Management Science

Publication details, including instructions for authors and subscription information:
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https://pubsonline.informs.org/journal/mnsc

MANAGEMENT SCIENCE
Articles in Advance, pp. 1–30
ISSN 0025-1909 (print), ISSN 1526-5501 (online)

Accelerability vs. Scalability: R&D Investment Under Financial Constraints and Competition

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Received: November 29, 2019
Revised: February 12, 2022
Accepted: May 31, 2022
Published Online in Articles in Advance: October 7, 2022

https://doi.org/10.1287/mnsc.2022.4503

Abstract. I develop a continuous-time model to examine how the interaction between competition and financial constraints affects firms' research and development (R&D) strategies. The model integrates two key characteristics of R&D investment: accelerability (i.e., higher R&D intensity leads to faster discovery) and scalability (i.e., higher R&D intensity leads to higher project payoff). I find that firms react strategically to their rivals' financial constraints when making investment decisions in a duopoly R&D race. In particular, firms respond positively to the R&D intensity of an unconstrained rival, while they respond in a hump-shaped fashion to the R&D intensity of a constrained rival. As a result, a constrained firm can preempt an unconstrained competitor in market equilibrium. Accelerability is necessary for such pre-emption to occur, and scalability generally reduces its likelihood. Comparison with a monopoly benchmark shows that the economic mechanism differs from over-investment induced by financial constraints alone. The model also generates new implications regarding how project characteristics and cash flow risks impact R&D decisions.

History: Accepted by Gustavo Manso, finance.

Supplemental Material: The online appendix is available at https://doi.org/10.1287/mnsc.2022.4503.

Keywords: accelerability • scalability • financial constraints • R&D race • over-investment • pre-emption

1. Introduction

Research and development (R&D) investment has always been an important corporate decision, and it is becoming more so in the current time of never-ending technology races. Comparing with capital investment, R&D projects often take longer, measured in years if not decades, before bearing the fruits of innovation. The maturity uncertainty and the innate technical risk make R&D decisions complex. Meanwhile, innovative firms, especially the smaller and younger ones, often face financial constraints (Hall 2002), and are subject to winner-takes-all competitions (modeled in Loury 1979 and Weeds 2002). These facts lead to a relevant question in the economics and finance literature: How do financial constraints interact with competition in affecting firms' R&D decisions?

To provide a theoretical framework for analyzing firms' R&D decisions in the presence of both financing frictions and innovation competition, I simplify the setting to focus on a single decision variable, that is, a fixed level of R&D investment per period, which is also referred to as "investment intensity" or "investment rate." A novel feature of the study is to combine the *scalability* of innovation projects (often recognized in traditional investment models) with their *accelerability* (typical in patent race models). The dynamic and strategic nature of the investment decision causes nontrivial tradeoff for an innovative firm, which leads to the specific research questions of this paper: How does the impact of financial constraints on a firm's R&D investment rate depend on the characteristics of innovation technologies? How do firms with the same or different financing constraints compete in R&D projects, and how do their strategic interactions vary with the R&D project characteristics?

To answer these questions, I build a continuous-time model of R&D investment. An all-equity firm with stochastic cash flows faces a now-or-never R&D investment opportunity and decides its investment intensity to maximize the firm value. The innovation process is stochastic, and both the speed of discovery and the value of innovation are weakly increasing in the investment rate that is fixed for the duration of the project. If a firm is financially constrained, it is unable to continue funding the investment once it drains the internal funds, and it has to terminate the project with no scrap value. Meanwhile, a competition on the R&D

project is modeled as a winner-takes-all speed contest of duopoly: only the first firm that makes a discovery gets all the market share while the other firm gets nothing. As mentioned already, I distinguish two typical aspects of R&D project characteristics: *accelerability* versus *scalability*. An *accelerable project* can be expedited by more intensive investment. Consider a pharmaceutical company searching the best chemical compound for a drug. Hiring more technicians will likely help find the most suitable compound sooner. A *scalable project*'s expected payoff can be raised by more intensive investment. One example is an R&D project by a semiconductor company that aims at improving the reliability and functionality of its products. More R&D investment helps the company earn more profits and get a larger market share at the end of R&D when its products are released. However, the speed of the process is perceived to be restricted by Moore's Law. Moore's Law predicts that the computer chip performance would roughly double every 18 months because of shrinking transistor dimensions and other improvements, and it has been widely used to guide the R&D and production planning in the semiconductor industry. Innovation projects in practice differ in their accelerability and scalability and their benchmark speed and discovery payoff.

From the baseline monopoly model with nondeferrable projects, I find that financial constraints can increase a firm's R&D investment intensity (Corollary 1). Intuitively, financial constraints impose a risk that a firm may run out of money when its valuable project is still in progress and has to terminate the project and forgo any potential future cash flows associated with a successful discovery. If the R&D project is accelerable, a constrained firm (FC) may find it optimal to invest more heavily (comparing with an unconstrained one (UC)) to expedite the discovery and increase the survival probability of the project. The over-investment can be optimal for a constrained firm even though the higher burning rate of internal cash flows makes its financing constraints bind earlier. This result complements real options models that show financial frictions can cause accelerated investment (Boyle and Guthrie 2003, Lyandres and Zhdanov 2010, Bolton et al. 2019). The economic mechanism remains that financial frictions render a firm less patient. There is a slight difference with the aforementioned studies: they show the effect of financing frictions through its erosion to the value of the option to delay investment, I emphasize the incentive to increase the probability of project survival to retain the project value and avoid abandonment.

The study of my R&D race model uncovers interesting features regarding the interaction of financial constraints and competition. I find that the R&D competition affects firms' investment intensities in equilibrium differently, depending on the participating firms' financial constraints as well as the R&D project characteristics. This can be understood through firms' responses to the investment intensities of their rivals. For example, on an accelerable project, a firm reacts positively to the investment rate of an unconstrained rival. The rival's discovery rate affects the firm's decision through changing the discount rate of future cash flows; thus, it has the same impact as the risk-free rate. On the contrary, a firm reacts to a constrained competitor's investment with a hump-shaped pattern of its own investment. In other words, as a financially constrained competitor increases its R&D investment, its rival firm in the race (regardless of being financially constrained or not) first uses a more aggressive response of R&D investment strategy, followed by a pull-back of the investment as the constrained competitor's investment increases further. The part of the reaction with the negative slope can be explained by a strategic motive. A high investment level of the constrained rival raises the cost of competing head-to-head with it, but waiting on the sideline for the rival to bust and quit can turn out to be an optimal decision. Project scalability preserves these patterns, which disappear (and be flat) if the project is only scalable and not accelerable. Comparing to the investment models with competition (Grenadier 2002, Weeds 2002, Novy-Marx 2007) that generally focus on the value erosion of competition and the motive from the fear-to-lose, my model shows strategic interaction can lead to not-fear-to-lose when facing a financially constrained competitor.

The aforementioned patterns of best responses hold in all three kinds of races: a *UCUC race* between two unconstrained firms, an *FCUC race* between a constrained and an unconstrained firms, and an *FCFC race* between two constrained firms. I find that, as a result of those best response patterns, an FC firm can pre-empt its UC rival in the equilibrium of an accelerable FCUC race. Intuitively, a UC firm may choose not to escalate the speed contest in the equilibrium but instead invests less intensively and waits for the FC firm to drop out of the race, so it can achieve discovery with a relatively low cost. This result is related to the weaker status of the FC competitor in an FCUC race: a UC firm can still achieve the innovation after its FC rival firm drops out of the race, but an FC firm's only possibility of achieving an innovation is by winning the race against its UC rival. This new form of pre-emption may help explain "the standard folklore that smaller firms are more aggressive about entering new markets or launching new products than bigger, safer, and less financially constrained firms" (Boyle and Guthrie 2003, p. 2144) in a setting with innovation competition.

My model on R&D investment provides a unified framework of examining the "termination risk" on R&D projects. The risk of having to terminate a valuable project acts like the Sword of Damocles, which is a significant concern for any innovative firms. Although it is related to the maturity uncertainty and technical risk in recent R&D models (Berk et al. 2004, Malamud and Zucchi 2019), a careful inspection of the sources of the termination including that from the market competition, or financing frictions in combination with cash flow risks is useful. For example, contrasting to the negative impact of a termination risk from cash flow volatility (diffusion risk) on investment under certain conditions, I show in the model discussion of Section 4.1 that a jump risk on cash flow raises investment intensity of an FC firm. Intuitively, catastrophic events on internal cash flow cause the project termination regardless of how much internal funds or liquidity the firm has. The jump risk thus effectively alleviates the negative impact of heavier investment (through making an earlier burn-out caused by a higher investment rate a lesser concern) and gives the FC firm an extra incentive to invest. This is similar to the effect of a larger discount rate in a model with exogenous competition (Hackbarth et al. 2014). Furthermore, my duopoly race model shows that the effect of the termination risk from competition depends on the characteristics of both competing firms' cash flows, financing capacity, and the R&D project's characteristics. As a tradeoff to retain model tractability, the paper is silent on optimal liquidity management.

I also discuss the implications of relaxing a few key assumptions in the model. For example, I argue that the winner-takes-all is essential to deliver the model results and suggest that the race model is most appropriate for studying R&D investment, as opposed to any capital investment. In addition, I provide some evidence on the utilization of external financing for firms that are able to tap the financial markets and fill the funding gap between the internal cash flow and the R&D investment with a cost. The analysis shows that the R&D investment intensity of such a firm is in between an unconstrained firm and a strictly constrained firm in the main model.

The race model in the paper integrates two kinds of investment models to some extent: those that study R&D races or investment with competition (Grenadier 2002, Weeds 2002, Novy-Marx 2007, Meng 2008) and those that study how financially constrained firms invest and abandon projects (Boyle and Guthrie 2003, Lyandres and Zhdanov 2010, Bolton et al. 2019). The former ones typically feature unconstrained firms, and the latter ones focus on stand-alone firms. This paper contributes to at least three strands of literature besides the ones mentioned already. First, by providing a new understanding of investment level decisions, this paper contributes to the corporate investment literature and complements endogenous timing models (McDonald and Siegel 1986, Boyle and Guthrie 2003, Bolton et al. 2011, Hugonnier et al. 2015) in which "the investment level is not a choice variable" (Gu 2016). Second, by introducing strategic interactions among R&D competitors with different financing constraints, this paper can enlighten more in-depth studies in the booming literature on the interaction of finance and industrial organization (Lambrecht 2001, Phillips and Zhdanov 2013, Hackbarth et al. 2014, Malamud and Zucchi 2019). Third, by recognizing the consequence of investment rate decisions on both the timing and scale of innovation for the first time as I am aware, the paper provides a new analytical tool to the growing literature on innovation and entrepreneurship, as well as the large extant literature on R&D and productivity or macroeconomics (Bloom 2007, Brown et al. 2009, Aw et al. 2011, Doraszelski and Jaumandreu 2013).

The theoretical framework in this paper generates new empirical implications regarding R&D strategies. For example, it is more likely to observe higher R&D investment rates by financially constrained firms (1) when they are racing against an unconstrained firm; (2) on projects with accelerability; (3) when they face a looming challenge that can potentially wipe out their funding sources for the R&D project, or a large decline of the funding cash flow; and (4) in a winner-takes-all race.

This paper has broader applications in the finance and economics research. For example, it demonstrates the relevance of cross-industry studies in examining the real effects of financial market frictions on corporate innovation. A potentially fruitful way of separating industries is by examining the accelerability and scalability of the new technologies in those industries. This can add a new line of research to the growing empirical literature studying the role of finance in the innovation process (Hellmann and Puri 2000, Lerner et al. 2011, Manso 2011, Tian and Wang 2014, Nanda and Rhodes-Kropf 2017). In addition, researchers may explain the time-varying composition of young and private firms making successful innovation in the economy by examining how the characteristics of innovation change over time.

The paper proceeds as follows. Section 2 presents a baseline monopoly model. Section 3 presents the R&D race model for three types of duopoly races. Section 4 discusses model extensions and the few key assumptions in the main model. Section 5 concludes the paper. The appendices contain all proofs and some additional figures.

2. Baseline Monopoly Model

Consider an all-equity firm, with assets-in-place (AIP) that generate a cash flow X_t (≥ 0) at time t that follows a geometric Brownian motion (GBM)[1]:

$$dX_t = \mu X_t dt + \sigma X_t dZ_t, \tag{1}$$

where $Z = \{Z_t; 0 < t < \infty\}$ is a standard Brownian motion, μ and σ are constants.

The firm is run by a risk-neutral agent who maximizes the firm value when making decisions. Upon the arrival of a nondeferrable one-time innovation opportunity, the agent decides whether to start a project then, and if she does, she chooses the firm's R&D investment expenditure per period ("investment intensity" or "investment rate," denoted as R), which needs to be paid at each t. The decision on R can potentially affect both the scale of the project payoff, and the speed of a successful innovation. Specifically, the project generates a random one-time payoff $\tilde{u}(R)$ at an uncertain discovery time τ_d, which is modelled as the first arrival of a Poisson process[2] with an intensity parameter $\lambda(R)$. The expected payoff is $u(R) = \mathbb{E}(\tilde{u}(R))$. Suppose both $\lambda(R)$ and $u(R)$ are weakly increasing and concave, and the R&D investment has decreasing returns to scale, that is, $\partial^2(u(R)\cdot\lambda(R))/\partial R\partial R < 0$, to cap the optimal investment intensity. For the ease of discussion, I define two kinds of R&D projects accordingly.

Definitions. A scalable project is a project whose payoff can be scaled up by higher investment intensity, that is, $u'(R) \equiv \partial u(R)/\partial R > 0$. An accelerable project is a project whose discovery can be expedited by higher investment intensity, that is, $\lambda'(R) \equiv \partial\lambda(R)/\partial R > 0$.

The two kinds of projects are not mutually exclusive. A project can be both scalable and accelerable, that is, $u'(R) > 0$ and $\lambda'(R) > 0$. Going beyond the extant research on R&D investment, this model captures both the scalability (of the final outcome) and the accelerability (of the discovery speed) of the input-output relationship for innovation investment. The former is often seen in capital investment models, in the form of a Cobb-Douglas production function, and the latter is often used to characterize projects for firms in a patent race. The novelty of modelling both the scalability and accelerability of R&D investment leads to the disentangle of financial constraints' effects on R&D investment through different aspects of project characteristics. When the functional forms of $u(R)$ and $\lambda(R)$ are needed in this paper, I use the following power functions[3]:

$$u(R) = A\cdot R^{\beta}, \quad \lambda(R) = \eta\cdot R^{\gamma}, \quad \text{with } \beta,\gamma \in [0,1) \text{ and } \beta+\gamma < 1. \tag{2}$$

The scaling factor $A > 0$ and the speed factor $\eta > 0$ are constants. I interpret β as the *project scalability*, which measures the degree to which the project is scalable, and interpret γ as the *project accelerability*, which measures the degree to which the project is accelerable.[4] Both β and γ are constants. I label $\{A,\beta,\eta,\gamma\}$ as the *technology parameters* or *tech-parameters* of the model. To have the sensible property that $\partial u/\partial\beta > 0$ and $\partial\lambda/\partial\gamma > 0$, I impose a lower bound on R: $R \geq \underline{R} \equiv 1$.

I compare two kinds of monopoly firm's investment decisions: an FC firm and a UC firm. The latter can issue new equity at no extra cost, whereas the former cannot. Whether the firm is financially constrained or not is exogenous to the model, and financial constraints are defined based on the cost of accessing financial markets (as in Kaplan and Zingales 1997 and Bolton et al. 2011).[5]

To keep the model simple yet powerful enough to develop the intuition, I make several assumptions. (1) There is no fixed cost of starting the now-or-never R&D project. (2) The investment rate is fixed once chosen (and until either the cash flow is exhausted for a constrained firm or the innovation becomes successful). (3) The R&D cash flows do not affect the AIP cash flows nor does the R&D project has any effect on the firm's financing ability, before or after the discovery (Boyle and Guthrie 2003).[6] (4) The financial constraints are extreme, such that an FC firm has no access to external financial market whatsoever. (5) The firm does not retain cash and instead pays out the residual of the AIP cash flow after the R&D investment cost in each period, that is, $X_t - R$. (6) If the project is discontinued before maturity, the scrap value is zero.[7] (7) The FC firm does not abandon an ongoing project unless it is forced to do so as a result of funding shortages, that is, once $X_t < R$.[8]

The assumptions (1), (4), and (5) are discussed in Section 4. The smooth investment assumption in (2) is crucial for solving the model analytically, and it is used in the literature (Malamud and Zucchi 2019), although it makes the model not fully dynamic. Studies such as Brown and Petersen (2011) and Liu et al. (2021) provide empirical evidence on the R&D investment being smooth. R&D smoothing is often regarded as a firm's reaction toward very large adjustment costs of R&D investment (Himmelberg and Petersen 1994, Hamermesh and Pfann 1996, Hall 2002). The substantial adjustment costs after cutting R&D temporarily, which usually involves firing quality workers include the time and efforts needed to find the right talents who then require a great amount of firm- and project-specific training; disruption to the R&D teams which experience the repeated turnover of workers; the potential dissemination of critical proprietary information from the fired R&D workers to the competitors, and so on.

2.1. Monopoly Firm's Problem

The agent maximizes the firm value by choosing an R&D strategy $\{\mathbb{1}_{\text{invest}}, R\}$. A monopoly firm's problem conditional on investing in the project can be described as follows, with X_0 representing AIP cash flow at the project arrival:

FC firm :

$$\sup_{R\in(0,X_0]} \mathbb{E}\left[\int_0^{\infty} e^{-rt}X_t dt + \int_0^{\tau_d\wedge\tau_c} e^{-rt}(-R)dt + e^{-r\tau_d}\tilde{u}\,\mathbb{1}_{\{\tau_d<\tau_c\}}\right], \tag{3}$$

UC firm :

$$\sup_{R>0} \mathbb{E}\left[\int_0^{\infty} e^{-rt}X_t dt + \int_0^{\tau_d} e^{-rt}(-R)dt + e^{-r\tau_d}\tilde{u}\right]. \tag{4}$$

The first terms in Equation (3) and Equation (4) represent the firm value from AIP cash flows. The second terms are the present value of the future R&D cost.

The FC firm stops paying for the investment at the earliest time (denoted as "$\wedge$") of (1) the R&D discovery at τ_d, and (2) the time that its funding shortages occur at $\tau_c \equiv \inf\{t : X_t < R\}$. The UC firm keeps investing in the R&D project until discovery happens. The last terms are the present value of the project payoff, which for the FC firm is only realized if the project reaches discovery before the financial constraints bind, that is, $\tau_d < \tau_c$. Provided assumption (3), the monopoly firm's problem is equivalent to maximizing the R&D project value. Lemma 1 presents the project values with its proof in Appendix A.1.

Lemma 1. *The R&D project values for a UC monopoly firm and an FC monopoly firm are*

$$V_{uc} = \sup_{R>0} \frac{u(R)\lambda(R) - R}{\lambda(R) + r}, \tag{5}$$

$$V_{fc}(X) = \sup_{R\in(0,X]} \frac{u(R)\lambda(R) - R}{\lambda(R) + r}\left(1 - \left(\frac{X}{R}\right)^{\alpha}\right), \tag{6}$$

respectively, $\alpha = 1/2 - \mu/\sigma^2 - \sqrt{(1/2 - \mu/\sigma^2)^2 + 2(\lambda(R) + r)/\sigma^2}$, *X is the AIP cash flow at the project arrival.*

The project value for a UC monopoly in Equation (5) is the present value of a perpetuity, with per-period payments of $u\lambda - R$ and a discount rate of $\lambda + r$. The investment intensity R affects V_{uc} through the expected payoff and the cost per period, as well as the discount rate. To an FC monopoly firm, the risk of having to abandon the project because of the shortage of funds reduces the project value proportionally by $(X/R)^{\alpha}$. Alternatively, the term $1 - (X/R)^{\alpha}$ in Equation (6) can be interpreted roughly as the probability of the project's survival from abandonment ($\Pr(\tau_d < \tau_c)$), or the pricing kernel for the cash flow stream of the project. This term shows that R also affects V_{fc} through its distance to the AIP cash flow, as well as the speed at which X_t drops to R (via α). Equations (5) and (6) indicate an upper bound for R: $u\lambda - R \geq 0 \Rightarrow R \leq \bar{R}$. Thus, when using Equation (2), the ranges of the investment intensities are

$$\begin{aligned} R_{uc} &\in [\underline{R}, \bar{R}_{uc}] = \left[1, (A\eta)^{\frac{1}{1-(\beta+\gamma)}}\right], \\ R_{fc} &\in [\underline{R}, \bar{R}_{fc}] = \left[1, \min\left\{(A\eta)^{\frac{1}{1-(\beta+\gamma)}}, X\right\}\right]. \end{aligned} \tag{7}$$

Propositions 1 and 2 present the comparative statics of the optimal investment intensities for the UC and FC monopoly, respectively, conditional on investment taking place and given (2). The proofs are in Appendices A.2 and A.3.

Proposition 1. (i) *If* $u' = 0$ *and* $\lambda' = 0$, *then* $R^*_{uc} = \underline{R}$. (ii) *If* $u' > 0$ *and/or* $\lambda' > 0, \partial R^*_{uc}/\partial A > 0$. (iii) *If* $u' > 0$, $\partial R^*_{uc}/\partial \beta > 0$. (iv) *If* $\lambda' = 0$ *and* $u' > 0$, $\partial R^*_{uc}/\partial \eta > 0$. (v) *If* $\lambda' = 0$, $\partial R^*_{uc}/\partial r = 0$. *If* $\lambda' > 0$, $\partial R^*_{uc}/\partial r > 0$.

Intuitively, if the investment intensity only affects the cost of running the project, but neither the project payoff nor the speed of discovery, it must be optimal to choose the lowest possible rate of investment. If the investment rate affects either the payoff scale or the speed or both, then a higher scaling factor ($A \uparrow$) motivates more intense investment for the UC monopoly ($R^*_{uc} \uparrow$). This is because either the scalability or the accelerability induces the complementarity between A and R_{uc}, in the sense that a higher A increases the marginal return to having a higher R_{uc}. The scalability β has a positive impact on R^*_{uc} for the same reason, provided a project is scalable. However, if the project is accelerable ($\lambda' > 0$), then even in the simplest case of a UC monopoly, the effects of the speed factor η and the project accelerability γ on R_{uc} are ambiguous. The directions of the effects depend on the comparison of two opposing forces, both via changing the expected time to discovery. Take the speed factor η for an example. A higher η (1) increases the marginal value of expanding R_{uc} via raising the expected effective payoff per period, whilst (2) raises the marginal cost of expanding R_{uc} through a heavier discount rate by shortening the time to discover. If $\lambda' = 0$ instead, then η has an unambiguous and positive impact on R^*_{uc} as a result of the absence of the second force (provided $u' > 0$). R^*_{uc} increases with the risk free rate if the project is accelerable, because a higher r reduces the marginal cost of expanding R_{uc} through the discount rate. If $\lambda' = 0$, then changes in r does not affect R^*_{uc}.

Proposition 2. (i) *Proposition* 1(i) *holds for* R^*_{fc}. (ii) *If* $\lambda' = 0$ *and* $u' > 0$, *then* $\partial R^*_{fc}/\partial X > 0$. (iii) *If* $\lambda' > 0$, *then* $\partial R^*_{fc}/\partial X > 0$ *when X is small, but* $\partial R^*_{fc}/\partial X < 0$ *when X is large.* (iv) *If* $\lambda' > 0$, *then* $\partial R^*_{fc}/\partial \mu < 0$ *when X is large. If* $\lambda' = 0$ *and* $u' > 0$, *then* $\partial R^*_{fc}/\partial \mu > 0$ *when X is large.* (v) *Regardless of* $\lambda' = 0$ *or not,* $\partial R^*_{fc}/\partial r > 0$. (vi) *If* $\lambda' = 0$ *and* $u' > 0$, *then* $\partial R^*_{fc}/\partial \eta > 0$ *if X is large, and* $\partial R^*_{fc}/\partial \eta < 0$ *if X is small.*

Comparing to a UC monopoly, some additional effects of R_{fc} on V_{fc} arise through the changes to the likelihood of project's survival. On the one hand, choosing a higher R_{fc} (recall $R_{fc} \leq X$) makes the burn-out of internal capital more imminent, regardless of the tech-parameters. If $\lambda' = 0$ and $u' = 0$ (as in Proposition 2(i)), a higher R_{fc} not only increases the cost of investment but also brings forward the funding shortages while having no benefit whatsoever. Therefore, it is optimal for an FC firm to invest at the minimal level $\underline{R}$. Conversely, if the project is accelerable ($\lambda' > 0$), a higher R_{fc} also speeds up the discovery and effectively delays the burn-out of internal capital relative to the project discovery. These two effects work against each other to impact the project value.

Consider Proposition 2, (ii) and (iii), which regard the impact of AIP cash flow X on R^*_{fc}. X affects R_{fc} only through the probability of project survival; thus, I focus on discussing the effect of X on the marginal return of increasing R_{fc} via the two channels mentioned in the last paragraph. Apparently, when $\lambda' = 0$, the second channel is absent, and the marginal return of a higher R_{fc} through changing the probability of project survival is always negative. By relaxing the financial constraints an FC firm faces, a higher AIP cash flow ($X\uparrow$) makes the marginal return of a higher R_{fc} less negative. Therefore, $\partial R^*_{fc}/\partial X > 0$. Proposition 2(iii) reveals that once the project accelerability is turned on ($\lambda' > 0$), the complementarity/substitutability between X and R_{fc} depends on the level of X. If X is small, indicating the financial constraints are more likely to bind soon, then a higher X increases the marginal return of increasing R_{fc}. However, as the constraints become less tight by a large enough X ($X\uparrow\uparrow$), the second channel of increasing R_{fc} (i.e., to speeds up the discovery relative to the burnout) dominates the first, causing the combined effect of increasing R_{fc} on V_{fc} to be positive. In such circumstances, a further increase in X leads to a decreasing marginal return of a higher R_{fc}, by making $\partial V_{fc}/\partial R_{fc}$ less positive. Intuitively, it is difficult to speed up the discovery relative to burnout if the expected discovery speed is already very fast. The nonmonotonic effect of X on $\partial R^*_{fc}/\partial X$ leads to the possibility of over-investment ($R^*_{fc} > R^*_{uc}$), stated more formally in Corollary 1.

Proposition 2(v) states that the risk-free rate always increases an FC firm's R&D investment in the monopoly benchmark. The proof in Appendix A.3 shows distinct reasons for that on projects with and without accelerability. If the project is not accelerable ($\lambda' = 0$), a higher investment expenditure R_{fc} makes it more difficult for the project to survive; thus, the marginal return of a higher R_{fc} through the survival probability term is negative. A higher r effectively delays the burnout time relative to innovation discovery, making the marginal return of a higher R_{fc} less negative. Hence, $\partial R_{fc}/\partial r > 0$ for $\lambda' = 0$. On the contrary, if the project is accelerable, the accelerability acts against the aforementioned force and weakens the effect of a higher r on the marginal return of an increasing R_{fc} but not enough to outweigh it. Plus, the additional effect of r on the marginal return of an increasing R_{fc} through the perpetuity term is positive (as for a UC firm shown in Proposition 1), leading to the combined positive impact of the risk-free discount rate r on R_{fc}. Proposition 2(v) relates to a few other results in the paper, such as the increasing best response of a FC firm's investment intensity toward its UC rival in Section 3.2, as well as the positive effect of a jump risk in AIP cash flow on R in Section 4.1.

Proposition 2(vi) shows that if X is small and the project is not accelerable, then the complementarity between the speed factor η and R_{fc} can be overturned. It is because the constraints are more likely to bind during project development; thus, a higher η does not increase the marginal return of a higher R_{fc}. Although not stated formally in the proposition, the proof in Appendix A.4 shows the R_{fc} increases with the diffusion risk σ^2 under certain conditions and it is in direct contrast with positive the effect of a jump risk on R_{fc} as detailed in Section 4.1.

Corollary 1 compares investment rates of an FC firm and a UC firm, with the first and last statements proved in Appendix A.4 and the possibility of over-investment evidenced by many numerical exercises using Equation (2) as the functional forms for the discovery payoff u and rate λ.

Corollary 1. *If $\lambda' = 0$ and $u' > 0$, then $R^*_{fc} < R^*_{uc}$. If $\lambda' > 0$, then financial constraints can induce over-investment, that is, $R^*_{fc} > R^*_{uc}$. Over-investment is more likely for an FC firm with large enough X and a low μ and on projects with a small to moderate γ.*

Table 1 lists the parameter values used in the baseline numerical exercise, where the over-investment ($R^*_{fc} > R^*_{uc}$) is present: $R^*_{fc} = 8.02$ and $R^*_{uc} = 6.79$. The baseline firm's AIP cash flows expect to decline 20% annually with a 30% annual volatility. The parameter values reflect the high uncertainty typically associated with innovative firms' internal cash flows or future funding, and are close to a few previous studies (Morellec and Schürhoff 2011, Hackbarth et al. 2014) except the growth rate of AIP μ.[9] The baseline values for the R&D project technology parameters (i.e., η, γ, A, β) are chosen ad hoc, with the simple goal of having reasonable investment intensity and expected discovery time at the optimum in the monopoly benchmark and in the race model equilibrium of Section 3.[10]

Corollary 1 closely resembles earlier work regarding the effect of funding risk on growth options. For example, the first statement relates to Myers (1977), which shows the probability of default leads to under-investment when it is impossible to accelerate the exercise of growth options. The statement on over-investment joins earlier work (Boyle and Guthrie 2003, Lyandres and

Table 1. Baseline Parameter Values for Numerical Solutions

Parameter	Value
Discount rate	$r = 0.05$
Discovery rate ($\lambda = \eta R^\gamma$)	$\eta = 0.05$, $\gamma = 0.7$
Expected project payoff ($u = AR^\beta$)	$A = 100$, $\beta = 0.01$
AIP cash flow at the project's arrival	$X = 100$
Growth/decline rate of AIP cash flow	$\mu = -0.2$
Volatility of AIP cash flow	$\sigma = 0.3$
(ext.) The jump risk of AIP cash flow	$\lambda_j = 0.1$

Zhdanov 2010, Bolton et al. 2019),[11] which suggests financial constraints make a firm endogenously more impatient. However, in this model with now-or-never investment opportunities, the economic mechanism is not exactly that financial frictions lower the value of the option to delay, as in the aforementioned studies. The intuition is that by investing more intensively in an accelerable project, a constrained firm may increase its chance of retaining the project value, instead of paying R&D expenditures regularly but getting nothing back in the end. It complements studies on investment timing decisions and show similar effects of future funding uncertainty on the investment level decision. On the one hand, a higher cash burning rate on the R&D investment leads to an earlier instance of project abandonment caused by funding shortages. On the other hand, by investing more heavily each period, the firm may be able to push the discovery fast enough to survive from the earlier burnout of funds. If by speeding up the project, the constrained firm gets a higher expected value, then it optimally invests more aggressively comparing with the first best.

Corollary 1 also clarifies the conditions on AIP cash flow and the project characteristics that motivate over-investment. A low X makes the threat of project abandonment from funding burnout so imminent that the project survival is highly unlikely, even with a faster discovery. A higher growth rate of AIP cash flow lowers the FC firm's incentive to speed up the innovation process to escape the burn-out. A high level of accelerability motives the UC firm to speed up its discovery, more so relative to an FC firm. All three forces push against over-investment.

Going back to the R&D project initiation decision, the firm optimally starts a nondeferrable innovative project if its value, which can be calculated using Lemma 1 with the optimal R, is positive. Ceteris paribus, a UC firm is more likely to initiate an investment than an FC firm because the UC firm's project value is always higher.

3. Model with an R&D Race

Built on the baseline monopoly model, I further study the Nash equilibrium of duopoly competition between two firms (call them Firm 1 and Firm 2) and examine how equilibrium R&D investment rates (R_1, R_2) depend on the competing firms' financial constraints and the type of projects. I compare three scenarios: (1) both firms are financially unconstrained (a UCUC race); (2) one firm is financially constrained and the other is not (an FCUC race); and (3) both firms are financially constrained (an FCFC race).

Following the literature on patent race models (Loury 1979, Dasgupta and Stiglitz 1980, Weeds 2002, Meng 2008), I assume the competition is a winner-takes-all speed contest. Only the firm that makes the discovery first is rewarded with future profit flows related to the project. All the other assumptions in the baseline monopoly model remain. The firms determine their R&D investment rates R_1 and R_2 simultaneously with complete information to maximize their firm values. Each firm's expected time to a successful discovery and the expected value of the future cash flows associated with the discovery depend only on its own choice of R, that is, $\lambda_i(R_i)$, $u_i(R_i)$. For simplicity, I assume the two firms face the same functional forms of λ and u as in Equation (2) but may differ in the tech-parameters.[12] Meanwhile, the firm can also differ in their AIP cash flows parameters, that is, $\mathrm{d}X_{i,t} = \mu_i X_{i,t}\mathrm{d}t + \sigma_i X_{i,t}\mathrm{d}Z_t$, $i \in \{1, 2\}$.

Going beyond the patent race models, I assume that if one firm drops out of the race because of financial constraints, the other firm carries on with its R&D effort and becomes the winner once it makes the discovery. If both firms abandon their projects before making a discovery, then there is no winner of the race. The case that both firms make the discovery at the same time is not considered because of its zero probability. Similar to financial constraints, the race competition also imposes a termination risk to firms: a firm's successful innovation makes its competitor's R&D project obsolete, and some may interpret it as an *obsolescence risk*. However, the termination risk from competition results from firms' strategic interactions in the R&D race and is thus endogenous in the model (unlike Eisdorfer and Hsu 2011, Hackbarth et al. 2014, and Gu 2016). Examining the three types of races in the following subsections separately helps to reveal the dependence of firms' strategic interactions in the race on both firms' financial constraints.

3.1. Race Between Two UC Firms

In a *UCUC race*, both firms keep paying the R&D expenditure until one participating firm becomes successful in its R&D pursuit first, and it gets the project payoff $\tilde{u}_i$. The slower firm gets nothing. For Firm i, its optimization problem can be written as

$$\sup_{R_i > 0} \mathbb{E}\left[\int_0^{\tau_i \wedge \tau_{-i}} \mathrm{e}^{-rt}(-R)\mathrm{d}t + \mathrm{e}^{-r\tau_i}\tilde{u}\mathbb{1}_{\{\tau_i < \tau_{-i}\}}\right]. \qquad (8)$$

When the project is accelerable but not scalable, that is, $\lambda'(R) > 0$ and $u'(R) = 0$, it corresponds to a typical patent race model with two participating firms. The project values are provided in Lemma 2, with the proof in Appendix A.5.

Lemma 2. *In a UCUC race, firms' project values are*

$$V_1 = \frac{u_1 \cdot \lambda_1 - R_1}{r + \lambda_1 + \lambda_2}, \qquad V_2 = \frac{u_2 \cdot \lambda_2 - R_2}{r + \lambda_1 + \lambda_2}, \qquad (9)$$

where (R_1, R_2) *is a pair of R&D investment intensities,* $u_i(R_i)$ *is Firm i's expected value of future project cash flows* *upon a successful discovery, and $\lambda_i(R_i)$ is Firm i's discovery rate $i \in \{1,2\}$.*

The patent race literature (Reinganum 1989) recognizes the effect of competition on firm value when competing firms are financially unconstrained, in the same spirit of Lemma 2, and with the number of rivals being more general.[13] Competition effectively increases the discount rate (comparing with Equation (5) in Lemma 1), via the rival firm's discovery rate. It is evident, as a departure from the patent race models, that if the project is not accelerable ($\lambda' = 0$), then having competition is equivalent to introducing an exogenous termination risk for a UC firm, and it reduces the marginal benefit and marginal cost of investment equally and thus has no impact on the investment intensity. When $\lambda' > 0$ as in patent race models, Proposition 3 shows the best responses are increasing, regardless of project scalability on which the patent race models are silent. The existence of Nash equilibrium is guaranteed, and the proof is in Appendix A.6.

Proposition 3. *In a UCUC race, a firm responds positively to the R&D intensity of its unconstrained rival if the project is accelerable, that is,* $\mathrm{d}R_i^*(R_{-i})/\mathrm{d}R_{-i} > 0$ *if* $\lambda' > 0$. *If* $\lambda' = 0$, *then* $\mathrm{d}R_i^*(R_{-i})/\mathrm{d}R_{-i} = 0$. *Regardless of scalability/accelerability, there exists at least one symmetric pure strategy Nash equilibrium.*

A UC firm responds positively to its UC rival's R&D rate when racing in accelerable projects, and it is a direct implication of the UCUC race being a log-supermodular game (Milgrom and Roberts 1990, Vives 1999). The two firms' investment rates are strategic complements, meaning a more intense investment of a rival firm increases the marginal return of the own firm's investment intensity. The strategic complementarities result from the speed contest without any financing considerations. Intuitively, the rival's earlier expected discovery makes a firm's marginal effort in R&D more worthwhile as it increases the chance to gain the whole market more significantly as opposed to ending up with nothing.

Proposition 3 clarifies that the widely accepted notion that competition enhances innovation only hold with accelerable projects ($\lambda' > 0$) in a winner-takes-all race. When the maturity of an innovation project is fixed ex ante, competition between two UC firms does not make them more aggressive even if the project is scalable. Instead, competition only reduces project values and thus may alter the project initiation decision. However, if the project is accelerable, the race can motivate R&D investment significantly. At the baseline as in Table 1, the competition increases a UC firm's investment from the monopoly level of $R_{uc}^* = 6.79$, to $R_{uc}^* = 35.5$ in the UCUC race. In Appendix A.7, I characterize a pure strategy symmetric equilibrium and provide the necessary and sufficient condition(s) for its uniqueness. Proposition 4 provides comparative statics of the UCUC equilibrium with respect to tech-parameters and r, and its proof is in Appendix A.8.

Proposition 4. *In a UCUC race, denote R_i^* as the equilibrium investment intensity $i \in \{1,2\}$:*

(1) *If $u' = 0$ and $\lambda' = 0$, then $R_1^* = R_2^* = \underline{R}$.*

(2) *If $u' > 0$ and $\lambda' = 0$, then $\partial R_i^*/\partial A_i, \partial R_i^*/\partial \beta_i, \partial R_i^*/\partial \eta_i, \partial R_i^*/\partial \eta_{-i}, \partial R_i^*/\partial r > 0$, $\partial R_i^*/\partial A_{-i}, \partial R_i^*/\partial \beta_{-i} = 0$.*

(3) *If $\lambda' > 0$ and $u' = 0$, then $\partial R_i^*/\partial A_i, \partial R_i^*/\partial A_{-i} > 0$, and $\partial R_i^*/\partial \eta_i, \partial R_i^*/\partial \gamma_i$, $\partial R_i^*/\partial \eta_{-i}, \partial R_i^*/\partial \gamma_{-i} > 0$ at least when γ_i is small.*

(4) *If $\lambda' > 0$ and $u' > 0$, then on top of the results in* (3), $\partial R_i^*/\partial \beta_i, \partial R_i^*/\partial \beta_{-i} > 0$.

This proposition states that a firm's equilibrium investment does not depend on its rival firm's scalability related parameters if the project is scalable but not accelerable. As $\lambda' > 0$, both firms' equilibrium investment rates increase in their own and the rival firms' tech-parameters. This is a direct consequence of the following two forces: both firms have increasing best responses as Proposition 3 shows, and a higher tech-parameter of a firm increases the marginal return on the investment intensity of that firm, leading to a more responsive best response of that firm toward its rival. Even without the change in its rival firm's response, which is also positive, both firms end up investing at higher levels in equilibrium.

3.2. Race Between an FC Firm (Firm 1) and a UC Firm (Firm 2)

Firms competing in innovation often vary in their financing abilities. It can appear in the form of a small firm competing against a large one or a young firm competing against a mature one. I abstract from the various scenarios and investigate an *FCUC race*, with Firm 1 being constrained and Firm 2 being unconstrained. Firms' problems are presented in Expressions (10) and (11). τ_i denotes Firm i's discovery time, and τ_{c_1} denotes the time that the FC firm has to abandon its project due to financial constraints, that is, $\tau_{c_1} = \inf\{t : X_{1,t} < R_1\}$. $\tilde{\tau}$ denotes the first discovery time with FC's possible burnout considered, that is, $\tilde{\tau} = \tau_1$ if $\tau_1 < \tau_2 \wedge \tau_{c_1}$ and $\tilde{\tau} = \tau_2$ if $\tau_1 > \tau_2 \wedge \tau_{c_1}$.

The FC firm's problem :

$$\sup_{R_1 \in (0, X_1]} \mathbb{E}\left[\int_0^{\tau_1 \wedge \tau_2 \wedge \tau_{c_1}} \mathrm{e}^{-rt}(-R)\mathrm{d}t + \mathrm{e}^{-r\tau_1}\tilde{u}\mathbb{1}_{\{\tau_1 < \tau_2 \wedge \tau_{c_1}\}}\right] \tag{10}$$

The UC firm's problem :

$$\sup_{R_2 > 0} \mathbb{E}\left[\int_0^{\tilde{\tau}} \mathrm{e}^{-rt}(-R)\mathrm{d}t + \mathrm{e}^{-r\tau_2}\tilde{u}\mathbb{1}_{\{\tau_1 > \tau_2 \wedge \tau_{c_1}\}}\right] \tag{11}$$

The FC firm only wins the race if it makes the discovery first and before running out of money in the process, that is, $\tau_1 < \tau_2 \wedge \tau_{c_1}$. It pays the R&D expenditure until one of the firms obtain discovery or itself cannot afford to invest any more, that is, the earliest of τ_1, τ_2 and τ_{c_1}. In contrast, the UC firm is in a more advantageous position: it wins the race either when it makes the discovery first or when the rival runs out of money before making its discovery, that is, $\tau_1 > \tau_2 \wedge \tau_{c_1}$.[14] The UC firm stops paying the R&D cost only when a discovery is made. Lemma 3 shows the project values, with its proof in Appendix A.9.

Lemma 3. *In an FCUC race, the project values for the two firms (FC is* 1*, UC is* 2*) are*

$$V_1(X_1) = V_1^d \cdot \left(1 - \left(\frac{X_1}{R_1}\right)^{\alpha_1'}\right), \tag{12}$$

$$V_2(X_1) = V_2^d \cdot \left(1 - \left(\frac{X_1}{R_1}\right)^{\alpha_1'}\right) + V_2^m \cdot \left(\frac{X_1}{R_1}\right)^{\alpha_1'}, \tag{13}$$

where (R_1, R_2) *is a pair of investment intensities with* $R_1 \in (0, X_1]$ *and* $R_2 > 0$. V_2^m *is the project value in a UC monopoly, and* V_i^d *is the project value in an UCUC duopoly, that is,*

$$V_2^m = \frac{u_2 \cdot \lambda_2 - R_2}{r + \lambda_2}, \quad V_i^d = \frac{u_i \cdot \lambda_i - R_i}{r + \lambda_2 + \lambda_1}, \tag{14}$$

$\alpha_1' = 1/2 - \mu_1/\sigma_1^2 - \sqrt{(1/2 - \mu_1/\sigma_1^2)^2 + 2(r + \lambda_2 + \lambda_1)/\sigma_1^2}$, *and* λ_i *and* u_i *are the discovery rate and expected project payoff of Firm i that depend on* R_i, $i \in \{1, 2\}$, *respectively.*

Both Equations (12) and (13) are intuitive. The only state variable for both competitors is the constrained firm's AIP cash flows X_1. The UC firm's cash flows do not affect its own investment decision and have no impact on its FC rival. There are two possibilities regarding the chronological order of the following two events: the earlier discovery among the two firms, and the FC firm being forced out of the race because of the shortage of its internal funds, that is, $\tau_1 \wedge \tau_2$ versus τ_{c_1}. The term $1 - (X_1/R_1)^{\alpha_1'}$ can be roughly understood as the probability (or the state price density associated with the possibility) that the earlier discovery among the two firms happens before FC's burnout, that is, $\Pr(\tau_1 \wedge \tau_2 \le \tau_{c_1})$. Relatedly, the term $(X_1/R_1)^{\alpha_1'}$ is roughly the probability of burnout happening first, that is, $\Pr(\tau_{c_1} < \tau_1 \wedge \tau_2)$. Because of the two possibilities, the market structure at the first discovery is uncertain: it is a duopoly in the first scenario and a UC monopoly in the second scenario. Correspondingly, the project value for the UC firm in Equation (13) is a weighted average of the values in the two scenarios. The weights are the probabilities, and the value is the same as a duopoly firm in the UCUC race in the first scenario (see Equation (9) in Lemma 2) and the same as a UC monopoly in the second scenario (see Equation (6) in Lemma 1). Likewise, the FC firm's value in Equation (12) is a weighted average of FC firm's project values, with the project value being zero in the second scenario. Corollary 2 presents the properties of firms' best responses in the FCUC race, with the functional forms of u and λ specified in Equation (2), and its proof in Appendix A.10.

Corollary 2. *In an FCUC race (FC-1, UC-2), if* $\lambda' = 0$, *then* $\mathrm{d}R_i^*(R_{-i})/\mathrm{d}R_{-i} = 0$. *If* $\lambda' > 0$, *then* $\mathrm{d}R_1^*(R_2)/\mathrm{d}R_2 > 0$, *and* $\mathrm{d}R_2^*(R_1)/\mathrm{d}R_1$ *is hump-shaped as long as X is not too large (i.e., it is not that* $X \gg (A\eta)^{\frac{1}{1-(\beta+\gamma)}}$*). Regardless of scalability/accelerability, there exists a unique pure strategy Nash equilibrium.*

For the ease of the discussion, I use the subscript "fc" and "uc" in replacement of "1" and "2." The first part of Corollary 2 is trivial. Without project accelerability (i.e., $\lambda' = 0$), the rival firm's investment rate, conditional on investing, does not impact a firm's R&D project value. Thus, $\mathrm{d}R_i^*(R_{-i})/\mathrm{d}R_{-i} = 0$. Corollary 2 also points out the distinct best responses of the two firms when $\lambda' > 0$, that is, a monotonically increasing $R_{fc}^*(R_{uc})$ versus a hump-shaped $R_{uc}^*(R_{fc})$. The UC firm's investment R_{uc} affects the FC firm's project value through the discovery rate λ_{uc}; λ_{uc} influences the FC firm's value in the same way as the risk-free rate, which always intensifies the FC firm's investment, as Proposition 2(v) shows.[15] This positive impact of an FCUC race on the FC firm's investment is similar to the impact of a UCUC race on a UC firm's investment, although the former is milder because of the FC firm's additional concern of the early burnout of internal funds.

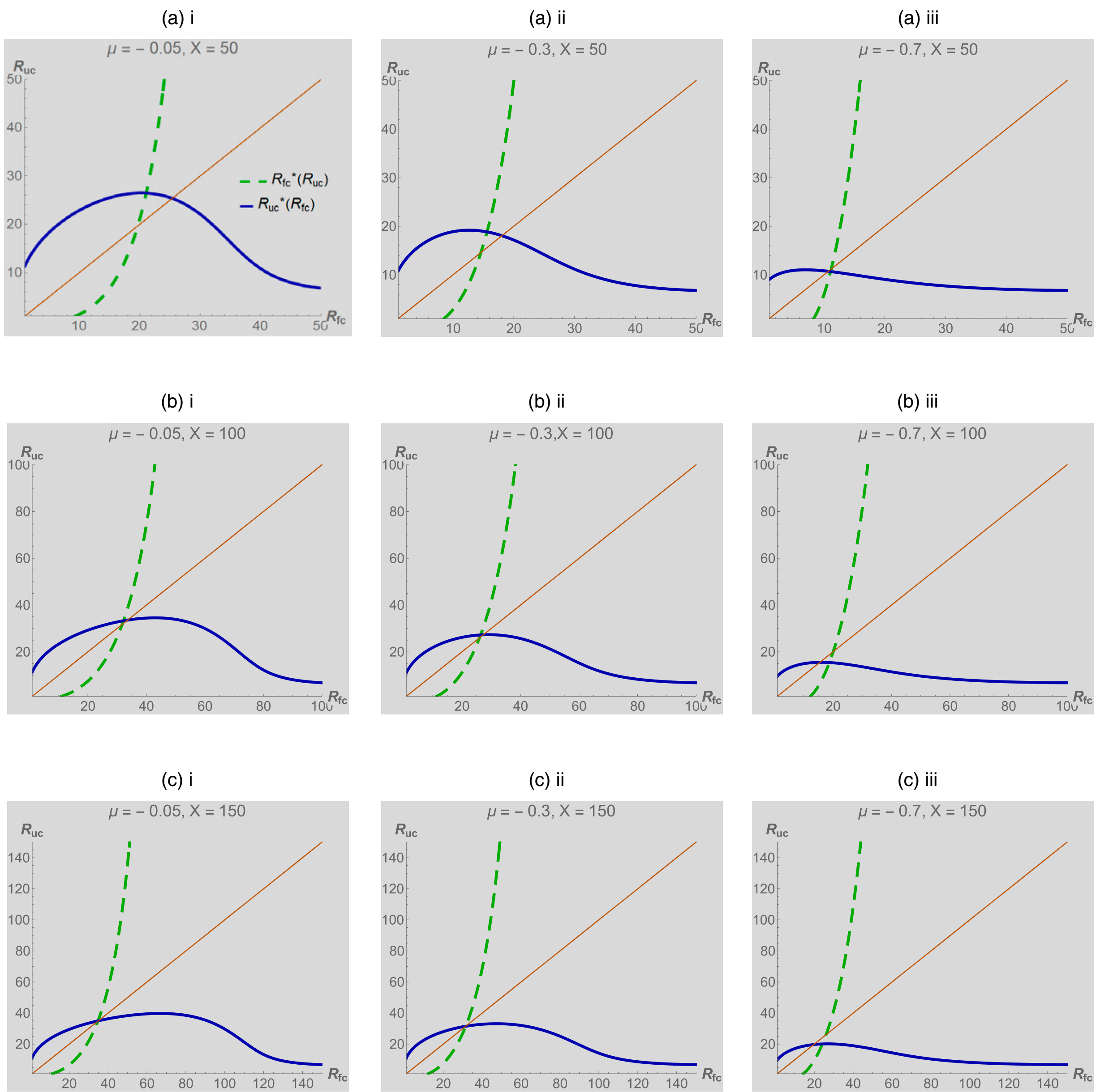
The hump-shaped $R_{uc}^*(R_{fc})$ demonstrates the UC firm's strategic considerations in an FCUC race on an accelerable project. To the UC firm, when its FC rival's investment is low relative to its AIP cash flow,[16] it is unlikely for the FC rival to have binding constraints before any discovery, making the race resembles a UCUC race. The conventional positive impact of competition in patent race models arises as a result. However, as the FC rival's investment increases, especially to a level that is close to its AIP cash flow (i.e., $R_{fc} \to X_{fc}$), it becomes highly likely that the FC rival will go bust soon, leaving the UC firm the only one to make a discovery. Meanwhile, the cost of competing head-to-head against an aggressive FC rival to win the race becomes prohibitively high for the UC firm. Therefore, the UC firm optimally pulls back its R&D investment rate and sits on the sideline of the innovation project, investing at nearly the monopoly level while waiting for the FC firm to exhaust its funds and leave the race.

3.2.1. Pre-Emption and Comparative Statics. In the context of the model, one firm pre-empts another in an R&D race when it achieves the discovery first.[17]

With pre-emption regarded in this way, I find through numerical exercises that an FC firm may pre-empt a UC firm in a R&D race if $\lambda' > 0$, with some examples shown in Figure 1 and Figure 2 (i.e., those equilibria that are under the 45-degree lines). Pre-emption by an FC firm in a FCUC race is not a mere extension of the over-investment result in the baseline monopoly model. Instead, the FC pre-emption results from the combination of two effects. (1) Motivated by the competition, the FC firm acts more aggressively than at the monopoly benchmark, trying to get the first discovery and reap the payoff. The incentive to invest is stronger as its UC rival intensifies the R&D investment. (2) As the FC firm becomes more competitive by raising its investment rate, the UC firm reacts positively to the race at first, but as the FC firm's burnout becomes more imminent, the UC firm acts in a laid-back fashion and retreats to its monopoly strategy.

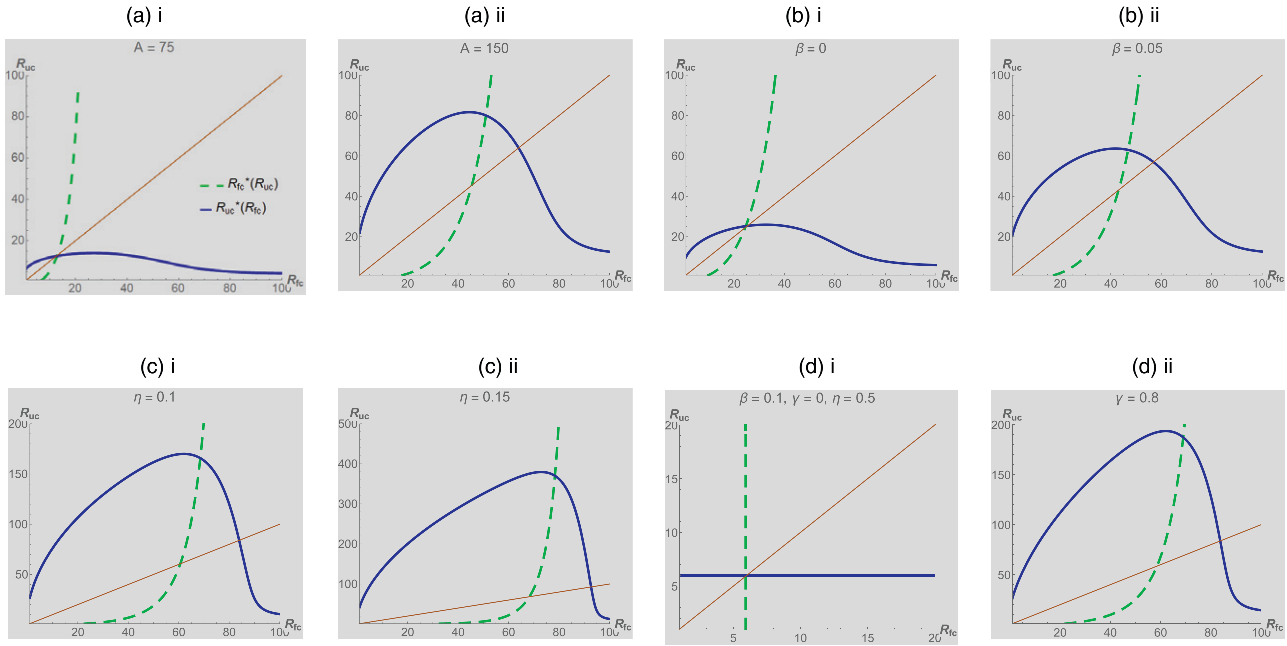
Figures 1 and 2 plot the best responses $R^*_{fc}(R_{uc})$, $R^*_{uc}(R_{fc})$ and equilibrium investment rates (R_1, R_2) in an FCUC race around the baseline values in Table 1. The subplots in Figure 1 change one AIP cash flow parameter for the FC competitor at a time, and the subplots in Figure 2 change one tech-parameter for both firms at a time. In

Figure 1. (Color online) Best Responses and the Equilibrium in an FCUC Race: AIP Cash Flow Changes



Notes. The blue solid lines plot the UC firm's best response to the investment intensity of the FC firm, that is, $R^*_{uc}(R_{fc})$. The green dashed lines plot $R^*_{fc}(R_{uc})$. The orange lines represent $R_{fc} = R_{uc}$. R_{fc} is on the x axis, and R_{uc} is on the y axis. Except those labeled on the subplots, all other model parameters are set at the values in Table 1.

Figure 2. (Color online) Best Responses and the Equilibrium in an FCUC Race: Technology Parameter Changes



Notes. The blue solid lines plot the UC firm's best response to the investment intensity of the FC firm, that is, $R^*_{uc}(R_{fc})$. The green dashed lines plot $R^*_{fc}(R_{uc})$. The orange lines represent $R_{fc} = R_{uc}$. R_{fc} is on the x axis, and R_{uc} is on the y axis. Except those labeled on the subplots, all other model parameters are set at the values in Table 1.

Figure 1, X increases by the rows, and μ decreases by the columns of the subplots. This exercise and many other unreported ones demonstrate a lower μ makes the FC pre-emption more likely. Examination of the figure reveals a different intuition from the over-investment in the monopoly model (Corollary 1). Instead of speeding up the FC firm's discovery as in the monopoly model, a lower μ makes the FC firm act less aggressively in the race on the contrary. This force is evident from the milder slope of $R^*_{fc}(R_{uc})$ as $\mu \downarrow$, and by it alone, UC pre-emption would emerge. Nevertheless, a faster declining AIP cash flow of the FC rival also makes the UC firm less concerned of the race and thus acts more like a monopoly firm as shown via the lower hump of $R^*_{uc}(R_{fc})$. Together, the FC pre-emption is more likely as $\mu \downarrow$ because the UC firm's reaction to the race dominates. In addition, a higher X makes FC pre-emption more likely, and it is driven by the increasingly aggressive response of the FC firm to its UC rival.

In Figure 2, (b).i shows that turning off the project scalability does not change the best responses of the two firms qualitatively. Plot (d).i confirms that firms do not respond to their rivals' strategies without project accelerability. More importantly, technology parameters, regardless of whether scale or speed related, all motivate both firms to respond more actively to their rival's investment decisions. This can be seen from the more curved $R^*_{uc}(R_{fc})$ and an increased slop of $R^*_{fc}(R_{uc})$. As I change the tech-parameters for both firms simultaneously, their positive effects on the UC firm's best response dominate those on the FC firm's best response, making the UC pre-emption more likely as a result.

3.3. Race Between Two FC Firms

In an *FCFC race*, both firms in the R&D race are financially constrained. This is relevant for races between innovative firms that already have some products that generate cash flows but are still facing large frictions in the financial market possibly because of information asymmetry. These firms can be at the forefront of new technologies and have the human capital that is essential for making a breakthrough in the relevant technology area, but the huge technological uncertainties deter external investors from getting involved. The firms are aware of the financing restrictions of the race participants, making the game a strategic one. As a firm determines the R&D investment rate R_i, it considers the direct impact of R_i on its own success rate λ_i, the discovery scale u_i, and the probability of reaching the innovation success before running out of funds (i.e., $\mathbb{E}\mathbb{1}\{\tau_{i,d} < \tau_{i,c}\}$). In addition, the firm considers the indirect impact of R_i through its rival firm's investment decision, that is, $R_{-i}(R_i)$, taking into account the similar considerations of its rival in choosing R_{-i}. Intuitively, both firms' assets-in-place cash flows (X_1 and X_2) affect each firm's project value, and I assume X_1 and X_2 are independent.

The race starts if both firms decide to invest in the innovative project upon the arrival of such an R&D

opportunity. The project development continues until a firm reaches a first discovery (before running out of money) or both firms drop out of the race because of financial constraints, whichever happens first. If one firm runs out of the internal funds before either firm's project reaches discovery, then the competition ends because this firm has to drop out of the race. After that, the other firm keeps investing in the project until it faces shortage of funds and must abandon the project, or its R&D project becomes successful, whichever happens first. I keep the assumption that both firms cannot adjust their investment intensities throughout the process.

Two firms' problems are symmetric. Take Firm 1's investment decision as an example. I use the Hamilton-Jacobi-Bellman (HJB) equation with boundary conditions to characterize the dynamics of project values at the optimum. The HJB equation on Firm 1's project value V_1 can be written as

$$rV_1dt = \mathbb{E}\mathscr{D}V_1 - R_1dt + \lambda_1 \times (u_1 - V_1)dt + \lambda_2 \times (0 - V_1)dt, \tag{15}$$

where V_1 represents $V_1(X_1, X_2 \mid R_1, R_2)$; $\mathbb{E}\mathscr{D}V_1 = ((1/2)(\partial^2 V_1/\partial X_1^2)\sigma_1^2 X_1^2 + (1/2)(\partial^2 V_1/\partial X_2^2)\sigma_2^2 X_2^2 + (\partial V_1/\partial X_1)\mu_1 X_1 + (\partial V_1/\partial X_2)\mu_2 X_2)dt$, and there are no cross terms as a result of the independence assumption of the two firms' AIP cash flows. Because the firms are financially constrained, I simplify the analysis of $V_1(X_1, X_2)$ by focusing on the domain of $(X_1 \in [R_1, +\infty], X_2 \in [R_2, +\infty])$ while setting $V_1(X_1, X_2) = 0$ for any $X_1 < R_1$. Using the definition of α in Lemma 1 of the FC monopoly, with added subscript i, that is, $\alpha_i = 1/2 - \mu_i/\sigma_i^2 - \sqrt{(1/2 - \mu_i/\sigma_i^2)^2 + 2(\lambda_i + r)/\sigma_i^2}$, and define V_i^m, V_i^d, α_i' in the same way as Lemma 3 of the FCUC race, Four Dirichlet boundary conditions on the partial differential equation of Equation (15) can be written as[18]:

$$V_1(X_1 = R_1, X_2 > R_2) = 0, \tag{16}$$

$$V_1(X_1 > R_1, X_2 = R_2) = V_1^m \cdot \left(1 - \left(\frac{X_1}{R_1}\right)^{\alpha_1}\right), \tag{17}$$

$$\lim_{X_1\to\infty} V_1(X_1, X_2 > R_2) = V_1^d \cdot \left(1 - \left(\frac{X_2}{R_2}\right)^{\alpha_2'}\right) + V_1^m \cdot \left(\frac{X_2}{R_2}\right)^{\alpha_2'}, \tag{18}$$

$$\lim_{X_2\to\infty} V_1(X_1 > R_1, X_2) = V_1^d \cdot \left(1 - \left(\frac{X_1}{R_1}\right)^{\alpha_1'}\right). \tag{19}$$

The explanations for the boundary conditions are linked to the previous sections of the paper. First, a project is worthless if the firm is forced to abandon it at $X_1 = R_1$ (see Equation (16)). Second, the project value equals that of an FC monopoly firm when the rival drops out of the race because of financial constraints, that is, $X_2 = R_2$ (see Equation (17), and it resembles Equation (6) of Lemma 1). Third, the project value equals that of a UC duopoly firm in an FCUC race when the firm's internal funds are so high as if it is not financially constrained, that is, $X_1 \to \infty$ (see Equation (18), and it resembles Equation (13) of Lemma 3). Fourth, the project value equals that of an FC duopoly firm in an FCUC race if its rival is not concerned of financial constraints, that is, $X_2 \to \infty$ (see Equation (19), and it resembles Equation (12) of Lemma 3). Firm 2's problem can be represented similarly.

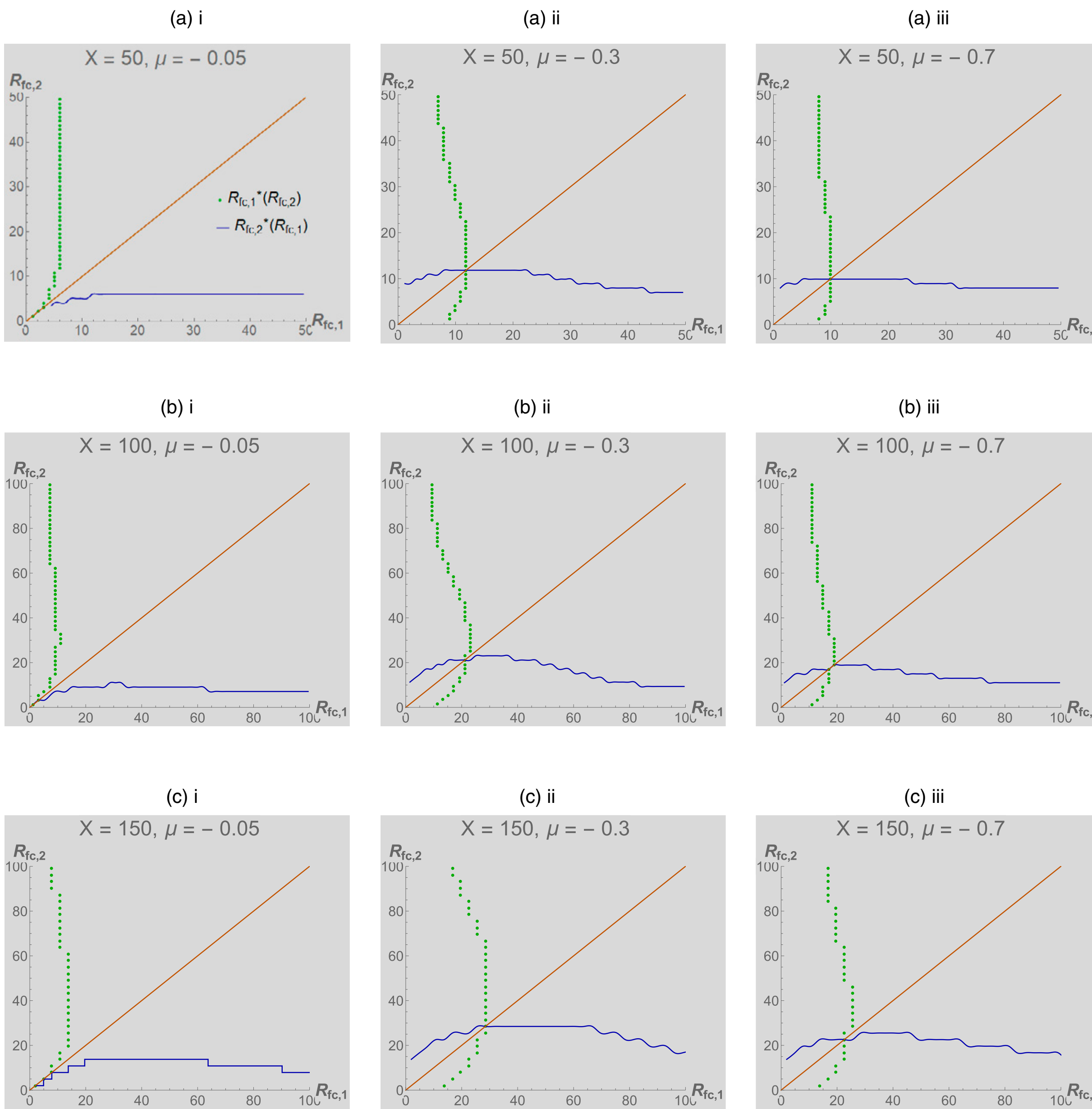
I use numerical methods to solve the PDEs of project values V_1 and V_2 given (R_1, R_2) and then obtain firms' responses to the investment rate of their rivals by finding the strategies that maximize project values. The details are described in Appendix A.11. The exercises performed suggest that the fixed point for firms' best response correspondences exists and is unique under a wide range of model parameters. Figures 3 and 4 illustrate some examples of FCFC equilibria as I change the AIP cash flow parameters (X and μ), and the tech-parameters (A, β, η, and γ) one at a time for both FC firms in the race.

Several interesting observations emerge from these exercises and many other unreported ones. First, all plots show that both firms have hump-shaped best responses to their rivals' investment intensities if $\lambda' > 0$ (as long as X_1 and X_2 are not too large to make firms unconstrained). They suggest that FC firms respond to their FC rivals in a similar way to how a UC firm responds to an FC rival in an FCUC race. From many numerical exercises that allow X_1 and X_2 to differ, I conjecture that it is the rival firm's financial constraints that induce the hump-shaped response. Second, unlike in an FCUC race where the hump-shaped $R_{uc}^*(R_{fc})$ becomes less curved as the AIP cash flow declines faster ($\mu \downarrow$), the curvature of $R_{fc,1}^*(R_{fc,2})$ in the FCFC race does not respond monotonically with respect to μ. It becomes more curved as the rate of decline increases first ($\mu \downarrow$) but less curved as the cash flow drops more sharply ($\mu \downarrow\downarrow$). This nonmonotonicity is tied to the fact that the reacting firm itself is financially constrained. Third, similar to an FCUC race, higher values of tech-parameters curve up the hump of the best responses by providing a stronger incentive to invest in the project. However, because of the firm's own constraints, the positive effect of the tech-parameters on the response in the FCFC race is not as significant as it is for a UC firm in an FCUC race.

3.4. Equilibrium Investment and Values: Comparison Among Races

To compare R&D intensities in the races that are analyzed in Sections 3.1 to 3.3, I show the best responses and equilibria of the three races at the baseline in Figure 5. The figure (and many unreported numerical exercises) reveal two distinct patterns of duopoly R&D races on accelerable projects. (1) Regardless of a firm's financial (un)constraints, competing against a UC rival leads to more intensive R&D investment in

Figure 3. (Color online) Best Responses and the Equilibrium in an FCFC Race: AIP Cash Flow Changes

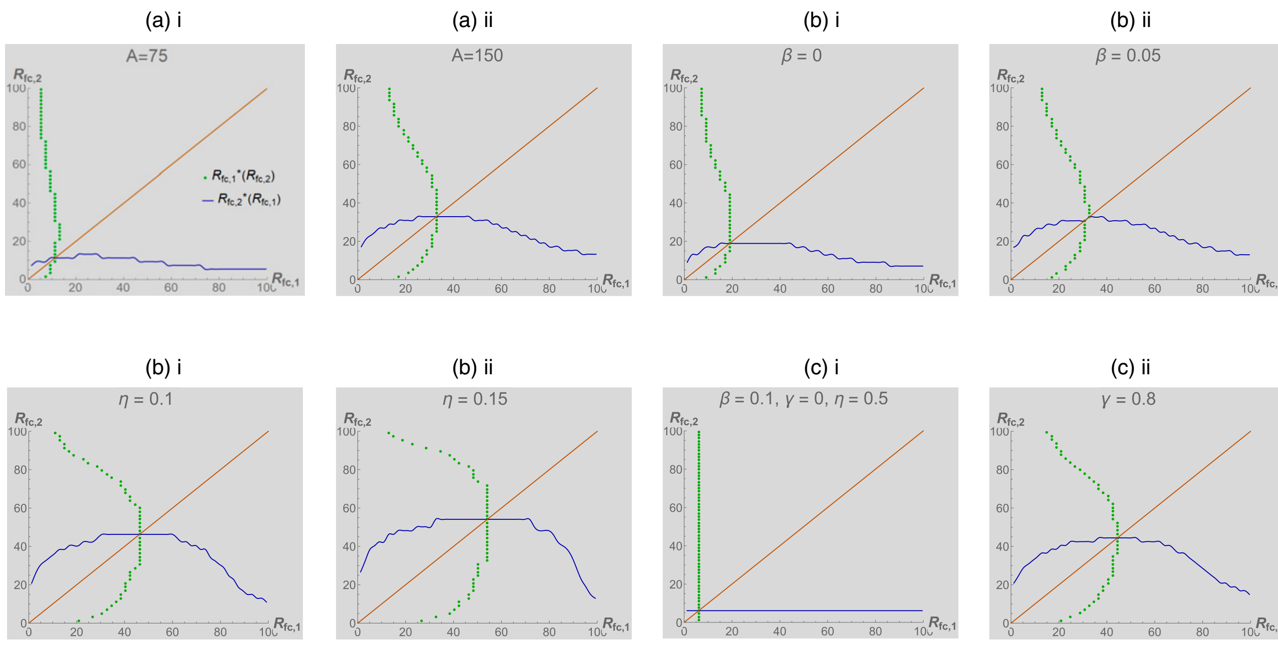


Notes. All model parameters values follow Table 1, except the ones labeled in the subplots. The blue lines plot Firm 2's best response to the investment intensity of Firm 1, that is, $R^*_{fc,2}(R_{fc,1})$. The green lines plot $R^*_{fc,1}(R_{fc,2})$. The orange lines represent $R_{fc,1} = R_{fc,2}$. $R_{fc,1}$ is on the x axis, and $R_{fc,2}$ is on the y axis.

equilibrium than competing against an FC rival. A firm has to forgo any potential profits associated with the project once its UC rival, which never abandons the project, makes a discovery. Therefore, the firm is motivated to expedite its discovery to lower the probability of losing the contest. (2) A firm responds in a hump-shaped fashion to the investment rate of its FC rival. As an FC rival firm increases its investment intensity and burns its internal funds faster, it is perceived to have a higher likelihood of project abandonment. Consequently, competing head-to-head against such a rival becomes less appealing.

Table 2 lists the equilibrium investment rates and project values under different market structures (monopoly or in an R&D race) around the baseline parameters, depending on whether the project is only scalable (Panel A) or only accelerable (Panel B) or both (Panel C). Table 2 confirms that competition increases investment intensity, regardless of financial constraints, except for a UC firm in a race on a project with scalability alone. Table 2 also provides support to the intuition that competing against a UC rival is more fierce than competing against an FC rival, inducing more intense investment. For example, an FC firm invests at $R^*_{fc,fcuc} = 28.8$ (more than three times of

Figure 4. (Color online) Best Responses and the Equilibrium in an FCFC Race: Technology Parameter Changes



Notes. The blue lines plot Firm 2's best response to the investment intensity of Firm 1, that is, $R^*_{fc,2}(R_{fc,1})$. The green lines plot $R^*_{fc,1}(R_{fc,2})$. The orange lines represent $R_{fc,1} = R_{fc,2}$. $R_{fc,1}$ is on the x axis, and $R_{fc,2}$ is on the y axis. All model parameters values follow Table 1, except the ones labelled in the subplots.

its monopoly level) in an FCUC race, as opposed to investing at $R^*_{fc,fcfc} = 21.4$ (between two to three times of its monopoly level) in an FCFC race. A UC firm invests at $R^*_{uc,ucuc} = 35.5$ (more than five times of its monopoly level) in a UCUC race, as opposed to investing at $R^*_{uc,fcuc} = 30.1$ (about four times of its monopoly level) in an FCUC race. Meanwhile, Table 2 suggests the over-investment incentive from increasing the probability of project survival in the monopoly model can be overshadowed by competition. This is evident by the absence of FC pre-emption in the FCUC race of Panels B and C while the over-investment by an FC monopoly is present. Regarding values, firms experience a large drop (more than half at the baseline) in the project value as a result of competition, regardless of whether they are financially constrained or not. Such value drops are more significant if the race is against a UC rival compared to an FC rival.[19]

4. Model Discussion

4.1. Downward Jump Risk in AIP Cash Flow

The diffusion process associated with dZ_t in Equation (1) of the baseline represents uncertainty from a firm's daily operations, and σ^2 captures the conventional cash flow risk. However, innovative firms can often be exposed to a more extreme cash flow risk, which I call "jump risk" or "catastrophe risk."[20] Such risks are exogenous to firms' operations and are related to *obsolescence risk* of the innovation opportunity in some investment models (Hackbarth et al. 2014). In what

Figure 5. (Color online) All Three Races at the Baseline

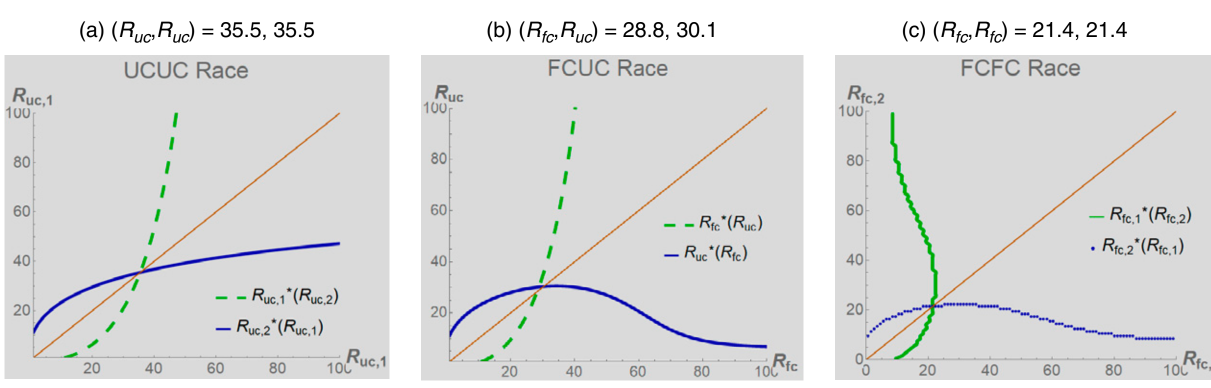


Table 2. Investment Intensities and Project Values

	Monopoly	UCUC race	FCUC race	FCFC race
Panel A: With project scalability alone ($\gamma = 0, \beta = 0.1, \eta = 0.5$): $\tau_d = 2$ (years)				
FC firm	($\mathbf{R_{fc} = 5.3}$, $V_{fc} = 97.1$)	n/a	(**5.9**, 51.2)	(**5.7**, 51.3)
UC firm	($\mathbf{R_{uc} = 6.0}$, $V_{uc} = 97.8$)	(**6.0**, 51.2)	(**6.0**, 51.3)	n/a
Panel B: with project accelerability alone ($\beta = 0$): $\tau_{d,fc} = 5$ (years), $\tau_{d,uc} = 5.7, \tau_{ucuc} = 1.9, \tau_{fcuc} = 2.1, \tau_{fcfc} = 2.6$.				
FC firm	($\mathbf{R_{fc} = 7.2}$, $V_{fc} = 45.9$)	n/a	(**24.7**, 21.9)	(**18.9**, 23.9)
UC firm	($\mathbf{R_{uc} = 6.0}$, $V_{uc} = 51.2$)	(**29.0**, 21.5)	(**25.2**, 23.1)	n/a
Panel C: With both accelerability and scalability: $\tau_{d,fc} = 4.7$ (years), $\tau_{d,uc} = 5.2, \tau_{ucuc} = 1.7, \tau_{fcuc} = 1.8, \tau_{fcfc} = 2.3$				
FC firm	($\mathbf{R_{fc} = 8.0}$, $V_{fc} = 47.4$)	n/a	(**28.8**, 22.3)	(**21.4**, 24.6)
UC firm	($\mathbf{R_{uc} = 6.8}$, $V_{uc} = 52.6$)	(**35.5**, 21.7)	(**30.1**, 23.8)	n/a

Notes. This table presents investment intensities R (in bold), project values V, and the expected (first) discovery time τ_d (in years) of the monopoly model in Section 2 and the race model in Section 3. The parameter values are set as in Table 1, if not otherwise specified. Multiple parameter values differ from the baseline in Panel A to have comparable magnitudes for the investment intensities in Panels B and C (but not comparable for values or the time to discover). n/a, not applicable.

follows, I examine the effect of a *jump risk* on an FC firm's investment rate and assume the AIP cash flow follows a combined geometric Brownian motion/jump process (as in chapter 5.B of Dixit and Pindyck 1994):

$$dX_t = \mu X_t dt + \sigma X_t dZ_t - X_t dq_1. \tag{20}$$

The component $-X_t dq_1$ captures an extreme downward jump risk on the AIP cash flow and represents a negative shock which wipes out all future cash flow from AIP. I assume the time of the potential catastrophic event follows an exponential distribution with the parameter λ_j, that is, it is expected in $1/\lambda_j$ years. When the catastrophe hits, an FC firm can no longer fund an ongoing R&D project and must terminate it. Consistent with the notion of a firm being unconstrained in the model, a UC firm is capable of raising funds through external financial markets and continuing the project, although it has no AIP left.

4.1.1. Jump Risk in the Monopoly Model. With the jump risk represented by $\lambda_j > 0$, Equation (A.48) in Appendix A.12 presents a monopoly firm's problem, and Lemma A.1 states the project values for an FC firm and a UC firm. Although the UC firm's project value remains the same as in Lemma1, λ_j changes V_{fc} in two ways. It raises the discount rate (Dixit and Pindyck 1994) and changes the project survival probability. Consistent with Merton (1976) and McDonald and Siegel (1986), the jump risk causes a sudden ruin of the project and effectively increases the interest rate. Furthermore, Proposition A.1 in Appendix A.12 shows that, unlike the ambiguous effect of cash flow risk σ, the jump risk always increases an FC monopoly firm's investment intensity R^*_{fc}, regardless of the project accelerability or scalability (but cannot be neither, as it is trivial).

Both the diffusion risk σ and jump risk λ_j of AIP cash flow can cause project termination for an FC firm, but the mechanisms differ. Lowering investment intensity delays project termination caused by the diffusion risk, but it does not impact the termination induced by the jump risk. On the contrary, a catastrophe risk in AIP effectively reduces the cost of financial constraints by lowering the concern of hitting the constraints early with a high level of investment. As a result, the jump risk motivates investment and can exacerbate the over-investment by an FC monopoly firm. For example, at the baseline as in Table 1, with a catastrophic event expected in 10 years (i.e., $\lambda_j = 0.1$), R^*_{fc} increases from 8.02 to 13.8, comparing with $R^*_{uc} = 6.79$ that remains the same after introducing the jump risk.

4.1.2. Jump Risk in an FCUC Race. I focus on the analysis of an FCUC race.[21] Lemma A.2 in Appendix A.12 presents project values in an FCUC race with the presence of a jump risk. Equation (A.56) shows that λ_j affects V_{fc} in the same way as the risk-free rate r; thus, it has the same positive effect on the FC firm's investment in the race as r on an FC monopoly (see Proposition 2). Besides affecting the discount rate, the jump risk has additional effects on V_{uc}. It effectively lowers the probability of the FC rival firm abandoning the project because of the diffusion risk. It also increases the UC firm's payoff in the scenarios where the FC rival would manage to make the discovery before the burnout of internal funds if the jump risk were absent but has to terminate the project because of the jump risk. A catastrophe pushes the FC rival out of the race, leaving the UC firm the only one working on the innovation project.

Through numerical exercises, I find that the jump risk on an FC competitor's cash flow changes the equilibrium investment rates mainly through altering the UC firm's response to its FC rival's investment. Figure B.1 shows comparable examples to Figure 5(b) with added jump risk with the intensity represented by λ_j. When the jump risk becomes more imminent ($\lambda_j \uparrow$), the UC firm acts less responsively to its FC rival, that is, the hump-shape of $R^*_{uc}(R_{fc})$ becomes less curved. Consequently, the jump risk makes the FC pre-emption more likely. For example, at the baseline as in Table 1, having a catastrophic event expected in 10 years (i.e., $\lambda_j = 0.1$), changes the equilibrium from $(\overline{R}_{fc} = 28.84, \overline{R}_{uc} = 30.12)$ to $(\overline{R}_{fc} = 29.4, \overline{R}_{uc} = 24.4)$ in the FCUC race.

4.2. Winner-Takes-All or Not

Winner-takes-all is a standard assumption in models on R&D investment with competition (Weeds 2002). It has the natural interpretation from the patent race literature that multiple firms strive for developing a new technology, and the successful innovation by one firm leads to a patent with exclusive rights of using the technology and eliminates all possible profits for the other firms. I relax this assumption from Section 3 and examine if the results change qualitatively.[22]

Suppose a firm does not have to drop out of the race when its rival reaches the discovery first. Instead, the firm can carry on investing in the project with its chosen investment intensity, and the remaining time to success still follows the same exponential distribution with parameter λ_i. Upon making the first discovery, the winner earns a monopoly profit of π_i^m per period until a second discovery is made by the other firm, after which the two firms split the market and each earns a duopoly profit flow of π_i^d. Firms make decisions on R_i simultaneously at the project arrival, and upon one firm reaching the first success, the other firm decides whether to continue pursuing the project or not. The competition between the two firms is similar to a typical capacity competition. In addition, if the second firm is an FC firm and it runs out of money before the discovery, then the competition ends with the first firm being the monopoly forever. Denote $u_i^m = \pi_i^m / r$ and $u_i^d = \pi_i^d / r$. Suppose the monopoly and duopoly profits are related to firms' investment decisions, and $u_i^m(R_i) = A^m \cdot R_i^\beta$ and $u_i^d(R_i) = A^d \cdot R_i^\beta$, with $A^d < A^m$.

4.2.1. UCUC Competition Without Winner-Takes-All.

In a UCUC competition with the winner-takes-all assumption removed, a firm optimally carries on investing after a rival firm makes the first discovery if the continuation value is positive, that is, continue investing if $u_i^d \lambda_i - R_i > 0$, and stop otherwise. If stopping is optimal, then the equilibrium is the same as in the setting of winner-takes-all. If continuation is optimal, then Firm 1's problem can be written as

$$V_1 = \sup_{R_1} \mathbb{E}\left\{ \int_0^{\tau_1} -R_1 e^{-rt} dt + 1_{\{\tau_1 < \tau_2\}} e^{-r\tau_1} u_1^m \right.$$
$$\left. + 1_{\{\tau_1 < \tau_2\}} e^{-r\tau_2} (u_1^d - u_1^m) + 1_{\{\tau_1 > \tau_2\}} e^{-r\tau_1} u_1^d \right\}. \quad (21)$$

The first term differs from the winner-takes-all case setting in Equation (8) as the R&D expenditure is paid until the firm's own discovery, regardless of which firm makes the first discovery. The second term is the same as Equation (8). The third and fourth terms are absent in the winner-takes-all setting, with the third term describing the reduction in the future payoff upon a rival's discovery and the last term describing the value of duopoly profits conditional on it is the second to discover.

Lemma A.3 in Appendix A.13 presents the project values. One immediate implication is that, in a UCUC competition where the order of discoveries does not matter (i.e., $u^m = u^d$ in the model), the firm's project value and its problem are identical to a UC monopoly, and firms do not respond to the investment rates of their rivals. Instead, the equilibrium investment rates remain at the monopoly level. Proposition A.2 in Appendix A.13 shows that the best responses in the UCUC competition without winner-takes-all are downward-sloping on accelerable projects, which is opposite to the winner-takes-all setting (see Proposition 3).[23] The difference is caused by a much higher possibility of reaping the gains of investment if it is profitable to be the second, as well as the shortened period to earn monopoly profits for the first firm that makes the discovery. As a result, the competition becomes less fierce, and a rival firm's investment no longer increases the marginal return of the firm's own investment, breaking down the complementarity between the investment of the two firms. From this striking comparison of firms' responses in a simple UCUC race, I argue that the winner-takes-all setting is key in driving the results of the race model, making the model most applicable to R&D investment.

Figure B.2 illustrates a few examples of best responses and equilibria of a UCUC race without the winner-takes-all assumption. They show how the equilibrium investment rates depend on the relative magnitudes of the monopoly and duopoly profit parameters A^m and A^d, as well as whether the project is scalable or accelerable or both. Plots (a)–(c) confirm that the larger the difference is between A^m and A^d, the lower the equilibrium R&D rates are, and the more responsive a firm is to the investment rate of its rival. Comparing with the winner-takes-all scenario (see Plot (d) as an equivalent graph), both firms invest less intensively without winner-takes-all. Plot (e) demonstrates that turning off project scalability for accelerable projects reduces equilibrium investment but does not eliminate the two firms' interactions. Plot (f) confirms the intuition that firms do not respond to the investment rate of its rival if $\lambda' = 0$.

4.3. Costly Access to External Financing

The extreme assumption on firms' financial constraints, coupled with the policy of no earnings accumulation, force an FC firm to drop an ongoing R&D project once the AIP cash flow drops below the R&D intensity ($X_t <$

R). To understand the impact of this assumption, I modify the monopoly model and assume the firm can finance its cash flow shortfall at a cost $h(X_t; R, \delta)$, with the cost parameter δ measuring the extent to which external financing is expensive.[24] Call this a firm with costly external financing (a CEF firm). The goal is to check if the firm's R&D investment is monotonic in δ. If so, it provides some evidence that firms' investment in a more realistic setting with costly external finance is between the two polar cases of a UC and an FC firm.

The firm's decision in this modified model is twofold: (1) the investment intensity R and (2) the abandonment threshold $\underline{X}$ at which the firm stops the project. Before the project abandonment, the firm pays out dividend $X_t - R$ if it is positive and finances the gap $R - X_t$ otherwise. Denote τ_d and τ_c as the Poisson discovery time and project abandonment time ($\tau_c = \inf\{t : X_t \le \underline{X}\}$); the firm's problem can be written as the expression of (A.68) in Appendix A.14. The project value is

$$V(X_t) = \sup_{\underline{X}, R} \mathbb{E}\left\{\int_0^{\tau_c \wedge \tau_d} (-R - h(X_t)\mathbb{1}_{\{X_t < R\}}) \cdot e^{-rt} \mathrm{d}t + \mathbb{1}_{\{\tau_d < \tau_c\}} e^{-r\tau_d} \tilde{u}\right\}. \tag{22}$$

Lemma A.4 in Appendix A.14 presents the conditions on a CEF firm's optimal investment. Proposition A.3 in the appendix confirms the intuition that the firm delays its project abandonment further if it can access cheaper external financing. Table B.1 lists the optimal $(R, \underline{X})$ for various combinations of the AIP cash flow X and the external financing cost parameter δ, with a cost function of $h = \delta \cdot (R - X_t)^2$. All other parameter values are set as in Table 1. Consistent with the comparative statics of an FC monopoly, the column of $\delta = 1$ or $\delta = 2$ shows that a CEF firm increases its investment intensity as AIP cash flow increases. New from this analysis, the rows in Table B.1 show that the CEF firm increases investment intensity with the cost of external financing at the baseline, and $R_{uc} < R_{cef} < R_{fc}$.

4.4. Some Other Assumptions

Firms do not save in the model, which follows earlier literature on corporate investment such as Hennessy et al. (2007). This assumption of no liquidity management helps keep the model parsimonious and trackable, arguably without compromising the main insights. Undeniably, cash holding policy and liquidity management are important (Brown and Petersen 2011; Bolton et al. 2013, 2019; Hugonnier et al. 2015) and relevant for R&D firms (Schroth and Szalay 2010, Ma et al. 2020). I argue that the qualitative results in both the monopoly model and the race model are likely to hold if the firm(s) can save, with probable changes in magnitudes. In the extreme case where the cash-carrying cost is prohibitively high, shareholders prefer not to save in the form of retaining cash inside the firm. As the cost or retaining cash reduces, a financially constrained firm accumulates some internal funds to increase the likelihood of project survival and invest at a rate closer to an unconstrained firm. However, such cash accumulation does not completely eliminate the FC firm's incentive to speed up the project development if it is accelerable, because saving only delays the possible burn-out but cannot eliminate it. Plus, using savings to invest in R&D, as opposed to receiving dividends each period, is risky for shareholders. Unlike capital investment, an R&D process can have huge uncertainty on its maturity. Saving all internal cash flows and spending them on the R&D investment may lead to nothing in return, especially in a race. The analysis of costly external financing in Section 4.3 shows a limited utilization of external financing, which mirrors the limited cash accumulation. If the agency cost of free cash flow (Jensen 1986) is introduced, which is reasonable given the intangibility nature of the R&D capital, it can further dampen the incentive to save.

The analyses throughout the paper emphasize the investment intensity decision, instead of the project initiation. It is because the latter decision is straightforward with the traditional net present value rule. If the innovative project requires an initial investment cost $\kappa > 0$, then the firm carries out the project upon its availability if the project value V exceeds the fixed cost ($V > \kappa$). Because the value of a project depends on the firm's financial constraints among other things, this entry cost affects an FC firm differently than a UC firm.

5. Final Remarks

Using a parsimonious continuous-time model, I examine the interactive impacts of financial constraints and competition on corporate R&D investment. The over-investment (in the R&D level decision) caused by financial constraints in a monopoly setting (Corollary 1) complements earlier work that shows acceleration in the investment timing decision (Boyle and Guthrie 2003, Lyandres and Zhdanov 2010, Bolton et al. 2019). Moreover, the R&D race model in the paper uncovers novel patterns of firms' responses to its rival's R&D investment. The increasing response to an unconstrained firm's investment intensity and the hump-shaped response to a financially constrained firm's investment intensity can be used to explain the FC pre-emption in the equilibrium of an FCUC race (Corollary 2). Firms' heterogeneity in their financing capabilities is thus an important factor in understanding R&D races, and this model adds to others that examine the effect of competition on investment (Weeds 2002, Grenadier 2002, Novy-Marx 2007).

By incorporating both project accelerability (standard in the patent race literature) and project scalability (widely considered in the investment literature), this paper shows that the characteristics of innovation projects can significantly impact a firm's R&D decisions with the presence of financial frictions and competition (Propositions 1–4). This study calls for more careful empirical investigations on R&D investment by considering the characteristics of the innovation technology and financial constraints of firms in the race.

A few directions for related future research can be promising. One regards endogenous choice of innovation technology by firms with different financing frictions, especially if firms can trade off between the discovery speed and the innovation scale. Another potential regards the possible extension to a general equilibrium setting in which firms with varying financing frictions compete in R&D races, and the examination of the welfare implications of financial constraints may be fruitful. Studying the optimal liquidity management policy for R&D firms that face financial constraints and competition is also important.

Acknowledgments

This paper is based on two chapters of my doctoral dissertation at the University of Maryland. I am deeply grateful to my committee members Richmond Mathews, Mark Loewenstein, Pete Kyle, and Liu Yang for valuable advice and continual encouragement. The paper has benefited greatly by the comments from the anonymous referee, anonymous associate editor, department editor (Gustavo Manso) of *Management Science*, as well as discussions with Laurent Frésard, Daniel Vincent, Nagpurnanand Prabhala, Gerard Hoberg, Vojislav Maksimovic, Wei Li, Wen Chen, Cecilia Bustamante, Julien Cujean, Rajshree Agarwal, Brent Goldfarb, Kebin Ma, Lailai Zhu, Katie Moon, Neng Wang, Graeme Guthrie, Andrea Gamba, Elizabeth A.Whalley, Shrihari Santosh, Christopher Mace, Ayako Yasuda, Kose John, John Thanassoulis, Erwan Morellec, Jonathan Berk, Vladimir Vladimirov, Stefano Rossi, Jean-Charles Rochet, Tanju Yorulmazer, Hui Chen, Jian Feng, and seminar participants at numerous institutions and conferences.

Appendix A. Proofs and Derivations

A.1. Proof of Lemma 1

The proof for Lemma A.1 in Appendix A.12 is more general. Setting $\lambda_j = 0$ in Appendix A.12 gives the proof.

A.2. Proof of Proposition 1

Proof. For the UC monopoly, if $u' = \lambda' = 0$, then R only appears in the numerator and as a cost term. Thus, the optimal R is the lowest R needed to maintain the project, and it is not affected by any parameters in the model. Otherwise, I use monotone comparative statics on the optimal R_{uc}. I first take a log transformation for the objective function (project value), which does not change R^*_{uc}. The range of the investment intensity R_{uc} in (7) satisfies the ascendancy property with regard to all of the parameters analyzed. Next, check whether the increasing differences of $\ln V_{uc}(R_{uc}, s)$ hold. I omit the subscripts from now on in this proof:

$$\ln V(R) = \ln(u\lambda - R) - \ln(\lambda + r) \Rightarrow \frac{\partial \ln V}{\partial R} = \frac{(u\lambda - R)'}{u\lambda - R} - \frac{\lambda'}{\lambda + r}, \tag{A.1}$$

and $0 < \beta + \gamma < 1$, $\partial u'/\partial A = (\beta/R)(\partial u/\partial A)$, $\partial u'/\partial \beta = (\beta/R)(\partial u/\partial \beta) + u/R$.

Regarding A and β:

$$\begin{aligned}\frac{\partial^2 \ln V}{\partial R \partial A} &= \frac{\left(\lambda' \dfrac{\partial u}{\partial A} + \lambda \dfrac{\partial u'}{\partial A}\right)(\lambda u - R) - (\lambda u - R)' \lambda \dfrac{\partial u}{\partial A}}{(\lambda u - R)^2} \\ &= \frac{\lambda}{(\lambda u - R)^2} \frac{\partial u}{\partial A} \cdot (1 - \beta - \gamma) > 0,\end{aligned} \tag{A.2}$$

$$\frac{\partial^2 \ln V}{\partial R \partial \beta} = \frac{\lambda}{(\lambda u - R)^2} \frac{\partial u}{\partial \beta} \cdot (1 - \beta - \gamma) + \frac{u\lambda}{R}(\lambda u - R)^{-1} > 0 \quad \text{if} \quad \beta \neq 0. \tag{A.3}$$

Regarding η:

$$\frac{\partial^2 \ln V}{\partial R \partial \eta} = \left(\frac{u(1 - \beta - \gamma)}{(\lambda u - R)^2} - \frac{r\gamma}{R(\lambda + r)^2}\right) \times \frac{\partial \lambda}{\partial \eta}. \tag{A.4}$$

If $\lambda' = 0$, thus $\gamma = 0$, then $\partial^2 \ln V/\partial R \partial \eta > 0$. Otherwise, Expression (A.4) cannot be signed.

Regarding γ:

$$\frac{\partial^2 \ln V}{\partial R \partial \gamma} = \left(\frac{u \cdot (1 - \beta - \gamma)}{(\lambda u - R)^2} - \frac{r\gamma}{R(\lambda + r)^2}\right) \cdot \frac{\partial \lambda}{\partial \gamma} + \frac{u\lambda r + \lambda R}{R(u\lambda - R)(\lambda + r)}. \tag{A.5}$$

If $\lambda' = 0$, thus $\gamma = 0$, $\partial^2 \ln V/\partial R \partial \gamma$ is not defined. Otherwise, Expression (A.5) cannot be signed. If $\lambda' > 0 \Rightarrow \gamma > 0$, then $\partial^2 \ln V/\partial R \partial \eta > 0$ implies $\partial^2 \ln V/\partial R \partial \gamma > 0$ because the latter has an additional and positive term.

Regarding r: If $\lambda' = 0$ and $u' > 0$, then $u'\lambda = 1 \Rightarrow R^*_{uc} = (1/A\beta\lambda)^{\frac{1}{\beta - 1}}$, so $\partial R^*_{uc}/\partial r = 0$. Otherwise

$$\frac{\partial^2 \ln V}{\partial R \partial r} = \frac{\lambda'}{(\lambda + r)^2} > 0 \quad \text{if} \quad \lambda' > 0. \tag{A.6}$$

According to Topkis's theorem (Topkis 1978; or theorem 2.3 in Vives 1999 or Athey 2002), the increasing differences of the $\ln V(R; s)$, which is an increasing transformation of $V(R; s), s \in \{A, \eta, \beta, \gamma\}$, coupled with the ascendancy property of the action domain $[\underline{R}(s), \bar{R}(s)]$, gives us the comparative statics in the proposition. □

A.3. Proof of Proposition 2

Proof. Define

$$\zeta = 1 - \left(\frac{X}{R}\right)^{\alpha}, Ⓐ = \left(\frac{1}{2} - \frac{\mu}{\sigma^2}\right)^2 + \frac{2(\lambda + \mathrm{r})}{\sigma^2}. \tag{A.7}$$

Use α' as the notation for the first- order derivative with regard to R (as opposed to that in Lemma 3),

$$\alpha = \frac{1}{2} - \frac{\mu}{\sigma^2} - \sqrt{Ⓐ}, \quad \alpha' = -\frac{\lambda'}{\sigma^2 \sqrt{Ⓐ}}. \tag{A.8}$$

If $u' = \lambda' = 0$, then R_{fc} appears only in the cost term and in ζ of V_{fc}, both having a negative impact on V_{fc}. Thus, $R^*_{fc} = \underline{R}$ is optimal, and it is not influenced by any parameters in the model. Otherwise, the range of R_{fc} in (7) satisfies the ascendancy property with regard to all of the parameters analyzed. I use the monotone comparative statics and check the cross derivatives of the log-transformation of the project value $\ln V_{fc}(R,s)$. I omit the subscripts from this point on in the proof.

$$\ln V(R) = \ln(u\lambda - R) - \ln(\lambda + r) + \ln\zeta \Rightarrow \frac{\partial \ln V}{\partial R} = \frac{(u\lambda - R)'}{u\lambda - R} - \frac{\lambda'}{\lambda + r} + \frac{\zeta'}{\zeta} \tag{A.9}$$

Regarding A, β: Denote $f = \zeta'/\zeta$. Expression (A.9) differs from Expression (A.1) only by f, thus

$$\frac{\partial f}{\partial A} = \frac{\partial f}{\partial \beta} = 0 \Rightarrow \frac{\partial^2 \ln V}{\partial R \partial A} > 0, \text{ and } \frac{\partial^2 \ln V}{\partial R \partial \beta} > 0 \quad \text{if} \quad u' > 0. \tag{A.10}$$

Regarding X: Given $\zeta' = (\zeta - 1)(\alpha' \ln X/R - \alpha/R)$, $\partial\zeta/\partial X = -\alpha/X(X/R)^{\alpha}$, $\partial\zeta'/\partial X = \zeta - 1/X[\alpha'(\alpha \ln X/R + 1) - \alpha^2/R]$, and $\zeta - 1 < 0$:

$$\frac{\partial f}{\partial X} = \frac{\frac{\partial \zeta'}{\partial X}\zeta - \zeta'\frac{\partial \zeta}{\partial X}}{\zeta^2} = \frac{(\zeta - 1)\left(\alpha'\left(\alpha \ln \frac{X}{R} + \zeta\right) - \frac{\alpha^2}{R}\right)}{\zeta^2 X} > 0$$

$$\Rightarrow \text{sign}\left(\frac{\partial^2 \ln V}{\partial R \partial X}\right) = \text{sign}\left(\frac{\alpha^2}{R} - \alpha'\left(\alpha \ln \frac{X}{R} + \zeta\right)\right). \tag{A.11}$$

If $\lambda' = 0 \Rightarrow \alpha' = 0 \Rightarrow \partial^2 \ln V/\partial R \partial X > 0$. If $\lambda' > 0 \Rightarrow \alpha' < 0$, then as $X \uparrow\uparrow$, $\partial^2 \ln V/\partial R \partial X < 0$, and as $X \to R$, $\partial^2 \ln V/\partial R \partial X > 0$.

Regarding μ: notice $\partial\alpha/\partial\mu = 1/\sigma^2[\textcircled{A}^{-1/2}(1/2 - \mu/\sigma^2) - 1] < 0$, and

$$\frac{\partial \alpha'}{\partial \mu} = \frac{\lambda'}{2\sigma^2}\textcircled{A}^{-\frac{2}{3}} \times \underbrace{\frac{\partial \textcircled{A}}{\partial \mu}}_{\oplus \text{ if } \mu \le \frac{\sigma^2}{2}} > 0, \quad \frac{\partial \zeta}{\partial \mu} = \frac{\partial \zeta}{\partial \alpha}\frac{\partial \alpha}{\partial \mu} > 0, \tag{A.12}$$

$$\frac{\partial^2 \ln V}{\partial R \partial \mu} = \frac{\partial^2 \ln \zeta}{\partial R \partial \mu} = \frac{\partial \frac{\zeta'}{\zeta}}{\partial \mu} = \frac{\frac{\partial \zeta'}{\partial \mu}\zeta - \zeta'\frac{\partial \zeta}{\partial \mu}}{\zeta^2}. \tag{A.13}$$

The numerator of the expression after the last equal sign of Expression (A.13) can be rewritten as

$$\left(\alpha' \ln \frac{X}{R} - \frac{\alpha}{R}\right)\frac{\partial \zeta}{\partial \mu} + \zeta(\zeta - 1)\frac{\partial\left(\alpha' \ln \frac{X}{R} - \frac{\alpha}{R}\right)}{\partial \mu}. \tag{A.14}$$

In Equation (A.14), <0 if X is large and $\lambda' > 0$. It is because the first term is negative when $X \uparrow\uparrow$, and the second term

$$\zeta(\zeta - 1)\frac{\partial\left(\alpha' \ln \frac{X}{R} - \frac{\alpha}{R}\right)}{\partial \mu} = \zeta(\zeta - 1)\ln \frac{X}{R}\frac{\partial \alpha'}{\partial \mu} - \frac{\zeta(\zeta - 1)}{R}\frac{\partial \alpha}{\partial \mu} < 0. \tag{A.15}$$

If $\lambda' = 0 \Rightarrow \alpha' = 0$, and $u' > 0$, then the numerator of the expression after the last equal sign of (A.13) can be rewritten as

$$-\frac{\alpha}{R}\frac{\partial \zeta}{\partial \alpha}\frac{\partial \alpha}{\partial \mu} - \frac{\zeta(\zeta - 1)}{R}\frac{\partial \alpha}{\partial \mu} = -\frac{1}{R}\frac{\partial \alpha}{\partial \mu}(\zeta - 1)\left(\alpha \ln \frac{X}{R} + \zeta\right) > 0 \text{ if } X \text{ is large.} \tag{A.16}$$

Regarding σ^2: for simplicity replace σ^2 with the notation of σ for this comparative statics analysis. Only the last term of Equation (A.9) is relevant and the numerator of the expression on $\partial^2 \ln V/\partial R \partial \sigma$, which is similar to Equation (A.14), can be rewritten as

$$\left(\alpha' \ln \frac{X}{R} - \frac{\alpha}{R}\right)\frac{\partial \zeta}{\partial \sigma} + \zeta(\zeta - 1)\frac{\partial\left(\alpha' \ln \frac{X}{R} - \frac{\alpha}{R}\right)}{\partial \sigma}.$$

A set of sufficient conditions for the first term to be negative are if X is large, and $\mu < \textcircled{A}^{-1/2} \cdot (1/2 - \mu/\sigma - \lambda - r)$. A sufficient condition for the second term to be negative is $\textcircled{A} > (1/2 - \mu/\sigma) - (\lambda + r)/\sigma$. These conditions can be satisfied at the same time, and when they are satisfied $\partial^2 \ln V/\partial R \partial \sigma < 0$.

Regarding r: Focus on the last term of Expression (A.9):

$$\frac{\partial f}{\partial r} = \frac{\frac{\partial \zeta'}{\partial r}\zeta - \zeta'\frac{\partial \zeta}{\partial r}}{\zeta^2} = \frac{\left(\alpha' \ln \frac{X}{R} - \frac{\alpha}{R}\right)\frac{\partial \zeta}{\partial r} + \zeta(\zeta - 1)\frac{\partial\left(\alpha' \ln \frac{X}{R} - \frac{\alpha}{R}\right)}{\partial r}}{\zeta^2}, \tag{A.17}$$

if $\lambda' = 0 \Rightarrow \alpha' = 0$, the numerator of Equation (A.17) can be rewritten as

$$\frac{\zeta - 1}{R}\left[-\alpha \ln \frac{X}{R} + \zeta\right]\frac{\partial \alpha}{\partial r} > 0, \tag{A.18}$$

with $\partial\alpha/\partial r < 0$ and $\partial\zeta/\partial r = \partial\zeta/\partial\alpha \times \partial\alpha/\partial r = (\zeta - 1)\ln X/R \times \partial\alpha/\partial r$ used in the derivation. Using monotone comparative statics, and together with Proposition 1, $\partial R^*_{fc}/\partial r > 0$ for $\lambda' = 0$. This is in contrast with $\partial R^*_{uc}/\partial r = 0$ when $\lambda' = 0$.

If $\lambda' > 0$, by using $\alpha' < 0, \partial\zeta/\partial r > 0, \partial\alpha'/\partial r = \lambda'/\sigma^4 \textcircled{A}^{-32} > 0$, the two terms associated with α' in the numerator of Equation (A.17) can be signed:

$$\alpha' \ln \frac{X}{R} \cdot \frac{\partial \zeta}{\partial r} + \zeta(\zeta - 1)\frac{\partial \alpha'}{\partial r} \cdot \ln \frac{X}{R} < 0. \tag{A.19}$$

Because $\partial\alpha/\partial r = -(1/2)\textcircled{A}^{-1/2}(\partial\textcircled{A}/\partial r)$, then $\partial\alpha'/\partial r = -\lambda'/\sigma^2 \cdot \partial\alpha/\partial r\, \textcircled{A}^{-1}$; thus, Equation (A.19) can be rewritten as

$$\ln \frac{X}{R}(\zeta - 1)\frac{\partial \alpha}{\partial r}\left[\alpha' \ln \frac{X}{R} - \zeta\frac{\lambda'}{\sigma^2}\textcircled{A}^{-1}\right] < 0. \tag{A.20}$$

The sign of (Equation (A.20) + Equation (A.18)) gives us the sign of the effect of r on increasing the marginal return of R_{fc} from the project survival probability ζ term. The summation of those two terms is

$$\frac{\partial \alpha}{\partial r}(\zeta - 1)\left(\frac{1}{R}\left[-\alpha \ln \frac{X}{R} + \zeta\right] + \alpha' \ln \frac{X}{R} - \zeta\frac{\lambda'}{\sigma^2}\textcircled{A}^{-1}\right). \tag{A.21}$$

Given $\lambda' = \lambda\gamma/R$, $\lambda/\sigma^2\textcircled{A}^{-1} < 1/2$, $\textcircled{A}^{1/2} > -\alpha$, and replace α' by Equation (A.8), the previous expression can be rearranged to have the sign depend on $(-\alpha \ln X/R + \zeta) \cdot (1/R - \gamma/2R)$, which is positive as $\gamma < 1$. The term $\lambda'/(\lambda + r)^2$ is also positive; thus, the sign of $Eq.(A.21)/\zeta^2 + \lambda'/(\lambda + r)^2$ is positive, and $\partial R^*_{fc}/\partial r > 0$ for $\lambda' > 0$.

Regarding η: Its effect on the first two terms of Equation (A.9) is the same as in the UC monopoly, thus let's focus on the last term:

$$\frac{\partial f}{\partial \eta} = \frac{\frac{\partial \zeta'}{\partial \eta}\zeta - \zeta'\frac{\partial \zeta}{\partial \eta}}{\zeta^2} = \frac{\left(\alpha' \ln \frac{X}{R} - \frac{\alpha}{R}\right)\frac{\partial \zeta}{\partial \eta} + \zeta(\zeta - 1)\frac{\partial\left(\alpha' \ln \frac{X}{R} - \frac{\alpha}{R}\right)}{\partial \eta}}{\zeta^2}. \tag{A.22}$$

If $\lambda' = 0$, then $\alpha' = 0$. Because $\partial\zeta/\partial\eta = \partial\zeta/\partial\alpha \cdot \partial\alpha/\partial\eta = (\zeta - 1)\ln(X/R) \cdot \partial\alpha/\partial\eta$ and $\partial\alpha/\partial\eta < 0$:

$$\frac{\partial f}{\partial \eta} = \frac{-\alpha\dfrac{\partial\zeta}{\partial\eta} - \zeta(\zeta-1)\dfrac{\partial\alpha}{\partial\eta}}{R\zeta^2} = \frac{\left(-\alpha\ln\left(\dfrac{X}{R}\right) - \zeta\right)(\zeta-1)\dfrac{\partial\alpha}{\partial\eta}}{R\zeta^2} \Rightarrow \operatorname{sign}\left(\frac{\partial f}{\partial\eta}\right) = \operatorname{sign}\left(-\alpha\ln\left(\frac{X}{R}\right) - \zeta\right).$$

As $X\uparrow\uparrow$, $\partial f/\partial\eta > 0$, and as $X \to R$, $\partial f/\partial\eta < 0$ and can overturn the positive effect of η on R for a UC firm, causing $\partial R^*_{fc}/\partial\eta < 0$. However, if $\lambda' > 0$, two components of the numerator of Equation (A.22) are

$$\frac{\partial\zeta}{\partial\eta} = \left(\frac{X}{R}\right)^{\alpha}\ln\left(\frac{X}{R}\right)\text{Ⓐ}^{-1/2}\frac{\partial\lambda}{\partial\eta}/\sigma^2, \quad \frac{\partial\alpha}{\partial\eta} = -\frac{\text{Ⓐ}^{-1/2}}{\sigma^2}\frac{\partial\lambda}{\partial\eta},$$
$$\frac{\partial\alpha'}{\partial\eta} = \frac{\gamma}{R\sigma^2}\text{Ⓐ}^{-1/2}\frac{\partial\lambda}{\partial\eta}\left(\frac{\lambda}{\sigma^2\text{Ⓐ}} - 1\right),$$
$$\frac{\partial\left(\alpha'\ln\dfrac{X}{R} - \dfrac{\alpha}{R}\right)}{\partial\eta} = \ln\frac{X}{R}\frac{\gamma}{R\sigma^2}\text{Ⓐ}^{-1/2}\frac{\partial\lambda}{\partial\eta}\left(\frac{\lambda}{\sigma^2\text{Ⓐ}} - 1\right) + \frac{\text{Ⓐ}^{-1/2}}{\sigma^2}\frac{\partial\lambda}{\partial\eta}\frac{1}{R}$$
$$= \frac{\text{Ⓐ}^{-1/2}}{\sigma^2}\frac{\partial\lambda}{\partial\eta}\frac{1}{R}\left(\ln\frac{X}{R}\left(\frac{\lambda}{\sigma^2\text{Ⓐ}} - 1\right)\frac{\gamma}{\sigma^2} + 1\right).$$

The numerator of Equation (A.22) thus equals

$$\text{Ⓐ}^{-1/2}\frac{1}{\sigma^2}\frac{\partial\lambda}{\partial\eta}\left[\left(\alpha'\ln\frac{X}{R} - \frac{\alpha}{R}\right)\left(\frac{X}{R}\right)^{\alpha}\ln\left(\frac{X}{R}\right) + \zeta(\zeta-1)\left(\ln\frac{X}{R}\left(\frac{\lambda}{\sigma^2\text{Ⓐ}} - 1\right)\frac{\gamma}{\sigma^2} + 1\right)\frac{1}{R}\right]. \tag{A.23}$$

Equation (A.23) cannot be signed in general because, when the first term tends to be positive, then second term is likely to be negative and vice versa. The sign of $\partial^2\ln V/\partial R\partial\eta$ is likely to be nonmonotonic (and the numerical exercises confirm it).

Regarding γ: focus again on the last term:

$$\frac{\partial f}{\partial\gamma} = \frac{\left(\alpha'\ln\dfrac{X}{R} - \dfrac{\alpha}{R}\right)\dfrac{\partial\zeta}{\partial\gamma} + \zeta(\zeta-1)\dfrac{\partial\left(\alpha'\ln\dfrac{X}{R} - \dfrac{\alpha}{R}\right)}{\partial\gamma}}{\zeta^2}. \tag{A.24}$$

If $\lambda' = 0$, then it is meaningless to discuss $\partial^2\ln V/\partial R\partial\gamma$ because $\gamma = 0$. Otherwise,

$$\frac{\partial\alpha}{\partial\gamma} = -\frac{\text{Ⓐ}^{-1/2}}{\sigma^2}\frac{\partial\lambda}{\partial\gamma}, \quad \frac{\partial\zeta}{\partial\gamma} = \left(\frac{X}{R}\right)^{\alpha}\ln\left(\frac{X}{R}\right)\text{Ⓐ}^{-1/2}\frac{\partial\lambda}{\partial\gamma}/\sigma^2,$$
$$\frac{\partial\alpha'}{\partial\gamma} = \frac{\gamma}{R\sigma^2}\text{Ⓐ}^{-1/2}\frac{\partial\lambda}{\partial\gamma}\left(\frac{\lambda}{\sigma^2\text{Ⓐ}} - 1\right) - \frac{\lambda}{\sigma^2 R}\text{Ⓐ}^{-\frac{1}{2}}.$$

Therefore,

$$\frac{\partial\left(\alpha'\ln\dfrac{X}{R} - \dfrac{\alpha}{R}\right)}{\partial\gamma} = \ln\frac{X}{R}\left(\frac{\gamma}{R\sigma^2}\text{Ⓐ}^{-1/2}\frac{\partial\lambda}{\partial\gamma}\left(\frac{\lambda}{\sigma^2\text{Ⓐ}} - 1\right) - \frac{\lambda}{\sigma^2 R}\text{Ⓐ}^{-\frac{1}{2}}\right) + \frac{\text{Ⓐ}^{-1/2}}{\sigma^2}\frac{\partial\lambda}{\partial\gamma}\frac{1}{R}$$
$$= \frac{\text{Ⓐ}^{-1/2}}{\sigma^2}\frac{\partial\lambda}{\partial\gamma}\frac{1}{R}\left(\ln\frac{X}{R}\left(\frac{\lambda}{\sigma^2\text{Ⓐ}} - 1\right)\frac{\gamma}{\sigma^2} + 1\right) - \ln\frac{X}{R}\frac{\lambda}{\sigma^2 R}\text{Ⓐ}^{-\frac{1}{2}}.$$

The numerator of Equation (A.24) equals the following, which cannot be signed generally:

$$\left(\alpha'\ln\frac{X}{R} - \frac{\alpha}{R}\right)\left(\frac{X}{R}\right)^{\alpha}\ln\left(\frac{X}{R}\right)\text{Ⓐ}^{-1/2}\frac{\partial\lambda}{\partial\gamma}/\sigma^2 + \zeta(\zeta-1)\left[\frac{\text{Ⓐ}^{-1/2}}{\sigma^2}\frac{\partial\lambda}{\partial\gamma}\frac{1}{R}\left(\ln\frac{X}{R}\left(\frac{\lambda}{\sigma^2\text{Ⓐ}} - 1\right)\frac{\gamma}{\sigma^2} + 1\right) - \ln\frac{X}{R}\frac{\lambda}{\sigma^2 R}\text{Ⓐ}^{-\frac{1}{2}}\right].$$

The expression $\partial R^*_{fc}/\partial\gamma$ is nonmonotonic, and it is again the case that if $\partial^2\ln V/\partial R\partial\eta > 0$, then $\partial^2\ln V/\partial R\partial\gamma > 0$ for sure.

Similar to the proof of Proposition 1, I use Topkis's theorem (Topkis 1978 or theorem 2.3 in Vives 1999) to get the comparative statics from the increasing differences and the ascendancy property of the action domain when the cross partial derivatives $\partial^2\ln V/\partial R\partial s$ can be signed. □

A.4. Proof of the First and Last Statements of Corollary 1

Proof. For the first statement: the proof in Appendix A.3 that shows $\partial R^*_{fc}/\partial X > 0$ when $\lambda' = 0$ can be used directly. The relevant part is Equation (A.11) and the analysis that follows plus the application of monotone comparative statics. As $X\uparrow\uparrow$, R^*_{fc} converges to R^*_{uc}. The monotonic (and positive) effect of X on R^*_{fc} thus gives us $R^*_{uc} > R^*_{fc}$. An alternative way of proving it is to check the first-order conditions for FC versus UC monopoly firms. For FC, an internal solution satisfies

$$\frac{\partial\left(1 - \left(\dfrac{X}{R}\right)^{\alpha}\right)}{\partial R} \times \frac{u(R)\lambda - R}{\lambda + r} + \frac{\partial\dfrac{u(R)\lambda - R}{\lambda + r}}{\partial R} \times \left(1 - \left(\frac{X}{R}\right)^{\alpha}\right) = 0. \tag{A.25}$$

The first of the two terms is negative, because $u(R)\lambda > R$ (for a firm to start the project), and the derivative of $1 - (X/R)^{\alpha_1}$ with regard to R is always negative. Given $1 - (X/R)^{\alpha_1} > 0$, it has to be $u'(R)\lambda - 1 > 0$ at R^*_{fc} for Equation (A.25) to hold. From a UC firm's problem, $u'(R)\lambda - 1 = 0$ at R^*_{uc}. The concavity of f indicates $R^*_{fc} < R^*_{uc}$. The second-order derivative shows that a sufficient condition for V_{fc} to be concave is $\alpha < -1$.

For the last statement: the proof in Appendix A.3 that shows $\partial R^*_{fc}/\partial\mu < 0$ if X is large and $\lambda' > 0$ can be used directly. The relevant part is Equation (A.13) to Equation (A.15). Both X and μ do not affect R^*_{uc}, but as $\mu\downarrow$ and X being large, $R^*_{fc}\uparrow$ and can exceed R^*_{uc}. Appendix A.3 demonstrates the relation between R^*_{fc} and γ is nonmonotonic. A small to moderate γ is required as $\lambda' > 0$ is necessary. □

A.5. Proof of Lemma 2

Proof. The two firms' problems are symmetric, so proving the project value for Firm 1 is sufficient:

$$\sup_{R_1}\mathbb{E}\left(\mathbb{1}_{\{\tau_1<\tau_2\}}e^{-r\tau_1}\tilde{u}_1 - \int_0^{\tau_1\wedge\tau_2}R_1e^{-rt}dt\right)$$
$$= \sup_{R_1}\mathbb{E}\left(\int_0^{\tau_2}\lambda_1\tilde{u}_1e^{-(r+\lambda_1)\tau_1}d\tau_1 - \int_0^{\tau_2}R_1e^{-(r+\lambda_1)t}dt\right)$$
$$= \sup_{R_1}\mathbb{E}\int_0^{\infty}(\lambda_1u_1 - R_1)\mathbb{1}_{\{t<\tau_2\}}e^{-(r+\lambda_1)t}dt$$
$$= \sup_{R_1}\frac{u_1\lambda_1 - R_1}{\lambda_1 + \lambda_2 + r}.$$

Alternatively, use the Bellman equation (i.e., the required rate of return for the investment equals the expected rate of capital gain minus the flow payment plus the expected probability weighted payoff at project discovery and subtract the expected probability weighted loss at rival's discovery) to get

$$rV_1 = \mathbb{E}\mathscr{D}\, V_1 - R_1 + \lambda_1(u_1 - V_1) + \lambda_2(0 - V_1).$$

Because the project value does not depend on its own or its rival's cash flow, and the project value is not time dependent, $\mathbb{E}\mathscr{D}\, V_1 = 0$. Thus, V_1 is as described in the lemma. □

A.6. Proof of Proposition 3

I first prove that with the project accelerability ($\lambda' > 0$), the UCUC race is a (strict) log-supermodular game (Milgrom and Roberts 1990, Vives 1999). A two-player one-dimension game is log-supermodular if the action sets are compact, and the smooth payoff function $\pi_i(a_i, a_j)$ has the following property: $\ln \pi_i(a_i, a_j)$ has increasing differences in actions (a_i, a_j), that is,

$$\pi_i \cdot \frac{\partial^2 \pi_i}{\partial a_i \partial a_j} - \frac{\partial \pi_i}{\partial a_i} \cdot \frac{\partial \pi_i}{\partial a_j} \geq 0. \tag{A.26}$$

Proof. Because of the symmetry of the game, to prove the UCUC race strictly log-supermodular, it is sufficient to show the following inequality for Firm 1:

$$V_1 \cdot \frac{\partial^2 V_1}{\partial R_1 \partial R_2} - \frac{\partial V_1}{\partial R_1} \cdot \frac{\partial V_1}{\partial R_2} > 0 \tag{A.27}$$

From the project values in Lemma 2:

$$\frac{\partial V_1}{\partial R_2} = -\frac{u_1\lambda_1 - R_1}{(r + \lambda_1 + \lambda_2)^2} \frac{\partial \lambda_2}{\partial R_2},$$

$$\frac{\partial V_1}{\partial R_1} = \frac{(r + \lambda_2) \cdot (u\lambda_1 - R_1)' - (\lambda_1 - R_1\lambda_1' - u_1'\lambda_1^2)}{(r + \lambda_1 + \lambda_2)^2},$$

$$\frac{\partial^2 V_1}{\partial R_1 \partial R_2} = \frac{\partial \lambda_2}{\partial R_2}\left(\frac{(u_1\lambda_1 - R_1)'}{(r + \lambda_1 + \lambda_2)^2} - \frac{2}{(r + \lambda_1 + \lambda_2)^3} \cdot [(r + \lambda_2) \cdot (u\lambda_1 - R_1)' - (\lambda_1 - R_1\lambda_1' - u_1'\lambda_1^2)]\right).$$

Putting them together:

$$\begin{aligned}
&V_1 \frac{\partial^2 V_1}{\partial R_1 \partial R_2} - \frac{\partial V_1}{\partial R_2}\frac{\partial V_1}{\partial R_1} \\
&= V_1 \cdot \frac{\partial \lambda_2}{\partial R_2}\left(\frac{(u_1\lambda_1 - R_1)'}{(r + \lambda_1 + \lambda_2)^2} - \frac{2}{(r + \lambda_1 + \lambda_2)^3} \cdot [(r + \lambda_2) \cdot (u\lambda_1 - R_1)' - (\lambda_1 - R_1\lambda_1' - u_1'\lambda_1^2)]\right) \\
&\quad + \frac{u_1\lambda_1 - R_1}{(r + \lambda_1 + \lambda_2)^2}\frac{\partial \lambda_2}{\partial R_2} \cdot \frac{(r + \lambda_2) \cdot (u\lambda_1 - R_1)' - (\lambda_1 - R_1\lambda_1' - u_1'\lambda_1^2)}{(r + \lambda_1 + \lambda_2)^2} \\
&= \frac{\partial \lambda_2}{\partial R_2} \cdot \frac{1}{(r + \lambda_1 + \lambda_2)^4}((u_1\lambda_1 - R_1)(u_1\lambda_1 - R_1)' \cdot (r + \lambda_1 + \lambda_2) \\
&\quad -(u_1\lambda_1 - R_1) \cdot [(r + \lambda_2) \cdot (u_1\lambda_1 - R_1)' - (\lambda_1 - R_1\lambda_1' - u_1'\lambda_1^2)]) \\
&= \frac{\partial \lambda_2}{\partial R_2} \cdot \frac{u_1\lambda_1 - R_1}{(r + \lambda_1 + \lambda_2)^4}((u_1\lambda_1 - R_1)' \cdot \lambda_1 + (\lambda_1 - R_1\lambda_1' - u_1'\lambda_1^2)) \\
&= \frac{\partial \lambda_2}{\partial R_2} \cdot \frac{u_1\lambda_1 - R_1}{(r + \lambda_1 + \lambda_2)^4}(u_1'(\lambda_1^2 - \lambda_1^2) + \lambda_1'(u_1\lambda_1 - R_1)) \\
&= \frac{\partial \lambda_2}{\partial R_2} \cdot \frac{(u_1\lambda_1 - R_1)^2}{(r + \lambda_1 + \lambda_2)^4}\lambda_1' > 0 \text{ if } \lambda_1' > 0 \text{ and } \frac{\partial \lambda_2}{\partial R_2} > 0.
\end{aligned} \tag{A.28}$$

Equation (A.28) shows the cancellation of u_1', which suggests that regardless of the sign of u_1', this is a (strict) log-supermodular game as long as the project is accelerable. The application of theories on supermodular games can be extended by considering increasing transformations of the payoff, including log-supermodular games (Milgrom and Roberts 1990, Vives 2007). Thus, by Topkis's theorem (Topkis 1978, and relatedly, Shannon 1995), Firm i's best response is increasing in its rival's action, that is, $\mathrm{d}R_i(R_-i)/\mathrm{d}R_{-i} > 0$. By Tarski's fixed point theorem (Tarski 1955 and in the review of Amir 2005), the existence of a pure-strategy equilibrium of the game is guaranteed. Alternatively, applying Topki's theorem stated as theorem 2.5 in Vives (1999) proves the existence of equilibrium.

For a scalable-only project (i.e., $u' > 0, \lambda' = 0$), it is straightforward to see the equilibrium investment rates satisfy $\lambda_i u_i' = 1$, for $i \in \{1, 2\}$. Because of the monotonicity of u_i', the equilibrium is unique. □

A.7. Pure Nash Equilibrium in UCUC Race in Section 3.1

A.7.1. Characterization of a Symmetric Pure Nash Equilibrium.

The investment intensities in a symmetric pure Nash equilibrium satisfy the first-order condition of Equation (A.30).

Proof. Focus on Firm 1's value in Equation (9), we have

$$\frac{\partial V_1}{\partial R_1} = \frac{(r + \lambda_2) \cdot (u_1\lambda_1 - R_1)' - (\lambda_1 - R_1\lambda_1' - u_1'\lambda_1^2)}{(r + \lambda_1 + \lambda_2)^2}. \tag{A.29}$$

At a symmetric equilibrium, $R_1 = R_2, \lambda_1 = \lambda_2$. The condition for a symmetric equilibrium can be written as the following, with subscripts removed to be general for both firms:

$$Ⓗ = \left.\frac{\partial V_1}{\partial R_1}\right|_{\lambda_2 = \lambda_1 = \lambda} = \frac{(r + \lambda) \cdot (u\lambda - R)' - (\lambda - R\lambda' - u'\lambda^2)}{(r + 2\lambda)^2} = 0. \tag{A.30}$$

It can be rewritten as

$$\begin{aligned}
(r + 2\lambda) \cdot (u\lambda - R)' = (u\lambda - R) \cdot \lambda \Rightarrow R - u\lambda \\
= \frac{(r + 2\lambda)(R - u\lambda)'}{\lambda'},
\end{aligned} \tag{A.31}$$

or with the functional forms in Equation (2):

$$R - A\eta R^{\gamma+\beta} = \frac{(r + 2AR^\gamma)(1 - A\eta(\beta + \gamma)R^{\beta+\gamma-1})}{\eta\gamma R^{\gamma-1}}. \tag{A.32}$$

Because UCUC race is a log-supermodular game, the existence of the pure strategy equilibrium is ensured by Topkis's theorem as mentioned in Appendix A.6. The second-order condition for a symmetric equilibrium is

$$Ⓖ = \frac{(u\lambda)''\lambda' + \lambda''(R - u\lambda)'}{(r + 2\lambda)\lambda'} < 0. \tag{A.33}$$

□

A.7.2. Uniqueness of the UCUC Equilibrium.

The symmetric Nash equilibrium is unique with Conditions (A.34), (A.35), and (A.39). With the functional forms of u and λ in Equation (2), the conditions that ensure the uniqueness of the equilibrium are Conditions (A.36) and (A.39).

Proof. If $H(R = \underline{R} = 1) > 0, H(R = \bar{R}) < 0,$ with H decreases in R and $G < 0$, then the R that solves $H = 0$ is unique. The first two conditions are

$$H(R = \underline{R} = 1) = \frac{(r + 2\lambda)(u\lambda - R)' - (u\lambda - R)\lambda'}{(r + 2\lambda)^2}\Bigg|_{R=\underline{R}=1} > 0, \tag{A.34}$$

$$H(R = \bar{R}) = \frac{(r + \lambda)\cdot(u'\lambda + u\lambda' - 1) - (\lambda - R\lambda' - u'\lambda^2)}{(r + 2\lambda)^2}\Bigg|_{u\lambda=R} < 0. \tag{A.35}$$

With the assumptions in Equation (2), Conditions (A.34) and (A.35) can be rewritten as

$$(r + 2\eta)(A\eta(\gamma + \beta) - 1) > (A\eta - 1)\gamma\eta, \tag{A.36}$$

$$\frac{(r + \lambda)\cdot(\beta + \gamma - 1) - (\lambda - \gamma\lambda - \beta\lambda)}{(r + 2\lambda)^2} = \frac{(r + 2\lambda)\cdot(\beta + \gamma - 1)}{(r + 2\lambda)^2} < 0 \quad \text{(guaranteed)}. \tag{A.37}$$

For the third condition, because

$$\frac{\partial H}{\partial R} = \frac{(u\lambda - R)'\lambda'}{(r + 2\lambda)^2} + G, \tag{A.38}$$

The condition for $\partial H/\partial R \le 0$ is not weaker than the condition for $G \le 0$. Use $(u\lambda - R)' = \lambda - R\lambda' - u'\lambda^2/r + \lambda$,

$$\frac{\partial H}{\partial R} = \frac{(u\lambda)''(r + \lambda)}{(r + 2\lambda)^2} + \frac{R\lambda''\lambda + \lambda'\lambda - R\lambda'^2}{(r + \lambda)(r + 2\lambda)^2} + \frac{Rr\lambda'' + u''\lambda^2(r + \lambda) + u'\lambda'\lambda(2r + \lambda)}{(r + \lambda)(r + 2\lambda)^2} < 0. \tag{A.39}$$

If $u' = 0$, then Condition (A.39) is guaranteed by decreasing hazard rate elasticity (Nti 1999). However, when $u' > 0$, Condition (A.39) is needed to ensure a unique symmetric equilibrium of the UCUC race. □

A.8. Proof of Proposition 4

Proof. The proof of Proposition 4(1) is omitted, as it is obvious. If $u' > 0$ and $\lambda' = 0$, the rival's investment does not affect the firm value; thus, both firms invest at their monopoly level in the equilibrium (with r being replaced by $r + \lambda_{-i}$), and comparative statics in Proposition 1 apply for own firm tech-parameters. The rival firm's tech-parameters only matter if they affect the discount rate via λ_{-i}, that is, η_{-i}.

$\lambda' > 0$: Given this is a strict log-supermodular game (see Appendix A.6), the equilibrium investment intensities (R_1^*, R_2^*) increase in a model parameter s if the value function $\ln V_i(R_i, R_{-i}; s)$ has increasing differences in (R_i, s), or $\partial^2 \ln V_i/\partial R_i \partial s \ge 0$ (theorem 2.3 and 2.4 in Vives 1999). Proving it for one firm (i.e., $\partial^2 \ln V_i/\partial R_i \partial s \ge 0$) is sufficient to get comparative statics for both firms (i.e., $\partial R_i^*/\partial s > 0$ and $\partial R_{-i}^*/\partial s > 0$) because the game is log-supermodular. Because of the symmetry of the game, let's just check V_1 for the parameters A_1, β_1, η_1, and γ_1. Use the notation "′"to denote the first order derivative regarding R_1:

$$\frac{\partial \ln V_1}{\partial R_1} = \frac{(\lambda_1 u_1 - R_1)'}{\lambda_1 u_1 - R_1} - \frac{\lambda_1'}{\lambda_1 + \lambda_2 + r}.$$

With the functional forms of u and λ from Equation (2), I check whether $\partial^2 \ln V_1/\partial R_1 \partial s \ge 0$ holds for own firm's project characteristics (this is almost identical to the proof of Proposition 1 in Section A.2, so the proof is succinct):

$$\frac{\partial^2 \ln V_1}{\partial R_1 \partial A_1} = \frac{\lambda_1}{(\lambda_1 u_1 - R_1)^2}\frac{\partial u_1}{\partial A_1}\cdot(1 - \beta - \gamma) > 0, \tag{A.40}$$

$$\frac{\partial^2 \ln V_1}{\partial R_1 \partial \beta_1} = \frac{\lambda_1}{(\lambda_1 u_1 - R_1)^2}\frac{\partial u_1}{\partial \beta_1}\cdot(1 - \beta - \gamma) + \frac{u_1\lambda_1}{R_1}(\lambda_1 u_1 - R_1)^{-1} > 0 \ \text{ for } \ \beta > 0, \tag{A.41}$$

$$\frac{\partial^2 \ln V_1}{\partial R_1 \partial \eta_1} = \left(\frac{u_1(1 - \beta - \gamma)}{(\lambda_1 u_1 - R_1)^2} - \frac{(\lambda_2 + r)\gamma}{R_1(\lambda_1 + \lambda_2 + r)^2}\right)\cdot\frac{\partial \lambda_1}{\partial \eta_1} > 0 \ \text{ at least for small } \ \gamma, \tag{A.42}$$

$$\frac{\partial^2 \ln V_1}{\partial R_1 \partial \gamma_1} = \left(\frac{u_1}{(\lambda_1 u_1 - R_1)^2}\cdot(1 - \beta - \gamma) - \frac{(\lambda_2 + r)\gamma}{R_1(\lambda_1 + \lambda_2 + r)^2}\right)\times\frac{\partial \lambda_1}{\partial \gamma_1} + \frac{\lambda_1(u_1\lambda_2 + u_1 r + R_1)}{R_1(\lambda_1 + \lambda_2 + r)(\lambda_1 u_1 - R_1)} > 0 \ \text{ at least for small } \gamma. \tag{A.43}$$

The derivations show that the key drives for the increasing comparative statics in the UCUC race are the decreasing returns to scale, that is, $\beta + \gamma < 1$, and $\partial \lambda_i/\partial \eta_i > 0$, $\partial \lambda_i/\partial \gamma_i > 0$. □

A.9. Proof of Lemma 3

The proof for Lemma A.2 in Appendix A.12 is more general. Setting $\lambda_j = 0$ in Appendix A.12 proves Lemma 3.

A.10. Proof of Corollary 2

Proof. Regarding the monotonicity of $R_1^*(R_2)$, that is, $R_{fc}^*(R_{uc})$, write the value function in Equation (12) as $V_1(R_1, R_2; X_1) = (u_1 \cdot \lambda_1 - R_1/r + \lambda_2 + \lambda_1) \times (1 - (X_1/R_1)^{\alpha_1'})$. Obviously, if $\lambda' = 0$, V_1 does not depend on R_2; thus, $\partial R_1^*/\partial R_2 = 0$. If $\lambda' > 0$, then R_2 affects V_1 through the term of λ_2 (and the term of α_1' via λ_2). Observe that r and λ_2 always appear together in V_1 in the form of summation, the directional effect of λ_2 on R_1^* is the same as the effect of r on R_1^*. $\partial R_{fc}/\partial r > 0$ from Proposition 2; therefore, $\partial R_1/\partial \lambda_2 > 0$ and $\partial R_1/\partial R_2 = \partial R_1/\partial \lambda_2 \times \partial \lambda_2/\partial R_2 > 0$.

Regarding $R_2^*(R_1)$, rewrite $V_2(R_1, R_2; X_1) = B + C$:

$$V_2(R_1, R_2; X_1) = \underbrace{\frac{\lambda_2 u_2 - R_2}{\lambda_2 + \lambda_1 + r}\cdot\left(1 - \left(\frac{X_1}{R_1}\right)^{\alpha_1'}\right)}_{B} + \underbrace{\frac{\lambda_2 u_2 - R_2}{\lambda_2 + r}\cdot\left(\frac{X_1}{R_1}\right)^{\alpha_1'}}_{C}. \tag{A.44}$$

It is obvious that if $\lambda' = 0$, V_2 does not depend on R_1; thus, $\partial R_2^*/\partial R_1 = 0$. Next, I prove that if $\lambda' > 0, \partial R_2^*(R_1)/\partial R_1 < 0$ when R_1 is large (and X_1 is not too large, i.e., it is not that $X \gg (A\eta)^{\frac{1}{1-(\beta+\gamma)}}$). The strict concavity of the natural logarithmic function implies $\ln V_2 < \ln Ⓑ + \ln Ⓒ$. Thus, $\partial^2 \ln V_2/\partial R_1 \partial R_2 \le \partial^2 \ln Ⓑ/\partial R_1 \partial R_2 + \partial^2 \ln Ⓒ/\partial R_1 \partial R_2$. Monotonicity of the same function implies $\partial^2 \ln V_2/\partial R_1 \partial R_2 \ge \partial^2 \ln Ⓑ/\partial R_1 \partial R_2$ if $Ⓒ > 0$. Applying $\partial f(x)^{g(x)}/\partial x = f(x)^{g(x)} (g' \ln f + (g/f) f')$ gives $\partial(1-(X_1/R_1)^{\alpha_1'})/\partial R_1 = -(X_1/R_1)^{\alpha_1'}(\partial \alpha_1'/\partial R_1) \ln(X_1/R_1) - (\alpha_1'/R_1))$ and $\partial\left(1-(X_1/R_1)^{\alpha_1'}\right)/\partial R_2 = -(X_1/R_1)^{\alpha_1'} \cdot \ln(X_1)/(R_1 \partial \alpha_1'/\partial R_2)$. With $Ⓜ \equiv (1/2 - \mu/\sigma^2)^2 + 2(\lambda + r)/\sigma^2$ defined, and recall $R_1 \le X_1$ for the constrained firm,

$$\frac{\partial^2 \ln Ⓑ}{\partial R_1 \partial R_2} = \frac{\lambda_1' \lambda_2'}{(\lambda_1+\lambda_2+r)^2} + \frac{\partial\left(\frac{\partial \alpha_1'}{\partial R_1} \ln \frac{X_1}{R_1} - \frac{\alpha'}{R_1}\right)}{\partial R_2} - \frac{\partial \frac{\left(\frac{\partial \alpha_1'}{\partial R_1} \ln \frac{X_1}{R_1} - \frac{\alpha'}{R_1}\right)}{1-\left(\frac{X_1}{R_1}\right)^{\alpha_1'}}}{\partial R_2}, \tag{A.45}$$

$$= \frac{\lambda_1' \lambda_2'}{(\lambda_1+\lambda_2+r)^2} + \ln \frac{X_1}{R_1} \frac{\lambda_1' \lambda_2'}{\sigma^4} Ⓜ^{-\frac{2}{3}} + \frac{\lambda_2'}{\sigma^2} Ⓜ^{-\frac{1}{2}} \frac{1}{R_1}$$

$$- \frac{\ln \frac{X_1}{R_1} \frac{\lambda_1' \lambda_2'}{\sigma^4} Ⓜ^{-\frac{2}{3}} + \frac{\lambda_2'}{\sigma^2} Ⓜ^{-\frac{1}{2}} \frac{1}{R_1}}{\left(1-\left(\frac{X_1}{R_1}\right)^{\alpha_1'}\right)} + \frac{\left(\frac{\partial \alpha_1'}{\partial R_1} \ln \frac{X_1}{R_1} - \frac{\alpha'}{R_1}\right) \times \left(\frac{X_1}{R_1}\right)^{\alpha_1'} \cdot \ln \frac{X_1}{R_1} \frac{\partial \alpha_1'}{\partial R_2}}{\left(1-\left(\frac{X_1}{R_1}\right)^{\alpha_1'}\right)^2}, \tag{A.46}$$

$$\begin{cases} \ge 0 \text{ if } R_1 \text{ is small (e.g. } R_1 \to \underline{R}), \\ < 0 \text{ if } R_1 \to X_1 (\text{i.e.} \ln X_1/R_1 \to 0), \text{ and } X_1 \in [\underline{R}_1, \bar{R}_1]. \end{cases} \tag{A.47}$$

Meanwhile,

$$\frac{\partial^2 \ln Ⓒ}{\partial R_1 \partial R_2} = \ln \frac{X_1}{R_1} \frac{\partial^2 \alpha_1'}{\partial R_2 \partial R_1} - \frac{\partial \alpha_1'}{\partial R_2} \frac{1}{R_1}$$
$$= \ln \frac{X_1}{R_1} \frac{\lambda_1' \lambda_2'}{\sigma^4} Ⓜ^{-\frac{2}{3}} + \frac{\lambda_2'}{\sigma^2} Ⓜ^{-\frac{1}{2}} \frac{1}{R_1} > 0.$$

Recall the economic interpretation of Ⓑ is the probability weighted project value in the scenario of $\tau_1 \wedge \tau_2 \le \tau_{c1}$, and Ⓒ corresponds to the scenario of $\tau_1 \wedge \tau_2 > \tau_{c1}$. As $R_1 \to X_1$, τ_{c1} happens almost instantly, which means the significance of Ⓒ in V_2 becomes negligible, and $\partial^2 \ln V_2/\partial R_1 \partial R_2 \le \partial^2 \ln Ⓑ/\partial R_1 \partial R_2 + \partial^2 \ln Ⓒ/\partial R_1 \partial R_2 < 0$. The decreasing differences coupled with the nonincreasing property of the action domain $R_2 \in [\underline{R}_2(R_1), \bar{R}_2(R_1)]$ give the decreasing $R_2^*(R_1)$ when R_1 is large (Amir 2005). However, if $(A\eta)^{1/1-(\beta+\gamma)} \ll X$, that is, the upper bound of the range of R_{fc} that makes the project value positive is much smaller than X, so that $R_1 \to X_1$ is impossible on the domain of $R_2^*(R_1)$, then $R_2^*(R_1)$ is monotonically increasing on its domain. Finally, the monotonicity of $R_2^*(R_1)$ is obvious if R_1 is small, and it is from $\partial^2 \ln V_2/\partial R_1 \partial R_2 \ge \partial^2 \ln Ⓑ/\partial R_1 \partial R_2 > 0$ as $Ⓒ > 0$ plus the ascendancy property of the action domain (Topkis 1978). The seemingly contradiction of the decreasing/ascendancy property of the same action domain is because the domain does not depend on the rival's investment.

Next, proving the existence of at least one pure strategy equilibrium is straightforward by using the shapes of the best responses in an FCUC race. Note that $R_i^*(R_j)$ is close to Firm i's monopoly investment intensity when $R_j = \underline{R}_j$. If X is not too large that $R_2^*(R_1)$ follows the hump shape, then it is obvious that the two best response functions have exactly one intersection. If X is large such that $R_2^*(R_1)$ is monotonically increasing on the domain of $R_1 \in [\underline{R}_1, \bar{R}_1]$, then in the same spirit of Taski's fixed point theorem, the existence of equilibrium is guaranteed. Without changes in convexity/concavity of the best response on its domain, the uniqueness of equilibrium is obtained. This equilibrium can be graphically shown to be stable. □

A.11. Numerical Procedures

When the analytical expressions for firm values are available, the usual software can be used to find the optimal investment intensities numerically. To solve the equilibrium of the FCFC race in Section 3.3, I used COMSOL Multiphysics software (v. 6.0. www.comsol.com. COMSOL AB, Stockholm, Sweden) with the following steps. More details can be found in the online appendix.

1. Given a pair of (R_1, R_2), solve $V_1(X_1, X_2 \mid R_1, R_2)$ on the whole domain of (X_1, X_2) numerically, using the PDE in Equation (15) and the boundary conditions in Equation (16) to Equation (19). Record $V_1(X_1^0, X_2^0 \mid R_1, R_2)$, where X_1^0, X_2^0 represent a point of interest, for example, the pair of baseline AIP cash flows shown in Table 1.
2. Fix R_2, change R_1 to any $\hat{R}_1$. Repeat Step 1, and record $V_1(X_1^0, X_2^0 \mid \hat{R}_1, R_2)$. Keep repeating this step, and find the optimal R_1 for the given R_2: $R_1^* = \arg\max_{R_1} V_1(R_1 \mid X_1^0, X_2^0, R_2)$.
3. Change R_2, follow Steps 1 and 2 for this new R_2. Repeat for a wide range of R_2 and find Firm 1's optimal choice each time. This leads to the best responses $R_1^*(R_2)$. Obtain $R_2^*(R_1)$ similarly.
4. Find the pure Nash equilibrium $(\bar{R}_1, \bar{R}_2)$ by finding the fixed point of the best response correspondences such that $R_2^*(\bar{R}_1) = \bar{R}_2$ and $R_1^*(\bar{R}_2) = \bar{R}_1$.

A.12. Results and Proofs of Section 4.1 with a Jump Risk in AIP Cash Flow

Redefine τ_c in Equation (3) of the monopoly model as $\tau_c \equiv \inf\{t : X_t < \underline{X}\}$, where $\underline{X}$ is the abandonment threshold of an FC monopoly $\underline{X} \ge R_{fc}$. The monopoly firm's problem can be written as

$$\begin{aligned} \text{An FC monopoly}: \quad & \sup_{R \in (0, \underline{X}], \underline{X}} \mathbb{E}\Bigg[\int_0^{\tau_j} e^{-rt} X_t dt \\ & + \int_0^{\tau_d \wedge \tau_c \wedge \tau_j} e^{-rt}(-R)dt + e^{-r\tau_d} \tilde{u} \mathbb{1}_{\{\tau_d < \tau_c \wedge \tau_j\}}\Bigg] \\ \text{A UC monopoly}: \quad & \sup_{R>0} \mathbb{E}\Bigg[\int_0^{\tau_j} e^{-rt} X_t dt \\ & + \int_0^{\tau_d} e^{-rt}(-R)dt + e^{-r\tau_d} \tilde{u}\Bigg]. \end{aligned} \tag{A.48}$$

Lemma A.1. *With a jump risk that wipes out all future AIP cash flow and happens at a first Poisson arrival with intensity λ_j, the monopoly R&D project values are*

$$V_{uc}(X) = \sup_{R>0} \frac{u(R)\lambda(R) - R}{\lambda(R) + r}, \tag{A.49}$$

$$V_{fc}(X) = \sup_{R \in (0, X]} \frac{u(R)\lambda(R) - R}{\lambda(R) + \lambda_j + r}\left(1 - \left(\frac{X}{R}\right)^{\alpha}\right), \tag{A.50}$$

for a UC firm and an FC firm, respectively, where $\alpha = 1/2 - \mu/\sigma^2 - \sqrt{(1/2-\mu/\sigma^2)^2 + 2(\lambda(R)+\lambda_j+r)/\sigma^2}$.

Proof. With the jump risk, the AIP cash flows follow a mixed Poisson-Wiener process of the form $dX/X = \mu dt + \sigma dZ + dq_j$, where dq_j takes the value of -1 with probability λdt and 0 with probability $1 - \lambda dt$. By Itô's lemma, the expected change in the project value given any choice of R is

$$\mathbb{E}\mathscr{D}V = \left\{\frac{\partial V}{\partial X}\mu X + \frac{1}{2}\frac{\partial^2 V}{\partial X^2}\sigma^2 X^2 + \lambda_j[V(0) - V]\right\}dt.$$

At the optimal strategy, the required rate of return for investing in the project should equal to the expected rate of capital gain minus the flow payment to the project and plus the expected payoff of the project at discovery while taking the discovery probability into consideration. The HJB equation on the project value can be written as

$$rV = \frac{\mathbb{E}\mathscr{D}V}{dt} - R + \lambda(u - V). \tag{A.51}$$

A.12.1. UC. Equation (A.49) is from Equation (A.51) by setting $V_X = 0$, $V_{XX} = 0$, and $V(0) = V$. Equation (A.49) can also be obtained by calculating $\sup_{R>0}\mathbb{E}(e^{-r\tau_d}\tilde{u} - \int_0^{\tau_d} Re^{-rt}dt)$.

A.12.2. FC. Rewrite Equation (A.51) as

$$(r + \lambda + \lambda_j)V = \frac{\partial V}{\partial X}\mu X + \frac{1}{2}\frac{\partial^2 V}{\partial X^2}\sigma^2 X^2 + \lambda u - R. \tag{A.52}$$

The solution to the ordinary differential equation follows the form of $V(X) = A_1 X^{\alpha_1} + A_2 X^{\alpha_2} + (u\lambda - R)/(r + \lambda + \lambda_j)$, where α_1, α_2 are the solutions of the quadratic function $\frac{1}{2}\sigma^2\alpha(\alpha - 1) + \mu\alpha - (r + \lambda + \lambda_j) = 0$. Suppose $\alpha_1 < 0$ and $\alpha_2 > 0$. The project value is subject to the following boundary conditions:

$$\lim_{X\to\infty} V(X) = \frac{u\lambda - R}{r + \lambda + \lambda_j}, \quad V(X = \underline{X}) = 0. \tag{A.53}$$

The first no-bubble condition gives us $A_2 = 0$, and the second value matching condition at the abandonment threshold gives us the value of A_1, which is a function of R. Together, and using α to replace α_1 for the ease of notation,

$$V_{fc}(X) = \sup_{\{R,\underline{X}\}} \frac{u\lambda - R}{\lambda + r + \lambda_j}\left(1 - \left(\frac{X}{\underline{X}}\right)^{\alpha}\right). \tag{A.54}$$

Given any R, provided that the project discovery is random and memoryless, the project value is higher if it lasts longer. The expression $\underline{X}^*_{fc} = R$ from Equation (A.54) thus leads to Equation (A.50). □

Proposition A.1. *A downward jump risk on* X_t *motivates an FC monopoly firm to invest more intensively, that is,* $\partial R^*_{fc}/\partial\lambda_j \geq 0$, *regardless of the project type. If* $\lambda' > 0$, *a higher jump risk makes over-investment more likely.*

Proof. If $u' > 0$ and $\lambda' = 0$, the jump risk acts exactly like the risk-free rate r. This can be seen by comparing Equation (A.50) in Lemma A.1 with Equation (6) in Lemma 1. The proof of Proposition 2 in Appendix A.3 shows $\partial R_{fc}/\partial r > 0$ for a project with $u' > 0$ and $\lambda' = 0$. Alternatively, the implicit function theorem can be used. □

Lemma A.2. *In an FCUC race with a jump risk on AIP cash flow, the project values for the two firms (FC is* 1, *UC is* 2*) are*

$$V_1(X_1) = \frac{u_1\lambda_1 - R_1}{r + \lambda_2 + \lambda_1 + \lambda_j}\left(1 - \left(\frac{X_1}{R_1}\right)^{\alpha}\right), \tag{A.55}$$

$$V_2(X_1) = \frac{u_2\lambda_{d,2} - R_2 + \lambda_j V_2^m(R_2)}{r + \lambda_2 + \lambda_1 + \lambda_j}\left(1 - \left(\frac{X_1}{R_1}\right)^{\alpha}\right) + V_2^m(R_2)\left(\frac{X_1}{R_1}\right)^{\alpha}, \tag{A.56}$$

where V_2^m *is defined in* Lemma 3, *and* $\alpha = 1/2 - \mu_1/\sigma_1^2 - \sqrt{(1/2-\mu_1/\sigma_1^2)^2 + 2(r+\lambda_2+\lambda_1+\lambda_j)/\sigma_1^2}$.

Proof. For FC, Firm 1's project value during the race satisfies the following HJB equation at the optimum:

$$rV_1 dt = \mathbb{E}\mathscr{D}V_1 - R_1 dt + \lambda_1(u_1 - V_1)dt + \lambda_2(0 - V_1)dt, \tag{A.57}$$

where $\mathbb{E}\mathscr{D}V_1(X_1) = \{(\partial V_1/\partial X_1)\mu_1 X_1 + 1/2(\partial^2 V_1/\partial X_1^2)\sigma_1^2 X_1^2 + \lambda_j[V_1(0) - V_1]\}dt$ and $V_1(0) = 0$. The solution of the corresponding ordinary differential equation (ODE) on V_1 follows the form of $V_1(X_1) = c_1 X_1^{\alpha_1} + c_2 X_1^{\alpha_2} + (\lambda_1 u_1 - R_1)/(r + \lambda_1 + \lambda_2 + \lambda_j)$, where α_1, α_2 are the roots of the quadratic function $(1/2)\sigma_1^2\alpha(\alpha - 1) + \mu_1\alpha - (r + \lambda_1 + \lambda_2 + \lambda_j) = 0$. The project value is subject to the two boundary conditions:

$$\lim_{X_1\to\infty} V_1(X_1) = \frac{\lambda_1 u_1 - R_1}{r + \lambda_2 + \lambda_1 + \lambda_j}, \quad V_1(X_1 \to R_1) = 0.$$

Equation (A.55) follows from applying the boundary conditions on the general solution of $V_1(X_1)$.

For UC, the project value satisfies the HJB equation at the optimum:

$$rV_2 dt = \mathbb{E}\mathscr{D}V_2 - R_2 dt + \lambda_2(u_2 - V_2)dt + \lambda_1(0 - V_2)dt + \lambda_j(V_2^m - V_2)dt, \tag{A.58}$$

where $\mathbb{E}\mathscr{D}V_2(X_1) = \{(\partial V_2/\partial X_1)\mu_1 X_1 + (1/2)(\partial^2 V_2/\partial X_1^2)\sigma_1^2 X_1^2 + \lambda\ j[V_2(X_1 = 0) - V_2(X_1)]\}dt$ and $V_2(X_1 = 0) = V_2^m$ as in Equation (14) of Lemma 3. The ODE can be solved with the two boundary conditions:

$$\lim_{X_1\to\infty} V_2(X_1) = \frac{u_2\lambda_2 - R_2 + \lambda_j V_2^m}{r + \lambda_2 + \lambda_1 + \lambda_j}, \tag{A.59}$$

$$V_2(X_1 = R_1) = V_2^m. \tag{A.60}$$

With λ_j, the UC firm's project value in the FCUC race does not converge to that in a UCUC race if X_1 is very high (see Equation (A.59)). This is because the jump terminates the FC firm's project even if the FC firm has a high AIP cash flow. Equation (A.60) is a value matching condition that the UC firm recovers its monopolistic project value when its FC rival abandons the project. Solving V_2 yields Equation (A.56). □

A.13. Competition Without Winner-Takes-All in Section 4.2

I present the results for the UCUC competition without winner-takes-all here. In the online appendix, I present firms' project values in the FCUC competition without winner-take-al via their HJB equations and the boundary conditions.

Lemma A.3. *In a non–winner-takes-all UCUC competition, if it is not worthwhile to continue investing in the project after the rival firm gets the discovery first, that is, $u_i^d\lambda_i - R_i \le 0$, then project values are the same as in Lemma 2. Otherwise,*

$$V_i = -\frac{R_i}{\lambda_i + r} + u_i^m\left(\frac{\lambda_1+\lambda_2}{\lambda_1+\lambda_2+r} - \frac{\lambda_{-i}}{r+\lambda_{-i}}\right) + u_i^d\left(\frac{\lambda_i}{r+\lambda_i} + \frac{\lambda_{-i}}{\lambda_{-i}+r} - \frac{\lambda_1+\lambda_2}{r+\lambda_1+\lambda_2}\right), \quad (A.61)$$

where u_i^m and u_i^d are the present value of the future monopoly and duopoly profits, that is, $u_i^m = \pi_i^m/r$, $u_i^d = \pi_i^d/r$, π_i^m or π_i^d is the per period profit upon discovery, $i \in \{1,2\}$.

Proof. Term by term of Equation (21):

$$\mathbb{E}\int_0^{\tau_1} -R_1e^{-rt}dt = \int_0^{\infty} f(\tau_1)\int_0^{\tau_1} -R_1e^{-rt}dtd\tau_1 = -\frac{R_1}{\lambda_1+r}, \quad (A.62)$$

$$\mathbb{E}1_{\{\tau_1<\tau_2\}}e^{-r\tau_1}u_1^m = u_1^m\int_{\tau_2=0}^{\infty}\int_{\tau_1=0}^{\tau_2} e^{-r\tau_1}f(\tau_1)d\tau_1f(\tau_2)d\tau_2 = \frac{\lambda_1}{\lambda_1+\lambda_2+r}u_1^m, \quad (A.63)$$

$$\mathbb{E}1_{\{\tau_1<\tau_2\}}e^{-r\tau_2}(u_1^d-u_1^m) = (u_1^d-u_1^m)\int_{\tau_1=0}^{\infty}\int_{\tau_2=\tau_1}^{\infty} e^{-r\tau_2}f(\tau_2)d\tau_2f(\tau_1)d\tau_1 = (u_1^d-u_1^m)\frac{\lambda_2\lambda_1}{(r+\lambda_2)(r+\lambda_1+\lambda_2)},$$

which can also be written as

$$(u_1^d-u_1^m)\lambda_2\cdot\left(\frac{1}{r+\lambda_2} - \frac{1}{r+\lambda_1+\lambda_2}\right).$$

$$\mathbb{E}1_{\{\tau_1>\tau_2\}}e^{-r\tau_1}u_1^d = u_1^d\int_{\tau_2=0}^{\infty}\int_{\tau_1=\tau_2}^{\infty} e^{-r\tau_1}f(\tau_1)d\tau_1f(\tau_2)d\tau_2 = u_1^d\frac{\lambda_1\lambda_2}{\lambda_1+r}\frac{1}{r+\lambda_1+\lambda_2}, \quad (A.64)$$

which can also be written as

$$u_1^d\lambda_1\left(\frac{1}{r+\lambda_1} - \frac{1}{r+\lambda_1+\lambda_2}\right). \quad (A.65)$$

Putting Equation (A.62) to Equation (A.64) together,

$$V_1 = -\frac{R_1}{\lambda_1+r} + \frac{\lambda_1}{\lambda_1+\lambda_2+r}u_1^m + (u_1^d-u_1^m)\lambda_2 \cdot\left(\frac{1}{r+\lambda_2} - \frac{1}{r+\lambda_1+\lambda_2}\right) + u_1^d\lambda_1\left(\frac{1}{r+\lambda_1} - \frac{1}{r+\lambda_1+\lambda_2}\right), \quad (A.66)$$

which can be rearranged by the terms of u_1^m and u_1^d to get Equation (A.61) for $i = 1$. With $u_1^m = u_1^d$, or $\lambda_2 = 0$, then the project values is the same as it is for a UC monopoly firm, that is, $V_1 = (u_1^m\lambda_1 - R)/(r+\lambda_1)$. One should not use $u_1^d = 0$ to check V_1 for the case for winner-takes-all, because $u_1^d\lambda_1 - R_1 > 0$ is required for Equation (A.61). □

Proposition A.2. *In a non–winner-takes-all UCUC competition, if it is worthwhile to continue investing in the project after the rival firm gets discovery first, then the best responses $R_1^*(R_2)$, $R_2^*(R_1)$ are decreasing functions. A Nash equilibrium exists, and it is unique.*

Proof. The cross-derivative of a firm's project value with respect to both firms' investment intensities can be written as

$$\frac{\partial^2 V_1}{\partial R_1\partial R_2} = \left[\frac{\partial(u_1^m - u_1^d)}{\partial R_1}\cdot\left(\frac{r}{(r+\lambda_1+\lambda_2)^2} - \frac{r}{(\lambda_2+r)^2}\right) - \frac{2r(u_1^m-u_1^d)}{(\lambda_1+\lambda_2+r)^3}\right]\cdot\frac{\partial\lambda_2}{\partial R_2}. \quad (A.67)$$

Given that both u_1^m and u_1^d increase with R_1 and monopoly profits are more than duopoly profits, the first term is negative. The term after the minus sign is positive. Together, that gives us $\partial^2V_1/\partial R_1\partial R_2 < 0$; that is, V_1 satisfies decreasing differences in (R_1, R_2). By the standard monotone comparative statics and with the regular conditions on V_1 easily verified, the best response functions are decreasing functions. Because of the symmetry of the problem, and the convexity of the best responses, there can only be one intersection of the best response graphs, and thus the equilibrium is unique. □

Equation (A.67) indicates the only cases in which a firm in a non–winner-takes-all UCUC competition does not respond to its rival's strategy are when $u_1^d = u_1^m$ or when its rival firm's success rate does not depend on the investment intensity, that is, $\partial\lambda_2/\partial R_2 = 0$.

A.14. Costly External Financing in Section 4.3

The problem faced by a CEF monopoly firm is

$$\sup_{\underline{X}, R}\ \mathbb{E}\left\{\int_0^{\tau_c\wedge\tau_d}(X_t - R)\cdot e^{-rt}dt + \mathbb{1}_{\{\tau_d<\tau_c\}}u\cdot e^{-r\tau_d} - \int_0^{\tau_c\wedge\tau_d}h(X_t;R)\mathbb{1}_{\{X_t<R\}}\cdot e^{-rt}dt + \int_{\tau_c\wedge\tau_d}^{\infty}X_t\cdot e^{-rt}dt\right\}.$$

The project value V before discovery or abandonment follows:

$$(r+\lambda)V = \mu XV_X + \frac{1}{2}\sigma^2X^2V_{XX} + \lambda u - R - g(X), \quad (A.68)$$

where $g(X) = h(X;R,\delta)\cdot\ \mathbb{1}_{\{X<R\}}$, and I've omitted the subscript t of X_t. Assume $\partial h/\partial\delta > 0$. For example, $h(X;R) = \delta\cdot(R-X_t)^n$ with $n > 1$. A requirement for Proposition A.3 is $n < \alpha_2 + 1$.

Lemma A.4. *With an external financing cost $g(\cdot)$, the CEF firm optimally abandons an ongoing R&D project at the threshold $\underline{X}^*(R)$, which satisfies*

$$\int_{\underline{X}^*(R)}^{\infty}\frac{(R+g(s)-\lambda u)}{s^{\alpha_2+1}}\mathrm{d}s = 0. \quad (A.69)$$

The firm's optimal investment intensity R^*, conditional on investment, maximizes the project value:

$$R^*(X) = \underset{R>0}{\operatorname{argmax}}\ \frac{2}{(\alpha_2-\alpha_1)\sigma^2} \times\left[\int_{\underline{X}^*(R)}^{X}(R+g(s)-\lambda u)\cdot\left(\frac{X^{\alpha_2}}{s^{\alpha_2+1}} - \frac{X^{\alpha_1}}{s^{\alpha_1+1}}\right)\mathrm{d}s\right], \quad (A.70)$$

where $\alpha_1 = (1/2) - \mu/\sigma^2 - \sqrt{((1/2)-(\mu/\sigma^2))^2 + 2(\lambda+r)/\sigma^2}$, and $\alpha_2 =$ $(1/2) - (\mu/\sigma^2) + \sqrt{((1/2)-(\mu/\sigma^2))^2 + 2(\lambda+r)/\sigma^2}$.

Proof. Equation (A.68) is a second-order linear ODE. I follow Boyce and DiPrima (2000) (see theorem 3.7.1. in the 12th edition of the book) and Liu and Loewenstein (2002) to solve V. This method states that if the functions p, q, and h are continuous on an open interval, and if the differentiable functions $y_1(x)$ and $y_2(x)$ are linearly independent solutions of the homogeneous equation $y'' + p(x)y' + q(x)y = 0$ corresponding to the nonhomogeneous equation $y'' + p(x)y' + q(x)y = m(x)$, then a particular solution of the nonhomogeneous equation is

$$Y(x) = -y_1(x)\int_{x_1^*}^{x} \frac{y_2(s)m(s)}{W(y_1,y_2)(s)}\mathrm{d}s + y_2(x)\int_{x_2^*}^{x} \frac{y_1(s)m(s)}{W(y_1,y_2)(s)}\mathrm{d}s,$$

where $W(y_1,y_2)(x)$ is the Wronskian of $y_1(x)$ and $y_2(x)$, that is, $W(y_1,y_2)(x) = y_1y_2' - y_1'y_2$, and x_1^*, x_2^* are a conveniently chosen point on the open interval that the functions p, g, and h are defined.

To apply the method, recall that the general solution for Equation (A.68) is

$$V(X) = c_1X^{\alpha_1} + c_2X^{\alpha_2} + V_p(X), \tag{A.71}$$

where α_1, α_2 are defined in the lemma, and X^{α_1}, X^{α_2}, $2(R + g(R-x) - \lambda u)/X^2\sigma^2$ correspond to y_1, y_2, and m in the method. The method gives us a particular solution as follows:

$$\begin{aligned} V_p(X) = &-X^{\alpha_1}\int_{x_1^*}^{X} \frac{2(R+g(s)-u\lambda)}{(\alpha_2-\alpha_1)s^{\alpha_1+1}\sigma^2}\mathrm{d}s \\ &+ X^{\alpha_2}\int_{x_2^*}^{X} \frac{2(R+g(s)-\lambda u)}{(\alpha_2-\alpha_1)s^{\alpha_2+1}\sigma^2}\mathrm{d}s. \end{aligned} \tag{A.72}$$

For convenience, set both of the lower bounds at the abandonment level $\underline{X}$, that is, $x_1^* = x_2^* = \underline{X}$. Then by plugging in the particular solution and collecting terms of X^{α_1} and X^{α_2}, Equation (A.71) can be written as

$$\begin{aligned} V(X) = &X^{\alpha_1}\left(c_1 - \int_{\underline{X}}^{X} \frac{2(R+g(s)-\lambda u)}{(\alpha_2-\alpha_1)s^{\alpha_1+1}\sigma^2}\mathrm{d}s\right) \\ &+ X^{\alpha_2}\left(c_2 + \int_{\underline{X}}^{X} \frac{2(R+g(s)-\lambda u)}{(\alpha_2-\alpha_1)s^{\alpha_2+1}\sigma^2}\mathrm{d}s\right). \end{aligned} \tag{A.73}$$

Three boundary conditions are used to solve the optimal abandonment threshold $\underline{X}$, and c_1 and c_2 in the value function. The first is based on the value of a UC monopoly, and the next two are value-matching and smooth-pasting conditions for value functions at the optimal abandonment threshold.

$$\lim_{X\to\infty} V(X) = \frac{u\lambda - R}{\lambda + r} \tag{A.74}$$

$$V(\underline{X}) = 0 \tag{A.75}$$

$$\left.\frac{dV(X)}{dX}\right|_{X=\underline{X}} = 0. \tag{A.76}$$

From Equation (A.74), as long as $\lim_{X\to\infty}\int_{\underline{X}}^{X} 2(R+g(s)-\lambda u)/(\alpha_2-\alpha_1)s^{\alpha_2+1}\sigma^2\mathrm{d}s$ has a finite limit,[25] the coefficient associated with the term X^{α_2} is zero as $X\to\infty$. Therefore,

$$c_2 = -\int_{\underline{X}}^{\infty} \frac{2(R+g(s)-\lambda u)}{(\alpha_2-\alpha_1)s^{\alpha_2+1}\sigma^2}\mathrm{d}s. \tag{A.77}$$

To verify c_2 in Equation (A.77) satisfies Equation (A.74), plug the expression of c_2 in Equation (A.73) and take the limit of $X\to\infty$:

$$\begin{aligned} \lim_{X\to\infty} V(X) &= \lim_{X\to\infty}\left\{c_1X^{\alpha_1} - X^{\alpha_1}\int_{\underline{X}}^{X} \frac{2(R+g(s)-\lambda u)}{(\alpha_2-\alpha_1)s^{\alpha_1+1}\sigma^2}\mathrm{d}s\right. \\ &\quad - X^{\alpha_2}\int_{\underline{X}}^{\infty} \frac{2(R+g(s)-\lambda u)}{(\alpha_2-\alpha_1)s^{\alpha_2+1}\sigma^2}\mathrm{d}s \\ &\quad \left. + X^{\alpha_2}\int_{\underline{X}}^{X} \frac{2(R+g(s)-\lambda u)}{(\alpha_2-\alpha_1)s^{\alpha_2+1}\sigma^2}\mathrm{d}s\right\} \\ &= \lim_{X\to\infty}\left\{-\underbrace{X^{\alpha_1}}_{\to 0}\underbrace{\int_{\underline{X}}^{X} \frac{2(R+g(s)-\lambda u)}{(\alpha_2-\alpha_1)s^{\alpha_1+1}\sigma^2}\mathrm{d}s}_{\to\infty \text{ if } \alpha_1+1<0}\right. \\ &\quad \left. - \underbrace{X^{\alpha_2}}_{\to\infty}\underbrace{\int_{X}^{\infty} \frac{2(R+g(s)-\lambda u)}{(\alpha_2-\alpha_1)s^{\alpha_2+1}\sigma^2}\mathrm{d}s}_{\to 0}\right\}. \end{aligned} \tag{A.78}$$

By applying L'Hôpital's rule on Equation (A.78), and provided that $\alpha_1\alpha_2 = -2(r+\lambda)/\sigma^2$, Equation (A.74) is verified:

$$\begin{aligned} &\lim_{X\to\infty} V(X) \\ &\overset{(H)}{=} \lim_{X\to\infty}\left\{-\frac{\dfrac{2(R+g(R-X)-\lambda u)}{(\alpha_2-\alpha_1)X^{\alpha_1+1}\sigma^2}}{-\alpha_1X^{-\alpha_1-1}} - \frac{\dfrac{2(R+g(R-X)-\lambda u)}{(\alpha_2-\alpha_1)X^{\alpha_2+1}\sigma^2}}{-\alpha_2X^{-\alpha_2-1}}\right\} \\ &= \frac{2(R-\lambda u)}{(\alpha_2-\alpha_1)\alpha_1\sigma^2} - \frac{2(R-\lambda u)}{(\alpha_2-\alpha_1)\alpha_2\sigma^2} = \frac{u\lambda - R}{\lambda + r}. \end{aligned}$$

Equation (A.75), gives us the solution for c_1:

$$V(\underline{X}) = c_1\underline{X}^{\alpha_1} + c_2\underline{X}^{\alpha_2} + 0 = 0 \Rightarrow c_1 = -c_2\underline{X}^{\alpha_2-\alpha_1}. \tag{A.79}$$

Applying the first fundamental theorem of calculus on Equation (A.76) leads to the following equation, with the second-order condition $\partial^2V(X,\underline{X};R)/\partial\underline{X}^2\big|_{\underline{X}^*} < 0$ ensuring the optimality of $\underline{X}$:

$$\begin{aligned} &c_1\alpha_1\underline{X}^{\alpha_1-1} + c_2\alpha_2\underline{X}^{\alpha_2-1} - \underline{X}^{\alpha_1}\frac{2(R+g(R-\underline{X})-\lambda u)}{(\alpha_2-\alpha_1)\underline{X}^{\alpha_1+1}\sigma^2} \\ &\quad + \underline{X}^{\alpha_2}\frac{2(R+g(R-\underline{X})-\lambda u)}{(\alpha_2-\alpha_1)\underline{X}^{\alpha_2+1}\sigma^2} = 0 \end{aligned}$$

$$\text{use Eq.(A.79)} \quad \Rightarrow -c_2\underline{X}^{\alpha_2-\alpha_1}\alpha_1\underline{X}^{\alpha_1-1} + c_2\alpha_2\underline{X}^{\alpha_2-1} + 0 = 0$$

$$\Rightarrow c_2(-\alpha_1+\alpha_2)\underline{X}^{\alpha_2-1} = 0 \tag{A.80}$$

Equation (A.80) suggests two possibilities: (1) $\underline{X} = 0$ and (2) $c_2 = 0$. I discard the first one because it is not sensible for the CEF firm to never abandon the project regardless of how costly the external financing is. With $c_2 = 0$, Equation (A.79) gives us $c_1 = 0$. To solve the optimal abandonment threshold given any choice of investment intensity R, I get from Equation (A.77):

$$-\int_{\underline{X}}^{\infty} \frac{2(R+g(s)-\lambda u)}{(\alpha_2-\alpha_1)s^{\alpha_2+1}\sigma^2}\mathrm{d}s = 0 \Rightarrow \int_{\underline{X}}^{\infty} \frac{(R+g(s)-\lambda u)}{s^{\alpha_2+1}}\mathrm{d}s = 0. \tag{A.81}$$

Equation (A.81) indicates that the threshold should be a function of the investment scale $\underline{X}(R)$. Moreover, given that g is nonincreasing in X_t and is positive only when $X_t < R$, Equation (A.81) suggests that the CEF firm is willing to endure negative cash flows with costly financing being possible and $\underline{X} < R$. With $c_1 = c_2 = 0$ plugged into Equation (A.73), the project value before success and abandonment is

$$V(X) = X^{\alpha_2}\int_{\underline{X}}^{X}\frac{2(R+g(s)-\lambda u)}{(\alpha_2-\alpha_1)s^{\alpha_2+1}\sigma^2}\,\mathrm{ds} - X^{\alpha_1}\int_{\underline{X}}^{X}\frac{2(R+g(s)-\lambda u)}{(\alpha_2-\alpha_1)s^{\alpha_1+1}\sigma^2}\,\mathrm{ds}$$
$$= \frac{2}{(\alpha_2-\alpha_1)\sigma^2}\left[\int_{\underline{X}}^{X}(R+g(s)-\lambda u)\left(\frac{X^{\alpha_2}}{s^{\alpha_2+1}} - \frac{X^{\alpha_1}}{s^{\alpha_1+1}}\right)\mathrm{ds}\right]. \quad \text{(A.82)}$$

□

Proposition A.3. *The abandonment threshold increases with the cost of external financing, that is,* $\partial\underline{X}/\partial\delta > 0$.

Proof. Rewrite Equation (A.69) as $f(g(X; R, \delta), \underline{X}) = 0$. With the conditions of the implicit function theorem satisfied, and given a fixed R that is relevant for an CEF firm (i.e., $\lambda u > R$):

$$\frac{\partial\underline{X}}{\partial\delta} = -\frac{\frac{\partial f}{\partial\delta}}{\frac{\partial f}{\partial\underline{X}}} = -\frac{\int_{\underline{X}^*(R)}^{\infty}\frac{1}{s^{\alpha_2+1}}\frac{\partial g(\cdot)}{\partial\delta}\mathrm{ds}}{-\frac{R+g(\underline{X},\delta)-\lambda u}{\underline{X}^{\alpha_2+1}}} = -\frac{\oplus}{-\oplus} > 0 \quad \text{(A.83)}$$

The numerator of the third expression in Equation (A.83) is positive because each element in the integral is nonnegative and some are positive. The denominator has a negative sign in the front as $\underline{X}$ is the lower end of the interval over which the integral is taken. The term $R + g(\underline{X}, \delta) - \lambda u/\underline{X}^{\alpha_2+1}$ is positive because this is the largest term of all the integrands, and all the integrands add up to zero. □

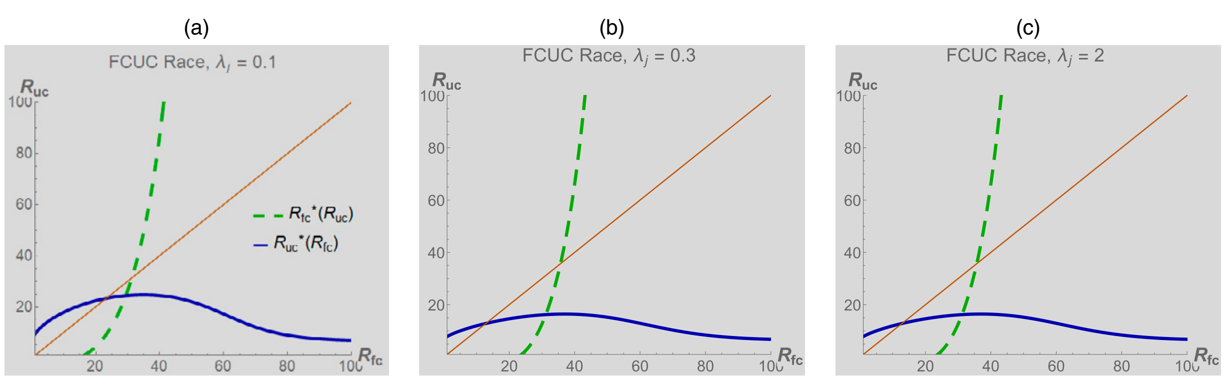
Appendix B. Additional Tables and Figures

Table B.1. Investment Intensities and Abandonment Thresholds in Section 4.3

	$\delta = 0.001$	$\delta = 1$	$\delta = 2$
$X = 75$	$R = 6.75$, $\underline{X} = 0$	$R = 6.85$, $\underline{X} = 2.80$	$R = 7.08$, $\underline{X} = 3.94$
$X = 100$	$R = 6.79$, $\underline{X} = 0$	$R = 7.22$, $\underline{X} = 3.08$	$R = 7.47$, $\underline{X} = 4.24$
$X = 150$	$R = 6.80$, $\underline{X} = 0$	$R = 7.46$, $\underline{X} = 3.27$	$R = 7.72$, $\underline{X} = 4.45$

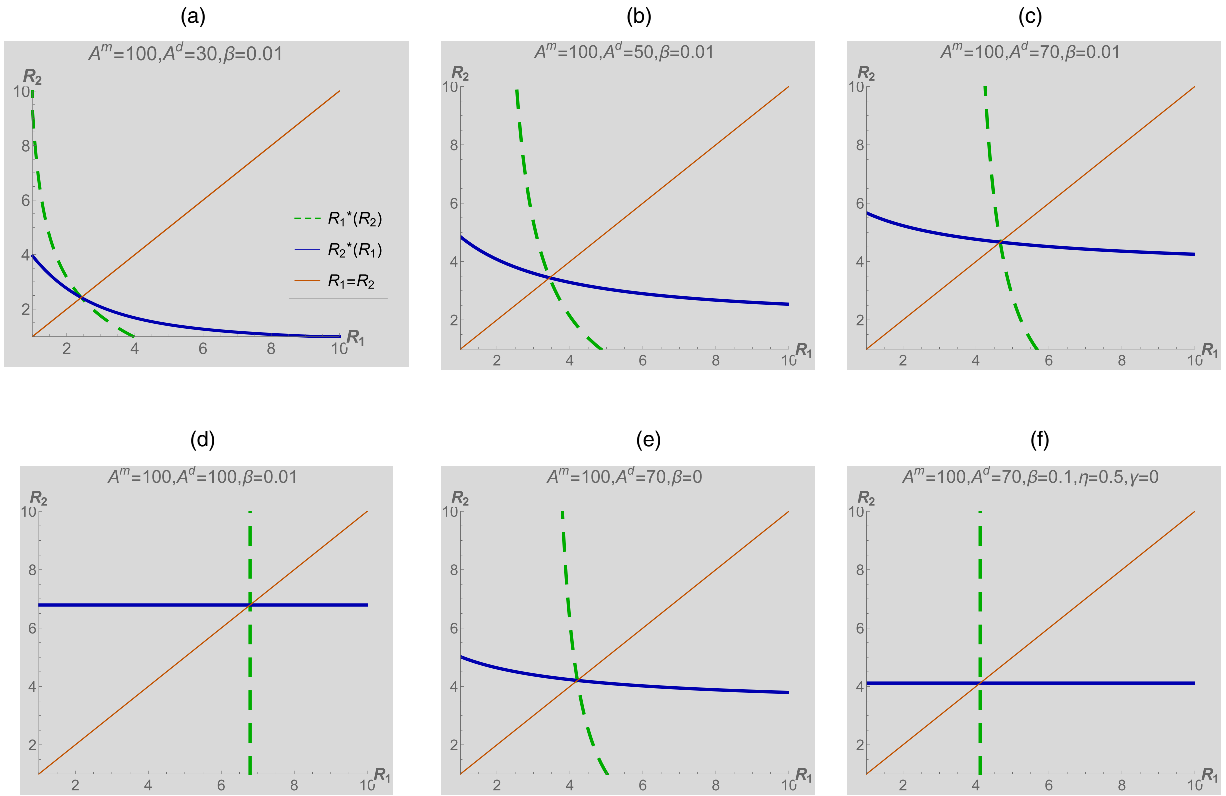
Notes. This table reports the investment intensity R and the abandonment threshold $\underline{X}$ on the AIP cash flow for a monopoly firm that is subject to external financing cost of $g(X) = \delta\cdot(R - X_t)^2\ \mathbb{1}_{\{R<X_t\}}$. The parameter values are set as in Table 1; $R_{uc} = 6.79$, $R_{fc} = 8.02$ in the baseline monopoly model.

Figure B.1. (Color online) FCUC Race with the Jump Risk in AIP Cash Flow



Notes. The figures plot the best responses and equilibrium of an FCUC race with $\lambda_j > 0$ on AIP cash flow. The plot description is similar to that of Figure 1.

Figure B.2. (Color online) UCUC Competition Without Winner-Takes-All



Notes. These are examples of the best responses and equilibria in a UCUC competition without the winner-takes-all assumption (Section 4.2). The blue solid lines plot Firm 2's best response to the investment rate of Firm 1, that is, $R_2^*(R_1)$, with R_1 on the x axis. The green dashed lines plot $R_1^*(R_2)$ with R_2 on the y axis. The orange lines represent $R_1 = R_2$. All other parameters, except the ones labeled, are set at the values in Table 1.

Endnotes

[1] Equation (1) can alternatively be interpreted as stochastic cash flows from a financier, so the model can apply not only to an established firm that has a new R&D investment opportunity, but also to a startup that uses venture capital funding to develop its first big idea.

[2] This implies that there is no cumulative learning in the model (see Berk et al. 2004 and Childs and Triantis 1999 for R&D models with learning), and τ_d follows an exponential distribution. R can also be interpreted as the searching intensity in the search models (Mortensen 1986). More-intense searching leads to speedier discovery and/or better search outcome.

[3] Power functions satisfy both (1) the nonincreasing discovery rate elasticity needed in patent race models to ensure the stability of equilibrium (Nti 1999) and (2) the decreasing returns to scale assumption needed to limit the optimal investment intensity. In addition, the constant discovery rate elasticity ($R\lambda'/\lambda = \gamma$) and the constant payoff elasticity ($Ru'/u = \beta$) from the power function help simplify model derivations.

[4] This model can be used more generally to study the tradeoff between a better-but-slower discovery versus a worse-but-speedier one (i.e., $\beta\gamma < 0$), but it is not the focus of this paper.

[5] I do not interpret the financial constraints as collateral constraints (as in Li 2011 and Rampini and Viswanathan 2013); thus, the liquidation of AIP does not prohibit a UC firm from getting external financing for its innovation project. This is relevant for the discussion in Section 4.1 regarding the jump risk in AIP cash flows.

[6] This is to exclude the investment incentive to relax financing constraints (Almeida et al. 2011), plus the main results will not change qualitatively if there is a moderate cannibalization cost (Hackbarth et al. 2014), that is, a negative effect on the AIP cash flows from a successful innovation.

[7] The assumption seems extreme, but the huge uncertainty during innovative project development and the exclusiveness of accumulated knowledge make it very difficult to evaluate the resale value of underdeveloped intangible assets.

[8] The proof of Lemma A.1 in Appendix A.12 shows that a monopoly firm never abandons an ongoing project unless it has to.

[9] A wide range of parameter values of μ are checked to ensure the findings in the paper are relevant for industries at large, instead of only the industries with sharply declining profits.

[10] At the baseline optimum, an FC (UC) monopoly firm spends around 8% (6.8%) of its instantaneous AIP cash flows on R&D, and the project value is around 16% (18%) of what its AIP are worth, at the time of the project initiation. The discovery takes about five years and slightly longer for UC than for FC monopoly.

[11] Boyle and Guthrie (2003) find that the threat of future funding shortfalls encourages acceleration of investment if the benefits of delay are outweighed by the risk of losing the ability to finance the project. Bolton et al. (2019) take a step further and show that financial constraints erode option value and lead to earlier investment, particularly, in projects with frontloaded cash flows. With risky debt financing, Lyandres and Zhdanov (2010) show the probability of default leads to the effect of accelerated investment via reducing the value of the option to wait.

[12] It implies that the project types are the same for the two competing firms (i.e., $\text{sign}(\lambda_1') = \text{sign}(\lambda_2')$, $\text{sign}(u_1') = \text{sign}(u_2')$). The model is flexible to allow $A_1 \neq A_2$, $\beta_1 \neq \beta_2$, $\eta_1 \neq \eta_2$, and $\gamma_1 \neq \gamma_2$.

[13] Patent race models are often used to study the impact of the number of competing firms on firm profits and the expected time of new technology introduction but are not concerned with the effect of financial constraints in such competition.

[14] The assumption that firms in the race do not actively abandon their ongoing projects simplifies the analysis. Otherwise, a firm wins the race if its success happens before its own project abandonment and its rival's success.

[15] Equation (12) differs from Equation (6) in Lemma 1 in that all the r terms are replaced by $\lambda_2 + r$. Proposition 2(v) shows the effect of r regardless of accelerability, whereas $\lambda' > 0$ is necessary here because it is through the discovery rate that the rival's project accelerability is relevant, without which R_{uc} does not matter for V_{fc}.

[16] The condition $X \gg (A\eta)^{1/1-(\beta+\gamma)}$ translates to the AIP cash flow at the project arrival being higher than 15 times of the optimal FC monopoly investment at the baseline.

[17] Alternatively, a pre-emption in an FCUC race can be understood as a firm investing more aggressively than it would in a UCUC race or in a monopoly. Also see Fudenberg et al. (1983) for a dynamic setting of pre-emption, in which the relative position during the race matters for firms' decisions.

[18] There are two more Dirichlet boundary conditions available to use: (1) $\lim_{X_1 \to \infty} V_1(X_1, X_2 = R_2) = \lambda_1 u_1 - R_1/r + \lambda_1$ and (2) $\lim_{X_1 \to \infty, X_2 \to \infty} V_1(X_1, X_2) = \lambda_1 u_1 - R_1/\lambda + \lambda_2 + r$. However, these two turn out to be converging cases for the ones already listed and are thus redundant.

[19] Numerical exercises on a wide range of parameters suggest $V_{fc,fcfc} > V_{uc,fcuc} > V_{fc,fcuc} > V_{uc,ucuc}$ if the project is accelerable. It implies that given a fixed rival, a firm's project value is higher if the firm is constrained than if it is unconstrained. The seemingly counter-intuitive result is related to how the rival firm reacts to the competition.

[20] A few examples of a catastrophe include (1) a car manufacturer is found to have fatal defects in its models, (2) a smartphone company is blown by a recall crisis, or (3) a pharmaceutical firm loses its dominant status in a market when its patent expires and its competitor successfully manufactured a generic drug. The diffusion and the jump processes are independent of each other.

[21] The UCUC race is not affected by λ_j, whereas the analytical solutions for the FCFC race with λ_j are not feasible.

[22] I focus on the UCUC race, and I describe firms' problems in the FCUC race without the winner-takes-all assumption in the online appendix, but solving that model is beyond the scope of this paper.

[23] It is conditional on being worthwhile to keep investing in an R&D project after a rival firm makes a first discovery as the proposition shows. The existence of the equilibrium is guaranteed in this two-player game (Vives 1999).

[24] One interpretation of the cost function $h(\cdot)$ is that it captures a cash bribery to the existing equity holders when it issues new equity or a flotation cost at issuance. The term $h(\cdot)$ is increasing and convex. The same assumptions are used in papers regarding financial constraints (Kaplan and Zingales 1997, Hennessy and Whited 2007).

[25] If g is a polynomial consisting the highest degree of h, then a sufficient condition for having a finite limit is $\alpha_2 > h$. With $h = 2$ in the functional form of g, this corresponds to $\mu < 0$ with all other parameters at the benchmark and R at the lower boundary.

References

Almeida H, Campello M, Weisbach MS (2011) Corporate financial and investment policies when future financing is not frictionless. *J. Corporate Finance* 17:675–693.

Amir R (2005) Supermodularity and complementarity in economics: An elementary survey. *Southern Econom. J.* 71(3):636–660.

Athey S (2002) Monotone comparative statics under uncertainty. *Quart. J. Econom.* 117:187–223.

Aw BY, Roberts MJ, Xu DY (2011) R&D investment, exporting, and productivity dynamics. *Amer. Econom. Rev.* 101:1312–1344.

Berk J, Green R, Naik V (2004) Valuation and return dynamics of new ventures. *Rev. Financial Stud.* 17:1–35.

Bloom N (2007) Uncertainty and the dynamics of R&D. *Amer. Econom. Rev.* 97:250–255.

Bolton P, Chen H, Wang N (2011) A unified theory of Tobin's q, corporate investment, financing, and risk management. *J. Finance* 66: 1545–1578.

Bolton P, Chen H, Wang N (2013) Market timing, investment, and risk management. *J. Financial Econom.* 109:40–62.

Bolton P, Wang N, Yang J (2019) Investment under uncertainty with financial constraints. *J. Econom. Theory* 184:104912.

Boyle GW, Guthrie GA (2003) Investment, uncertainty, and liquidity. *J. Finance* 58:2143–2166.

Boyce W, DiPrima R (2000) *Elementary Differential Equations and Boundary Value Problems*, 7th ed. (Wiley, New York).

Brown JR, Petersen BC (2011) Cash holdings and R&D smoothing. *J. Corporate Finance* 17:694–709.

Brown JR, Fazzari SM, Petersen BC (2009) Financing innovation and growth: Cash flow, external equity, and the 1990s R&D boom. *J. Finance* 64:151–185.

Childs PD, Triantis AJ (1999) Dynamic R&D investment policies. *Management Sci.* 45:1359–1377.

Dasgupta P, Stiglitz J (1980) Uncertainty, industrial structure, and the speed of R&D. *Bell J. Econom.* 11(1):1–28.

Dixit AK, Pindyck R (1994) *Investment Under Uncertainty* (Princeton University Press, Princeton, NJ).

Doraszelski U, Jaumandreu J (2013) R&D and productivity: Estimating endogenous productivity. *Rev. Econom. Stud.* 80:1338–1383.

Eisdorfer A, Hsu P-H (2011) Innovate to survive: The effect of technology competition on corporate bankruptcy. *Financial Management* 40:1087–1117.

Fudenberg D, Gilbert R, Stiglitz J, Tirole J (1983) Preemption, leapfrogging, and competition in patent races. *Eur. Econom. Rev.* 22:3–31.

Grenadier SR (2002) Option exercise games: An application to the equilibrium investment strategies of firms. *Rev. Financial Stud.* 15:691–721.

Gu L (2016) Product market competition, R&D investment, and stock returns. *J. Financial Econom.* 119:441–455.

Hackbarth D, Mathews R, Robinson D (2014) Capital structure, product market dynamics, and the boundaries of the firm. *Management Sci.* 60:2971–2993.

Hall BH (2002) The financing of research and development. *Oxford Rev. Econom. Policy* 18:35–51.

Hamermesh DS, Pfann GA (1996) Adjustment costs in factor demand. *J. Econom. Literature* 34:1264–1292.

Hellmann T, Puri M (2000) The interaction between product market and financing strategy: The role of venture capital. *Rev. Financial Stud.* 13:959–984.

Hennessy CA, Whited TM (2007) How costly is external financing? Evidence from a structural estimation. *J. Finance* 62: 1705–1745.

Hennessy CA, Levy A, Whited TM (2007) Testing Q theory with financing frictions. *J. Financial Econom.* 83:691–717.

Himmelberg CP, Petersen BC (1994) R&D and internal finance: A panel study of small firms in high-tech industries. *Rev. Econom. Statist.* 76(1):38–51.

Hugonnier J, Malamud S, Morellec E (2015) Capital supply uncertainty, cash holdings, and investment. *Rev. Financial Stud.* 28:391–445.

Jensen MC (1986) Agency cost of free cash flow, corporate finance, and takeovers. *Amer. Econom. Rev.* 76(2):76.

Kaplan SN, Zingales L (1997) Do investment-cash flow sensitivities provide useful measures of financing constraints? *Quart. J. Econom.* 112:169–215.

Lambrecht BM (2001) The impact of debt financing on entry and exit in a duopoly. *Rev. Financial Stud.* 14:765–804.

Lerner J, Sorensen M, Strömberg P (2011) Private equity and long-run investment: The case of innovation. *J. Finance* 66:445–477.

Li D (2011) Financial constraints, R&D investment, and stock returns. *Rev. Financial Stud.* 24:2974–3007.

Liu D, Li Z, He H, Hou W (2021) The determinants of R&D smoothing with asset sales: Evidence from R&D-intensive firms in China. *Internat. Rev. Econom. Finance* 75:76–93.

Liu H, Loewenstein M (2002) Optimal portfolio selection with transaction costs and finite horizons. *Rev. Financial Stud.* 15:805–835.

Loury GC (1979) Market structure and innovation. *Quart. J. Econom.* 93:395–410.

Lyandres E, Zhdanov A (2010) Accelerated investment effect of risky debt. *J. Banking Finance* 34:2587–2599.

Ma L, Mello AS, Wu Y (2020) First-mover advantage, time to finance, and cash holdings. *J. Corporate Finance* 62:101584.

Malamud S, Zucchi F (2019) Liquidity, innovation, and endogenous growth. *J. Financial Econom.* 132:519–541.

Manso G (2011) Motivating innovation. *J. Finance* 66:1823–1860.

McDonald R, Siegel D (1986) The value of waiting to invest. *Quart. J. Econom.* 101:707–727.

Meng R (2008) A patent race in a real options setting: Investment strategy, valuation, CAPM beta, and return volatility. *J. Econom. Dynamic Control* 32:3192–3217.

Merton RC (1976) Option pricing when underlying stock returns are discontinuous. *J. Financial Econom.* 3:125–144.

Milgrom P, Roberts J (1990) Rationalizability, learning, and equilibrium in games with strategic complementarities. *Econometrica* 58:1255–1277.

Morellec E, Schürhoff N (2011) Corporate investment and financing under asymmetric information. *J. Financial Econom.* 99:262–288.

Mortensen DT (1986) Job search and labor market analysis. Ashenfelter O, Layard R, eds. *Handbook of Labor Economics*, vol. 2 (Elsevier, Amsterdam), 849–919.

Myers SC (1977) Determinants of corporate borrowing. *J. Financial Econom.* 5:147–175.

Nanda R, Rhodes-Kropf M (2017) Financing risk and innovation. *Management Sci.* 63:901–918.

Novy-Marx R (2007) An equilibrium model of investment under uncertainty. *Rev. Financial Stud.* 20:1461–1502.

Nti KO (1999) Stability in the patent race contest of Lee and Wilde. *Econom. Theory* 14:237–245.

Phillips G, Zhdanov A (2013) R&D and the incentives from merger and acquisition activity. *Rev. Financial Stud.* 26:34–78.

Rampini A, Viswanathan S (2013) Collateral and capital structure. *J. Financial Econom.* 109:466–492.

Reinganum JF (1989) The timing of innovation: Research, development, and diffusion. Schmalensee R, Willig RD, eds. *Handbook of Industrial Organization*, vol. 1 (Elsevier, Amsterdam), 849–908.

Schroth E, Szalay D (2010) Cash breeds success: The role of financing constraints in patent races. *Rev. Finance* 14:73–118.

Shannon C (1995) Weak and strong monotone comparative statics. *Econom. Theory* 5:209–227.

Tarski A (1955) A lattice-theoretic fixed point theorem and its applications. *Pacific J. Math.* 5(2):285–309.

Tian X, Wang TY (2014) Tolerance for failure and corporate innovation. *Rev. Financial Stud.* 27:211–255.

Topkis DM (1978) Minimizing a submodular function on a lattice. *Oper. Res.* 26:305–321.

Vives X (1999) *Oligopoly Pricing: Old Ideas and New Tools* (MIT Press, Cambridge, MA).

Vives X (2007) Supermodularity and supermodular games. https://media.iese.edu/research/pdfs/OP-07-18-E.pdf.

Weeds H (2002) Strategic delay in a real options model of R&D competition. *Rev. Econom. Stud.* 69:729–747.