Systemic risk in financial networks: a data-driven machine learning analysis

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Abstract

The purpose of this paper is to assess the role of financial variables and network topology as determinants of systemic risk (SR). The SR, for different levels of the initial shock, is computed for institutions in the Brazilian interbank market by applying the *differential DebtRank* methodology. The financial institution(FI)-specific determinants of SR are evaluated through two machine learning techniques: XGBoost and random forest. Shapley values analysis provided a better interpretability for our results. Furthermore, we performed this analysis separately for banks and credit unions. We have found the importance of a given feature in driving SR varies with i) the level of the initial shock, ii) the type of FI, and iii) the dimension of the risk which is being assessed – i.e., potential loss caused by (systemic impact) or imputed to (systemic vulnerability) the FI. Systemic impact is mainly driven by topological features for both types of FIs. However, while the importance of topological features to the prediction of systemic impact of banks increases with the level of the initial shock, it decreases for credit unions. Concerning systemic vulnerability, this is mainly determined by financial features, whose importance increases with the initial shock level for both types of FIs.

1. Introduction

This paper is related to the literature on network-based models of systemic risk (SR). In these models, SR is the result of a shock propagation throughout a network of interconnected financial institutions (FI). Here, we apply machine learning (ML) techniques to assess the role of financial and topological variables as determinants of SR. ML techniques are able to capture complex non-linear relationships among variables.

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We believe such a feature is important for our task as many studies show networks can amplify shocks in non-linear ways.[1]

We find the main drivers of SR vary with the size of the initial shock, as well as the dimension of the risk – i.e., whether the risk refers to the potential loss which may be caused by or imputed to the financial institution (FI). Due to the remarkable differences between banks and credit unions, we perform our analysis separately for each type of FI. Our results also indicate the main determinants of SR are different for banks and credit unions.

In financial markets, SR and contagion are two intertwined concepts. A concise definition of SR can be found in [1]: "The risk that (i) an economic shock such as market or institutional failure triggers (through a panic or otherwise) either the failure of a chain of markets or institutions or a chain of significant losses to financial institutions, (ii) resulting in increases in the cost of capital or decreases in its availability, often evidenced by substantial financial-market price volatility" (p. 204). Therefore, contagion is one of the key ingredients of systemic risk. This is the mechanism through which an idiosyncratic event limited to an individual component could propagate throughout the whole system, developing into a system-wide impact.[2] Moreover, contagion unveils the two quantitative dimensions of SR, *impact diffusion* and *impact susceptibility* [6]. Impact diffusion measures the potential harm that one institution could cause to the economy, while the impact susceptibility measures the likelihood that random negative events end up causing losses to an institution. Therefore, they capture different aspects of SR and hence complement each other.

Identifying systemically relevant FIs – both in terms of impact diffusion and susceptibility perspectives – is crucial not only methodologically, but also from the financial regulation viewpoint. Policy instruments aiming at addressing systemic risk have been headed towards systemically important FIs. Their purpose is to minimize the probability and the costs of a financial crisis. The Dodd-Frank Act is concerned about the regulation of systemically relevant firms and sectors ([7]). The new Basel III agreement requires an additional capital surcharge on domestic and global systemically important banks, as defined by the Financial Stability Board (FSB) and the Basel Committee on Banking Supervision (BCBS) [8, 9]. For this reason, the proper detection of systemically important institutions and their determinants is a key concern to ensuring financial stability.

The driving factors of SR depend on how it is measured. There are two approaches aiming at computing SR: the market-based approach and the network-based approach. The market-based approach is underpinned on the premise that banks are strongly disciplined by the market ([10]). Hence, there is a strong relationship between SR and market values. The main shortfall of this approach is that it neglects interconnections

[1] Indeed, in traditional econometric models, based on linear regression, these non-linear linkages are often neglected. These non-linear relationships can be important in the study of the determinants of complex phenomena in a network approach.

[2] Contagion may take place through several transmission mechanisms, sometimes with two or more operating at the same time. Some examples are: i) losses engendered by fire sales in face of common asset exposures [2], bank runs due to confidence crisis [3], and default cascade among banks connected through debt obligations [4]. For a thorough review, see [5].

between FIs, which are taken into consideration within the network-based approach. These interconnections proved to be an important driver of SR in the 2007-2008 financial crisis. The network-based approach assesses how an initial shock propagates throughout a network of FIs interconnected through some kind of vulnerability link (debt obligations, common asset exposures, etc.), resulting in some kind of aggregate fragility (such as credit risk and liquidity risk).

Within the market-based approach, both the triggering event and its impact are evaluated in terms of some market value, such as stock prices or credit default swaps (CDS) spreads. For instance, the *marginal expected shortfall*, or MES ([11]), is defined as the expected net equity return of a bank when the market is at its 5% worst performance level in a year. Other examples of market-based measures of SR include the ΔCoVaR ([12]), the *distress insurance premium* – DIP ([13]), the Lehar's indicator ([14]), and the SRISK ([15]).[3]

Studies relying on market-based measures (e.g., [17, 18, 19, 20, 21, 22, 23]) frequently pose banks' asset size as positively related to SR. Alternatively, banks' equity (or equity-to-assets ratio) is pointed as a mitigating factor of systemic relevance. Other elements with a positive effect on banks' SR include engagement in non-traditional banking activities, higher leverage, lower liquidity, higher non-performing loans ratio, and more government support. However, [18] claim the bank-specific determinants of SR are often unique to each crisis and depend on the characteristics of the regulatory regime.

The network-based approach has emerged as an important ally for the analysis of SR. It allows for the estimation of SR as the result of an initial shock in a given component propagating through a network of interlinked components. The initial shock is usually represented by a depletion in the agent's economic value (e.g., equity). In network models, there are two approaches of shock propagation: i) the Eisenberg and Noe (E-N) approach ([24]), in which contagion is triggered by a complete depletion of the agent's resources, and ii) the *distress* approach, in which contagion is triggered by a partial loss of economic value ([25, 26, 27]).

The E-N approach is useful in the modeling of catastrophic events, such as the bankruptcy of big banks. A key limitation of this approach is that, under this contagion trigger, a shock engenders a significant SR only when combined with other conditions. The E-N approach is unable to reproduce crises driven by the transmission of small shocks among highly interconnected FIs, as in the 2007-2008 financial crisis. The *distress* methodology overcomes this problem. It holds two differences regarding the previous approach: i) first, it considers potential rather than real losses; ii) second, the contagion trigger may be a partial (not necessarily a complete) depletion of the agent's resources. When these two mechanisms are activated, interconnectedness gains a much more prominent role in propagating shocks throughout financial networks.

In financial network models, the links can be represented by debt obligations ([28]), common asset exposures ([29]), ownership relationships ([30]), derivative contracts ([31]), liquidity risk ([32, 33]), among others. A fruitful literature has employed the network approach in the assessment of SR mainly in the

[3]For a comprehensive review on market-based measures of SR, see [16].

interbank market (e.g., [28, 4, 34, 25]). However, it was also successfully applied to other contexts, such as payment systems ([35]), production networks [36, 37], multilayer financial networks ([32, 33]), and bipartite bank-credit networks ([38]).

The main advantage of the network approach is that it allows to understand how the topological features of the underlying financial network contribute as amplifying or attenuating drivers of SR. Among financial regulators, expressions as "too-interconnected-to-fail" and "too-systemic-to-fail" have been used in parallel to the term "too-big-to-fail". It reflects the general consensus that the most systemically important FIs are not necessarily the biggest ones. Assessing data from U.S. institutions from August 2007 to June 2010, [25] have shown the correlation between *DebtRank* and asset size is lower than 0.4. Moreover, this correlation decreases towards the peak of the crisis. Interconnectedness, other than size, should be taken into account when assessing systemic relevance.[4] The complex networks literature has a large body of research devoted to designing measures that capture local to global topological patterns within the network. This paper employs these measures to understand how the network structure drives SR.

Interconnectedness is related to how important and influential an FI is in the whole financial network. It can be naturally captured by the concept of *centrality* according to the complex network literature. There are at least three classical measures of centrality: the degree (the number of direct neighbors of a given node), the betweenness centrality (the fraction of shortest paths[5] going through a given node), and the closeness centrality (the average of the shortest path length from a given node to every other node in the network). Other measures of centrality include, for instance, the eigenvector centrality ([43]), the subgraph centrality ([44]), the PageRank centrality ([45]), and the communicability centrality ([46]).

Centrality measures have been successfully applied to the identification of systemically important banks. Assessing two Mexican financial networks, [47] have found the contagion ranking and the centrality ranking are highly correlated in the top 15 positions in both networks (the interbank market and the payment systems network). In the study of [48], the centrality measures (degree, betweenness, closeness, and Bonacich centrality) performed very well in identifying systemically relevant institutions in the Turkish interbank market. In a simulation exercise, [49] have found the cascade depth – the number of failed nodes as a consequence of single failure in one of the nodes – is negatively correlated with node degree, but positively correlated with betweenness centrality and local rank. Our paper contributes to this literature by bringing supervised ML algorithms to understand the main drivers of systemic risk.

The aim of this paper is to assess the role of both topological and non-topological features as drivers of SR.

[4]A study commissioned by the International Monetary Fund, the Bank for International Settlements, the Financial Stability Board and the G20 ([39]) has found interconnectedness is the second most important factor in the determination of the systemic importance of FIs. Although size is the most important factor, it is not the only dimension that prevails when establishing the systemic importance of FIs.

[5]The shortest path between two nodes is the one in which the sum of the weights of the constituent edges is minimized. There are some excellent textbooks the reader unfamiliar with the complex networks concepts may refer to, as [40], [41] and [42].

We compute the systemic relevance of institutions in the Brazilian interbank market through the *differential DebtRank* methodology ([27]). The triggering event, or initial shock, is represented by an equity loss of individual institutions in a given fraction. We simulate different initial losses and analyze the importance of topological and non-topological features in explaining the contagion losses arising from the triggering events. We analyze both from the perspective of inflicting losses (systemic impact) and the likelihood of being recipient of losses initiated by any other institution in the network (systemic vulnerability). To analyze the importance of topological and non-topological features in shaping SR, we employ two ML techniques: XGBoost and random forest. Moreover, we perform this task separately for banks and credit unions since they have very different business models.[6] Finally, further insights were brought about by computing the Shapley values. This provides information not only on the size, but also on the direction of the effect of a given feature.

Among the potential explanatory variables, there are financial and topological variables. The topological features assessed in our study are the following centrality measures: degree, clustering coefficient, closeness centrality, betweenness centrality, k-core, and PageRank. As for the financial variables, we use all relevant financial information available in our data set: total assets, equity, return on equity, interbank assets-to-equity ratio, and interbank liabilities-to-equity ratio.

Our results can be summarized as follows: the importance of a given feature in driving SR varies with i) the level of the initial shock, ii) the type of FI, and iii) the dimension of the risk (impact or vulnerability) which is being assessed. Systemic impact is mainly driven by topological features for both types of FIs. However, while the importance of topological features to the prediction of systemic impact of banks increases with the level of the initial shock, it decreases for credit unions. Concerning systemic vulnerability, this is mainly determined by financial features, whose importance increases with the initial shock level for both types of FIs.

The PageRank is the most important driver of the systemic impact of banks. Moreover, this importance increases with the level of the initial shock. This measure reflects not only the in-degree (number of lenders) of the node, but also that of its direct and indirect connections. Hence, a shock in an FI with a high PageRank will spam through a large number of FIs, causing a large impact in the whole system. On the other hand, the systemic impact of credit unions is driven by a combination of topological (closeness centrality and PageRank) and financial (interbank liabilities-to-equity ratio and total assets) variables. The higher the level of the initial shock, the higher the relevance of these two financial features. Interbank assets-to-equity ratio is the main driver of systemic vulnerability for both banks and credit unions, especially for higher levels of the initial shock. Thus, while the impact of an FI is mainly driven by its centrality, its vulnerability depends on how much it is exposed to other FIs in the financial network.

[6] The main differences of credit unions from banks are: i) credit unions are not profit-oriented and ii) they conduct their business activities solely with their members. To more details, see, e.g., [50].

Our contribution to the literature is threefold. First, we address SR considering different levels of initial shock. As discussed in [51], traditional centrality measures assess networks from a static point of view. A bank which is very central – that is, systemically relevant – in a financial network at a high level of external risk (which, in our framework, is represented by the initial fraction of equity loss) will not necessarily be central at a lower level of external risk, and vice versa. Therefore, a change in the external level of risk can make an FI more or less systemically relevant. It is worth investigating whether not only the systemic relevance of individual institutions, but also its determinants, are affected by the level of external risk. This result is also related to the study of [52], that assessed the importance of the shock size as determinant of the relationship between interconnectedness and SR. The authors showed a more (less) interconnected financial network brings a higher stability to the financial system under sufficiently small (large) negative shocks. We go one step further by showing that the role of financial and other topological features, other than interconnectedness, in driving SR changes with the shock size.

Second, to our best knowledge, the application of ML methods to the identification of systemically important financial institutions is a novelty. The relationship between SR and its determinants can be expressed by the equation $Y_i = f(\mathbf{X}_i) + \epsilon$. Our interest is in the optimal prediction of Y_i instead of the interpretability of *f*, which is often the case of microeconometric models that use linear models such as OLS or panel fixed effects. We believe networks encode complex financial relationships among FIs. In this way, non-linear models could largely improve the model's estimation quality. Hence, we employ ML methods for this task.

Third, we clearly disentangle the two dimensions of SR. Most measures consider the SR posed by an institution as positively correlated to its loss given a distress in the system ([22]). However, this statement is not true in many situations. For instance, suppose there is an institution acting mainly as a borrower in the interbank market. Its default would cause great distress in the system: a great number of institutions would not receive their debt obligations. Nonetheless, it would not be so impacted by the default of other institutions, as it has few borrowers.

Besides the literature on bank-specific determinants of SR, our research is also related to studies tackling which network properties are more relevant to the dynamics of the system (e.g., [53, 54, 55, 56]). For instance, in studies devoted to the dynamics of disease spreading [55, 54], the purpose is to predict the stationary value of Y_i, the share of infected nodes when the disease is seeded at node i. They concluded the degree distribution is crucial for the existence of a vanishing threshold. Studies on other dynamical processes, as synchronization phenomena ([53]) and rumor spreading ([57]), reached similar conclusions. Even though the financial network is exogenous, the spreading of an initial shock can be considered a dynamical process. By counting each time a shock propagates from one node to its neighbors as one time step, this process spans for a certain number of periods T. After this time, the outcome of this process – in our case, the aggregate loss of economic value – reaches a stationary value.

This paper proceeds as follows. Methodological issues and the data set are discussed in Section 2. Section

3 brings the results. Finally, conclusions take Section 4.

2. Methodology and data set

2.1. The data set

Our data set comprises quarterly information from March 2012 through December 2015 on the institutions participating in the Brazilian interbank market. We considered financial conglomerates or individual FIs belonging to the Brazilian banking sector (classified from "b1" through "b4" according to the Central Bank of Brazil's classification system). Institutions with negative net worth were excluded. The number of institutions in our sample on each date varies from 839 to 950.

Next, we build the network formed by the net exposures of these institutions in the interbank market. In this network, we consider all types of unsecured financial instruments registered in the Central Bank of Brazil. The main types of financial instruments are credit, capital, foreign exchange operations, and money markets. These operations are registered and controlled by different custodian institutions: Cetip[7] (private securities), the Central Bank of Brazil's Credit Risk Bureau System (SCR)[8] (credit-based operations), and the BM&FBOVESPA[9] (swaps and options operations).

We calculate the following node centrality measures on our interbank network: degree (K), clustering coefficient (C), closeness centrality (CC), betweenness centrality (B), PageRank (PR), and k-core (KC). Our network is directed. Thus, two centrality measures (K and CC) are computed for both incoming and outgoing edges, being differentiated by the suffixes "in" and "out". The incoming (outgoing) edges refer to the relationships an institution takes part as a borrower (lender) in the interbank market.

In addition to these centrality measures, we collected some financial information on the institutions in our sample: total assets, net worth, and return on equity.[10] We also computed the interbank assets/liabilities-

[7]Cetip is a depositary of mainly private fixed income, state and city public securities, and other securities. As a central securities depositary, Cetip processes the issue, redemption, and custody of securities, as well as, when applicable, the payment of interest and other events related to them. The institutions eligible to participate in Cetip include commercial banks, multiple banks, savings banks, investment banks, development banks, brokerage companies, securities distribution companies, goods and future contracts brokerage companies, leasing companies, institutional investors, non-financial companies (including investment funds and private pension companies) and foreign investors.

[8]SCR is a very thorough data set that records every single credit operation within the Brazilian financial system worth 200BRL or above. Up to June 30th, 2016, this lower limit was 1,000BRL. Therefore, all the data we are assessing have been retrieved under this rule. SCR details, among other things, the identification of the bank, the client, the loan's time to maturity and the parcel that is overdue, modality of loan, credit origin (earmarked or non-earmarked), interest rate, and risk classification of the operation and the client.

[9]BM&FBOVESPA is a privately-owned company that was created in 2008 through the integration of the Sao Paulo Stock Exchange (Bolsa de Valores de São Paulo) and the Brazilian Mercantile & Futures Exchange (Bolsa de Mercadorias e Futuros). As Brazil's main intermediary for capital market transactions the company develops, implements and provides systems for trading equities, equity derivatives, fixed income securities, federal government bonds, financial derivatives, spot FX, and agricultural commodities. On March 30th, 2017, BM&FBOVESPA and Cetip merged into a new company named B3.

[10]This information was retrieved from `https://www3.bcb.gov.br/ifdata`.

to-equity ratio. The set of variables that will be explored as potential determinants of SR are presented in Table 1, and the correlation among them is depicted in Figure 1. We can observe there are expressive correlations between topological variables (e.g., Kin and PR), financial variables (e.g., NW and TAS), and between different types of variables (e.g., Kout and TAS).

Type	Variable	Acronym
Financial	Total assets	TAS
	Net worth	NW
	Return on equity	ROE
	Interbank assets-to-equity ratio	IBA
	Interbank liabilities-to-equity ratio	IBL
Topological	Degree	Kin/Kout
	Clustering coefficient	C
	Closeness centrality	CCin/CCout
	Betweenness centrality	B
	PageRank	PR
	k-core	KC

Table 1: Potential determinants assessed in the study.

2.2. Systemic impact and vulnerability

We compute our metrics of SR following the *differential DebtRank* methodology ([27]. The exposure network of the interbank market is represented by $\mathbf{A} \in N \times N$, where N is the number of banks and A_{ij} is the asset invested by i at j. At period 0, we impose an exogenous shock on FI j, reducing its equity by a fraction of ζ. It will cause a subsequent loss $L_{ij}(1)$ to its creditors, indexed by i, equal to $\mathbf{A}_{ij}\zeta$. At period 2, j's creditors will propagate this loss to their creditors in a similar fashion, and so on. Formally, we have

$$L_{ij}(t) = min\left(A_{ij}, L_{ij}(t-1) + \mathbf{A}_{ij}\frac{[L_j(t-1) - L_j(t-2)]}{E_j}\right), \tag{1}$$

$$L_i(t) = min\left(E_i, L_i(t-1) + \sum_j \mathbf{A}_{ij}\frac{[L_j(t-1) - L_j(t-2)]}{E_j}\right), \tag{2}$$

in which $t \geq 0$ and E_j is FI j's equity. Thus, when an FI j suffers an additional loss equal to fraction ζ of its equity, it will impose a loss to its creditors that corresponds to ζ times their exposures towards j. Observe that equity positions as well as the exposure network are time-invariant, i.e., they are taken as exogenous. The propagation considers stress differentials rather than stress absolute values (hence the methodology's name) to avoid double-counting. Two more restrictions apply:

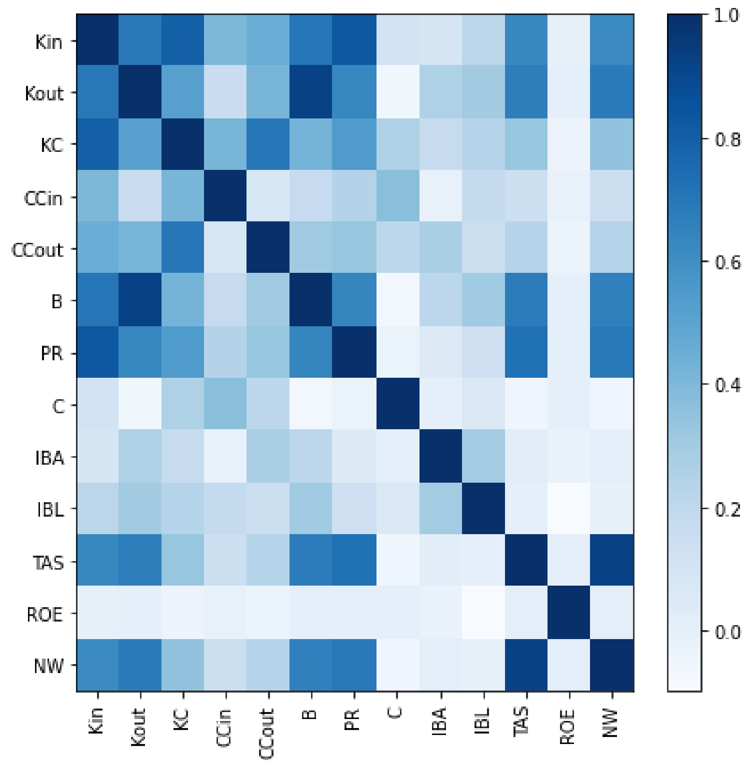

Figure 1: Correlation between the potential determinants.

- L_{ij} cannot be greater than A_{ij}, i.e., j cannot impose to i a loss greater than i's exposures towards j. When $L_{ij} = A_{ij}$, j stops imposing losses on i;
- L_i cannot be greater than E_i, i.e., i's losses cannot be greater than its equity. When $L_i = E_i$, i stops propagating losses to other FIs.

The system converges after a sufficiently large number of periods $T \gg 1$. Then we have the final matrix of losses $\mathbf{L}^{j,\zeta} \in N \times 1$, where $L_i^{j,\zeta}$ is the total loss suffered by agent i after an initial shock of size ζ at agent j.

We repeat this process for the other FIs. Finally, we compute our two measures of SR. We define the *systemic impact* (SI) of bank i as

$$SI_{i\zeta} = \frac{\sum_j \left[L_j^{i,\zeta} - L_j^{i,\zeta}(0) \right]}{\sum_j E_j}, \tag{3}$$

where $L_j^{i,\zeta}(0) = \zeta E_j$ if $j = i$ and 0 otherwise. Our second measure, the *systemic vulnerability* (SV), is represented by the following equation:

$$SV_{i\zeta} = \frac{1}{N} \sum_j \frac{L_i^{j,\zeta} - L_i^{j,\zeta}(0)}{E_i}. \tag{4}$$

Therefore, $SI_{i\zeta}$ measures the fraction of the aggregate FIs' equity which is lost as a consequence of an initial shock of size ζ at FI i's equity. On the other hand, $SV_{i\zeta}$ refers to the average i's equity loss when the other FIs are reduced by ζ. Observe the following:

- We remove the initial shock from the computation of the SR measures, as we are interested only in the losses caused by the contagion;
- We also compute $SI_{i\zeta}$ for the FI that suffered the initial shock. Due to network cyclicality, a shock propagated by a given FI can hit it back. For the same reason, we include the loss imposed by an FI on itself in the calculation of $SV_{i\zeta}$.

2.3. Random forest and XGBoost

After the computation of the systemic risk measures, we employ two machine learning techniques – XGBoost ([58]) and random forest ([59]) – to assess their determinants. Both are ensemble learning methods that can be used for both classification and regression. In this case, they are employed for regression tasks.

Random forest (RF) operates by constructing several decision trees.[11] It returns the average prediction of the individual decision trees. XGBoost (XB) is an optimization algorithm that works with an ensemble of weak predictors (usually, decision trees) and creates a more efficient predictor model. At each boosting stage, the XB algorithm attempts to increase the performance of the predecessor model by including a new estimator.

The purpose is to estimate a predicted output $\hat{y}_i$ from an observed output y_i and a vector of explanatory variables X_i. In this paper, the output to be predicted are the systemic risk measures $SI_{i\zeta}$ and $SV_{i\zeta}$, and the explanatory variables are those listed in Table 1. Both models are trained and validated through a process known as *repeated k-fold cross-validation.* The data set (the observed output and the explanatory variables) is split into k different parts (folds). $k-1$ folds are used in the development of the model. Then, the model is trained on the remaining fold: the predicted output $\hat{y}_i$ and the observed output y_i of the remaining fold are used to compute score measures, such as the root mean squared error (RMSE). Each fold is used as the testing data set. In this paper, we applied a repeated k-fold cross-validation with $k = 5$ and 10 repetitions. Hence, a total of 50 regressions are run.

The RMSE is used to tune the number of estimators of both methods. In the RF, the number of estimators is the number of decision trees in each forest. In the XB, this is the number of boosting stages to be performed. The number of estimators varies within a grid of ascending values. For each of these values, the regressions are run and the average score is computed. The number of estimators is chosen so that increasing it does not improve the performance of the method. We performed the tuning within the grid [30, 50, 70, 100, 300, 500]. After this procedure, we set the value of both parameters as 50.

[11] On decision trees, see, e.g., [60]

3. Results

We computed $SI_{i\zeta}$ and $SV_{i\zeta}$ varying the value of ζ in the interval (0.1,1] with step 0.1. Hence, there are 20 dependent variables for each observation. We excluded 76 outliers out of 14,467 observations, with $SV_{i\zeta} > 10$. These are small FIs, most of them credit unions, highly leveraged as lenders in the interbank market between March 2012 and September 2013.

We applied the two ML techniques – RF and XB – to predict $SV_{i\zeta}$ only using the observations with positive assets in the interbank market. The reason is that, if an FI did not grant loans, it is not vulnerable to other FI's defaults. Hence, its vulnerability is zero by definition. Similarly, we performed the ML analysis to predict $SI_{i\zeta}$ only using the observations with positive liabilities. We also performed the analysis separately for banks (FIs classified as b1, b2, or b4) and credit unions (FIs classified as b3C or b3S). Credit unions have many differences from banks. Unlike most FIs, credit unions are not profit-oriented. Their business activities (receive deposits or shares and grant loans) are conducted solely with their members, which are also their owners. Managers of credit unions do not receive bonuses and any surplus is distributed among their members-owners ([50]). Due to these distinctive characteristics, credit unions are also expected to have specific SR drivers.

The differences between banks and credit unions are also evidenced in our sample. Credit unions are much more numerous than banks, but are smaller in terms of assets and equity. Banks are much more interconnected and leveraged (both in terms of assets and liabilities) in the interbank market. However, while banks act mainly as lenders in the interbank market (the out-degree is greater than the in-degree), credit unions act mainly as borrowers. All this implies that banks are more vulnerable to shocks in other FIs and shocks in banks cause a higher impact in the whole system.

3.1. Systemic impact

The systemic impact of the banks is mainly driven by the PageRank (Figure 2). PageRank is a centrality measure specially designed for directed graphs and it is computed recursively. The PageRank of an FI is positively impacted by its in-degree (number of lenders), but also by the in-degree of its direct and indirect neighbors, weighted by a dumping factor (the further away the neighbor, the smaller its impact on the FI's PageRank). Therefore, a shock in an FI with a high PageRank is expected to propagate through a high number of other FIs.

Both methods provide similar outcomes. After PageRank, total assets appears as the second most important feature for small values of the initial shock. As ζ increases, so the relevance of the PageRank and total assets become less important. Considering the aggregate relevance of financial and topological variables (Figure 3), we can observe that the latter become more important drivers of the systemic impact of banks as ζ increases.

As expected, the systemic impact of credit unions is driven by features different from those of banks. Closeness centrality and PageRank appear as the main topological features driving the systemic impact of

	Bank	Credit union
Number per period	128.75	775.44
Kin	11.39	1.59
Kout	16.04	0.73
IBA	1.43	0.32
IBL	1.81	0.99
NW*	3.82	0.03
TAS*	50.30	0.15
V = 0.1	0.19	0.07
V = 1	0.57	0.26
S = 0.1	0.48	0.18
S = 1	1.88	0.28

*: in BRL billions.

Table 2: Average value of some variables by type of FI.

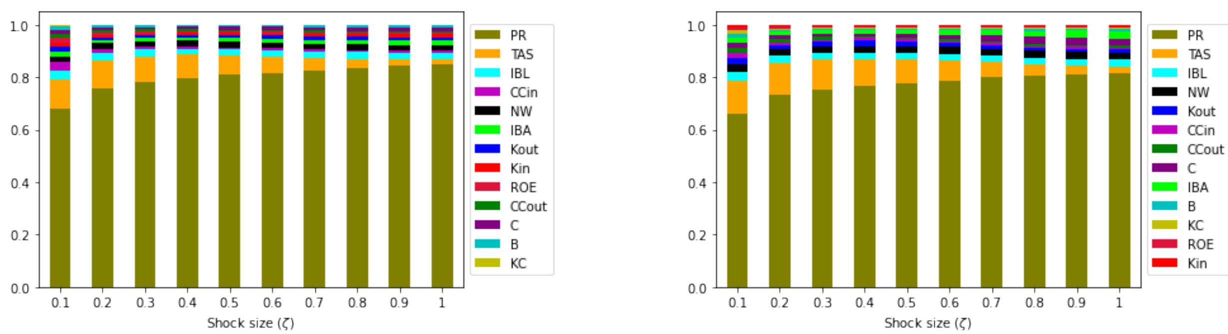

Figure 2: Importance of the features to the prediction of the systemic impact of the banks obtained through RF (left) and XB (right).

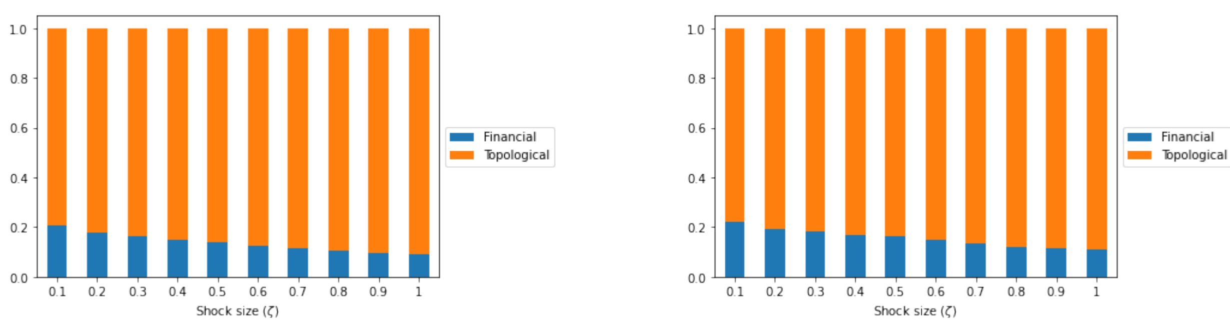

Figure 3: Aggregate importance of financial and topological features to the prediction of the systemic impact of the banks obtained through RF (left) and XB (right).

credit unions. Closeness centrality is related to physical proximity. Nodes with high closeness centrality have the shortest average distance (as measured by the shortest path) to all other nodes in the network. Both

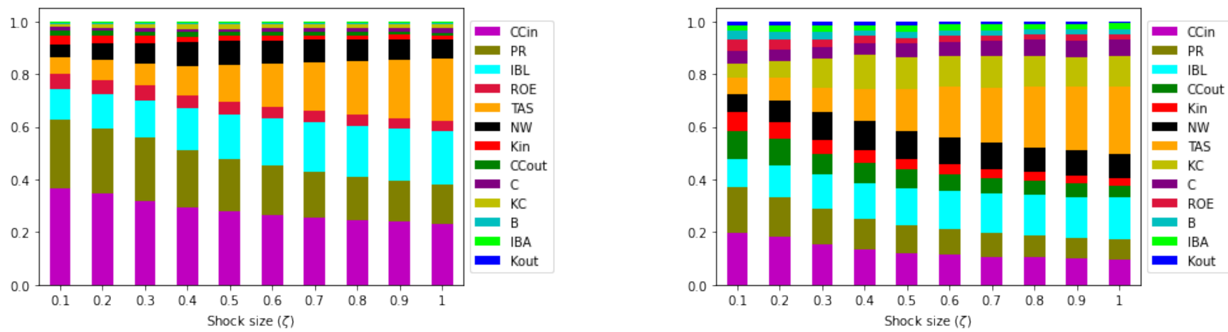

Figure 4: Importance of the features to the prediction of the systemic impact of the credit unions obtained through RF (left) and XB (right).

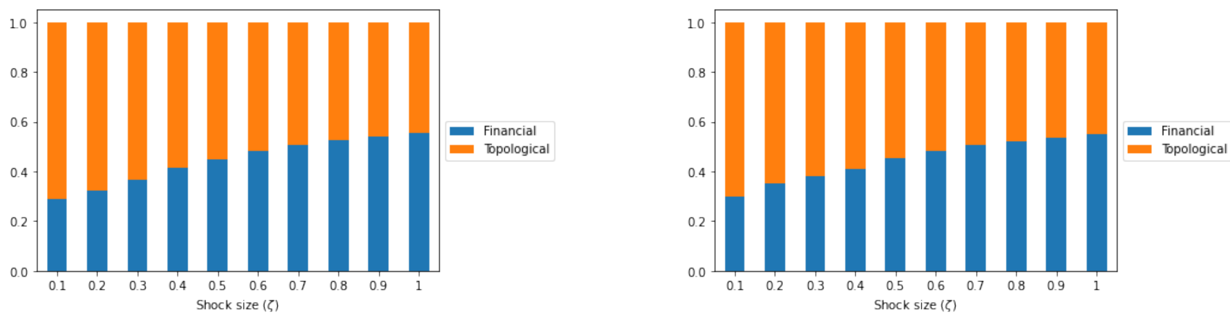

Figure 5: Aggregate importance of financial and topological features to the prediction of the systemic impact of the credit unions obtained through RF (left) and XB (right).

methods attach a higher importance to two financial features – interbank liabilities-to-equity ratio and total assets – as far as ζ increases. Unlike the case of banks, the aggregate importance of the financial variables increases with ζ, although the topological variables are the main drivers of systemic impact for credit unions for any value of the initial shock. All these considerations can be seen in Figures 4 and 5.

3.2. Systemic vulnerability

Unlike the case of systemic impact, the vulnerability of FIs is mainly driven by financial variables, in particular by the interbank assets-to-equity ratio. Thus, an FI's systemic vulnerability essentially depends on its exposure in the interbank market. However, this feature alone is not enough to predict the FIs' systemic vulnerability, mainly for smaller values of ζ. The aggregate impact of the other financial and topological features is non-negligible. XB attaches a smaller importance to interbank assets (and to financial features in general) than RF. Moreover, financial variables appear to be more important for credit unions than for banks. These considerations are depicted in Figures 6-9.

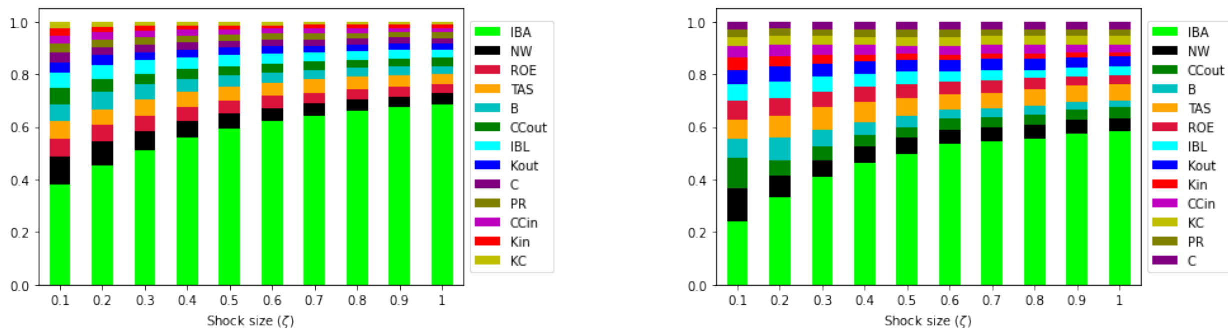

Figure 6: Importance of the features to the prediction of the systemic vulnerability of the banks obtained through RF (left) and XB (right).

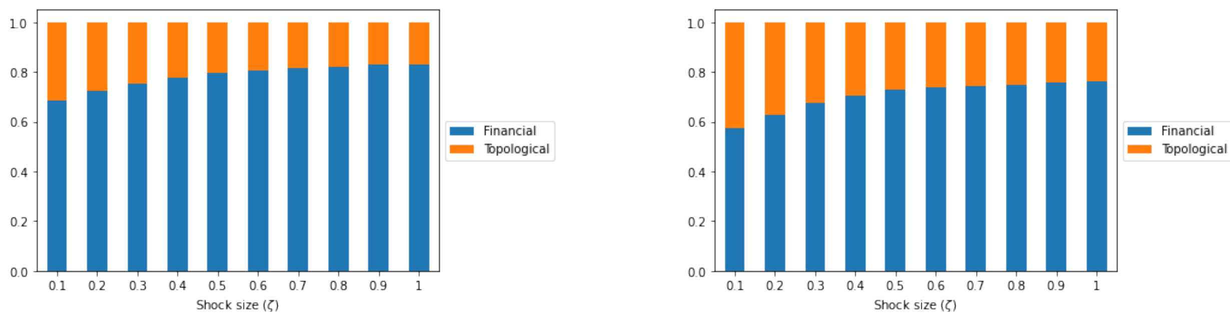

Figure 7: Aggregate importance of financial and topological features to the prediction of the systemic vulnerability of the banks obtained through RF (left) and XB (right).

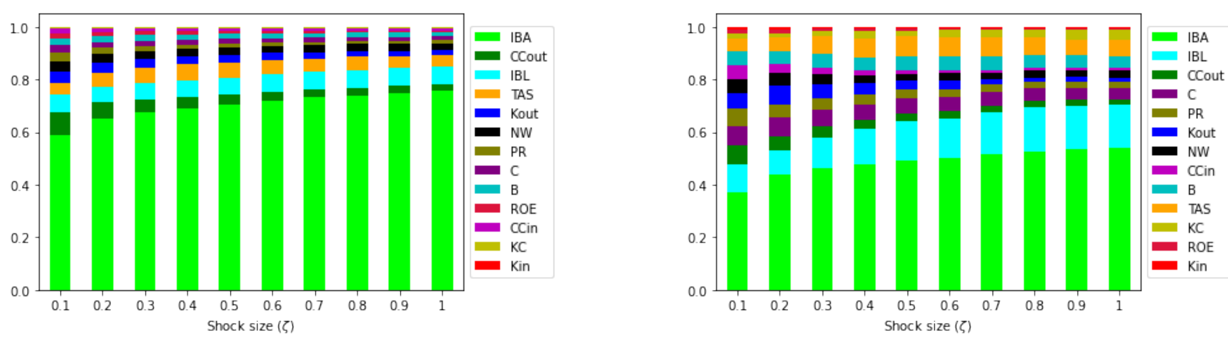

Figure 8: Importance of the features to the prediction of the systemic vulnerability of the credit unions obtained through RF (left) and XB (right).

Comparing Figures 3 and 7, we can observe an important asymmetry. The aggregate importance of topological features in driving the systemic impact of banks varies between 0.8 and 0.9. Notwithstanding, the aggregate importance of financial features in driving systemic vulnerability is smaller, in the range 0.6-0.8 (depending on ζ and the method used). We observe a similar asymmetry in the case of credit unions, namely, the importance of financial variables driving systemic vulnerability is higher than the importance of topological variables driving systemic impact (Figures 5 and 9).

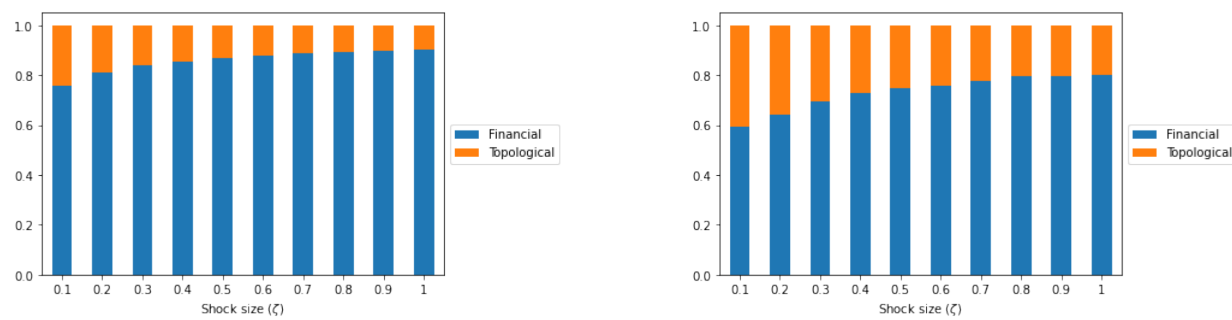

Figure 9: Aggregate importance of financial and topological features to the prediction of the systemic vulnerability of the credit unions obtained through RF (left) and XB (right).

3.3. Shapley values

In order to go further on the interpretability of our results, we resort to the computation of Shapley values. This approach is originated from the coalition games theory ([61, 62]). Besides providing additional evidence on features' importance, Shapley values can also inform whether a given feature is positively or negatively correlated to the systemic risk measure. We compute Shapley values through the SHAP (SHapley Additive exPlanation) framework proposed by [63]. The authors propose an explainer model g aiming at predicting an output using a set of M features as inputs. The predicted value for a given data-instance is given by

$$g(z') = \phi_0 + \sum_{i=1}^{M} \phi_i z'_i, \tag{5}$$

where z' is a binary variable indicating whether feature i was included in the model or not. Therefore, the SHAP value ϕ_i indicates in which extent the feature i shifts the predicted value up or down from a given mean output ϕ_0. [63] showed that, under certain properties (local accuracy, missingness, and consistency), ϕ_i corresponds to the Shapley value of the game theory. The SHAP value of feature i is given by

$$\phi_i = \sum_{S \subseteq M n i} \frac{|S|!(|M| - |S| - 1)!}{M!} [F(S \cup \{i\}) - F(S)]. \tag{6}$$

Therefore, the SHAP value of feature i for a given data-instance computes the difference between the predicted value of the instance using all features in S plus feature i, $F(S \cup \{i\})$, and the prediction excluding feature i, $F(S)$. This is weighted and summed over all possible feature vector combinations of all possible subsets S.[12] We then proceed as follows:

- Compute the SHAP value according to the explainer models (RF and XB), considering our systemic risk measures as the output to be predicted;

[12]For details on the calculation of SHAP values, see, e.g., [63] and [64].

- Compute the average absolute SHAP value over all data-instances. This will inform the size of the feature importance in driving the output;

- Multiply the average absolute SHAP value by the sign of the correlation between the feature value and the SHAP value. This will show whether the feature is positively or negatively correlated to the output.

As in the previous section, for both models (RF and XB), we implement a k-fold cross-validation with $k = 5$ and 10 repetitions. The results are presented in Figures 10-13 and roughly corroborate those of the previous subsection. The systemic impact of banks is mainly (positively) determined by the PageRank. However, this effect is nonlinear regarding the size of the initial shock. The maximum impact of PageRank on banks' systemic impact is observed at a shock size $\zeta_{max} < 1$. The importance of the interbank liabilities-to-equity ratio, net worth, and interbank assets-to-equity ratio increases monotonically with ζ. As expected, while the first two variables have a positive impact on banks' systemic impact, the last one impacts it negatively.

The relative importance of total assets, interbank liabilities-to-equity ratio, and net worth in driving the systemic impact of credit unions increases monotonically with ζ. The most important topological variables in determining the systemic impact of credit unions are the closeness centrality (in) and PageRank. The impact of the former is positive, whereas the latter has a negative impact in most cases (according to the RF model, the PageRank has a negative effect on the systemic impact of credit unions for small values of ζ).

The vulnerability of both banks and credit unions, according to both methods, is mainly driven by the interbank assets-to-equity ratio. Its impact is positive. However, there is an important difference between banks and credit unions. While the effect of the feature increases monotonically with the initial shock size in the former case, it has a maximum at a level of ζ below 1 in the latter one.

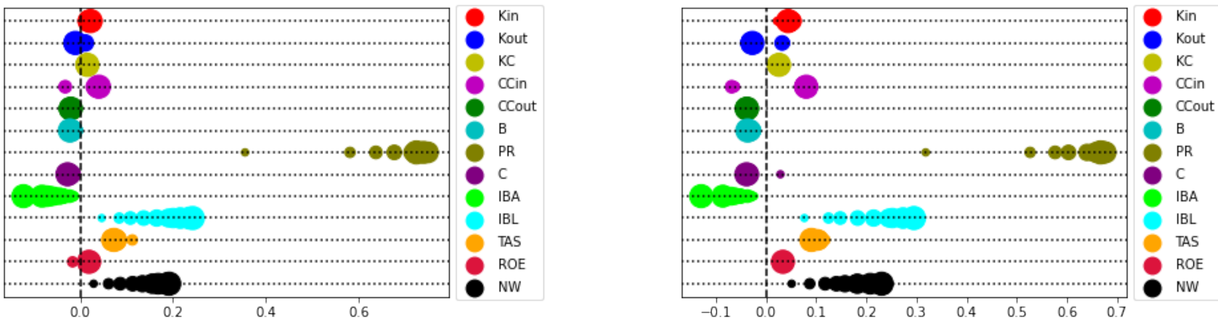

Figure 10: Average absolute SHAP values multiplied by the sign of the correlation between SHAP values and the feature values. The predicted output is the systemic impact of the banks obtained through RF (left) and XB (right). Dots size is proportional to the size of the initial shock (ζ).

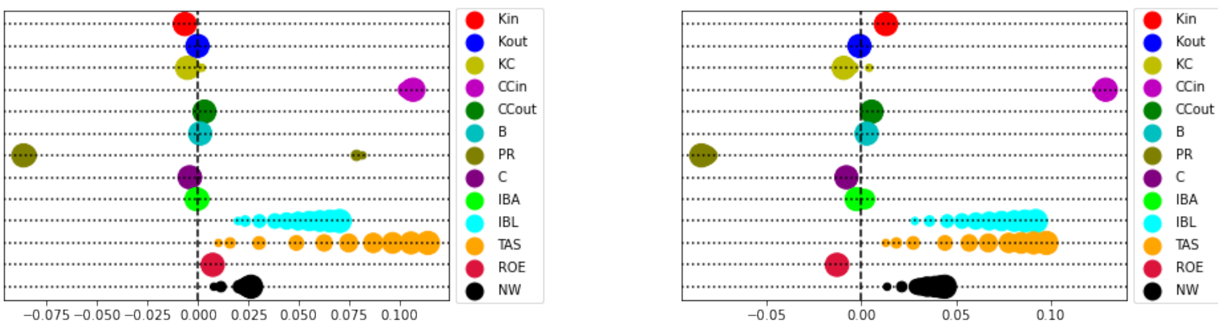

Figure 11: Average absolute SHAP values multiplied by the sign of the correlation between SHAP values and the feature values. The predicted output is the systemic impact of the credit unions obtained through RF (left) and XB (right). Dots size is proportional to the size of the initial shock (ζ).

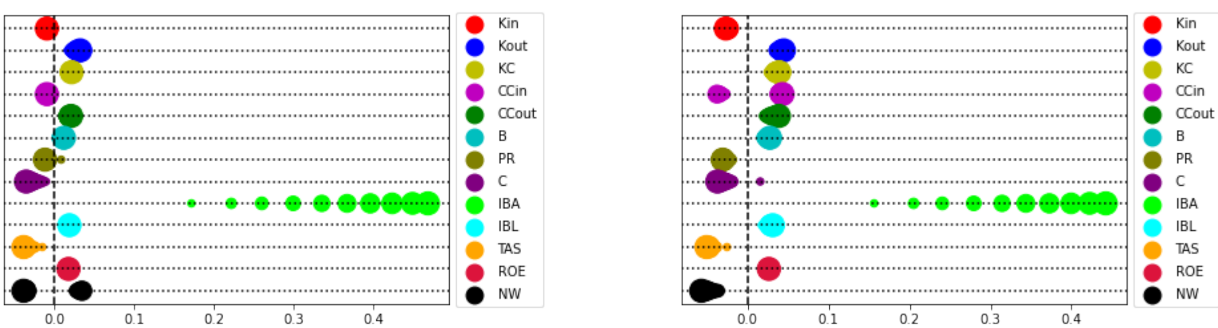

Figure 12: Average absolute SHAP values multiplied by the sign of the correlation between SHAP values and the feature values. The predicted output is the systemic vulnerability of the banks obtained through RF (left) and XB (right). Dots size is proportional to the size of the initial shock (ζ).

4. Concluding remarks

In this study, we assessed the role of financial and topological features as drivers of SR. Our data set comprises quarterly information on FIs in the Brazilian interbank market between March 2012 and December 2015. We computed the SR in its both dimensions – systemic impact and systemic vulnerability – for different levels of the initial shock. We performed this task using the *differential DebtRank* methodology. To assess

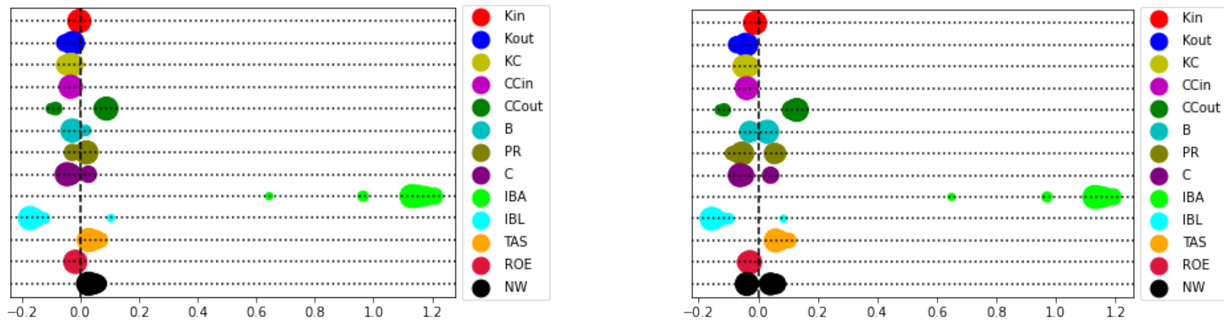

Figure 13: Average absolute SHAP values multiplied by the sign of the correlation between SHAP values and the feature values. The predicted output is the systemic vulnerability of the credit unions obtained through RF (left) and XB (right). Dots size is proportional to the size of the initial shock (ζ).

the relevance of each feature, we used two machine learning techniques: random forest and XGBoost. As banks and credit unions have different characteristics, we carried out this last step separately for each type of FI. We also computed the Shapley values employing these two techniques as explainer models. Shapley values inform not only on the size of the effect of a given feature on the SR, but also on the direction of this effect – that is, whether the feature is positively or negatively correlated to the SR measure.

We have found that the drivers of SR depend on the dimension of the risk that is being assessed. Topological features are the most important drivers of the systemic impact. PageRank appears as the main determinant of systemic impact for banks. In the case of credit unions, the most important topological features are closeness centrality, and PageRank. On the other hand, financial variables are the main determinants of systemic vulnerability. Interbank assets-to-equity ratio figures as the most important driver of systemic vulnerability for both types of FIs, although the role of other variables cannot be neglected mainly for small levels of initial shock.

Another interesting finding is that the importance of a given feature in driving SR varies with the level of the initial shock. In general terms, the importance of topological features on the prediction of systemic impact of the banks increases for higher levels of the initial shock. For credit unions, the opposite happens. Financial variables become more relevant, although topological variables play a more important role for any value of initial shock. Moreover, the importance of financial features as drivers of systemic vulnerability increases with the initial shock level for both types of FIs.

Finally, our results show that different types of FIs have different key drivers of SR. Interbank assets-to-equity ratio is the main driver of systemic vulnerability for both banks and credit unions. However, while the systemic impact of banks is mainly determined by the PageRank, the systemic impact of credit unions is driven by a combination of topological and financial variables.

This study brings an important contribution to the literature on the determinants of systemic risk. We show that the drivers of systemic risk depend on at least three aspects: the dimension of the risk – the loss *suffered* or *caused* by the FI -, the size of the initial shock on the system, and the type of the FI. It also provides insights to policymakers aiming at targeting systemically important FIs. Finally, it sheds some light on the dynamical process concerning the spread of shock in financial networks.

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