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Testing for stationarity in heterogeneous panel data in the presence of cross section dependence[*]

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Abstract

The panel variant of the KPSS tests developed by Hadri (2000) for the null of stationarity suffers from size distortions in the presence of cross section dependence. However, applying the bootstrap methodology we find that these tests are approximately correctly sized.

JEL Classification: C12; C15; C22; C23

Keywords: Heterogeneous dynamic panels, Monte Carlo, bootstrap, unit root tests, cross section dependence.

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1 Introduction

A number of alternative procedures have been proposed to test for the presence of unit roots in dynamic heterogeneous panels, see e.g. Im, Pesaran, and Shin (2003) (IPS) and Maddala and Wu (1999) (MW). These authors test the null hypothesis of a unit root against the alternative of at least one stationary series, by using the (Augmented) Dickey-Fuller (ADF) statistic across the cross-sectional units of the panel. By contrast, Hadri (2000) proposed an *LM* procedure to test the null hypothesis that all of the individual series are stationary (either around a mean or around a trend) against the alternative of at least a single unit root in the panel. The two *LM* tests proposed by Hadri (2000) are panel versions of the test developed by Kwiatkowski, Phillips, Schmidt, and Shin (1992). A critical assumption underlying these tests is that of cross section independence among the individual time series in the panel and both the MW and IPS tests exhibit severe size distortions in the presence of cross-sectional dependence.

This paper investigates the performance of the Hadri (2000) tests in the presence of cross-section dependence. Based on Monte Carlo simulations we find that the Hadri (2000) tests also suffer from severe size distortion, and we use the bootstrap method to allow for potential cross-sectional dependency. The plan of the paper is as follows. Section 2 briefly reviews the Hadri (2000) approach to unit root testing in panels. Section 3 presents the design of the Monte Carlo simulation and the main results.

2 The Hadri tests

Hadri (2000) proposes residual based Lagrange Multiplier tests for the null hypothesis that the time series for each cross section unit, i, are stationary around a level or around a deterministic time trend, against the alternative of at least a single unit root. Following Hadri (2000), consider the models:

$$y_{it} = r_{it} + \varepsilon_{it}, \tag{1}$$

and

$$y_{it} = r_{it} + \beta_i t + \varepsilon_{it,} \tag{2}$$

where r_{it} is a random walk,

$$r_{it} = r_{it-1} + u_{it}, \tag{3}$$

and ε_{it} and u_{it} are mutually independent normal distributions. Also, ε_{it} and u_{it} are $i.i.d.$ across i and over t, with $E\left[\varepsilon_{it}\right] = 0$, $E\left[\varepsilon_{it}^2\right] = \sigma_\varepsilon^2 > 0$, $E\left[u_{it}\right] = 0$, $E\left[u_{it}^2\right] = \sigma_u^2 \geq 0$, $t = 1, ..., T$ and $i = 1, ..., N$.

Let $\hat{\varepsilon}_{it}^\mu$ ($\hat{\varepsilon}_{it}^\tau$) be the residuals from the regression of y_i on an intercept, for model (1), (on an intercept and a linear trend term, for model (2)). Let $\hat{\sigma}_{\varepsilon\mu}^2$ ($\hat{\sigma}_{\varepsilon\tau}^2$) be a consistent estimator of the error variance (corrected for degrees of freedom) from the appropriate regression, which are given by:

$$\hat{\sigma}_{\varepsilon\mu}^2 = \frac{1}{N(T-1)} \sum_{i=1}^N \sum_{t=1}^T \hat{\varepsilon}_{it}^{\mu\,2},$$

and

$$\hat{\sigma}_{\varepsilon\tau}^2 = \frac{1}{N(T-2)} \sum_{i=1}^N \sum_{t=1}^T \hat{\varepsilon}_{it}^{\tau\,2}.$$

Also, let S_{it}^l be the partial sum process of the residuals,

$$S_{it}^l = \sum_{j=1}^t \hat{\varepsilon}_{ij}^l, \quad l = \mu, \tau.$$

Then the LM statistic is:

$$LM_l = \frac{\dfrac{1}{N} \sum_{i=1}^N \dfrac{1}{T^2} \sum_{t=1}^T S_{it}^{l\,2}}{\hat{\sigma}_\varepsilon^2}, \quad l = \mu, \tau.$$

Hadri (2000) considers the standardised statistics:

$$Z_\mu = \frac{\sqrt{N}\left(LM_\mu - \xi_\mu\right)}{\zeta_\mu} \Rightarrow N(0,1), \tag{4}$$

and

$$Z_\tau = \frac{\sqrt{N}\left(LM_\tau - \xi_\tau\right)}{\zeta_\tau} \Rightarrow N(0,1). \tag{5}$$

The mean and the variance of the random variable Z_μ are $\xi_\mu = \frac{1}{6}$ and $\zeta_\mu^2 = \frac{1}{45}$, respectively. The mean and the variance of the random variable Z_τ are $\xi_\tau = \frac{1}{15}$ and $\zeta_\tau^2 = \frac{11}{6300}$, respectively.

3   Monte Carlo simulations and results

We use Monte Carlo simulations to examine the effects of cross-section dependence on the size and power of Z_μ and Z_τ. Following Hadri (2000), to investigate the size of Z_μ and Z_τ the data-generating process (DGP) for model (1) is:

2

$$y_{it} = \alpha_i + \varepsilon_{it},$$

while for model (2) the corresponding DGP is given by:

$$y_{it} = \alpha_i + \beta_i t + \varepsilon_{it},$$

where ε_t are $i.i.d.$ $N(0,1)$ under the null hypothesis. We generate $\alpha_i \sim U[0,10]$ and $\beta_i \sim U[0,2]$, where U stands for a uniform distribution; α_i and β_i are generated once and then fixed in all replications. We consider the cases of $N = (15, 25, 50, 100)$ and $T = (15, 25, 50)$, where the first 100 time observations for each cross-sectional unit are discarded. A total of 2,000 replications are used in computing the empirical size and power of the tests at the 5% nominal level.

The Monte Carlo experiments of Hadri (2000) demonstrate that these tests have good size properties for T and N sufficiently large and this observation is consistent with our own findings (which are not reported here to save space).

However, even for relatively large N and T, and in line with the results on both the IPS and the MW panel unit root tests, the tests of Hadri (2000) suffer from severe size distortions in the presence of cross-sectional dependence, as can be seen in Table 1, where following O'Connell (1998) we assume that the variance-covariance matrix of the disturbance vector $\varepsilon_t = [\varepsilon_{1t}, \varepsilon_{2t}, ..., \varepsilon_{Nt}]$ has the form:

$$E\left(\varepsilon_t \varepsilon_t'\right) = \Omega = \begin{pmatrix} 1 & \omega & \dots & \omega \\ \omega & 1 & \dots & \omega \\ \vdots & \vdots & \ddots & \vdots \\ \omega & \omega & \dots & 1 \end{pmatrix}, \qquad \omega < 1.$$

In the Monte Carlo experiments, $\omega = (0.3, 0.5, 0.7, 0.9)$, with $\omega = 0.0$ being the case of cross-sectional independence considered by Hadri (2000). We note that the size distortion increases as the strength of the cross-sectional dependence, ω, increases.

To correct the size distortion caused by cross-sectional dependence, we follow Maddala and Wu (1999) and more recently Chang (2004), who considered bootstrapping unit root tests. To implement this approach in the context of the Hadri tests, we bootstrap the residuals from either a regression of y_i on a constant for the Z_μ test or y_i on a constant and a trend for the

3

Z_τ test. As suggested by Maddala and Wu (1999) (p.646), we resample the residuals with the cross-section index fixed, so that we preserve the cross-correlation structure of the error term. Results are based on 2,000 Monte Carlo replications, with 100 bootstrap replications used to generate the bootstrap distributions of the tests. The major conclusion that one can draw is that bootstrapping the errors seems to largely correct for the over-size problem (Table 1), although for $T = 15$ and $T = 25$ there is evidence to suggest that even the bootstrapped test statistics are slightly over-sized.

Finally, to investigate the empirical power of the Hadri tests we assume that $\lambda = \sigma_u^2/\sigma_\varepsilon^2 = (0.001, 0.1, 1, 10, 100)$, with $\lambda = \infty$ corresponding to a pure random walk in y_t; notice that $\lambda = 0$, i.e. $\sigma_u^2 = 0$, corresponds to the case where y_t is stationary (these parameter values are similar to those used by KPSS and Hadri). The results reported in Table 2 show that for given T, N and ω power increases with λ. We note that for fixed T, N and λ, as the degree of cross-sectional dependency, ω, increases, power falls.

Table 1. Empirical size of the Hadri tests with cross section dependence

T	N	ω	Z_μ statistic N(0,1)	Bootstrap	Z_τ statistic N(0,1)	Bootstrap
15	15	0.3	9.80	5.75	6.30	5.35
		0.5	14.45	6.20	10.90	5.05
		0.7	18.10	6.30	14.35	5.30
		0.9	21.10	6.25	17.85	5.05
15	25	0.3	12.10	6.75	9.05	5.65
		0.5	16.55	6.50	14.50	6.00
		0.7	20.10	6.60	19.35	6.10
		0.9	22.80	6.30	22.80	6.15
15	50	0.3	16.15	6.35	12.25	4.65
		0.5	21.60	6.35	18.85	5.45
		0.7	23.85	6.25	24.20	5.80
		0.9	25.00	6.25	26.25	6.20
15	100	0.3	18.15	5.55	15.65	5.55
		0.5	24.05	5.70	22.95	4.90
		0.7	27.70	5.90	27.05	5.15
		0.9	29.10	5.75	30.35	6.00
25	15	0.3	11.80	6.60	10.45	6.80
		0.5	16.50	5.85	15.95	5.80
		0.7	19.05	6.50	19.65	5.50
		0.9	20.40	6.25	21.50	5.55
25	25	0.3	11.25	5.40	12.15	6.05
		0.5	15.70	5.30	17.40	6.05
		0.7	18.90	5.60	22.05	6.60
		0.9	21.30	4.95	25.85	6.10
25	50	0.3	17.90	6.95	16.35	6.10
		0.5	23.00	6.60	22.35	6.30
		0.7	24.50	6.80	26.85	6.50
		0.9	26.25	6.95	28.90	5.95
25	100	0.3	16.95	4.55	19.60	4.60
		0.5	22.65	4.95	24.65	4.40
		0.7	25.40	5.05	27.10	4.80
		0.9	27.60	4.75	27.65	5.10
50	15	0.3	9.30	4.95	10.50	5.55
		0.5	14.35	5.40	15.10	5.80
		0.7	17.25	5.85	19.60	6.00
		0.9	19.65	6.15	23.40	6.10
50	25	0.3	11.65	4.85	12.20	4.65
		0.5	16.10	4.50	17.45	4.50
		0.7	19.75	5.05	20.55	4.55
		0.9	22.15	4.70	24.05	4.95
50	50	0.3	15.95	5.55	16.20	4.90
		0.5	19.95	5.25	22.35	4.65
		0.7	22.30	4.85	25.00	4.60
		0.9	23.50	4.80	26.65	4.10
50	100	0.3	18.85	4.90	19.35	4.40
		0.5	23.25	4.70	24.20	5.25
		0.7	27.15	5.40	27.45	5.20
		0.9	27.70	5.55	29.65	4.95

The 95% confidence intervals for 5% signficance levels based on 2,000 replications are (4.04,5.96).

Table 2. Power of bootstrapped Hadri tests with cross section dependence

T	N		Z_μ statistic					Z_τ statistic				
					λ					λ		
			0.001	0.01	0.1	1	100	0.001	0.01	0.1	1	100
15	15	0.3	5.90	10.35	64.25	99.95	100.00	5.50	6.30	19.95	95.10	100.00
		0.5	6.20	9.35	58.55	99.95	100.00	5.05	6.10	17.15	93.40	100.00
		0.7	6.90	8.95	51.75	99.90	100.00	5.40	5.95	14.75	91.90	100.00
		0.9	6.40	8.65	44.00	99.80	100.00	5.25	5.85	13.05	89.20	100.00
15	25	0.3	6.65	11.65	75.25	100.00	100.00	5.80	7.00	22.40	98.50	100.00
		0.5	6.80	9.45	66.65	100.00	100.00	5.95	7.20	19.60	97.30	100.00
		0.7	7.15	8.85	56.75	100.00	100.00	6.25	7.35	16.70	95.25	100.00
		0.9	6.55	8.35	48.15	100.00	100.00	6.30	7.15	15.80	92.75	100.00
15	50	0.3	6.75	10.90	83.40	100.00	100.00	4.75	5.65	24.80	99.80	100.00
		0.5	6.85	9.25	72.40	100.00	100.00	5.55	6.45	19.70	99.40	100.00
		0.7	6.65	8.80	60.10	100.00	100.00	5.85	6.80	17.40	98.55	100.00
		0.9	6.55	8.20	48.65	100.00	100.00	6.35	7.10	16.40	96.45	100.00
15	100	0.3	5.90	10.30	89.30	100.00	100.00	5.60	6.55	24.85	99.95	100.00
		0.5	5.85	9.15	77.65	100.00	100.00	5.05	5.95	20.85	99.90	100.00
		0.7	6.25	8.45	63.30	100.00	100.00	5.35	6.30	16.85	99.05	100.00
		0.9	6.05	7.85	50.60	100.00	100.00	6.05	7.15	15.80	97.50	100.00
25	15	0.3	7.30	26.45	98.65	100.00	100.00	6.90	10.15	69.50	100.00	100.00
		0.5	7.30	18.45	97.70	100.00	100.00	5.95	8.55	57.35	100.00	100.00
		0.7	7.00	14.00	96.35	100.00	100.00	5.55	7.40	45.15	100.00	100.00
		0.9	6.45	11.10	94.40	100.00	100.00	5.85	6.75	36.30	100.00	100.00
25	25	0.3	6.20	25.25	99.75	100.00	100.00	6.30	9.95	80.25	100.00	100.00
		0.5	5.95	15.15	99.50	100.00	100.00	6.55	8.85	65.20	100.00	100.00
		0.7	5.65	11.70	98.55	100.00	100.00	6.65	8.45	47.00	100.00	100.00
		0.9	5.25	9.55	96.75	100.00	100.00	6.30	7.65	36.40	100.00	100.00
25	50	0.3	8.05	29.05	100.00	100.00	100.00	6.40	9.55	89.45	100.00	100.00
		0.5	7.20	17.95	99.95	100.00	100.00	6.40	8.25	70.05	100.00	100.00
		0.7	7.65	12.80	99.80	100.00	100.00	6.50	8.15	51.60	100.00	100.00
		0.9	7.35	11.50	99.30	100.00	100.00	6.15	7.55	39.20	100.00	100.00
25	100	0.3	5.10	25.60	100.00	100.00	100.00	4.85	8.35	94.20	100.00	100.00
		0.5	5.55	14.05	100.00	100.00	100.00	4.55	6.85	72.55	100.00	100.00
		0.7	5.45	10.65	99.90	100.00	100.00	5.00	6.25	51.50	100.00	100.00
		0.9	5.40	9.55	99.50	100.00	100.00	5.15	6.30	35.95	100.00	100.00
50	15	0.3	10.05	88.25	100.00	100.00	100.00	6.80	28.95	100.00	100.00	100.00
		0.5	8.25	77.05	100.00	100.00	100.00	6.55	17.75	99.90	100.00	100.00
		0.7	7.20	61.65	100.00	100.00	100.00	6.20	13.35	99.70	100.00	100.00
		0.9	6.90	46.80	100.00	100.00	100.00	6.80	11.90	98.80	100.00	100.00
50	25	0.3	9.80	95.15	100.00	100.00	100.00	5.80	29.50	100.00	100.00	100.00
		0.5	7.05	83.70	100.00	100.00	100.00	5.05	15.85	99.95	100.00	100.00
		0.7	6.75	65.00	100.00	100.00	100.00	4.75	11.35	99.95	100.00	100.00
		0.9	5.95	47.30	100.00	100.00	100.00	5.20	10.40	99.45	100.00	100.00
50	50	0.3	10.40	99.40	100.00	100.00	100.00	5.55	33.65	100.00	100.00	100.00
		0.5	7.30	89.95	100.00	100.00	100.00	5.45	18.15	100.00	100.00	100.00
		0.7	6.15	69.55	100.00	100.00	100.00	4.95	11.65	100.00	100.00	100.00
		0.9	5.95	47.15	100.00	100.00	100.00	4.55	9.85	99.90	100.00	100.00
50	100	0.3	9.15	99.80	100.00	100.00	100.00	5.50	32.25	100.00	100.00	100.00
		0.5	6.95	93.55	100.00	100.00	100.00	5.75	14.90	100.00	100.00	100.00
		0.7	6.65	72.35	100.00	100.00	100.00	5.45	12.20	100.00	100.00	100.00
		0.9	6.85	48.45	100.00	100.00	100.00	5.20	10.65	99.95	100.00	100.00

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