 3, 5, and 6: For all parameters within Case I, the* *investor invests a positive amount given by ζ^* up to his budget constraint, where*

$$\zeta^* = \frac{(Y_0 - Y_r)}{(p - Y_b)} \frac{1 + \Omega^\gamma \Phi^\gamma}{\Omega - \Omega^\gamma \Phi^\gamma} \tag{2.11}$$

with $\Omega = \frac{Y_g - p}{p - Y_b} > 0$, $\Phi = \frac{\pi}{1-\pi} > 0$ and $\gamma = \frac{1}{1-\beta} > 2$. He does so despite negative expected returns.

Proof. See Appendix 2.8.1. ■

Ω is the ratio of the outcome in the good state and the absolute of the outcome in the bad state. $\Omega > 1$ implies that the gains in the good state exceed the losses in the bad state. Φ is given by the ratio of the subjective probability for the good state and the subjective probability of the bad state. $\Phi < 1$ implies that the good state is perceived as less likely. Given that $\pi < 0.5$, I find $\Phi < 1$. Following the definitions above, $\Omega\Phi$ is the probability weighted outcome ratio. A $\Omega\Phi < 1$ implies a negative subjective expected value, which is equivalent to Assumption 6.

γ provides the elasticity of intertemporal substitution. A larger β implies a higher elasticity of intertemporal substitution and, hence, lower cost for a suboptimal distribution of consumption across periods. In this setting, this is equivalent to lower cost for an uneven distribution across the states of nature.

Lemma 1 shows the importance of loss recovery. When incentivized by even a remote chance to recover losses, the investor will invest a positive amount even if he expects negative returns.

Corollary 1 *The optimal investment can be decomposed into a loss recovery component ζ_{Min} and an investment component IC:*

$$\zeta^* = \zeta_{Min} + IC, \tag{2.12}$$

where

$$\zeta_{Min} = \frac{Y_0 - Y_r}{Y_g - p} > 0 \quad \textit{and} \quad IC = \frac{\Omega^{\gamma-2}\Phi^{\gamma}(1+\Omega)(Y_0 - Y_r)}{(1-\Omega^{\gamma-1}\Phi^{\gamma})(p - Y_b)} > 0. \tag{2.13}$$

The recovery component is the amount necessary in order for the investor to have a chance to recover his initial losses. The investment component is given by the remainder.

Proof. See Appendix 2.8.2. ■

The loss recovery component ζ_{Min} results from risk seeking behavior i.e. the convex part of the utility function. The investment component, in contrast, results from the concave part which corresponds to risk aversion.

As mentioned before, the optimization problem is only correctly specified if the optimal ζ^* in Equation (2.11) fulfills the minimum ζ condition in Equation (2.8). Equation (2.12) shows that this condition is fulfilled for all ζ^* as $IC > 0$. Let us now take a closer look on the effect of price p. Given that I am analyzing the unraveling of a bubble, it makes sense to look at what happens when the price falls further. It can be shown that:

Corollary 2 *A lower price leads to a larger investment component and a lower loss recovery component of* ζ^**.*

Proof. See Appendix 2.8.3. ■

The risky asset becomes more profitable, due to the lower price. Hence, less investment is needed to recover prior losses. At the same time, a more lucrative investment is more attractive and the investment component rises. The overall effect on the optimal demand is of primary interest which results from the interplay of the two effects as illustrated in Figure 2.4. The solid black line is the minimum ζ_{Min} as in Equation (2.8). The dashed black line illustrates the optimum ζ^* as in Equation (2.11) and the dashed gray line shows the upper bound for ζ given by the budget constraint. The decline in the loss recovery component dominates for high prices as the investment remains unprofitable. However, for sufficiently low prices, the investment component takes over and rapidly increases with falling prices driving the overall demand up.

Figure 2.5 illustrates the effect of a change in the parameter values of π, Y_g, Y_0, and Y_r, respectively. In the upper two cases, an increase in π and Y_g, respectively, leads the risky asset to be more lucrative. Hence, for this specification the investment component drives the demand even for higher values of p. In the bottom left case, an increase in the reference value leads to an upward shift of both minimum and optimal demand. This is the case as the investor is loss driven and a larger loss leads to a higher willingness to invest. Similarly, a larger loss due to a lower Y_r shifts both minimum and optimal demand up. Additionally, the budget constraint is more restricting due to the lower recovery.

2.4.2 Case II: Only partial recovery of losses possible

Now consider the case, where it is impossible for the investor to recover his losses fully. The expected utility function is consequently given by

$$\begin{aligned} E(U) &= \pi u^{-}(x_g) + (1-\pi)u^{-}(x_b) \\ &= -\pi[-\zeta(Y_g - p) + (Y_0 - Y_r)]^{\beta} - (1-\pi)[\zeta(p - Y_b) + (Y_0 - Y_r)]^{\beta}. \end{aligned} \tag{2.14}$$

The investor's optimization in Case II leads to the following lemma:

Lemma 2 *Given Assumptions 1, 2, 3, 4, 5, and 6: Whenever the investor has no chance to regain his losses he either invests all he can or nothing.*

Proof. See Appendix 2.8.4. ■

The intuition for this result is straightforward. The investor is risk seeking in his optimization in Case II. If the terms of the investment are unacceptable, the investor will not invest anything. This is e.g. the case when the investment implies a certain loss. If the terms are acceptable, the investor will invest all his disposable wealth into the asset in order to have a chance to get as close as possible to recovering his prior losses. He perceives the riskiness of the asset positively, as an increase in the spread of good and bad states gets him, *ceteris paribus*, closer to recovering his losses.

Combining the two cases and by that Lemmas 1 and 2 leads to the following: If the investor chooses to re-enter the market, he will invest ζ^* up to his budget constraint.

$$\zeta^* = \begin{cases} \frac{(Y_0-Y_r)}{(p-Y_b)}\frac{1+\Omega^\gamma\Phi^\gamma}{\Omega-\Omega^\gamma\Phi^\gamma} & \text{if } \frac{(Y_0-Y_r)}{(p-Y_b)}\frac{1+\Omega^\gamma\Phi^\gamma}{\Omega-\Omega^\gamma\Phi^\gamma} \leq \frac{Y_r}{p} \\ \frac{Y_r}{p} & \text{if } \frac{(Y_0-Y_r)}{(p-Y_b)}\frac{1+\Omega^\gamma\Phi^\gamma}{\Omega-\Omega^\gamma\Phi^\gamma} > \frac{Y_r}{p}. \end{cases} \tag{2.15}$$

In a final step, it is necessary to evaluate when the investor will re-enter, i.e. when the utility from investing is larger than the utility from abstaining from the market.

2.5 The decision to re-enter

The focus of the analysis is the investor's willingness to re-enter the market given that he reconsiders his absence from the market at a specific point in time. In order to determine this, I compare the investor's utility from remaining outside the market and re-entering.[8]

An investor compares re-entering and investing ζ^* to the certain loss when abstaining the market. For re-entering to be optimal, the following condition needs to be fulfilled

$$E(U(\zeta^*)) \geq E(U(\zeta = 0)).$$

Lemma 3 *Given Assumptions 1 and 6: The condition for re-entering and hence for*

[8]In the following sections, I am focusing on interior solutions for the optimal investment, i.e. cases not bound by the budget constraint. Bound cases are considered in Appendix 2.8.9.

positive demand is given by

$$\pi \left(\frac{IC}{\zeta_{Min}} \right)^{\beta} \left[\left(\frac{1}{\Omega^{\beta}\Phi} \right)^{\gamma} - 1 \right] - 1 \leq 0. \tag{2.16}$$

Proof. See Appendix 2.8.5. ■

Lemma 3 describes the moment when an investor is willing to re-enter the market. Any investor, for whom this condition is not fulfilled, will not re-enter the market upon reconsidering his portfolio. It is important to keep in mind that this model focuses on a bear market. Consequently, this paper aims at answering the question what happens when the price falls and whether this leads to a sudden increase in demand. Lemma 1 and Lemma 3 lead to the following conclusion:

Proposition 1 *When p falls, the "potential" demand, i.e. the desired amount invested upon reconsideration, jumps from zero to the optimal level.*

Proof. See Appendix 2.8.6. ■

The implication here is that this sudden jump can be responsible for the temporary reversal of the downward trend. The size of the jump in demand is given by the optimal demand in Equation (2.11) as the investor has no holding prior to re-entering.

$$\zeta^* = \frac{(Y_0 - Y_r)}{(p - Y_b)} \frac{1 + \Omega^{\gamma}\Phi^{\gamma}}{\Omega - \Omega^{\gamma}\Phi^{\gamma}}$$

Figure 2.6 depicts Proposition 1 for different parameter values. The solid line is given by the investor's demand. The jump in "potential" demand occurs once Equation (2.16) holds. Given a high price, the investor's optimal demand is zero. As the price falls over time, it reaches the point when demand jumps to over 56% of what the investor held at the height of the bubble. From this point onwards, the demand increases continuously with a falling price. The discontinuity in the demand function already hints the potential for an upward jump in the price. If the investor faces falling prices, from one moment to the other, he is willing to invest large amounts into an asset which he sold off previously. At this point, the expected return of the asset is still negative.

The four graphs in Figure 2.7 highlight the *ceteris paribus* effect of an increase in the probability of the good state π, a better outcome in the good state Y_g, a higher reference value Y_0, and a decrease in the recovery price Y_r, respectively, compared to Figure 2.6. Doubling the probability π leads to a similar size in the jump, yet the jump occurs earlier, i.e. for higher p. This is intuitive as a higher π implies a more profitable investment. The same is true for an increase in the good outcome. In the latter case, however, the investment reaches only around 34% of the prior holdings.

The two bottom graphs show an increase in the reference value and a decrease in the recovery rate, respectively. Both are equivalent to a higher loss with respect to the reference value and hence have the same effect. The only difference is that a change in Y_r lowers the budget constraint, restricting the slope of the demand for low p. The reaction to a change in the initial loss leads to the main result of the paper. Using Lemmas 1 and 3 it follows that:

Proposition 2 *The decision to re-enter is independent of the initial loss; however, the size of the potential investment positively depends on it.*

Proof. See Appendix 2.8.7. ■

A group of investors with similar expectations about the asset's true value are willing to re-enter the market at a similar time, independently of their prior losses and reference value. However, the size of each investor's investment will differ. The above implies that the timing of the reinvestment is also independent of the disposable wealth.[9]

The results agree with Berkelaar and Kouwenberg (2009) on that the heterogeneity of investors with regard to wealth and reference value matters. In this setting, however, the implications are quite different. While the heterogeneity is important to

[9]This is true, as long as the disposable wealth is sufficient to allow for the optimal investment. If this is not given, the re-entering will be delayed.

explain the size of individual demand, the timing is not affected. This implies that from a macro perspective, the heterogeneity matters much less as the individual demand jumps at the same time leading to a jump in aggregate demand. Given the importance of the timing in this setting, the next section goes into it in more detail.

2.6 Equilibrium price

The above analysis has so far considered the CPT investor in isolation. This section takes the previous results further by looking at the interaction of multiple CPT and risk-averse (RA) investors. The model proposed here is not a standard general equilibrium model, but describes how the market reaches its equilibrium.

The layout of the model is summarized as follows. In period 0, the CPT investors update their rational expectations about the pay-offs of the asset and trade with the RA investors before the latter update their own expectations. The RA investors do not know the exact demand function of the CPT investors. Therefore, in period 1 the RA investors submit sell orders anticipating the maximum possible demand of the CPT investors to a given price. The anticipated maximum demand, however, assumes the demand function of the CPT investors to be continuous.[10] This creates a disparity between the anticipated demand and the actual demand due to the jump in the latter. Consequently, there are two possible cases. If the actual demand falls short of the anticipated maximum demand, the RA investors submit further sell orders in the next periods until the market is in equilibrium.[11] If the actual demand exceeds the anticipated demand, this implies that the orders submitted by the RA investors imply too low a price. It follows that the price will rise in the following period in order to bring the market into equilibrium. This reversal in the price is what is described as a *dead cat bounce*. This set-up is formalized

[10]One option is that the RA investors simply ignore the influence of the perceived loss by the CPT investors. I keep the function general in order not to pose strong restrictions.

[11]One can think of this as the RA investors submitting orders with a fix price and quantity.

below.

Assumption 7 *The total amount of investors and assets is normalized to unity.* n *investors are risk-averse (RA), while* $1-n$ *are CPT investors, following the optimization behavior described above. At the start of the bear market, a total amount* x *of the asset is held by RA investors and* $(1-x)$ *is held by CPT investors.*

The number and holdings of RA versus CPT investors allows to differentiate between different asset classes. While traditional currency or commodity markets may be dominated by sophisticated RA investors, cryptocurrencies may be thought of as CPT dominated. This idea is in line with Bianchi and Dickerson (2018) who argue that cryptocurrency markets see higher trading volume by retail investors while the number of institutional investors and hedge funds is lower than in other markets. For brevity, I normalize the updated pay-off of the asset to

Assumption 8 *The pay-off of the asset in the good state is normalized to 1, while the bad state's pay-off is set to 0 such that*

$$Y_g = 1 \qquad \textit{and} \qquad Y_b = 0. \tag{2.17}$$

Assumption 9 *Assume that all CPT investors receive information that the asset is overvalued before the RA investors do. The CPT investors are hence able to sell all their holdings to price* p_0 *to the RA investors, where* p_0 *is determined by the RA investors expectations before receiving the update.*

The results do not necessarily depend on the strict form of this assumption. Similar to the assumption that the CPT investor liquidates all holdings at price Y_r, this assumption simplifies the model in order to focus on the motivation of the CPT investor. A more gradual sell-off is also possible and would likely lead to very similar results.[12]

[12] An alternative set-up for this would be that the CPT investor does not sell-off his holdings but has

After learning about the updated pay-off structure of the asset, the RA investors' demand for the asset is determined by the maximization of their utility function.

Assumption 10 *Each RA investor maximizes a CRRA utility function given by*

$$U_{RA} = \pi(X_g)^\alpha + (1-\pi)(X_b)^\alpha, \tag{2.18}$$

where $1-\alpha$ is the degree of risk aversion of the RA investor, X_g is the wealth in the good state, and X_b is the wealth in the bad state.

In contrast to the CPT, the CRRA utility function is taking into account the total wealth, rather than the change in wealth.

Assumption 11 *The total wealth of the RA investor in the good state is given by*

$$\begin{aligned} X_g &= \frac{x}{n}Y_g + \Delta(Y_g - p) + W + (Yg - p_0)\frac{1-x}{n} \\ &= \frac{x}{n} + \Delta(1-p) + W + (1-p_0)\frac{1-x}{n} \\ &= \frac{1}{n} + \Delta(1-p) + W - p_0\frac{1-x}{n}. \end{aligned} \tag{2.19}$$

In the bad state, this is equivalent to

$$\begin{aligned} X_b &= \frac{x}{n}Y_b + \Delta(Y_b - p) + W + (Y_b - p_0)\frac{1-x}{n} \\ &= -\Delta p + W - p_0\frac{1-x}{n}. \end{aligned} \tag{2.20}$$

The total wealth in the good state decomposes as follows: $\frac{x}{n}Y_g$ is the final pay-off of the initial holdings of the individual RA investor. It is given by the total amount of

savings that can be used for loss driven investment. In this case, the assumption of CPT investors to be fast, would not be necessary. As the loss driven behavior is the key driver of this model, such a set-up is likely to lead to very similar results, however with different budget constraints. An additional feature of this set-up would be that the observed dynamics resemble more closely the disposition effect described by e.g. Shefrin and Statman (1985). However, the main result that (perceived) loss drives the bear market rally remains in place.

the asset held by RA investors x, divided by the number of RA investors n. The amount purchased by an individual RA investor is given by Δ. p corresponds to the price he is paying. W is given by the exogenous, independent wealth of the RA investor and $(Yg - p_0)\frac{1-x}{n}$ is the profit from buying all of the CPTs' asset holdings. In the bad state, the total wealth is derived equivalently.

The maximization of the RA investors' utility results in an optimal change in demand Δ^*. For this analysis, it makes sense to consider the total supply.

Lemma 4 *The total supply is given by the total amount of assets to be sold by the n RA investors*

$$\begin{aligned} S(p) = -n\Delta^* &= -n\frac{(W - p_0\frac{1-x}{n})(\Omega^\epsilon\Phi^\epsilon - 1) - \frac{1}{n}}{p(\Omega + \Omega^\epsilon\Phi^\epsilon)} \\ &= \frac{1 - (Wn - p_0(1-x))(\Omega^\epsilon\Phi^\epsilon - 1)}{p(\Omega + \Omega^\epsilon\Phi^\epsilon)}, \end{aligned} \tag{2.21}$$

where $\epsilon = \frac{1}{(1-\alpha)}$ is one over the risk aversion parameter. The supply function $S(p)$ is a concave function increasing in price p.

This paper does not restrict short selling for RA investors, as they are likely to be more sophisticated. However, such a restriction would not significantly alter the results. When choosing how much to supply, the RA investors anticipate the total maximum demand for a given price and supply the anticipated (maximum) equilibrium amount. The logic is as follows. In a given period, the suppliers observe the demand at the current price. The highest demand they deem possible in the next period would follow a function which is zero at the current price but increases as the price falls in the next period. Why would the RA investors go for the maximum demand? Assume that the RA investors collectively under-anticipated the total demand. In this case, each of the suppliers would have had an incentive to supply more. In contrast, if the RA investors assumed the total demand to be too high, none of the suppliers would have profited from supplying less. The RA investors, therefore supply in order to meet the maximum demand they deem possible. Thereby, the RA investors are more careful than in standard equilibrium models.[13]

Assumption 12 *Given a price p_t in the current period and a price p_{t+1} in the next period, the anticipated maximum demand is given by $\Upsilon_{p_t}(p_{t+1})$, with $\Upsilon_{p_t}(p_t) = 0$, $\frac{\partial \Upsilon_{p_t}(p_{t+1})}{\partial p_{t+1}} < 0$ and $\frac{\partial^2 \Upsilon_{p_t}(p_{t+1})}{\partial p_{t+1} \partial p_{t+1}} > 0$. Once the suppliers observe a positive demand, they can anticipate the actual demand.*

The definition of $\Upsilon_{p_t}(p_{t+1})$ is kept fairly independent on purpose in order to keep the results general but interpretable. If the actual demand is equal to the anticipated maximum demand, the market will be in equilibrium. This is the standard case considered in general equilibrium models. If the demand remains non-positive, the suppliers will repeat the step above, using the anticipated maximum demand function $\Upsilon_{p_{t+1}}(p_{t+2})$. If the demand is positive but below the supply, it is optimal for the suppliers to lower the price further. They will lower it more cautiously however taking into account the current level of demand.

Figure 2.8 illustrates the procedure described above. The x-axis shows the price and a move from right to left implies a devaluation. As the price falls, the anticipated maximum demand is increasing until it reaches the supply function at price p_1. This is the maximum price, which the suppliers expect to lead to an equilibrium. They hence choose to supply the amount $S(p_1)$ in this period. As there is no demand, the suppliers use the anticipated maximum demand function once more in the next period, leading to the supply of $S(p_2)$.

This describes the development of the supply by the RA investors. In the next step, let us consider the equilibrium in the market given by the (actual) aggregated

[13] In order to not make it optimal to submit sell orders for the whole supply, we could assume that there are marginal costs to submitting orders. This would lead the suppliers to again go for the anticipated maximum demand, given that they are confident enough about this being the maximum.

demand and the total supply. The total demand of the CPT investors given Assumptions 7, 8, and 9 is given by

$$
\begin{aligned}
D(p) = (1-n)\zeta^*(p) &= (1-n)\left(\frac{1-x}{1-n}\right)\left(\frac{Y_0-p_0}{p}\right)\frac{(1+\Omega^\gamma\Phi^\gamma)}{(\Omega-\Omega^\gamma\Phi^\gamma)} \\
&= (1-x)\left(\frac{Y_0-p_0}{p}\right)\frac{(1+\Omega^\gamma\Phi^\gamma)}{(\Omega-\Omega^\gamma\Phi^\gamma)}.
\end{aligned} \tag{2.22}
$$

The efficient price p^* is hence given by $D(p^*) = S(p^*)$. Figure 2.9 illustrates the equilibrium. Let us denote the price at the re-entry of the CPT investors by p_R. As highlighted in Proposition 2, this point is independent of the initial loss and will hence be the same for heterogeneous investors.

The next part combines the anticipated maximum demand and the actual demand function. Figure 2.10 illustrates an example where the model exhibits a *dead cat bounce*. At the first intersection of the anticipated maximum demand and the supply function, at price p_1, the actual demand is zero. At the second intersection at p_2, the actual demand exceeds the anticipated demand, due to the jump at p_R. It follows that in period 1, there is no trade and the price is set to p_1. In period 2, the price falls to price p_2 and $\Upsilon_{p_1}(p_2)$ units are traded. As demand exceeds supply at this point, the price will increase over the next periods until it reaches the equilibrium level p^*. Consequently, after falling to p_2, the price experiences a reversal rising by $p^* - p_2$. This is the size of the *dead cat bounce*.

Similarly, Figure 2.11 shows an example, which does not lead to a reversal. In period 1, the price reaches p_1. The actual demand given by the black line is not equal to the supply, however, it is positive. Consequently, $D(p_1)$ of the asset is traded. Having now observed the actual demand, the suppliers will supply only as much in the next period as is necessary to meet the actual demand at price p^*. There is no overshooting in the price and no reversal.

When generalizing this, a clear condition for the *dead cat bounce* appears. Whether there is a *dead cat bounce*, depends on the relative size of the equilibrium price p^*, the re-entry price p_R, the point where the supply function is zero $S^{-1}(0)$ and the intersection of the anticipated maximum demand with the supply function (p_t). This is illustrated in figure 2.12.

Lemma 5 *There exists a dead cat bounce, iff the intersection of the anticipated maximum demand with the supply function falls between the price for zero demand and the equilibrium price:*

$$S^{-1}(0) < p_t < p^* \tag{2.23}$$

In contrast, there is no dead cat bounce, if the intersection of the anticipated maximum demand with the supply function falls between the equilibrium price and the re-entry price:

$$p^* \leq p_t \leq p_R \tag{2.24}$$

The size of the dead cat bounce is determined by the distance between the equilibrium price and the intersection: $p^* - p_t$.

The first implication to be drawn from this is that the size of the *dead cat bounce* cannot exceed $p^* - S^{-1}(0)$. Apart from this, the results are very sensitive to the starting value when the RA investors begin to anticipate the CPT investors' demand as well as the functional form of the anticipated demand. For this reason, it is more meaningful to state the results in terms of probabilities and expectations dependent on the starting value. The model extension in this section leads to a series of testable predictions.

Proposition 3 *Given the interaction of multiple CPT and RA investors:*

(a) *The higher the reference value Y_0 of the CPT investors, the higher is the probability of a dead cat bounce and the larger its expected size.*

(b) *The larger the share of RA investors n, the smaller is the possible dead cat bounce.*

(c) *A lower share of RA investors n or a lower share of initial holdings held by RA investors x can increase the probability that the dead cat bounce happens in a later period.*

Proof. See Appendix 2.8.8. ■

Proposition 3a) states, a higher reference value of the CPT investors, e.g. due to more optimism at the build up of the bubble, increases the probability of a *dead cat bounce* as well as its expected size. This is due to the same effect observed in Proposition 2. A higher reference value leads to a higher perceived loss and thereby to a larger demand upon re-entering. The larger jump in demand means, that the difference between the anticipated maximum demand and the actual demand is larger. This leads to a larger reversal and also increases its probability.

According to Proposition 3b), a larger share of RA investors reduces the size of a possible *dead cat bounce*. The reason for this is that the amount of the asset held by each RA investor $\frac{x}{n}$ is smaller compared to his wealth w. Due to this, the RA investors are overall willing to take more risk. This increases the supply for the asset by RA investors when the asset has negative expected returns in which case the RA investors engage in short selling. This makes the aggregate supply more price sensitive in this area. Consequently, the difference between the prices for which the supply meets the anticipated maximum demand and the actual demand shrinks and the possible *dead cat bounce* shrinks as well.

Similarly, Proposition 3c) states a lower share of RA investors decreases the supply of the asset and increases the price where the supply meets the anticipated maximum demand for a given period. This in turn increases the probability that the CPT investors do not yet re-enter the market as the price is too high. Consequently, given there is going to be a *dead cat bounce*, the probability that we see a reversal in a later period increases. If the RA investors held a lower share before the beginning of the bear market (a lower x), this decreases each RA investors total wealth equally in the good and in the bad state as can be seen in Assumption 11. Due to their risk aversion, this implies that a lower initial share x leads to a larger punishment for the difference in outcomes. This decreases the amount a RA investor is willing to sell in excess of his holdings (short selling). This in turn increases the price for which the supply curve and the anticipated maximum demand intersect. As above, this is equivalent to saying the probability that there is a *dead cat bounce* in a later period increases conditional on there being a *dead cat bounce* at all.

2.7 Conclusion

Unraveling bubbles in capital, currency, and commodity markets often experience temporary reversals of the downward trend, also known as *dead cat bounce* or *bear market rally*. This is the first paper to offer a theoretical explanation for a phenomenon which is largely recognized in financial markets.

According to investopedia, a *dead cat bounce* "can be a result of traders or investors closing out short positions or buying on the assumption that the security has reached a bottom". In contrast, this paper shows how preferences according to cumulative prospect theory (CPT) can explain the temporary reversal of the downward trend in unraveling bubbles. CPT preferences lead an investor to take high risk and make unprofitable investments in hope to recover losses experienced after the burst of the bubble. This leads to a jump in demand for the individual investor. The decision to re-enter is independent of investors' reference value and prior loss. Due to this, a group of investors with the same expectations with regard to the asset will re-enter the market at the same time, leading to a large aggregate jump in demand. In addition, this implies that even under heterogeneous expectations, the release of minor information can have a major impact on the price. Due to the discontinuous demand functions, a minor adjustment in the expected return can lead a group of investors to re-enter the market at the same time. Thereby, seemingly unimportant information can trigger a bear market rally even under heterogeneous expectations.

The model proposed here leads to several testable results. A lower share of unsophisticated investors (CPT investors) is associated with a smaller *dead cat bounce*. A higher share of unsophisticated investors or a higher share of holdings by unsophisticated investors during the peak of the bubble increase the probability that a price reversal happens at a later point in time, i.e. that there is a longer period between the peak of the bubble and the *dead cat bounce*. Finally, the larger the optimism during the build-up of the bubble, the higher the probability of a *dead cat bounce* and the larger its expected size.

This paper provides a new mechanism explaining the temporary reversal of the downward trend in unraveling bubbles. There are several other papers that have shown mechanisms which are likely to interact with the one above. Some, such as fire sales described by Miller and Stiglitz (2010) work against the reversal and hence highlight the size of the demand necessary to match the downward pressure. Others, such as herd behavior and limited arbitrage are likely to support the reversal once it started. Kaizoji and Sornette (2008) and Kaizoji (2010a,b) analyze the effect of herd behavior in financial markets. On the one hand, herding puts additional pressure on the price once it starts falling, on the other hand, it is also likely to amplify the reversal. Abreu and Brunnermeier (2003) show that arbitrageurs may forgo betting against a bubble when uncertain about the timing of the burst. This bubble riding behavior is likely to appear during the reversal as well, prolonging its existence further.

Further research is clearly necessary in order to better understand the downward dynamics in financial markets. More specifically, the results in this paper provide several areas for further research. Firstly, a more dynamic setting is necessary to see how the mechanism shown in this paper interacts with the intertemporal optimization as well as investors herding and fire sales. A further analysis in a multi-asset setting is also likely to provide valuable insights. Additionally, it is crucial that we gain better insights into the empirical side of the reversal, *dead cat bounce*, or *bear market rally* in the aftermath of financial bubbles. The existing research on bear market rallies is limited to the stock market. The commonness of the phenomenon in FX as well as commodity markets raises the question to what extent the reversals differ from one another. Further, the analysis indicates that reversals may differ depending on whether they occur in the middle of the fall in prices or mark the end of the unraveling.

This paper is a first step in the theoretical analysis of unraveling bubbles. Further analysis of downward dynamics deserve more attention in research and promise valuable insights for policymakers, researchers, and investors.

2.8 Appendix

2.8.1 Proof of Lemma 1

The maximization problem in Case I is given by

$$\max_{\zeta} E(U), \tag{2.25}$$

which leads to the first order condition

$$\begin{aligned}\frac{\partial E(U)}{\partial \zeta} =& \pi\beta[\zeta(Y_g - p) - (Y_0 - Y_r)]^{\beta-1}(Y_g - p) \\ & - (1-\pi)\beta[\zeta(p - Y_b) + (Y_0 - Y_r)]^{\beta-1}(p - Y_b) = 0 \end{aligned} \tag{2.26}$$

$$\begin{aligned} \Rightarrow \qquad \Omega\Phi[\zeta(Y_g - p) - (Y_0 - Y_r)]^{\beta-1} &= [\zeta(p - Y_b) + (Y_0 - Y_r)]^{\beta-1} \\ \Omega\Phi[\zeta(p - Y_b) + (Y_0 - Y_r)]^{1-\beta} &= [\zeta(Y_g - p) - (Y_0 - Y_r)]^{1-\beta} \\ \Omega^{\gamma}\Phi^{\gamma}[\zeta(p - Y_b) + (Y_0 - Y_r)] &= [\zeta(Y_g - p) - (Y_0 - Y_r)] \\ \zeta((Y_g - p) - (p - Y_b)\Omega^{\gamma}\Phi^{\gamma}) &= (Y_0 - Y_r)(1 + \Omega^{\gamma}\Phi^{\gamma}) \\ \zeta^* &= \frac{(Y_0 - Y_r)}{(p - Y_b)}\frac{1 + \Omega^{\gamma}\Phi^{\gamma}}{\Omega - \Omega^{\gamma}\Phi^{\gamma}} \end{aligned}$$

with $\Omega = \frac{Y_g - p}{p - Y_b} > 0$, $\Phi = \frac{\pi}{1-\pi} > 0$ and $\gamma = \frac{1}{1-\beta} > 2$.

The second order condition is given by

$$\begin{aligned}\frac{\partial^2 E(U)}{\partial \zeta \partial \zeta} =& \pi\beta(\beta-1)[\zeta(Y_g - p) - (Y_0 - Y_r)]^{\beta-2}(Y_g - p)^2 \\ & - (1-\pi)\beta(\beta-1)[\zeta(p - Y_b) + (Y_0 - Y_r)]^{\beta-2}(p - Y_b)^2 < 0 \end{aligned}$$

$$
\begin{aligned}
\Rightarrow \quad & \Omega^{\frac{2}{2-\beta}}\Phi^{\frac{1}{2-\beta}}[\zeta(p-Y_b)+(Y_0-Y_r)] > [\zeta(Y_g-p)-(Y_0-Y_r)] \\
& \zeta(\Omega-\Omega^{\frac{2}{2-\beta}}\Phi^{\frac{1}{2-\beta}})(p-Y_b) < (Y_0-Y_r)(1+\Omega^{\frac{2}{2-\beta}}\Phi^{\frac{1}{2-\beta}}).
\end{aligned}
$$

Using ζ^* from the first order condition leads to

$$
\begin{aligned}
\frac{Y_0-Y_r}{p-Y_b}\frac{1+\Omega^\gamma\Phi^\gamma}{\Omega-\Omega^\gamma\Phi^\gamma}(\Omega-\Omega^{\frac{2}{2-\beta}}\Phi^{\frac{1}{2-\beta}})(p-Y_b) &< (Y_0-Y_r)(1+\Omega^{\frac{2}{2-\beta}}\Phi^{\frac{1}{2-\beta}}) \\
\frac{1+\Omega^\gamma\Phi^\gamma}{\Omega-\Omega^\gamma\Phi^\gamma}(\Omega-\Omega^{\frac{2}{2-\beta}}\Phi^{\frac{1}{2-\beta}}) &< (1+\Omega^{\frac{2}{2-\beta}}\Phi^{\frac{1}{2-\beta}}) \\
-\Omega^{\frac{2}{2-\beta}}\Phi^{\frac{1}{2-\beta}}+\Omega^{\gamma+1}\Phi^\gamma &< \Omega^{(1+\frac{2}{2-\beta})}\Phi^{\frac{1}{2-\beta}}-\Omega^\gamma\Phi^\gamma \\
\Omega^\gamma\Phi^\gamma &< \Omega.
\end{aligned}
$$

It is straightforward to show, that this second order condition is equivalent to Inequality (2.5) which follows from Assumption 6:

$$
\begin{aligned}
\Omega^\gamma\Phi^\gamma &< \Omega \\
\Phi\Omega^\beta &< 1 \\
\frac{\pi}{1-\pi}\left(\frac{Y_g-p}{p-Y_b}\right)^\beta &< 1 \\
\pi(Y_g-p)^\beta-(1-\pi)(p-Y_b)^\beta &< 0.
\end{aligned}
$$

It follows that ζ^* is the optimal demand under Case I.

Finally, let us consider the upper bound. Given the single extreme value shown by the first order condition, the optimal value must be given by the upper bound, whenever ζ^* exceeds the upper bound given by the budget constraint $\frac{Y_r}{p}$.

It follows for Case I, that the optimal demand is either given by ζ^* or full investment. ■

2.8.2 Proof of Corollary 1

Corollary 1 follows from a simple transformation of Equation (2.11):

$$\begin{aligned}
\zeta^* &= \frac{(Y_0 - Y_r)}{(p - Y_b)} \frac{1 + \Omega^\gamma \Phi^\gamma}{\Omega - \Omega^\gamma \Phi^\gamma} \\
&= \frac{(Y_0 - Y_r)}{(Y_g - p)} \Omega \frac{1 + \Omega^\gamma \Phi^\gamma}{\Omega - \Omega^\gamma \Phi^\gamma} \\
&= \frac{(Y_0 - Y_r)}{(Y_g - p)} \frac{1 + \Omega^\gamma \Phi^\gamma - \Omega^{\gamma-1} \Phi^\gamma + \Omega^{\gamma-1} \Phi^\gamma}{1 - \Omega^{\gamma-1} \Phi^\gamma} \\
&= \frac{(Y_0 - Y_r)}{(Y_g - p)} \left(1 + \frac{\Omega^{\gamma-1} \Phi^\gamma (1 + \Omega)}{1 - \Omega^{\gamma-1} \Phi^\gamma}\right) \\
&= \frac{(Y_0 - Y_r)}{(Y_g - p)} + \frac{(Y_0 - Y_r)}{(Y_g - p)} \frac{\Omega^{\gamma-1} \Phi^\gamma (1 + \Omega)}{1 - \Omega^{\gamma-1} \Phi^\gamma} \\
&= \zeta_{Min} + \frac{(Y_g - p) \Omega^{\gamma-2} \Phi^\gamma (1 + \Omega)(Y_0 - Y_r)}{(p - Y_b)(1 - \Omega^{\gamma-1} \Phi^\gamma)(Y_g - p)} \\
&= \zeta_{Min} + \frac{\Omega^{\gamma-2} \Phi^\gamma (1 + \Omega)(Y_0 - Y_r)}{(1 - \Omega^{\gamma-1} \Phi^\gamma)(p - Y_b)} \\
&= \zeta_{Min} + IC.
\end{aligned}$$

Further, given Inequality (2.5), it can be shown that

$$\begin{aligned}
\pi (Y_g - p)^\beta - (1 - \pi)(p - Y_b)^\beta &< 0 \\
\Phi \Omega^\beta &< 1 \\
\Omega^{\gamma-1} \Phi^\gamma &< 1.
\end{aligned}$$

This implies that $IC > 0$ must hold. ■

2.8.3 Proof of Corollary 2

The loss recovery component $\zeta_{Min} = \frac{(Y_0 - Y_r)}{(Y_g - p)}$ is given by Equation (2.8). It follows

$$-\frac{\partial \zeta_{Min}}{\partial p} < 0.$$

Hence, as the price p falls, the loss recovery component declines as well.

From the proof in Appendix 2.8.2 it is clear that IC can be written as

$$IC = \frac{(Y_0 - Y_r)}{(Y_g - p)} \frac{\Omega^{\gamma-1}\Phi^{\gamma}(1+\Omega)}{1-\Omega^{\gamma-1}\Phi^{\gamma}}.$$

For simplicity, let us denote

$$\Psi = \frac{\Omega^{\gamma-1}\Phi^{\gamma}(1+\Omega)}{1-\Omega^{\gamma-1}\Phi^{\gamma}}.$$

It follows that $IC = \zeta_{Min}\Psi$. This allows us to analyze the components separately

$$\begin{aligned}
-\frac{\partial\Psi}{\partial p} &= -\frac{\partial\Omega}{\partial p}\Phi^{\gamma}\left(\frac{\left(\gamma\Omega^{\gamma-1}+(\gamma-1)\Omega^{\gamma-2}\right)\left(1-\Omega^{\gamma-1}\Phi^{\gamma}\right)+\left(\Omega^{\gamma}+\Omega^{\gamma-1}\right)(\gamma-1)\Omega^{\gamma-2}\Phi^{\gamma}}{\left(1-\Omega^{\gamma-1}\Phi^{\gamma}\right)^2}\right) \\
&= -\frac{\partial\Omega}{\partial p}\frac{\Phi^{\gamma}}{\left(1-\Omega^{\gamma-1}\Phi^{\gamma}\right)^2}\left((\gamma-1)\Omega^{\gamma-2}+\Omega^{\gamma-1}\left(\gamma-\Omega^{\gamma-1}\Phi^{\gamma}\right)\right) \\
&= -\frac{\partial\Omega}{\partial p}\frac{\Phi^{\gamma}\Omega^{\gamma-2}}{\left(1-\Omega^{\gamma-1}\Phi^{\gamma}\right)^2}\left(\gamma-1+\gamma\Omega-\Omega^{\gamma}\Phi^{\gamma}\right). \qquad (2.27)
\end{aligned}$$

As $\frac{\partial\Omega}{\partial p} = \frac{\partial\frac{Y_g-p}{p-Y_b}}{\partial p} < 0$, $\gamma > 2$, and Inequality (2.5) holds, it follows that

$$-\frac{\partial\Psi}{\partial p} > 0. \qquad (2.28)$$

Hence, a fall in the price leads to an increase in Ψ. As $-\frac{\partial\zeta_{Min}}{\partial p} < 0$, this implies two contrary forces within the investment component. It can be shown that

$$-\frac{\partial IC}{\partial p} = -\frac{\partial \zeta_{Min}}{\partial p}\Psi - \zeta_{Min}\frac{\partial \Psi}{\partial p} \tag{2.29}$$

$$= -\frac{Y_0 - Y_r}{(Y_g - p)^2}\Phi^{\gamma}\frac{\left(-2(\gamma-1)\Omega^{\gamma} - (\gamma-2)\Omega^{\gamma-1} - \Omega^{2\gamma-2}\Phi^{\gamma} - \Omega^{\gamma+1}\left(\gamma - \Omega^{\gamma-1}\Phi^{\gamma}\right)\right)}{\left(1-\Omega^{\gamma-1}\Phi^{\gamma}\right)^2} \tag{2.30}$$

$$> 0. \tag{2.31}$$

A lower price leads to a larger investment as the effect on Ψ dominates the effect on ζ_{Min}. ■

2.8.4 Proof of Lemma 2

The first order condition for the investor's maximization problem in Case II is given by

$$\begin{aligned}\frac{\partial E(U)}{\partial \zeta} =& \pi\beta[-\zeta(Y_g - p) + (Y_0 - Y_r)]^{\beta-1}(Y_g - p)\\ &- (1-\pi)\beta[\zeta(p - Y_b) + (Y_0 - Y_r)]^{\beta-1}(p - Y_b) = 0\end{aligned}$$

$$\begin{aligned}\Rightarrow \qquad \Phi\Omega[\zeta(p - Y_b) + Y_0 - Y_r]^{1-\beta} &= [-\zeta(Y_g - p) + Y_0 - Y_r]^{1-\beta}\\ (\Phi\Omega)^{\gamma}\left[\zeta(p - Y_b) + Y_0 - Y_r\right] &= [-\zeta(Y_g - p) + Y_0 - Y_r]\\ \zeta[(Y_g - p) + (p - Y_b)\left(\Phi\Omega\right)^{\gamma}] &= (Y_0 - Y_r)[1 - (\Phi\Omega)^{\gamma}]\\ \hat{\zeta}^* &= \frac{Y_0 - Y_r}{p - Y_b}\frac{1 - (\Phi\Omega)^{\gamma}}{\Omega + (\Phi\Omega)^{\gamma}}.\end{aligned} \tag{2.32}$$

The second order condition is given by

$$\begin{aligned}\frac{\partial^2 E(U)}{\partial\zeta\partial\zeta} =& -\pi\beta(\beta-1)[-\zeta(Y_g - p) + (Y_0 - Y_r)]^{\beta-2}(Y_g - p)^2\\ &- (1-\pi)\beta(\beta-1)[\zeta(p - Y_b) + (Y_0 - Y_r)]^{\beta-2}(p - Y_b)^2 < 0.\end{aligned}$$

Taking into account the result for the first order condition in Equation (3.1) we get

$$\begin{aligned}
-\Phi\Omega^2 \left[-\hat{\zeta}^*(Y_g - p) + (Y_0 - Y_r)\right]^{\beta-2} &> \left[\hat{\zeta}^*(p - Y_b) + (Y_0 - Y_r)\right]^{\beta-2} \\
-\Phi\Omega^2 \left[-\frac{Y_0 - Y_r}{p - Y_b} \frac{1-(\Phi\Omega)^\gamma}{\Omega + (\Phi\Omega)^\gamma}(Y_g - p) + (Y_0 - Y_r)\right]^{\beta-2} &> \\
&\left[\frac{Y_0 - Y_r}{p - Y_b} \frac{1-(\Phi\Omega)^\gamma}{\Omega + (\Phi\Omega)^\gamma}(p - Y_b) + (Y_0 - Y_r)\right]^{\beta-2} \\
-\Phi\Omega^2 \left[\frac{1-(\Phi\Omega)^\gamma + \Omega + (\Phi\Omega)^\gamma}{\Omega + (\Phi\Omega)^\gamma}\right]^{2-\beta} &> \left[\frac{-\Omega + (\Phi\Omega)^\gamma\,\Omega + \Omega + (\Phi\Omega)^\gamma}{\Omega + (\Phi\Omega)^\gamma}\right]^{2-\beta} \\
-\Phi\Omega^2 \left[1+\Omega\right]^{2-\beta} &> \left[(\Phi\Omega)^\gamma (1+\Omega)\right]^{2-\beta} \\
-1 &> (\Phi\Omega)^\gamma\,\Omega^{-1}. \qquad (2.33)
\end{aligned}$$

From Equation (2.33) it becomes clear that the second order condition for a local maximum cannot be fulfilled as the right-hand-side cannot be negative. Hence, there is no local maximum for the expected utility in Case II.

It is straightforward to see that the second order condition for a local minimum must be fulfilled as $-1 < (\Phi\Omega)^\gamma\,\Omega^{-1}$. Hence Equation (3.1) always describes the minimum of the subjective expected utility. Consequently, the optimal demand is given by the corner solutions.

Following Assumption 6, the investment has a negative subjective expected return, i.e. $\Phi\Omega < 1$. It follows that the minimum $\hat{\zeta}^*$ is given by a positive investment. Consequently, there are feasible ζ to both sides of the minimum. This and the fact that Equation (3.1) describes the only extreme point imply that the investor needs to decide between investing the minimal amount $\zeta = 0$, i.e. abstaining from the market, and investing the maximum amount $\zeta = \frac{Y_r}{p}$.

The intuition here is as follows. On the one hand, a higher investment leads to higher expected losses, which makes investments less attractive. On the other hand, a larger investment implies more risk which together with the risk seeking preferences of the investor lead the asset to be more attractive. Depending on which of the effects is stronger, the investment is either zero or equal to the budget constraint. ■

2.8.5 Proof of Lemma 3

For the investor, the crucial question is whether to abstain or to invest. Consequently, it comes down to whether the utility of buying ζ^* is larger or equal than the utility of not investing.

Re-entering with ζ^* implies full recovery of the initial loss as in Equation (2.6) while abstaining from the market implies a certain loss. In order for re-entering to be optimal, it must thus hold that:

$$
\begin{aligned}
E(U(\zeta^*)) &\geq E(U(\zeta = 0)) \\
\pi \left[\zeta^*(Y_g - p) - (Y_0 - Y_r)\right]^\beta - (1-\pi)\left[\zeta^*(p - Y_b) + (Y_0 - Y_r)\right]^\beta &\geq -(Y_0 - Y_r)^\beta \\
\pi \left[\Omega \frac{1 + \Omega^\gamma \Phi^\gamma}{\Omega - \Omega^\gamma \Phi^\gamma} - 1\right]^\beta - (1-\pi)\left[\frac{1 + \Omega^\gamma \Phi^\gamma}{\Omega - \Omega^\gamma \Phi^\gamma} + 1\right]^\beta &\geq -1 \\
\pi \left[\frac{\Omega + \Omega^{\gamma+1}\Phi^\gamma - \Omega + \Omega^\gamma \Phi^\gamma}{\Omega - \Omega^\gamma \Phi^\gamma}\right]^\beta - (1-\pi)\left[\frac{1 + \Omega^\gamma \Phi^\gamma + \Omega - \Omega^\gamma \Phi^\gamma}{\Omega - \Omega^\gamma \Phi^\gamma}\right]^\beta &\geq -1 \\
\pi \left[\frac{(1+\Omega)\,\Omega^\gamma \Phi^\gamma}{\Omega - \Omega^\gamma \Phi^\gamma}\right]^\beta - (1-\pi)\left[\frac{1+\Omega}{\Omega - \Omega^\gamma \Phi^\gamma}\right]^\beta &\geq -1 \\
\pi \left[\frac{(1+\Omega)\,\Omega^{\gamma-1} \Phi^\gamma}{1 - \Omega^{\gamma-1} \Phi^\gamma}\right]^\beta - (1-\pi)\left[\frac{1+\Omega}{\Omega - \Omega^\gamma \Phi^\gamma}\right]^\beta &\geq -1 \\
\pi \left(\frac{IC}{\zeta_{Min}}\right)^\beta - (1-\pi)\left[\frac{1+\Omega}{\Omega - \Omega^\gamma \Phi^\gamma}\right]^\beta &\geq -1 \\
\pi \left(\frac{IC}{\zeta_{Min}}\right)^\beta - (1-\pi)\left[\frac{(1+\Omega)}{\Omega\left(1 - \Omega^{\gamma-1}\Phi^\gamma\right)} \frac{\left(\Omega^{\gamma-1}\Phi^\gamma\right)}{\left(\Omega^{\gamma-1}\Phi^\gamma\right)}\right]^\beta &\geq -1 \\
\left(\frac{IC}{\zeta_{Min}}\right)^\beta \left[\pi - (1-\pi)\left(\frac{1}{\Omega\Phi}\right)^{\beta\gamma}\right] &\geq -1 \\
\left(\frac{IC}{\zeta_{Min}}\right)^\beta \left[\pi - (1-\pi)\Phi\left(\frac{1}{\Omega^\beta \Phi}\right)^{\gamma}\right] &\geq -1 \\
\pi \left(\frac{IC}{\zeta_{Min}}\right)^\beta \left[\left(\frac{1}{\Omega^\beta \Phi}\right)^{\gamma} - 1\right] - 1 &\leq 0.
\end{aligned}
$$

This condition must be fulfilled for a positive demand in case of an interior solution. ■

2.8.6 Proof of Proposition 1

Proposition 1 consists of two components. First, a lower price leads to a positive demand by the investor. Additionally, Proposition 1 states that there is a jump in the demand.

According to Lemma 3, positive demand for an investor within Case I is dependent on Condition (2.16) being fulfilled

$$\pi \left(\frac{IC}{\zeta_{Min}}\right)^{\beta} \left[\left(\frac{1}{\Omega^{\beta}\Phi}\right)^{\gamma} - 1\right] - 1 \leq 0.$$

As in Appendix 2.8.3, I denote $\Psi = \frac{IC}{\zeta_{Min}}$ leading to

$$\pi \Psi^{\beta} \left[\left(\frac{1}{\Omega^{\beta}\Phi}\right)^{\gamma} - 1\right] - 1 \leq 0.$$

To see how a lower p influences this condition, consider the negative derivative of the left-hand-side with respect to p.

$$-\frac{\partial LHS}{\partial p} = \pi\beta \frac{\left(\Omega^{-\beta\gamma}\Phi^{-\gamma} - 1\right)}{\Psi^{1-\beta}} \frac{\partial \Psi}{\partial p} - \frac{\pi \Psi^{\beta}\beta\gamma}{\left(\Omega^{-\beta\gamma-1}\Phi^{-\gamma}\right)} \frac{\partial \Omega}{\partial p}.$$

The first derivative of Ψ is given by Equation (2.27)

$$\frac{\partial \Psi}{\partial p} = \frac{\partial \Omega}{\partial p} \frac{\Phi^{\gamma}\Omega^{\gamma-2}}{\left(1 - \Omega^{\gamma-1}\Phi^{\gamma}\right)^2} \left(\gamma - 1 + \gamma\Omega - \Omega^{\gamma}\Phi^{\gamma}\right).$$

It follows that

$$\begin{aligned}
-\frac{\partial LHS}{\partial p} &= -\pi\beta \frac{\partial \Omega}{\partial p} \left(\frac{\left(\Omega^{-\beta\gamma}\Phi^{-\gamma} - 1\right)}{\Psi^{1-\beta}} \frac{\Omega^{\gamma-2}\Phi^{\gamma}}{\left(1 - \Omega^{\gamma-1}\Phi^{\gamma}\right)^2} \left(\gamma - 1 + \Omega\gamma - \Omega^{\gamma}\Phi^{\gamma}\right) - \frac{\Psi^{\beta}\gamma}{\left(\Omega^{-\beta\gamma}\Phi^{-\gamma}\right)} \right) \\
&= -\frac{\pi\beta}{\Psi^{1-\beta}} \frac{\partial \Omega}{\partial p} \left(\frac{\left(\Omega^{-1} - \Omega^{\gamma-2}\Phi^{\gamma}\right)}{\left(1 - \Omega^{\gamma-1}\Phi^{\gamma}\right)^2} \left(\gamma - 1 + \Omega\gamma - \Omega^{\gamma}\Phi^{\gamma}\right) - \Psi\gamma \left(\Omega^{-\beta\gamma-1}\Phi^{-\gamma}\right) \right) \\
&= -\frac{\pi\beta}{\Psi^{1-\beta}} \frac{\partial \Omega}{\partial p} \left(\frac{\left(\gamma - 1 + \Omega\gamma - \Omega^{\gamma}\Phi^{\gamma}\right)}{\left(1 - \Omega^{\gamma-1}\Phi^{\gamma}\right)\Omega} - \frac{\left(\Omega^{\gamma-1}\Phi^{\gamma}(1+\Omega)\right)}{\left(1 - \Omega^{\gamma-1}\Phi^{\gamma}\right)} \gamma \left(\Omega^{-\beta\gamma-1}\Phi^{-\gamma}\right) \right) \\
&= -\frac{\partial \Omega}{\partial p} \frac{\pi\beta \left(\gamma - 1 + \Omega\gamma - \Omega^{\gamma}\Phi^{\gamma} - \gamma\left(\Omega^{\gamma}\Phi^{\gamma} + \Omega^{\gamma+1}\Phi^{\gamma}\right)\left(\Omega^{-\beta\gamma-1}\Phi^{-\gamma}\right)\right)}{\Psi^{1-\beta}\left(1 - \Omega^{\gamma-1}\Phi^{\gamma}\right)\Omega} \\
&= -\frac{\partial \Omega}{\partial p} \frac{\pi\beta \left(\gamma - 1 + \Omega\gamma - \Omega^{\gamma}\Phi^{\gamma} - \gamma\left(1 + \Omega\right)\right)}{\Psi^{1-\beta}\left(\Omega - \Omega^{\gamma}\Phi^{\gamma}\right)} \\
&= \frac{\partial \Omega}{\partial p} \frac{\pi\beta \left(1 + \Omega^{\gamma}\Phi^{\gamma}\right)}{\Psi^{1-\beta}\left(\Omega - \Omega^{\gamma}\Phi^{\gamma}\right)}.
\end{aligned}$$

Given that $\frac{\partial \Omega}{\partial p} < 0$, $\Psi = \frac{IC}{\zeta_{Min}} > 0$, and $\Omega > \Omega^\gamma \Phi^\gamma$, it follows that

$$-\frac{\partial LHS}{\partial p} < 0.$$

A decline in the price p lowers the left-hand-side of Condition (2.16) and hence, makes it more likely to be fulfilled. Lemma 3 states, that whenever this condition is fulfilled, the investor demands a positive amount.

Let us now consider the second component. Lemma 1 implies $\zeta^* > 0$. Given that an abstention from the market leads to zero demand, there must be a jump in demand once Condition (2.16) is fulfilled. ■

2.8.7 Proof of Proposition 2

Following Lemma 1, the optimal demand by the investor is given by

$$\zeta^* = \frac{(Y_0 - Y_r)}{(p - Y_b)} \frac{1 + \Omega^\gamma \Phi^\gamma}{\Omega - \Omega^\gamma \Phi^\gamma}.$$

The demand is consequently positively dependent on the prior loss with respect to the reference value $(Y_0 - Y_r)$.

Lemma 3 provides the condition for re-entering

$$\pi \left(\frac{IC}{\zeta_{Min}} \right)^\beta \left[\left(\frac{1}{\Omega^\beta \Phi} \right)^\gamma - 1 \right] - 1 \leq 0.$$

Keeping in mind that $\frac{IC}{\zeta_{Min}} = \frac{\Omega^{\gamma-1}\Phi^\gamma(1+\Omega)}{1-\Omega^{\gamma-1}\Phi^\gamma}$, this condition is independent of the prior loss $Y_0 - Y_r$.

It follows that the decision whether to re-enter is independent of the prior loss and the reference value, while the amount demanded increases linearly. ■

2.8.8 Proof of Proposition 3

a) The higher the reference value of the CPT investors, the higher is the probability of a *dead cat bounce* and the larger its expected size:
It is straightforward to show that

$$\frac{\partial D(p)}{\partial Y_0} = \frac{(1-x)}{p} \frac{(1+\Omega^{\gamma}\Phi^{\gamma})}{\Omega - \Omega^{\gamma}\Phi^{\gamma}} > 0 \tag{2.34}$$

$$\frac{\partial p_R}{\partial Y_0} = 0 \tag{2.35}$$

$$\frac{\partial S(p)}{\partial Y_0} = 0. \tag{2.36}$$

The demand function becomes steeper, while the price of the jump p_R and the supply function ($S(p)$) remain unchanged. Given this, the equilibrium price p^* for which $D(p^*) = S(p^*)$ increases while the rest remains the same, leading to a larger *dead cat bounce* as $\frac{\partial(p^* - p_t)}{\partial Y_0} = \frac{\partial p^*}{\partial Y_0} > 0$. Furthermore, as p^* increases, $S^{-1}(0)$ and p_R remains the same. As illustrated in figure 2.12, the area which leads to a DCB hence increases and the area for no DCB shrinks. ∎

b) The larger the share of RA investors n, the smaller is the possible *dead cat bounce*:
An increase in the share of RA investors has no effect on the actual demand function

$$\frac{\partial D(p)}{\partial n} = 0$$

it is therefore reasonable to assume that the curvature of the anticipated demand function Υ does also not react. As

$$\frac{\partial S(p)}{\partial n} = \frac{w(1 - \Omega^{\epsilon}\Phi^{\epsilon})}{p(\Omega + \Omega^{\epsilon}\Phi^{\epsilon})} \qquad \text{and} \qquad \frac{\partial^2 S(p)}{\partial n \partial p} > 0, \tag{2.37}$$

the supply curve increases with n for high prices ($1 > \Omega\Phi$) and decreases for low prices ($1 < \Omega\Phi$). Let us consider the case where there exists a DCB, i.e. $p^* > p_t$. We know from above that $\frac{\partial D(p)}{\partial n} = \frac{\partial \Upsilon_{p_t}(p_{t+1})}{\partial n} = 0$, consequently, the change in the intersection of $S(p)$ with $D(p)$ and $\Upsilon_{p_t}(p_{t+1})$, respectively, given a higher n depends purely on the change in the supply function. Given that $\frac{\partial^2 S(p)}{\partial n \partial p} > 0$, it must be true that $\frac{\partial (p^* - p_t)}{\partial n} < 0$ for a given p_t. It follows that the size of the DCB $p^* - p_t$ decreases as n increases, if there is a DCB. ■

c) A lower share of RA investors n or a lower share of initial holdings held by RA investors x increases the probability that the *dead cat bounce* happens in a later period: As illustrated in Figure 2.12, if $p_R \geq p_t \geq p^*$, there is no DCB. A $p_t > p_R$ would imply that there is no trade in this period and the suppliers adjust their offers according to p_{t+1}. This p_{t+1} can then either fall in the DCB or no DCB area. Let us consider the effect of a change in p_t on the possibility of a DCB occurring in a later period. A DCB can only occur in a later period ($t + 1$,...) if $p_t > p_R$. Hence, an increase of p_t over this threshold would make it possible for a DCB to occur in the next period. A decrease in p_t would not affect the probability of a DCB in the next period as long as $p_t < p_R$. As mentioned before,

$$\frac{\partial S(p)}{\partial n} = \frac{w(1 - \Omega^\epsilon \Phi^\epsilon)}{p(\Omega + \Omega^\epsilon \Phi^\epsilon)}. \tag{2.38}$$

As $\frac{\partial S(p)}{\partial n} > 0$ for negative expected return ($1 > \Omega\Phi$), the supply function in this region shifts upwards. This implies, that the anticipated maximum demand and the supply function intersect for a small p_t if they intersect in this region. It must hence be true that $\frac{\partial p_t}{\partial n} < 0$ for negative expected return. If it holds that $\pi(1 - p_R) - (1 - \pi)p_R < 0$, i.e. if the CPT investors are willing to re-enter the market with negative expected returns, then $\frac{\partial S(p_R)}{\partial n} > 0$. Further this implies that a lower n could push p_t over the threshold p_R and thereby delay a potential DCB. Consequently, a lower n can increase the probability that the DCB occurs in a later period.

The same logic also applies to a change in the initial share x of the asset held by RA investors.

$$\frac{\partial S(p)}{\partial x} = \frac{p0(1-\Omega^{\epsilon}\Phi^{\epsilon})}{p(\Omega+\Omega^{\epsilon}\Phi^{\epsilon})} \tag{2.39}$$

As $\frac{\partial S(p)}{\partial x} > 0$ for negative expected return $(1 > \Omega\Phi)$, it must be true that $\frac{\partial p_t}{\partial x} > 0$ in this case. Hence, a higher x could push p_t over the threshold p_R and thereby delay a potential DCB. ■

2.8.9 Corner solutions in the optimization

The following analysis focuses on cases where the budget constraint is binding i.e. the investor cannot invest his optimal amount.

By definition, in Case I the investor is still able to recover his initial loss despite the budget constraint. In Case II, he is unable to recover the loss fully. In either of the two, he has a lower utility compared to the unbound case. *Ceteris paribus*, he will therefore not re-enter at the marginal point where an unbound investor would.

The investor will re-enter if and only if the utility of re-entering exceeds the utility of not investing. For Case I this implies the following condition:

$$\begin{aligned} E\left(u^{+}\left(\zeta = Y_r/p\right)\right) &\geq E\left(u^{-}\left(\zeta = 0\right)\right) \\ \pi\left[\frac{Y_r}{p}(Y_g - p) - (Y_0 - Y_r)\right]^{\beta} - (1-\pi)\left[\frac{Y_r}{p}(p - Y_b) + (Y_0 - Y_r)\right]^{\beta} &\geq -\left[Y_0 - Y_r\right]^{\beta} \\ -\pi\left[\frac{Y_r Y_g}{p} - Y_0\right]^{\beta} + (1-\pi)\left[Y_0 - \frac{Y_r Y_b}{p}\right]^{\beta} &\leq \left[Y_0 - Y_r\right]^{\beta}. \end{aligned} \tag{2.40}$$

To see the reaction of the condition to a further fall in the price, consider the negative first derivative of the left-hand-side.

$$-\frac{\partial LHS}{\partial p} = -\pi\beta\left[\frac{Y_rY_g}{p} - Y_0\right]^{\beta-1}\left(-\frac{Y_rY_g}{p^2}\right) + (1-\pi)\beta\left[Y_0 - \frac{Y_rY_b}{p}\right]^{\beta-1}\frac{Y_rY_b}{p^2}$$
$$> 0. \tag{2.41}$$

A lower price increases the left-hand-side of Condition (2.40). Hence a lower p can lead the condition to be fulfilled.

The same can be shown for Case II:

$$E\left(u^-\left(\zeta = Y_r/p\right)\right) \geq E\left(u^-\left(\zeta = 0\right)\right)$$
$$-\pi\left[-\frac{Y_r}{p}(Y_g - p) + (Y_0 - Y_r)\right]^\beta - (1-\pi)\left[\frac{Y_r}{p}(p - Y_b) + (Y_0 - Y_r)\right]^\beta \geq -\left[Y_0 - Y_r\right]^\beta$$
$$\pi\left[1 - \frac{Y_r}{p}\frac{Y_g - p}{Y_0 - Y_r}\right]^\beta + (1-\pi)\left[1 + \frac{Y_r}{p}\frac{p - Y_b}{Y_0 - Y_r}\right]^\beta \leq 1, \tag{2.42}$$

$$\begin{aligned}-\frac{\partial LHS}{\partial p} =& \pi\beta\left[1 - \frac{Y_r}{p}\frac{Y_g - p}{Y_0 - Y_r}\right]^{\beta-1}\left(\frac{Y_r}{(Y_0 - Y_r)}\frac{Y_g}{p^2}\right)\\ &+ (1-\pi)\beta\left[1 + \frac{Y_r}{p}\frac{p - Y_b}{Y_0 - Y_r}\right]^{\beta-1}\left(\frac{Y_r}{(Y_0 - Y_r)}\frac{Y_b}{p^2}\right)\\ >& 0.\end{aligned} \tag{2.43}$$

A fall in the price may be sufficient for an unbound investor to re-enter the market, while an investor bound by his budget constraint would not invest. When the price falls further, the investor's budget constraint relaxes and the investment becomes more profitable. Hence, the investor will enter the market for lower p, i.e. at a later point in time.

It is straightforward to see, that both Conditions (2.40) and (2.42) depend on the initial loss and hence, the reference value and the disposable wealth. This has to be the case, as the budget constraint is given by the disposable wealth and the case differentiation depends on the initial loss.

Whenever Conditions (2.40) and (2.42), respectively, are fulfilled, the investor re-enters the market. In the cases I consider here, the investor dedicates all his disposable wealth to the investment whenever he re-enters. This implies that the demand jumps from zero to the budget constraint Y_r/p. Hence, there is still a sudden increase in demand.

Figure 2.13 illustrates the case where the investor has no chance to fully recover his initial losses. This case is identical to the one depicted in Figure 2.6, only that there is a larger initial loss due to a higher reference value Y_0 and a lower recovery price Y_r.

Again, the solid line is given by the demand while the dashed line illustrates Assumption 6. Additionally, the dotted line illustrates Condition (2.9). For a price p to the left of this line, the initial loss can be recovered and hence it follows Case I. At this point, the investor continues investing all his disposable wealth.

Due to the higher loss, the investor is willing to invest a much larger amount. At the marginal point, the highest price he is willing to invest, he is demanding nearly 120% of what he possessed before the burst.

2.8.10 Change in results if $\lambda \neq 1$

This section outlines the implications of a loss aversion parameter λ different from unity. As mentioned in Section 2.3, the experimental evidence indicates $\lambda \approx 2.25$, however, this does not change the central results.

The investor's expected utility, equivalent to Equation (2.7), is now given by

$$E(U) = \pi[\zeta(Y_g - p) - (Y_0 - Y_r)]^\beta - (1-\pi)\lambda[\zeta(p - Y_b) + (Y_0 - Y_r)]^\beta$$

leading to

$$\zeta^* = \frac{(Y_0 - Y_r)}{(p - Y_b)} \frac{1 + \Omega^\gamma \hat{\Phi}^\gamma}{\Omega - \Omega^\gamma \hat{\Phi}^\gamma}$$

with $\Omega = \frac{Y_g - p}{p - Y_b} > 0$, $\hat{\Phi} = \frac{\pi}{1-\pi}\frac{1}{\lambda} > 0$ and $\gamma = \frac{1}{1-\beta} > 0$.

Note that the results is the same as in Equation (2.11) apart from $\hat{\Phi}$. $\hat{\Phi}$ is the ratio of the subjective probability for the good state and the subjective probability of the bad state (Φ) weighted by the inverse of the loss aversion parameter. The higher the loss aversion parameter, the lower the optimal demand.

The second order condition is accordingly given by

$$\Omega^{\gamma-1}\hat{\Phi}^\gamma < 1$$

or

$$\pi(Y_g - p)^\beta - (1-\pi)\lambda(p - Y_b)^\beta < 0$$

and must be fulfilled as Inequality (2.5) is given.

It follows, that the decomposition in Corollary 1 remains valid only that

$$\zeta^* = \zeta_{Min} + \hat{IC}$$

with $\hat{IC} = \zeta_{Min}\hat{\Psi}$ and $\hat{\Psi} = \frac{\Omega^{\gamma-1}\hat{\Phi}^\gamma(1+\Omega)}{1-\Omega^{\gamma-1}\hat{\Phi}^\gamma} > 0$.

Corollary 2 remains valid as

$$
\begin{aligned}
-\frac{\partial \hat{IC}}{\partial p} &= -\frac{\partial \zeta_{Min}}{\partial p}\hat{\Psi} - \zeta_{Min}\frac{\partial \hat{\Psi}}{\partial p} \\
&= -\frac{Y_0 - Y_r}{(Y_g - p)^2}\hat{\Phi}^\gamma \frac{\left(-2(\gamma-1)\Omega^\gamma - (\gamma-2)\Omega^{\gamma-1} - \Omega^{2\gamma-2}\hat{\Phi}^\gamma - \Omega^{\gamma+1}\left(\gamma - \Omega^{\gamma-1}\hat{\Phi}^\gamma\right)\right)}{\left(1-\Omega^{\gamma-1}\hat{\Phi}^\gamma\right)^2} \\
&> 0.
\end{aligned}
$$

In case I, the loss aversion parameter appears in both states of the expected utility as both states lead to a loss with respect to the reference value. It follows that

$$
\begin{aligned}
E(U) &= \pi u^-(x_g) + (1-\pi)u^-(x_b) \\
&= -\pi\lambda[-\zeta(Y_g - p) + (Y_0 - Y_r)]^\beta - (1-\pi)\lambda[\zeta(p - Y_b) + (Y_0 - Y_r)]^\beta.
\end{aligned}
$$

The optimization leads to the minimum

$$
\hat{\zeta}^* = \frac{Y_0 - Y_r}{p - Y_b}\frac{1-(\Phi\lambda\Omega)^\gamma}{\Omega + (\Phi\lambda\Omega)^\gamma},
$$

which is equivalent as the one in Equation (3.1). Similarly the second order condition is the same as well. Hence, the results for Case II do not change.

The decision to re-enter given by Lemma 3 is now defined by

$$
\begin{aligned}
E(U(\zeta^*)) &\geq E(U(\zeta = 0)) \\
\pi\left[\zeta^*(Y_g - p) - (Y_0 - Y_r)\right]^\beta - (1-\pi)\lambda\left[\zeta^*(p - Y_b) + (Y_0 - Y_r)\right]^\beta &\geq -\lambda(Y_0 - Y_r)^\beta \\
\hat{\Psi}^\beta\left[\frac{\pi}{\lambda} - (1-\pi)\left(\frac{1}{\Omega^\gamma\hat{\Phi}^\gamma}\right)^\beta\right] &\geq -1 \\
(1-\pi)\hat{\Phi}\hat{\Psi}^\beta\left[1 - \frac{1}{\hat{\Phi}}\left(\frac{1}{\Omega^\gamma\hat{\Phi}^\gamma}\right)^\beta\right] &\geq -1 \\
(1-\pi)\hat{\Phi}\hat{\Psi}^\beta\left[\left(\frac{1}{\Omega^\beta\hat{\Phi}}\right)^\gamma - 1\right] - 1 &\leq 0.
\end{aligned}
$$

While the expression in Lemma 3 changes, it is straightforward to see that the conclusions from it given by Propositions 1 and 2 remain valid.

Hence, I conclude that the results are not affected by the simplification of setting the loss aversion parameter $\lambda = 1$.

2.8.11 On assumption 3

Assumption 3 is not necessary for the results in this paper to hold. However, the assumption allows for one parameter to be omitted and requires one less case differentiation. Denoting by X_0 the initial holdings, it follows that

$$
\begin{aligned}
E(U) =& \pi\left[\zeta(Y_g - p) - (Y_0 - Y_r)X_0 + (1 - X_0)\right]^\beta \\
&- (1-\pi)\left[\zeta(p - Y_b) + (Y_0 - Y_r)X_0 - (1 - X_0)\right]^\beta \\
\zeta^* =& \frac{(Y_0 - Y_r)X_0 - (1 - X_0)}{(p - Y_b)} \frac{1 + \Omega^\gamma\Phi^\gamma}{\Omega - \Omega^\gamma\Phi^\gamma}.
\end{aligned}
$$

This is the case as long as $\zeta(Y_b - p) - (Y_0 - Y_r)X_0 + (1 - X_0) < 0$. If this is not given, the investor is not loss driven and therefore will not invest when facing negative expected returns. A lower initial investment X_0 makes this assumption less likely to be fulfilled.

2.8.12 Partial liquidation

If the investor does not fully liquidate his position at the height of the bubble, the optimization problem is as follows

$$\begin{aligned} E(U) =& \pi\left[-Y_0 + \zeta Y_g - (\zeta - (1-S))p + Y_r S\right]^\beta \\ & - (1-\pi)\left[-\left(-Y_0 + \zeta Y_b - (\zeta - (1-S))p + Y_r S\right)\right]^\beta \\ =& \pi\left[\zeta(Y_g - p) + (p - Y_0)(1-S) + (Y_r - Y_0)S\right]^\beta \\ & - (1-\pi)\left[\zeta(p - Y_b) - (p - Y_0)(1-S) - (Y_r - Y_0)S\right]^\beta , \end{aligned}$$

where S is the portion of the portfolio which the investor sells at the height of the bubble. ζ is given by the total amount upon re-entering including the retained portion of the initial holdings $(1-S)$. All of the initial holdings lead to a perceived loss of Y_0. The final holdings lead to a payoff of either Y_g or Y_b. For the sold portion the investor receives Y_r, while the non-sold portion $(1-S)$ leads to savings in purchasing cost p. The optimization leads to

$$\zeta^* = \frac{(Y_0 - p) - (Y_r - p)S}{(p - Y_b)} \frac{1 + \Omega^\gamma \Phi^\gamma}{\Omega - \Omega^\gamma \Phi^\gamma}.$$

When setting the sold portion S to unity, this entails the previous case of

$$\zeta^* = \frac{(Y_0 - Y_r)}{(p - Y_b)} \frac{1 + \Omega^\gamma \Phi^\gamma}{\Omega - \Omega^\gamma \Phi^\gamma}.$$

Given that $Y_r > p$, the total optimal demand ζ^* decreases with a higher sold portion S.

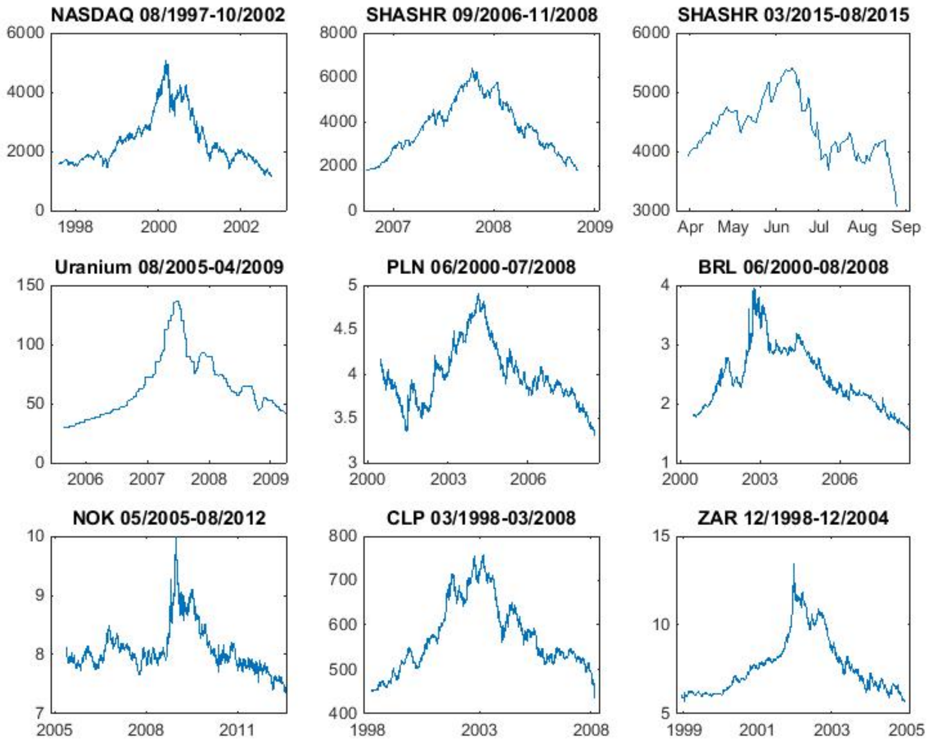


Figure 2.1: Bursting bubbles followed by *bear market rallies*
This figure illustrates nine financial bubbles with one or multiple *bear market rallies* in stock indices, commodities, and currencies: NASDAQ, Shanghai stock index (SHASHR), Uranium, Polish Zloty (PLN), Brazilian Real (BRL), Norwegian Crown (NOK), Chilean Peso (CLP), and South African Rand (ZAR)

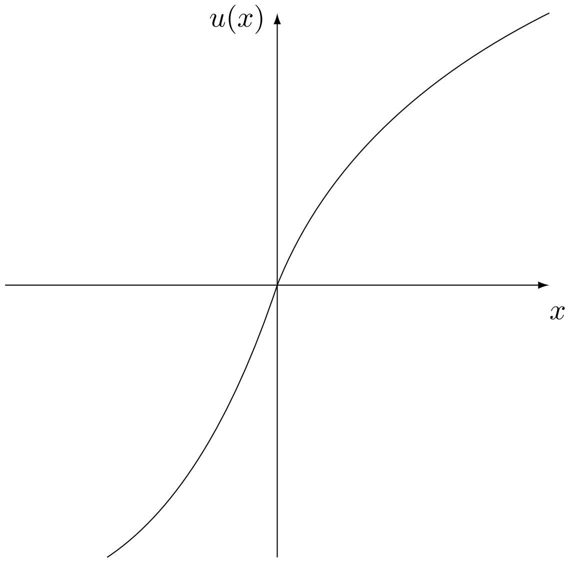


Figure 2.2: Cumulative prospect theory (CPT) utility function
The figure provides an illustration of a utility function according to cumulative prospect theory.

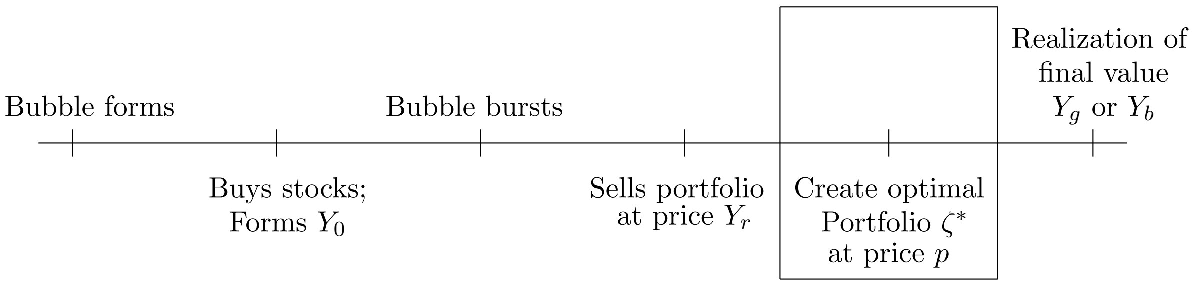


Figure 2.3: Time line
The box surrounds the area explicitly incorporated into the model. The investor's actions are shown below the line, while the market development is above.

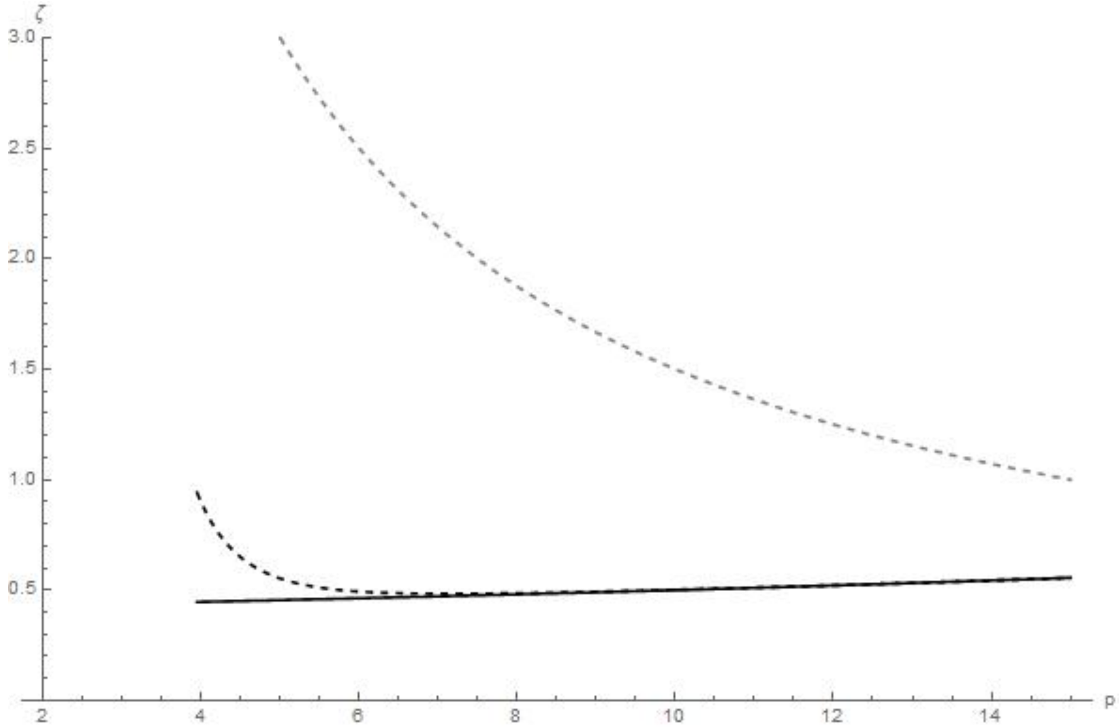


Figure 2.4: Minimum, maximum, and optimal ζ I
The solid black line is the minimum ζ_{Min} as in Equation (2.8). The dashed black line illustrates the optimum ζ^* as in Equation (2.11). The dashed gray line shows the upper bound for ζ given by the budget constraint. The parameter values are given by: $Y_g = 60$, $Y_r = 15$, $Y_0 = 40$, $Y_b = 1$, $\pi = 0.05$ and $\beta = 0.8$. As in the analysis, I set $\lambda = 1$.

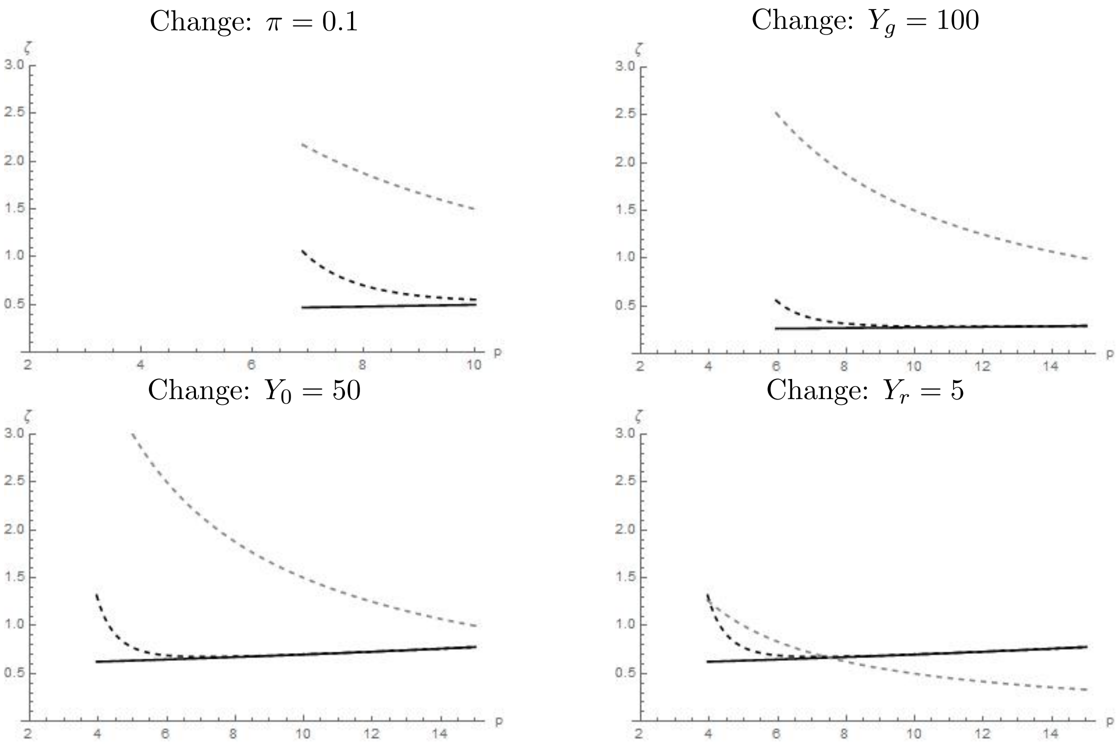


Figure 2.5: Minimum, maximum, and optimal ζ II
The images illustrate ζ as in Figure 2.4 for different parameter values. The solid black line is the minimum ζ_{Min} as in Equation (2.8). The dashed black line illustrates the optimum ζ^* as in Equation (2.11). The dashed gray line shows the upper bound for ζ given by the budget constraint. The 4 graphs illustrate the effect of a change in π, Y_g, Y_0 and Y_r, respectively, compared to the benchmark case in Figure 2.4.

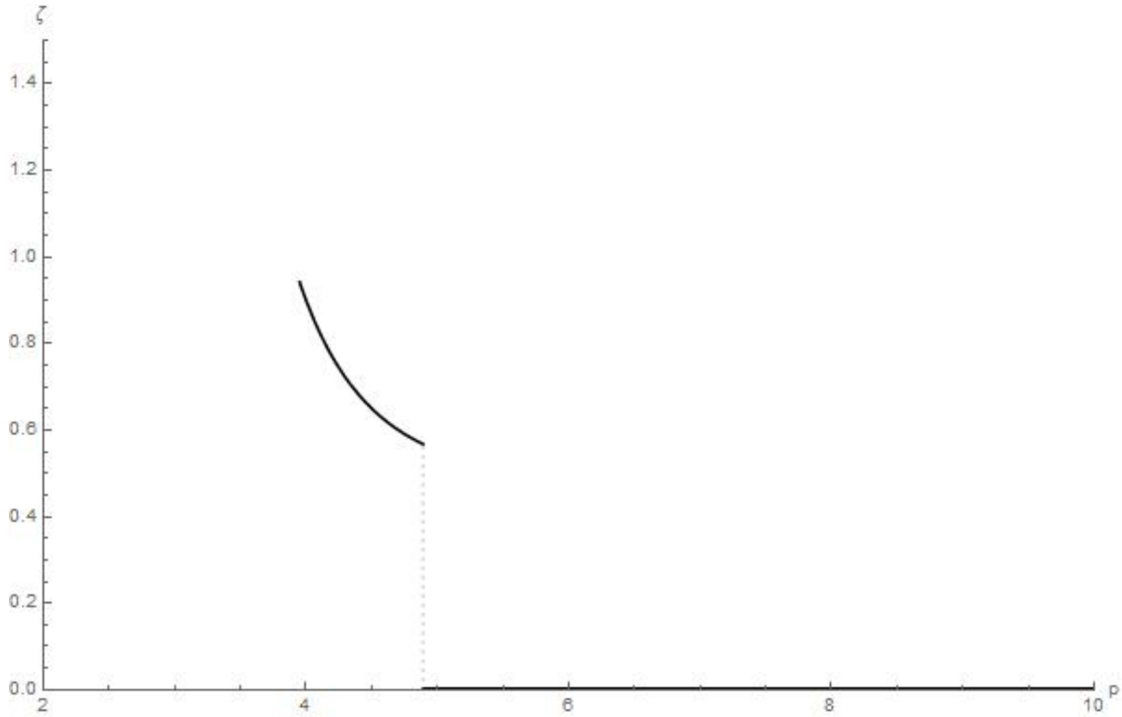


Figure 2.6: Proposition 1: Jump in demand
The images illustrate the optimal investment. The black line illustrates the optimum ζ^* as in Equation (2.15) bound by the budget constraint The graph is based on the parameters $Y_g = 60$, $Y_0 = 40$, $Y_r = 15$, $Y_b = 1$, $\beta = 0.8$, and $\pi = 0.05$. As in the analysis λ is set to unity.

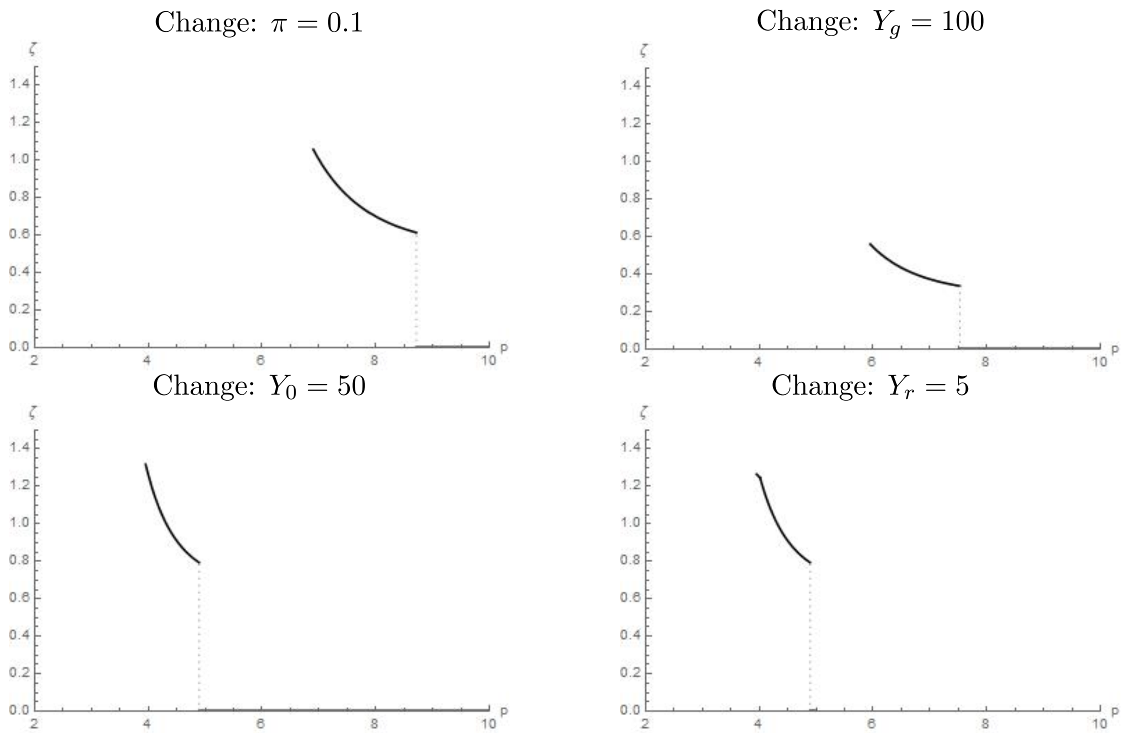


Figure 2.7: Jump in demand with varying parameters
The images illustrate the optimal investment for different parameters. The black line illustrates the optimum ζ^* as in Equation (2.15) bound by the budget constraint The images result from a change in *pi*, Y_g, Y_0, and Y_r, respectively, compared to the image in Figure 2.6.

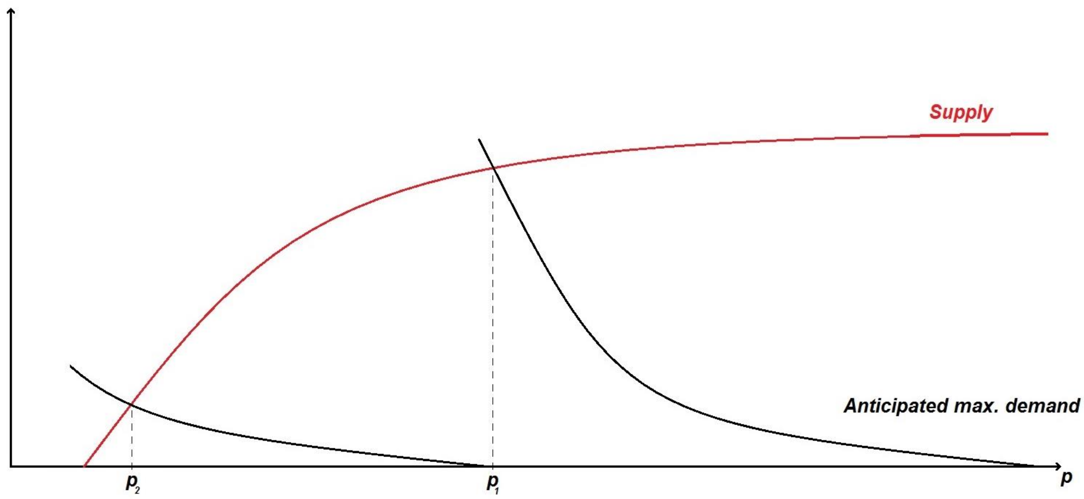


Figure 2.8: Anticipated maximum demand
The red line illustrates the supply function $S(p)$. The black line illustrates the development of the anticipated maximum demand in two steps.

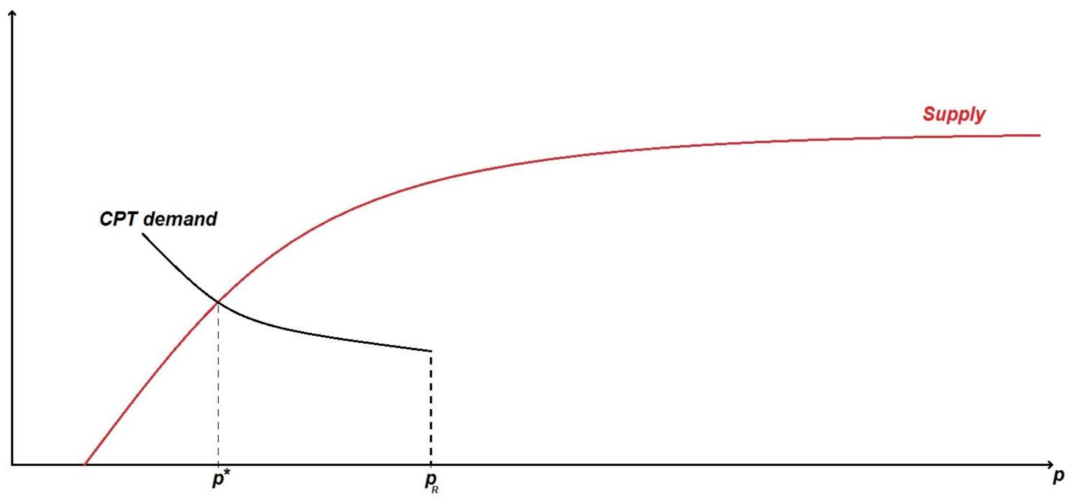


Figure 2.9: Equilibrium price
The red line illustrates the supply function $S(p)$. The black line illustrates the aggregated optimal demand by the CPT investors.

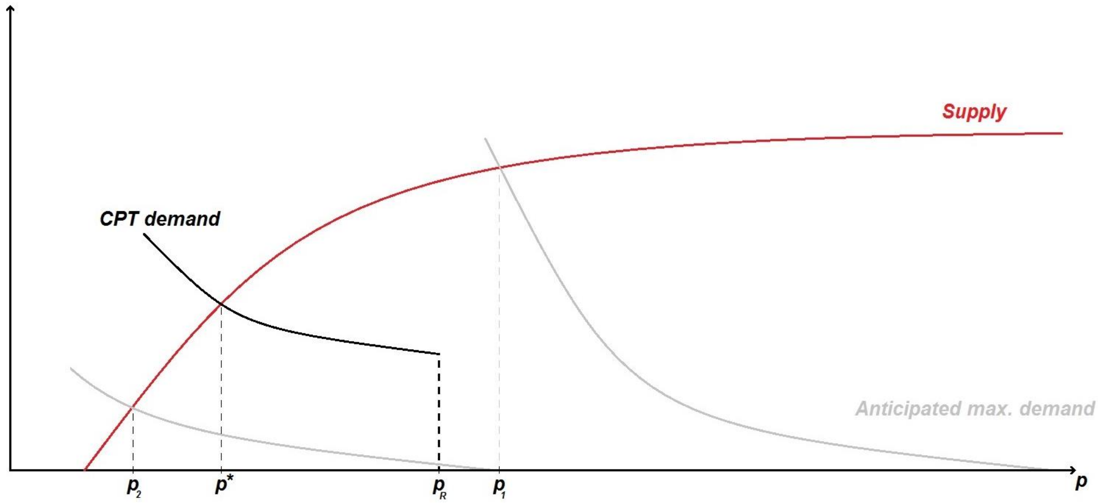


Figure 2.10: Example with *dead cat bounce*
The red line illustrates the supply function $S(p)$. The black line illustrates the aggregated optimal demand by the CPT investors. The grey line illustrates the development of the anticipated maximum demand in two steps.

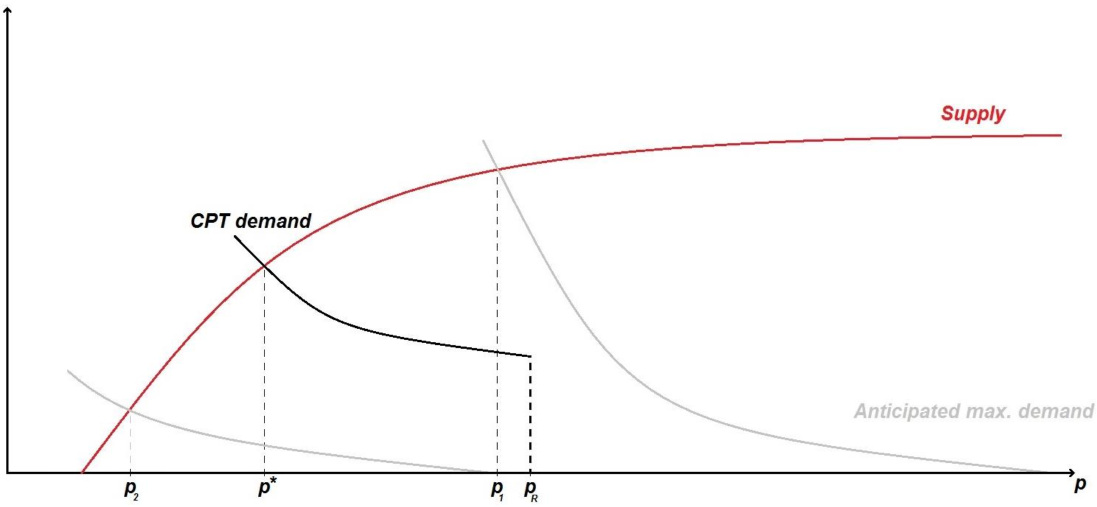


Figure 2.11: Example without *dead cat bounce*
The red line illustrates the supply function $S(p)$. The black line illustrates the aggregated optimal demand by the CPT investors. The grey line illustrates the development of the anticipated maximum demand in two steps.

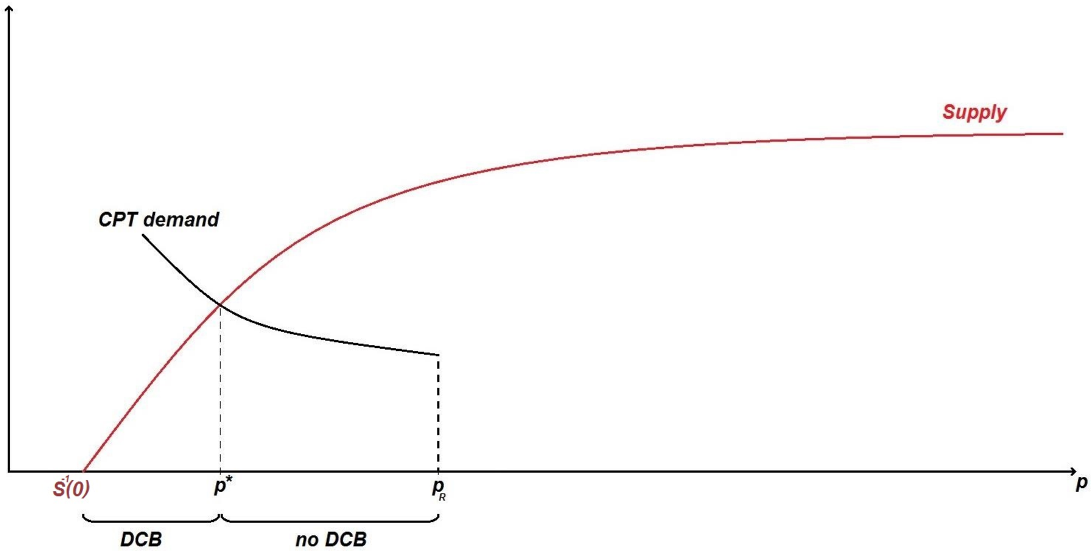


Figure 2.12: Condition for a *dead cat bounce*
The red line illustrates the supply function $S(p)$. The black line illustrates the aggregated optimal demand by the CPT investors. If the price p_t falls in the area marked *DCB*, this will lead to a *dead cat bounce*. Similarly, if p_t lies in the *no DCB* area, there will be no reversal.

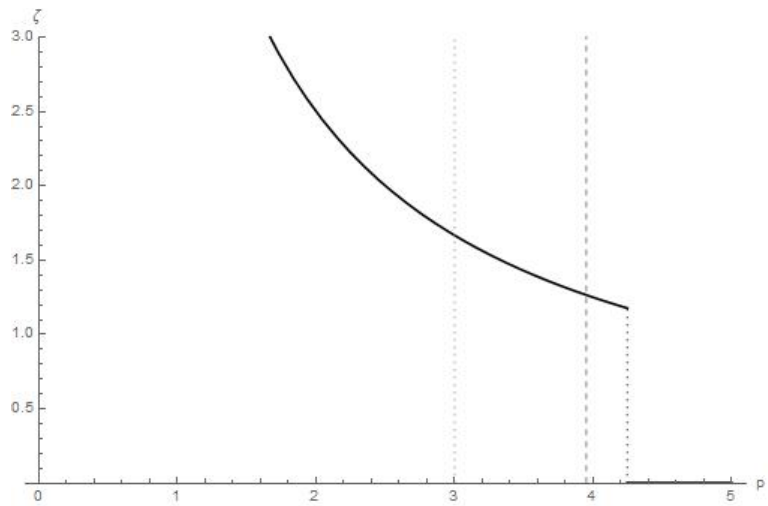


Figure 2.13: Jump in demand in Case II
The solid line shows the demand. The dashed line illustrates Assumption 6 which is only fulfilled to the right of the line. For p to the left of the dotted line, the investor can recover his initial loss, i.e. Case I. The parameters are chosen as follows: $Y_g = 60$, $Y_0 = 100$, $Y_r = 5$, $Y_b = 1$, $\beta = 0.8$ and $\pi = 0.05$. As in the analysis, I set $\lambda = 1$.

Chapter 3

Price Discovery and Toxic Arbitrage

3.1 Introduction

In recent years, large parts of the financial system have experienced increased fragmentation.[1] As assets are traded in multiple venues, this raises the question where price discovery takes place and how liquidity and information spreads through the market as a whole.[2] Standard techniques for measuring price discovery are designed for high liquidity environments and as fragmentation continues, they will face an increased number of markets for which this is not always given.

In this paper, I use a new theoretical framework to derive a measure of information shares which is more robust when applied to markets subject to low liquidity. It therefore provides a useful addition to the analysis of multi-venue trading. The measure builds upon earlier work on toxic arbitrage by Foucault et al. (2016). Toxic arbitrage opportunities are caused by price deviations between markets due to information arrival in one market and asynchronous price adjustment in the other. Such arbitrage opportunities therefore provide useful insights into the information dynamics of markets. This is the first paper to take advantage of the direction of toxic arbitrage opportunities, to relate this to price discovery, or to use the concept of toxic arbitrage in order to analyze such dynamics.

The contribution to the literature is twofold. Firstly, I introduce a model which highlights the importance of the direction of toxic arbitrage opportunities. This allows me to explore the connection between the informational structure of markets, their spreads and the frequency of toxic arbitrage opportunities. High spreads in one market serve as a protection not only against informed traders in this market but also against arbitrageurs incentivized by informed traders in the other market. Thereby, higher spreads in one market lead to fewer toxic arbitrage opportunities initiated by information arriving in the other market. This connection needs to be taken into account when using the

[1] see e.g. Gomber et al. (2016).
[2] see e.g. Tse et al. (2006), Brogaard et al. (2014), and Hendershott and Menkveld (2014).

informational content of the frequency of toxic arbitrage opportunities. Similarly, higher transaction fees in either market reduce incentives for informed traders to create toxic arbitrage opportunities. Lastly, the distribution of the impact of information arriving in one market will also need to be taken into account. These connections are important for any future work using toxic arbitrage opportunities.

My second contribution and the main contribution of this paper is the introduction of a model based Toxic Arbitrage Information Share which corrects for the spreads, transaction fees, and informational structures of both markets. This measure is fully theoretically derived and does not share computational similarities with the standard price discovery shares by Hasbrouck (1995), Gonzalo and Granger (1995), or their extensions. Due to this, the Toxic Arbitrage Information Share avoids the criticism brought forward e.g. by Gomber et al. (2016). Additionally, the Toxic Arbitrage Information Share provides approximate confidence intervals which allow to evaluate the precision of the measure on a given day. In comparison, Hasbrouck (1995) provides approximate bounds without a proper point estimate. Gonzalo and Granger (1995) does not allow for confidence intervals.

To emphasize these contributions, I first compare the Toxic Arbitrage Information Share to standard procedures using simulated data. The performance of the Toxic Arbitrage Information Share speaks for its theoretical backing and the simulations show that it performs comparably or better than standard procedures. In a second step, I apply the Toxic Arbitrage Information Share to a unique data set combining US dollar/Brazilian real futures traded at the Chicago Mercantile Exchange (CME) and the Bolsa de Valores, Mercadorias e Futuros de São Paulo (BMF). The interaction of these two markets is an interesting case to apply the Toxic Arbitrage Information Share. The products traded are almost perfect substitutes with a sufficient overlap in trading hours. At the same time, trading is clearly segmented due to high market entry costs and regulation. Furthermore, one would expect relevant information for the exchange rate to originate in both markets and, as Anand et al. (2011) highlight, geographical proximity matters for price discovery. Finally, the futures market is known to be the driver of price discovery for Brazilian real (Ventura and Garcia (2012)). Comparing the results from the Toxic Arbitrage Information Share with the results using the Hasbrouck information share and the Gonzalo Granger component share, I find that the three information shares find similar median values. This makes this paper the first to provide empirical evidence that there is indeed a close relationship between toxic arbitrage and information arrival.

In contrast to other procedures, the Toxic Arbitrage Information Share provides a less volatile and more persistent estimate. In line with the assessment by market participants, the Toxic Arbitrage Information Share finds not a single day where CME is dominating price discovery.[3]

While this set-up is an interesting application of the new measure, it is clearly not the only possible application. For an appropriate usage, two markets need to have overlapping trading hours and the products must represent equivalent assets for arbitrage to be possible. The measure is hence applicable to a range of cross-listed stocks and assets traded in multiple venues including currencies, commodities, and derivatives. While low liquidity environments highlight the benefits of the new measure compared to standard approaches, higher liquidity can be expected to lead to more arbitrage opportunities and hence more precise estimates.

The remainder of this paper is structured as follows. The next section provides a brief overview of the related literature. In Section 3.3, I derive a theoretical model and develop the toxic arbitrage based information share. Section 3.4 introduces a procedure of estimating the measure including approximate confidence intervals. In Section 3.5, I run several simulations in order to evaluate the performance of the procedure. Sections 3.6 and 3.7 provide a description of the data set used and the empirical application of

[3]According to discussions with stakeholders of the USD/BRL futures market such as market makers, brokers, traders, and specialists from BMF and CME, market participants consider CME to be irrelevant for price discovery in the USD/BRL futures market.

the Toxic Arbitrage Information Share. Section 3.8 concludes the paper.

3.2 Literature

This paper forms part of the wide literature of multi-venue trading and connects the concept of toxic arbitrage introduced by Foucault et al. (2016) with the literature on price discovery crucially shaped by Hasbrouck (1995) and Gonzalo and Granger (1995). Research on multi-venue trading has profited greatly by the the improved availability of high quality high frequency data and, at the same time, has become more relevant due to an increased fragmentation across trading venues. With the continuing trend towards more market segmentation, future research on multi-venue trading is likely to face the challenge of a higher relevance of potentially low liquidity markets.

There is a substantial literature around arbitrage opportunities as a feature of market inefficiency and market frictions. Gromb and Vayanos (2012) provide a broad overview of the theoretical literature. The presence of arbitrageurs can be beneficial or harmful to market participants (see e.g. Copeland and Galai (1983) and Gromb and Vayanos (2002)). Foucault et al. (2016) put this ambiguity at the core of their paper, differentiating between toxic arbitrage and non-toxic arbitrage. While toxic arbitrage is information driven and leads to welfare loss as the arbitrageur takes advantage of the market makers' latency, non-toxic arbitrage serves as a channel to balance excess liquidity across markets and is therefore welfare enhancing. In this paper, I use this distinction, but rather than focusing on welfare effects, I take advantage of the informational content of toxic arbitrage opportunities in order to analyze price discovery.

O'Hara (2003) highlights that price discovery is one of the principle purposes of markets. Given that similar or related assets are often traded in multiple venues, a large part of the literature has therefore focused on the analysis of price discovery in order to attribute information shares to different venues. The standard approaches for determining price discovery are the information share going back to Hasbrouck (1995) and the permanent component share by Gonzalo and Granger (1995). Both approaches are based on the Vector Error Correction Model proposed by Engle and Granger (1987).[4] These approaches are the clear benchmark for any analysis of price discovery, but as papers such as Narayan and Smyth (2015) highlight, there are limitations due to their underlying assumptions. Some of these limitations are addressed by adaptations in papers such as Yan and Zivot (2010) and Dias et al. (2016). However, extensions to the standard approaches still rely on complete limit order books and high frequency in trades which are not necessarily given in low liquidity markets. In this paper, I therefore depart from the VECM based price discovery analysis and propose a measure which is based on the frequency of toxic arbitrage opportunities.

By using individual events to infer price discovery, this paper is closely related to Muravyev et al. (2013). The authors study disagreement events between the equity and options market and find that the options market generally adjusts in order to eliminate disagreements. They take this as evidence that options market quotes do not drive price discovery. In this paper, I take this idea further and generalize it by creating an event based price discovery measure.

Most of the multi-venue literature focuses on equity markets. However, there is also a substantial literature looking at arbitrage and price discovery in foreign exchange markets (see e.g. Tse et al. (2006), Akram et al. (2008), Ranaldo (2009), and Poskitt (2010)). Due to its application to foreign exchange futures, this paper is also contributing to this part of the literature. Ventura and Garcia (2012) find that the futures market in São Paulo accounts for most of the price discovery compared to the spot market. This paper is the first to look at the futures market in Chicago and its relevance for price discovery for the Brazilian real. That the Toxic Arbitrage Information Share does not need high liquidity may be especially beneficial for the analysis of central versus local

[4] For a further description of how the procedures are related, see e.g. Baillie et al. (2002)

trading of currencies, a topic of high political interest.

The next section develops the theoretical model which provides a framework for the following analysis and the construction of the Toxic Arbitrage Information Share.

3.3 Model

This section provides the theoretical foundation for analyzing the connection between toxic arbitrage and price discovery. The model is inspired by Foucault et al. (2016). Similar to their paper, I am differentiating between the arrival of liquidity traders which potentially leads to non-toxic arbitrage opportunities and informed traders potentially leading to toxic arbitrage opportunities. While Foucault et al. (2016) focus on the connection between latency and toxic arbitrage, my focus is on the direction and informational impact of toxic arbitrage opportunities. The direction is determined by where the information which leads to the toxic arbitrage opportunity arrives first. In order to focus on price discovery, the model presented here is a simplification of a general equilibrium model including liquidity traders in both markets. The only effect liquidity traders have is to make the market makers existence profitable and allow to endogenize the spreads. The results in this paper and the definition of the Toxic Arbitrage Information Share are not affected by this simplification. A reason for this is that all results are conditional on the observed spreads. Consequently, endogenizing the spreads and the market makers' behavior does not affect the results of the model and has no benefits here.

There are two markets B and C trading the same asset, three dates ($t \in \{0, 1, 2\}$), two market makers, one in each market, an informed trader in each market, and an arbitrageur between markets. In total, there are five agents. The central point of this set-up is that the arbitrageur is the only market participant able and willing to trade in both markets due to market entry costs. Furthermore, her orders move faster than information on the prices in each market. Figure 3.1 provides the time line of the model.

The description below is in reverse order.

At $t = 2$ the final value $\theta = \mu + \epsilon$ of the asset is realized, where ϵ is the variation in the fundamental value of the asset following a symmetric distribution around zero. The expected value of the asset is hence $E(\theta) = \mu + E(\epsilon) = \mu$. Market maker $j \in \{B, C\}$ is specialized in market j and is only active in this market. Due to the physical distance between the market places, the information is not instantaneously available in both markets potentially leading to short lived arbitrage opportunities.

At $t = 1$, market makers simultaneously post ask price a_j and bid price b_j for $j \in \{B, C\}$ such that:

$$a_j = \mu + \frac{S_j}{2}, \qquad \text{and} \qquad b_j = \mu - \frac{S_j}{2}, \tag{3.1}$$

where S_j is market maker j's bid-ask spread. In this simplification of the model, the spreads are taken as exogenous, however, all results also hold in the general equilibrium case. Quotes are for a fixed quantity, normalized to 1.

There are two possible events in this model. With probability δ_b, an informed trader arrives in market B and with probability $(1 - \delta_b)$ an informed trader arrives in market C. Each of these events has the potential of causing a toxic arbitrage opportunity. The informed trader observes ϵ, the change in the fundamental value at time $t = 1$. Hence, he uses this information to trade before the information becomes public knowledge at time $t = 2$. This news arrival via the informed trader can lead to what Foucault et al. (2016) describe as toxic arbitrage.[5] Toxic arbitrage opportunities are not considered welfare improving. The arbitrageur uses his lower latency in order to trade against stale trades by the other market maker.

The price impact of the informed trader's information is given by ϵ. I do not assume that the distribution of ϵ is independent of where the informed trader arrives.

[5] It is a central difference between the model used by the authors and the one used in this paper, that not all information arrival leads to toxic arbitrage opportunities.

Hence, informed traders in market B may have higher or lower price impact information than informed traders arriving in market C. Let us denote by n_j the price impact of the information which the informed trader j holds and by n_B^+ the absolute size of the price impact.

Say, an informed trader arrives in market B with information that the asset is overpriced ($n_B < 0$). It follows, that the true value of the asset which is only observed by the informed trader is given by $\theta = \mu - n_B^+$. The informed trader has two ways to taking advantage of his information. He can execute a sell market order and trade directly against market maker B's best bid. This is the traditional way of informed trading as in e.g. Kyle (1985). Additionally, the informed trader also has the option of posting a sell limit order to be picked up by the arbitrageur. The informed trader will take advantage of both options as long as it is profitable to do so. In the next step, let us look at the profitability condition in more detail.

When using a market order, the informed trader trades at price $b_B = \mu - \frac{S_B}{2}$ leading to the following profits for the informed trader and market maker B, respectively,

$$Profit_{Inf_M} = b_B - \left(\mu - n_B^+\right) - c = \left(\mu - \frac{S_B}{2}\right) - \left(\mu - n_B^+\right) - c \tag{3.2}$$

$$= n_B^+ - \frac{S_B}{2} - c, \tag{3.3}$$

$$Profit_{MM_B} = \left(\mu - n_B^+\right) - b_B = \mu - n_B^+ - \left(\mu - \frac{S_B}{2}\right) \tag{3.4}$$

$$= -\left(n_B^+ - \frac{S_B}{2}\right), \tag{3.5}$$

where c is given by transaction fees per transaction. The fees are assumed to be equal across venues. The informed trader arriving in market B will only use a market order if this results in a profit.[6] Let us denote by $\underline{n}_B$, the price impact of information which

[6]The results for positive information with price impact n_B^+ are equivalent.

leads to a zero profit. It follows that

$$\begin{aligned}\underline{n}_B - \frac{S_B}{2} - c &= 0 \\ \underline{n}_B &= \frac{S_B}{2} + c. \end{aligned} \tag{3.6}$$

For all information where $n_B^+ < \underline{n}_B$, the informed trader arriving in market B will not execute a market order. More generally, if the price impact in absolute terms (n_B^+) does not exceed the threshold, the informed trader will not execute a market order. This shows that both transaction fee c as well as the spread S_B make it harder for the informed trader to use his private information. Both c and S_B serve as a protection for market maker B.

Additionally, the informed trader has the option of using a sell limit order. He does so in order to incentivize the arbitrageur to trade against him. This means that the informed trader posts a limit order at the highest price which still makes it profitable for the arbitrageur to trade. This price denoted by a_B^T needs to be lower than the best ask a_B by market maker B so that the informed trader's limit order is now the best ask in market B.

The price a_B^T is hence given by

$$\begin{aligned} a_B^T &= a_B - \gamma \\ &= \mu + \frac{S_B}{2} - \gamma, \end{aligned} \tag{3.7}$$

where $\gamma > 0$. The informed trader chooses γ in order to incentivize the arbitrageur to trade against her limit order. Consequently, γ is the price improvement the informed trader needs to offer compared to the market maker's best price in order to make it profitable for the arbitrageur to trade. The arbitrageur will buy at this price if he can make a profit by simultaneously selling the asset in market C. Due to the trades in both markets, the arbitrageur incurs a total cost of $2c$. The profit of the arbitrageur is then given by

$$\begin{aligned}
Profit_{Arb} &= a_B^T - b_C - 2c \\
&= \left(\mu - \frac{S_C}{2}\right) - \left(\mu + \frac{S_B}{2} - \gamma\right) - 2c \\
&= \gamma - \frac{S_C + S_B}{2} - 2c. \qquad (3.8)
\end{aligned}$$

The informed trader will choose the smallest γ which still leads the arbitrageur to engage in arbitrage, i.e.

$$\gamma = \frac{S_C + S_B}{2} + 2c, \qquad (3.9)$$

leading to $Profit_{Arb} = 0$. In this case, the informed trader uses the arbitrageur as a channel to trade against market maker C. γ can also be thought of as the price of the arbitrageurs' service of providing a channel for indirectly trading against the market maker in the other market. Using a limit order results in the following profits

$$\begin{aligned}
Profit_{Arb} &= b_C - a_B^T - 2c = \left(\mu - \frac{S_C}{2}\right) - \left(\mu + \frac{S_B}{2} - \gamma\right) - 2c \\
&= -\frac{S_C + S_B}{2} + \gamma - 2c = 0, \qquad (3.10) \\
Profit_{Inf_L} &= a_B^T - (\mu - n_B^+) - c \\
&= \left(\mu + \frac{S_B}{2} - \gamma\right) - (\mu - n_B^+) - c \\
&= n_B^+ - \frac{S_C}{2} - 3c, \qquad (3.11) \\
Profit_{MM_C} &= (\mu - n_B^+) - b_C = (\mu - n_B^+) - \left(\mu - \frac{S_C}{2}\right) \\
&= -\left(n_B^+ - \frac{S_C}{2}\right). \qquad (3.12)
\end{aligned}$$

Given a sufficiently large price impact of the information, each market maker

j consequently incurs a loss of $-\left(n_B^+ - \frac{S_j}{2}\right)$. The informed trader is trading directly against market maker B and uses the arbitrageur to trade indirectly against market maker C. Combining the two sub-profits of the informed trader in Equations (3.3) and (3.11) leads to a total profit of

$$\begin{aligned} Profit_{Inf} =& Profit_{Inf_M} + Profit_{Inf_L} \\ =& \left(n_B^+ - \frac{S_B}{2} - c\right) + \left(n_B^+ - \frac{S_C}{2} - 3c\right) \\ =& 2n_B^+ - \frac{S_B + S_C}{2} - 4c. \end{aligned} \tag{3.13}$$

However, following the same logic as above, the informed trader arriving in market B will only use a limit order if it is profitable to do so. The profit for the informed trader of using a limit order is given in Equation (3.11) by

$$Profit_{Inf_L} = n_B^+ - \frac{S_C}{2} - 3c.$$

Following the same logic as before, the informed trader will only use a limit order if his profit is positive. Hence, I define the informational impact which leads to a zero profit from a limit order as $\underline{n}_B^*$. For all information where $n_B^+ < |\underline{n}_B^*|$, the informed trader will not post a limit order in market B. It follows that

$$\begin{aligned} \underline{n}_B^* - \frac{S_C}{2} - 3c &= 0 \\ \underline{n}_B^* &= \frac{S_C}{2} + 3c. \end{aligned} \tag{3.14}$$

Figure 3.2 illustrates the optimal decision making of the informed trader. A high spread in both markets relative to the information impact n_B^+ implies that the informed trader takes no action as shown in the upper right corner. A high spread in market B and a low spread in market C leads to the informed trader only choosing a limit order. The reverse implies that the optimal choice is only to use a market order. If both spreads are sufficiently narrow compared to the price impact of the information, the informed trader will use both order types.

Let us now consider the case where an informed trader arrives in market C. The probability of this happening is given by $(1 - \delta_b)$. The solutions for this case are symmetric to the case where the informed trader arrives in market B. It follows that

$$\underline{n}_B = \frac{S_B}{2} + c \qquad \underline{n}_C = \frac{S_C}{2} + c \tag{3.15}$$

$$\underline{n}_B^* = \frac{S_C}{2} + 3c \qquad \underline{n}_C^* = \frac{S_B}{2} + 3c. \tag{3.16}$$

The lowest profitable impact of news for a market order in market j is given by $\underline{n}_j$. $\underline{n}_j^*$ is the lowest profitable impact of news for a limit order. Hence, $\underline{n}_j$ is the threshold for informed traders arriving in market j to trade profitably against market maker j. $\underline{n}_j^*$ is the threshold for informed traders arriving in market j to use a limit order to be picked up by the arbitrageur. Consequently, $\underline{n}_j^*$ will determine the number of toxic arbitrage opportunities. Say the price impact of an informed trader's information arriving in market B is between $\underline{n}_B$ and $\underline{n}_B^*$. In this case, the informed trader will only execute a market order but not submit a limit order. The market order will take up liquidity and potentially widen the spread by emptying the best price in market B. This will not lead to an arbitrage opportunity. Arbitrage opportunities can only be caused by the submission of limit orders as a limit order narrows the spread in a market. If the spread becomes sufficiently narrow, this opens an arbitrage opportunity. Market orders can only widen the spread and therefore cannot lead to arbitrage opportunities.

These thresholds hold given that both limit and market orders incur the same transaction fees. Under make-take fees, where liquidity providers receive a fee rather than paying one, the trade-off naturally changes. Rather than having to take into account three times the fee, the informed investor would calculate with a mix of make and take fees. Overall, this is likely to make limit orders more attractive and, *ceteris paribus*, lead to more arbitrage opportunities. While this is not the focus of this paper, the theoretical model presented here provides a valuable foundation for such analysis of the link between fee structure and information dynamics across venues. Compared to the spreads in each venue, the fees are generally very small. Due to this, the overall impact of the fee structure on the Toxic Arbitrage Information Share will be small as well.

Comparing $\underline{n}_j$ and $\underline{n}_j^*$ as displayed in Equations (3.15) and (3.16), I find

$$\underline{n}_j = \frac{S_j}{2} + c \quad < \quad \underline{n}_k^* = \frac{S_j}{2} + 3c. \tag{3.17}$$

$\underline{n}_j$ is smaller than $\underline{n}_k^*$ for $j \neq k$ and $j, k \in \{B, C\}$. This implies that it is easier for market maker j to deter an informed trader arriving in market k from using a limit order than to deter an informed trader arriving in (his) market j from using a market order. It is easier for a market maker to defend himself against informed traders in other markets than against informed traders in the same market. The reason for this is that an informed trader using a limit order to incentivize the arbitrageur needs to compensate the arbitrageur for his transaction costs. In presence of make-take fees, this does not necessarily hold. The fee structure across venues hence has a direct impact on the informational dynamics between them. In the next step, I use my model in order to find the information share using the relative share of toxic arbitrage opportunities.

3.3.1 Toxic Arbitrage Information Share

The probability for seeing a toxic arbitrage opportunity initiated in market B and C, respectively, is given by

$$Prob_{B/C} = \delta_b * Prob\left(n_B^+ > \underline{n}_B^*\right) \tag{3.18}$$

$$Prob_{C/B} = (1 - \delta_b) * Prob\left(n_C^+ > \underline{n}_C^*\right). \tag{3.19}$$

This is the probability that the informed trader arrives in the initiating market multiplied by the probability that the price impact of the information is sufficiently large for the informed trader to use a limit order.

It follows that the relative frequency of toxic arbitrage opportunities is given by

$$\begin{aligned} Tox &= \frac{Prob_{B/C}}{Prob_{B/C} + Prob_{C/B}} \\ &= \frac{1}{1 + \left(\frac{(1-\delta_b)}{\delta_b}\right)\left(\frac{Prob\left(n_C^+ > \underline{n}_C^*\right)}{Prob\left(n_B^+ > \underline{n}_B^*\right)}\right)}. \end{aligned} \tag{3.20}$$

This is the percentage of toxic arbitrage opportunities which is initiated by informed traders arriving in market B. The ratio Tox clearly differs from the true information share which can be thought of as the cumulative price impact of information arriving in this market relative to the overall information arriving. The latter is given by

$$\begin{aligned} IS^* &= \frac{\delta_b E\left(n_B^+\right)}{\delta_b E\left(n_B^+\right) + (1-\delta_b) E\left(n_C^+\right)} \\ &= \frac{1}{1 + \left(\frac{(1-\delta_b)}{\delta_b}\right)\left(\frac{E\left(n_C^+\right)}{E\left(n_B^+\right)}\right)}. \end{aligned} \tag{3.21}$$

For brevity, let us introduce the following notation

$$\Delta_b = \left(\frac{(1-\delta_b)}{\delta_b}\right), \quad \Psi = \left(\frac{Prob\left(n_C^+ > \underline{n}_C^*\right)}{Prob\left(n_B^+ > \underline{n}_B^*\right)}\right), \quad \Gamma = \left(\frac{E\left(n_C^+\right)}{E\left(n_B^+\right)}\right), \tag{3.22}$$

where Δ_b indicates how likely it is for news to arrive in market C compared to marked B. Ψ is the probability that news arriving in market C lead to a toxic arbitrage opportunity, relative to the probability of news arriving in market B to do so. A Ψ larger than unity implies that it is more likely for an informed trader arriving in market C to cause a toxic arbitrage opportunity than for an informed trader arriving in market B. Γ is given by the expected impact of news arriving in market C relative to the expected price impact of news arriving in market B. A Γ larger than unity implies that information from informed traders arriving in market C have a higher expected absolute price impact than information from traders arriving in market B. Using this notation, I solve Tox in Equation (3.20) for the unobservable Δ_b

$$Tox = \frac{1}{1+\Delta_b\Psi} \tag{3.23}$$

$$\Rightarrow \quad \Delta_b = \frac{(1-Tox)}{\Psi Tox} \tag{3.24}$$

and substitute this in Equation (3.21) for the true information share

$$\begin{aligned} IS^* &= \frac{1}{1+\Delta_b\Gamma} \\ &= \frac{1}{1+\frac{(1-Tox)}{\Psi Tox}\Gamma}. \end{aligned} \tag{3.25}$$

Equation (3.25) provides a way to correct the difference between the relative toxic arbitrage share and the true information share. In line with the idea that toxic arbitrage opportunities are caused by information arrival, more information arriving in market B leads to a higher Tox. A higher Tox in turn is associated with a higher information share IS^*. Following Equation (3.17), a lower spread in market B leads to a lower threshold $\underline{n}_C^*$ for information arriving in market C to cause a toxic arbitrage opportunity. A lower $\underline{n}_C^*$ increases $Prob\left(n_C^+ > \underline{n}_C^*\right)$ and thereby Ψ. Consequently, a lower spread in market B is associated with a higher IS^* given Tox. More intuitively, a higher spread in market B makes it harder for informed traders in market C to take advantage of their information using limit orders. Therefore, the information is less likely to cause an arbitrage opportunity and Tox will underestimate the true information share IS^*.

This specification of IS^* in Equation (3.25) provides us with a toxic arbitrage based information share which only depends on three parameters: Tox, Ψ and Γ. All of these parameters can be approximated by observable information and consequently it is possible to calculate this Toxic Arbitrage Information Share. It is worth noting that the identity in Equation (3.25) holds, independently of the distribution of information price impacts. The only underlying assumptions are that the expected average price impact is zero and that the distribution is symmetric.

In the next section, I propose a procedure for estimating the Toxic Arbitrage Information Share.

3.4 Estimation procedure

In order to estimate the Toxic Arbitrage Information Share, I need estimates for the three parameters Tox, Ψ and Γ. The first step of the following procedure regards the identification of toxic arbitrage opportunities. The second step addresses the estimation of Ψ and Γ.

3.4.1 Classification of toxic arbitrage opportunities

An arbitrage opportunity is given when the spread between the best bid in one market and the best ask in the other exceeds the costs of taking advantage of the arbitrage opportunity. Hence, it must be given that

$$b_k - a_j > 2c_k + 2c_j \text{ where } k, j \in \{C, B\}, k \neq j, \tag{3.26}$$

where c_k and c_j are the transaction fees per one asset traded in market k and j, respectively. We need to take into account two transactions in each market as the arbitrageur also incurs costs when closing his position in each market after the arbitrage opportunity ceased.

Building on the description of asynchronous price adjustment by Schultz and Shive (2010), Foucault et al. (2016) introduce the concept of toxic arbitrage as arbitrage opportunities due to information arriving in one of the markets. As shown in the model above, such arbitrage opportunities are expected to be initiated by a price movement in the market where the information arrives as the informed trader posts a limit order. Toxic arbitrage opportunities end with a price movement in the market which did not initiate it, as the price in that market adjusts due to arbitrage. Arbitrage due to information implies that the resulting price movement is permanent. Foucault et al. (2016) focus on this fact by classifying toxic arbitrage opportunities as arbitrage opportunities resulting in price movements in both markets in the same direction, i.e.

$$(b_{C;post} - b_{C;pre})\,(a_{B;post} - a_{B;pre}) > 0 \text{ and} \tag{3.27}$$

$$(a_{C;post} - a_{C;pre})\,(b_{B;post} - b_{B;pre}) > 0, \tag{3.28}$$

where $b_{j;post}$ and $a_{j;post}$ are the bid and ask prices in market $j \in \{C, B\}$ after the arbitrage opportunity ends and $b_{j;pre}$ and $a_{j;pre}$ are the bid and ask prices just before the arbitrage opportunity begins. I am slightly deviating from this definition for two reasons. Firstly, there is a significant amount of arbitrage opportunities where one of these prices does not exist, i.e. where the arbitrage opportunity starts with the first limit order on one side of the limit order book or where the arbitrage opportunity ends with the emptying of one side of the limit order book, especially in CME. In such cases, I am unable to compute the price impact of the arbitrage opportunity. Secondly, when focusing on the price on one side of the limit order book, i.e. bid or ask prices, for a given arbitrage opportunity, the identification of the arbitrage opportunities may be falsified by a change in the spreads in one or both markets during the existence of the arbitrage opportunity. This could be taken care of by taking into account the mid-price which, however, brings us back to the problem with partially empty limit order books. In order to avoid these problems, I focus on the chronology of price changes. As mentioned before, a toxic arbitrage opportunity ends due to a price movement in the market which did not cause the deviation in price. For each arbitrage opportunity, I hence record which market's price movement opens the arbitrage opportunity and which market's price movement closes it. In line with the logic by Foucault et al. (2016), if these two markets are not identical, this arbitrage opportunity is classified as toxic. This means that the price deviation was caused by information arriving in one market and closed by the asynchronous price adjustment in the other market. If these two markets are identical, I classify this arbitrage opportunity as non-toxic, i.e. as liquidity driven. For the remainder of this paper, I am using the chronology of price movements for the identification of toxic arbitrage opportunities.

3.4.2 Estimation of the Toxic Arbitrage Information Share

Tox is given by the ratio of toxic arbitrage opportunities

$$Tox = \frac{\#Toxic_{BMF/CME}}{\#Toxic_{BMF/CME} + \#Toxic_{CME/BMF}}. \tag{3.29}$$

In order to get an approximation for the relative probability of seeing a toxic arbitrage opportunity in market C relative to market B (Ψ) and the average price impact of information arriving in market C relative to market B (Γ), I need to make an assumption about the distribution of the price impacts in each market. Similar to standard econometric techniques, I assume a normal distribution of price impacts in each market

$$n_B \sim \mathcal{N}(0,\, \sigma_B^2) \qquad \text{and} \qquad n_C \sim \mathcal{N}(0,\, \sigma_C^2), \tag{3.30}$$

where σ_j is the standard deviation of the price impact of information arriving in market j. The mean price impact is zero. In order to calculate σ_B, I am using the observed permanent price impacts of information arriving in market B given by the change in the mid-price. Let us define n_j^+ as the absolute price impact of information arriving in market j given by

$$n_j^+ = |p_{B;1min} - p_{B;pre}|, \tag{3.31}$$

where $p_{B;1min}$ is the mid-price in market B one minute after the arbitrage opportunity closed and $p_{B;pre}$ is the mid-price in market B just before the arbitrage opportunity opened. For each toxic arbitrage opportunity, I can estimate this price impact if both mid-prices exist. I am using the mid-prices in market B due to its higher liquidity.

Given the assumption of normality, the distribution of positive observed price impacts is essentially a truncated normal distribution. The threshold of the truncation is given by the spread S_C and the transaction fees for all relevant transactions. There are five relevant transactions in every case.[7] The expected value of the truncated normal distribution subject to this threshold is illustrated in Figure 3.3.

Formally, the expected value of the truncated normal distribution is given by

$$E(n_B^+|n_B > \underline{n}_B^*) = \frac{\sigma_B \phi\left(\frac{\underline{n}_B^*}{\sigma_B}\right)}{\left(1 - \Phi\left(\frac{\underline{n}_B^*}{\sigma_B}\right)\right)}, \tag{3.32}$$

where $\phi(\cdot)$ is the probability density function of the standard normal distribution and $\Phi(\cdot)$ is its cumulative distribution function

$$\phi(x) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}x^2\right), \qquad \Phi(x) = \frac{1}{2}\left(1 + erf(x/\sqrt{2})\right). \tag{3.33}$$

The average value of the observed positive price impacts should be asymptotically equal to the expected value of the truncated normal distribution subject to this threshold. By

[7] As in the calculation of the arbitrage opportunities, I need to take into account all transaction fees which the arbitrageur is facing. The arbitrageur needs to sell the asset after correction of the arbitrage opportunity and hence needs to take into account the costs for these trades as well. Additionally, it is necessary to include the costs for the submission of a limit order by the informed trader. This makes five transactions in total. As transaction fees are usually low compared with the spread, the transaction fees play only a negligible role.

setting $E(n_B^+|n_B > \underline{n}_B^*)$ equal to the mean of all positive price impacts of B/C arbitrage opportunities, I am hence able to derive σ_B.[8] In order to do so, I numerically solve for σ_B which solves the equation

$$mean(n_B^+) = \frac{\sigma_B \phi\left(\frac{\underline{n}_B^*}{\sigma_B}\right)}{\left(1 - \Phi\left(\frac{\underline{n}_B^*}{\sigma_B}\right)\right)}. \tag{3.34}$$

This provides us with an estimate of σ_B, the standard deviation of price impacts of information arriving in market B. Given the assumption of a normal distribution with zero mean, the distribution of price impacts is fully identified. I can now use this distribution to estimate the denominators of Ψ and Γ as shown in Equation (3.22). The denominator of Ψ is the probability for seeing a B/C arbitrage opportunity conditional on that an informed trader arrives in market B. This is equal to one minus the cumulative distribution function of a normal distribution given σ_B

$$\begin{aligned} Prob\left(n_B^+ > \underline{n}_B^*\right) &= 1 - Prob\left(n_B^+ < \underline{n}_B^*\right) \\ &= 1 - \Phi\left(\frac{\underline{n}_B^*}{\sigma_B}\right). \end{aligned} \tag{3.35}$$

The denominator of Γ is the expected positive price impact of information arriving in market B, which is equivalent to the expected value of a normal distribution truncated at zero

$$\begin{aligned} E\left(n_B^+\right) &= E\left(n_B | n_B > 0\right) \\ &= \frac{\sigma_B \phi\left(0\right)}{\left(1 - \Phi\left(0\right)\right)} \\ &= \frac{2\sigma_B}{\sqrt{2\pi}}. \end{aligned} \tag{3.36}$$

[8] I am using the absolute price impact of all B/C arbitrage opportunities here. Given the assumption that the underlying distribution of price impacts is symmetric around zero, this does not influence the results but provides us with more observed price impacts.

This is illustrated in Figure 3.4.

Following the same steps as above, I can also derive the numerators of Ψ and Γ by determining the standard deviation of the distribution of price impacts of information arriving in market C. This leads to

$$Prob\left(n_C^+ > \underline{n}_C^*\right) = 1 - \Phi\left(\frac{\underline{n}_C^*}{\sigma_C}\right) \qquad \text{and} \qquad E\left(n_C^+\right) = \frac{2\sigma_C}{\sqrt{2\pi}}. \tag{3.37}$$

From here, I am able to identify Ψ and Γ as

$$\Psi = \frac{1 - \Phi\left(\frac{\underline{n}_C^*}{\sigma_C}\right)}{1 - \Phi\left(\frac{\underline{n}_B^*}{\sigma_B}\right)} \qquad \text{and} \qquad \Gamma = \frac{\sigma_C}{\sigma_B}. \tag{3.38}$$

When combining this with Tox as in Equation (3.29), I am able to fully identify IS^* using Equation (3.25). I denote the resulting measure as Toxic Arbitrage Information Share ($TAIS$)

$$\begin{aligned} TAIS &= IS^* \\ &= \frac{1}{1 + \frac{(1-Tox)}{\Psi Tox}\Gamma} \\ &= \left(1 + \frac{(1-Tox)}{Tox}\Psi^{-1}\Gamma\right)^{-1} \\ &= \left(1 + \frac{(1-Tox)}{Tox}\frac{1 - \Phi\left(\frac{\underline{n}_B^*}{\sigma_B}\right)}{1 - \Phi\left(\frac{\underline{n}_C^*}{\sigma_C}\right)}\frac{\sigma_C}{\sigma_B}\right)^{-1}, \end{aligned} \tag{3.39}$$

where the thresholds are given by $\underline{n}_B^* = S_C/2 + 5c$ and $\underline{n}_C^* = S_B/2 + 5c$.[9] It is worth noting here that the transaction costs are generally negligible compared with half-spreads. Hence, half-spreads are the drivers of the adjustment.

In order to calculate the Toxic Arbitrage Information Share, I need to take into

[9] As mentioned before, the thresholds are the sum of the half-spread and the total transaction fees involved in the creation and resolution of an arbitrage opportunity.

account that the spreads are not constant over time. Following the same logic as above, the detailed approach is presented in Appendix 3.9.1.

3.4.3 Confidence intervals

Equation (3.39) shows the point estimate of the Toxic Arbitrage Information Share. The measure is based on the ratio Tox and the precision of the measure crucially depends on the total number of observed toxic arbitrage opportunities. As a next step, I am deriving a "density" estimate from this point estimate. We can think of the occurrence of a toxic arbitrage opportunity initiated in market B given that a toxic arbitrage opportunity occurs as a success in a Bernoulli trial. The probability of k successes in n trials is hence given by

$$P(k) = \binom{n}{k} p^k (1-p)^{n-k}, \tag{3.40}$$

where p is the probability of success. In my set-up, I observe the number of "successes" k i.e. the number of toxic arbitrage opportunity initiated in market B, as well as the total number of toxic arbitrage opportunities n. Rather than finding the probability of k successes given the probability p, it is necessary to derive the probability of success p given the number of observed successes k. Put differently, I am looking for the density function of $P(p|k)$. Following Bayes' rule, I find that

$$P(p|k) \propto P(k|p)P(p). \tag{3.41}$$

Using the non-informative Jeffreys prior for the binomial proportion p given by a Beta(0.5, 0.5) distribution[10], I am able to derive the posterior distribution of p given n and k. This allows me to estimate values of p which bound the 95% confidence interval: $p_{.025}$ and

[10] A detailed description of the non-informative Jeffreys prior and its application is given by Gelman et al. (1995)

$p_{.975}$. Using these values in Equation (3.39), rather than the ratio of toxic arbitrage opportunities Tox, I can estimate a realistic 95% confidence interval between $TAIS_{.025}$ and $TAIS_{.975}$ for the Toxic Arbitrage Information Share.

$$TAIS_{.025} = \left(1 + \frac{(1-p_{.025})}{p_{.025}} \frac{1-\Phi\left(\frac{\underline{n}_B^*}{\sigma_B}\right)}{1-\Phi\left(\frac{\underline{n}_C^*}{\sigma_C}\right)} \frac{\sigma_C}{\sigma_B}\right)^{-1} \tag{3.42}$$

$$TAIS_{.975} = \left(1 + \frac{(1-p_{.975})}{p_{.975}} \frac{1-\Phi\left(\frac{\underline{n}_B^*}{\sigma_B}\right)}{1-\Phi\left(\frac{\underline{n}_C^*}{\sigma_C}\right)} \frac{\sigma_C}{\sigma_B}\right)^{-1}. \tag{3.43}$$

These bounds fully capture the uncertainty around the probability of observing a toxic arbitrage opportunity initiated in market B rather than C given the set of arbitrage opportunities we observe. However, there is still a degree of uncertainty around the measures of σ_j and $\underline{n}_j^*$ for $j \in \{B, C\}$. This uncertainty is not directly captured here and hence may lead to a moderate underestimation of these bands. When using these confidence intervals for the $TAIS$, it makes sense to also use the median of this interval as the point measure. In this case, it implies a posterior mean of $\frac{k+0.5}{n+1}$. While very close to the measure in Equation (3.39) given by $Tox = \frac{k}{n}$, the median measure provides more conservative estimates as the information share never reaches zero or one.

In the next section, I use this estimation procedure in order to compare the performance of standard procedures with the Toxic Arbitrage Information Share.

3.5 Simulations

The Toxic Arbitrage Information Share provides a model based measure of information arriving in each market. It is reasonable to expect this to be similar in size as other measures of price discovery. The standard procedures for price discovery are given by the information share (IS) introduced by Hasbrouck (1995) and the permanent component share (CS) by Gonzalo and Granger (1995).

As highlighted by Baillie et al. (2002), the computation of the Gonzalo Granger and Hasbrouck procedures are closely related as both are based on Vector Autoregression (VAR). Yan and Zivot (2010) and Baillie et al. (2002) highlight the importance of choosing the frequency when using these procedures. A lower frequency is expected to lead to wider bands of the Hasbrouck information share, while higher frequency data may be affected more by microstructure noise. It is one of the central advantages of the Toxic Arbitrage Information Share that the frequency of the data does not matter. It is worth noting that the Toxic Arbitrage Information Share purely focuses on the arrival of information in each market and works in expectation. The information shares introduced by Gonzalo and Granger (1995) and Hasbrouck (1995) as well as their numerous extensions have a less narrow definition. The Gonzalo Granger permanent component share measures the relative influence of the underlying equilibrium price on each market. It is not straightforward, however, how a market would be able to adjust a price towards the long-run equilibrium without some sort of information arrival. When looking at illiquid markets, there are some problems with data availability. Of course, all procedures are unable to take into account periods where one of the limit order books is empty. However, the Toxic Arbitrage Information Share is more flexible in this regard as it can handle limit order books which are partially empty. Narayan and Smyth (2015) provide an overview of the assumptions and limitations of the standard procedures resulting from a simple implementation of the VAR approach. For example, the VAR system depends on the correct identification of the lag structure and that the structure does not change over the estimation period. The VAR is also more prone to errors due to missing observations during the day which interrupt the time series. In these regards, the Toxic Arbitrage Information Share proposed here is more flexible and robust. For a detailed description and implementation of the standard procedures see e.g. Tse et al. (2006), Chen and Gau (2010), and Yan and Zivot (2010).

In the remainder of this section, I am running several simulations in order to compare the performance of the Toxic Arbitrage Information Share with the Hasbrouck information share as well as the Gonzalo Granger permanent component share. The first simulations are based on the theoretical framework introduced in Section 3.3. The purpose of these simulations is to see if the estimation procedure outlined above does in fact lead to efficient results under the assumptions of the model. In order to evaluate the robustness of the Toxic Arbitrage Information Share to other price dynamics, I am additionally using a more neutral structure in the spirit of Hasbrouck (2002) adapted to reflect lower liquidity.

3.5.1 Model-based simulation

The following simulations incorporate the idea of the arbitrageur as a channel to trade in other markets. The exact simulation model is provided in Appendix 3.9.2.

It follows a four period sequence. In the first period, the informed trader acts according to the innovation to the efficient price. As in the model in Section 3.3, the informed trader chooses limit orders and market orders optimally. In the second period, the arbitrageur acts if there is an arbitrage opportunity. In periods three, the market makers learn about the innovation and react if the informed trader arrived in that market. In the last period, the other market maker learns about the innovation in the efficient price, adjusts his prices, and brings the market back to equilibrium. After this, the sequence begins again. The data is simulated with market C as the less liquid market having a spread twice as wide as market B. The Toxic Arbitrage Information Share is estimated following the steps outlined in Section 3.4 using the resulting best bid and offer[11]. The Hasbrouck and Gonzalo Granger procedure are estimated using the resulting mid-price.

Figure 3.5 illustrates the results for different underlying information shares each using 100 simulations. The x-axis shows the underlying information share of market

[11] In these simulations, there is no difference between the two point measures discussed in the previous section.

B while the y-axis is given by the measured information shares. The dashed black 45 degree line hence marks an unbiased estimate.

The red line shows the median Toxic Arbitrage Information Share ($TAIS$) resulting from the simulations. The red shaded error bands show the area engulfing 90% of the estimates. The results for the permanent component share (CS) are given in blue. When using the procedure introduced by Hasbrouck (1995), the information share depends on the ordering of the variables and can differ substantially. In the two variable case, the two possible orderings provide what is generally considered an upper and a lower bound. Hasbrouck (2002) shows that all permutations need to be taken into account. The two green shaded areas illustrate the error bands around the upper and lower bound of the Hasbrouck information share (IS), respectively. The green lines are given by their median values.

The Toxic Arbitrage Information Share provides an unbiased estimate for all underlying information shares. The error bands have a maximum spread of five percentage points. That the $TAIS$ performs well in these simulations speaks in favor of the estimation procedure to provide an appropriate empirical representation of the theoretically derived measure. The permanent component share performs similarly well, though with a tendency to overestimate the importance of the more liquid market for low information shares. As the Hasbrouck information share (green) does not provide a single value, it can only be evaluated using two metrics. Are the bounds unbiased in that the true value does not exceed the bounds? Are the bounds narrow enough to provide useful information? In these simulations, the Hasbrouck information share shows a tendency to overestimate the importance of the more important market. The bounds only hold for values between 40 and 60%. The lower bound fails to provide an orientation for high values and the upper bounds fails for low values of the underlying information share. Given that the simulations are based on the theoretical model, it is not surprising to see the Toxic Arbitrage Information Share to perform better than other measures. However, the exercise provides two main insights. Firstly, the proposed procedure provides a valid approach to estimating the $TAIS$. Secondly, the precision of the standard procedures can be negatively affected by differing error correction dynamics.

The precision of the Toxic Arbitrage Information Share crucially depends on the number of toxic arbitrage opportunities in the sample. By restricting the number of arbitrage opportunities we observe, we can get an idea of the precision of the measure. Figure 3.6 shows the results for this exercise using 100 simulations with the number of arbitrage opportunities on the x-axis and the estimated $TAIS$ on the y-axis. The red line depicts the median $TAIS$ for a given number of toxic arbitrage opportunities. 95% of the estimated $TAIS$ lie within the red shaded area. The black dotted line shows the true information share. When using less than ten toxic arbitrage opportunities, it becomes clear that the measure can hardly provide a useful estimate. This is especially true when only using one arbitrage opportunity where the measure consistently underestimates the result when the true information share is .9. However, when using at least ten toxic arbitrage opportunities the measure quickly converges.

While Figure 3.6 shows the precision of the $TAIS$ measure, Figure 3.7 illustrates the precision of the bounds. The red lines show the median bounds of the 95% confidence interval. The shaded area illustrates where 95% of the bound estimates lie. The result is similar to Figure 3.6 as it shows that the bands are very wide for less than ten observations. However, I find that the bounds appropriately capture the uncertainty of the measure. Given an information share of .9 as illustrated in Figure 3.7a, the true information share is within the bounds in 89% of simulations. Figure 3.7b shows the results for an information share of .8. Here, the true information share lies within the bounds 91% of simulations. As explained in Section 3.4.3, the measure of the bounds takes into account only one effect of the low number of observations. Consequently, it is not surprising that the confidence intervals are not perfectly reliable. However, they provide an important indication for the precision of the measure.

After concluding that the estimation procedure accurately represents the theoretical measure, it is important to evaluate how the measure performs when moving away from the mechanism outlined in the model. Therefore, the next step is to see how the measure performs given different price dynamics.

3.5.2 Simulation II

The following simulations display a structure in the spirit of Hasbrouck (2002). Rather than modeling the adjustment to price changes by providing a theoretical error correction mechanism, these simulations are based on a simple lagged adjustment. Simulations more in line with the idea of informed traders using the arbitrageur as a channel are likely to improve the performance of the Toxic Arbitrage Information Share.

The efficient price m_t is given by a modified random walk

$$m_t = m_{t-1} + \hat{u}_t \tag{3.44}$$

$$\hat{u}_t = \mathbb{1}_{t,p} u_t, \tag{3.45}$$

where u_t are price innovations following a discretized normal distribution $u_t \sim \mathcal{N}(0,\, \sigma_u^2)$.[12] In order to take into account periods with no price innovation, I introduce a binomially distributed dummy $\mathbb{1}_{t,p}$ which is one with probability p and zero otherwise. If $p = 0.4$, this implies that there is a change in the efficient price 40% of the time and 60% of the time there is not. Similarly, the efficient price in market $j \in \{B, C\}$ is given by

$$m_{j,t} = m_{t-1} + \hat{u}_{j,t}. \tag{3.46}$$

The set of price innovations in market B ($\hat{u}_{B,t}$) is a subset of the overall price innovations

[12] In order to take into account that prices can only change in multiples of the tick-size, I am discretizing all random variables in the simulation by rounding them to 2 digits. This is equivalent to saying that the tick-size is 0.01. The discretization is important for this exercise in order to ensure that there is a limited number of spread sizes, which in turn is necessary for the calculation of the Toxic Arbitrage Information Share.

$\hat{u}_t$. This subset affects market B immediately, while the remaining set of $\hat{u}_t$ only affects the mid-price in market B with a lag via m_{t-1}. Formally, this can be described as

$$\hat{u}_{B,t} = \mathbb{1}_{t,p_B} \hat{u}_t, \tag{3.47}$$

where $\mathbb{1}_{t,p_B}$ is a binomially distributed dummy which is 1 with probability p_B. Whenever an innovation is not immediately affecting the mid-price in market B, it is immediately affecting the mid-price in market C. Hence, $\hat{u}_{C,t}$ is given by

$$\hat{u}_{C,t} = (1 - \mathbb{1}_{t,p_B})\hat{u}_t. \tag{3.48}$$

This implies that p_B is the information share of market B. Finally, the observed best prices in market j are given by

$$a_{j,t} = m_{j,t} + \frac{S_{j,t}}{2} \tag{3.49}$$

$$b_{j,t} = m_{j,t} - \frac{S_{j,t}}{2}. \tag{3.50}$$

The following simulations are based on 10,000 observations with $p = 0.5$ and $\sigma_u = 1$. In order to evaluate the precision of the different price discovery measures, I estimate them for varying p_B. The Hasbrouck information share and the permanent component share are based on the observed mid-price

$$obs_j = m_{j,t} \tag{3.51}$$

$$= b_{j,t} + \frac{a_{j,t} - b_{j,t}}{2}. \tag{3.52}$$

The Toxic Arbitrage Information Share is estimated using the observed best bid and offer $a_{j,t}$ and $b_{j,t}$.

Figure 3.8 illustrates the results for the $TAIS$ (red), the CS (blue), and the

IS (green) when simulating the above price series 100 times for different values of p_B. Again, the simulations are based on $S_{B,t} = 1$ and $S_{C,t} = 2$.

For intermediate values, the upper and lower estimates of the Hasbrouck information share correctly encompass the 45 degree line with a spread of around 20 percentage points. However, for information shares below 30% and above 70%, the bounds fail increasingly to provide actual limits. For shares lower than 20% and in excess of 80%, the true information share is out of bounds around 85% of the time. This is in line with the results from the previous simulation though to a lesser extent. The permanent component share (blue) appears unable to differentiate information shares between 15% and 85%. For more extreme values, it still significantly overestimates the share of the less important market. This is in line with the conclusion by Hasbrouck (2002) that the permanent component share can provide misleading results. In this simulation, the permanent component share is strongly affected by the inertia of the price series due to $p = .5$. This is similar to the effect of using observations on a high frequency and again highlights the importance of the choice of the frequency especially in low liquidity environments.

The $TAIS$ (red) performs better than the permanent component share in that its estimates are closer to the true value given by the 45 degree line. However, it shows a bias towards an information share of around 55%. The reason for this is that the $TAIS$ corrects for the size of the spreads in accordance with the theoretical model. According to the model dynamics in section 3.3, a higher spread in one market leads to an decrease in toxic arbitrage opportunities initiated in the other market. As the spread in market C is larger, we expect a smaller fraction of informed traders in market B to use limit orders. This leads to the ratio of toxic arbitrage opportunities (Tox) to be lower than the true information share.

In order to balance this, the $TAIS$ corrects the Tox upwards. Equation (3.53) shows the definition of $TAIS$ as given in Equation (3.39) under the additional assumptions of $\sigma_B = \sigma_C = 1$ and $c = 0$ as given in the simulated data,

$$TAIS = \left(1 + \frac{(1 - Tox)}{Tox} \frac{1 - \Phi\left(S_C\right)}{1 - \Phi\left(S_B\right)}\right)^{-1} \tag{3.53}$$

$$= \left(1 + 0.14 \frac{(1 - Tox)}{Tox}\right)^{-1}. \tag{3.54}$$

Without this correction, the $TAIS$ would be equal to the Tox, symmetric and intersect the diagonal at the 50%. The larger the role of the mechanism described in Section 3.3 is, the smaller would we expect the bias of the $TAIS$ to be. It speaks for the $TAIS$, that it performs better than the permanent component share even though the simulations are designed to its disadvantage. The results in Figure 3.8 show that none of the methods provides an unbiased measure but that they provide complementary insights.

3.5.3 Simulation III

The final price generating process follows the process in Simulation II while introducing missing observations in the bid and ask prices of market C

$$a_{j,t} = \begin{cases} m_{j,t} + \frac{S_{j,t}}{2} & \text{with probability} \quad 1 - p_{a,j} \\ \{\} & \text{with probability} \quad p_{a,j} \end{cases} \tag{3.55}$$

$$b_{j,t} = \begin{cases} m_{j,t} - \frac{S_{j,t}}{2} & \text{with probability} \quad 1 - p_{b,j} \\ \{\} & \text{with probability} \quad p_{b,j} \end{cases}, \tag{3.56}$$

where $p_{b,j}$ and $p_{a,j}$ is the probability for a missing value in the best bid and ask of market j, respectively.

The simulations are based on 10,000 observations with $p = 0.5$, $\sigma_u = 1$, $S_{B,t} = 1$, $S_{C,t} = 2$, $p_{a,B} = p_{b,B} = 0$, and $p_{a,C} = p_{b,C} = 0.1$. Hence, ten percent of the bid and ask prices, respectively, of market C are missing.

Figure 3.9 illustrates how the change affects the Hasbrouck information share.

The spread between the upper and lower bound is in excess of 50 percentage points for most of the simulations and 40 percentage points at its minimum. It thereby does not allow for any meaningful conclusion about the information dynamics. The permanent component share appears to perform better under these circumstances. Paradoxically, by introducing missing values in the mid-price series, the share is less affected by the inertia of the price series as missing observations are dropped. A similar effect would also be achieved when reducing the observation frequency. Despite this, the measure continues to show a bias towards a more balanced information share. The $TAIS$ is largely unaffected by the missing observations. If we take out the correction of the $TAIS$ highlighted in Equation (3.53), the $TAIS$ and the permanent component share would be identical in this set-up. The reason for this is, that the necessity for the correction, which is a central feature of the $TAIS$, is not given if the price adjustment follows the naive mechanism in this simulation. It again speaks for the estimation procedure behind the $TAIS$, that this correction is the only source for its worse performance in this simulation. The more important the mechanism outlined in the model above, the better is the $TAIS$ expected to perform.

In summary, I find that the estimation procedure outlined in Section 3.4 provides an estimate consistent with its underlying measure. The closer the price dynamics in the data are to the mechanism outlined in Section 3.3, the better the Toxic Arbitrage Information Share performs. However, even in absence of this mechanism, the $TAIS$ provides valuable insights compared to the standard procedures especially in low liquidity markets.

Neither of the three procedures provides a perfect measure under low liquidity conditions and in absence of the mechanism highlighted in the theoretical deviation. In the simulations above, the usefulness of the Hasbrouck information share suffers either under incorrect bounds (Simulation II) or excessively wide spreads (Simulation III). The permanent component share is biased towards a more balanced result and is greatly affected by inertia in the price series. The $TAIS$ provides the best estimate for information shares where the more liquid market is dominating price discovery ($p_B > 55\%$), but overestimates the importance of the more liquid market for lower values. This suggests that the three measures are complementary for analyzing price discovery dynamics. The bounds of the Hasbrouck procedure provide a good orientation as long as the spread do not explode but should not be taken literally. The results suggest that one needs to take into account the permanent component share's bias towards balanced results. If it shows that one market dominates the price discovery, the numeric value should be seen as lower estimate. In absence of strategic use of limit orders, the Toxic Arbitrage Information Share suffers from a bias towards more balanced results as well. The more important the strategic use of limit orders is, the smaller this bias will be. This implies that the $TAIS$ also needs to be interpreted with caution but balances out some of the shortcomings of the other two measures. With regard to the precision of the $TAIS$, I find that an increase in the number of toxic arbitrage opportunities quickly improves the performance of the Toxic Arbitrage Information Share. It appears reasonable to focus on samples with at least ten toxic arbitrage opportunities.

In the remainder of this paper, I apply this new measure to a unique high frequency data set of foreign exchange futures. Section 3.6 provides a description of the data while Section 3.7 shows the empirical application.

3.6 Description of market set-up and data

For the application of the Toxic Arbitrage Information Share, I am using a unique data set combining high frequency data from Chicago Mercantile Exchange (CME) and Bolsa de Valores, Mercadorias e Futuros de São Paulo (BMF). BMF is the largest stock exchange in Latin America (IMF (2016)) and was the sixth largest derivative market worldwide in 2015 (Statista). The most traded derivatives on BMF are US dollar futures denominated in Brazilian real (BRL). Due to the heavy regulation on the spot exchange market, Ventura and Garcia (2012) find that US dollar futures are responsible for most of the price discovery in Brazilian real, the most traded South American currency (BIS (2016)). CME is the world's largest marketplace for derivatives and offers a Brazilian real futures contract with almost identical conditions to the contract traded in BMF. The BRL futures in CME are denominated in US-dollar and consequently, a long position in CME's BRL futures is equivalent to a short position in BMF's USD futures. Table 3.1 provides a short overview of the contracts.

For both contracts, the last business day of each month is the last trading day for the contract expiring in the consecutive month. The contract expiring in the next month is generally the most traded contract in both markets and, hence, the contract I am focusing on at each point in time.

The trading volume of this contract in BMF is several times higher than in CME. Figure 3.10 shows the daily trading volume in USD 1 billion in BMF (blue line) and CME (black line) from November 2011 to February 2014. The data set used in this paper does not provide quote data for the months between October 2012 and October 2013 and hence these months are excluded in the subsequent analysis. This is the area between the two dashed vertical lines. I am including this period in this graph for comparison purposes. Only days with trades in both markets are included and the last active day of each month is excluded. I find that the volume in CME in terms of contracts is on average 6.2% of the BMF volume. In March 2012, CME reaches an average of 25.6% relative to BMF and around 12% in April and May 2012. This period is discussed further in Section 3.7. As shown in Figure 3.10, this increase is mostly due to spikes in CME activity. Over these 28 months, the exchange rate was on average USD/BRL 2.06 and saw its minimum and maximum at 1.70 and 2.45, respectively. This implies that the contract size in BMF (USD 50,000) over this period was on average roughly equal to the CME contract in USD terms (BRL 100,000). The depreciation of the Brazilian real over this period implies, however, that the size of the CME contract in USD terms decreases over time. At the end of the period, a single CME contract covers an amount equivalent to only USD 42,808.22.

Figure 3.11 shows the share of average trading volume per hour in BMF (blue line) and CME (green line), respectively. The x-axis is given by the time of day in São Paulo. There is almost no trade in CME outside the BMF trading hours. CME is most active upon the opening of BMF, while BMF sees its peak in traded contracts at 11am. Both markets' activity decreases over the course of the day with a local peak at 3pm. The last trading hour in BMF appear to see a stronger decline in BMF than CME.

An important property of the market for USD/BRL futures is that there are high costs for foreign firms to gain access to BMF. During the time considered in this paper, Brazilian law required non-resident investors to have a legal and fiscal representative present in Brazil.[13] According to market participants in Brazil, CME's market entry costs make it unattractive for most Brazilian investors to trade in CME directly. Additionally, over the horizon of our sample, the Brazilian government levies a tax on international transactions and financial products. This *Imposto sobre operaçẽs financeiras* can serve as an additional deterrent for investors to be active in both markets. Together, this implies that the markets, while trading close to equivalent products, serve different sets of traders.[14] Again, based on the information from market participants, some traders serve as arbitrageurs connecting both markets while market makers, especially in CME, closely track movements in BMF in order to limit their exposure to arbitrageurs.

BMF data is directly provided by the exchange with millisecond precision. The data set includes all trades and quotes. CME data is provided by Thomson Reuters and includes all trades as well as changes in the best bid and ask prices with millisecond

[13]Source: www.bmfbovespa.com.br

[14]It may also explain, why CME is able to provide a market in this product despite the far lower trading volume.

time stamps. The analysis focuses on the dynamics of the best bid and ask prices in both markets. Due to the inverse denomination of the contracts, a purchase of a CME contract is equivalent to the selling of a BMF contract. For the remainder of this paper, I invert the prices in CME in order to create time series with the same price denomination, meaning

$$b_C = \frac{1}{\hat{a}_C} \qquad \text{and} \qquad a_C = \frac{1}{\hat{b}_C}, \tag{3.57}$$

where $\hat{b}_C$ and $\hat{a}_C$ are the best bid and ask prices denominated in USD for one BRL as reported by CME. The prices I use from here onward are b_C and a_C, the implied best bid and ask prices in BRL for one USD. I also adjusted the BMF prices in order to reflect the price for one USD instead of the contract quotation for USD 1,000.

The data horizon in this paper is from November 2011 to September 2012 and from November 2013 to February 2014 resulting in 248 trading days where both markets are open. As mentioned above, the gap in the data horizon is due to data availability. The data contains observations for 9,817,200 seconds. 8,662,015.50 (88.2%) of these show positive spreads in both markets.[15] 99.82% of the observations with positive spreads have a higher spread in CME than in BMF. However, this does not necessarily imply that it is not sensible for liquidity or informed traders to open arbitrage opportunities in BMF as Section 3.7 shows.

3.7 Empirical analysis

The empirical Section of this paper consists of two parts. Firstly, I use the model in Section 3.3 in order to determine the effective transaction fees in the markets. I am using these transaction fees to locate the arbitrage opportunities and classify them into toxic

[15]Negative spreads are a common problem when using high frequency data and observations with negative spreads are generally excluded from the analysis.

and non-toxic arbitrage opportunities. In a second step, I calculate the Toxic Arbitrage Information Share and compare it to the Hasbrouck information share as well as the Gonzalo Granger permanent component share.

3.7.1 Transaction fees

In order to identify the set of arbitrage opportunities, it is crucial to have a realistic estimate of the effective costs which an arbitrageur faces. While the trading venues CME and BMF provide information about transaction fees, these greatly depend on the type of investor and investment behavior. Further, it is necessary to allow for the possibility that traders will not engage in arbitrage without a minimum profit or without covering additional costs. The fees in BMF range between USD 0.11 and USD 0.44 per contract for high frequency traders depending on the total number of contracts traded. In CME, I find fees between USD 0.10 and USD 0.56. In what follows, I am using an implication of the model in order to obtain an estimate of the effective transaction costs excluding the spreads. According to the model, toxic arbitrage opportunities are created by strategically placed limit orders by informed traders. He chooses the smallest price improvement γ which still leads the arbitrageur to engage in arbitrage, leading to the γ as solved for in Equation (3.9)

$$\gamma = \frac{S_C + S_B}{2} + 2c. \tag{3.58}$$

This implies a size of the arbitrage opportunity of $2c$, the total transaction costs of the arbitrageur to take advantage of the arbitrage opportunity. Liquidity traders are omitted in the reduced form of the model presented here, for brevity. However, following the same logic as for the informed trader, a liquidity trader can use a limit order to incentivize the arbitrageur. By doing so, the liquidity trader creates a non-toxic arbitrage opportunities and the same logic with regard to the size of the arbitrage opportunity applies. Generalizing this idea, implies that there is no reason for a size difference across types of arbitrage. The reason for this is that limit orders are used to incentivize the arbitrageur to trade and the arbitrageurs incentivization does not differ for informed or liquidity trades. Toxic arbitrage opportunities and non-toxic arbitrage opportunities are hence expected to have the same size. By size, I denote the difference between best bid and ask price across markets enabling arbitrage. For an arbitrageur to break even, he has to cover the transaction fees in both markets. Further, the profit also needs to cover the transaction fees when liquidating his position in both markets. Hence, his total costs involve four individual transactions.[16] The size of the arbitrage opportunity must exceed these costs

$$b_k - a_j > 2c_k + 2c_j \text{ where } k, j \in \{C, B\}, k \neq j, \tag{3.59}$$

where c_k and c_j are the transaction fees per Brazilian real traded in market k and j, respectively. I now take the implication from the model as a condition which has to hold given the correct transaction fees: Toxic and non-toxic arbitrage opportunities must on average be equal in size. I am choosing the lowest transaction fees for which this holds true. The estimation is under the assumption that the relevant costs for the arbitrageur have not changed over the time horizon. According to conversations with CME and BMF, this is true at least for the nominal transaction fees in both markets.

While the transaction fees do not change over time, the relative size of the contracts in both markets changes with the exchange rate as they are denominated in BRL and USD, respectively. I am correcting for this change in contract size as it also affects the effective transaction fees per USD in each market as nominal transaction fees are charged per contract traded. A depreciation in the exchange rate implies that the

[16]Say an arbitrageur buys in both markets at time $t = 1$. For these two trades, he needs to pay transaction fees. However, these traders also lead to a long position in each market. In order to close these positions when the price has returned to equilibrium at $t = 2$, he sells in both markets which again leads to transaction fees. There are four transactions in total, each incurring fees.

contract in CME which covers BRL 100,000 is worth less in USD terms and therefore covers a different value compared to BMF which has its contract size fixed in USD. The condition for an arbitrage opportunity hence becomes

$$b_i - a_j > 2c_B + 2c_C \left(\frac{E_t}{2}\right) \text{ where } i, j \in \{C, B\}, i \neq j, \tag{3.60}$$

where E_t is the exchange rate USD/BRL and $\left(\frac{E_t}{2}\right)$ balances out the change in relative size of the contracts.

I find, that given transaction fees of on average USD 0.275 per contract in each market, there is no significant difference in size between toxic and non-toxic arbitrage opportunities. This number is well in range of transaction fees which are given by the trading venues. It is sensible to expect the effective transaction costs of an arbitrageur in excess of the half spreads to be larger than the minimal nominal fees of around USD 0.11. In case of BMF, this implies effective transaction fees of USD 5.50 per USD 1 million traded. Consequently, I use this estimate of the transaction fees to identify the set of arbitrage opportunities. It is worth noting that while an estimate for the transaction fees matters for the identification of arbitrage opportunities, their size is generally negligible compared to the spreads.

3.7.2 Arbitrage opportunities

Table 3.3 shows the number of arbitrage opportunities and their total length. Using the transaction fees above, I find a total of 14,268 arbitrage opportunities with a total duration of 105,495.65 seconds. This is equivalent to 1.22% of the sample. Of these 14,268 arbitrage opportunities, 98 are not used for the subsequent analysis as they either start or end with a negative spread or end with the end of the trading day. Two thirds of these arbitrage opportunities would be taken advantage of by executing sell orders in each market. I denote these opportunities as Type 1. The rest can be taken advantage of by executing buy orders in both markets, which I call Type 2 arbitrage. It is interesting to note that Type 1 arbitrage opportunities are not only more common but also appear to have a longer average duration as they make for 71% of the total duration of identifiable arbitrage opportunities.

Figure 3.12 shows the total number of arbitrage opportunities per week. 44% of these are found in March 2012. Five days (13th, 14th, 15th, 19th and 21st of March) have over 500 arbitrage opportunities each. These five days account for 38% of all arbitrage opportunities in our sample. It is worth noting that this is the same time where Figure 3.10 shows spikes in CME trading activity. This period is most likely to be explained by a change in the *Imposto sobre operações financeiras* (IOF) on March 15, 2012. The IOF is a federal transaction tax in Brazil levied on credit, foreign exchange, insurance, and securities transactions. While it is important to note that these days are extraordinary, they do not affect the estimation of the information shares on other days as the measure for each day is estimated separately.

Table 3.4 shows the classification of the identified arbitrage opportunities using the chronology of price changes. CME/CME and BMF/BMF denote non-toxic arbitrage opportunities initiated in CME and BMF, respectively. Such arbitrage opportunities are expected to be caused by liquidity shocks. CME/BMF and BMF/CME are toxic arbitrage opportunities caused by price relevant information in CME and BMF, respectively.

As shown in Table 3.4, 1,011 (7.1%) of the arbitrage opportunities are toxic and initiated in CME. 5976 (42.2%) are toxic arbitrage opportunities initiated in BMF. This implies that half of all arbitrage opportunities are toxic and most are caused by information arriving in BMF. The non-toxic arbitrage opportunities make 23.3% and 27.4%, respectively. There are 247 days in my sample which have at least one arbitrage opportunity. Table 3.5 provides summary statistics for these days.

There are five days which see a full limit order book for less than 8 hours (28,800 seconds), two of these are due to CME and three are due to BMF. The median time without a full limit order book in CME is 3.5 minutes per day. In BMF the median time is less than a minute. As indicated before, the mean spread in CME is consistently higher than in BMF. The minimum number of trades in CME is none compared to 13,520 traded contracts in BMF. The total duration of arbitrage opportunities in one day varies between 0.01 seconds and 20,010 seconds (5 hours and 33 minutes). The total number of arbitrage opportunities for days with at least one arbitrage opportunity varies between 1 and 1,457. 229 out of 247 days see at least one toxic arbitrage opportunity. On 228 out of 247 days, there is at least one non-toxic arbitrage opportunity.

3.7.3 Toxic Arbitrage Information Share

The summary statistics of the arbitrage opportunities in Table 3.4 show that most of the toxic arbitrage opportunities (5,976 compared to 1,011) are caused by information arriving in BMF rather than in CME. However, as highlighted in Section 3.3, the frequency of toxic arbitrage opportunities alone does not provide an appropriate measure of information arrival. In order to calculate the information share, I require measures for the right-hand-side parameters of Equation (3.39): Tox, Ψ and Γ.

When using the distribution of price impacts for both B/C and C/B arbitrage opportunities, I use the permanent price impact of the information events on BMF mid-prices here, due to BMF's higher liquidity. For the transaction fees, I use the same estimate as in the determination of the arbitrage opportunities, i.e. USD 5.50 per USD 1 million. It is worth noting that in this application the effect of the transaction fees on $TAIS$ is marginal compared to the spreads and does not affect the results.

In what follows, I am focusing on days with at least ten toxic arbitrage opportunities in order to keep the error around the Toxic Arbitrage Information Share sufficiently small. This leads to 106 days or over 42% of the total number of days with at least on arbitrage opportunity.[17] The results do not change when including all observations.[18]

[17]This is equivalent to 35% of the total sample.

[18]In the subsequent analysis, 8 of these days will drop out as it is not possible to generate the day spe-

The spreads in both markets, S_C and S_B, are observable as long as open bid and open ask orders exist. As before, I follow the procedure as introduced in Section 3.4.

The first row in Table 3.6 provides the summary statistics for the ratio of toxic arbitrage opportunities. The mean value of close to 90% already suggests a much higher share of information arriving in BMF. However, as highlighted before, this measure is biased due to the different price impact of information in both markets and the different spreads.

Following the numerical approximation in Equation (3.34) for both BMF and CME and solving for the standard deviations of the price impacts of arriving information leads to a median σ_B and σ_C of 0.0011 and 0.00086, respectively. Hence information arriving in BMF has on average a larger price impact than information arriving in CME. The correlation of the two market's standard deviations over time is 0.5. Together with the lower spreads in BMF, this suggests that the percentage of BMF initiated toxic arbitrage opportunities Tox is underestimating the information share of BMF. Using the approximated σ_B and σ_C per day, I can now estimate the daily Ψ.

Using Tox, daily Ψ as well as σ_B and σ_C, I am able to determine the bias given by the difference between the toxic arbitrage ratio and the unbiased information share. The bias is presented in the second row. As suspected, the Toxic Arbitrage Information Share is almost always higher than Tox. The mean bias of -.047 implies that the ratio of toxic arbitrage opportunities underestimates the percentage of information entering BMF by 4.7 percentage points on average. The bias ranges from an underestimation by 61.2 percentage points to an overestimation by 13.9 percentage points. Rows three to five provide the lower band ($Toxic_{0.025}$), mean measure ($TAIS$), and the upper band ($Toxic_{0.975}$) of the Toxic Arbitrage Information Share. I find that the $TAIS$ for BMF is between 42.2% and 99.8%. This means, that between 42.2% and 99.8% of price relevant information arrives first in BMF. The median Toxic Arbitrage Information Share is 97.3%

cific standard deviation of the information arriving in CME. An alternative to dropping the observation would be to use standard deviations from the day before or after.

while the mean is 94.1%. This is very much in line with the information I received from conversations with market participants in Brazil who expect no gain in information from observing the prices in CME. The lower bound of the confidence interval ($TAIS_{.025}$) has an absolute minimum of 19%, yet most of the values are above 80% leading to a mean value of 87%. The upper bound has a minimum of 63% with most values close to 100%. The sixth row provides the spread of the 95% confidence interval serving as a good indication for the precision of the measure. The median spread is 7 percentage points.

The information share based on toxic arbitrage opportunities provides a model based measure of information arriving in each market. As in the simulations, it is reasonable to expect this to be similar in size as other measures of price discovery. The standard procedures for price discovery are again given by the information share (IS) introduced by Hasbrouck (1995) and the permanent component share (CS) by Gonzalo and Granger (1995). In the remainder of this section, I first look at the summary statistics of the information shares in order to compare their overall properties. In a second step, I compare the behavior of these measures over time in order to get a better idea of their stability.

The first row of Table 3.7 again provides the summary statistics for the Toxic Arbitrage Information Share ($TAIS$). Rows two to four provide the summary of the permanent component share (CS) and the bounds of the Hasbrouck information share (IS), respectively. Following Hasbrouck (1995, 2003), I use secondly frequency of the quoted mid-price in order to estimate the daily component and information shares. While Hasbrouck (1995) uses best quotes as in the application here, some studies use average transaction price in given intervals, generally between 1 second and 5 minutes. Even when using 5 minute intervals, there will not be a single trade in around 50% of the observations for CME. Following this procedure leads to IS and CS price discovery shares with even more unrealistically high information shares for CME.

The Toxic Arbitrage Information Share is close to the Gonzalo Granger component share. The difference between the two measures in median and mean is .3 and 2.5 percentage points, respectively. The median and mean of the lower bound for the Hasbrouck information share is 92.8% and 77.7%, respectively, and hence lower than either CS other $TAIS$. This is what we would expect if all measures work sufficiently well. Similarly, the median and mean upper bound are close to but above the median and mean $TAIS$. The spread of the Hasbrouck measure has a low median value of 5.8 percentage points. However, for individual days, the range extents to up to 70 percentage points. The fact that the three procedures yield similar results serves as a confirmation of the logic put forward in Foucault et al. (2016) that toxic arbitrage opportunities are in fact information driven. The difference in the summary statistics are mostly driven by individual days where the standard procedures find most of the price discovery to take place in CME.

The summary statistics in Tables 3.6 and 3.7 show that the values for the different price discovery measures generally point in the same direction. However, it is not only of interest which market dominates price discovery but also how the shares change over time and whether individual days see a higher importance of a less important market. Figures 3.13a and 3.13b illustrate the development of these shares over time. As in Table 3.6, I am only looking at days with at least ten toxic arbitrage opportunities. Figure 3.13a illustrates the evolution of $TAIS$ (red line) and the permanent component share (blue line). The black vertical line marks the data gap between October 2012 and October 2013. I find that both measures are close to 100% most of the time. The component share is more volatile and shows several instances where CME has a larger price discovery share than BMF. There is only one instance where the Toxic Arbitrage Information Share is below 50%. At this time, the permanent component share also shows a strong drop, though significantly more extreme. For the other drops in the permanent component share, there is no equivalent reaction. Instead there are two further instances where the $TAIS$ drops below 75% without a drop in CS. The red shaded area shows the 95% confidence interval of the $TAIS$. As mentioned above, the confidence intervals need to be interpreted with caution but provide a good indication for the precision of the measure. The drops in CS where $TAIS$ remains high cannot be attributed to an increase in the imprecision of the $TAIS$. The lower band drops on around ten days below 75%, only three of which coincide with a drop of the CS below that level.

The red line in Figure 3.13b again illustrates the toxic arbitrage based information share for days. The green line shows the mid-point of the bounds of the Hasbrouck information share (IS). The green area illustrates the range between the upper and the lower estimate from the Hasbrouck procedure. The upper bound remains close to 100% most of the time. For the first 70 observations there is only one day where it drops below 75%. On this day, the $TAIS$ also drops. On the other days, where the $TAIS$ drops, we also see a drop in the lower bound of the IS and hence of its midpoint. The lower bound drops more often resulting in large spreads for the Hasbrouck information share. Similarly to what we observe with the CS, drops in the upper bound of the IS where $TAIS$ remains high, cannot be explained by the imprecision of the latter. Figure 3.13a and b illustrate that different measures lead to different results on individual days. Overall, only the downward spikes of the $TAIS$ in the beginning of the sample fail to find similar evidence in either of the other measures. Several times we observe spikes in the standard measures, without a similar reaction in the $TAIS$. However, in these instances the spread in the Hasbrouck information share is usually wide, indicating less precision in the measure. On those days, we generally observe some overlap between the $TAIS$ confidence interval and the IS interval. There are five days where we see a downward spike in the IS with narrow spread without an equivalent reaction of $TAIS$. While these days see downward spikes in CS as well, the difference between the two measures is substantial.

The five days in March 2012 which see a larger number of arbitrage opportunities and unusual volume in CME are untypical in that three of them pose some of the few instances when the $TAIS$ is significantly below the other two measures. When excluding these days given by the third, fourth, fifth, seventh, and ninth observation in Figure 3.13, the impression that the standard measures have a tendency to see price discovery as more balanced is even stronger.

While both standard measures broadly rely on the same assumptions, they can differ substantially. The correlation coefficient between the permanent component share and Hasbrouck's midpoint is 85%. Yet, the CS lies outside the latter's bounds 20% of the time. While Hasbrouck always shows a drop when Gonzalo Granger does, there are several more outliers for Hasbrouck. Figure 3.14 shows how these observations relate to the difference between $TAIS$ and Hasbrouck. The x-axis is given by the distance between the bounds of the Toxic Arbitrage Information Share and the Hasbrouck information share bounds where zero implies that the bounds overlap The y-axis shows to what extent the permanent component share lies outside Hasbrouck's bounds.

There is a clear positive correlation between the two measures' disagreement with the Hasbrouck information share. The days where we see the largest difference between Hasbrouck's bounds and the permanent component share, the $TAIS$ also shows its largest difference. This suggests that these days remain difficult to evaluate.

In summary, all three measures clearly show that BMF dominates price discovery on almost all days. The Hasbrouck procedure shows large spreads between its bound on a quarter of the days with most of the volatility coming from movements in the lower bound. The upper bound remains close to 100% for most of the time which is in line with the results for the Toxic Arbitrage Information Share. The permanent component share is highly correlated with the bounds of the Hasbrouck information share, however, in a fifth of the cases the measures disagree with the CS being outside the latter's bounds. These are also the cases where I find the largest disagreement between the $TAIS$ and the Hasbrouck procedure.

When comparing these results with the simulations in Section 3.5, we find the following. The permanent component share has a tendency to show that the price discovery is more balanced than it actually is when involving low liquidity markets. At the same time, Hasbrouck only provides an upper and a lower bound which can provide a general idea of the dynamics. However, under low liquidity conditions, the bounds have a tendency to widen extremely. Additionally, in the case of one market dominating price discovery, we find that the bounds are not always reliable even though the resulting error is usually not extreme. Under such circumstances, the $TAIS$ is a useful complement in the analysis of price discovery dynamics.

Apart from the biases and imprecisions illustrated in the simulations, part of the disagreement of the measures may be due to their focus. The Toxic Arbitrage Information Share is based on extreme events while the standard procedures are designed to accommodate the whole range of shocks. If the distribution of shocks is not well approximated by a normal distribution, the results are likely to differ. In that sense, the $TAIS$ may provide a more relevant measurement for traders and regulators focusing on large price movements. However, this would not help explain the discrepancy between the two VECM based measures.

3.8 Conclusion

This paper introduces a model to analyze the relationship of information arrival and toxic arbitrage opportunities in a two market setting. Using this model, I derive a measure of information share which is based on the occurrence of toxic arbitrage opportunities and corrects for distortions due to differences in the informational structure, the spreads, and transaction fees in both markets. In order to test the properties of the toxic arbitrage based information share, I run a set of simulations as well as using a unique data set of US dollar/Brazilian real futures traded in Chicago and São Paulo. Both allow me to evaluate the performance of the Toxic Arbitrage Information Share in a market setting with low liquidity in one of the markets, namely CME.

As the fragmentation of financial markets continues, low liquidity markets are likely to play a more important role in price discovery. The widely used price discovery procedures by Hasbrouck (1995) and Gonzalo and Granger (1995) are not designed for such market environments potentially leading to biased results and spurious conclusions. By not relying on the standard assumptions of VECM based approaches, the Toxic Arbitrage Information Share ($TAIS$) is less affected by price inertia and partially empty limit order books. The simulations as well as application show that the $TAIS$ is a valuable complement to the standard procedures for the analysis of price discovery involving low liquidity markets.

The median estimate for the Toxic Arbitrage Information Share is close to the standard price discovery shares. The difference is mostly driven by downward spikes in the latter. Generally, the conclusion from the information shares are in line with the opinions of market participants in BMF and CME.

In summary, my results suggest that the Toxic Arbitrage Information Share is a good addition to the analysis of information shares and price discovery especially when involving low liquidity markets. Being derived in complete separation of standard approaches, the Toxic Arbitrage Information Share avoids most of the problems of VAR based information shares.

3.9 Appendix

3.9.1 Approximation of the standard deviation

In order to correctly approximate the standard deviation of the price information in market j, I need to take into account that the spreads are not constant over time. This makes it necessary to be more explicit about the nature of S_j.

I denote by $S_{j,pre}$ the spread in market j just before an arbitrage opportunity is initiated in the other market. Hence, $S_{B,pre}$ is the spread in market B just before information arrives in market C. This spread in market j together with the transaction fees determine the threshold whether information arrival in the other market leads to a toxic arbitrage opportunity. In order to determine the underlying distribution of price impacts from the observed toxic arbitrage opportunities, the different thresholds need to be taken into account. For each $S_{i,B} \in S_{B,pre}$, I am numerically approximating[19] the value of $\sigma_{i,C}$, the standard deviation of the price impacts from information arriving in market C, following

$$mean(n_C^+ | S_{B,pre} = S_{i,B}) = \frac{\sigma_{i,C}\phi\left(\frac{\underline{n}_{i,C}^*}{\sigma_{i,C}}\right)}{\left(1 - \Phi\left(\frac{\underline{n}_{i,C}^*}{\sigma_{i,C}}\right)\right)}, \tag{3.61}$$

where $\underline{n}_{i,C}^*$ is the threshold for information arriving in market C to cause a toxic arbitrage opportunity given a spread of $S_{i,B}$ in market B. All of the resulting estimates $\sigma_{i,C}$ are approximations of the same σ_C of the underlying distribution. In order to take all of these into account, I calculate the weighted average of the standard deviations

$$\sigma_C = \sum \frac{w_i}{\sum w_i} \sigma_{i,C}. \tag{3.62}$$

The weights w_i are given by the number of C/B toxic arbitrage opportunities at each

[19] I am following the procedure by Soetaert and Herman (2009).

$S_{i,B}$. Following the same procedure, I am estimating σ_B. Using these estimates for the standard deviations, it is possible to estimate Γ as

$$\Gamma = \frac{\sigma_C}{\sigma_B}. \tag{3.63}$$

Γ is constant as long as the informational structure of the markets (σ_C and σ_B) does not change. Given the standard deviations of the price impact of information, the next step is to calculate Ψ given by

$$\Psi = \left(\frac{Prob\left(n_C^+ > \underline{n}_C^*\right)}{Prob\left(n_B^+ > \underline{n}_B^*\right)} \right). \tag{3.64}$$

Ψ depends on the thresholds $\underline{n}_C^*$ and $\underline{n}_B^*$ and therefore changes with the spreads in both markets. Let us denote by Ψ_t the value of Ψ at a specific point in time t. The spreads at this time are given by $\sigma_{t,C}$ and $\sigma_{t,B}$. For each t, I can calculate Ψ_t as given by

$$\Psi_t = \frac{1 - \Phi\left(\frac{\underline{n}_{t,C}^*}{\sigma_C}\right)}{1 - \Phi\left(\frac{\underline{n}_{t,B}^*}{\sigma_B}\right)}, \tag{3.65}$$

where $\underline{n}_{t,C}^*$ and $\underline{n}_{t,B}^*$ are the thresholds given spreads $S_{t,C}$ and $S_{t,B}$, respectively.[20] I then use the average of the Ψ_t as an estimate for the Ψ

$$\Psi = \frac{1}{T} \sum_{t=1}^{T} \Psi_t. \tag{3.66}$$

[20] Alternatively, one can take the average of $Prob\left(n_C^+ > \underline{n}_{t,C}^*\right)$ and $Prob\left(n_B^+ > \underline{n}_{t,B}^*\right)$ separately, before dividing one by the other. The results differ due to Jensen's inequality, however, the difference is negligible.

3.9.2 Theory based simulations

The following simulations incorporate the idea of the arbitrageur as a channel to trade in other markets. The data is generated in a four period sequence with

$$t \in \{..., 1, 1\frac{1}{4}, 1\frac{2}{4}, 1\frac{3}{4}, 2, 2\frac{1}{4}, ...\}. \tag{3.67}$$

Whenever $t \in \mathbb{Z}$, where $\mathbb{Z}$ is the set of integers, the markets are in equilibrium.

In what follows, I denote $\mathbb{T}_s$ as s periods after the equilibrium period. Hence $\mathbb{T}_0 = \mathbb{Z}$ and $\mathbb{T}_s = \mathbb{Z} + \frac{s}{4}$ for $s \in \{1, 2, 3\}$.

In the first period after an equilibrium, i.e. $t \in \mathbb{T}_1$, the informed trader acts according to the innovation to the efficient price $\hat{u}_t$. In periods $t \in \mathbb{T}_2$, the arbitrageur acts if there is an arbitrage opportunity. In periods $t \in \mathbb{T}_3$ the market makers learn about $\hat{u}_t$ if the informed trader arrived in that market. In periods $\mathbb{T}_0$, the other market maker learns about $\hat{u}_t$ and adjust their prices. At this point both markets are back to equilibrium.

The efficient price m_t evolves according to

$$m_t = m_{t-1} + \hat{u}_t \tag{3.68}$$

$$\hat{u}_t = \mathbb{1}_{t \in \mathbb{T}_1} u_t. \tag{3.69}$$

where u_t are price innovations following a discretised normal distribution $u_t \sim \mathcal{N}(0, \sigma_u^2)$. In order to take into account that prices can only change in multiples of the ticksize, I am discretising all random variables in the simulation by rounding them to 2 digits. This is equivalent to saying that the ticksize is 0.01. [21] The dummy $\mathbb{1}_{t \in \mathbb{T}_1}$ adjusts for

[21] The discretisation is important for this exercise in order to ensure that there is a limited number of spread sizes, which in turn is necessary for the calculation of the Toxic Arbitrage Information Share.

the fact that there is only a change in the efficient price in period t if $t \in \mathbb{T}_1$. Hence,

$$\mathbb{1}_{t \in \mathbb{T}_1} = \begin{cases} 1 & \text{for} \quad t \in \mathbb{T}_1 \\ 0 & \text{for} \quad t \notin \mathbb{T}_1 \end{cases} . \tag{3.70}$$

The best bid and ask in market j in equilibrium is given by

$$a_{j,t} = m_{t-1} + \frac{S_j}{2} \tag{3.71}$$

$$b_{j,t} = m_{t-1} - \frac{S_j}{2} \tag{3.72}$$

where S_j is the spread in market j. The true information share is given p_B, which is the probability that the informed trader in market B observes the price innovation $\hat{u}_t$ at time t. Whenever the informed trader in market B does not observe the innovation, the informed trader in C does. Given that he observes the innovation, following the logic of the model in Section 3.3, the informed trader j submits a market order iff $\hat{u}_t > \frac{S_j}{2}$. Iff $\hat{u}_t > \frac{S_k}{2}$ where $k \neq j$, the informed trader will use a limit order. As in the model, the informed trader can use both orders if both conditions are fulfilled. In the following, I describe the price dynamics for a negative price innovation, i.e. $\hat{u}_t < 0$, observed by the informed trader in market B. The dynamics for a positive innovation are analogous.

To illustrate the sequence of events, let $t = 0$ denote the equlibrium state, i.e. $t \in \mathbb{T}_0$. Consequently, the efficient price change arrives at $t = 1/4$. The baseline scenario is that the price innovation is too small to be profitable for the informed trader i.e. that $\hat{u}_{1/4} \leq \min\left(\frac{S_B}{2}, \frac{S_C}{2}\right)$. In this case, the best bid and offer (BBO) remain unchanged for two periods until at $t = 3/4$ the market maker in B observes the new efficient price and adjusts his BBO to the new equilibrium level

$$a_{B,3/4} = m_{1/4} + \frac{S_B}{2} \tag{3.73}$$
$$b_{B,3/4} = m_{1/4} - \frac{S_B}{2}. \tag{3.74}$$

Market maker C adjusts her prices in the new equilibrium period i.e. $t = 1$

$$a_{C,1} = m_{1/4} + \frac{S_C}{2} \tag{3.75}$$
$$b_{C,1} = m_{1/4} - \frac{S_C}{2}. \tag{3.76}$$

Letting market maker in B observe the price innovation a period before market maker C makes it easier for the standard procedures to measure p_b. If both adjust their prices together in the baseline scenario, the result for the $TAIS$ will not change.

If $\hat{u}_{1/4} > \frac{S_B}{2}$, the informed trader submits a sell market order, bringing the best bid in market B to the new equilibrium

$$b_{B,1/4} = m_{1/4} - \frac{S_B}{2}. \tag{3.77}$$

Apart from this change, the dynamics are the same as in the baseline scenario.

In line with the model, the informed trader submits a sell limit order, if $\hat{u}_{1/4} > \frac{S_C}{2}$. In order to create a profitable arbitrage opportunity, the limit order needs to be at a price $a_{B,1} < b_{C,0}$. Given the assumed tick size of 0.01 in both exchanges, the new best offer in market B is hence

$$a_{B,1} = b_{C,0} - 0.01. \tag{3.78}$$

As this opens a profitable arbitrage opportunity, the arbitrageur will trade in both markets leading to an adjustment in market C

$$b_{C,2} = a_{B,1}. \tag{3.79}$$

The price in market B does not adjust as the informed trader has no incentive to post a limit order with only a limited quantity.

If the informed trader uses both limit and market orders, the dynamics outlined above a complementary. The simulations are run for different true information shares p_b each with 2,500 price innovation events resulting in 10,000 observations ($t \in [1:2,500]$). The spread for market C is twice the spread for market B ($S_B = 1$,$S_B = 2$) and $\sigma_u = 1$

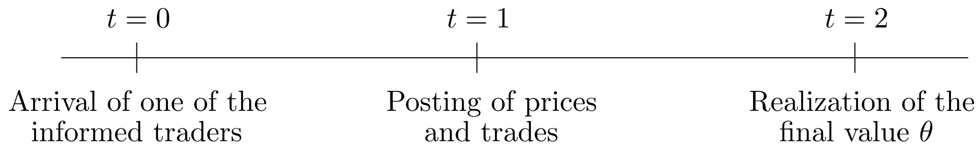


Figure 3.1: Time line

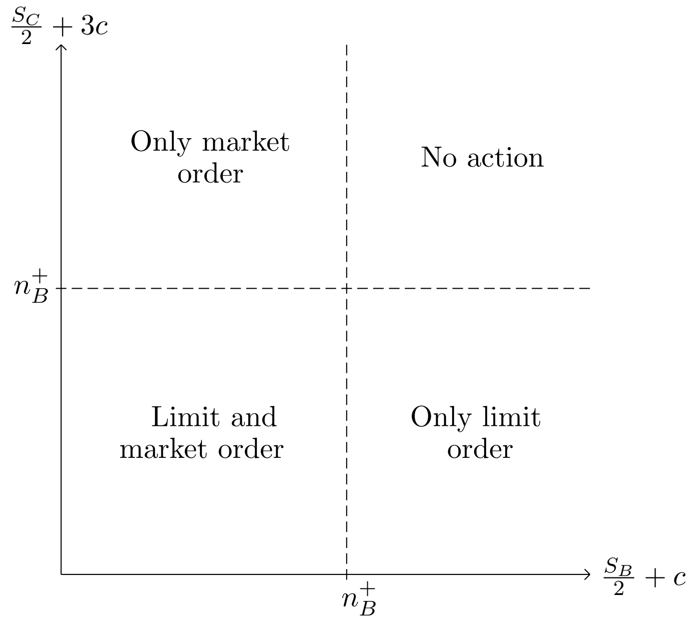


Figure 3.2: Choice of informed trader B
The figure illustrates the choice of the informed trader to use market and limit orders conditional on the size of transaction fees c and spreads in both markets.

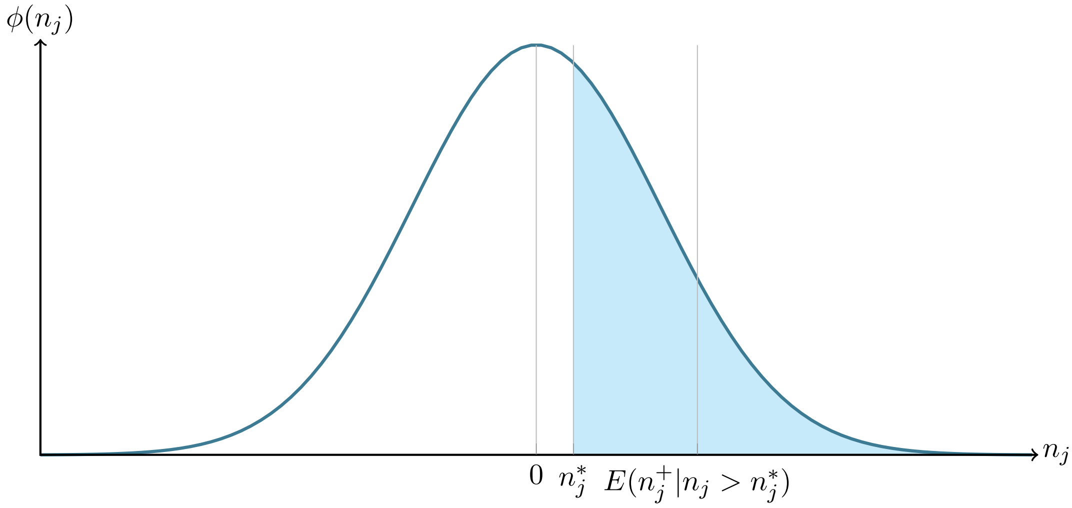


Figure 3.3: Truncated normal distribution
The graph shows the expected value $E(n_j^+|n_j > n_j^*)$ of a truncated normal distribution with a truncation at n_j^*.

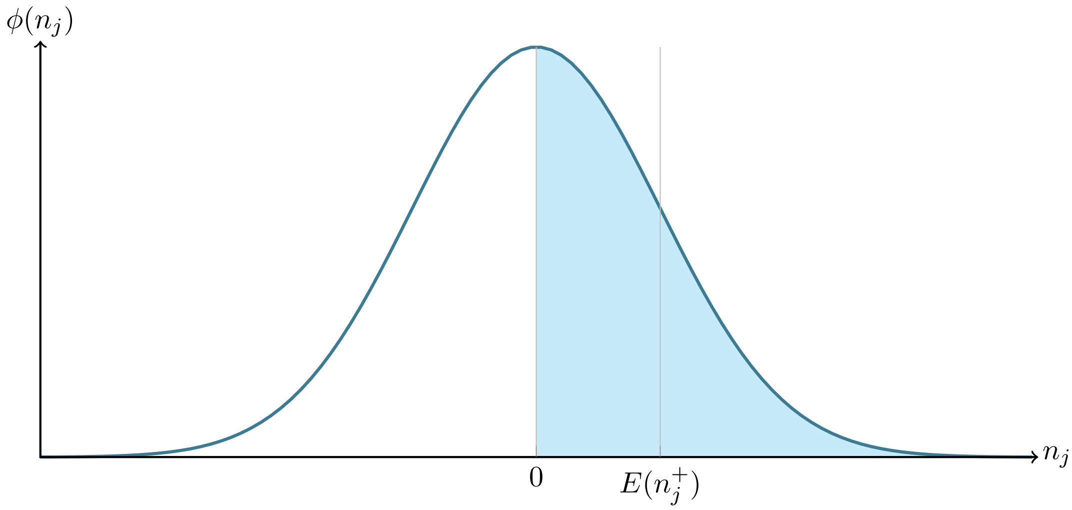


Figure 3.4: Normal distribution truncated at zero
The graph shows the expected value $E(n_j^+)$ of a truncated normal distribution with a truncation at zero.

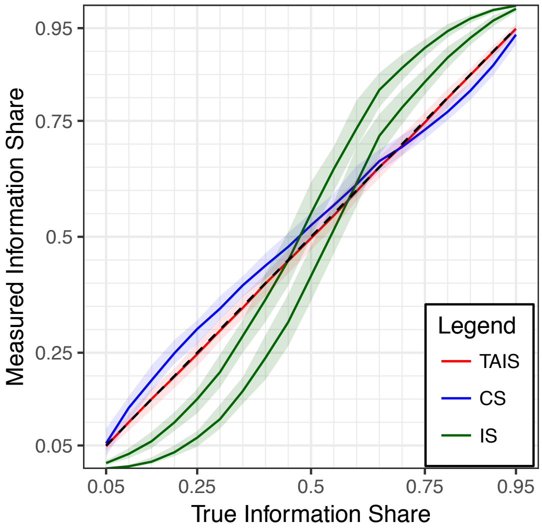
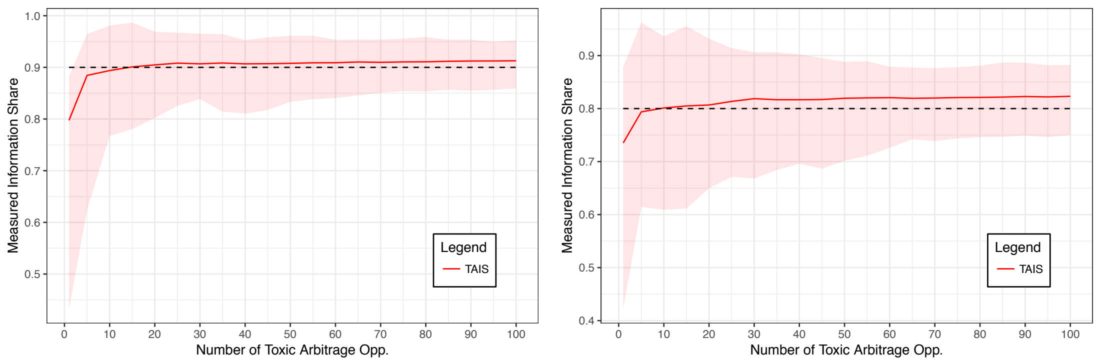


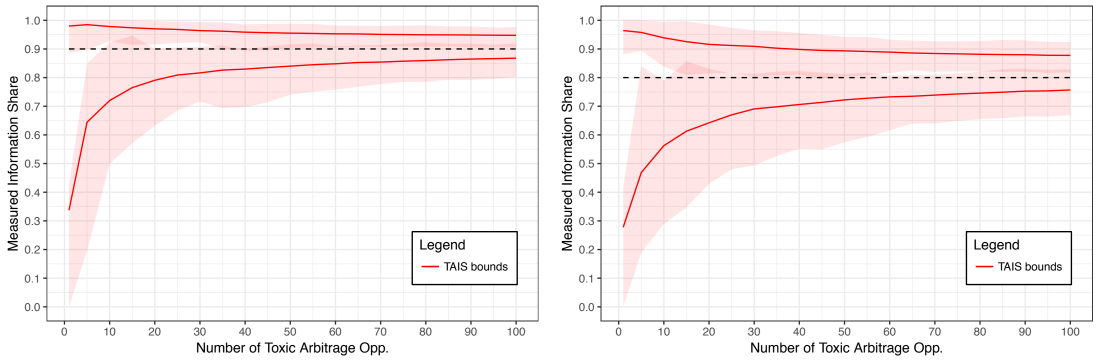
Figure 3.5: Model-based simulation
The red line is given by the estimate of the $TAIS$ while the blue line is given by the mid-price based estimates of the permanent component share. The green lines are given by the upper and lower bounds of the Hasbrouck information share. The respective shaded area engulfing 90% of the estimates.



(a) Information Share: 0.9

(b) Information Share: 0.8

Figure 3.6: Precision of model-based simulation
The figures provide the estimates of the $TAIS$ for different numbers of toxic arbitrage opportunities. 95% of the estimated $TAIS$ lie within the red shaded area. The left image is based on a information share of 0.9, while the right image is based on an information share of 0.8.



(a) Information Share: 0.9 (b) Information Share: 0.8

Figure 3.7: Model-based simulation: $TAIS$ bounds
The figures provide the estimates of the upper and lower bounds of the $TAIS$ for different numbers of toxic arbitrage opportunities. 95% of the estimated $TAIS$ lie within the red shaded area. The left image is based on a information share of 0.9, while the right image is based on an information share of 0.8.

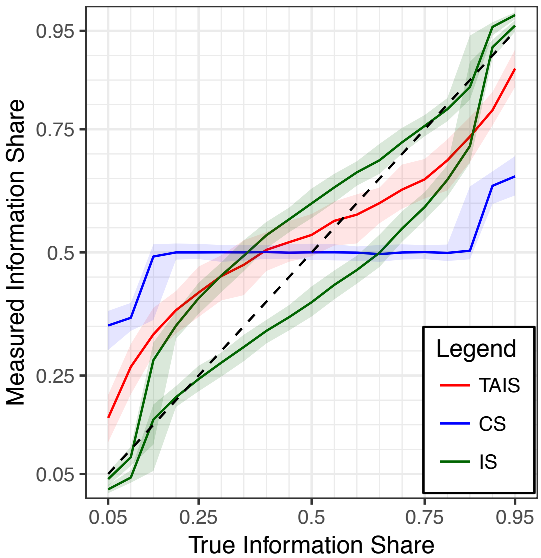


Figure 3.8: Simulation II
The red line is given by the estimate of the $TAIS$ while the blue line is given by the mid-price based estimates of the permanent component share. The green lines are given by the upper and lower bounds of the Hasbrouck information share. The respective shaded area engulfing 90% of the estimates.

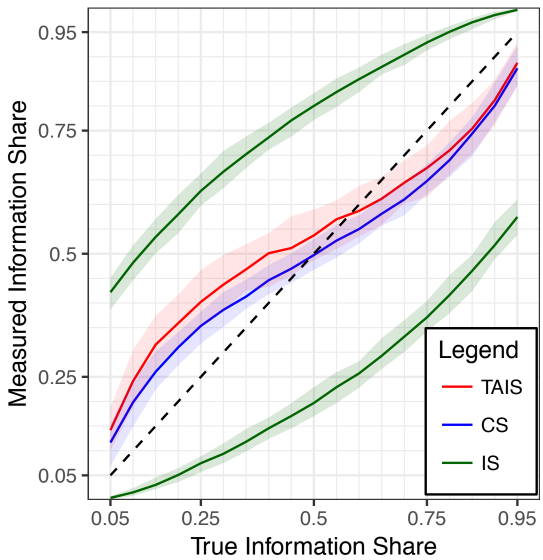


Figure 3.9: Simulation III
The red line is given by the estimate of the $TAIS$ while the blue line is given by the mid-price based estimates of the permanent component share. The green lines are given by the upper and lower bounds of the Hasbrouck information share. The respective shaded area engulfing 90% of the estimates.

	CME	**BMF**
Location	Chicago (USA)	São Paulo (Brazil)
End of trading	last business day of month	last business day of month
Contract size	100,000 BRL	50,000 USD
Quotation	USD per BRL	BRL per USD1,000.00
Tick size	USD 0.05 per BRL1,000.00	BRL 0.5 per USD1,000.00
Transaction fees	in USD	in USD
Trading hours	5pm-4pm CT	9am-6pm BRT

Table 3.1: Summary statistics of BRL-USD futures market
CT describes Chicagos time zone while BRT is the Brazilian time zone. The difference between the two is between two and four hours as both are affected by daylight savings time.

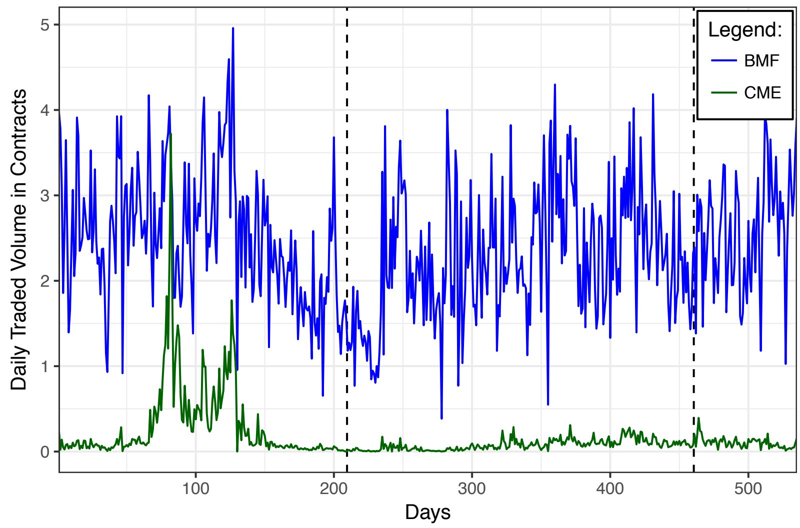


Figure 3.10: Daily traded volume in BMF and CME
Average trading volume per day in CME and BMF in USD 1 billion. The daily volume for CME includes trades outside of BMF trading hours. Only days with trades in both markets are included. The last active day of each month is excluded. The figure is based on the contract expiring in the following month.

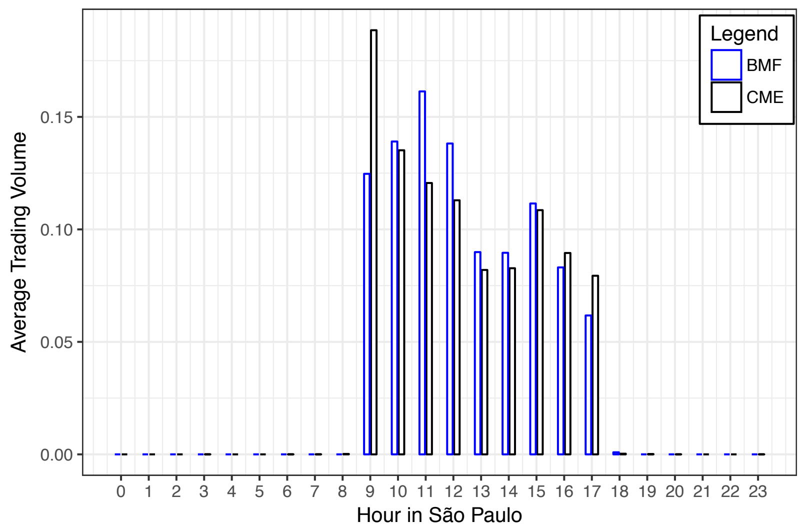


Figure 3.11: Hourly traded volume in BMF and CME in percent
The graph is based on the contract expiring in the following month. The blue (green) bars illustrates the share of trading volume per hour in BMF (CME). Only days with trades in both markets are included. The last active day of each month is excluded.

	Seconds	Percent
Total	9,817,200.00	
Positive spread	8,662,015.50	88.23%
$\mathbf{S_C > S_B}$	8,646,173.87	99.82%
$\mathbf{S_C < S_B}$	15,841.63	0.18%

Table 3.2: Spreads
The table shows the total time both markets are open in the sample as well as the time with positive spreads in both markets. The third (fourth) row shows the share of time when CME has a wider (narrower) spread than BMF.

	Total	Identifiable	Type 1 (Sell)	Type 2 (Buy)
Number	14,268	14,170	9,250	4,920
Total time (sec)	105,495.65	104,690.27	74,188.97	30,501.30
Total time	29:18:15.65	29:04:50.27	20:36:28.97	8:28:21.30
% of pos. spread	1.22%	1.21%	0.86%	0.35%

Table 3.3: Total arbitrage opportunities
The table shows the number of arbitrage opportunities, the total time arbitrage opportunities exist, and the percentage of time arbitrage opportunities exist relative to the total time with positive spread. 98 arbitrage opportunities are not identifiable. Type 1 (2) arbitrage opportunities would be taken advantage of by selling (buying) in both markets.

	Number	Percentage
CME/CME	3,304	23.3%
CME/BMF	1,011	7.1%
BMF/CME	5,976	42.2%
BMF/BMF	3,879	27.4%

Table 3.4: Arbitrage classification
The table shows the number and share of arbitrage opportunities by type. CME/CME are non-toxic arbitrage opportunities which are initiated and closed by price moves in CME.

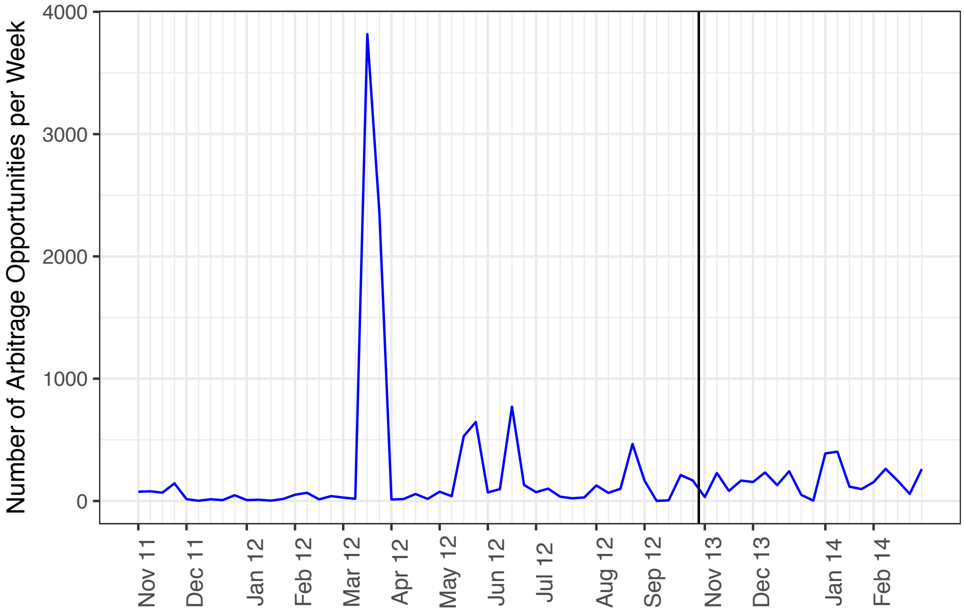


Figure 3.12: Weekly number of arbitrage opportunities
The figure shows the total number of arbitrage opportunities on a weekly bases. The black vertical line marks the data gap between October 2012 and October 2013.

	Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
Full LOB CME	0	31,640	32,190	30,880	32,380	32,400
Full LOB BMF	7.66	32,350	32,350	31,340	32,360	32,400
Mean Spread CME	0.0012	0.0019	0.0027	0.0055	0.0040	0.2002
Mean Spread BMF	0.0005	0.0005	0.0005	0.0006	0.0006	0.0021
Contr. Traded CME	0	4,966	9,876	26,110	23,690	372,200
Contr. Traded BMF	13,520	195,200	254,900	250,200	301,800	496,000
Arb. Duration	0.01	11.32	112.60	423.80	303.50	20,010.00
Number of Arbs.	1	5	20	57.37	44.50	1,457.00
CME/BMF	0.00	0.00	0.00	4.09	1.00	297.00
BMF/CME	0.00	2.00	5.00	24.19	15.00	728.00
CME/CME	0.00	0.00	1.00	13.38	4.50	644.00
BMF/BMF	0.00	2.00	8.00	15.70	19.00	206.00

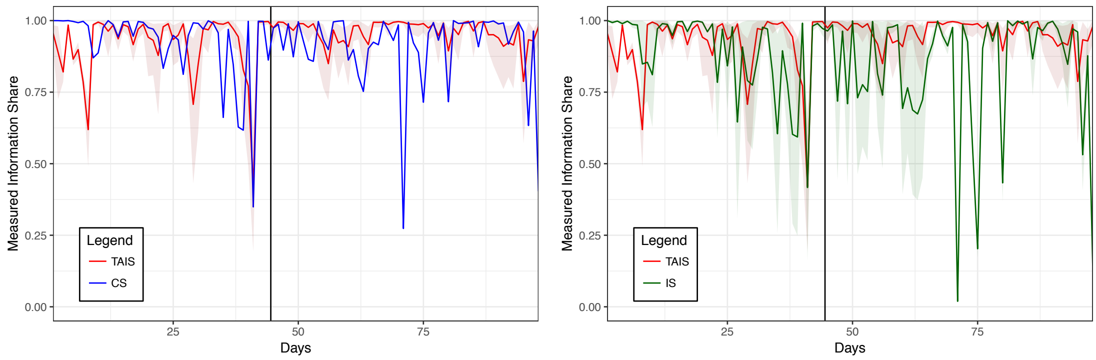
Table 3.5: Daily summary statistics
The table displays summary statistics for days with at least one arbitrage opportunity. Full LOB CME and Full LOB BMF is the number of seconds in a day with a full limit order book in CME and BMF, respectively. A full limit order book means, that there are both best bid and ask prices available. Mean Spread is the mean value of the spread in each market. The CME spread is the implicit spread given the transformation of CME prices. Contr. Traded is the number of traded contracts in each market. Arb. Duration is the total duration of arbitrage opportunities in a day in seconds.

	Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
Tox	0.1600	0.8648	0.9444	0.8944	1.0000	1.0000
Bias	-0.6118	-0.0631	-0.0186	-0.0469	0.0058	0.1389
$\mathbf{TAIS}_{.025}$	0.1926	0.8172	0.9149	0.8709	0.9610	0.9921
TAIS	0.4217	0.9292	0.9728	0.9413	0.9889	0.9975
$\mathbf{TAIS}_{.975}$	0.6290	0.9847	0.9971	0.9785	1.0000	1.0000
Spread	0.0071	0.0361	0.0688	0.1076	0.1509	0.5068

Table 3.6: Unbiased information share based on toxic arbitrage (#tox$\geq$ 10)
The table shows the summary statistics for several variables. Tox is given by the percentage of toxic arbitrage opportunities initiated in BMF, Bias describes the difference between Tox and the $TAIS$ measure given by the mean value of the confidence interval. Rows three to five provide the lower bound, mean, and upper bound of the $TAIS$ 95% confidence interval. The sixth row shows the summary statistics of the spread between $TAIS_{.975}$ and $TAIS_{.025}$.

	Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
TAIS	0.4217	0.9292	0.9728	0.9413	0.9889	0.9975
CS	0.2736	0.8996	0.9698	0.9168	0.9945	1.0000
$\textbf{IS}_{mid}$	0.0192	0.7858	0.9607	0.8612	0.9876	1.0000
$\textbf{IS}_{low}$	0.0057	0.5919	0.9278	0.7773	0.9772	1.0000
$\textbf{IS}_{up}$	0.0328	0.9815	0.9985	0.9485	1.0000	1.0000
$\textbf{IS}_{spread}$	0.0000	0.0187	0.0581	0.1713	0.3038	0.6984

Table 3.7: Comparison of information shares (#tox$\geq$ 10)
The table shows the summary statistics of the Toxic Arbitrage Information Share ($TAIS$), permanent component share (CS), and Hasbrouck information share (IS) for days with at least 10 toxic arbitrage opportunities. For the Hasbrouck procedure, the table provides the upper and lower bound as well as the spread between these. CS and IS are based on secondly frequency.



(a) Permanent component share

(b) Hasbrouck information share

Figure 3.13: Price discovery over time
The red and blue lines provides the $TAIS$ and permanent component share estimate, respectively, per day with at least ten toxic arbitrage opportunities. The red shaded area shows the 95% confidence bands for the $TAIS$. The green line is given by the mid point of the Hasbrouck information share bounds with the shaded area being the area between the bounds. The black vertical line marks the data gap between October 2012 and October 2013.

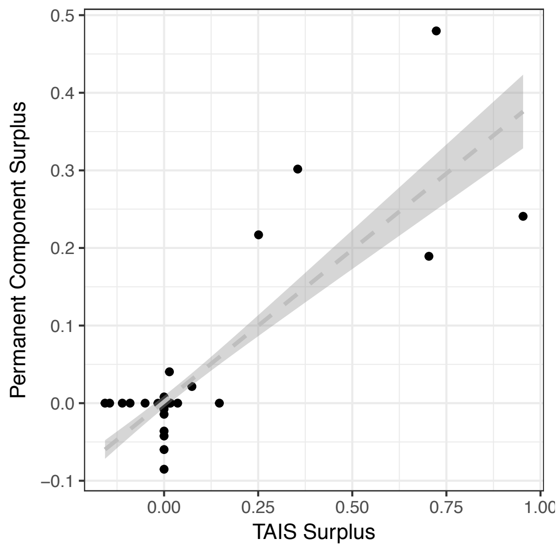


Figure 3.14: Out of bound observations
The x-axis is given by the distance between the bounds of the Toxic Arbitrage Information Share and the Hasbrouck information share bounds where zero implies that the bounds overlap The y-axis shows to what extent the permanent component share lies outside Hasbrouck's bounds.

Chapter 4

FX Exposure and Foreign Ownership

4.1 Introduction

Exchange rates are a central feature in international finance, not only as currencies are an asset in their own right but also due to their interaction with all other asset classes. While reasons for the importance of exchange rates are abound, the empirical literature finds, more often than not, that exchange rate risk plays a minor role in financial markets. Regardless of whether firms import or export, their returns are generally expected to be linked to exchange rates. However, individual stocks' exchange rate exposure is found to be surprisingly low.[1] Similarly, currency risk seems to play a minor role in explaining the equity home bias. This is the first paper to illustrate the importance of exchange rate risk and indirect currency hedging for within country differences in investment decisions. The resulting differences in foreign and domestic portfolios can help explain some of the conflicting results on the importance of exchange rate risk especially in the home bias literature.

Domestic investors are exposed to changes in the exchange rate via the returns from domestic stocks. If a stock is positively correlated with the exchange rate, as one would expect from an importing firm, the investor benefits from an appreciation in the exchange rate.[2] In addition to this indirect exposure, foreign investors are subject to currency risk, as they have to convert realized returns into their reference currency. This is the classical currency risk. The combination of both exposures leads to a quadratic effect which, even under frictionless hedging, leads to differences in the optimal portfolios of domestic and foreign investors.

Using a simple theoretical framework, I illustrate how domestic and foreign investors are differently affected by stocks' FX exposure. While the effect can be reduced using simple exchange rate hedging, it never fully disappears. Given optimal hedging, foreign investors invest more into stocks with positive FX exposure compared to domes-

[1]See e.g. Bartram and Bodnar (2007).

[2]The exchange rate is defined as units of foreign currency per domestic currency.

tic investors. In contrast, if hedging is limited due to frictions, foreign investors prefer negative FX exposure to implicitly hedge currency risk. Furthermore, when there are costs or frictions to hedging, the effect of FX exposure on the optimal portfolio can be substantial. In a second step, I use a set of simulations to test if foreign investors would want to adjust their portfolio weights with regard to stocks' FX exposure when facing strong frictions to hedging. Foreign investors overweight stocks with negative FX correlation compared to domestic investors.[3] In line with the theory, this is optimal as such stocks provide an implicit hedge against exchange rate risk. In a final step, I test the hypotheses derived from the model and simulations with a data set of 21 developing and developed countries. I find that within country differences in foreign ownership are partly explained by stocks' FX exposure and that the relationship is in line with the predictions.

Foreign ownership is generally associated with negative FX correlation, in line with the idea of implicit hedging. This effect holds true for developed countries throughout the sample period. In case of emerging markets, the negative link can be observed for the pre-crisis period, however, it is not present after the financial crisis. This indicates additional frictions in the post-crisis environment. In the overall sample, the dynamics are not only driven by implicit hedging, but also by a desire by foreign investors to limit FX correlation in either direction. The results help explain why currency risk is found to be more important for explaining the bond than the equity home bias. In addition, the results in this paper cast doubt on the usefulness of the standard home bias measure as used in the literature. The FX correlation further promises better results in international capital asset pricing models.

The results in this paper are based on the cross section of foreign ownership in stocks within a country by controlling for both time and country fixed effects. Any alternative explanation would have to explain why foreign ownership within a country

[3]Such stocks are more likely to be exporting than importing firms.

differs at a specific point in time. The results are hence unlikely to be driven by shifts in international capital flows or differences in how countries restrict foreign investors. In the presence of such restrictions, the results presented here are expected to be a lower bound for the effect. All regressions include a series of firm level controls to make sure that the reported effects are indeed due to firms' exchange rate exposure rather than other firm characteristics.

Home bias has received a lot of attention over past decades. In most studies home and foreign bias are used synonymously, as over-investment in domestic assets implies aggregate under-investment in foreign assets.[4] Most of the literature focuses on equity home bias, beginning with a range of studies of individual countries and later shifting to comparisons across countries and investment flows. The latter was substantially facilitated by the Coordinated Portfolio Investment Survey (CPIS) conducted by the IMF. A smaller, more recent literature extends the analysis to the bond home bias (e.g. Maggiori et al. (2018), Fidora et al. (2007)). The macro finance literature has looked at foreign investments for different countries and has identified several drivers of aggregate home bias. The majority sought to find rational and behavioral drivers ranging from transaction costs (Glassman and Riddick (2001)) to corporate governance (Dahlquist et al. (2003)) to information barriers and lack of familiarity (Ahearne et al. (2004), Portes and Rey (2005)).[5] More recent studies, such as Levy and Levy (2014), question the benefits of international diversification and conclude that the home bias is the result of optimal behavior. Given the increasing integration of international financial markets and the reduction in informational frictions, one would expect the home bias to fade over time. However, Levy and Levy (2014) do not find any indication that the home bias is decreasing, arguing that the increasing correlation of asset returns over time is a crucial factor limiting the benefits from international diversification.

[4]A notable exception is Bekaert and Wang (2009), which focuses on the degree of foreign bias by a given country to different countries.

[5]While having a macro perspective, Dahlquist et al. (2003) is a notable exception in the use of firm data on a wide panel of countries.

An additional factor regularly investigated as a driver of the home bias is exchange rate risk. The compelling reasoning is that exchange rate risk is a key difference between the positions of foreign and domestic investors and hence provides a potential explanation. Thapa and Poshakwale (2012) analyze the role of country-specific equity market characteristics in explaining foreign investment across countries. They find that exchange rate volatility has no significant effect. In the bond market, Maggiori et al. (2018) find that the currency in which a bond is denominated almost fully explains the bond home bias in their data. Similarly, Fidora et al. (2007) find that real exchange rate volatility helps explain home bias and that the effect is substantially larger for bonds than equity.

Due to the availability of the Coordinated Portfolio Investment Survey, most studies focus on country-level aggregate measures of home bias. In line with this, the theoretical literature often considers national indices (e.g. Glassman and Riddick (1996)) when determining potential drivers. They implicitly assume that it is optimal for foreign and domestic investors to hold the same portfolio of assets within a country and that they only differ in the overall weight of each country's portfolio. This is based on results such as Sercu (1980) who assumes that currency risk can be perfectly hedged. More recently, Boermans and Vermeulen (2016) find empirically that currency denomination matters for explaining home bias by different groups of investors for both equities and bonds.

On the asset level, there is a large literature investigating the connection between asset returns and exchange rate movements. Apart from a range of international asset pricing models using exchange rates or exchange rate volatility, a strand of the literature starting with Adler and Dumas (1984) investigates the co-movement of each asset's returns with currency returns. The resulting exchange rate exposure puzzle describes the surprising observation that a much lower number of firms see significant FX exposure than previously expected. Bartram and Bodnar (2007) provide an extensive survey of the literature, showing that different approaches help to reduce the discrepancy between expectations and observations. However, a substantial gap still remains. Rather than to explain this puzzle, this paper uses the concept of FX exposure introduced by this literature to shed light on how the home bias in stocks within a country is affected by exchange rate movements. Starting with Jorion (1990), the FX exposure literature measures FX exposure in excess of market returns. As this paper focuses on the investors' rather than the firms' perspective, a broader definition is more appropriate. Therefore, I depart from the FX exposure literature by using the FX correlation of a stocks' return as the total FX exposure.

Hau and Rey (2006) argue that while the exchange rate may influence asset prices, the reverse should also be true. Gains in a country's equity market relative to another country lead to a higher exposure to the country's risk for investors, incentivizing international investors to rebalance their portfolio away from it. Uncovered equity parity (UEP) hence states that excess capital gains in a country are followed by a devaluation of the country's currency. Both Hau and Rey (2006) and Curcuru et al. (2014) find evidence in line with the resulting negative correlation of index returns and currency value, however the latter argue that this is not due to risk management but rather due to investors rebalancing toward more lucrative investments. In addition, Cenedese et al. (2016) look at the cross section of international equity returns and document systematic violations of the uncovered equity parity. This paper is related to this literature as the focus is once more on the co-movement of asset and currency returns. While focusing on the cross section of individual assets rather than the index level, this paper does not depend on any assumptions on the validity of UEP. The results presented describe differences between stocks in the same country and hence a rebalancing away from the country specific risk does not affect their validity. However, insights from this analysis may help explain why the literature struggles to come to a conclusion on the validity of UEP as foreign and domestic investors hold different portfolios and hence have different risk loadings.

This paper contributes to the existing literature in two ways. First, this is the first paper to investigate the effect of a macroeconomic driver, namely the exchange rate, on within country differences in foreign ownership. I show that FX exposure matters in determining foreign ownership between firms in a given country. Second, as FX exposure differs across firms, the (optimal) portfolio of a foreign investor differs from a domestic investor's portfolio. This is a violation of the assumption of equal optimal portfolios which is the basis for current measures of home and foreign bias. The departure from the global portfolio assumption has far reaching consequences for asset pricing, financial stability, and the use of foreign currency denominated debt.

The rest of this paper is structured as follows. Section 4.2 introduces the theoretical model highlighting the key dynamics to consider. In Section 4.3, I use a set of simulations to look at the dynamics in a more complex setting and derive the testable implications described in Section 4.4. Section 4.5 provides information on the data used and its properties while Section 4.6 shows the empirical results. Section 4.7 concludes the paper.

4.2 Theory

In this model, there are a domestic (d) and a foreign (f) investor who make a portfolio choice at time $t = 0$ and optimize their returns at $t = 1$. The exchange rate at time $t = 0$ is 1 and $S \sim N(1, \sigma_s^2)$ at $t = 2$, implying that $(S - 1)$ is the relative change in the exchange rate. It is defined as foreign currency per unit of domestic currency. In both the domestic and the foreign market, there exists a risk free asset with return r_d and r_f, respectively. Furthermore, there are two risky assets in the domestic market. The return of asset 1 is $r_1 \sim N(\mu_1, \sigma_1^2)$ and uncorrelated with the exchange rate. The return of asset 2 is given by $r_2 = \hat{r}_2 + a(S - 1)$, where $\hat{r}_2 \sim N(\mu_2, \hat{\sigma}_2^2)$ is uncorrelated with either r_1 or S. It follows for asset 2 that $r_2 \sim N(\mu_2, \sigma_2^2)$ with $\sigma_2^2 = \hat{\sigma}_2^2 + a^2\sigma_S^2$. The expected return of each asset is assumed to be positive. The parameter a determines the correlation coefficient ρ of the return of asset 2 with the exchange rate as

$$\begin{aligned}\rho &= \frac{cov[r_2, S]}{sd[S]sd[r_2]} \\ &= \frac{a\sigma_S}{\sqrt{(\hat{\sigma}_2^2 + a^2\sigma_S^2)}}.\end{aligned} \tag{4.1}$$

A positive (negative) ρ is associated with a firm profiting from an appreciation (depreciation) in the domestic currency as would be expected from an importing (exporting) firm. As discussed by Bodnar et al. (2002) as well as Bartram et al. (2010), many firms reduce their FX exposure by hedging, using foreign currency debt, or pass-though of FX exposure to costumers. Hence, the interpretation with regard to importing and exporting firms is not absolute.

It follows that we can express the volatility of asset 2 as

$$\sigma_2^2 = \hat{\sigma}_2^2 \frac{1}{1-\rho^2}. \tag{4.2}$$

In summary, the two risky assets are uncorrelated with each other and only asset 2 is correlated with the exchange rate with correlation coefficient $\rho \in (-1, 1)$.

Both the domestic and the foreign investor choose their portfolio in order to maximize their utility following a basic mean-variance optimization with regard to the excess portfolio return R_j in local currency with $j \in [d, f]$

$$U_j\left[w_{j,1}, w_{j,2}, w_{j,S}\right] = E\left[R_j(w_{j,1}, w_{j,2}, w_{j,S})\right] - \frac{1}{2}\delta Var\left[R_j(w_{j,1}, w_{j,2}, w_{j,S})\right], \tag{4.3}$$

where $w_{j,1}$ and $w_{j,2}$ are the weights of investor j on assets 1 and 2, respectively. The hedging demand $w_{j,S}$ denotes investment by investor j in the non-local risk-free asset with return r_k where $k \in [d, f]$ and $k \neq j$. The risk aversion parameter is given by $\delta > 0$ and is assumed to be equal for both investors. The optimization with regard to local currency returns and the correlation of an asset with the exchange rate are the crucial differences to standard asset pricing models[6].

First, let us consider the return for the domestic investor. The details of the optimization are provided in the appendix. The domestic investor's local currency return is given by

$$R_d = w_1(1 + r_1) + w_2(1 + r_2) + w_3\frac{1}{S}(1 + r_f) + (1 - w_1 - w_2 - w_3)(1 + r_d) - 1. \quad (4.4)$$

As I am focusing on the effect of asset-currency correlation, I assume the risk-free rate in both markets to be zero. This eliminates carry trade strategies from the models and leaves investments into the risk-free asset to be purely for hedging purposes. It follows

$$R_d = w_1 r_1 + w_2 r_2 + w_3 \left(\frac{1}{S} - 1 \right). \quad (4.5)$$

At this point, I face the common problem that $\frac{1}{S}$ does not follow a normal distribution. However, as standard deviations of exchange rates are generally small, the difference between $\frac{1}{S} - 1$ and $(1 - S)$ is sufficiently small to be ignored[7]. Taking into account the definition of r_2, it follows

$$R_d = w_1(r_1) + w_2(\hat{r}_2) + (w_2 a - w_3)(S - 1). \quad (4.6)$$

[6] One argument for using local currency returns is that wealth and fund managers generally provide information about nominal returns to their clients. They are hence incentivized to optimize with regard to local currency returns.

[7] Put differently, w_3 can be seen as a derivative of the exchange rate S rather than the weight on buying the foreign risk free asset.

The definition of the random variables r_1, $\hat{r}_2$, and S lead to

$$E\left[R_d\right] = w_1\mu_1 + w_2\mu_2 \tag{4.7}$$

$$Var\left[R_d\right] = w_1^2\sigma_1^2 + w_2^2(\hat{\sigma}_2^2 + a^2\sigma_S^2) + w_3^2\sigma_S^2 - 2w_2w_3a\sigma_S^2. \tag{4.8}$$

Setting the first derivative of the domestic investor's return with regard to w_3 to zero leads to

$$w_3 = w_2 a. \tag{4.9}$$

The domestic investor uses the currency hedge in order to eliminate the risk resulting from the correlation between asset 2 and the exchange rate. The weights of asset 1 and 2 follow as

$$w_1 = \frac{1}{\delta}\frac{\mu_1}{\sigma_1^2} \tag{4.10}$$

$$\begin{aligned} w_2 &= \frac{1}{\delta}\frac{\mu_2}{\hat{\sigma}_2^2} \\ &= \frac{1}{\delta}\frac{\mu_2}{\sigma_2^2(1-\rho^2)}. \end{aligned} \tag{4.11}$$

The domestic investor's weights are not affected by ρ as he is able to hedge the correlation between asset 2 and the exchange rate completely. This results the weights for both assets to only depend on idiosyncratic risk despite the additional volatility of asset 2 for $|\rho| > 0$. As one would expect, the weight increases with the expected return of the asset and decreases in with the risk aversion parameter δ as well as the non-hedgeable risk of the asset.

In case of the foreign investor, the return is given by

$$R_f = w_1 S(1 + r_1) + w_2 S(1 + r_2) + w_3 S(1 + r_d) + (1 - w_1 - w_2 - w_3)(1 + r_f) - 1. \tag{4.12}$$

Setting the risk-free rates equal to zero results in

$$R_f = (w_1 + w_2 + w_3)(S - 1) + w_1 S r_1 + w_2 S(\hat{r}_2 + a(S - 1)). \tag{4.13}$$

The exchange rate affects the return of the foreign investor twice. As for the domestic investor, there is the currency risk due to the FX correlation of asset 2. Additionally, she faces currency risk when transferring the returns made in the domestic currency into her own currency. Details on the optimization are provided in the appendix. For the expected return and its variance, it follows

$$E[R_f] = w_1 \mu_1 + w_2(\mu_2 + a\sigma_S^2) \tag{4.14}$$

and

$$Var[R_f] = Var[(w_1 + w_2(1 - a) + w_3)S + w_1 S r_1 + w_2 S \hat{r}_2 + w_2 a S^2]. \tag{4.15}$$

The optimization leads to

$$w_3 = -w_1(1 + \mu_1) - w_2(1 + \mu_2 + a). \tag{4.16}$$

As shown in Equation (4.16), if $a = 0$, the foreign investor hedges the expected amount she needs to convert at $t = 2$. For $a \neq 0$ the additional hedged amount is equal to the total hedging amount by the domestic investor, i.e. the hedging of asset 2's FX correlation. Thereby, the foreign investor also chooses w_3 in order to eliminate the correlation between the portfolio and the exchange rate as

$$Cov[R_f, S] = (w_1(1+\mu_1) + w_2(1+\mu_2+a) + w_3)\sigma_S^2 \tag{4.17}$$
$$= 0. \tag{4.18}$$

Furthermore, the optimization leads to

$$w_1 = \frac{1}{\delta}\frac{\mu_1}{\sigma_1^2 + \sigma_1^2\sigma_S^2} \tag{4.19}$$
$$w_2 = \frac{1}{\delta}\frac{\mu_2 + \rho\sigma_2\sigma_S}{\sigma_2^2\left(1+\sigma_S^2 - \rho^2(1-\sigma_S^2)\right)}. \tag{4.20}$$

The weight of asset 1 decreases not only with the volatility σ_1 but also the volatility of the exchange rate σ_S. Interestingly, ρ does not affect the weight invested in asset 1. The dynamics in the weight on asset 2 is more complex. Everything else equal, given a fix absolute correlation coefficient $|\rho|$, the foreign investor prefers a positively correlated asset to a negative correlation.

Now, let us look at the share of foreign ownership in the two assets. I define eFO_2 as the excess foreign ownership share of asset 2 over asset 1 or

$$\begin{aligned} eFO_2 &= FO_2 - FO_1 \\ &= \frac{w_{f,2}}{w_{f,2}+w_{d,2}} - \frac{w_{f,1}}{w_{f,1}+w_{d,1}} \\ &= \frac{\rho\sigma_S\left((1-\rho^2)(1+\sigma_2^2)\sigma_2 - 2\rho\sigma_S\mu_2\right)}{\left(2+\sigma_S^2\right)\left((1-\rho)\rho^2\sigma_S\sigma_2 + 2(1-\rho^2)\mu_2 + (1+\rho^2)\mu_2\sigma_S^2\right)}, \end{aligned} \tag{4.21}$$

where FO_j is asset j's foreign ownership share.

The relationship between the correlation coefficient ρ and excess foreign ownership in that asset is non-linear. Figure 4.1a) illustrates this relationship. As indicated above, if $\rho = 0$ the excess foreign return is zero as both investors have the same relative weights. A negative correlation between the asset and the exchange rate implies a negative eFO_2, i.e. asset 2 has a lower foreign ownership share than asset 1. In case of a positive ρ, the excess foreign ownership can be either positive or, in very extreme cases, negative. Interestingly, the excess foreign ownership of asset 2 does not depend on the characteristics of asset 1. It is also worth noting that the magnitude of the effect is small. A change from $\rho = -.5$ to $\rho = .5$ implies 1% difference in excess foreign ownership. The driving force for the upward slope is the quadratic term $w2a(S^2 - S)$ resulting from asset 2's correlation with the exchange rate. Due to this, the investor is not able to fully hedge the currency risk as it is non-linear. Given $\rho > 0$ the investors profits more from appreciation in the exchange rate than she suffers from depreciation. For $\rho < 0$, appreciations lead to a higher loss than the gains during depreciations. This is in contrast to e.g. Sercu (1980) where all currency risk is fully hedged as assets are not correlated with the exchange rate.

4.2.1 Portfolio diversification without hedging

The model above illustrates the effect of foreign exchange exposure on differences in domestic and foreign portfolio weights. It allows for the use of foreign exchange positions in order to hedge the FX exposure. However, Levich et al. (1999) show that most institutional investors do not fully hedge their FX positions. In their survey, the authors find that over a third of institutional investors do not explicitly manage FX exposure at all. There are multiple reason why investors may not (fully) hedge their currency risk the most notable being transaction costs, capital requirements, and other market frictions. In order to take this into account, this extension of the model assumes that neither investor hedges their position, i.e. $w_3 = 0$. A more general solution for optimization with hedging costs is given in the appendix. The two cases presented here are the extremes nested in this general solution.

Without hedging, the optimal weights for the domestic investor are given by the discount factor multiplied by the asset's return over the variance of the asset, i.e.

$$w_1 = \frac{1}{\delta}\frac{\mu_1}{\sigma_1^2} \qquad \text{and} \qquad w_2 = \frac{1}{\delta}\frac{\mu_2}{\sigma_2^2}. \tag{4.22}$$

While the result for the weight on asset 1 is the same as in Equation (4.10) in the previous set-up, the weight on asset 2 is lower compared to Equation (4.11) as the investor is not able to hedge the correlation of the asset with the exchange rate.

The closed form solution for the foreign investor's weights without currency hedging is fairly complex. They solve the following equation system resulting from the optimization:

$$w_1 = \frac{\frac{1}{\delta}\mu_1 - w_2\sigma_S^2(1 + a + \mu_2 + \mu_1(1 + a + \mu_2))}{\sigma_1^2 + \sigma_S^2(\sigma_1^2 + (1 + \mu_1)^2)} \tag{4.23}$$

$$w_2 = \frac{\frac{1}{\delta}(\mu_2 + a\sigma_S^2) - w_1\sigma_S^2(1 + \mu_1 + a(1 + \mu_1) + \mu_2(1 + \mu 1))}{2a^2\sigma_S^4 + \sigma_2^2 + \sigma_S^2(1 + a^2 + \sigma_2^2 + 2\mu_2 + \mu_2^2 + 2a(1 + \mu_2))}. \tag{4.24}$$

The excess foreign ownership in this case is illustrated in Figure 4.1b). Surprisingly, the relationship between the correlation coefficient ρ and the excess foreign ownership eFO_2 is almost fully inverted compared to the case of full hedging in Figure 4.1a). It is worth noting, that the scale in the non-hedged case is a magnitude higher. This implies that even in a mixed scenario, this relationship is likely to dominate. The negative slope for intermediate ρ is due to the fact that it is optimal for the foreign investor to use an asset's FX correlation to implicitly hedge currency risk. As ρ gets closer to unity, a increases infinitely and drives up the volatility of the asset. Consequently, neither investor is willing to hold asset 2 and the excess foreign ownership converges to zero for extreme ρ. This leads to the convex shape for extremely positive and the concave shape for extremely negative ρ.

The model above focuses on nominal returns ignoring benefits from the diversification from country specific shocks. A foreign investor may prefer to invest into importing firms as the help diversify away from shocks to the foreign country by exposing her to shocks in the domestic country. Similarly, a domestic investor may prefer exposure to an exporting firm following the same reasoning. This would emphasize the slope in Figure 4.1a).[8] Eun et al. (2017) argue that the benefits from international diversification are higher than generally thought if investors hold stocks with a more "local" exposure. This implies that rather than focusing on either importers or exporters, investors benefit more from holding stocks which are uncorrelated with the exchange rate. This behavior is equivalent to having a bell shape in Figures 4.1a) and b).

The next section looks at a set of simulations in order to see to what extent these dynamics remain in a more complex setting of a large number of correlated assets.

4.3 Simulation

It is needless to say that the model above is a stark simplification of the problem investors face. The central reason for this basic set-up and the difference to common models on international diversification is the correlation between the exchange rate and asset returns. As an investor diversifies risk over a range of assets, the covariance matrix of the assets is crucial in determining the weights. As a consequence, a higher weight of a single asset due to FX correlation will not only affect the weight of this asset relative to the rest of the portfolio but all weights relative to each other. This creates complex knock-on affects in determining the differences in weights between domestic and foreign investors. A central question addressed in this section is whether this degree of complexity makes it impossible to determine the effect of FX exposure on individual asset weights. In order to see how the basic intuition of the results hold up in a more realistic set-up, this section provides a set of simulations. The main purpose of the simulations is twofold. First, the simulations illustrate the complexity of the problem which foreign investors

[8] As shown in the empirical results, there is little evidence for this to be the case.

face when considering even low exchange rate exposure of a set of assets. Secondly, the results provide a more sophisticated basis for testable hypotheses.

The simulations provide several insights into the dynamics of international diversification. First, for foreign investors using the optimal domestic portfolio can lead to a close to optimal Sharpe ratio with an optimal hedging position, however the hedging necessary to achieve this can be substantial. In the simulations, the hedging position makes up on average around 30% of the portfolio. Second, if currency risk is not hedged as is indicated in the empirical literature, adjusting the optimal domestic weights with regard to the FX exposure leads to an increase in the Sharpe ratio.

As basis for the simulations, I am using a set of 30 stocks. The local currency return of stock i in a month is given by

$$r_i = \beta_{m,i} r_m + \beta_{s,i} r_s + \epsilon_i, \tag{4.25}$$

where r_m is the market return and r_s is the exchange rate return. The returns are defined as

$$\begin{bmatrix} r_m \\ r_s \end{bmatrix} \sim N\left(\begin{bmatrix} \bar{r}_m \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_m^2 & \sigma_m \sigma_s \rho_m \\ \sigma_m \sigma_s \rho_m & \sigma_s^2 \end{bmatrix} \right), \tag{4.26}$$

$$\begin{bmatrix} \epsilon_i \\ \epsilon_j \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix} \right), \tag{4.27}$$

and

$$\begin{bmatrix} \beta_{m,i} \\ \beta_{s,i} \end{bmatrix} \sim N\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0.2 & 0 \\ 0 & 0.0025 \end{bmatrix} \right). \tag{4.28}$$

Following historic data for the German stock index DAX, I choose the remaining parameters to be $\bar{r}_m = .005$, $\sigma_m^2 = .004$, and $\sigma_s^2 = .014$. The exchange rate is based on the euro/US dollar rate. I denote this set of parameters as set A. Additionally, I use three further sets of parameters as shown in Table 4.1.

Let us consider the German example and think of the domestic investor as European and the foreign investor as American. The euro zone investor chooses weights for the euro returns while the US investor chooses weights to maximize the return in US dollar. The risk free rate is set to zero as all returns are excess returns. In a first step, I am comparing the optimal sharp ratios of a foreign and domestic mean-variance investor given the ability to hedge exchange rate risk freely. As in the previous section, the central difference for the two investors is the fact that exchange rate fluctuations affect the US investor both via the FX exposure of individual stocks and the exposure due to the conversion into US dollar.

Generally, the weights for US investors w_{US} are highly correlated with the weights for euro zone investors w_{EU} with a correlation coefficient consistently above .99, as one would expect. However, the absolute magnitude of the weights varies widely. Depending on the correlation of the market return with the exchange rate ρ_m, the weights differ on average by between 1.0 and 4.4 percentage points.[9] In comparison, the mean absolute weight of a US investor varies between 4.6 and 2.8 percent. This implies, that it is not optimal for US investors to invest into the same portfolio as domestic investors.

Perfect hedging by the US investor while using the euro zone investor's weights leads to the best expected results only inferior to the true optimal weights for the US investor with optimal hedging. Given the magnitude of the differences when investors do not hedge their positions as shown in the previous section and the empirical evidence on hedging behavior, it is reasonable to expect that a lack of hedging determines the dynamics. Let us assume that the US investor does not use his optimal weights but

[9] The estimates are based on $\rho_m \in [-.5, .5]$.

the euro zone investor's weights. I compare this with a set of strategies which change individual weights in the portfolio depending on each stock's FX correlation. The idea here is to see if changing the domestic portfolio taking into account the FX correlation can lead to a better Sharpe ratio for the US investor.

The adjustment in the weight is denoted by θ, such that the international investor's weights are given by

$$w_{US,j} = w_{EU,j} + \theta(\rho_j - \bar{\rho}), \tag{4.29}$$

where ρ_j is the FX correlation of asset j. The normalization using the mean FX correlation $\bar{\rho}$ is necessary to ensure the sum of all weights to be unity. For each simulation, the possible adjustment parameters are rank by their Sharpe ratio and the θ with the highest mean rank is considered the optimal θ^*.

The following results are based on a set of 200 simulations each with 10,000 observations. Figure 4.2 illustrates the results for the simulations. The x-axis shows the FX correlation of the market returns ρ_m while the y-axis shows the optimal adjustment coefficient θ^*. The dots show the results for the four sets of parameters A to D connected by smoothed lines. For almost all settings, the coefficient θ^* is negative. Hence, for a US investor investing more into negatively correlated stocks and less in positively correlated stocks leads to better results than using the euro zone investor's portfolio weights. This is in line with the results in the previous section of using assets' FX correlation for implicit hedging. The coefficient is only marginally positive for low negative market FX correlation ($\rho_m \in [-.3, -.15]$) when both market and exchange rate volatility is low. Whenever the exchange rate is more volatile, as in examples C and D, the coefficient is strongly negative. Hence, we would expect that international investors would take the FX correlation more into account for emerging market stocks, overweighting those with negative FX correlation. In order to illustrate the magnitude of this effect, consider a market with a zero FX correlation and a stock with an FX correlation of -.2. Compared to a domestic investor, an international investor would invest 1.6 percentage points more into this stock given the parameters of set A resembling the German stock market. Given sets C (D), the adjustment is around 11.6 (10.5) percentage points. Table 4.2 shows the results for multiple sets of simulations finding them to be stable. The next section provides a set of testable hypotheses based on the simulations as well as the theoretical model.

4.4 Hypotheses

The rest of the paper focuses on the testing of the following set of hypotheses. The first hypothesis follows the literature in assuming that domestic and foreign investors should hold the same portfolios. If investors do not take into account stocks' FX correlation, the portfolio weight would be unaffected by FX correlation as well. If individual investors do not differentiate stocks based on their FX correlation, the share of foreign ownership of the stocks would in turn also not be affected.

Hypothesis 1 *The FX correlation of stocks has no influence on the relative share of foreign ownership within a country.*

In contrast, the second hypothesis follows the logic outlined in the previous sections and the reasoning that a lack of hedging would dominate the dynamics. If investors take the FX correlation into account, a higher FX correlation is associated with a lower weight in foreign investors' portfolio compared to domestic investors' weights.

Hypothesis 2 *A higher FX correlation coincides with a lower share in stocks' foreign ownership.*

The simulation results illustrated in Figure 4.2 show that more volatile markets are predicted to see a stronger (negative) relationship between assets' FX exposure and their foreign ownership. Given that emerging markets are generally expected to be more volatile, this leads to the following hypothesis.

Hypothesis 3 *The link between FX correlation and foreign ownership is stronger in emerging compared to developed markets.*

In the model above, a crucial assumption is that each investor optimizes returns in their local currency. Given the volatility of purchasing power of many, especially developing countries' currencies, it is reasonable to expect that investors may rather optimize their position in terms of more stable currencies. Hence, the relationship between foreign ownership and FX correlation would be stronger when using a basket of reserve currencies rather than a GDP weighted basket.

Hypothesis 4 *A firm's foreign ownership share is more strongly linked to correlation of the firm's returns with a basket of reserve currencies than with a more general currency basket.*

Alternatively to the logic outlined above, foreign investors may simply avoid firms with high absolute FX correlation in order to avoid this exposure. This would also be in line with the logic proposed by Eun et al. (2017) that investors should prefer stocks with "local" rather than "global" exposure.

Hypothesis 5 *A firm's foreign ownership decreases with absolute FX correlation.*

The next section introduces the data set used to test these hypotheses.

4.5 Data

In order to test the hypotheses above, I use firm level data compiled from Datastream. The data set consists of 21 countries with observations from Jan 2001 to Dec 2014. Table 4.3 provides the descriptive statistics for each country. Financial firms and those with a stock price below the equivalent of USD 2 are excluded. The measure for foreign ownership used here is "NOSHFR" in Datastream, which is the percentage of total shares in issue held by institutions domiciled in countries other than that of the firm. While several firms have no foreign ownership, the dataset also includes a large number of firms with no information of foreign ownership. These are consequently also excluded in the analysis. The observations of the three most represented countries, USA, Japan, and Korea, make up 38.3%, 22.4%, and 9.8%, respectively, of the sample. In terms of market capitalization in the sample, the shares are 54.7%, 7.4%, and 2.7%, respectively. In comparison and according to WorldBank data for 2014, the shares of global market capitalization are 41.6%, 6.9%, and 1.9%. The most underrepresented country is China with only 1.29% market capitalization in the sample compared to a 9.5% share in the global market. The statistics are based on the observations for the firms from each country for the period between Jan 2006 and Dec 2014 as this period is ultimately used for the analysis. MV is the market valuation in US dollar and *Turn* is the monthly turnover in billion of local currency. The share of free floating shares in foreign ownership (FO) is shown as the average and standard deviation for the firms which have at least some foreign ownership. Finally, *share DO* provides the average share of firms in a given country which were held solely by domestic investors. From the descriptive statistics it is clear that the assumption of foreign investors holding the market portfolio does not hold. Apart from Hong Kong, for over half of the observations from each country, domestic investors hold all floating shares. For USA, Japan, China, and Turkey the share is over 90%. Even when looking at the firms with some foreign ownership, there is a large difference between firms in each country. The average foreign ownership in this sample is 4.5%. 79.3% of the firms have no foreign ownership. When excluding these firms, the average foreign ownership is 21.8%.

In this paper, I am using the term FX exposure in order to describe the contemporaneous correlation between the monthly returns of an exchange rate index and the returns of a firm in local currency. This definition comes from the theoretical model introduced above where this FX exposure is given by the correlation coefficient $\rho = cor\left[r_2, (S-1)\right]$. As e.g. Bartram and Bodnar (2007) show, the literature on the FX exposure puzzle focuses on a firm's exposure to exchange rate movements after taking into account the exposure to market risk. The classical exchange rate exposure $\beta_{i,S}$ is hence given by

$$r_{i,t} = \alpha_i + \beta_{i,M} r_{M,t} + \beta_{i,S} r_{S_T,t} + \epsilon_{i,t}, \tag{4.30}$$

where $r_{i,t}$ is the stock return of firm i at time t, $r_{M,t}$ is the market return, and $r_{S_T,t}$ is the return of a trade-weighted exchange rate basket. The trade-weighted basket is generally used as an approximation for the extent to which a firm is exposed to each currency. I depart from this in three important ways. First, I use a GDP weighted basket of 26 major currencies to approximate the wealth that can be invested by investors from each currency area. Even if country A does not trade much with country B, country B's investors still have the potential to invest in companies in country A. The second difference is that I am not correcting for market risk. In line with the theory, an investor cares about the total FX exposure of a stock rather than the FX exposure of a stock in excess of the market's FX exposure. Hence, a correction for market risk in this context is not only unnecessary but undesirable. Jorion (1990) considered both approaches but found that the ranked correlation of both measures is close to unity. The difference is consequently a matter of absolute size, rather than relative ranking. However, as I use the measure in a second step regression, the absolute size matters here. Finally, rather than running a linear regression of the local currency returns on the exchange rate basket return, I use the correlation coefficient as in the theoretical model. Given that

$$\begin{aligned} \beta &= \frac{cov\left[r_{i,t}, r_{S,t}\right]}{\sigma_S^2} \\ &= \rho \frac{\sigma_r}{\sigma_S}, \end{aligned} \tag{4.31}$$

it is straightforward to see that the regression coefficient β of a simple linear regression is increasing with the volatility of the stock σ_r. The correlation coefficient ρ in contrast is bound between $(-1, 1)$. The regression coefficient β provides the nominal explanatory power of the exchange rate, i.e. by how much does the return of the asset change per unit of exchange rate return. In contrast, the correlation coefficient ρ provides the relative explanatory power given by how much of the total return is explained by exchange rate movements. It is more closely related to the R^2 of a regression. Consequently, the size of the correlation coefficient ρ is more informative when looking at the cross section of stocks as a higher correlation coefficient implies a stronger connection between the exchange rate and the asset's return.

Table 4.4 provides the summary statistics for firms' FX correlation per country. The FX correlation is calculated with annualized monthly returns for five year rolling windows[10]. For most countries, the mean and median FX exposure is positive. USA and Hong Kong are the notable exceptions. South Africa sees the largest absolute mean exposure with .35. The spread between the 25 and 75 percentile is between .08 and .28 for each country with an average of .18. Hence, there is a wide variation in correlations for the observations in each country.

[10] Additional results with two year rolling windows lead to similar results.

4.6 Empirical analysis

The empirical part of the analysis focuses on the share of foreign ownership as dependent variable. Naturally, the share of foreign ownership $FO_{t,i}$ of firm i at time t is given by a bounded interval $FO_{t,i} \in [0, 1]$. In order to correct for this in the regression set-up and to avoid biases in the estimation, I am using the logistic transformation of this variable

$$lFO_{t,i} = log\left(\frac{\frac{FO_{t,i}(n-1)+0.5}{n}}{1 - \frac{FO_{t,i}(n-1)+0.5}{n}}\right), \tag{4.32}$$

where n is given by the total number of observations. Hypothesis 1 states that the FX exposure of a stock has no impact on the share of foreign ownership. I test this using the regression

$$lFO_{t,i} = \alpha_0 + \theta\rho_{t,i} + \alpha_X X_{t,i} + \epsilon_{t,i}, \tag{4.33}$$

where $\rho_{t,i}$ is the correlation coefficient between firm i's monthly stock return and returns in the exchange rate basket over the five years leading up to time t. The exchange rate basket is a GDP weighted basket of 26 mayor currencies. X_i is a set of control variables on the firm level. The control variables are given by log market valuation (MV), volatility of annualized monthly returns over the past five years (Vol), book-to-market ratio (BM), and log turnover in the current month (Turn). Column (1) of Table 4.9 shows the regression results for Equation (4.33) using the full sample. All regressions include time and country fixed effects as well as firm and time clustered standard errors. The reason for using country and time fixed effects is to eliminate country-wide fluctuations of FX ownership shares in a given market. These are likely to occur due to changes in the monetary policy of this or other countries. Because the focus of the analysis is on the effect of FX correlation on the differences between firms within a country at a given time, firm fixed effects are not used. Differences between firms are however used when clustering the standard error. For the overall regression, Hypothesis 1 that FX correlation has no impact on foreign ownership is strongly rejected. In line with Hypothesis 2, the coefficient for ρ is significantly negative.

Column (2) and (3) shows the regression results for developed and emerging markets, separately. Following the Columbia University EMGP definition of emerging markets, the latter are given by China, Taiwan, Korea, Hong Kong, Thailand, India, Poland, Turkey, South Africa, and Singapore. The developed markets are USA, Japan, Canada, Germany, UK, France, Italy, Belgium, Australia, Switzerland, and Denmark. The results for developed markets in Column (2) is in line with the results for Column (1). In contrast, the results for emerging markets show no significant relationship between FX correlation and foreign ownership. This is in contrast with Hypothesis 2 and the model's predictions. Hypothesis 3 states that emerging markets are expected to see a stronger negative coefficient than developed markets. Following the results in Table 4.9, this Hypothesis is strongly rejected.

Table 4.7 shows the results for each individual country. Despite the lower number of observations, the coefficient for FX correlation is significantly different from zero for ten out of 21 countries. 80% of the significant coefficients are negative in line with Hypothesis 2. Of the eleven developed markets, not a single country rejects Hypothesis 2. In summary, there is consistent evidence that FX correlation is associated with differences in foreign ownership within a country at least for developed markets. This evidence is contrary to the common assumption that foreign investors strive to hold the same portfolio as domestic investors.

Due to the logistic transformation, the coefficients of the regression are not straightforward to interpret. To provide an idea of the magnitude, consider the case of the mean US firm in this data set with mean log market valuation, volatility, book-to-market ratio, and log turnover. Due to the large number of zero foreign ownership firms and the large weight put on observations close to the bounds in a logistic transformation, the measured effect of a change in ρ for the mean firm is close to zero. In order to correct for that, it makes sense to focus on firms with at least some foreign ownership. Table 4.8 shows the results when only including observations with non-zero foreign ownership. In the case of the mean US firm in this subset, a change in ρ by -.18 from .08 to -.10 increases foreign ownership from 12.5% to 13.0%. The change from .08 to -.10 is the difference between the third and first quartile of FX correlation among these firms. In case of the mean Japanese firm, a change from .23 to .04 implies a change in foreign ownership from 8.9% to 9.3%. Put differently, a decrease in the FX correlation by .18 leads to a 3.7% higher investment position by foreign investors in case of the mean US firm and 4.5% in the mean Japanese firm. When looking at the median US and Japanese firm, the results are almost identical.

Departing from the assumption that each investor maximizes with regard to their local currency returns, it is possible that foreign investors especially in emerging markets maximize with regard to a basket of reserve currencies. Following this logic, Hypothesis 4 states that using FX correlation with regard to reserve currencies sees a stronger effect than when using a general currency basket. I am using the definition of the IMF's special drawing rights (SDR) valid from January 2006 to December 2010 as basket of reserve currencies. For the whole sample, SDR are pegged to US dollar, euro, British pound, and Japanese yen.[11] For consistency, I am using the same weights throughout the period fixed at 44%, 34%, 11%, and 11%, respectively. Columns (4)-(5) in Table 4.9 show the results. Similar to the previous regressions, the coefficient for FX correlation is significantly negative for the whole sample and the sub-sample of developed markets but insignificant for emerging markets. In line with Hypothesis 4, the coefficients in all three samples are more negative compared to Columns (1)-(3), however the difference is not significant. Similarly, the explanatory power of the regressions using the reserve

[11] In November 2015, the IMF decided to add the Chinese yuan to the currency basket coming into effect in October 2016. The weights for SDR between 2006 and 2011 are constant. The change in weights prior 2006 and after 2010 are small and do not affect the results.

currency basket is only marginally higher.

Contrary to the logic outlined in Section 4.2, foreign investors may avoid firms with high absolute FX correlation and prefer firms which either hedge their own exposure or are generally not exposed to FX movements. In order to test this explanation denoted as Hypothesis 5, Table 4.10 provides the results when including absolute FX correlation $|\rho|$. For the full sample in column (1), both the coefficient for the absolute term and the linear term are highly significant at -.408 and -.485, respectively, supporting a mix of both Hypothesis 2 and 5. For firms with negative FX correlation, the two coefficients cancel out and there is no significant relationship. For positively correlated firms, a higher FX correlation is associated with a lower foreign ownership share. The results for developed markets in column (2) are in line with the overall results. Column (3) shows the results for emerging markets with insignificant coefficients for ρ and $|\rho|$. Hence, there is some evidence to suggest that foreign investors both have a tendency to prefer stocks with negative FX correlation and at the same time limit exposure to FX correlation altogether. While the prior is in line with Hypothesis 2, the latter is in line with Hypothesis 5.

Regulators have reacted to the financial crisis by changing how international investors deal with risk including currency risk. While law makers in different countries acted differently, it raises the question if the collective of international investors have changed their behavior significantly. In order to assess this, the next set of regressions includes only observations before June 2007 and after December 2009 with the prior being pre- and the latter being post-crisis. Columns (4)-(6) show the results when including ρ_{post}, the FX correlation of a given firm in the post-crisis period. Hence, ρ captures the effect of pre-crisis FX correlation while ρ_{post} is given by the change between pre- and post-crisis. For all three, the overall sample as well as the subsets of developed and emerging markets, the pre-crisis coefficient is significantly negative, implying a preference by foreign investors for stocks with negative FX correlation in line with Hypothesis 2. The coefficient for ρ_{post} is insignificant for the overall sample and developed markets. In case of emerging markets, the negative coefficient for ρ is canceled out by a positive ρ_{post} for the post-crisis period. Hence, the overall coefficient for ρ in the post-crisis period is insignificant. The result for the pre-crisis period is not only in line with Hypothesis 2 for a positive ρ coefficient but also in line with Hypothesis 3, that the effect for emerging markets is significantly stronger. Hence, while the results for the pre-crisis period are very much in line with the idea that foreign investors prefer stocks with negative FX correlation, the behavior for emerging markets in the post-crisis period does not.

Consequently, it appears that either post-crisis regulation or the changed market environment has changed investors preference for negative FX correlation in emerging markets. There are at least two possible explanations for this. As post-crisis regulations made hedging more expensive, foreign investors may treat FX correlation differently compared to the pre-crisis period, however higher hedging costs would be expected to make implicit hedging more important. Alternatively, other factors may have become more important to foreign investors leading to FX correlation to become secondary and ultimately insignificant. Given that FX correlation is a natural component to be taken into account as it directly affects the return distribution, the decrease in its importance is an indication for frictions in international capital markets.

Due to the estimation of the correlation coefficient between the stock return and the exchange rate basket, each firm's FX correlation is fairly persistent over time. This is the case as they are calculated over a five-year rolling window. In order to show that the persistence in the correlation coefficients is not driving the results, Figure 4.3 illustrates the coefficient for the FX correlation when running cross sectional regressions per month using country fixed effects and firm level control variables. The grey band represents the 5% significance interval. The coefficient is negative and significantly different from zero for 59 of the 109 months. For the remaining months, the coefficient is not significantly different from zero. 98% of these months are between May 2006 and May 2010. To some extent this coincides with the financial crisis. Similarly, when only looking at developed markets as in Figure 4.4, 73 months see significantly negative coefficients, while for one month it is significantly positive. Hence, despite the persistence in the calculated correlation coefficients, the results do not appear to be driven by this. Figures 4.3 and 4.4 show that the relationship generally holds in the cross section, which is in line with Hypothesis 2.

As a final step, I further assess how the relationship between FX correlation and foreign ownership develops over time for a subset of countries. Figure 4.5 illustrates the development of the coefficients calculated over one year periods with monthly fixed effect for the three largest markets: USA, Japan, and China. A dashed line indicates that the coefficient is not significantly different from zero using a 10% significance level. The same control variables are used as before. As in the regressions above, the coefficients are overall negative. The US coefficient shows some significantly positive observations around the financial crisis. From 2010 onward, it is consistently significantly negative. The positive coefficient during the financial crisis implies that foreign investors had a preference for firms with positive FX correlation such as importers. There is a similar pattern for Japan, with a period of positive coefficients during the financial crisis followed by negative coefficients between 2011 and 2013, before turning positive in 2014. A possible explanation for this is that exposure to reserve currencies such as US dollar and Japanese yen was seen as more desirable during the financial crisis. Additionally, exposure to importing firms implies a stronger exposure to country specific shocks. Hence, foreign investors chose a higher exposure to US and Japan specific shocks after the beginning of the financial crisis. In case of China, the coefficient is initially highly negative before reaching a similar negative level as Japan and the US in 2011. The path that the coefficient for China depicts appears to track the establishment of the Chinese yuan as a major currency.

The analysis shows broad consistency in the negative results for the three countries at least after the crisis. The absolute size and change of the coefficients needs to be interpreted with caution, however, due to the logistic transformation.

4.6.1 Home bias measures

Under the assumption that domestic and foreign investors hold the same portfolio in each country, their overall optimal portfolio is generally considered identical. This is expressed in standard home bias measures as e.g. by Sercu and Vanpée (2007) given by

$$HomeBias = DomesticHoldings - \frac{HomeCapitalization}{WorldCapitalization}. \tag{4.34}$$

Domestic and foreign investors are expected to invest the same fraction of their portfolio, given by the size of the domestic market relative to world capitalization, into the domestic market.

However, as shown in the descriptive statistics, foreign investors avoid many firms altogether or are barred from investing in these firms. As highlighted in the empirical analysis, FX exposure is one driver for the differences in foreign ownership between firms in a given country. Therefore, home bias cannot merely be measured by differences in aggregated domestic and foreign holdings. A true measure needs to take into account the risk profile of foreign investors' holdings. This reasoning goes beyond the exposure to currency risk.

To illustrate this, consider the following example. As foreign investors put a higher weight on firms with negative FX correlation, they are more exposed to shocks to exporting firms than importing firms. Given that exporting firms do not provide much diversification away from shocks to the global economy, the benefit of investing in the domestic market is lower. Similar to Levy and Levy (2014), this raises questions about the benefits of international diversification for investors. However, in contrast to Levy and Levy (2014), the FX exposure of firms is a central driver of the dynamics and the results indicate that a more appropriate measure of home bias is needed. Such a measure needs to take into account the portfolio of foreign investors rather than the aggregate holdings. This is especially true when considering currency risk as a driver of home bias.

The measure of currency risk also needs to be revisited. The home bias literature finds conflicting results for the importance of currency risk. Thapa and Poshakwale (2012) find that currency risk plays only a minor role in explaining the home bias in equity markets. In contrast, Maggiori et al. (2018) conclude that currency differences are the main driver of the bond home bias going so far as to refer to it as home currency bias. The measurement of currency risk is likely to be a key reason for this result. Thapa and Poshakwale (2012) use real exchange rate volatility as their currency risk measure. Thereby, they only capture the general conversion risk and do not take into account the effect of indirect currency hedging via the FX correlation. This would lead to an overestimation (underestimation) of currency risk for assets which are negatively (positively) correlated with the exchange rate. Maggiori et al. (2018) profit from the fact that the bond market provides a more complete measure of currency risk as they use differences in currency denomination. By comparing the investment into local versus foreign currency denominated debt, all aspects of currency risk are captured. Hence, the dramatic difference in their conclusions may partly be due to an unprecise measurement of currency risk in equity markets rather than a difference in investor preferences.

4.7 Conclusion

This paper investigates the influence of individual stocks' exchange rate exposure on the difference in portfolio allocation by domestic and international investors measured by foreign ownership. It is the first paper to look at the effect of exchange rates on within country differences in foreign ownership. Thereby, the paper contributes among other to the macro finance literature on home bias by showing how foreign currency exposure can help explain the within country differences in foreign bias.

Using a basic theoretical framework, I find that the general assumption of foreign and domestic investors to hold each country's market portfolio is inappropriate when considering an assets' exchange rate exposure and a desire to optimize with respect to nominal returns. While marginal when allowing for perfect hedging, the effect becomes substantial when considering frictions such as transaction costs, capital requirements, or capital controls. The basic model provides intuition for the dynamics. In order to capture the full complexity of choosing optimal portfolio weights, I use a set of simulations for more realistic predictions. Under restricted hedging, foreign investors generally prefer higher weights on stocks with negative FX correlation compared to domestic investors' optimal weights.

I test these predictions using data for 21 countries and find that firm level FX exposure helps to explain within country foreign ownership differences in line with the simulations. Foreign investors generally prefer firms with negative FX correlation which helps to implicitly hedge currency risk. Hence, foreign investors prefer to hold shares of exporting firms rather than importing firms. In the pre-crisis period, this effect was significantly larger for emerging markets as one would expect. In the post-crisis period, however, there is no significant link between foreign ownership and FX correlation in emerging markets. For developed markets, I do not observe a reduction in the effect. Additionally, there is evidence that foreign investors prefer absolute FX correlation to be limited.

This paper provides the first evidence for the importance of exchange rate exposure in order to explain differences in foreign ownership within a country. Thereby, it opens the path for further analysis on the targets of international investment flows both within and between countries as well as the interaction of exchange rates and asset prices. The importance of FX exposure for international portfolio allocation raises doubts on current measures of home bias in international finance and may help explain the persistence of the home bias puzzle.

While the focus of this paper is on the connection between currency risk and within country differences in foreign ownership, it provides relevant insights for international asset pricing as well. It proposes the FX correlation as the driver of cross sectional differences in demand and hence prices rather than the use of currency beta. In addition, I argue for the use of FX correlation without controlling for market risk as the market return will be correlated with the exchange rate as well. While the market risk needs to be accounted for in asset pricing, the calculation of the currency factor should not be in excess of the market's FX correlation but rather include it.

4.8 Appendix

4.8.1 Parameters a and ρ

The parameters a and the correlation coefficient ρ are closely related

$$\begin{aligned}
\rho &= \frac{cov[r_2, S]}{sd[S]sd[r_2]} \\
&= \frac{a\sigma_S^2}{\sigma_S\sqrt{(\hat{\sigma}_2^2 + a^2\sigma_S^2)}} \\
&= \frac{a\sigma_S}{\sqrt{(\hat{\sigma}_2^2 + a^2\sigma_S^2)}} \\
\Rightarrow \qquad a &= \frac{\rho\hat{\sigma}_2}{\sigma_S\sqrt{(1-\rho^2)}}. \qquad (4.35)
\end{aligned}$$

It follows that we can express the volatility of asset 2 as

$$\sigma_2^2 = \hat{\sigma}_2^2\frac{1}{1-\rho^2}. \qquad (4.36)$$

4.8.2 Domestic investor with hedging

The return of the domestic investor is given by

$$\begin{aligned}
R_d &= w_1(1+r_1) + w_2(1+r_2) + w_3\frac{1}{S}(1+r_f) + (1-w_1-w_2-w_3)(1+r_d) - 1 \\
&= r_d + w_1(r_1 - r_d) + w_2(r_2 - r_d) + w_3\left(\frac{1}{S} - 1\right) + w_3\left(\frac{1}{S}r_f - r_d\right) \qquad (4.37)
\end{aligned}$$

As I am focusing on the effect of asset-currency correlation, I assume the risk-free rate in both markets to be zero. This eliminates carry trade strategies from the models and leaves investments into the risk-free asset to be purely for hedging purposes.

$$R_d = w_1r_1 + w_2r_2 + w_3\left(\frac{1}{S} - 1\right) \qquad (4.38)$$

At this point, I face the common problem that $\frac{1}{S}$ does not follow a normal distribution. However, as standard deviations of exchange rates are generally small, the difference between $\frac{1}{S}-1$ and $(1-S)$ is sufficiently small to be ignored. Put differently, w_3 can be seen as a derivative of the exchange rate S rather than the weight on buying the foreign risk free asset.

$$\begin{aligned} R_d &= w_1(r_1) + w_2(r_2) + w_3\left(1-S\right) \\ &= w_1(r_1) + w_2\left(\hat{r}_2 + a(S-1)\right) + w_3\left(1-S\right) \\ &= w_1(r_1) + w_2(\hat{r}_2) + (w_2 a - w_3)(S-1) \end{aligned} \tag{4.39}$$

The definition of the random variables r_1, r_2, and S lead to

$$E\left[R_d\right] = w_1\mu_1 + w_2\mu_2 \tag{4.40}$$

$$\begin{aligned} Var\left[R_d\right] =& w_1^2\sigma_1^2 + w_2^2\hat{\sigma}_2^2 + (w_2 a - w_3)^2\sigma_S^2 + 2w_1w_2Cov[r_1,\hat{r}_2] \\ &+ 2w_1(w_2a - w_3)Cov[r_1, S] + 2w_2(w_2a - w_3)Cov[\hat{r}_2, S] \\ =& w_1^2\sigma_1^2 + w_2^2\hat{\sigma}_2^2 + (w_2 a - w_3)^2\sigma_S^2 \\ =& w_1^2\sigma_1^2 + w_2^2(\hat{\sigma}_2^2 + a^2\sigma_S^2) + w_3^2\sigma_S^2 - 2w_2w_3a\sigma_S^2. \end{aligned} \tag{4.41}$$

For the first derivative of the domestic investor's optimization problem with respect to w_3 it follows that

$$\begin{aligned} \frac{\partial U}{\partial w_3} &= 0 \\ -\delta\left(w_3\sigma_S^2 - w_2a\sigma_S^2\right) &= 0 \\ w_3 &= w_2 a. \end{aligned} \tag{4.42}$$

The domestic investor uses the currency hedge in order to eliminate the risk resulting from the correlation between asset 2 and the exchange rate. Further, it follows that

$$\begin{aligned} \frac{\partial U}{\partial w_1} &= 0 \\ \mu_1 - \delta \left(w_1 \sigma_1^2 \right) &= 0 \\ w_1 &= \frac{1}{\delta} \frac{\mu_1}{\sigma_1^2} \end{aligned} \tag{4.43}$$

and

$$\begin{aligned} \frac{\partial U}{\partial w_2} &= 0 \\ \mu_2 - \delta \left(w_2 (\hat{\sigma}_2^2 + a^2 \sigma_S^2) - w_3 a \sigma_S^2 \right) &= 0 \\ \mu_2 - \delta \left(w_2 (\hat{\sigma}_2^2 + a^2 \sigma_S^2) - (w_2 a) a \sigma_S^2 \right) &= 0 \\ w_2 &= \frac{1}{\delta} \frac{\mu_2}{\hat{\sigma}_2^2}. \end{aligned} \tag{4.44}$$

4.8.3 Foreign investor with hedging

In case of the foreign investor, the return R_f is as follows. Again, I omit the index f of the weights for brevity:

$$\begin{aligned} R_f &= w_1 S(1 + r_1) + w_2 S(1 + r_2) + w_3 S(1 + r_d) + (1 - w_1 - w_2 - w_3)(1 + r_f) - 1 \\ &= r_f + (w_1 + w_2 + w_3)(S - 1) + w_1 S r_1 + w_2 S r_2 + w_3 S r_d - (w_1 + w_2 + w_3) r_f \\ &= r_f + (w_1 + w_2 + w_3)(S - 1)(1 + r_f) + w_1 S(r_1 - r_f) + w_2 S(r_2 - r_f) + w_3 S(r_d - r_f). \end{aligned} \tag{4.45}$$

Setting the risk-free rates equal to zero results in

$$
\begin{aligned}
R_f &= (w_1 + w_2 + w_3)(S - 1) + w_1 S r_1 + w_2 S r_2 \\
&= (w_1 + w_2 + w_3)(S - 1) + w_1 S r_1 + w_2 S(\hat{r}_2 + a(S - 1)). \qquad (4.46)
\end{aligned}
$$

The exchange rate affects the return of the foreign investor twice. As for the domestic investor, there is the exchange rate risk due to the FX correlation of asset 2. Additionally, she faces currency risk when transferring the returns made in the domestic currency into her own currency. For the expected return and its variance follows

$$
\begin{aligned}
E[R_f] &= w_1 E[S r_1] + w_2 E[S \hat{r}_2] + w_2 (a E[S^2] - a E[S]) \\
&= w_1 E[S] E[r_1] + w_2 E[S] E[\hat{r}_2] + w_2 (a E[S^2] - a E[S]) \\
&= w_1 \mu_1 + w_2 \mu_2 + w_2 (a(1 + \sigma_S^2) - a) \\
&= w_1 \mu_1 + w_2 (\mu_2 + a \sigma_S^2) \qquad (4.47)
\end{aligned}
$$

and

$$
\begin{aligned}
Var[R_f] &= Var[(w_1 + w_2 + w_3) S + w_1 S r_1 + w_2 S \hat{r}_2 + w_2 a S^2 - w_2 a S] \\
&= Var[(w_1 + w_2(1 - a) + w_3) S + w_1 S r_1 + w_2 S \hat{r}_2 + w_2 a S^2]. \qquad (4.48)
\end{aligned}
$$

For brevity, denote $\lambda = (w_1 + w_2(1-a) + w_3)$

$$
\begin{aligned}
Var[R_f] =&\lambda^2\sigma_S^2 + w_1^2 Var[Sr_1] + w_2^2 Var[S\hat{r}_2] + w_2^2 a^2 Var[S^2] \\
&+ 2\lambda w_1 Cov[S, Sr_1] + 2\lambda w_2 Cov[S, S\hat{r}_2] + 2\lambda w_2 a Cov[S, S^2] \\
&+ 2w_1 w_2 Cov[Sr_1, S\hat{r}_2] + 2w_1 w_2 a Cov[Sr_1, S^2] + 2w_2^2 a Cov[S\hat{r}_2, S^2] \\
=&\lambda^2\sigma_S^2 + w_1^2(E[S^2 r_1^2] - E[S]^2 E[r_1]^2) + w_2^2(E[S^2\hat{r}_2^2] - E[S]^2 E[\hat{r}_2]^2) \\
&+ w_2^2 a^2(E[S^4] - E[S^2]^2) + 2\lambda w_1(E[S^2 r_1] - E[S]E[Sr_1]) \\
&+ 2\lambda w_2(E[S^2\hat{r}_2] - E[S]E[S\hat{r}_2]) + 2\lambda w_2 a(E[S^3] - E[S^2]E[S]) \\
&+ 2w_1 w_2(E[S^2 r_1\hat{r}_2] - E[Sr_1]E[S\hat{r}_2]) + 2w_1 w_2 a(E[S^3 r_1] - E[Sr_1]E[S^2]) \\
&+ 2w_2^2 a(E[S^3\hat{r}_2] - E[S\hat{r}_2]E[S^2]).
\end{aligned} \tag{4.49}
$$

Given the definitions of the variables and their independence, we find that

$$
\begin{aligned}
E[S^2] &= (1+\sigma_S^2) & E[S^2 r_j r_k] &= E[S^2]\mu_j\mu_k \\
E[S^3] &= (1+3\sigma_S^2) & E[S^2 r_j] &= E[S^2]\mu_j \\
E[S^4] &= (1+6\sigma_S^2+3\sigma_S^4) & E[S^3 r_j] &= E[S^3]\mu_j \\
E[r_j^2] &= (\mu_j^2+\sigma_j^2) & E[S^2 r_j^2] &= E[S^2]E[r_j^2] \\
E[Sr_j] &= \mu_j. & &
\end{aligned}
$$

For the optimization it follows that

$$
\begin{aligned}
\frac{\partial U}{\partial w_3} &= 0 \\
0 &= -\delta\left(w_3\sigma_S^2 + w_1(\sigma_S^2 + \mu\sigma_S^2) + w_2(\sigma_S^2 + \mu\sigma_S^2 + a\sigma_S^2)\right) \\
w_3 &= -w_1(1+\mu_1) - w_2(1+\mu_2+a)
\end{aligned} \tag{4.50}
$$

and

$$
\begin{aligned}
& \frac{\partial U}{\partial w_1} = 0 \\
\Rightarrow \qquad & w_1 = \frac{1}{\delta} \frac{\mu_1}{\sigma_1^2 + \sigma_1^2 \sigma_S^2} && (4.51) \\
& \frac{\partial U}{\partial w_2} = 0 \\
\Rightarrow \qquad & w_2 = \frac{1}{\delta} \frac{\mu_2 + a\sigma_S^2}{\hat{\sigma}_2^2 + \hat{\sigma}_2^2 \sigma_S^2 + 2a^2 \sigma_S^4}. && (4.52)
\end{aligned}
$$

Using the definition of a, it follows that

$$
\begin{aligned}
w_2 = & \frac{1}{\delta} \frac{\mu_2 + \left(\frac{\rho \hat{\sigma}_2}{\sigma_S \sqrt{(1-\rho^2)}} \right) \sigma_S^2}{\hat{\sigma}_2^2 + \hat{\sigma}_2^2 \sigma_S^2 + 2 \left(\frac{\rho \hat{\sigma}_2}{\sigma_S \sqrt{(1-\rho^2)}} \right)^2 \sigma_S^4} \\
= & \frac{1}{\delta} \frac{(1-\rho^2)\mu_2 + \sqrt{(1-\rho^2)}\rho \hat{\sigma}_2 \sigma_S}{(1-\rho^2)\hat{\sigma}_2^2 + (1+\rho^2)\sigma_S^2 \hat{\sigma}_2^2}. \qquad (4.53)
\end{aligned}
$$

When formulating the weight on asset 2 in terms of its overall risk σ_2 as in Equation (4.36), it further follows that

$$
w_2 = \frac{1}{\delta} \frac{\mu_2 + \rho \sigma_2 \sigma_S}{\sigma_2^2 \left(1 + \sigma_S^2 - \rho^2 (1 - \sigma_S^2) \right)}. \qquad (4.54)
$$

4.8.4 Investor with hedging costs

Investors are likely to not fully hedge their positions due to several market frictions and regulatory requirements. In order to capture this, the following model imposes a cost of $\frac{c}{2}$ for buying the risk free asset from the country where the investor is not located. The cost increases quadratically in order to capture both the fact that put and call positions are both affected as well as that regulatory requirements are likely to matter more as positions get larger.

$$R_d = w_1(r_1) + w_2(\hat{r}_2) + (w_2 a - w_3)(S - 1) - w_3^2 \frac{c}{2} \tag{4.55}$$

The definition of the random variables r_1, r_2, and S lead to

$$\begin{aligned}
E\left[R_d\right] =& w_1\mu_1 + w_2\mu_2 - w_3^2\frac{c}{2} && (4.56)\\
Var\left[R_d\right] =& w_1^2\sigma_1^2 + w_2^2\hat{\sigma}_2^2 + (w_2 a - w_3)^2\sigma_S^2 + 2w_1w_2Cov[r_1, \hat{r}_2]\\
&+ 2w_1(w_2 a - w_3)Cov[r_1, S] + 2w_2(w_2 a - w_3)Cov[\hat{r}_2, S]\\
=& w_1^2\sigma_1^2 + w_2^2\hat{\sigma}_2^2 + (w_2 a - w_3)^2\sigma_S^2\\
=& w_1^2\sigma_1^2 + w_2^2(\hat{\sigma}_2^2 + a^2\sigma_S^2) + w_3^2\sigma_S^2 - 2w_2w_3a\sigma_S^2. && (4.57)
\end{aligned}$$

The expected return is reduced by the hedging costs, while the variance is not affected. For the first derivative of the domestic investor's optimization problem with respect to w_3 it follows that

$$\begin{aligned}
\frac{\partial U}{\partial w_3} &= 0\\
-w_3 c - \delta(w_3\sigma_S^2 - w_2 a\sigma_S^2) &= 0\\
w_3 &= w_2\frac{a}{\frac{c}{\delta\sigma_S^2} + 1}. && (4.58)
\end{aligned}$$

When hedging is free, the domestic investor uses the currency hedge in order to eliminate the risk resulting from the correlation between asset 2 and the exchange rate. As the costs c increase, the hedging position linearly decreases. The weight on asset 1 remains unchanged compared to the case of no hedging costs with

$$\begin{aligned}\frac{\partial U}{\partial w_1} &= 0\\ w_1 &= \frac{1}{\delta}\frac{\mu_1}{\sigma_1^2}.\end{aligned} \tag{4.59}$$

It also follows that

$$\begin{aligned}\frac{\partial U}{\partial w_2} &= 0\\ \mu_2 - \delta\left(w_2(\hat{\sigma}_2^2 + a^2\sigma_S^2) - w_3 a\sigma_S^2\right) &= 0\\ w_2 &= \frac{1}{\delta}\frac{\mu_2}{\hat{\sigma}_2^2 + a^2\sigma_S^2(1 - \frac{1}{\frac{c}{\delta\sigma_S^2}+1})}.\end{aligned} \tag{4.60}$$

With no hedging costs, the domestic investor only takes into account the idiosyncratic risk of asset 2 in order to determine its weight. As the cost of hedging increases, the domestic investor increasingly takes into account the hedgeable risk as well.

For the foreign investor, the optimization problem is equivalent. The expected return is reduced by $w_3^2\frac{c}{2}$ while the variance does not change. For the first derivative of the foreign investor's optimization problem with respect to w_3 it follows that

$$\begin{aligned}\frac{\partial U}{\partial w_3} &= 0\\ w_3 &= -\frac{w_1(1+\mu_1) + w_2(1+a+\mu_2)}{(\frac{c}{\delta\sigma_S^2}+1)}.\end{aligned} \tag{4.61}$$

When hedging is free, the foreign investor uses the currency hedge in order to eliminate the correlation between her return and the exchange rate. As the costs c increase, the hedging position linearly decreases. Further, it follows that

$$\begin{aligned}\frac{\partial U}{\partial w_1} &= 0 \\ w_1 &= \frac{\mu_1(1+\frac{c}{\sigma_S^2\delta}) - cw_2(1+a+\mu_2)(1+\mu_1)}{(1+\sigma_S^2)\sigma_1^2\delta + c(\frac{\sigma_1^2}{\sigma_S^2} + (\sigma_1^2 + (1+\mu_1)^2))}\end{aligned} \tag{4.62}$$

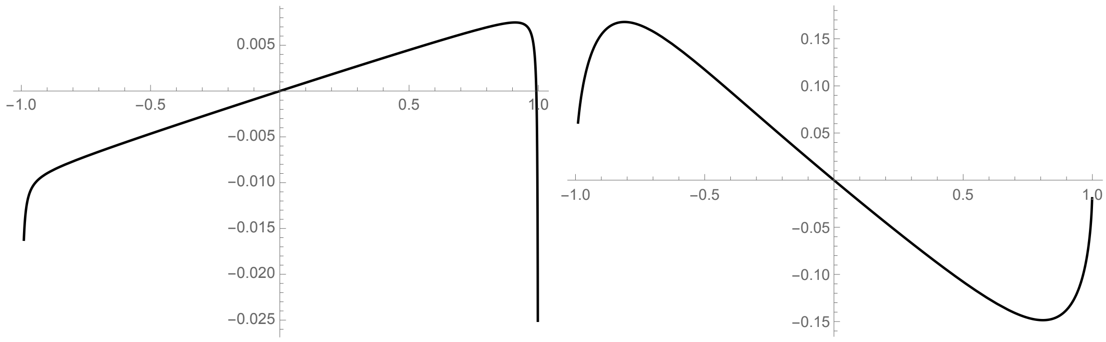
and

$$\begin{aligned}\frac{\partial U}{\partial w_2} &= 0 \\ \mu_2 - \delta\left(w_2(\hat{\sigma}_2^2 + a^2\sigma_S^2) - w_3 a\sigma_S^2\right) &= 0 \\ w_2 &= \frac{(\frac{\mu_2}{\delta} + w_3 a\sigma_S^2)}{(\hat{\sigma}_2^2 + a^2\sigma_S^2)}\end{aligned} \tag{4.63}$$

It becomes clear that the resulting terms, for each weight are highly complex.

Set	A	B	C	D
σ_m^2	.004	.004	.016	.016
σ_s^2	.014	.056	.014	.056

Table 4.1: Parameters
Four sets of simulation parameters. σ_m is the standard deviation of market return and σ_s is the standard deviation of the exchange rate. The parameters of set A are based on the German stock market.



(a) With hedging (b) Without hedging

Figure 4.1: Theory results
The x-axis is given by asset 2's FX correlation ρ. The y-axis shows excess foreign ownership eFO_2 as given in Equation (4.21) and as it would result from Equations (4.23) and (4.24).

ρ_m	θ_1^*	θ_2^*	θ_3^*	θ_4^*	θ_5^*
.3	-.205	-.20	-.20	-.195	-.185
0	-.07	-.055	-.075	-.075	-.065
-.3	.00	.00	.00	.00	-.01

Table 4.2: Four simulation results for parameter set A
The results are based on five sets of 200 simulations each, each in turn with 10,000 observations.

Country	# Firms	MV		Turn		FO		share DO
		avg	sd	avg	sd	avg	sd	
USA	2,293	5,295	21,116	0.95	3.9	1.58	7.2	0.91
Japan	1,484	1,220	6,466	0.001	0.006	1.21	4.5	0.90
France	301	3,495	12,022	0.37	1.5	5.79	15.4	0.74
UK	289	4,037	14,468	56.0	220	8.84	13.6	0.51
Germany	231	4,610	14,545	0.008	0.033	9.63	21.9	0.69
Australia	70	7,705	16,256	0.46	1.1	4.84	11.0	0.72
Switzerland	134	7,577	26,729	0.42	1.5	6.82	15.9	0.67
Italy	69	4,188	14,357	0.77	3.5	4.54	13.4	0.79
Denmark	74	2,516	7,595	0.025	0.073	4.82	12.0	0.76
Canada	5	90	63	0.15	0.28	7.19	9.4	0.51
Belgium	60	2,710	13.548	0.14	0.54	16.61	25.3	0.55
China	544	2,334	3,115	0.081	0.096	0.77	5.3	0.97
Korea	739	1,041	6,319	0.0001	0.0004	2.25	7.3	0.86
Taiwan	105	5,881	12,238	0.013	0.019	1.62	4.5	0.84
Hong Kong	56	18,518	44,138	0.076	0.18	26.19	27.2	0.37
Thailand	42	5,037	6,224	0.006	0.010	9.94	18.6	0.74
India	177	3,984	8,235	0.004	0.008	10.00	20.2	0.70
Turkey	34	963	1,585	0.044	0.084	2.60	12.9	0.95
South Africa	79	4,096	6,907	0.03	0.06	3.31	11.0	0.81
Poland	66	620	1,739	0.011	0.044	15.80	27.2	0.66
Singapore	17	6,699	9,951	0.17	0.26	12.62	22.9	0.57

Table 4.3: Descriptive statistics
This table provides the descriptive statistics of the firms in each country in this data set # Firms provides the number of firms, MV is given by the market valuation in USD, and Turn provides turnover in bn LCY. FO provides the percentage of foreign ownership in a firm given that foreign ownership is not zero. Share DO is the share of firms fully in domestic ownership.

Country	25 perc.	Mean	Median	75 perc.	Mean abs.
USA	-0.099	-0.001	-0.005	0.095	0.118
Japan	0.049	0.146	0.153	0.245	0.176
France	0.055	0.116	0.151	0.211	0.161
UK	0.110	0.209	0.198	0.279	0.210
Germany	0.035	0.107	0.135	0.207	0.156
Australia	0.163	0.241	0.252	0.325	0.255
Switzerland	0.102	0.198	0.228	0.336	0.234
Italy	0.075	0.128	0.170	0.238	0.177
Denmark	0.050	0.109	0.135	0.186	0.148
Canada	0.048	0.191	0.194	0.325	0.206
Belgium	0.067	0.152	0.188	0.288	0.200
China	-0.081	0.003	-0.001	0.076	0.098
Korea	0.014	0.098	0.111	0.195	0.142
Taiwan	-0.029	0.076	0.065	0.174	0.130
Hong Kong	-0.133	-0.046	-0.054	0.031	0.109
Thailand	0.017	0.101	0.121	0.204	0.140
India	-0.023	0.056	0.060	0.143	0.106
Turkey	0.021	0.058	0.035	0.098	0.098
South Africa	0.271	0.366	0.344	0.438	0.349
Poland	0.039	0.096	0.104	0.157	0.126
Singapore	-0.010	0.078	0.104	0.198	0.128

Table 4.4: Summary of FX exposure
This table provides the quantiles of the correlation between the monthly return on a GDP weighted exchange rate basket of 26 mayor currencies and firms' returns. Additionally, the table provides the mean as well as the mean absolute value of the correlation. The correlations are calculated over six month windows.

	(1)	(2)	(3)	(4)	(5)	(6)
	All	DM	EM	All	DM	EM
MV	0.098**	-0.070	0.680***	0.101**	-0.068	0.686***
	(0.044)	(0.048)	(0.099)	(0.044)	(0.048)	(0.099)
Vol	-0.037***	27.4***	-0.016***	-0.039***	27.4***	-0.017***
	(0.005)	(0.435)	(0.004)	(0.005)	(0.433)	(0.004)
ρ	-0.604***	-0.753***	0.101	-0.734***	-0.879***	-0.112
	(0.171)	(0.187)	(0.502)	(0.168)	(0.186)	(0.488)
BM	-0.017	-0.004	-0.254***	-0.017	-0.004	-0.252***
	(0.018)	(0.019)	(0.090)	(0.018)	(0.019)	(0.090)
Turn	0.020	0.121***	-0.304***	0.018	0.120***	-0.308***
	(0.029)	(0.031)	(0.072)	(0.029)	(0.031)	(0.072)
Obs.	613,411	500,199	113,212	613,411	500,199	113,212
R^2	0.1176	0.1151	0.1440	0.1178	0.1154	0.1441

Table 4.5: Regression: FX correlation and foreign ownership
The table shows the regression results with foreign ownership subject to logistic transformation as dependent variable. Time and country fixed effects are used and the standard errors are double clustered over time and firm. The coefficient for Vol is given in 10^{-12}. In Columns (4)-(6), the FX correlation is calculated using the SDR weighted basket of reserve currencies.

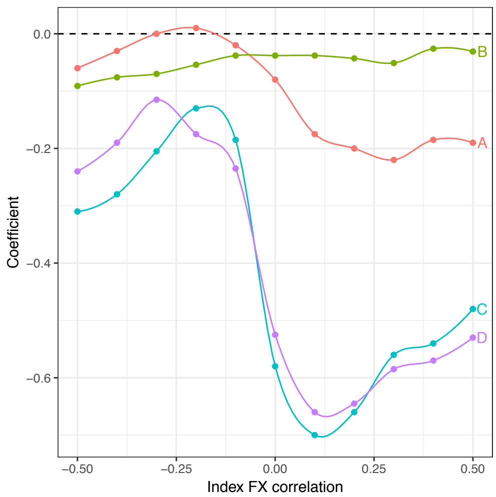


Figure 4.2: Simulation results
Optimal adjustment parameter θ^* plotted against index FX correlations. The four sets of simulations are based on the parameters in Table 4.1. Set A is based on the German stock market. The parameters are given in Table 4.1.

	(1)	(2)	(3)	(4)	(5)	(6)
	All	DM	EM	All	DM	EM
MV	0.102**	-0.066	0.683***	0.117**	-0.045	0.651***
	(0.044)	(0.048)	(0.099)	(0.045)	(0.050)	(0.101)
Vol	-0.037***	27.3***	-0.015***	-0.037***	-2110***	-0.025***
	(0.005)	(0.438)	(0.005)	(0.005)	(426)	(0.005)
ρ	-0.408**	-0.531***	0.302	-1.054***	-0.922***	-2.234***
	(0.177)	(0.191)	(0.486)	(0.242)	(0.254)	(0.824)
$\lvert\rho\rvert$	-0.485**	-0.570**	-0.434			
	(0.220)	(0.237)	(0.608)			
ρ_{post}				0.359	-0.016	2.485***
				(0.301)	(0.326)	(0.926)
BM	-0.017	-0.004	-0.254***	-0.018	0.001	-0.309***
	(0.018)	(0.019)	(0.090)	(0.023)	(0.019)	(0.096)
Turn	0.018	0.118***	-0.305***	-0.004	0.096***	-0.321***
	(0.029)	(0.032)	(0.071)	(0.030)	(0.033)	(0.074)
Obs.	613,411	500,199	113,212	446,877	362,402	84,475
R^2	0.1177	0.1153	0.1441	0.1378	0.1372	0.1566

Table 4.6: Regression: Absolute FX correlation and post-crisis
The table shows the regression results with foreign ownership subject to logistic transformation as dependent variable. Time and country fixed effects are used and the standard errors are double clustered over time and firm. Columns (1) and (4) use all countries. Columns (2) and (5) include the eleven developed markets as defined by Columbia University EMGP. Columns (3) and (6) include the ten emerging markets. The coefficient for Vol is given in 10^{-12}. In columns (4)-(6), only observations before June 2007 and after December 2009 are used, with the latter being considered post crisis.

	MV	Vol	ρ	BM	Turn	Obs.	R^2
USA	-0.291***	0.000	-0.437*	0.134***	0.192***	235,096	0.0076
Japan	0.245***	0.000***	-0.816**	-0.210**	0.024	137,688	0.0268
France	0.044	-0.000	-0.700	-0.022	0.295**	31,163	0.0397
UK	0.148	-0.000	-2.800**	0.182	-0.050	31,242	0.1944
Germany	0.131	0.000	-1.777*	-0.050	-0.208	23,968	0.0126
Australia	-1.318***	0.025	0.504	-1.414	1.117***	6,794	0.0824
Switzerland	-0.138	0.000***	-3.996***	-0.007	0.348	13,310	0.0406
Italy	0.390	-0.000	-0.803	-0.347	-0.077	7,184	0.0300
Denmark	0.053	0.001***	-2.915	-0.414	0.306	7,271	0.0814
Canada	-1.425	-0.045	12.75	1.734	-0.332	443	0.2777
Belgium	0.315	0.000*	0.729	1.038	-0.021	6,040	0.0177
China	0.020	-0.000	-1.076	0.581	-0.007	12,541	0.0132
Korea	0.664***	-0.000***	1.160*	-0.032	-0.199***	59,829	0.0474
Taiwan	0.371	-0.000***	0.837	-2.090***	-0.412	3,552	0.0567
Hong Kong	-2.214*	0.000***	2.737	-0.939	1.607***	4,119	0.1144
Thailand	-0.674	0.001	-8.592**	2.074	0.308	2,068	0.2779
India	-0.208	-0.000***	2.750	-2.904***	-0.382	13,487	0.2131
Turkey	0.795*	-0.000	-0.641	-0.489	-0.360	3,079	0.1229
South Africa	0.332	-0.015	-7.666***	-0.152	0.386	6,937	0.1269
Poland	2.348***	-0.000***	-8.102***	-0.519	-0.893***	6,164	0.2551
Singapore	-0.329	0.005	7.610*	1.593	-0.184	1,436	0.0746

Table 4.7: Regression per country
The table shows the regression results with foreign ownership subject to logistic transformation as dependent variable. Time fixed effects are used and the standard errors are double clustered over time and firm.

	(1)	(2)	(3)	(4)
	USA	Japan	UK	Korea
MV	0.262***	-0.002	0.234***	-0.045
	(0.087)	(0.055)	(0.065)	(0.057)
Vol	773	-0.012***	350***	-0.710***
	(652)	(0.003)	(74.8)	(0.076)
ρ	-0.231	-0.265*	-1.04***	-0.323
	(0.249)	(0.142)	(0.266)	(0.246)
BM	0.009	-0.149**	0.039	-0.134
	(0.008)	(0.023)	(0.035)	(0.084)
Turn	-0.257***	-0.041*	-0.203***	-0.072*
	(0.056)	(0.023)	(0.042)	(0.043)
Obs.	20,950	14,371	15,165	8.582
R^2	0.0905	0.0330	0.0994	0.0658

Table 4.8: Regression per country with some foreign ownership
The table shows the regression results with foreign ownership subject to logistic transformation as dependent variable. Time fixed effects are used and the standard errors are double clustered over time and firm. Only observations with at least some foreign ownership are used.

	(1)	(2)	(3)	(4)	(5)	(6)
	All	DM	EM	All	DM	EM
MV	0.223***	0.080	0.736***	0.171***	-0.006	0.645***
	(0.045)	(0.050)	(0.089)	(0.039)	(0.045)	(0.073)
Vol	10.5***	17.9***	-12.3*	-0.030***	4.18***	-0.032***
	(0.873)	(0.243)	(7.18)	(0.004)	(0.219)	(0.002)
ρ	-0.387***	-0.471***	-0.135	-0.160*	-0.130	-0.089
	(0.082)	(0.093)	(0.186)	(0.084)	(0.114)	(0.135)
BM	-0.006	0.012	-0.182***	-0.007	0.002	-0.138**
	(0.016)	(0.018)	(0.063)	(0.016)	(0.018)	(0.056)
Turn	0.061**	0.154***	-0.275***	0.034**	0.137***	-0.203***
	(0.027)	(0.029)	(0.065)	(0.026)	(0.030)	(0.052)
Obs.	797,980	663,329	134,651	821,050	615,973	205,077
R^2	0.1201	0.1206	0.1370	0.1325	0.1282	0.1628

Table 4.9: Regression: FX correlation and foreign ownership II
The table shows the regression results with foreign ownership subject to logistic transformation as dependent variable. Time and country fixed effects are used and the standard errors are double clustered over time and firm. The coefficient for Vol is given in 10^{-12}. In Columns (1)-(3), Vol and ρ are calculated using 24-month windows. Columns (4)-(6) are based on relaxed restrictions for non-US countries with a the minimum stock price of USD 1.

	(1)	(2)	(3)	(4)	(5)	(6)
	All	DM	EM	All	DM	EM
MV	0.224***	0.082	0.739***	0.254***	0.118**	0.728***
	(0.044)	(0.050)	(0.089)	(0.046)	(0.051)	(0.090)
Vol	10.4***	17.9***	-12.8***	-18.9***	-7250	-11.6***
	(0.871)	(0.246)	(4.33)	(7.09)	(277)	(4.31)
ρ	-0.388***	-0.432***	0.009	-0.789***	-0.868***	-0.362
	(0.083)	(0.092)	(0.184)	(0.159)	(0.172)	(0.361)
$\lvert\rho\rvert$	-0.139	-0.150	-0.379*			
	(0.098)	(0.111)	(0.222)			
ρ_{post}				0.726***	0.807***	0.083
				(0.187)	(0.206)	(0.452)
BM	-0.006	0.012	-0.183***	-0.003	0.020	-0.192***
	(0.016)	(0.018)	(0.063)	(0.018)	(0.017)	(0.064)
Turn	0.060**	0.153***	-0.277***	0.066**	0.160***	-0.278***
	(0.027)	(0.029)	(0.065)	(0.028)	(0.030)	(0.065)
Obs.	797,980	663,329	134,651	631,429	525,532	105,897
R^2	0.1201	0.1206	0.1370	0.1350	0.1377	0.1425

Table 4.10: Regression: Absolute FX correlation and post-crisis II
The table shows the regression results with foreign ownership subject to logistic transformation as dependent variable. Time and country fixed effects are used and the standard errors are double clustered over time and firm. Columns (1) and (4) use all countries. Columns (2) and (5) include the eleven developed markets as defined by Columbia University EMGP. Columns (3) and (6) include the ten emerging markets. The coefficient for Vol is given in 10^{-12}. In columns (4)-(6), only observations before June 2007 and after December 2009 are used, with the latter being considered post crisis. Vol and ρ are calculated using 24-month windows.

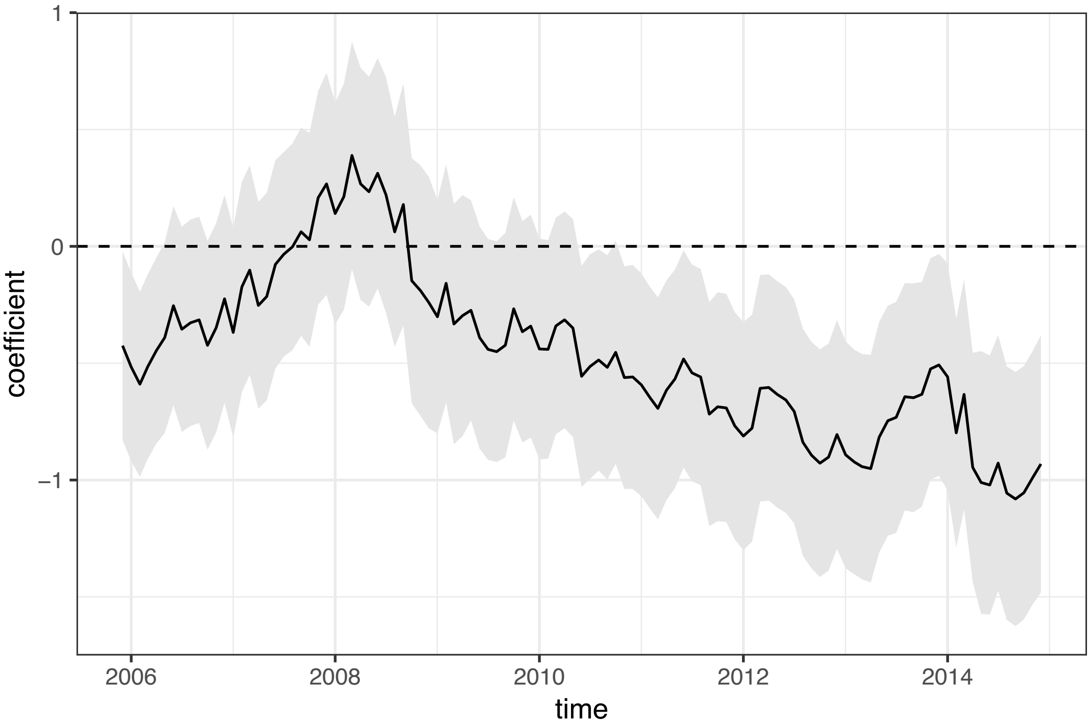


Figure 4.3: Coefficient $\hat{\theta}$ for per month regression
The figure shows the coefficient $\hat{\theta}$ for monthly regressions using the same control variables as in the main regressions including country fix effects. The grey band marks the 5% significance interval.

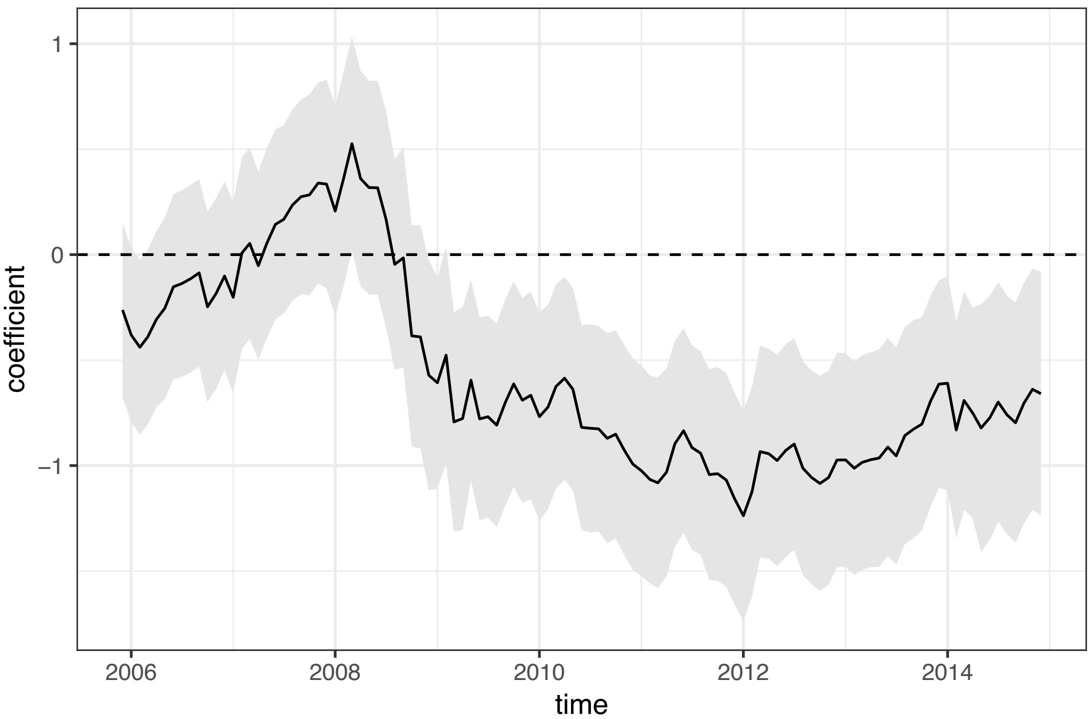


Figure 4.4: Coefficient $\hat{\theta}$ for per month regression in developed markets
The figure shows the coefficient $\hat{\theta}$ for monthly regressions using the same control variables as in the main regressions including country fix effects. Only developed markets are used in the regression. The grey band marks the 5% significance interval.

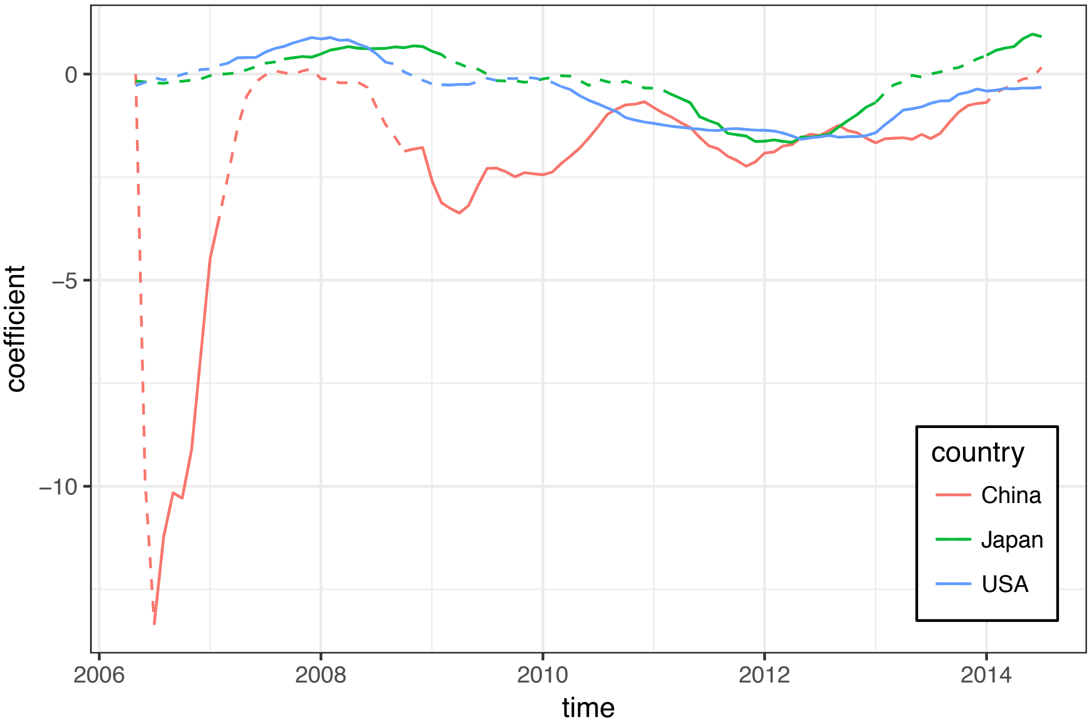


Figure 4.5: Rolling window coefficient $\hat{\theta}$ for major markets
The figure shows the coefficient $\hat{\theta}$ for one year rolling window regressions for USA, Japan, and China. The control variables and specifications are the same as in the main regressions. The dashed lines mark insignificant coefficients.

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