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# Bilateral Trading in Networks\*

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## Abstract

In many markets goods flow from initial producers to final customers travelling through many layers of intermediaries and information is asymmetric. We study a dynamic model of bargaining in networks that captures these features. We show that the equilibrium price demanded over time is non-monotonic and exchange is reminiscent of fire-sale and hot-potato trade dynamics. Traders who intermediate the object arise endogenously and attain a profit. This profit depends on their network location. We reflect on inefficiencies introduced by strategic intermediation in networks.

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# 1 Introduction

We study strategic intermediation in markets characterised by incompleteness of trading opportunities, dispersed information, and resale. A finite number of risk-neutral traders are connected in a trading network; a link from trader  $i$  to trader  $j$  means that  $i$  can negotiate with  $j$  at zero cost. The absence of a link subsumes a prohibitive trading cost. One arbitrary trader is the producer of a single valuable object. A trader's valuation for the object can be either low or high; it is private information and independent of other traders' valuations. In each round of trade, the current owner of the object either consumes it, in which case the game ends, or makes a take-it-or-leave-it offer to a connected trader that she chooses. The trader who receives the offer either accepts or rejects it. After this decision is taken, a new round of trade starts. The time horizon is infinite; traders have a common discount factor, and they maximise their net discounted surplus.

As a prime example, consider markets for agricultural goods in developing countries. Local producers access only a limited number of traders, who operate in geographically close markets; these traders, in turn, access other traders, who operate in markets located farther away from the original producers. Local producers could access such distant markets, but the lack of infrastructure and the difficulties of raising capital make such journeys infeasible. The structure of who can trade directly with whom— the trading network— is a technology that can be altered with large investments, like building a bridge or a road. But, in the short run and as a first approximation, it can be seen as fixed and known to market participants. Furthermore, negotiations are often bilateral, and products are exchanged for cash between intermediaries en route from local producers. Finally, each trader has precise information about the demand of the local market in which they operate— i.e., they know their valuation for the object. But in developing countries, markets are dispersed and the communication infrastructures are poor. The implication is that asymmetric information among traders about the state of the demand in each local market is pervasive.<sup>1</sup>

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<sup>1</sup>This description is a brief summary of the stylised facts of markets in developing countries, which have

Incompleteness of trading opportunities, dispersed market information, and resale are also prominent features of over-the-counter financial markets. Products such as foreign currencies, swaps, forward rate agreements, and exotic options are often traded via bilateral negotiation in OTC markets. These securities are subject to counter-party risk, and, therefore, bonds of trusts and information flows between firms/dealers are particularly important. This gives rise to trading costs that can be heterogeneous across pairs of traders. Furthermore, private observability of customer order flow and individual liquidity shocks are sources of asymmetric information among dealers. A large amount of inter-dealer trading is often observed, a phenomenon also referred to as hot-potato trading.<sup>2</sup>

Our analysis addresses the following questions: How does the underlying network structure affect the way market participants set prices? What are the network locations that provide larger payoffs to traders? What inefficiencies might be introduced by asymmetric information and incompleteness of trading relationships? In Section 3, we answer these questions for the class of multi-layer networks. These networks capture environments in which there is a natural direction of trade from upstream to downstream traders. In the rest of this introduction, we present the results for general networks (contained in Section 4), and, when appropriate, we comment on additional insights that are obtained from the analysis of multi-layer networks.

The set of weak-Markov perfect Bayesian equilibria (henceforth “equilibria”) that we

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been documented in several empirical studies, such as Fafchamps and Minten (1999),(2001), Jensen (2007) Aker (2010), Svensson and Yanagizawa (2009), and Treb (2014). These papers focus on a variety of goods, ranging from non-storable goods, such as fish in Jensen (2007), to highly storable goods, such as grain in Aker (2010).

<sup>2</sup>We refer to Lyons (1997) for a seminal paper on hot-potato trading. Li and Schurhoff (2012) study the trade of US municipal bonds in the OTC market. They document that bonds move from the municipality through an average of six inter-dealer trades. They also document that there is systematic price dispersion across dealers, with earlier dealers maintaining systematically larger margins. We refer to Allen and Babus (2009) for a survey of networks in financial markets.

characterise has a simple structure (Propositions 2-4). A high-value trader who acquires the object consumes it. In contrast, a low-value trader engages in a sequence of offers to other connected traders until the object is sold (unless, at some point, her own consumption value is higher than the discounted resale value of the object). All of her offers except the last come at prices that only traders with a high value are willing to accept. We refer to these offers as *consumption offers* because, once accepted, the object is consumed. Refused consumption offers are followed by an offer that low and high-value traders accept. We refer to these offers as *resale offers* because they come at a price equal to the expected revenue that the low-value trader obtains from reselling to other traders (i.e., their *resale value*).

We show that the equilibrium sequence of asking prices is non-monotonic in time (Proposition 5). Prices in resale offers are decreasing in time because, as time passes, all traders become more pessimistic about the total expected demand in the network; moreover, since sellers have all the bargaining power, they are able, when passing on the object, to extract in full the resale value of the subsequent owner. However, prices in consumption offers spike, as sellers are attempting to exploit their positional power in the network to appropriate the surplus of connected traders with high value. The equilibrium price dynamic, then, is reminiscent of fire-sale and hot-potato trade dynamics: a low-value trader buys the object at a price that equals the expected sale price; if she fails to sell it at a high a price, she is “forced” to sell it at a price lower than the one she paid. Since when consumption offers are accepted the game ends, the fact that prices in resale offer are declining implies that the realized sequence of transaction prices is also declining, with the possible exception of the last one.

We then investigate how the network location of a trader affects her payoffs. Only traders who receive a resale offer can make a positive profit. We call these traders *dealers*. Low-value dealers break-even in expected value: they buy at a price which equal their resale value. High-value dealers make a positive profit: they acquire the object at a price lower than their consumption value. Dealers arise endogenously, depending on their network location. In

multi-layer networks the dealers are the traders that lie on the path of the network that maximises the number of traders connected to the nodes in that path. In general networks, we show that if a high-value dealer is *essential* in connecting the local producer to another trader, then the former obtains a higher expected payoff.<sup>3</sup>

Finally we show, within the class of multi-layer networks, that strategic intermediation introduces inefficiencies. In particular, when traders are sufficiently patient, the equilibrium trading path does not always maximise ex-ante allocative efficiency. At each point in time, in equilibrium, the owner of the object makes consumption offers to her trading partners and then makes a final resale offer to the trading partner with the highest resale value. This is the trading partner from which it departs the path that maximises the number of traders connected to nodes in that path. However, it may be different from the path that would maximise ex-ante efficiency (i.e., the path that maximises the sum of the number of traders connected to nodes in that path *and* the number of nodes in the path).

A central message in our paper is that the interplay between asymmetric information and strategic intermediation in networks leads to spatial price dispersion (the object is priced differently across network locations), and allocative inefficiency. These predictions are in line with most of the empirical work studying markets in developing countries. Moreover, recent empirical studies have provided compelling evidence that the introduction of information technology in developing countries— e.g., the availability of mobile phones— has reduced asymmetric information across markets and, as a consequence, has sharply decreased spatial price dispersion and inefficiencies— see e.g., Jensen (2007), Aker (2010), and Svensson and Yanagizawa (2009). In our framework, we could think of such an innovation as leading to a setting in which either the demand of each local market (the private valuation of the trader) becomes common knowledge among traders, or the initial producer can directly access all other traders. In both cases, the equilibrium predicts constant prices across locations, larger

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<sup>3</sup>A trader  $i$  is essential to connect  $j$  to the initial producer if trader  $i$  lies in every path from the initial owner to trader  $j$ .

profits to the initial producer, and allocative efficiency.

Understanding how networks affect trade is a central question in economics. A large literature has focused on buyer-seller networks—see Manea (2015) for a survey of this literature. A few recent papers have studied strategic intermediation in networks under complete information.<sup>4</sup> We contribute to this literature by examining, for the first time, a dynamic model of trade in the presence of asymmetric information. The prevalence of asymmetric information in decentralised markets motivates this extension, and, as we discussed above, our results provide novel predictions that are consistent with existing empirical evidence.<sup>5</sup>

Our paper also relates to bilateral bargaining models with one-sided asymmetric information.<sup>6</sup> Most of this literature focuses on one seller and one buyer—e.g., Hart (1989); exceptions are Fudenberg, Levine and Tirole (1987) and De Fraja and Muthoo (2000) in which there are multiple buyers. We extend the analysis to allow for resale; indeed, both Hart (1989) and De Fraja and Muthoo (2000) are a special case of our model. The possibility of resale generates new questions. It also creates new difficulties, as it implies that the continuation value of a trader depends on the all vector of beliefs about other traders' types through the chain of resale opportunities. As a consequence, some methods of analysis

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<sup>4</sup>Blume et al. (2009), Choi, Galeotti and Goyal (2013), Nava (2009), and Gale and Kariv (2007), (2009) focus on markets in which intermediaries post prices. In Kotowski and Leister (2014), negotiation occurs via auctions. Manea (2014) studies a model of bilateral bargaining with random selection of proposer. Gofman (2011) also considers trade with asymmetric information but does not explicitly model the underlying bargaining protocol. Siedlarek (2015) considers multilateral bargaining. We refer to Condorelli and Galeotti (2015) for a survey of models of strategic intermediation in networks.

<sup>5</sup>Our approach, which combines both incomplete information and an explicit network structure, stands in contrast to recent models of trading that employ the random-matching approach pioneered by Rubinstein and Wolinsky (1985)—e.g., Duffie, Garleanu and Pedersen (2005), Satterthwaite and Shneyerov (2007), and Golosov, Lorenzoni and Tsyvinski (2009).

<sup>6</sup>Classical papers in this literature are Ausubel and Deneckere (1989), Fudenberg and Tirole (1983), Fudenberg, Levine and Tirole (1985), Sobel and Takahshi (1983), Cramton (1984), Gul, Sonnenschein and Wilson (1986), and Chatterjee and Samuelson (1987).

that are used in the classical bilateral bargaining model cannot be easily transposed to our setting.

Finally, our paper is related to the literature on the effect of resale on market outcomes. Calzolari and Pavan (2006) consider a simple seller-intermediary-buyer architecture. Haile (2003), Garratt and Troger (2006), Krishna and Hifalir (2008), Jehiel and Moldovanu (1999) and Zheng (2002) analyse a complete network and their focus on auctions and inter-bidder resale. We work with a general network and explore a different set of questions.

## 2 The Network Trading Game

A *trading network* is a directed graph  $G = (N, E)$ , where  $N = \{1, \dots, n\}$  is the set of *traders*, and  $E \subseteq 2^{N \times N}$ , the set of edges, represents potential trading relationships. If  $ij \in E$ , we say that  $j$  is a *trading partner* of  $i$ —i.e., trader  $i$  can sell to trader  $j$ ; the set of  $i$ 's trading partners is  $N_i = \{j \in N \setminus \{i\} : ij \in E\}$ .

There is a single unit of an indivisible good, the *object*, initially owned by one of the traders,  $s^0$ . Each trader  $i$  has a private valuation for the object,  $v_i \in \{v_L, v_H\}$ , where  $0 < v_L < v_H$ . The (common) prior probability that  $v_i = v_H$  is  $\mu_i \in (0, 1)$ , and values are independently distributed. We restrict attention to networks in which there is at least one (directed) path from  $s^0$  to each  $i \neq s^0$ .

There is an infinite number of rounds. Starting with  $t = 0$ , each round of trade is as follows: (i) The current owner of the object, denoted  $s^t$  (i.e., the *seller*), chooses to either consume the object or not and, in the latter case, makes a take-it-or-leave-it offer to one of her trading partners,  $b^t \in N_{s^t}$  (i.e., the *buyer*), at some price  $p^t \in \mathbb{R}$ ; (ii)  $b^t$  decides whether to accept the offer or reject it. The game ends if  $s^t$  consumes the object. If  $s^t$  does not consume, the game proceeds to round  $t + 1$  and  $s^{t+1} = b^t$  if  $b^t$  accepted the offer; otherwise,  $s^{t+1} = s^t$ . We assume perfect observation—i.e., players observe all actions.

The  $t$ -period payoff to the seller is  $v_{s^t}$  if she consumes,  $p^t$  if her offer is accepted, and 0 oth-



erwise; buyer  $b^t$  obtains  $-p^t$  if she accepts, and 0 otherwise. All other players get 0. Traders have a common discount factor  $0 < \delta < 1$  and they maximise their net discounted surplus.

We call the multi-stage extensive-form game with observed actions, independent types, and incomplete information that we have described above the *network trading game*. Strategies and mixed strategies are defined in the usual way.

Throughout the paper, we restrict attention to *weak-Markov perfect Bayesian equilibria* (henceforth referred to as wMPBE). A perfect Bayesian equilibrium is a strategy profile and a belief system such that the strategies are sequentially rational given the belief system, and the belief system is consistent with Bayesian updating whenever applicable, including at off-path histories. As is standard, we further assume that a deviation by player  $i$  cannot influence beliefs about other players; that any two players must always have the same belief about a third player; and that degenerate beliefs are never updated.

A wMPBE is a perfect Bayesian equilibrium in which the acceptance strategy depends only on buyers' private information, the price asked, the identity of the seller, and the state of beliefs. Instead, the consumption decision and the offer made by a seller may depend, in addition to her private information, on the state of beliefs and the offer in the previous round (if made by that seller), but not on earlier history.<sup>7</sup>

In the next two sections we present our results. We postpone to section 5 a discussion of the main assumptions and their role in our analysis.

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<sup>7</sup>In a strong-Markov PBE the seller's strategy only depends on the state of beliefs at every history. It is well known that strong-Markov PBE do not always exist in bargaining games with incomplete information. For this reason, the literature has focused on wMPBE, see Fudenberg, Levine and Tirole (1985), and Fudenberg, Levine and Tirole (1983).

### 3 Multi-layer Networks

A multi-layer network is an acyclic network with the additional property that for every two distinct downstream competitors— i.e.,  $j$  and  $j'$  with  $ij \in E$  and  $ij' \in E$ — there is no a path from  $j$  to  $j'$  or from  $j'$  to  $j$ . Directed trees rooted at  $s^0$  are examples of multi-layer networks. An example of a multi-layer network that is not a tree is depicted in Figure 1.

There are two main reasons for paying special attention to multi-layer networks. First, these networks capture environments in which there is a natural direction of trade, from upstream to downstream traders, as is typical in most supply chains in manufacturing, retail, and agriculture. Second, in multi-layer networks the buyers that a seller can reach either directly or indirectly— i.e. for which there is a path from the seller to the buyer— cannot have received offers in the past. Therefore, the continuation payoff of a buyer upon accepting an offer is independent of the information that has been revealed in previous rounds. This simplification allows us to construct equilibria recursively. The explicit characterization that we obtain illustrates how the possibility of resale, together with asymmetric information, leads to novel and rich economic insights. Most of the results we emphasise carry over in general networks, as we show in the next section.<sup>8</sup>

#### 3.1 Construction of an equilibrium

We now show how to construct an equilibrium in an arbitrary multi-layer network. We focus on the case in which  $\mu_i = \mu$  for all  $i \neq s^0$ . We use the network of Figure 1 to illustrate the different steps. We refer the reader to the online Appendix A for formal statements.

**First Step: Partition.** We first construct a partition  $\{L_0, L_1, \dots, L_{k^*}\}$  of  $N$  that creates an ordering of traders based on their resale opportunities. Each trader in  $L_0$  has no resale opportunities. Each trader in  $L_1$  can resell only to a subset of traders belonging to  $L_0$ ; each

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<sup>8</sup>Multi-layer networks, or variants of it, are the focus of analysis of recent models of trading in network with resale, such as Gale and Kariv (2009), Gofman (2011), Manea (2014) and Kotowski and Leister (2014).

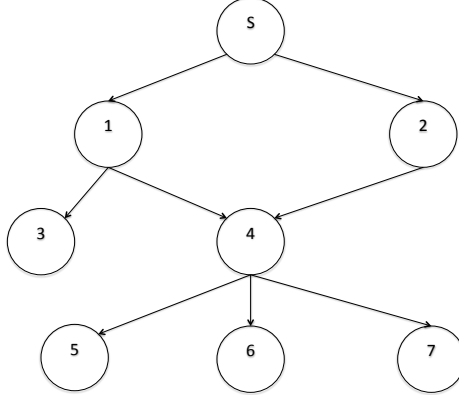


Figure 1: An Example of a multi-layer network

trader in  $L_2$  can resell to some traders in  $L_1$ , and, potentially, also to some traders in  $L_0$ , and so on till we exhaust traders and reach the initial seller,  $L_{k^*} = \{s^0\}$ .

Formally, we construct the following sets recursively: first, we construct  $L_0 \equiv \{i \in N : N_i = \emptyset\}$ ; then, for  $k \geq 1$ , we construct  $L_k \equiv \{i \in N \setminus \cup_{y=0}^{k-1} L_y : N_i \subseteq \cup_{y=0}^{k-1} L_y\}$ ; we stop when we reach  $k^*$  such that  $L_{k^*} = \{s^0\}$ . The fact that  $G$  is a multi-layer network implies that there exists a finite  $k^* \geq 1$  and that  $\{L_0, \dots, L_{k^*}\}$  is a partition of  $N$  (see Lemma A in online Appendix A). In Figure 1, we have that  $k^* = 3$ , and  $L_0 = \{3, 5, 6, 7\}$ ,  $L_1 = \{4\}$ ,  $L_2 = \{1, 2\}$  and  $L_3 = \{s\}$ .

**Second Step: auxiliary game and recursive construction.** Since traders in  $L_0$  have no resale opportunities, they consume the object as soon as they acquire it. We denote by  $R_i = v_L$  the equilibrium payoff of low-value trader  $i \in L_0$  in the auxiliary game, where  $i$  is the owner of the object. Assume now that  $i \in L_1$  owns the object. It is clear that  $i \in L_1$  consumes if she has a high value. If she has a low value, however, the strategic situation that trader  $i$  faces is a multi-lateral bargaining problem where each buyer  $b \in N_i \subseteq L_0$  has private valuation  $v_b \in \{\delta R_b, \delta v_H\}$  and the probability that  $v_b = \delta v_H$  is  $\mu$ . Proposition A in online Appendix A establishes that, in this auxiliary game, there exists an equilibrium in which trader  $i$  consumes the object if  $v_L \geq \delta[\mu v_H + (1 - \mu)v_L]$ . Otherwise, trader  $i$

screens all buyers by asking, in sequence,  $\delta v_H$  and, upon rejection, she consumes the object. Buyers, anticipating the seller's behaviour, accept offers up to their valuation. We refer to the resulting equilibrium payoff of low-value trader  $i$  in this auxiliary game as  $i$ 's resale value and we denote it by  $R_i$ . Note that  $R_i \geq v_L$ .

We now move up through the layers and iterate this reasoning. Suppose that low-value trader  $i \in L_2$  owns the object. Since each  $i$ 's trading partner  $b$  belongs to  $L_0 \cup L_1$ , we know, from the above considerations, that the continuation payoff of high-value and low-value  $b$ , by purchasing the object from  $i$ , is  $\delta v_H$  and  $\delta R_b$ , respectively. Hence, low-value trader  $i$  faces a multi-lateral bargaining problem where each buyer  $b \in N_i$  has private valuation  $v_b \in \{\delta R_b, \delta v_H\}$  and the common prior that  $v_b = \delta v_H$  is  $\mu$ . If  $\delta R_b \leq v_L$  for all  $b \in N_i$ , we are in a situation that is analogous to the one described in the previous paragraph. When  $\delta R_b > v_L$  for some  $b$ , Proposition A in online Appendix A establishes that there is an equilibrium in which, trader  $i$  makes an offer at a price  $\delta R_{b^*}$  to the buyer  $b^*$  with the highest resale value (i.e.,  $b^* = \arg \max_{b \in N_i} R_b$ ), if  $R_{b^*} > \mu v_H + (1 - \mu)\delta R_{b^*}$ . Otherwise, trader  $i$  makes an offer at  $\delta v_H$ , in sequence, to all buyers  $b \neq b^*$ . Each of these offers is accepted by high-value buyers and rejected by low-value buyers. When all these offers are rejected, it becomes common knowledge that all buyers  $b \neq b^*$  have a low value. From this point onwards, the game is analogous to a classical bilateral bargaining game with one-sided asymmetric information— e.g., see Ausubel and Deneckere (1989), and Fudenberg, Levine and Tirole (1985). In particular, since the buyer has only two valuations, the (generically) unique equilibrium is characterised in Hart (1989).<sup>9</sup>

We denote by  $R_i$  the resulting  $i$ 's equilibrium payoff of trader  $i$  in this auxiliary game. Once we have derived  $R_i$  for all  $i \in L_2$ , we move to  $L_3$ . We iterate until we reach  $L_{k^*}$  and we derive  $R_{s_0}$  which, in fact, is the expected payoff of the initial seller at the equilibrium that

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<sup>9</sup>Along the equilibrium path, seller  $i$  makes a sequence of offers at a price higher than  $\delta R_{b^*}$  and high-value buyer  $b^*$  accepts, with positive probability, each of these offers. Every time that an offer is rejected, the seller updates downward her belief that  $v_{b^*} = \delta v_H$ . When this belief is sufficiently low, the seller makes an offer at  $\delta R_{b^*}$ , and  $b^*$  accepts this offer with probability one.

we have constructed.

**Sufficiently patient traders.** When traders are sufficiently patient, the (low-value) seller, in each auxiliary game, never consumes in the first period—see Proposition B online Appendix A. In fact, there exists  $\bar{\delta} < 1$  so that for all  $\delta \in [\bar{\delta}, 1)$ , the following holds. First, a low-value seller  $i \in L_1$  makes a sequence of offers at  $\delta v_H$ ; each of these offers is accepted by high-value buyers, and, upon rejection of all offers,  $i$  consumes; the resale of  $i \in L_1$  is, therefore,

$$R_i = \delta v_H \mu \sum_{x=1}^{|N_i|} (1 - \mu)^{x-1} \delta^{x-1} + (1 - \mu)^{|N_i|} \delta^{|N_i|} v_L.$$

Second, a low-value seller  $i \in L_k$  with  $k \geq 2$  makes a sequence of offers at  $\delta v_H$  to all buyers different from the highest resale buyer  $b^* = \arg \max_{j \in N_i} R_j$ , and only high-value buyers accept these offers.<sup>10</sup> If all these offers are rejected,  $i$  engages in bilateral bargaining with  $b^*$ . Hence, for all  $i \in L_k$ ,

$$R_i = \delta v_H \mu \sum_{x=1}^{|N_i|-1} (1 - \mu)^{x-1} \delta^{x-1} + (1 - \mu)^{|N_i|-1} \delta^{|N_i|-1} \Pi_{ib^*}(\delta),$$

where  $\Pi_{ib^*}(\delta)$  is the payoff of seller  $i$  in the bilateral bargaining game with  $b^*$ . From Hart (1989) and Ausubel and Deneckere (1989), we know that when  $\delta$  approaches one, the payoff of seller  $i$ ,  $\Pi_{ib^*}(\delta)$ , converges to the resale value of  $b^*$ . Hence, the resale values of all traders  $i \in L_k$ , with  $k = 0 \dots k^*$ , as  $\delta$  approaches one, converge to:

$$R_i = \begin{cases} v_L, & \text{if } i \in L_0 \\ v_H - (1 - \mu)^{|N_1|} [v_H - v_L], & \text{if } i \in L_1 \\ v_H - (1 - \mu)^{|N_i|-1} [v_H - \max_j R_j] & \text{if } i \in L_k, k \geq 2. \end{cases} \quad (1)$$

To illustrate, consider Figure 1. When  $\delta$  approaches 1, we have that resale values converge to  $R_4 = v_H - (1 - \mu)^3 [v_H - v_L]$ ,  $R_1 = v_H - (1 - \mu) [v_H - R_4] > R_2 = v_H - R_4$  and  $R_s = v_H - (1 - \mu) [v_H - R_1]$ . Along the limit equilibrium, the seller asks a price  $v_H$  to 2 and,

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<sup>10</sup>Note that when  $\delta$  is sufficiently high,  $\delta R_j > v_L$  for all  $j \in L_k$  with  $k \geq 1$ .

upon rejection, sells the object to 1 and makes a profit equal to  $R_1$ . Trader 1 consumes if she has a high-value. If she has a low value, asks a price  $v_H$  to trader 3 and, upon rejection, sells the object to 4 and makes a profit of  $R_4$ . Trader 4 consumes if she has a high value. If she has a low value, she asks a price  $v_H$  to all her buyers, and, upon rejection, consumes.

### 3.2 Discussion of main insights

We first describe the equilibrium properties we have derived and then discuss their economic content. For simplicity of exposition, we focus on the limit case of  $\delta$  approaching one.

There are two types of offers along the equilibrium path: *resale offers* and *consumption offers*. A resale offer is an offer that both a high-value and a low-value trader accept. A consumption offer is at a price that makes a high-value trader indifferent between accepting and rejecting the offer, whereas a low-value trader strictly prefers to reject the offer. A buyer that accepts a consumption offer always consumes. In Figure 1, traders 1 and 4 receive a resale offer with positive probability, while traders 2, 3, 5, 6 and 7 receive only consumption offers.

The prices associated with resale offers are declining in time. Resale offers reflect the expected surpluses that future trades generate. For example, the resale offer of the initial seller to trader 1 converges to  $R_1 = v_H - (1 - \mu)^4(v_H - v_L)$ , while the resale offer that 1 makes to 4, converges to  $R_4 = v_H - (1 - \mu)^3[v_H - v_L]$ . Expected surpluses decrease as the object flows downstream from the initial seller because later traders have fewer resale opportunities than earlier traders; and upstream traders, having all the bargaining power, are able to fully extract the resale values of downstream ones. However, the price sequence is, in general, non-monotonic because each consumption offer is at a price that is higher than every price asked in future resale offers. This reflects the idea that, once a trader becomes an owner, she can exploit her local market power against some of her trading partners.

Finally, only traders who, along the equilibrium path, receive a resale offer with positive probability can obtain positive expected profit. We call these traders *dealers*. Low-value

dealers obtain zero profit, because they buy the object at a price that is equal to their discounted resale value. High-value dealers obtain a positive expected profit from buying at a price lower than their valuation and consuming. Furthermore, earlier high-value dealers obtain a higher expected payoff than later high-value dealers. Although earlier dealers acquire the object at a higher price than later dealers, the former have a higher probability of acquiring the good than the latter. The second effect dominates the former. The decline of the price associated with resale offers incorporates only the decline in the expected demand due to the rejection of consumption offers, whereas the difference in the probability of acquiring a good also accounts for the possibility that dealers consume the good themselves. In Figure 1, traders 1 and 4 are dealers, and high-value trader 1's expected payoff is  $(1 - \mu)^5[v_H - v_L]$ , which is higher than high-value trader 4's expected payoff of  $(1 - \mu)^6[v_H - v_L]$ .

The aforementioned properties extend to general networks. However, the structure of multilayer networks allows us to provide sharper equilibrium results with regard to network location and equilibrium payoffs, as we now illustrate.

For a path  $(j_0, j_1, \dots, j_m, j_{m+1})$  from the initial seller  $j_0 = s^0$  to a trader  $j_{m+1} \in L_0$ , we let  $q$  be  $(j_1, \dots, j_m)$ ; we denote by  $\mathcal{Q}$  all such sequence  $q$ . We also let  $k(q) = |N_s| + \sum_{j \in q} |N_j|$  be the sum of outward links generated by  $q$ , and we let  $d(q) = m$  be the number of traders in  $q$ . Note that  $k(q)$  is the number of traders that are potentially offered the good, should the trading route be, effectively, given by  $q$ . For instance, in a sparse network as the line (i.e.,  $E = \{s^0 2, 23, 34, \dots, n-1n\}$ ) there is only one sequence  $q = \{2, \dots, n\}$ ,  $k(q) = n - 2$  and  $d(q) = n - 2$ . Instead, in the star (i.e.,  $E = \{s^0 2, \dots, s^0 n\}$ ) the set  $\mathcal{Q}$  is empty; in this case, we set  $k = n - 1$ , and  $d = 0$ . In Figure 1,  $\mathcal{Q} = \{(1, 4), (2, 4)\}$ ,  $k(1, 4) = 7$ ,  $k(2, 4) = 6$ , and  $d(1, 4) = d(2, 4) = 2$ .

**Proposition 1.** *Consider a multi-layer network in which  $q^* = \arg \max_{q \in \mathcal{Q}} k(q) - d(q)$  is unique. There exists a  $\bar{\delta} < 1$  such that for all  $\delta \in [\bar{\delta}, 1)$ , there exists an equilibrium in which  $q^*$  is the set of dealers, the equilibrium payoffs of the initial seller converges, as  $\delta$  goes to 1, to  $\Pi_s = v_H - (1 - \mu)^{k(q^*) - d(q^*)}[v_H - v_L]$ , and the equilibrium payoff of high-value dealer  $j_l^*$*

converges to  $\Pi_{j_l^*} = [v_H - \Pi_s](1 - \mu)^l$ , for all  $l = 1 \dots m$ .

The proof of the proposition is in the online Appendix A. As traders become sufficiently patient, the object travels along path  $q^*$ , which maximises the number of traders that are offered the object, minus the number of dealers,  $d(q^*)$ . In Figure 1,  $q^* = (1, 4)$ .  $k(q^*) - d(q^*)$  is the total number of traders who receive a consumption offer and, therefore, measures the aggregate local monopoly power that is exercised in the network. This characterisation implies that the dealers are not necessarily traders with many connections. They are, instead, traders that give access to downstream networks where a few traders are connecting to many traders. The payoff of the initial seller increases with a change in the network that increases  $k(q^*) - d(q^*)$ ; dealers' payoff is declining with their distance from the initial seller.<sup>11</sup>

We conclude this section with two remarks. The first remark illustrates possible inefficiencies that equilibrium play can introduce in models of trading in networks. Consider the multi-layer network in Figure 3.2. Note that  $q^* = (1, 4)$  and, therefore, as  $\delta$  is sufficiently high, there is an equilibrium in which  $(1, 4)$  are dealers, and since  $k(1, 4) = 6$ , the ex-ante expected total surplus generated converges to  $v_H - (v_H - v_L)(1 - \mu)^6$ , as  $\delta$  goes to one.<sup>12</sup> Note, however, that ex-ante total surplus could have been higher if the object flew from  $s$  to 11 along  $q^{**} = \{2, 6, 7, 8, 9, 10\} = \arg \max_{q \in Q} k(q)$ , with offers to 1 and 5. In fact,  $k(q^{**}) = 8$ , and the ex-ante expected total surplus would have been  $v_H - (v_H - v_L)(1 - \mu)^8$ .

Although along  $q^{**}$ , there is a higher number of traders that can receive an offer relative to  $q^*$ , traders  $\{6, 7, 8, 9, 10, 11\}$  along  $q^{**}$  have no local monopoly power. This results in a low resale value of trader 2. The alternative path  $q^*$  is short, and each trader has substantial

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<sup>11</sup>A few examples serve as an illustration. When the network is sparse, as in the line,  $k(q^*) - d(q^*) = 0$ , the profit of the initial seller equals her valuation, and the  $l$ th dealer obtains  $[v_H - v_L](1 - \mu)^l$ . When the network is a star, the initial seller extracts all surplus.

<sup>12</sup>In this example, we have  $L_0 = \{3, 5, 11, 12\}$ ,  $L_1 = \{4, 10\}$ ,  $L_2 = \{1, 9\}$ ,  $L_3 = \{8\}$ ,  $L_4 = \{7\}$ ,  $L_5 = \{6\}$ ,  $L_6 = \{2\}$ ,  $L_7 = \{s\}$ .



local monopoly power, leading to a high resale value for trader 1.<sup>13</sup>

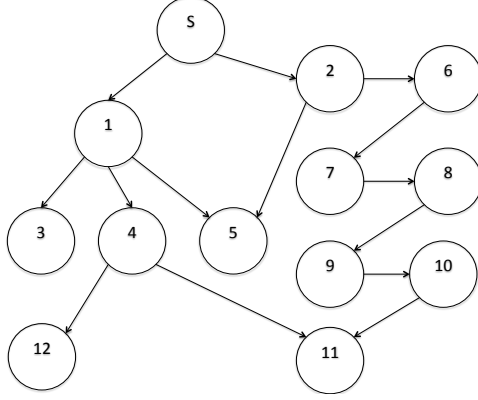


Figure 2: A remark on inefficiency

The second remark compares our findings with the equilibrium properties of a network trading game in which there is complete information about traders' valuations. As a concrete example, consider the trading network in Figure 1, and suppose that it is common knowledge that traders  $\{s, 1, 2, 4\}$  have a low value and that traders  $\{4, 5, 6, 7\}$  have value  $v_H$ . When  $\delta^2 v_H > v_L$ , along the equilibrium path,  $s$  sells to 1 at  $\delta^2 v_H$ , who buys and resell to trader 3 at  $\delta v_H$ . Note that the discounted price is constant over time; only the initial seller makes a positive profit and the outcome is allocative efficient.

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<sup>13</sup>Manea (2014) studies a similar model with two main differences: there is complete information and, in each round of trade, there is a random selection of proposer. The fact that the seller does not have full bargaining power—i.e., the selection of proposer is random—combined with strategic intermediation leads to inefficiencies. The inefficiency that we emphasise is, in contrast, derived by the combination of asymmetric information and strategic intermediation.

## 4 Equilibria in General Trading Networks

This section develops equilibrium properties for general trading networks. All proofs are in the Appendix. In a wMPBE, the acceptance strategy of a buyer depends only on her private information, the price asked, and the identity of the seller. This implies that at the beginning of each round of trade, a seller's equilibrium continuation payoff depends only on her private information and the state of beliefs.<sup>14</sup> We call *resale value* the continuation payoff of a low-value seller  $i$ , and we denote this quantity as  $R_i$ , omitting reference to beliefs.

**Proposition 2.** *In every wMPBE:*

- 1) *A high-value seller consumes the object.*
- 2) *A seller never makes an offer to buyer  $b$  at a price strictly below her discounted resale value,  $\delta R_b$ . Hence, a low-value trader, with the exception of the initial seller, makes zero profit.*

The first part of Proposition 2 can be interpreted as stating that in every wMPBE, there cannot be speculative bubbles. Because monetary transfers always cancel out, the total payoff that traders can jointly attain in this game cannot exceed the consumption value of a high-value trader. Therefore, a high-value seller can obtain a higher payoff by delaying consumption only if some other trader makes a negative expected payoff. This is, however, impossible because a trader can always reject all offers and obtain zero payoff. This result also guarantees that, at every round of trade, bilateral bargaining is with one-sided incomplete information.

The second part of the proposition states that a low-value trader cannot expect to make a positive profit from buying and reselling. Sellers are perfectly informed of the resale value

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<sup>14</sup>As a consequence, in equilibrium a seller never makes an offer that is refused with probability one. Otherwise in a wMPBE she would also find optimal to make the same offer in the next round.

of low-value traders and, since they hold all bargaining power, they never offer the object at a price below that. Hence, there are no arbitrage opportunities for low-value sellers.

Dynamic bargaining games with more than two players typically have multiple wMPBE—e.g., Fudenberg, Levine and Tirole (1987), and De Fraja and Muthoo (2000). Proposition 2 illustrates a number of properties that hold for all wMPBE, but we need to impose a further restriction in order to characterise in greater detail the price dynamic and the effect of a trader’s network location on her equilibrium payoff.

**Definition 1.** *A regular equilibrium is a wMPBE that satisfies the following conditions:*

- a. Skimming-Property: if at a certain (public) history low-value buyer  $b$  is indifferent between accepting and rejecting an offer, then that offer at the same (public) history is accepted with probability one by high-value buyer  $b$ .*
- b. Indifference-Property: At all (public) histories, if the supremum price that a high-valuation buyer accepts with probability one is higher than her discounted resale value, then the high-valuation buyer is indifferent between accepting and rejecting that price.<sup>15</sup>*

**Proposition 3.** *A regular equilibrium exists for every network trading game.*

In the classical bilateral bargaining problem with one buyer, the existence of an equilibrium is proved by induction on beliefs. In the two-value case, an equilibrium is first found for games in which the belief that the buyer has high value is low. Then, an equilibrium is shown to exist for a higher belief threshold, and so on.<sup>16</sup> The above logic does not extend to

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<sup>15</sup>More formally, fix a wMPBE. Let buyer  $\lambda_i^H(h \oplus (p, i))$  be the probability with which buyer  $i$  accepts the offer at price  $p$  at history  $(h \oplus (p, i))$  when his valuation is  $v_H$ . Let  $\mu(h)$  be the belief profile at history  $h$ . The equilibrium satisfies the indifference property if for all histories  $h$ ,  $p^* := \sup\{p : \lambda_i^H(h \oplus (p, i)) = 1\} \geq \delta R_i(\mu(h))$  implies that  $\delta v_H - p^* = \delta V_i^H(h_r^*)$ , where  $h_r^*$  is the history that ensues when an offer  $p^*$  following  $h$  is refused, and  $V_i^H$  represents the continuation payoff of the high-value trader  $i$ . Note that, since the equilibrium is wMPBE,  $\mu(h) = \mu(h')$  implies that  $\lambda_i^H(h \oplus (p, i)) = \lambda_i^H(h' \oplus (p, i))$ .

<sup>16</sup>See Ausubel and Deneckere (1989), Fudenberg, Levine and Tirole (1985), and Hart (1989) for details.

network trading games. With multiple buyers and resale, the behavior of a seller and a buyer negotiating over a certain offer depends not only on the common belief about the valuation of the informed party, but also on the common beliefs about the valuations of other traders. Indeed, a buyer may purchase the object to resell it at a later date to another trader, who can purchase the object to resell it to yet another trader at an even later date, and so on. Thus, the acceptance strategy of a buyer and the optimal offer for a seller depend on the entire profile of beliefs through the resale possibilities.

We have, therefore, pursued a different strategy. Instead of doing an induction on the vector of beliefs, a multi-dimensional object, our induction argument is on time. First, we prove that each network trading game with a finite horizon has a regular equilibrium. Then, using the fact that regular equilibria always end in finite time (see the ensuing proposition 4), we complete the proof by an argument analogous to that of Chatterjee and Samuelson (1987). Despite its simple logic, the proof is fraught with technical difficulties and relegated to online Appendix B.

We now motivate our definition of regularity. The skimming property ensures that, for each buyer, the common belief that she has high value decreases when she refuses an offer. This, in turn, is key to proving that the game always ends in finite time, a result that we use throughout. In bargaining games in which the payoff of buyers can accrue only by consuming the object (or in which resale values are not influenced by past offers as in multi-layer networks), the skimming property holds in every equilibrium. The reason is that high-value buyers lose more from delaying consumption than low-value buyers. Therefore, when a low-value trader is willing to accept and consume, the high-value trader has a strictly incentive to do the same. In our model, the willingness to pay of a low-value buyer today or at a future date may not be the same because past history influences resale opportunities. Hence, the simple argument provided above fails, and we have not been able to prove that the skimming property holds in every wMPBE.

The indifference condition allows us to prove that, in every regular equilibrium, offers are

at a price that makes either the low-value or the high-value buyer indifferent between accepting and rejecting. The indifference property is always satisfied in classic bargaining games with no resale. In the two-value case, it is guaranteed by the fact that the equilibrium payoff correspondence is monotonic in trader's beliefs. That is, the more likely it is that the buyer has a high value, the tougher is the seller, and the lower is the buyer's equilibrium payoff. However, when a seller faces multiple buyers and buyers can resell, a buyer's continuation payoff depends on the entire vector of beliefs, and we cannot guarantee that her equilibrium payoff correspondence is monotonic in the buyer's own belief. As a consequence, we cannot rule out equilibria in which the high-value buyer accepts up to a price that does offer her a strictly positive profit, but rejects with probability one any slightly higher price.

Summing up, focusing on regular equilibria allows us to rely on the following ancillary results.

**Proposition 4.** *In a regular equilibrium:*

- 1) *There exists  $T^* < \infty$  such that consumption takes place before round  $T^*$  in any on-path terminal history.*
- 2) *Each offer made to trader  $i$  is either a resale offer at price  $\delta R_i$  – i.e., an offer such that the low-value trader is indifferent between accepting and rejecting it – or a consumption offer – i.e., an offer such that the high-value trader is indifferent between accepting and rejecting it.*

Parts 1 and 2 of Proposition 4 are instrumental in proving our main propositions in the next section. In particular, the fact that the game ends in finite time guarantees that the sequence of offers is finite, and we can always find a last offer for every trader. This is important to use induction on the equilibrium trading path. The indifference condition allows us to link prices in consumption offers with the continuation payoff in the case of refusal, which embodies prices in future offers. Without this property, it is doubtful whether one could say something meaningful about the price dynamic.

## 4.1 Price dynamics

In a regular equilibrium, a high-value trader who acquires the object consumes it. A low-value trader who acquires the object makes a sequence of offers until the object is sold (unless, at some point, the discounted resale value of the object is lower than her own consumption value). In particular, the owner starts by making a sequence of consumption offers to some of her neighbors. If these offers are rejected, she makes a resale offer, which is accepted. Hence, an equilibrium terminal history of the game can be summarised by a list of consumption offers and resale offers, culminating in consumption.

For an arbitrary equilibrium terminal history, let  $p_\ell^s$  indicate the  $\ell$ th consumption offer that the  $s$ th seller makes,  $r_s^{s-1}$  indicate the resale offer that the  $s-1$ th seller makes to the  $s$ th seller, and  $(p_1^1, p_1^2, \dots, r_2^1, p_2^1, p_2^2, \dots, p_2^3, \dots)$  be the entire list of consumption and resale offers. The resale offer that seller  $s-1$  makes to  $s$  is

$$\begin{aligned} r_s^{s-1} &= \delta R_s \\ &= \delta [\alpha_1 p_1^s + \delta(1 - \alpha_1) \alpha_2 p_2^s + \dots + \Pi_{i=1}^k (1 - \alpha_i) \delta^{k-1} r_{s+1}^s], \end{aligned} \tag{2}$$

where  $\alpha_k$  indicates the probability that the consumption offer  $p_k^s$  is accepted.<sup>17</sup> In words, the discounted resale value of a trader who acquires the object equals, in equilibrium, the expected discounted sale price.

**Proposition 5.** *In a regular equilibrium, the following conditions hold along all terminal histories:*

- 1) *The discounted price of resale offers is decreasing in time – i.e., if  $r^t$  is the resale offer at  $t$  and  $r^{t'}$  is the resale offer at  $t'$ , with  $t' > t$ , then  $\delta^t r^t > \delta^{t'} r^{t'}$ .*

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<sup>17</sup>In general, a seller could be indifferent among different offers, in which case there may be equilibria at which the seller randomises. In these equilibria, the sequence of consumption and resale offers, along the equilibrium path, follows a stochastic process determined by the sellers' equilibrium strategies. Equation (2) must hold for every possible sequence of offers.

- 2) *The price asked in every consumption offer is greater than the first ensuing resale offer – i.e., if  $p^t$  is the consumption offer at  $t$  and  $r^{t'}$  is the resale offer at  $t' > t$ , and every offer from  $t + 1$  to  $t' - 1$  is a consumption offer, then  $p^t > r^{t'}$ .*

The first part of Proposition 5 illustrates that prices in resale offers embody all the public information available on the value of the object and that traders, over time, become more pessimistic about this fundamental. Consider, in fact, two consecutive resale offers, the first at period  $t$  and the second at period  $t' > t$ . The trader who receives the resale offer at  $t'$  knows that all consumption offers from period  $t + 1$  to period  $t' - 1$  have been rejected. When a trader rejects a consumption offer, all other traders update downward their belief that she has a high consumption value; thus, the trader who receives the resale offer at period  $t'$  is more pessimistic than the trader who receives the resale offer at period  $t$ . Hence, in our bargaining game where sellers make all the offers, the price asked in resale offers, evaluated at a given fixed date, declines over time.<sup>18</sup>

The result that consumption offers are above the ensuing resale offers reflects the ability of sellers to use their local bargaining power to demand a high price from some of their trading partners, before passing the object to another dealer.

A consequence of Proposition 5 is the non-monotonicity in the price demanded over time. This follows by combining two observations. First, equilibrium discounted resale values decline over time (part 1 of Proposition 5). Second, in equilibrium, the discounted resale value of an agent equals the expected discounted price at which she will sell the object (equation 2).

Finally, we note that, while demanded prices are non-monotonic in time, actual transaction prices decline as time passes, with the possible exception of the last accepted offer. In fact, after an offer is accepted, trading will continue only if a low-value trader acquires the

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<sup>18</sup>We remark that proving this proposition is not straightforward because the resale value of seller  $s + 1$  depends potentially on the entire profile of beliefs and can be influenced by the set of consumption offers made by seller  $s$ .

object. Hence, all accepted offers, possibly except the last one, must be resale offers.

## 4.2 Payoff ranking

We now discuss equilibrium payoff across traders, excluding the initial seller. In view of Proposition 4, we know that, in a regular equilibrium, trader  $i$  will: (i) either never receive an offer, or (ii) only receive consumption offers, or (iii) receive a combination of both consumption and resale offers. It is clear that if a trader receives no offer, she obtains zero profit. Assume now that trader  $i$  receives only consumption offers, and focus on the last consumption offer that trader  $i$  receives; this offer exists because the game ends in finite time. By Proposition 4, this offer must leave high-value trader  $i$  indifferent between accepting and rejecting it; since it is the last offer she receives, her continuation payoff from rejecting is zero, and, therefore, it must come at a price of  $\delta v_H$ . This implies that all previous offers must be at a price of  $\delta v_H$ , as well. Trader  $i$ 's payoff is, therefore, zero.

Hence, only traders who receive a resale offer with positive probability can make a positive profit. We call these traders *dealers*. Proposition 2 tells us that in every WMPBE, all low-value dealers obtain zero profit. Hence, consider a high-value dealer. The resale value of every trader is bounded above by  $\delta v_H$ , and, therefore, a resale offer is never at a price higher than  $\delta^2 v_H$ . This implies that, conditional on receiving the resale offer, a high-value dealer must obtain at least  $\delta v_H(1 - \delta) > 0$ . Hence, in a regular equilibrium, high-value dealers obtain a positive expected payoff. We now examine how the position of a dealer along the equilibrium path affects her payoff at the start of the game.

**Proposition 6.** *In a regular equilibrium, if all offers to trader  $j$  are always preceded by an earlier resale offer to dealer  $i$ , then the expected utility of high-value dealer  $i$  is strictly greater than that of trader  $j$ .*

Note that the payoff of a high-value dealer is determined by the probability of receiving her first resale offer, as well as the price of the resale offer. Since the price of resale offers



declines along the equilibrium path, conditional on receiving the resale offer, later high-value dealers obtain a higher surplus. However, earlier high-value dealers have a higher probability of receiving the resale offer. Recall that the resale offer to trader  $i$  comes at a price that is equal to the expected discounted sale price, should trader  $i$  accept the resale offer and then resell the object. Hence, the decline in the price of resale offers offsets the expected demand of the traders that receive consumption offers, but it does not incorporate the possibility that dealers may consume the object. As a consequence, the price differential between later and earlier high-value dealers does not compensate them for the decrease in the probability of obtaining the offer.

Proposition 6 has a sharp implication for the relation between the location of traders in the trading network and their payoffs, as shown in the next corollary. We say that trader  $j$  is *essential* for trader  $i$  if  $j$  belongs to every path from the initial seller to trader  $i$ . Trader  $i$  is an *end-trader* if  $ij \notin E$  for all  $j$  (e.g., in a multi-layer network traders in  $L_0$  are end-traders).

**Corollary 1.** *In a regular equilibrium:*

- 1) *Every end-trader obtains zero profit.*
- 2) *If trader  $j$  is essential for trader  $i$ , then high-value trader  $j$  obtains a higher expected profit than high-value trader  $i$ .*

The corollary points out the importance of the location of a trader in a trading network. It emphasizes that traders who are essential in connecting other traders to the initial owner obtain a payoff advantage. Among the essential traders, the ones who are located more upstream in the network (i.e., closer to the seller) enjoy higher surplus. As we illustrated in Section 3, sharper results can be obtained when more structure is imposed on the architecture of the trading network.

## 5 Discussion of the Main Assumptions

The assumptions that we employ are all relatively standard in the literature, and were chosen to build the simplest possible model that incorporates resale and asymmetric information. Nonetheless, a few of them deserve further discussion.

We assume actions are observable. This assumption is at odds with the idea that information is dispersed across traders, but relaxing it would entail considerable technical difficulties. Suppose that only the buyer who received the offer could observe it. Then, as the game proceeds, two traders may end up with different beliefs about the value of a third trader. For instance, if  $s$  has made an offer to  $b_1$ , she may have a belief about the value of  $b_1$  different than the one held by  $b_2$ , who may conjecture that  $b_1$  has only received the offer with a certain probability. Higher-order beliefs would then start to play a role in the analysis. The assumption can be partially relaxed in the class of multi-layer networks, where we could assume that the actions in any round are observed only by the neighbors of the seller.<sup>19</sup>

The assumption that each trader has only two possible values assures that bargaining takes place under one-sided asymmetric information. This is necessary to progress in the analysis, given the the additional intricacies that the opportunities of resale introduce.<sup>20</sup> However, a minimal feasible generalisation of our information structure consists in assuming that traders are heterogeneous with regard to their low value (i.e.,  $v_i \in \{v_L^i, v_H\}$  with  $v_L^i < v_H$  for all  $i \in N$ ). All our results carry over under this specification. The willingness to pay of a low-value buyer is the discounted continuation payoff she obtains by purchasing the object:  $\delta v_L$  if she consumes or her discounted resale value if larger. Since resale values are

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<sup>19</sup>In the equilibrium that we characterize the behavior of buyers is not influenced by the belief they have about traders who moved earlier in the game, with the possible exception of the seller and her neighbors.

<sup>20</sup>Zheng (2002) studies a dynamic optimal auction model with resale. The initial seller and all the buyers are part of a complete network. He focuses on the attainability of Myerson's second-best revenue and restricts attention to a special class of value distributions. Commenting on the difficulties that he encountered, Zheng notes that "extending the positive result beyond the special case is difficult if not impossible."

generally different across traders, our current analysis already deals with heterogeneity in the willingness to pay of low-value buyers.

For the sake of tractability we also assume that the seller makes all the offers: allowing the informed party to make offers increases her signaling possibilities substantially. While providing a formal analysis is outside the scope of this paper, we recognize that our results hinge on this assumption.<sup>21</sup> Nonetheless, we expect them to be robust to giving a certain amount of bargaining power to buyers (except perhaps in limit case of no discounting). In fact, the fundamental force toward price decline that we have uncovered (i.e., the decrease in everybody's resale value as consumption offers are being made and refused) would still be present with alternating-offer bargaining.

Finally, in our model, any two agents have the same information about the valuation of any third agent. However, an alternative natural assumption would be that each trader knows her valuation and, in addition, the valuation of her closest trading partners, but remains uncertain about the valuation of all other traders; (with this information structure being common knowledge). For multi-layer networks, we can slightly modify the construction derived in Section 3.1 to characterise an equilibrium in this environment.<sup>22</sup>

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<sup>21</sup>Suppose the game has alternating-offer and an initial seller  $s$  is facing a single buyer  $b$ . Let  $v_L \equiv 0$ . If  $s$  is sufficiently pessimistic about the value of  $b$ , then she will bargain exclusively with the low-value  $b$ , and will only extract a share of the available gains from trade (e.g.,  $\delta R_b / (1 + \delta)$  assuming Rubinstein's bargaining game). Hence, for sufficiently high discount factor,  $s$ 's resale value will be lower than the discounted resale value of  $b$ . Also note that the payoff of  $s$  could be lower than that of  $b$ . See Ausubel, Cramton and Deneckere (2002) for the analysis of the buyer-seller game.

<sup>22</sup>To see this, consider Figure 1 and assume that  $\delta v_H > v_L$ . Low-value trader 4 can ask and get  $\delta v_H$  if one of her trading partner has high value; otherwise, she consumes, and gets  $v_L$ . Suppose that low-value trader 1 has the object. Again, if either 3 or 4 has a high value, then trader 1 obtains  $\delta v_H$ . If they both have a low value, then trader 1 faces a bargaining game where trader 3 has valuation  $\delta v_L$  with probability one, whereas trader 4 has valuation  $\delta v_L$  with probability  $(1 - \mu)^3$  and has valuation  $\delta v_H$  otherwise. By deriving the equilibrium in this subgame in the standard way, we can compute how much the initial seller  $s$  expects low-value trader 1 to be willing to pay in the event that all of 1's trading partners have a low value. The

## 6 Concluding Remarks

Our model serves as a useful abstraction of many decentralised markets in which trade proceeds from a producer to a final customer, travelling through many possible layers of intermediation.

Two features of our characterisation are in stark contrast to results from models with complete information, such as the ones we discussed in the introduction. First, the sequence of prices at which the object is exchanged is decreasing in time (with the possible exception of the last offer). Dealers only break-even in expectation, while they realize a loss every time they need to transfer the object to another dealer. Instead, with complete information, dealers know exactly the price at which they will sell and therefore non-decreasing prices are required to guarantee the profitability of their trade.

Second, the object will often cycle in the network: along the equilibrium path, a trader can buy and resell multiple times. The key observation is that, as traders become more patient, the resale value of a trader who can directly or indirectly access (at least) a trader who has high-value with positive probability is higher than the consumption value of any low-value trader. Hence, the object tends to move toward traders whose value is still uncertain. It follows that, in a strongly connected network, as traders become fully patient, the object will keep circulating until it becomes common knowledge that all traders in the network have low value – at which point the current owner will consume. This immediately implies that, in the limit as the discount factor goes to one, the object will indeed cycle with some probability in all networks where cycling is required for the object to visit all traders.<sup>23</sup>

Hence, our results complement the insights originating from models with complete in-

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method to construct the equilibrium then follows the same logic that we developed in Section 3.1.

<sup>23</sup>Remarkably, the discussion above suggests that the limit outcome of a strongly connected network trading game would be ex-post efficient – and therefore exhibit the standard Coasian dynamics – if there existed a uniform bound, independent of the discount factor, on the number of rounds in which the game terminates.

formation and can help to shed light on data where decreasing exchange prices and cycles are observed. For instance, consider the May 6 2010 Flash Crash, a US trillion-dollar stock market crash that lasted about 36 minutes. According to various sources (see e.g., Kirilenko et al. (2014)), the crash was characterised by a high-frequency of inter-dealer trading (hot-potato trading), with prices for a set of securities quickly decreasing in time, before finally rebounding.

## 7 Appendix

### Proof of Proposition 2.

**First part.** We first note that, at each round  $t$ , the total  $t$ -period payoff (i.e., sum of payoffs over all players) is  $v_i$  if there is consumption by player  $i$  and 0, otherwise. Consequently, regardless of the realization of types, the maximal discounted sum of total payoffs from period  $t + 1$  onwards is  $\delta^{t+1}v_H$  (i.e., when consumption takes place at period  $t + 1$ ).

Next, assume, in contradiction with the proposition, that there exists an equilibrium where player  $i$  owns the object at period  $t$ , his type is  $v_H$ , and he does not consume. To be an equilibrium, it must be that player  $i$ 's equilibrium payoff from period  $t$  onwards is at least  $\delta^t v_H$ , since he can consume the object at period  $t$ . However, given that the total payoff at period  $t$  is 0 (since player  $i$  does not consume the object) and the maximal discounted sum of total payoffs from period  $t + 1$  onwards is  $\delta^{t+1}v_H$ , it must be that at least one type of some player expects a strictly negative equilibrium payoff from period  $t$  onwards. This is impossible, as each type of each player can guarantee herself a payoff of at least 0 by rejecting all offers.

**Second part.** The following notation simplifies the exposition: (i) for a given equilibrium,  $V_i^H(h)$  (or  $V_i^L(h)$ ) represents the continuation payoff of player  $i$  with value  $v_H$  (or  $v_L$ ) at history  $h$ ; (ii) the operator  $\oplus$  represents history concatenation; and (iii) in combining histories, a generic offer is denoted  $(i, p)$  where  $i$  the player who receives the offer and  $p$  is

the asked price; consumption is denoted  $c$  and non consumption  $nc$ ; finally acceptance of an offer is denoted  $a$  and rejection  $na$ ; (iv) we denote by  $\mu_i(h)$  the (common) probability that  $i$  has high-value at (public) history  $h$  and we denote by  $\mu(h) = (\mu_1(h), \dots, \mu_n(h))$  the (common) profile of beliefs.

In what follow, we show that in a wMPBE, at any history  $h$  where  $s$  is a seller,  $s$  never makes an offer  $(i, p)$ , to some  $i \in N_s$ , such that  $p < V_i^L(h \oplus nc \oplus (i, p) \oplus a)$ . The following lemma is instrumental in proving this result.

**Lemma 1.** *In a wMPBE, if trader  $i$  is not the current owner at  $h$  and  $\mu_i(h) = 1$ , then  $V_i^L(h) = V_i^H(h) = 0$ .*

**Proof of Lemma 1** We show that no seller will ask to  $i$  a price strictly below  $\delta v_H$  starting from  $h$ . Take a seller starting from  $h$  and suppose that she has a high-value. In light of the first part of Proposition 2, the seller can obtain  $\delta v_H$  by consuming in the next round and therefore she will make no offer below  $\delta v_H$  (unless such offer was refused with probability one, which can't happen in a wMPBE: since an offer rejected with probability one does not affect belief, it would be optimal for the seller to repeat it in the next round, and this is clearly impossible).

Next, we consider low-value sellers. Let  $\underline{p}$  be the infimum price that is ever offered to  $i$  from  $h$  onward at both on-path and off-path histories. We show that  $\underline{p} = \delta v_H$ .

By way of contradiction, suppose that  $\underline{p} < \delta v_H$ . By definition of  $\underline{p}$ , there must exists an history  $h'$  such that  $i$  is offered  $p' \in [\underline{p}, \delta v_H - \delta(\delta v_H - \underline{p})]$ . Note that  $\delta v_H - \delta(\delta v_H - \underline{p})$  is the price that makes player  $i$  of type  $v_H$  indifferent between accepting that price and consume tomorrow, or rejecting and accept  $\underline{p}$  tomorrow and then consume. By construction of  $\delta v_H - \delta(\delta v_H - \underline{p})$ , it is a strict best response of high-value player  $i$  to accept the offer at  $p'$ . Therefore the seller that makes offer  $(i, p')$  obtains an expected payoff of  $p'$  because she anticipates that the offer is accepted with probability one, and that  $i$  consumes the object; recall that  $\mu_i(h) = 1$  and therefore  $\mu_i(h') = 1$  because degenerate beliefs are never updated. But note that the seller can strictly increase her profits by making offer  $(i, p' + \varepsilon)$ , with

$p' + \varepsilon < \delta(\delta v_H - \underline{p})$ ; in fact, player  $i$  has also a strict incentive to accept this offer. This concludes the proof of Lemma 1. ■

Now, for a player  $i \in N$  we define the set  $H(i)$  as the set of histories, including off-path histories, such that: (a) player  $i$  receives an offer at  $h \in H(i)$  and (b) no prior (observable) deviation has taken place at the consumption stage. It is important to note that at any history  $h \in H(i)$ , the seller that makes the offer to  $i$ , say  $s$ , has  $\mu_s(h) = 0$ . This holds by combining the three facts: part (b) in the definition of  $H(i)$ , high-value owners are always expected to consume (from part 1 of Proposition 2), and degenerate beliefs are never updated.

Let  $p(h)$  be the price asked in  $h$ , and let  $\bar{u} = \sup_{h \in H(i)} \{V_i^L(h \oplus (i, p(h)) \oplus a) - p(h)\}$  be the maximal payoff attainable by the low-value player  $i$  from accepting an offer. We show that  $\bar{u} = 0$ .

By way of contradiction, assume that  $\bar{u} > 0$ . Note that the maximal payoff attainable by  $i$  by refusing some offer is at most  $\delta\bar{u}$ . Second, observe that by definition of  $\bar{u}$  as a supremum, there exists  $\bar{h} \in H(i)$  and offer  $(i, p(\bar{h}))$  such that  $\delta V_i^L(\bar{h} \oplus nc \oplus (i, p(\bar{h})) \oplus a) - p(\bar{h}) \in (\delta\bar{u}, \bar{u})$ .

We next argue that  $V_i^L(\bar{h} \oplus nc \oplus (i, p) \oplus a) = V_i^L(\bar{h} \oplus nc \oplus (i, p(\bar{h})) \oplus a)$  for any  $p$ . This follows because the continuation payoff of the low-value  $i$  in  $\bar{h} \oplus nc \oplus (i, p) \oplus a$  only depends on the beliefs over traders different from  $i$ . In fact, if she consumes, her payoff is  $v_L$ . If she does not consume, the belief of the other traders with regard to  $i$  becomes degenerate (because of part 1 Proposition 2) and, therefore, it stays constant in any future on-path history. Furthermore, since  $\bar{h} \in H(i)$ , we have that  $\mu_s(\bar{h}) = 0$ , and therefore this belief is not affected by the seller posting a different price.

Hence, there exists  $p' \in (p(\bar{h}), V_i^L(\bar{h} \oplus nc \oplus (i, p(\bar{h})) \oplus a) - \delta\bar{u})$ , which is accepted with probability one by the low-value player  $i$ , as it provides a payoff larger than  $\delta\bar{u}$ . It follows that  $p(\bar{h})$  and  $p'$  are also accepted by the high-value player  $i$  with probability one, as refusal would imply  $\mu_i(\bar{h} \oplus nc \oplus (i, p(\bar{h})) \oplus na) = 1$  and provide zero payoff to  $i$  in light of Lemma 1.

But then the seller can strictly increase her payoff at  $\bar{h}$  if she makes offer  $(i, p')$ , instead

of the postulated optimal offer  $(i, p(\bar{h}))$ . This is the case because both  $p'$  and  $p(\bar{h})$  are accepted with probability one by player  $i$ , and the continuation payoff of the seller, following acceptance of  $(i, p(\bar{h}))$  or following acceptance of  $(i, p')$  must be the same. In fact, in a weak-Markov equilibrium the continuation payoff of the seller following an offer to  $i$  which is accepted can only depend on  $i$  (i.e., the new seller) and on the state of beliefs at the moment in which  $i$  acquires the object. We have already argued that the belief about the seller cannot change following the two different offers because  $\mu_s(\bar{h}) = 0$ . The belief about  $i$  is identical in both cases because  $i$  accepts with probability one both offers. ■

### Proof of Proposition 4

**First part.** The proof of this part follows closely the proof for an analogous statement in Fudenberg, Levine and Tirole (1985) (see Lemma 3). For a related proof in the case of a seller bargaining with multiple players see Lemma 2 in DeFraja and Muthoo (2000).

By way of contradiction, assume that there is a set  $H^\infty$  of infinite public histories that has strictly positive probability in the support of the equilibrium.

First, note that if at some generic beginning-of-period history  $h$  we have  $\mu_i(h) = 0 \forall i \in N$ , then the game ends at  $h$  with probability one. Consumption takes place at  $h$  since the maximum attainable continuation payoff for any type in any equilibrium where there is no consumption at  $h$  is equal to  $\delta v_L$ .

Then, for any  $h \in H^\infty$ , define  $\{h^0(h), h^1(h), \dots\}$  as the beginning-of-period sub-histories of  $h$  (i.e.,  $h^0(h) = \emptyset$  and  $h^t(h)$  is an history which includes all events that took place in  $h$  from period 0 up until and including the acceptance decision in round  $t - 1$ ). The skimming property implies that  $\mu_i(h^t(h))$  is weakly decreasing in  $t$ . By the argument above, and remembering that degenerate beliefs never change, for any  $h \in H^\infty$ , there exists (maximal) non-empty  $Z(h) \subset N$  such that  $\forall i \in Z(h)$  and  $t \geq 1$ , we have  $\mu_i(h^t(h)) > 0$ .

Fix  $h \in H^\infty$  and  $i \in Z(h)$ , then  $\forall(\epsilon, k)$  with  $k \geq 1$  and  $\epsilon > 0$  there exists  $t_i^h(\epsilon, k)$  such that  $\forall t > t_i^h(\epsilon, k)$ ,  $\mu_i(h^{t+k}(h)) - \mu_i(h^t(h)) < \epsilon$ . If this was not the case, there would exist



a time period  $T_i^*$  such that  $\mu_i(h^{T_i^*}) = 0$  (for details see DeFraja and Muthoo (2000), page 863). Then, let  $t^*(\epsilon, k) = \sup_{h \in H^\infty} \sup_{i \in Z(h)} \{t_i^h(\epsilon, k), \tilde{t}^h\}$ , where  $\tilde{t}^h$  is the period such that, for all  $t \geq \tilde{t}^h$  we have  $\mu_j(h^t(h)) = 0$  for all  $j \neq Z(h)$ . If the supremum is not finite, we select  $t^*(\epsilon, k)$  arbitrarily large, in such a way that the effect on expected payoffs of histories that exceed the bound is made negligible. Without loss of generality assume that there are only infinite histories lasting beyond  $t^*(\epsilon, k)$ .

Next, let  $Z = \cup_{h \in H^\infty} Z(h)$  and suppose that all offers to  $i \in Z$  between  $t > t^*(\epsilon, k)$  and  $t+k$  along  $h$  come at prices above the resale value of the low-value. (Otherwise by the skimming property player  $i$  of high-value would accept with probability one and consume, contrary to the stated assumption). Note that then the probability with which the high-value  $i$  accepts an offer between  $k$  and  $t+k$  along  $h \in H^\infty$  is less than or equal to  $\mu_i(h^{t+k}(h)) - \mu_i(h^t(h))$ . This follows from Bayesian updating given that the offers must come at price that the low-value refuses with probability one.

We are now ready to conclude the proof by contradiction. We now argue that for any  $h \in H^\infty$  there exists a period at which the owner of the good consumes (i.e., consumption almost surely takes place, hence the set  $H^\infty$  has zero probability). In fact, suppose that at some node  $h^{t^*(\epsilon, k)}(h)$  in history  $h \in H^\infty$ , the seller's continuation payoff is bounded from above by  $\delta v_H(1 - (1 - \epsilon)^z) + \delta^k v_H$  where  $z$  is the numbers of traders in  $Z$ . Then for all  $\epsilon$  small and  $k$  large enough this continuation would be strictly lower than  $v_L$  which is the payoff from consuming. (Given that there is discounting and equilibrium per-period payoffs are bounded, this follows even in case  $t^*(\epsilon, k)$  has been selected arbitrarily large). A contradiction to the hypothesis that the game continues beyond  $t^*(\hat{\epsilon}, \hat{k})(h)$ .

To conclude the proof we now verify that the payoff of the seller is bounded above by  $\delta v_H(1 - (1 - \epsilon)^z) + \delta^k v_H$ . To see this observe that: (i) any offer that high-value traders accept with some probability, between between  $t^*(\epsilon, k)$  and  $t^*(\epsilon, k) + k$ , regardless of the history, comes at a price at most equal to  $\delta v_H$ ; (ii) the probability that such offers are accepted is at most  $\epsilon$  for each  $i \in Z$  between  $t^*(\epsilon, k)$  and  $t^*(\epsilon, k) + k$ , regardless of the history; (iii) offers

to a low-value traders, if any, come to traders with low-value with probability one and at a price equal to the continuation payoff of such traders, which can only make the bound less tighter.

**Second part.** First recall that, along the path, a seller never makes an offer that is rejected with probability one. Next, note that part 2 of Proposition 2 implies that if the seller makes an offer to  $i$ , then she will ask either her discounted resale value, or a strictly higher price. If she asks the discounted resale value, then, by definition, low value trader  $i$  is indifferent between accepting and rejecting the offer. If the seller asks a price strictly higher than  $i$ 's discounted resale, then the low-value trader  $i$  rejects it with probability one. Since the seller never makes offers that are rejected with probability one, the high-value trader  $i$  must accept the offer with strictly positive probability. If the high-value trader accepts with probability one, then the indifference property of regular equilibria guarantees the result. If the high-value buyer mixes between acceptance and rejection, then the result is true in light of the indifference condition for mixing. ■

**Proof of Proposition 5.** We prove part 2). Part 1) follows by part 2) and equation (2) from the main text. The proof is by induction. We discuss the base case and then move to the induction step. We focus on histories that terminate with consumption in finite time.

As our induction *base case*, we show that there exists  $t^*$  such that the statement is true for any offer made from round  $t^*$  onward. By part 1 of Proposition 4 there exists a  $T^*$  that is the maximum round in which consumption takes place. Let  $t^* + 1 < T^*$  be the latest round in which a resale offer is made in equilibrium. Let  $r^*$  be the price asked in the unique resale offer made in that round. (The argument is analogous if (i) the seller randomizes among different resale offers in  $t^* + 1$  or (ii) there are multiple on-path histories on which a resale offer is made in the  $t^* + 1$  period.) We show that, along the equilibrium path, for all consumption offers the price  $p^*$  asked in round  $t^*$  must be strictly greater than  $r^*$ . Assuming that  $p^*$  is followed by  $r^*$  with probability one (the conclusion holds a fortiori otherwise), there are two possibilities. First, the offer in  $t^*$  goes to a player  $j$  which is different than the player that receives offer at

price  $r^*$ ; in this case,  $p^* = \delta v_H$  because player  $j$  is indifferent between accepting and rejecting and there are no further resale offers after  $t^* + 1$ ; therefore  $p^* = \delta v_H > r^* = \delta R^*$ , because  $v_H > R^*$ . Second, the offer in  $t^*$  goes to the same player that receives the offer at price  $r^*$ ; in this case, since  $p^*$  is a consumption offer, by part 2 of Proposition 4 the indifference condition for the high-value dictates that  $\delta v_H - p^* = \delta(\delta v_H - r^*)$ . To see this point observe that, by the skimming property, an high-value player must accept a resale offer. We conclude that  $p^* = \delta v_H(1 - \delta) + \delta r^* > r^*$ , given that  $\delta v_H > r^* = \delta R^*$ .

The *induction step*, consists in showing that the statement is valid for any consumption offer  $(i, p)$  in round  $t$  made by some seller  $s$  after some history  $h^t$ , given that it is true for any consumption offer made from round  $t+1$  onward. Note that since  $(i, p)$  is a consumption offer, consumption by the high-value follows acceptance. Henceforth, to simplify the exposition, we focus on the case in which no other consumption offer is made to  $i$  following a rejection of offer  $(i, p)$  and consumption offers are accepted with probability one, but with extra notational burden the proof extends to this case as well.

The following notation is used in the rest of the proof. Starting with the rejection of offer  $(i, p)$  made by seller  $s$  and, restricting attention to the equilibrium path, consider all the different sequences of offers leading to a resale offer being made by  $s$ . In those cases in which there are on-path terminal histories where  $s$  makes more than one resale offer, then we construct a single sequence, truncating it at the first resale offer that  $s$  makes. The set of all such sequences is denoted  $\Sigma$ . For a given  $\sigma \in \Sigma$ , call  $r(\sigma)$  the price quoted in the last offer of  $\sigma$  (which is a resale offer),  $t(\sigma)$  the length of the sequence, and call  $1 - \beta(\sigma)$  the probability that the resale offer  $r(\sigma)$  occurs in equilibrium, evaluated conditional on refusal of offer  $(i, p)$ .<sup>24</sup>

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<sup>24</sup>To fix ideas, suppose that after  $(i, p)$  is refused,  $s$  consumes with probability  $x_1$  and otherwise makes an offer  $(j, p)$ ; that such offer is accepted with probability  $x_2$  and otherwise refused; that in case of refusal  $s$  makes offer  $(j', p')$  with probability  $x_3$  and offer  $(j'', p'')$  with the remaining probability, and that both of these offers are resale offers, both accepted with probability one. Then  $\Sigma \equiv \{\sigma_1, \sigma_2\}$  where  $\sigma_1 = \{(j, p), (j', p')\}$  and  $\sigma_2 = \{(j, p), (j'', p'')\}$ ,  $t(\sigma_1) = t(\sigma_2) = 2$ , and  $1 - \beta(\sigma_1) = (1 - x_1)(1 - x_2)x_3$ ,  $1 - \beta(\sigma_2) = (1 - x_1)(1 - x_2)(1 - x_3)$ .

Analogously, we denote by  $\Sigma_i$  the set of sequences of offers starting with the rejection of offer  $(i, p)$  and terminating with a resale offer to  $i$ . In those cases in which there are on-path terminal histories where  $i$  receives more than one resale offer, then we construct the set  $\Sigma_i$  truncating each sequence at the first resale offer to  $i$ . For any given  $\sigma \in \Sigma_i$  we call  $r^i(\sigma)$  the price of the resale offer to  $i$  that ends the sequence, we call  $t(\sigma)$  the length of the sequence, and we call  $(1 - \beta(\sigma))$  the probability that the offer that ends the sequence occurs according to the equilibrium, evaluated after the refusal of  $(i, p)$ . Observe that both  $\Sigma$  and  $\Sigma_i$  are either empty or well defined, given that the game ends in finite time.

We can begin the analysis, by noting that if  $\Sigma$  is empty then the statement is trivially true. Similarly, if  $\Sigma_i$  is empty, then trader  $i$ , upon rejecting  $(i, p)$ , does not receive any further offer and obtains a continuation of 0. The indifference condition with respect to consumption offer  $(i, p)$  implies, then, that  $p = \delta v_H$ . Since  $\delta v_H > r(\sigma)$  for all  $\sigma \in \Sigma$ , we obtain that  $p > r(\sigma)$  for all  $\sigma \in \Sigma$ .

For the case in which  $\Sigma_i$  is non empty, the induction argument is concluded by means of the two following lemmas. Lemma 2 provides a lower bound of the price  $p$  associated to consumption offer  $(i, p)$ . This lower bound is obtained by using the indifference property of regular equilibria. Lemma 3 provides an upper bound of the first resale offer following refusal of  $(i, p)$ . This uses our induction hypothesis.

**Lemma 2.** *There exists  $\hat{\sigma} \in \Sigma_i$  such that  $p > \delta v_H \beta(\hat{\sigma}) + (1 - \beta(\hat{\sigma}))r^i(\hat{\sigma})$*

**Proof of Lemma 2.** In a regular equilibrium, the high-value trader  $i$  is indifferent between accepting and rejecting the consumption offer  $(i, p)$ . Therefore, recalling that the high-value  $i$  must accept any resale offer with probability one and that his payoff is zero whenever she does not receive a resale offer (as in that case he only obtains offers at  $\delta v_H$ ), we have

$$\delta v_H - p = \int_{\Sigma_i} (1 - \beta(\sigma)) \delta^{t(\sigma)} (\delta v_H - r^i(\sigma)) d\sigma.$$

We conclude that there exists  $\hat{\sigma} \in \Sigma_i$  such that:

$$\delta v_H - p \leq \delta (1 - \beta(\hat{\sigma})) \delta^{t(\hat{\sigma})} (\delta v_H - r^i(\hat{\sigma})). \quad (3)$$

and therefore

$$\begin{aligned} p &\geq \delta v_H - \delta(1 - \beta(\hat{\sigma}))\delta^{t(\hat{\sigma})}(\delta v_H - r^i(\hat{\sigma})) \\ &> \delta v_H - (1 - \beta(\hat{\sigma}))(\delta v_H - r^i(\hat{\sigma})). \end{aligned}$$

where the first inequality is a rewriting of inequality (3), and the second inequality follows by replacing  $\delta^{t(\hat{\sigma})} < 1$  with 1. ■

**Lemma 3.** *For all  $\sigma \in \Sigma$ , we have  $r(\sigma) \leq \delta v_H \beta(\hat{\sigma}) + (1 - \beta(\hat{\sigma}))r^i(\hat{\sigma})$ , where  $\hat{\sigma} \in \Sigma_i$  is derived from Lemma 1.*

**Proof of Lemma 3.** For any  $\sigma \in \Sigma$ , let  $V_s^L(\sigma)$  be the continuation payoff of the seller following rejection of  $(i, p)$ , computed assuming that when each of her offer is refused she makes offers according to  $\sigma$ .<sup>25</sup>

It is immediate to observe that  $V_s^L(\sigma) = V_s^L(\sigma')$  for any  $\sigma, \sigma' \in \Sigma$  (recalling that  $\Sigma$  only contains sequences of on-path offers). In fact, the low-value seller  $s$  expects no further surplus in the continuation game ensuing after any of her offer is accepted and both  $r(\sigma)$  and  $r(\sigma')$  are accepted with probability one. Hence, if, by contradiction,  $V_s^L(\sigma) > V_s^L(\sigma')$ , the seller would strictly prefer to implement the sequence of offers  $\sigma$ , and therefore  $\sigma'$  could not be part of the equilibrium.

Next, by the induction hypothesis, every price that  $s$  offers, following the rejection of offer  $(i, p)$  until her resale offer  $r(\sigma)$ , is a consumption offer and is strictly larger than  $r(\sigma)$ . Therefore,  $V_s^L(\sigma) \geq r(\sigma)$ .

We can then conclude that for all  $\sigma \in \Sigma$  we have

$$r(\sigma) \leq V_s^L(\sigma) = V_s^L(\hat{\sigma}) \leq \delta v_H \beta(\hat{\sigma}) + (1 - \beta(\hat{\sigma}))r^i(\hat{\sigma}),$$

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<sup>25</sup>To pursue the example in the previous footnote,  $V_s^L(\sigma_1) = x_1 v_s + (1 - x_1)x_2 p + (1 - x_1)(1 - x_2)p'$  and  $V_s^L(\sigma_2) = x_1 v_s + (1 - x_1)x_2 p + (1 - x_1)(1 - x_2)p''$ .

where the last inequality follows because we have computed the upper bound of  $V_s^L(\hat{\sigma})$  by assuming that if acceptance of an offer (or consumption by  $s$ ) takes place along  $\hat{\sigma}$  then the highest possible and undiscounted surplus is collected by  $s$ . ■

The two lemmas imply that  $p > r(\sigma)$  for every  $\sigma \in \Sigma$  and conclude the induction step and the proof of part 2 of the Proposition. ■

**Proof of Proposition 6.** We maintain the definition of  $\Sigma_i$  which has been introduced in the proof of Proposition 5, with the proviso that the set is now defined at the empty history (beginning of the game) with an initial seller denoted  $s$  with  $s = s^0$ . For  $\sigma \in \Sigma_i$ , the definitions of  $r^i(\sigma)$ ,  $(1 - \beta(\sigma))$  and  $t(\sigma)$  are borrowed from the same proof.

In addition, we need the following notation. For each  $\sigma \in \Sigma_i$ , define the set  $\Gamma_j(\sigma)$  to include all the sequences of offers that start after the resale offer  $(i, r^i(\sigma))$  and that end with a resale offer to player  $j$  (again, focus on the first resale offer to  $j$ ). Note that  $\Gamma_j(\sigma)$  may be empty for some  $\sigma \in \Sigma_i$ . For any  $\gamma \in \Gamma_j(\sigma)$  call  $r_\sigma^j(\gamma)$  the resale offer received by  $j$  at the end of the sequence  $\gamma$ , let  $t_\sigma(\gamma)$  be the length of the sequence and  $1 - \beta_\sigma(\gamma)$  be the probability, evaluated after the acceptance of  $r^i(\sigma)$ , that the game reaches offer  $r_\sigma^j(\gamma)$ .

We can now write the interim utility of the high-value  $i$  as

$$V_i^H(\emptyset) = \int_{\Sigma_i} (1 - \beta(\sigma)) \delta^{t(\sigma)} (\delta v_H - r^i(\sigma)) d\sigma. \quad (4)$$

Similarly, noting that all offers received by  $j$  must originate through some sequence  $\sigma \in \Sigma_i$ , we can write:

$$V_j^H(\emptyset) = \int_{\Sigma_i} (1 - \beta(\sigma)) \delta^{t(\sigma)} \left( \int_{\Gamma(\sigma)} (1 - \beta_\sigma(\gamma)) \delta^{t_\sigma(\gamma)} (\delta v_H - r_\sigma^j(\gamma)) d\gamma \right) d\sigma. \quad (5)$$

Note that we are implicitly ignoring consumption offers that  $i$  and  $j$  may receive along the various sequences of offers in virtue of the indifference property.

Next, we observe that, for any  $\sigma \in \Sigma_i$  and  $\gamma(\sigma) \in \Gamma_j(\sigma)$ :

$$r^i(\sigma) < \beta_\sigma(\gamma) \delta v_H + (1 - \beta_\sigma(\gamma)) \delta^{t_\sigma(\gamma)} r_\sigma^j(\gamma). \quad (6)$$

This inequality follows by exploiting two facts. First, every seller intermediating the object between the initial resale offer and the final resale offer must be indifferent between the different sequences of offers that she may implement by randomization (see the discussion in the proof of proposition 5). Second, the right hand side of the above expression is (more than) the total expected surplus that can be generated along the sequence  $\gamma(\sigma)$ . The strictness of the inequality is guaranteed by the fact that when  $i$  receives her first resale offer she will accept and consume, which happens with positive probability. (The proposition will hold a fortiori in the case in which the high-value  $i$  has accepted a previous consumption offer with probability one.)

For every  $\sigma \in \Sigma_i$  and any  $\gamma \in \Gamma_j(\sigma)$ , using inequality (6) we can write:

$$\begin{aligned}\delta v_H - r^i(\sigma) &> (1 - \beta_\sigma(\gamma))(\delta v_H - \delta^{t_\sigma(\gamma)} r_\sigma^j(\gamma)) \\ &> (1 - \beta_\sigma(\gamma))\delta^{t_\sigma(\gamma)}(\delta v_H - r_\sigma^j(\gamma)),\end{aligned}$$

where the first inequality follows from subtracting  $\delta v_H$  to both sides of inequality (6), and the second by multiplying  $\delta v_H$  by the factor  $\delta^{t_\sigma(\gamma)} < 1$ .

Using expression (4) for  $V_i^H(\emptyset)$ , expression (5) for  $V_j^H(\emptyset)$  and  $\delta v_H - r^i(\sigma)$  as an upper bound, we obtain the desired conclusion

$$\begin{aligned}V_j^H(\emptyset) &= \int_{\Sigma_i} (1 - \beta(\sigma))\delta^{t(\sigma)} \left( \int_{\Gamma(\sigma)} (1 - \beta_\sigma(\gamma))\delta^{t_\sigma(\gamma)}(\delta v_H - r_\sigma^j(\gamma))d\gamma \right) d\sigma \\ &< \int_{\Sigma_i} (1 - \beta(\sigma))\delta^{t(\sigma)} \left( \int_{\Gamma(\sigma)} (\delta v_H - r^i(\sigma))d\gamma \right) d\sigma \leq V_i^H(\emptyset). \quad \blacksquare\end{aligned}$$

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