

Supplementary information for

Core Electron Binding Energies in Solids from Periodic All-Electron Δ -Self-Consistent-Field Calculations

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Core hole localization in $2\times 2\times 1$ supercell of magnesium

An isosurface plot of the vacant core eigenstate ($\text{Mg } 1\text{s}$ core hole in a $2\times 2\times 1$ supercell of magnesium) is shown in Figure S1. The empty core orbital is found to be localized at one of the magnesium atoms and its wavefunction has almost perfect spherical symmetry.

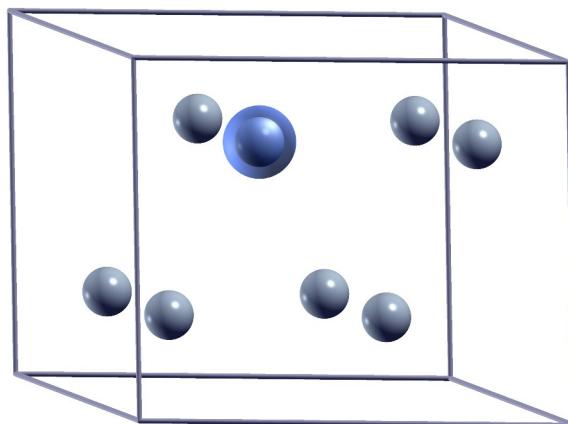


Fig. S1. A localized core hole in a $2\times 2\times 1$ supercell of magnesium. An isosurface of the probability density of the empty core eigenstate (isovalue = 1×10^{-6}) in the converged calculation of $E_{N-1, ch}$ is shown in blue. The positions of the Mg atoms are shown in gray.

We have also examined the electronic structure of the core hole final state using Mulliken analysis (Mulliken, R. S. *J. Chem. Phys.* **1955**, *23*, 1833-1840.) The results are given in Table S1. The per-atom charge analysis indicates that the magnesium atom with a core hole carries a positive charge that is somewhat less than unity, and the rest of the positive charge is distributed among the remaining Mg atoms. A closer look at the populations of the different angular momentum channels indicates that in fact, the occupancy of basis functions of p-symmetry is higher for the atom with a core hole. This is indicative of screening of the core hole by the sea of conduction electrons. The per-atom spin analysis indicates that almost the entire spin of the total system is carried by the atom with a core hole, and the spin polarization arises from s states. This is consistent with the removal of one electron from a localized s-orbital. We note that the precise values of the Mulliken populations are dependent on the chosen basis set: they provide a reasonable qualitative description of the electronic structure of the core hole state, but they cannot be interpreted as a quantitative measure of the extent of screening.

Table 1. Mulliken analysis of the electronic structure of a Mg $2\times 2\times 1$ supercell with a localized Mg 1s core hole

Atom	Electrons	Per-atom charge analysis				Per-atom spin analysis			
		Total	s ($l = 0$)	p ($l = 1$)	d ($l = 2$)	Total	s ($l = 0$)	p ($l = 1$)	d ($l = 2$)
1	11.28	0.72	3.69	7.09	0.45	-1.13	-1.00	-0.08	0.00
2	11.96	0.04	4.77	6.78	0.37	0.01	0.01	0.00	0.00
3	11.98	0.02	4.79	6.78	0.37	0.01	0.01	0.00	0.00
4	11.96	0.04	4.77	6.78	0.37	0.01	0.01	0.00	0.00
5	11.98	0.02	4.79	6.78	0.37	0.01	0.01	0.00	0.00
6	11.96	0.04	4.77	6.78	0.37	0.01	0.01	0.00	0.00
7	11.98	0.02	4.79	6.78	0.37	0.03	0.03	0.01	0.00
8	11.91	0.09	4.75	6.81	0.31	0.00	0.00	0.00	0.00

Experimental core electron binding energies

For metals and graphite, the binding energies are given relative to the Fermi level. For insulators, values referenced to the valence band maximum are reported.

Li 1s in lithium metal

Shek, M. et al. <i>Surf. Sci.</i> 1990 , <i>234</i> , 324-334.	55.1 eV
Contour, J. et al. <i>J. Microsc. Spectr. Elec.</i> 1979 , <i>4</i> , 483-491.	54.6 eV
Wertheim, G. et al. <i>Solid State Commun.</i> 1980 , <i>33</i> , 1127-1130.	54.9 eV
Kowalczyk, S. P. et al. <i>Phys. Rev. B</i> 1973 , <i>8</i> , 3583.	54.8 eV
Average:	54.85 eV

Be 1s in beryllium metal

Powell, C. J. <i>Appl. Surf. Sci.</i> 1995 , <i>89</i> , 141-149. *	111.85 eV
* recommended reference value based on three different measurements	
Average:	111.85 eV

Na 1s in sodium metal

Kowalczyk, S. P. et al. <i>Phys. Rev. B</i> 1973 , <i>8</i> , 3583.	1071.7 eV
Barrie, A. et al. <i>J. Electron. Spectrosc.</i> 1975 , <i>7</i> , 1-31.	1071.8 eV
Citrin, P. H. <i>Phys. Rev. B</i> 1973 , <i>8</i> , 5545.	1071.76 eV
Average:	1071.75 eV

Na 2p in sodium metal

Kowalczyk, S. P. et al. <i>Phys. Rev. B</i> 1973 , <i>8</i> , 3583.	30.4 eV
Barrie, A. et al. <i>J. Electron. Spectrosc.</i> 1975 , <i>7</i> , 1-31.	30.6 eV
Citrin, P. H. <i>Phys. Rev. B</i> 1973 , <i>8</i> , 5545.	30.52 eV
Average:	30.51 eV

Mg 1s in magnesium metal

Jennison, D. R. et al. <i>Solid State Phys.</i> 1984 , <i>17</i> , 3701-3710.	1303.2 eV
Yoshimura, K. et al. <i>Jpn. J. Appl. Phys.</i> 2007 , <i>46</i> , 4260-4264.	1303.5 eV
Ley, L. et al. <i>Phys. Rev. B</i> 1975 , <i>11</i> , 600.	1303.0 eV
Peng, X. D. et al. <i>Surf. Sci.</i> 1988 , <i>195</i> , 103-114.	1303.3 eV
Darrah Thomas, T. et al. <i>Phys. Rev. B</i> 1986 , <i>33</i> , 5406.	1303.2 eV
Average:	1303.24 eV

Mg 2p in magnesium metal

Powell, C. J. <i>Appl. Surf. Sci.</i> 1995 , <i>89</i> , 141-149. *	49.79 eV
* recommended reference value based on three different measurements	
Average:	49.79 eV

C 1s in graphite

Kieser, J. et al. <i>Appl. Phys.</i> 1976 , <i>9</i> , 315.	284.31 eV
Johansson, G. et al. <i>J. Electron. Spectrosc.</i> 1973 , <i>2</i> , 295-317.	284.3 eV
Xie, Y. et al. <i>Surf. Sci. Spectra</i> 1992 , <i>1</i> , 367-372.	284.64 eV
Hamrin, K. et al. <i>Phys. Scr.</i> 1970 , <i>1</i> , 277-280.	284.3 eV
Estrade-Szwarcopf, H. et al. <i>J. Phys. Chem. Solids</i> 1992 , <i>53</i> , 419-436.	284.5 eV
Average:	284.41 eV

Be 1s in BeO

Hamrin, K. et al. <i>Phys. Scr.</i> 1970 , <i>1</i> , 277-280.	109.8 eV
Koh, D. et al. <i>J. Vac. Sci. Technol. B</i> 2019 , <i>37</i> , 041206.	110.2 eV
Average:	110.00 eV

O 1s in BeO

Hamrin, K. et al. <i>Phys. Scr.</i> 1970 , <i>1</i> , 277-280.	527.7 eV
Average:	527.70 eV

B 1s in hex-BN

Hamrin, K. et al. <i>Phys. Scr.</i> 1970 , <i>1</i> , 277-280.	188.4 eV
Henck, H. et al. <i>Phys. Rev. B</i> 2017 , <i>95</i> , 085410.	188.3 eV
Average:	188.35 eV

N 1s in hex-BN

Hamrin, K. et al. <i>Phys. Scr.</i> 1970 , <i>1</i> , 277-280.	396.1 eV
Henck, H. et al. <i>Phys. Rev. B</i> 2017 , <i>95</i> , 085410.	395.9 eV
Average:	396.00 eV

C 1s in diamond

Gaowei, M. et al. <i>Appl. Phys. Lett.</i> 2012 , <i>100</i> , 201606.	284.44 eV
McFeely, F.R. et al. <i>Phys. Rev. B</i> 1974 , <i>9</i> , 5268.	283.8 eV
Kono, S. et al. <i>Jpn. J. Appl. Phys.</i> 2014 , <i>53</i> , 05FP03.	284.01 eV
Maier, F. et al. <i>Phys. Rev. B</i> 2001 , <i>64</i> , 165411.	283.9 eV
Average:	284.04 eV

Si 2p in β-SiC

Bermudez, V. M. <i>J. Appl. Phys.</i> 1988 , <i>63</i> , 4951-4959.	99.1 eV
King, S. W. et al. <i>J. Electron. Mater.</i> 1999 , <i>28</i> , L34-L37.	99.3 eV
Average:	99.20 eV

C 1s in β-SiC

Bermudez, V. M. <i>J. Appl. Phys.</i> 1988 , <i>63</i> , 4951-4959.	281.9 eV
Waldrop, J. R. et al. <i>Appl. Phys. Lett.</i> 1990 , <i>56</i> , 557-559.	281.45 eV
King, S. W. et al. <i>J. Electron. Mater.</i> 1999 , <i>28</i> , L34-L37.	281.3 eV
Average:	281.55 eV

Si 2p in silicon

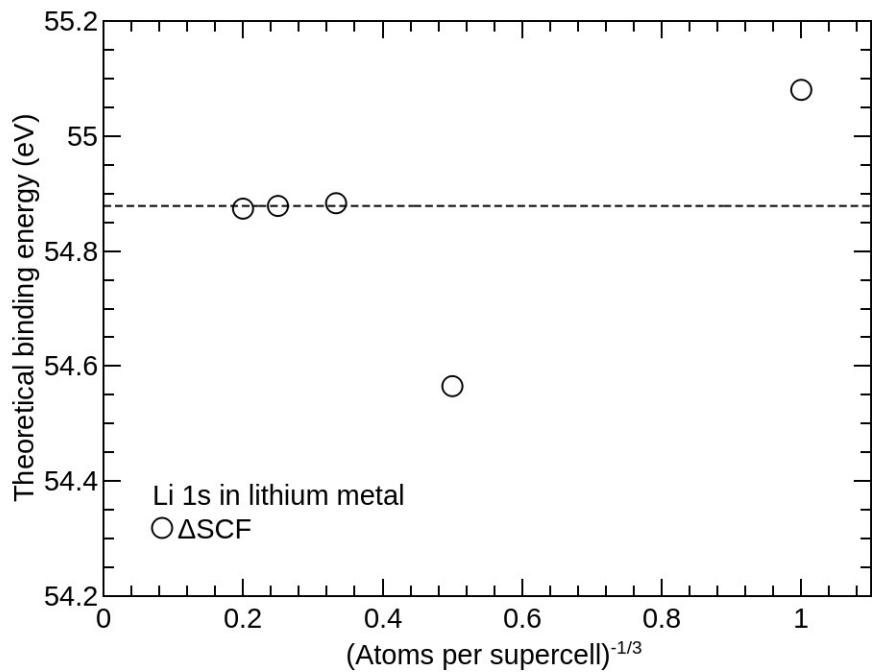
Yu, E. T. et al. <i>Appl. Phys. Lett.</i> 1990 , <i>56</i> , 569-571.	98.95 eV
Puthenkovilakam, R. et al. <i>Appl. Phys. Lett.</i> 2004 , <i>84</i> , 1353-1355.	99.1 eV
Average:	99.03 eV

High frequency (optical) dielectric constants

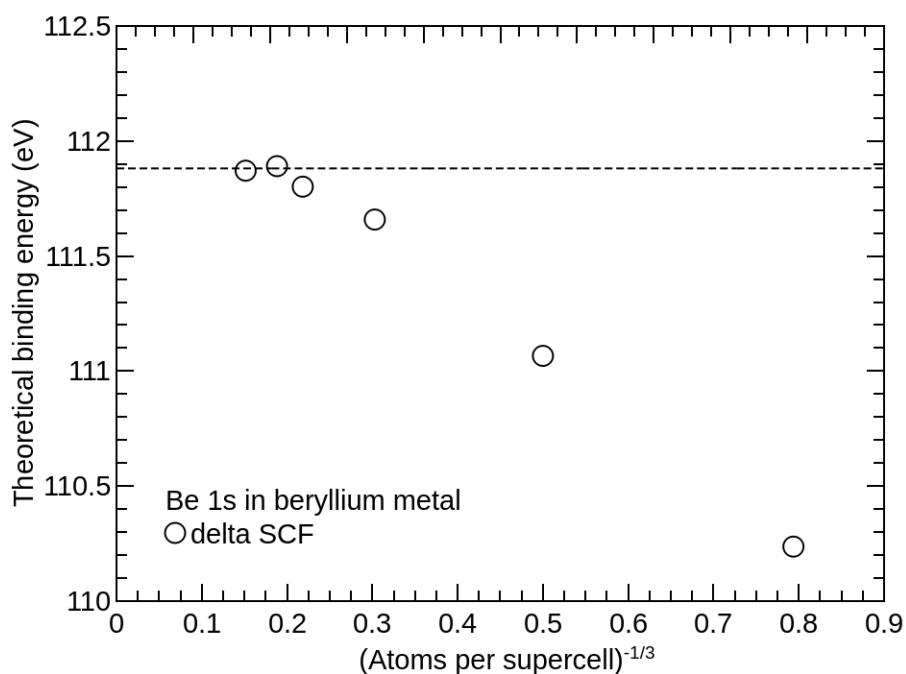
Material	Reference	ϵ_r
BeO	Gaskins, J. T. et al. <i>ECS J. Solid State Sci. Technol.</i> 2017 , <i>6</i> , N189-N208.	2.9
hex-BN	Geick, R et al. <i>Phys. Rev.</i> 1966 , <i>146</i> , 543-547. *	4.67
	* averaged over in-plane and out-of-plane directions.	
Diamond	Madelung, O. et al. <i>Semiconductors: Group IV Elements, IV-IV and III-V Compounds. Part a – Lattice Properties</i> ; Springer-Verlag: Berlin, Heidelberg, Germany, 2001.	5.7
β -SiC	Patrick, L. et al. <i>Phys. Rev. B</i> 1970 , <i>2</i> , 2255.	6.52
Silicon	Madelung, O. et al. <i>Semiconductors: Group IV Elements, IV-IV and III-V Compounds. Part a – Lattice Properties</i> ; Springer-Verlag: Berlin, Heidelberg, Germany, 2001.	11.7

Extrapolation of calculated core electron binding energies

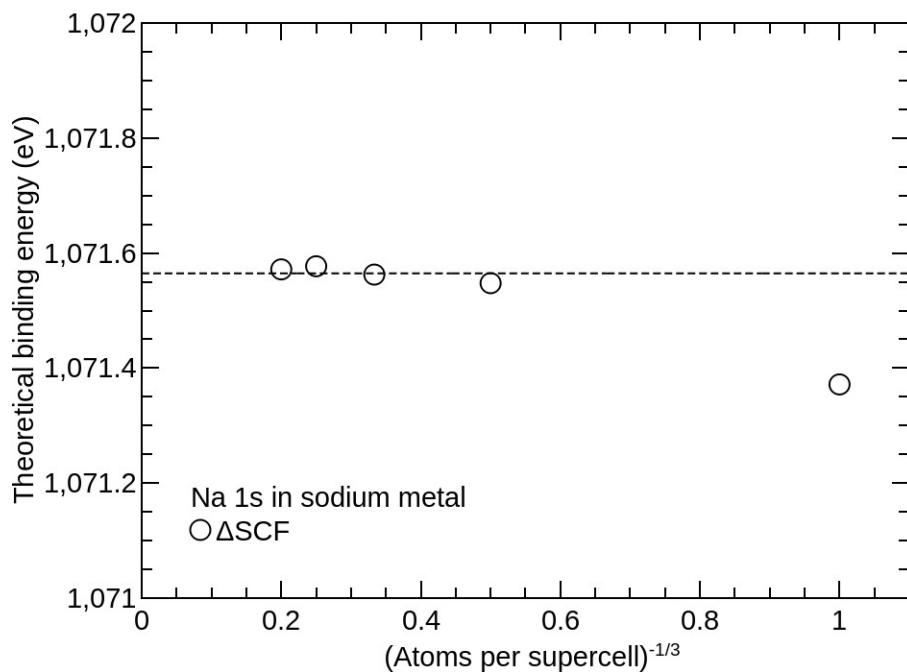
Li 1s in lithium metal



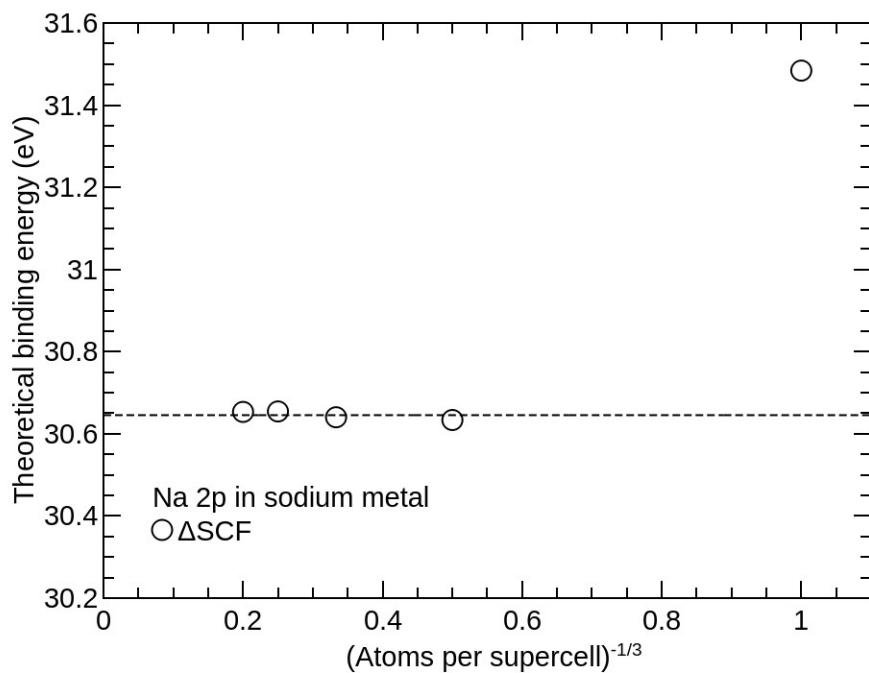
Be 1s in beryllium metal



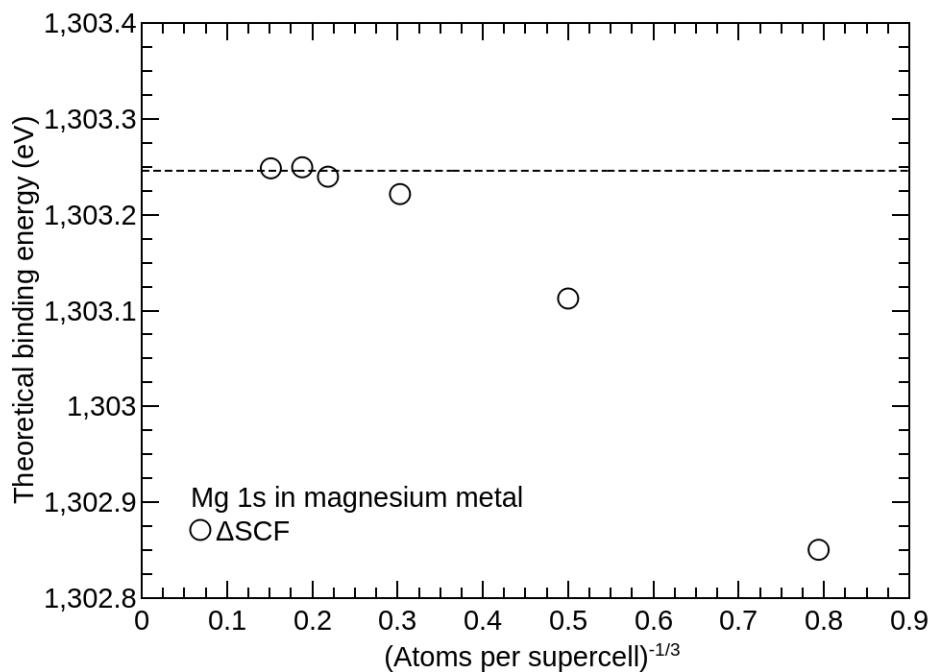
Na 1s in sodium metal



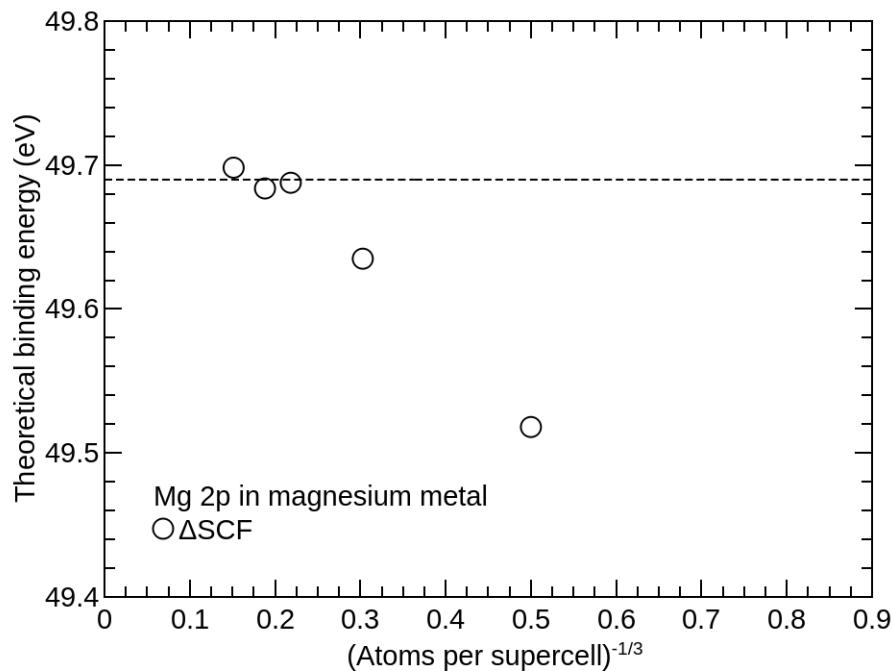
Na 2p in sodium metal



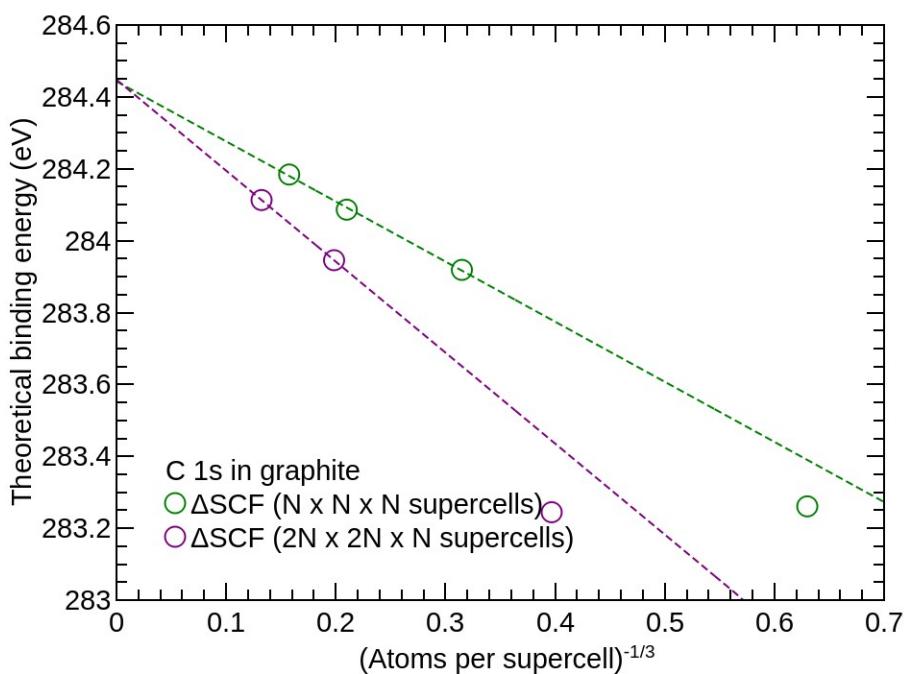
Mg 1s in magnesium metal



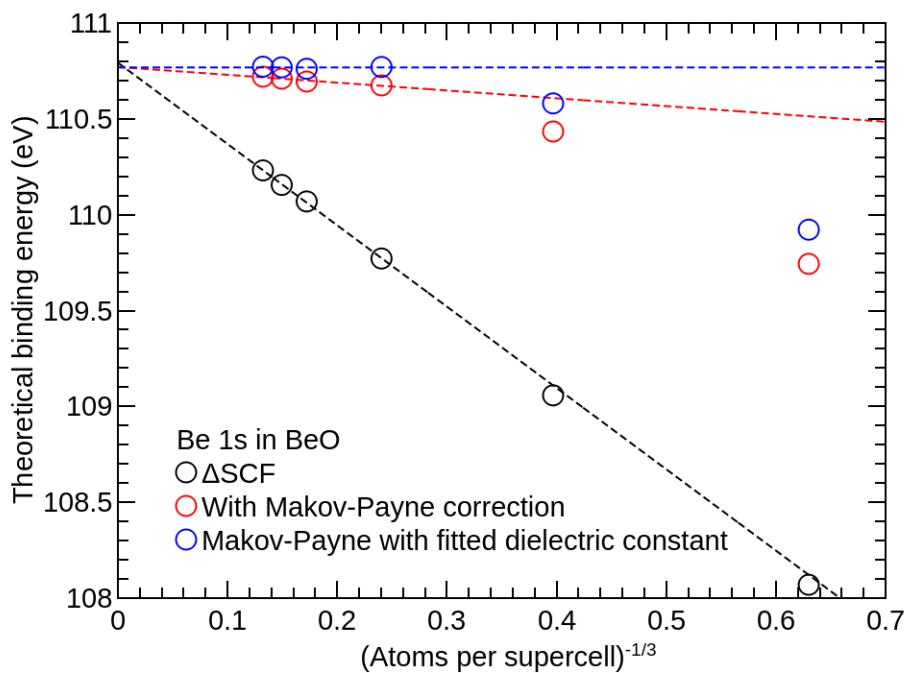
Mg 2p in magnesium metal



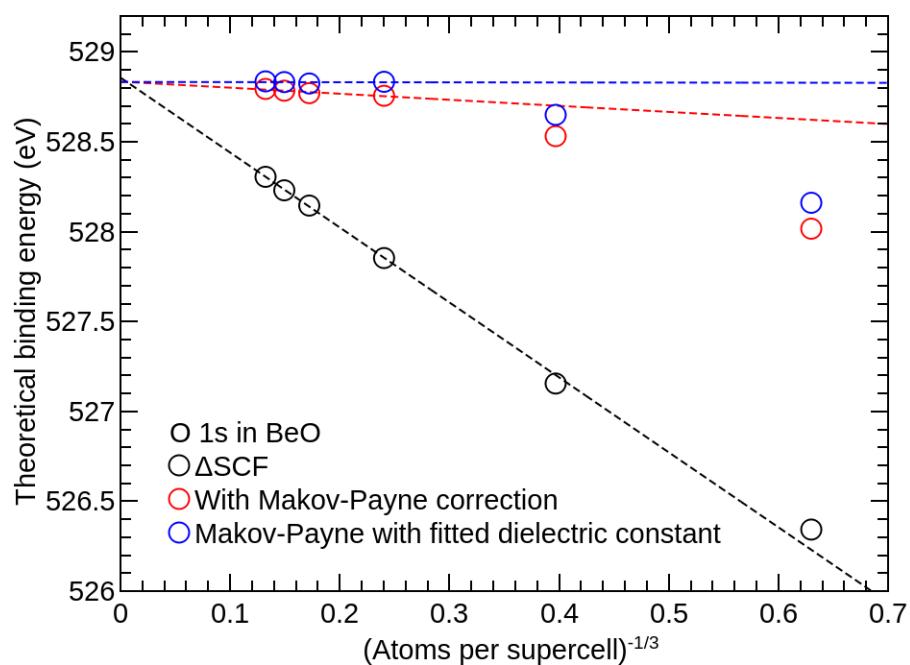
C 1s in graphite



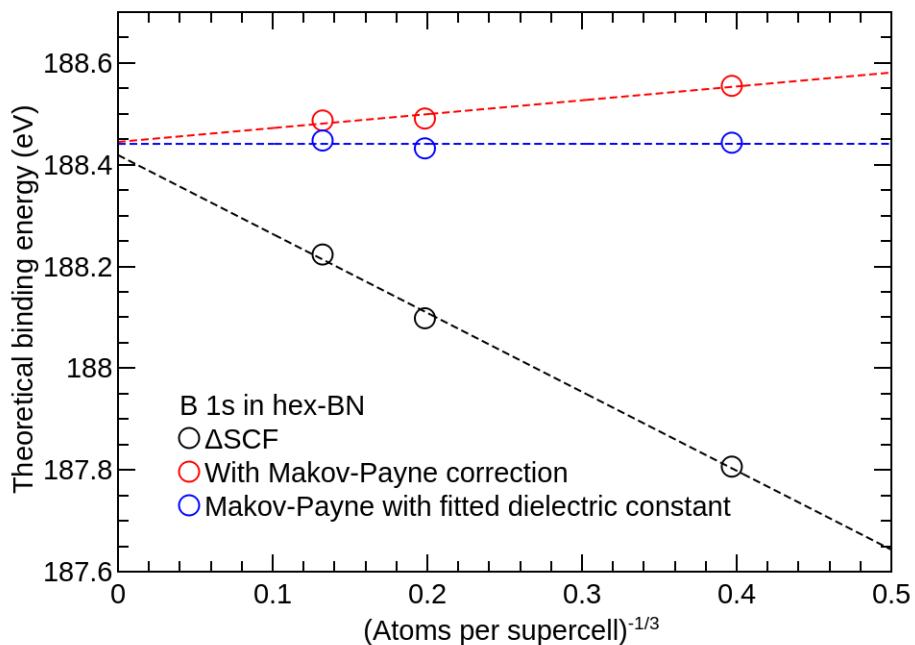
Be 1s in BeO



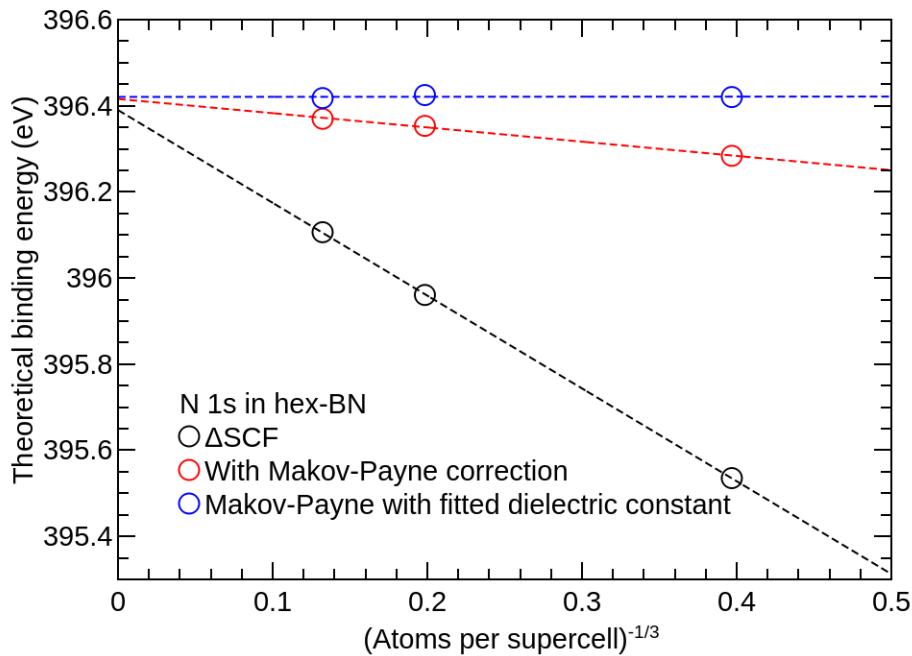
O 1s in BeO



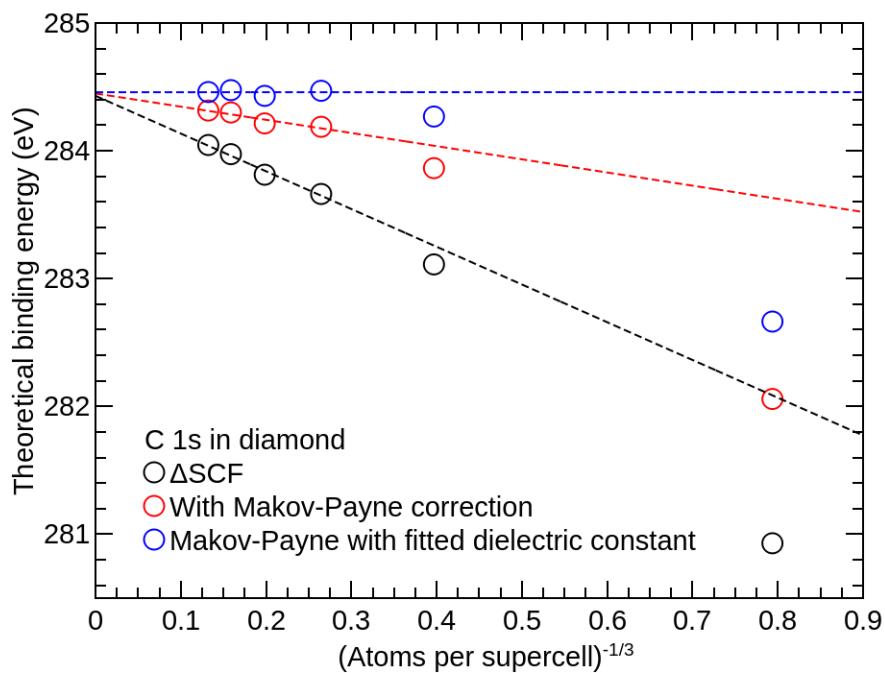
B 1s in hex-BN



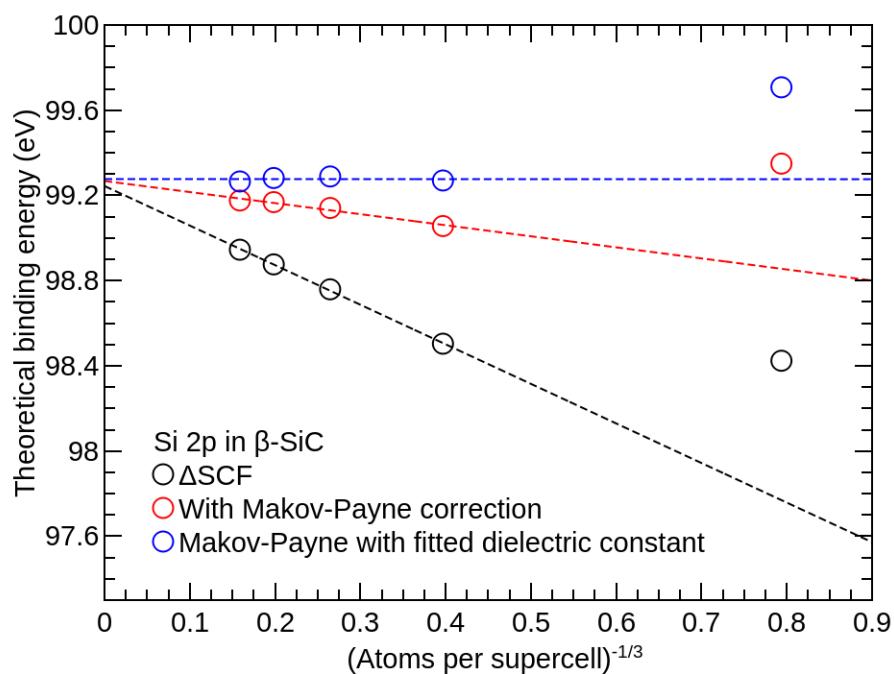
N 1s in hex-BN



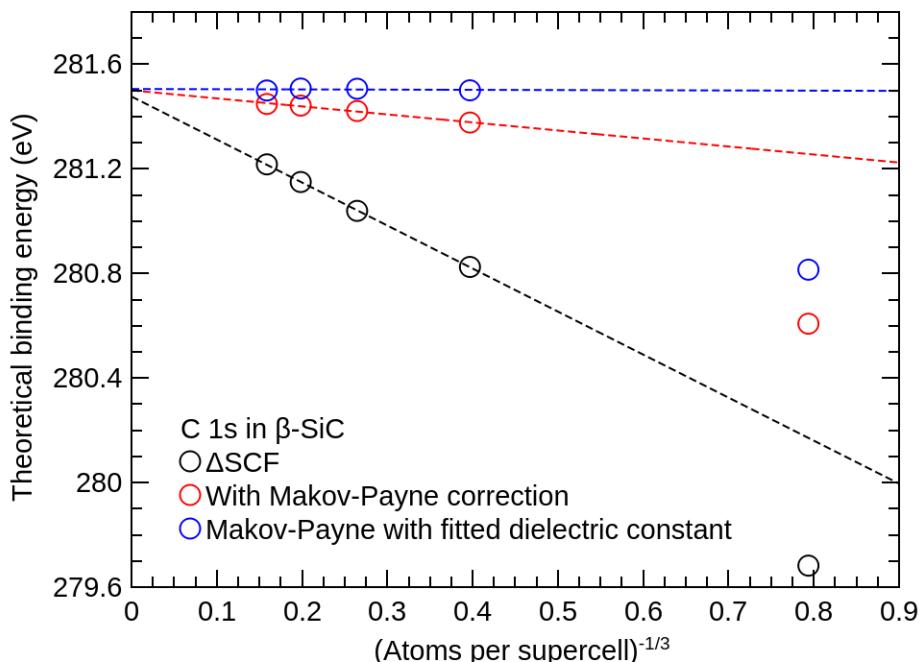
C 1s in diamond



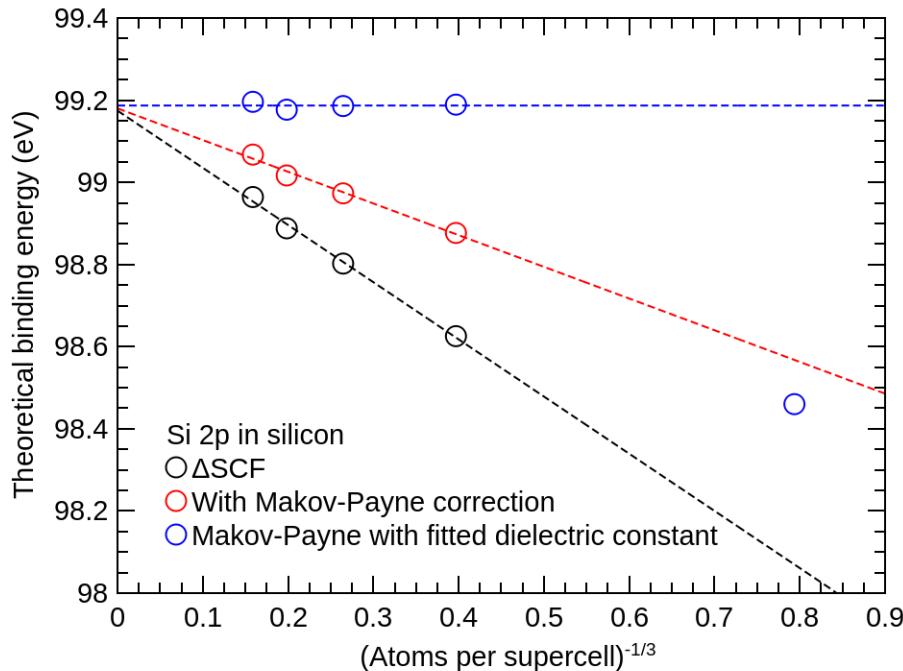
Si 2p in β -SiC



C 1s in β -SiC



Si 2p in silicon



k-point grids, finite size corrections, and numerical results

Finite size corrections are only given (and applied) for insulators. “MP corr. BE” stands for the calculated binding energy with the Makov-Payne correction using the experimental (optical) dielectric constant. “MP (fit ϵ) corr. BE” stands for the calculated binding energy with the Makov-Payne correction using a fitted dielectric constant that makes the binding energy independent of supercell size for the largest supercells.

Li 1s in lithium metal

Supercell	Atoms	k-grid	Δ SCF (eV)
$1 \times 1 \times 1$	1	$30 \times 30 \times 30$	55.08
$2 \times 2 \times 2$	8	$15 \times 15 \times 15$	54.57
$3 \times 3 \times 3$	27	$10 \times 10 \times 10$	54.88
$4 \times 4 \times 4$	64	$8 \times 8 \times 8$	54.88
$5 \times 5 \times 5$	125	$6 \times 6 \times 6$	54.87
Extrapolated value:			54.88

Be 1s in beryllium metal

Supercell	Atoms	k-grid	ΔSCF (eV)
$1\times 1\times 1$	2	$48\times 48\times 32$	110.24
$2\times 2\times 1$	8	$24\times 24\times 32$	111.07
$3\times 3\times 2$	36	$16\times 16\times 16$	111.66
$4\times 4\times 3$	96	$12\times 12\times 11$	111.80
$5\times 5\times 3$	150	$10\times 10\times 11$	111.89
$6\times 6\times 4$	288	$8\times 8\times 8$	111.87
Extrapolated value:			111.88

Na 1s in sodium metal

Supercell	Atoms	k-grid	ΔSCF (eV)
$1\times 1\times 1$	1	$30\times 30\times 30$	1071.37
$2\times 2\times 2$	8	$15\times 15\times 15$	1071.55
$3\times 3\times 3$	27	$10\times 10\times 10$	1071.56
$4\times 4\times 4$	64	$8\times 8\times 8$	1071.58
$5\times 5\times 5$	125	$6\times 6\times 6$	1071.57
Extrapolated value:			1071.56

Na 2p in sodium metal

Supercell	Atoms	k-grid	ΔSCF (eV)
$1\times 1\times 1$	1	$30\times 30\times 30$	31.48
$2\times 2\times 2$	8	$15\times 15\times 15$	30.63
$3\times 3\times 3$	27	$10\times 10\times 10$	30.64
$4\times 4\times 4$	64	$8\times 8\times 8$	30.65
$5\times 5\times 5$	125	$6\times 6\times 6$	30.65
Extrapolated value:			30.65

Mg 1s in magnesium metal

Supercell	Atoms	k-grid	ΔSCF (eV)
$1\times 1\times 1$	2	$48\times 48\times 32$	1302.85
$2\times 2\times 1$	8	$24\times 24\times 32$	1303.11
$3\times 3\times 2$	36	$16\times 16\times 16$	1303.22
$4\times 4\times 3$	96	$12\times 12\times 11$	1303.24
$5\times 5\times 3$	150	$10\times 10\times 11$	1303.25
$6\times 6\times 4$	288	$8\times 8\times 8$	1303.25
Extrapolated value:			1303.25

Mg 2p in magnesium metal

Supercell	Atoms	k-grid	ΔSCF (eV)
$1\times 1\times 1$	2	$48\times 48\times 32$	50.16
$2\times 2\times 1$	8	$24\times 24\times 32$	49.52
$3\times 3\times 2$	36	$16\times 16\times 16$	49.63
$4\times 4\times 3$	96	$12\times 12\times 11$	49.69
$5\times 5\times 3$	150	$10\times 10\times 11$	49.68
$6\times 6\times 4$	288	$8\times 8\times 8$	49.70
Extrapolated value:			49.69

C 1s in graphite

Supercell	Atoms	k-grid	ΔSCF (eV)
$1\times 1\times 1$	4	$36\times 36\times 12$	283.26
$2\times 2\times 2$	32	$18\times 18\times 6$	283.92
$3\times 3\times 3$	108	$12\times 12\times 4$	284.09
$4\times 4\times 4$	256	$9\times 9\times 3$	284.18
$2\times 2\times 1$	16	$18\times 18\times 12$	283.24
$4\times 4\times 2$	128	$9\times 9\times 6$	283.95
$6\times 6\times 3$	432	$6\times 6\times 4$	284.11
Extrapolated value:			284.44

Be 1s in BeO

O 1s in BeO

B 1s in hex-BN

N 1s in hex-BN

C 1s in diamond

Si 2p in β -SiC

C 1s in β -SiC

Supercell	Atoms	k-grid	$q^2\alpha/2L$ (eV)	Δ SCF (eV)	MP corr. BE (eV)	MP (fit ϵ) corr. BE (eV)
$1 \times 1 \times 1$	2	$48 \times 48 \times 48$	6.03	279.68	280.61	280.81
$2 \times 2 \times 2$	16	$24 \times 24 \times 24$	3.60	280.82	281.38	281.50
$3 \times 3 \times 3$	54	$16 \times 16 \times 16$	2.49	281.04	281.42	281.51
$4 \times 4 \times 4$	128	$12 \times 12 \times 12$	1.91	281.15	281.44	281.51
$5 \times 5 \times 5$	250	$10 \times 10 \times 10$	1.51	281.22	281.45	281.50
Extrapolated values:				281.48	281.50	281.51
Experimental dielectric constant = 6.52, fitted dielectric constant = 5.33						

Si 2p in silicon

Supercell	Atoms	k-grid	$q^2\alpha/2L$ (eV)	Δ SCF (eV)	MP corr. BE (eV)	MP (fit ϵ) corr. BE (eV)
$1 \times 1 \times 1$	2	$36 \times 36 \times 36$	5.28	97.45	97.90	98.46
$2 \times 2 \times 2$	16	$18 \times 18 \times 18$	2.94	98.63	98.88	99.19
$3 \times 3 \times 3$	54	$12 \times 12 \times 12$	2.00	98.80	98.97	99.19
$4 \times 4 \times 4$	128	$9 \times 9 \times 9$	1.51	98.89	99.02	99.18
$5 \times 5 \times 5$	250	$7 \times 7 \times 7$	1.21	98.96	99.07	99.20
Extrapolated values:				99.17	99.18	99.19
Experimental dielectric constant = 11.7, fitted dielectric constant = 5.22						

Relaxed structures

All structures were relaxed in FHI-aims using the DFT with the exchange-correlation functional SCAN and the default “tight” basis sets and integration grids. Variable-cell relaxation with fixed angles between the unit cell vectors were performed, until all forces were below 0.005 eV/ \AA . In some cases, numerical stability issues were observed during structural relaxation (mismatch of real and predicted energy gain between successive relaxation steps). Numerical stability issues with the SCAN functional have been previously noted, e.g. Bartok et al. *J. Chem. Phys.* **2019**, *150*, 161101. To overcome this issue, the numerical grids were tightened as follows: the “radial multiplier” was increased by a factor of two, one additional localized angular grid division was uncommented, and the outermost angular grid was increased to match the uncommented value. The relaxed structures are given below, in the FHI-aims geometry.in format. All quantities are given in units of \AA ngström.

k-point grids used in geometry relaxation

Lithium metal: $12 \times 12 \times 12$

Beryllium metal: $18 \times 18 \times 12$

Sodium metal: $12 \times 12 \times 12$

Magnesium metal: $18 \times 18 \times 12$

Graphite: $18 \times 18 \times 9$

BeO: $18 \times 18 \times 12$

hex-BN: $18 \times 18 \times 8$

Diamond: $18 \times 18 \times 18$

β -SiC: $18 \times 18 \times 18$

Silicon: $18 \times 18 \times 18$

Lithium metal

lattice_vector	3.48166314	-0.00000000	0.00000000
lattice_vector	0.00000000	3.48166513	0.00000000
lattice_vector	-0.00000000	0.00000000	3.48166512
atom	-0.00000000	0.00000000	0.00000000 Li
atom	1.74083158	1.74083256	1.74083256 Li

Beryllium metal

lattice_vector	2.26150031	0.00827131	0.00000000
lattice_vector	-1.12335037	1.96244733	0.00000000
lattice_vector	0.00000000	0.00000000	3.57104216
atom	0.00188679	1.31275315	2.67828164 Be
atom	1.13626315	0.65796560	0.89276052 Be

Sodium metal

lattice_vector	4.19260705	-0.00000000	0.00000000
lattice_vector	-0.00000000	4.19260951	0.00000000
lattice_vector	0.00000000	0.00000000	4.19260949
atom	0.00000002	0.00000002	0.00000002 Na
atom	2.09630350	2.09630473	2.09630472 Na

Magnesium metal

lattice_vector	3.16084026	0.01677186	0.00000000
lattice_vector	-1.56577757	2.74565180	0.00000000
lattice_vector	0.00000000	0.00000000	5.16338856
atom	0.00607990	1.83830438	1.29063688 Mg
atom	1.58898278	0.92411917	3.87275156 Mg

Graphite

lattice_vector	1.22503480	2.12182291	0.00000000
lattice_vector	1.22503516	-2.12182275	0.00000000
lattice_vector	-0.00000000	-0.00000000	-6.90943766
atom	0.00000051	0.00228806	-5.18207532 C
atom	-0.00000058	-0.00228823	-1.72736234 C
atom	1.22503564	-0.70503486	-5.18207532 C
atom	1.22503436	0.70503513	-1.72736233 C

BeO

lattice_vector	1.34620893	2.32221793	-0.00000000
lattice_vector	1.34620938	-2.32221782	0.00000000
lattice_vector	-0.00000000	-0.00000000	-4.36284836
atom	1.34620922	-0.77673125	0.00114554 Be
atom	1.34620905	0.77673129	-2.18027864 Be
atom	1.34620923	-0.77302324	-2.71221950 O
atom	1.34620904	0.77302328	-0.53079533 O

hex-BN

lattice_vector	2.49434294	0.00322099	0.00000000
lattice_vector	-1.24441379	2.16179992	-0.00000000
lattice_vector	-0.00000000	0.00000000	6.75674636
atom	0.03056985	-0.01400160	3.37837068 B
atom	-0.02873139	1.45627531	0.00000250 B
atom	-0.03128564	0.01468785	0.00000250 N
atom	0.03312410	1.42758586	3.37837068 N

Diamond

lattice_vector	2.50282191	-0.00738214	-0.00521996
lattice_vector	1.24494967	2.17114328	-0.00524220
lattice_vector	1.24565517	0.71917935	2.04918084
atom	0.62435420	0.36039068	0.25500547 C
atom	4.36907272	2.52254984	1.78371314 C

β -SiC

lattice_vector	2.17088417	2.17088417	-0.00607491
lattice_vector	-0.00607102	2.17089000	2.17089000
lattice_vector	2.17088420	-0.00607489	2.17088420
atom	-0.00056996	-0.00057059	-0.00057060 Si
atom	1.08449429	1.08449541	1.08449543 C

Silicon

lattice_vector	-0.00000000	2.71693088	2.71693088
lattice_vector	2.71693342	-0.00000000	2.71693342
lattice_vector	2.71693596	2.71693596	0.00000000
atom	-0.00000007	-0.00000012	0.00000019 Si
atom	1.35846741	1.35846684	1.35846588 Si

Total energies of positively charged supercells: effect of spin

hex-BN				Lithium			
Supercell	E _{tot} (no spin)	E _{tot} (N _{up} -N _{down} =1)	Difference	Supercell	E _{tot} (no spin)	E _{tot} (N _{up} -N _{down} =1)	Difference
1×1×1	-4331.66	-4331.86	-0.20	2×2×2	-3275.81	-3275.78	0.03
2×2×1	-17356.00	-17356.00	0.00	3×3×3	-11068.96	-11068.95	0.01
3×3×1	-39062.17	-39062.16	0.01	4×4×4	-26244.79	-26244.79	0.00
4×4×2	-138904.32	-138904.33	-0.01	5×5×5	-51264.33	-51264.33	0.00
5×5×2	-217041.96	-217041.96	0.00				
6×6×2	-312543.51	-312543.51	0.00				
6×6×3	-468818.76	-468818.76	0.00				

* all energies are given in eV. For lithium, the spin-polarized calculation for the 1×1×1 supercell did not converge, so this datapoint is omitted.

Basis sets

In FHI-aims, both the integration grid and the basis functions need to be defined for each type of atom in each calculation. In this work, four types of settings were used for different atoms, as described below.

- For the atom with a core hole, the “special” basis sets given below were used. This are based on the “tight” default settings, with additional core basis functions to allow the remaining core electrons to relax in the presence of the core hole.
- For the nearest neighbours of the atom with a core hole, the “tight” default settings were used.
- For the second nearest neighbours of the atom with a core hole, the “intermediate” default settings were used.
- For all other atoms, the “light” default settings were used.
- The species defaults mentioned above (default integration grids and basis sets) refer to the ones provided with FHI-aims version 201231, originally published in Blum et al.
Comput. Phys. Commun. **2009**, *180*, 2175.
- The same basis sets were always used for evaluating $E_{N-1,\text{ch}}$ and $E_{N-1,\text{ground}}$

Lithium

```
#####
# "Core" basis functions and numerical settings for Li atom.
# Based on "tight" defaults (V. Blum, 2009).
#
#####
species      Li_core
# global species definitions
nucleus      3
mass         6.941
#
l_hartree    6
#
cut_pot      4.5  1.8  1.0
basis_dep_cutoff 1e-4
#
radial_base   29  7.0
radial_multiplier 2
angular_grids specified
division     0.4484 110
division     0.5659 194
division     0.6315 302
division     0.6662 434
# division    0.8186 590
# division    0.9037 770
# division    6.2760 974
# outer_grid   974
outer_grid    434
#####
#
# Definition of "minimal" basis
#
#####
# valence basis states
valence      2 s  1.
#
# ion occupancy
ion_occ      1 s  2.
#####
#
# Suggested additional basis functions. For production calculations,
# uncomment them one after another (the most important basis functions are
# listed first).
#
# Constructed for dimers: 1.80 A, 2.25 A, 2.75 A, 3.50 A, 4.50 A
#
#####
```

```
# "First tier" - improvements: -189.23 meV to -6.35 meV
    hydro 2 p 1.6
    hydro 2 s 2
    hydro 3 d 2.6
# "Second tier" - improvements: -4.69 meV to -0.41 meV
    hydro 3 p 4.6
    hydro 2 p 1.8
    hydro 3 s 6.2
    hydro 4 d 4.7
    hydro 4 f 4.1
# "Third tier" - improvements: -0.20 meV to -0.15 meV
#     hydro 4 d 0.95
#     hydro 3 p 6.2
#     hydro 3 s 1.7
# Additional basis functions for atom with a core hole
    hydro 1 s 4.0
    hydro 1 s 6.0
    hydro 2 s 5.0
```

Beryllium

```
#####
# "Core" basis functions and numerical settings for Be atom.
# Based on "tight" defaults (V. Blum, 2009).
#
#####
species      Be_core
# global species definitions
nucleus      4.0
mass         9.012182
#
l_hartree    6
#
cut_pot      4.0 2.0 1.0
basis_dep_cutoff 1e-4
#
radial_base   31 7.0
radial_multiplier 2
angular_grids specified
division     0.4283 110
division     0.4792 194
division     0.5061 302
division     0.7227 434
# division   0.8724 590
# division   0.9555 770
# division   2.9770 974
# outer_grid  974
outer_grid   434
#####
#
# Definition of "minimal" basis
#
#####
# valence basis states
valence      2 s  1.999
valence      2 p  0.001
# ion occupancy
ion_occ      2 s  1.
#####
#
# Suggested additional basis functions. For production calculations,
# uncomment them one after another (the most important basis functions are
# listed first).
#
# Constructed for dimers: 1.75 A, 2.0 A, 2.375 A, 3.00 A, 4.00 A
#
```

```
#####
# "First tier" - improvements: -677.26 meV to -34.75 meV
  ionic 2 p auto
  hydro 3 s 2.9
  hydro 3 d 3.5
# "Second tier" - improvements: -16.34 meV to -1.26 meV
  hydro 3 p 3.1
  hydro 4 d 4.7
  hydro 3 p 2.4
  hydro 4 f 7.6
  hydro 2 s 2.9
# "Third tier" - improvements: -0.27 meV to -0.05 meV
#   hydro 2 p 8.2
#   hydro 5 g 10.8
#   hydro 4 f 7
#   hydro 3 s 2.3
#   hydro 4 d 3.8

# Additional basis functions for atom with a core hole
  hydro 1 s 5.0
  hydro 1 s 7.0
  hydro 2 s 6.0
```

Boron

```
#####
# "Core" basis functions and numerical settings for B atom.
# Based on "tight" defaults (V. Blum, 2009).
#
#####
species      B_core
# global species definitions
nucleus      5.0
mass         10.811
#
l_hartree    6
#
cut_pot      4.0 2.0 1.0
basis_dep_cutoff 1e-4
#
radial_base   32 7.0
radial_multiplier 2
angular_grids specified
division     0.3742 110
division     0.5197 194
division     0.5753 302
division     0.7664 434
# division   0.8392 770
# division   1.6522 974
# outer_grid  974
outer_grid   434
#####
#
# Definition of "minimal" basis
#
#####
# valence basis states
valence      2 s 2.
valence      2 p 1.0
# ion occupancy
ion_occ      2 s 1.
#####
#
# Suggested additional basis functions. For production calculations,
# uncomment them one after another (the most important basis functions are
# listed first).
#
# Constructed for dimers: 1.25 A, 1.625 A, 2.5 A, 3.5 A
#
#####
```

```
# "First tier" - improvements: -710.52 meV to -92.39 meV
  hydro 2 p 1.4
  hydro 3 d 4.8
  hydro 2 s 4
# "Second tier" - improvements: -33.88 meV to -2.20 meV
  hydro 4 f 7.8
  hydro 3 p 4.2
  hydro 3 s 3.3
  hydro 5 g 11.2
  hydro 3 d 5.4
# "Third tier" - improvements: -1.28 meV to -0.36 meV
  hydro 2 p 4.7
  hydro 2 s 8.4
  hydro 4 d 5.8
# "Fourth tier" - improvements: -0.25 meV to -0.12 meV
#   hydro 3 p 2.2
#   hydro 3 s 3
#   hydro 4 f 9.8
#   hydro 5 g 12.8
#   hydro 4 d 10
# Further functions
#   hydro 4 f 14
#   hydro 3 p 12.4

# Additional basis functions for atom with a core hole
hydro 1 s 9.0
hydro 1 s 7.0
hydro 1 s 3.0
hydro 2 s 5.0
hydro 2 p 6.0
```

Carbon

```
#####
# "Core" basis functions and numerical settings for C atom.
# Based on "tight" defaults (V. Blum, 2009).
#
#####
species      C_core
#   global species definitions
nucleus      6.0
mass         12.0107
#
l_hartree    6
#
cut_pot      4.0 2.0 1.0
basis_dep_cutoff 1e-4
#
radial_base   34 7.0
radial_multiplier 2
angular_grids specified
  division  0.2187 50
  division  0.4416 110
  division  0.6335 194
  division  0.7727 302
  division  0.8772 434
#  division  0.9334 590
#  division  0.9924 770
#  division  1.0230 974
#  division  1.5020 1202
#  outer_grid 974
#  outer_grid 434
#####
#
# Definition of "minimal" basis
#
#####
#   valence basis states
valence      2 s 2.
valence      2 p 2.0
#   ion occupancy
ion_occ     2 s 1.
ion_occ     2 p 1.
#####
#
# Suggested additional basis functions. For production calculations,
# uncomment them one after another (the most important basis functions are
# listed first).
```

```

#
# Constructed for dimers: 1.0 Å, 1.25 Å, 1.5 Å, 2.0 Å, 3.0 Å
#
#####
# "First tier" - improvements: -1214.57 meV to -155.61 meV
hydro 2 p 1.7
hydro 3 d 6
hydro 2 s 4.9
# "Second tier" - improvements: -67.75 meV to -5.23 meV
hydro 4 f 9.8
hydro 3 p 5.2
hydro 3 s 4.3
hydro 5 g 14.4
hydro 3 d 6.2
# "Third tier" - improvements: -2.43 meV to -0.60 meV
hydro 2 p 5.6
hydro 2 s 1.4
hydro 3 d 4.9
hydro 4 f 11.2
# "Fourth tier" - improvements: -0.39 meV to -0.18 meV
#     hydro 2 p 2.1
#     hydro 5 g 16.4
#     hydro 4 d 13.2
#     hydro 3 s 13.6
#     hydro 4 f 17.6
# Further basis functions - improvements: -0.08 meV and below
#     hydro 3 s 2
#     hydro 3 p 6
#     hydro 4 d 20

# Additional basis functions for atom with a core hole
hydro 1 s 10.0
hydro 1 s 8.0
hydro 1 s 4.0
hydro 2 s 6.0

```

Nitrogen

```
#####
# "Core" basis functions and numerical settings for N atom.
# Based on "tight" defaults (V. Blum, 2009).
#
#####
species      N_core
#   global species definitions
nucleus      7
mass         14.0067
#
l_hartree    6
#
cut_pot      4.0  2.0  1.0
basis_dep_cutoff 1e-4
#
radial_base   35 7.0
radial_multiplier 2
angular_grids specified
  division 0.1841 50
  division 0.3514 110
  division 0.5126 194
  division 0.6292 302
  division 0.6939 434
#  division 0.7396 590
#  division 0.7632 770
#  division 0.8122 974
#  division 1.1604 1202
#  outer_grid 974
outer_grid   434
#####
#
# Definition of "minimal" basis
#
#####
#   valence basis states
valence     2 s  2.
valence     2 p  3.
#
#   ion occupancy
ion_occ     2 s  1.
ion_occ     2 p  2.
#####
#
# Suggested additional basis functions. For production calculations,
# uncomment them one after another (the most important basis functions are
# listed first).
```

```

#
# Constructed for dimers: 1.0 Å, 1.1 Å, 1.5 Å, 2.0 Å, 3.0 Å
#
#####
# "First tier" - improvements: -1193.42 meV to -220.60 meV
    hydro 2 p 1.8
    hydro 3 d 6.8
#    hydro 3 s 5.8
# "Second tier" - improvements: -80.21 meV to -6.86 meV
    hydro 4 f 10.8
#    hydro 3 p 5.8
    hydro 1 s 0.8
    hydro 5 g 16
    hydro 3 d 4.9
# "Third tier" - improvements: -4.29 meV to -0.53 meV
    hydro 3 s 16
    ionic 2 p auto
    hydro 3 d 6.6
    hydro 4 f 11.6
# "Fourth tier" - improvements: -0.75 meV to -0.25 meV
#    hydro 2 p 4.5
#    hydro 2 s 2.4
#    hydro 5 g 14.4
#    hydro 4 d 14.4
#    hydro 4 f 16.8
# Further basis functions - -0.21 meV and below
#    hydro 3 p 14.8
#    hydro 3 s 4.4
#    hydro 3 d 19.6
#    hydro 5 g 12.8

# Additional basis functions for atom with a core hole
    hydro 1 s 11.0
    hydro 1 s 9.0
    hydro 1 s 5.0
    hydro 2 s 8.0
    hydro 2 s 10.0
    hydro 2 p 6.5
    hydro 3 s 6.2
    hydro 3 p 6.2

```

Oxygen

```
#####
# "Core" basis functions and numerical settings for O atom.
# Based on "tight" defaults (V. Blum, 2009).
#
#####
species          O_core
#   global species definitions
nucleus          8
mass              15.9994
#
l_hartree        6
#
cut_pot          4.0  2.0  1.0
basis_dep_cutoff 1e-4
#
radial_base      36  7.0
radial_multiplier 2
angular_grids specified
division    0.1817  50
division    0.3417  110
division    0.4949  194
division    0.6251  302
division    0.8014  434
#   division    0.8507  590
#   division    0.8762  770
#   division    0.9023  974
#   division    1.2339  1202
#   outer_grid  974
outer_grid      434
#####
#
# Definition of "minimal" basis
#
#####
#   valence basis states
valence          2  s  2.
valence          2  p  4.
#
#   ion occupancy
ion_occ          2  s  1.
ion_occ          2  p  3.
#####
#
# Suggested additional basis functions. For production calculations,
# uncomment them one after another (the most important basis functions are
# listed first).
```

```

#
# Constructed for dimers: 1.0 Å, 1.208 Å, 1.5 Å, 2.0 Å, 3.0 Å
#
#####
# "First tier" - improvements: -699.05 meV to -159.38 meV
hydro 2 p 1.8
hydro 3 d 7.6
hydro 3 s 6.4
# "Second tier" - improvements: -49.91 meV to -5.39 meV
hydro 4 f 11.6
hydro 3 p 6.2
hydro 3 d 5.6
hydro 5 g 17.6
hydro 1 s 0.75
# "Third tier" - improvements: -2.83 meV to -0.50 meV
ionic 2 p auto
hydro 4 f 10.8
hydro 4 d 4.7
hydro 2 s 6.8
# "Fourth tier" - improvements: -0.40 meV to -0.12 meV
#     hydro 3 p 5
#     hydro 3 s 3.3
#     hydro 5 g 15.6
#     hydro 4 f 17.6
#     hydro 4 d 14
# Further basis functions - -0.08 meV and below
#     hydro 3 s 2.1
#     hydro 4 d 11.6
#     hydro 3 p 16
#     hydro 2 s 17.2

# Additional basis functions for atom with a core hole
hydro 1 s 12.0
hydro 1 s 10.0
hydro 1 s 6.0
hydro 2 s 10.0
hydro 2 p 8.0
hydro 2 p 6.0
hydro 3 d 8.0

```

Sodium

```
#####
# "Core" basis functions and numerical settings for Na atom.
# Based on "tight" defaults (V. Blum, 2009).
#
#####
species      Na_core
# global species definitions
nucleus      11
mass         22.98976928
#
l_hartree    6
#
cut_pot      4.5      2.0  1.0
basis_dep_cutoff 1e-4
#
radial_base   40 7.0
radial_multiplier 2
angular_grids specified
division     0.5925 110
division     0.7843 194
division     1.0201 302
division     1.1879 434
# division   1.3799 590
# division   1.4503 770
# division   7.0005 974
# outer_grid 974
outer_grid   434
#####
#
# Definition of "minimal" basis
#
#####
# valence basis states
valence      3 s  1.
valence      2 p  6.
#
# ion occupancy
ion_occ      2 s  2.
ion_occ      2 p  6.
#####
#
# Suggested additional basis functions. For production calculations,
# uncomment them one after another (the most important basis functions are
# listed first).
#
# Constructed for dimers: 2.0 A, 2.5 A, 3.0 A, 3.75 A, 4.5 A
```

```

#
#####
# "First tier" - improvements: -60.09 meV to -10.02 meV
hydro 2 p 1.2
hydro 3 s 1.8
hydro 3 d 3.8
# "Second tier" - improvements: -2.94 meV to -1.27 meV
hydro 4 p 3.1
hydro 3 s 10
hydro 4 f 6.2
hydro 4 d 1.3
# "Third tier" - improvements: -0.83 meV to -0.07 meV
# hydro 3 d 7.8
# hydro 3 p 2.3
# hydro 5 g 9.6
# hydro 4 p 0.85
# hydro 5 f 1.8
# hydro 2 s 0.6
# Further basis functions that fell out of the optimization - noise level...
# hydro 5 g 0.1
# hydro 4 d 3.4
# hydro 4 s 0.1

# Additional basis functions for atom with a core hole

hydro 1 s 15.0
hydro 1 s 13.0
hydro 1 s 9.0
hydro 2 s 13.5
hydro 2 s 11.5
hydro 2 p 10.5
hydro 2 p 8.5
hydro 3 d 7.0
hydro 3 p 8.0

```

Magnesium

```
#####
# "Core" basis functions and numerical settings for Mg atom.
# Based on "tight" defaults (V. Blum, 2009).
#
#####
species      Mg_core
# global species definitions
nucleus      12
mass         24.3050
#
l_hartree    6
#
cut_pot      5.0      2.0  1.0
basis_dep_cutoff 1e-4
#
radial_base   40 7.0
radial_multiplier 2
angular_grids specified
division     0.5421  50
division     0.8500  110
division     1.0736  194
division     1.1879  302
division     1.2806  434
# division   1.4147  590
# division   1.4867  770
# division   1.6422  974
# division   2.6134 1202
# outer_grid  974
outer_grid   434
#####
#
# Definition of "minimal" basis
#
#####
# valence basis states
valence      3 p  0.001
valence      3 s  1.999
# ion occupancy
ion_occ      2 s  2.
ion_occ      2 p  6.
#####
#
# Suggested additional basis functions. For production calculations,
# uncomment them one after another (the most important basis functions are
# listed first).
```

```

#
# Constructed for dimers: 2.125 Å, 2.375 Å, 2.875 Å, 3.375 Å, 4.5 Å
#
#####
# "First tier" - improvements: -230.76 meV to -21.94 meV
hydro 2 p 1.5
ionic 3 d auto
hydro 3 s 2.4
# "Second tier" - improvements: -5.43 meV to -1.64 meV
hydro 4 f 4.3
hydro 2 p 3.4
hydro 4 s 11.2
hydro 3 d 6.2
# "Third tier" - improvements: -0.92 meV to -0.22 meV
hydro 2 s 0.6
hydro 3 p 4.8
hydro 4 f 7.4
hydro 5 g 6.6
hydro 2 p 1.6
hydro 3 d 1.8
# "Fourth tier" - improvements: -0.09 meV to -0.05 meV
hydro 4 p 0.45
hydro 5 g 10.4
hydro 2 s 12.4
hydro 4 d 1.7

# Additional basis functions for atom with a core hole
hydro 1 s 16.0
hydro 1 s 14.0
hydro 1 s 10.0
hydro 2 s 14.5
hydro 2 s 12.5
hydro 2 p 11.5
hydro 2 p 9.5
hydro 3 d 8.0
hydro 3 p 9.0

```

Silicon

```
#####
# "Core" basis functions and numerical settings for Si atom.
# Based on "tight" defaults (V. Blum, 2009).
#
#####
species      Si_core
# global species definitions
nucleus      14.
mass         28.0855
#
l_hartree    6
#
cut_pot      4.0      2.0  1.0
basis_dep_cutoff 1e-4
#
radial_base   42 7.0
radial_multiplier 2
angular_grids specified
division     0.4121  50
division     0.7665  110
division     1.0603  194
division     1.2846  302
division     1.4125  434
# division   1.4810  590
# division   1.5529  770
# division   1.6284  974
# division   2.6016 1202
# outer_grid  974
outer_grid   434
#####
#
# Definition of "minimal" basis
#
#####
# valence basis states
valence      3 s  2.
valence      3 p  2.
# ion occupancy
ion_occ      3 s  1.
ion_occ      3 p  1.
#####
#
# Suggested additional basis functions. For production calculations,
# uncomment them one after another (the most important basis functions are
# listed first).
```

```

#
# Constructed for dimers: 1.75 Å, 2.0 Å, 2.25 Å, 2.75 Å, 3.75 Å
#
#####
# "First tier" - improvements: -571.96 meV to -37.03 meV
hydro 3 d 4.2
hydro 2 p 1.4
hydro 4 f 6.2
ionic 3 s auto
# "Second tier" - improvements: -16.76 meV to -3.03 meV
hydro 3 d 9
hydro 5 g 9.4
hydro 4 p 4
hydro 1 s 0.65
# "Third tier" - improvements: -3.89 meV to -0.60 meV
ionic 3 d auto
hydro 3 s 2.6
hydro 4 f 8.4
hydro 3 d 3.4
hydro 3 p 7.8
# "Fourth tier" - improvements: -0.33 meV to -0.11 meV
#     hydro 2 p 1.6
#     hydro 5 g 10.8
#     hydro 5 f 11.2
#     hydro 3 d 1
#     hydro 4 s 4.5
# Further basis functions that fell out of the optimization - noise
# level... < -0.08 meV
#     hydro 4 d 6.6
#     hydro 5 g 16.4
#     hydro 4 d 9

# Additional basis functions for atom with a core hole

hydro 2 p 15.0
hydro 2 p 12.0
hydro 2 p 5.0
hydro 2 s 14.0
hydro 1 s 16.0
hydro 1 s 9.0

```

Sample control.in files

Below, the contents of the control.in files, excluding the species definitions, for the calculation of the C 1s core electron binding energy in a $3\times 3\times 3$ supercell of β -SiC are given. In particular:

- The charge_1 control.in file is used to calculate the total energy of the ground state of the N-1 electron system
- The init_part_1 and init_part_2 control.in files are used in successive runs to localize a core orbital onto a particular atom, and next, to create a localized core hole. In these runs, the nuclear charge of the “target” atom for localizing a core hole is increased by 0.1 e. In init_part_1, restart files are written. In init_part_2, restart files are read in and written out.
- The hole_run control.in file is used to calculate the total energy of the core hole state. In hole_run, restart files are read in.
- The point_charge_in_a_box control.in file is used to calculate the total energy of a system with just a point charge in a periodic box, with a uniform compensating background. This value is used for applying the Makov-Payne correction. In practice, the point charge is artificially created by inserting a hydrogen atom with the electron removed (except for a tiny fraction of an electron as the present version of FHI-aims does not permit calculations with an electron count of exactly zero.)

charge_1 control.in

```
xc                  dfauto scan
spin                collinear
default_initial_moment 0.0
relativistic        zora scalar 1e-12

k_grid              16 16 16

preconditioner      kerker off

override_illconditioning .true.

charge              1.0
```

init_part_1 control.in

```
xc                  dfauto scan
spin                collinear
default_initial_moment 0.0
relativistic        zora scalar 1e-12

k_grid              16 16 16

restart_write_only   restart_file
restart_save_iterations 20
KS_method           serial

preconditioner      kerker off

override_illconditioning .true.

charge              0.1
```

init_part_2 control.in

```
xc                  dfauto scan
spin                collinear
default_initial_moment 0.0
relativistic        zora scalar 1e-12

k_grid              16 16 16

restart              restart_file
KS_method           serial

preconditioner      kerker off

override_illconditioning .true.

charge              1.1
force_occupation_projector 28 1 0.0 28 54
sc_iter_limit       1
```

hole_run control.in

```
xc                      dfauto scan
spin                     collinear
default_initial_moment 0.0
relativistic             zora scalar 1e-12

k_grid                  16 16 16

restart_read_only        restart_file
KS_method                serial

preconditioner           kerker off

override_illconditioning .true.

charge                  1.0
force_occupation_projector 28 1 0.0 28 54

output                  mulliken
```

point_charge_in_a_box control.in

```
xc                      dfauto scan
spin                     none
relativistic             none
KS_method                serial

charge                  0.99999999

k_grid                  1 1 1
```