Two-Hop Connectivity to the Roadside in a VANET Under the Random Connection Model

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Abstract—In this paper, we compute the expected number of vehicles with at least one two-hop path to a fixed roadside unit (RSU) in a multi-hop, one-dimensional vehicular ad hoc network (VANET) where other cars can act as relays. The pairwise channels experience Rayleigh fading in the random connection model, and so exist, with probability function of the mutual distance between the cars, or between the cars and the RSU. We derive exact equivalents for the expected number of cars with a two-hop connection to the RSU when the car density ρ tends to zero and infinity, and determine its behaviour using an infinite oscillating power series in ρ, which is accurate for all regimes. We also corroborate those findings to a realistic situation, using snapshots of actual traffic data. Finally, a normal approximation is discussed for the probability mass function of the number of cars with a two-hop connection to the RSU.

Index Terms—Vehicular networks, end-to-end connectivity, random connection model, stochastic geometry, mobility traces.

I. Introduction

The requirement for wireless communication technologies that sustain reliable connectivity between vehicles in smart motorways will become essential. A key element of the road infrastructure will be the roadside unit (RSU) with mounted sensors and wireless connectivity. The RSU will receive a variety of messages from vehicles not necessarily connected to each other, fuse the combined information and broadcast it to the vehicles [1]. In this scenario, it is important that the broadcast reaches as many vehicles as possible. A practical solution to achieve this objective is to combine both Vehicle-to-Vehicle (V2V) and Vehicle-to-Infrastructure (V2I) communication, establishing connections between the vehicles and the RSU in a multi-hop fashion. 3GPP Release 16 has introduced in the 5G NR the sidelink air interface to support, among other use cases, the range extension for transportation safety [2]. It is therefore a natural step to start with the simplest case of multi-hop connectivity to the RSU with $k = 2$ hops.

Motivated by this emerging scenario, the objective of this article is to develop our understanding of the RSU multi-hop communication range, i.e., how many vehicles are within k hops of the RSU, as a function of the traffic density ρ cars per unit length of road. In this case, we adopt a *protocol-and-interference-free* approach, where the vehicles are modeled as
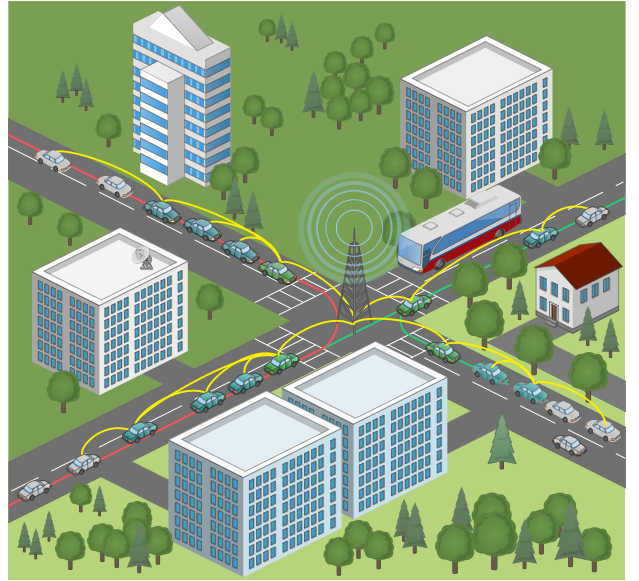
Fig. 1. A snapshot of an infrastructure-based VANET model with multi-hop connectivity. Green cars have single-hop connections to the RSU, blue cars are two-hop neighbours and gray cars are more than two hops away from the RSU. Every car is represented by a vertex in the graph model and an edge between two vertices exists if the two cars are connected to each other.

the vertices of a one-dimensional (1D) soft random geometric graph (RGG) [3]. The reason is that we aim to identify the fundamental limits of multi-hop communication in VANETs, in particular, the simplest case, two-hop connectivity, between the vehicles and the RSU. We study the number of cars within $k = 2$ hops to a single, fixed RSU, denoted as N_2, in a random geometric vehicular network built on a quasi-1D line, i.e., a multi-lane motorway provided that the connection range is much larger than the road width, which is the case for the nominal range values of C-V2X and DSRC/ITS-G5.

The main technical contribution of our work is an expression for the scaling of $\mathbb{E}[N_2]$ with density

$$\mathbb{E}[N_2] \simeq 2\rho\sqrt{\log\left(\rho\sqrt{\pi/2}\right)}, \tag{1}$$

as $\rho \to \infty$. This is actually a counter-intuitive result, as one might have surmised that the quantity $\mathbb{E}[N_2]$ would grow proportionally to ρ^2. We also derive an exact expression for $\mathbb{E}[N_2]$, given by Eq. (7) and we compare these analytic predictions to snapshots of real motorway traffic, showing how variations in the underlying stationary Poisson Point Process

(PPP) model of the cars does not affect the results, and observe a normal approximation property for N_2.

Two recent major works deal with the random connection model in the V2V and V2I settings [4], [5]. Ng et al. in [4] present analytic formulas for the probability of two-hop connectivity, which gives the probability that all vehicles in a VANET are within two hops, given that the vehicles connect pairwise with probability p. This is developed for the general case of an arbitrary number of hops to the roadside by Zhang et al. in [5]. The study in [6] calculates the probability that a node in a 1D PPP has a two-hop path to the RSU. However, the distribution of the number of nodes with a two-hop path is not treated therein, neither asymptotic equivalents for ultra-dense and sparse networks are derived, as we will do in this paper. In references [7] and [8] the location of a node is fixed and the distribution of the number of k-hop paths to the RSU is computed. This is different than the number of nodes in the graph with a k-hop path to the RSU, which is treated in the present paper for $k = 2$. Note that a node may have several two-hop paths to the RSU, e.g., more than one relay to the RSU in the case of two-hop paths. Finally, it is worth mentioning that the connectivity analysis and the results of this paper are applicable for each road segment (or line) of two-dimensional road network models such as the Poisson and Manhattan line processes [9], [10]. The only conditions is that the distribution of vehicles along each line must follow the 1D homogeneous PPP and the connection function must be a stretched exponential, as we will shortly discuss.

In the rest of this paper, we set up the VANET deployment model and the random connection model in Section II. In Section III, we state the main results of this paper. In Section IV, we corroborate our scaling laws with experimental evidence provided from synthetic motorway traffic data. Finally, conclusions and future steps are discussed in Section V.

II. SYSTEM MODEL

We consider a *random connection model*, which is an RGG $G_H(\mathcal{X})$ built on a stationary, homogeneous PPP $\mathcal{X}$ with flat intensity $\rho > 0$ on an interval $V \subset \mathbb{R}$ centered at the origin. The nodes (or vertices) of the graph represent vehicles. In addition, we add a vertex at the location $u \in \mathbb{R}$ representing the RSU. See Fig. 1. The edges of the graph represent wireless communication links (known in physics as *soft connectivity* [12]) with the connection function $H(r)$ modeling "Rayleigh fading" for any V2V or V2I communication link at distance r. The selection of Rayleigh fading is relevant to our system model; Channel measurements have indicated that the narrowband small scale fading in V2V communication resembles Rayleigh for link distances larger than 50 m [11].

The connection function $H(r) = \mathbb{P}(x \leftrightarrow y) \in [0, 1]$ gives the probability that two nodes $x, y \in \mathbb{R}$ at distance $r = |x-y|$ in the graph are connected by an edge, resembling the long-term proportion of time that a Rayleigh fading channel at distance separation r is in coverage [17]. It is a stretched exponential of the link distance r and has been widely used to study connectivity in soft RGGs [3], [12]

$$H(r) = e^{-\beta r^\eta}, \ \beta > 0, \tag{2}$$

where the parameter β depends on the signal-to-noise (SNR) decoding threshold at the receiver and the exponential form captures the fact that in Rayleigh channels the distribution of the received signal power within a symbol's duration is exponential.

In the next section, we will take $\beta = 1$ for brevity, and we will focus on the case with $\eta = 2$, which is associated with free space wireless propagation path. Our analysis stays the same for other values of $\beta > 0$, while the assumption for $\eta = 2$ will allow us to express multi-dimensional integrals in a closed-form and obtain important performance insights for multi-hop connectivity. It is also noted that to have a reliable two-hop path to the RSU, the values of the connection function for both hops (V2V and V2I) must be higher than or equal to a threshold. From this perspective, a *decode-and-forward* relay is essentially assumed here.

Mecke formula: We close this section with the statement of the Mecke formula, which will be used throughout the paper. It is an extension of Campbell's formula to sum over tuples of a point process. Often, and throughout this paper, we sum indicator functions over tuples of nodes of a point process, and evaluate the typical value of the sum over all graphs. We refer the reader to [3, Lemma 2.3] for the following lemma.

Lemma II.1 (Mecke Formula)**.** *Let $n \geq 1$. For any measurable real-valued function g defined on the vertex set V of a graph $\mathcal{G}$, the following relation holds:*

$$\mathbb{E}\left[\sum_{X_1 \in \mathcal{X}} g(X_1)\right] = \rho \int_V \mathbb{E}\left[g(x_1)\right] \mathrm{d}x_1 \tag{3}$$

where $\mathcal{X}$ is a PPP with intensity $\rho > 0$ on V.

III. RESULTS

This section presents the main technical results of our work. We start by introducing the preliminaries and then address the problem of determining the statistics of the number of vehicles on the road with a two-hop path connection to the RSU.

A. Preliminaries

In what follows, we let

$$N_1(x, u) = \sum_{z \in \mathcal{X}} \mathbf{1}(x \rightarrow z \rightarrow u) \tag{4}$$

denote the number of distinct two-hop paths between vertices at x and u in V, and $\mathbf{1}(\cdot)$ being the indicator function. The following lemma is used throughout this article.

Lemma III.1. *The number $N_1(x, u)$ of two-hop paths between two given vertices $x, u \in V$ has a Poisson distribution.*

Proof. A proof of this result can be found in [8, Section 2] by computing the moments of $N_1(x, u)$ using the Mecke formula and non-flat partitions. $\square$

Next, we compute the mean number of two-hop paths which join two vertices of the graph, starting with the case of a finite interval of width $|V| < \infty$, which will later become the real line $\mathbb{R}$.

Proposition III.1. *When we consider a domain of finite volume $|V|$, the probability of existence of at least one two-hop path between $x, u \in V$ is given by*

$$\mathbb{P}(x \to u) = 1 - \exp\left(-\frac{\rho}{2}\sqrt{\frac{\pi}{2}}e^{-(x-u)^2/2}\times\right.$$
$$\left.\left(\text{erf}\left(\frac{|V| - (x-u)}{\sqrt{2}}\right) + \text{erf}\left(\frac{|V| + x - u}{\sqrt{2}}\right)\right)\right), \quad (5)$$

where $\text{erf}(x) = (2/\sqrt{\pi})\int_0^x e^{-t^2}\,\mathrm{d}t$ *is the error function.*

Proof. The mean of $N_1(x, u)$ is given by the Campbell's theorem for point processes as follows

$$\mathbb{E}[N_1(x, u)] = \rho \int_V \mathbb{E}\left[\mathbf{1}(x \to z \to u)\right]\mathrm{d}z$$
$$= \rho \int_V H(x, z)H(z, u)\mathrm{d}z$$
$$= \rho \int_{-|V|/2}^{|V|/2} e^{-(x-z)^2 - (z-u)^2}\mathrm{d}z$$

Then, use the void probability of the Poisson distribution

$$\mathbb{P}(x \leftrightarrow u) = 1 - e^{-\mathbb{E}[N_1(x,u)]}. \quad (6)$$

to obtain Eq. (5). $\qquad\square$

Proposition III.2 below, which treats the infinite length case $V = \mathbb{R}$, can be obtained either from Proposition III.1 by letting $|V| \to \infty$ in Eq. (5), or from Lemma III.1.

Proposition III.2. *The probability of existence of at least one two-hop path between x and u in $\mathbb{R}$ is given by*

$$\mathbb{P}(x \leftrightarrow u) = 1 - \exp\left(-\rho\sqrt{\frac{\pi}{2}}e^{-(x-u)^2/2}\right).$$

Proof. Here, we have

$$\mathbb{E}[N_1(x, u)] = \rho \int_{-\infty}^{\infty} \mathbb{E}\left[\mathbf{1}(x \to z \to u)\right]\mathrm{d}z$$
$$= \rho \int_{-\infty}^{\infty} e^{-(x-z)^2 - (z-u)^2}\mathrm{d}z$$
$$= \rho\sqrt{\frac{\pi}{2}}e^{-(x-u)^2/2},$$

and again use the void probability of the Poisson distribution. $\qquad\square$

It is worth noting that for small distances $|x - u|$, the two vertices can also have a single-hop connection.

B. Main result

Next, we assume that the RSU is located at u and apply Proposition III.2 to analyze the mean number of vertices with two-hop connectivity to the RSU.

Theorem III.1. *Let N_2 be the number of vertices with two-hop connectivity to the RSU in a specific realisation of $G_H(\mathcal{X})$. The mean number of vertices with at least one two-hop path to the RSU is given by*

$$\mathbb{E}[N_2] = \rho\sqrt{2\pi}\sum_{k=1}^{\infty}(-1)^{k-1}\frac{\left(\rho\sqrt{\pi/2}\right)^k}{k!\sqrt{k}}. \quad (7)$$

In addition, we have the exact equivalents

$$\mathbb{E}[N_2] \simeq 2\rho\sqrt{2\ln\rho + 2\ln\sqrt{\pi/2}} \quad (8)$$

as $\rho \to \infty$, and

$$\mathbb{E}[N_2] \simeq \pi\rho^2 \quad (9)$$

as $\rho \to 0$.

Proof. We represent the number N_2 as the sum

$$N_2 = \sum_{x \in \mathcal{X}} \mathbf{1}(x \leftrightarrow u). \quad (10)$$

Next, we find via Lemma II.1 and Proposition III.2, that the expected value of this number is

$$\mathbb{E}[N_2] = \mathbb{E}\left[\sum_{x \in \mathcal{X}} \mathbf{1}(x \leftrightarrow u)\right]$$
$$= \rho \int_{-\infty}^{\infty} \mathbb{E}\left[\mathbf{1}(x \leftrightarrow u)\right]\mathrm{d}x$$
$$= \rho \int_{-\infty}^{\infty} \mathbb{P}(x \leftrightarrow u)\mathrm{d}x$$
$$= \rho \int_{-\infty}^{\infty} \left(1 - \exp\left(-\rho\sqrt{\pi/2}e^{-x^2/2}\right)\right)\mathrm{d}x.$$

Let us define $\alpha = \rho\sqrt{\pi/2}$ and

$$f(\alpha) = \int_{-\infty}^{\infty} \left(1 - \exp\left(-\alpha e^{-x^2/2}\right)\right)\mathrm{d}x.$$

The integral in the expression of $f(\alpha)$ above can be expanded in a Taylor series about $\rho = 0$ and integrated term by term to obtain the infinite oscillating sum

$$f(\alpha) = -\int_{-\infty}^{\infty} \sum_{k=1}^{\infty} \frac{(-\alpha)^k}{k!}e^{-kx^2/2}\mathrm{d}x, \quad (11)$$

and by swapping the order of the summation and the integration, and then integrating one time, we obtain Eq. (7).

Next, we derive the asymptotic expressions. By doing a change of variable first $v = x/\sqrt{2\log\alpha}$ and then by splitting the integral, we have

$$f(\alpha) = \sqrt{2\log\alpha}\int_{-\infty}^{\infty}\left(1 - \exp\left(-\alpha^{1-v^2}\right)\right)\mathrm{d}v$$
$$= 2\sqrt{2\log\alpha}\int_0^1 \left(1 - \exp\left(-\alpha^{1-v^2}\right)\right)\mathrm{d}v +$$
$$2\sqrt{2\log\alpha}\int_1^{\infty}\left(1 - \exp\left(-\alpha^{1-v^2}\right)\right)\mathrm{d}v. \quad (12)$$

Using the bound

$$0 \le 1 - e^{-\alpha^{1-v^2}} \le e\alpha^{1-v^2}, \quad (13)$$

which is valid as $0 \le \alpha^{1-v^2} \le 1$ when $v \ge 1$ and $\alpha \ge 1$, the second integral in Eq. (12) can be bounded via

$$0 \le \int_1^{\infty}\left(1 - e^{-\alpha^{1-v^2}}\right)\mathrm{d}v \le \alpha e \int_1^{\infty}\alpha^{-v^2}\mathrm{d}v$$
$$\le \alpha e \int_1^{\infty}ve^{-v^2\log\alpha}\mathrm{d}v = \frac{e}{2\log\alpha}. \quad (14)$$

Hence, as $\alpha \to \infty$ the second integral in Eq. (12) goes to zero. Furthermore, by the limit $\lim_{\alpha \to \infty} \alpha^{1-v^2} = \infty$, $v \in [0,1)$, and dominated convergence, the first integral in Eq. (12) satisfies

$$\lim_{\alpha \to \infty} \int_0^1 \left(1 - e^{-\alpha^{1-v^2}}\right) \mathrm{d}v = 1. \qquad (15)$$

We can therefore conclude that $f(\alpha) \simeq 2\sqrt{2\log\alpha}$ as $\alpha \to \infty$. The equivalent expression as α tends to zero follows by truncation of Eq. (7) at the first term. $\square$

For completeness, we also state the generalization of the Theorem III.1 for arbitrary $\beta > 0$ without proof.

Proposition III.3. *The mean number of vertices with at least one two-hop connection to the RSU is*

$$\mathbb{E}[N_2] = \rho\sqrt{\frac{2\pi}{\beta}} \sum_{k=1}^{\infty} (-1)^{k-1} \frac{\left(\rho\sqrt{\pi/(2\beta)}\right)^k}{k!\sqrt{k}}. \qquad (16)$$

In addition, we have the following exact asymptotics

$$\mathbb{E}[N_2] \simeq 2\rho\sqrt{(2/\beta)\ln\rho + (2/\beta)\ln\sqrt{\pi/(2\beta)}}, \qquad (17)$$

as $\rho \to \infty$, and as $\rho \to 0$

$$\mathbb{E}[N_2] \simeq \pi\rho^2/\beta. \qquad (18)$$

Theorem III.1 is validated against simulations in Fig. 2. We see that the asymptotic expressions remain very accurate also for realistic values of ρ, while the approximation for $\rho \to 0$ is accurate only for very sparse networks. In Fig. 3 we observe that even for relatively low densities, a Gaussian distribution approximates well the probability mass function (PMF) of N_2, i.e., after rescaling

$$\mathbb{P}\left(\frac{N_2 - \mathbb{E}[N_2]}{\sqrt{\mathrm{Var}[N_2]}} \le x\right) \simeq \Phi(x), \qquad x \in \mathbb{R}, \qquad (19)$$

as $\rho \to \infty$, with Φ the cumulative distribution function of the standard normal distribution. Observing normal approximation is common in the case of independent trials. Here, the events "a node x has a two-hop path to the RSU", for each $x \in \mathcal{X}$, are dependent, nevertheless, the dependency is sufficiently short range to lead to a Gaussian distribution in the dense limit. A proof for the Gaussian convergence is not a triviality and is left for future work.

IV. VALIDATION WITH SYNTHETIC TRACES

Next, we assess the sensitivity of our model against the Poisson assumption for the distribution of vehicles along a motorway. Using synthetic mobility traces [13]–[16], we simulate the distribution of the number of vehicles with at least one two-hop link to the RSU, and compare this distribution (also referred to as the empirical distribution) to a Gaussian with mean equal to the parameter calculated in Eq. (7). Overall, the spatial distribution of vehicles, at a snapshot of time, along a motorway, is not exactly Poisson. Therefore, it is important we quantify the sensitivity of the mean calculated in Eq. (7) to small perturbations to the deployment model.
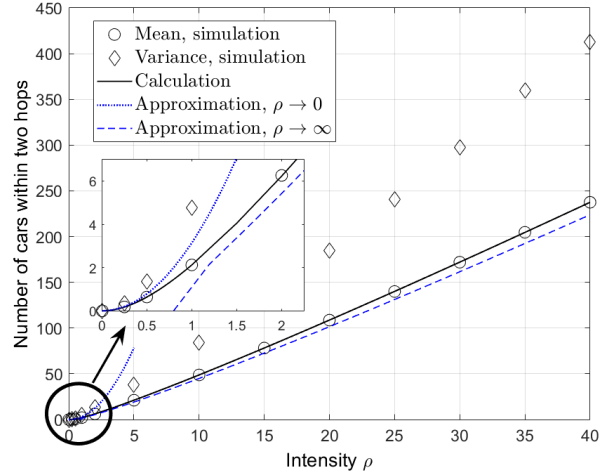

Fig. 2. The mean number of vehicles with a two-hop path connection to the RSU calculated using Eq. (7) is compared against $20\,000$ Monte Carlo simulation runs over a line segment $\mathcal{X} \subset [-10, 10]$ with the connection function $H(r) = \exp(-r^2)$. The simulated variance is depicted too.
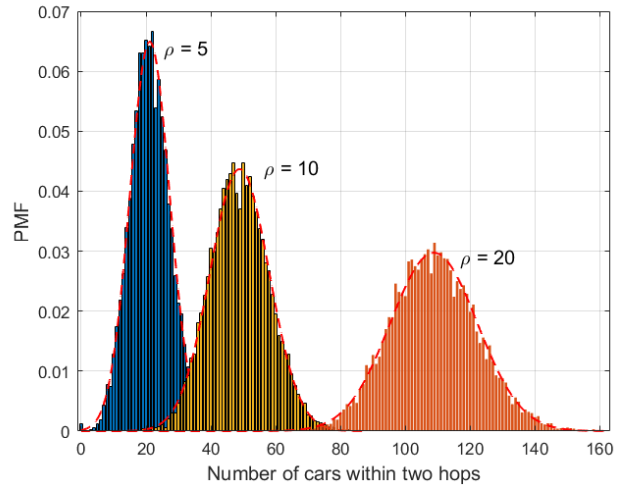

Fig. 3. Counts of nodes with two-hop connectivity to the RSU for various densities ρ of cars and illustration of the Gaussian distribution fit using Eq. (19). The mean of the Gaussian is calculated from Eq. (7) and the simulated variance is used to generate the dashed red lines. See the caption of Fig. 2 for the rest of the parameter settings.

Thanks to [13], [14], 1200 consecutive snapshots of road traffic along a 10-km three-lane motorway are publicly available. The associated data files contain the horizontal location and occupied lane for all the vehicles over the snapshots. The time granularity is set to one second, hence, during the simulation time, there are slight variations in the intensity of vehicles, see the solid line in the inset of Fig. 4.

We first generate the empirical distribution of headway distances for each snapshot. To do that we project all vehicles onto a single line, which does not introduce much error, because the communication range is expected to be much larger than the width of the road. In each simulation run, we use inversion sampling of the empirical CDF to cover a line segment of 10 km with vehicles. The set of vehicles is denoted

expanded in a power series to provide an infinite oscillating series expression for $\mathbb{E}[N_2]$, which, truncated to the first term, shows that the two-hop connectivity grows quadratically with traffic density, when the traffic is low. On the other hand, we have shown that $\mathbb{E}[N_2] = O(\rho\sqrt{\log\rho})$ as $\rho \to \infty$, which indicates that the number of two-hop neighbors increases very slowly with the density when the traffic is high. Finally, we carry out simulations confirming that the typical number of vehicles in the two-hop range of an RSU is an accurate representation of an interference-free network of moving cars, at a random instance of time, and further, that a Gaussian central limit theorem is also present. We have also identified the following non-trivial problem as promising direction for future work: *Calculate the distribution of the number of k-hop paths between a node at x and the RSU, which will enable us to compute the expected number of vehicles $\mathbb{E}[N_k]$ with a k-hop path to the RSU.*
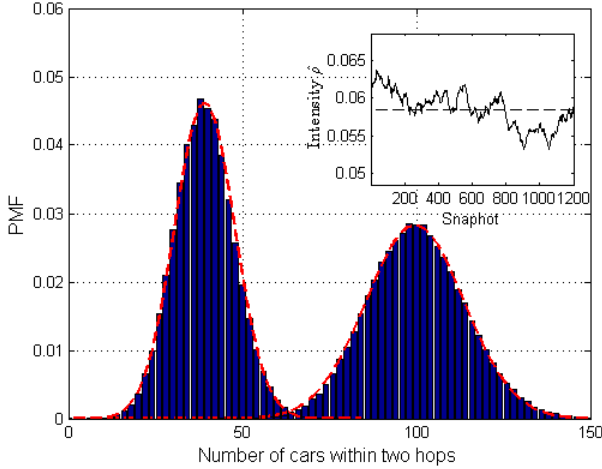

Fig. 4. The empirical PMF (blue bars), of the number of vehicles with a two-hop connection to the RSU averaged over 1200 snapshots. For each snapshot we generated 100 independent spatial configurations of vehicles by sampling its empirical CDF of inter-vehicle distances. Two values for the parameter β are considered: $\beta = 5 \times 10^{-5}$ and $\beta = 10^{-5}$. The simulated mean values for the number of vehicles with a two-hop path to the RSU is 39.51 and 100.60, respectively. The estimated intensity of vehicles $\hat{\rho}$ for each snapshot is depicted in the inset (solid line). The mean intensity of vehicles averaged over all considered snapshots is equal to $\hat{\rho} = 0.0585\,\mathrm{m}^{-1}$, see the dashed line in the inset. After substituting $\hat{\rho} = 0.0585$ into Eq. (7), we obtain the following values for the expected number of vehicles with a two-hop connection to the RSU: 39.12 vehicles for $\beta = 5 \times 10^{-5}$ and 99.60 vehicles for $\beta = 10^{-5}$. The red dashed line corresponds to the Gaussian approximation for the distribution of N_2 using a mean equal to $\mathbb{E}[N_2]$ calculated from Eq. (7) and variance obtained by the simulations.

by $\mathcal{Y}$, and the origin, where the RSU is located, is taken in the middle of road segment. For each vehicle $y \in \mathcal{Y}$, we generate a random number distributed uniformly in $[0, 1]$. After comparing it with the value of the connection function $e^{-\beta r^2}$, where r is the distance between the vehicle and the RSU, we can identify the set of vehicles $\mathcal{Y}_1$, which have a single-hop connection to the RSU. Then, we search over the vehicles $y \in \mathcal{Y}\backslash\mathcal{Y}_1$ and separate those with a single-hop connection to at least one of the vehicles in $\mathcal{Y}_1$. They become the elements of the set $\mathcal{Y}_2$, which consists of the vehicles with a two-hop path to the RSU. To complete the set $\mathcal{Y}_2$, we need to add to it the elements of the set $\mathcal{Y}_1$ which also have a two-hop connection to the RSU.

Both histograms in Fig. 4 illustrate that Eq. (7) yields a very good estimate of the simulated mean number of vehicles with a two-hop path to the RSU. Also, the quality of the Gaussian fit for the distribution of the number of two-hop neighbors to the RSU, discussed in Eq. (19), is good even though the actual deployment of vehicles does not follow a PPP and the density of vehicles varies with time.

V. CONCLUSIONS

Motivated by V2I communications in a VANET topology, we investigate the notion of k-hop connectivity to a fixed RSU, specifically, in the random connection model on a 1D PPP with connection function $H(r) = \exp(-\beta r^2)$, and with $k = 2$. We calculate the typical number of cars with a two-hop path connection to the RSU, denoted by $\mathbb{E}[N_2]$. This is then

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