# *Supplemental Material for*
# Microscopic details of stripes and bubbles in the quantum Hall regime

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In Fig. S1 we show the variation in $\nu(\vec{r})$ for three different densities at fixed magnetic field $B$. These results are identical to Fig. 1 of the main text for $\nu_\downarrow(\vec{r})$ but also show $\nu_\uparrow(\vec{r})$ in the left column. The choice of colors is as in Refs. [1, 2]. In Fig. S2(a) and (c) one can see that for pure Hartree interaction there is no stripe formation. The density modulation in $\nu(\vec{r})$ is much less than in Figs. **??** and S1 and roughly follows the random potential shown in Fig. S3. Furthermore, the charge density modulation in the spin-up and spin-down levels "repel" each other due to the Hartree interaction. In Fig. S2(b) and (c) we show the situation without interaction. Clearly, there is also no stripe formation. The $\nu(\vec{r})$ modulation rather closely follows the random potential of Fig. S3. The charge densities in the spin-up and spin-down levels do not influence each other and follow nearly identically the disorder potential because of any missing interaction.

Fig. S4 complements Fig. **??** by showing in addition the Hall resistance $R_{xy}$.

In order to show that the presence of the remnants of the LL wave functions around each stripe is significant, we perform the calculations of $I_\nu(f)$ for three test patterns for $\nu(\vec{r})$. The results are given in Fig. S5. We find that only the variation given by $\nu(\vec{r})$ as calculated in HF can reproduce essential global features of the experimental NMR results presented in Ref. [3].

In Fig. S6 we show the behavior of $I_\nu(f)$ for stripes and bubble-like charge density waves. As in Fig. S5, the HF results for $\nu(\vec{r})$ lead to a reasonable qualitative agreement with the non-interacting model used in Ref. [3]. Nevertheless, the details around, e.g., $\nu = 2.5$ are rather different, highlighting the importance of interactions. We note that normally stripes appear only starting with filling factor $\nu = 4.5$. This is known also experimentally, but for experimental reasons the authors of Ref. 3 could not go to that filling factor. Instead, they used filling factor $\nu = 2.5$ and forced, by using an in-plane component of the magnetic field, the electron system to form a stripe pattern. Clearly, there is no need for our simulations to also model this experimental "trick". In order to compare the effect of stripe patterns on the NMR Knight shift, we therefore use the stripe pattern in the "correct" range $\nu = 4$–$5$. The result in Fig. S6 (b) has striking similarities but seems indeed a bit richer in features than the experimental curve for $\nu = 2$–$3$. However, since the Knight shift spectrum looses its local information due to the spatial integration, we can also evaluate the range $\nu = 2$–$3$ as shown in Fig. S6. Indeed the agreement with the experiments of Ref. 3 becomes even better in this filling factor range. We can still see in total 3 peaks, two of them clearly separated and a third one as a shoulder on the high frequency flank, just as shown for the experiments in Fig. 2b of Ref. 3. This makes the agreement almost perfect for the experiments as shown in Fig. S7 albeit not with the non-interacting modelling of Fig. 2c [3]. Filling factor 6–7 requires much more computing time and, although a most interesting question, there are currently no experiments available for comparison.

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## SUPPLEMENTARY REFERENCES

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[2] J. Oswald and R. A. Römer, Physical Review B **96** (2017), 10.1103/PhysRevB.96.125128.
[3] B. Friess, V. Umansky, L. Tiemann, K. von Klitzing, and J. H. Smet, Physical Review Letters **113**, 076803 (2014).

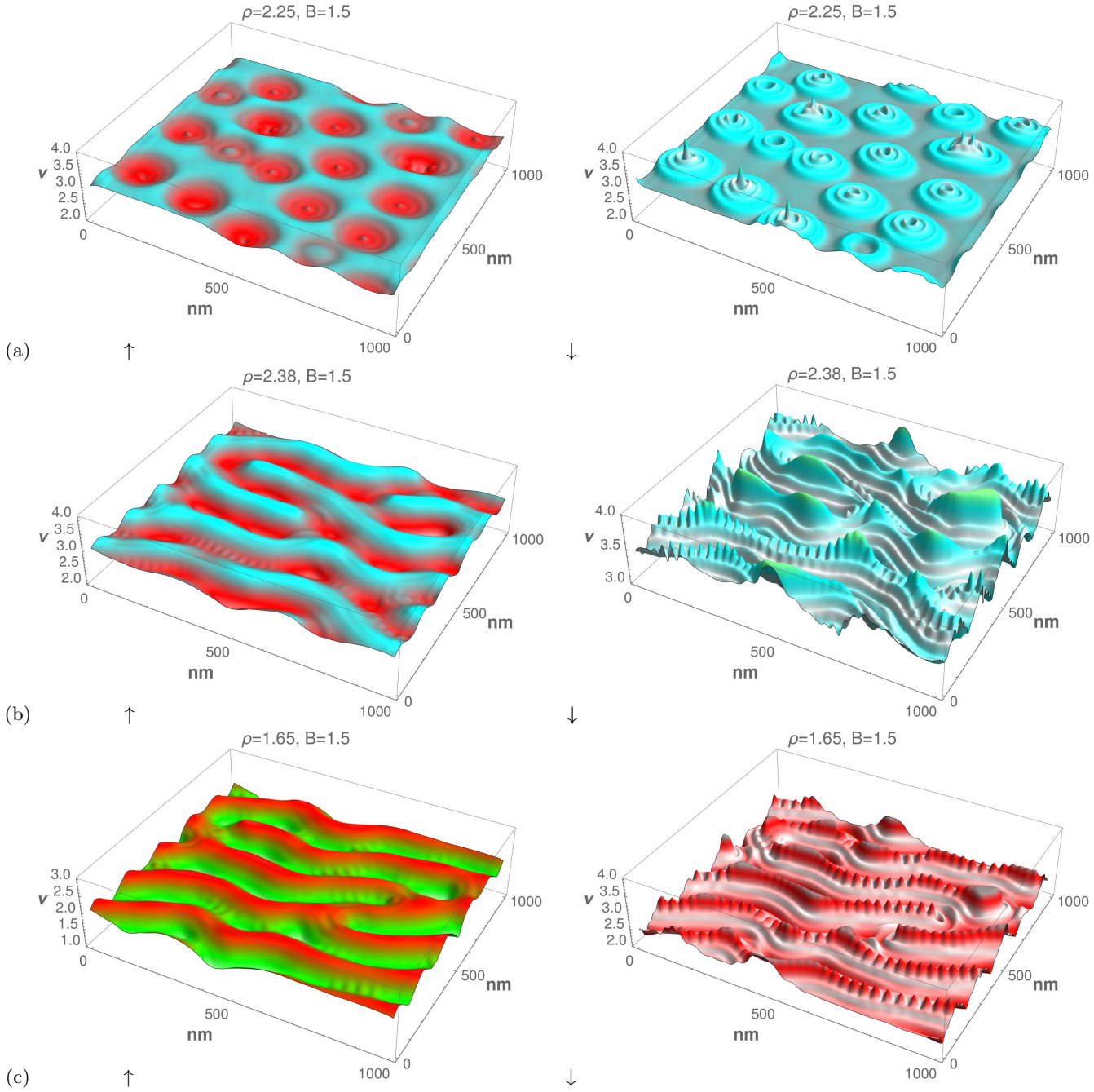


FIG. S1. Lateral carrier density distribution mapped on the filling factor scale $\nu(\vec{r})$ for different total filling factors (a) $\nu = 6.20$, (b) $\nu = 6.54$ and (c) $\nu = 4.54$. The left and right columns contains results for $\nu_\uparrow$ and $\nu_\downarrow$, respectively. The color shades represent the filling factor range, where green indicates the second LL for $\nu_{\downarrow,\uparrow} = 1 \rightarrow 2$, red the third LL, light blue the fourth and yellow denoting LL 5. These colours are as in Refs. 1 and 2 and the value indicated for $\rho$, $B$ denote the electron density in units of $10^{11}$ cm$^{-2}$ and 1 Tesla, respectively.

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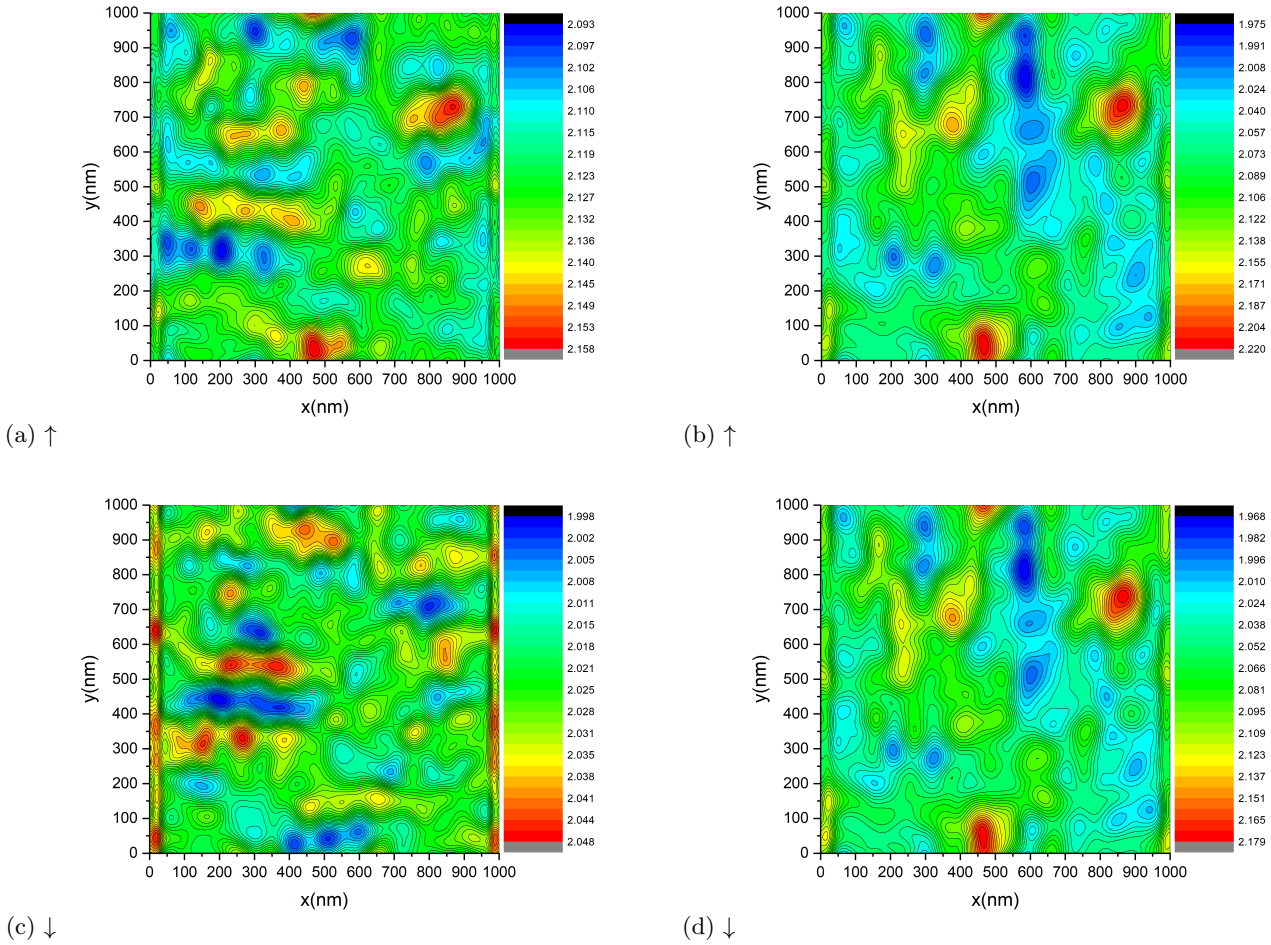
Hartree non-interacting




(a) ↑ (b) ↑

(c) ↓ (d) ↓

FIG. S2. Lateral charge density of the partly filled top Landau levels at total filling factor $\nu = 4.5$. The top row with (a) and (b) gives the spin-down levels while to bottom row shows that spin-down levels in (c) and (d). The left column with (a) and (c) has been calculated for pure Hartree interaction while the right column with (b) and (d) shows a non-interacting situation. The color shades represent the filling factor $\nu(\vec{r})$ as given in the scales and the lines denote equal heights in $\nu(\vec{r})$.

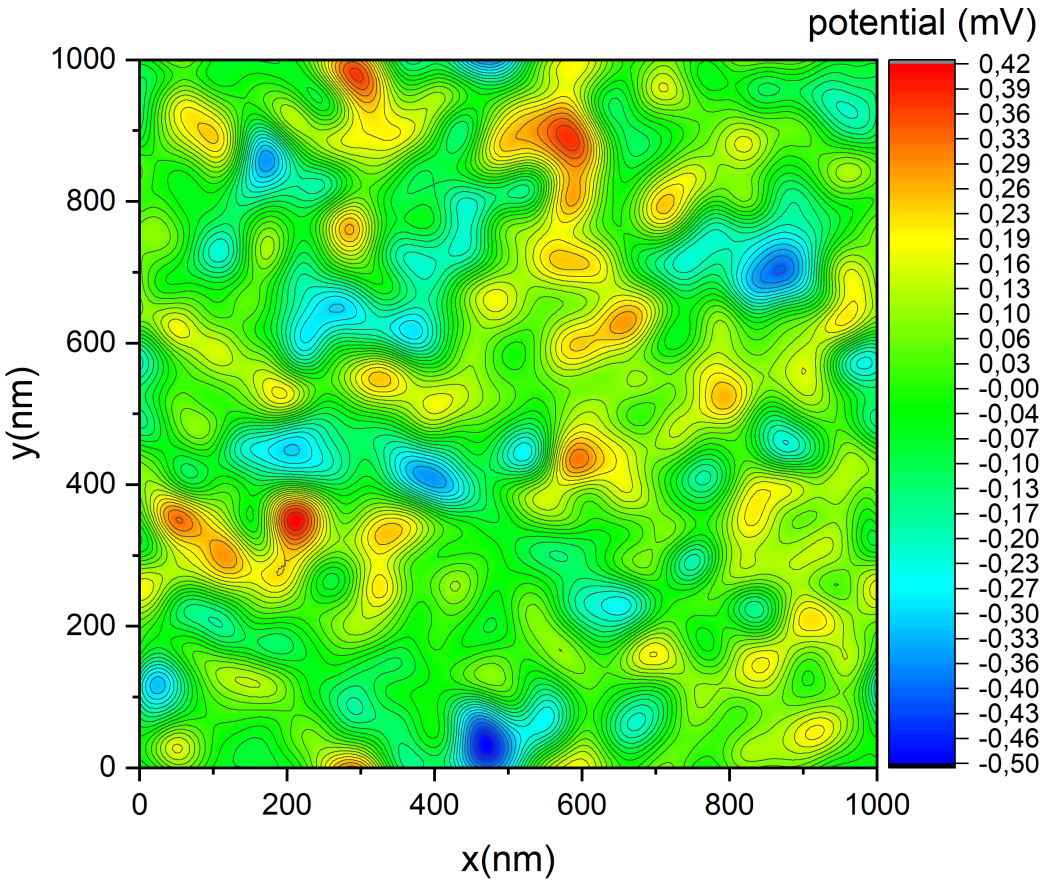



FIG. S3. Lateral random disorder potential $V(\vec{r})$ visualized in a false color plot. The lines denote equipotentials while the potential energies are indicated by the colors as in the color scale provided.

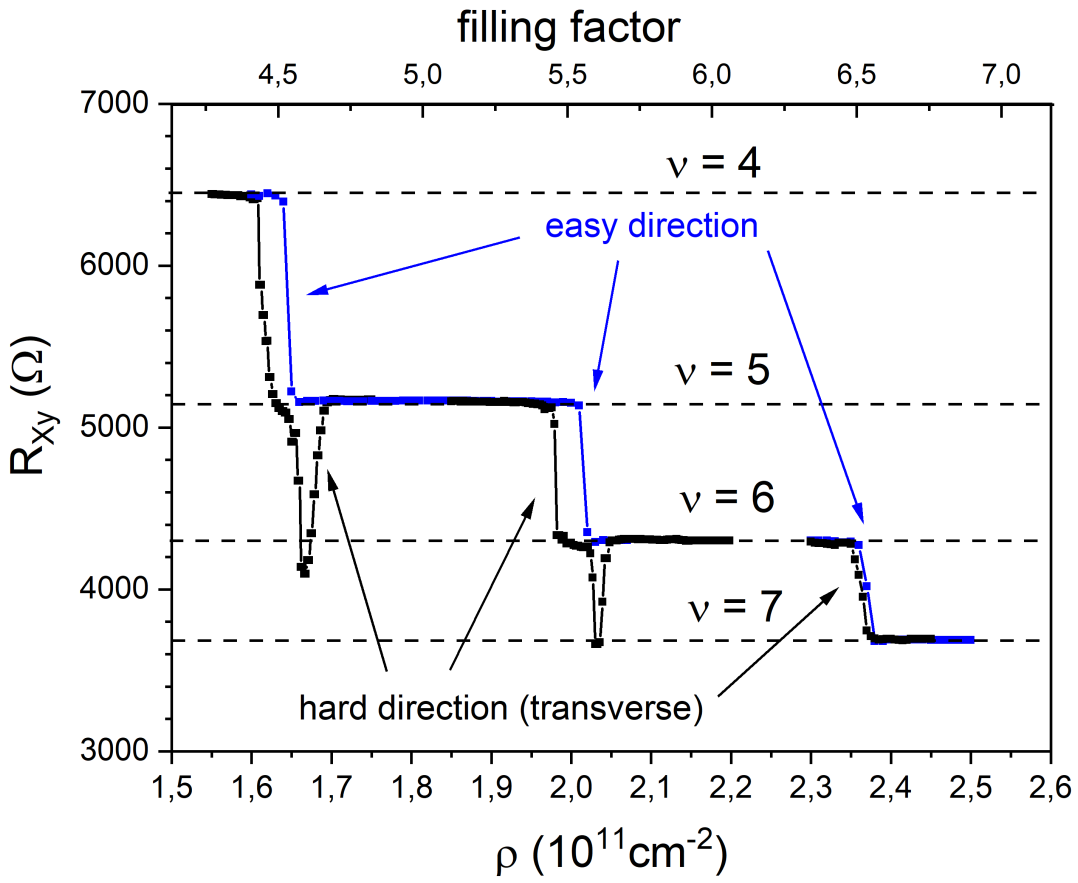



FIG. S4. Hall resistance $R_{xy}$ for the longitudinal resistance $R_{xx}$ shown in Fig. 2 with the same set of parameters.

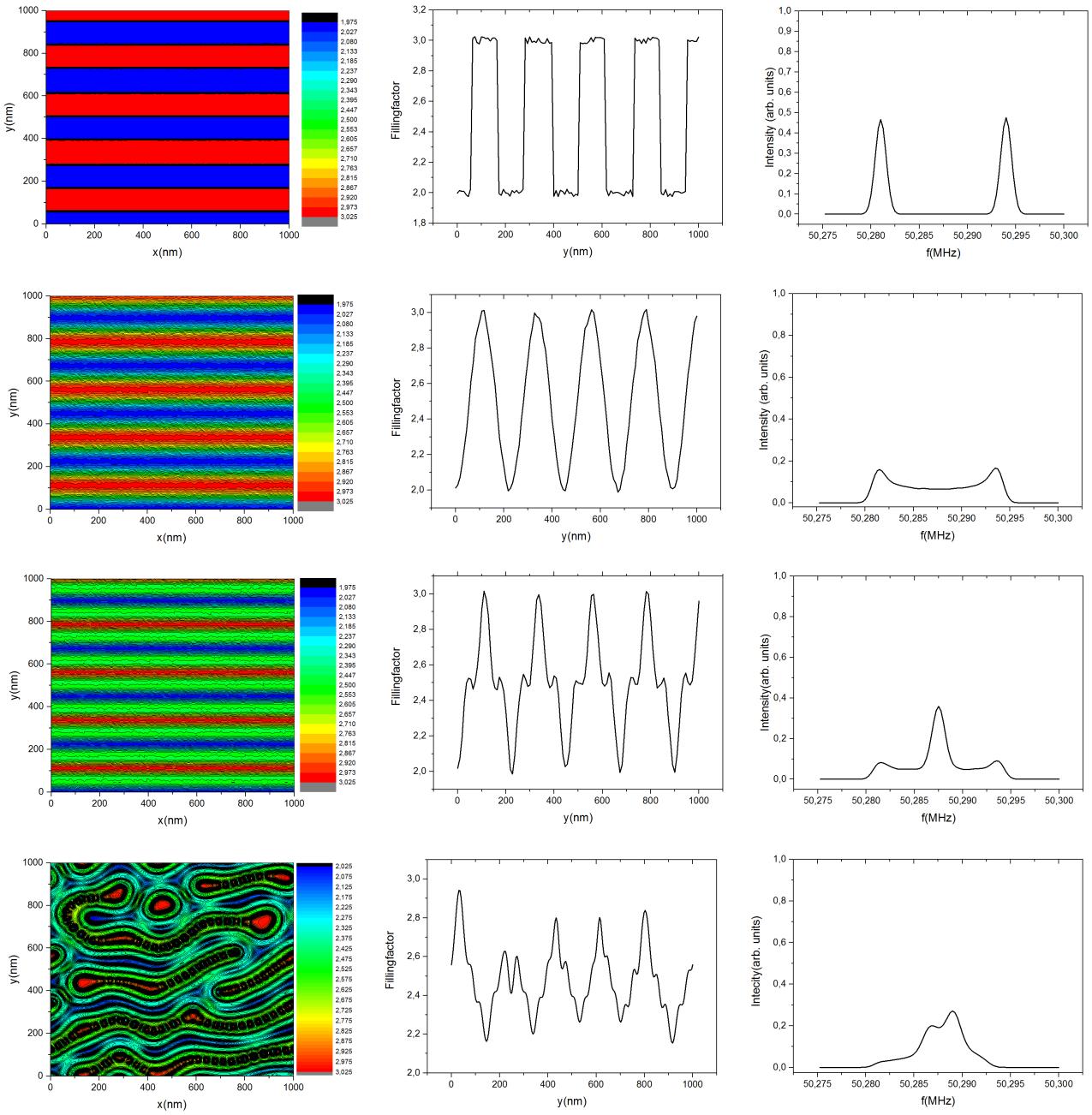



FIG. S5. Calculations of the NMR intensity $I_\nu(f)$ for 4 different stripe-like variations of $\nu(\vec{r})$ at $\nu = 4.5$. Rows 1-4 corresponds to (a) a simply square modulation, (b) a sinusoidal modulation, (c) a sinusoidal variations with left and right shoulders, (d) a modulation as computed from HF. The first column shows the spatial variations in $\nu(\vec{r})$ as given by the color scales. The second column represent a typical cross-section for each situation and the third column shows the estimated NMR intensity $I_{4.5}(f)$.

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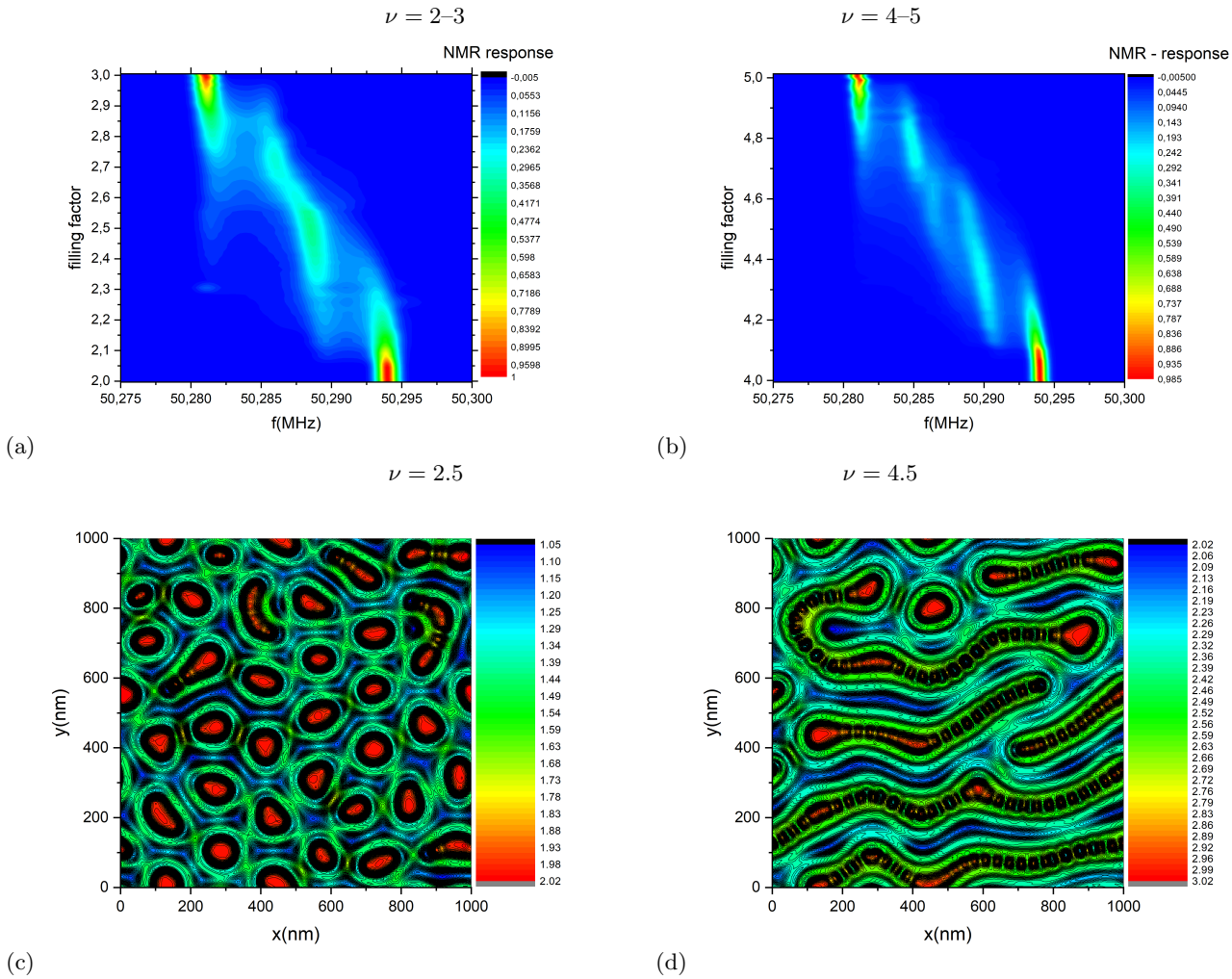


FIG. S6. NMR intensities $I_\nu(f)$ for (a) $\nu = 2-3$ and (b) $\nu = 4-5$ and local $\nu(\vec{r})$ at (c) $\nu = 2.5$ and (d) $\nu = 4.5$ in left and right columns, respectively. (b+d) The right column reproduces results already shown in Figs. **??** and **??**. The color shades represent $I_\nu(f)$ and $\nu(\vec{r})$ as given by the scales. Lines in (c+d) connect equal height in $\nu(\vec{r})$.

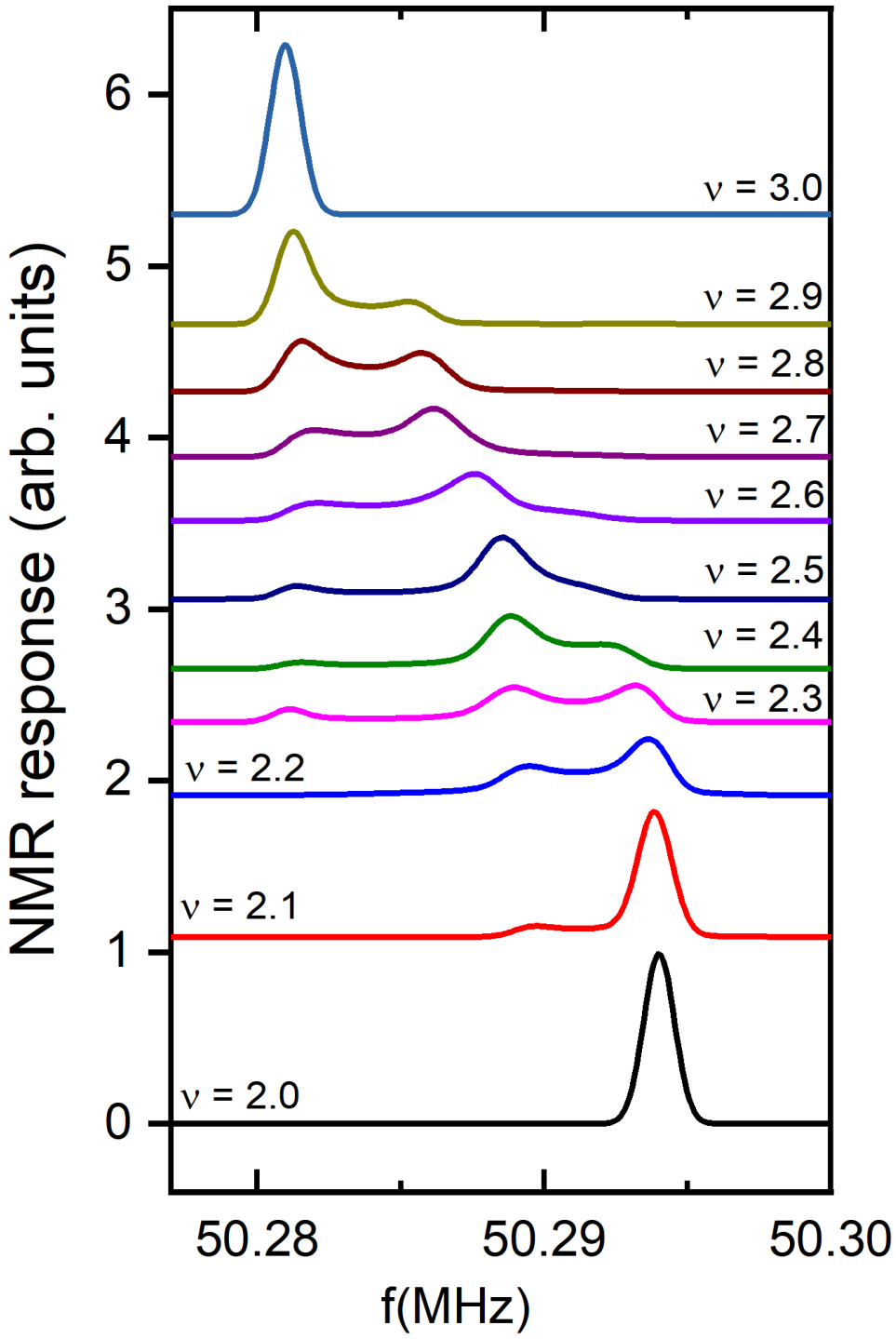


FIG. S7. NMR intensities $I_\nu(f)$ as shown in Fig. **??**(a) but for the filling factor range $\nu = 2 - 3$.