Modelling hydraulic fracture propagation in heterogeneous porous media

Modelling Hydraulic Fracture Propagation in Heterogeneous Porous Media

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Abstract

Predicting propagation of fluid-driven fractures in heterogeneous porous materials is critical for design of engineering operations to sustainably use underground energy resources. In this paper, a computational model is presented using the phase-field theory and in the context of finite strains kinematics. The framework is built upon the use of variational principle to minimise the energy of the fracturing heterogeneous porous system. The material properties are distributed statistically over the domain to consider the heterogeneity of the porous material, and the nonlinearity of flow generated within the cracked zone is modelled by considering the effect of inertial flow. This numerical framework is verified using a benchmark example, and its application is tested by simulating a pressurised cavity in a heterogeneous reservoir to identify the influential parameters in the propagation path, including the degree of heterogeneity in the mechanical properties of the porous media and flow characteristics.

Key words: *nonlinear finite element method; phase-field fracture; heterogeneous porous media*

1. Introduction

It is well understood from experimental studies on the hydraulic fracturing that the propagation of pressurised cracks is the result of interaction between several processes, namely viscous flow within cracks, fluid leak-off into the porous rock, and surface creation in solid skeleton [1]. The mathematical models have been developed to formulate the behaviour of propagating fractures in porous media during years, although the necessary assumptions have been considered for the sake of simplification such as impermeable elastic medium, or non-deforming cracking body. The numerical methods paved the way to include the complexities of the real-world problem of fluid-driven fracturing in porous media. One of the most versatile methods for modelling cracks in continuous media is the phase-field theory which is based on the Griffith's theory and the variational calculus to minimise the energy of a fracturing body by the creation of new surfaces within the volume [2].

This study is dedicated to the simulation of the propagation of fluid-driven fractures in heterogeneous nonlinear settings using the phase-field modelling. This method provides the availability to predict the cracking path in heterogeneous materials in a straightforward manner [3, 4]. The effect of heterogeneity in both mechanical and hydraulic properties of porous materials has been recognized as an influential parameter in the crack pattern and the initiation of cracks [5]. The proposed mathematical framework is capable of modelling the non-Darcy fluid regime within the fracture network which has been proved as an important factor in modelling the fractured wells subjected to high fluid injection rate. The nonlinear hydromechanical behaviour is modelled based on the macroscopic approach to the theory of porous media, and the phase-field theory is applied with the use of a degradation function [6] affecting the effective stresses in the deforming solid phase to model fractures. An automatic mesh refinement technique is also implemented to increase the stability and accuracy of the prediction of the hydraulic fracture pattern in heterogeneous domains, to which the material properties are assigned spatially using Weibull distribution.

2. Mathematical framework

The phase-field modelling of cracks in porous media can be formulated by assuming a damage parameter (s) in the medium. The energy functional $\mathbb{W}$ is then defined as the summation of bulk energy and surface energy corresponding to the solid phase of the fracturing porous material.

$$\mathbb{W} = \int_{\Omega_0} \left((s^2 + \kappa_\varepsilon)\Psi^+_{\text{eff}}(\boldsymbol{C}) + (1+\kappa_\varepsilon)\Psi^-_{\text{eff}}(\boldsymbol{C}) \right) dV + \frac{G_c}{2}\int_{\Omega_0} \left(\varepsilon \|\nabla_X(s)\|^2 + \frac{1}{\varepsilon}(1-s)^2 \right) dV \tag{1}$$

In the above equation, $\boldsymbol{C} = \boldsymbol{F}^T \cdot \boldsymbol{F}$ is the right Cauchy-Green deformation tensor, $\boldsymbol{F}$ is the deformation gradient, G_c is the critical energy release rate, and l_0 is the length-scale parameter to model the diffusive crack. The weak form of crack evolution problem in the reference configuration is derived as stated below by minimising ($\delta\mathbb{W} = 0$) with respect to the damage parameter s.

$$\int_{\Omega_0} 2\Psi^+_{\text{eff}}(\boldsymbol{C})\, s \cdot \psi \, dV + 2l_0 G_c \int_{\Omega_0} \boldsymbol{GRAD}(\psi) \cdot \boldsymbol{GRAD}(s)\, dV - \frac{G_c}{2l_0}\int_{\Omega_0} (1-s)\cdot \psi \, dV \\ - 2l_0 G_c \int_{\partial\Omega_0} \psi\, \boldsymbol{GRAD}(s) \cdot \boldsymbol{N} dA = 0 \tag{2}$$

The hydromechanical behaviour of the deforming porous medium is modelled by updating displacement and pore pressure fields $(\boldsymbol{u}, p)$. When hydraulic or mechanical loading is applied on the body. The above equation is solved by giving a fixed state of $(\overline{\boldsymbol{u}}, \overline{p})$ to calculate the effective energy functional $\Psi^+_{\text{eff}}(\boldsymbol{C})$. In the context of finite strains, the second Piola-Kirchhoff effective stress tensor $\boldsymbol{S}'$ and the nonlinear Green-Lagrange strain tensor $\boldsymbol{E}$, which are energy conjugates on the reference configuration Ω_0 construct the energy functional.

$$\boldsymbol{S}' = (s^2 + \kappa_\varepsilon)\frac{\partial \Psi^+_{\text{eff}}(\boldsymbol{C})}{\partial \boldsymbol{C}}\frac{\partial \boldsymbol{C}}{\partial \boldsymbol{E}} + (1+\kappa_\varepsilon)\frac{\partial \Psi^-_{\text{eff}}(\boldsymbol{C})}{\partial \boldsymbol{C}}\frac{\partial \boldsymbol{C}}{\partial \boldsymbol{E}} \tag{3}$$

$$\boldsymbol{E} = \frac{1}{2}(\boldsymbol{C} - \boldsymbol{I}) \tag{4}$$

The balance of linear momentum in the material form and the mass conservation law in the spatial form are utilised to model the nonlinear poroelastic behaviour of the materials. These governing equations are written over the reference volume dV of body Ω_0 as

$$\rho_0 \boldsymbol{G} + DIV(\boldsymbol{P}) = 0 \tag{5}$$

$$\dot{J} + div(\boldsymbol{v}^r) + \frac{1}{M}\left(\dot{p} - \dot{J}\frac{p}{J}\right) = 0 \tag{6}$$

where, $\boldsymbol{P}$ is the first Piola-Kirchhoff total stress, $\rho_0 \boldsymbol{G}$ is the body forces in the reference configuration, the Jacobian is $J = \det(\boldsymbol{F})$, M is the Biot modulus of the material, p is the Kirchhoff pore pressure in the reference volume, and $\boldsymbol{v}^r$ is the Darcy velocity. Notice that the operators DIV and div are defined with respect to the material and spatial points respectively. Finally, the coupled weak forms of the hydromechanical model in an updated lagrangian finite element format is written as

$$\int_{\Omega_0} \boldsymbol{grad}(\boldsymbol{\eta}) : \mathbb{C}^\tau : \boldsymbol{grad}(\boldsymbol{u})\; dV - \int_{\Omega_0} (\alpha p)\, div(\boldsymbol{\eta})\, dV - \int_{\Omega_0} \boldsymbol{\eta} \cdot \rho_0 \boldsymbol{G}\, dV - \int_{\partial\Omega_{0_\sigma}} \boldsymbol{\sigma} \cdot \boldsymbol{\eta}\, dA = 0 \tag{7}$$

$$\int_{\Omega_0} \omega \cdot \dot{J} dV + \frac{1}{M}\int_{\Omega_0} \omega(J\dot{p} - \dot{J}p) dV + \int_{\Omega_0} J\boldsymbol{grad}(\omega) \cdot \boldsymbol{v}^r dV - \int_{\partial\Omega_{0_q}} \omega \boldsymbol{q} dA = 0 \tag{8}$$

In the Eq. (7), $\boldsymbol{\tau}'$ is the Kirchhoff effective stress tensor as defined in $\boldsymbol{\tau} = \boldsymbol{\tau}' - \alpha p \boldsymbol{I}$, where α is Biot's coefficient. The geometrical nonlinearity is considered in the formulations using a Neo-Hookean hyperelastic material for solid skeleton to define the stress-strain behaviour as

$$\mathbb{C}^\tau_{\text{ijkl}} = \frac{1}{J}\boldsymbol{F}_{\text{iA}}\boldsymbol{F}_{\text{jB}}[\lambda(\boldsymbol{C}^{-1}_{\text{AB}}\boldsymbol{C}^{-1}_{\text{CD}}) + (\mu - \lambda \ln J)(\boldsymbol{C}^{-1}_{\text{AC}}\boldsymbol{C}^{-1}_{\text{BD}})]\boldsymbol{F}_{\text{kC}}\boldsymbol{F}_{\text{lD}} \tag{9}$$

The constitutive relationship to govern the fluid flow both inside the formed cracks and the porous undamaged material is implemented to consider the inertial effect of turbulent flow within the cracks when needed. By the propagation of crack and the increase in the porosity of cracked region, the fluid inside the crack tends to flow faster, and the Reynold's number exceeds the limit corresponding to Darcy-type flow. As a result, the fluid velocity $\boldsymbol{v}^r$, in the Eq. (8), is updated in the iterative Newton-Raphson algorithm in the nonlinear hydromechanical model to model the turbulent flow in the crack.

3. Numerical simulations and results

The method is verified by simulating the propagation of a hydraulic fracture caused by the fluid injection into the middle of a $1m^3$ cubic sandstone sample in [7]. The same homogeneous material properties and geometry is used in this study, and the results approve that the same trend for the pressure drop, also consistent with experimental results, near the fluid injection point can be seen (Fig 1-a). The nonlinear flow inside the crack found to be an influential factor in the uniform distribution of the fluid pressure along the crack. It can be seen in (Fig. 1-b) that the nonlinear behaviour is activated after the formation of a considerable length of the crack. It can be understood that the changes of cracked zone pressure in time is more consistent when the nonlinear flow is considered in the analysis.

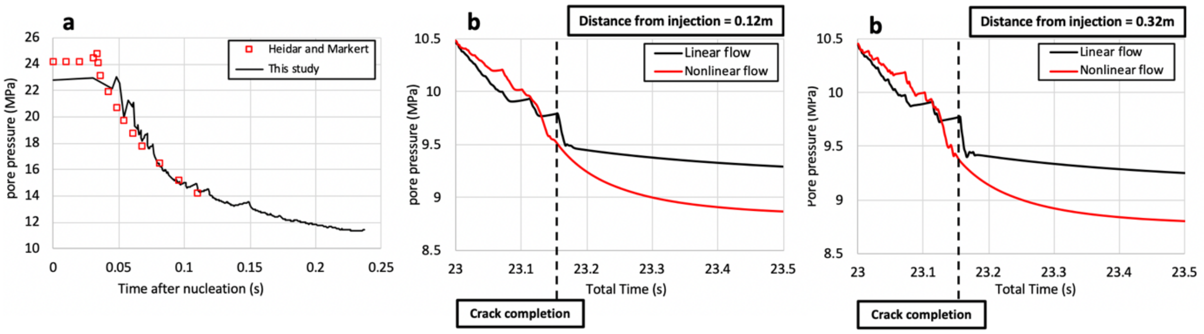

Fig. 1. (a) shows changes of pressure by crack propagation; (b) the effect of using the nonlinear flow equation.

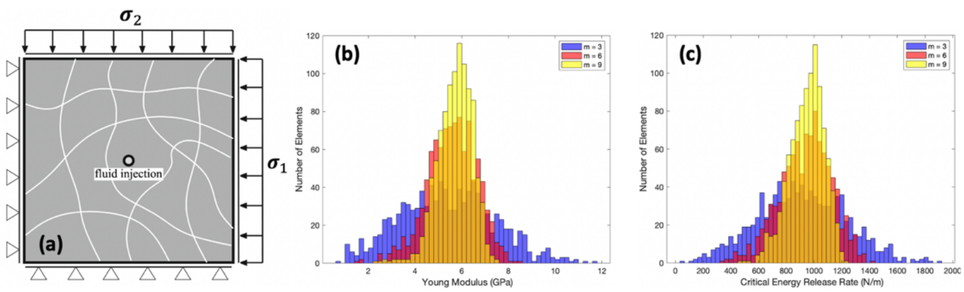

Fig. 2. (a) is the boundary value problem subjected to far-field stresses; (b) and (c) depict the heterogeneity distribution around mean values $E = 6GPa$ and $G_c = 1000\ N/m$.

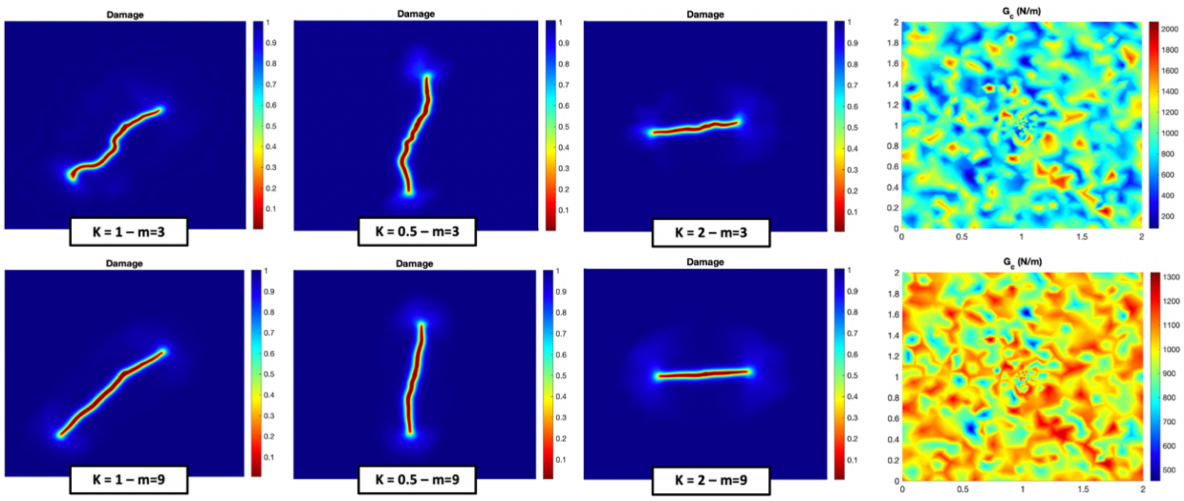

Fig. 3. Cracking patterns in heterogeneous medium. Young modulus and critical energy release rate are distributed over the domain using Weibull method, and the distribution of G_c with shape parameters m = 3 and m = 9 are shown above.

To show the applicability of the method in spatially heterogeneous materials, a plane strain boundary value problem of dimensions $2m \times 2m$, shown in the Fig. (2-a), is subjected to an increasing water injection rate of $dQ/dt = 6t\ cm^3/s$ with time in a cavity in the middle of the domain. All the external boundaries are considered as draining boundaries, and the effect of far-field stresses on the hydraulic fracture propagation is studied for three stress ratios $K = \sigma_1/\sigma_2$. The young modulus E and the critical energy release rate G_c are randomly distributed over the domain using Weibull distribution method with a cross correlation (1:1). The distribution of these parameters around their mean values are shown in

Fig. 2 for a sample of size 1000 elements from a highly heterogeneous material to a nearly homogeneous material. In Fig. 3, the effect of far-field stresses on the crack direction and the effect of heterogeneity on the cracking pattern are shown for different stress ratios $(1, 0.5, 2)$ and two degrees of heterogeneity. The effect of heterogeneity is also investigated by setting the stress ratio $K = 2$ and tracing the hydraulic fracturing patterns and the changes of pore pressure inside the crack in Fig 4. By increasing the degree of heterogeneity in the material, the crack is developed in a specific direction depending on the most favourable path to release energy. The graph in Fig. 4 shows that increasing the heterogeneity of the system results in demanding less energy to cause fracture propagation. It can be seen that the hydraulic fracture starts growing in the highly heterogeneous case $(m = 3)$ in a lower maximum pressure and lower injection rate, meaning a lower level of fluid energy given to the system.

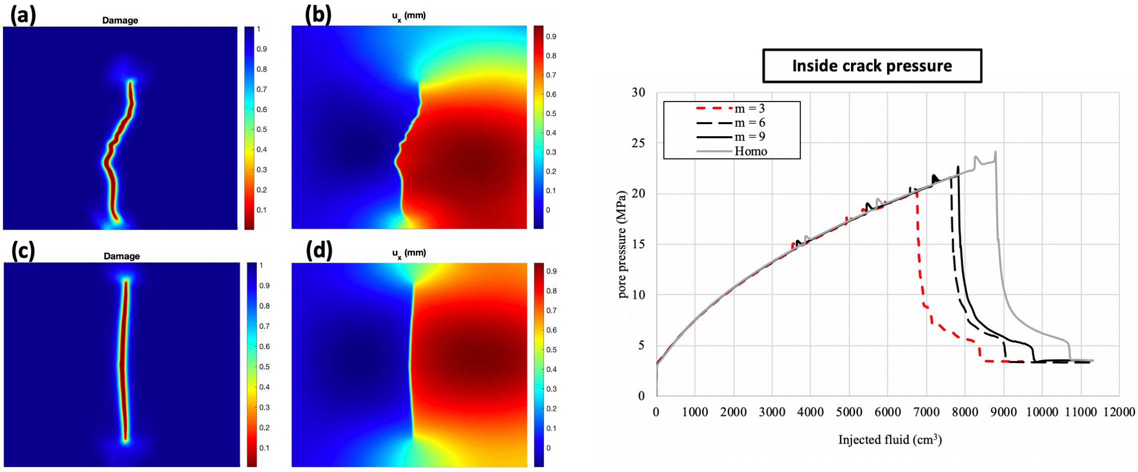

Fig. 4. (a-b) and (c-d) are snapshots of the crack path and displacement field before crack completion for highly heterogeneous and fully homogeneous cases respectively. The graph shows the fluid pressure evolution in the crack.

4. Conclusion

The phase-field modelling of hydro-fractures is found to be a highly versatile method to simulate cracking in heterogeneous porous media. The formulation proposed in this study provides the basis for the nonlinear hydromechanical analysis of porous media considering the inertial flow in the fractured zone. The heterogeneity in material properties is found affecting the hydraulic fracturing process by lowering the energy demand and the creation of tortuous paths in the medium. The state of principal stresses is identified as the most important factor in determination of the hydraulic fracture path in heterogeneous materials, although the heterogeneity causes the local bends in the propagation path due to the searching for the most favourable way for releasing energy and the creation of crack.

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