A Theory of Power Wars*

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Abstract

We present a theory of war onset and war duration in which power is multidimensional and can evolve through conflict. The resources players can secure without fighting are determined by their *political power*, while the ability of appropriating resources with violence is due to their *military power*. When deciding whether to wage a war, players evaluate the consequences on the current allocation of resources as well as on the future distribution of military and political power. We deliver three main results: a key driver of war is the *mismatch* between military and political power; dynamic incentives may amplify static incentives, leading forward-looking players to be more belligerent; and a war is more likely to last for longer if political power is initially more unbalanced than military power and the politically under-represented player is militarily advantaged. Our results are robust to allowing the peaceful allocation of resources to be a function of both political and military power. Finally, we provide empirical correlations on inter-state wars that are consistent with the theory.

Keywords: Formal Model; International Relations; Causes of War; Dynamic Game; War Onset; War Duration; Balance of Power; Power Mismatch; Power Shift; Civil Wars; Inter-State Wars

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1 Introduction

The international relations debates on the importance of the balance of *power* for peace preservation are endless. At the same time, when scholars talk about balance versus preponderance of power, they almost always refer to a single measure of power, namely, *relative military strength*. Similarly, the formal theory literature focuses on this same notion of power when it points to asymmetric information about power and lack of commitment in the future use of power as the rationalist explanations for conflict. In this paper, we argue that considering simultaneously multiple notions of relative power and how they evolve over time can uncover important elements to advance our understanding of conflict onset and its duration.

While relative military strength shapes players' *outside option* and, thus, it is an important driver of players' relative bargaining advantage, other elements also concur to determine a negotiation's outcome. *(Political) Institutions*, that is "humanly devised constraints that structure political, economic and social interactions" (North 1991), represent an additional key determinant of negotiated outcomes. In fact, a large literature in political science has emphasized that protocols for the production and adoption of policy proposals have profound effects on outcomes. Dozens of formal models have shown how specific allocations of proposal rights (i.e., the ability to initiate new policies), veto rights (i.e., the ability to block proposed policies) and voting weights affect political outcomes (Shapley and Shubik 1954, Winter 1996, McCarty 2000, Cox and McCubbins 2001, 2002, 2005, Tsebelis 2002, Snyder, Ting and Ansolabehere 2005). Similarly, institutions can impose hard constraints on negotiations' outcomes. For example, they can establish the status quo in case of lack of an agreement, what coalitions can be formed, whether transfers can be used, what is contractable, and to what extent some members of the polity can appropriate resources at the expense of others. In this paper, we use the term *relative political power* to capture the advantage conferred to a player by the existing political institutions — for example, the relative control of the political bodies governing the allocation of resources in peace.

Our key premise is that institutions — even if they can evolve and be adjusted over time — are *persistent*, namely, inherited from past interactions. Changing formal institutions usually requires protracted negotiations and involves a large degree of coordination among players.[1] Moreover, even when formal institutions do change, the interaction between formal and informal institutions — or between *de jure* and *de facto* political power — might lead to persistence in political and economic outcomes (Acemoglu and Robinson 2008). At the same time, conflict can be a catalyst of institutional change and can lead to a rapid adjustment in political power which is unachievable by peaceful means: after a war, the winner typically improves its relative military strength (because it lost fewer weapons or soldiers than the opponent in combat) but it also has a chance to obtain a greater control of political institutions. In other words, conflict determines a swift shift in both military and political power. As a consequence, countries (in interstate disputes) or ethnic groups (in domestic disputes) decide whether to wage war based not only on the effects on the current allocation of resources, but crucially also on the future distribution of relative powers: the value of winning a war is augmented by the degree to which victory affects future political power (which determines the future flow of benefits in peace), as well as future military power (which determines the future outside option, that is, the likelihood of winning future wars).

This paper presents a dynamic model to shed light on how these evolving multidimensional forces shape the trade-off between peace and war in international relations. Our two-period model can be described informally as follows: given any initial distribution of military and political power and after observing the cost of conflict in the first period, players choose whether to wage war or not. If peace prevails, players receive the flow of benefits determined by the initial political/institutional power balance. If, instead, war takes place, both players suffer the cost. The war may be *decisive* or not: if war is decisive, the winner appropriates all available resources in both periods and the game ends; if war is not decisive,

[1]For example, in many democracies, constitutional amendments require a qualified majority in the legislature and, possibly, the approval of local legislatures or a referendum (Lutz 1994, Lijphart 1999).

both military and political power evolve (in favor of the winner), players receive the flow of benefits determined by their evolved political power, and the game proceeds to a second period with the initial distribution of military and political power determined by the outcome of the war fought in the previous period. After observing the cost of conflict in this second period, players choose again whether to accept the peaceful distribution of resources (determined by their current relative political power) or, else, go to a decisive war, hoping to appropriate all resources (minus the cost of conflict).

We first characterize how the initial distribution of powers and the magnitude of power shifts after conflict affect war onset and its duration in the benchmark model where the allocation of resources in peace is determined exogenously by political power. Then, we extend the analysis to allow for the allocation of resources in a period to be determined endogenously by bargaining. Our main result is that — in the static (i.e., when players are myopic) as well as in the dynamic case (i.e., when players are forward looking) — the key cause of war is what we call the *mismatch* between military and political power: the probability of a first-period war does not only depend on whether military power is balanced, but, crucially, on whether the two types of powers are balanced with respect to one another and on whether the powers' mismatch will be exacerbated or reduced by an indecisive war. As the chance of indecisive wars varies, we show exactly how the mismatch remains a key determinant of the incentive to fight and how the powers' mismatch interacts with the more standard pure military imbalance. In particular, even when a first-period war is never decisive, and, thus, players are certain to interact for two periods, the expected evolution of the mismatch determines the chance of war in the first period: winning a first battle certainly matters in terms of appropriating additional resources in that period, but, importantly, it also matters in terms of changing the power balance and increasing one's own military and political power going forward.

In sum, the broader message that our simple two-period model delivers is that the mismatch between political and military power as well as its evolution over time are key deter-

minants of war incentives. This is true regardless of players' time horizon but even more so when the rivals are patient and forward looking. Our extension shows that this message remains true in a standard bargaining framework where the allocation of resources in a period can be contracted upon under the threat of violence. In this case, war will not occur in the second and last period of the game as, without uncertainty, an agreement which avoids destruction of resources can always be found. However, bargaining breakdowns and war may occur in the first period of the game, as the evolution of military and political powers and their future use in case of an indecisive war cannot be contracted ex-ante.

Our model delivers predictions also on the duration of wars: we capture duration of wars by studying the equilibrium probability of a second war conditional on a war taking place in the first period. As we show, war duration depends in a nuanced way on the balance of military power, the balance of political power, and the magnitude of power shifts after conflict. In particular, a war is more likely to continue (i.e., the static incentive for war increases) if the politically under-represented player is the militarily advantaged player and if political power is more unbalanced than military power.

Our marginal contribution with respect to existing formal models in international relations is twofold: we introduce a multidimensional concept of power; and we highlight that power arrangements evolve over time, with conflict acting as a catalyst for rapid change. In this framework, not only the current mismatch between powers but also its *evolution over time* are key determinant of the chance of war (and of war duration). Most formal models of international relations are inherently static as they treat the decision to go to war as a game-ending move: once the players decide to fight, their strategic interaction ends and each player receives a payoff that reflects the distribution of military power.[2] In the early 2000s, a wave of formal models studied the (more realistic) setup where conflict does not end with a single battle always wiping out the loser (Filson and Werner 2002, Slantchev 2003, Powell 2004). We share with these models the idea that a war can potentially last for multiple

[2]See Fearon (1995), Powell (2002), Jackson and Morelli (2011) for surveys of rationalist theories of war.

battles. At the same time, these models differ from ours in significant ways: contrary to our framework, players care about the division of a single flow of resources and the game ends as soon as these resources are assigned; power to allocate these resources is crucially unaffected by conflict, namely, battles in these models simply affect beliefs about relative military strength, as well as both players' capacity to sustain another battle.

More generally, the existing formal literature on the insurgence and duration of wars takes players' relative power as a fixed parameter. There are only two strands of literature where power is in some sense endogenous. The first strand is the literature on strategic militarization, where players choose their military endowment in anticipation of war (Powell 1993, Jackson and Morelli 2009, Meirowitz and Sartori 2009). The second strand, closer to our paper, focuses more explicitly on power shifts. In Powell (2012, 2013) and Debs and Monteiro (2014) wars can occur to block an anticipated power shift in favor of one player (which could be caused, for example, by a consolidation of power or a militarization strategy by the ruling faction or hegemon state). In these models, dynamic incentives over the evolution of power increase the players' incentive to wage war, as occurs in our model. Contrary to our model, where (military and political) power shifts are triggered by conflict, these models assume that changes in (military) power are triggered by acquiescence to a peaceful bargaining agreement. More importantly, both models consider a unidimensional notion of power. Namely, only what we call *military power* can change over time and can be affected by conflict, whereas what we call *political power* is exogenous and fixed over time.[3,4]

The paper is organized as follows: in Section 2 we introduce our model. Section 3 contains the equilibrium predictions. Sections 4 and 5 show robustness of the main results from Section 3 when we allow the allocation of resources in a period to be determined by

[3]For example, in Powell (2012, 2013), the player favored by the potential shift in military power can make a take-it-or-leave-it offer to the opponent and, thus, can extract all the surplus from avoiding conflict. This agenda setting advantage is fixed and unaffected by conflict.

[4]Another dynamic model of endogenous power evolution is Fearon (1996), which considers an infinite horizon, complete information game in which two states bargain over resources and the current allocation of resources determines the future military balance. Contrary to our model, war is always decisive and eliminates the loser, thus ending the interaction.

bargaining. Section 6 presents empirical correlations which provide support for some of our assumptions and are consistent with the predictions of the theory. Section 7 concludes.

2 Model

Consider the standard problem of two players (ethnic groups or countries, modeled as unitary actors), A and B, having to share a per period surplus (normalized to 1), which may come from exploitation of the local resources or control of the state. The two players' *relative power* is multidimensional: at the beginning of period $t = \{1, 2\}$, player A has a share of *military power* equal to $m_t \in [0, 1]$ and a share of *political power* equal to $p_t \in [0, 1]$; player B's military and political power in the same period are, respectively, $(1 - m_t)$ and $(1 - p_t)$.

In each period, the two players sequentially choose whether to wage war against the opponent or not, in random order. If neither player attacks, there is peace and the outcome is determined by relative political power in the current period, that is, by the allocation of resources dictated by peace-time institutions: A consumes p_t and B consumes $(1 - p_t)$. If, instead, either player attacks, a war occurs and the outcome is determined by relative military power in the current period: A wins with probability m_t and B wins with probability $(1 - m_t)$.

We define $c_t \in [0, 1]$ as the cost of war in period t suffered by both players. This cost is a random variable, drawn from a uniform distribution with support $[0, 1]$ and observed by both players at the beginning of every period t. This means that players know the realization of c_t but only know the distribution of c_{t+1} when deciding whether to attack or not in period t.

The war is always decisive in the second period. We define $k \in [0, 1]$ as the first-period *decisiveness of war*, namely, the probability that the war is decisive in the first period. If $k = 1$, a first-period war is always decisive: it has one clear winner that obtains all the political and military power, $(p_t = 1, m_t = 1)$, for the two periods. If $k = 0$, a first-period war is never decisive: it only has a partial winner and the spoils of war obtained in the first

period are

$$p_{t+1} = \begin{cases} p_t + b_t & \text{if } A \text{ wins} \\ p_t - b_t & \text{if } A \text{ loses} \end{cases}$$

After an indecisive war, the game continues to a second period. In this second period, the power and military structures are determined by the outcome of the first-period war:

$$m_{t+1} = \begin{cases} m_t + a_t & \text{if } A \text{ won} \\ m_t - a_t & \text{if } A \text{ lost} \end{cases} \quad , \quad p_{t+1} = \begin{cases} p_t + b_t & \text{if } A \text{ won} \\ p_t - b_t & \text{if } A \text{ lost} \end{cases}$$

In other words, if the first period is peaceful, the distribution of military and political power at the beginning of the second period is unchanged. On the other hand, if conflict breaks out, the war alters the distribution of military and political power in favor of the winning side (either totally or partially). If there is a war in the second period, it is always decisive. We assume power shifts have the following properties:

1. **Boundedness:** political and military powers always remain within the boundaries of the interval $[0, 1]$.

2. **Anonymity:** the extent to which power shifts favor the winning side does not depend on the winner's identity but is only a function of starting military and political power (im)balance.

3. **Symmetry:** positive and negative power shifts are symmetric, that is, they have the same magnitude.

4. **Independence:** the power shift in one dimension does not depend on the power shift in the other dimension.

5. **Power Ranking Preservation:** in case of conflict, the ranking of A's relative powers is preserved regardless of the conflict's outcome, that is, $m_t > p_t \Longrightarrow m_{t+1} > p_{t+1}$.[5]

For ease of exposition, in the remainder of the paper we make the following assumption.

Assumption 1 *At the beginning of the game, A is the politically under-represented player that is, $(m_1 - p_1) \geq 0$.*

Note that this assumption is without loss of generality: the analysis for the case where $(m_1 - p_1) < 0$ is identical up to the players' labels. Finally, the following definitions will prove helpful in discussing the intuition behind the results presented in the following Section.

Definition 1 *Let $M_t = (m_t - p_t) \in [0, 1]$ be the degree of A's political under-representation or, in other words, the mismatch between relative military and political powers in period t.*

Definition 2 *Let $\Omega_t = (a_t - b_t)$ be the difference between the change in m_t and the change in p_t after an indecisive war occurs in period t. In other words, Ω_t is the shock to the powers' mismatch after an indecisive war occurs in period t.*

Endowed with these definitions, we can express the mismatch in period $t+1$ compactly, as follows:

$$M_{t+1} = \begin{cases} M_t + \Omega_t & \text{if } A \text{ wins} \\ M_t - \Omega_t & \text{if } A \text{ loses} \end{cases}$$

Note that Ω_t can be either positive or negative, depending on the initial degree of power (im)balance in the two dimensions. Regardless of the sign of Ω_t, the power ranking preservation property of power shifts implies that, if $M_t \geq 0$, then $M_{t+1} \geq 0$. This means that, if A is the politically under-represented player at the beginning of the game, it will be the politically under-represented player in both periods, regardless of whether an indecisive conflict occurs in the first period and, when it does, of the winner's identity.

[5] Restricting attention to the 725 conflicts in our dataset for which information is available on the distribution of relative military and economic power at the beginning of the conflict and at the end of the conflict, for 719 out of 725 conflicts the property holds.

3 Results

3.1 Probability of War in Static Game

Consider first the static game (or the last period in the dynamic game). When players only care about their current consumption, they decide whether to attack or not by comparing the share of resources assigned to them by peace-time institutions — which depends on relative political power in the current period — and the expected consumption after conflict — which is a function of relative military power and the cost of war in the current period. In particular, A prefers a peaceful outcome if and only if:

$$p_t > m_t - c_t \quad \Longleftrightarrow \quad c_t > m_t - p_t = M_t$$

Similarly, B prefers peace if and only if:

$$(1 - p_t) > (1 - m_t) - c_t \quad \Longleftrightarrow \quad c_t > p_t - m_t = -M_t$$

which is always the case, since $c_t \geq 0$ and $M_t \geq 0$.

Combining the two conditions above, we have war when:

$$c_t \leq m_t - p_t = M_t$$

The probability of war (before the realization of c_t) is, thus:

$$\Pr\left(c_t \leq M_t\right) = M_t$$

Proposition 1 *In the absence of concerns for the future, only the politically under-represented player has an incentive to attack and the ex-ante probability war occurs is equal to the mismatch between political and military power, that is, M_t.*

When Assumption 1 is violated (that is, when $m_t < p_t$ and, thus, B is the politically under-represented player), a similar analysis shows that A never attacks and that the chance B attacks is equal to $(p_t - m_t) = -M_t$, that is, the mismatch between political and military power.

3.2 Value Functions (Expected Utilities from Power Allocations)

In the second period, a war is always decisive. This means that the equilibrium strategies and the associated probability of war for the static case apply to the second period of the two-period game. Thus, we can use them to compute the *value functions* of state (m_2, p_2) — that is, the expected utilities players A and B derive from power allocations (m_2, p_2), evaluated at the end of the first period in the case there was peace or an indecisive war.

$$\begin{aligned} V^A(m_2, p_2) &= (1 - M_2)\, p_2 + M_2 \int_0^{M_2} \frac{m_2 - c}{M_2} dc = (1 - M_2)\, p_2 + M_2 \left(m_2 - \frac{M_2}{2} \right) \\ V^B(m_2, p_2) &= (1 - M_2)\,(1 - p_2) + M_2 \left((1 - m_2) - \frac{M_2}{2} \right) \end{aligned}$$

In both expressions, the first term is the value of the state in peace — that is, a player's relative political power — weighted by the probability of peace in that state, $1 - M_2$. The second term is the value of the state in war, weighted by the probability of war in that state, M_2. The value of the state in war consists of the expected consumption after war — that is, a player's relative military power — minus the expected cost of war, $M_2/2$.

We can further simplify the two players' value functions and write them as follows:

$$V^A(m_2, p_2) = p_2 + \frac{(M_2)^2}{2}, \quad V^B(m_2, p_2) = (1 - p_2) - 3\frac{(M_2)^2}{2}$$

The value functions are essential elements of the dynamic incentive to wage war in the first period of the dynamic game so they are worth a close look. We highlight some of their properties. First, the value function of A, the politically under-represented player,

is increasing in the powers' mismatch, while the value function of B, the politically over-represented player is decreasing in the powers' mismatch. The intuition is the following. Regardless of its relative military power (and, thus, its chance to win a war), the politically under-represented player expects a gain in consumption from conflict (with respect to its consumption in peace), even when taking into account the cost of war. Both this expected gain — which is equal to the mismatch minus the expected cost of war, that is, $M_2 - M_2/2$ — and the chance of realizing it — which is equal to the probability of war, that is, M_2 — grow with the magnitude of the mismatch. On the other hand, the politically over-represented player expects a loss in consumption from conflict (with respect to its consumption in peace). Both this expected loss and the chance of realizing it are increasing in the mismatch. Second, the value functions of both players are always increasing in a player's relative military power: for any level of A's relative political power, A prefers a larger m_2 and, thus, a larger mismatch, while B prefers a lower m_2 and, thus, a smaller mismatch. Third, the value function of A, the politically under-represented player, is increasing in its own relative political power. On the other hand, the value function of B, the politically over-represented player, is not monotonic in its own relative political power: for a given m_2, V^B is increasing in $(1 - p_2)$ if and only if $(1 - p_2) < (1 - m_2) + \frac{1}{3}$. Hence, if $(1 - m_2) < 2/3$, B's value function is decreasing in $(1 - p_2)$ for high enough $(1 - p_2)$. Intuitively, if B's relative military power is sufficiently small, B prefers not to have too much political power as A might engage in a war that B is likely enough to lose. Finally, total expected welfare in the second period is equal to:

$$W(m_2, p_2) = V^A(m_2, p_2) + V^B(m_2, p_2) = 1 - M_2$$

which intuitively goes to 1, the total per period surplus, only when the chance of war is zero, that is, when there is no mismatch. It then decreases as the chance of war increases.

3.3 Probability of War in Dynamic Game

In the first period, A prefers to attack B rather than maintain peace if and only if:

$$\begin{aligned} EU_A(Peace) &< EU_A(War) \\ p_1 + V^A(m_1, p_1) &< -c_1 + km_1(2) + (1-k)\begin{pmatrix} m_1\left(p_1 + b_1 + V^A(m_1 + a_1, p_1 + b_1)\right) + \\ (1-m_1)\left(p_1 - b_1 + V^A(m_1 - a_1, p_1 - b_1)\right) \end{pmatrix} \end{aligned}$$

In case of peace, A consumes p_1 today and its future military and political power coincide with the current ones; thus, its expected utility from the second period is given by the value function evaluated at (m_1, p_1). In case of war, A's expected consumption today is $-c_1$ and its future military and political power will shift depending on the war outcome; if A wins (which happens with probability m_1) the win can be outright (with chance k) and he will obtain 2 (1 over two periods), or A 's win can be partial (with chance $(1-k)$), namely, A's military and political power grow and he obtains $(p_1 + b_1)$ plus his expected utility from the second period, which is given by the value function evaluated at $(m_1 + a_1, p_1 + b_1)$; if, instead, A loses (which happens with probability $1 - m_1$), A's loss might be total (giving 0 for 2 periods) or partial, namely, A's military and political power decrease and he obtains $(p_1 - b_1)$ plus his expected utility from the second period, which is given by the value function evaluated at $(m_1 - a_1, p_1 - b_1)$.

Substituting the value functions, we have:

$$c_1 < -2p_1 - \frac{(M_1)^2}{2} + km_1(2) + (1-k)\begin{pmatrix} m_1\left(2(p_1 + b_1) + \frac{(M_1+\Omega_1)^2}{2}\right) \\ + (1-m_1)\left(2(p_1 - b_1) + \frac{(M_1-\Omega_1)^2}{2}\right) \end{pmatrix}$$

In other words, A has an incentive to wage war against B if and only if the realized cost of war in the first period, c_1, is below a threshold, $\overline{c}_A$.

Similarly, B prefers to attack A rather than maintain peace if and only if:

$$1-p_1+V^B(m_1,p_1) < -c_1+2k\,(1-m_1)+(1-k)\left(\begin{array}{c} m_1\left(1-p_1-b_1+V^B(m_1+a_1,p_1+b_1)\right)+ \\ (1-m_1)\left(1-p_1+b_1+V^B(m_1-a_1,p_1-b_1)\right)\end{array}\right)$$

Namely:

$$c_1 < -2(1-p_1)+3\frac{(M_1)^2}{2}+2k\,(1-m_1)+(1-k)\left(\begin{array}{c} m_1\left(2(1-p_1-b_1)-3\frac{(M_1-\Omega_1)^2}{2}\right) \\ +(1-m_1)\left(2(1-p_1+b_1)-3\frac{(M_1+\Omega_1)^2}{2}\right)\end{array}\right)$$

Hence, B has an incentive to wage war if and only if c_1 is below a threshold, $\overline{c}_B$.

Straightforward calculations yield the following proposition which is key to characterize the incentives for war in the dynamic game.

Proposition 2 *The cost thresholds below which A and B, respectively, prefer to wage war in the first period are:*

$$\overline{c}_A \;:\; = k\left(2M_1-\frac{M_1^2}{2}\right)+(1-k)\left[(2m_1-1)\,(2b_1)+\frac{1}{2}\left(E\left(M_2^2\right)-M_1^2\right)\right] \tag{1}$$

$$\overline{c}_B \;:\; = -k\left(2M_1-\frac{3}{2}M_1^2\right)-(1-k)\left[(2m_1-1)\,(2b_1)+\frac{3}{2}\left(E\left(M_2^2\right)-M_1^2\right)\right] \tag{2}$$

Since the cost of war is non-negative, player A has a potential incentive to attack (that is, there exists a realization of c_1 such that this player prefers war to peace) only if $\overline{c}_A$ is positive, and likewise for player B. For positive values of its cost threshold, the ex-ante probability that a player attacks (that is, the probability that the realization of c_1 is below this threshold) is increasing in the cost threshold. In both equations, the term proportional to (k) represents the *static incentive* to attack, which emerges when the first-period war is decisive. Note how this *only depends* on the mismatch M_1. The term proportional to $(1-k)$ represents the *dynamic incentive to attack*, which emerges when the first-period war is not decisive. For example, focusing on player A and looking at equation (1):

1. The term $\left(2M_1 - \frac{M_1^2}{2}\right)$ represents the expected gain in consumption from a decisive first-period war relative to peace. Since $M_1 \in [0,1]$, this term is always positive.

2. The term $(2m_1 - 1)(2b_1)$ represents the expected gain in consumption from an indecisive first-period war followed by a peaceful second period. This term is positive if and only if $m_1 \geq 1/2$, that is, if player A is militarily advantaged and, thus, political power is more likely than not to shift to its advantage.

3. The term $\frac{E\left(M_2^2\right) - M_1^2}{2}$ represents the expected gain in consumption from an indecisive first-period war followed by a second-period war (weighted by the probability a second-period war occurs). This term is positive if and only if war is expected to increase the square of the powers' mismatch, regardless of whether A is militarily advantaged or not. As discussed in Section 3.2, this dynamic incentive is due to the fact that the politically under-represented player expects a gain in consumption from conflict even when he is unlikely to win.

Lemma 1 *If the mismatch is expected to increase* $(E(M_2) > M_1)$ *so is its square* $(E(M_2^2) > M_1^2)$

Proof. Note that:

$$E\left(M_2^2\right) - M_1^2 = 2(2m_1 - 1)\Omega_1 M_1 + \Omega_1^2 = 2(E(M_2) - M_1)M_1 + \Omega_1^2$$

■

As a result of Lemma 1, the results we state in terms of the expected *square of* the mismatch could be equally stated, more succinctly, in terms of the mismatch itself.

How do dynamic considerations change players' incentives to attack the opponent? Consider player A, the under-represented or marginalized player. The term in square brackets of the RHS of equation (1) above represents A's dynamic incentive to attack. When this term is positive, dynamic considerations amplify player A's static incentive to attack (and, thus,

the potential evolution of powers triggered by violence leads to a greater ex-ante probability of war). This happens for sure when A is militarily advantaged and $E\left(M_2^2\right) > M_1^2$, that is, when the square of the mismatch is expected to increase after a first-period indecisive war.

Proposition 3 *The politically under-represented player, A, has a static incentive to attack which is always positive and increasing in the mismatch, M_1. Moreover if A is militarily advantaged ($m_1 > 1/2$), then A has a dynamic incentive to attack which is positive and, thus, amplifies the static incentive. Lastly, the dynamic incentive to attack is increasing in the expected increase in the (square of the) mismatch after an indecisive first-period war.*

Proof. In equation (1), the term proportional to k is always positive. Let's consider the term proportional to $(1-k)$.

$$\begin{aligned}(2m_1-1)2b_1 + \frac{E\left[(M_2)^2\right] - (M_1)^2}{2} &= (2m_1-1)(2b_1 + M_1\Omega_1) + \frac{(\Omega_1)^2}{2} \\ &= (2m_1-1)((2-M_1)b_1 + M_1 a_1) + \frac{(\Omega_1)^2}{2}\end{aligned}$$

The LHS is increasing if the expected (square of, thus the) mismatch is increasing. Moreover, when $m_1 > 1/2$ all terms in the RHS are always positive since $M_1 \in [0,2]$. Thus, A has an incentive to attack for low enough war costs. ■

Corollary 1 *When player A is militarily advantaged ($m_1 > 1/2$), the ex-ante probability of first-period war is always positive, even when A is not under-represented ($M_1 = 0$).*

The above corollary shows that a military advantage from the under-represented side is sufficient to have a positive chance of war at the outset. However, military advantage is neither necessary for war nor the sole incentive to wage war. Below, we expand on the result from Proposition 3 above, outlining some special cases.

Special Case 1: Always Decisive War ($k = 1$).

Proposition 4 *When war is always decisive ($k = 1$): a) only the politically under-represented player has an incentive to wage war in the first period; b) the probability of first-period war depends only on the mismatch, M_1, and is strictly increasing in the mismatch.*

Proof. Consider $k = 1$. We have:

$$\overline{c}_A = M_1 \left(\frac{4 - M_1}{2} \right) \geq 0, \quad \overline{c}_B = -M_1 \left(\frac{4 - 3M_1}{2} \right) \leq 0$$

and $\overline{c}_A$ is increasing in $M_1 \in [0, 1]$ ■

Special Case 2: Never Decisive War ($k = 0$).

Proposition 5 *When war is never decisive ($k = 0$), the incentive to attack in the first period depends only on the expected change of the (square of the) mismatch after a first-period war and on who is militarily advantaged: a) If the (square of the) mismatch is expected to increase and the under-represented player is military advantaged ($m_1 > 1/2$), then only this player has an incentive to attack; b) If the (square of the) mismatch is expected to decrease and the over-represented player is military advantaged ($m_1 < 1/2$) then only this player has an incentive to attack.*

Proof. Consider $k = 0$. We have:

$$\overline{c}_A = (2m_1 - 1)(2b_1) + \frac{1}{2}\left(E\left(M_2^2\right) - M_1^2\right), \quad \overline{c}_B = -(2m_1 - 1)(2b_1) - \frac{3}{2}\left(E\left(M_2^2\right) - M_1^2\right) \quad (3)$$

■

Special Case 3: No Military Advantage ($m_1 = 1/2$).

Proposition 6 *When no player has military advantage ($m_1 = 1/2$): a) the over-represented player has never an incentive to wage war in the first period; b) the under-represented player has an incentive to wage war in the first period if the (square of the) mismatch is expected to*

increase after an indecisive first-period war; c) this incentive is always present, even when the first period war is never decisive $(k = 0)$.

Proof. Consider the case of no military advantage: $m_1 = 1/2$. In this case we have:

$$\overline{c}_A := k\left(2M_1 - \frac{M_1^2}{2}\right) + (1-k)\frac{1}{2}\left(E\left(M_2^2\right) - M_1^2\right) > 0 \tag{4}$$

■

3.4 War Duration: Does War Feed War?

In addition to investigating the likelihood of war onset as a function of the initial allocation of military and political power, our framework allows us to make progress on an important question in international relations, namely, does war provoke more war? In our model, a second-period war is always decisive, but does such a war become more or less likely after a first-period war? The following proposition characterizes *war duration* as a function of initial conditions:

Proposition 7 *A war is more likely to happen in the second period following an indecisive war in the first period (relative to peace in the first period) if and only if the mismatch is expected to increase, and the more so the more the mismatch is expected to increase.*

When players only care about their current division of resources, they decide whether to attack or not by comparing the share of resources assigned to them by peace-time institutions — which depends on relative political power in the current period — and the expected consumption after conflict — which is a function of relative military power and the cost of war in the current period. As a result, as discussed in Section 3.1, the probability of a second period war depends only on the size of the mismatch. The power shift specification introduced in Section 3.5 allow us to obtain more specific results.

3.5 Power Shift Specification

We now introduce a further assumption on power shifts when the war is indecisive, which allows us to obtain more specific results. The *power shifts* are a function $f : [0,1]^2 \to [0,1]^2$ which maps (m_t, p_t), that is, the initial values of military and political power, to (m_{t+1}, p_{t+1}), that is, the new values of military and political power. We make the following assumption on f which we use for the results below:

Assumption 2 *Power shifts take the following form:*

$$m_{t+1} = \begin{cases} m_t + a_t & \textit{if } A \textit{ wins} \\ m_t - a_t & \textit{if } A \textit{ loses} \end{cases} \quad , \quad p_{t+1} = \begin{cases} p_t + b_t & \textit{if } A \textit{ wins} \\ p_t - b_t & \textit{if } A \textit{ loses} \end{cases}$$

where $a_t = g\,(1/2 - |m_t - 1/2|)$ *and* $b_t = g\,(1/2 - |p_t - 1/2|)$*, for any* $g \in (0,1)$*.*

The functional form we adopt satisfies the properties listed in Section 2. The only non-trivial property is the *power ranking preservation* property, as proven by the following lemma.

Lemma 2 *In case of conflict, the ranking of A's relative powers is preserved regardless of the conflict's outcome, that is,* $m_t > p_t \Longrightarrow m_{t+1} > p_{t+1}$*.*

Proof. See Appendix. ■

Let us highlight some properties regarding the magnitude of the power shifts.

- **Role of g:** As g increases, power shifts increase in magnitude; when g approaches 0, there is no change in either power after conflict; when g approaches 1, the shift is maximal and equal to $\min\{(1 - m_t), m_t\}$ for military power and to $\min\{(1 - p_t), p_t\}$ for political power.

- **Hegemony vs. Balance:** For any given g, the power shift in any dimension is larger for intermediate initial values of A's relative power in that dimension (that is, military or political *power balance*), and fades to zero for extreme initial values of A's relative power in that dimension (that is, military or political *power imbalance*, namely *hegemony*). This captures the idea that the scope for a change in relative power depends on the initial conditions and can be different in different dimensions.

Lastly we can define:

Definition 3 *Let $\mu_t = \left|m_t - \frac{1}{2}\right|$ be the degree of imbalance in military power in period t. Similarly, let $\pi_t = \left|p_t - \frac{1}{2}\right|$ be the degree of imbalance in political power in period t. When $\mu = 0$ ($\pi = 0$) there is perfect balance of military (political) power, whereas if $\mu = 1/2$ ($\pi = 1/2$) one player has hegemony in military (political) power.*

This implies that:

$$\Omega = g\left(\pi - \mu\right)$$

Using Assumption 2, we can obtain further results. First, we sharpen our result regarding the probability of a first-period war when no player is politically under-represented.

Proposition 8 *When no player is politically under-represented ($M_1 = 0$): a) only the militarily advantaged player has an incentive to wage war in the first period — i.e., the probability A (B) wages war in the first period is positive if and only if $m_1 > 1/2$ ($m_1 < 1/2$); (b) only the dynamic incentive matters, namely, the probability the militarily advantaged player wages war in the first period is strictly decreasing in k and strictly increasing in the magnitude of the shift in political power after an indecisive war, b_1.*

Proof. Given Assumption 2, if $M_1 = 0$, then $\Omega_1 = 0$. We have:

$$\bar{c}_A = -\bar{c}_B = (1-k)\left(2m_1 - 1\right)2b_1$$

■

Most importantly, we can sharpen our result about war duration, namely, whether war feeds more war, as a function of initial conditions:

Proposition 9 *A war is more likely to happen in the second period following an indecisive war in the first period (relative to peace in the first period) if and only if political power is initially more unbalanced than military power and the politically under-represented player is militarily advantaged. Moreover, the larger these relative imbalances are the larger the expected war duration is.*

Proof. The probability of war in the last period is equal to the mismatch M.

The (expected) mismatch change is:

$$\begin{aligned} E[M_2] - M_1 &= [m_1(M_1 + \Omega_1) + (1 - m_1)(M_1 - \Omega_1)] - M_1 \\ &= (2m_1 - 1)\,\Omega_1 = g\,(2m_1 - 1)\,(\pi_1 - \mu_1) \end{aligned}$$

Note that the case $m_1 < 1/2$ and $\pi_1 < \mu_1$ does not exist, since $m_1 > p_1$. Thus, the expected mismatch increases if and only if $m_1 > 1/2$ and $(\pi_1 - \mu_1) > 0$, and the more so the larger these quantities/imbalances are. ■

The intuition behind this result is that, when the initial allocation of military power is more balanced than the initial allocation of political power, then military power will change more than political power after an indecisive war. As a consequence, the powers' mismatch will grow if the politically under-represented player wins a war and will decrease if it loses it. When the politically under-represented player is militarily advantaged, and thus expected to win, an indecisive war will likely continue to a second period. This is due to the fact that the mismatch between political and military power is expected to persist (in fact, it is expected to grow) and a successful conflict will not reduce the grievance of the politically under-represented player (in fact, it will exacerbate it).

Finally, we characterize the chance of a second war conditional on the initial degree of power imbalances and crucially on *who wins the first war,* which addresses the question of why some conflicts may last longer than others.

Proposition 10 *If $\pi_1 > \mu_1$, then the incentive to wage a second war following an indecisive war in the first period grows when A, the politically under-represented player, wins the first conflict. On the other hand, if $\pi_1 < \mu_1$, then the incentive to wage a second war grows when B, the politically over-represented player, wins the first conflict.*

Proof. Following a first war, a second war is more likely if the mismatch increases, that is, when $M_2 > M_1$. $\Omega_t \geq 0$ if and only if $\pi_t \geq \mu_t$, that is, if and only if political power is more unbalanced than military power and, thus, by our assumptions on the technology of power shifts (Assumption 2), the change in political power after a war is smaller than the change in military power. When $\Omega_t > 0$ ($\Omega_t < 0$), war increases the mismatch if the politically under-represented player wins (loses) and reduces it otherwise. Thus, if $\pi_t \geq \mu_t$, then $\Omega_t \geq 0$ and a second war is more likely if the politically under-represented player wins. When, instead, $\pi_t < \mu_t$ we have $\Omega_t < 0$ and a second war is more likely if the politically over-represented player wins. ■

4 Bargaining on Allocation of Resources

The general idea we aimed to capture in our core model in Section 2 is that the flow of benefits received by players is not simply a function of relative military strength but also of relative political power or, in other words, political institutions which determine a bargaining advantage for one of the two players (for any given balance of military power). Our model represented this notion in the starkest possible way, by assuming that the allocation of resources in peace or after an indecisive war is perfectly correlated with a variable we labeled political power. Beyond leading to a tractable model, using the starkest possible assumption

also makes the underlying incentives as crisp as possible. Nonetheless, there are several possible formalizations of this concept.

The goal of this Section is to micro-found p and to show that, even if we allow players to bargain over the allocation of resources in a period, conflict still happens with positive probability and the mismatch between powers is still crucial for the chance of war. In particular, we show that the reduced-form assumption from our base model can be micro-founded with alternating-offer bargaining à la Rubinstein (1982) where players have no chance to impose an agreement with violence and relative political power represents the chance of making the first proposal (that is, agenda setting rights). In fact, this model shows how the resources allocated to a player in a period are a mere *rescaling* of that player's relative political power p.[6] Namely, when players are somewhat patient and, thus, making multiple offers is not too costly, the flow of benefits to a player is strictly increasing in its relative political power and the player who is politically advantaged gets the lion's share of the pie.

Consider the following augmented version of our core model from Section 2. There are two periods. The initial distribution of power is $m_1 \in [0, 1]$, $p_1 \in [0, 1]$, and the cost of war in this period, $c_1 \in [0, 1]$, is drawn from a uniform distribution and observed by both players. Then A and B decide sequentially whether to attack or not.

If either player attacks and war is decisive, no bargaining is needed and the winner gets the entire pie in both periods. On the other hand, if no player attacks or if either player attacks and war is indecisive, players bargain à la Rubinstein (1982) to determine the flow of benefits in this period: players alternate deterministically in making offers and current political power, p_1, determines the chance of being the agenda-setter in the first bargaining round – that is, A makes the first offer with probability p_1. In each bargaining round, the receiver can accept the offer or reject the offer, move to a further round of bargaining and make a counter-proposal. Bargaining continues until an agreement is reached and players

[6]Indeed, allocated resources and political power coincide when players become infinitely impatient (i.e. when making more than one offer becomes prohibitively costly).

discount the future with a common factor $\delta \in [0, 1)$. After an agreement is reached, players move to the second period of the game. If no player attacks, the distribution of power at the beginning of the bargaining stage is the same as the initial one and the probability A makes the first offer is $p_2 = p_1$. On the other hand, if either player attacks and war is indecisive, the distribution of power changes in favor of the winner before the bargaining stage — $m_2 = m_1 \pm a$ and $p_2 = p_1 \pm b$ — and A makes the first offer with the probability determined by the war outcome.

The second period is exactly like the first period with a possibly different initial allocation of power — $m_2 \in [0, 1]$, $p_2 \in [0, 1]$ — inherited from the first period, and war being always decisive and with a realized cost $c_2 \in [0, 1]$

This model micro-founds the assumption that the flow of benefits to A in a period is proportional to its relative political power. Namely, calling $x(p, \delta)$ the expected share to A when its political power at the beginning of the bargaining sub-game is p and before the identity of the first-mover is realized, we have

Proposition 11 *$x(p, \delta)$ is a linear rescaling of p and coincides with p as δ goes to 0.*

Proof. Consider the unique SPE outcome of the bargaining subgame. As shown by Rubinstein (1982), we have

$$x(p, \delta) = p\left(\frac{1-\delta}{1-\delta^2}\right) + (1-p)\left(\frac{\delta(1-\delta)}{1-\delta^2}\right) = p\left(\frac{1-\delta}{1+\delta}\right) + \frac{\delta}{1+\delta}$$

which gives our result. Note that $x(p, \delta)$ is strictly increasing in p for any $\delta \in [0, 1)$ and goes to p as δ goes to 0. ■

5 Extension: Bargaining in the Shadow of Power

In this Section we expand the results obtained in the previous section and we assume that the flow of benefits in each period is determined through alternating-offer bargaining where,

in each stage of bargaining, the non-proposing player has three available actions: he can accept the offer, reject the offer and make a counteroffer, or reject the offer and impose an agreement with violence. As in the model from Section 4, a player's relative political power represents its chance of making the first proposal. In this case, the (endogenous) flow of benefits to a player is strictly increasing in both its military and political power rather than being only a function of political power (as in the models from Section 2 and Section 4). In this game of complete information, this leads to no war in the second/last period of the game, but, interestingly, the chance of war in the first-period remains positive: the bargaining breakdown is due to dynamic considerations, and the mismatch between military and political power remains a key driver of conflict.

The setup of this model is identical to the model presented in Section 4 (and, in particular, political power still represents the chance of being the first mover in the bargaining sub-game) with two exceptions. First, c_t, the cost of war in period t, is drawn uniformly from $[0, C]$ with $C \in (0, 1)$.[7] Second, as in Powell (1996), in each round of bargaining, the receiver can accept the offer, reject the offer and make a counter-offer or, crucially, reject the offer and try to impose a settlement with violence. In the last case, each player obtains the entire pie with probability equal to its current military power and both players suffer cost c_t.[8]

As discussed in Powell (1996) and in Osborne and Rubinstein (1990) — if players are sufficiently impatient (and, thus, the threat of using violence to impose an agreement rather than making a counteroffer is credible) — in the unique SPE outcome of this bargaining sub-game, an agreement is reached immediately and the first proposer offers the opponent exactly its outside option (that is, the chance of imposing a settlement successfully minus

[7] In particular, in order to ensure that both players' outside options are always non-negative and to avoid dealing with corner cases, we assume $C \leq \min\{m_1, 1 - m_1\} - a_1$.

[8] In the Appendix, we consider an alternative model where players do not have the chance to fight before bargaining on the allocation of resources in a period (thus, their political and military power in the bargaining sub-game equals the political and military power at the beginning of that period) but they can try to impose a settlement with violence while bargaining (and this conflict triggers a shift in political and military power at the beginning of the following period). We show that, as in the models we discuss in the paper, bargaining breakdown is possible (for similar dynamic considerations) and the initial mismatch between powers is relevant for the chance of a first-period war.

the cost of doing so).[9]

Denote by $x(p, m, c)$ the expected share to A before the identity of first mover is realized but after c is observed; we have

$$x(p, m, c) = p\,(m + c) + (1 - p)\,(m - c) = m + (2p - 1)c$$

which is strictly increasing in both m and p for any $c > 0$ (that is, as long as the cost of imposing a settlement is positive). Denote with $x(p, m)$ the expected share to A before the identity of the first mover and c are realized and observed; we have

$$x(p, m) = E_c\left[x(p, m, c)\right] = m + (2p - 1)\frac{C}{2}$$

which is strictly increasing in both m and p for any $C > 0$ (that is, as long as the cost of imposing a settlement can be positive). In the rest of this Section, we use this characterization of the bargaining sub-game outcome to solve the broader game with backward induction.

5.1 Period 2

A and B decide whether to attack or not after observing the realization of c_2.

A's and B's expected utility from not attacking are, respectively

$$\begin{aligned} x(m_2, p_2, c_2) &= m_2 + (2p_2 - 1)c_2 \\ 1 - x(m_2, p_2, c_2) &= 1 - m_2 + (1 - 2p_2)c_2 \end{aligned}$$

[9] Following a reasoning similar to the one in Section 4.2 of Osborne and Rubinstein (1990), it can be shown that this is the case if both players' outside options are larger than $\frac{\delta}{1-\delta}$, that is, the discounted utility from being the next proposer in the bargaining game where violence cannot be used. When δ is sufficiently large, resorting to violence is never rational for either player and the game is strategically equivalent to bargaining without the chance to impose a settlement, that is, to the model we discussed in Section 4.

A is better off not attacking if and only if

$$\begin{aligned} m_2 + (2p_2 - 1)c_2 &\geq m_2 - c_2 \\ 2p_2c_2 &\geq 0 \end{aligned}$$

which is always satisfied.

B is better off not attacking if and only if

$$\begin{aligned} 1 - m_2 + (1 - 2p_2)c_2 &\geq 1 - m_2 - c_2 \\ 2(1 - p_2)c_2 &\geq 0 \end{aligned}$$

which is always satisfied.

Thus, there is never war in the second period. If we denote with $v_i(m_2, p_2)$ player i's expected utility from the second period when military and political power at the beginning of the second period are, respectively, m_2 and p_2, we have

$$\begin{aligned} v_A(m_2, p_2) &= m_2 + (2p_2 - 1)E[c_2] = m_2 + (2p_2 - 1)\frac{C}{2} \\ v_B(m_2, p_2) &= 1 - m_2 - (2p_2 - 1)E[c_2] = 1 - m_2 - (2p_2 - 1)\frac{C}{2} \end{aligned}$$

5.2 Period 1: A's Incentive to Wage War

Denote by $x(m_1, p_1, c_1)$ the share to A in the bargaining that follows the decision not to attack. A's expected utility from not attacking is

$$\begin{aligned} U_A[\text{Peace}] &= x(m_1, p_1, c_1) + v_A(m_1, p_1) \\ &= m_1 + (2p_1 - 1)c_1 + m_1 + (2p_2 - 1)\frac{C}{2} \\ &= 2m_1 + (2p_1 - 1)\left(c_1 + \frac{C}{2}\right) \end{aligned}$$

Denote by Δ_x the magnitude of the change in the expected share from bargaining after a war (with respect to bargaining without a war; this change is positive if A is the winner and negative otherwise); and denote by Δ_v the magnitude of the change in the value function after a war (with respect to the value function without a war; this change is positive if A is the winner and negative otherwise). We have

$$\Delta_x = a + 2b(c_1), \quad \Delta_v = a + 2b\left(\frac{C}{2}\right)$$

A's expected utility from attacking is

$$\begin{aligned}
U_A[\text{War}] &= -c_1 + k(2m_1) + (1-k)\left[x(m_1, p_1, c_1) + v_A(m_1, p_1) + (2m_1 - 1)(\Delta_x + \Delta_v)\right] \\
&= -c_1 + k(2m_1) + (1-k)\left[2m_1 + (2p_1 - 1)\left(c_1 + \frac{C}{2}\right) + (2m_1 - 1)\left(2a + 2b\left(c_1 + \frac{C}{2}\right)\right)\right] \\
&= -c_1 + 2m_1 + (1-k)\left[(2p_1 - 1)\left(c_1 + \frac{C}{2}\right) + (2m_1 - 1)\left(2a + 2b\left(c_1 + \frac{C}{2}\right)\right)\right]
\end{aligned}$$

A prefers peace to war if and only if

$$2m_1 + (2p_1 - 1)\left(c_1 + \frac{C}{2}\right) \geq -c_1 + 2m_1 + (1-k)\left[\begin{array}{c} (2p_1 - 1)\left(c_1 + \frac{C}{2}\right) + \\ (2m_1 - 1)\left(2a + 2b\left(c_1 + \frac{C}{2}\right)\right) \end{array}\right]$$

$$c_1 \geq k\left[(1 - 2p_1)\left(c_1 + \frac{C}{2}\right)\right] + (1-k)\left[(2m_1 - 1)\left(2a + 2b\left(c_1 + \frac{C}{2}\right)\right)\right]$$

Thus, the cutoff for war c_A is:

$$c_A := \frac{k\left[(1 - 2p_1)\left(\frac{C}{2}\right)\right] + (1-k)\left[(2m_1 - 1)\left(2a + 2b\left(\frac{C}{2}\right)\right)\right]}{1 - k(1 - 2p_1) - (1-k)(2m_1 - 1)2b}$$

When this cutoff is positive, a war initiated by A happens with positive probability. Note that this cutoff is strictly decreasing in p_1 and strictly increasing in m_1.

5.3 Period 1: B's Incentive to Wage War

Denote by $x(m_1, p_1, c_1)$ the share to B in the bargaining which follows the decision not to attack. B's expected utility from not attacking is

$$\begin{aligned} U_B[\text{Peace}] &= x(m_1, p_1, c_1) + v_B(m_1, p_1) \\ &= 1 - m_1 - (2p_1 - 1)c_1 + 1 - m_1 - (2p_1 - 1)\frac{C}{2} \\ &= 2\,(1 - m_1) - (2p_1 - 1)\left(c_1 + \frac{C}{2}\right) \end{aligned}$$

Using the same notation as above, B's expected utility from attacking is

$$\begin{aligned} U_B[\text{War}] &= -c_1 + k(2\,(1 - m_1)) + (1 - k)\left[x(m_1, p_1, c_1) + v_B(m_1, p_1) - (2m_1 - 1)(\Delta_x + \Delta_v)\right] \\ &= -c_1 + 2\,(1 - m_1) + (1 - k)\left[-(2p_1 - 1)\left(c_1 + \frac{C}{2}\right) - (2m_1 - 1)\left(2a + 2b\left(c_1 + \frac{C}{2}\right)\right)\right] \end{aligned}$$

B prefers peace to war if and only if

$$2\,(1 - m_1) - (2p_1 - 1)\left(c_1 + \frac{C}{2}\right) \geq -c_1 + 2\,(1 - m_1) - (1 - k)\left[\begin{array}{c} (2p_1 - 1)\left(c_1 + \frac{C}{2}\right) + \\ (2m_1 - 1)\left(2a + 2b\left(c_1 + \frac{C}{2}\right)\right) \end{array}\right]$$

The cost cutoff below which B prefers to attack is

$$c_B := \frac{k\left[(2p_1 - 1)\left(\frac{C}{2}\right)\right] + (1 - k)\left[(1 - 2m_1)\left(2a + 2b\left(\frac{C}{2}\right)\right)\right]}{1 - k(2p_1 - 1) - (1 - k)(1 - 2m_1)2b}$$

When this cutoff is positive, a war initiated by B happens with positive probability. Note that this cutoff is strictly increasing in p_1 and strictly decreasing in m_1.

5.4 Result

We finally come to our main result that highlights how the mismatch matters for war/peace.

Proposition 12 *The chance of war is zero with perfect balance of powers* $(m_1, p_1) = (1/2, 1/2)$ *, but the more military advantaged a player is and/or the lower political advantage a player has, the higher its incentives to attack and, thus, the chance of war.*

Proof. In perfect balance of powers, $(m_1, p_1) = (1/2, 1/2)$, we have zero chance of war because:

$$c_A(1/2, 1/2) = c_B(1/2, 1/2) = 0$$

But for A for instance, the cutoff $c_A(m_1, p_1)$ is strictly decreasing in p_1 and strictly increasing in m_1, thus increasing the mismatch from this perfect balance situation, either by increasing m_1 or decreasing p_1, makes c_A, thus the probability of war, positive. The larger this mismatch becomes, either by increasing m_1 or decreasing p_1, the larger the chance of war. ■

6 Some Empirical Observations

In this section, we display some illustrative correlations between powers' mismatch and interstate war onset and incidence, leaving for future work the possibility to establish causality.

Our interstate war dataset includes 325 country dyads for the period 1960–2011. Since most interstate wars are between contiguous countries, our dataset only includes dyads that share either a land boarder or river border, or are located within 24 nautical miles of each other by sea (intersection of territorial waters).[10] Information on our dependent variables, interstate conflict onset and conflict incidence, for the years 1816–2011, come from the Dyadic Militarized Interstate Disputes Version 3 dataset (Maoz 2005). A challenge is to find valid proxies of military and political power. As a proxy of the relative military power (m) of a country A with respect to the other country B in the dyad, we take the troop number of A divided by the sum of A's and B's troop numbers, using data on countries' military personnel

[10] Information on territorial contiguity comes from the COW Direct Contiguity Dataset Version 3.2 (Stinnett, Tir, Schafer, Diehl, and Gochman 2002) which lists all the country dyads from 1816 to 2016 and their territorial relationship.

from the COW National Material Capabilities Dataset (NMC, Singer, Bremer, and Stuckey 1972).[11] As a proxy of the relative political power of country A with respect to B (p) we take the GDP of A divided by the sum of GDP of A and B, using data from the World Bank National Accounts Data. Our main independent variable, the *mismatch* between military and political power, is then constructed as the absolute value of the difference between these two variables. Finally, as data from the Dyadic MID is directed (both A vs B and B vs A), we created a non-directed dataset to test the model where country "A" is the potential challenger, that is, the country with $m > p$, in accordance with the theoretical framework developed in the paper.

Following Cunningham and Lemke (2013)—who show that interstate and intrastate wars are explained by similar factors—we use three sets of control variables, one set drawn from the interstate wars literature and two sets drawn from the civil wars literature. A first set of correlates is from the existing literature on the onset of interstate wars: the number of allies each state in the dyad has; the number of interstate "enduring rivals" each state in the dyad has; the number of direct land neighbors each state in the dyad has; an indicator of whether either country in the dyad is a major power as coded by COW.[12] A second set of correlates is drawn from Hegre and Sambanis (2006) who produce a list of the most robust predictors of civil war onset. Borrowing from their findings, our second set of control variables includes the following: a dummy indicating whether the state experienced a previous interstate conflict; gross domestic product per capita; logged population; and Polity III 0-10 democracy score.[13] Finally, the third set of control variables includes insurgency conditions and ethnicity, as suggested by the canonical study by Fearon and Laitin (2003). The specific correlates are the following: the percentage of the two states' territory that is mountainous, whether the

[11] In Appendix C, we provide a validity test of this simple proxy for military power.

[12] These variables come from the replication data for Cunningham and Lemke (2013). Their original source is EUGene (Bennett and Stam 2000), with the exception of the number of allies and rivals which come from Diehl and Goertz (2001).

[13] These variables also come from the replication data for Cunningham and Lemke (2013). Their source for gross domestic product per capita and for population is Gleditsch (2002).

Model	Variables	Conflict Onset	Conflict Incidence
No Controls	Mismatch	8.89***	10.22***
	Intercept	0.05***	0.07***
	Model χ^2	28.49***	26.86***
	Observations	10,585	10,585
Controls from COW	Mismatch	8.61***	11.09***
	Allies A	0.99	1.00
	Allies B	0.90**	0.80***
	Rivals A	1.16*	1.11
	Rivals B	1.60***	2.04***
	Neighbors A	1.02	1.02
	Neighbors B	1.00	1.01
	Major Power	0.89	0.81
	Intercept	0.04***	0.06***
	Model χ^2	140.34***	69.44***
	Observations	9,147	9,147
Controls from Hegre & Sambanis (2006)	Mismatch	11.97***	13.43***
	Prior War A	1.03	1.57**
	Prior War B	1.67*	4.00***
	GDP Per Capita A	0.95*	0.95*
	GDP Per Capita B	0.98	0.99
	Log of Population A	1.20***	1.16**
	Log of Population B	1.07	1.05
	Democracy A	1.01	0.99
	Democracy B	1.04	1.06
	Intercept	0.00***	0.01***
	Model χ^2	75.50***	65.80***
	Observations	8,218	8,218
Controls from Fearon & Laitin (2003)	Mismatch	8.03***	9.99***
	Log of Mountains A	1.21**	1.22*
	Log of Mountains B	1.04	1.02
	Discontinuous Territory A	2.29***	2.43**
	Discontinuous Territory B	0.92	0.84
	New State A	0.76	0.94
	New State B	1.16	1.04
	Instability A	1.11	1.02
	Instability B	1.07	1.00
	Ethnic Fractionalization A	1.02	0.75
	Ethnic Fractionalization B	0.97	0.92
	Intercept	0.03***	0.05***
	Model χ^2	47.82***	38.71***
	Observations	7,879	7,879

Table 1: Logit regressions of onsets and incidence of interstate conflict. Mismatch constructed as absolute value of (Military Ratio – GDP Ratio). Cell entries are odds ratios. Robust standard errors clustered by countries' dyad. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

two states have non-continuous territory, whether they are a new state (a dummy equal to 1 for the first two years after independence), a measure of instability (a dummy equal to 1 when the state's regime type score has changed by three or more in any of the previous three years), and a measure of ethnic fractionalization.[14]

Table 1 presents the results for three separate specifications. Following Cunningham and Lemke (2013), we avoid combining all variables in one analysis because of the dubious usefulness of kitchen-sink models.[15] In Table 1, we report odds ratios, interpreted in the following way: odds ratios below 1 mean that increasing values of the independent variable make conflict onset less likely, while values above 1 mean that conflict onset is more likely. Table 1 implies strongly that the mismatch between political and military power is positively and significantly correlated with interstate war onset and with interstate war incidence, even when controlling for all the other relevant factors.

7 Conclusion

This paper proposes a simple dynamic theory of war onset and duration that emphasizes the critical role of the mismatch between military and political power and of its evolution over time. Our core model is based on two central assumptions. First, in the absence of conflict, the distribution of resources in a given period is determined by the current political institutions which cannot be easily modified with peaceful bargaining. Second, conflict catalyzes change in both political institutions and military endowments. Our model points out a non-monotonicity in the role of balance of military power as this no longer is a sufficient statistic for war or peace: the cases where both powers are balanced and the cases in which

[14]Following Fearon and Laitin (2003), Cunningham and Lemke (2013) also include in this third set of controls an indicator of whether a country is an oil exporter. In our current dataset, this information is available only for a subset of observations, and for this reason in Table 1 we report results without these variables. Including these additional two dummies does not change the results.

[15]See also Achen (2002) and Ray (2003), who document how misleading regression models with many regressors can be. A model that includes all the available independent variables gives similar results.

one group is proportionally more powerful in both spheres should be equally unlikely to display wars ex-ante, but the case with balanced military powers could have larger duration in case a first war erupts. The case of balanced military power *together with* unequal political power can be explosive (under some conditions) both in terms of ex ante probability of war and in terms of war duration. Our results are robust to allowing the allocation of resources in a period to be determined endogenously through bargaining and we offer some preliminary empirical evidence.

The Houthis in Yemen are a good example of a politically under-represented group with significant military strength.[16] The civil war they started in 2014 can be attributed to both an enduring political and economic discrimination (see Shuja al-Deen 2019) and their increased strength due to pro-Saleh wings of the country's armed services turning on the Houthi side. President Hadi's government is not only facing a centrist conflict with the Houthis, but he also needs to face a secessionist movement with a now much stronger Southern Transition Council (STC) helped also by the U.A.E., making again a clear case of powers' mismatch (see Salisbury 2018). Having to face a centrist challenge by the Houthis and a secessionist challenge by the STC, the government is made weaker on each front and unable to make concessions to one without upsetting the other, hence avoiding the mismatch on both sides would be close to impossible even if Hadi wanted. This combination of bargaining difficulties that perpetuate and exacerbate the mismatches makes the Yemen case an unfortunate showcase for why mismatches can persist and wars can be hard to stop.

Understanding that, in reality, the mismatch between military and political powers is more important than the balance of power debate focusing on a one dimensional notion of power should be important not only in international and national relations, but also, potentially, in other subfields of political economic studies. In future research, it would be interesting to extend the framework of this paper to study social and family conflicts,

[16]Salisbury (2017) gives a detailed account of the much higher relative strength of Houthis with respect to their relative political power. See also, for example, Fattah 2010, and Salmoni et al. 2010, for the various insurgency wars before the bargaining attempt made by President Hadi.

democratic politics dynamics, and even institutional design, given that no existing theory of institutional design takes into account that the optimal set of institutions may depend on the current distribution of the other types of power that do not depend on the institutions being chosen.

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A Proof of Lemma 2

Assume $m_t > p_t$. We need to show that $m_t + a_t > p_t + b_t$ and that $m_t - a_t > p_t - b_t$. We first show that $m_t + a_t > p_t + b_t$.

$$\begin{aligned} m_t + a_t &> p_t + b_t \\ m_t + g\left(1/2 - |m_t - 1/2|\right) &> p_t + g\left(1/2 - |p_t - 1/2|\right) \\ (m_t - p_t) &> g\left(|m_t - 1/2| - |p_t - 1/2|\right) \end{aligned}$$

There are three cases: (a) $1/2 > m_t > p_t$, (b) $m_t > p_t > 1/2$, (c) $m_t > 1/2 > p_t$.

In case (a), we have:

$$(m_t - p_t) > g\left((1/2 - m_t) - (1/2 - p_t)\right) = g\left(p_t - m_t\right)$$

which is satisfied, since $m_t > p_t$ and $g > 0$.

In case (b), we have:

$$(m_t - p_t) > g\left((m_t - 1/2) - (p_t - 1/2)\right) = g\left(m_t - p_t\right)$$

which is satisfied, since $g < 1$.

In case (c), we have:

$$(m_t - p_t) > g\left((m_t - 1/2) - (1/2 - p_t)\right) = g\left(m_t - (1 - p_t)\right)$$

which is satisfied, since $(1 - p_t) > p_t$ and $g < 1$.

We now show that $m_t - a_t > p_t - b_t$.

$$\begin{aligned} m_t - a_t &> p_t - b_t \\ m_t - g\left(1/2 - |m_t - 1/2|\right) &> p_t - g\left(1/2 - |p_t - 1/2|\right) \\ (m_t - p_t) &> g\left(|p_t - 1/2| - |m_t - 1/2|\right) \end{aligned}$$

There are three cases: (a) $1/2 > m_t > p_t$, (b) $m_t > p_t > 1/2$, (c) $m_t > 1/2 > p_t$.

In case (a), we have:

$$(m_t - p_t) > g\left((1/2 - p_t) - (1/2 - m_t)\right) = g\left(m_t - p_t\right)$$

which is satisfied, since $g < 1$.

In case (b), we have:

$$(m_t - p_t) > g\left((p_t - 1/2) - (m_t - 1/2)\right) = g\left(p_t - m_t\right)$$

which is satisfied, since $m_t > p_t$ and $g > 0$.

In case (c), we have:

$$(m_t - p_t) > g\left((1/2 - p_t) - (m_t - 1/2)\right) = g\left((1 - m_t) - p_t\right)$$

which is satisfied, since $(1 - m_t) < m_t$ and $g < 1$. ■

B Alternative Model with Bargaining

Consider the baseline model from Section 2 but — instead of assuming that peace-time consumption is proportional to relative political power — assume that, in each period, players can bargain over the allocation of that period's flow of resources before attacking or not as

in the "canonical or standard model of the origins of war" discussed in Powell (2002). In this alternative model, the bargaining breakdown occurs purely because of dynamic considerations: the incentive to attack in the first period is driven by the expected gain in relative military and political power at the beginning of the second period (which, in turn, positively affects the expected share of the resources in the second-period peaceful bargaining agreement). Nonetheless, as in the models we discussed in the paper, conflict happens with positive probability and the mismatch between powers is still relevant for the chance of war.

B.1 Static Game and Value Functions

Since the game is one of complete information (i.e., players' outside options are common knowledge), then, not surprisingly, there is never conflict in the second and last period of the game. Denote with $x_2 \in [0, 1]$ the allocation to player A in period 2. A prefers a peaceful agreement to war if and only if:

$$x_2 \geq m_2 - c_2 \tag{5}$$

Similarly, B prefers a peaceful agreement to war if and only if:

$$\begin{aligned} 1 - x_2 &\geq (1 - m_2) - c_2 \\ x_2 &\leq m_2 + c_2 \end{aligned} \tag{6}$$

Since $m_2 + c_2 \geq m_2 - c_2$, $m_2 + c_2 \geq 0$ and $m_2 - c_2 \leq 1$, there is always at least one feasible bargaining agreement, $x_2 \in [0, 1]$, such that

$$m_2 + c_2 \geq x_2 \geq m_2 - c_2$$

In other words, there is always at least one feasible bargaining agreement, $x_2 \in [0, 1]$,

such that both players prefer peace to war. In fact, any

$$x_2 \in [\max\{0, m_2 - c_2\}, \min\{1, m_2 + c_2\}]$$

is a bargaining agreement which is feasible and dissuades both players from attacking. The exact point on this Pareto frontier where the second-period peaceful agreement lies depends on the players' relative bargaining power (which, in turn, can be a function of the details of the bargaining protocol employed). In this extension, we embody a player's relative bargaining power (or, in other words, the extent to which the player is able to appropriate the peace-time surplus) with his relative political power at the beginning of period 2, that is, p_2 for player A and $1 - p_2$ for player B. One way to micro-found this is to assume that the player recognized to be the agenda setter makes a take-it-or-leave-it offer to the opponent and that the chance of being recognized as the proposer is equal to a player's political power.[17]

We denote with $v_A(m_2, p_2)$ player A's expected utility from the second period when military and political power at the beginning of the second period are, respectively, m_2 and p_2. To avoid dealing with corner cases, we assume that, in the second period, both players can be made indifferent between a peaceful agreement and war with a non-negative transfer for any realization of the cost of war. This implies the following assumption:

Assumption 3 *$c_2 \sim U[0, C]$ where $C \leq \{\min\{m_1, (1 - m_1)\} - a_1 \ \in (0, 1/2)$.*

Then, in a peaceful bargaining agreement, both players will be granted at least their outside option (that is, their expected utility from war) and the peace-time surplus (that is, the total utility loss from war, $2c$) will be shared according to their relative bargaining

[17] As discussed in Section 5, assuming that counter-offers are possible and that players alternate as proposers would lead to the same results as long as players discount an agreement which takes multiple rounds of proposals with a sufficiently low factor.

power.

$$
\begin{aligned}
v_A(m_2, p_2) &= E[m_2 - c_2 + p_2(2c_2)] \\
&= m_2 + (2p_2 - 1)E[c_2] \\
&= m_2 + Cp_2 - \frac{C}{2} \qquad (7) \\
v_B(m_2, p_2) &= E[1 - m_2 - c_2 + (1 - p_2)(2c_2)] \\
&= 1 - m_2 - (2p_2 - 1)E[c_2] \\
&= 1 - m_2 - Cp_2 + \frac{C}{2} = 1 - v_A(m_2, p_2) \qquad (8)
\end{aligned}
$$

Note that, in this formulation, the players' relative political power — p_2 and $(1 - p_2)$ — are the players' Nash bargaining weights. These weights can also be micro-founded as the result of a non-cooperative bargaining game. For example, we would obtain the same expected values from second-period bargaining if, at the beginning of the second period, each player was selected to make a take-it-or-leave-it offer to the other player with a probability equal to his relative political power. The expected value from the second period is increasing in a player's relative military power (which determines his outside option) and in a player's relative political power (which determines his ability to appropriate the peace-time surplus).

B.2 Dynamic Game

For ease of notation, we can get rid of subscripts and denote m_1 with m and p_1 with p. Let $b \in (0, 1/2)$, $a \in (0, 1/2)$ and $\Delta(m, p)$ be, respectively, the change (in favor of the winner) of relative political power, relative military power and the expected utility from the second period in case of war in the first period.

Using the results from the previous section we have:

$$\begin{aligned}\Delta(m,p) &= v_A(m+a,p+b) - v_A(m,p) \\ &= m+a+C(p+b) - \frac{C}{2} - m - Cp - \frac{C}{2} \\ &= a + Cb \end{aligned} \tag{9}$$

Let the transfer to A in the first-period bargaining agreement be $x_1 \in [0,1]$.

A prefers a peaceful agreement to war if and only if:

$$x_1 + v_A(m,p) \geq -c_1 + k(2m) + (1-k)\begin{pmatrix} m(p+b+v_A(m,p)+\Delta(m,p))+ \\ (1-m)(p-b+v_A(m,p)-\Delta(m,p)) \end{pmatrix} \tag{10}$$

Namely, defining K as below, we have:

$$x_1 \geq -c_1 + K \tag{11}$$

where:

$$\begin{aligned} K &= k(2m - v_A(m,p)) + (1-k)\left(p + (2m-1)(b+\Delta(m,p))\right) \\ &= k\left(m - Cp + \frac{C}{2}\right) + (1-k)\left(p + (2m-1)((1+C)b+a)\right) \end{aligned}$$

Similarly, B prefers a peaceful agreement to war if and only if:

$$1 - x_1 + v_B(m,p) \geq -c_1 + 2k(1-m) + (1-k)\begin{pmatrix} m(1-p-b+v_B-\Delta(m,p))+ \\ (1-m)(1-p+b+v_B(m,p)+\Delta(m,p)) \end{pmatrix}$$

Namely $x_1 \leq c_1 + K$. In sum, if there is a feasible bargaining agreement, $x_1 \in [0,1]$, such that

$$K + c_1 \geq x_1 \geq K - c_1$$

then, there is no war in the first period. Since $K + c_1 \geq K - c_1$, war can be avoided with a bargaining agreement if these two conditions are simultaneously satisfied:

$$K + c_1 \geq 0, \qquad K - c_1 \leq 1$$

Assumption 4 *Assume A is the militarily advantaged player, that is, $m \geq 1/2$.*

Under this assumption, we have $K + c_1 \geq 0$ for any $c_1 \in [0, 1]$, and the first condition is always satisfied. Thus, there is no first-period war if and only if the second condition

$$K - c_1 \leq 1 \tag{12}$$

is satisfied for any $c_1 \in [0, 1]$. This means that we have war with positive probability if and only if $K > 1 + c_1$ for some $c_1 \in [0, 1]$. Thus, there is war with positive probability if and only if $K > 1$. Proposition 13 summarizes the above discussion.

Proposition 13 *When players can bargain over the allocation of resources in peace, then only if $K > 1$ there can be a first-period war and its probability is:*

$$Pr[War] = \max\{0, K - 1\} \tag{13}$$

B.3 Special Cases

We now analyze some special cases to clarify the forces at stake.

B.3.1 Special Case 1: Always Decisive War ($k = 1$)

Proposition 14 *When players can bargain over the allocation of resources in peace and war is always decisive ($k = 1$), there is a positive probability of first-period war if and only if the mismatch between military and political power is sufficiently large; the probability of*

first-period war is weakly increasing in the mismatch between military and political power (strictly increasing in the region of parameters where war happens with positive probability).

Proof. There is war with positive probability if and only if

$$m - Cp > 1 - \frac{C}{2} \tag{14}$$

Since the LHS goes to 1 as m goes to 1 and p goes to 0 while the RHS is bounded away from 1 (since C is strictly positive), there are initial allocations of relative military and political power, (m, p), which conduce to a positive probability of war in the first period. In particular, there is a positive probability of war only if $m \geq p$ (to see this, note that the inequality can never be satisfied when $p = m$ and that the LHS is decreasing in p so the inequality is even harder to be satisfied when $p > m$). Indeed, there is a positive probability of war if and only if the mismatch between military and political power, $m - p$ is sufficiently large. When this is the case, the probability of war in the first period is

$$\Pr[\text{War}] = m - Cp + \frac{C}{2} - 1$$

which is strictly increasing in the mismatch between military and political power. ■

B.3.2 Special Case 2: Never Decisive War ($k = 0$)

Proposition 15 *When players can bargain over the allocation of resources in peace and war is never decisive ($k = 0$), there is a positive probability of first-period war if and only if the militarily advantaged player has a sufficiently large political power and the power shifts in case of war are sufficiently large.*

Proof. There is war with positive probability if and only if

$$p + (2m - 1)((1 + C)b + a) \quad > \quad 1 \tag{15}$$

and, in this case, the probability of war is

$$\Pr[\text{War}] \quad = \quad p + (2m-1)((1+C)b + a) - 1 \tag{16}$$

which is increasing in p, m, a and b. ■

So for example under the power shift specification from Section 2.5, we have that if $p \leq 1/2$, we have $a = g(1-m)$ and $b = gp$, so the probability of war is

$$\Pr[\text{War}] = \max\{0, p + (2m-1)g(p(1+C) + 1 - m) - 1\} \tag{17}$$

which is strictly increasing in p, C, g; it is also strictly increasing in m if $m < \frac{(1+C)}{2}p + \frac{3}{4}$. Whereas if $p \geq 1/2$, we have $a = g(1-m)$ and $b = g(1-p)$, so the probability of war is

$$\Pr[\text{War}] = \max\{0, p + (2m-1)((1+C)g(1-p) + g(1-m)) - 1\} \tag{18}$$

which is strictly increasing in p, C, g; it is strictly increasing in m if m and p are sufficiently close to 1/2 and strictly decreasing in m if m and p are sufficiently close to 1.

B.3.3 Special Case 3: No Military Advantage ($m_1 = 1/2$)

Proposition 16 *When players can bargain over the allocation of resources in peace and no player is militarily advantaged ($m_1 = 1/2$), there is never a first-period war.*

Proof. There is a first-period war with positive probability if and only if

$$\begin{aligned} k\left(\frac{1}{2} - Cp + \frac{C}{2}\right) + (1-k)p &> 1 \\ k(1+C)\left(\frac{1}{2} - p\right) + p &> 1 \end{aligned}$$

which is not satisfied for any $p \in [0,1]$. ■

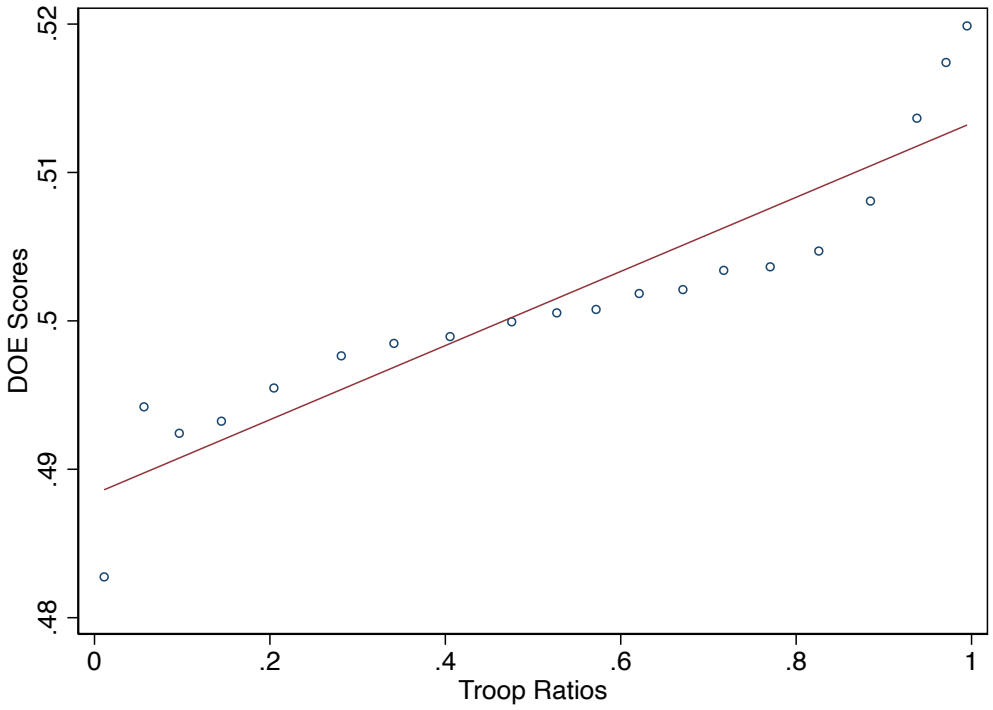

Figure 1: Binscatter DOE vs. Troop Ratio

C Validity of Proxy for Relative Military Power

To test the validity of our military power proxy, it was measured against some relevant developments in the literature. A recent paper by Carroll and Kenkel (2019) applied a superlearner algorithm to militarized dispute data to create a Dispute Outcome Expectation score. This score seeks to provide probabilities on three outcomes: A wins, B wins, or stalemate. As our model does not contemplate stalemate as an ex-ante possibility, we created a ratio between the relative winning probabilities to compare to our troop ratio proxy. The correlation between the two measures, as shown by Table 2, is nearly 0.6, indicating that our measure is indeed a good proxy for military power. Moreover, the figure displays a binscatter plot of the two measures, demonstrating the positive relationship between the two.

	DOE	Mil. Prop
DOE	1	
Mil.Prop	0.5603	1

Table 2: Correlation between the two military measures