Essays in Information Economics

by

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Declaration: I declare that all the following work was carried out by me. Except the following:

Chapter 2 I co-authored with Giulio Trigilia.

To the memory of Zura Koroglishvili, Zulia Elizbarashvili, Valiko Kachlishvili and Nikolozi Koroglishvili

1 The First Chapter: Introduction

During my doctorate studies, I got particularly interested in financial and information economics. I started the first project with my co-author G. Trigilia who helped me to understand deeper many concepts from mechanism design. The joint work is presented in the second chapter. My interest in information economics and information design led me to the second project which I did on my own. The third chapter is devoted to the problem of information design in a competitive environment. Below I present abstracts of these two projects.

1.1 Strategic default, investment and the resolution of financial distress

In recent years, the U.S. experienced an increase in the share of default events that are resolved out-of-court, as well as a reduction in bankruptcy-related costs. This trend raises the question as to what drives the frequency with which defaults turn into bankruptcies. We propose a theory based on three pillars: first, bankruptcy is costlier than out-of-court restructuring; second, creditors cannot commit to take defaulting borrowers to court; third, firms have private information about the value of their assets, outside investors only learn them only upon bankruptcy. Creditor's bargaining power upon default decreases with bankruptcy costs and it increases with the frequency of strategic default – that is, default by firms which could have honored their obligations. When bankruptcy costs decrease, creditors obtain higher recovery rates out-of-court and therefore firms have lower incentives to default strategically. As a result, bankruptcy can occur less frequently.

1.2 Communication Mechanisms in Competition

I examine how a market mediator can help market players to have an incentive compatible communication which leads to a more efficient market outcome. I introduce the mediator whose function is a collection of financial information from the players and sharing this information with other market players. The crucial assumption is that information disclosure to the mediator by the participants is voluntary, so there is no third party who forces the market players to disclose private information. Given that the mediator can commit to a reporting policy to the bidders, I characterise the information reporting policy that the mediator should adopt to minimize the probability of investment in unprofitable projects by the uninformed players. I show that manipulation of the first order beliefs of the uninformed player is not sufficient to extract information from the informed player. The main insight of the paper suggests that allowing the player who shared information to know the degree to which his information was shared with other participants will incentivise information disclosure.

2 The Second Chapter: Strategic default, Investment and the Resolution of Financial Distress

2.1 Introduction

In recent years, private restructuring of distressed securities witnessed a comeback in the U.S., surging from 10% of default events around 2007 to 40% in 2016.[1] The trend coincided with an increasingly faster and less costly bankruptcy procedure - a legal process through which entities who cannot repay debts to creditors may seek relief from their debts. For instance, the average Chapter 11 bankruptcy case took seven months to resolve in 2017, less than half than that of 2013.[2] How to reconcile a negative relation between bankruptcy-related costs and the use of bankruptcy as a means to resolve financial distress episodes? Is it consistent with the view that private restructuring out-of-court is the response to costly bankruptcy (e.g., Jensen (1991)). And more generally, what could be driving different types of default resolutions across firms and jurisdictions?

We provide a tentative answer to these questions in a model where firms are heterogeneous in their solvency probabilities, and operate in environments characterized by two key parameters: a risk-free rate and a bankruptcy cost.[3] Firms borrow from competitive investors under symmetric information, but two frictions arise afterwards: (i) the borrower privately observes the realized return on the invested capital, while others can observe it only when bankruptcy takes place; (ii) investors cannot credibly commit that they will take defaulting firms to court: all parties knows that upon default they will decide whether or not to restructure their claims out-of-court. As a result, the securities issued induce a Bayesian game played after returns realize, in which borrowers have some bargaining power vis à vis their lenders until they miss a payment.

[1]See Moody's 'A Closer Look at Distressed Exchanges' (2017). The International Swaps and Derivatives Association (ISDA) 2014 Master Agreement, which sets standards for credit default swap contracts, includes in a 'default event' both in-court and out-of-court resolutions of distress. Similar broad definitions are adopted by credit rating agencies, such as Moody's, Fitch and Standard & Poor's.

[2]See Reuter's Ever-shorter U.S. bankruptcies have creditors scrambling, February 2017.

[3]Introducing additional default costs which do not depend on whether bankruptcy occurs or not would not have any qualitative effect in our model; we normalize such costs to zero. As a consequence, our bankruptcy cost could also be interpreted as capturing any incremental loss of firm value accruing when out-of-court restructuring fails and bankruptcy takes place.

For a given capital structure, the incentives to reject an out-of-court reorganization plan and take a defaulting firm to court increase with the expected recovery rate from bankruptcy, which depends positively on the expected value of the creditor's claims upon default, and negatively on bankruptcy costs. In turns, the expected value of the creditor's claims upon default depends on the odds that default is strategic – that is, that the defaulting firm could have honored its obligation – as opposed to being due to insolvency. The higher these odds are, the higher the expected value of the creditor's claims upon default and the stronger the incentive for creditors to resolve distress episodes in court.

Our first result is that optimal securities provide incentives for borrowing firms to strategically default on their debts, even if this is anticipated at the investment stage and if other feasible securities could have prevented strategic default from taking place (or reduce its incidence). Strategic default is optimal because it increases the expected recovery rate, alleviating the limited commitment problem faced by the creditors, who cannot commit that they will take defaulting borrowers to court.

At the *extensive margin*, the marginal firm that invests defaults strategically with positive probability. For this firm, investment would be impossible absent strategic default, because bankruptcy costs are larger than the recovery rate under truth-telling. At the *intensive margin*, the frequency of strategic default depends on the probability that firms are solvent in a non-monotonic fashion. Locally, more profitable firms are less likely to default strategically. However, there exists a profitability threshold at which the probability of strategic default jumps up, before declining again. The intuition behind this regime shift is as follows. As firms' profitability rises, the effect strategic default has on the dollar value of the equilibrium out-of-court restructuring plan offered by the borrowers compounds, because the probability that borrowers are in a high state conditional on them reporting a low state mechanically rises. That is, the set of out-of-court restructuring plans that creditors can credibly enforce without destroying incentives ex post expands. So, the highest incentive compatible credit spread rises, enabling the creditors to break even at a lower bankruptcy probability – i.e., accepting more often the out-of-court plan – and lowering the deadweight losses.

Empirically, strategic default accounts for some of the cross-sectional variation in recovery rates. The US regulated utilities offer a recent example. According to Moody's, they "experienced an average recovery rate of approximately 90 percent. This high recovery rate is likely driven by utilities' often observed behavior of strategically choosing to default, when asset values are still relatively high, in order to seek rate relief from regulators".[4] Favara et al. (2012) argue that firms in countries that favor shareholders in reorganization have lower equity betas due to strategic default. Garlappi and Yan (2011) find a manifestation of strategic default in the fact that shareholders' recovery is often positive. Davydenko and Strebulaev (2007) find that strategic default affects credit spreads especially for risky firms facing high liquidation costs.[5] Blouin and Macchiavello (2017) document frequent strategic default by firms in developing countries.[6] Finally, the empirical literature on recovery rates established that the frequency of recovery rates is bimodal, with large mass on extreme (high or low) recoveries, and little mass for intermediate recoveries, which is consistent with the presence of strategic default.[7]

Through our characterization of optimal strategic default, we pin down the expected default frequency and the frequency with which default is resolved out-of-court, both of which are now endogenous. This is in contrast with both credit risk models (e.g., Fan and Sundaresan (2000) and Davydenko and Strebulaev (2007)), where the Coase theorem holds and it is assumed that private renegotiation fails some of time for exogenous reasons, and with previous models of default resolution under asymmetric information (e.g., Giammarino (1989)), where contracts are exogenous and so is the probability of default. We will discuss at length these differences in the literature review of Section 2.2.

[4]Quote from Moody's Ultimate Recovery Database Special Comment, issued in April 2007. Importantly, utilities strategically default because of limited commitment by the regulator, who could not commit to give no "rate relief". Our paper is about private creditors, not regulators, but the underlying commitment issues are similar.

[5]Interestingly, they observe that strategic default increases both default probabilities and recovery rates, which may justify its relatively low effect on credit spreads for investment grade firms.

[6]Recent work also documents strategic default in the mortgage market. Gerardi, Herkenhoff, Ohanian, and Willen (2017) find that 30% of mortgage payers who defaulted after the 2008 crisis were acting strategically. The magnitude is consistent with both Guiso, Sapienza, and Zingales (2013) – who also find variation across social groups – and Artavanis and Spyridopoulos (2018) – who show that financial and legal sophistication positively affect strategic default rates.

[7]See both Schuermann (2004) and Altman and Kalotay (2014).

Contrary to the argument in Jensen (1991), optimal securities induce a frequency of private resolutions of default that *decreases* with both bankruptcy costs and the risk-free rate. The intuition is that higher bankruptcy costs reduce the maximum cash flows that can be extracted by creditors out-of-court, which feeds back into lower incentive compatible repayment outside of default (or *credit spread*). In turns, lower credit spreads and higher haircuts out-of-court imply lower return to the creditors, who need more frequent bankruptcies to be willing to invest in the firm. We formally show in the main text that, locally, the probability of private resolution of default is decreasing with bankruptcy costs.[8] Because this comparative static is with respect to bankruptcy *costs*, not to its degree of creditors' friendliness, it relates to Ponticelli and Alencar (2016), who study patterns of default across more or less congested courts in Brazil, or to Djankov, Hart, McLiesh, and Shleifer (2008), who find that richer countries take less to resolve default and save the going concern 75% of the times, against the 17% of the times of lower income countries.[9]

Turning to credit spreads, we show that they monotonically increase in both the risk-free rate and the bankruptcy costs. The risk-free rate affects spreads both directly (because they are defined as returns to creditors outside default net of the risk-free rate), and indirectly, as a higher risk-free rate implies a higher opportunity cost of funds for the creditors. The indirect effect always dominates: transferring a risky dollar from the borrower to the creditor comes at a deadweight cost, and so it is more expensive that transferring a risk-free dollar. As for bankruptcy costs, the effect is similar and it is due to the larger wedge between a risk-free dollar and a risky dollar that a wider bankruptcy cost brings about. Both effects are consistent with the evidence about credit spreads.[10]

Finally, our model has implications for the distribution of recovery rates conditional on a type of default event. Out-of-court, recovery rates fall with

[8]See Lemma 6 and Corollary 2 in the main text of this chapter.

[9]However, it does not directly relate to Claessens and Klapper (2005) finding that link creditors' friendliness of bankruptcy regimes to bankruptcy use.

[10]Longstaff, Mithal, and Neis (2005) attribute around three quarters of the cross-sectional variation in firms' CDS spreads to default risk. Estimates of the cost of default conditional on default happening range around 20-25% of firm value, but due to selection bias these defaults should be more likely to have a low default cost. For the average firm, Glover (2016) estimates that they could be as high as 45%.

bankruptcy costs, because these costs measure the inverse of creditors' bargaining power upon default. Higher costs imply weaker creditors' bargaining power and therefore higher equilibrium haircuts observed out-of-court. As for recoveries in bankruptcy, there are two distinct effects. On the one hand, higher bankruptcy costs mechanically lower recovery rates, because they increase the deadweight losses. On the other, changes in bankruptcy costs induce endogenous changes in the incentives for borrowers to strategically default, which might more than offset the former effect. Indeed, we find that recoveries in bankruptcy are non-monotonic, and they are the lowest for intermediate ranges of bankruptcy costs, as in these ranges strategic default never occurs. These predictions are – to the best of our knowledge – new in the literature, and could explain what drives the hump-shaped (bimodal) recovery rates cross-sectional distribution.[11] In our model, recovery rates out-of-court are higher than recovery rates in court, on average, which is consistent with the evidence. For instance, Moody's observes that 'the recovery rate averages 70% for senior unsecured bonds in a distressed exchange compared with 40% in a bankruptcy default. A likely explanation is that companies that initiated distressed exchanges were under less credit stress than those that underwent payment defaults or bankruptcies'.[12]

Because in our baseline model creditors can contract on payment both in and out-of-court, and are only limited in their ability to commit to bankrupt defaulting firms, we extend our model to cover cases where commitment problems are more severe, and payments outside of bankruptcy are all subject to potential renegotiation. As in Gale and Hellwig (1989), now multiple equilibria may arise and they are sustained by some arbitrary off-equilibrium belief system. We concentrate attention to the set of equilibria that can be implemented by a uniform off-equilibrium belief that a deviation comes from the high type. We will discuss how and when this is with loss of generality. Our main results are qualitatively unchanged in this extension, but there are two important differences. First, now the equilibrium strategic default is always the minimum required for investment

[11] See especially Gilson, John, and Lang (1990), Schuermann (2004), Jankowitsch, Nagler, and Subrahmanyam (2014) and Altman (2006).

[12] Large differences exist also across types of bankruptcy, as shown by Bris, Welch, and Zhu (2006) in a comparison of Chapter 7 liquidation against Chapter 11 reorganization in the US. See also Bernstein, Colonnelli, and Iverson (2016). Internationally, Davydenko and Franks (2008) find that the median recovery rate is 92% in the UK, 67% in Germany, and 56% in France.

to be sustainable. Second, optimal securities are not necessarily fully collateralized. The intuition is that there is now a tight link between credit spreads and collateral values both in and out of default states, and so fully-collateralized securities might bring about excessively high credit spreads, which can only be sustained by higher-than-optimal frequency of bankruptcy.

A final extension of our model addresses issues related to the cardinality of the type space. In particular, it asks whether our results are robust to the introduction of additional states, and whether strategic default incentives are monotone in the type space. Due to the many constraints and the complex non-linear behavior of our objective function, it is very hard to derive analytic properties of the solution for models with more than two states of nature. Therefore, we resort to numerical simulations. Our main results are as follows. First, deadweight losses and credit spreads are increasing function of the risk-free rate and the bankruptcy cost, confirming the findings from the baseline model. Second, strategic default takes place at the optimal contracts. Interestingly, though, it is not monotonic in the firm's returns: firms with high realized returns might default more often than firms with intermediate returns. This sheds some light on the many complications involved in solving our model, and further clarify why we insist on a relatively simple version that can be fully derived and understood analytically.

2.2 Related literature

The relevant theoretical work can be divided, with the cost that comes with simplification, in two streams. We partition our literature review accordingly.

Security design. The most relevant work is in security design and optimal contracting. Starting with Townsend (1979), Costly-State-Verification (CSV) papers have emphasized imperfect observability of returns as an important concern driving firms' capital structure choices, and provides a micro-foundation to bankruptcy as certification. In the original CSV model, creditors have full commitment power and verification is deterministic. The model has been extended to random verification by Border and Sobel (1987) and Mookherjee and Png (1989), and to limited commitment by Gale and Hellwig (1989), Khalil and

Parigi (1998) and Krasa and Villamil (2000).

Gale and Hellwig (1989) considered a CSV model with exogenous recovery rates from verification. In contrast, we have fixed investment and endogenous recovery rates. Khalil and Parigi (1998) focus on variable investment and fully mixed equilibria, in the spirit of inspection games. In contrast, we have fixed investment and we show that the optimal contract may involve no strategic default (i.e., partial mixing). Krasa and Villamil (2000) consider a different game where both parties pay out-of-pocket for bankruptcy ex post, and they allow for side payments. They show that deterministic verification is optimal, and when limited commitment does not bind the optimal contract resembles straight debt.[13] In contrast, our optimal allocation is implemented by random verification (generically) and this is what induces strategic default in equilibrium.

Interestingly, the development of credit derivatives, and in particular credit default swaps, inspired a recent literature that studies how the incentives for borrowers and lenders to renegotiate their debts in-court, as opposed to out-of-court. As Bolton and Oehmke (2011) show, credit default swaps strengthen the bargaining power of creditors ex post and therefore might improve investment conditions. They also might generate an overinsurance problem, though, so a trade-off arises. Unlike Bolton and Oehmke (2011), we do not assume that out-of-court restructuring is costly, and that it makes future cash flows verifiable. In our case, when the firms negotiate out-of-court the value of the assets is never observed by outsiders. Only when bankruptcy takes place, is the value revealed to outsiders (and the court), either due to liquidation at market prices or due to the extensive disclosure requirements that come with bankruptcy filings.

A related, though distinct literature applies static games to the analysis of renegotiation upon default. Giammarino (1989) first emphasized that asymmetric information at the renegotiation stage could be an important determinant of the frequency of out-of-court restructuring, as opposed to in-court bankruptcies. However, Giammarino takes the initial securities issued as given, and therefore is subject to the Lucas critique. In contrast, we consider optimal securities and

[13] With one important difference from Townsend: while in Townsend messages are *direct*, and this is without loss of generality because of the revelation principle, Krasa and Villamil (2000) sustain their optimal debt by an indirect message space that contains just two messages: default or no-default.

derive predictions on this frequency based on deep parameters of the economy. This is important, especially as Davydenko and Franks (2008) document large differences in private contracting across countries: weaker creditors' protection correlates with larger collateralization and shorter maturity. François and Raviv (2017) propose a different direction, emphasizing heterogeneous beliefs about the results of a formal restructuring plan and about judicial discretion as potential sources of the different outcomes of default events. In our model, beliefs are symmetric.

Credit risk models. An alternative stream of literature, which has arguably been more central in deriving empirical predictions about strategic default, develops structural models of credit risk. Merton (1974) replicates risky debt by a portfolio of risk-less debt and a put option, linking default risk explicitly to the volatility of a firm's return. This approach implies that default probabilities are inversely related to recovery rates, which does not necessarily happen in our model due to strategic default. Leland (1994) introduces endogenous default boundaries, default-related costs and strategic default in Merton's style structural models. The boundary is pinned down by a smooth-pasting condition. Importantly, there is no uncertainty about the value of the firm's asset upon default: the boundary is public information. In contrast, our results are entirely driven by the asymmetry of information ex post between insiders and outsiders.

Given the empirical regularity that borrowers strategically default on their debts, it is not a surprise that structural credit risk models considered the consequences of strategic default on credit spreads. In particular, Fan and Sundaresan (2000) build strategic default opportunities in a Leland (1994) type model. Strategic default is incentivized by the possibility of out-of-court renegotiation, which allows the parties to save on bankruptcy costs. Because the parties have symmetric information at the renegotiation stage, the Coase theorem holds and we should never observe bankruptcy in equilibrium in this type of model, which is counterfactual. To rule out this outcome, these models assume that with some *exogenous* probability renegotiation fails.[14] An important contribution of our paper to this literature is to offer a model that makes this

[14] See also the Appendix to Davydenko and Strebulaev (2007), Favara, Schroth, and Valta (2012) and Antill and Grenadier (2017).

probability endogenous. The cost of making this probability endogenous is that, while we derive several comparative static predictions, we lose the quantitative implications these models have. We hope that future work can bridge this gap.

2.3 Setup

A Borrower (B) is endowed with a project, and seeks financing from a creditor (C). Both parties are risk-neutral. The project requires \$1 to be implemented, and yields return $\tilde{x} \in \{L, H\}$. C has an alternative investment opportunity that yields a risk-less return r. Let $L < r < H$, so that the project is risky and losses occur in the low state.[15] Let $\Pr.[\tilde{x} = H] \equiv p \in (0, 1)$. B privately observes the realized state x; does not observe the realized state, but can verify it at an exogenous cost $\mu \geq 0$. The timing of the game is as follows:

t=0 B issues securities, trying to sell them to C, who is competitive

- If C buys the securities, the proceedings of \$1 get invests it in the project. Investment is assumed to be an observable and verifiable action
- Otherwise, the game ends. B has a payoff equal to zero, whereas C invests the dollar at the risk-free rate of return r

t=1 B privately observes the realized return x, and publicly reports it to be $m \in \{L, H\}$.[16] B can randomize the report she makes about the state. The associated contractual payments from B to C are denoted by the function $s_m : \{L, H\} \to \mathbb{R}$.[17] C has two options:

- To accept the report and receive the payment s_m, in which case B retains a payoff equal to $x - s_m$ and there are no deadweight losses

[15] We choose to work with a binary state space for two reasons. First, because we want to allow for stochastic monitoring *and* – later on – limited commitment, which are hard to handle with more than two states. Second, because we are interested in the comparative statics of the problem, which cannot be fully determined with more states. See Border and Sobel (1987) and Mookherjee and Png (1989).

[16] As there are only two states, the restriction to direct messages is without loss of generality. No communication corresponds to the case where, regardless of the realized x, both types are expected to send the same message m.

[17] We think of $s(\cdot)$ as describing both: (i) the face value of debt in the high state; (ii) the restructured value of C's claims in the low state.

- To reject the payment and verify the state x at a cost μ. After verification, C can enforce payments based on both the report m and the true state x, denoted by the function $c : \{L, H\}^2 \to \mathbb{R}$. In the event of verification, B receives a payoff of $x - c_{m,x}$ while C obtains $c_{m,x} - \mu$. Whenever two subscripts are used, the first refers to the message m and the second to the state x.[18]

We restrict attention to contracts that satisfy *limited liability*, namely such that $0 \leq s_m \leq m$ and $0 \leq c_{m,x} \leq x$, for all m and x. We label such contracts as *feasible*.

We denote the probability that the creditors take a defaulting borrower to court (i.e., they verify the state) at $t-1$ by the function $b_m : M \to [0, 1]$. Notice that creditors are allowed to randomize their verification decision. This possibility was not explicitly considered in Townsend (1979), and was introduced in costly-state-verification models by Border and Sobel (1987) and Mookherjee and Png (1989). Random verification will be central in the derivation of all our main results. Before proceeding to the analysis, it is useful to define what a credit spread and a recovery rate are in our setup.

Definition 1. *Given a risk-free rate r and a repayment from B to C when the firm does not default s_H, corporate Credit Spreads (CS) are denoted by:*

$$CS := s_H - r.$$

Definition 2. *Given a bankruptcy cost μ, an out-of-court repayment from B to C upon default s_L, and a repayment in bankruptcy c_L or C_H, depending on the realized state, the Recovery Rate (RR) for creditors is given by:*

$$RR := \begin{cases} c_H - \mu, & \textit{with frequency} \quad \dfrac{\text{Pr.[Strategic Default]} \times p \times b}{1 - p + \text{Pr.[Strategic Default]} \times p} \\ s_L, & \textit{with frequency} \quad (1 - b) \\ c_L - \mu, & \textit{with frequency} \quad \dfrac{(1-p) \times b}{1 - p + \text{Pr.[Strategic Default]} \times p} \end{cases}$$

The definition of recovery rates implicitly assumes that strategic default

[18] For instance, s_L corresponds to the case of $m = L$ absent verification; $c_{H,L}$ corresponds to $m = H$ when the true state is verified to be low, and so on.

happens only in the high state, a property that is relatively obvious but will be proved formally in our analysis, and not taken for granted. Both recovery rates and credit spreads will depend on exogenous variables (that is, p, r, μ, L, H), on contractual variables (that is, c_H, c_L and s_L, s_H) and on endogenous equilibrium variables (that is, b and Pr.[Strategic Default]). The rest of the paper is split in two conceptual parts. In the first, we derive the optimal contracts as a function of the exogenous variables, and anticipating the nature of the equilibrium response they will induce, in terms of both verification probabilities and strategic default. Next, we study how credit spreads, recovery rates and other statistics of interest vary as the exogenous parameters change. To start, we characterize the full commitment benchmark.

2.4 Commitment benchmark

In this part, we present the full commitment benchmark, where C can commit to take a defaulting B to court. We allow for the non-deterministic verification probability as was mentioned before. The contribution of this part to the existing literature is a full characterisation of an optimal contract under the objective of minimisation of expected verification probability. [19]

If C can commit to verifying the state, the revelation principle holds and we can restrict attention to direct truthful contracts. Two observations (which we formally prove in the Appendix) help in simplifying the problem and the notation: (i) it is never optimal to verify when the high state is reported by B, so we know that $b_H = 0$ and we can relabel $b_L \equiv b$; (ii) because $b_H = 0$, we have one degree of freedom in setting $c_{H,x}$. So, we set it equal to zero and relabel $c_{L,L} \equiv c_L$ and $c_{L,H} \equiv c_H$. It is well known that the optimal contract minimizes the deadweight losses and at the optimal contract C receives an expected return equal to r – a property we formally show in the Appendix. Therefore, optimal

[19] As a comparison, Border and Sobel (1987) consider revenue maximisation as an optimality criterion and charachterise the set of efficient mechanisms. The difference with respect to Mookherjee and Png (1989) is the fact that they consider a risk-averse principal.

contracts solve the following program:

$$\min_{b,c_L,c_H,s_L,s_H} (1-p)b \ \text{ s.t.}$$

$$(IC) \quad s_H \leq bc_H + (1-b)s_L$$

$$(PC) \quad ps_H + (1-p)[b(c_L - \mu) + (1-b)s_L] = r$$

$$(FCs) \; b \in [0,1], \ 0 \leq c \leq x, \ 0 \leq s \leq x.$$

where IC stands for Incentive Constraint, PC for Participation Constraint and FCs for Feasibility Constraints. The IC states that when B is in the high state, he needs to be weakly better off by reporting it truthfully (and paying s_H) than by reporting a low state and pay either c_H, if he is caught cheating, or s_L is there is no verification by C. The PC ensures that C gets an expected return equal to r from her investment in the project, net of the expected bankruptcy costs. The FCs guarantee that verification probabilities are interior, and that all payments satisfy limited liability from both sides.

Notice that c_L enters both PC and the deadweight loss function, because it is transferred to C on-the-equilibrium path. In contrast, c_H is off-equilibrium; it serves as an incentive device to prevent strategic default by the high type ex-post. Lemma 1 derives a few properties of optimal contracts that simplify the analysis. First, it shows that verification must take place with positive probability. If not, C can never attain an expected return equal to r (Claim 1, a). Second, it shows that to compensate creditors for the default risk they are taking, credit spreads must be strictly positive (Claim 1, b). Third, it states that the incentive constraint must be binding (Claim 2) and that downside protection has to be maximal (Claim 3).

Lemma 1. *Optimal contracts under full commitment satisfy the following properties:*

1. *Investment takes place only if:*
   - (a) *Bankruptcy occurs on-the-equilibrium path (i.e., $b > 0$)*
   - (b) *Credit spreads are positive (i.e., $CS > 0$, or equivalently $s_H > r$)*
2. *At any optimal contract, the incentive constraint must be binding*

3. *Any optimal contract is equivalent to one with with maximal downside protection (i.e., $s_L = L$). Moreover, whenever $b < 1$ any contract with $s_L \neq L$ is suboptimal*

Proof. See the Appendix. $\square$

Lemma 1 implies that the IC reads $s_H = bH + (1-b)L$. Plugging IC into PC yields:

$$b(pH + (1-p)[c-\mu]) + (1-b)L = r \tag{2.1}$$

From equation (2.1), we obtain the necessary condition for investment $pH + (1-p)[L-\mu] \geq r$, which tells that contract when verification occurs with probability one upon default, and C is a senior claimant upon verification, must be feasible for investment to take place. Solving (2.1) for b yields:

$$b = \frac{r-L}{pH - L + (1-p)[c-\mu]} \tag{2.2}$$

Nothing guarantees that there exists a feasible value for c such that the fraction lies between zero and one, so we have to consider corner cases as well. For now, assume such value exists. Substituting (2.2) into the objective function we can rewrite our problem as:

$$\min_{c\geq 0} \frac{r-L}{pH - L + (1-p)[c-\mu]} \quad \text{s.t. } 0 \leq c \leq L.$$

The sign of the first derivative is negative, which implies that we should set $c = L$.

Observation 1. *Investment takes place if and only if $(1-p)\mu + r \leq pH + (1-p)L$. The optimal contract sets $c_L = s_L = L$, $c_H = H$, $s_H = bH + (1-b)L$ and $b = \frac{r-L}{p(H-L)-(1-p)\mu}$.*

Proof. Trivial from the above argument. The investment condition comes from $b \leq 1$. Also, observe that $b \geq 0 \iff p(H-L) - (1-p)\mu \geq 0$, which always holds provided that $(1-p)\mu + r \leq pH + (1-p)L$, as required in the Proposition. $\square$

A few lessons can be learned from Observation 1. First, the optimal security resembles a fully collateralized standard debt contract. Differently from

Townsend (1979), verification is generically interior.[20] We do not find random verification as an inherently non-realistic feature of allocations. In reality, default does not always trigger costly procedures such as monitoring, verification or bankruptcy – depending on the interpretation, which is to some degree subjective and specific to the application considered. What is not quite convincing is that whenever C verifies, she is choosing an ex post suboptimal course of action, because there is no residual uncertainty in equilibrium about the value of B's assets conditional on the report being L (i.e., default). Indeed, this property makes the commitment allocation infeasible under limited commitment, as we shall see.

The second lesson one can learn from Observation 1 is that asymmetric information reduces the amount of projects that can be financed. In fact, whenever $p(H-L)+L \in [r, r+(1-p)\mu)$ the borrower cannot raise funds from C under incomplete information, while the project would still have positive net present value under full information. The investment condition $(1-p)\mu + r \leq pH + (1-p)L$ may appear strange at first sight, because it states that investment takes place if and only if the expected proceeds from the project exceed r plus the cost of verifying the low message with probability one under truth-telling. However, we know that at the optimal contract generically we have $b < 1$. To make sense of the condition, consider equation (2.1), which consists of the participation constraint after a binding IC has been plugged in. Taking the derivative with respect to b yields $pH - L + (1-p)(c-\mu)$. Because it is optimal to set $c = L$, the expression simplifies to $p(H-L) - (1-p)\mu$ which is always positive when the investment condition holds. In words, the expected feasible repayment from B to C increases with b, so the highest payout occurs when $b = 1$. If that is not enough for C to get r in expectation, then investment cannot take place.

A final point worth stressing about 1 is that the verification probability – and so the deadweight loss – increases in the risk-free rate r, as it governs the minimum payout necessary to guarantee that C receives her required rate or return.

[20]It reaches one only when the investment condition binds, which occurs for a measure zero set of parameter values.

2.5 Limited Commitment

Moving to limited commitment, we should consider the possibility that when B is in state H, for instance, she reports with some probability that the state is L instead.[21] Denote the probability that a borrower in state $i = L, H$ reports it truthfully by $(1 - d_i)$, for some $d_i \in [0, 1]$. It is straightforward to see that d_L will always be zero, so that we can restrict attention to d_H only, and relabel it d for ease of notation. According to the terminology used in the introduction, d denotes the probability that B strategically defaults conditional on her being in the high state. The unconditional probability of strategic default will be $p \times d$. Under truth-telling, the Verification Constraint (which we refer to as VC and ensures that C's strategy is ex post optimal) reads:

$$s_L \leq \mathbb{E}[c|s_L] - \mu, \text{ with equality when } b \in (0, 1). \tag{VC}$$

The VC constraint ensures that C finds it weakly optimal to verify the state of the world when a low message is reported by B. In this section, because of truth-telling, the constraint simplifies and we have $\mathbb{E}[c|s_L] = c_L$.

Lemma 2. *Generically, the optimal contract under commitment does not satisfy VC.*

Proof. First, consider the generic case where the b of Observation 1 is strictly less than one – equivalently, whenever the investment condition holds as a strict inequality. In this case, simply note that while the contract requires $b > 0$, we have $s_L = L > L - \mu = c - \mu$, so C never wants to verify ex-post. In the (non-generic) case where the investment condition holds with equality, one has the freedom of setting s_L as low as possible, subject to the feasibility requirement that $s_L \geq 0$. As a consequence, setting $s_L = 0$ we get that the allocation implemented by the optimal contract of Observation 1 can be replicated under limited commitment if and only if $L \geq \mu$, which may or may not hold. □

2.5.1 Preventing strategic default under limited commitment

Lemma 2 shows that limited commitment concerns have pervasive effects: the commitment solution is not implementable under limited commitment. It re-

[21] This is because the revelation principle can no longer be invoked.

mains an open question whether optimal contracts adjust by allowing for strategic default, or whether they prevent it with a different allocation of cash flow rights. We break the analysis in two parts: first, we consider the optimal contracts that prevent strategic default under limited commitment; then, we consider the optimal contract under strategic default and compare the two. Lemma 3 allows us to simplify the analysis. The Lemma shows that at the optimal contract that prevents strategic default, both the incentive constraint (Claim 1), and the verification constraint (Claim 3) are binding. In addition, it shows that investing under truth-telling is feasible only if $\mu \leq L$, as otherwise the verification constraint cannot be satisfied by a feasible pair c_L, s_L.

Lemma 3. *Optimal contracts that prevent strategic default under limited commitment have the following properties:*

1. *At any optimal contract, the incentive constraint must be binding*
2. *For investment to be implementable, bankruptcy costs must be relatively low: $L \geq \mu$.*
3. *Generically, when $pH + (1-p)(L-\mu) > r$, at any optimal contract VC binds. Otherwise, in the non-generic case where $pH + (1-p)(L-\mu) = r$, there exists an optimal contract where VC binds.*

Proof. See the Appendix. □

In light of Lemma 3, and because we are implementing under truth-telling, we can again set $c_H = H$ without loss of generality – it if off-equilibrium. Optimal contracts solve:

$$\min_{c,s_H,s_L} b \text{ s.t.}$$

$$\begin{aligned}
&(IC) \quad s_H = bH + (1-b)s_L \\
&(VC) \quad s_L = c - \mu \\
&(PC) \quad ps_H + (1-p)[b(c-\mu) + (1-b)s_L] = r \\
&(FCs) \quad b \in [0,1],\ 0 \leq c \leq L,\ 0 \leq s_i \leq i,\ \text{for } i \in \{L, H\}.
\end{aligned}$$

Plugging both IC and VC into PC yields:

$$b = \frac{r - c + \mu}{p(H - c + \mu)}$$

The derivative with respect to c is negative, so we set $c = L$ and Proposition 1 ensues.

Proposition 1. *Investment takes place if and only if* $(1-p)\mu + r \leq pH + (1-p)L$ *and* $L \geq \mu$. *The optimal contract sets* $c_L = L = s_L + \mu$, $c_H = H$, $s_H = bH + (1-b)(L-\mu)$ *and* $b = \frac{r-L+\mu}{p(H-L+\mu)}$.

Proof. Trivial from the above reasoning. Note that $b > 0$ for all parameter values. □

Although nothing guarantees that preventing strategically default is optimal under limited commitment (as we shall prove, it is often suboptimal), it is worth noticing a few properties of the optimal contracts that achieve this goal. At the *extensive margin*, limited commitment reduces the set of creditors that can invest without strategically defaulting. Indeed, all firms facing bankruptcy-related costs larger than L won't be able to invest without strategically defaulting on their claims in the high state. At the *intensive margin*, instead, limited commitment triggers an increase in the frequency with which default translates into bankruptcy, which rises the associated deadweight losses. Absent strategic default, the probability of default in both cases is $(1-p)$, which implies that we can simply consider whether we can rank the probability that default triggers bankruptcy b across cases. The next Lemma shows that, as one would have expected, they can be ranked.

Lemma 4. *The optimal contract that prevents strategic default under limited commitment features strictly higher probability that default triggers verification (i.e.,* b*) than the optimal contract under full commitment does.*

Proof. Just for this proof, denote the optimal verification probability under commitment (in Observation 1) by b_1 and that under limited commitment and no strategic default (in Proposition 1) by b_2. We have that:

$$\begin{aligned} b_2 \geq b_1 &\iff \frac{r-L+\mu}{p(H-L+\mu)} \geq \frac{r-L}{p(H-L)-(1-p)\mu} \\ &\iff \underbrace{\left(1+\frac{\mu}{r-L}\right)}_{>0 \text{ because } r>L>0 \text{ and } \mu>0} \times \underbrace{\left(\frac{p(H-L)-(1-p)\mu)}{p(H-L+\mu)}\right)}_{>0 \text{ whenever the investment condition holds}} > 0 \end{aligned}$$

□

In the next subsection we turn attention to the optimal contracts under limited commitment, which may or may not involve strategic default.

2.5.2 Optimal contract under limited commitment

For notational convenience, define the posterior probability that the state is high after the low message is observed by $p(d) \equiv \frac{pd}{(1-p+pd)}$. Optimal contracts solve:

$$\min_{\{c_i,s_i\}_{i=L}^{H}} b(1-p+pd) \tag{2.3}$$

subject to:

$$s_H \leq bc_H + (1-b)s_L, \text{ with equality whenever } d \in (0,1) \tag{IC}$$

$$s_L \leq p(d)c_H + (1-p(d))c_L - \mu, \text{ with equality whenever } b \in (0,1) \tag{VC}$$

$$p\big[(1-d)s_H + d\big(b(c_H-\mu)+(1-b)s_L\big)\big] + (1-p)\big[b(c_L-\mu)+(1-b)s_L\big] = r \tag{PC}$$

$$0 \leq c_i \leq x_i,\ b \in [0,1],\ d \in [0,1], \text{ and } 0 \leq s_i \leq m_i \tag{FCs}$$

The Verification Constraint (i.e., VC) guarantees that C's action is optimal ex post, given the posterior she holds after having observed the message sent by B.

Notice that the objective function now reflects a change in perspective: optimal contracts do not minimize the probability of default per se, but rather the probability that bankruptcy (i.e., verification) occurs. The direct effect of strategic default on the deadweight losses is clearly positive – that is, strategic default increases the deadweight losses by increasing the odds of B defaulting. However, there is an indirect effect as well that links b to d, so that the overall impact of strategic default on the deadweight losses can go either way. We will discuss when and why strategic default can decrease b, and how this indirect effect could dominate the direct effect, after having derived a few preliminary properties of optimal contracts.

Before proceeding, note that the necessary condition for investment in Observation 1, i.e. $pH + (1-p)L \geq r + (1-p)\mu$, remains necessary here. In addition, Lemma 5 proves that both the incentive constraint (Claim 1) and the verification constraint (Claim 3) must be binding. To show the latter, an intermediate step is to prove that $c_H > s_L$ (Claim 2). Finally, it shows that optimal contract have full collateralization (Claim 4) and it provides another necessary condition for investment to take place (Claim 5).

Lemma 5. *Optimal contracts under limited commitment have the following properties:*

1. *At any optimal contract, the incentive constraint must be binding*
2. *At any optimal contract, $c_H > s_L$*
3. *At any optimal contract where $d > 0$, VC must be binding*
4. *Any optimal contract features full collateralization: $c_H = H$ and $c_L = L$*
5. *Investment is implementable only if $p(H-L) + L \geq \mu$*

Proof. See the Appendix. □

Plugging both IC and VC into PC and solving for b yields:

$$b(d) = \frac{r - (p(d)H + (1-p(d))L - \mu)}{p(1-d)(H - (p(d)H + (1-p(d))L - \mu))}$$

As a result of the previous Claims, our problem becomes:

$$\min_{d} \; b(d)(1 - p + pd) \tag{2.4}$$

subject to:

$$0 \leq b(d) \leq 1, \; d \in [0,1], \quad \text{and } 0 \leq p(d)H + (1-p(d))L - \mu \leq L \qquad \text{(FCs)}$$

To tackle this problem, we take the following approach: first, we solve a relaxed problem without feasibility constraints on the verification probability b;

then we check if the solution we found for the relaxed problem is feasible. Even without the feasibility constraints on b, we still have feasibility constraint on s_L. Potentially, they may be bind for some degree of strategic default d. What helps us in the analysis is the strict monotonicity of such feasibility constraints with respect to d. Indeed, because $s_L(d) = p(d)H + (1 - p(d))L - \mu$ is a strictly increasing function of d, it follows that the feasibility constraints on $s_L(d)$ are also linear in d, and we can write them as $\underline{d} \leq d \leq \bar{d}$, where

$$\underline{d} = \frac{(1-p)(\mu - L)}{p(H-\mu)}$$

$$\bar{d} = \frac{(1-p)\mu}{p(H-L-\mu)}$$

Proposition 2. *A solution to the relaxed problem exists if and only if $pH + (1-p)L \geq \mu$.*

1. *If $L \geq \mu$, there exist two cases:*
   (a) *If $p < p_1$ strategic default is prevented by optimal contracts: $d = 0$ and $b = b_0$;*
   (b) *If $p \geq p_1$ optimal contracts induce high strategic default: $d = \bar{d}$ and $b = b(\bar{d})$;*
2. *If $L < \mu$, then we must have $d > 0$ and there exist two cases:*
   (a) *If $p < p_2$ optimal contracts induce low strategic default: $d = \underline{d}$ and $b = b(\underline{d})$;*
   (b) *If $p \geq p_2$optimal contracts induce high strategic default: $d = \bar{d}$ and $b = b(\bar{d})$.*

where $p_1 \equiv \frac{\mu+r-L}{H-r}$, $p_2 \equiv \frac{\mu}{H-L} + \left(\frac{H}{H-r} - \frac{H}{H-L}\right)$. We also have that $p_1 < p_2$.

Proof. In this proof, we concentrate on the case when $(1-p)\mu + r < pH + (1-p)L$. If the expression above holds as equality, then there exists only one feasible contract: $b = 1$, $d = 0$ and $s_H = H$. First, we investigate the behavior of our objective function $b(d)(1 - p + pd)$. As it is shown in Appendix, the derivative of this function potentially has two roots: d_1 and d_2. However, it turns out that d_2 does not belong to the $[0, 1]$ range for any parameter configuration. Another property which is shown in the Appendix is that if $\mu + r > pH + (1-p)L$, then $\lim_{d\to 1-0} b(d)(1-p+pd) = +\infty$. In this case, the derivative of the deadweight loss function is always positive and its graph is strictly increasing in the $[0,1]$ interval. In this case, one always chooses the left corner of the graph. As the analysis in this parameter region is trivial, we pay more attention to the other, more interesting case below. If $\mu+r < pH+(1-p)L$, then $\lim_{d\to 1-0} b(d)(1-p+pd) = -\infty$. In this parameter region, as shown in the Appendix, it is possible to have at most one local extremum point in $(0,1)$ interval. According to *Lemma* 2 in the Appendix, if $d_1 \in (0,1)$ and if d_1 is local extremum, then it must be a local maximum. So, there are two cases to consider in the parameter region where $\mu+r < pH+(1-p)L$: the case where the local maximum $d_1 \in (0,1)$, that where there is no local maximum points in the $(0,1)$ region. In any event, we shall compare the feasible left and right corners of the deadweight loss function to determine, where the graph achieves its minimum point. The feasible left corner of our graph will depend on the difference between L and μ. If $L \geq \mu$, then left root coincides with point $d = 0$. Otherwise, the minimum feasible d is determined by condition $s_L(d) = 0$ and it equals: $\frac{(1-p)(\mu-L)}{p(H-\mu)} > 0$. As for the right corner, $d = 1$ is infeasible, because there is no finite non-negative verification probability that would satisfy PC. So, the right corner is pinned down by the condition $s_L(d) = L$ and it equals: $1 > \frac{(1-p)\mu}{p(H-L-\mu)} > 0$ Feasibility of this root is guaranteed by the parameter region we are considering: $\mu + r < pH + (1-p)L$. A direct comparison of the feasible left corner and right corner in appropriate parameter regions pins down values of p_1 and p_2. □

After having derived a solution for the relaxed program, we now return to the full problem, to check whether the verification probability associated with our solution is indeed feasible.

Proposition 3. *If the solution to the relaxed problem does not satisfy feasibility, then there is no feasible contract that sustains investment.*

Proof. The solution obtained in Proposition 2, subparts $(1.a)$, $(1.b)$ and $(2.b)$ all satisfy the constraint $b(d^*) \leq 1$. So, feasibility holds in these regions. The only case where the optimal contract found in Proposition 2 might not be feasible is $(2.a)$. In this parameter region, we are interested in the case of

$p < p_2$. The condition $b(d) \leq 1$ is not satisfied by $\underline{d}$ if and only if: $p < \bar{p} = \frac{H(\mu-L)+r(H-\mu)}{H(H-L)} < p_2$. To prove that there is no feasible contract when $p < \bar{p}$, observe that equation $b(d) = 1$ has only one non-negative root and it is equal to: $d_0 = \frac{pH+(1-p)L-(1-p)\mu-r}{p\mu}$. We can establish that $d_0 < \underline{d}$ if $p < \bar{p}$. So, if $d_0 < \underline{d}$, then $\forall d \geq \underline{d} : b(d) > 1$. The last part comes from the fact if there was $d \in [\underline{d}, 1)$ such that $b(d) \leq 1$, then $b(d) = 1$ would have more than one root, because we know that $b(\underline{d}) > 1$. But this is impossible due to continuity of the function $b(d)$ over the $[0, 1)$ interval. □

Proposition 3 adds an additional necessary condition for investment, relative the one stated in Proposition 2: $p \geq \bar{p}$. This condition is relevant only if $L < \mu$, otherwise it would be redundant, as summarized in the following Corollary which gives necessary and sufficient conditions required for investment to be sustainable under limited commitment.

Corollary 1. *Investment under limited commitment takes place if and only if: (i)* $pH + (1-p)L \geq \mu + \max\{0, r - p\mu\}$*; and (ii)* $p \geq \bar{p}$ *while* $L \leq \mu$*.*

The max operator takes into account that both the condition in Proposition 2 and that derived for the full commitment case (see Observation 1) need to hold, and these cannot be ranked a priori because which one binds depends on the relation between r and $p\mu$, which is indeterminate given our assumptions. It is useful to compare the conditions stated in Corollary 1 with those in Proposition 1, which were: $(1-p)\mu + r \leq pH + (1-p)L$ and $L \geq \mu$. Allowing for strategic default relaxes the constraint $L \geq \mu$, allowing for investment to occur in this case whenever p is sufficiently high. The intuition behind this result is as follows. Strategic default increases the expected recovery rate, which is necessary for VC to be satisfied. However, it also needs to have a beneficial (i.e., negative) effect on b, because the marginal condition was coming from the constraint $b \leq 1$. This requires p to be sufficiently high.

While Corollary 1 and Proposition 2 – cases (2.a) and (2.b) – jointly clarify the role played by strategic default at the extensive margin, we are also interested in its effect on investment terms, at the intensive margin. As stated in Proposition 2, when p is small – as in case (1.a) – then strategic default does not occur and optimal contracts implement truth-telling. For higher p we move to case (1.b), in which even if B could invest under truth-telling, she finds it too expensive. This is because the negative effect of strategic default on the probability that default triggers bankruptcy (i.e., b) more than compensates the increase it brings about in the probability that B defaults. Namely, firms in (1.b) feature a probability of default larger than p and, at the same time, a lower probability that default triggers bankruptcy b. The combined effect lowers the expected bankruptcy costs faced by these firms. To further analyze the effect various parameters have on the likelihood and consequences of strategic default, the next section derives several comparative statics of interest.

2.6 Comparative Statics

The main advantage of having analytical solutions and a relatively simple model is that we are able to explicitly derive several comparative statics of interest. Some will be more expected, others less so. Figure 2.1 plots the objective function – that is, the expected deadweight losses from investment – across three cases: (i) the full commitment case; (ii) the limited commitment case under truth-telling; and (iii) the limited commitment case without the truth-telling requirement. Few properties are worth noticing.

As it should be expected, the deadweight losses increase with bankruptcy costs and are strictly lower under full commitment than they are under limited commitment. Deadweight losses are lower because the creditors can commit that they are going to take defaulting borrowers to court a certain amount of times, even if this is (generically) ex post inefficient – by Lemma 2. As a consequence, they can set both s_L and c_L as maximal, without worrying about the verification constraint.

The deadweight losses are the lowest at the optimal contract that prevents strategic default under full commitment. Again, this is expected, as we have two additional constraints relative to the commitment case (i.e., $d = 0$ and the verification constraint), and one additional constraint relative to the limited commitment case (i.e., $d = 0$). In this case, investment takes place if and only if $\mu \leq 3$, and when it does the deadweight losses are relatively large. Strategic default helps in two regions. First at the *intensive margin*, when $\mu \leq 2$, in which case one can compute that $p \geq p_1$ and we have strategic default $\bar{d} > 0$

induced by the optimal contract. In this region, although feasible, preventing strategic default is needlessly costly for the firm ex ante: a strictly positive amount of strategic default lowers the expected deadweight losses. To see why this happens, observe that the derivative of the deadweight loss function with respect to d reads:

$$\frac{\partial DWL}{\partial d} = \mu\left(\underbrace{p \times b(d)}_{\text{Direct Effect}} + \underbrace{\frac{\partial b(d)}{\partial d}(1 - p + p \times d)}_{\text{Indirect Effect}}\right)$$

While the direct effect of strategic default is always to increase the deadweight losses, because for a given b the chances of observing the low message increase with it, the indirect effect has an ambiguous sign. Namely, the probability of verification may well fall with strategic default, and this is what our results hinge on.

The second region in which strategic default helps is at the *extensive margin*, where $\mu \in [3, 4]$. In this case, investment would be impossible absent strategic default, because of the verification constraint. This case is comparable, for instance, to that studied in Bolton and Oehmke (2011) – absent credit derivatives. A final observation related to Figure 2.1(a) is that the objective function is continuous in μ, although not differentiable because of the multiple regimes-switches occurring.

Another interesting angle is to study how the deadweight losses change as the risk-free rate changes. From Figure 2.1(b), we observe that the value from strategic default is concentrated in environments where the risk-free rate is low. In such cases, implementing investment under truth-telling proves extremely costly – the distance between the commitment and truthful deadweight losses shrinks monotonically with r. As r increases, the slope of the deadweight loss function under commitment is flatter than that of the optimal contract undr limited commitment, which quickly converges towards that of truth-telling and merges with it for $\mu \geq 4$.

Now, we can turn attention to the credit spreads. Figure 2.1(c) shows that spreads are uniformly lower under commitment and higher under limited commitment and truth-telling. At the intensive margin, strategic default strictly

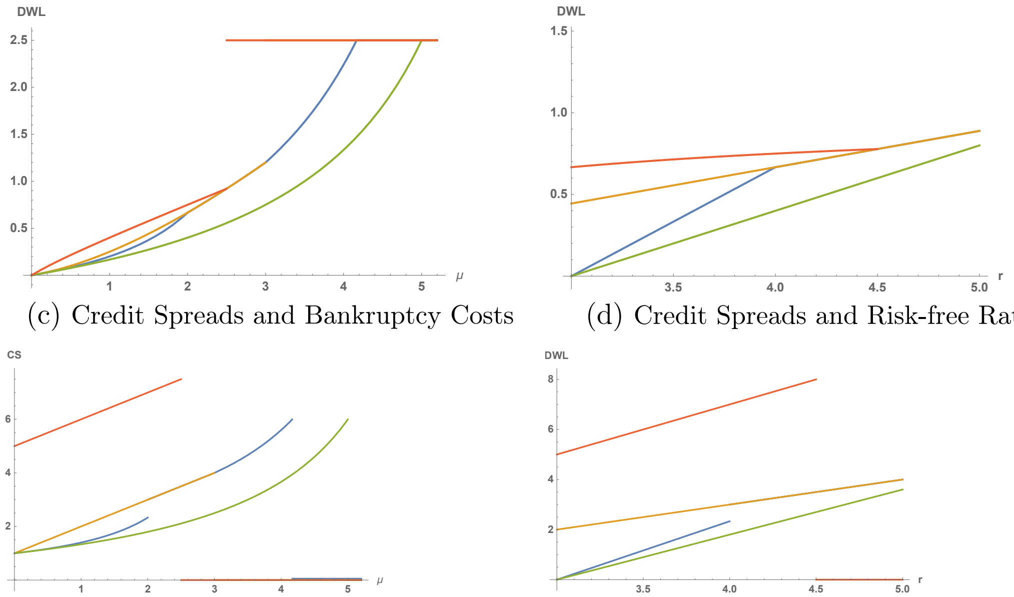
Figure 2-1: Losses, Credit Spreads and the Economy

(a) Losses and Bankruptcy Costs

(b) Losses and Risk-free Rates

(c) Credit Spreads and Bankruptcy Costs

(d) Credit Spreads and Risk-free Rate



Green: full commitment case; Orange: limited commitment and truth-telling; Blue: limited commitment. Parameter values: $p = 0.5$, $\mu = 2$, $L = 3$, $H = 10$.

lowers the credit spreads whenever $\mu \leq 2$, and they jump up at $\mu = 2$ when the regime switches from implementing $\bar{d}$ to implementing $d = 0$. Although they are continuous for $\mu \geq 2$ – provided that investment is implementable – credit spreads are not a differentiable function of μ in this range either, because of another regime swith from $d = 0$ to $d = \underline{d}$ which occurs at $\mu = 3$.

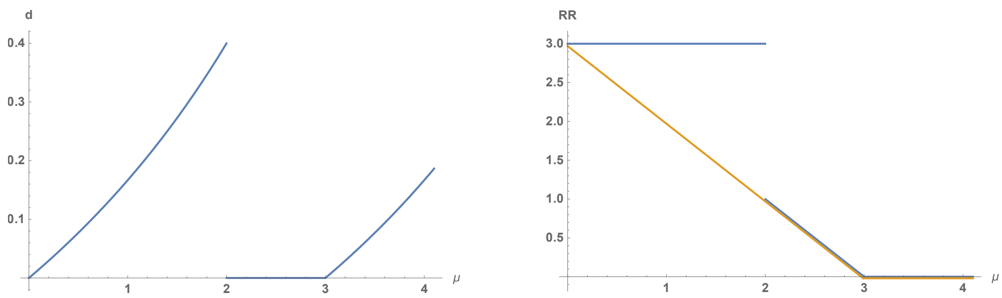
As shown in Figure 2.1(d), and consistent with the empirical evidence, credit spreads increase with the risk-free rate r, which proxies for the required rate of return of competitive investors to finance the project. Under limited commitment, credit spreads discontinuously jump up at a critical level of r – in our numerical example, at $r = 4$.

Figure 2.2(a) plots the frequency of strategic default as a function of the bankruptcy cost μ, clarifying the reasons behind the observed jump in credit spreads. The non-monotonicity of strategic default in the costs of bankruptcy is, to our knowledge, a new result of this paper relative to the existing literature, although it need not hold for all parameter values.

Turning to the model's implications for recovery rates, 2.2(b) plots the out-of-court recovery rate as a function of the bankruptcy costs μ. Strategic default

Figure 2-2: Some Economic Effects of Bankruptcy costs

(a) Strategic Default (b) Recovery Rates Out-of-Court



Orange: limited commitment and truth-telling; Blue: limited commitment.
Parameter values: $p = 0.5$, $r = 4$, $L = 3$, $H = 10$.

for low values of μ generates higher recovery rates than truthful reporting. In particular, the equilibrium amount of strategic default is such that recovery rates are flat in μ, until we switch regime and, for bankruptcy costs $\mu \geq 3$, recovery rates out-of-court drop discretely and then keep falling continuously throughout. This is consistent with the empirical evidence in Djankov et al. (2008), but it highlights an important, yet often overlooked, point. Because when bankruptcy costs increase recovery rates out-of-court decrease, consistently with the story that creditors have lower bargaining power vis à vis their debtors, it is not obvious that higher bankruptcy costs should be associated with more restructuring out-of-court (as suggested in Jensen (1991)). We now turn attention to this empirical prediction and study if it is consistent with our model.

To this end, Figure 2.4 plots the frequencies of each possible default event as a function of the costs of bankruptcy μ. Recall that, upon default, three events might occur in the model: (i) default is resolved out-of-court, by means of a private restructuring of claims; (ii) default triggers bankruptcy and the firm is insolvent; (iii) default triggers bankruptcy but the firm is solvent – i.e., the default event was strategic. As the Figure 2.3 shows, the frequency of an out-of-court restructuring falls as bankruptcy costs increase.[22] This is because high bankruptcy costs imply lower value of equilibrium out-of-court restructuring plans, which in turns induces high incentives to strategic default and needs to be countered, for investment to take place, by a higher probability that default

[22] Corollary 2 will formally show this fact.

is resolved in-court. This prediction is consistent with the fact that private restructuring of debt is particularly relevant in countries where bankruptcy costs are not prohibitively high, such as the US. However, it still needs to be tested in the data – to our knowledge.

As for the recovery rates in bankruptcy, it is non-monotonic: it is high when bankruptcy costs are low, because of strategic default at the intensive margin; it falls in intermediate ranges of μ, because now truthful reporting becomes optimal; finally, it increases at the extensive margin when bankruptcy costs are high, and absent strategic default the parties would not be able to invest. In general, upon bankruptcy the recovery rates distribution is predicted to be bi-modal, with a peak on the low and a peak on the high recovery state. With more than two states, predictions do not change, but the distribution will become increasingly spread out, as it typically is in the data.

In Proposition 2 we derived threshold for the solvency probability p – we labelled them p_1 and p_2 – such that strategic default increases above the thresholds. However, it remains to be studied the local changes in strategic default as the main parameters of interest increase. The next Lemma answers such question.[23]

Lemma 6. *Locally, the optimal level of strategic default behaves as follows:*

- *It increases with the verification cost μ:*

  1. $\frac{\delta \bar{d}}{\delta \mu} = \frac{(1-p)(H-L)}{p(H-L-\mu)^2} > 0$
  2. $\frac{\delta \underline{d}}{\delta \mu} = \frac{(1-p)(H-L)}{p(H-\mu)^2} > 0$

- *It falls with the solvency probability p:*

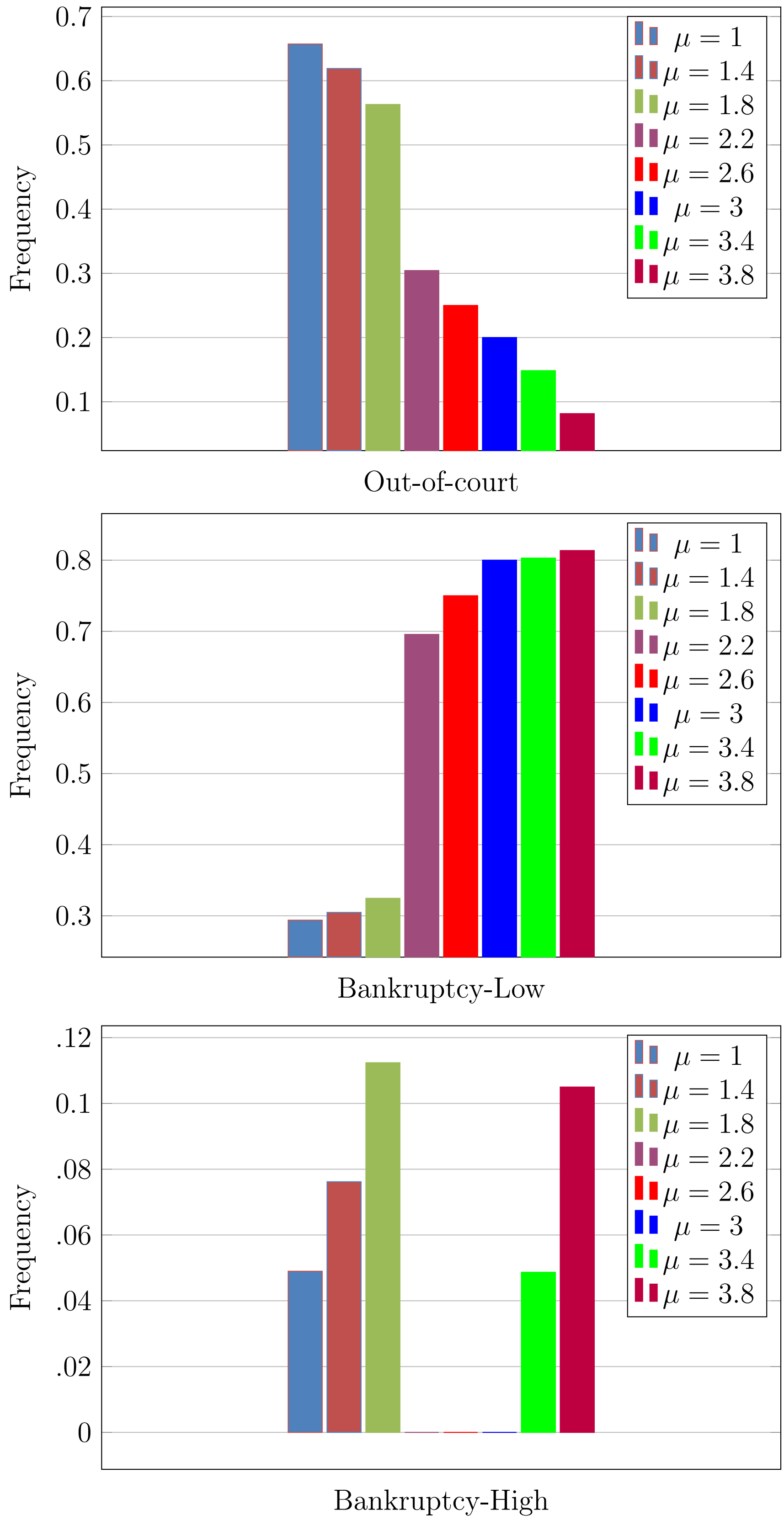
  1. $\frac{\delta \bar{d}}{\delta p} = \frac{-\mu}{p^2(H-L-\mu)^2} < 0$
  2. $\frac{\delta \underline{d}}{\delta p} = \frac{-(\mu-L)}{p^2(H-\mu)^2} < 0$

- *It falls with the high-state return H:*

  1. $\frac{\delta \bar{d}}{\delta H} = \frac{-\mu(1-p)}{p(H-L-\mu)^2} < 0$
  2. $\frac{\delta \underline{d}}{\delta H} = \frac{-(1-p)(\mu-L)}{p(H-\mu)^2} < 0$

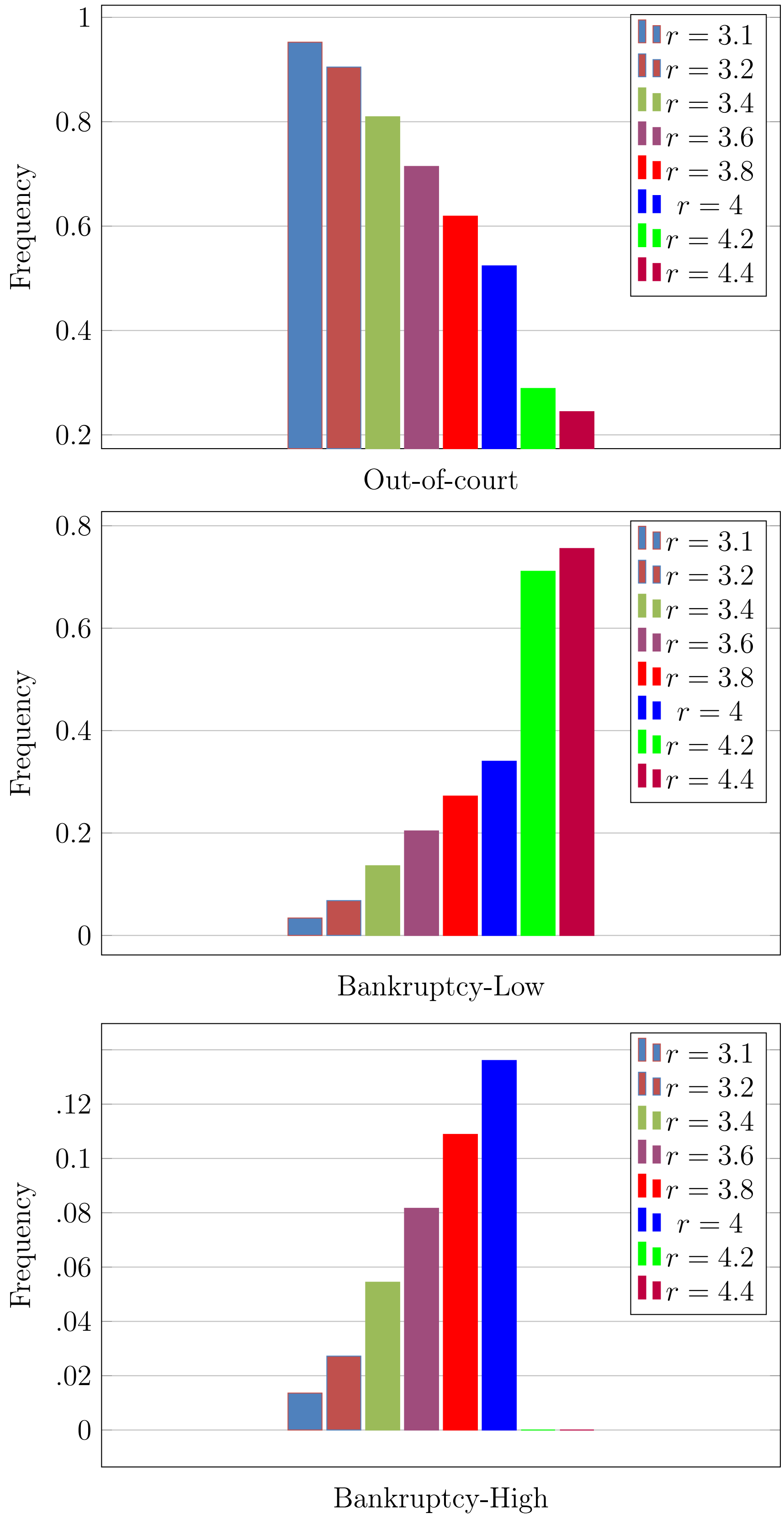
[23] We omit the proof because it simply consists in taking a few first derivatives.

Figure 2-3: Default Events Frequencies and Bankruptcy Costs



Parameter values: $p = 0.5$, $r = 4$, $L = 3$, $H = 10$.

Figure 2-4: Default Events Frequencies and Risk-free Rates



Parameter values: $p = 0.5$, $\mu = 2$, $L = 3$, $H = 10$.

- *It does not vary with the risk-less rate r.*

A few lessons can be learned from Lemma 6. First, consider the effect of μ, which acts as our modeling proxy for bankruptcy-related costs, and can be taken as a measure of the degree of creditors' protection provided by the institutional environment in which investment takes place. Locally, an increase in μ brings about an increase of strategic default instances. This is because a higher μ means a weaker bargaining position for the creditors ex post, which itself translates into lower incentive compatible returns on the securities they hold absent verification. For the creditors to break even, borrowers must default more frequently in equilibrium. However, as bankruptcy costs increase there is a regime shift at the thresholds p_1 and p_2, at which the frequency of strategic default falls. As a consequence, the effect is overall non-monotonic.

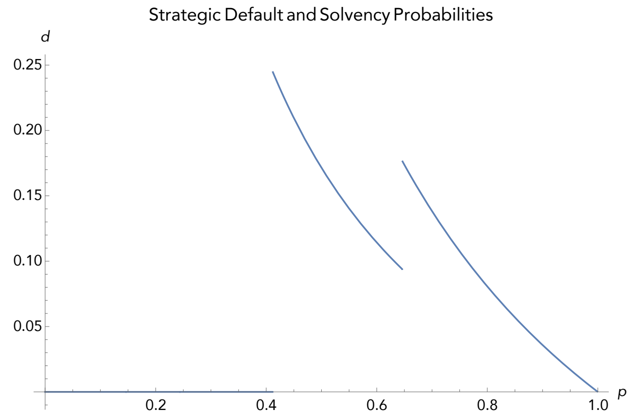
In addition, we have a formal statement about the local behaviour of the probability that bankruptcy will take place after default. It immediately follows from the first part of Lemma 6 .

Corollary 2. *Locally, the level of bankruptcy probability is increasing with the bankruptcy costs :* $\frac{\delta b}{\delta \mu} > 0$

Second, consider the effect of p on strategic default, which is shown in Figure 2.5. When the solvency probability p is sufficiently small, then there is either no investment (if $L \leq \mu$), or investment takes place without inducing incentives for the borrower to strategically default. As p rises, we hit the threshold $p \geq \bar{p}$, above which strategic default enables investment to take place. Above $\bar{p}$, further local increases in solvency probabilities lower the frequency of strategic default in equilibrium monotonically, until we hit the threshold at which we shift from $\underline{d}$ to $\bar{d}$. The probability of borrowers strategically defaulting jumps up, for then falling monotonically again. As a result, overall the effect of p on d is also non-monotonic.

Finally, although it appears surprising that r does not locally affect strategic default, it should be noted that it affects the thresholds p_1 and p_2, so determining at which parameter configuration the regime shift occurs. In particular, the effect of r on the thresholds is positive.

Figure 2-5: Strategic Default and Solvency Probability



Parameter values: $\mu = 2$, $L = 0.8$, $r = 3$, $H = 9$

2.7 No Commitment

In this section, we analyze the case where creditors cannot contract on any mapping that includes reported states, in the spirit of Gale and Hellwig (1989). In the previous analysis, creditors could not contract on verification b, but they still could contract on payments absent verification s_i for $i \in \{L, H\}$. It remains to be understood what happens when also the function s_i cannot be contracted upon. As usual, we begin by establishing some useful properties of optimal contracts for this case. Notice that a contract still exists, in that it can specify a function $c_x : \{L, H\} \to \mathbb{R}_+$. Also, we cannot a priori rule out that the both ex post types – i.e., $x = L$ and $x = H$ – use mixed strategies, so we need to return to our indexed notation d_i and b_i, where $i \in \{L, H\}$.

When s_i is not contractible, the ex post game changes. Now, first the borrower offers a repayment s_i. We will start our analysis with three levels of repayments $i \in \{L, M, H\}$ and we show in the Appendix that for any number of repayments $i \in \{s_1, s_2, ..., s_n\}$, it is without loss of generality to concentrate on two levels of repayment $i \in \{L, H\}$.

Upon observing s_i, the creditor updates her belief about the realized return, and chooses whether or not to verify the state and be able to enforce payments c_x accordingly. Importantly, there are infinite possible off-equilibrium payments. If the creditor observes payment other than s_i for $i \in \{L, M, H\}$, Bayes' rule cannot be used to update the beliefs and therefore the posterior is arbitrary. In the absence of a better criterion, we will characterize equilibria and optimal contract for the case where, out of equilibrium, creditors believe that the offer comes from the high type with probability one. Of course, this choice is with loss of generality, but so would any other choice be. See Gale and Hellwig (1989) for a discussion of the problem, and of the potential refinements. The general problem we consider in this section reads as follows:

$$\min_{c_L, c_H} \; b_L(1-d_L)(1-p) + b_M(pd_H + (1-p)d_L)$$

subject to:

$$b_L c_L + (1-b_L)s_L \leq b_M c_L + (1-b_M)s_M, \quad \text{with equality if } d_L \in (0,1) \tag{IC_L}$$

$$s_H \begin{cases} \leq b_M c_H + (1-b_M)s_M & \text{if } d_H = 0 \\ = b_M c_H + (1-b_M)s_M & \text{if } d_H \in (0,1) \\ \geq b_M c_H + (1-b_M)s_M & \text{if } d_H = 1 \end{cases} \tag{$IC_{H,1}$}$$

$$s_H \leq b_L c_H + (1-b_L)s_L \tag{$IC_{H,2}$}$$

$$s_M \leq p(d)c_H + (1-p(d))c_L - \mu, \quad \text{with equality if } b_M \in (0,1) \tag{VC_0}$$

$$s_L \leq c_L - \mu, \quad \text{with equality if } b_L \in (0,1) \tag{VC_1}$$

$$s_H \geq c_H - \mu, \tag{VC_2}$$

$$p[(1-d_H)s_H + d_H(b_M(c_H-\mu) + (1-b_M)s_M)] + (1-p)\times$$
$$\times [(1-d_L)(b_L(c_L-\mu) + (1-b_L)s_L) + d_L(b_M(c_L-\mu) + (1-b_M)s_M)] = r, \tag{PC}$$

$$0 \leq c_L \leq L, \text{ and } 0 \leq c_H \leq H, \tag{FCs}$$

where the posterior belief $p(d)$ reads:

$$p(d) = \frac{pd_H}{(1-p)d_L + pd_H}$$

About the program, three comments are due. First, the only contractual variables are c_L and c_H, as everything else will be induced as part of a bayesian equilibrium played ex post. Second, the objective function now comprises of both verification upon the low offered repayment, and verification upon the intermediate offered repayment. Strategic default might arise only in the latter case (see Lemma 7: Claim 8 in the Appendix for a formal discussion). Finally, the problem has several additional constraints. Let us review them one by one, and explain their origin and meaning before proceeding to the analysis.

Among the constraints to the problem, IC_L guarantees that when the firm is in the low state it does not find it strictly beneficial to claim it is in the middle state. $IC_{H,1}$ and $IC_{H,2}$ guarantee that when the firm is in the high state it finds it weakly beneficial to report according to a specific d_H. Then, there are three 'verification constraints': VC_0 ensures that there is verification upon medium reported returns, given the posterior belief that the state is high $p(d)$. This constraint was also present before. VC_1 and VC_2 ensure that the creditor finds it weakly optimal to verify in the low and in the high state. While VC_1 is intuitive, and it just reflects the fact that the low report only comes in equilibrium from the low type, VC_2 needs further clarification. Notice that now nothing prevents the creditor from verifying also upon observing the high report. This is because reported states are not contractible (they are not even observable to third parties). As a consequence, the high payment needs to be such that the creditor does find it weakly optimal *not to verify* in this case – hence, the inequality is flipped. To conclude the problem, we have our usual Participation Constraint PC and the feasibility constraints on c_L and c_H (i.e., the FCs). As done in the previous sections, we proceed by proving a set of preliminary claims that simplify the problem.

Lemma 7. *Optimal contracts that prevent strategic default under non contractible communication have the following properties:*

1. *Without loss, we can restrict attention to contract such that VC_1 binds*
2. *At any optimal contract VC_2 binds*
3. *For investment to take place, it must be that $c_H - \mu > L$*
4. *If $b_M = 1$, then we must have a separating equilibrium where $s_M = s_L = c_L - \mu$*
5. *Without loss, we can restrict attention to contract such that VC_0 binds*
6. *Without loss, we can restrict attention to contract such that IC_L binds*

7. *At any optimal contract $b_L \geq b_M$, with strict inequality when $s_L \neq s_M$*

8. *If $IC_{H,1}$ holds with equality, than $IC_{H,2}$ always holds strictly*

9. *At any optimal contract $s_H \leq b_M c_H + (1 - b_M) s_M$*

10. *At any optimal contract $IC_{H,1}$ binds*

Proof. See the Appendix. □

The Lemma presents the result in the 'right' sequence, in the sense that to prove claim (2), we use claim (1), and so on. It states that all the VCs and the ICs are binding, and that the verification probability monotonically falls with the reported states. Now we are equipped to characterize the solution to our problem.

Lemma 8. *Any equilibrium where there are three repayment levels $\{s_L, s_M, s_H\}$ is dominated by another equilibrium with two repayment levels $\{s_L, s_H\}$.*

Proof. See the Appendix. □

The proof to Lemma 8 is technical, and we defer its discussion to the Appendix. However, it allows us to greatly simplify the problem and to derive a solution to it. Our minimization program can be rewritten as follows:

$$\min_{c_L, c_H} \; b(pd + (1-p))$$

subject to:

$$s_H = bc_H + (1-b)s_L, \quad \text{with equality if } d \in [0,1] \tag{IC}$$

$$s_L = p(d)c_H + (1-p(d))c_L - \mu, \quad \text{with equality if } b_M \in [0,1] \tag{VC}$$

$$p[(1-d)s_H + d(b(c_H - \mu) + (1-b)s_L)] + (1-p)[(b(c_L - \mu) + (1-b)s_L)] = r \tag{PC}$$

$$0 \leq c_L, s_L \leq L, \text{ and } 0 \leq c_H, s_H \leq H \tag{FCs}$$

Notice that we can dispense any reference to the medium repayment, and we return to a familiar VC that works for the low repayment and ensures that verification does sometimes happen there. The following Proposition characterizes optimal contracts and the allocations they implement.

Proposition 4. *The solution of above problem exists iff* $pH + (1-p)L \geq \mu + r$

1. *If* $L \geq \mu$*:*

   $d = 0, b = \frac{r-L+\mu}{(r-L+\mu)+p\mu}, c_L = L, c_H = L + \frac{b}{1-b}\mu, s_L = L - \mu$

2. *If* $L < \mu$*, then we must have* $d > 0$ *and there exist two cases:*

   (a) *If* $p < \frac{pH-r}{\mu}$ *then* $d = \frac{\mu(1-p)}{r+(1-p)\mu}, b = \frac{r}{r+\mu p(1-d)}, c_L = 0, c_H = \frac{r+\mu p(1-d)}{(1-d)p}, s_L = 0$.

   (b) *If* $p \geq \frac{pH-r}{\mu}$*, then* $d = \frac{pH-r}{pH}, b = \frac{H-r-\mu}{H-r-\mu p(1-d)}, c_L = \frac{r+\mu-pH}{(1-p)}, c_H = H, s_L = 0$.

Proof. See the Appendix. □

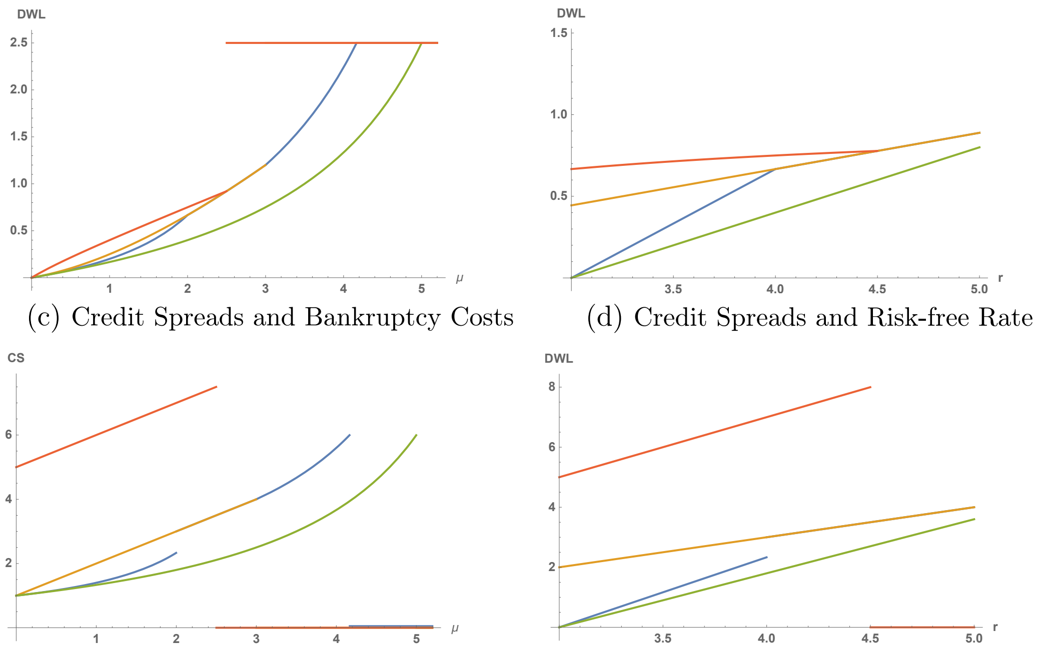
Importantly, in Proposition 4 the only contractual variables are the repayments upon bankruptcy c_L and c_H. Those are taken as given in the closest paper that studies a similar model, that is Gale and Hellwig (1989). However, such restriction is not without loss of generality, as in this case these variables change dramatically across parameters. While in case (1) debt is secured, as it had been in the previous limited commitment case, in part (2) debt is unsecured, and it only pays off in the high state if strategic default occurs. In addition, c_H is no longer equal to H in parts (1) and (2,a) – i.e., the maximum value consistent with limited liability. In these cases, c_H is always interior now, and it governs credit spreads. To establish this link is another novel contribution of our work.

The deadweight losses and credit spreads increase with bankruptcy costs and with the risk-free rate, as before. Credit spreads are also higher, reflecting further constraints to the economic allocation of resources. Figures 2.6(a) - 2.6(d) revisit our previous plots by adding the no commitment case. As shown in the figures, lack of commitment to transfers severely affects the extensive margin, as now the underinvestment problem kicks in at a much lower threshold of bankruptcy costs.

A final remark: a reader might be disturbed by the apparent lack of multiple equilibria. This is solely a function of our arbitrary assumption that off-equilibrium beliefs are that any deviation comes from the high type. This is a simplification, and leaves aside many interesting issues that Gale and Hellwig (1989) discuss and explore at length. However, it has a natural motivation in

Figure 2-6: Losses, Credit Spreads and the Economy (revisited)

(a) Losses and Bankruptcy Costs (b) Losses and Risk-free Rates



(c) Credit Spreads and Bankruptcy Costs (d) Credit Spreads and Risk-free Rate

Green: full commitment case; Orange: limited commitment and truth-telling; Blue: limited commitment; Red: no commitment. Parameter values: $p = 0.5$, $\mu = 2$, $L = 3$, $H = 10$.

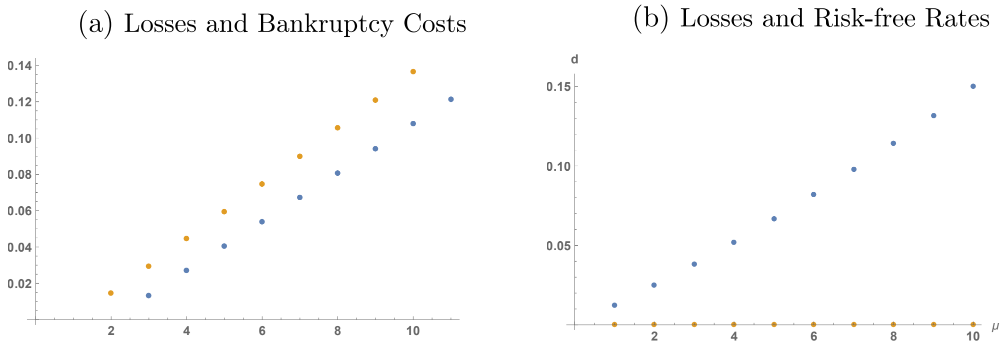
our setting: any different off-equilibrium belief would further constrain the economic allocation, inducing weakly larger deadweight losses and credit spread. We just focus on the lower bound, and shows how it compares to the other cases.

2.8 Beyond two states

In the final extension to our model, we finally move beyond two states (default; no default), by introducing a third 'medium' state between the low and the high previously considered. The goal of this extension is twofold. First, we will show that our qualitative results do not depend on the cardinality of the state space, which would not surprise readers familiar with costly-state-verification models. Second, we will clarify why we restricted attention to the two-state case, instead of writing a more general model from the start. The three-state case proves very hard to analyze, and the numerical simulations will shed some light on the reasons behind such complications.

Figure 2.7(a) plots the deadweight losses for the commitment and the limited

Figure 2-7: Losses, Credit Spreads and the Economy (revisited)



Orange: strategic default in the high state d_H; Blue: strategic default in the medium state d_M. Parameter values: $p_H = 0.5$, $p_M = 0.2$, $r = 2$, $L = 1$, $M = 3$, $H = 6$.

commitment cases across values of μ. The displayed dots are the results of a numerical simulation. Once again, the deadweight losses are increasing in μ. Limited commitment reduces the efficiency of the implemented allocations in a similar fashion as it did in the two-state case.

Importantly, Figure 2.7(b) shows that strategic default need not be a monotonic function of the realized returns: the chances that strategic default happens from firms in the high state are larger than the chances it happens in the medium state (which, although very low, are never zero in our simulations). Further, although it cannot be seen in the figure, strategic default might remain a non-monotonic function of μ: it is not monotonic in the medium state. These results clarifies some of the technical difficulties involved in analytically solving our model with more than two states.

2.9 Conclusions

We considered a costly-state-verification model where (i) creditors can randomize their verification strategies, but (ii) they cannot commit to a mapping from reported states to verification policies in the contract. We find both assumptions realistic. As for randomization, in practice we observe that out-of-court restructuring plans are only sometimes successful in avoiding costly bankruptcy. As for limited commitment, there are limited ways to commit to a certain restructuring protocol at the investment stage.

These two assumptions jointly generate incentives for borrowers to strategically default on their debts at the optimal contract. Strategic default increases the bargaining power creditors have ex post, at the restructuring stage, by rising the expected recovery rate from bankruptcy. As a consequence, it serves two purposes. At the extensive margin, it enables firms operating in countries with weak creditors' protection and high bankruptcy costs to invest. At the intensive margin, it enables firms to save on expected bankruptcy costs by reducing the probability that default triggers costly bankruptcy.

To sum up, bankruptcy-related costs and limited commitment on the creditors' side jointly imply that: (i) borrowers may find it optimal to default strategically on their debts; (ii) strategic default is necessary for investment to take place in environments characterized by high bankruptcy costs; (iii) even when strategic default could be avoided by choosing some feasible capital structures, firms and investors may prefer alternatives that induce strategic default with positive probability in an effort to reduce the deadweight losses associated with bankruptcy and maximize firm value; (iv) when strategic default is associated to higher firm value, we should observe both a higher default frequency and a higher probability of successful restructuring of out-of-court; (v) therefore, a higher probability of default does not necessarily imply a lower firm value.

We wish that these findings might inform future empirical end theoretical work on the costs and benefits of strategic default in financial markets, and to enrich credit risk models with an endogenous fraction of default events that are predicted to resolve out-of-court.

2.10 Appendix

2.10.1 Full problem in the commitment case

We consider the problem an optimal contract solves when the creditor can commit to a verification strategy contractually. As was mentioned in Section 3, the repayment upon verification is a function of both the message and the state. The verification probability $b(m_i)$ is a function of the message. In addition, we have two incentive constraints: one for the low state and the second for the high state. The full problem reads:

$$\max_{b(m_i),c(m_{i,j}),s(m_i)} \; p(b(m_H)(H-c_{H,H})+(1-b(m_H))(H-s_H))+$$
$$(1-p)(b(m_L)(L-c_{L,L})+(1-b(m_L))(L-s_L))$$
subject to:
$$b(m_L)c_{L,L}+(1-b(m_L))s_L \leq b(m_H)c_{H,L}+(1-b(m_H))s_H \qquad (IC_L)$$
$$b(m_H)c_{H,H}+(1-b(m_H))s_H \leq b(m_L)c_{L,H}+(1-b(m_L))s_L \qquad (IC_H)$$
$$p(b(m_H)(c_{H,H}-\mu)+(1-b(m_H))s_H)+$$
$$+(1-p)(b(m_L)(c_{L,L}-\mu)+(1-b(m_L))s_L) \geq r \qquad (PC)$$
$$b(m_i)\in[0,1],\; 0\leq c_{m_i,j}\leq x_i,\; 0\leq s_i\leq x_i \text{ for } i,j\in\{L,H\} \qquad (FC)$$

The first thing to notice is the fact that at an optimal contract PC must hold as an equality. Suppose not, then there exists another contract which gives higher profit to the borrower. Take all repayments $c(m_i,j)$, $s(m_i)$ and scale it down by the fraction $\phi\in(0,1)$: such that $c(m_i,j)'=\phi c(m_i,j)$ and $s(m_i)'=\phi s(m_i)$. The fraction ϕ is chosen in such a way that PC holds as equality at the new contract:

$$\phi=\frac{r}{p(b(m_H)(c_{H,H}-\mu)+(1-b(m_H))s_H)+(1-p)(b(m_L)(c_{L,L}-\mu)+(1-b(m_L))s_L)}$$

This deviation is feasible, because it satisfies IC, and PC now holds as an equality. Also it decreases the total transfer from the borrower to the creditor. So, the utility of the borrower will be higher at the new contract, proving that PC binds.

Second, since PC binds at the optimal contract , we can rewrite our objective function by plugging PC to the original objective function. The resulting expression reads:

$$U_B = pH + (1-p)L - r - (pb(m_H) + (1-p)b(m_L))\mu$$

The first three terms are constants and denote the expected net profit that can be obtained from the project. They do not affect the solution, so our problem can be re-written as follows:

$$\min_{b(m_i),c(m_i,j),s(m_i)} (pb(m_H) + (1-p)b(m_L))\mu \text{ s.t.}$$
$$(IC_L),(IC_H),(PC),(FCs)$$

This formulation stresses the fact that the maximization of the borrower's utility is equivalent to minimization of the deadweight losses associated with the verification.

The third observation we make is that $b(m_H)c_{H,H} + (1-b(m_H))s_H > r$. Otherwise, it is easy to check that a contract which does not satisfy this condition is infeasible, because it would violate PC.

The forth observation is that $b(m_H) = 0$. This can be proved by contradiction. Imagine that in the optimal contract $b(m_H) > 0$, then we could propose a new contract that yields higher utility to the borrower. Set $b(m_H)' = 0$, $s'_H = b(m_H)c_{H,H} + (1-b(m_H))s_H$. It is possible to set such a level of s'_H, because $r < b(m_H)c_{H,H} + (1-b(m_H))s_H \leq H$. IC_H and PC are unchanged at the new contract, so the only constraint to check is IC_L. The constraint reads:

$$b(m_L)c_{L,L} + (1-b(m_L))s_L < s'_H$$

The inequality follows from the fact that the RHS of the expression above is strictly less than r, so the borrower has strict incentives to tell the truth in the low state. So, the new contract satisfies IC_L as well. This concludes the proof that $b(m_H) = 0$. Because $b(m_H) = 0$ makes the choice of $c_{H,H}$ and $c_{H,L}$ irrelevant, we can simplify the notation and relabel $c_{L,H} = c_H$ and $c_{L,L} = c_L$.

The fifth observation is the fact that at the optimal contract IC_L is slack. This follows from the fact that at any optimal contract $b(m_H) = 0$. If IC_L is satisfied with equality, then it means that $s_H \leq L$, which contradicts feasibility. So, if we consider the set of contract where $b(m_H) = 0$, then we can omit IC_L.

The sixth observation is the following one: the punishment for the strategic defaulter must be $c_H = H$.

Proof. We consider two cases: $b < 1$ and $b = 1$. When $b < 1$, if we assume that in the optimal contract $c_H < H$, there always exists a feasible contract $c'_H = c_H + \epsilon$, $b' = b - \delta$ and $s'_H = b'c'_H + (1 - b')L$ where $\delta > 0$ and $\epsilon = \frac{\delta(pc_H + (1-p)(c_L - \mu))}{p(b-\delta)} > 0$. The positive sign of the ϵ follows from the same arguments as in Claim 3 of Lemma 1. So, we have feasible contract with a lower verification probability, reaching a contradiction.

For the case when $b = 1$, we observe that $b = 1$ is optimal if and only if $pH + (1 - p)(L - \mu) = r$. If instead $pH + (1 - p)(L - \mu) > r$, then there exists a contract with $b < 1$. This follows from the fact that if $b = 1$ and $c_i = x_i$, then the borrower compensates the creditor strictly more than r. The borrower can slightly decrease b still satisfying PC. In the other direction, if $pH + (1 - p)(L - \mu) = r$ then $b = 1$ and $c_i = x_i$ is the only feasible contract, so it should be optimal. As a result, whenever at the optimal contract $b = 1$, then it follows necessarily that $c_H = H$. □

This leads to the reduced formulation of the problem, which we consider in the main text.

2.10.2 Proof of Lemma 1.

Proof. Claim 1: Start with part (a), and suppose by contradiction that $b = 0$. IC reads $s_H \leq s_L$. But, by feasibility constraints, $s_L \leq L$, so PC cannot be satisfied because $L < r$. As for part (b), suppose by contradiction that $s_H \leq r$. Then, PC reads $ps_H + (1 - p)[b(c_L - \mu) + (1 - b)s_L] = r$. But, by feasibility, $c_L \leq L$ and $s_L \leq L$, so PC is not satisfied.

Claim 2: Suppose by contradiction that IC is slack: $s_H < bH + (1-b)s_L$. PC reads $ps_H + (1-p)[b(c_L - \mu) + (1-b)L)] = r$. There exists another contract $s'_H = s_H + \epsilon$ and $b' = b - \delta$, where $\epsilon = \frac{(1-p)\delta((c_L-\mu)-s_L)}{p}$ and $\delta > 0$. For δ small enough, the new contract satisfies IC, while PC holds by the construction of ϵ as a function of δ. As a result, we obtain a new contract with $b' < b$. This reduction in b is feasible due to Claim 1. So, the deadweight losses decrease and we reach a contradiction.

Claim 3: First, we will consider the case when $b < 1$. Plugging IC into PC yields: $b(pc_H + (1-p)(c_L - \mu)) + (1-b)s_L = r$. This expression determines the set of feasible contracts. Suppose that at the optimal contract $s_L < L$, then we can propose a new contract $s'_L = s_L - \epsilon$ and $b' = b - \delta$, where $\delta > 0$ and $\epsilon = \frac{(pc_H+(1-p)(c_L-\mu))\delta}{1-(b-\delta)} > 0$. The condition that guarantees that $\epsilon > 0$ is given by: $pc_H + (1-p)(c_L - \mu) > r$ (as required by feasibility). The repayment in the high state is $s'_H = b'c_H + (1-b')s'_L$. The new contract achieves lower verification probability, reducing the deadweight losses. So, s_L cannot be less than L.

If $b = 1$, we have $s_H = H$ and $pc_H + (1-p)(c_L - \mu) = r$. As s_L is off-equilibrium, we can set it equal to L without loss of generality. $\square$

2.10.3 Proof of Lemma 3.

Proof. Claim 1: The proof is identical to the one for the commitment case (see Lemma 1).

Claim 2: If the inequality does not hold, there will not exist a feasible c_L that gives a weakly positive payoff to the creditor upon verification, and, because $s_L \geq 0$, the VC cannot be satisfied by any feasible pair (s_L, c_L).

Claim 3: We know that $b > 0$ is needed for the investment to take place, so VC reads $s_L \leq c - \mu$, with equality whenever $b \in (0,1)$. We are left with one case: $b = 1$ and $s_L < c - \mu$. We know from the Proposition 1 that $b = 1$ is optimal if and only if $pH + (1-p)(L - \mu) = r$. This contract is feasible also in the case of limited commitment, because we know that $L \geq \mu$. Now, we consider the case when $pH + (1-p)(L-\mu) > r$, $b = 1$ and VC reads $s_L < c_L - \mu$. First, from PC either $c_H < H$ or $c_L < L$. Suppose that $c_L < L$ and consider a deviation to the new contract where $b = 1 - \epsilon$, $c'_L = c_L + \delta$, $s'_L = c_L + \delta - \mu$ and $s'_H = (1-\epsilon)c_H + \epsilon s'_L$. For every $\epsilon > 0$ there exists $\delta = \frac{\epsilon p(c_H - s'_L)}{(1-p)} > 0$, such that

PC holds. The last inequality comes from the fact that at any feasible contract $c_H > r$. From the definitions of s'_H and s'_L, both IC and VC hold, and VC holds with equality. So, we found a new contract where b is lower and VC binds. The same reasoning holds if $c_H < H$: $b' = 1 - \epsilon$, $c'_H = c_H + \epsilon(c_H - c_L + \mu)$, $s'_H = s_H$ and $s'_L = c_L - \mu$. $\square$

2.10.4 Proof of Lemma 5.

Proof. Claim 1: We proceed by contradiction. If IC does not bind, then we must have $d = 0$. We have to consider two cases, depending on whether or not VC binds. First, suppose it binds. The contract has to satisfy the following set of constraints:

$$s_H < bc_H + (1-b)s_L, \tag{IC}$$

$$s_L = c_L - \mu, \tag{VC}$$

$$ps_H + (1-p)\big[b(c_L - \mu) + (1-b)s_L\big] = r \tag{PC}$$

Consider now the following deviation: $s'_H = s_H$, $s'_L = s_L$, $c'_L = c_L$, $c'_H = c_H$, $b' = b - \delta$. By the Archimedian property of real numbers and the fact that in the original contract $b > 0$, $\exists \delta > 0$ such that IC is satisfied (strictly) for some $b' > 0$. Both VC and PC are unchanged. So, the new contract lowers the deadweight losses, leading to contradiction.

As for the second case, suppose that VC is not binding. Now, we have $b = 1$ and the contract has to satisfy:

$$s_H < c_H, \tag{IC}$$

$$s_L < c_L - \mu, \tag{VC}$$

$$ps_H + (1-p)\big[c_L - \mu\big] = r \tag{PC}$$

Consider now the following deviation: $s'_H = s_H - \varepsilon$, $s'_L = s_L$, $c'_L = c_L$, $c'_H = c_H$, $b' = 1 - \delta$, where $\varepsilon = \frac{\delta(1-p)(c_L - \mu - s_L)}{p} > 0$. By the Archimedian property and the fact that in original contract $b > 0$ and $s_H < H$, $\exists \delta > 0$ at which the new contract satisfies PC, VC and IC. As in the previous case, the new contract lowers the deadweight losses, reaching a contradiction. So, we conclude that at any optimal contract IC must be binding.

Claim 2: By contradiction, consider a contract where $c_H \leq s_L$ and IC binds. From IC we know that $s_H = bc_H + (1-b)s_L$. It follows that $s_H \in [0, s_L]$. The PC constraint after plugging IC reads: $p\big[(1-d)s_H + d\big(b(c_H-\mu)+(1-b)s_L\big)\big] + (1-p)\big[b(c_L-\mu)+(1-b)s_L\big] = r$. The LHS of this equation is at most equal to L, whereas the RHS is equal to $r > L$. So, there does not exist $b \in [0,1]$, such that PC holds. We conclude that there does not exist a feasible contract where $c_H \leq s_L$.

Claim 3: Suppose by contradiction that VC is slack. By Claim 1, we know that IC binds. Moreover, we restrict attention to $d > 0$ because the case of $d = 0$ was considered in the previous section. If VC is slack, then we must have that $b = 1$ and any feasible contract must satisfy the following set of constraints:

$$s_H = c_H, \tag{IC}$$

$$s_L < p(d)c_H + (1-p(d))c_L - \mu, \tag{VC}$$

$$p\big[(1-d)s_H + d\big(c_H-\mu\big)\big] + (1-p)\big[c_L-\mu\big] = r \tag{PC}$$

Consider now the following deviation: $s'_H = s_H - \delta$, $s'_L = s_L$, $c'_L = c_L$, $c'_H = c_H - \delta$, $b' = 1$, $d' = d - \varepsilon$, where $\delta = \mu\varepsilon$. By the Archimedian property of real numbers and the fact that in the original contract $c_H > L$, $\exists\varepsilon > 0$ such that VC holds as strict inequality, PC is unchanged because of the way δ was defined, and IC also holds. As at the original contract $d > 0$, such a deviation is feasible and it reduces the deadweight losses, reaching a contradiction and proving that VC binds.

Claim 4: From the previous analysis, we established that at the optimal contract IC and VC both bind. Moreover, because the case of $d = 0$ was covered in the previous section, we restrict attention to $d > 0$ here. Once again, we prove the claim by contradiction. Suppose that $c_L < L$. VC reads: $s_L = p(d)c_H + (1-p(d))c_L - \mu$ and combining PC with IC we get: $b = \frac{r-s_L}{p(1-d)(c_H-s_L)}$. Consider now the following deviation: $s'_H = b'c_H + (1-b')s_L$, $s'_L = s_L$, $c'_L = c_L + \varepsilon$, $c'_H = c_H$, $b' = \frac{r-s_L}{p(1-d')(c_H-s_L)}$, $d' = d - \delta$, where $\delta = \frac{\varepsilon(1-p)}{p(c_H-s_L-\mu)}$. From the Archimedian property of real numbers and Claim 1, $\exists\varepsilon > 0$ such that $\delta > 0$, IC is satisfied, VC is unchanged because of our definition of δ, and PC holds because $b' < b$. As we just found a feasible contract where $b' < b$ and $d' < d$, the deadweight losses must be lower than at the original contract we started, reaching a contradiction.

As for the case of $c_H < H$, the logic is very similar. The deviation from d consists in subtracting $\delta = \frac{\varepsilon}{(c_H - s_L - \mu + \varepsilon)}$, which is positive number given that $\varepsilon > 0$. In addition, $c'_H = c_H + \varepsilon$ and $b' = \frac{r - s_L}{p(1-d')(c'_H - s_L)} < \frac{r - s_L}{p(1-d)(c_H - s_L)} = b$. The objective function at the new contract is strictly less than the contract we started from, reaching a contradiction.

Claim 5: To derive the necessity of this condition, one should look at VC: $s_L \leq p(d)c_H + (1 - p(d))c_L - \mu$, with equality whenever $b \in (0, 1)$. The RHS reaches a maximum at the point $c_H = H$, $c_L = L$, $d = 1$ and it is equal to: $p(H - L) + L - \mu$. If this expression is strictly less than zero, then no feasible contract can induce the investments in the project. □

2.10.5 Additional details of Proposition 2

The objective function is a continuous function everywhere where it is defined:

$$DWL(d) = \frac{r - (p(d)H + (1 - p(d))L - \mu)}{p(1 - d)(H - (p(d)H + (1 - p(d))L - \mu))}(1 - p + pd)$$

As can be seen from the denominator of the expression above, the graph is unbounded at two points: $d = 1$ and $d = \frac{pH + (1-p)L - (1-p)\mu - H}{p\mu} < 0$. There is only one non-negative point where the graph can reach zero:

$$d_0 = \frac{pH + (1 - p)L - (1 - p)\mu - r}{p\mu}$$

We are interested in how the graph behaves when we approach to 1 from the left. As there is only one non-negative root of the DWL function and we know that $d = 1$ is the only non-negative vertical asymptote, it follows that if the root $d_0 > 1$, then $\lim_{d \to 1-0} b(d)(1 - p + pd) = +\infty$. If $d_0 < 1$, then $\lim_{d \to 1-0} b(d)(1 - p + pd) = -\infty$. Finally, if $d_0 = 1$, then $\lim_{d \to 1-0} b(d)(1 - p + pd) = 0$, because the quadratic term in the numerator vanishes faster than the linear term in denominator.

The derivative of the DWL function with respect to d is given by the expression below:

$$DWL'(d) = \frac{\begin{array}{r}(\mu + (-1+d)\mu p)^2 + \mu(1 + (-1+d)p)(r + (-1+d)pr - (-1+p+dp)H + 2(-1+p)L)+ \\ + (-1+p)(H-L)((-1+(-1+d)^2p^2)r + L - p((-1+(-1+d)^2p)H + L))\end{array}}{(1-d)^2 p(\mu + (-1+d)\mu p - (-1+p)(H-L))^2}$$

The numerator of the expression is a quadratic function of d, which can have two roots:

$$d_1 = \frac{((-1+p)p(\mu^2 + \mu(r-L) + p(r-H)(H-L)) - \sqrt{(-1+p)^2p^2(r-H)(H-L)^2(\mu + r - pH + (-1+p)L)}}{(p^2(\mu^2 + \mu(r-H) + (-1+p)(r-H)(H-L)))}$$

$$d_2 = \frac{((-1+p)p(\mu^2 + \mu(r-L) + p(r-H)(H-L)) + \sqrt{(-1+p)^2p^2(r-H)(H-L)^2(\mu + r - pH + (-1+p)L)}}{(p^2(\mu^2 + \mu(r-H) + (-1+p)(r-H)(H-L)))}$$

The discriminant of the numerator of the DWL' is non-negative if and only if $pH + (1-p)L \geq \mu + r$. If the discriminant is strictly negative, then the graph of the DWL function is always increasing in the $[0, 1)$ range. If the discriminant is non-negative, then it is possible to have at most two local extrema of our graph. If d_2 exists, then it is easy to show that it lies outside (0,1) range.

Next Lemma will determine which type of extremum is the point d_1, conditional on lying in the $(0, 1)$ range.

2.10.6 Type of Local Extremum

Lemma 9. *If there is a local extremum of the DWL function d^* in $(0, 1)$ range then it must be a local maximum.*

Proof. If a local extremum of the DWL function exists, then the derivative of the DWL function at that point should exist and it should be zero. This happens only in a specific range of parameters, which satisfy the following inequality: $pH + (1-p)L \geq \mu + r$, this corresponds to the case when the discriminant of the numerator of the DWL' is non-negative. Then, we know that $\lim_{d \to 1-0} b(d)(1 - p + pd) = -\infty$ if $pH + (1-p)L > \mu + r$ or $\lim_{d \to 1-0} b(d)(1 - p + pd) = 0$ if $pH + (1-p)L = 0$. The second limit follows from the fact that a quadratic function goes to zero faster than a linear function. If $pH + (1-p)L > \mu + r$, then d^* cannot be local minimum, because otherwise there should exist a point x

($d^* < x < 1$), such that x is local maximum. This contradicts the fact that there can be only one local extremum point in the $(0,1)$ range. If $pH+(1-p)L = \mu+r$, then $DWL(d^*) > 0$. If $DWL(d^*) \leq 0$, then there is no feasible contract available (because the DWL function has only one non-negative root), which contradicts the investment conditions derived in Proposition 1. □

2.10.7 Proof of Lemma 7.

Proof. Claim 1: Suppose not, i.e. $s_L < c_L - \mu$. It follows that we must have $b_L = 1$ Consider deviating to a contract where $s'_L = c_L - \mu$ and keep $b'_L = 1$. All other constraints remain unchanged, and so is the deadweight loss function, proving the claim.

Claim 2: Suppose not, i.e. $s_H > c_H - \mu$. There exists another payment $s'_H = c_H - \mu - \epsilon$ such that, upon receiving s'_H the creditor still does not verify. So, the deviation is profitable for the borrower, proving the claim.

Claim 3: This follows immediately from Claim 2: because in the high state the creditor receives exactly $c_H - \mu$, if this quantity is less than $L < r$ then PC cannot hold.

Claim 4: If $b_M = 1$, there cannot be strategic default (i.e., $d_H = 0$), because by defaulting the high-state borrower is always verified, and so repays $c_H > c_H - \mu = s_H$, which is the repayment under truth-telling. But if $d_H = 0$, then we must have $s_M = s_L = c_L - \mu$ and there can only be a separating equilibrium, proving the claim.

Claim 5: Suppose by contradiction that VC_0 is slack, i.e. $s_M < p(d)c_H + (1-p(d))c_L - \mu$. Then, it must be that $b_M = 1$, which by Claim 4 implies that $s_M = c_L - \mu = p(d)c_H + (1-p(d))c_L - \mu$, reaching a contradiction.

Claim 6: Suppose that $d_L = 1$, in which case it could be that $b_L c_L + (1 - b_L)s_L < b_M c_L + (1 - b_M)s_M$. As b_L and s_L are off-equilibrium, we can set them so that the constraint holds with equality without loss of generality. Of course, if $d_L = 0$ then we have a separating equilibrium and b_M can be set arbitrarily so that the constraint holds with equality again, proving the claim.

Claim 7: First, observe that s_M is a convex combination of s_L and $c_H - \mu > L \geq s_L$, which implies that $s_M \geq s_L$. It follows that $b_L \geq b_M$. In addition, when $d_H > 0$, we have $s_M > s_L$ which implies that $b_L > b_M$.

Claim 8: Recall that by Claim 6, IC_L is binding. Define the function $T := b_L c_L + (1 - b_L)s_L - b_M c_L - (1 - b_M)s_M = 0$, where the equality follows from Claim 6. Taking the derivative with respect to c_L yields: $\partial T/\partial c_L = b_L - b_M > 0$. Plugging s_H from the binding $IC_{h,1}$ into $IC_{H,2}$, and observing that $c_H - \mu > L \geq c_L$, proves the claim. This claim proves that there is no equilibrium when the High type of borrower strategically pools upon s_L.

Claim 9: Suppose not, i.e. $s_H > b_M c_H + (1 - b_M)s_M$. It follows that $d_H = 1$, and therefore for financing to take place it must be that $p(d)c_H + (1 - p(d))c_L - \mu > L$. As a consequence, $b_M = 1$, which implies that $d_H = 0$: a contradiction.

Claim 10: Suppose not, i.e. $s_H = c_H - \mu < b_M c_H + (1 - b_M)s_M$. Then we must have a separating equilibrium where $d_L = d_H = 0$ and $s_M = s_L = c_L - \mu$. In addition, PC reads: $p(c_H - \mu) + (1 - p)(b_L(c_L - \mu) + (1 - b_L)s_L) = r$. Consider deviating to $b'_L = b_L - \epsilon$, such that $IC_{H,1}$ still holds. As $c_L - \mu = s_L$, PC does not change, proving our claim. $\square$

2.10.8 Contract space without commitment

In this part of the appendix, we will discuss the set of feasible contracts in the non commitment section. We will show that two repayment levels equilibrium will be without loss of generality $\{s_L, s_H\}$.

Imagine there are three levels of repayments, where s_M is a new repayment level. Following Claim 8 of Lemma 7, we will consider the case when s_L is proposed only by the Low type. We will consider the case when s_M is a pooling repayment which is chosen (if s_M repayment was separating, then this case is trivial) by both types with probabilities d_H and d_L.

We will propose a new equilibrium where both the creditor and the borrower will be weakly better off. Consider a deviation from the three repayment equilibrium, so that both type of the borrower now put the weights d_H and d_L to repayments s_H and s_L. IC and VC will be intact, so the incentive constraints will be satisfied. Now, the creditor will additionally collect $pd_H(c_H - \mu)$ from the High type and $(1 - p)d_L(c_L - \mu)$ from the Low type, but will not collect $((1-p)d_L + pd_H)(p(d)(c_H - \mu) + (1 - p(d))(c_L - \mu))$ from the pooling repayment s_M. This quantities are the same, because $p(d) = \frac{pd_H}{(1-p)d_L + pd_H}$. We showed that the creditor will collect the same amount of money in the new equilibrium.

Lemma 8 shows that if the borrower will redistribute the default weights d_H and d_L to states H and L, then the value of the DWL function will be lower. This proves that for any three repayment equilibrium exists a two repayment equilibrium where the DWL is lower. The proof can be generalized to more than three repayments by induction.

2.10.9 Proof of Lemma 8.

Proof. From Lemma 7, we can conclude that all constraints in the problem of the borrower except $IC_{H,2}$ hold as equalities. This will allow us to express the verification probabilities b_M and b_L.

$$b_M = \frac{(c_H - r - \mu)d_L}{c_H d_L - r d_L + d_H \mu p - d_L \mu p}$$

$$b_L = \frac{(c_H - r - \mu)(d_L(1-p) + d_H p)}{(1-p)(d_L(c_H - r) + \mu p(d_H - d_L))}$$

So, DWL function will look next:

$$DWL = (1-p)(1-d_L)b_L(d_L, d_H) + ((1-p)d_L + p d_H)b_M(d_L, d_H)$$

The derivative of the DWL function is positive with respect to d_H for any level of d_L, so the optimal level of d_H should be zero. This suggests that it is in the interest of the borrower not to have the equilibrium repayment s_M. This Lemma shows the incentives of the borrower to separate the types when it is possible($L \geq \mu$) . □

2.10.10 Proof of Proposition 4.

Proof. We will rewrite IC, VC and PC constraints in the next form:

$$c_H = c_L + \frac{b(1-p+pd)}{(1-b)(1-p)}\mu$$

$$s_L = c_L + \frac{pbd - (1-p)(1-b)}{(1-b)(1-p)}\mu$$

$$b = \frac{(1-p)(r - c_L + \mu)}{(1-p)(r - c_L + \mu) + p(1-p+pd)\mu}$$

First, we will solve the relaxed problem, when we do not take into account the limited liability constraint on s_L. The DWL function will be next:

$$b(c_L, d)(1-p+pd) = \frac{(1-p)(r-c_L+\mu)}{(1-p)(r-c_L+\mu)+p(1-p+pd)\mu}(1-p+pd)$$

The derivative of the above function with respect to d is positive and is negative with respect to c_L. So, it is optimal to set $d = 0$ and $c_L = L$. Resulting c_H will be feasible because of the investment condition. And resulting s_L will be in the feasible range iff $L \geq \mu$. So, we have a solution for the parameter region $L \geq \mu$.

Next, we consider the parameter region $L < \mu$. First, we consider the case when c_H is binding from above(it cannot be binding from below, otherwise it contradicts the investment condition). We will have next DWL function:

$$b(d)(1-p+pd) = \frac{H-\mu-r}{H-r-\mu p(1-d)}$$

As in the previous case, the derivative with respect to d will be negative, so we can set d as low as s_L becomes binding. The condition that guarantees that the resulting d is feasible is given by $p \geq \frac{pH-r}{\mu}$.

The final case we consider is when s_L is binding from below. We will have next expression for $b = \frac{r}{r+\mu p(1-d)}$. Also, in this case the derivative of DWL with respect to d is negative, so the minimum possible level we can set d is such that the limited liability constraint on c_L is binding. The condition that guarantees the feasibility of c_H is $p < \frac{pH-r}{\mu}$. $\square$

3 The Third Chapter: Communication Mechanisms in Competition

3.1 Introduction

Imagine two firms competing with each other for the right to finance a risky project. Consider also the third party which cares about the efficiency of such investments and has only the information channel of interaction with the players. If one competitor is better informed about the project than the other one, would it be possible for the third party to persuade a better informed competitor to share part of his information about the project with a less informed one in order to reduce the potential losses from inefficient investments? This paper studies communication mechanisms that can be used in order to facilitate an information flow between asymmetrically informed competitors in the static setting for the sake of alleviating the problem of inefficient investments.

Information in markets is distributed unevenly - some market players may have a better access to information sources than others. The asymmetric distribution of information may create the problem of inefficient distribution of resources.[24] There are various agencies in the markets which try to tackle the inefficiency problems caused by the asymmetric distribution of information. The role of these agencies is a facilitation of information exchange among the market players. Some of these agencies are private and they sell their services to the market players. In some countries, governments take an active role in the regulation of the information exchange among market players.[25]

Generally, market agencies incentivise market players to reveal their private information either by the pricing policy or by the regulation mechanism of the state. I do not consider any mechanism which is based on the pricing policy. In addition, I also do not consider any regulatory restrictions that oblige market players to report their information.

In terms of dynamics, there can be different incentives of market players to have information communication. The most important one is an information

[24]See Akerlof (1970), Spence (1973)

[25]The examples of private agencies are credit bureaus Equifax, Experian and TransUnion in the American financial market. In Poland, Iran and other countries the market agencies that collect and disseminate information are public.

exchange incentive: a market player shares information with other players expecting that in the future they will share their information with him if they will be better informed. However, if there is a short-term investor who is inclined to a specific project, then the information exchange incentive is absent. This leads to another feature of the model - the static framework allows to abstract from such an obvious incentive as the information exchange one.

In the model, I consider two market players who compete for the right to finance a risky project which requires fixed indivisible investment costs. One of the players is better informed about the outcome of the project than the other. Both players simultaneously announce their proposed shares in the project and the lowest one wins the right to finance it. With some probability the project will fail and the player who invested in the project will get nothing. In this setting the problem of inefficient investments will arise - a less informed player will invest in the project with a positive probability even in the case when a more informed player gets a signal indicating that the project will fail.

In order to alleviate the problem of inefficient investments, I propose several classes of communication mechanisms which allow information to flow from the informed player to the uninformed one. I consider a mediator who collects private information from the player on a voluntarily basis and commits to send messages to the players based on the information collected. The objective of the mediator is efficiency - he chooses among communication mechanisms to minimise the level of inefficient investments by the uninformed player. If the mediator chooses a mechanism which induces the informed player to reveal his information, then there will be a transmission of information which may lead the uninformed player to take better decisions given new information.

The general approach to this problem leads to the revelation principle. Following its logic, the mediator collects private information from the players in an incentive compatible way and the players optimally choose the actions recommended by the mediator.[26] This approach brings the issue of intractability. The main reason is the fact that the strategy space of each player is a continuum set. This fact creates a continuum of incentive compatibility constraints which make the problem so hard. Instead, I approach the problem with a more restrictive

[26]Myerson (1982): Without loss of generality, one can concentrate on this set of communication mechanisms.

but tractable perspective: I consider subclasses of communication mechanisms with finite message sets. So, the mediator will send the players a signal about the outcome of the project rather than recommending which action to take. This subclass of mechanisms seems natural.[27] Examining the structure of these subclasses is a step towards the understanding of the structure of the general communication mechanisms set.

The first set of results of the paper asserts the informed player would never reveal his private information if the mediator commits to send positive amount of information to the uninformed player. In other words, the equilibrium effect of an information transmission to the uninformed player works against the informed player. More information makes the uninformed player to bid more aggressively and this negatively affects the informed player's utility. Whether information transmitted to the uninformed player is private or public will not matter for the result.

In the second part of the paper, I analyse the set of communication mechanisms where the mediator privately sends messages to both of the players. The message to the informed player signals the type of the uninformed player. This signal is the main tool which allows the mediator to incentivise the informed player to reveal the private information. The intuition behind this result is next: if the informed player knows that the project is not profitable, than sharing information about it will not change his profits, since in any case he will not participate in the bidding. On the other hand, if the project is profitable, than lying about it will give the informed player less information about a bidding behaviour of the uninformed player. So, the informed player trades his information about the profitability of the project for the information about the bidding behaviour of his competitor. In turn, more information about the uninformed player leads to a weaker competition for the project which leads to higher utility for the informed agent.

Another result of the second part of the paper gives the minimum bound on the cardinality of the message set the mediator uses for the informed player. If it is less than this bound, a message to the informed agent will not be informative enough in the equilibrium to incentivise him to reveal private information. It is

[27] Obviously, a recommended action is also a signal about the outcome of the project.

the bound on the informativeness of the signal to the informed player.

The third part of the paper analyses the set of communication mechanisms where a message to the uninformed player is private and is public to the informed player. When the message to the informed player is public, the mediator will employ a different tool for the punishment of the informed player in the case of lying.[28] Due to the public nature of the message to the informed player, the mediator by sending such a message changes the belief of the uninformed player about the outcome of the project. So, the incentives of the informed player to misreport his private information will depend on the behaviour of the uninformed player after this deviation. More aggressive behaviour of the uninformed player gives additional incentives to the informed agent to reveal private information truthfully. If the informed agent will decide to lie, the mediator will send a public message signaling the uninformed player about a good outcome of the project. In turn, this induces the uninformed player to bid more aggressively and reduces the utility of the informed player.

If one wants an information revelation from the informed agent under an informative reporting policy [29] then either the cardinality of the informed agent's message space must be increased or messages to the informed player must have a public component.

The last part of the paper considers different extensions of the model. I prove that the problem of inefficient investments is not a first price auction specific. Also, I discuss the case of verifiable messages and prove that the mediator will not play any role.

Related Literature. The paper is related to two different strands of literature: auctions with asymmetrically informed bidders and information design. The first strand of literature was started by Engelbrecht-Wiggans, Milgrom, and Weber (1983), they showed that, in equilibrium, an uninformed bidder would get a zero expected profit and a perfectly informed player would get a positive profit in a first price sealed-bid auction. In my setting with the mediator, there will exist cases where an informed bidder will have a second order uncertainty about an uninformed bidder.[30] I show the conditions under which the uninformed

[28]When the informed player misreports private information.

[29]To differentiate from the uninformative policy of the mediator, where an information revelation comes for free.

[30]This happens in the case of private messages, when the uninformed player gets some

player will get a positive profit in equilibrium. Engelbrecht-Wiggans and Weber (1983) consider common-value sequential auction between a perfectly informed bidder and an uninformed bidder. They find that in equilibrium, the uninformed bidder's pay-off exceeds the informed bidder's pay-off when the horizon is long enough. In the static setting I consider, the equilibrium payoff of an informed bidder always exceeds the one of uninformed.

The paper is also related to the information design literature. The paper of Kamenica and Gentzkow (2004) studies a single agent persuasion problem. In my paper, there are two players to persuade, so there is a need to track higher order beliefs of the players. The paper of Bergemann and Morris (2019) examines a general problem of information design in games. They do not consider a problem of information design in the first price auction setting. The paper of Bergemann, Brooks, and Morris (2017) considers various information structures in a first price auction and characterise the lowest winning-bid distribution. In my paper, I am charachterising optimal information structures from the viewpoint of efficiency.

3.2 Framework

3.2.1 Environment

I consider three risk-neutral players - an informed player (IP), an uninformed player (UIP) and an entrepreneur. The entrepreneur has a project with an uncertain outcome which requires fixed indivisible amount of investments I. There are two possible outcomes of the project $\theta \in \Theta = \{0, 1\}$ [31]. The entrepreneur does not know the outcome of the project ex ante, as well as the players. I denote by p_0 a prior probability the outcome of the project is 1. The probability p_0 is common knowledge.

The IP has an informational advantage: he gets an informative signal about the outcome of the project. The signal is a function $s : \Theta \to \Delta\{0, 1\}$. It is correlated with the state θ. I denote by $s(\theta)$ the probability of a signal realisation equal to 1 given the state.

information about the profitability of the project.

[31] I will also use the term state to denote the outcome of the project.

$$s(1) := p(s = 1|\theta = 1)$$

$$s(0) := p(s = 1|\theta = 0)$$

The condition which shows that the signal is informative:

$$s(1) > s(0)$$

The signal realisation is private and is only observable by the informed player. The structure of the signal is common knowledge.

Since the entrepreneur does not have opportunity to finance the project himself, he runs a first price sealed-bid auction for the right to finance the project. Each player bids his proposed share in the project. The lower bid wins the auction and gives the right to claim corresponding share in the outcome of the project. The players may choose not to participate in the auction. I call this situation as proposing an empty contract. Non-participation of a given player is not observable by the other player at the moment of bidding.

The timing of the game:

1. The informed player observes a signal realisation s about the outcome of the project.

2. The players simultaneously bid α_i where $i \in \{I, UI\}$. The bid may be an empty contract.

3. The winner invests in the project.

4. The outcome of the project is realised and the winner gets his own share in the outcome.

This game will be denoted as the Investment Game.

3.2.2 Analysis of the Investment Game

The posterior beliefs of the informed player about the state after observing the signal realisation equal to 1 or 0 will be denoted:

$$p'(1) := p(\theta = 1|s = 1) = \frac{s(1)p_0}{s(1)p_0 + s(0)(1 - p_0)}$$

$$p'(0) := p(\theta = 1|s = 0) = \frac{(1 - s(1))p_0}{(1 - s(1))p_0 + (1 - s(0))(1 - p_0)}$$

I denote cumulative distribution functions of the players by $F_I(x)$ and $F_{UI}(x)$ when they use mixed strategies in equilibrium.

The equilibrium characterisation of the Investment Game is presented below.[32]

Claim 1. ***The equilibrium characterisation of the Investment Game***

1) If $p_0 < I$ and $p'(1) < I$, in equilibrium the project is not financed.

2) If $p_0 < I$ and $p'(1) \geq I$, in equilibrium the project is financed only by the informed player after the signal realisation $s = 1$. The winning bid is $\alpha_I = 1$. The uninformed player does not participate in the auction.

3) If $p_0 \geq I$ and $p'(0) < I$, there exists the unique equilibrium.

If the signal realisation is 0, the informed player bids an empty contract and if the signal realisation is 1, he mixes in the interval $[\frac{I}{p_0}, 1]$ according to the next cumulative distribution function:

$$F_I(x) = \begin{cases} \frac{p_0 x - I}{p_0(x - I)}, & \textit{if } x < 1 \\ 1, & x=1 \end{cases} \tag{3.1}$$

The uninformed player bids an empty contract with probability $1 - q$ and with probability q mixes in the interval $[\frac{I}{p_0}$, 1] according to the next cumulative distribution function:

[32]Nash Equilibrium is used as an equilibrium concept.

$$F_{UI}(x) = \frac{p'(1)x - I - u_I}{q(p'(1)x - I)}$$

, where u_I is the expected utility of the informed player after the signal realisation $s = 1$.

4) If $p_0 \geq I$ and $p'(0) \geq I$, in equilibrium the project is financed only by the informed player. The equilibrium bids will be $\alpha_I = \alpha_{UI} = \frac{I}{p_0}$. The entrepreneur always chooses the informed player to finance the project when the bids of the players are equal.

The equilibrium characterisation of the Investment Game and the argument about the uniqueness of equilibrium are in the Appendix. The important point in the above proposition is the problem of inefficient investments, which occurs only in the third case. In this case, the UIP will invest in an unprofitable project with a positive probability after the IP gets the signal realisation $s = 0$. If the signal realisation was public, then this case would never occur. From this point, I will concentrate only on the parameter region 3, since it is of primary interest.

3.3 Communication

3.3.1 Communication Mechanisms

In this section, there will be definitions of communication mechanisms - systems which allow for an incentive compatible communication among the players. Informally, one can interpret a communication mechanism as a mediator who collects private information from the players on a voluntary basis and can commit to send messages to the players as a function of the collected information. In the paper, I will frequently use the mediator as an interpretation of communication mechanisms.

First, following Myerson (1982), I will formally define a communication mechanism (system) and then I will restrict attention to some subclasses of communication mechanisms.

Definition 3. *$(M, A, p(a|m))$ is a communication mechanism (system) of the Investment Game, where $M = (M_I, M_{UI})$ are sets from which the players send messages, $A = (A_I, A_{UI})$ are sets from which the players get messages and*

$p(a|m) : M \to \Delta A$ is a rule which links each pair of messages (m_I, m_{UI}) to the conditional distribution function $p(a_I, a_{UI}|m_I, m_{UI})$.[33]

With the use of communication mechanisms, the original Investment Game will change, since the strategy space of the players will now include the message sets (M_I, M_{UI}) and the reporting function $p(a|m)$ may change the first and higher order beliefs of the players. I will call this new game as an Extended Investment Game. Obviously, the structure of the game will be influenced by the choice of a communication mechanism. Next, I consider the set of direct communication mechanisms and the revelation principle.

Definition 4. *A direct communication mechanisms is a communication mechanism such that $M = (S_I)$, $A = (\mathcal{A}_{\mathcal{I}}, \mathcal{A}_{\mathcal{UI}})$, where S_I is a set of types of the informed player and $\mathcal{A}_{\mathcal{I}}$ and $\mathcal{A}_{\mathcal{UI}}$ are action sets of the players.*

Claim 2. ***(Myerson(1982))***
It is without loss of generality to consider only the set of truthful obedient equilibria of the set of direct communication mechanisms. Where a truthful equilibrium $(m_I(s), \alpha)$ is an equilibrium of an Extended Investment Game induced by a direct communication mechanism, where the informed player finds optimal to report his type truthfully $m_I(s) = s$. In addition, an equilibrium is obedient if the players follow the recommendation of the reporting rule $\alpha = p(a|m_I(s))$.

The first constraint imposed by the truthful reporting will be denoted as an incentive compatibility (IC) constraint. The second constraint is an obedience constraint which tells that each player is better off if follows the recommendation of the mediator.

The reporting policy of the mediator $p(a|m)$ can be considered as a correlated signal sent to the players. A recommendation to the player i: $p(a_i|m)$ delivers the information both about the type of the other player and about the action of the other player.

A truthful obedient equilibrium with a corresponding reporting rule in a direct communication mechanism is a Bayesian Correlated Equilibrium. [34] Since, the full characterisation of the set of Bayesian Correlated Equilibria proved to be

[33] I will use the terms "recommendation", "reporting function" and "reporting policy" to denote the rule $p(a|m)$.
[34] See Bergemann and Morris (2019).

very hard task in the setting with continuum action sets, I will consider the set of **simple communication mechanism**, where $M = (S_I)$ and $A = (A_I, A_{UI})$ given that cardinalities of the sets A_I and A_{UI} are the same as the cardinality of the set S_I. This restriction limits the set of available mechanisms, but it gives more tractability and a natural interpretation of the reporting policy $p(a|m)$ as a signal about the state rather than action recommendation.[35] If at least one of cardinalities of the finite sets A_I and A_{UI} is more than the one of S_I, these mechanisms will be called **discrete communication mechanisms**.

A reporting policy of a simple communication mechanism will take the next form $p : (S_I) \to \Delta(A_I \times A_{UI})$. I will use the set $\{G, B\}$ for A_{UI} and $\{L, H\}$ for A_I. Where the message "G" will be linked with the realisation of the signal $s = 1$ and the message "B" with the realisation of the signal $s = 0$.

The following corollary allows to concentrate only on the set of truthful equilibria of simple communication mechanisms.

Corollary 3. ***Myerson(1982)***
It is without loss of generality to ignore the set of non-truthful equilibria $(m(s), \alpha) \in (S, \Delta\mathcal{A})$ *of simple communication mechanisms.*

3.3.2 The Second Best Reporting Policy

The second best reporting policy is a reporting policy of a communication mechanism which maximises the sum of the utilities of the players and the entrepreneur under the constraint that the mediator has the same information about the signal s as the informed player.

Claim 3. *The fully revealing reporting policy* $p(G|s = 1) = 1$ *and* $p(B|s = 0) = 1$ *of a simple communication mechanism is the second best reporting policy.* [36]

Proof. If the mediator is informed as well as the IP, he will use whole his information power to minimise the probability of investment in the project by the UIP after the signal realisation $s = 0$.[37] The fully revealing reporting policy does it perfectly. Once the posterior of the UIP about the state is below the

[35] As was mentioned earlier, a recommended action also has information about the state.
[36] $p(m_{UI}|s)$ denotes a reporting probability to the UIP of the message m_{UI} when a message to the IP is integrated out.
[37] It is easy to show that this problem is equivalent to the maximisation of the sum of the utilities of the players and the entrepreneur.

investment costs I the UIP will bid an empty contract with probability 1. This will drop the probability of inefficient investments by the UIP to the same level as for the IP. $\square$

For the fully revealing reporting policy, the utility of IP will be zero, since the full information revelation of the mediator completely destroys the information rent of the IP.

3.4 Communication Results

3.4.1 One side communication

In this section, I will consider two subsets of simple communication mechanisms: the first is the case when the informed player always gets uninformative messages and the second is when the message to the informed player is perfectly correlated with the message to the uninformed player. The reporting policies of such communication mechanisms will be called pure private and pure public reporting policies since they correspond to the cases of private and public reporting to the UIP.

In the case when the mediator sends messages m_{UI} to the UIP, I will use the reporting policy $p(m_{UI}|s)$. I will consider only consistent reporting policies[38]:

$$p(G|s=1) \geq p(G|s=0)$$

In addition, I will outline the set of informative reporting policies among consistent ones - for which the above inequality holds strictly.

Following Kamenica and Gentzkow (2004), I can represent the reporting policy $p(m_{UI}|s)$ in terms of the distribution of posteriors of the UIP $(p_{UI}(B), p_{UI}(G))$. Each posterior is an updated belief of the UIP about the state θ after a message from the mediator[39]:

$$p_{UI}(m_{UI}) := p(\theta = 1|m_{UI})$$

[38]Concentration on consistent policies is without loss of generality, since if there is a reporting function such that the above inequality does not hold, it is possible to change the labels of the messages "G" and "B" with each other, so the inequality above eventually will hold.

[39]The condition that guarantees the feasibility of a pair of the posteriors is: $(1 - p(G))p_{UI}(B) + p(G)p_{UI}(G) = p_0$, where $p(G)$ is an unconditional probability of sending the message "G" to the UIP by the mediator.

First, I will consider pure public reporting policies. In this case, the IP will perfectly know a message sent to the UIP by the mediator. This corresponds to public simple communication mechanisms where $p(m_I = m_{UI}|m_{UI}) = 1$.

Claim 4. *There does not exist a truthful equilibrium of an Extended Investment Game for any informative public simple communication mechanism.*

The proof in the Appendix is divided into two parts. First, I find all equilibria of an Extended Investment Game for a pure public reporting assuming that the mediator knows as much as the IP.[40] Second, I find the reporting policies which are compatible with incentives of the IP to reveal his information about the signal realisation. This approach will be used also in the subsequent claims.

The intuition behind this result is very simple: since the information flow from the IP to the UIP makes the later bid more aggressively in equilibrium, then the former will never have an incentive to truthfully report the information to the mediator. Any information transmission will decrease the utility levels of the IP.

The second case is a case with pure private reporting to the UIP, when the IP does not observe the message the UIP received from the mediator. This corresponds to private simple communication mechanisms where $p(m_I|G) = p(m_I|B)$.

Claim 5. *There does not exist a truthful equilibrium of an Extended Investment Game for any informative private simple communication mechanism.*

The proof of the above claim is in the Appendix. The equilibrium effect of an informative reporting policy plays against the IP. More information makes the UIP bid even more aggressively than in the previous case, since now the UIP knows that the IP has uncertainty about the message the opponent received. This, in turn, destroys incentives of the IP to share the information with the mediator if the later is committed to the informative reporting policy.

The results of the above claims leads to the consideration of more complicated reporting policies of the mediator. With a reacher mechanism set, there may be a hope to create incentives for the IP to reveal his information.

[40] I call it an equilibrium search part.

3.4.2 General reporting policies: Private messages

Due to the fact that there is no role for the mediator in the cases of pure public and pure private reporting, I will consider the whole set of simple communication mechanisms with a pair of reporting functions $p(m_{UI}|s)$ and $p(m_I|m_{UI}, s)$ to the UIP and to the IP accordingly. A pair of $(p(m_{UI}|s),\ p(m_I|m_{UI}, s))$ will give a joint distribution function which is a general reporting rule of a simple communication mechanism:

$$p(m_I, m_{UI}|s) = p(m_I|m_{UI}, s)p(m_{UI}|s)$$

In this part, I assume that a message sent to the IP will be private[41]. With the private reporting to the IP, there will be different types of the IP from the perspective of the UIP.

Before trying to find equilibria for different reporting policies, it is worth to think about the reporting policies in this setting, namely, the private reporting policy to the IP. I will use the consistency condition for these policies:

$$p(H|m_{UI} = G, s) \geq p(H|m_{UI} = B, s) \qquad \forall s$$

As in the previous section, this is without loss of generality. The definition of the informative reporting policy will now also include the informativeness of the reporting policy to the IP (when the above inequality is strict at least after one signal realisation).

The IP will have a posterior about the message sent to the UIP, which will depend on both a signal realisation and a message received from the mediator.[42]

Next, I am partially characterising the set of truthful equilibria of informative simple communication mechanisms. It is important to understand the structure of this set in order to find necessary conditions that an optimal information reporting policy satisfies.

Claim 6. *In any truthful equilibrium of an informative simple communication*

[41] Obviously, the message to the UIP should also be private, otherwise we are in the case of pure public policies.

[42] In addition, for every signal realisation s, the condition that an expected posterior of a message m_{UI} is equal to the prior should hold: $p(m_{UI}|s) = p(L|s)p(m_{UI}|L, s) + p(H|s)p(m_{UI}|H, s) \quad \forall s$

mechanism:

1) Both types of the IP are bidding non-empty contracts mixing in their bidding sets X_I^H *and* X_I^L*, where* $minX_I^L = maxX_I^H$*. Both types of the IP get positive utility in expectation with the "L" type getting weakly higher utility.*

2) If the "B" type of the UIP is bidding then $minX_{UI}^B = maxX_{UI}^G$*. The "B" type of the UIP will always get zero utility. If the "B" type of the UIP is bidding, then necessarily the "G" type of the UIP is bidding and gets positive utility.*

3) The Low type of the IP will never bid according to the degenerate probability distribution.

The claim gives a list of properties every truthful equilibrium should satisfy given an informative reporting policy. The first part of the claim outlines the fact that after the message "L" the IP will bid more aggressively rather than after "H". It is clear that the posterior of the IP that the message "G" was sent to the UIP will drop after "L", which will make the IP more "optimistic". Surprisingly, if the "B" type of the UIP is bidding, he will bid more aggressively than the "G" type. It is explained by the fact that the marginal rate of substitution between a bid and a probability of winning is higher for the "B" type.

The crucial point for the third part is the same level of expected utility of the different types of the IP. If the IP after the "L" message is bidding according to the degenerate distribution, then from Claim 6, the bid should be $\alpha_I^L = 1$. In this case, the IP gets the same expected utility both after "L" and "H" messages. By misreporting the signal realisation, the IP will boost the probability of the "B" message, which will rise his expected utility.

The interest of the mediator is to minimise the probability of investments in the project by the UIP after the signal realisation $s = 0$. The main constraint that the mediator takes into account is (IC) which tells that the IP has incentives to report the signal realisation truthfully[43]. The other constraints are feasibility and consistency constraints. (IC) is formulated in a general way, since the form of the expected utility function of the IP after the signal realisation $s = 1$ will depend on the concrete reporting policy. The problem of the mediator is given below:

[43]Obviously (IC) is formulated with respect to signal realisation $s = 1$, since if $s = 0$ there is always weak incentives to report truthfully.

$$\min_{p(m_{UI}|s),p(m_I|m_{UI},s)} p(G|s=0)q_G + p(B|s=0)q_B$$

subject to:

$$u_I(m=1|s=1) \geq u_I(m=0|s=1) \tag{IC}$$

$$p(G|s=1) \geq p(G|s=0),\ p(H|m_{UI}=G,s) \geq p(H|m_{UI}=B,s)\ \forall s \tag{CC}$$

$$p(m_{UI}|s) \in [0,1],\ p(m_I|m_{UI},s) \in [0,1] \tag{FC}$$

Next claim finds an optimal reporting policy and proves that there is no role for the mediator.

Claim 7. ***Characterisation of an optimal reporting policy***

1) Exists an optimal reporting policy, where the reporting function to the IP after the signal realisation $s=0$ is uninformative: $p(L|G,s=0) = p(L|B,s=0)$.

2) The uninformative reporting function to the UIP is optimal: $p(G|s=1) = p(G|s=0)$.

The first part of the claim follows from the logic that the RHS of (IC) which is the utility of the IP in the case of misreport of the signal should be as low as possible - it gives the mediator additional freedom in choice of reporting policies. The part of the reporting function $p(m_I|m_{UI}, s=0)$ can be chosen in such a way to maximise the incentives of the IP to report the signal realisation truthfully. The uninformative reporting policy after $s=0$ will do this job, it optimally punishes the IP in the case of misreport.

The second part of the proof is crucial. It tells that the equilibrium forces [44] do not allow the mediator to use the information power fully. It means there does not exist an equilibrium of an Extended Investment Game such that different types of the IP strictly prefer all of their optimal actions to the ones of the other type. For all of the equilibrium types found, it is not the case: both

[44] Namely the property that the mixing sets of the types of the IP X_I^H , X_I^L and $X_I^L \cup X_I^H$ are connected sets.

of the types of the IP have a common optimal action $\alpha_I = t$.[45] In this respect, the equilibrium forces do not allow the mediator to send an enough informative message m_I such that different types of the IP have disjoint optimal actions sets. If different types of the IP find a common optimal action then there is not enough spread between the posteriors $p(G|m_I, s = 1)$ about the type of the UIP. The reason is that for a sufficiently informative message m_I which induces different optimal actions sets for different types of the IP, there will not exist an equilibrium.[46]

3.4.3 A Positive Result: Strategy Space

The impossibility result for the simple communication mechanisms gives a clue how to construct a communication mechanism which will increase an efficiency of investments in the Investment Game.

I consider now discrete communication mechanisms with $M_{UI} = \{B, G\}$ and $M_I = \{L, M, H\}$. The reason why I consider such a mechanism is the possibility that one type of the IP will not have an optimal action which will be also optimal for the other type. The most important point is allowing the IP to utilise the information provided by the mediator. To overcome the equilibrium forces which do not allow to send sufficiently informative messages to the IP, I need to make different types of the IP to have mixing regions which do not coincide even in a single point. Very different mixing regions of different types of the IP will correspond to the better use of the signal m_I by the IP.

The first part of Claim 7 will hold also in this case - the punishment in the case of the misreport by the IP should be an uninformative message m_I:

$$p(m_I|G, s = 0) = p(m_I|B, s = 0)$$

Below, I present an example of an informative reporting policy of a discrete communication mechanism which alleviates the problem of inefficient investments.

[45] t is an equilibrium threshold of different types of the IP. See the Appendix.

[46] If X_I transforms to a non-connected set due to a sufficiently informative m_I, there will be a profitable deviation of the "G" type of the UIP.

An example of an incentive compatible informative reporting policy.

Consider the case of the fully informative signal $p'(1) = 1$ and the parameter values $p_0 = 0.6$ and $I = 0.56$. The reporting policy is given below:

$$p(H|G, s = 1) = 0.5 \quad p(M|G, s = 1) = 0.4$$

$$p(H|B, s = 1) = 0.08 \quad p(M|B, s = 1) = 0.1$$

$$p(G|s = 1) = 0.25 \quad p(G|s = 0) = 0.06$$

This reporting policy supports modified type 4 equilibrium from Claim 6. In this equilibrium "G" type of the UIP bids in $[\gamma, 1]$ and the "B" type bids an empty contract with probability 1. The "L" type of the IP bids unity with probability 1, the "H" type bids in $[\gamma, t]$ and the "M" type in $[t, 1]$. The resulting probability of the inefficient investments $p(G|s = 0) = 0.06$, which is less than the probability of inefficient investments in the absence of the mediator $q = \frac{p_0 - I}{1 - I} = 0.08$.

This example shows a positive role of the mediator in the class of discrete communication mechanisms.

3.4.4 Public messages

The next step in the analysis of the Investment Game is a consideration of public reporting policies which may give the mediator additional tool to increase efficiency in this framework. The reporting rule will have a public component $p(m_I|m_{UI}, s)$. So, the mediator sends a public message m_I to the IP given a private message m_{UI} to the UIP.

There are two possible effects of such reporting policies on the outcome of the Investment Game. With the public reporting policy the posterior of the UIP will change after the mediator sends a message to the IP. This gives less control over the posteriors of the UIP which can play in any direction. The other effect of the public reporting is an equilibrium effect. And it is also not clear where it will drive the outcome of the Investment Game.

From the technical point of view a communication mechanism with a public reporting policy is a discrete communication mechanism. The reason is that the UIP will get two messages: one is a private message from the mediator and the second message is perfectly correlated with the message m_I sent to the IP. So, the cardinality of the M_{UI} grows to 4.

I characterise the set of truthful equilibria of communication mechanisms with public reporting. The first part comes from Claim 5, where instead of $p(m_{UI}|s = 1)$, I have $p(m_{UI}|m_I, s = 1)$, so the IP has additional information about the message sent to the UIP. In addition, the UIP will update the posterior about the state given the message to the IP, since this message may have some information about the signal realisation.

Claim 8. *In any truthful equilibrium in the case of public reporting:*

1) The IP always bids a non-empty contract and gets strictly positive utility levels after the signal realisation $s = 1$. The utility of the IP after "L" is not always higher than after "H".

2) If the "B" type of the UIP is bidding then $minX_I^B = maxX_I^G$. The "B" type of the UIP will always get zero utility. If the "B" type of the UIP is bidding, then necessarily the "G" type of the UIP is bidding and gets positive utility.

3) Both after the "H" and "L" messages, the UIP cannot bid a non-empty bid with probability one.

4) Non-bidding behaviour of the UIP is higher after "H" than after "L": $(1 - q)(H) > (1 - q)(L)$ iff $p(H|s = 1) > p(H|s = 0)$.

All equilibria in this subclass of mechanisms are obtained when $p_{UI}(m_{UI})$ is replaced with $p_{UI}(m_{UI}, m_I)$ and $p(G|s)$ with $p(G|m_I, s)$ in Claim 5. The parametric conditions will stay intact up to m_I. The first two properties of the claim automatically follow from Claim 5. The third and the forth properties are proved in the Appendix.

The essence of the above claim is the fact that the mediator punishes the deviation of the IP from reporting $s = 1$ to $s = 0$ by more aggressive bidding behaviour of the UIP. For example, if the bidding probability of the UIP is higher after "H" message, then, in the case of deviation of the IP, the mediator will try to send message "H" with a higher probability. If the IP decides to misreport the realisation of the signal, he will face the different frequency of the actions of the UIP which will serve as a punishment. It is important to notice that the punishment in the case of a misreport of the signal realisation is not necessarily a less informative reporting rule to the IP.

There exist examples of an incentive compatible informative public reporting policy that alleviate the problem of inefficient investments.

3.4.5 Comparison of public reporting and private reporting

It is worth to think about the difference between private and public reporting policies. The crucial point in the comparison is the mechanism which extract information from the IP. In the case of public reporting, the mediator will punish the IP by choosing a public reporting policy that changes the action distribution of the UIP not in favour of the IP. Since, the probability distribution of the message m_I changes with the report of the IP about the signal realisation, the bidding behaviour of the UIP will be affected by the fact of misreporting. The fact of misreporting may induce the UIP to bid more aggressively and this, in turn, will reduce the utility of the IP. This is an exact mechanism of punishment of the IP in the case of lying in the public reporting case.

In the case of the private reporting, the mechanism of inducing the IP to submit the correct signal realisation is different. The message m_I is observable only by the IP, so less correlation between m_I and m_{UI} in case of misreporting may induce the IP to bid less precisely. In turn, less precise bidding will reduce the rent of the IP. So, in the case of private reporting the way of punishment of the informed player works through the information channel. The incentives to report truthfully (about the profitability of the project) may be explained also in a different way: the informed player will give a piece of his information about the project in exchange of a weaker competition from the uninformed player. This is achieved through a more informative signal to the informed agent about the bidding behavior of the uninformed agent in case the former reports to the mediator that the project is profitable. In case, the project is not profitable, than the informed agent is weakly better off to report the state, since in any case he will not take part in the auction. As a result, a weaker competition which serves as an incentivisation mechanism to share the information will increase the profits of the informed agent.

3.5 Extensions

3.5.1 The Second Price Auction

The inefficiency of resource distribution that arises in the framework may be a first price auction specific. So, there is a hope that in another format of an auction such inefficiency may not be present. In this extension, I analyse what happens in the case of a second price auction.

In the second price auction, the lowest bid wins and the winner gets share of the higher bid. There should be also a specification what happens if a player bids an empty contract. In case one of the bidders bid empty contract, then the other one should get maximum share of 1. In the case both of the players bid empty contract, then the project will not be financed. This is a natural adaptation of the second price auction to the setting.

Claim 9. *If $I < p(s=0)$, then there does not exist an equilibrium where the uninformed player bids an empty contract with certainty.*

Proof. Assume there exists such an equilibrium. Then in this equilibrium, the IP would bid α_I after the signal realisation of $s=1$ and an empty contract after the signal realisation of $s=0$. So the IP would win the auction after the $s=1$ and otherwise the project would not be financed. The UIP may deviate from the strategy and to undercut the IP by bidding $\alpha_I - \epsilon$. In this case, the utility of the UIP would be:

$$u_{UI}(\alpha_I - \epsilon) = p_0(s(1)\alpha_I + (1-s(1))1) + (1-p_0)(s(0)\alpha_I + (1-s(0))1) - I =$$

$$= p(s=1)\alpha_I + p(s=0) - I$$

The condition that guarantees that the above expression is weakly less than zero :

$$\alpha_I \leq \frac{I - p(s=0)}{p(s=1)}$$

Since, the right hand side of the equation can be negative, then such a profitable deviation of the UIP will exists in some cases. So, an equilibrium where the UIP bids an empty contract may not exist. □

Remark: I do not allow the negative bids in this framework. The cost of allowing it is the fact that there will exist equilibria where one of the player is bidding $-\infty$ and wins the project for sure. In such equilibria, the entrepreneur will get zero utility which suggests that he will never choose such a format.

3.5.2 The case of verifiable messages

I consider the case of verifiable messages when the IP gets an evidence about the signal realisation he received. Then, the IP cannot lie about the signal realisation, but he can choose not to report anything to the mediator.

Claim 10. *For the case of verifiable messages, the Second Best Reporting Policy is feasible.*

Proof. In the case when the realisation of signal is equal to 0, the IP has weak incentives to report his signal realisation truthfully, since he is indifferent with respect to action of the other player, and he knows that, in any case, he will not participate in the auction. After the realisation of the signal equal to 1, the IP has a choice of sending an empty message or a truthful report to the mediator. Any choice of the IP will be interpreted as a message equal to the signal realisation $s = 1$. It is the unraveling result. The mediator will get the information about the signal realisation "for free" and will truthfully reveal it to the UIP. The Second Best Reporting Policy will be feasible.

□

As the proof shows, the mediator has the second best reporting policy when the IP is subject to verifiable information. The conflict of interests between the different types of the IP creates a possibility for the truthful information revelation. The result of the claim will be overturned if the mediator does not have a power to choose an equilibrium of the continuation-game starting after the signal realisation $s = 0$. In this case, the result is not robust to the next logic: the IP will think that if after the signal realisation $s = 0$, he would reveal the outcome of the signal, then he would lose a possibility not to reveal the information in the case of the signal realisation $s = 1$. Losing this possibility diminishes the information rent of the IP, since it affects his information advantage. The IP after the realisation of the signal $s = 0$ will choose the equilibrium in which he does not reveal the information to the mediator.

Remark. The same result as in the above claim may be achieved if there is a direct communication between the players and the uninformed player can choose an equilibrium of the continuation-game. So, the role of the mediator is completely determined by his ability to choose an equilibrium of the continuation-game after the signal realisation $s = 0$.

3.6 Conclusion

In this paper, I investigate the role of communication mechanisms in a competitive setting. The emphasis of my analysis is devoted to the information role such mechanisms play in order to alleviate the problem of inefficient distribution of resources due to asymmetric distribution of information.

I investigate several classes of communication mechanisms: one side communication, private reporting and public reporting. The negative results with respect to one side communication and simple communication mechanisms directed me to consider more sophisticated discrete mechanisms which allow incentive compatible communication.

Although my analysis partially characterises an optimal reporting policy, the claims identify the main driving forces of information revelation in private and public reporting. In addition, my analysis revealed the difference between private and public reporting rules. For the private case the information punishment that the mediator uses allow the informed player to share some information with the uninformed player. For the public reporting, the mediator uses a punishment method which forces the uninformed player to bid more aggressively.

The results obtained in this paper inform the future research about the methods one can use to approach questions of information design in competitive environments. Also one will be more informed about the problems and challenges in the analysis of these topics.

3.7 Appendix

3.7.1 Proof of Claim 1

Proof. **1)** $\mathbf{p_0} < \mathbf{I}$ and $\mathbf{p'(1)} < \mathbf{I}$. It is easy to see that the expected revenue from the project is less than the cost of investment I for both of the players, so the decision of non-investment is optimal.

2)$\mathbf{p_0} < \mathbf{I}$ and $\mathbf{p'(1)} \geq \mathbf{I}$. The signal to the informed player is informative enough, so after the realisation $s = 1$, the posterior belief is (weakly) above the investment costs I. In this case the IP knows that he is the only one who will bid a non-empty contract. This allows him to bid the highest possible bid $\alpha_I = 1$. The UIP does not invest for the same reason as in case 1.

3) $\mathbf{p_0} \geq \mathbf{I}$ and $\mathbf{p'(0)} \leq \mathbf{I}$. The UIP's prior is above the investment costs I. There is a rationale for the UIP to bid a non-empty contract.

In equilibrium none of the players will bid according to the pure strategies. If you are bidding according to the pure strategy then necessarily you are the IP and you bid $\alpha_I \leq \frac{I}{p_0}$ in order not to be undercut. But, in this case the UIP will bid en empty contract, which makes the initial bid of the IP suboptimal.

First, I give an equilibrium of the Investment Game. Second, I will sketch a proof why it is the unique equilibrium.

The IP bids a non-empty contract iff he receives the signal realisation $s = 1$. After the signal realisation $s = 1$, the IP mixes in the interval $[\frac{I}{p_0}, 1]$ according to F_I. The UIP bids an empty contract with $1 - q$ and with q mixes in the interval $[\frac{I}{p_0}, 1]$.

The expected utility of the informed player after the signal realisation $s = 1$ will be:

$$U_I(s = 1) = ((1 - q) + q(1 - F_{UI}(\alpha_I)))(p'(1)\alpha_I - I)$$

For any equilibrium action α_I of the IP, his utility should be the same and should be equal to u_I. This condition alongside with the regularity conditions on the probability bounds give the cumulative distribution function and the bidding probability of the UIP with the utility level of the IP.

$$F_{UI}(x) = \frac{p'(1)x - I - \bar{u}_I}{q(p'(1)x - I)}$$

$$q = \frac{p'(1)(p_0 - I)}{p_0(p'(1) - I)}$$

$$u_I = p'(1)\frac{I}{p_0} - I$$

The expected utility function of the UIP will be equal to:

$$U_{UI} = p_0(s(1)(1 - F_I(\alpha_{UI})) + (1 - s(1)))(\alpha_{UI} - I) +$$

$$+(1 - p_0)(s(0)(1 - F_I(\alpha_{UI})) + (1 - s(0)))(-I)$$

In equilibrium, the utility level of the UIP is equal to zero. The zero utility condition will give the cumulative distribution function for the informed player.

$$F_I(x) = \begin{cases} \frac{p_0 x - I}{p_0 s(1)x - I(p_0 s(1) + (1-p_0)s(0))}, & \text{if x < 1} \\ 1, & \text{x=1} \end{cases}$$

The condition $\lim_{x \to 1-0} F_I(x) \leq 1$ follows from the fact $p'(0) \leq I$. This means that the IP puts an atom on the $\alpha_I = 1$.

The cumulative distribution functions of the players constitute a mixed strategy equilibrium of the game.

Uniqueness.

To prove that the above equilibrium is unique, I will need to establish several observations. I denote by X_I and X_{UI} the mixing regions of the players.

To avoid the problem of non-measurable sets and inability to prescribe probabilities to these sets, I concentrate on mixing regions of each players consisting of at most countable number of disjoint open intervals and at most countable number of atomic points.

The first observation states that in any equilibrium $supX_{UI} = supX_I = 1$ and, in addition, the IP will have an atom at point $\alpha_I = 1$ and the UIP does not have an atom at $\alpha_{UI} = 1$.

First, I show that $supX_{UI} = supX_I$. Without loss of generality, assume that $supX_{UI} < supX_I$. It means that either IP has an atom at $supX_I$ or an open interval $(a, supX_I) \subset X_I$ for some $a > supX_{UI}$. There cannot be such an open interval, since the IP would shift any mass from $(supX_{UI}, supX_I)$ to the point $\alpha_I = 1$. It is a profitable deviation of the IP. Imagine that there is a atom at

$supX_I$. In this case, the UIP will have always an incentive to undercut the IP, since moving a mass from $(supX_{UI} - \gamma, supX_{UI}) \subset X_{UI}$ [47] to $supX_I - \epsilon$ for small enough $\epsilon, \gamma > 0$ will be profitable for the UIP. A contradiction.

Second, imagine that the IP does not have an atom at $supX_I$. If the UIP has an atom at $supX_I$, then his expected utility will be negative. The reason is that by bidding $\alpha_{UI} = supX_I$, the UIP will win only after $s = 0$. If the UIP does not have an atom, then for α_{UI} arbitrary close to $supX_I$, the expected utility will be negative for the same reason. This gives us a contradiction. If the UIP has an atom at $\alpha_{UI} = supX_{UI}$, then the IP will always have an incentive to undercut the UIP by bidding $\alpha_I = supX_I - \epsilon$ instead of $\alpha_I = supX_I$. So, it cannot occur in equilibrium.

Third, imagine $supX_{UI} = supX_I < 1$. Then, there will always be a deviation for the IP - he will put his atom at $\alpha_I = 1$ and gain strictly higher utility. A contradiction. The first observation is proved.

The second observation states that in any equilibrium $X_I = X_{UI} \cup \{1\}$. I assume the opposite, so, without loss of generality, exists either an atom $a \in X_I$ and $a \notin X_{UI} \cup \{1\}$ or exists an an interval $(t_1, t_2) \subset X_I$ and $(t_1, t_2) \not\subset X_{UI} \cup \{1\}$.

First, consider the case of an atom $a \in X_I$ and $a \notin X_{UI} \cup \{1\}$. Then, there will exist an element $a + \epsilon \in X_{UI} \cup \{1\}$ for any arbitrary small ϵ, since $a \neq 1$. [48] By moving a weight from $(a, a + \epsilon)$ to $(a, a - \epsilon)$, the UIP will be strictly better off, since for an infinitesimal decrease in the winning bid the increase of the winning probability will be finite and positive.

Second, consider the case of an interval $(t_1, t_2) \subset X_I \setminus \{1\}$ and $(t_1, t_2) \not\subset X_{UI}$. Obviously, there will exist the $b = inf\{x \in X_{UI} | x \geq t_2\}$. this case, the IP will be better off if he puts a probability weight from the (t_1, t_2) to $b - \epsilon$ for $\epsilon > 0$ small enough. In this way, the IP will undercut the UIP and earn additional utility. This proves the second observation.

The third observation states that, in any equilibrium, the mixing regions are connected sets. Without loss of generality, imagine that X_I is not connected, than $\exists (t_1, t_2) \not\subset X_I$ [49] and $b, c \in X_I$ such that $c < t_1 < t_2 < b$. Obviously, $\exists l = inf\{x \in X_I | x \geq t_2\}$ and $\exists m = sup\{x \in X_I | x \leq t_1\}$ such that $m < t_1 <$

[47] Or, an atom from $supX_{UI}$.

[48] If such an element does not exist, then see the second case.

[49] The case when there is only one point instead of an interval is trivial.

$t_2 < l$. Then, there is a rationale for the IP to move a probability weight from $(m-\epsilon, m)$[50] to the point $a-\gamma$ with small enough $\gamma > 0$ and $\epsilon > 0$ for the next reason: with this move the IP will get a higher share in the project with the same probability of winning in the auction.[51] This leads to the fact that the mixing sets of the players are connected.

The forth point is the fact that the only atom in the mixing regions is due to the IP when he bids $\alpha_I = 1$. Imagine that there is an atom $a \in X_I$, then there will be an incentives for the UIP to deviate from all his bids $\alpha_{UI} \in [a, a+\epsilon]$ to the point $a-\gamma$ with small enough $\gamma > 0$. The point where this logic does not work is $\alpha_I = 1$. The other points where the UIP cannot undercut the IP are $\alpha_I \leq \frac{I}{p_0}$, but they will not be played in equilibrium. The same reasoning is true for the UIP. The point $\alpha_{UI} = 1$ was discussed in the observation 1.

The fifth observation is $minX = \frac{I}{p_0}$. Imagine $minX > \frac{I}{p_0}$, then the UIP would get a positive utility and would never bid an empty contract. There will not exist an equilibrium where the UIP is bidding an empty contract with zero probability, since the IP in that case will never bid $\alpha_I = 1$.

These five claims establish the fact that the mixing region of each player should be the set $[\frac{I}{p_0}, 1]$. The equilibrium I found above is the only possible equilibrium in this region.

4) $\mathbf{p_0 \geq I}$ **and** $\mathbf{p'(0) > I}$. The IP will bid a non-empty contract after any signal realisation . In equilibrium, the UIP and the IP after any signal realisation will bid $\frac{I}{p_0}$ and the entrepreneur will always choose the IP to allocate the project.[52] The utility of the informed player after signal realisation s will be $(\frac{p'(s)I}{p_0} - I) > 0$ and zero for the UIP. For the IP there is no sense of decreasing or increasing the bid, since in either case the utility will be less than the one above. For the UIP there will not be any profitable deviations as well. There will not exist an equilibrium in mixed strategies similar to the 3rd part of the proof because $p'(0) > I$, which will make $\lim_{x \to 1-0} F_I(x) > 1$.

□

[50] Or, m if it has an atom.

[51] Here, I am using the previous observation.

[52] Any other allocation of the project will destroy the equilibrium, since it gives the incentives to the IP to undercut the UIP in this case.

3.7.2 Proof of Claim 4

Proof. If the IP observes m_{UI}, then from Claim 1, I know the equilibrium of an Extended Investment Game given a reporting policy $p(m_{UI}|s)$. [53] The equilibrium form and the uniqueness follows from Claim 1, where instead of a prior p_0, there will be a posterior of the UIP $p_{UI}(m_{UI})$. The expected utility of the IP after the signal realisation $s = 1$ will be:

$$U_I = ((1-p(G|s=1))(1-q^B(p_{UI}(B)))+p(G|s=1)(1-q^G(p_{UI}(G))))(p'(1)-I)$$

, where $1 - q_{m_{UI}}$ is the probability the UIP bids an empty contract after the message m_{UI}.

Given that for every consistent reporting policy $p_{UI}(B) \leq p_{UI}(G)$, the expected utility of the IP is decreasing with $p(G|s = 1)$. To satisfy (IC) constraint of the IP, the utility under truth-telling should be (weakly) higher than under misreporting. It is obvious from the utility function of the IP that to satisfy (IC) the next condition is necessary:

$$p(G|s = 1) \leq p(G|s = 0)$$

There is only one consistent reporting policy which is satisfying above inequality - the uninformative reporting policy:

$$p(G|s = 1) = p(G|s = 0)$$

So, the only way for the mediator to induce truth-telling from the IP is sending uninformative messages to the UIP, which proves that the mediator has no role. $\square$

3.7.3 Proof of Claim 5

Proof. The proof is structured in the same manner as the one for Claim 4. First, I find all equilibria for every consistent reporting policy dropping (IC) and, second, I find the reporting policies that are incentive compatible.

[53] Assuming that (IC) constraint will be considered later in the proof.

There will be some preliminary observations. I will denote by the X_{UI}^G and X_{UI}^B the mixing sets of the different types of the UIP. I define $X_{UI} = X_{UI}^G \cup X_{UI}^B$. Following the Uniqueness part of Claim 1, it is easy to show that at any equilibrium in the case of pure private reporting policies: 1) $supX_{UI} = supX_I = 1$, 2) $X_{UI} \cup \{1\} = X_I$, 3) X_{UI} and X_I are connected sets 4) Only possible probability atom is $\alpha_I = 1$. In addition, $minX_{UI}^B = maxX_I^G$ whenever the "B" type bids a non-empty contract with positive probability. If $minX_{UI}^B < maxX_I^G$, then there will be a profitable deviation of the "B" type of the UIP. The observation from Claim 1 that $minX_{UI} = minX_I = \frac{I}{p_0}$ is not a necessarily condition, since the possibility to have different types of the UIP allows the "G" type in some cases to have a positive utility in equilibrium.

For the part of the equilibrium search, I will analyse two cases $p_{UI}(B) < I$ and $p_{UI}(B) \geq I$.

Case 1: $p_{UI}(B) < I$.

The UIP after the message "B" will bid an empty contract, since his posterior is less than the costs of investments.

1) I consider the first type of equilibrium. In this equilibrium the UIP after "G" message will bid in range $[\frac{I}{p_{UI}(G)}, 1]$ with probability q_G and an empty contract with $1 - q_G$. The IP will bid in the range $[\frac{I}{p_{UI}(G)}, 1]$.

The expected utility of the UIP after the message "G" will be:

$$U_{UI}^G = p_{UI}(G)(s(1)(1 - F_I(x)) + (1 - s(1)))(x - I) +$$

$$+(1 - p_{UI}(G))(s(0)(1 - F_I(x)) + (1 - s(0)))(-I)$$

Since, the expected utility of the UIP should be 0, the resulting cumulative distribution function of the IP will be:

$$F_I(x) = \begin{cases} \frac{p_{UI}(G)x - I}{p_{UI}(G)s(1)x - I(p_{UI}(G)s(1) + (1 - p_{UI}(G))s(0))}, & \text{if x} < 1 \\ 1, & \text{x=1} \end{cases} \tag{3.2}$$

$\lim_{x \to\ 1-0} F_I(x) < 1$ follows from $p'(0) < I$. The expression $p_{UI}(G)s(1) + (1 - p_{UI}(G))s(0)$ will be denoted as $E_G s$.

The expected utility of the IP after the signal realsiation $s = 1$ will be:

$$U_I(s=1) = (1-p(G|s=1)+p(G|s=1)(1-q_G+q_G(1-F_{UI}(\alpha_I)))(p'(1)\alpha_I - I)$$

The resulting cumulative distribution function of the UIP will be:

$$F_{UI}^G(x) = \frac{p'(1)x - I - u_I}{q_G p(G|s=1)(p'(1)x - I)}$$

With the utility level of the IP:

$$u_I = p'(1)(\frac{I}{p_{UI}(G)}) - I$$

And the feasibility condition on $q_G = \frac{p'(1)(p_{UI}(G)-I)}{p(G|s=1)(p_{UI}(G)(p'(1)-I))}$ gives:

$$p(G|s=1) \geq \frac{p'(1)(p_{UI}(G)-I)}{p_{UI}(G)(p'(1)-I)}$$

I call the above condition a parametric restriction, since from the viewpoint of the players the reporting policy is a parameter of the model.

2)The second type of equilibrium involves the UIP to earn a positive utility and bid in the range $[\gamma, 1]$.

Following the argument from part 1), the cumulative distribution function of the IP will be:

$$F_I(x) = \begin{cases} \frac{p_{UI}(G)x - I - u_{UI}^G}{p_{UI}(G)s(1)x - I(p_{UI}(G)s(1)+(1-p_{UI}(G))s(0))}, & \text{if } 0 \leq \text{x} < 1 \\ 1, & \text{x=1} \end{cases} \tag{3.3}$$

The cumulative distribution function of the UIP will have the same form as in the previous case with $q_G = 1$.

γ will be determined from the feasibility conditions on the cumulative distribution function of the UIP:

$$\gamma = \frac{I + (p'(1) - I)(1 - p(G|s=1))}{p'(1)}$$

$$u_I = p'(1)\gamma - I$$

A condition on γ that guarantee the minimum feasible bid of the UIP equals at least $\frac{I}{p_{UI}(G)}$ will result in:

$$p(G|s=1) \leq \frac{p'(1)(p_{UI}(G)-I)}{p_{UI}(G)(p'(1)-I)}$$

The two parametric conditions listed in the above sub-cases span whole parametric space. This completes the first case.

Case 2: $p_{UI}(B) \geq I$.

The first type of equilibrium will remain intact with the identical parametric restriction. If the expected utility of the UIP after "G" is zero, then utility of the UIP after "B" will be less than zero if he is bidding a non-empty contract.

The second type of equilibrium is similar to the second type of the first case if I add the no-bidding condition of the "B" type. The reason why the UIP after "B" will not bid is because his best choice is bidding $\alpha_{UI} = 1$ which gives the UIP non-positive utility under the condition:

$$\frac{p'(1)(p_{UI}(B)-I)(p_{UI}(G)s(1)-IE_G s)}{p_{UI}(G)(p'(1)-I)(p_{UI}(B)s(1)-IE_B s)} \leq p(G|s=1)$$

There is the third type of equilibrium in this case. After the message "B", the UIP bids empty contract with probability $1-q_B$ and bids in the region $[t,1]$ with probability q_B and after the message "G", he bids in the region $[\gamma, t]$. The IP randomizes in $[\gamma, 1]$.

The IP should mix according to the next cumulative distribution function to make each type of the UIP indifferent among the bids in each region:

$$F_I(x) = \begin{cases} \frac{p_{UI}(G)x-I-u_{UI}^G}{p_{UI}(G)s(1)x-IE_G s}, & \text{if } \gamma \leq \text{x} \leq \text{t} \\ \frac{p_{UI}(B)x-I}{p_{UI}(B)s(1)x-IE_B s}, & \text{if } t < \text{x} < 1 \end{cases} \tag{3.4}$$

The utility level u_{UI}^G will be pinned down from the boundary condition on the distribution function $F_I(0) = 0$:

$$u_{UI}^G = p_{UI}(G)\gamma - I$$

In addition, there should be a continuity condition on the distribution function $F_I(x)$ at point t. With the continuity condition neither type of the UIP has incentives to deviate:

$$u_{UI}^G = (p_{UI}(G)t - I) - (p_{UI}(B)t - I)\frac{p_{UI}(G)xs(1) - IE_G s}{p_{UI}(B)xs(1) - IE_B s}$$

From the perspective of the IP if he will not distinguish the different types of the UIP, there will exist an aggregate cumulative distribution function the UIP. The aggregate cumulative distribution function that makes the IP indifferent among the bids in his mixing set is:

$$F_{UI}(x) = \frac{p'(1)x - I - u_I}{p'(1)x - I} = p(G|s=1)F_{UI}^G(x) + p(B|s=1)(q_B F_{UI}^B(x))$$

The fact that the mixing regions of each type of the UIP are disjoint [54] gives the unique distribution functions for each type of the UIP:

$$F_{UI}^G(x) = \frac{p'(1)x - I - u_I}{p(G|s=1)(p'(x)x - I)}$$

$$F_{UI}^B(x) = \frac{(1 - p(G|s=1))(px - I) - u_I}{(1 - p(G|s=1))(p'(x)x - I)}$$

The conditions $F_{UI}^B(1) = 1$ and $F_{UI}^G(t) = 1$ give:

$$t = \frac{p'(1)\gamma - Ip(G|s=1)}{p'(1)(1 - p(G|s=1))}$$

$$1 - q_B = \frac{p'(1)\gamma - I}{(p'(1) - I)(1 - p(G|s=1))}$$

The feasibility conditions on γ and t will define the parametric region of this equilibrium type which may intersect with the parametric regions found in the sub-case 1 and sub-case 2. In addition, all these parametric regions do not cover parametric space completely. For some reporting policies there will not exist an equilibrium, which is a constraint on the set of reporting policies the mediator can use.

This completes the second case.

[54] I do not take into account the common element t, since one can exclude it from the mixing set of the "B" type without loss of generality.

Remark. I considered three free parameters for any reporting policy: $p_{UI}(B)$, $p_{UI}(G)$ and $p(G|s=1)$. But, there are only two free parameters because the third one is a function of the other two. So, given $p_{UI}(B)$ and $p_{UI}(G)$, the parameter $p(G|s=1)$ will be pinned down.

$$p(G|s=1) = \frac{(p_{UI}(G) - p'(0))(p_0 - p_{UI}(B))}{(p'(1) - p'(0))p(s=1)(p_{UI}(G) - p_{UI}(B))}$$

$$p(G|s=0) = \frac{(p'(1) - p_{UI}(G))(p_0 - p_{UI}(B))}{(p'(1) - p'(0))p(s=0)(p_{UI}(G) - p_{UI}(B))}$$

After the equilibrium analysis I can write the expected utility of the IP in the general way:

$$U_I(s=1) = ((1 - p(G|s=1))(1 - q_B) + p(G|s=1)(1 - q_G))(p'(1) - I)$$

In the second part, I will analyse the set of the reporting policies which are consistent and incentive compatible.

First, the consistency implies that the signal is weakly informative:

$$p_{UI}(G) \geq p_{UI}(B)$$

or

$$p(G|s=1) \geq p(G|s=0)$$

It is clear from the form of $U_I(s=1)$ function that it is decreasing with $p(G|s=1)$ and (IC) will not be satisfied under any consistent reporting policy except the uninformative one.

□

3.7.4 Proof of Claim 6

Proof. As was in Claim 5, there will be some preliminary observations that will simplify the analysis. I will denote by the X^G_{UI} and X^B_{UI} the mixing sets of the different types of the UIP and by the X_I^H and X_I^L the mixing sets of the different types of the IP. I define $X_{UI} = X_{UI}^G \cup X_{UI}^B$ and $X_I = X_I^H \cup X_I^L$. Following the Uniqueness part of Claim 1 and Claim 5, it is easy to show that at any equilibrium induced by a simple communication mechanism: 1) $supX_{UI} = supX_I = 1$, 2) $X_{UI} \cup \{1\} = X_I$, 3) X_{UI} and X_I are connected sets,4) The only probability atom is $\alpha_I = 1$, 5)$minX_{UI}^B = maxX_I^G$ whenever the "B" type bids a non-empty contract. In addition, $minX_I^L = maxX_I^H$ will hold, since a higher posterior of the "L" type about the message $m_{UI} = B$, forces him to bid more aggressively than the "H" type. [55] Given these observations, I proceed to the equilibrium search part.

First, I consider the case $\mathbf{p_{UI}(B) < I}$

There will be four different types of equilibria depending on the parameter region.

1) The IP bids unity after the message "L", and mixes in the region $[\frac{I}{p_{UI}(G)}, 1]$ after the message "H". The UIP mixes in the region $[\frac{I}{p_{UI}(G)}, 1]$ with probability q_G and bids an empty contract with $1-q_G$. He gets zero utility in the equilibrium.

For the UIP who received the message "G", the utility will be zero:

$$U_{UI}^G = p_{UI}(G)(s(1)(p(H|G, s=1)(1-F_I^H(\alpha_{UI}))+(1-p(H|G, s=1))))+(1-s(1)))(\alpha_{UI}-I)+$$

$$+(1-p_{UI}(G))(s(0)(p(H|G, s=1)(1-F_I^H(\alpha_{UI}))+(1-p(H|G, s=1)))+(1-s(0)))(-I) = 0$$

The cumulative distribution function that makes the UIP to have a zero utility in expectation is:

$$F_I^H(x) = \begin{cases} \frac{p_{UI}(G)x-I}{p(H|G,s=1)(p_{UI}(G)s(1)x-I(p_{UI}(G)s(1)+(1-p_{UI}(G))s(0)))}, & \text{if x} < 1 \\ 1, & \text{x=1} \end{cases} \quad (3.5)$$

[55]If this condition does not hold, then there will be a profitable deviation of the "L" type of the IP. Essentially, it comes from the single crossing condition.

The parameter restriction which follows from the fact that $\lim_{x\to\ 1-0} F_I^H(x) \leq 1$:

$$p(H|G, s=1) \geq \frac{p_{UI}(G) - I}{p_{UI}(G)s(1) - I(p_{UI}(G)s(1) + (1-p_{UI}(G))s(0))}$$

The utility of the IP after receiving the message "H" and the signal realisation $s = 1$:

$$U_I^H(s=1) = ((1-p(G|H, s=1)) + p(G|H, s=1)((1-q_G) + q_G(1-F_G))))(p'(1)\alpha_I - I)$$

The cumulative distribution function for the UIP that makes the "H" type of the IP indifferent among his actions $\alpha_I \in [\frac{I}{p_{UI}(G)}, 1]$:

$$F_{UI}^G(x) = \frac{p'(1)x - I - u_I^H}{p(G|H, s=1)q_G(p'(1)x - I)}$$

where

$$u_I^H = \frac{I(p'(1) - p_{UI}(G))}{p_{UI}(G)}$$

$$q_G = \frac{p'(1)(p_{UI}(G) - I)}{p_{UI}(G)(p'(1) - I)}$$

Above quantities are pinned down by the fact that $F_{UI}^G(1) = 1$ and $F_{UI}^G(\frac{I}{p_{UI}(G)}) = 0$.

The second parameter restriction for this equilibrium type which follows from the fact that $q_G \leq 1$ is:

$$p(G|H, s=1) \geq \frac{p'(1)(p_{UI}(G) - I)}{p_{UI}(G)(p'(1) - I)}$$

This parameter restriction tells that the message m_I to the IP should be enough informative, such that it allows the IP to have randomized strategy in the equilibrium. If this condition is not met, then there is a profitable deviation of the IP $\alpha_I = 1$.

The expected utility of the IP after the signal realisation $s = 1$ is:

$$u_I(s=1) = (1 - p(G|s=1) + p(G|s=1)(1-q_G))(p'(1) - I)$$

2) The IP bids unity after the message "L", and mixes in the region $[\gamma, 1]$ after the message "H". The UIP mixes in the region $[\gamma, 1]$ and gets non-negative utility. The UIP never bids an empty contract.

The cumulative distribution function of the IP that makes the UIP indifferent is:

$$F_I^H(x) = \begin{cases} \frac{p_{UI}(G)x - I - u_{UI}^G}{p(H|G,s=1)(p_{UI}(G)s(1)x - I(p_{UI}(G)s(1)+(1-p_{UI}(G))s(0)))}, & \text{if x} < 1 \\ 1, & \text{x=1} \end{cases} \tag{3.6}$$

And the parameter restriction follows from the fact that $\lim_{x \to 1-0} F_I^H(x) \leq 1$:

$$p(H|G, s=1) \geq \frac{p_{UI}(G)(1-\ \gamma)}{p_{UI}(G)s(1) - I(1-p_{UI}(G))s(0) - Ip_{UI}(G)s(1)}$$

‘

The cumulative distribution function of the UIP :

$$F_{UI}^G(x) = \frac{p'(1)x - I - u_I^H}{p(G|H, s=1)(p'(1)x - I)}$$

γ will be pinned down by $F_{UI}(1) = 1$ and will be equal:

$$\gamma = \frac{p'(1) - p(G|H, s=1)(p'(1) - I)}{p'(1)}$$

With the parameter restriction following from the fact that $\gamma \geq \frac{I}{p_{UI}(G)}$:

$$p(G|H, s=1) \leq \frac{p'(1)(p_{UI}(G) - I)}{p_{UI}(G)(p'(1) - I)}$$

The expected utility of the IP after the signal $s = 1$ is:

$$u_I(s=1) = (1 - p(G|s=1))(p'(1) - I)$$

3) The IP mixes in the region $[t, 1]$ after the message "L", and mixes in the region $[\frac{I}{p_{UI}(G)}, t]$ after the message "H". The UIP mixes in the region $[\frac{I}{p_{UI}(G)}, 1]$ with probability q_G and bids an empty contract with $1 - q_G$.

As was in Claim 5, there will be an aggregate bidding function of the IP from the perspective of the UIP. The aggregate cumulative distribution function that makes the IP indifferent among the bids in his mixing set is:

$$F_I = \frac{p_{UI}(G)x - I}{p_{UI}(G)s(1)x - IE_G s}$$

This aggregate function should satisfy next equation:

$$F_I = p(H|G, s = 1)F_I^H + (1 - p(H|G, s = 1))F_I^L$$

Given that each type of the IP mixes in the disjoint sets [56] , I can uniquely pin down the cumulative distribution function of each type of the IP:

$$F_I^H(x) = \frac{p_{UI}(G)x - I}{p(H|G, s = 1)(p_{UI}(G)s(1)x - IE_G s)}, \text{if } \quad \frac{I}{p_{UI}(G)} \leq x \leq t$$

$$F_I^L(x) = \begin{cases} \frac{p_{UI}(G)x - I - p(H|G,s=1)(p_{UI}(G)s(1)x - IE_G s)}{(1-p(H|G,s=1))(p_{UI}(G)s(1)x - IE_G s)}, & \text{if t <x} \leq 1 \\ 1, & \text{if x=1} \end{cases} \tag{3.7}$$

To make each type of the IP indifferent among the bids in the corresponding regions, the UIP should bid according to the next cumulative distribution function:

$$F_{UI}(x) = \begin{cases} \frac{p'(1)x - I - u_I^H}{q_G p(G|H,s=1)(p'(1)x - I)}, & \text{if } \frac{I}{p_{UI}(G)} \leq \text{x} \leq \text{t} \\ \frac{p'(1)x - I - u_I^L}{q_G p(G|L,s=1)(p'(1)x - I)}, & \text{if } t < \text{x} \leq 1 \end{cases} \tag{3.8}$$

With the expected utility of the "L" type of the IP after the signal realisation $s = 1$:

$$u_I^L = (1 - q_G p(G|L, s = 1))(p'(1) - I)$$

[56] I do not take into account the common element t, since one can exclude it from the mixing set of the "L" type without loss of generality.

(q_G, t) will be pinned down from the continuity of the F_{UI}^G at t [57] and from the fact that $F_I^L(t) = 0$.

$$q_G = \frac{p'(1)(p_{UI}(G)p(G|H, s=1)(1-t) + p_{UI}(G)p(G|L, s=1)t - p(G|L, s=1)I)}{(p'(1)-I)p_{UI}(G)p(G|H, s=1)p(G|L, s=1)}$$

$$t = \frac{I(1-p(H|G, s=1)E_G s)}{p_{UI}(G)(1-p(H|G, s=1)s(1))}$$

The set of parameters of this equilibrium type will have an intersection with the set of parameters of the equilibrium types 1 and 2.

The expected utility of the IP after the signal realisation $s = 1$ will be:

$$u_I(s=1) = p(H|s=1)(p'(1)\frac{I}{p_{UI}(G)} - I) +$$

$$+p(L|s=1)(1-p(G|L, s=1) + p(G|L, s=1)(1-q_G))(p'(1)-I) =$$

$$= p(B|s=1)(p(H|B, s=1)(p'(1)\frac{I}{p_{UI}(G)} - I) + p(L|B, s=1)(p'(1)-I))+$$

$$+p(G|s=1)(p(H|G, s=1)(p'(1)\frac{I}{p_{UI}(G)} - I) + p(L|G, s=1)(1-q_G)(p'(1)-I))$$

4) The IP mixes in the region $[t, 1]$ after the message "L", and mixes in the region $[\gamma, t]$ after the message "H". The UIP mixes in the region $[\gamma, 1]$ and gets a non-negative utility. The UIP never bids an empty contract.

The only thing which will change in the cumulative distribution function of the UIP (in comparison to the sub-case 3) is an absence of probability $1 - q_G$ of an empty bid:

$$F_{UI}^G(x) = \begin{cases} \frac{p'(1)x - I - u_I^H}{p(G|H, s=1)(p'(1)x - I)}, & \text{if } \gamma < \text{x} < \text{t} \\ \frac{p'(1)x - I - u_I^L}{p(G|L, s=1)(p'(1)x - I)}, & \text{if } t < \text{x} < 1 \end{cases} \tag{3.9}$$

The conditions on the cumulative distribution function $F_{UI}^G(1) = 1$ and $F_{UI}^G(\gamma) = 0$ will give two conditions on the utility levels of the IP:

$$u_I^H = p'(1)\gamma - I$$

[57] This condition guarantees that there is no deviations of the "L" type of the IP.

$$u_I^L = (1 - p(G|L, s = 1))(p'(1) - I)$$

Another condition that guarantees no deviation of the IP is a continuity of the cumulative distribution function $F_{UI}(x)$ at the point t which gives an expression for γ:

$$\gamma = \frac{p(G|H, s = 1)u_I^L + p'(1)p(G|L, s = 1)t - p(G|H, s = 1)(p'(1)t - I)}{p'(1)p(G|L, s = 1)}$$

The conditions on the cumulative function of the IP $F_I^H(\gamma) = 0$ and $F_I^L(t) = 0$ will give:

$$u_{UI}^G = p_{UI}(G)\gamma - I$$

$$t = \frac{\gamma - \frac{I}{p_{UI}(G)}p(H|G, s = 1)E_G s}{(1 - p(H|G, s = 1)s(1))}$$

The feasibility restrictions on (γ, t) are defining the set of parameter values that has an intersection with the set of parameters of the equilibrium types 1 and 2.

The expected utility of the IP after the signal realisation $s = 1$ will be:

$$u_I(s = 1) = p(H|s = 1)(p'\gamma - I) + (1 - p(H|s = 1))(1 - p(G|L, s = 1))(p'(1) - I) =$$

$$= p(B|s = 1)(p(H|B, s = 1)(p'(1)\gamma - I) + p(L|B, s = 1)(p'(1) - I)) +$$

$$+ p(G|s = 1)(p(H|G, s = 1)(p'(1)\gamma - I))$$

The second case is $\mathbf{p_{UI}(B) \geq I}$.

The first and the third types of equilibria from the previous case with their parametric regions will not change, since "B" type of the UIP will not have any deviations from a non-bidding strategy.

For the equilibria types 2 and 4, I need to add another restrictions on the parameters which guarantee that there are no deviations from the "B" type.

The additional parameter restriction for the equilibrium type 2 will be:

$$\frac{p'(1)(p_{UI}(B)-I)(p_{UI}(G)s(1)-IE_Gs)}{p_{UI}(G)(p'(1)-I)(p_{UI}(B)s(1)-IE_Bs)} \leq p(G|H, s=1)$$

And for the equilibrium type 4:

$$\gamma \leq \frac{I}{p_{UI}(G)} + \frac{I(p_{UI}(G)-p_{UI}(B))(s(1)-s(0))(1-I)}{p_{UI}(G)(p_{UI}(B)s(1)-IE_Bs)}$$

There will exist equilibria of other types.

In the fifth type of equilibrium, the IP bids unity after the message "L", and mixes in the region $[\gamma, 1]$ after the message "H". The UIP mixes in the region $[t, 1]$ after the message "B", and mixes in the region $[\gamma, t]$ after the message "G".

The expected utility of the IP after the signal $s = 1$ will be:

$$u_I(s=1) = (1 - p(G|s=1))(1-q_B)(p'(1)-I)$$

In the sixth type of equilibrium, The IP mixes in the region $[t_I, 1]$ after the message "L", and mixes in the region $[\gamma, t_I]$ after the message "H". The IP mixes in the region $[t_U, 1]$ after the message "B", and mixes in the region $[\gamma, t_U]$ after the message "G". The expected utility of the IP after the signal $s = 1$ is:

$$u_I(s=1) = p(B|s=1)(p(H|B, s=1)(p'(1)\gamma - I) + p(L|B, s=1)(1-q_B)(p'(1)-I)) +$$

$$+p(G|s=1)(p(H|G, s=1)(p'(1)\gamma - I))$$

I do not explicitly analyse these types of equilibria in the claim.

The first and the second parts of the claim follow from the preliminary observations and the equilibria types. Preliminary observations also exclude a possibility of other types of equilibria.

The third part of the proof. For the first type of equilibrium the expected utility of the IP after the signal realisation equal to 1 is:

$$u_I(s=1) = (1 - p(G|s=1) + p(G|s=1)(1-q_G))(p'(1)-I)$$

I will consider the deviation of the IP such that he submits the signal realisation $s = 0$ when the true realisation is $s = 1$. I will consider the deviation of the IP such that his actions after "H" and "L" will remain the same (as in the case when $s = 1$).[58] The utility of the IP under the mentioned deviation will be:

$$u_I(m = 0|s = 1) = (1 - p(G|s = 0) + p(G|s = 0)(1 - q_G))(p'(1) - I)$$

which is strictly higher than the original utility under truth-telling, because for the informative reporting policy $p(G|s = 0) < p(G|s = 1)$ should hold. The cases for the equilibrium types two and five will be similar. So, it cannot be the case that the "L" type is bidding according to the degenerate probability distribution.

□

3.7.5 Proof of Claim 7

Proof. 1) I consider the expected utility of the IP under the deviation $m = 0$ if the true signal realisation is $s = 1$. The reporting function $p(m_I|m_{UI}, s = 0)$ is important only for the punishment of the IP if he decides to deviate from the truth-telling. In the case of the signal realisation $s = 0$ this reporting function is payoff irrelevant. So, the primary motive of the mediator is to punish the deviator as much as possible. One can consider the choice of an optimal action by the IP in the case of deviation from truth-telling as a decision problem where m_{UI} serves as a state which perfectly predicts the action of the UIP and m_I serves as a signal about the state. So, the uninformative signal $p(m_I|m_{UI}, s = 0)$ will be an optimal punishment for the IP in the case of deviation from truth-telling. Fully uninformative reporting policy $p(m_I|m_{UI}, s = 0)$ may not be the only optimal punishment.

2) Without loss of generality consider type 3 of equilibrium. Under truth-telling when $s = 1$, $\alpha_I = t$ is an optimal action for both types of the IP, so the expected utility of the IP under truth-telling:

[58]This may be a sub-optimal deviation of the IP.

$$u_I(m=1|s=1) = (p(G|s=1)(1-F_{UI}(t)) + (1-p(G|s=1)))(t-I)$$

Under the deviation $(m=0)$, the utility will be at least:

$$u_I(m=0|s=1) = (p(G|s=0)(1-F_{UI}(t)) + (1-p(G|s=0)))(t-I)$$

The utility under lying is strictly higher due to $p(G|s=0) < p(G|s=1)$ for an informative reporting policy. The only case when it is equal is the case of the uninformative signal $p(G|s=0) = p(G|s=1)$. For the equilibrium types 4 and 6, the logic is similar. The crucial step in the proof is the fact that $\alpha_I = t$ is an optimal action for both of the types of the IP.

□

3.7.6 Proof of Claim 8

Proof. **The third part.** Imagine that the second type of the equilibrium is induced both after "H" and "L". Then under informative reporting policy, the IP would gain if he lies about signal realisation $s=1$. Since $p(G|s=1) > p(G|s=0)$, there will always exist such a deviation.

The forth part. Define:

$$(1-q)(m_I) := p(G|m_I, s=1)(1-q_G) + p(B|m_I, s=1)(1-q_B)$$

. Imagine that $(1-q)(H) > (1-q)(L)$, then if $p(H|s=1) < p(H|s=0)$ there will always exists a deviation of the IP from truth-telling. The IP will say $m=0$ about the signal realisation and bid $\alpha_I = 1$, his utility will be strictly higher because of more frequent non-bidding behaviour after "H".

□

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