Design models for predicting shear resistance of studs in solid concrete slabs based on symbolic regression with genetic programming

Vitaliy V. Degtyarev[1a], Stephen J. Hicks[*2], and Jerome F. Hajjar[3b]

[1] New Millennium Building Systems, Columbia, SC, USA
[2] School of Engineering, University of Warwick, Coventry, CV4 7AL, UK
[3] Department of Civil and Environmental Engineering, Northeastern University, Boston, MA, USA

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Abstract. Accurate design models for predicting the shear resistance of headed studs in solid concrete slabs are essential for obtaining economical and safe steel-concrete composite structures. In this study, symbolic regression with genetic programming (GPSR) was applied to experimental data to formulate new descriptive equations for predicting the shear resistance of studs in solid slabs using both normal and lightweight concrete. The obtained GPSR-based nominal resistance equations demonstrated good agreement with the test results. The equations indicate that the stud shear resistance is insensitive to the secant modulus of elasticity of concrete, which has been included in many international standards following the pioneering work of Ollgaard et al. In contrast, it increases when the stud height-to-diameter ratio increases, which is not reflected by the design models in the current international standards. The nominal resistance equations were subsequently refined for use in design from reliability analyses to ensure that the target reliability index required by the Eurocodes was achieved. Resistance factors for the developed equations were also determined following US design practice. The stud shear resistance predicted by the proposed models was compared with the predictions from 13 existing models. The accuracy of the developed models exceeds the accuracy of the existing equations. The proposed models produce predictions that can be used with confidence in design, while providing significantly higher stud resistances for certain combinations of variables than those computed with the existing equations given by many standards.

Keywords: headed studs; shear resistance; steel-concrete composite structures; reliability; machine learning; symbolic regression; genetic programming

1. Introduction

Steel-concrete composite structures rely on the longitudinal shear transfer between the two materials, achieved by the shear connectors attached to the steel components and encased in concrete. Various types of shear connectors have been used in construction (Mujagić et al. 2007; Shariati et al. 2012; Pavlović et al. 2013), with the welded headed studs being the most common due to the speed and simplicity of their installation and the reliability of their performance.

Many researchers have experimentally studied the load-slip performance of welded studs in solid concrete slabs using push tests. Test results formed the basis for several design models of the stud shear resistance proposed over time, some of which have been adopted in design standards. Probably the most well-known is the design model developed by Ollgaard et al. (1971), which was based on regression analyses of 48 push test results; this design model was subsequently adopted by AISC 360 (1986) and was also adapted for use in Eurocode 4 (EN 1994-1-1 2004, EN 1994-2 2005). However, comparisons of the existing design models with a larger pool of test data demonstrate that there is room for improvement to achieve more accurate resistance predictions, whilst maintaining the reliability requirements of the design standards (Pallarés and Hajjar 2010; Hicks 2017; Bonilla et al. 2018).

Pallarés and Hajjar (2010) assessed the predicted strength given by the AISC 360 (2005), EN 1994-1-1 (2004), ACI 318 (2008), and PCI (2004) design equations for stud shear connectors in solid slabs against 391 monotonic and cyclic test results. They found in particular that the AISC 360 (2005) formula for the steel failure mode was accurate only if a resistance factor was included. The stud shear strength governed by concrete strength did not need to be checked when: a resistance factor of 0.65 was applied to the steel strength formula; the studs were not subject to concrete breakout failure; normal weight concrete is used; and the stud height-to-diameter ratio (h/d) in excess of 5 is provided. In the North American design context, they also concluded that the EN 1994-1-1 (2004)

*Corresponding author, Ph.D., Professor
E-mail: stephen.j.hicks@warwick.ac.uk
[a] Ph.D., P.E., S.E.
E-mail: vitaliy.degtyarev@newmill.com
[b] Ph.D., P.E., CDM Smith Professor and Department Chair
E-mail: jf.hajjar@northeastern.edu

equations provided conservative predictions for steel and concrete failure modes when the partial factor was applied.

Hicks (2017) conducted reliability analyses of the EN 1994-1-1 (2004), Oehlers and Johnson (1987), and Döinghaus (2001) design models for stud shear resistance in solid normal weight concrete slabs in accordance with EN 1990 (2005). From considering a database of 242 push-out tests, the results from this study showed that the design equations for the concrete failure mode of all three considered models did not satisfy the Eurocode reliability requirements. In addition, it was demonstrated that the models could be extended over a wider range of concrete strengths and stud diameters currently permitted in European design practice. Modifications to all three models were proposed to ensure that they delivered the target partial factor required by EN 1994-1-1 (2004) for the design resistance of a headed stud.

Bonilla et al. (2018) evaluated the accuracy of the stud shear resistance predictions for solid slabs by the AISC 360 (2010), EN 1994-1-1 (2004), GB50017 (2003), JSCE (2007), and AS-2327.1 (2003) design models against experimental data. They concluded that the AISC 360 (2010) and JSCE (2007) equations might give unconservative results for stud diameters of 25, 27, and 30 mm. The EN 1994-1-1 (2004), GB50017 (2003), and AS-2327.1 (2003) models produced overly conservative results for stud diameters from 13 to 22 mm and less conservative results for stud diameters of 25, 27, and 30 mm.

This study describes an application of symbolic regression with genetic programming (GPSR) to experimental data to formulate new descriptive equations for predicting the nominal and design shear resistance of the studs. GPSR is a machine learning (ML) technique that seeks to find underlying relationships between dependent and independent variables of a dataset in the form of a mathematical expression by applying genetic operations and evaluating the performance of each generation of the functions until the desired performance has been achieved.

GPSR is a variant of genetic programming (GP) (Wang et al. 2019, Ari and Alagöz 2021, Zhang et al. 2021), which has been successfully applied for solving different structural engineering problems. GP-based descriptive equations for predicting material properties at ambient and elevated temperatures were proposed by Ozbay et al. (2008), Sarıdemir (2010), Özcan (2012), Jafari and Mahini (2017), Velay-Lizancos et al. (2017), Naser (2019), and İpek and Mermerdaş (2020). In reinforced concrete, GP-based predictive equations were developed for the shear strength of elements (Aval et al. 2017, Chaabene and Nehdi 2021, Gondia et al. 2020, Pérez et al. 2010, Shahnewaz and Alam 2020, Solhmirzaei et al. 2020, and Naser 2020), the strength of beam-column and slab-column joints (Jeon et al. 2014, Jeong et al. 2021a, Mansouri et al. 2021), column capacity (Lim et al. 2016), bond strength (Güneyisi et al. 2013, Golafshani et al. 2015), transfer length (Jeong et al. 2021b), fire-induced concrete spalling (Naser and Salehi 2020), as well as bond properties and bending capacity of fiber-reinforced polymer-strengthened and reinforced concrete members (Naser 2020). In steel structures, GP was employed for deriving equations for bending capacity of castellated beams and circular tubes (Gandomi et al. 2011, Shahin and Elchalakani 2014), flexural overstrength factor and available rotation capacity of steel beams (Güneyisi et al. 2013, D'Aniello et al. 2014, 2015), beam resistance under localized loads (Graciano et al. 2021), compressive capacity of perforated tubular members (Hernández et al. 2018), damage evaluation of steel columns under blast loads (Momeni et al. 2020), and plasticity models (Bomarito et al. 2021). GP has also found applications for predicting the capacity of concrete-filled steel columns in compression (Güneyisi and Nour 2019, Nour and Güneyisi 2019, Naser et al. 2021, İpek and Güneyisi, 2022).

The abovementioned studies demonstrated that GP is an efficient technique capable of deriving new empirical equations with improved performance. Therefore, it was hypothesized that GPSR could discover hidden relationships between variables and produce improved predictive models of the stud shear resistance in solid concrete slabs. This study had the following objectives: 1) to derive new equations for predicting the nominal shear resistance of studs in normal weight concrete (NWC) and lightweight concrete (LWC) via GPSR; 2) to evaluate the reliability of the obtained equations in accordance with European and US design practices; 3) to develop models for predicting the design shear resistance of studs that meet the target reliability index; and 4) to compare the performance of the developed design models with several existing models, including those currently used in international design standards. The novelty of the present work consists of improved GPSR-based descriptive equations for predicting the shear resistance of studs in both NWC and LWC solid slabs and subsequent refinement of the GPSR-based equations for use in design from reliability analyses. The proposed models can be applied in the design of composite bridge beams which commonly use solid concrete slabs, or in the design of composite columns using concrete-encased and concrete-filled steel sections. However, the models are not directly applicable to studs in steel decking, which are commonly found in composite beams in modern multi-storey buildings, owing to the reduction in resistance caused by the geometry of the deck profile. Notwithstanding this, given that most international design standards provide reduction factor equations that are multiplied to the resistance of studs embedded in solid concrete slabs, the GPSR-based descriptive equations offer insights on the importance of some of the variables that have historically been used in code-defined design models. In addition, the models will provide a basis for future investigations on whether it is entirely appropriate that a simple reduction factor can adequately capture the performance of studs in decking.

2. Test databases

Two databases of push-out test results for headed stud shear connectors embedded in solid NWC and LWC slabs were used in this study (Hicks 2021a, Hicks 2021b). The databases contain 242 NWC and 90 LWC test results and include the mean measured shear resistance per stud, P_{em}, along with the following mean measured material and

geometric properties: concrete compressive strength, f_{cm}; secant modulus of elasticity of concrete, E_{cm}; ultimate tensile strength of studs, f_{um}; stud shank diameter, d_m; diameter of the weld collar, d_{dom}; the height of the weld collar, h_{wm}; the overall height of stud after welding, h_m; stud height-to-diameter ratio, h_m/d_m; and concrete density (LWC database only). Nominal properties of the independent variables are also presented in the databases, which are based on the specified properties reported in the tests, or evaluated from considering the statistical properties assumed in European and international product standards.

The test databases include independent variables in the following ranges:

- NWC: 16.61 MPa $\leq f_{cm} \leq$ 115.83 MPa, 15.1 GPa $\leq E_{cm} \leq$ 46.5 GPa, 426 MPa $\leq f_{um} \leq$ 675 MPa, 12.7 mm $\leq d_m \leq$ 31.8 mm, 21.0 mm $\leq d_{dom} \leq$ 44.5 mm, 3.0 mm $\leq h_{wm} \leq$ 8.6 mm, 70 mm $\leq h_m \leq$ 200 mm, and 3.00 $\leq h_m/d_m \leq$ 9.09.
- LWC: 20.48 MPa $\leq f_{cm} \leq$ 55.71 MPa, 10.41 GPa $\leq E_{cm} \leq$ 19.44 GPa, 407 MPa $\leq f_{um} \leq$ 600 MPa, 12.7 mm $\leq d_m \leq$ 22.2 mm, 17.0 mm $\leq d_{dom} \leq$ 29.0 mm, 3.0 mm $\leq h_{wm} \leq$ 6.0 mm, 51 mm $\leq h_m \leq$ 114 mm, 2.67 $\leq h_m/d_m \leq$ 8.00, and 1410 kg/m^3 $\leq$ density $\leq$ 1970 kg/m^3.

Distributions of the database variables are illustrated in Figs. 1 and 2, which demonstrate that the test databases cover a wide range of design parameters used in composite steel-concrete construction.

Figs. 1 and 2 also show that only a small number of tests are available for the following ranges of the variables:

- NWC: f_{um} > 600 MPa, d_m < 16 mm and d_m > 25 mm, and 3.9 < h_m/d_m and h_m/d_m > 6.
- LWC: f_{cm} > 40 MPa, f_{um} > 530 MPa, d_m > 20 mm, h_m/d_m > 6, and density < 1550 kg/m^3 and density > 1850 kg/m^3.

Future experimental studies on specimens with the variables in these ranges would be beneficial for extending the knowledge and improving design models.

Figs. 3 and 4 present correlation matrices for the database variables. For NWC, the stud shank diameter, d_m, has the highest positive correlation with P_{em}, characterized by a correlation coefficient of 0.77. In descending order, the weld collar diameter, d_{dom}, concrete elastic modulus, E_{cm}, and concrete compressive strength, f_{cm}, have correlation coefficients of 0.68, 0.64, and 0.62 with P_{em}, respectively. The weld collar height, h_{wm}, and stud height, h_m, demonstrate an even smaller correlation with P_{em}, with a coefficient of correlation of 0.52. The stud height-to-diameter ratio, h_m/d_m, and ultimate tensile strength of studs, f_{um}, show practically no correlation with P_{em}. For LWC, d_m, d_{dom}, and h_{wm} have the strongest positive correlations with P_{em}, characterized by correlation coefficients of 0.87, 0.86, and 0.80, respectively. All other variables demonstrate relatively weak correlations with P_{em}. According to EN ISO 13918 (2017), the weld collar diameter and height depend on the stud shank diameter, which is reflected by strong correlations between d_m and d_{dom}, and d_m and h_{wm}, in both databases.

It should be noted that the correlation matrices are based on a linear correlation of each variable considered separately from the others. They are unable to capture complex nonlinear relationships between the variables and the interaction effects of several variables. The correlation matrices give insights into the variables that might affect the

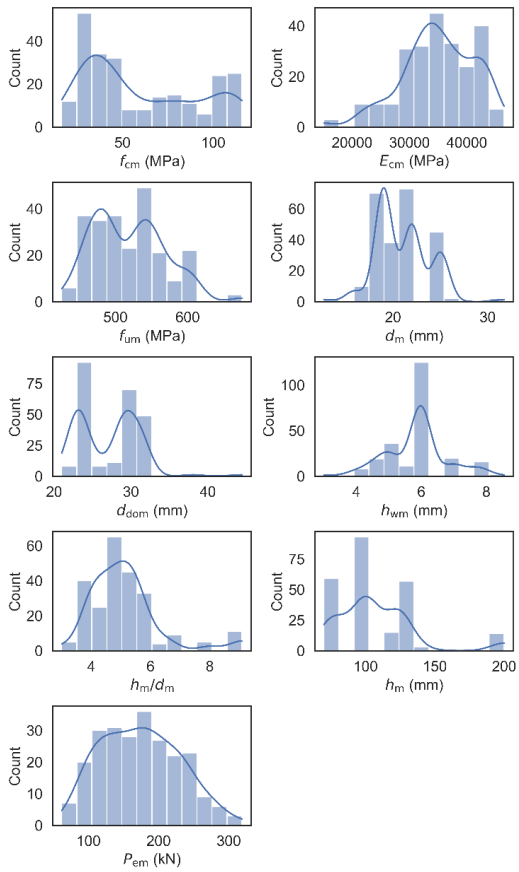

Fig. 1 Distributions of NWC database variables

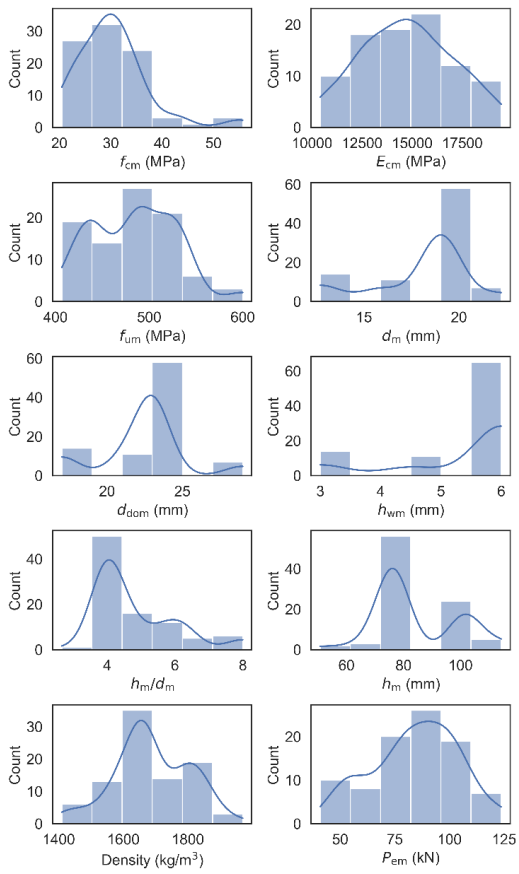

Fig. 2 Distributions of LWC database variables

stud resistance most significantly but do not paint the complete picture of the relationships between the independent and dependent variables.

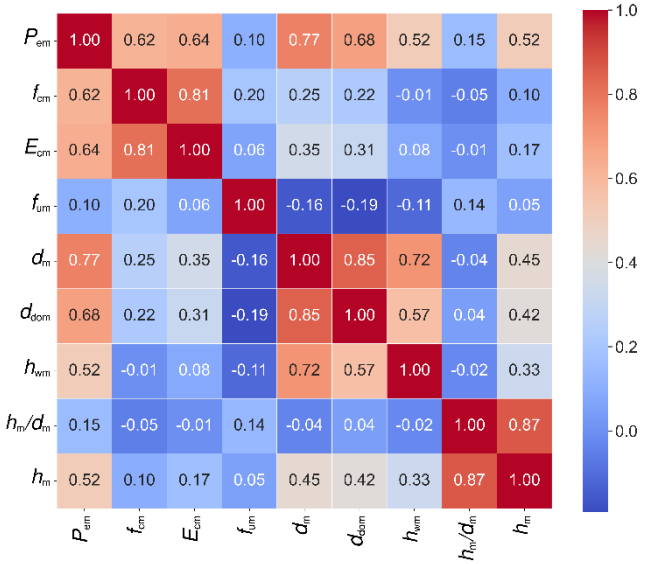

Fig. 3 Correlation matrix of NWC database variables

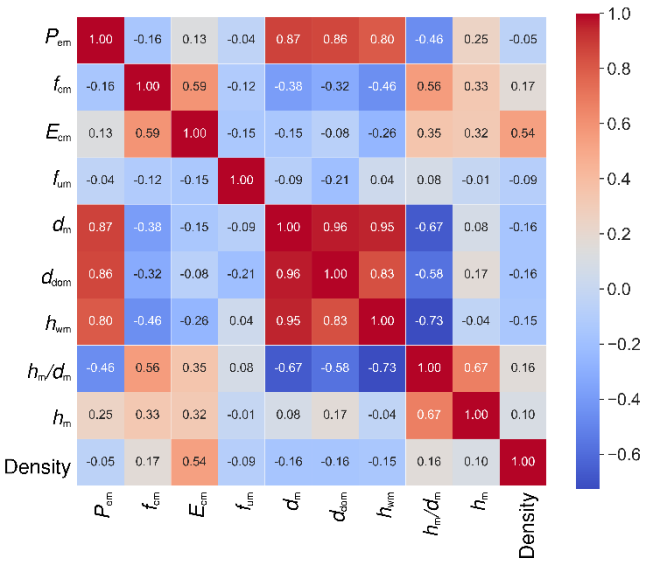

Fig. 4 Correlation matrix of LWC database variables

3. Symbolic regression with genetic programming

ML algorithms are effective at discovering and revealing hidden relationships between variables. The advantage of GPSR over other ML models is that they produce descriptive models in the form of easily comprehended mathematical expressions, as opposed to predictions which often cannot be easily explained and interpreted by humans (Naser 2021).

GPSR, proposed by Koza (1992), is a type of regression analysis that searches over the space of all possible mathematical expressions for the one that produces the best predictions for a given dataset. In this study, GPSR analyses were performed using *gplearn* (Stephens 2016), an open-source Python-based library, with genetic programming implemented to solve symbolic regression problems.

Fig. 5 demonstrates a flow chart of the algorithm. The analysis starts with creating a population of random functions represented as tree structures consisting of a mix of variables, constants, and functions. The variables are the independent variables of the dataset. The user specifies the functions considered by the algorithm. They include addition, subtraction, multiplication, division, square root, and others. The number of functions in the first and following generations, or population size, is a hyperparameter specified by the user at the beginning of the analysis.

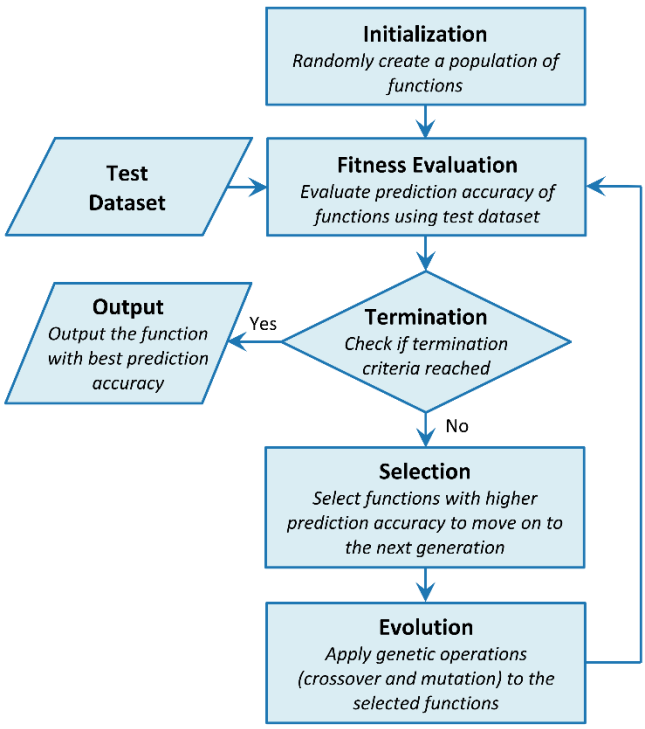

Fig. 5 Flow chart of GPSR

The prediction accuracy, or fitness, of the initial random functions is evaluated against the dataset. Mean absolute error, mean squared error or root-mean-squared error can be used as prediction accuracy metrics. Functions with the best fitness value evolve into the next generation. The function selection process is done via tournaments, where the population of functions is randomly split into smaller subsets. The functions compete between themselves, and the fittest function from each subset is selected to move into the next generation.

The algorithm's next step consists of genetic operations, such as crossover and mutation, performed on the selected functions. In crossover, the functions are randomly mixed. Mutation involves random replacing of parts of the functions, which are referred to as subtrees. The processes of fitness evaluation, selection, and evolution are repeated for each generation of functions until the maximum number of generations specified by the user is reached or when the fitness metric of at least one function in the population is smaller than the specified stopping value.

Several hyperparameters must be specified at the beginning of the analysis, including the following:

- population size (the number of functions in each generation),
- the maximum number of generations,
- tournament size (the number of functions that will compete to become a part of the next generation),
- stopping criteria (the fitness metric value required to stop evolution early),
- initial depth (the range of tree depths for the initial generation of random functions),
- the function set for building and evolving functions,
- the fitness metric used for fitness evaluation,
- parsimony coefficient, which is used to penalize large functions by adjusting their fitness to be less favorable in the selection,
- the probability of performing crossover and mutation on a tournament winner.

Optimal values of the hyperparameters that result in the desired algorithm performance are usually obtained via multiple trials. The reader is referred to the *gplearn* documentation (Stephens 2016) for more information about the GPSR method.

Multiple analyses of the stud resistance test data were performed in this study to find optimal hyperparameters that produce descriptive equations with the best performance. The original independent variables of the database and their combinations, such as $f_{um}A_m$, $h_{wm}d_{dom}f_{cm}$, $f_{cm}A_m$, (where $A_m=\pi d_m^2/4$), and others, were considered. Special attention was paid to ensure that the developed equations had reasonable forms and sizes to be convenient for hand calculations. The equation size was controlled by carefully selecting the initial function depth and parsimony coefficient values. Many equations were obtained from the analyses. Their performance was carefully evaluated to select the simplest equations producing good agreements with the test data.

4. Descriptive equations for predicting the nominal shear resistance of studs in solid concrete slabs

Two descriptive equations for predicting the nominal (mean) shear resistance of headed studs, P_n, were selected from the conducted GPSR analyses. The equations are referenced as SRN1 (Eq. (1)) and SRN2 (Eq. (2)).

$$P_n = 1.1\lambda\sqrt[4]{f_{cm}f_u^3\frac{h}{d}}\frac{\pi d^2}{4} \qquad \text{(SRN1)} \quad (1)$$

$$P_n = (1.1-0.1\eta)\sqrt[4]{f_{cm}f_u^3\left(\frac{h}{d}-\eta\right)}\frac{\pi d^2}{4} \qquad \text{(SRN2)} \quad (2)$$

where f_u, d, and h are the nominal ultimate tensile strength, shank diameter, and height of the stud; λ is the concrete type factor taken as 1.00 for NWC and 0.84 for LWC; η is the concrete type coefficient taken as 0 for NWC and 1 for LWC.

It can be observed that SRN1 and SRN2 models are identical for NWC and differ for LWC. Fig. 6 shows

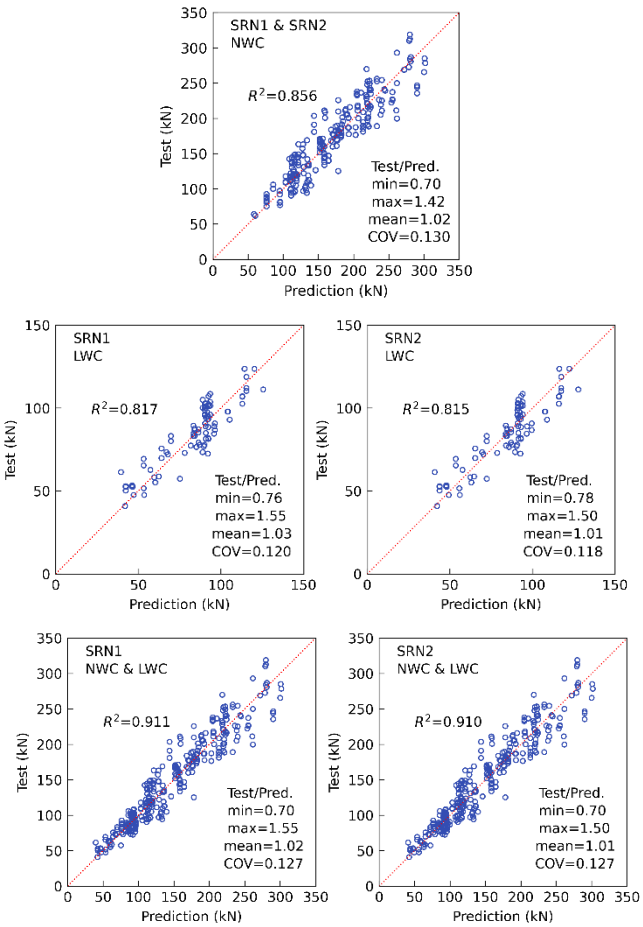

Fig. 6 Performance of the developed equations for predicting the nominal shear resistance of studs

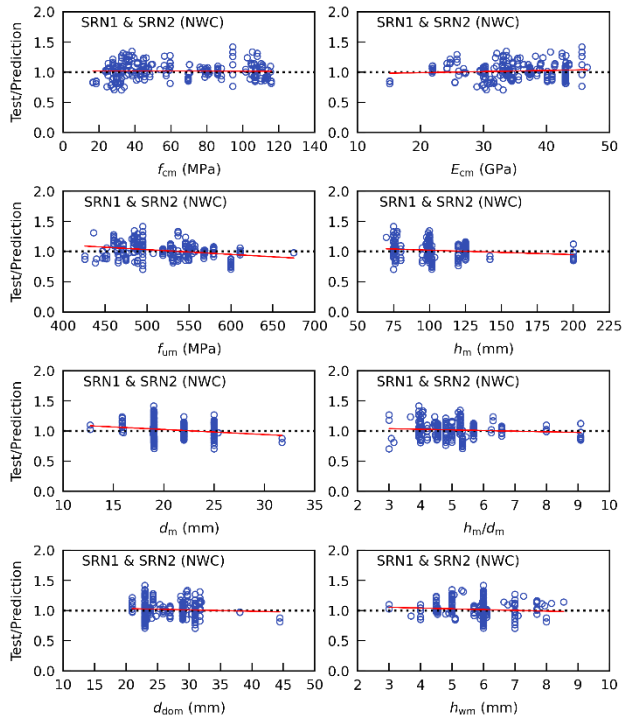

Fig. 7 Test-to-prediction ratios versus NWC database variables for the nominal resistance equations

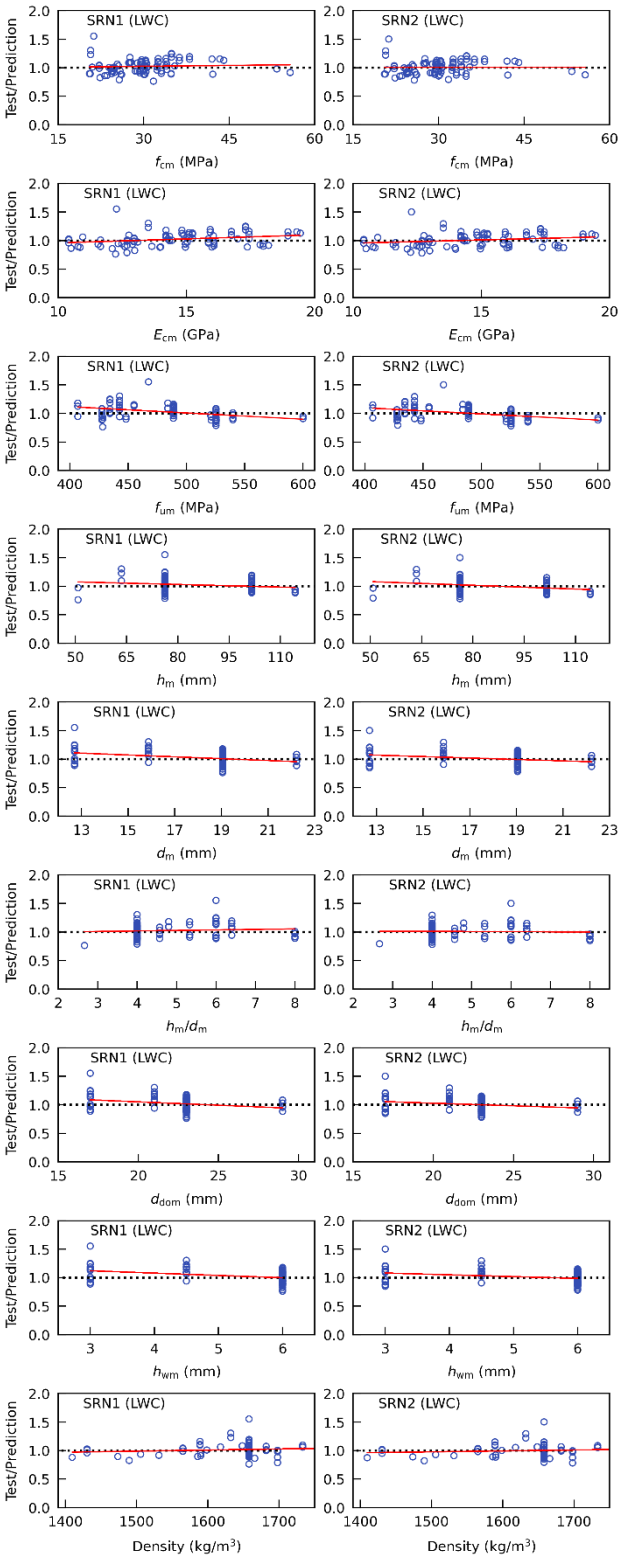

Fig. 8 Test-to-prediction ratios versus LWC database variables for the nominal resistance equations

comparisons of the stud shear resistance predicted by models SRN1 and SRN2 with the experimental data from the NWC, LWC, and combined NWC and LWC databases. The equation variables were taken as the mean measured values from the test database. The presented comparisons demonstrate that the stud resistance predictions by the developed equations agreed reasonably well with the test results for NWC and LWC. Both models produced comparable performance metrics.

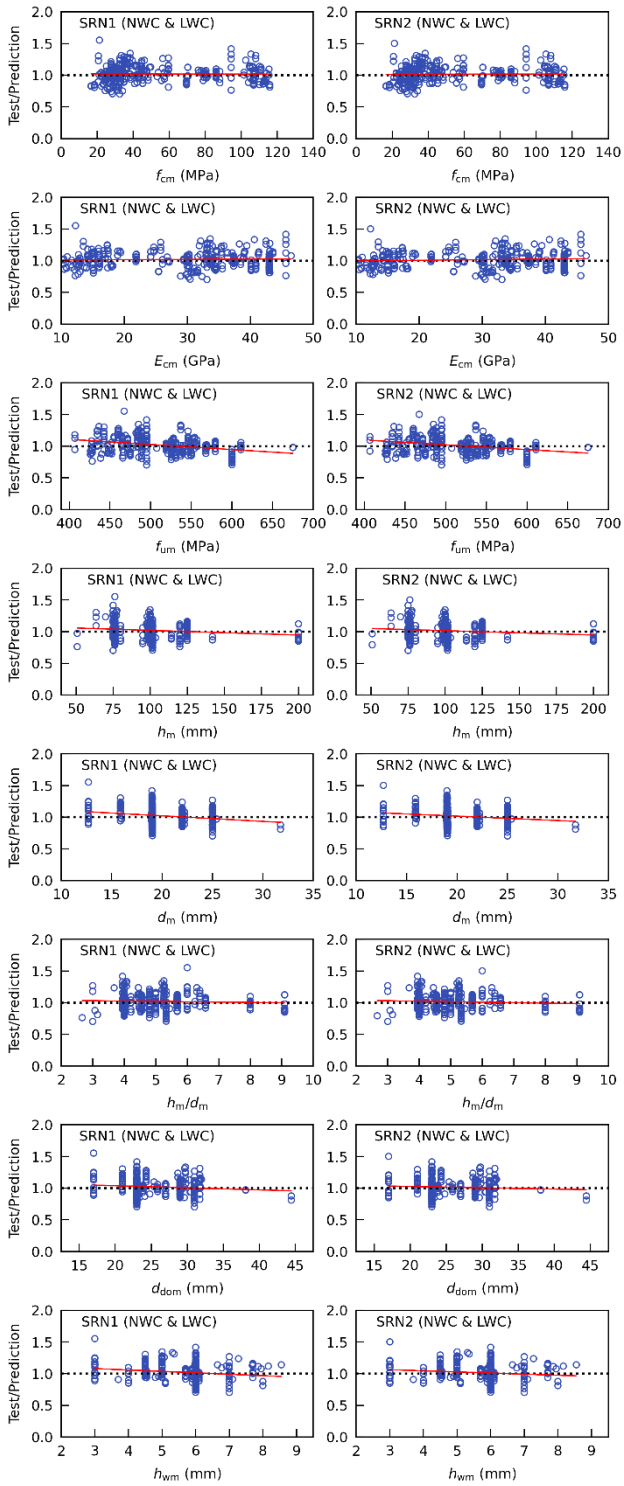

Fig. 9 Test-to-prediction ratios versus NWC and LWC combined database variables for the nominal resistance equations

Figs. 7-9 show test-to-prediction ratios as functions of the test database variables for each developed model for the NWC, LWC, and combined NWC and LWC data. The figure

demonstrates that all models produced relatively consistent predictions of the stud resistance for the entire ranges of the test dataset variables. It can also be seen that predictions of some models might be improved by introducing additional coefficients, which are functions of the independent variables. For example, coefficients that are functions of d and f_u could be added to the models to improve their performance (the former is consistent with that found by Hicks 2017, who proposed a reduction factor based on stud shank diameter for studs in NWC). That, however, was not done in the present work to preserve the simplicity of the equations, which demonstrate decent prediction accuracy in the original form without any additional modifications.

In the proposed models, the differences between NWC and LWC are accounted for by coefficients λ and η. Several existing stud resistance models presented in Section 7 include E_{cm}, which allows for distinguishing between NWC and LWC with equal compressive strengths. Figs. 8 and 9 demonstrate a weak correlation between the test-to-prediction ratios for both developed models and E_{cm}, which justifies the absence of this variable in the proposed models. Stud resistance proposal #4 by Pallarés and Hajjar (2010) presented in Section 7 is based on a similar approach. It includes an LWC reduction coefficient and does not include E_{cm}. It can also be observed from Fig. 8 that the LWC density has a weak correlation with the test-to-prediction ratios for the proposed models. This finding is important as, following its introduction within the empirical equations proposed by Ollgaard et al. (1971), the secant modulus of elasticity is widely used within the design models for stud resistance in many international standards (see Section 7).

5. Reliability analyses of the models according to the European and US design practices

The information presented in the previous section shows that models SRN1 and SRN2 can predict the nominal (mean) shear resistance of studs in solid concrete slabs reasonably well. The models, however, have to be calibrated via reliability analyses to ensure that a sufficiently low probability of failure is achieved. Design standards from around the world contain different calibration requirements. In this study, the reliability of the developed equations was evaluated in accordance with both European and US design practices.

5.1 Eurocodes

In Europe, steel-concrete composite structures are designed in accordance with Eurocode 4 (EN 1994-1-1 2004, EN 1994-2 2005), which recommends the partial factor for shear connectors, γ_V, of 1.25. In the interest of harmonization across the Eurocodes, this value was considered appropriate for all connection components (bolts, rivets, welds, etc.), and the resistance to fracture of a steel cross-section in tension. The design resistance is based on a probability of failure that does not exceed $P_f = 1.2 \times 10^{-3}$ (EN 1990 2002, Hicks 2017). This value corresponds to the target reliability index, $\beta = 3.8$ for a 50-year reference period multiplied by the First Order Reliability Method (FORM) sensitivity factor for resistance, α_R, of 0.8, resulting in the adjusted target reliability index of $\alpha_R\beta = 3.04$. The reliability analyses for evaluating the design resistance should follow the method of EN 1990 Annex D, which consists of the following steps:

1) A design model for the theoretical resistance, r_t, is developed based on test results.

$$r_t = g_{rt}(\underline{X}), \tag{3}$$

where $g_{rt}(\underline{X})$ is resistance function of the basic variables $\underline{X}$.

2) Theoretical resistances are compared with experimental resistances, r_e, by plotting an r_e-r_t diagram.

3) The mean value of correction factor, b, is estimated as follows:

$$b = \frac{\sum r_e r_t}{\sum r_t^2} \tag{4}$$

4) The coefficient of variation, V_δ, of the error terms, δ_i, is estimated from Eqs. (5)-(9).

$$\delta_i = \frac{r_{ei}}{b r_{ti}} \tag{5}$$

$$\Delta_i = \ln(\delta_i) \tag{6}$$

$$\bar{\Delta} = \frac{1}{n}\sum_{i=1}^{n} \Delta_i \tag{7}$$

$$s_\Delta^2 = \frac{1}{n-1}\sum_{i=1}^{n}(\Delta_i - \bar{\Delta})^2 \tag{8}$$

$$V_\delta = \sqrt{\exp(s_\Delta^2) - 1} \tag{9}$$

where the subscript i denotes values for the ith test specimen and n is the total number of test specimens.

5) The coefficient of variation of the theoretical resistance, V_{rt}, is estimated from EN 1990 equations (which are challenging to implement for complex functions) or, more conveniently, through Monte Carlo simulation (Hicks 2017).

6) The coefficient of variation, V_r, is estimated as follows

$$V_r = \sqrt{V_\delta^2 + V_{rt}^2} \tag{10}$$

7) The characteristic resistance, R_k, is obtained from Eq. (11).

$$R_k = b g_{rt}(\underline{X}_m)\exp(-k_\infty \alpha_{rt} Q_{rt} - k_n \alpha_\delta Q_\delta - 0.5Q^2) \tag{11}$$

where $\underline{X}_m$ is the array of mean values of the basic variables; k_n is the characteristic fractile factor for the number of tests, n with a probability of 0.05; $k_\infty = 1.64$; $\alpha_{rt} = Q_{rt}/Q$ is the weighting factor for the theoretical resistance; $\alpha_\delta = Q_\delta/Q$ is the weighting factor for the error terms; and $Q_{rt} = \sqrt{\ln(V_{rt}^2 + 1)}$, $Q_\delta = \sqrt{\ln(V_\delta^2 + 1)}$, and $Q = \sqrt{\ln(V_r^2 + 1)}$ are the standard deviations of the theoretical resistance, error terms, and resistance, respectively.

8) The design resistance, R_d, is determined from Eq. (12)

$$R_d = b g_{rt}(\underline{X}_m)\exp(-k_{d,\infty}\alpha_{rt} Q_{rt} - k_{d,n}\alpha_\delta Q_\delta - 0.5Q^2) \tag{12}$$

where $k_{d,n}$ is the design fractile factor for the number of tests, n, with a probability of failure of 1.2×10^{-3}; and $k_{d,\infty} =$

$\alpha_R\beta = 3.04$.

9) The partial factor, γ_M, is determined as follows.

$$\gamma_M = \frac{R_k}{R_d} \tag{13}$$

10) The corrected partial factor, γ^*_M, is obtained from Eq. (14).

$$\gamma_M^* = \frac{R_n}{R_d} = k_c\gamma_M \tag{14}$$

where R_n is the nominal resistance determined from the design model using nominal values of the variables and $k_c = R_n/R_k$.

This procedure was used by Hicks (2017) to evaluate the reliability of the stud resistance design models of Eurocode 4, Oehlers and Johnson (1987), and Döinghaus (2001). The study showed that all three models needed modifications to satisfy the reliability requirements with the partial factor of 1.25. The modified equations were proposed by Hicks (2017).

In the present study, the required partial factors for the developed nominal stud resistance models were determined per the EN 1990 method described above. The Monte Carlo simulation performed for estimating V_{rt} was based on 10,000 random values generated for each nominal material and geometric property from the test database (Hicks 2021). Log-normal distributions with the parameters listed hereafter were assumed for all properties. The values given below represent the mean of the ratios of the actual to nominal values of the variables.

1) Concrete compressive strength, f_{cm}: the mean of 1.0 and the coefficient of variation (CoV) of $8/(1.64f_{cm})$ (EN 1992-1-1 2004).

2) Ultimate tensile strength of studs, f_u: the mean of 1.1 and CoV of 0.05 (Hicks 2017).

3) Stud shank diameter, d: the mean of 1.0 and CoV of $0.231/d$.

4) Height of stud after welding, h: the mean of 1.0 and CoV of $1.012/h$.

The CoV values for d and h were determined based on the tolerances published in ISO EN 13918 (2017).

Table 1 Reliability analysis results for the nominal resistance models per EN 1990

Model	b	V_δ	V_{rt}	V_r	k_c	$\gamma^*_M=\gamma_V$
NWC (n=242)						
SRN1	1.001	0.131	0.058	0.143	1.09	1.33
SRN2	1.001	0.131	0.058	0.143	1.09	1.33
LWC (n=90)						
SRN1	1.004	0.118	0.070	0.137	1.12	1.37
SRN2	0.989	0.116	0.070	0.135	1.13	1.38
NWC & LWC (n=332)						
SRN1	1.002	0.128	0.061	0.142	1.09	1.34
SRN2	1.000	0.127	0.061	0.141	1.10	1.34

The reliability analyses were separately performed for the NWC, LWC, and combined LWC and NWC data. The reliability analysis results for each nominal resistance model are summarized in Table 1. Both models need a partial factor, γ_V, higher than 1.25 to provide the design resistance with the required reliability level. Therefore, the nominal resistance models SRN1 and SRN2 must be modified to meet the reliability requirements of EN 1990 (2002) and EN 1994-1-1 (2004).

It should also be acceptable to use the SRN1 and SRN2 models with a partial factor larger than 1.25 (e.g., 1.40) for the design resistance predictions. However, according to Annex D of EN 1990 the partial factor that is applied should be taken from the appropriate Eurocode. Therefore, to enable the models to be used directly in design, the recommended value of 1.25 was taken from Eurocode 4 and a reduction coefficient applied. As well as satisfying the EN 1990 requirements, the adoption of this approach also facilitates direct comparison with the performance of the existing design equations. As discussed hereafter, both approaches give approximately the same design shear resistance.

5.2 US design practice

AISC 360 (2016) governs the design of steel-concrete composite structures in the US. It includes shear strength provisions for steel headed stud anchors in both composite beams and other composite components. A composite component is defined as a member, connecting element or assemblage in which steel and concrete elements work as a unit in the distribution of internal forces; composite beams with solid slabs or formed steel deck are excluded from this definition. This distinction determines how the resistance factor is applied in design.

The AISC 360 nominal shear strength formula for studs in composite beams considers the stud material and concrete strengths, but does not include a resistance factor. The required reliability is provided by the resistance factors applied to the composite beam strength (Mujagić and Easterling 2009, Pallarés and Hajjar 2010). Therefore, the resistance factors for composite beams must, in theory, be calibrated for each new stud shear strength equation considered, which was beyond the scope of the present study.

In contrast, a resistance factor of 0.65 is applied directly to the nominal shear strength of a stud anchor in other composite components. AISC 360 gives a shear strength formula based on the stud material only and references ACI 318 for concrete breakout strength calculations, where applicable. The AISC 360 provisions for composite components have not been evaluated in the present work.

The resistance factor for a stud strength model can be computed using Eq. (15) recommended by Ravindra and Galambos (1978).

$$\phi_v = \frac{R_m}{R_n} e^{(-\alpha\beta V_R)} \tag{15}$$

where R_m/R_n is the average ratio between the experimental and predicted values, α=0.55, β is the target reliability index, and $V_R = \sqrt{V_F^2 + V_P^2 + V_M^2}$, where V_F is the coefficient of variation on fabrication (stud dimensions), V_P is the coefficient of variation of R_m/R_n, and V_M is the coefficient of variation of the material properties. The V_F and V_M values of 0.05 and 0.09, respectively, were used in this study, following the recommendations of Pallarés and Hajjar (2010). In their work on stud connectors, these authors considered a reliability index of β= 3.0 and 4.0; the former value was considered to be delivered by composite beams, whilst the

latter was adopted to permit studs to be used in a wider range of applications.

The R_m/R_n and V_P values were determined for the proposed nominal shear strength models using two approaches: 1) based on the mean measured concrete strength, $f'_{cr}=f_{cm}$, and stud tensile strength, f_{um}; and 2) based on the specified concrete strength, f'_c, and the stud nominal tensile strength of f_u=450 MPa (65 ksi). The f'_{cr} values were converted into f'_c per the ACI 301 (2016) recommendations. The 450 MPa (65 ksi) value was selected because studs with such a nominal tensile strength are commonly used in the US (AISC SCM 2016). The NWC and LWC test databases include 9 and 29 tests, respectively, with f_{um} < 450 MPa, which were excluded from the resistance factor calculations using the second approach.

The resistance factors computed for the target reliability indices of β = 3.0 and 4.0 (Pallarés and Hajjar 2010) are presented in Table 2. It can be noted that the resistance factor values based on the nominal properties and β=4.0 agree well with the partial factor value recommended by Eurocode 4 ($\phi_v=1/\gamma_V=1/1.25=0.80$) and exceed the AISC 360 resistance factor for studs in composite components.

Table 2 Resistance factors for the nominal strength models per US design practice

Model	Concrete/stud strength	R_m/R_n	V_P	ϕ_v	
				β= 3.0	β= 4.0
NWC (n=242 for $f'_{cr}=f_{cm}$, f_{um} and n=233 for f'_c, f_u=450 MPa)					
SRN1	$f'_{cr}=f_{cm}$, f_{um}	1.018	0.130	0.77	0.71
	f'_c, f_u=450 MPa	1.200	0.118	0.93	0.85
SRN2	$f'_{cr}=f_{cm}$, f_{um}	1.018	0.130	0.77	0.71
	f'_c, f_u=450 MPa	1.200	0.118	0.93	0.85
LWC (n=90 for $f'_{cr}=f_{cm}$, f_{um} and n=61 for f'_c, f_u=450 MPa)					
SRN1	$f'_{cr}=f_{cm}$, f_{um}	1.025	0.120	0.79	0.72
	f'_c, f_u=450 MPa	1.197	0.095	0.95	0.88
SRN2	$f'_{cr}=f_{cm}$, f_{um}	1.008	0.118	0.78	0.71
	f'_c, f_u=450 MPa	1.178	0.095	0.94	0.87
NWC & LWC (n=332 for $f'_{cr}=f_{cm}$, f_{um} and n=294 for f'_c, f_u=450 MPa)					
SRN1	$f'_{cr}=f_{cm}$, f_{um}	1.020	0.127	0.78	0.71
	f'_c, f_u=450 MPa	1.200	0.114	0.93	0.86
SRN2	$f'_{cr}=f_{cm}$, f_{um}	1.015	0.127	0.78	0.71
	f'_c, f_u=450 MPa	1.196	0.114	0.93	0.85

6. Design shear resistance of headed studs in solid concrete slabs

Reduction coefficients were applied to nominal resistance models SRN1 and SRN2 (Eqs. (1) and (2)) to provide the reliability level required by the Eurocodes. Required magnitudes of the reduction coefficients for each model were determined by trials. The resulting design resistance models SRD1 and SRD2 are described by Eqs. (16) and (17). As can be seen from comparing the nominal and design resistance models, a reduction coefficient of 0.9 was necessary for both models to achieve the required reliability with the partial factor of 1.25.

$$P_{\mathrm{Rd}} = \lambda \sqrt[4]{f_{\mathrm{ck}} f_{\mathrm{u}}^3 \frac{h}{d} \frac{\pi d^2}{4} \frac{1}{\gamma_{\mathrm{V}}}} \qquad \text{(SRD1)} \quad (16)$$

$$P_{\mathrm{Rd}} = (1 - 0.1\eta) \sqrt[4]{f_{\mathrm{ck}} f_{\mathrm{u}}^3 \left(\frac{h}{d} - \eta\right)} \frac{\pi d^2}{4} \frac{1}{\gamma_{\mathrm{V}}} \qquad \text{(SRD2)} \quad (17)$$

Reliability analyses of the design models performed in accordance with European and US design practices described in Section 5 are summarized in Tables 3 and 4, respectively.

Table 3 Reliability analysis results for the design resistance models per EN 1990

Model	b	V_δ	V_{rt}	V_r	k_c	$\gamma^*_M=\gamma_V$
NWC (n=242)						
SRD1	1.101	0.131	0.058	0.143	0.99	1.21
SRD2	1.101	0.131	0.058	0.143	0.99	1.21
LWC (n=90)						
SRD1	1.104	0.118	0.070	0.137	1.02	1.24
SRD2	1.098	0.116	0.070	0.135	1.02	1.24
NWC & LWC (n=332)						
SRD1	1.102	0.128	0.061	0.142	1.00	1.22
SRD2	1.101	0.127	0.061	0.141	0.99	1.21

Table 4 Resistance factors for the design strength models per US design practice

Model	Concrete/stud strength	R_m/R_n	V_P	ϕ_v	
				β= 3.0	β= 4.0
NWC (n=242 for $f'_{cr}=f_{cm}$, f_{um} and n=233 for f'_c, f_u=450 MPa)					
SRD1	$f'_{cr}=f_{cm}$, f_{um}	1.119	0.130	0.85	0.78
	f'_c, f_u=450 MPa	1.321	0.118	1.00	0.94
SRD2	$f'_{cr}=f_{cm}$, f_{um}	1.119	0.130	0.85	0.78
	f'_c, f_u=450 MPa	1.321	0.118	1.00	0.94
LWC (n=90 for $f'_{cr}=f_{cm}$, f_{um} and n=61 for f'_c, f_u=450 MPa)					
SRD1	$f'_{cr}=f_{cm}$, f_{um}	1.128	0.120	0.87	0.80
	f'_c, f_u=450 MPa	1.317	0.095	1.00	0.97
SRD2	$f'_{cr}=f_{cm}$, f_{um}	1.120	0.118	0.86	0.79
	f'_c, f_u=450 MPa	1.309	0.095	1.00	0.96
NWC & LWC (n=332 for $f'_{cr}=f_{cm}$, f_{um} and n=294 for f'_c, f_u=450 MPa)					
SRD1	$f'_{cr}=f_{cm}$, f_{um}	1.122	0.127	0.86	0.78
	f'_c, f_u=450 MPa	1.320	0.114	1.00	0.94
SRD2	$f'_{cr}=f_{cm}$, f_{um}	1.120	0.127	0.86	0.78
	f'_c, f_u=450 MPa	1.318	0.114	1.00	0.94

The partial factors smaller than 1.25 obtained for SRD1 and SRD2 demonstrate that the reliability of the design models meets or exceeds the reliability level required by Eurocodes. It can also be noticed that the ratios of the partial factors for the

SRD1 and SRD2 models (see Table 3) to those for the SRN1 and SRN2 models (see Table 1) are approximately equal to the reduction coefficient of 0.9 applied to the nominal resistance models. Therefore, the design stud resistances obtained from the SRN1 and SRN2 models with the partial factors from Table 1 are approximately equal to those from the SRD1 and SRD2 models with the partial factors from Table 3. Conversely, when considering US design practice, the resistance factors, ϕ_v, were relatively high for both models, especially when the nominal values of the concrete and stud resistances were used. The resistance factors for the proposed models are higher than the AISC 360 resistance factor of 0.65 for the stud shear strength in composite components.

Following the ranges of the variables in the test databases, the applicability of the proposed design models should be limited by the following values:

- NWC: 20 MPa $\leq f_{cm} \leq$ 115 MPa (12 MPa $\leq f_{ck} \leq$ 90 MPa), 450 MPa $\leq f_u \leq$ 600 MPa, 16 mm $\leq d \leq$ 25 mm, $3 \leq h/d \leq 9$;
- LWC: 24 MPa $\leq f_{cm} \leq$ 58 MPa (16 MPa $\leq f_{ck} \leq$ 50 MPa), 450 MPa $\leq f_u \leq$ 600 MPa, 13 mm $\leq d \leq$ 22 mm, $3 \leq h/d \leq 8$.

7. Comparisons of the developed models with the existing descriptive equations

The developed models for predicting the shear resistance of studs in solid concrete slabs were compared with the following existing models:

- Eurocode 4 (EN 1994-1-1 2004 and EN 1994-2 2005),
- Eurocode 4 modified (Hicks 2017),
- AISC (AISC 360 2016),
- AS/NZS (AS/NZS 2327 2017 and AS/NZS 5100.6 2017),
- JSCE (2017),
- Oehlers and Johnson modified (Hicks 2017),
- Döinghaus modified (Hicks 2017),
- Konrad et al. (2020),
- Hanswille and Porsch (2007), and
- Pallarés and Hajjar (2010).

The equations proposed by Xue et al. (2008) and Bonilla et al. (2012) were also considered but not included in the comparisons because their applicability ranges were considerably narrower than the ranges of the variables in the test databases.

The existing equations are summarized in Table 5, with the consistent notation used for each variable. When two equations are given for an existing model, the design resistance is taken as the smaller value computed from each equation. The AISC 360 (2016) and Pallarés and Hajjar (2010) equations give the nominal stud resistance, P_n, with no resistance factors applied. All other existing equations predict the design resistance, P_{Rd}. Therefore, two separate comparisons of the proposed equations were performed: models SRN1 and SRN2 were compared with the existing nominal resistance equations, whereas models SRD1 and SRD2 were compared with the existing design resistance equations. The design resistance models were compared in terms of characteristic resistance, $P_{Rk} = P_{Rd}\gamma_V$, or $P_{Rk} = P_{Rd}\gamma_b$ for the JSCE model.

Pallarés and Hajjar (2010) evaluated C_v values of 0.65, 0.75, and 1. C_v=1 was used in this study because it produced better agreements of the predictions with the experimental data than two other values of C_v.

Table 5 Existing descriptive equations

Source	Equations	
EC 4	$P_{Rd} = 0.8 f_u (\pi d^2/4)/\gamma_V$	(18)
	$P_{Rd} = 0.29 \alpha d^2 \sqrt{f_{ck} E_{cm}}/\gamma_V$	(19)
EC 4 modified	$P_{Rd} = 0.94 \eta f_u (\pi d^2/4)/\gamma_V$	(20)
	$P_{Rd} = 0.25 d^2 \sqrt{f_{ck} E_{cm}}/\gamma_V$	(21)
AISC 360	$P_n = 0.5 \frac{\pi d^2}{4} \sqrt{f_c' E_c} \leq 0.75 \frac{\pi d^2}{4} f_u$	(22)
AS/NZS	$P_{Rd} = 0.70 d^2 f_u/\gamma_V$	(23)
	$P_{Rd} = 0.29 d^2 \sqrt{f_{ck} E_{cm}}/\gamma_V$	(24)
JSCE	$P_{Rd} = (\pi d^2/4) f_u/\gamma_b$	(25)
	$P_{Rd} = \left(31 \frac{\pi d^2}{4} \sqrt{\frac{h}{d} \frac{f_{ck}}{1.3}} + 10000\right) \Big/ \gamma_b$	(26)
Oehlers and Johnson modified	$P_{Rd} = 3.2 f_u \frac{\pi d^2}{4} \left(\frac{E_{cm}}{E_{sc}}\right)^{0.4} \left(\frac{f_{ck}}{f_u}\right)^{0.35} \frac{1}{\gamma_V}$	(27)
Döinghaus modified	$P_{Rd} = (0.84 f_u (\pi d^2/4) + 1.63 f_{ck} d_{do} h_w)/\gamma_V$	(28)
	$P_{Rd} = 0.25 d^2 \sqrt{f_{ck} E_{cm}}/\gamma_V$	(29)
Konrad et al.	$P_{Rd} = \left[326 \frac{d_{do} h_w}{2} \left(\frac{f_{ck}}{30 \frac{N}{mm^2}}\right)^{2/3} + 220 d^2 \left(\frac{f_{ck}}{30 \frac{N}{mm^2}}\right)^{2/3} \left(\frac{f_u}{500 \frac{N}{mm^2}}\right)^{1/2}\right] \frac{1}{\gamma_V}$	(30)
	$P_{Rd} = \left[313 \frac{d_{do} h_w}{2} \left(\frac{f_{ck}}{30 \frac{N}{mm^2}}\right)^{2/3} + 240 d^2 \left(\frac{f_u}{500 \frac{N}{mm^2}}\right)\right] \frac{1}{\gamma_V}$	(31)
Hanswille and Porsch	$P_{Rd} = 0.83 f_u (\pi d^2/4)/\gamma_V$	(32)
	$P_{Rd} = 0.245 \alpha d^2 \sqrt{f_{ck} E_{cm}}/\gamma_V$	(33)
Pallarés and Hajjar #1	$P_n = 17 \frac{\pi d^2}{4} (f_c')^{0.45} (E_c)^{0.04} \leq C_v \frac{\pi d^2}{4} f_u$	(34)
Pallarés and Hajjar #2	$P_n = 6.2 \frac{\pi d^2}{4} (f_c' E_c)^{0.2} \leq C_v \frac{\pi d^2}{4} f_u$	(35)
Pallarés and Hajjar #3	$P_n = 18 \frac{\pi d^2}{4} (f_c')^{0.5} (h)^{0.2} \leq C_v \frac{\pi d^2}{4} f_u$	(36)
Pallarés and Hajjar #4	$P_n = 9 \lambda (f_c')^{0.5} (d)^{1.4} (h)^{0.6} \leq C_v \frac{\pi d^2}{4} f_u$	(37)

Notes:
1. $\alpha = 0.2\,(h/d + 1)$ for $3 \leq h/d \leq 4$ and $\alpha = 1$ for $h/d > 4$.
2. $\eta = 1.25 - 3d/337$.
3. $f_u \leq 500$ MPa in Eqs. (18), (20), (23), (27), and (28); $f_u \leq 620$ MPa in Eq. (32); and $f_u \leq 740$ MPa in Eqs. (30) and (31).
4. γ_V=1.25 and $\gamma_b = 1.3$.
5. E_{sc} is the modulus of elasticity of the stud material.
6. λ is a factor taken as 0.75, 0.85, and 1 for all-lightweight, sand-lightweight, and normal weight concrete, respectively.
7. Units: kips, inches in Eqs. (22), and (34)-(37); N, mm in all other equations.

The P_{Rk} and P_n values computed with the proposed

and existing models were compared with the P_{em} values from the test databases. Separate comparisons were conducted for the NWC, LWC, and combined NWC and LWC data. The following metrics were used to evaluate the model performance: root-mean-square error (*RMSE*) (Eq. (38)), mean absolute error (*MAE*) (Eq. (39)), mean absolute percentage error (*MAPE*) (Eq. (40)), and the coefficient of determination (R^2) (Eq. (41)). The minimum, maximum, mean, and coefficient of variation values of the test-to-prediction ratios were also determined for each model.

$$RMSE = \sqrt{\frac{1}{n}\sum_{\mathrm{i}=1}^{\mathrm{n}}(y-x)^2} \tag{38}$$

$$MAE = \frac{1}{n}\sum_{\mathrm{i}=1}^{\mathrm{n}}|y-x| \tag{39}$$

$$MAPE = \frac{100}{n}\sum_{\mathrm{i}=1}^{\mathrm{n}}\left|\frac{y-x}{y}\right| \tag{40}$$

$$R^2 = \left[\frac{\sum_{\mathrm{i}=1}^{\mathrm{n}}(x-\bar{x})(y-\bar{y})}{\sqrt{\sum_{\mathrm{i}=1}^{\mathrm{n}}(x-\bar{x})^2\sum_{\mathrm{i}=1}^{\mathrm{n}}(y-\bar{y})^2}}\right]^2 \tag{41}$$

where n is the number of samples, y is the experimental resistance value (P_{em}), x is the predicted resistance value (P_{Rk} or P_{n}), $\bar{x}$ and $\bar{y}$ are the means of the x and y values.

Tables 6 and 7 summarize the performance metrics for each model. For NWC, the prediction accuracy of the developed models exceeded the accuracy of most of the existing models. Only the JSCE and Pallarés and Hajjar #4 models showed slightly better performance metrics than the developed design and nominal resistance models, respectively.

For LWC, the proposed models demonstrate better prediction accuracies than the existing models. The mean values of the test-to-prediction ratios for the JSCE (2007) and Konrad et al. (2020) models were smaller than one, which indicates that these models produce unconservative results for LWC (this may be because LWC was not considered within the development of these two models). For the combined NWC and LWC data, only the JSCE (2007) model shows some performance metrics better than those for the proposed models. However, the mean value of the test-to-prediction ratio for the JSCE (2007) model is very close to unity (1.057), which makes its compliance with the reliability requirements questionable.

It should be noted that all existing models, except Oehlers and Johnson modified, consist of two equations: for the stud material and concrete strengths. For high-strength concrete, the shear resistance is limited by the stud strength. Conversely, the developed models consist of one equation with no upper limit. They show good agreement with the test results and allow for a continuous moderate stud resistance increase when concrete strength increases. This feature of the proposed equations should result in higher shear resistance values, whilst still delivering the required reliability index.

Fig. 10 presents test-to-prediction ratio distributions for the design resistance models for the combined NWC and LWC data. The design shear resistance values, P_{Rd}, were computed using the nominal values of the test database variables. The distributions illustrate the safety and conservatism of the predictions by each design model. The developed design models produce safe predictions with the lowest CoV values of the test-to-prediction ratios.

Table 6 Performance metrics of the existing and proposed design resistance models

Model	RMSE (kN)	MAE (kN)	MAPE (%)	R^2	Test-to-Prediction Ratio			
					min	max	mean	CoV
NWC								
EC4	51.8	43.7	23.8	0.765	0.832	1.862	1.329	0.148
EC4M	41.4	36.5	21.6	0.838	0.773	1.810	1.290	0.153
AS/NZS	40.7	33.6	18.7	0.780	0.666	1.671	1.229	0.142
JSCE	24.1	19.1	11.1	0.880	0.802	1.593	1.101	0.116
OJM	52.1	46.6	27.9	0.802	0.845	1.936	1.419	0.167
DM	38.7	34.0	20.5	0.853	0.773	1.810	1.272	0.157
K	35.0	29.6	16.4	0.854	0.690	1.552	1.191	0.120
HP	46.6	41.8	24.4	0.873	0.937	1.847	1.346	0.140
SRD1	27.3	22.2	12.8	0.856	0.774	1.557	1.119	0.130
SRD2	27.3	22.2	12.8	0.856	0.774	1.557	1.119	0.130
LWC								
EC4	23.5	22.1	26.7	0.812	0.946	2.571	1.389	0.152
EC4M	31.7	30.4	36.4	0.791	1.015	2.982	1.608	0.157
AS/NZS	23.5	22.1	26.7	0.791	0.875	2.571	1.386	0.157
JSCE	12.2	9.2	11.6	0.762	0.705	1.256	0.938	0.116
OJM	34.7	33.7	40.3	0.858	1.139	2.977	1.700	0.131
DM	31.7	30.4	36.4	0.791	1.015	2.982	1.608	0.157
K	10.6	8.0	10.3	0.842	0.592	1.813	0.989	0.152
HP	32.8	31.6	38.0	0.812	1.119	3.043	1.645	0.152
SRD1	12.0	9.9	11.8	0.817	0.840	1.708	1.128	0.120
SRD2	11.7	9.6	11.4	0.815	0.869	1.668	1.120	0.118
NWC & LWC								
EC4	45.9	37.9	24.6	0.858	0.832	2.571	1.345	0.150
EC4M	39.0	34.9	25.6	0.902	0.773	2.982	1.376	0.186
AS/NZS	36.9	30.5	20.9	0.868	0.666	2.571	1.271	0.157
JSCE	21.5	16.5	11.2	0.907	0.705	1.593	1.057	0.135
OJM	48.0	43.1	31.2	0.876	0.845	2.977	1.495	0.177
DM	36.9	33.0	24.8	0.910	0.773	2.982	1.363	0.192
K	30.4	23.7	14.7	0.883	0.592	1.813	1.136	0.150
HP	43.3	39.1	28.1	0.922	0.937	3.043	1.427	0.172
SRD1	24.1	18.9	12.6	0.911	0.774	1.708	1.122	0.127
SRD2	24.1	18.8	12.4	0.911	0.774	1.668	1.120	0.127

Notes: EC4 = Eurocode 4, EC4M = Eurocode 4 modified, AS/NZS = AS/NZS 2327, OJM = Oehlers and Johnson modified, DM = Döinghaus modified, K = Konrad et al., and HP = Hanswille and Porsch.

The mean values of the test-to-prediction ratios for the developed models are one of the lowest, indicating higher design resistances computed with the developed models. Only the JSCE (2007) model shows a smaller mean test-to-prediction ratio than the proposed models, but the compliance of this model with the reliability requirements for LWC is questionable, as was discussed previously.

Figs. 11-14 present P_{Rd}/d^2 and P_n/d^2 values computed with the proposed and existing design resistance models for NWC and LWC as functions of concrete strength for different stud tensile strengths, f_u, and the stud height-to-diameter ratios, h/d. The JSCE (2007) and Konrad et al. (2020) models were not included in Fig. 12 for LWC because they produced unconservative predictions for LWC, with the mean test-to-prediction ratios less than unity, as discussed previously.

Table 7 Performance metrics of the existing and proposed nominal resistance models

Model	RMSE (kN)	MAE (kN)	MAPE (%)	R^2	Test-to-Prediction Ratio			
					min	max	mean	CoV
				NWC				
AISC	50.4	41.9	22.2	0.689	0.687	1.934	1.281	0.174
PH1	34.9	29.9	17.8	0.839	0.772	1.617	1.214	0.142
PH2	54.7	48.2	27.0	0.786	0.803	1.892	1.388	0.135
PH3	27.1	22.7	13.8	0.871	0.763	1.510	1.143	0.133
PH4	21.6	16.0	9.3	0.855	0.713	1.451	1.023	0.123
SRN1	21.6	17.1	10.4	0.856	0.703	1.416	1.018	0.130
SRN2	21.6	17.1	10.4	0.856	0.703	1.416	1.018	0.130
				LWC				
AISC	9.8	8.0	10.5	0.847	0.646	1.898	1.071	0.144
PH1	10.8	8.8	11.1	0.810	0.641	1.847	1.092	0.131
PH2	18.2	16.7	20.5	0.831	0.777	2.026	1.267	0.127
PH3	10.9	8.1	9.9	0.801	0.658	1.786	1.028	0.140
PH4	12.8	10.8	12.8	0.687	0.786	1.590	1.085	0.144
SRN1	8.8	7.5	9.4	0.817	0.763	1.553	1.025	0.120
SRN2	8.9	7.6	9.5	0.815	0.782	1.502	1.008	0.118
				NWC & LWC				
AISC	43.4	32.7	19.0	0.792	0.646	1.934	1.224	0.186
PH1	30.3	24.2	16.0	0.879	0.641	1.847	1.181	0.147
PH2	47.6	39.6	25.2	0.858	0.777	2.026	1.355	0.139
PH3	23.8	18.7	12.7	0.899	0.658	1.786	1.112	0.142
PH4	19.6	14.6	10.2	0.908	0.713	1.590	1.040	0.133
SRN1	19.0	14.5	10.1	0.911	0.703	1.553	1.020	0.127
SRN2	19.0	14.6	10.2	0.910	0.703	1.502	1.015	0.127

Notes: AISC = AISC 360, PH1, PH2, PH3, and PH4 = Pallarés and Hajjar Eqs. (34), (35), (36), and (37), respectively.

Relatively wide ranges of the design and nominal resistance predictions by different models can be observed from Figs. 11-14. The shapes of the presented curves of the stud resistance normalized by d^2 for the proposed models generally correspond to the curves for the existing descriptive equations. They show a more significant increase in stud shear resistance as the concrete strength increases up to around 30 MPa and 40 MPa for NWC and LWC, respectively. For concretes beyond these values, an increase in concrete strength results in a more moderate increase in stud shear resistance.

As discussed previously, SRN1/SRD1 and SRN2/SRD2 models are identical for NWC, which is reflected by the graphs in Figs. 11 and 13. For LWC, the proposed models produce slightly different stud resistances. For h/d=3, SRN1/SRD1 models give greater stud resistances than those computed with SRN2/SRD2 models. When h/d increases, SRN2/SRD2 models produce higher stud resistances than SRN1/SRD1.

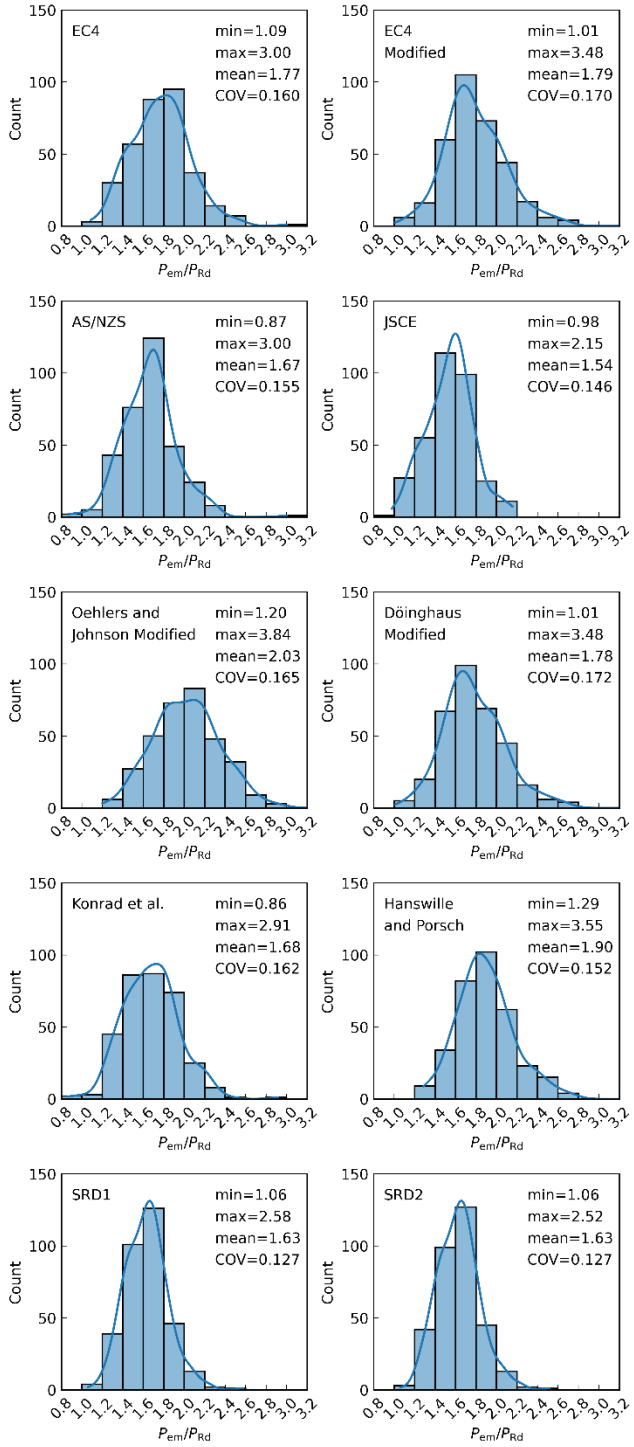

Fig. 10 Test-to-prediction ratio distributions using the existing and proposed design models for the combined NWC and LWC database

It can also be seen that the proposed models give higher stud shear resistances than the existing models for many

combinations of f_{ck}, f_u, and h/d, especially for high-strength concretes, high-strength studs, and studs with higher height-to-diameter ratios. The increased resistance for larger h/d ratios may be of particular interest to designers as many international standards do not directly exploit this effect within their provisions.

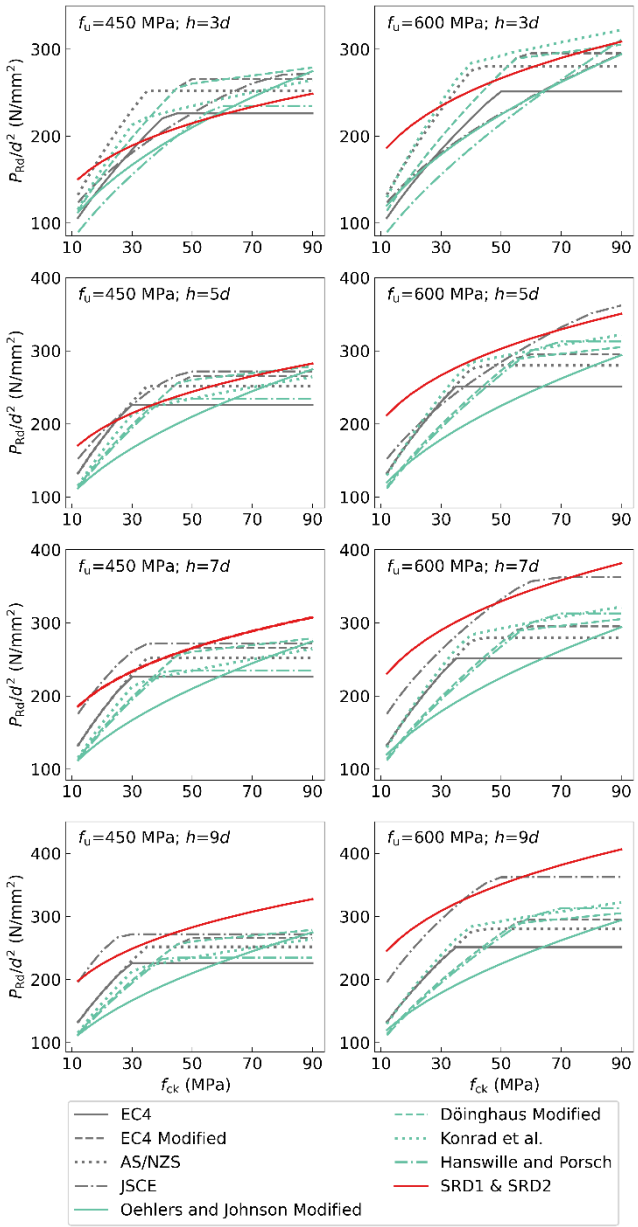

Fig. 11 Comparisons of design shear resistances of studs in NWC predicted by the existing and proposed models

From the present comparison, it is recommended that the SRD2 model would be a worthy candidate for use in design in that, not only does it deliver the target reliability for both NWC and LWC, it also provides much more competitive design resistances than many of the existing design models.

8. Conclusions

This paper has presented new models for predicting the

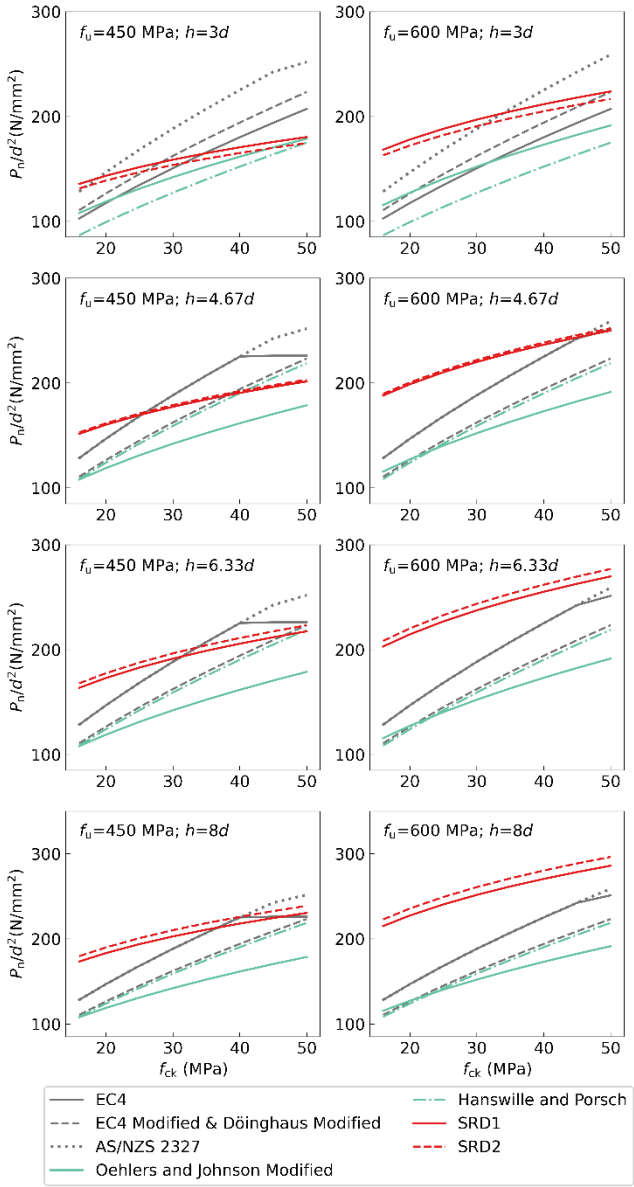

Fig. 12 Comparisons of design shear resistances of studs in LWC (1800 kg/m^3) predicted by the existing and proposed models

shear resistance of headed studs in solid concrete slabs made of normal and lightweight concrete. The models were obtained by applying symbolic regression with genetic programming to experimental data with 242 normal weight concrete samples and 90 lightweight concrete samples. Each proposed model consists of one relatively simple equation, which is a nonlinear function of stud and concrete strengths, stud shank diameter, and stud height-to-diameter ratio.

The test-to-prediction ratios for the developed models showed practically no correlation with the secant modulus of elasticity of concrete and concrete density, indicating that the stud shear resistance is insensitive to these parameters based on the available test data. This finding is important as, following its introduction within the empirical equations proposed by Ollgaard et al. (1971), the secant modulus of elasticity is widely used within the design models for stud resistance in many international standards.

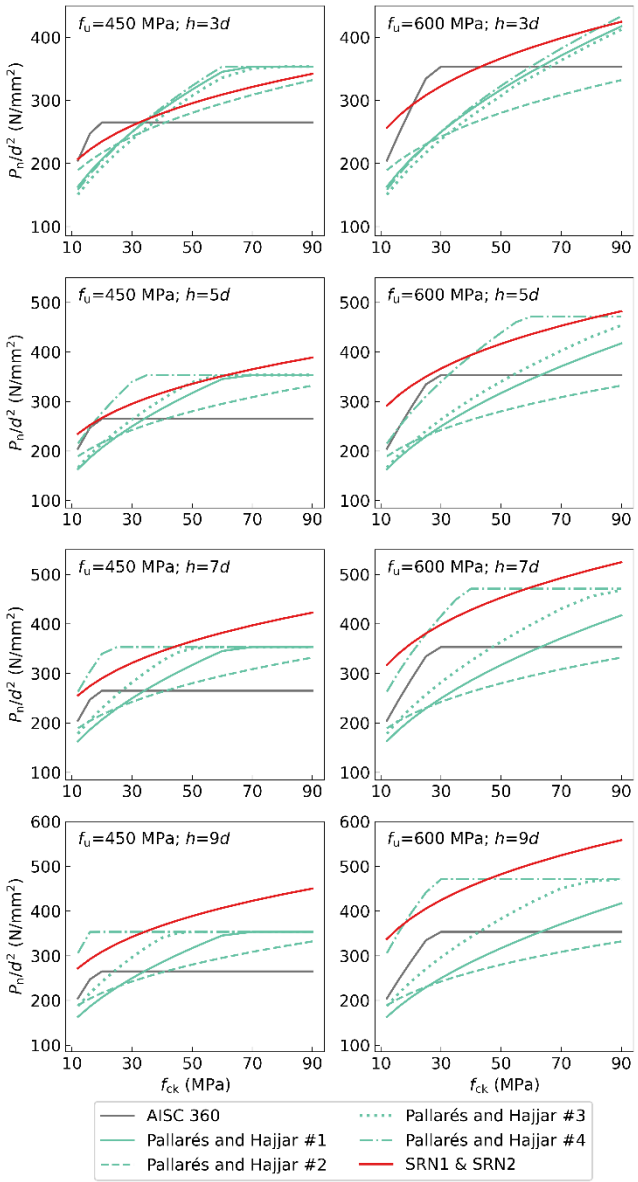

Fig. 13 Comparisons of nominal shear resistances of studs in NWC predicted by the existing and proposed models

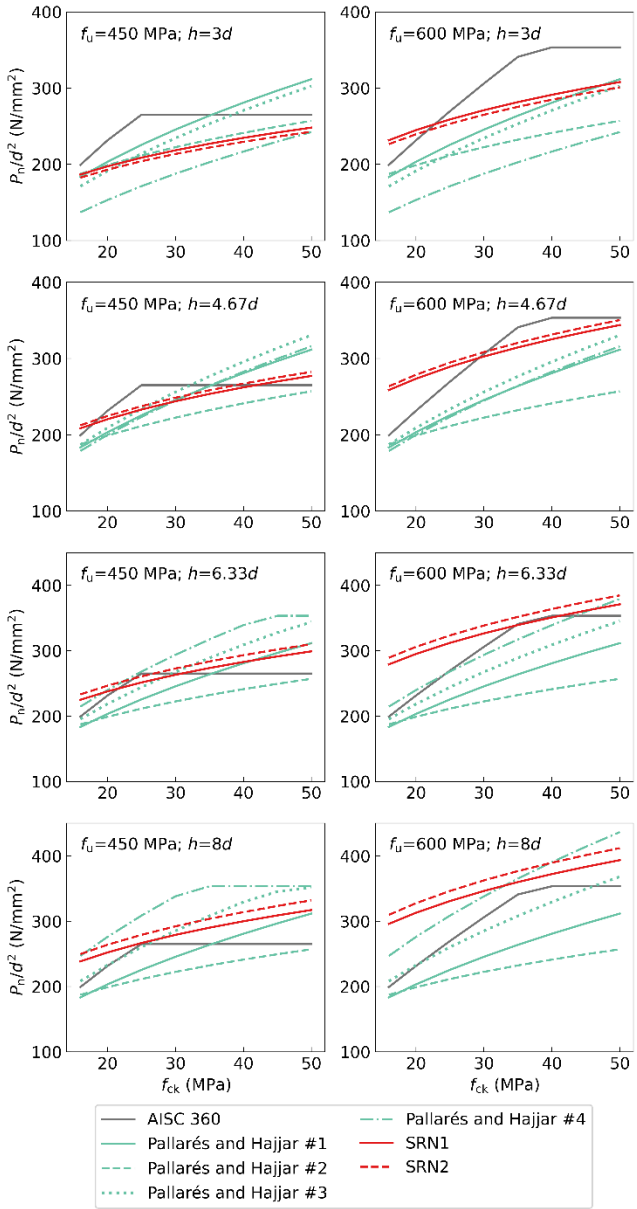

Fig. 14 Comparisons of nominal shear resistances of studs in LWC (1800 kg/m^3) predicted by the existing and proposed models

The proposed models produce higher shear resistances for studs with larger height-to-diameter ratios, which is not currently a feature of the provisions given in international standards.

The obtained GPSR-based nominal strength equations were subsequently refined for use in design from reliability analyses to provide the reliability level required by the Eurocodes with the partial factor of 1.25. Resistance factors for the proposed models were also determined in accordance with US design practice. The nominal and design stud resistances produced by the developed models were compared with those predicted by 13 existing models. The proposed models showed the highest accuracy in predicting the nominal and design shear resistances for the combined normal and lightweight concrete data than the existing descriptive equations. The developed models produce higher stud shear resistance than the existing models for many combinations of f_{ck}, f_u, and h/d, especially for high-strength concretes, high-strength studs, and studs with higher height-to-diameter ratios. The latter finding may be of particular interest to designers as many international standards do not directly exploit this effect within their provisions. From the present study, it is recommended that the SRD2 model should be considered as a worthy candidate for possible implementation within future design standards on steel-concrete composite construction.

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