Internet Appendix for
"Rent extraction with securities plus cash"

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ABSTRACT

This Internet Appendix provides additional analyses supporting the main text. In Section I, we characterize the optimal equity-plus-cash mechanism for the one-bidder, three-type case when $s(v)$ is not concave. In Section II, we analyze informal auctions. In Section III, we investigate a simple setting of two-sided private information.

[1]Liu, Tingjun and Dan Bernhardt, Internet Appendix for "Rent extraction with securities plus cash," *Journal of Finance*, [DOI STRING]. Please note: Wiley is not responsible for the content or functionality of any supporting information supplied by the authors. Any queries (other than missing material) should be directed to the authors of the article.

I. Optimal Equity-Plus-Cash Mechanism When s(V) Is Not Concave (One-bidder, Three-type Case)

Proposition IA1: When $\tau > 0$, the optimal equity-plus-cash mechanism has these features:

(i) If there are sufficiently few intermediate type 2s so that either

$$f_1 \geq f_3 \frac{V_3 + s_3 - (V_2 + s_2)}{V_2 + s_2 - (V_1 + s_1)} \text{ and } \tau f_3 \frac{(V_3 - V_2)(V_2 - V_1)}{V_3 + s_3 - (V_1 + s_1)} \geq f_2 s_2 \tag{IA1}$$

or

$$f_1 < f_3 \frac{V_3 + s_3 - (V_2 + s_2)}{V_2 + s_2 - (V_1 + s_1)} \text{ and } \tau f_1 \frac{(V_3 - V_2)(V_2 - V_1)}{V_3 + s_3 - (V_1 + s_1)} \geq f_2 s_2, \tag{IA2}$$

then the seller excludes type 2s and extracts all surplus from types 1 and 3, earning expected profit

$$\Pi_s = f_1 s_1 + f_3 s_3. \tag{IA3}$$

(ii) With more type 2s, so that neither (IA1) nor (IA2) hold, and with

$$f_1 \geq f_3 \frac{V_3 + s_3 - (V_2 + s_2)}{V_2 + s_2 - (V_1 + s_1)}, \tag{IA4}$$

so that type 1s are relatively more abundant than type 3s, the seller extracts all surplus from types 1 and 2, and leaves rents to type 3, earning expected profit

$$\Pi_s = f_1 s_1 + f_2 s_2 + f_3 s_3 - \tau f_3 \frac{(V_3 - V_2)(V_2 - V_1)}{V_3 + s_3 - (V_1 + s_1)}. \tag{IA5}$$

If, instead, inequality (IA4) is reversed, then the seller extracts all surplus from types 2 and 3, and leaves rents to type 1, earning expected profit

$$\Pi_s = f_1 s_1 + f_2 s_2 + f_3 s_3 - \tau f_1 \frac{(V_3 - V_2)(V_2 - V_1)}{V_3 + s_3 - (V_1 + s_1)}. \tag{IA6}$$

(iii) In all cases, the optimal mechanism can be implemented by a single contract with $p = 1$ and $e \in (0, 1)$. More generally, the optimal mechanism can be implemented by multiple contracts so that the higher type pays (weakly) higher cash and less equity share.

Proof of Proposition IA1: First, note that for any i, if $p_i > 0$, equation (30) in the main text yields $\pi_{ii} \geq 0$; where if $p_i = 0$, then the details of contract i do not affect (29), (30), or (31). Thus, we can assume $\pi_{ii} \geq 0$ for all i. Set $j = 2$ in (29). Then by $p_1 \leq 1$, $p_2 \leq 1$ and $\pi_{ii} \geq 0$,

$$p_1 \pi_{11} \geq \max\{p_2 \pi_{12}, 0\}, \tag{IA7}$$

and

$$p_3 \pi_{33} \geq \max\{p_2 \pi_{32}, 0\}. \tag{IA8}$$

By (27),

$$
\begin{aligned}
& \frac{V_3+s_3-(V_2+s_2)}{V_3+s_3-(V_1+s_1)}\pi_{12}+\frac{V_2+s_2-(V_1+s_1)}{V_3+s_3-(V_1+s_1)}\pi_{32}-\pi_{22} \\
= \;& \frac{V_3+s_3-(V_2+s_2)}{V_3+s_3-(V_1+s_1)}\left(\pi_{12}-\pi_{22}\right)+\frac{V_2+s_2-(V_1+s_1)}{V_3+s_3-(V_1+s_1)}\left(\pi_{32}-\pi_{22}\right) \\
= \;& \frac{V_3+s_3-(V_2+s_2)}{V_3+s_3-(V_1+s_1)}\left[\left(1-e_2\right)\left(v_1-v_2\right)+V_2-V_1\right] \\
& +\frac{V_2+s_2-(V_1+s_1)}{V_3+s_3-(V_1+s_1)}\left[\left(1-e_2\right)\left(v_3-v_2\right)+V_2-V_3\right] \\
= \;& \frac{V_3+s_3-(V_2+s_2)}{V_3+s_3-(V_1+s_1)}\left(V_2-V_1\right)-\frac{V_2+s_2-(V_1+s_1)}{V_3+s_3-(V_1+s_1)}\left(V_3-V_2\right) \\
= \;& \frac{1}{V_3+s_3-(V_1+s_1)}\left(\left(V_3-V_2+s_3-s_2\right)\left(V_2-V_1\right)-\left(V_2-V_1+s_2-s_1\right)\left(V_3-V_2\right)\right) \\
= \;& \tau, \qquad \text{(IA9)}
\end{aligned}
$$

where τ is defined in (26). By (IA9) and $\pi_{22}\geq 0$, we have

$$
\frac{V_3+s_3-(V_2+s_2)}{V_3+s_3-(V_1+s_1)}\pi_{12}+\frac{V_2+s_2-(V_1+s_1)}{V_3+s_3-(V_1+s_1)}\pi_{32}\geq \tau. \qquad \text{(IA10)}
$$

Rewrite the seller's expected profit (31) as

$$
\begin{aligned}
\Pi_s &= f_1p_1\left(s_1-\pi_{11}\right)+f_2p_2\left(s_2-\pi_{22}\right)+f_3p_3\left(s_3-\pi_{33}\right) \\
&\leq f_1p_1s_1+f_2p_2s_2+f_3p_3s_3-p_2\left(f_1\max\left\{\pi_{12},0\right\}+f_2\pi_{22}+f_3\max\left\{\pi_{32},0\right\}\right) \\
&\leq f_1s_1+f_2p_2s_2+f_3s_3-p_2\left(f_1\max\left\{\pi_{12},0\right\}+f_3\max\left\{\pi_{32},0\right\}\right) \\
&= f_1s_1+f_3s_3+p_2\left(f_2s_2-\pi^*\right) \qquad \text{(IA11)}
\end{aligned}
$$

where the first inequality follows from (IA7) and (IA8), the second inequality follows from $p_1 \leq 1$, $p_3 \leq 1$, and $\pi_{22} \geq 0$, and

$$\pi^* \equiv f_1 \max\{\pi_{12}, 0\} + f_3 \max\{\pi_{32}, 0\}. \tag{IA12}$$

Next we bound π^* from below.

CLAIM:

$$\pi^* \geq \begin{cases} \tau f_3 \frac{V_3+s_3-(V_1+s_1)}{V_2+s_2-(V_1+s_1)} & \text{if} \quad f_1 \geq f_3 \frac{V_3+s_3-(V_2+s_2)}{V_2+s_2-(V_1+s_1)} \\ \tau f_1 \frac{V_3+s_3-(V_1+s_1)}{V_3+s_3-(V_2+s_2)} & \text{if} \quad f_1 < f_3 \frac{V_3+s_3-(V_2+s_2)}{V_2+s_2-(V_1+s_1)} \end{cases} \tag{IA13}$$

To prove the claim, we consider three cases.

Case 1: Suppose that

$$0 \leq \pi_{12} \leq \tau \frac{V_3 + s_3 - (V_1 + s_1)}{V_3 + s_3 - (V_2 + s_2)}. \tag{IA14}$$

Then (IA10) yields $\pi_{32} \geq 0$, and (IA12) yields

$$\begin{aligned} \pi^* &= f_1 \pi_{12} + f_3 \pi_{32} \\ &\geq f_1 \pi_{12} + f_3 \left(\frac{V_3 + s_3 - (V_1 + s_1)}{V_2 + s_2 - (V_1 + s_1)} \tau - \frac{V_3 + s_3 - (V_2 + s_2)}{V_2 + s_2 - (V_1 + s_1)} \pi_{12} \right) \\ &= \frac{V_3 + s_3 - (V_1 + s_1)}{V_2 + s_2 - (V_1 + s_1)} f_3 \tau + \left(f_1 - f_3 \frac{V_3 + s_3 - (V_2 + s_2)}{V_2 + s_2 - (V_1 + s_1)} \right) \pi_{12}, \end{aligned}$$

where the inequality follows from (IA10). Under (IA14), if $f_1 \geq f_3 \frac{V_3+s_3-(V_2+s_2)}{V_2+s_2-(V_1+s_1)}$ then

$$\begin{aligned} \pi^* &\geq \frac{V_3 + s_3 - (V_1 + s_1)}{V_2 + s_2 - (V_1 + s_1)} f_3 \tau \\ &= \tau f_3 \frac{V_3 + s_3 - (V_1 + s_1)}{V_2 + s_2 - (V_1 + s_1)} \end{aligned}$$

and if $f_1 < f_3 \frac{V_3+s_3-(V_2+s_2)}{V_2+s_2-(V_1+s_1)}$, then

$$\begin{aligned}
\pi^* &\geq \frac{V_3+s_3-(V_1+s_1)}{V_2+s_2-(V_1+s_1)} f_3 \tau + \left(f_1 - f_3 \frac{V_3+s_3-(V_2+s_2)}{V_2+s_2-(V_1+s_1)}\right)\left(\tau \frac{V_3+s_3-(V_1+s_1)}{V_3+s_3-(V_2+s_2)}\right) \\
&= \tau f_1 \frac{V_3+s_3-(V_1+s_1)}{V_3+s_3-(V_2+s_2)}.
\end{aligned}$$

This proves the claim for Case 1.

Case 2: $\pi_{12} < 0$. Then (IA10) yields $\pi_{32} > \tau \frac{V_3+s_3-(V_1+s_1)}{V_2+s_2-(V_1+s_1)}$, which we substitute into (IA12) to obtain

$$\pi^* \geq f_3 \tau \frac{V_3+s_3-(V_1+s_1)}{V_2+s_2-(V_1+s_1)}, \tag{IA15}$$

which satisfies the first line of (IA13). Next, suppose that $f_1 < f_3 \frac{V_3+s_3-(V_2+s_2)}{V_2+s_2-(V_1+s_1)}$. Then plugging $f_3 > f_1 \frac{V_2+s_2-(V_1+s_1)}{V_3+s_3-(V_2+s_2)}$ into (IA15) yields the second line of (IA13). This proves the claim for Case 2.

Case 3: $\pi_{12} > \tau \frac{V_3+s_3-(V_1+s_1)}{V_3+s_3-(V_2+s_2)}$. Plugging this condition into (IA12) yields

$$\pi^* \geq f_1 \tau \frac{V_3+s_3-(V_1+s_1)}{V_3+s_3-(V_2+s_2)}, \tag{IA16}$$

so the second line of (IA13) is trivially satisfied. Next, suppose that $f_1 \geq f_3 \frac{V_3+s_3-(V_2+s_2)}{V_2+s_2-(V_1+s_1)}$. Plugging this condition into (IA15) yields the second line of (IA13). This proves the claim for Case 3.

Next, from (IA13) and (IA11),

$$\Pi_s \leq \begin{cases} f_1 s_1 + f_2 s_2 + f_3 s_3 - \tau f_3 \frac{V_3+s_3-(V_1+s_1)}{V_2+s_2-(V_1+s_1)} & \text{if} \quad f_1 \geq f_3 \frac{V_3+s_3-(V_2+s_2)}{V_2+s_2-(V_1+s_1)} \text{ and } \tau f_3 \frac{V_3+s_3-(V_1+s_1)}{V_2+s_2-(V_1+s_1)} < f_2 s_2 \\ f_1 s_1 + f_3 s_3 & \text{if} \quad f_1 \geq f_3 \frac{V_3+s_3-(V_2+s_2)}{V_2+s_2-(V_1+s_1)} \text{ and } \tau f_3 \frac{V_3+s_3-(V_1+s_1)}{V_2+s_2-(V_1+s_1)} \geq f_2 s_2 \\ f_1 s_1 + f_3 s_3 & \text{if} \quad f_1 < f_3 \frac{V_3+s_3-(V_2+s_2)}{V_2+s_2-(V_1+s_1)} \text{ and } \tau f_1 \frac{V_3+s_3-(V_1+s_1)}{V_3+s_3-(V_2+s_2)} \geq f_2 s_2 \\ f_1 s_1 + f_2 s_2 + f_3 s_3 - \tau f_1 \frac{V_3+s_3-(V_1+s_1)}{V_3+s_3-(V_2+s_2)} & \text{if} \quad f_1 < f_3 \frac{V_3+s_3-(V_2+s_2)}{V_2+s_2-(V_1+s_1)} \text{ and } \tau f_1 \frac{V_3+s_3-(V_1+s_1)}{V_3+s_3-(V_2+s_2)} < f_2 s_2 \end{cases} \tag{IA17}$$

Now consider the following single contract with $p = 1$ and (a) if either (IA1) or (IA2) holds then

$$e = \frac{s_3 - s_1}{V_3 + s_3 - V_1 - s_1}, c = V_T + s_1 - \frac{eV_1}{1-e},$$

(b) if neither (IA1) nor (IA2) holds, and $(IA4)$ holds, then

$$e = \frac{s_2 - s_1}{V_2 + s_2 - V_1 - s_1}, c = V_T + s_1 - \frac{eV_1}{1-e},$$

and (c) if neither (IA1) nor (IA2) holds, and $(IA4)$ is reversed, then

$$e = \frac{s_3 - s_2}{V_3 + s_3 - V_2 - s_2}, \quad c = V_T + s_3 - \frac{eV_3}{1-e}.$$

It is simple to show that this contract satisfies the properties stated in the proposition that in case (i), bidder types 1 and 3 receive zero rents and type 2 would receive strictly negative profit and hence not participate; and in case (ii) all bidder types receive nonnegative expected profit, and type 3 earns strictly positive rent when $(IA4)$ holds, while type 1 earns strictly positive rent when $(IA4)$ is reversed. The seller's expected profit achieves the upper bound on Π_s specified in (IA17), so the mechanism is optimal and (IA3), (IA5), and (IA6) hold.

Next, consider the following menu of two contracts with $p = 1$ for both contracts, and

$$e_1 \geq \frac{s_3 - s_1}{V_3 + s_3 - V_1 - s_1}, c_1 = V_T + s_1 - \frac{e_1 V_1}{1 - e_1}, \tag{IA18}$$

$$e_3 \leq \frac{s_3 - s_1}{V_3 + s_3 - V_1 - s_1}, c_3 = V_T + s_3 - \frac{e_3 V_3}{1 - e_3} \tag{IA19}$$

if either (IA1) or (IA2) holds; and

$$e_{23} = \frac{s_2 - s_1}{V_2 + s_2 - V_1 - s_1}, c_{23} = V_T + s_2 - \frac{e_{23} V_2}{1 - e_{23}}$$

$$e_1 \geq e_{23},\ c_1 = V_T + s_1 - \frac{e_1 V_1}{1 - e_1}$$

if neither (IA1) nor (IA2) holds, and ($IA4$) holds;

$$e_{12} = \frac{s_3 - s_2}{V_3 + s_3 - V_2 - s_2}, c_{12} = V_T + s_2 - \frac{e_{12} V_2}{1 - e_{12}}$$

$$e_3 \leq e_{12},\ c_3 = V_T + s_3 - \frac{e_3 V_3}{1 - e_3}$$

if neither (IA1) nor (IA2) holds, and ($IA4$) is reversed. The index "23" means that types 2 and 3 receive the same contract; index "12" has an analogous interpretation. It is easy to show that this menu of contracts induces the same acceptance-rejection decision from the bidder and acheives the same revenue as the earlier single contract. Thus, this menu also implements the optimal mechanism. We now show that a higher type pays (weakly) more cash and a smaller equity share. In the case in which either (IA1) or (IA2) holds, (IA18) and (IA19) yield $e_3 \leq e_1$, and

$$\begin{aligned}
c_3 - c_1 &= s_3 - \frac{e_3 V_3}{1-e_3} - \left(s_1 - \frac{e_1 V_1}{1-e_1}\right) \\
&= s_3 + V_3 - \frac{V_3}{1-e_3} - \left(s_1 + V_1 - \frac{V_1}{1-e_1}\right) \\
&\geq s_3 + V_3 - \frac{V_3}{1-e_3} - \left(s_1 + V_1 - \frac{V_1}{1-e_3}\right) \\
&\geq s_3 + V_3 - s_1 - V_1 - \frac{V_3 - V_1}{1 - \left(\frac{s_3 - s_1}{V_3 + s_3 - V_1 - s_1}\right)} = 0.
\end{aligned}$$

One can similarly show for the other two cases that the higher type pays (weakly) higher cash and less equity. ∎

II. Informal Auctions

In this appendix, we consider the possibility that the acquirer can select the cash-equity mix to offer and the target cannot reject any offer that leaves it with non-negative expected profit. When the target cannot commit in any way, even to set a reserve price the results is an informal auction: bidders are free to choose the cash-equity combination, and the seller picks the most attractive bid combination ex post.

LEMMA IA1: There is no pooling unless the bid is pure cash: for any V_1 and V_2, if $c(V_1) = c(V_2)$ and $e(V_1) = e(V_2)$, then either $V_1 = V_2$ or $e(V_1) = e(V_2) = 0$.

Proof of Lemma IA1: Suppose instead that $e(V_1) = e(V_2) > 0$, and that multiple types bid $\{c(V_1), e(V_2)\}$. Denote the set of all such types by τ. Then the monetary value that the target assigns to the bid is $c(V_1) + e(V_2)\mathrm{E}[s(V)|V \in \tau]$. Now the highest type in τ can strictly benefit by deviating to a pure cash bid of dollar amount $c(V_1)+e(V_2)\mathrm{E}[s(V)|V \in \tau]$, because the target will assign the same monetary value for this cash bid as for this bidder's equilibrium bid. Therefore, the probability of winning is the same, but the bidder pays strictly less (because if it used equity, its equity payment would be $e(V_2)\,s(V) > e(V_2)\,E\,[s(V)|V \in \tau]$), a contradiction. ■

In light of the lemma, for any V, the monetary value that the target assigns to $\{c(V), e(V)\}$ is $c(V) + e(V)s(V)$. Next, when a bidder of type V decides on a bid, it has the option to mimic a type $V' = V - dV$ just below it. Such a deviation has two effects. First, the deviation reduces the probability of winning to that of type V'. Second, the deviation changes the expected payment if it wins from $c(V) + e(V)s(V)$ to $c(V') + e(V')\,s(V)$. On the margin, these two effects must balance out (or else there is a profitable deviation). Type V can also deviate to a cash bid of amount $c(V') + e(V')\,s(V')$. Since the seller values this cash bid the same as the bid $\{c(V'), e(V')\}$ by type V', the marginal effect on the probability of winning is the same as if the bidder deviates to $\{c(V'), e(V')\}$. However, unless $e(V') = 0$, the monetary value would be strictly less because $s(V) > s(V')$. Thus, type V would profit by deviating to a pure cash bid unless $e(V') = 0$. Consequently, an equilibrium only involves cash bids.

III. Two-sided Private Information

We provide a qualitative analysis in a setting with two bidder types and two seller types. Let the bidder's possible standalone values and synergies be (V_{Ai}, s_i), for $i = 1, 2$, with $0 < V_{A1} < V_{A2}$ and $0 < s_1 < s_2$. Let $f_{Ai} > 0$ be the probability of a type i bidder, where $f_{A1} + f_{A2} = 1$. Let the seller's possible standalone values be V_{Ti}, $i = 1, 2$, where $0 < V_{T1} < V_{T2}$. Let $f_{Tj} > 0$ be the probability of a type j seller, where $f_{T1} + f_{T2} = 1$. Each seller type offers a menu of contracts, $\{c_i, e_i; p_i\}_{i=1,2}$, one for each bidder type. When a bidder selects contract i, it wins with probability $p_i \in [0, 1]$; and when the bidder wins, it pays cash c_i and equity share $e_i \in [0, 1]$.

Denote the menu of contracts offered by a type j seller by $\{c_{ji}, e_{ji}; p_{ji}\}_{i=1,2}$. The equilibrium is pooling if seller types 1 and 2 offer the same menu—that is, if $c_{1i} = c_{2i}$, $e_{1i} = e_{2i}$, and $p_{1i} = p_{2i}$ for $i = 1, 2$. The equilibrium is separating otherwise.

Given the menu of contracts $\{c_k, e_k; p_k\}_{k=1,2}$ offered by the seller, each type i bidder forms beliefs about the seller's expected standalone value, V_T. Denote these beliefs by

$$\theta_i(\{c_k, e_k; p_k\}_{k=1,2}) \in [V_{T1}, V_{T2}]\,,\ i = 1, 2. \tag{IA20}$$

The expected (net) profit of a type i bidder that chooses contract $\{c_k, e_k; p_k\}$ is

$$\Pi = p_k\left((1 - e_k)\left(V_{Ai} + s_i + \theta_i - c_k\right) - V_{Ai}\right). \tag{IA21}$$

On an equilibrium path, $\theta_i = E\left[V_T |\, \{c_k, e_k; p_k\}_{k=1,2}\right]$. Thus, in a pooling equilibrium,

$$\theta_i(\{c_{jk}, e_{jk}; p_{jk}\}_{k=1,2}) = E\left[V_T\right] \text{ for } j = 1,2 \text{ and } i = 1,2; \tag{IA22}$$

and in a separating equilibrium in which a type j seller offers menu $\{c_{jk}, e_{jk}; p_{jk}\}_{k=1,2}$,

$$\theta_i(\{c_{jk}, e_{jk}; p_{jk}\}_{k=1,2}) = V_{Tj} \text{ for } j = 1,2 \text{ and } i = 1,2. \tag{IA23}$$

Let $\Pi_{i,k,j}$ be the expected profit of a type i bidder when it chooses contract k in the menu:

$$\Pi_{i,k,j} = p_{jk}\left(\left(1 - e_{jk}\right)\left(V_{Ai} + s_i + \theta_i - c_{jk}\right) - V_{Ai}\right), \tag{IA24}$$

where θ_i satisfies (IA22) or (IA23). Incentive compatibility for a type i bidder requires

$$\Pi_{i,i,j} \geq \Pi_{i,k,j} \text{ for all } i,\ j \text{ and } k \neq i, \tag{IA25}$$

Note that this trivially holds in a pooling equilibrium. Individual rationality requires

$$\Pi_{i,i,j} \geq 0 \text{ for all } i, j. \tag{IA26}$$

The equilibrium expected profit of a type i bidder, integrated over the two seller types, is $\Pi_{b,i} = \sum_{j=1}^{2} f_{Tj}\Pi_{i,i,j}$. To obtain the unconditional equilibrium expected profit of bidders, integrate over their types to obtain $\Pi_b = \sum_{i=1}^{2} f_{Ai}\Pi_{b,i}$. To obtain the expected profit of a type j seller from offering the menu offered by a type k seller, integrate over the two buyer types:

$$\pi_{s,j,k} = \sum_{i=1}^{2} f_{Ai} p_{ki} \left(e_{ki} \left(V_{Ai} + s_i + \theta_i - c_{ki} \right) + c_{ki} - V_{Tj} \right), \tag{IA27}$$

where $\theta_i = V_{Tk}$ if the equilibrium is separating, and $\theta_i = E\left[V_T\right]$ if the equilibrium is pooling.

In equilibrium, a type j seller's expected profit is $\pi_{s,j,j}$. Incentive compatibility requires that it not be profitable for a type j seller to offer the menu offered by a type $k \neq j$ seller:

$$\pi_{s,j,j} \geq \pi_{s,j,k} \text{ for } k \neq j \text{ and both } j. \tag{IA28}$$

Unlike the bidder, which can only choose between the two contracts offered, a seller can deviate by offering any arbitrary menu of contracts. The optimality of the mechanism for a seller requires that the expected profit of each seller type weakly exceed what she can get from offering any other menu. Complications arise in imposing this requirement because, for any menu, the set of equilibria and equilibrium payoffs depend on the possible off-equilibrium-path beliefs. We impose a minimal (and necessary) requirement for seller optimality: for any off-equilibrium-path offer $\left\{c'_{ji}, e'_{ji}; p'_{ji}\right\}_{i=1,2}$ made by a type j seller, if her expected profit is at least $\Pi'_{s,\min}$ for every bidder belief that satisfies (IA20), then $\Pi_{s,j} \geq \Pi'_{s,\min}$.

PROPOSITION IA2: Suppose that

$$\frac{s_2 - s_1}{V_{A2} - V_{A1}} > \frac{f_{A1}}{(1 - f_{A1})}, \tag{IA29}$$

and there is sufficient information asymmetry on V_T that

$$V_{T2} - V_{T1} > \phi\left(V_{A1}, V_{A2}, s_1, s_2, f_{A1}, f_{T1}\right), \tag{IA30}$$

where the function ϕ is defined in equation (IA41) of the proof. Then the bidder's expected profit is strictly positive in any equilibrium: $\Pi_b > 0$.

Proposition IA2 reflects the intuition that when information asymmetry about a seller's standalone value is sufficiently high, full extraction by the seller is impossible even in a pooling equilibrium because the high type seller's rents would be too low, providing it incentives to deviate. Failure of full extraction need not imply that a bidder will earn positive rents, because a seller may sell only to one bidder type and exclude the other type. In such a case, the bidder earns no rents even though the seller does not extract full rents. Condition (IA29) rules out such a case by ensuring that it is not optimal for the seller to exclude a type 2 bidder and only sell to a type 1 bidder. Condition (IA29) holds as long as the probability of a type 1 bidder, f_{A1}, is not too high. This requirement is not that restrictive: with $n > 2$ bidder types, the condition ensuring that the bidder earns strictly positive rents is still that the probability of a low type 1 bidder is not too high, which is naturally satisfied when n is large.

Proof of Proposition IA2. From (IA24),

$$\Pi_{1,2,j} - \Pi_{2,2,j} = p_{j2}\left(\left(V_{A2} - V_{A1}\right) - \left(1 - e_{j2}\right)\left(\left(V_{A2} - V_{A1}\right) + s_2 - s_1\right)\right).$$

From bidder incentive compatibility, $\Pi_{1,1,j} \geq \Pi_{1,2,j}$, so the difference in the equilibrium expected profits of bidder types 1 and 2, conditional on seller type j, satisfies:

$$\Pi_{1,1,j} - \Pi_{2,2,j} \geq p_{j2} \left((V_{A2} - V_{A1}) - (1 - e_{j2}) \left((V_{A2} - V_{A1}) + s_2 - s_1 \right) \right).$$

Summing over seller types yields

$$\Pi_{b,1} - \Pi_{b,2} \geq \sum_{j=1}^{2} f_{Tj} p_{j2} \left((V_{A2} - V_{A1}) - (1 - e_{j2}) \left((V_{A2} - V_{A1}) + s_2 - s_1 \right) \right).$$

Defining

$$\Delta \equiv p_{22} \left(V_{A2} - V_{A1} \right) - p_{22} \left(1 - e_{22} \right) \left((V_{A2} - V_{A1}) + s_2 - s_1 \right), \tag{IA31}$$

$\Pi_{b,2} \geq 0$ yields

$$\Pi_{b,1} \geq \sum_{j=1}^{2} f_{Tj} p_{j2} \left((V_{A2} - V_{A1}) - (1 - e_{j2}) \left((V_{A2} - V_{A1}) + s_2 - s_1 \right) \right) \geq f_{T2} \Delta. \tag{IA32}$$

Incentive compatibility for a type $j = 1$ seller similarly yields $\pi_{s,1,1} \geq \pi_{s,1,2}$. By (IA27),

$$\pi_{s,1,2} - \pi_{s,2,2} = \sum_{i=1}^{2} f_{Ai} p_{2i} \left(1 - e_{2i} \right) \left(V_{T2} - V_{T1} \right).$$

Thus, the expected profit of type 1 seller exceeds that of type 2 seller by at least

$$\begin{aligned}\pi_{s,1,1} - \pi_{s,2,2} &\geq (V_{T2} - V_{T1}) \sum_{i=1}^{2} f_{Ai} p_{2i} (1 - e_{2i}) \\ &\geq (V_{T2} - V_{T1}) f_{A2} p_{22} (1 - e_{22}) .\end{aligned}$$

Re-arranging yields $p_{22} (1 - e_{22}) \leq \frac{\pi_{s,1,1} - \pi_{s,2,2}}{(V_{T2} - V_{T1}) f_{A2}}$. Substituting this inequality into the last term in (IA31) yields

$$\begin{aligned}\Delta &\geq p_{22} (V_{A2} - V_{A1}) - \frac{\pi_{s,1,1} - \pi_{s,2,2}}{(V_{T2} - V_{T1}) f_{A2}} ((V_{A2} - V_{A1}) + s_2 - s_1) \\ &\geq p_{22} (V_{A2} - V_{A1}) - \frac{\pi_{s,1,1}}{(V_{T2} - V_{T1}) f_{A2}} ((V_{A2} - V_{A1}) + s_2 - s_1) . \quad \text{(IA33)}\end{aligned}$$

The seller's unconditional equilibrium expected profit, summed over its two types, is

$$\begin{aligned}\pi_s &= f_{T1} \pi_{s,1,1} + f_{T2} \pi_{s,2,2} \quad \text{(IA34)} \\ &\geq f_{T1} \pi_{s,1,1} . \quad \text{(IA35)}\end{aligned}$$

Because a seller's unconditional expected profit cannot exceed the full extraction amount,

$$\pi_{s,1,1} \leq f_{A1} s_1 + f_{A2} s_2,$$

which, by (IA35), yields $\pi_{s,1,1} \leq \frac{f_{A1} s_1 + f_{A2} s_2}{f_{T1}}$. Substituting this inequality into (IA33) yields

$$\Delta \geq p_{22} (V_{A2} - V_{A1}) - \frac{f_{A1} s_1 + f_{A2} s_2}{(V_{T2} - V_{T1}) f_{A2} f_{T1}} ((V_{A2} - V_{A1}) + s_2 - s_1) . \quad \text{(IA36)}$$

Next, we bound p_{22}. If a seller type offers a menu consisting of the single contract with $\left(c, e = 1 - \frac{V_{A2}}{V_{A2}+s_2+V_{T1}-c}; p = 1\right)$, where c is sufficiently negative, then by (IA21), this contract will yield both bidder types non-negative expected profit. Hence, both bidder types will accept the offer. When c is sufficiently negative, the seller's expected profit approaches $f_{A2}s_2 + f_{A1}(V_{A1} + s_1 - V_{A2})$. Thus, optimality of the mechanism requires

$$\pi_{s,j,j} \geq f_{A2}s_2 + f_{A1}(V_{A1} + s_1 - V_{A2}), \ j = 1, 2. \tag{IA37}$$

Suppose the equilibrium is separating. Then conditional on a type 2 seller, the maximum welfare surplus is $s_1 + p_{22}s_2$. The bidder's individual rationality condition yields $\pi_{s,2,2} \leq s_1 + p_{22}s_2$, which, by (IA37), yields

$$s_1 + p_{22}s_2 \geq f_{A2}s_2 + f_{A1}(V_{A1} + s_1 - V_{A2}). \tag{IA38}$$

Substitute for $f_{A2} = 1 - f_{A1}$, define $\delta \equiv \frac{s_2 - s_1}{V_{A2} - V_{A1}} - \frac{f_{A1}}{(1-f_{A1})}$ and solve the inequality for

$$p_{22} \geq \frac{\delta(V_{A2} - V_{A1})(1 - f_{A1})}{s_2}. \tag{IA39}$$

Note that $\delta > 0$ from the premise of the proposition in (IA29).

Now consider a pooling equilibrium. Both seller types will offer the same menu, so $p_{21} = p_{22}$. Unconditional on the seller type, the maximum welfare surplus is again $s_1 + p_{22}s_2$. Individual rationality of bidders yields that a seller's unconditional expected equilibrium profit satisfies $\pi_s \leq s_1 + p_{22}s_2$, where $\pi_s = f_{T1}\pi_{s,1,1} + f_{T2}\pi_{s,2,2}$. Then (IA37) and (IA34) yield (IA38) and (IA39).

Thus, (IA39) holds in both separating and pooling equilibria. Plugging (IA39) into (IA36) yields

$$\Delta \geq \frac{\delta\left(V_{A2}-V_{A1}\right)^{2}\left(1-f_{A1}\right)}{s_{2}}-\frac{f_{A1} s_{1}+f_{A2} s_{2}}{\left(V_{T2}-V_{T1}\right) f_{A2} f_{T1}}\left(\left(V_{A2}-V_{A1}\right)+s_{2}-s_{1}\right). \tag{IA40}$$

This yields $\Delta > 0$ when (IA30) holds—that is, when $V_{T2} - V_{T1} > \phi$, where

$$\phi \equiv \frac{\left(f_{A1} s_{1}+f_{A2} s_{2}\right) s_{2}\left(\left(V_{A2}-V_{A1}\right)+s_{2}-s_{1}\right)}{f_{A2} f_{T1}\left(V_{A2}-V_{A1}\right)^{2}\left(1-f_{A1}\right) \delta}. \tag{IA41}$$

By (IA32) and $\Pi_{b,2} \geq 0$, the proposition follows. ∎