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Dynamic Electoral Competition with Voter Loss-Aversion and Imperfect Recall

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Abstract: This paper explores the implications of voter loss-aversion and imperfect recall for the dynamics of electoral competition in a simple Downsian model of repeated elections. We first establish a benchmark result: when the voters' reference point is forward-looking, there are a continuum of rational expectations equilibria (REE). When voters are backward-looking i.e. the reference point is last period's recalled policy, interesting dynamics only emerge when voters have imperfect recall about that policy. Then, the interplay between the median voter's reference point and political parties' choice of platforms generates a dynamic process of polarization (or de-polarization). Under the assumption that parties are risk-neutral, platforms monotonically converge over time to a long-run equilibrium, which is always a REE. When parties are risk-averse, dynamic incentives also come into play, and generally lead to more policy moderation, resulting in equilibria that are more moderate than the most moderate REE.

KEYWORDS: electoral competition, repeated elections, loss-aversion, imperfect recall advantage
JEL CLASSIFICATION: D72, D81

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1 Introduction

This paper explores the implications of voter loss-aversion and imperfect recall for the dynamics of electoral competition in an otherwise quite standard Downsian model with probabilistic voting where parties care both about policies and office.

Our analysis is motivated by two stylized facts about voter behaviour. First, there is now considerable evidence that citizens place greater weight on negative news than on positive when evaluating candidates for office, or the track records of incumbents. In the psychology literature, this is known as negativity bias.[1] For example, several studies find that U.S. presidents are penalised electorally for negative economic performance but reap fewer electoral benefits from positive performance (Bloom and Price, 1975; Lau, 1985; Klein, 1991). Following Lockwood and Rockey (2020), we will interpret this behaviour as the outcome of voter loss-aversion.[2]

The second stylized fact is *imperfect recall* by voters of past policy platforms and policies implemented, for which there is considerable evidence. For example, Lodge et al. (1995), in a review of the literature, state that the "evidence is overwhelming" that "citizens cannot remember many details of election campaigns". However, this does not mean that past policies do not affect voter's current decisions. Lodge et al. (1989, 1995), in a series of experiments, find evidence of a boundedly-rational process of candidate evaluation, where voters are responsive to campaign information, via an overall evaluation of the candidate, formed at the time they process the information, "but are unable, for good reasons, to recollect accurately the considerations that enter into their evaluations". (Lodge et al. (1995), p309).

In this paper, we explore the implications of loss-aversion with both backward-looking and forward-looking reference points. We first show that if voters have forward-looking reference points, in the sense that their reference point correctly anticipates the equilibrium outcome, there are a continuum of equilibria, which we call rational expectations equilibria (REE). This non-uniqueness is driven by the fact that in equilibrium, party platforms are at the boundary between the median voter's gain and loss domains, so the loss for a party from moving towards its ideal point is greater than the gain from an equivalent move towards the median voter's ideal point.

We then explore the how the dynamics of electoral competition unfold when voters

[1]See for example, the survey on negativity bias by Baumeister et al. (2001).

[2]Similar asymmetries have also been identified in the UK and other countries. For example, for the UK, Soroka (2006) finds that citizen pessimism about the economy, as measured by a Gallup poll, is much more responsive to increases in unemployment than falls. Kappe (2013) uses similar data to explicitly estimate a threshold or reference point value below which news is "negative", and finds similar results. Nannestad and Paldam (1997) find, using individual-level data for Denmark, that support for the government is about three times more sensitive to a deterioration in the economy than to an improvement. Soroka and McAdams (2015) argue that this negativity bias on the part of voters is an example of a more general bias whereby suggest that humans respond more to negative than to positive information, and they link this bias to loss-aversion.

have backward-looking reference points i.e. where their reference point is last period's equilibrium policy. Here, we distinguish between two separate effects. First, the position of the median voter's reference point will affect the trade-off between policy position and election probability for either party, the *trade-off* effect. Second, if parties are risk-averse, they care about the variance of equilibrium policies, and in this case, they have a *dynamic incentive* to manipulate the median voter's reference point in the following period. This makes the analysis of multi-period equilibrium intractable, although some results can be obtained for two periods (see section 5 below). So, for most of the paper, we close down the dynamic incentive by assuming that parties are risk-neutral over platforms.

In this case, we first establish that if voters can perfectly recall the previous period's winning platform, the dynamics are relatively uninteresting; the platforms of both parties converge to an REE platform in one step, and for a wide range of initial reference points, the outcome is completely determined just by this reference point: other parameters do not affect the time path.

We then consider the more interesting and realistic case where voters can only imperfectly recall last period's platform. Technically, this smooths the political parties' payoffs; then, the interplay between the median voter's reference point and political parties' choice of platforms generates a gradual process of polarization (or depolarization) of preferences. In fact platforms converge monotonically to a limiting value which depends amongst other things, on the degree of voter loss-aversion and the bias in voter recall. Our model predicts that this dynamic process of party polarization can occur *even though* the ordinal preferences of voters, i.e. their ideal points, do not change at all. So, our model is consistent with the fact that while evidence for elite polarization in the US over the last four decades is very strong, there is much less evidence of polarization at the voter level, at least on the issues. This evidence is further discussed below.

One feature of the equilibrium in both the case of perfect and imperfect recall is that platforms at all periods are bounded within a narrow range, in the sense that they lie within the range of possible REE platforms. In the last section, we analyse a two-period version of the model where parties are risk-averse and thus have dynamic incentives. In this case, with two elections, conditional on a given initial (period zero) policy, equilibrium platforms are more moderate in the first period than in the second. If the initial policy is close enough to the median voter's ideal point, first-period policy can be more moderate than the most moderate REE platform. So, with dynamic incentives, equilibrium policies do not have to lie in the REE range.

Related Literature

This paper is related to an early unpublished paper by Lockwood and Rockey (2015), and subsequent papers by Alesina and Passarelli (2019) and Lockwood and Rockey (2020). In Lockwood and Rockey (2015, 2020), analyses of a similar model to the one studied in this paper were undertaken, where voters have loss-aversion, but perfect recall. In Lockwood and Rockey (2020), the one-period and two-period versions of the model with perfect recall were studied. The focus of Lockwood and Rockey (2020) was however, mainly on how platforms adjusted in the second period to shifts in voter preferences. Lockwood and Rockey (2015) was an early version of that paper, and also had some results on dynamic incentives. So, this paper extends their analysis to the multi-period case with imperfect recall, and importantly, also provides a benchmark in the form of the REE.

Alesina and Passarelli (2019) also study a two-period model of electoral competition with loss-aversion. However, their model is rather different to the standard Downsian model. There is a one-dimensional policy describing the scale of a project, which generates both costs and benefits for the voter. In this setting, for loss-aversion to play a role, the benefits and costs of the project must be evaluated relative to separate reference points. This is because if loss-aversion applies to the net benefit from the project, the status quo cannot affect the ideal point of any voter. We do not need this construction, because in our setting, the voters compare the utility from policy positions to party valences. Finally, their model assumes a backward-looking reference point throughout.

Alesina and Passarelli (2019) prove persistence in policies; if (for example) the right-wing party wins the election, then in the following period, *both* parties' equilibrium platforms will be further to the right. Unlike this paper, they only consider two periods, so they cannot study the long-run behaviour of this process.[3] Also, reflecting the fact that their model is very different to ours, the dynamics in our model (which is much closer to a classic Downsian model) are qualitatively completely different. In fact, due to the symmetry of our model, the second period platforms of both parties are *independent* of which party wins the election in the first period.[4] Rather, the inter-temporal dependence between equilibrium platforms is in the the *polarisation* dimension; for example, in the two period case, the amount of polarization in the second period is increasing in the polarization on the first period, and this is also true in the case with an arbitrary number of periods.

This paper is also more loosely related to the literature studying repeated elections in a Downsian setting where parties can make ex ante policy commitments, and where there is some kind of linkage between periods in the economic or political environment. This literature can be helpfully sub-divided by considering whether it is preferences, the economic environment, or the political environment that provides the linkage.

In our paper, the linkage is clearly through preferences. So, perhaps the closest to our paper is the recent paper by Callander and Carbajal (2020) where the inter-temporal linkage comes in the form of an adjustment process for voter preferences; voters move their ideal points in the direction of the party that they voted for in the previous period. This adjustment is, in their words, "a smooth generalization of cognitive dissonance theory"

[3] In the introduction, they describe this persistence as a "cycle", but technically, as only two periods are studied, they cannot establish whether the time-path for platforms is cyclic or monotonic.

[4] Both parties' policies in equilibrium are equidistant from the median voter's ideal point.

whereby voters adjust their preferences to avoid conflict with their decisions. Therefore, as in our paper, the inter-temporal linkages arise because of a behavioral modification to standard voter preferences. However, our mechanism is arguably more subtle, as the ideal points of the voters do not "chase" the party platforms. Rather, voter ideal points are fixed, and it is is the voter reference point, which determines the location of curvature in the election probability, which changes with platforms. Also, unlike Callander and Carbajal (2020), we are able to analyse the case where parties are forward-looking, rather than myopic.[5]

Papers where there is an inter-temporal linkage in the economic environment include Battaglini (2014), who studies a dynamic theory of electoral competition in which two office-motivated parties choose platforms including the level of public debt, which creates the linkage. In his model, however, parties are solely office-motivated, so there is full platform convergence in equilibrium, and the focus is on how this platform deviates from Pareto-efficiency. Also related are two papers where first period policy decisions by the winning party change induced preferences over taxes or other fiscal policies in the second period. Biais and Perotti (2002) studies the effect of privatization in building support for the right-wing party at the second election, and Prato (2018) has a model where voters learn about an aggregate shock via home-ownership in the first period, which changes their induced preferences over a tax rate in the second period.

Papers where the political environment provides the linkage include Nunnari and Zápal (2017); Forand (2014) which assume that if a party wins the election, it is then committed to its winning platform for as long as in remains in power. Forand (2014) finds that incumbents' reduced policy flexibility leads to alternation in power and policy dynamics that converge to alternations at policies equally preferred by the median voter.[6]

Finally, several recent papers explore the implications of voter loss-aversion in rather different political settings to the one that we study here. Passarelli and Tabellini (2017) study a model of political protest where protest is partially motivated by policy payoffs relative to what is seen as a fair reference point. However, in that paper, the reference point is exogenously determined by a definition of what the voters deem a "fair" policy. Grillo (2016) shows that with loss aversion, truthful communication to the voters about

[5] Also somewhat related is a seminal paper by Duggan (2000) who studied a model where individual citizens have heterogeneous policy preferences and also care about office, and can pre-commit to policy positions before elections. Every period the incumbent faces a challenger in an election, where the latter is randomly selected from the population. There is no inter-temporal linkage in preferences as such, but the equilibrium evolution of platforms over time depends on the preference type of the incumbent and/or challenger. This model has since been extended in various directions. to allow for term limits (Bernhardt et al., 2004), multi-dimensional policy spaces (Banks and Duggan, 2008), for political parties who can choose candidates, (Bernhardt et al., 2009), and to the case where candidates also differ in valence (Bernhardt et al., 2011).

[6] A second, less closely related class of papers with a political linkage are models of dynamic legislative bargaining with an endogenous status quo and farsighted players (Baron, 1996; Kalandrakis, 2006; Diermeier and Fong, 2011; Bowen et al., 2014; Dziuda and Loeper, 2016). In those models, the status quo policy in the legislature is the winning policy proposal from the previous period.

valence is possible in equilibrium. Grillo and Prato (2020) study a model of democratic backsliding where citizens' retrospective assessment of politicians depend on reference points that are endogenous to incumbent behavior. These last two papers are more closely related to what we do because they assume forward-looking reference points. However, these models are quite different to the standard Downsian model of electoral competition, and so we believe that our paper is the first to solve for the equilibrium with forward-looking reference points in that setting.

Evidence on Elite and Non-Elite Polarization

Empirical work on political polarization has distinguished between *elite* polarization, particularly that of politicians, and *mass* polarization. The evidence is clear, that whether computed on the basis of words or deeds American political elites are increasingly polarized. Gentzkow et al. (2019) analyse the corpus of Congressional speeches and show that measured on the basis of differences in language used, partisanship has been consistently increasing since the 1994 election. Analyses based on deeds, particularly voting patterns, find similar evidence although they tend to identify earlier periods of polarization too.[7] Hare and Poole (2014) provide evidence based on common-space (DW-Nominate) estimates of members of congress' positions that polarization has increased since around 1980. McCarty et al. (2009) document a similar pattern in state legislatures. Bonica (2014) suggests that polarization in the Senate is driven by a small number of key events such as the Affordable Care Act, or the run-up to the Iraq war.

Less clear is the extent to which such elite polarization reflects increased mass polarization. Fiorina et al. (2005, p9) argued that claims of an increasingly polarized US electorate "…rests on misinterpretation of election returns …" and that, crucially, that "There is little evidence that Americans' ideological or policy *positions* are more polarized today than they were two or three decades ago, although their *choices* often seem to be." However, Abramowitz and Saunders (2008) and Gentzkow (2016) point out that on key issues, voters are increasingly polarized. That is, while previously voters may have lent rightwards on some issues, and leftwards on others, and voted for both Democratic and Republican candidates that such beliefs and behaviour is increasingly rare.[8]

To summarise, there is robust evidence of increased party polarization over the last 40 years, and there is evidence of increased partisanship amongst those aligned to a party, but there is little evidence of polarization of voters' ideal points.

The remainder of the paper is organised follows. Section 2 sets out the model, section 3 analyses the rational expectations benchmark and section 4 considers repeated elections with a backward-looking reference point. Section 5 considers dynamic incentives, and

[7] Poole and Rosenthal (1984) is an early paper documenting increased polarization in the Senate.

[8] Moreover, Iyengar and Westwood (2015), Webster and Abramowitz (2017) and Iyengar et al. (2019) argue that polarization has moved beyond differences over policy and that today members of each party hold strongly negative feelings about members of the other, so called emphaffective polarization.

section 6 concludes.

2 The Model

2.1 The Environment

There are two parties L and R, and a finite set of voters N who interact over periods $t = 0, 1, 2, \ldots, T$. The number of voters, n, is odd. At $t = 1, \ldots, T$, the two parties, L and R, choose platforms $x_{L,t}, x_{R,t}$ in the policy space $\Re$. At $t = 0$, the platform x_0 is predetermined. Parties are assumed to be able to commit to implement these platforms. Thus, the basic framework is Downsian competition. Parties are also described by a party characteristic $v_{R,t}$, $v_{L,t}$. Our primary interpretation of this will be as valence, although it could capture other things such as the charisma of the candidate, etc. The distinctive feature of our model is that voters have loss-aversion over platforms, with the reference point of a voter being their recollection of the winning platform of the previous period, as described in more detail below.

2.2 Order of Events and Information Structure

Within period t, the order of events is as follows. First, parties L, R simultaneously choose their platforms. Then, the valences $v_{R,t}, v_{L,t}$ are drawn. The difference $v_t = v_{L,t} - v_{R,t}$ is assumed to be uniformly distributed with support $[-\frac{1}{2\rho}, \frac{1}{2\rho}]$. As we will see, the parameter ρ measures the responsiveness of the median voter to policy changes by the parties. Then, all voters vote simultaneously for one party or the other. We will assume that voters do not play weakly dominated strategies; with only two alternatives, this implies that they vote sincerely.

This timing implies that the current valence of both parties is not known at the point when platforms are chosen. For the challenger, this is quite plausible; parties may not fully know their competence in office when they have been out of office for some time. This assumption is also made for the incumbent to maintain symmetry of the model.

Finally, note that from a modeling point of view, the purpose of this timing assumption is a standard one; it makes the outcome of the election uncertain for the two political parties, thus preventing complete convergence in equilibrium to the median voter's ideal point.

2.3 Voter Payoffs

We assume that "ordinary" or intrinsic utility over platforms $x \in \Re$ for voter i is of the absolute value form $u_i(x) = -|x - x_i|$. We also rank voters by their ideal points i.e. $-1 < x_1 < x_2 < \cdots < x_n < 1$. We assume that voter $m = \frac{n+1}{2}$ has an ideal point $x_m = 0$.

As we shall see shortly, this voter will be the median voter in the usual sense i.e. will be decisive in any election.

Following Kőszegi and Rabin (2006),Kőszegi and Rabin (2007), we specify the gain-loss utility over policy for voter i at date $t = 1, 2$ as;

$$u_i(x_t; r_t) = \begin{cases} u_i(x_t) - u_i(r_t), & u_i(x_t) \geq u_i(r_t) \\ \lambda(u_i(x_t) - u_i(r_t)), & u_i(x_t) < u_i(r_t) \end{cases} \tag{1}$$

where r_t is the reference platform. That is, the parameter $\lambda > 1$ measures the degree of loss-aversion. The assumption that λ is the same for all voters is made just for convenience, and could be relaxed. The payoff from party $K = L, R$ at date t with platform x_t and valence v_t is assumed additive in both terms i.e.

$$u_i(x_{K,t}; r_t) + v_{K,t} \tag{2}$$

It remains to specify the reference platform r_t. In the next section, we study an expectations-based reference platform as a benchmark. For the remainder of the paper, we will assume that voters are "backward looking" in that $r_t = \tilde{x}_{t-1}$, where $\tilde{x}_{t-1}$ is the *recalled* equilibrium policy i.e. the platform of the winning party in the previous period.

We model imperfect recall by assuming that the actual winning platform at $t - 1$ is scaled by a random factor ε_t i.e.

$$\tilde{x}_{t-1} = \varepsilon_t x_{t-1},$$

where ε_t is has a continuous distribution with support $[0, \infty)$, distribution function F, and a mean of $1 + b$, where $b > -1$ is the degree of bias in the recall. Also, ε_m is the median value of ε; we allow for skewed distributions, such as the exponential i.e. $1 + b \neq \varepsilon_m$. If $\varepsilon_t \equiv 1$, we have the case of *perfect recall.*

2.4 Win Probabilities

Here, we characterize the probability p_t that party R wins the election. We have assumed that all voters do not use weakly dominated strategies, implying that they vote sincerely. So, from (2), any voter i will vote for party R at t, given platforms $x_{L,t}, x_{R,t}$, if and only if

$$u_i(x_{R,t}; \varepsilon_t x_{t-1}) \geq v_t + u_i(x_{L,t}; \varepsilon_t x_{t-1}) \tag{3}$$

Note that even with loss-aversion, the policy payoffs $u_i(x; x_0)$ are single-peaked in x for a fixed x_0. It follows immediately that the median voter is decisive. So, from now on, we can focus only on the median voter, and we can therefore drop the "m" subscripts, so $u(x; r)$ refers to the median voter's payoff.

The probability that party R wins the election is the probability that the median voter votes for R. Now, parties do not observe the actual recalled reference point, but they know x_{t-1}, so from (3), p_t is just the probability that v_t is less than the expectation of the utility difference for the median voter of $u(x_{R,t};\varepsilon_t x_{t-1}) - u(x_{L,t};\varepsilon_t x_{t-1})$. From the uniform distribution of v_t, as long as $p_t \in (0,1)$, we have

$$p_t = \frac{1}{2} + \rho\left(Eu(x_{R,t};\varepsilon_t x_{t-1}) - Eu(x_{L,t};\varepsilon_t x_{t-1})\right) \tag{4}$$

where the expectation is taken with respect to ε_t. So, we see that the greater ρ, the more responsive is the election probability to platform changes. A sufficient condition for $p_t \in (0,1)$ is then:[9]

A1. $\frac{1}{2} > \rho\lambda$

2.5 Party Payoffs

As is standard, parties have a payoff to holding office, denoted M. Parties also have policy preferences, with the L party having an ideal point of -1, and party R an ideal point of 1. Generally, these can be expressed as $u_R(x) = u\left(|x-1|\right),\ u_L(x) = u(|x+1|)$, where u is a symmetric, concave function with a maximum at zero. For example, $u_R = -\left|x-1\right|^{\alpha},\ \alpha \leq 1$ is a class of such functions. An important special case is where $\alpha = 1$, i.e. absolute value preferences, in which case parties are risk-neutral over lotteries over policies in $[-1,1]$.

In any period t, expected payoffs for the parties are calculated in the usual way as the probability of winning, times the policy payoff plus M, plus the probability of losing, times the resulting policy payoff i.e.

$$\begin{aligned} \pi_{R,t} &= p_t(u_R(x_{R,t}) + M) + (1-p_t)u_R(x_{L,t}) \\ \pi_{L,t} &= p_t u_L(x_{R,t}) + (1-p_t)(u_R(x_{L,t}) + M) \end{aligned} \tag{5}$$

We assume that parties are forward looking, with a discount factor δ.

Finally, we want to rule out the uninteresting case where the incentives to converge to the median voter's ideal point, zero, are so large that parties do not choose different platforms in equilibrium. So, we will assume:

A2. $\frac{1}{2}u'_R(0) > \rho\lambda M$

This assumption has the following interpretation: the benefit from a small increase in x_R from zero, at the equilibrium election probability, one half, exceeds the expected loss from a lower probability of holding office, which is proportional to M.

Finally, we wish to ensure that equilibrium always exists in the one-shot version of this game, no matter how the reference point is specified. Clearly, from (5), party payoffs

[9] This is because $Eu(x_{R,t};\varepsilon_t x_{t-1})$ is bounded above by zero, and $Eu(x_{L,t};\varepsilon_t x_{t-1})$ is bounded below by $Eu(-1;0) = -\lambda$, so p_t is bounded above by $\frac{1}{2} + \rho\lambda$; A1 then ensures that p_t is bounded above by 1.

are continuous in $x_{R,t}, x_{L,t}$. Also, party R (resp.L) will never want to choose a platform below -1 (above 1), so without loss of generality, we can constrain the platforms to lie in the compact set $[-1, 1]$. So, by the Glicksberg-Fan theorem, we only require that for any r_t, $\pi_{R,t}, \pi_{L,t}$ are concave in $x_{R,t}, x_{L,t}$ respectively. This concavity is established in the Appendix. Given all this, in what follows, we can, without loss of generality, characterize equilibrium using the first-order conditions for the parties choice of x_L, x_R.

3 Rational Expectations Equilibrium

Here, we study equilibrium in this model when the reference point of the median voter coincides with the *actual equilibrium* of the game. Such an equilibrium is both of intrinsic interest and also provides a natural benchmark when we study repeated elections with a backward-looking reference point. We will call this a *rational expectations* equilibrium.[10] To do this, we focus on a static version of the game, and thus drop all subscripts. In general, the equilibrium outcome is a lottery $\{p, 1-p\}$ over the outcomes $\{x_R, x_L\}$, so we define a stochastic reference point r^e to be a lottery $\{p^e, 1-p^e\}$ over the outcomes $\{x_R^e, x_L^e\}$, where "e" denotes the expectation of the part of the median voter.

We assume that this stochastic reference point is taken as fixed by parties when choosing policies. This is a standard assumption in models of economy policy formation where the trade-offs facing the policy-maker depend of private-sector beliefs. It is also in the spirit of a Personal Equilibrium as defined in Kőszegi and Rabin (2006), although of course here, the beliefs pertain not to the decision-makers (the parties) but to the environment faced by the decision-makers.[11]

The expected utility of the median voter from any platform is

$$Eu(x; r^e) = p^e u(x; x_R^e) + (1-p^e) u(x; x_L^e) \tag{6}$$

Combining (1) and (6), we can obtain an explicit formula for $Eu(x; r^e)$, given in (A.3) in the Appendix. This is straightforward to calculate but involves four different cases, depending on the relationship of $|x|$ to $|x_R|, |x_L|$. Then, from (4), in this setting, the probability that party R wins the election is

$$p = \frac{1}{2} + \rho \left(Eu(x_R; r^e) - Eu(x_L; r^e) \right) \tag{7}$$

Then, using (5), we can express the payoffs of the parties as $\pi_R(x_R, x_L; r^e)$, $\pi_L(x_R, x_L; r^e)$, where p is defined in (7). Then, the platform x^* is a *rational expectations equilibrium* if

[10] In the literature on loss-aversion, this is sometimes known as equilibrium with an expectations-based reference point (Gneezy et al. (2017)).

[11] There is also a small related literature on non-cooperative games with loss-aversion: in Shalev (2000), for example, an equilibrium definition where players take their own reference points as exogenous is proposed.

and only if (i) $\{x^*, -x^*\}$ are mutual best responses for the parties given r^e; (ii) r^e is the lottery $\{\frac{1}{2}, \frac{1}{2}\}$ on $\{x^*, -x^*\}$.

Define the critical values:

$$\frac{1}{2}u_R'(x^+) - \rho\left(u_R(x^+) + M - u_R(-x^+)\right) = 0 \tag{8}$$

$$\frac{1}{2}u_R'(x^-) - \rho\lambda\left(u_R(x^-) + M - u_R(-x^-)\right) = 0 \tag{9}$$

respectively. It is easy to check that these have unique solutions x^+, x^- respectively, with $0 < x^- < x^+$; this is proved as part of the proof of Proposition 1. The interpretation of these values is that (8) is the first-order condition describing the equilibrium x^+ when median voter utility is in the gain domain, and (9) is the first-order condition describing the equilibrium x^- when median voter utility is in the loss domain. The reason why there is lower polarization when the game is played in the loss domain of the voter (i.e. $x^- < x^+$) is simply that then, the median voter penalizes movement in the platform away from his ideal point more heavily.

Then, we have:

Proposition 1. *There is a continuum of symmetric rational expectations equilibria, where* $x_R = x^*,\ x_L = -x^*,\ x^- \leq x^* \leq x^+$.

The intuition for this is the following. At equilibrium, by definition, platforms are at the dividing line between the loss and gain domains of the median voter, for both realizations of r^e. So, if party R moves away from the ideal point of the median voter, it incurs a larger electoral penalty (i.e. a bigger reduction in p) than it gains by moving toward the ideal point of the median voter by the same amount. This asymmetry sustains multiple equilibria. The existence of multiple equilibria with expectation-based reference points is not surprising; Kőszegi and Rabin (2006) found such a multiplicity of personal equilibria in single-person decision problems.

We close this section with two further observations. First, the equilibrium outcome is a lottery over $\{x^*, -x^*\}$ with equal probabilities. If the party payoffs are strictly concave, both parties strictly prefer the equilibria with smaller variance, and so the Pareto-dominant equilibrium is $x^* = x^-$. If party payoffs are of the absolute value form, parties are indifferent over all equilibria. This observation will help use solve for the dynamic equilibrium in the absolute value case.

Second, when parties are risk-neutral i.e. have in the absolute value preferences as in (11) below, we can explicitly solve (8) and (9) to get

$$x^- = \frac{1}{4\rho\lambda} - \frac{M}{2},\ x^+ = \frac{1}{4\rho} - \frac{M}{2} \tag{10}$$

So, one can see, as expected, that the length of the interval of equilibria, $\frac{1}{4\rho} - \frac{1}{4\rho\lambda}$ is increasing in the amount of loss-aversion.

4 Multi-Period Electoral Competition

4.1 Dynamic Incentives

In this section, we study repeated elections with a backward-looking reference point. A major complication in this case is the possible existence of dynamic incentives. The problem is the following. Generally, when the outcome is x_t at t, this helps determine r_{t+1}. Now, if the equilibrium expected payoff at $t+1$ depends directly on r_{t+1}, then forward-looking parties will take account of the effect of their choice of x_t on their equilibrium expected payoff at $t+1$ when choosing their actions at t. While interesting, dynamic incentives make the characterization of the path of platforms analytically intractable, particularly the long-run platform as $T \to \infty$.

However, this problem can be finessed if we assume party payoffs mirror those of the voters, being of the absolute value form i.e. of the form

$$u_R(x) = -\left|x-1\right|, \; u_L(x) = -\left|x+1\right|. \tag{11}$$

Then, the expected payoffs from symmetric equilibrium in (for example) the final period are

$$\frac{M}{2} + \frac{1}{2}\left(u_K(x_T^*) + u_K(-x_T^*)\right), \; K = R, L \tag{12}$$

as each of $x_T^*, -x_T^*$ occurs with equal probability. But, as long as $x_T^* < 1$, from (11),

$$u_R(x) + u_R(-x) = u_L(x) + u_L(-x) = -2 \tag{13}$$

Then, we can combine (12), (13) to conclude that the expected payoffs from symmetric equilibrium in the final period are simply $\frac{M}{2} - 1$ for both parties, and thus independent of x_{T-1}. Then, by backward induction, the continuation payoff from *any* t onwards is independent of x_{t-1}. So, to conclude, with absolute value preferences for parties, we can solve for the political equilibrium as a sequence of static problems, where only the median voter's reference point is varying over time.

4.2 Perfect Recall

In this section, we set $\varepsilon_t \equiv 1$. In this case, we can apply Proposition 1 from Lockwood and Rockey (2020), who study this case for a one-shot election. In particular, because we have eliminated dynamic incentives, this result carries over to every period, and so we can state the following result.

Proposition 2. *If parties are risk-neutral and voters have perfect recall, there is a unique equilibrium* $x_{R,t} = -x_{L,t} = x_t^*, t = 1, \ldots, T$. *The equilibrium platforms at* x_t^* *solve*

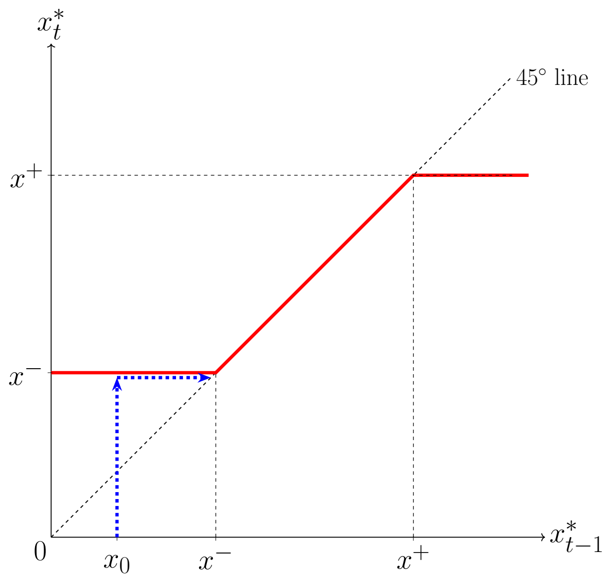
$x_{R,t} = -x_{L,t} = x_t^*,$ *where*

$$x_t^* = \begin{cases} x^+, & x_{t-1}^* > x^+ \\ x_{t-1}, & x^- \leq x_{t-1}^* \leq x^+ \\ x^-, & x_{t-1}^* < x^- \end{cases} \tag{14}$$

This result says that if the previous period's platform, x_{t-1}^*, lies between x^- and x^+, the range of values for the rational expectations equilibrium, then the current platform is just equal to the the previous period's. If the previous period's platform is outside this range, the current platform moves to the closest rational expectations equilibrium value. The intuition is rather similar to that for the rational expectations equilibrium. If $x^- \leq x_{t-1}^* \leq x^+$, party R has neither an incentive to increase or decrease x_R. If on the other hand (for example) $x_{t-1}^* \geq x^+$, the current reference point is in the gain domain, and either party can increase its payoff by moderating its platform slightly to attract more votes.

Figure 1 shows the evolution of platforms over time. There is "one-step" convergence to a rational expectations equilibrium. In particular, whatever x_0, x_1^* will move to some value in the interval $[x^-, x^+]$ and then stay there in all subsequent periods. So, the long-run equilibrium is *completely predetermined* by the initial condition, and thus cannot be affected by other parameters of the model. This is a drawback, insofar as in this case, we cannot explain the long-run outcome as depending on the underlying parameters of the model λ, b, M, ρ.

Figure 1: Equilibrium Platform Dynamics with Perfect Recall



4.3 Imperfect Recall

Here, to lighten notation, let $|x_{t-1}| = s$. This is the absolute value of the previous period's winning platform and is the state variable connecting payoffs in different periods. The payoff from the recalled value of the status quo is $|\varepsilon_t s| = \varepsilon s$, where, again to lighten notation, we set $\varepsilon_t \equiv \varepsilon$. So, using this fact, and (1), we see that

$$u(x; \varepsilon s) = \begin{cases} \lambda(\varepsilon s - |x|), & |x| > \varepsilon s \\ \varepsilon s - |x|, & |x| \leq \varepsilon s \end{cases}. \tag{15}$$

So, taking expectations in (15), for all $x \in \Re$, we have[12]

$$Eu(x; \varepsilon s) = \lambda \int_0^{|x|/s} (\varepsilon s - |x|) f(\varepsilon) d\varepsilon + \int_{|x|/s}^{\infty} (\varepsilon s - |x|) f(\varepsilon) d\varepsilon \tag{16}$$

In other words, when s is small in absolute value relative to x, the realization of ε places the recalled reference platform εs in the loss domain and thus the payoff is multiplied by the loss aversion parameter λ. On the other hand, when x_0 is large in absolute value relative to x, the realization of ε places the recalled reference platform εs in the gain domain and thus the payoff is not multiplied by the loss aversion parameter λ.

Armed with the formula (16), we can then calculate the probability p that party R wins from (4), and in turn, we can characterize the symmetric equilibrium x^*. The details are straightforward and are included in the proof of the Proposition 3. Here, we just provide some intuition by studying how the reference point affects the trade-off between the election probability and the platform for party R. Note that from (4), we have:

$$\frac{\partial p}{\partial x_R} = \rho E u'(x_R; \varepsilon s) \tag{17}$$

where the prime denotes the derivative of $Eu(x_R; \varepsilon s)$ with respect to x_R. It is easy to compute this derivative from (16), noting that $|x_R| = x_R$ for party R, which gives

$$\frac{\partial p}{\partial x_R} = -\left(\rho(\lambda - 1)F\left(\frac{x_R}{s}\right) + 1\right) \tag{18}$$

The key point here is that the trade-off between x_R and p depends smoothly on last period's platform s: this is the *trade-off* effect referred to in the introduction. Note also that the size of the trade-off is decreasing in absolute value as s increases. So, the "penalty" in terms of a lower election probability is smaller, the larger is the initial reference point. It is this feature that creates the dynamic linkage between periods. This feature also distinguishes p from an ordinary non-linear function of x, which could arise e.g. from a

[12] When $s = 0$, we can take the limit as $s \to 0$ in (16) to get $Eu(x; 0) = \lambda \int_0^{\infty} (\varepsilon s - |x|) f(\varepsilon) d\varepsilon$.

non-uniform distribution of v.

Given these facts, we would expect that the equilibrium platforms at t to depend smoothly on x_{t-1}, and also be increasing in x_{t-1}, from the penalty argument. This is in fact, exactly what we find:

Proposition 3. *There is a unique equilibrium $x_{R,t} = -x_{L,t} = x_t^*, t = 1, ..T$. The equilibrium platform x_t^* at t solves*

$$x_t^* = \phi(x_{t-1}^*) \tag{19}$$

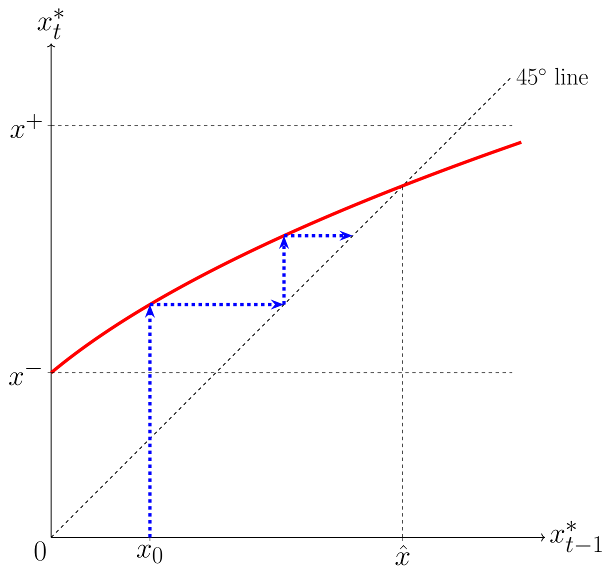
where $x = \phi(s)$ is the unique solution to

$$\frac{1}{2} - \rho\left[(\lambda - 1)F\left(\frac{x}{s}\right) + 1\right](2x + M) = 0 \tag{20}$$

For any positive $s, \phi(s)$ lies between the values x^- and x^+, with $lim_{s\to 0}\ \phi(s) = x^-$, $lim_{s\to\infty}\ \phi(s) = x^+$. Moreover, $\phi(s)$ is strictly increasing in s.

The evolution of the equilibrium path over time is shown in Figure 2. Because $\phi(.)$ is strictly increasing, there is clearly monotonic convergence to a unique long-run platform $\hat{x}$ that solves $\hat{x} = \phi(\hat{x})$. This process is the one described in the introduction to this paper. For example, Figure 2 shows the case where starting from a relatively moderate historically determined platform x_0, both parties have an incentive to choose more polarized platforms $x_{R,1} = -x_{L,1} = x_1^* > x_0$. This in turn leads to a an outward shift in the expected value of median voter's reference point, which creates an incentive for further polarization in the parties' platforms, and so on. A reverse process of depolarization would obviously occur if the initial platforms were extreme.

Figure 2: Equilibrium Platform Dynamics with Imperfect Recall



Note that the unique long-run platform $\hat{x}$ solves $\hat{x} = \phi(\hat{x})$. Solving (20) for $\hat{x}$, noting that in this case $F(x/s) = F(1)$, we then get:

$$\hat{x} = \frac{1}{4\rho((\lambda - 1)F(1) + 1)} - \frac{M}{2} \tag{21}$$

Note from A2 that $\hat{x}$ is strictly positive.[13] To interpret this long-run equilibrium, note that we can write

$$F(1) - 0.5 = \underbrace{F(1) - F(1 + b)}_{\text{bias}, \ -\beta} + \underbrace{F(1 + b) - 0.5}_{\text{skewness}, \ \kappa} \tag{22}$$

So, $\beta > 0$ if the voters have a positive bias in recall i.e. $b > 0$, and vice-versa. The skewness parameter is positive if ε is skewed to the right i.e. $E\varepsilon = 1 + b > \varepsilon_m$, where ε_m is the median value of ε. see that the steady state platform is strictly positive by A2.

Then combining (21), and (22), we get:

$$\hat{x} = \frac{1}{4\rho((\lambda - 1)(0.5 + \kappa - \beta) + 1)} - \frac{M}{2}$$

So, we can summarize as follows:

Proposition 4. *With imperfect recall, the equilibrium platforms converge monotonically over time to the long-run equilibrium $\hat{x}$. The long-run equilibrium platforms are less polarized, the larger is loss-aversion λ, or the skewness of ε. The long-run equilibrium platforms are more (less) polarized if there is positive (negative) bias b in recall.*

The intuition for these results is fairly straightforward. The higher the degree of loss-aversion, the more the voters dislike polarization of platforms, and so the less polarization there will be in long-run equilibrium. If there is positive bias in recall, the recalled reference platform from last period will other things equal, be larger, so electoral competition is more likely to occur in the gain domain for voters, leading to more polarization in the long run. The reverse applies if there is negative bias in recall. Finally, the skewness of ε matters for the long-run equilibrium; for example, if ε is skewed to the right, more than half the realisations of ε will be below the mean, which for a given mean, makes it relatively likely that the median voter will evaluate the platforms using a smaller reference point. This in turn means that electoral competition is more likely to take place in the loss domain, leading to a smaller $\hat{x}$.

[13] As $F(1) < 1$, $\hat{x} > \frac{1}{4\rho\lambda} - \frac{M}{2}$, and $\frac{1}{4\rho\lambda} > \frac{M}{2}$ by A2.

4.4 The Rate of Convergence

Here, we ask how the rate of convergence varies with key behavioral parameters λ and b in the imperfect recall case. By the rate of convergence, we mean how fast the percentage deviation of the equilibrium from its long-run value, $d_t = (x_t^* - \hat{x})/\hat{x}$, tends to zero. Here it is impossible to establish clear analytical results, because both numerator and denominator in d_t depend on model parameters, and so we rely on numerical simulations. We assume that ε is exponentially distributed i.e. $F(x) = 1 - e^{-x/(1+b)}$, where $E\varepsilon = 1+b$, so b is the bias. Also, note that the variance is $(1+b)^2$. In this case, from Proposition 3, the dynamic relationship between x_{t-1} and x_t is given by the implicit equation:

$$\frac{1}{2} - \rho\left[(\lambda-1)\left(1-\exp\left(-\frac{x_t}{x_{t-1}(1+b)}\right)\right)+1\right](2x_t+M) = 0 \tag{23}$$

The results showing how the convergence rate d_t varies with parameters λ and b are shown in Figure 3 below. We allow the initial absolute value of the platform x_0 to be either above ($x_0 = 0.9$) or below ($x_0 = 0.1$) the long-run value. Panels (a) and (b) show that the greater degree of loss-aversion, the slower is convergence to the long-run value $\hat{x}$, and this is qualitatively independent of x_0. On the other hand, the effect of bias b on convergence depends on whether initial absolute value of the platform x_0. If it is high relative to $\hat{x}$, increased bias speeds convergence. If it is low relative to $\hat{x}$, increased bias slows convergence.

5 Dynamic Incentives

We now turn to ask how dynamic incentives change the nature of the polarization process. As already discussed, dynamic incentives arise when parties are strictly risk-averse, and so we assume this in what follows. From now on, for tractability, we assume that there are only two periods, excluding period 0 i.e. $T = 2$. We consider the imperfect recall case, for two reasons. First, it is more realistic, and second, because party payoffs are differentiable, the results are easier to state and prove.

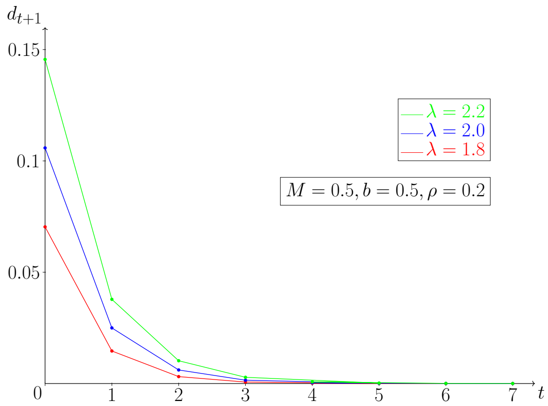
As in the case of absolute value preferences, in period 2, the second-period platforms are characterized by a mapping from the state variable $s = |x_1|$ of the form $x_2^* = \phi_2(s)$. In this case, as shown in the Appendix, $x = \phi_2(s)$ is the unique solution to

$$\frac{1}{2}u_R'(x) - \rho\left[(\lambda-1)F\left(\frac{x}{s}\right)+1\right](u_R(x)+M-u_R(-x)) = 0$$

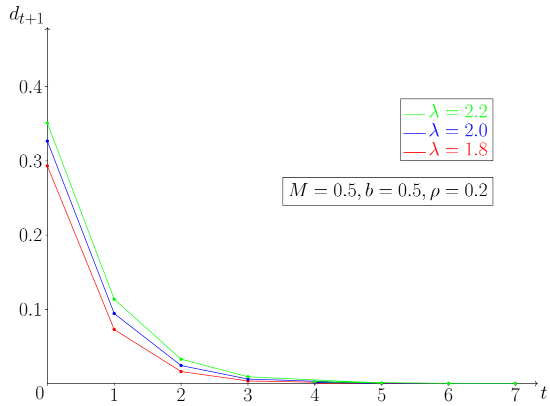
Second-period continuation payoffs for the two parties are thus

$$V_K(x) = \frac{1}{2}(M + u_K(\phi_2(x)) + u_K(-\phi_2(x)),\ K = R, L$$

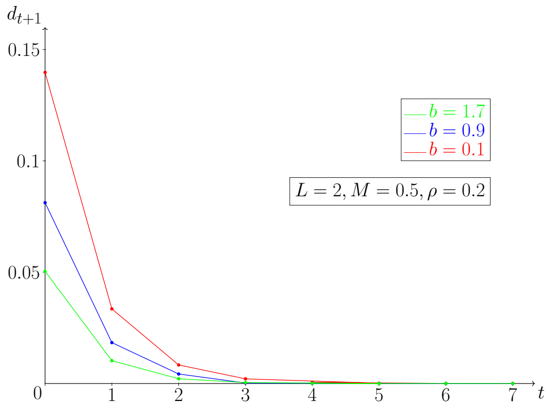
Figure 3: Convergence Rates



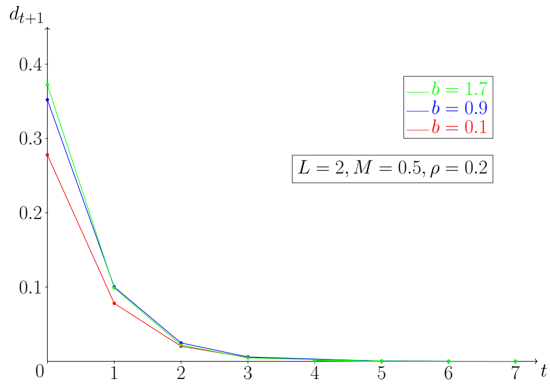
(a) Convergence rate d_{t+1} as λ varies with $x_0 = 0.9$.



(b) Convergence rate d_{t+1} as λ varies with $x_0 = 0.1$.



(c) Convergence rate d_{t+1} as b varies with $x_0 = 0.9$.



(d) Convergence rate d_{t+1} as b varies with $x_0 = 0.1$.

Figure 4: Convergence rate with imperfect recall and $x_0 = 0.9$.

Then, we can write payoffs for the two parties in the first period as:

$$\Pi_R(x_R, x_L) = p(u_R(x_R) + M) + (1-p)u_R(x_L) + \delta\left[pV_R(x_R) + (1-p)V_R(x_L)\right] \tag{24}$$

$$\Pi_L(x_R, x_L) = pu_L(x_R) + (1-p)(u_L(x_L) + M) + \delta[pV_L(x_R) + (1-p)V_L(x_L)] \tag{25}$$

where to simplify notation, we drop time subscripts from all first-period variables so $x_{R,1} = x_R$, $x_{L,1} = x_L$, etc. We will study symmetric Nash equilibrium in first-period platforms in the game with these payoffs.[14] Generally, for existence of equilibrium, it is sufficient that $\Pi_K(x_R, x_L)$ is concave in x_K.[15] Showing concavity of the payoffs is not straightforward, due to the continuation payoffs, and so we just assume concavity at this point.

To develop intuition, note that the dynamic incentive for party R to choose a more moderate platform is at $t = 1$ is the following. Consider a small increase in x_R at period 1; this has the following effect on the second-period continuation payoff:

$$\delta V_R'(x_R) = \frac{\delta}{2}(u_K'(\phi_2(x_R)) - u_K'(-\phi_2(x_R))\phi_2'(x_R) < 0 \tag{26}$$

where $u_K'(-\phi_2(x)) > u_K'(\phi_2(x))$ because of the strict concavity of u_R. So, there is a dynamic incentive to reduce x_R, relative to the one-shot game, in order to reduce the variance of next period's platforms. This suggests that in general, for a given initial x_0, there should be less polarization in equilibrium than with no dynamic incentives. The obvious benchmark here is the one-period game (formally, where $\delta = 0$ in (24)). In fact, it is easy to get the following result:

Proposition 5. *For any given x_0, the equilibrium x_1^* is strictly less polarized than the equilibrium in the one-period game. If $x_0 \simeq 0$, the equilibrium x_1^* can be strictly less polarized than the smallest rational expectations equilibrium x^-.*

The first part of this result says that there is *dynamic moderation* - the first period platforms are less polarized than in either the one-period case or the two-period case without risk-aversion. This result is not entirely new - similar results were proved in Lockwood and Rockey (2015) for the version of this model with perfect recall (Proposition 3 in that paper), and in Alesina and Passarelli (2019) in the setting of their rather different model (Proposition 6 in that paper). What is new in Proposition 5 is a demonstration that with dynamic incentives, the time-path of the equilibrium platforms need not lie in the set of rational-expectations equilibria; for a small enough initial platform, the first-

[14] These are defined in the usual way as a pair x_L, x_R where (i) $-x_L^* = x_R^* = x_1^*$; (ii) x_R^* is a best response to x_L^* and vice versa, given the payoffs (24), (25) and where p is defined by (4).

[15] The payoffs are continuous in both x_R, x_L and as already discussed, without loss of generality, we can assume $x_K \in [-1, 1]$, so if the concavity condition holds, there is an equilibrium in pure strategies by the Glicksberg-Fan theorem.

period platforms can be more moderate than the most moderate possible equilibrium with rational expectations.

We turn now to consider the welfare consequences of dynamic incentives. We focus on the welfare of the median voter. If ideal points are symmetrically distributed around zero, this is also the average or total welfare of the entire population at a symmetric equilibrium where $x_{R,t} = -x_{L,t} = x_t^*$. The per period expected utility of the median voter at symmetric equilibrium is clearly decreasing in the amount of polarization in equilibrium, as the median voter has a concave payoff, symmetric around zero. Moreover, it is easy to show that the higher δ in (24), the smaller is $\phi_1(.)$ and so the first-period equilibrium is always strictly less polarized. So, we can summarize:

Proposition 6. *In the two-period game, the more forward-looking are the political parties, i.e. the higher δ, the better off is the median voter. If the distribution of ideal points is symmetric, all voters are better off the higher is δ.*

6 Conclusions

This paper has explored the implications of voter loss-aversion for the dynamics of electoral competition in a simple Downsian model of repeated elections. When the voters' reference point is forward-looking, there are a continuum of rational expectations equilibria (REE). When voters are backward-looking i.e. the reference point is last period's recalled policy, interesting dynamics only emerge when voters have *imperfect recall* about that policy. Then, the interplay between the median voter's reference point and political parties' choice of platforms generates a dynamic process of polarization (or de-polarization). Under the assumption that parties are risk-neutral, platforms monotonically converge over time to a long-run equilibrium, which is always a REE. When parties are risk-averse, dynamic incentives also come into play, and generally lead to more policy moderation, resulting in equilibria that are more moderate than the most moderate REE.

References

Abramowitz, Alan I. and Kyle L. Saunders, "Is Polarization a Myth?," *The Journal of Politics*, apr 2008, *70* (2), 542–555.

Alesina, Alberto and Francesco Passarelli, "Loss Aversion in Politics," *American Journal of Political Science*, 2019.

Banks, Jeffrey S. and John Duggan, "A Dynamic Model of Democratic Elections in Multidimensional Policy Spaces," *Quarterly Journal of Political Science*, 2008, *3* (3), 269–299.

Baron, David P., "A Dynamic Theory of Collective Goods Programs," *American Political Science Review*, jun 1996, *90* (2), 316–330.

Battaglini, Marco, "A dynamic theory of electoral competition," *Theoretical Economics*, 2014, *9* (2), 515–554.

Baumeister, Roy F., Ellen Bratslavsky, Catrin Finkenauer, and Kathleen D. Vohs, "Bad is stronger than good.," *Review of General Psychology*, 2001, *5* (4), 323–370.

Bernhardt, Dan, Larissa Campuzano, Francesco Squintani, and Odilon Câmara, "On the benefits of party competition," *Games and Economic Behavior*, 2009, *66* (2), 685–707.

_ , Odilon Câmara, and Francesco Squintani, "Competence and ideology," *Review of Economic Studies*, apr 2011, *78* (2), 487–522.

_ , Sangita Dubey, and Eric Hughson, "Term limits and pork barrel politics," *Journal of Public Economics*, 2004, *88* (12), 2383–2422.

Biais, Bruno and Enrico Perotti, "Machiavellian privatization," *American Economic Review*, 2002, *92* (1), 240–258.

Bloom, Howard S and H Douglas Price, "Voter response to short-run economic conditions: The asymmetric effect of prosperity and recession," *American Political Science Review*, 1975, *69* (04), 1240–1254.

Bonica, Adam, "The Punctuated Origins of Senate Polarization," *Legislative Studies Quarterly*, feb 2014, *39* (1), 5–26.

Bowen, T. Renee, Ying Chen, and Hülya Eraslan, "Mandatory versus Discretionary Spending: The Status Quo Effect," *American Economic Review*, oct 2014, *104* (10), 2941–2974.

Callander, Steven and Juan Carlos Carbajal, "Cause and Effect in Political Polarization: A Dynamic Analysis," 2020.

Diermeier, Daniel and Pohan Fong, "Legislative Bargaining with Reconsideration," *The Quarterly Journal of Economics*, may 2011, *126* (2), 947–985.

Duggan, John, "Repeated Elections with Asymmetric Information," *Economics & Politics*, 2000, *12* (1), 109–119.

Dziuda, Wioletta and Antoine Loeper, "Dynamic Collective Choice with Endogenous Status Quo," *Journal of Political Economy*, aug 2016, *124* (4), 1148–1186.

Fiorina, Morris P, Samuel J Abrams, and Jeremy C Pope, *Culture war? The myth of a polarized America*, New York: Pearson Longman, 2005.

Forand, Jean Guillaume, "Two-party competition with persistent policies," *Journal of Economic Theory*, 2014, *152* (1), 64–91.

Gentzkow, M, "Polarization in 2016," *Toulouse Network of Information Technology whitepaper*, 2016, pp. 1–23.

Gentzkow, Matthew, Jesse M. Shapiro, and Matt Taddy, "Measuring Group Differences in High-Dimensional Choices: Method and Application to Congressional Speech ," *Econometrica*, 2019, *87* (4), 1307–1340.

Gneezy, Uri, Lorenz Goette, Charles Sprenger, and Florian Zimmermann, "The limits of expectations-based reference dependence," *Journal of the European Economic Association*, 2017, *15* (4), 861–876.

Grillo, Edoardo, "The hidden cost of raising voters' expectations: Reference dependence and politicians' credibility," *Journal of Economic Behavior & Organization*, 2016, *130*, 126–143.

_ **and Carlo Prato**, "Reference points and democratic backsliding," *American Journal of Political Science*, 2020.

Hare, Christopher and Keith T. Poole, "The polarization of contemporary American politics," *Polity*, 2014, *46* (3), 411–429.

Iyengar, Shanto and Sean J. Westwood, "Fear and Loathing across Party Lines: New Evidence on Group Polarization," *American Journal of Political Science*, jul 2015, *59* (3), 690–707.

_ **, Yphtach Lelkes, Matthew Levendusky, Neil Malhotra, and Sean J. Westwood**, "The Origins and Consequences of Affective Polarization in the United States," *Annual Review of Political Science*, may 2019, *22* (1), 129–146.

Kalandrakis, Tasos, "Proposal Rights and Political Power," *American Journal of Political Science*, apr 2006, *50* (2), 441–448.

Kappe, Roland, "Asymmetric Evaluations : Government Popularity and Economic Performance in the United Kingdom -," 2013, (September), 13–15.

Klein, Jill G., "Negativity Effects in Impression Formation: A Test in the Political Arena," *Personality and Social Psychology Bulletin*, 1991, *17* (4), 412–418.

Kőszegi, B. and M. Rabin, "A Model of Reference-Dependent Preferences," *The Quarterly Journal of Economics*, November 2006, *121* (4), 1133–1165.

Kőszegi, Botond and Matthew Rabin, "Reference-Dependent Risk Attitudes," *American Economic Review*, September 2007, *97* (4), 1047–1073.

Lau, Richard R., "Two Explanations for Negativity Effects in Political Behavior," *American Journal of Political Science*, February 1985, *29* (1), 119.

Lockwood, Ben and James Rockey, "Negative Voters: Electoral Competition with Loss-Aversion," 2015.

_ and _ , "Negative Voters? Electoral Competition with Loss-Aversion," *Economic Journal, forthcoming*, 2020.

Lodge, Milton, Kathleen M McGraw, and Patrick Stroh, "An impression-driven model of candidate evaluation," *American Political Science Review*, 1989, *83* (2), 399–419.

_ , Marco R Steenbergen, and Shawn Brau, "The responsive voter: Campaign information and the dynamics of candidate evaluation," *American political science review*, 1995, *89* (2), 309–326.

McCarty, Nolan, Keith T. Poole, and Howard Rosenthal, "Does gerrymandering cause polarization?," *American Journal of Political Science*, 2009, *53* (3), 666–680.

Nannestad, Peter and Martin Paldam, "The grievance asymmetry revisited: A micro study of economic voting in Denmark, 1986–1992," 1997.

Nunnari, Salvatore and Jan Zápal, "Dynamic elections and ideological polarization," *Political Analysis*, 2017, *25* (4), 505–534.

Passarelli, Francesco and Guido Tabellini, "Emotions and political unrest," *Journal of Political Economy*, 2017, *125* (3), 903–946.

Poole, Keith T. and Howard Rosenthal, "The Polarization of American Politics," *The Journal of Politics*, 1984, *46* (4), 1061–1079.

Prato, Carlo, "Electoral competition and policy feedback effects," *The Journal of Politics*, 2018, *80* (1), 195–210.

Shalev, Jonathan, "Loss aversion equilibrium," *International Journal of Game Theory*, 2000, *29* (2), 269–287.

Soroka, Stuart and Stephen McAdams, "News, Politics, and Negativity," *Political Communication*, 2015, *32* (1), 1–22.

Soroka, Stuart N., "Good news and bad news: Asymmetric responses to economic information," *Journal of Politics*, 2006, *68* (2), 372–385.

Webster, Steven W. and Alan I. Abramowitz, "The Ideological Foundations of Affective Polarization in the U.S. Electorate," *American Politics Research*, 2017.

A Appendix: Proofs of Propositions

Proof that π_K is concave in x_K. We only need prove this for party R. From (5), generally,

$$\frac{\partial \pi_R}{\partial x_R} = pu_R'(x_R) + \frac{\partial p}{\partial x_R}\left(u_R(x_R) + M - u_R(-x_L)\right) \tag{A.1}$$

So, from (A.1):

$$\frac{\partial \pi_R}{\partial x_R} = pu_R''(x_R) + 2\frac{\partial p}{\partial x_R}u_R'(x_R) + \frac{\partial^2 p}{\partial x_R^2}\left(u_R(x_R) + M - u_R(-x_L)\right) \tag{A.2}$$

Now, from (17), (18), we see that $\frac{\partial p}{\partial x_R} < 0$. Also, $u_R''(x_R) \leq 0$. So, the first two terms in (A.2) are negative, and the sum of the two is always strictly negative. Also, as $x_R \geq 0 \geq x_L$, the term $u_R(x_R) + M - u_R(-x_L)$ is strictly positive. So, for concavity, it is sufficient that $\frac{\partial^2 p}{\partial x_R^2} \leq 0$. Now, using (17), (18), it is easy to calculate that

$$\frac{\partial^2 p}{\partial x_R^2} = -\frac{\rho(\lambda - 1)}{|x_0|} f\left(\frac{x_R}{|x_0|}\right) < 0$$

as required. □

Proof of Proposition 1. (a) First, we show that (8) ,(9) have unique positive solutions $0 < x^- < x^+$. Define

$$\phi(x, \alpha) \equiv \frac{1}{2}u_R'(x) - \rho\alpha\left(u_R(x) + M - u_R(-x)\right),\ \alpha = 1, \lambda$$

Note that

$$\phi_x = \frac{1}{2}u_R''(x) - \rho\alpha\left(u_R'(x) + u_R'(-x)\right) < 0$$

This is because (i) as u_R is concave, $u_R''(x) \leq 0$; (ii) as w.l.o.g. $x \leq 1$, $u_R'(x), u_R'(-x) > 0$. So, it follows that (8) ,(9) have unique solutions. Moreover, $0 = \phi(x^-, \lambda) < \phi(x^-, 1)$, so as $\phi_x < 0$ and $\phi(x^+, 1) = 0$, it must be that $x^+ > x^-$. Also, $\phi(0, \lambda) = \frac{1}{2}u_R'(0) - \rho\lambda M$ which is strictly positive by A2, so this implies $x^- > 0$.

(b) We now characterize equilibrium. Combining (1) and (6), we can obtain an explicit formula for $Eu(x; r^e)$:

$$Eu(x; r^e) = \begin{cases} p^e\left|x_R^e\right| + (1 - p^e)\left|x_L^e\right| - |x|, & |x| \leq \left|x_R^e\right|, \left|x_L^e\right| \\ \lambda\left(p^e\left|x_R^e\right| + (1 - p^e)\left|x_L^e\right| - |x|\right), & |x| > \left|x_R^e\right|, \left|x_L^e\right| \\ \lambda p^e\left(\left|x_R^e\right| - |x|\right) + (1 - p^e)\left(\left|x_L^e\right| - |x|\right), & \left|x_R^e\right| < |x| \leq \left|x_L^e\right| \\ p^e\left(\left|x_R^e\right| - |x|\right) + \lambda(1 - p^e)\left(\left|x_L^e\right| - |x|\right), & \left|x_L^e\right| < |x| \leq \left|x_R^e\right| \end{cases} \tag{A.3}$$

There are four different possibilities as there are four different ways in which the payoff $|x|$ can relate to the reference payoffs $\left|x_R^e\right|, \left|x_L^e\right|$.

We can now calculate a change in x_R changes p, taking r^e as given, and starting at the equilibrium $\{x^*, -x^*\}$. In this calculation, we must also assume that r^e is rational i.e. a lottery of $\left\{\frac{1}{2}, \frac{1}{2}\right\}$ on $\{x^*, -x^*\}$.

From (A.3), we see that a small increase in x_R from x^* to x' moves the median voter into the loss domain with respect to both $x^*, -x^*$ i.e. $|x'| > |x^*| = |-x^*|$. So, from (A.3), this change decreases p by $-\lambda\rho$. In the same way, from (A.3), we see that a small decrease in x_R from x^* to x' moves the median voter into the gain domain with respect to both $x^*, -x^*$ i.e. $|x'| < |x^*| = |-x^*|$. So, from (A.3), this change increases p by ρ.

So, a necessary and sufficient condition for neither an increase nor a decrease in x_R to increase π_R is that

$$\frac{1}{2}u_R'(x^*) - \rho\lambda\left(u_R(x^*) + M - u_R(-x^*)\right) \leq 0 \leq \frac{1}{2}u_R'(x^*) - \rho\left(u_R(x^*) + M - u_R(-x^*)\right) \tag{A.4}$$

This characterizes x^*.

(c) Now suppose that $x^* \in [x^-, x^+]$. Then as $\phi_x < 0$,

$$\phi(x^*, \lambda) \leq \phi(x^-, \lambda) = 0,\ \ \phi(x^*, 1) \geq \phi(x^+, 1) = 0$$

where the equalities follow from (8) ,(9). Consequently, $\phi(x^*, \lambda) \leq 0 \leq\ \phi(x^*, 1)$ which is a restatement of (A.4). In other words, any $x^* \in [x^-, x^+]$ satisfies (A.4) and is thus an equilibrium. By the same argument, any $x^* \notin [x^-, x^+]$ cannot be an equilibrium. □

Proof of Proposition 3. Here, we continue to drop time subscripts and define s as above. (i) The equilibrium $x_R = -x_L = x^*$ must described by the first-order condition for the choice of x_R by party R which can always be written as

$$\frac{\partial \pi_R}{\partial x_R} = \frac{1}{2}u_R'(x^*) + \frac{\partial p}{\partial x_R}\left(u_R(x^*) + M - u_R(-x^*)\right) = 0 \tag{A.5}$$

Using (11), this simplifies to

$$\frac{\partial \pi_R}{\partial x_R} = \frac{1}{2} + \frac{\partial p}{\partial x_R}\left(2x^* + M\right) = 0 \tag{A.6}$$

Then, combining (11) and (18), we get (20). So, (20) characterises the equilibrium x for a given s, as claimed.

(ii) We need to prove that (20) has a unique solution for fixed s. First, write (20) more compactly as $g(x; s) = 0$. Then we can write

$$g(0; s) = \frac{1}{2} - \rho\left[(\lambda - 1)F\left(0\right) + 1\right]M = \frac{1}{2} - \rho M > \frac{1}{2} - \lambda\rho M > 0 \tag{A.7}$$

where the last inequality follows from A2. Similarly, by inspection,

$$\lim_{x\to\infty} g(x; s) \leq \frac{1}{2} - \rho\lambda\left(2x' + M\right) < 0 \tag{A.8}$$

for x' large enough. Finally, note

$$g'(x; s) = -\rho\frac{\lambda - 1}{s}f\left(\frac{x}{s}\right)\left(2x + M\right) - \rho\left[(\lambda - 1)F\left(\frac{x^-}{s}\right) + 1\right] < 0 \tag{A.9}$$

So, $g(x; s)$ is strictly decreasing in x. Combining (A.7), (A.8), (A.9), we see that $g(x; s) = 0$ has a unique solution $x^* = \phi(s)$ strictly between x^- and x^+.

(iii) We need to show that $x = \phi(s)$ lies between x^- and x^+ as defined in (10), with $x^- = \phi(0)$. From (20), we get:

$$x = \frac{1}{4\rho\left((\lambda-1)z+1\right)} - \frac{M}{2}, \quad z = F\left(\frac{x}{s}\right) \tag{A.10}$$

But $0 \leq z \leq 1$ for all x, s, and also $z \to 1$ for fixed x as $s \to 0$. So by inspection of (A.10), (10), this certainly holds.

(iv) We need to show that x^* is increasing in s. We write (A.5) slightly differently as

$$\frac{\partial \pi_R}{\partial x_R} = \frac{1}{2} - \rho\left[(\lambda-1)F\left(\frac{x_R}{s}\right)+1\right](x_R + x_L + M) = 0$$

remembering that $x_R = -x_L = x^*$. Then as $\frac{\partial^2 \pi_R}{\partial x_R^2} < 0$, we see by standard arguments that

$$\text{sign } \frac{\partial x_R}{\partial s} = \text{sign } \frac{\partial \pi_R}{\partial x_R \partial s}$$

But

$$\frac{\partial \pi_R}{\partial x_R \partial s} = \rho(\lambda-1)\frac{x}{s^2} f\left(\frac{x}{s}\right)(u_R(x) + M - u_R(x)) > 0$$

as required. □

Proof of Proposition 5. As in the proof of Proposition 3, it must be that the first-period symmetric equilibrium platform $x_R = -x_L = x_1^*$ solves $\frac{\partial \Pi_R}{\partial x_R} = 0$. Using (24), and (18), this can be written

$$\frac{1}{2}u_R'(x_1^*) - \rho\left[(\lambda-1)F\left(\frac{x_1^*}{|x_0|}\right)+1\right](u_R(x_1^*) + M - u_R(-x_1^*)) + \delta V_R'(x_1^*) = 0 \tag{A.11}$$

As $x_0 \to 0$, $F\left(\frac{x}{|x_0|}\right) \to 1$, so from (A.11), x_1^* solves

$$\frac{1}{2}u_R'(x_1^*) - \rho\lambda\left(u_R(x_1^*) + M - u_R(-x_1^*)\right) + \delta V_R'(x_1^*) = 0$$

which from (26) implies

$$\frac{1}{2}u_R'(x_1^*) - \rho\lambda\left(u_R(x_1^*) + M - u_R(-x_1^*)\right) > 0 \tag{A.12}$$

But then from (A.12) and (A.12), plus the the fact (established in he proof of Proposition 1) that the expression on the left-hand of (A.12) is decreasing in x, it must be that $x_1^* < x^-$ for x_0 small enough. □