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Air flow over a flapping bird's wing at low Reynolds numbers: discussions in a teaching demonstration

Hollis Williams*

School of Engineering, University of Warwick, Coventry CV4 7AL, United Kingdom

E-mail: Hollis.Williams@warwick.ac.uk

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Abstract
Bird wings are frequently modelled in the aerodynamics literature under the assumption that they are static aerofoils. Air flows over real bird wings can violate this assumption, both because of the topology of the wing and the fact that real wings typically undergo flapping and twisting motions during flight. There are many computational and numerical studies of air flows on flapping bird's wings at low Reynolds numbers, but relatively few experimental studies due to the difficulty of modelling a realistic flapping motion. In this article, the flow on a real flapping bird wing is visualised in a teaching demonstration using high-speed photography. It is found that the main features of the flow can be observed and that the demonstration is a teaching aid for assisting student understanding of principles of aerodynamics. The possibility of incorporating the demonstration into a standard teaching course is discussed, with relevant teaching objectives highlighted. In conclusion, it is recommended that the demonstration could be useful as part of a course on aerodynamics, with further follow-up demonstrations and studies suggested.

Keywords: fluid dynamics, flow visualisations, biomechanics

S Supplementary material for this article is available online

(Some figures may appear in colour only in the online journal)

*Author to whom any correspondence should be addressed.

1. Introduction

The aerodynamics of a conventional aerofoil is a standard part of the syllabus for many physics and engineering students [1, 2]. An interesting question which often occurs to many students is how closely the air flow over a real bird or insect wing is actually modelled by an aerofoil section, or whether the presence of feathers and other features significantly modifies the flow. Real wings are typically not used to study flows because of experimental difficulties. The most thorough experimental study which did use real bird wings was that of Withers, who studied a wide variety of real bird wings in a wind tunnel, finding that the pattern of air flow over bird wings is indeed consistent with standard aerodynamic theory for aeroplane wings at low Reynolds numbers [3]. However, a weakness of the approach of Withers which should be mentioned is that he considered wings as aerofoils which are kept fixed in place like an aeroplane wing, neglecting the flapping, twisting and beating motions which occur for real wings. Besides bird flight, it is also known that the flight of insects depends on their ability to generate aerodynamic forces by flapping their wings [4–6].

Another complicating factor is that flapping flight is characterized by extremely unsteady evolution of vortical structures close to the wing, which necessitates sophisticated methods to experimentally measure instantaneous loads and three-dimensional pressure fields [7]. In reality, because of these complicated types of motions, studies of flows around flapping wings are typically based on numerical solutions and sophisticated computational fluid dynamics (CFD) simulations. These simulations often involve a quasi-steady assumption, which states that the forces on the flapping wing are equal to those in steady motion at the same instantaneous velocity [8, 9]. Another common approach is to solve the Navier–Stokes equations on a grid which deforms dynamically to imitate the deformations of a flapping wing. Studies of this kind found that the leading vortex attached to a wing is stable during a downstroke but highly unstable during the upstroke such that it sheds continuously [10]. Other numerical works have shed new insight into optimality of bird wingstrokes with respect to power consumption and elucidated the mechanism by which rotation of the wing generates different amounts of lift depending on whether it occurs near the end of a stroke [11, 12].

Besides this, models have shown influences on flapping flight from damping of angular velocity by a flapping counter-torque, spanwise flexibility of the wing, and flapping frequency of the wing [13–15]. Finally, it is clear that the body of a flapping insect or bird can also influence the flow or generate additional vortices. For example, Wan *et al* showed that during free forward flight of a cicada two new vortices are generated by the body of the cicada, one of which interacts with vortices from the edge of the wing [16]. This aspect of the problem is neglected in this article since a single wing is considered detached from the body of the bird.

As evidenced by the above summary, this is clearly a technical subject with many subtle modifications to the flow. Nevertheless, it is possible to exhibit the main features of both steady and flapping motion in a simple demonstration experiment using a bird wing, a wind tunnel and smoke for flow visualisation. It is proposed that the demonstration provides some physical intuition and that it is also effective in a classroom environment, stimulating further interest in fluid dynamics. This study can be seen as a simpler, pedagogical version of the direct flow visualisations of flapping wings which exist in the literature. These other studies mostly rely on particle image velocimetry, whereby the air around the wing is seeded with particles which are illuminated with a thin focussed laser sheet (thickness typically between 1 and 3 mm) [17–19]. The demonstration described here does not involve use of seeded particles or lasers and instead uses only smoke, but at the price of only being able to observe the key features of the wake in a rather qualitative way. Nevertheless, the demonstration is simple enough that it could be repeated by students, some of whom may wish to investigate the more sophisticated PIV methods.

2. Materials and methods

A simple experiment is described which can be used to observe the main features of the flow over a flapping bird wing. The experiment requires a wind tunnel. Engineering departments at universities will generally have wind tunnels which can be booked in advance for demonstrations if they are not being used by undergraduate students. It may also be possible to ask a graduate student if they have some free time to show a demonstrator how to start the tunnel and then vary the wind speed. A useful addition to the experiment is to have markings on the wall to show angles, so that one can quantitatively vary the angle of object placed in a flow (shown in figure 1 as the blue markings on the black background). The experiment requires a real bird wing. In the experiment described here, a pheasant wing with a length of 37 cm was used (where the length of the wing is denoted by l in table 1). It is not always a viable option to use real bird wings in experiments but it may be possible to borrow them from the zoology department at a university [3]. The basic experimental set-up without smoke is shown in figure 1.

The second key part of the experiment is injection of smoke into a point in the tunnel, enabling one to visualise flow fields for flows over objects. A typical way is to have a metal wire stretched across the tunnel which is soaked in a smoke-generating oil. The oil generates non-toxic white smoke when a current is run through the wire. This smoke is then carried through the rest of the tunnel by the air flow. Various types of oil can be ordered which have been designed specifically for this purpose (they are usually blends of alcohol, glycerol and glycol). The final optional component in the demonstration is a video camera to make recordings of the flow. In the demonstration described in this article, a high-speed video camera has been used as shown in figure 1 (a Photron FASTCAM SA-X2 high-speed camera at frame rate of 12 500 fps). This device is far too expensive to be available in a high school department, but note that the main features of the flow are quite coarse and can be observed using the camera on a typical smartphone.

In the demonstration, the basic idea is that the class demonstrator or tutor holds a wing by hand and then move it up and down with a slight twisting motion to roughly imitate a wing flapping on a bird. Students could also consider an oscillating clamper which imitates the same type of motion, although this is difficult because several types of motion are coupled together. The procedure for the demonstration is as follows: the wire is prepared and coated in oil. The instructor then climbs into the tunnel with the wing and makes sure that the orientation of the wing is corrected to be recorded by video cameras which are in place. In figure 1, one can see that there is a large panel which can be removed so that a person can climb into the tunnel. This can then be put back into place when the flow is started in the tunnel. Note that the demonstrator does need to be in the tunnel during the demonstration if the wing is being flapped by hand. This should not pose any dangers, especially at the very low wind speeds used in experiment described here. The smoke is also non-toxic and should not be harmful if breathed in over a short period of time. It may however be necessary to fill out a risk assessment form before the experiment can go ahead.

An air flow is then started in the tunnel. Once the flow has begun, a student outside the tunnel injects smoke continuously into the tunnel (usually by clicking a button which runs a current through the wire and heats it). To begin, the wind tunnel is started at a low wind speed corresponding to $U = 1$ m s^{-1} and the wing is held flat at an angle of attack $\alpha = 0°$ (these quantities will be described more formally in the next section). Once all students are happy

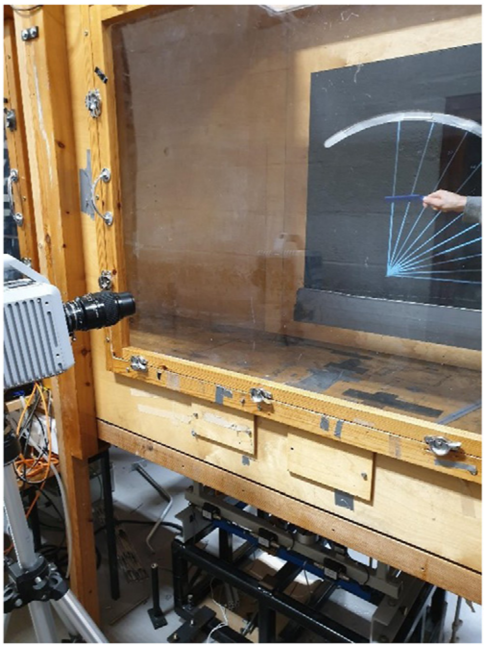

Figure 1. Image of the experimental wind tunnel set-up and high-speed camera. A piece of acrylic has been used as a reference. Note that the object is *not* clamped in place and that it is instead moved by hand to imitate a typical flapping-twisting motion.

Table 1. List of parameters used for the experiment.

Parameter	Value	Units
w	1.05	m
h	1.5	m
l	0.37	m
α	0, 15	deg
U	1	m s^{-1}
f	1.8	Hz

with this part of the demonstration and video recordings have been obtained, the wing is then manually flapped and twisted by hand to imitate the typical 'beating' motion which a flapping wing might undergo during the flight of a bird. This part might be difficult for a student to imitate, so it is advised that it be carried out by an instructor with knowledge of aerodynamics (specifically flapping flight). In the demonstration described in this article, a flapping frequency of around $f = 1.8$ Hz is used. The flapping frequency should be relatively slow so as to imitate a reasonable flapping frequency which might be observed in the natural world. It is assumed that most students would have intuition of what such a frequency would be from having seen birds in flight.

Once the demonstration has been completed, the smoke generator is switched off. The air flow is then also switched off and the panel is removed from the side so that the demonstrator

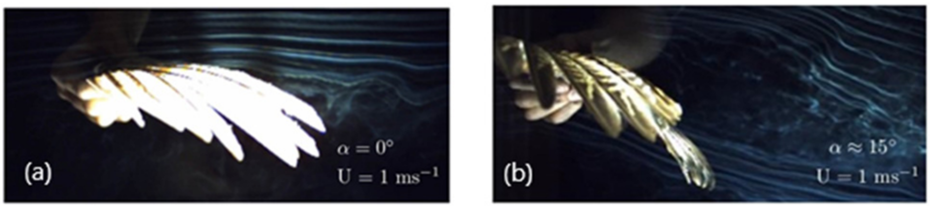

Figure 2. (a) Wing held flat in a low Reynolds number air flow with $\alpha = 0°$ and $U = 1$ m s^{-1}. (b) Wing held in the same flow at an angle with $\alpha \approx 15°$ and $U = 1$ m s^{-1}.

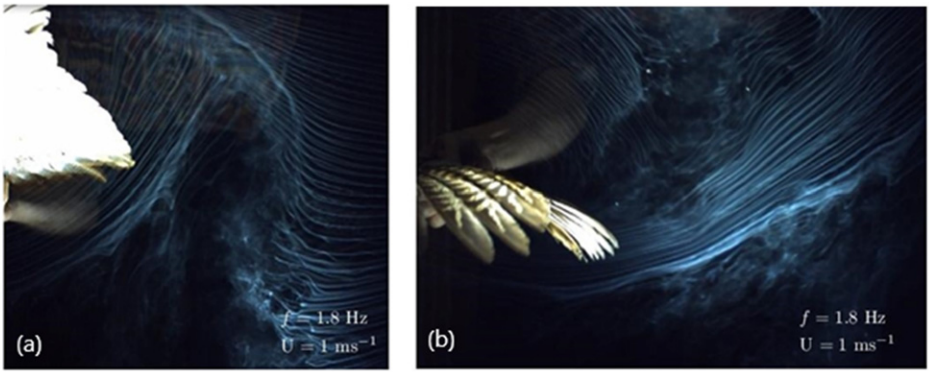

Figure 3. (a) Upstroke of a flapping wing in a low Reynolds number air flow, with $f = 1.8$ Hz and $U = 1$ m s^{-1}. (b) Downstroke of a flapping wing in the same flow, also with $f = 1.8$ Hz and $U = 1$ m s^{-1}.

can exit the tunnel. In table 1, the parameters used in the demonstration experiment are indicated, where w denotes the width of the tunnel, h is the height of the tunnel, l is the length of the wing, α is the angle of attack (to be defined shortly), U is the flow speed, and f is the flapping frequency during flapping flight. Students might like to vary these parameters to see how the structure of the flow is altered.

3. Results

Images are now provided which were obtained as a result of the experiment using the parameters detailed in table 1. In figure 2(a), the flow is shown when the wing is held flat in a low Reynolds number flow with angle $\alpha = 0°$ and $U = 1$ m s^{-1}. In figure 2(b), the modification to the flow is shown when the same wind speed is maintained, but the demonstrator now holds the wing at an angle. The results of the flapping motion in the same flow are shown in figure 3, where a flapping frequency of $f = 1.8$ Hz is imposed by the demonstrator. In figure 3(a), a typical upstroke is shown for the flapping motion, whereas figure 3(b) shows the following downstroke.

4. Discussion

It is proposed that the result of the demonstration could be used as a starting point for several possible teaching discussions regarding aerodynamics. Figure 2 can be used to summarise some basic points. Firstly, the flow can be altered significantly by changing the angle between the chord and the direction of the flow (this angle is known as the angle of attack, denoted by α). The angle of attack and the chord have been marked in the diagram in figure 4, where the chord is the line which connects the front leading edge with the trailing edge of the wing. Figure 4 also shows the directions of the lift force and drag force which act on a wing. The lift force L and drag force D acting on the wing for unit width are determined by the following equations:

$$L = C_L l \frac{\rho U^2}{2}, \tag{1a}$$

$$D = C_D l \frac{\rho U^2}{2}, \tag{1b}$$

where C_L is the lift coefficient, C_D is the drag coefficient, l is the chord length, ρ is the density of the fluid, and U is the flow speed. Both C_L and C_D are typically determined by experiment using a force balance device and depend on the object used (a force balance device can be seen in the bottom of figure 1). The value for the angle of attack at which the lift coefficient vanishes is called the zero lift angle. Measurements with a force balance device require mounting the wing in a fixed position, so they cannot be used for a flapping wing.

Note that in the right-hand image of figure 2, the air flow separates away from the upper edge of the wing when the angle of attack is increased, which causes a drop in the lift coefficient C_L. The value of α for which the lift coefficient reaches a maximum before starting to decrease is known as the stalling angle.

Secondly, students might assume that the low flow speed used in both figures 2 and 3 immediately implies that the flow has a low Reynolds number. The Reynolds number is defined to be the following dimensionless ratio

$$\mathrm{Re} = \frac{\rho U L}{\mu}, \tag{2}$$

where μ is the dynamic viscosity of the fluid and L is the width of the wind tunnel. In the demonstration, $U = 1\ \mathrm{m\,s^{-1}}$, $\rho = 1.225\ \mathrm{kg\,m^{-3}}$, $\mu = 1.84 \times 10^{-5}\ \mathrm{kg\,m^{-1}\,s^{-1}}$ and $l = 0.37$ m. Substituting in these values, it is found that $\mathrm{Re} \approx 6.9 \times 10^4$. This will be puzzling to many students, as this seems like an extremely large Reynolds number which is well above the threshold at which one expects to see turbulence in a flow in a pipe, and yet the flow over the wing shown in figure 2 is clearly not turbulent. By re-arranging the above equation, one sees that turbulence must appear even at lower flow speeds. The answer to this puzzle is that the turbulence is confined to the boundary layer, which appears at the walls and ceiling of the tunnel. The 'true' Reynolds number is actually determined by the length of this boundary layer and the main body of the flow is unperturbed. For this reason, the tunnel has to be very large so that there is no interaction with the boundary layer. As a rough rule of thumb, one might expect both the width and height of the tunnel to be four times greater than the characteristic length scale of the object in the flow for wall effects to be negligible. In this demonstration, the height of the tunnel is 150 cm and the width is 105 cm, which is reasonably close to the limit given by the rule of thumb as long as the wing is kept in the middle of the tunnel.

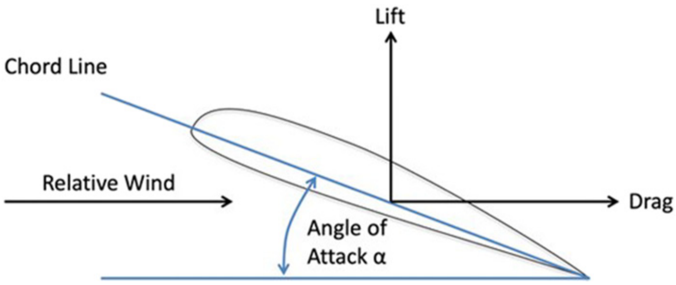

Figure 4. Diagram indicating the angle of attack α, lift force L, and drag force D. Note that the lift force is perpendicular to the drag force. Reproduced with permission from [20]. Credit: James Albright.

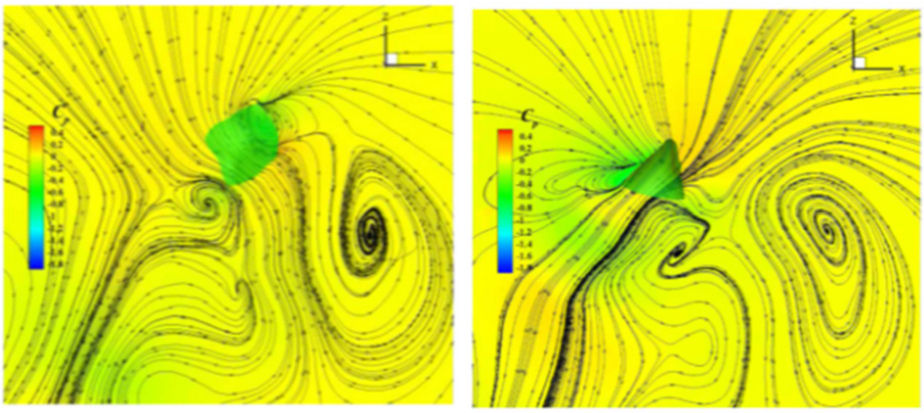

Figure 5. Contours of pressure coefficient distribution and streamlines on a plane near the middle of the initial wingspan during the beginning of a downstroke (left) and an upstroke (right). Reproduced from [10]. CC BY 4.0.

Thirdly, both figures 2 and 3 give a good opportunity to explain the related concepts of streaklines and streamlines. One could take this opportunity to point out to students that the smoke lines which one sees in the flow visualisation are streaklines, and not streamlines, as they might initially guess. A streakline is a line formed by a series of fluid particles which pass through a point in the stream one after another, whereas a streamline is quite different, being the curve such that the tangent at each point of the curve is in the direction of flow at that point. Nevertheless, the two are the same if the flow does not vary over time. In figure 5, the pressure coefficient distribution and streamlines for a flapping bird wing during the initial stage of a downstroke (left) and an upstroke (right) are reproduced (these were obtained using numerical simulations).

The pressure coefficient C_p shown in figure 5 is obtained by normalizing the pressure difference according to

$$C_p = \frac{p - p_{\mathrm{ref}}}{\frac{1}{2}\rho_{\mathrm{air}} U_{\mathrm{ref}}^2}, \tag{3}$$

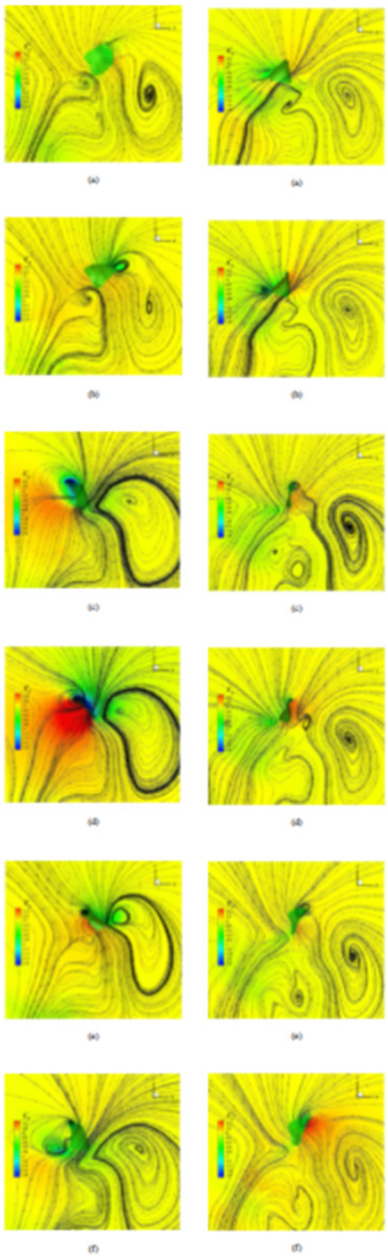

Figure 6. Contours of pressure coefficient distribution and streamlines on a plane near the middle of the initial wingspan during a downstroke (left) and an upstroke (right). Reproduced from [10]. CC BY 4.0.

where p is the static pressure, p_{ref} is the reference pressure (given by 1.013 25 Pa) and U_{ref} is the reference velocity [10].

Although the demonstration with smoke visualisation is qualitative, one can compare figure 3 with the numerical results shown in figure 5. In order for the complete motion of the downstroke and upstroke in the experiment to be visible, the full video footage has been included in video 2 of the supplementary materials (https://stacks.iop.org/EJP/43/055801/mmedia). For completeness, the full set of flow fields for each stage of the stroke in reference has also been reproduced, where the three-dimensional wing model is plotted in green [10]. In common with the experimental demonstration in figure 5, one sees that at the beginning of the downstroke a leading edge vortex forms which moves backward along the chord before separating away from the trailing edge. On the other hand, the upstroke has a smaller leading edge vortex which sheds continuously. The CFD results describe additional vortex structures during downstroke and upstroke which are not easily observable in the experimental demonstration. These vortices are visualised using the non-dimensional Q-criterion, where a positive value indicates the presence of vortices. In [10], this criterion is normalized to account for the three-dimensional wing model:

$$Q^* = \frac{Q}{(U_{\text{ref}}/\overline{c}_m)^2}, \tag{4}$$

where $\overline{c}_m$ is the mean chord length [21]. Note also that the flow imposed on the flapping wing is distinct from the flow generated by the wing, since the vortex structures around the wing have an effect on the aerodynamics of the wing (figure 6)..

Finally, research has shown that increasing the frequency of the wingbeats (known as flapping frequency f) can increase the maximum value of the ratio of the lift and drag coefficients at high altitudes [22]. A higher value for this ratio means that the wing produces lift with a lower drag force, hence the bird expends less energy. Video 2 shows a series of two consecutive wingstrokes. The two strokes are completed in a time period of 1.09 s and the frequency can be computed as

$$f = \frac{1}{T}, \tag{5}$$

where T is the period of the cycle. For the strokes shown in video 2, this gives a frequency $f = 1.8$ Hz. The frequencies in [22] are somewhat higher (4 and 5 Hz) due to air being hypodense and hypobaric at high altitude, so the flapping frequency shown in figure 3 would have an adverse effect on the aerodynamics of the wing at greater altitudes. However, the flows in [22] involve high speeds between 14 and 17 m s^{-1} and altitude changes which influence the density and viscosity of the air. The flow in this case is generally turbulent and is modelled numerically by the Reynolds-averaged Navier–Stokes equation for compressible flows, where the compressible momentum Navier–Stokes equation can be expressed as

$$\begin{aligned} \frac{Du}{Dt} &= \rho\left(\frac{\partial u}{\partial t} + u\cdot\nabla u\right) \\ &= -\nabla\overline{p} + \nabla\cdot\left\{\mu\left(\nabla u + (\nabla u)^{\mathrm{T}} - \frac{2}{3}(\nabla\cdot u)I\right)\right\} + \rho g, \end{aligned} \tag{6}$$

where g denotes the acceleration due to gravity, $\nabla\cdot$ is the divergence, p is the pressure, and an overline denotes time-averaging. Along with this, one has the usual continuity equation after assuming conversation of mass:

$$\frac{\partial\rho}{\partial t} + \nabla\cdot(\rho u) = 0. \tag{7}$$

Table 2. Summary of previous results.

Reference	Visualisation method	Type of wing	Live specimen?	Information obtained
This article	Smoke wire	Pheasant	No	Images of wake structure
Willmott *et al* [23]	Smoke wire	Hawkmoth	Yes	Images of wake structure
Bomphrey *et al* [18]	Smoke wire and 2D digital PIV	Hawkmoth	Yes	Wake structure, velocity and vorticity profiles
		Hawkmoth	Yes	
del Estal Herrero *et al* [19]	Large-scale PIV	Mechanical	No	Velocity and vorticity profiles

Leaving aside this difference, it is clear that the frequency used in figure 3 is of the right order of magnitude to represent a reasonably flapping frequency which would be seen in Nature, so no vortices are being created due to an unrealistic wingbeat frequency which would not be present in a real flapping flight.

At high altitudes, there is also a similar effect whereby increasing the amplitude from 50 to 60 cm was found to cause a large increase in the average lift force. The drag force is, however, increased by a similar amount [22]. It is harder to determine the amplitude of the wingstrokes seen in video 2 because a twisting motion is introduced alongside the beating motion, but the amplitude of the cycles looks to be of the same magnitude and so there are no modifications of the flow due to an unnaturally large stroke amplitude. Also note that although smoke flows over flapping wings have previously been performed in the literature, these involve a delicate experimental set-up which requires tethering of a live hawkmoth in the tunnel with stainless steel tubes [23]. Table 2 below summarises some of the major previous contributions and the contribution of this article. A review of previous results can be found at [17].

5. Teaching style

The demonstration shown in this article is primarily pedagogical, so the teaching benefits of the exercise are described here. Although the demonstration is qualitative, one has the main highlight that using simple equipment the experiment is able to re-create some of the qualitative findings of complicated CFD simulations described in [10] (at least regarding the topology of the wake). The other students who were present during the demonstration were somewhat sceptical that it might be possible to distinguish any vortex structures being shed from the wing during a flapping motion, so it was surprising that the main vortices in section 3 can be quite easily distinguished from the high-speed footage.

The first learning objective of the experiment is to motivate and demonstrate key concepts of aerodynamics which are necessary especially for those students who wish to undertake further study in engineering. This experiment illustrates many textbook concepts from fluid dynamics and aerodynamics which might otherwise seem quite dry, including angle of attack, streaklines and streamlines, the concept of a flow visualisation, and the Reynolds number (which also helps to motivate the general idea of a dimensional number which can be used to characterise a fluid system). A second more general objective is to show that idealised mathematical models can successfully capture complicated aspects of real world phenomena. A student might study diagrams of aerofoils in textbooks and wonder if the flow around such a simplified object gives a good approximation of the low Reynolds number flow around a real bird wing which has many complicated modifications such as feathers. One can see that the visualisation of the flow in figure 1(a) matches diagrams of flows around aerofoils in textbooks.

However, a related point is that these simplified models can break down quite quickly when one tries to model phenomena which are closer to those which are seen in the real world. Again, this experiment provides a good demonstration of this point, since real flight in the natural world involves flapping motion, and one sees that initiating a flapping motion modifies the flow field in a way which is very complicated and is typically studied used numerics [10]. Nevertheless, a simple experiment such as the one described here can still capture some of the key qualitative phenomena which are observed in the numerical simulations. Finally, it is certainly desirable to get students more involved in and thinking about experiments. This demonstration meets this objective, since students can assist in the demonstration, offer suggestions for improvements, learn how to use video analysis software, repeat the experiment with different parameters, and also think of their own experiments which they can carry out in the wind tunnel. A typical problem in high school and undergraduate physics course is that theory features prominently, whereas lab demonstrations are few and far between. The demonstration proposed here on the other hand should be relatively easy to organise whilst still being close to experiments which are done in real academic research.

Regarding teaching style and preparation, it is suggested that the demonstration be placed in the middle of an introductory course on fluid dynamics or mathematical methods in physics. As emphasised in this article, one of the main objectives is to motivate various basic concepts in aerodynamics (including Reynolds number, boundary layers, laminar flow, streamlines and angle of attack). These concepts can seem dry and academic when introduced in a teaching course, so it is beneficial to show them 'in action'. The students who were present during the demonstration agreed that the smoke visualisation helped to make classic fluid dynamics concepts more concrete and practical. Another objective is to use the flow over a flapping wing as an example of a complicated flow which more closely matches typical flows which one might find in the real world. The flow over a flapping wing is very complicated and is usually modelled numerically, but certain key features can be observed in a simple demonstration which qualitatively match the results from sophisticated CFD simulations.

There are also various teaching aids which can be used in the delivery of the demonstration. One particularly useful teaching aid used in the demonstration described in this article was the presence of the blue markings on the side of the wind tunnel which enable a quantitative determination of the angle of attack. To aid with visualisations, these markings could also be made fluorescent. This can also be combined with video analysis software to make more accurate determinations. If the demonstrator does not wish to study flapping motion and only wishes to motivate the usual low Reynolds number flow on aerofoils, then regular plastic aerofoils can also be used as a teaching aid to be compared with the real bird wing. In this case, both the wing and the aerofoil can be clamped into place. In this case, since the wing is clamped, there is no need for the demonstrator to climb into the tunnel. The other main teaching aid is the use of a smartphone or a high speed camera to make recordings of the flow. These are particularly useful as they can be played in the classroom by an instructor so that features of the flow can be analysed at leisure and compared with results obtained by more sophisticated analysis in the research literature [10]. Students could also make their own recordings using different parameters and then make a comparison in class with other videos.

The two main problems with preparing the demonstration are access to a wind tunnel and access to a high-speed video camera. Access to a wind tunnel is admittedly a problem, since wind tunnels at universities are typically heavily booked or already being used for a variety of undergraduate and postgraduate projects. It may be helpful to book several weeks in advance when asking for a demonstration, since the wire for the smoke set-up needs to be installed.

It may also help to request the demonstration outside of term time when the tunnel is not in such heavy use by university students. To maximise the impact of the demonstration, the tutor could ask at the beginning of the course if anyone might be interested in a trip to a wind tunnel halfway through term to get an idea of numbers of students attending. Since these factors might make access to a wind tunnel difficult or impossible, videos 1 and 2 have been included for the benefit of readers who are high school students, which show the complete flow visualisation used for figures 2 and 3.

Besides access to the tunnel, it is also required that someone be present who knows how to operate the tunnel and introduce the smoke for the flow visualisation. High school teachers could contact PhD students who are using the tunnel for thesis work and ask if they might have some free time in their schedule to operate the tunnel for the purpose of a demonstration. The high speed camera allows for a very nice visualisation of the flow, but it is not strictly necessary to observe the main features of the flow, which can be discerned with the naked eye. If students have access to a modern smartphone, these can usually record 400 frames per second.

6. Conclusion

In conclusion, a simple demonstration has been proposed which enables visualisation some of the features of air flow over a flapping-twisting bird wing and the associated wake structure. This type of flow has been intensively studied due to its importance as a realistic flow which is ubiquitous amongst insects and birds in Nature, so even simple qualitative visualisations of flapping flow are of value if they can provide some physical intuition about the flow. Given that there are several difficulties involved in visualising a flow over such an object, it is not obvious that such a simple set-up with smoke could elucidate any of the main features of the wake topology when a bird wing is artificially flapped by hand.

A natural next step which is missing in the article is to take the demonstration slightly further by changing variables and observing the changes to the wake structure. These include but are not restricted to flow speed and flapping frequency. Students could also try to identify the effect of wing shape on the flow by starting with simple shapes and then making incremental modifications which make the aerofoil somewhat closer to the geometry of a bird wing. Templates of three-dimensional objects can be made with relative ease using CAD software such as Fusion 360 which can then be exported to a 3D printer. This in itself would be a useful exercise for those students who wish to study engineering further, where CAD skills are highly in demand. 3D printers are usually available in engineering departments and it may be possible to book sessions using these. Elasticity can be controlled in 3D printers, so it may be possible to create wings with similar flexibility and elasticity properties to real bird wings. These can either be flapped by hand as in this article or attached to oscillating clamps and made to flap according to specified values.

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ORCID iDs

Hollis Williams https://orcid.org/0000-0003-3292-602X

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