# The Value of Being Lucky: Option Backdating and Non-diversifiable Risk[†]

Vicky Henderson[‡] Jia Sun [§] A. Elizabeth Whalley [¶]

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The practice of executives influencing their option compensation by setting a grant date retrospectively is known as backdating. Since executive stock options are usually granted at-the-money, selecting an advantageous grant date to coincide with a low stock price will be valuable to an executive. Empirical evidence shows that backdating of executive stock option grants was prevalent, particularly at firms with highly volatile stock prices. Executives who have the opportunity to backdate should take this into account in their valuation. We quantify the value to a risk averse executive of a lucky option grant with strike chosen to coincide with the lowest stock price of the month. We show the ex ante gain to risk averse executives from the ability to backdate increases with both risk aversion and with volatility, and is significant in magnitude. Our model involves valuing the embedded partial American lookback option in a utility indifference setting with key features of risk aversion, inability to diversify and early exercise.

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[‡]Department of Statistics, University of Warwick, Coventry, CV4 7AL. UK. Email: vicky.henderson@warwick.ac.uk
[§]China Credit Rating Co. Ltd, 28 Jin Rong Street, Beijing 100032, China. Email: sunjia@chinaratings.com.cn
[¶]Warwick Business School, University of Warwick, Coventry. CV4 7AL. UK. Email: elizabeth.whalley@wbs.ac.uk

# 1 Introduction

The practice of executives influencing their option compensation by setting a grant date retrospectively is known as backdating. Since options are usually granted at-the-money, selecting an advantageous grant date to coincide with a low stock price will be valuable to an executive. Backdating of option awards received significant attention in the financial press and led to in excess of 140 firms being the subject of investigations by the Securities and Exchange Commission (SEC). The US regulatory environment prior to 2002 allowed executives up to 45 days after the end of the fiscal year to report their option grants and thus there was significant scope for retrospective granting. Following the introduction of the Sarbanes-Oxley Act (SOX) in August 2002, they had to report within two business days of the grant. There is a substantial body of evidence documenting that backdating of option awards took place in the US, particularly prior to the tightening of SEC reporting rules and that it was particularly prevalent in firms with highly volatile stock prices. Heron & Lie (2009) estimate that 13.6% of option grants were manipulated in their study. See also Yermack (1997), Lie (2005), Heron & Lie (2007), Collins, Gong & Li (2005), Narayanan, Schipani & Seyhun (2007), Narayanan & Seyhun (2008) and Bebchuk, Grinstein & Peyer (2010). Although the tightening of reporting in 2002 reduced the opportunity and incidence of backdating, it remains important to understand the value to executives of backdating and why it occurred more frequently in firms with highly volatile stock prices. In fact, contrary to the belief that backdating would completely disappear in the post-SOX era, Narayanan & Seyhun (2008) find more than 20% of executives report their options late (after the two day rule) with 10% reporting more than a month late.

We adopt the modelling perspective that, ex ante, executives who have the opportunity to backdate should take this into account in their valuation of their option grants. Executives anticipate being able to choose the best grant date over a backdating window. This is consistent with the characterization of “lucky” grants of Bebchuk, Grinstein & Peyer (2010) as being at-the-money grants at the lowest stock price of the month. Thus, in contrast to the valuation of standard executive stock options, the value of the opportunity to backdate involves a lookback feature. Specifically, we value this option to backdate in a utility indifference setting which takes into account both the executive’s risk aversion and exposure to non-diversifiable risk and the American early exercise feature of option grants.

We demonstrate that not only does this approach show that the opportunity to backdate is very valuable - but that its value increases when the volatility of the firm’s stock price is higher - thus enabling us to explain the empirical finding that backdating was more prevalent in high volatility firms. In contrast, existing Black Scholes based estimates of value are unable to obtain this result.

We also highlight that, in contrast to standard American (or European) options where risk aversion reduces their value, the value of backdating is worth more as risk aversion increases.

We now explain our contributions in more detail. Firstly we show that risk aversion increases the value of backdating. The magnitude of gains the risk averse executive could expect from the opportunity to backdate are significant and therefore could have provided a motivation to undertake backdating of options. For our base level of risk aversion, percentage gains given a one month backdating window range between 7.2% to as much as 25.5%. We also consider a much shorter two day backdating window and find that there is still a 5% increase in value to the executive from the backdating opportunity. So, despite the much tighter post-SOX reporting rules, the opportunity for executives to benefit from backdating is still present.

Risk aversion is usually associated with a reduction in the value of risky contingent claims (eg. see the indifference pricing work of Musiela & Zariphopolou 2004, Oberman & Zariphopolou 2003, and also Lambert, Larcker & Verrechia 1991, Detemple & Sundaresan 1999, Miao & Wang 2007) and earlier exercise (Oberman & Zariphopolou 2003, Leung & Sircar 2009, Carpenter 1998, Bettis, Bizjak & Lemmon 2005, Carpenter, Stanton & Wallace 2010, 2019). Our finding in this paper that it instead increases the value of backdating is in sharp contrast. The reason is that non-diversifiable risk has a greater negative impact on American at-the-money (or non-backdated) options than those that are backdated or in-the-money.

In addition to the focus on the value to the risk averse executive, we show that the additional cost of the grant imposed on shareholders when risk averse executives have the opportunity to backdate is significant. We show that the additional costs to shareholders in the presence of backdating, however, are lower in magnitude than the size of the gain that the executive can expect from backdating.

Our second main finding is to show that incorporating the embedded lookback option, and taking account of non-diversifiable risk and early exercise can explain the greater frequency of backdating in high volatility firms. There is substantial evidence across the empirical literature (Bebchuk, Grinstein & Peyer 2010, Heron & Lie 2009, Bizjak, Lemmon & Whitby 2009) that firms with more volatile stock prices were more likely to engage in backdating. Intuition would suggest that executives at firms with high volatility potentially have more to gain from backdating as they have a greater chance of obtaining a date with a lower strike price. We will call this the expected strike discount effect. However, this intuition is incomplete, and Black Scholes option pricing arguments inform us that at-the-money call options are more sensitive to volatility than in-the-money calls. Thus for a given increase in volatility, a non-backdated option will have a larger increase in its Black Scholes value than an otherwise equivalent backdated option making the ability to engage in backdating less attractive to an executive. This moneyness-on-vega effect works in the opposite direction to the expected strike discount effect. Ex post computations using the Modified Black Scholes formula (see Appendix 6.3) and a fixed adjustment to strike by Walker (2007) and Dierker & Hemmer (2007) demonstrate that the moneyness-on-vega effect is dominant. European Black Scholes computations of Eikseth & Lindset (2011) also find this relationship. These papers thus obtain the opposite prediction to that documented empirically. We resolve this puzzle, and by doing so, demonstrate the importance of taking into account both the executive's exposure to non-diversifiable risk and their ability to exercise options early when assessing the benefit of backdating to executives. In our model, for our base level of risk aversion in Panel B of Tables 2 and 3, we see that the benefit of backdating is always increasing with stock price volatility, consistent with the evidence in the empirical literature. Why is this the case? When the executive is risk averse, the difference between the sensitivities of in and at-the-money options to volatility is much smaller than under Black Scholes and thus the moneyness-on-vega effect is reduced, and is typically dominated by the impact of the expected strike discount.

The methodology in our study extends the line of research on pricing and hedging in incomplete markets under utility indifference pricing (see for example, Henderson 2002, Musiela & Zariphopolou 2004, also Henderson & Hobson 2009). The agent's partial hedging/control problem is considered in tandem with American option exercise, resulting in mixed control/stopping. Applications to executive stock options include Detemple & Sundaresan (1999), Oberman & Zariphopolou (2003), Leung and Sircar (2009), Henderson (2007), Grasselli and Henderson (2009), Carpenter, Stanton & Wallace (2010), Henderson, Sun & Whalley (2014), and Leung & Wan (2015). Optimal stopping problems are solved via a free boundary problem and the associated Hamilton-Jacobi-Bellman equation. This paper extends these works to consider partial lookback American options in a utility indifference setting. Our problem is a hybrid European-American option where after the grant date, the option is American style and can be exercised. Prior to the grant date, the option is a non-standard European lookback, where the payoff at the grant date is a non-linear function of the strike, given by the utility value of the American option. The valuation of lookback options dates back to Goldman, Sosin & Gatto (1979), and Conze & Viswanathan (1991). More recent work, for example, Lai & Lim (2004), and Dai & Kwok (2005) and Kimura (2011) examines early exercise policies of American lookbacks, and Kou (2008) surveys computational methods.

There are remarkably few theoretical models of option backdating and surrounding issues. Stannard & Guthrie (2020) develop a model to formalize the managerial power view of option backdating whilst Dierker & Hemmer (2007) suggest that firms may have used backdating as an optimal response to distortions in the institutional environment. Eikseth & Lindset (2011) undertake an ex ante valuation under a Black Scholes setting, however, as we show later, such a model gives the opposite relationship with volatility than is documented in the empirical literature. There are also a number of empirical papers which estimate the ex post gains to executives from backdating using data and a fixed reduction in strike price (rather than our model which takes into account a probability distribution over possible prices). Prices are computed with a Modified Black Scholes formula which makes an ad hoc adjustment for early exercise, see Appendix 6.3 (Narayanan & Seyhum 2008, Narayanan, Schipani & Seyhum 2007).

## 2 Model

A company will have a board meeting at date $T_0$, at which options will be granted to senior executives. Given the majority of options are granted at-the-money, in the absence of backdating, our benchmark is that at-the-money call options are granted on date $T_0$. We will assume options have a life of $T$ years and are American style, and hence are exerciseable until maturity date $T_0+T$.[1]

Executives know they have the opportunity to backdate the option award and receive at-the-money options retrospectively, on an earlier date with a more favorable strike price. Following Bebchuk, Grinstein & Peyer (2010), where lucky grants are awarded at the lowest strike price of the month, we first consider a backdating window or lookback period of one month, ie. the window $[0, T_0]$ is one month. Executives can select the best date within the one month window on which their options are granted. Indeed, Microsoft were found to have been offering executives the chance to select their own grant date for at-the-money options within a one month window, see Forelle & Bandler (2006). We also consider a second scenario whereby executives only have a two-day backdating window to capture the post-SOX institutional environment. Denote the best strike by $J = \min_{0\leq u\leq T_0} S_u$, where $S$ is the stock price, and the date on which the minimum occurred by $0 \leq t_{\min} \leq T_0$.[2] Ex post, executives who decide to backdate report that they received at-the-money (strike J) options at $t_{\min}$. In fact, at the board meeting date, they have options which are necessarily in-the-money, since $J \leq S_{T_0}$. Ex ante, executives who have the opportunity to backdate should take this into account in their valuation. Our aim is to compare the ex ante value of the option grant if backdating can occur, to the benchmark value of the grant if backdating cannot

[1] We do not consider a vesting period during which options are not exerciseable. See for example Sircar & Xiong (2007) in a Black Scholes setting. Vesting would not change our key findings. It would slightly reduce all option values (and costs) but should not significantly alter relative benefits (and costs) of backdating or their relationship with Black Scholes estimates. We also do not treat option resetting (see Acharya et al. 2000) whereby companies may reset the option terms during the life of the option.

[2] Since we work in continuous time, the model allows the best strike to be chosen over the period $0 \leq t_{\min} \leq T_0$. In reality, intraday prices would not be possible, and the value of the opportunity would lessen slightly.

occur, both for a fixed size of option grant.

Executives are constrained in their ability to trade the stock $S$ which underlies their options. Insiders cannot short sell stock as they are prohibited by Section 16-c of the Securities and Exchange Act. Evidence of early exercise across all ranks of employees suggests they are constrained as to the hedging they can carry out (Bettis, Bizjak & Lemmon 2005, Carpenter, Stanton & Wallace 2019). This means they cannot fully hedge risk and face some non-diversifiable risk whilst they continue to hold unexercised options. Although the executive cannot trade the firm's stock, $S$, we allow them to partially hedge their exposure by taking a position in the market portfolio, $M$ and riskless bond with constant riskless rate $r$. Prices for the stock $S$ and market $M$ follow

$$\frac{dS}{S} = (\nu - q)dt + \eta dB \tag{1}$$

$$\frac{dM}{M} = \mu dt + \sigma dZ \tag{2}$$

where standard Brownian motions $B$ and $Z$ are defined on a probability space $(\Omega, \mathcal{F}, \mathcal{F}_u, \mathcal{P})$ where $\mathcal{F}_u$ is the augmented $\sigma$-algebra generated by $\{B_u, Z_u; 0 \leq u \leq t\}$ and their instantaneous correlation is $\rho \in (-1, 1)$. The volatility of stock returns, $\eta$, expected return on the stock, $\nu$, proportional dividend yield, $q \geq 0$, and expected return, $\mu$ and volatility of market returns, $\sigma$, are all constants. The mean stock return, $\nu$ is equal to the CAPM return for the stock, given its correlation with the market, $\nu = r + \beta(\mu - r); \beta = \rho\eta/\sigma$. Denote by $\theta_t$ the cash investment held in the market $M$ at time $t$. The executive's non-option wealth follows

$$dW_u = (rW_u + \theta_u(\mu - r))du + \theta_u \sigma dZ_u; W_t = w \tag{3}$$

given initial wealth $w$. Remaining non-diversifiable risk that cannot be hedged is represented by idiosyncratic volatility $(1 - \rho^2)\eta^2$, and is greater, the lower the (absolute value of) correlation between the stock and market.

The executive aims to maximize the expected utility of total wealth at the option maturity, $T_0 + T$, over choice of exercise time $\tau$ and outside investment in the market $\theta_u$ (satisfying the usual integrability condition $\mathbb{E} \int_t^{T_0+T} \theta_u^2 du)$ and riskless bonds. We assume the executive has constant absolute risk aversion (CARA) denoted by $U(x) = -e^{-\gamma x}; \gamma > 0$. Solving under other utilities would add a further dimension and would not alter our main findings. When the option is exercised, it is optimal to sell shares immediately. In the model, the cash proceeds are added to outside wealth and will continue to be invested optimally in the market and riskless bond until the maturity date.

## 2.1 Valuing the Option grant with the opportunity to backdate

We now value the option grant under this framework. Over the period $[T_0, T_0+T]$, the option grant with the opportunity to backdate consists of $N$ American calls with known strike $J$. We denote the value to the risk averse executive (at time $u \in [T_0, T_0+T]$ with current wealth $W_u$, current stock price $S_u$) of the option grant with the opportunity to backdate by $\mathcal{Y}_b(u, W_u, S_u; J)$ and set out how to value this grant in the next subsection. Over the backdating window $[0, T_0]$, the option grant cannot be exercised and has features in common with a lookback option. A (floating strike) European lookback call with maturity $T_0$ allows the holder to buy the stock at the minimum stock price over the option's life, $J$, and so has a payoff equal to the difference between the terminal stock price, $S_{T_0}$ and the strike, $J$. Our backdated option grant instead has a payoff at $T_0$ equal to $\mathcal{Y}_b(T_0, W_{T_0}, S_{T_0}; J)$, the value of the $N$ American calls, and hence has a payoff which is non-linear in $J$.

**Valuation after the board meeting date,** $[T_0, T_0+T]$

We work backwards in time from the maturity date $T_0+T$, and first consider times between $T_0$ and $T_0+T$ when the options are exercisable by the executive. By definition, the strike $J = \min_{0\le u\le T_0} S_u$ is known. The value to the executive of having $N$ exercisable strike $J$ options, optimal choice over outside investments $\theta_t$ and optimal choice over the exercise time of options,[3] $\mathcal{Y}_b(u, W_u, S_u; J)$ solves the complementarity problem: (for $T_0 \le u \le T_0+T$)

$$\mathcal{Y}_b(u, W_u, S_u; J) \ge \mathcal{M}(u, W_u + N(S_u - J)^+, T_0+T) \tag{4}$$

$$\frac{\partial \mathcal{Y}_b}{\partial t} + \sup_\theta\{\mathcal{L}\mathcal{Y}_b\} \le 0 \tag{5}$$

$$\left(\frac{\partial \mathcal{Y}_b}{\partial t} + \sup_\theta\{\mathcal{L}\mathcal{Y}_b\}\right)(\mathcal{M}(u, W_u + N(S_u - J)^+, T_0+T) - \mathcal{Y}_b(u, W_u, S_u; J)) = 0 \tag{6}$$

where the differential operator $\mathcal{L}$ is defined by

$$\mathcal{L} = \frac{\eta^2 s^2}{2}\frac{\partial^2}{\partial s^2} + (\nu - q)s\frac{\partial}{\partial s} + \rho\theta\sigma\eta s\frac{\partial^2}{\partial w \partial s} + \frac{\theta^2\sigma^2}{2}\frac{\partial^2}{\partial w^2} + [\theta(\mu - r) + rw]\frac{\partial}{\partial w}. \tag{7}$$

The inequality (4) says that the options stay alive whilst the value of continuing is greater than the exercise value, where the exercise value $\mathcal{M}(u, W_u + N(S_u - J)^+, T_0+T)$ is the value derived

[3] We effectively assume the option grant is exercised as a block, consistent with most models of ESO valuation (Carpenter, Stanton & Wallace 2010, amongst many others). Grasselli & Henderson (2009) study the impact of a portfolio of (identical) ESOs and show risk aversion leads to separate exercises. However, by assuming block exercise, we obtain a numerically tractable problem and we can compare the value of backdated versus non-backdated options. Non-block exercise will be of second-order importance, as even large changes in exercise thresholds have a relatively small effect on option values.

from optimally investing the exercise proceeds and any cash wealth until $T_0 + T$, given by (Merton 1971):

$$\mathcal{M}(t, w, \tilde{T}) = \sup_{\{\theta_s\}_{t \le s \le \tilde{T}}} \mathbb{E}U(W_{\tilde{T}} | W_t = w) = -e^{-\gamma w e^{r(\tilde{T}-t)}} e^{-\frac{(\mu-r)^2}{\sigma^2}(\tilde{T}-t)}. \tag{8}$$

We can now define boundary conditions at maturity and at $S = 0$ via:

$$\mathcal{Y}_b(T_0 + T, W_{T_0+T}, S_{T_0+T}; J) = \mathcal{M}(T_0 + T, W_{T_0} + N(S_{T_0} - J)^+, T_0 + T) \tag{9}$$

and $\mathcal{Y}_b(u, W_u, 0; J) = \mathcal{M}(u, W_u, T_0 + T)$.

The option grant is exercised when the value from continuing to hold it is sufficiently low that it equals the value from exercising and investing the payoff $N(S_u - J)$ optimally, and thus the optimal exercise time $\tau$ is characterized by:

$$\tau = \inf\{T_0 \le u \le T_0 + T : \mathcal{Y}_b(u, W_u, S_u; J) = \mathcal{M}(u, W_u + N(S_u - J)^+, T_0 + T)\}. \tag{10}$$

Due to the choice of CARA, we can scale out dependence on wealth. As for standard American options, we can describe the optimal exercise time $\tau$ as the first time the stock price reaches an exercise threshold

$$\tau = \inf\{T_0 \le u \le T_0 + T : S_u = S_b^*(u)\}. \tag{11}$$

We define the utility indifference value of the backdated option grant to the executive, $V_b(u, S_u; J)$; $u \in [T_0, T_0 + T]$, with strike $J = \min_{0 \le u \le T_0} S_u$, to be the cash equivalent which, when invested optimally, would give the same expected utility as is achieved by the option grant: $\mathcal{Y}_b(u, W_u, S_u; J) = \mathcal{M}(u, W_u + V_b(u, S_u; J), T_0 + T)$. We now solve the free boundary problem numerically using standard finite difference methods, see Appendix 6.1 for details. We also refer the reader to Oberman & Zariphopolou (2003) where existence and uniqueness of solutions in the viscosity sense is proved for finite maturity American options under CARA. Note although the strike $J = \min_{0 \le u \le T_0} S_u$ is known during $[T_0, T_0 + T]$, we will need the values at $T_0$ for each possible value of $J$, and hence we need a three-dimensional grid $(t, S, J)$.

**Valuation during the backdating window,** $[0, T_0]$

To value the option grant prior to $T_0$, we use conditioning to reduce the valuation to that of a European derivative with payoff at $T_0$ of $V_b(T_0, S_{T_0}; J_{T_0})$. We have

$$\mathcal{Y}_b(u, W_u, S_u; J_u) = \sup_{\{\theta_u\}_{0 \le u \le T_0}} \mathbb{E}\mathcal{M}(T_0, W_{T_0} + V_b(T_0, S_{T_0}; J_{T_0}), T_0 + T); \ 0 \le u \le T_0 \tag{12}$$

During the backdating window $u \in [0, T_0]$, $\mathcal{Y}_b$ solves a partial differential equation (pde), together with a boundary condition at $T_0$ involving known values $V_b(T_0, S_{T_0}; J_{T_0})$ computed earlier.

Again, using a standard transformation to remove wealth dependence, we solve the pde numerically using standard finite difference methods. Appropriate boundary conditions are: an upper bound for large $S$, a lower bound for $S = J$, and an implicit boundary condition for $J = 0$. See Appendix 6.1 for details.

## 2.2 Valuing the option grant without backdating: A benchmark

In the absence of backdating, the executive receives at-the-money American call options at $T_0$ with a $T$ year maturity. Prior to receiving the option grant at $T_0$, the executive has $N$ forward-starting at-the-money American calls with unknown stochastic strike $S_{T_0}$. At $T_0$, the option grant are simply American call options with a given strike because at $T_0$, the strike is known. We again work backwards in time, first valuing after the board meeting date.

**Valuation after the board meeting date,** $[T_0, T_0 + T]$

During the time period $u \in [T_0, T_0 + T]$, the grant is valued using the above framework with the restriction $J = S_{T_0}$, giving the indifference value $V_{nb}(u, S_u, S_{T_0})$. As described earlier in (11), we can give the optimal exercise time as the first time the stock price reaches an exercise threshold, denoted by $S^*_{nb}(u)$ in the absence of backdating.

**Valuation during the backdating window,** $[0, T_0]$

To value the option grant prior to $T_0$, we use conditioning to reduce the valuation to that of a European derivative with payoff at $T_0$ of $V_{nb}(T_0, S_{T_0}, S_{T_0})$. We have

$$\mathcal{Y}_{nb}(u, W_u, S_u, S_{T_0}) = \sup_{\theta_u; 0 \leq u \leq T_0} \mathbb{E}\mathcal{M}(T_0, W_{T_0} + V_{nb}(T_0, S_{T_0}, S_{T_0}), T_0 + T); \; 0 \leq u \leq T_0. \tag{13}$$

In Appendix 6.1, we give details of how to derive the following expression for the utility indifference value of the option grant:

$$V_{nb}(u, S_u, S_{T_0}) = -\frac{1}{\gamma(1-\rho^2)e^{r(T_0+T-u)}} \log \mathbb{E}[e^{-\gamma(1-\rho^2)V_{nb}(T_0, S_{T_0}, S_{T_0})e^{r((T_0+T)-T_0)}}]; \; 0 \leq u \leq T_0. \tag{14}$$

For existence and uniqueness arguments for exponential utility indifference pricing for European options, see Musiela & Zariphopolou (2004). Note, although it is usual to see the minimal martingale measure appear in the expression for the utility indifference price (see Musiela & Zariphopolou 2004) in our model, this is in fact simply equivalent to the original $\mathbb{P}$ measure due to the CAPM assumption whereby $\nu = r + (\mu - r)\rho\eta/\sigma$. We use numerical integration to compute the certainty equivalent value of the non-backdated option grant $V_{nb}(u, S_u, S_{T_0})$ for $u \in [0, T_0]$.

### 2.3 The Cost of the option grant to the company

As Hall & Murphy (2002) highlight, restricting the trading and hedging of executives who hold options creates a distinction between the cost to the company and value to the individual executive of option grants. In particular, the opportunity cost of an option to a company and its shareholders is the amount the company could receive if it were to sell a hedgeable option to an outsider rather than granting it to the executive. Shareholders are not restricted in their ability to diversify, and thus, the cost of options to them can be computed as if they were able to hedge perfectly. The cost or objective value of an option grant is the value of an equivalent tradeable American option, but with the exercise decision controlled by the executive (see Carpenter 1998, Bettis, Bizjak & Lemmon 2005 and Carpenter, Stanton & Wallace 2010 all in the absence of backdating). Tradable American option values can be calculated under Black Scholes model assumptions. In our model, the cost of a backdated option grant is its equivalent tradeable value, given the backdating decision, exercise decision and optimal partial hedging are optimized by the executive.

**Cost after the board meeting date,** $[T_0, T_0 + T]$

The cost to shareholders after $T_0$ of the $N$ American calls with strike $J$, satisfies the standard Black Scholes pde for stock prices below the executive's exercise threshold, $s \leq S_b^*(t)$

$$\frac{\partial C_b}{\partial t} + (r-q)s\frac{\partial C_b}{\partial s} + \frac{\eta^2 s^2}{2}\frac{\partial^2 C_b}{\partial s^2} - rC_b = 0 \tag{15}$$

with boundary conditions $C_b(u, 0, J) = 0, C_b(T_0+T, S_{T_0+T}, J) = N(S_{T_0+T} - J)^+$ and $C_b(u, S_b^*, J) = N(S_b^*(u) - J)^+$.

**Cost during the backdating window,** $[0, T_0]$

During $[0, T_0]$, $C_b(u, S_u; J)$ solves the Black Scholes pde in (15), together with a boundary condition at $T_0$. Again, we solve the pde numerically together with appropriate boundary conditions. Finally, the cost of the option grant in the absence of backdating, $C_{nb}$, is computed using the same methods as above, taking into account the corresponding no-backdating exercise threshold $S_{nb}^*(u); T_0 \leq u \leq T_0 + T$.

## 3 The Value and Cost of Backdating: Results

In this section we present results from numerically solving the models outlined in Section 2. Parameters are chosen as follows. We take our base case value of absolute risk aversion $\gamma = 0.2$ in line with those in the literature (Leung & Sircar 2009 and Miao & Wang 2007.) They are also comparable to the levels of relative risk aversion and outside wealth used by Carpenter, Stanton & Wallace (2010). Consider an executive with a grant of one million options with strike \$10, on stock worth \$10. If she has relative risk aversion of 4 and outside wealth of \$20 million, her absolute risk aversion is $\gamma = 0.2$. We also consider a very low value of absolute risk aversion $\gamma = 0.01$, to show how values and costs vary significantly from those under a Black Scholes model, even for executives who are only slightly risk averse. Option maturities for executives have been documented to be surprisingly homogeneous with a ten year length (Carpenter, Stanton & Wallace 2019), so we take $T = 10$.

We briefly summarize notation. We denote by $V_b = V_b(t, S_t, J)$ the utility indifference value of the option grant with the opportunity to backdate, with strike $J = \min_{u \leq T_0} S_u$; $V_{nb} = V_{nb}(t, S_t, S_{T_0})$ the utility indifference value of the benchmark non-backdated option grant with strike $S_{T_0}$; $C_b = C_b(t, S_t, J)$ the cost of the backdated option grant to well-diversified shareholders (under optimal exercise by executive); and $C_{nb} = C_{nb}(t, S_t, S_{T_0})$ the cost of the non-backdated option grant to well-diversified shareholders. Table 2 reports these values and costs for a backdating window of one month and for a range of values of riskfree rate, dividend yield, volatility and risk aversion. Table 3 (Panel B) considers a post-SOX two-day backdating window.

For comparison purposes with the extant literature we are also interested in the Black Scholes equivalents of the above. Table 1 reports the Black Scholes values, denoted by $V_b^{BS}$ and $V_{nb}^{BS}$, for the same set of parameters as used in Table 2. Panel A of Table 3 reports Black Scholes values for the same parameters as in Panel B of Table 3. Under Black Scholes, all risk is assumed hedgeable, and so option values to the unconstrained (risk neutral) executive and option costs to well-diversified shareholders are identical.

## 3.1 The magnitude of the ex ante gains to the executive from the opportunity to backdate

In this section we report on the magnitude of the gains that could be expected by the executive from having the opportunity to backdate the option grant. We first consider a backdating window of one month. (In fact, pre-SOX executives had much longer than one month to backdate, so our estimates are conservative.) Consider an executive with $N = 1$ million options on stock worth \$10. For an executive with our base level of risk aversion, and with stock parameters of $r = 0.05, q = 0$ and volatility 60%, the value of the grant in the absence of backdating is \$ 2.121 million. This increases by 17% to a value of \$ 2.48 million when the executive has the opportunity to backdate over a one month lookback window. For an executive with our base risk aversion level, Table 2 shows the percentage gains from the opportunity to backdate vary between 7.2% and 25.5% across our range of other parameters. Percentage gains or the relative gain from backdating are calculated by $\%\mathcal{G} = (V_b - V_{nb})/V_{nb}$. We can also consider the dollar gains from the opportunity to backdate,

$V_b - V_{nb}$. For the base parameters the dollar gain is around \$ 0.36 million.

How do these gains compare to those calculated in the absence of risk aversion? We compute the relative gain from backdating under the Black Scholes model, given by $\%\mathcal{G}^{BS} = (V_b^{BS} - V_{nb}^{BS})/V_{nb}^{BS}$. Table 1 displays these values for the benefit of backdating to executives under a Black Scholes framework for a one month backdating window. For the same base parameters as earlier, we find the comparable Black Scholes percentage gain is only around 3.2% or \$ 0.24 million in dollar terms. Across the same parameters as in Table 2, we see the percentage gains under Black Scholes are much smaller and vary between 2.2% and 8.2%.

Now consider a much shorter backdating window of two days. For an executive with our base level of risk aversion, and with stock parameters of $r = 0.05, q = 0$ and volatility 60%, from Table 3, the value of the grant in the absence of backdating is \$ 2.14 million. This increases by 5% to a value of \$ 2.25 million when the executive has the opportunity to backdate over a two day lookback window. The Black Scholes model would suggest a much smaller benefit of just over 1%. Panel B of Table 3 shows the values with and without the backdating opportunity and the percentage gains from the opportunity to backdate over two days vary between 2.3% and 7.5% across our range of other parameters. Across the same parameters, in Panel A we see the comparable percentage gains under Black Scholes are much smaller and vary between 0.65% and 2.9%. Despite the backdating window shrinking dramatically under the post-SOX rules, the benefit to the risk averse executive from having the opportunity to backdate was still present and our base parameters suggest these gains can be around 5%.

We now consider varying the length of the backdating window between two days and six months and observe in Figure 1 how the relative or percentage gain from backdating varies with the length of the window. For example, the figure of 17% for a one-month lookback window is plotted on the figure. The value of the option with the opportunity to backdate, and the percentage gain, both increase when the length of the lookback window increases, keeping all else fixed.

We ask the question: why are the gains to executives so much larger when risk aversion is taken into account? In fact, non-diversifiable risk reduces option values of both backdated and non-backdated options relative to their Black Scholes equivalents. We return to our earlier example to illustrate this. A risk averse executive with the base level of risk aversion and $r = 0.05, q = 0$ and volatility 60% placed a value of \$ 2.48 million on the grant if she could backdate over a one month window. However, an otherwise identical but risk neutral executive would have valued this opportunity at \$ 7.63 million.[4] The fact that an executive with exposure to non-diversifiable risk

[4]Similarly, the risk averse executive values the option grant in the absence of backdating at \$ 2.12 million whereas the risk neutral executive places a much higher value of \$ 7.4 million on this grant of options, see Tables 1 and 2.

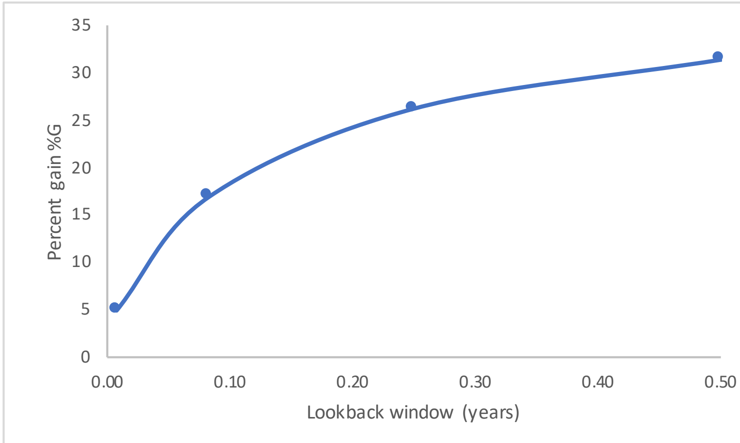


Figure 1: Percentage gain ($\%\mathcal{G}$) from backdating opportunity to a risk averse executive, with varying lengths of lookback window, $T_0$. On the x-axis is $T_0$ expressed in years. The option grant is $N = 1$ million American call options with a ten year maturity $T = 10$. Parameters are: $S_0 = 10$, risk aversion $\gamma = 0.2$, $r = 0.05, q = 0$, volatility $\eta = 0.6$, correlation $\rho = 0$.

places a much lower value on an option grant (in the absence of backdating) is well established - the presence of non-diversifiable risk represents a cost to the executive which remains until the options are exercised. This cost encourages the executive to exercise earlier, at a lower moneyness, than in a Black Scholes world. This reasoning carries over to the situation here where the executive has the opportunity to backdate.

To illustrate this point, Figure 2 plots exercise thresholds for these base case parameters and two values of risk aversion - a low value, $\gamma = 0.01$, and our base value, $\gamma = 0.2$. Thresholds are plotted over the period $[T_0, T + T_0]$ when the option is exercisable. We can indeed see that risk aversion lowers the threshold. Earlier works such as Oberman & Zariphopoulou (2003), Leung & Sircar (2009) contain some further comparative statics on such exercise thresholds for standard American options in a utility indifference setting.

Although risk aversion reduces the option values of both backdated and non-backdated options relative to their Black Scholes equivalents, there is a larger proportional reduction in value for at-the-money options. Thus there is a larger difference between the utility indifference value and the Black Scholes value of non-backdated options than for the backdated options. This means that the ratio $V_b/V_{nb}$ is larger than the equivalent Black Scholes ratio $V_b^{BS}/V_{nb}^{BS}$. Why is this the case? Call option time-value is maximized close-to-the-money so that far in-the-money call options have little time-value. Risk aversion impacts option values primarily through its effect on option time-value, since it only affects value whilst the executive continues to hold the exercisable option. Thus risk aversion has greater impact on at-the-money than in-the-money American options. In addition to

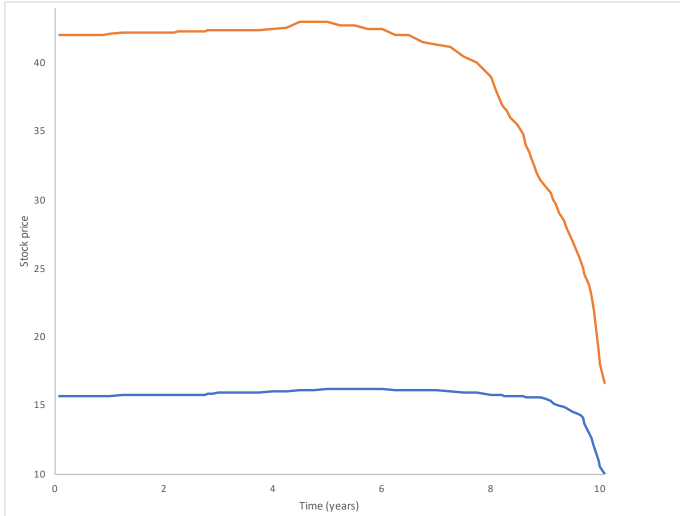


Figure 2: Exercise thresholds $S_b^*(u); u \in [T_0, T + T_0]$ for the base case option parameters and for values of risk aversion $\gamma = 0.01$ (higher threshold) and $\gamma = 0.2$ (lower threshold). The option grant is $N = 1$ million American call options with a ten year maturity $T = 10$ and a one month backdating window $T_0 = 1/12$. Strike is $J = min_{0 \le u \le T_0} S_u = 9.94$. Parameters are: $S_0 = 10$, $r = 0.05, q = 0$, volatility $\eta = 0.6$, correlation $\rho = 0$.

the discussion above, we also need to consider the impact of the lookback option inherent in the opportunity to backdate. Other things being equal, the longer the lookback period, the greater the expected strike discount on the backdated option and thus the further in-the-money it will be at $T_0$. Thus, the longer the backdating window, the greater the potential difference in moneyness.

As well as backdating being most valuable to an executive with higher risk aversion, we also see from Tables 2 and 3 that backdating is more valuable when a stock has a higher dividend yield and volatility. Each of these magnifies the impact of non-diversifiable risk on at-the-money relative to in-the-money American exercise options. Numerical investigations show that non-zero correlation reduces the benefit of backdating. For example, for our base level of risk aversion, and with $r = 0.05, q = 0$ and volatility 60%, with correlation of 0.15, the percentage gain from backdating over a one month window is 15.6%.

At this point it is worthwhile returning to compare our results to those computed by Eikseth & Lindset (2011) who also consider an ex ante model, but without executive risk aversion. Under their parameters of $S = \$1, r = 5\%, q = 2\%, \sigma = 30\%, T = 10$, with a one month window and without American early exercise, they obtain a gain in value from backdating of around 4%. (Under their extended model with infinitely lived options and executive departure, this gain will rise slightly, however they do not report numbers for a one month window so it is difficult to compare directly to

Black Scholes

| | | $\eta = 0.2$ | $\eta = 0.4$ | $\eta = 0.6$ | $\eta = 0.8$ |
|---|---|---|---|---|---|
| q = 0.0 | $V_b^{BS}$ | 4.695 | 6.255 | 7.632 | 8.532 |
| | $V_{nb}^{BS}$ | 4.531 | 6.038 | 7.398 | 8.351 |
| | $\%\mathcal{G}^{\mathcal{BS}}$ | **3.633%** | **3.591%** | **3.162%** | **2.167%** |
| q = 0.05 | $V_b^{BS}$ | 2.001 | 3.775 | 5.244 | 6.115 |
| | $V_{nb}^{BS}$ | 1.850 | 3.533 | 4.968 | 6.098 |
| | $\%\mathcal{G}^{\mathcal{BS}}$ | **8.192%** | **6.852%** | **5.550 %** | **2.788 %** |

Table 1: *Black Scholes values/costs of the option grant with and without the opportunity to backdate over a one month window.* Denote by $V_b^{BS}$ the Black Scholes value of the option grant with the opportunity to backdate, $V_{nb}^{BS}$ the Black Scholes value of the non-backdated grant, and $\%\mathcal{G}^{BS}$ the % gain to the risk neutral executive from having the opportunity to backdate. These also represent the costs to well-diversified shareholders. The option grant is $N = 1$ million American call options with a ten year maturity $T = 10$ and so all values and costs are expressed in millions of dollars. The Table varies volatility $\eta$ and dividend yield $q$ and we take $T_0 = 1/12$, $r = 0.05$ and $S_0 = 10$.

ours). Using their parameters, and with our base risk aversion, we can apply our model to estimate the gains from backdating over a one month window. We find the value of the grant in the absence of backdating is \$ 0.326 million. This increases to \$ 0.347 million when backdating over one month, giving a larger increase in value of 6.2%. We note that as we have shown previously, the effect of risk aversion on gains to backdating is stronger at higher volatility levels and thus for higher volatility, we would obtain larger differences.

### 3.2 The magnitude of the cost of backdating to shareholders

In this section, we analyze the cost of backdating to shareholders. We calculate the increased cost of the option grant from the perspective of well-diversified shareholders if executives have the opportunity to backdate as $\%\mathcal{C} = (C_b - C_{nb})/C_{nb}$. These additional costs are also significant in magnitude. For the firm employing our executive with base level of risk aversion and parameters $r = 0.05, q = 0, \eta = 0.6$, the cost to the firm of the grant in the absence of backdating is \$ 3.19 million. If the executive can backdate over a one month window, this cost rises to \$ 3.42 million, a proportional increase of 7.4%. Table 2 shows the percentage rises in compensation costs to the firm range from 4.67% to 11.8%, assuming our base level of risk aversion. We also see from Table 2 that backdating is most costly to shareholders when executives are more risk averse, and when a stock has a higher dividend yield and volatility. Furthermore, Table 3 shows the cost to shareholders remains significant in the post-SOX era, when executives only have a two day window in which to backdate, and particularly so when volatility is high.

Panel A: Risk averse with $\gamma = 0.01$

| | | $\eta = 0.2$ | $\eta = 0.4$ | $\eta = 0.6$ | $\eta = 0.8$ |
|---|---|---|---|---|---|
| q = 0.0 | $V_b$ | 4.324 | 5.092 | 5.729 | 6.057 |
| | $V_{nb}$ | 4.125 | 4.847 | 5.438 | 5.711 |
| | $\%\mathcal{G}$ | **4.811%** | **5.051%** | **5.346%** | **6.058%** |
| | $C_b$ | 4.703 | 5.8 | 6.696 | 7.195 |
| | $C_{nb}$ | 4.497 | 5.562 | 6.455 | 6.980 |
| | $\%\mathcal{C}$ | **4.581%** | **4.222%** | **3.737%** | **3.082%** |
| q = 0.05 | $V_b$ | 1.933 | 3.472 | 4.542 | 5.182 |
| | $V_{nb}$ | 1.776 | 3.221 | 4.218 | 4.805 |
| | $\%\mathcal{G}$ | **8.814%** | **7.790%** | **7.688 %** | **7.846%** |
| | $C_b$ | 1.992 | 3.731 | 5.056 | 5.932 |
| | $C_{nb}$ | 1.836 | 3.490 | 4.781 | 5.652 |
| | $\%\mathcal{C}$ | **8.551%** | **6.917%** | **5.769 %** | **4.961%** |

Panel B: Risk averse with $\gamma = 0.2$

| | | $\eta = 0.2$ | $\eta = 0.4$ | $\eta = 0.6$ | $\eta = 0.8$ |
|---|---|---|---|---|---|
| q = 0.0 | $V_b$ | 2.221 | 2.338 | 2.481 | 2.577 |
| | $V_{nb}$ | 2.072 | 2.092 | 2.121 | 2.100 |
| | $\%\mathcal{G}$ | **7.198%** | **11.781%** | **16.969%** | **22.714%** |
| | $C_b$ | 3.105 | 3.264 | 3.424 | 3.535 |
| | $C_{nb}$ | 2.966 | 3.086 | 3.189 | 3.218 |
| | $\%\mathcal{C}$ | **4.671%** | **5.765%** | **7.377%** | **9.841%** |
| q = 0.05 | $V_b$ | 1.345 | 1.927 | 2.241 | 2.427 |
| | $V_{nb}$ | 1.195 | 1.664 | 1.862 | 1.934 |
| | $\%\mathcal{G}$ | **12.541%** | **15.802%** | **20.326 %** | **25.491%** |
| | $C_b$ | 1.709 | 2.576 | 3.017 | 3.284 |
| | $C_{nb}$ | 1.566 | 2.355 | 2.742 | 2.938 |
| | $\%\mathcal{C}$ | **9.160%** | **9.391%** | **10.029 %** | **11.791%** |

Table 2: *Values to the risk averse executive and costs to shareholders of the option grant with and without the opportunity to backdate over a one month window.* Denote by $V_b$ the utility indifference value of the option grant with the opportunity to backdate, $V_{nb}$ the utility indifference value of the non-backdated grant, and $\%\mathcal{G}$ the % gain to the executive from the backdating opportunity. Also denote by $C_b$ the cost of the backdated option to shareholders, $C_{nb}$ the cost of the non-backdated option to shareholders and $\%\mathcal{C}$ the % cost to shareholders if executives have the opportunity backdate. The option grant is $N = 1$ million American call options with a ten year maturity $T = 10$ and so all values and costs are expressed in millions of dollars. The Table varies risk aversion $\gamma$, volatility $\eta$ and dividend yield $q$. Parameters unless otherwise stated are: $T_0 = 1/12$, $r = 0.05$, $S_0 = 10$, and correlation $\rho = 0$.

Panel A: Black Scholes

| | | $\eta = 0.2$ | $\eta = 0.4$ | $\eta = 0.6$ | $\eta = 0.8$ |
|---|---|---|---|---|---|
| q = 0.0 | $V_b^{BS}$ | 4.576 | 6.092 | 7.441 | 8.404 |
| | $V_{nb}^{BS}$ | 4.511 | 6.015 | 7.365 | 8.35 |
| | $\%\mathcal{G}^{\mathcal{BS}}$ | **1.44%** | **1.28%** | **1.03%** | **0.647%** |
| q = 0.05 | $V_b^{BS}$ | 1.897 | 3.618 | 5.069 | 6.196 |
| | $V_{nb}^{BS}$ | 1.844 | 3.542 | 4.989 | 6.123 |
| | $\%\mathcal{G}^{\mathcal{BS}}$ | **2.874%** | **2.146%** | **1.604%** | **1.24%** |

Panel B: Risk averse with $\gamma = 0.2$

| | | $\eta = 0.2$ | $\eta = 0.4$ | $\eta = 0.6$ | $\eta = 0.8$ |
|---|---|---|---|---|---|
| q = 0.0 | $V_b$ | 2.122 | 2.176 | 2.247 | 2.275 |
| | $V_{nb}$ | 2.074 | 2.099 | 2.1395 | 2.133 |
| | $\%\mathcal{G}$ | **2.314 %** | **3.668%** | **5.024%** | **6.657%** |
| | $C_b$ | 3.013 | 3.151 | 3.277 | 3.314 |
| | $C_{nb}$ | 2.968 | 3.097 | 3.207 | 3.222 |
| | $\%\mathcal{C}$ | **1.505%** | **1.75%** | **2.17%** | **2.863%** |
| q = 0.05 | $V_b$ | 1.248 | 1.755 | 1.995 | 2.114 |
| | $V_{nb}$ | 1.199 | 1.674 | 1.882 | 1.967 |
| | $\%\mathcal{G}$ | **4.087 %** | **4.839%** | **6%** | **7.473%** |
| | $C_b$ | 1.618 | 2.445 | 2.844 | 3.032 |
| | $C_{nb}$ | 1.571 | 2.377 | 2.762 | 2.931 |
| | $\%\mathcal{C}$ | **2.981%** | **2.875%** | **2.958%** | **3.459%** |

Table 3: *Values of the option grant with and without the opportunity to backdate over a* **two day** *window.* Panel A gives values under the Black Scholes model. These also represent costs to well diversified shareholders. Panel B considers a risk averse executive with the base parameter value of $\gamma = 0.2$. All other parameters are those used in Table 2.

How does the magnitude of the cost increase compare to that of the gains to the executive? Recall, in the absence of backdating, non-diversifiable risk exposure means that the indifference value of options to the executive is less than their objective cost to shareholders. However, when we measure the benefit of the opportunity to backdate relative to the increased cost to shareholders, we find the opposite is true. The additional cost to the firm's shareholders from backdating is lower than the magnitude of the gains that are expected by the risk averse executive. For example, our base case executive enjoyed a potential gain of 17% from having the opportunity to backdate, whilst the cost to the firm increased by only 7.4%. Furthermore, the additional cost to the firm is greater when the executive is risk averse than if the executive were risk neutral.

A similar intuition to that used earlier can be used to explain these relationships. First, observe that calculation of the objective cost of options involves using the optimal exercise threshold of the executive (see earlier). Thus for in-the-money options, those that are backdated, there is much less of a gap between value and cost, as they are both tied to the intrinsic value at the same threshold, and the threshold is closer for in-the-money than at-the-money options. This results in the ratio $V_b/V_{nb}$ exceeding $C_b/C_{nb}$, ie. the benefit of backdating to the executive is greater than the increased cost to shareholders. We now give an explanation as to why the increased cost to shareholders is greater if executives are risk averse than if they are risk neutral. Observe that although shareholders are assumed risk neutral, they use the exercise threshold of the risk averse executive, which is lower than that of a risk neutral executive. This translates into option costs being lowered for both backdated and non-backdated options, relative to Black Scholes costs, but as before, there is a greater proportional reduction in at-the-money costs. Thus there is a larger difference between the objective cost and Black Scholes cost for non-backdated options than for the backdated options, again because of the effect of risk aversion on option time value.

# 4 The impact of volatility on the ex ante value of the option to backdate

Empirical and anecdotal evidence suggests backdating was disproportionately prevalent in the technology sector. Motivated by anecdotal observations, Heron & Lie (2009) found that that tech firms and firms with high stock price volatility are significantly more likely to manipulate grants. The estimated fraction of unscheduled, at-the-money grants that are manipulated is 20.1% among non-high-tech firms and 32.0% among high-tech firms. Further, they estimate the fraction of unscheduled at-the-money grants that are manipulated to be 13.6% among firms with low volatility, 26.2% among firms with medium volatility and 29.0% among firms with high volatility. Controlling for CEO stock and option holdings, Bizjak, Lemmon & Whitby (2009) find stock price volatility to be positively associated with the likelihood of starting to backdate, and comment that "firms are more likely to be able to take advantage of backdating options when the stock price is more variable". Bebchuk, Grinstein & Peyer (2010) show that this is not only due to differences between high and low volatility firms, but that grant events were more likely to be lucky in months when the difference between the lowest and median stock price was greatest. They find that pre-SOX, 55% of all "lucky" grants were unexpectedly granted at the lowest price and thus can be estimated to have been manipulated. However, in the class of grants with the highest volatility quartile (decile), this fraction increases to 71% (76%).

Why is there such strong anecdotal and empirical evidence of a link between high volatility firms and backdating? Our model is able to shed light on this link because three key components of our model - an ex ante viewpoint, risk aversion and American exercise - play an important role. However, first we outline the reasoning that appears most often in the literature (see Heron & Lie 2009, Bizjak, Lemmon & Whitby 2009, Bebchuk, Grinstein & Peyer 2010). As alluded to in the discussion above, the explanation is that firms with higher volatility have greater opportunities to reduce the option's strike price via backdating. And the larger the expected strike price discount, the greater are the expected gains from backdating. (The expected strike price discount effect will be greater, the longer the lookback period over which the executive can backdate options.) In fact, although these authors do not acknowledge it, their reasoning is analogous to valuing the lookback option inherent in the option to backdate, as we have quantified in this paper. Although this argument is certainly true, there is also a counteracting effect. A comparison between the backdated and non-backdated option values at $T_0$ only considers the difference in their respective strikes. (The non-backdated option is granted at-the-money with strike $S_{T_0}$, whilst the backdated option has a lower strike $J$, as $J \leq S_{T_0}$, and so is in-the-money at $T_0$). However, before $T_0$ option values also depend on volatility. Black Scholes call option values are increasing in volatility (vega is positive) but their sensitivity to volatility depends upon moneyness. At-the-money call options are more sensitive to volatility than in-the-money options and so non-backdated options are more sensitive to volatility than backdated options. Thus for a given increase in volatility, a non-backdated option has a larger increase in value than an otherwise equivalent backdated option. This effect, which we call the moneyness-on-vega effect, works in the opposite direction to the expected strike price discount effect.

Walker (2007) (and Dierker & Hemmer 2007) recognized both the expected strike price discount and the moneyness-on-vega effect but demonstrated via "back-of-the-envelope" Modified Black Scholes calculations (with an ex post fixed strike adjustment and a six year expected life), that the benefit from backdating tends to be lower if volatility is higher. European Black Scholes computations of Eikseth & Lindset (2011) also find this relationship. (Eikseth & Lindset 2011 also find in an extended version of their model with an infinitely lived option and executive departure that the value of backdating is relatively insensitive to changes in volatility.) That is, each of these papers finds the opposite prediction to that found in the empirical literature.

In Tables 1 and 3 (A), we report the benefit from backdating for varying volatility, calculated under a Black Scholes model. The benefit from backdating is calculated as the proportional increase in Black Scholes value relative to the non-backdating benchmark value, ie. $\%\mathcal{G}^{BS} = (V_b^{BS} - V_{nb}^{BS})/V_{nb}^{BS}$. Note, in contrast to the use of Modified Black Scholes in the literature, our Black Scholes values take into account optimal exercise timing. However, for the wide range of parameters considered in the Table, we find that the benefit of backdating drops as volatility increases. Thus, consistent with the examples in the aforementioned papers, we find the moneyness-on-vega effect tends to dominate over the expected strike discount in a Black Scholes setting.

Up to now, we have shown that under Black Scholes (with either optimal exercise, or fixed expected life), we tend to obtain the opposite theoretical prediction to that observed empirically. How do we resolve this puzzle? Consider our utility indifference model and let us now revisit the moneyness-on-vega effect. When the executive is risk averse, an in-the-money American call option value is still less sensitive to volatility than an at-the-money American call, but the difference between their sensitivities is much smaller than under Black Scholes. Hence, whilst this causes the benefit of backdating to decrease with volatility, as it did under Black Scholes, it now decreases less than under Black Scholes. Why is this the case? It is well understood that risk aversion and non-diversifiable risk decreases option values and option vegas relative to Black Scholes values. Option vegas under the utility indifference pricing model can be negative for sufficiently far in-the-money options (see Carpenter, Stanton & Wallace 2010, Miao & Wang 2007, Henderson 2007). However, the American exercise feature places a lower bound on negative vega, and hence the option vegas vary within a tighter band than under the Black Scholes model.

Table 2 reports the benefit from the backdating opportunity for varying volatility, calculated under the utility-based model which takes into account the executive's exposure to non-diversifiable risk. We see that in Panel B, for our base level of risk aversion, the benefit from backdating is always increasing in volatility. Even in Panel A, with very low risk aversion, the backdating benefit is increasing in volatility in all-but-one parameter set. Strikingly, in Panel B of Table 3, we see that the benefit from backdating is increasing in volatility, even for just a two day backdating window.

Thus, taking account of non-diversifiable risk exposure in the model has altered the tradeoff between the expected strike discount effect and the effect of moneyness-on-vega. Tables 2 and 3 show that for most parameter values, the expected strike discount effect is now the dominant one, implying that the benefit from backdating increases with volatility.

We highlight here that the ability of our model to explain the relationship between volatility and backdating hinges on taking an ex ante view (valuing the lookback option) in combination with the inclusion of both non-diversifiable risk and the American exercise feature and hence necessitates consideration of a sophisticated model. The value of a European option is not tied to the intrinsic value at the exercise threshold and can go below intrinsic value. For equivalent European options in the utility-based model, it would not necessarily be the case that the difference in sensitivities of in and at-the-money calls is reduced, relative to Black Scholes. The implication of this is that just using a utility indifference model with a fixed maturity adjustment (rather than optimal exercise) will not be sufficient to predict the empirical relationship between backdating and volatility.

# 5 Conclusion

The importance of taking risk aversion and exposure of executives to non-diversifiable risk into account in valuing their stock option compensation was well known. This paper shows that this feature is of paramount importance in understanding the value of backdating to executives and has enabled us to resolve a long outstanding puzzle in the literature of why backdating has been more prevalent at firms with highly volatile stocks. We have provided a modeling framework and demonstrated that indeed, risk aversion and exposure to non-diversifiable risk, together with early exercise features of options, mean that the opportunity to backdate is particularly valuable to executives. We go further still in being able to demonstrate that the more risk averse an executive, the more valuable the option to backdate becomes. This is in sharp contrast to the well understood negative relation between risk aversion and utility indifference prices or values for European or American call options.

Furthermore, the additional costs to shareholders when executives backdate were significant and thus firms are likely to have underestimated the cost to shareholders of options during the period of time when backdating was common. However, we remark that in common with much of the literature, we have not modelled the additional indirect costs of backdating.

Prior to our research, there was a mismatch between the empirical finding that backdating is more prevalent in firms with highly volatile stocks and the predictions of the theoretical literature. We have shown that by considering features of the contracts such as American exercise, together with risk aversion and non-diversifiable risk, our theoretical model gives predictions which are consistent with the empirical evidence.

# 6 Appendix

## 6.1 Details of numerical solution of free boundary problems

*Valuing the option grant with backdating*

We use separation of variables and a power transformation via $\mathcal{Y}_b(u,w,s;J) = \mathcal{M}(u,w,T_0+T)\mathcal{H}_b(u,s;J)^{1/(1-\rho^2)}$ to restate (4), (5), (6) and (7) as

$$\mathcal{H}_b(u,S_u;J) \geq e^{-\gamma(1-\rho^2)(s-J)^+ e^{r(T_0+T-u)}} \tag{16}$$

$$\frac{\partial \mathcal{H}_b}{\partial t} + \{\tilde{\mathcal{L}}\mathcal{H}_b\} \leq 0 \tag{17}$$

$$(\frac{\partial \mathcal{H}_b}{\partial t} + \{\tilde{\mathcal{L}}\mathcal{H}_b\})(e^{-\gamma(1-\rho^2)(s-J)^+ e^{r(T_0+T-u)}} - \mathcal{H}_b(u,S_u;J)) = 0 \tag{18}$$

where the differential operator $\tilde{\mathcal{L}}$ is defined by

$$\tilde{\mathcal{L}} = \frac{\eta^2 s^2}{2}\frac{\partial^2}{\partial s^2} + (\nu - q)s\frac{\partial}{\partial s} \tag{19}$$

Transformed boundary conditions are $\mathcal{H}_b(T_0+T,s;J) = e^{-\gamma(1-\rho^2)(s-J)^+}$ and $\mathcal{H}_b(u,0;J) = 1$.

We next transform to the heat equation by defining new variables $x \in \mathbb{R}$ and $\tau \in [0, 0.5\eta^2(T_0 + T - u)]$ via $S = \delta e^x, J = \delta e^y, \xi = \frac{1}{2}(2(\nu-q)/\eta^2 - 1)$ and $\tau = \frac{1}{2}\eta^2(T_0+T-u)$. Write

$$\mathcal{H}_b(T_0+T-\tau/0.5\eta^2, \delta e^x; \delta e^y) = \delta u_b(x,y,\tau)e^{-\xi x - \xi^2\tau} \tag{20}$$

*During* $[T_0, T_0+T]$

The domain of $(x,y,\tau)$ is truncated from $\mathbb{R}\times\mathbb{R}\times[0,0.5\eta^2(T_0+T)]$ to a finite domain $[x_{min}, x'_{max}] \times [x_{min}, x_{max}] \times [0, 0.5\eta^2(T_0+T)]$. Let $U^n_{i,j}$ be the approximate value of $u_b(x,y,\tau)$ at node $(x_{min}+i\Delta x, x_{min}+j\Delta x, n\Delta\tau)$ on a uniform 3-d grid for $j = 0,1,2,...,k_{max}$, $n = 0,1,2,...,n_{max}$ and $i = j, j+1, ..., j_{max}$, $x'_{max} = x_{min} + j_{max}\Delta x$. We take $k_{max} = 2000$, $n_{max} = 10000$, $x_{min} = -3$, $x_{max} = 3$. $j_{max}$ is determined to ensure the exercise threshold is sufficiently far from the upper bound of the stock price. Given the upper bound of strike $S_{T_0,k_{max}}$, we increase $j_{max}$ until the index corresponding to the exercise threshold (anywhere in time) is less than $j_{max} - 50$. We undertook extensive testing for different values of each of these choices. We employ a Crank Nicolson finite difference approximation with the free boundary constraint enforced by a projected successive over relaxation algorithm (PSOR). For similar numerical schemes for standard American options we refer the reader to Wilmott et al (2005).

*During the backdating window* $[0, T_0]$

First, observe that $\mathcal{Y}_b(u, W_u, S_u, J)$ solves the pde

$$\frac{\partial \mathcal{Y}_b}{\partial t} + \sup_{\theta}\{\mathcal{L}\mathcal{Y}_b\} = 0 \tag{21}$$

where the differential operator is given in (7), and with boundary condition at $T_0$:

$$\mathcal{Y}_b(T_0, W_{T_0}, S_{T_0}; J_{T_0}) = \mathcal{M}(T_0, W_{T_0} + V_b(T_0, S_{T_0}, J_{T_0}), T_0 + T). \tag{22}$$

Similar boundary conditions to a lookback option are needed: an upper bound for large $S$, a lower bound for $S = J$, and an implicit boundary condition for $J = 0$. For the upper bound for large $S$, we need an approximation to the price, as it does not simply increase linearly in the underlying as in a Black Scholes setting. We thus calculate an approximation via numerical integration. We have for $u \in [0, T_0]$

$$\mathcal{H}_b(u, S_{max}; J) = \mathbb{E}[e^{-\gamma(1-\rho^2)e^{r(T_0-u)}V_b(T_0,S_{T_0};J)}] = \int_0^\infty e^{-\gamma(1-\rho^2)e^{r((T_0+T)-T_0)}V_b(T_0,S_{T_0};J)} f(S_{T_0}, T_0; S_{max}) dS_{T_0} \tag{23}$$

where $f(S_{T_0}, T_0; S_{max})$ is the lognormal density function:

$$f(S_{T_0}, T_0; S_{max}) = \frac{1}{S_{T_0}\eta\sqrt{T_0 - u}\sqrt{2\pi}} exp[-\frac{(ln(S_{T_0}/S_{max}) - (\nu - q - \frac{1}{2}\eta^2)(T_0 - u))^2}{2\eta^2(T_0 - u)}] \tag{24}$$

and $V_b(T_0, S_{T_0}; J)$ is the known value of a backdated call option with strike $J$, maturing at $T_0 + T$, calculated as described above.

For the lower bound for $S = J$, we use the fact that the utility value remains unchanged for a small change in $J$ when $S = J$. For the implicit boundary condition $J = 0$, the option value is equal to the stock price, which gives

$$u_b(x, x_{min}, \tau) = \frac{1}{\delta} e^{\xi x + \xi^2 \tau} e^{-\gamma(1-\rho^2)(\delta e^x)e^{2r\tau/\eta^2}} \tag{25}$$

We use an explicit finite difference approximation during the backdating window $[0, T_0]$ as it allows us to evaluate the boundary conditions, particularly the upper bound more easily.

*Valuing the option grant without backdating*

*During the backdating window* $[0, T_0]$

We begin with the representation in (13) for the value without backdating:

$$\mathcal{Y}_{nb}(u, W_u, S_u, S_{T_0}) = \sup_{\theta_u; 0 \le u \le T_0} \mathbb{E}\mathcal{M}(T_0, W_{T_0} + V_{nb}(T_0, S_{T_0}, S_{T_0}), T_0 + T);\ 0 \le u \le T_0. \tag{26}$$

This is a European style exponential utility indifference pricing problem, and we will appeal to earlier work of Henderson (2002), Musiela & Zariphopolou (2004) to derive an explicit utility indifference price. Note also the existence and uniqueness arguments for exponential utility indifference pricing for European options are given in earlier works.

First, observe that $\mathcal{Y}_{nb}(u, W_u, S_u, S_{T_0})$ solves the pde

$$\frac{\partial \mathcal{Y}_{nb}}{\partial t} + \sup_{\theta}\{\mathcal{L}\mathcal{Y}_{nb}\} = 0 \tag{27}$$

where the differential operator is given in (7), and with boundary condition at $T_0$:

$$\mathcal{Y}_{nb}(T_0, W_{T_0}, S_{T_0}; S_{T_0}) = \mathcal{M}(T_0, W_{T_0} + V_{nb}(T_0, S_{T_0}, S_{T_0}), T_0 + T). \tag{28}$$

Again, use a separation of variables via
$\mathcal{Y}_{nb}(u, w, s; S_{T_0}) = \mathcal{M}(u, w, T_0{+}T) g_{nb}(T_0{-}u, logs)$ with $g_{nb}(0, logS_{T_0}) = e^{-\gamma V_{nb}(T_0, S_{T_0}, S_{T_0}) e^{r(T_0+T-T_0)}}$.
We now employ a Hopf-Cole transformation (Henderson 2002, Musiela and Zariphopolou 2004) via $g_{nb}(T_0 - u, log\ s) = \mathcal{G}_{nb}(\tau, z + (r - 1/2\eta^2)\tau)^{1/(1-\rho^2)}$ giving

$$-\frac{\partial \mathcal{G}_{nb}}{\partial \tau} + \frac{1}{2}\eta^2 \mathcal{G}_{nb}'' = 0 \tag{29}$$

with solution

$$\mathcal{G}_{nb}(\tau, x) = \mathbb{E}(\mathcal{G}_{nb}(0, x + \eta B_\tau)) \tag{30}$$

where $B$ is a standard Brownian motion. Then we have

$$g_{nb}(T_0 - u, logS_u) = \left[\mathbb{E}(e^{-\gamma(1-\rho^2)V_{nb}(T_0, S_{T_0}, S_{T_0})e^{r(T_0+T-T_0)}})\right]^{1/(1-\rho^2)} \tag{31}$$

and finally

$$\mathcal{Y}_{nb}(u, W_u, S_u, S_{T_0}) = \mathcal{M}(u, W_u, T_0 + T)\left[\mathbb{E}(e^{-\gamma(1-\rho^2)V_{nb}(T_0, S_{T_0}, S_{T_0})e^{r(T_0+T-T_0)}})\right]^{1/(1-\rho^2)} \tag{32}$$

$$= \quad -e^{-(\mu-r)^2/\sigma^2(T_0+T-u)} e^{-\gamma W_u e^{r(T_0+T-u)}} \left[\mathbb{E}(e^{-\gamma(1-\rho^2)V_{nb}(T_0, S_{T_0}, S_{T_0})e^{r(T_0+T-T_0)}})\right]^{1/(1-\rho^2)} \tag{33}$$

The final step is to re-express (33) in terms of the utility indifference value of the option grant using $\mathcal{Y}_{nb}(u, W_u, S_u; S_{T_0} = \mathcal{M}(u, W_u + V_{nb}(u, S_u; S_{T_0}), T_0 + T)$ to give the expression for $V_{nb}(u, S_u; S_{T_0})$ given earlier in (14). This is then computed via numerical integration.

## 6.2 Robustness

We perform various robustness checks on our numerical algorithm. We experimented with grid size versus speed to ensure the stability of our results. There are also several checks we performed. First, we obtain convergence of values computed in the utility indifference model to their equivalent Black Scholes counterparts by taking limits as $\gamma$ approaches zero or $\rho^2$ approaches one. In particular, this enables us to test various values for $k_{max}$, the number of spatial points in the grid. Second, when dividends are zero, American call options are not exercised early, so the Black Scholes value of the non-backdated option is just the Black Scholes European forward-start, which has a closed-form expression. We calibrated prices in this limiting case. Third, we also tested the code used over $[0, T_0]$ for the case of a standard European lookback call option for which there is an explicit formula (Goldman, Sosin & Gatto 1979).

### 6.3 Modified Black Scholes formula

The Modified Black Scholes formula is used by Narayanan & Seyhum (2008), Narayanan, Schipani & Seyhun (2007), Walker (2007) and Bebchuk et al (2013) in their ex post estimates of the gains from backdating to executives. The Financial Accounting Standards Boards (FASB) accepts the use of the Black and Scholes formula with the options contractual term replaced by its average or expected life to approximate for early exercise. This approximation is used by the majority of firms in option valuation. Denote the expected life by $T_e \leq T$. Denote by $K$ the strike of the option. Then the Modified Black Scholes formula is given by:

$$S_0 e^{-qT_e} N(d_1) - K e^{-rT_e} N(d_2) \tag{34}$$

where $d_1 = (\ln(S_0/K) + (r - q + \eta^2/2)T_e)/(\eta\sqrt{T_e})$ and $d_2 = d_1 - \eta\sqrt{T_e}$.

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