Physics-Informed Neural Network Modeling of Soliton Pulses in Optical Communication Systems

Joshua Uduagbomen
School of Engineering
University of Warwick
Coventry, United Kingdom
Joshua.Uduagbomen@warwick.ac.uk

Subhash Lakshminarayana
School of Engineering
University of Warwick
Coventry, United Kingdom
Subhash.Lakshminarayana@warwick.ac.uk

Mark S. Leeson
School of Engineering
University of Warwick
Coventry, United Kingdom
Mark.Leeson@warwick.ac.uk

Tianhua Xu
School of Engineering
University of Warwick
Coventry, United Kingdom
Tianhua.Xu@warwick.ac.uk

***Abstract*—The nonlinear Schrödinger equation which models the pulse propagation in an optical fiber is solved using a physics-informed neural network for the case of soliton propagation. The prediction accuracy, measured against the exact solution (computed using the Runge-Kutta method), is found to be 2.223×10^{-3}.**

***Index Terms*—peregrine soliton, optical fiber communication, physics-informed neural network**

I. Introduction

For the transmission of very large amount of data over long distances and wide bandwidths with low latency, optical communications presently stand out and remains unchallenged as the key enabling technology and backbone of global modern telecommunications networks. Based on this context, the modeling of optical fibers is of high significance to evaluate the performance of long-haul wideband optical communication systems and networks, since it is extremely expensive and intractable to study such large-scale optical systems or networks in lab and field experiments. However, the simulation of the mix of dispersion and fiber nonlinearities, which was generally implemented using split-step Fourier method (SSFM), can result in a demanding computational procedure, especially when long transmission distances and wide bandwidths are considered [1], [2]. Meanwhile, neural networks (NNs) can be used to investigate the performance of an optical fiber communications system, and to realize this, we require numerical simulations in the forward propagation of optical pulses. Recent applications have seen deep learning being employed in optical communications for data fitting when given the SSFM generated input and output pulse pairs for training. However, much more can be achieved with NNs in nonlinear optics than just estimating the results from SSFM, which in itself has its own drawbacks [3]. The nonlinear Schrödinger equation (NLSE) can be solved directly using NNs evading the complexity of numerical methods.

II. Principle of PINNs

First we define a differential equation $u_t+\mathcal{N}[u]=0$, where u, u_t and $\mathcal{N}$ denote the solution, the derivative of the solution with respect to t and the nonlinear function respectively. Our objective is to solve the equation, and we adopt the PINN approach [4] where the solution and the nonlinear terms are approximated as the output of the NN as shown in Fig 1. To this end, we define a model $f := u_t + \mathcal{N}[u]$ and learn the shared parameters between the NNs, $u(t,x)$ and $f(t,x)$ by minimizing the mean squared error loss

$$MSE = \frac{1}{N_u}\sum_{i=1}^{N_u}|u({t_u}^i,{x_u}^i)-u^i|^2 + \frac{1}{N_f}\sum_{i=1}^{N_f}|f({t_f}^i,{x_f}^i)|^2, \tag{1}$$

where $\{{t_u}^i,{x_u}^i,u^i\}_{i=1}^{N_u}$ is the training data on u and $\{{t_f}^i,{x_f}^i\}$ are the collocation points for $f(t,x)$. The objective of the first loss term is to satisfy the network u and the second loss term is the physics-based regularisation term. This term is minimised to ensure the solution provided by the NN solves the underlying differential equations.

In the fibre-optics domain, the complex envelope of the optical field $A(z,\tau)$, can be represented in its rectangular form as $A(z,\tau)=u(z,\tau)+iv(z,\tau)$ [5], and the normalized NLSE can be written as $A_z+\frac{i}{2}A_{\tau\tau}-i|A|^2A=0$. This can be split into its real and imaginary parts as $f(u): u_z-\frac{1}{2}v_{\tau\tau}+(u^2+v^2)v$ and $g(v): v_z+\frac{1}{2}u_{\tau\tau}+(u^2+v^2)u$ respectively. Where τ, z, g and f is the time, normalized distance, the function governing $v(z,\tau)$ and the function governing $u(z,\tau)$ respectively. Resolving this satisfies the relation $NLSE(u,v)=f(u)+ig(v)=0$. A hyperbolic tangent activation function and a five-layer deep neural network with 100 neurons per layer has been used to represent the function $A(z,\tau)$. To constrain the NLSE, 20,000 discrete points have been sampled and these have been set as input along with 50 points of the initial pulse which are used to ensure that the initial pulse function approaches zero.

III. Results and Discussion

The peregrine soliton is the solution of the normalized NLSE given by [6]

$$i\frac{\partial A}{\partial z}+\frac{1}{2}\frac{\partial^2 A}{\partial^2\tau}+|A|^2A=0 \tag{2}$$

Its evolution has been investigated here using PINNs and the simulation results show the dynamical evolution of the pulse envelope under the impact of both chromatic dispersion and fiber nonlinearity. In Fig. 2, Fig. 3 and Fig. 4, the predicted solutions using PINN (green solid line) are compared with the exact solutions (purple dashed-line) obtained by solving the NLSE using the Runge-Kutta method with evolution starting from the initial pulse, $2sech(t)$. We have used this to verify

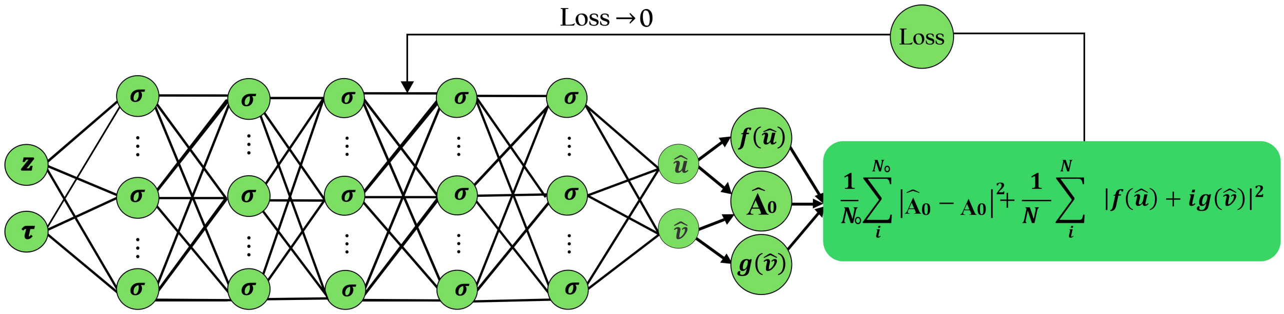

Fig. 1. The architecture of the PINN scheme showing the input layer, the hidden layer and the output layer. As part of the neural network, PINN encodes the governing equations [5].

the accuracy of the algorithm and the prediction error was measured as 2.223×10^{-3}. In Fig. 4, a strongly localized peak can be seen in the temporal profile and from the simulation results, two distinct phases can be observed. The first phase involves the rapid evolution of the pulse to approach a profile of an ultrashort compressed pulse followed by the return phase of broadening towards the initial state.

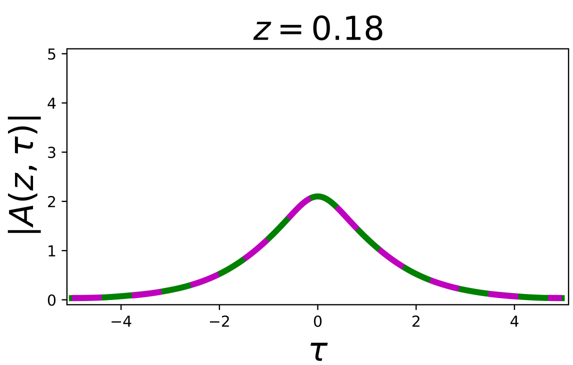

Fig. 2. Temporal profile showing the optical field at z = 0.18, a relatively short distance after z = 0

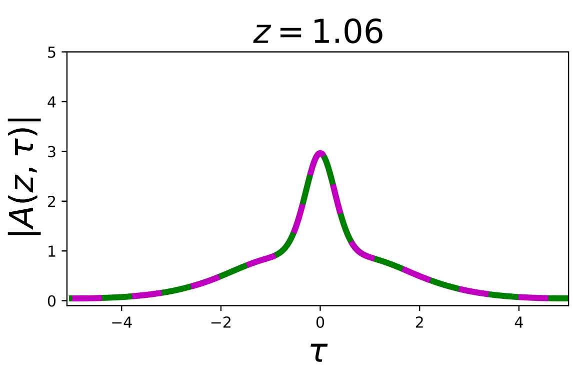

Fig. 3. Temporal profile showing the optical field when z = 1.06. Here the return phase of the pulse evolution after the point of maximum compression is captured

IV. Conclusion

A prediction accuracy of 2.223×10^{-3} was achieved after numerical simulations and this shows that PINNs are capable of accurately capturing and predicting the pulse propagation in

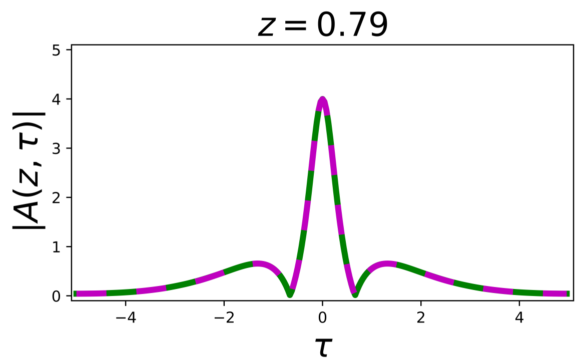

Fig. 4. Temporal profile showing the point of maximum compression during pulse evolution and the optical field when z = 0.79

an optical fiber and will ultimately be able to characterize other physical effects and model pulse propagation in wideband optical fiber communication systems.

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