Dark Knights:
The Rise in Firm Intervention by CDS Investors

András Danis Andrea Gamba *

January 20, 2023

ABSTRACT

There have been several cases in recent years where credit default swap (CDS) buyers and sellers intervene in the restructuring of a distressed firm. We show theoretically that this can increase firm value. Intervention by CDS buyers solves the commitment problem between equity- and debtholders but increases the probability of inefficient liquidation. Intervention by CDS sellers reduces the issue of excessive liquidation while keeping the benefits of CDS buyer intervention. Having both types of intervention decouples the commitment problem from the liquidation problem. Under certain assumptions, the so-called empty creditor problem can be solved and firm value reaches first-best.

Keywords: credit default swaps, CDS, empty creditor, bankruptcy, hedge fund activism
JEL classification: G33, G34

*Danis (corresponding author) is at the Department of Economics and Business, Central European University, Quellenstrasse 51, 1100, Vienna, Austria. Email: DanisA@ceu.edu. A large part of this paper was written while Danis was at Scheller College of Business, Georgia Institute of Technology. Gamba is at Warwick Business School, University of Warwick, andrea.gamba@wbs.ac.uk. We thank conference participants at the EFA Meeting 2019, Canadian Derivatives Institute (CDI) 2019, WFA 2020, IDC 2020, MFA 2020, AFA 2021, CFMR 2021, and the SFS Cavalcade 2021 for their feedback. We also thank the editor (Lukas Schmid), two anonymous referees, Patrick Augustin, Philip Bond, Sudheer Chava, Jonathan Cohn (WFA discussant), Jason Donaldson (AFA discussant), Matthias Efing (EFA discussant), Julian Franke, Vincent Glode (CDI discussant), Rich Mathews (Cavalcade discussant), Martin Oehmke (IDC discussant), Jovan Stojkovic (CFMR discussant), Heather Tookes, Zhe Wang (MFA discussant), Daniel Weagley, and Adam Zawadowski as well as seminar participants at Georgia Tech, the Office of the Comptroller of the Currency, University of Maryland, Tilburg University, Erasmus University, ESCP Paris, University of Luxembourg, ESSEC, EDHEC, and Copenhagen Business School for their comments. We thank Marti Subrahmanyam, Dragon Tang, and Sarah Wang for sharing their data with us. We thank Yue Hu for excellent research assistance.

Norske Skog, a company based in Norway and one of the world's leading paper producers, was suffering from years of declining sales leading up to 2016. In March of the same year, the company announced that it had raised both debt and equity financing from two hedge funds, Blackstone's GSO Capital Partners and Cyrus Capital Partners. The interesting part of the deal was that these hedge funds had previously sold Credit Default Swap (CDS) protection on the firm's debt. Because of their CDS positions, they had an economic incentive to support the distressed firm and to avoid bankruptcy. The deal was controversial because some investors had speculated on the failure of Norske Skog. Among these investors was BlueCrest Capital Management, who had purchased CDSs on the underlying firm.[1] BlueCrest was paying an expensive insurance premium but Norske Skog did not default, resulting in a loss for BlueCrest.

This is not the only case where CDS investors interfered with a corporate restructuring. Table 1 summarizes several cases that have occurred in recent years. This small sample of cases provides anecdotal evidence and does not substitute for rigorous empirical analysis. In some of these cases, the protection seller actively intervenes in a distressed firm's restructuring to avoid bankruptcy, while in others it is the protection buyer who tries to trigger a credit event to receive a payoff from a CDS contract. What the cases have in common is that a CDS investor with a large position tries to actively influence corporate restructuring. Also, all these cases occurred recently, between 2013 and 2019.

There is anecdotal evidence that these cases are just examples of a broader shift in how hedge funds participate in the CDS market.[2] The Wall Street Journal argues that activist investors are increasingly using long or short positions in CDSs to affect corporate deals.

Following the recent cases of CDS intervention, several commentators have called for a reform of the CDS market. Some even suggest shutting down the CDS market completely.[3] Regulators are concerned, too. The Commodity Futures Trading Commission (CFTC), in an unprecedented move, issued a public warning in 2018 that certain activities of CDS investors could be viewed as market manipulation.[4] In 2019, the CFTC acknowledged that it is concerned about CDS activism

[1] See Financial Times, "Hedge funds spar over Norske Skog debt restructuring", February 25, 2016.
[2] The Wall Street Journal, "Credit-Default Swaps Get Activist New Look", December 23, 2014.
[3] Financial Times, "Time to wipe out the absurd credit default swap market", May 11, 2018.
[4] The Wall Street Journal, "How Regulators Averted a Debacle in Credit Default Swaps", July 8, 2018.

and that this is a recent and growing phenomenon: "The earliest of these activities date back to 2006. During the following 10 years, we have observed a total of six instances of similar strategies. In the last two and a half year period, we have observed 14 strategies, seven of which have occurred in just the last six months."[5]

We examine whether CDS investor intervention is beneficial or harmful. In particular, we are interested in the effect on the underlying firm, and use firm value as a proxy for aggregate welfare. Table 2 provides an overview of the different types of CDS intervention. Out of the four types in Table 2, we analyze two types of CDS intervention in particular. First, we examine what happens if a firm's lender is allowed to purchase CDS protection, which creates an incentive to push the firm into bankruptcy. This lender is also called an empty creditor. Second, we analyze the effect of allowing a protection seller to intervene in the restructuring of a distressed firm, by injecting capital to avoid a credit event. We use a theoretical model to show that the popular conclusion is not necessarily valid. We find that two-sided intervention—by the protection buyer and seller—increases firm value instead of destroying it.

We start with a model where the firm's lender can purchase CDS protection but without the ability of the protection seller to intervene in a debt restructuring. This model builds on the well-known theory in Bolton and Oehmke (2011).[6] The economy consists of a firm, a lender, and a protection seller. The firm's owner faces multiple frictions like the inability to commit to repaying the debt in the future and bankruptcy costs. These frictions constrain the firm and keep investment below the first-best level.

Like in Bolton and Oehmke, the existence of a CDS market has both positive and negative effects on firm value. We extend their model by comparing these two effects to each other and calculating the net effect on firm value. On the one hand, the hedged bondholder can improve his payoffs in a future out-of-court debt restructuring. The equity holders have to offer him a

[5] CFTC public webcast titled "Credit Derivatives", published on July 10, 2019 at `https://youtu.be/Qqo9KR6JXaM`.

[6] Their theoretical framework has since been extended by Arping (2014), Kim (2016), Danis and Gamba (2018), Colonnello, Efing, and Zucchi (2019), Campello and Matta (2020), Bartram, Conrad, Lee, and Subrahmanyam (2021), and Wong and Yu (2021), among others. Empirical tests of the theory can be found in Subrahmanyam, Tang, and Wang (2014), Danis (2017), Guest, Karampatsas, Petmezas, and Travlos (2017), Batta and Yu (2018), Narayanan and Uzmanoglu (2018), Chang, Chen, Wang, Zhang, and Zhang (2019), Colonnello, Efing, and Zucchi (2019), Caglio, Darst, and Parolin (2019), Degryse, Gündüz, O'Flynn, and Ongena (2020), and Bartram, Conrad, Lee, and Subrahmanyam (2021).

higher debt payment, otherwise the bondholder would simply reject the offer which would trigger liquidation and a payment from the protection seller. This tough stance alleviates the commitment problem between equity holders and the lender. Ex ante, it decreases borrowing costs and increases firm value. On the other hand, if a lender has high demands in a debt renegotiation then there will be some states of the world without enough money in the firm to satisfy these demands. Ex ante, this increases the probability of costly liquidation, which increases borrowing costs and reduces firm value. The net effect on firm value can be either positive or negative and depends on parameter values.

We augment the active CDS buyer model by allowing the CDS seller to intervene in the firm's debt restructuring. To the best of our knowledge, this is the first model to incorporate intervention by both CDS buyers and sellers. The protection seller can reduce the firm's debt by injecting equity capital. The results of this small change in the model are large, as firm investment increases substantially. The reason is that the probability of liquidation drops as the protection seller injects enough equity to keep the firm alive. There is also a virtuous feedback effect on intervention by the protection buyer: knowing that the protection seller will intervene, the protection buyer can be even more aggressive, buy more CDS protection and be even tougher in renegotiation. All this reduces the cost of borrowing ex ante, which allows the firm to invest more. Under certain assumptions on parameter values, the probability of liquidation drops to zero and investment reaches first-best.

Another result is that even though the probability of liquidation is extremely low, the debt holder purchases a large amount of CDS contracts. This initially counter-intuitive outcome is an important feature of the equilibrium: It is precisely *because* the creditor purchases a lot of protection that the CDS seller has a strong incentive to save the firm if it is in distress. The lender understands this and buys more CDS protection ex ante.

A related result is that the protection seller charges a positive CDS spread upfront, even though a credit event is never triggered in equilibrium. At first sight, this might seem counter-intuitive and unfair. A casual observer might complain that the protection buyer pays an excessively high insurance premium. But the protection seller cannot make a loss in expectation, otherwise he would exit the market. The insurance premium is just compensation for the protection seller for saving the firm from liquidation. Actually, the seller is providing a valuable service to society, by avoiding costly liquidation.

Our main results are robust to several extensions. First, we assume more than one CDS seller. Second, we allow the CDS protection seller to inject debt instead of equity into the underlying firm. Third, we allow for an arbitrary distribution of bargaining power between equity holders and lenders in debt renegotiation.

Our model also provides novel testable predictions. For example, we predict that following an exogenous increase in the number of protection sellers, the probability of bankruptcy, and CDS spreads will increase, while investment and firm value will drop.

Our findings have important policy implications. The notion that intervention by CDS investors is unfair and reduces welfare—while intuitive—is not necessarily true. With the caveat that we only consider two types of CDS investor intervention, we show that such activism can actually increase firm value. Allowing intervention by both protection buyers and sellers could lead to lower CDS spreads, fewer bankruptcies, more investment, and higher firm value.

We contribute to multiple strands of the literature. Our anecdotal evidence suggests that there are different kinds of intervention by CDS investors. Some cases of intervention by protection buyers are already documented in Bolton and Oehmke (2011), but other types of CDS intervention seem to be new. In our theoretical analysis, we focus on two-sided CDS intervention and show that it increases firm value instead of destroying it.[7] We show that under certain assumptions on the distribution of the firm's future profitability, the benefits of an active protection seller can be so large that the firm reaches the first-best level of investment. Compared to the theoretical framework of Bolton and Oehmke (2011), this implies that the negative effect of a CDS market, which derives from the so-called empty creditor problem, is reduced to zero, and only the positive effects prevail. In other words, we show that the empty creditor problem can be at least partially solved by CDS intervention, and under certain parameter assumptions, it can be solved completely. We believe that this is a significant contribution to our current understanding of the costs and benefits of having a CDS market.

[7] In our anecdotal evidence in Table 1, capital injection by the protection seller can be found in the cases of Norske Skog, RadioShack, Matalan, McClatchy, and Thomas Cook.

There seems to be a difference between some of the model's predictions and certain findings in the empirical literature. Subrahmanyam, Tang, and Wang (2014) find that the introduction of a CDS market increases the probability of bankruptcy. However, we find that the introduction of a CDS market, together with two-sided CDS intervention, has no effect on the probability of liquidation. The reason is that our no-CDS model predicts a zero probability of default, and the introduction of a CDS market (together with two-sided intervention) also leads to no liquidation. As we show in Section 3.1, this is an artifact of the assumption of a single protection seller. With multiple protection sellers, it is possible that liquidation occurs in equilibrium, although we cannot calculate the precise probability of liquidation.

Another possibility to reconcile the difference is that there has been a change in CDS intervention over time. Subrahmanyam, Tang, and Wang (2014) use a relatively early sample period, while we want to explain CDS intervention in recent years. It is possible that intervention by protection buyers used to be the norm, and two-sided intervention only became wide-spread in recent years. If this is the case, then we would expect a large positive effect of the introduction of CDSs on credit risk in the early years, and a smaller positive effect in later years. Future empirical research should investigate whether the effect of CDSs on the credit risk has changed over time. To do so, researchers should not only compare CDS-firms to no-CDS firms, but also examine the behavior of credit risk within CDS-firms over time.

We also contribute to the literature on contracting mechanisms under frictions. With frictions like commitment problems and liquidation costs, the two affected parties (equity and debt) cannot use contracts to reach a satisfactory outcome. We show that a new arrangement of state-contingent cash flow rights and control rights between three parties (equity, debt, and CDS dealer) can overcome the problems created by liquidation costs and the lack of commitment. Allowing the debt holder to buy CDS protection allows him to overcome the commitment problem, but at the cost of increasing the liquidation cost problem. Having both intervention by the CDS buyer *and* the CDS seller allows the parties to decouple the commitment problem from the liquidation cost problem: They can keep the benefits of stronger bargaining by the debtholder, which solves the commitment problem, but without increasing the probability of inefficient liquidation. In this sense, the careful allocation of cash flow rights and control rights among the three parties allows us to reach an outcome that is socially more desirable than what can be achieved by bilateral contracting.[8]

Finally, we contribute to the legal literature, which identified the earliest cases of CDS intervention (see Lubben (2007) and Hu and Black (2008)). More recently, a second wave of legal scholars became interested in CDS intervention. There is a lot of disagreement in this literature on whether CDS activism is good or bad. Fletcher (2019) argues that it reduces welfare and should be strictly regulated, while Buccola, Mah, and Zhang (2020) argue the opposite. Our contribution is to highlight ex-ante effects on outcome variables such as debt issuance, investment, and hedging, as well as the importance of endogenously priced CDS contracts. Our model, which encompasses both ex-post and ex-ante effects, hopefully provides a comprehensive theoretical framework that other scholars can use.

1. Model with an active CDS buyer

We start with a model where only the CDS buyer can intervene in the underlying firm. We will then extend the model to allow for intervention by the protection seller. A real-life motivation for the first model, where the CDS seller cannot intervene, is that there is a large number of dealers, each with a tiny portion of the CDS contract, which means that none of them have an incentive to save the firm. Another interpretation of this model is a world where the regulator forbids dealers to interfere with the distressed firm.

We present a model that builds on Bolton and Oehmke (2011) and Danis and Gamba (2018). We make several simplifying assumptions, which allow us to derive closed-form solutions and to present the underlying mechanism most transparently. We relax several of these assumptions and derive some of our results in a much more general framework in the Internet Appendix.

All agents in the model are risk-neutral. We model a single firm, owned by an entrepreneur who makes investment and financing decisions to maximize her expected payoff at the beginning of the

[8]The only other paper we know that has looked at the effect of financial derivatives on contracting outcomes is Noldeke and Schmidt (1995). However, they focus on a different problem: They show that the famous hold-up problem of Hart and Moore (1988) can be solved by an option contract between the two parties.

period. The main driver of the model is the firm's profit shock, a continuous-state random variable. The probability of the end-of-period shock, z, is determined by the cumulative distribution function $\Gamma(z)$, where $z > 0$.

We denote by k the capital stock. To finance investment, the firm issues debt alongside a possible equity injection. The debt contract is an unsecured zero-coupon bond with face value b paid at the end of the period. Both k and b are non–negative. Debt financing is cheaper than equity financing because debt payments reduce the corporate income tax base. The corporate tax rate is $\tau \in [0, 1[$.[9] The risk-free interest rate and equity issuance costs are zero for simplicity. We relax both of these assumptions in the Internet Appendix.

The main benefit of debt financing is the tax shield. This raises the concern of whether reliance on debt financing has any welfare effects. However, our results are not dependent on this assumption. Any friction that creates a relative benefit for debt financing, such as equity issuance costs, is sufficient.

The firm's profit shock determines the asset value at the end of the period:

$$a(z, k) = zk^{\alpha}, \tag{1}$$

where $\alpha \in]0, 1[$ is the return-to-scale parameter.

After the realization of z, the owner decides whether to pay b in full, to renegotiate the debt by paying an amount b_r, or to file for bankruptcy. The owner cannot commit not to renegotiate the debt in the future. For simplicity, the whole firm value is lost in bankruptcy. In the Internet Appendix, we relax this assumption and allow for an arbitrary proportional bankruptcy cost.

For simplicity, we equate bankruptcy to liquidation. We do not distinguish between Chapter 11 bankruptcy (reorganization) and Chapter 7 bankruptcy (liquidation).[10]

[9] Since the debt is a zero-coupon bond, we assume for simplicity that the whole face value is deductible. Because this overstates the tax benefits of debt compared to the real world, we compensate by choosing a lower parameter value for the tax rate.

[10] For a recent paper that compares different kinds of distress resolution, see Donaldson, Morrison, Piacentino, and Yu (2021). However, the authors do not examine the role of CDS investors.

The liquidation value of the assets is always below the "going concern" value, a. Our model makes no prediction for firms where the liquidation value is higher than the going concern value.[11]

We assume a competitive market for insuring against credit risk. In particular, the debt holder can purchase a CDS from a dealer (protection seller) at the time the debt contract is issued. The lender (protection buyer) chooses the fraction h of the face value of debt covered by the CDS contract. The dollar amount, or notional amount, of insured debt is therefore hb. After observing h, the protection seller sets the CDS spread (the insurance premium) accordingly. The CDS spread is endogenously determined and the protection seller has rational expectations: he understands that selling CDS protection to the debt holder may change both the probability of default and the debt payoff in default and adjusts the CDS spread accordingly.

The debt holder chooses the hedge ratio, h, to maximize his expected payoff. Because we assume that the credit risk market is competitive, the CDS spread is fair (and the transaction has zero–NPV for the protection seller). In the first part of this section, h will be an arbitrary hedge ratio. We discuss later how the optimal hedge ratio is determined.[12]

The sequence of events is as follows. The firm owner chooses a capital level k and a face value of debt b. The debt holder observes the outcome of these decisions and chooses a hedge ratio h. At the end of the period, nature chooses a profitability shock z, and the owner decides between repaying the debt, renegotiating with the debt holder, or liquidating the firm. The timeline of events is shown in Figure 1.

We describe the payoffs to equity and debt as a function of the owner's default decision. The payoff to equity at the end of the period is $a-b$ if the debt is repaid, $a-b_r$ if the debt is successfully renegotiated, and 0 if the firm is liquidated. The corresponding payoff to debt is b when it is repaid in full and b_r in case of a successful renegotiation. In the case of liquidation, the debt holder does

[11] In the legal literature, Fletcher (2019) implicitly assumes that for all distressed firms the liquidation value exceeds the continuation value and concludes that CDS intervention is always welfare-reducing. By contrast, Buccola, Mah, and Zhang (2020) implicitly assume that distressed firms are economically viable and reach the opposite conclusion. Empirically, it is very difficult to distinguish between viable and unviable firms.

[12] There are no speculators in the CDS market in our model, and therefore no so-called naked CDS positions. The only agents who trade in the CDS market are the lender and the dealer. However, speculators would not change much in this framework. By definition, they do not own the debt of the underlying firm, so they cannot participate in any debt restructuring. Therefore, they would not have a direct effect on the probability of default or on the recovery rate in default.

not receive anything from the firm, due to liquidation costs, but he receives hb from the protection seller.

If the debt is renegotiated, the equity holder makes a take-it-or-leave-it offer to the bondholder. This assumption is relaxed in the Internet Appendix, where we solve a Nash bargaining game with an arbitrary distribution of bargaining power between the two parties.

The bargaining problem has two constraints, $a - b_r \geq 0$ and $b_r \geq hb$. The first constraint is that the owner's payoff after successful renegotiation, $a - b_r$, must be at least as large as her outside option. Similarly, the second constraint makes sure that the debt holder's renegotiation payoff b_r is not below his outside option. The bargaining problem, together with the constraints, is important for the results of the model. If the hedge ratio h is sufficiently high, then the outside option of the debt holder hb is so large that there is no b_r that can satisfy the two constraints. In that case renegotiation is not feasible. And even in cases where renegotiation is feasible, a higher hedge ratio increases the payoff to the debt holder, because the b_r that solves the bargaining game is increasing in the debt holder's outside option. Because of the simplifying assumption on the distribution of bargaining power, it is easy to see that the bargaining game is feasible if and only if $hb < a$. The solution, if it exists, is $b_r = hb$.

In the following, we assume that the optimal hedge ratio is $h^* \in [0, 1]$. We prove that this is true even in a much more general setting in the Internet Appendix.

We derive the owner's optimal default decision. If the firm's asset value a is above the threshold $a_R = hb$, it is optimal to renegotiate the debt. The new face value of debt in these states is $b_r = hb$. If the asset value is below a_R, it is optimal to liquidate the firm.

Interestingly, it is never optimal to repay the debt in full. This is because—due to our simplifying assumptions—there is no cost associated with renegotiation and the debt holder has no bargaining power. Also, since $h^* \in [0, 1]$ and $b_r = hb$, we have that $b_r \leq b$, i.e., the renegotiated face value of debt is always below the full face value. Therefore, renegotiation always dominates repayment for the owner. In the Internet Appendix, we allow for renegotiation costs and positive bargaining power for the debt holder, and show that in very high states of the world debt repayment is optimal. The main results, however, are similar to the simple model.

Figure 2 shows the optimal default decision graphically. Low asset values a lead to liquidation, while high asset values trigger renegotiation. Figure 2 also summarizes nicely the positive and negative effects of adding a CDS market to the economy together with the protection buyer's ability to actively participate in bargaining with the firm. On the one hand, if the hedge ratio increases from $h = 0$ to $h = h^*$, the payoff to the debt holder in a future renegotiation, b_r, increases. This is beneficial to the firm as well because the ex ante cost of borrowing goes down. On the other hand, the increase in the hedge ratio from $h = 0$ to $h = h^*$ pushes the threshold a_R up, which makes liquidation more likely and renegotiation less likely, reducing the expected payoff to the debt holder. Anticipating this outcome, the bondholder adjusts credit spreads upwards when the debt is issued.

The model encompasses also the case without a CDS market. Debt renegotiation is always feasible in that world because the only thing that would make it infeasible is the bondholder's hedging activity. For the same reason, the bondholder's outside option is zero, and since he has no bargaining power, the bargaining outcome is always $b_r = 0$. This leads to an extreme equilibrium where the debt is always renegotiated to $b_r = 0$, which implies that the debt holder never receives any payments ex post. The result is that the bondholder never wants to lend to the firm ex ante. The commitment problem between equity holders and the lender is so severe that the debt market breaks down.

The reason for this extreme outcome is the combined assumption of very high liquidation costs and very low bargaining power of the lender. In a debt renegotiation, the owner can credibly threaten to liquidate the firm, in which case the lender would receive a payoff of zero. Since debt renegotiation is a take-it-or-leave-it offer, the lender can do nothing but accept a debt payment of $b_r = 0$. As shown in the Internet Appendix, the outcome is less extreme in a more general model, where the bondholder receives a positive payoff in most states of the world.

We now turn to the owner's ex ante decision. The owner maximizes the cum-dividend value of equity (i.e., of the firm), defined as

$$
\begin{aligned}
V(k^*, b^*, h(k^*, b^*)) &= \max_{k,b} V(k, b, h(k, b)) \\
&= \max_{k,b} \left\{ m(k, b) - k + (1 - \tau) \int_{z_R}^{\infty} (a(z, k) - b_r) d\Gamma(z) \right\},
\end{aligned} \tag{2}
$$

where $m(k, b)$ denotes the equilibrium price of debt, which we will derive later on. The lower limit of the integral is $z_R = a_R/k^\alpha$. While a_R and b_r are a function of (b, h), in what follows we suppress these dependencies for brevity.

The lender maximizes his expected payoff, denoted by M, by choosing the hedge ratio:

$$
m(k, b) = \max_h \; M(k, b, h). \tag{3}
$$

The solution of the above program, $h = h(k, b)$, is the state-contingent optimal hedge ratio that is considered in the owner's program in (2).

To find M, the expected payoff to the debt holder, we first derive the fair price of the CDS contract. The credit event that triggers the CDS payment is bankruptcy/liquidation. An out-of-court debt restructuring does not trigger a CDS payment, in line with the Standard North American Contract (SNAC) of the International Swaps and Derivatives Association (e.g., Bolton and Oehmke, 2011). The price of credit protection for a given hedge ratio $h \in [0, 1]$ is the expectation of the net compensation from the protection seller:

$$
C(k, b, h) = \int_0^{z_R} hb \; d\Gamma(z). \tag{4}
$$

The expected payoff to the debt holder, including the payment from the protection seller in case of a credit event, but excluding the insurance premium $C(k, b, h)$, is

$$
\psi(k, b, h) = \int_0^{z_R} hb \; d\Gamma(z) + \int_{z_R}^{\infty} b_r \; d\Gamma(z). \tag{5}
$$

The expected value of the debt for a given hedge ratio, h, net of the cost of the CDS is

$$M(k,b,h) = \psi(k,b,h) - C(k,b,h).$$

After simplifying, the price of debt and the optimal h are found by solving the program

$$m(k,b) = \max_h \int_{z_R}^{\infty} b_r \; d\Gamma(z). \tag{6}$$

We assume that the profitability shock z follows a uniform distribution, $z \sim U[0, Z]$, which allows us to solve the model in closed form. Under this assumption, one can show that the optimal hedge ratio is $h^* = 1$.[13]

Next, we solve the owner's investment and financing problem in Equation (2). Using the optimal hedge ratio and substituting the equation for debt into Equation (2), firm value can be simplified to

$$V(k,b,h)|_{h=1} = -k + \int_{z_R}^{Z} zk^{\alpha} d\Gamma(z) - \tau \int_{z_R}^{Z} (zk^{\alpha} - b) d\Gamma(z), \tag{7}$$

where $z_R = b/k^{\alpha}$. For an arbitrary level of capital, the optimal amount of debt can be found by solving the first-order condition with respect to b, which yields

$$b^*(k) = \frac{Zk^{\alpha}\tau}{1+\tau}. \tag{8}$$

Using the previous result, one can solve the first-order condition with respect to k to find the optimal level of capital:

$$k^* = \left(\frac{\alpha Z}{2(1+\tau)} \right)^{\frac{1}{1-\alpha}}. \tag{9}$$

Note that the first-best level of capital would be

$$k_{FB} = \left(\frac{\alpha Z}{2} \right)^{\frac{1}{1-\alpha}}. \tag{10}$$

[13] It is sufficient to show that $\partial M/\partial h$ is positive for all $h \in [0,1]$ because of our assumption that $h^* \in [0,1]$. The derivative of Equation (6) with respect to h is $b(Zk^{\alpha} - 2hb)/Zk^{\alpha}$. This is trivially positive at $h = 0$. At $h = 1$, it is positive if and only if $b \leq Zk^{\alpha}/2$. We conjecture that $b \leq Zk^{\alpha}/2$ and verify that it is true below, in Equation (8), for all $\tau \in [0,1]$. Under this assumption, the derivative $\partial M/\partial h$ is also positive for all $h \in [0,1]$.

The equilibrium level of capital k^* is below the first-best level because of three frictions in the economy: taxes, bankruptcy costs, and the firm's lack of commitment to repaying the debt in the future.

It is useful to compare this equilibrium to a counterfactual outcome in a world without a CDS market and no intervention by a protection buyer. We have explained above that in such a no-CDS world the firm cannot issue any debt, or $b = 0$. It is easy to show that the optimal investment of the firm in this case is

$$k^*_{\text{no-CDS}} = \left(\frac{(1-\tau)\alpha Z}{2}\right)^{\frac{1}{1-\alpha}}. \tag{11}$$

This is always less than the optimal investment in a world with a CDS market. In other words, introducing a CDS market together with one type of CDS intervention allows the firm to move closer to the first-best investment level. This shows that CDS intervention can be beneficial for firms. It can allow the firm to find cheaper debt financing and to invest more. In this simple model, the benefit comes from improving the bondholder's payoff in renegotiation from $b_r = 0$ to $b_r = hb$. While there are also costs created by the introduction of a CDS market, namely through the increased probability of liquidation, the benefits outweigh the costs. The net effect on the firm is positive.

In the general model, the net effect of a CDS market on investment and firm value can be both positive or negative, depending on parameter values. However, the main message remains true: Allowing for the intervention of a CDS buyer—by having a lender who takes a stronger bargaining position in a future debt renegotiation—has some positive effects on the firm.

Next, we extend the model by allowing the protection seller to intervene in the underlying firm as well. As we will show, the results will be very different from the model with only one type of intervention. In particular, having two types of intervention removes the negative effect of a CDS market on firms, so the net effect on firm value is unambiguously positive.

2. Model with an active CDS seller

We extend the previous model by allowing the protection seller to intervene by injecting equity into the firm to reduce the debt. The interpretation of this model is that sometimes there are fewer protection sellers, so they have a stronger incentive to save the firm. Another interpretation is that the regulator allows CDS sellers to intervene in distressed firms. To fix ideas, we will think of the previous model as one with a continuum of protection sellers, and this extension as one with a single CDS seller.

The main change in the model is that after nature has chosen profitability z, the dealer makes a take-it-or-leave-it offer to the owner. The dealer offers to recapitalize the firm by reducing the face value of debt to a new level $b_n \leq b$, through an equity injection in the amount of $b - b_n$. In return, the dealer asks for an equity stake of $\theta \in [0, 1]$ in the restructured firm.[14] The owner can accept or reject the offer. After the debt restructuring, the equity holders[15] make the default decision (i.e., repay/renegotiate/liquidate). For simplicity, we assume that the debt holder can only purchase CDS protection but cannot sell it, or $h \geq 0$. The timeline of events is now slightly different, as depicted in Figure 3.

It might seem inconsistent that the protection seller makes a take-it-or-leave-it offer to the owner, which implies that the dealer has a lot of bargaining power, while at the beginning of the model he sells CDS contracts at a competitive price, which assumes very little bargaining power. We argue that it is plausible that the protection seller's bargaining power can vary across the two settings. After the CDS contract has been sold and after a low profit shock z has realized, the firm is known to be in financial distress. At this point in time, the dealer is the single natural bargaining party for the owner because the dealer has the strongest incentive to bail out the firm. This is different from the ex ante CDS market, where we implicitly assume that the cost of entry is so low

[14] One could also assume other types of financing such as debt. We choose equity financing for simplicity. With debt financing, the dealer would want to make sure that the new debt does not trigger a credit event. This could be achieved by providing a debt contract with a maturity that exceeds the maturity of the CDS contract. Alternatively, the protection seller can provide short-term debt, but require the underlying company to issue the debt through a separate legal entity (this is called Orphaned CDS in Table 2). We show in Section 3 that our main results hold with a debt injection. Empirically, we observe both equity injections (e.g., Norske Skog) and debt injections (e.g., RadioShack, Matalan, or McClatchy) in Table 1.

[15] We write equity holders (plural) because in some states of the world the original owner accepts the offer and the dealer also becomes an equity holder.

that the protection seller has to offer competitive CDS spreads in order to keep other dealers from entering.

It is useful to write down the terminal payoffs of each player in this game, for each possible outcome of the firm's default decision. Table 3 summarizes these payoffs. It shows that under debt repayment and renegotiation the firm's cash flow to equity is split between the initial owner, who receives a fraction $1-\theta$, and the dealer, who gets θ.

The first step to solve the model is to examine the equity holders' default decision. We know that renegotiation is feasible if there is a b_r such that the two conditions $a - b_r \geq 0$ and $b_r \geq hb$ are simultaneously satisfied, which is equivalent to the condition

$$hb \leq a. \tag{12}$$

Analogously to the active CDS buyer model, the renegotiated debt repayment is $b_r = hb$. Notice that even if debt is reduced from b to b_n, the renegotiation outcome depends on the original face value b. This is because the CDS contract was purchased on the original debt and because we assume high liquidation costs.[16] This feature of b_r is important for many of the results below.

To solve the model, we make the following conjecture: *The debt holder's optimal hedge ratio is* $h \leq 1$. This conjecture greatly simplifies the exposition of the model solution. We prove that this conjecture is correct in Appendix A.

If renegotiation is feasible, the equity holders prefer repayment to renegotiation if $a-b_n \geq a-b_r$, or equivalently, if $b_n \leq hb$. If renegotiation is infeasible, the equity holders prefer repayment to liquidation if $a - b_n \geq 0$, or $b_n \leq a$.

The optimal default decision of the equity holders depends on b_n and hb and is described in the following lemma. The proof is in Appendix B. The default decision is summarized graphically in Figure 4.

[16]This is explained in the proofs, in Footnote 25.

Lemma 1. *We can distinguish between two cases. If $b_n \leq hb$, the optimal decision is to liquidate if $a < b_n$ and to repay the debt if $a \geq b_n$. If $b_n > hb$, the optimal decision is to liquidate if $a < hb$ and to renegotiate if $a \geq hb$.*

The intuition for this result is the following. For brevity, we focus on the case where the debt is reduced substantially to $b_n \leq hb$. If the firm's asset value is below the reduced face value of debt, $a < b_n$, the equity holders choose to liquidate the firm and their payoff is zero, due to limited liability. If they repaid the debt instead, their payoff would be less than zero. Renegotiating the debt is not an option, because the debt holder wants to be paid at least hb, which is more than the total asset value of the firm. For high asset values, the owners opt for repaying the debt, because it is better than the prospect of renegotiation. In renegotiation, the debt holders want at least hb, but with repayment, the equity holders only need to pay b_n, with $b_n \leq hb$.

The most important observation about this lemma is that if the debt is reduced a lot, then liquidation occurs only in the states $a < b_n$, whereas if the debt is only reduced a little, then the firm files for bankruptcy in more states, $a < hb$. Since liquidation is socially costly, it is important for ex ante firm value to reduce the probability that it occurs in equilibrium.

Next, we find the owner's optimal decision on whether to accept the protection seller's take-it-or-leave-it offer. In equilibrium, the protection seller will propose an equity stake θ for himself that makes the owner indifferent between accepting and rejecting. As Figure 4 shows, the default decision and the resulting payoffs depend on the region in which a lies. For each case and each region, we determine θ by equating the owner's payoff under acceptance to his payoff under rejection.

The optimal choice of the equity stake θ is summarized in the following lemma. The proof is in Appendix C.

Lemma 2. *If $b_n \leq hb$, then the protection seller's equity stake is*

$$\theta = \begin{cases} 1 - \frac{a-hb}{a-b_n} & \text{if } a \geq hb, \\ 1 & \text{if } a < hb. \end{cases}$$

If $b_n > hb$, *then*

$$\theta = \begin{cases} 0 & \text{if } a \geq hb, \\ 1 & \text{if } a < hb. \end{cases}$$

In simple terms, the mechanism behind this result is as follows. For brevity, we only cover the first case, $b_n \leq hb$. If the asset value is high, then the owner has to decide between accepting the equity injection, in which case she will own a smaller share of a firm with a healthier debt load, or rejecting the offer, which would allow her to keep the whole firm, but with a higher debt burden. This tradeoff has an interior solution, which is $\theta = 1 - \frac{a-hb}{a-b_n}$. If the asset value is low, the owner would get zero if she rejected the offer, because the debt load of the firm relative to the asset value would be so high that the firm would have to be liquidated. Therefore, the protection seller can take the whole firm ($\theta = 1$) and get away with it.

Next, we solve the dealer's choice of how much equity to inject, which determines the new face value of debt b_n. The protection seller maximizes the algebraic sum of the expected value of his (negative) CDS payoff in liquidation, of his (positive) payoff as a future equity holder, and his (negative) payoff from injecting equity capital in the amount of $b - b_n$ into the firm. We do not include his cash inflow from the upfront insurance premium because that is sunk at this point.

The dealer's optimal choice of b_n is summarized in the following lemma. For ease of exposition, we assume that the hedge ratio is $h > 1/2$, which is true in equilibrium, as we will show below. The proof, which includes the derivation for an arbitrary hedge ratio, is in Appendix D.

Lemma 3. *For a given hedge ratio that satisfies* $h > 1/2$, *the new amount of debt is*

$$b_n = \begin{cases} b & \text{if } a < b - hb, \text{ followed by liquidation,} \\ a & \text{if } b - hb \leq a < hb, \text{ followed by repayment,} \\ b & \text{if } a \geq hb, \text{ followed by renegotiation.} \end{cases}$$

If firm value turns out to be very low, $a < b - hb$, then the protection seller does not reduce debt at all. This is because he would need to inject so much capital that it is not worth it for him to avoid the cash outflow associated with a credit event. If the asset value realization is in an intermediate range, $b - hb \leq a < hb$, then the protection seller reduces the debt to $b_n = a$. This is exactly the face value that avoids liquidation later. We know from Lemma 1 that liquidation only occurs if $a < b_n$. In other words, the protection seller injects just enough equity to avoid liquidation. This intuitively makes sense, because a liquidation would create a large cash outflow for the protection seller. He can avoid this large cash outflow by injecting a little bit of equity into the firm.

If the asset value is very high, $a \geq hb$, the protection seller does not inject any equity, which means the principal remains unchanged. The reason is that the protection seller has no incentive to reduce the debt: A debt reduction is costly because he has to inject equity into the firm. The protection seller only gains by acquiring a larger share θ of the firm (Lemma 2 shows that θ is higher if b_n is lower). However, these two effects exactly offset each other, so the protection seller is not better off by injecting equity into such a firm.

An interesting observation is that the threshold that determines how frequently liquidation will occur in the future, $b - hb$, is decreasing in the hedge ratio h. In other words, if the lender chooses a higher hedge ratio, he can reduce the probability of costly liquidation. This happens because purchasing more CDS protection increases the protection seller's incentive to intervene in the future by injecting equity. This will be important to understand the equilibrium later.

Next, we find the lender's expected payoff at the stage when he chooses the hedge ratio h. We assume that the CDS spread is set so that the protection seller breaks even *including* the contingent equity injection of $b - b_n$.

To derive the market value of debt, we need to take into account several different cash flows: debt repayment by the firm, recapitalization by the dealer, debt payoffs in renegotiation and liquidation (if any), cash inflows from the CDS contract (if any), and the cash outflow of the upfront CDS premium. Similarly to the previous lemma, we only present the case if the hedge ratio is $h > 1/2$, which will be true in equilibrium. The proof, which contains the derivation for an arbitrary hedge ratio h, is in Appendix E.

Lemma 4. *The expected payoff to the debt holder is*

$$M(k,b,h) = \int_0^{z_L} (\underbrace{0}_{\text{bond payoff}} + \underbrace{hb}_{\text{CDS payoff}}) d\Gamma(z) + \int_{z_L}^{z_R} [\underbrace{(b-a)}_{\text{recapitalization}} + \underbrace{a}_{\text{debt repaym.}}] d\Gamma(z)$$
$$+ \int_{z_R}^{Z} \underbrace{hb}_{\text{renegotiation}} d\Gamma(z) - \underbrace{\left[\int_0^{z_L} hbd\Gamma(z) + \int_{z_L}^{z_R} (b-a)d\Gamma(z)\right]}_{\text{CDS premium}},$$

where $z_L = (b - hb)/k^\alpha$ and $z_R = hb/k^\alpha$ are the thresholds from Lemma 3. The value of debt can be simplified to

$$M(k,b,h) = \int_{z_L}^{z_R} ad\Gamma(z) + \int_{z_R}^{Z} hbd\Gamma(z). \tag{13}$$

The first integral in the long equation covers the bad states of the world, $a < b - hb$. In these states, as we know from Lemma 3, the debt is not reduced and the firm files for bankruptcy. The second integral is over the intermediate states of the world, $b - hb < a < hb$, where we know from Lemma 3 that the debt b is reduced to $b_n = a$ and the remaining debt is repaid in full. The third integral covers the good states, $a \geq hb$, where Lemma 3 says that the debt is not reduced. This is followed by renegotiation. Finally, the term in square brackets is the upfront CDS premium paid to the protection seller. This consists of two terms because the protection seller has to be compensated for two types of losses: First, in bad states, the firm is liquidated, which triggers a credit event. Second, in intermediate states, the protection seller will inject equity into the firm.

An observation can be made about the cost of debt financing at this point. It is easy to show that $M < b$, which means the debt is not risk-free. However, the debt value in the active CDS buyer model was $M = \int_0^{z_R} 0d\Gamma(z) + \int_{z_R}^{Z} hbd\Gamma(z)$, which is strictly below debt value in the extended model. In other words, for fixed values of k, b, and h, the cost of debt financing is lower in the model in which the dealer can recapitalize the firm. The reason is that in the intermediate states of the world, the protection seller provides financing to the distressed firm and saves it from liquidation. This reduces expected liquidation costs ex ante, which reduces the cost of debt.

Finally, we solve the owner's ex ante investment and financing problem,

$$V(k^*, b^*, h(k^*, b^*)) = \max_{k,b} \Bigg\{ m(k,b) - k$$
$$+(1-\tau)\left[\int_{z_L}^{z_R} (1-\theta)(a(z,k) - b_n)d\Gamma(z) + \int_{z_R}^{Z} (1-\theta)(a(z,k) - b_r)d\Gamma(z)\right]\Bigg\}. \quad (14)$$

Under the assumption of a uniform distribution for the profitability shock z, one can show that the optimal hedge ratio is $h^* = 1$.[17] We know from the previous steps that $b_n = a$ in the first integral, that $b_r = hb = b$, that $\theta = 0$ in the second integral (from Lemmas 2 and 3) because $a = b_n = hb$, and that $z_R = b/k^\alpha$ and $z_L = 0$. After making these substitutions, firm value simplifies to

$$V(k,b,h)|_{h=1} = m(k,b) - k + (1-\tau)\int_{z_R}^{Z} (a(z,k) - b)d\Gamma(z),$$

The market value of debt $m(k,b)$ is given in Lemma 4. After substituting, firm value becomes

$$V(k,b,h)|_{h=1} = -k + \int_0^Z zk^\alpha d\Gamma(z) - \tau \int_{z_R}^{Z} (zk^\alpha - b)d\Gamma(z). \quad (15)$$

For an arbitrary level of capital k, the optimal amount of debt can be found by solving the first-order condition with respect to b, which yields

$$b^*(k) = Zk^\alpha. \quad (16)$$

This is strictly higher than the optimal amount of debt in the active CDS buyer model. Using the previous result, one can solve the first-order condition with respect to k to find the optimal level of capital:

$$k^* = \left(\frac{\alpha Z}{2}\right)^{\frac{1}{1-\alpha}}. \quad (17)$$

[17]It is sufficient to show that $\partial M/\partial h$ is positive for all $h \in [0,1]$, because of our conjecture that $h \leq 1$. For the case $h \leq 1/2$, M is given in the appendix, Equation (19), and the derivative with respect to h is $b - 2b^2h/(Zk^\alpha)$. It turns out that this is indeed positive at the equilibrium face value b^*, as we show later in Equation (16). For the case $h > 1/2$, M is given by Equation (13), and the derivative with respect to h is $b(b - 2hb + Zk^\alpha)/Zk^\alpha$. This is trivially positive at $h = 1/2$. At $h = 1$, it is positive if and only if $b \leq Zk^\alpha$. We will make the assumption $b \leq Zk^\alpha$ at this point and verify that it is true in Equation (16). Under this assumption, the derivative $\partial M/\partial h$ is positive for all $h \in [0,1]$.

This level of capital is higher than in the active CDS buyer model. The increase in investment is so large that the firm reaches the first-best level of investment (compare Equations (10) and (17)). In other words, the firm is able to achieve the same capital level as in a world with no frictions.

It is worthwhile to discuss why this high investment level is possible. The three frictions that constrain investment in this setting are the firm's inability to commit to repaying its debt, bankruptcy costs, and taxes. The lack of commitment is alleviated by the lender's ability to buy CDS protection. With our particular assumption on parameter values and the distribution of z, this problem can be solved completely by increasing the lender's renegotiation payoff to $b_r = hb = b$. As we know from Section 1, however, this creates a new problem, which is a higher likelihood of costly liquidation. This problem is solved by the protection seller's ability to inject equity into the firm. Again, with our parameter assumptions, this problem is fully solved as the probability of bankruptcy drops to zero. Finally, taxes become irrelevant as well, because the CDS market allows the firm to increase leverage so much that it can benefit from the maximum possible tax shield.[18]

Another observation is that even though the probability of liquidation is zero, the debt holder purchases a large amount of CDS contracts. In fact, he is fully hedged, as his hedge ratio is $h^* = 1$. This seems counter-intuitive, but the two outcomes are necessary to sustain the equilibrium of the model. It is precisely *because* the lender purchases a lot of protection that the CDS seller has a strong incentive to save the firm if it is in distress.

A related observation is that the protection seller charges a positive CDS spread upfront, even though a credit event is never triggered in equilibrium. At first sight, this might seem counter-intuitive and unfair. Casual observers might complain that the protection buyer is paying an insurance premium that is too high. But the protection seller is just being compensated for saving the firm from liquidation. He creates firm value by avoiding costly liquidation.

To close the comparison of the active CDS buyer model and the extended model, we examine the difference between firm value in the two models. The preceding analysis, particularly the positive effect of a single protection seller on investment, suggests that firm value will increase as well. This is indeed the case and we summarize these results in the following statement.

[18] Equation (16) shows that the face value of debt is so high that it pushes up the threshold z_R in Equation (15) to $z_R = b/k^\alpha = Z$, so the region $[z_R, Z]$ where the firm would have to pay taxes shrinks to zero.

Proposition 1. *(a) Ex ante firm value increases if there is a single protection seller in the CDS market who can intervene in financial distress.*

(b) The debt holder purchases a large amount of CDS contracts even though the probability of liquidation is zero.

(c) The insurance premium in the CDS market is positive, even though a credit event is never triggered in equilibrium.

The first statement can be seen from comparing Equations (7) and (15). These two equations show that for any capital k and debt b, firm value is higher with a single protection seller. Therefore, firm value must be higher at the optimal levels of capital and debt as well.

2.1. Testable predictions

By comparing the active CDS buyer model in Section 1 to the extended model in Section 2 we can derive multiple testable predictions.

If there is an exogenous shock, like a regulatory change, that prohibits the protection seller from offering to finance the underlying firm in distress, ex ante yield spreads increase (the market value of debt decreases), and the probability of liquidation increases as well. At the same time, firm value and investment decrease.

Another prediction is that if there is an exogenous increase in the number of protection sellers for the same underlying firm, then the yield spread and the probability of liquidation both increase. At the same time, firm value and investment decrease. We summarize these predictions, together with predictions with respect to the tax rate τ and the hedge ratio h, in Table 4.

There are two empirical challenges to implementing these tests. First, the CDS market is very opaque. CDS holdings data for protection buyers and sellers are not publicly available for researchers. This data, to the best of our knowledge, is only accessible to regulatory agencies such as the Federal Reserve Board or the Securities and Exchange Commission. Second, even if we could measure the concentration of protection sellers, we would need an exogenous shock that changes the level of concentration.

Therefore, we also derive predictions that are immediately testable with existing data, using the difference-in-differences methodology that is standard in the empirical CDS literature. We compare a firm with both an active CDS buyer and seller to a firm without a CDS contract traded on its debt. Table 5 shows that introducing a CDS contract with two-sided intervention increases investment, the face value of debt, the market value of debt, firm value, and book leverage, while not affecting the probability of liquidation in equilibrium. We also derive testable predictions with respect to the tax rate τ and the hedge ratio h.

3. Robustness

We show that our main results are robust to four extensions. First, we allow for more than one protection seller. Second, we allow the CDS protection seller to inject debt instead of equity into the underlying firm. Third, we relax the assumption that the equity holders have all the bargaining power versus the debt holders. Finally, we allow for the possibility that side-payoffs of equity holders are taken into account during the bargaining between equity holders and lenders.

3.1. Multiple CDS dealers

For simplicity, we assume that the protection buyer can purchase CDS contracts from two dealers. If the underlying firm ends up in financial distress, both dealers can inject equity into the firm. The two dealers play a non-cooperative game by taking each other's injection decisions as given.

We assume that the dealers have a disincentive to holding large positions and therefore charge an insurance premium that increases in the dollar amount of the hedge ratio. We do not model this disincentive explicitly, as it would be outside of the scope of the paper. But one can think of various micro-foundations for such an assumption. For example, the dealer might be subject to Value-at-Risk constraints that limit the size of a single position. One can call this the "risk management friction". The result of this assumption is that it is not optimal to buy only from one protection seller.

We show that there are two types of equilibria: (1) Neither of the dealers inject equity and the firm is liquidated, and (2) the dealers inject just enough equity to avoid liquidation and the remaining debt is repaid later.

The equity injection equilibrium is virtually identical to the equilibrium in the model with a single CDS dealer. This shows that our main results are robust to the existence of two protection sellers.

The liquidation equilibrium is new: It does not exist in the single-dealer model. In this sense, liquidation is more likely to occur in the two-dealer model than in the single-dealer model. This shows that the probability of bankruptcy is not necessarily zero in a model with dealer intervention and brings the predictions of the model closer to the empirical findings of Subrahmanyam, Tang, and Wang (2014). In particular, we show that liquidation is possible with two dealers, while it never occurs in a no-CDS world.

By extending the model to multiple dealers but keeping the rest of the model the same, one cannot calculate the exact probability with which liquidation occurs. However, the multiplicity of equilibria is similar to well-known models for bank runs (Diamond and Dybvig (1983)). As has been shown by Morris and Shin (2010) and Goldstein and Pauzner (2005), global games can be used to reduce multiple equilibria to a unique equilibrium. As a result, a bank run occurs with positive probability in equilibrium. Potentially, global games could be used in our context to obtain a unique equilibrium in which liquidation occurs with positive probability. However, solving the global game would be outside of the scope of the current paper. The model extension can be found in Section G of the Internet Appendix.

3.2. Debt vs. equity injection

We show that our main results are robust to the possibility that the CDS protection seller injects debt instead of equity into the distressed firm. This is important because, empirically, debt injections are quite common. Several of the firms in Table 1 have issued debt.

For the debt injection to work, it needs to alleviate financial distress for the underlying firm. There are various ways this can be accomplished in practice. One of these methods, which is what we implement also, is to extend the maturity of debt. In particular, we assume that the CDS dealer injects long-term debt into the firm. The proceeds of the debt issuance are used to reduce the outstanding face value of existing (short-term) debt.

Another assumption that we make is that the equity holder and the long-term lender (i.e., the CDS dealer) bargain together against the existing lender in a possible debt renegotiation. We argue that this is a plausible assumption since the CDS dealer empirically often has a lot of control over the underlying firm. This is true even if the dealer is not a shareholder and does not have formal voting rights. The dealer is often the cheapest source of financing for the underlying firm.[19] Since the dealer can always withhold the life-saving financing, he effectively has a lot of control over the underlying firm.

Under these assumptions, we show that the results are very similar to the case with an equity injection. In particular, the probability of liquidation is reduced to zero, and both investment and firm value attain the first-best value. The detailed derivation can be found in the Internet Appendix.

3.3. General bargaining power between debt and equity

In this extension, we relax the assumption that the equity holders have all the bargaining power vis-a-vis the debt holders. Mathematically, we introduce a parameter $q \in [0, 1]$ which measures the Nash bargaining power of debtholders.

[19]For anecdotal evidence, see for example the case of Radioshack, where three hedge funds offered cheap financing to the distressed firm. See "Credit-Default Swaps Get Activist New Look," December 23, 2014, The Wall Street Journal.

We show that with two-sided activism, by both the CDS buyer and seller, the firm reaches first-best investment and firm value, for all values of $q \in [0, 1]$. We further show that if there is only activism by the CDS buyer, then the introduction of a CDS market can have either a positive or negative net effect on firm value. However, even for the values of q for which the CDS-effect on firm value is negative, the firm reaches first-best under two-sided activism. This implies that the increase in firm value can be even larger than what we have seen in the simple model with $q = 0$. The detailed derivation can be found in the Internet Appendix.

We can also use this model extension to demonstrate the feedback effect of protection sellers on protection buyers. The optimal hedge ratio with only CDS buyer intervention is $h^* = 1 - q$. With two-sided intervention, this increases to $h^* = 1$. The intuition is the following: knowing that the protection seller will intervene, the protection buyer can be even more aggressive and buy more CDS protection. As a result, he will be even tougher in renegotiation, which helps to solve the commitment problem between equity and debt.

3.4. Side-payoffs of shareholders in debt renegotiation

With respect to the model where the CDS dealer can inject equity into the firm we should mention that we assume that when the shareholders of the firm decide whether to repay the debt, to renegotiate the debt, or to liquidate the firm, they only take into account the payoffs of their shares. While this is natural for the original owner of the firm, who only owns shares, the new owner (i.e., the CDS dealer) might own shares as well as a short CDS position. It is not obvious, a priori, whether one should also take into account the (negative) CDS payoffs in case of a liquidation in the outside option of Nash bargaining. We argue that it makes sense to use only payoffs from shares because, empirically, it might be hard for one shareholder to convince other shareholders during a debt renegotiation that one single shareholder's side payoffs should be taken into account. However, our results are robust to this alternative way of modeling the default decision, as we show in the Internet Appendix.

4. Discussion and policy implications

4.1. Coase Theorem

After allowing the protection seller to intervene, the efficient outcome of no liquidation is reached. Since there are no transaction costs associated with bargaining between the owner and the protection seller, this outcome seems to be an implication of the Coase Theorem. However, we argue that our results go beyond the Coase Theorem.

The Coase Theorem is about how ex post bargaining can lead to efficient outcomes. However, some of our most interesting results are not the efficient outcome ex post, but what happens ex ante. And even what happens ex post in our model is different from the simple Coasian prediction.

Ex ante, we show that the lender buys more CDS insurance upfront because he wants to give a strong incentive to the protection seller to intervene. In equilibrium, the ex ante probability of needing the insurance is zero, and yet the lender buys a lot of insurance. Finally, the firm borrows and invests more at time zero.

Ex post, our model is different from Coase as well. It is the lender who effectively causes the deadweight cost of liquidation, but the protection seller makes a payment to a different party—the owner—to avoid the inefficiency.

To make this distinction clearer, note that our results go through even if the equity injection by the dealer does not trigger a payment from the firm to the lender. All that we need is that the dealer makes a cash injection into the firm. Even if the cash is held on the balance sheet and not paid out to the lender, the net debt of the firm is reduced, default is avoided, and the results are the same as in Proposition 1.[20]

Our results are more reminiscent of the theoretical literature on how carefully chosen ex ante contracts can solve ex post hold-up problems or conflicts of interest.[21] For example, Aghion and Bolton (1992) show how a standard debt contract can solve a financing problem between an en-

[20]To see this, one can re-interpret our model as follows: a can be seen as the physical assets of the firm, i.e., without cash holdings, b can be interpreted as net debt, and b_n as the new net debt after the cash injection. If the firm makes default decisions based on net debt, then our equations and results remain the same.

[21]We thank Philip Bond for this insight.

trepreneur and an investor. Assigning control rights in a non-contingent way to the entrepreneur would prohibit her from raising sufficient financing. But giving state-contingent control rights to the investor solves that problem. In another example, Noldeke and Schmidt (1995) show that the famous hold-up problem of Hart and Moore (1988) can be solved by an option contract between the two parties, assuming that actions are verifiable.

Our model implies that adding a CDS contract to the lender's portfolio and allowing the protection seller to intervene solves two problems: (i) the problem that the owner cannot commit not to renegotiate the debt contract in good states of the world, and (ii) the excessive liquidation resulting from the fact that the lender becomes reluctant to renegotiate the debt.

4.2. Alternative contracts

One of the main results in Section 2 is that the probability of liquidation is zero in equilibrium. The reduction in expected bankruptcy costs is one of the main reasons for the increase in firm value going from the model in Section 1 to the model in Section 2. However, this raises the question of whether the same outcome could be reached through an alternative contractual arrangement. In particular, why do the owner and the lender not come to an agreement that avoids costly liquidation?

Our model actually allows for such negotiations between the two parties. If there were no CDS market, then the owner and the lender would both agree that avoiding liquidation is in both parties' interest and they would agree on debt renegotiation. This model is solved formally in Internet Appendix C. As a result, the probability of liquidation without a CDS market would be zero.[22]

However, the introduction of a CDS market and allowing CDS trading and intervention by the lender destroys that outcome. The lender wants to improve his ex post bargaining position, so he purchases CDS contracts ex ante. As a result, he demands so much in renegotiation that in certain states of the world debt renegotiation becomes infeasible, which leads to liquidation. Therefore,

[22] Even though liquidation never occurs in the no-CDS model, the outcome is far from efficient. The reason is that the probability of strategic default—or debt renegotiation in good states of the world—is much higher, which depresses firm value ex ante.

the owner and the lender cannot reach the efficient outcome of no liquidation through a bilateral arrangement.

4.3. Policy implications

Our analysis has several important policy implications. We show that having a protection seller who interferes with the debt restructuring of a financially distressed firm is not necessarily reducing firm value. This is not a trivial insight, as some market commentators argue that the recent cases of CDS investor intervention in Table 1 are evidence that the CDS market is absurd and dysfunctional.[23] Also, speculative protection buyers who would benefit from a credit event consider an unexpected intervention of a protection seller as unfair to them.

Our results suggest that the ability of a protection seller to interfere with debt restructuring actually improves firm value. Firm investment increases, the probability of liquidation goes down, and credit spreads narrow. Under certain assumptions on the distribution of future profitability and certain parameter values, firm investment can even achieve the first-best level, and the probability of liquidation can drop to zero.

The resulting equilibrium may seem unfair because the probability of liquidation is zero, but the protection buyer still pays a positive CDS spread. We argue that this is, in fact, necessary: The positive up-front CDS spread compensates the protection seller for his ex post cash injections into the distressed firm.

Our results also imply that a smaller number of protection sellers is better because it increases the incentives of the seller to save the distressed firm. This is in contrast to the intuitive notion that a large number of market participants is always better. Our result is analogous to theories on the disadvantages of dispersed bondholders (e.g., Gertner and Scharfstein, 1991). The result is policy-relevant today, as there are concerns that the inter-dealer CDS market is too concentrated.[24] To the extent that a smaller number of dealers is correlated with a smaller number of protection sellers, our results suggest that a concentrated inter-dealer market can have its benefits.

[23] E.g., Financial Times, "Time to wipe out the absurd credit default swap market", May 11, 2018.
[24] The Wall Street Journal, "Big banks agree to settle swaps lawsuit", September 12, 2015.

All these results are based on the assumption of symmetric information between protection buyers, protection sellers, and the underlying firm. In particular, we assume that everyone knows how many protection sellers there are and how much protection they have sold. If we relax the assumption of symmetric information, it is unclear whether the firm value increasing equilibrium still prevails. A protection seller might have an incentive to pretend ex ante that he cannot rescue the firm, to sell protection at a high CDS spread, and to rescue the firm ex post.

Empirically, it is not unlikely that this can happen. The CDS market is very opaque, and no regular investor knows how many protection sellers there are, how much protection they have sold, and whether they have deep pockets to inject cash into the underlying firm. Therefore, it is possible that regulation that improves the transparency of the CDS market, such as reporting requirements for large positions, can increase firm value. Other authors have proposed disclosure requirements in the CDS market as well (e.g., Bolton and Oehmke, 2011), although for different reasons. However, further research is needed to examine the role of asymmetric information.

Our model does not directly incorporate manufactured defaults, a special type of intervention by CDS investors. In narrowly tailored credit events (NTCEs), as they are formally called, typically the protection buyer offers cheap financing to a financially distressed firm. The catch is that the CDS buyer asks the firm to make a payment on existing debt contracts with a slight delay. This triggers a credit event in the CDS market, creating a payoff to the protection buyer. Section H of the Internet Appendix contains a detailed discussion of how our analysis relates to manufactured defaults.

5. Conclusion

We show that CDS intervention does not necessarily reduce firm value. This is true in spite of the seemingly unfair outcome that the protection buyer has to pay a CDS spread upfront that is high relative to the low probability of liquidation. We show that the lender, who is the typical protection buyer in our model, is not worse off by paying the CDS spread, because the seller will bail out the firm in the future, which is good for the lender. We also show that the lower probability of bankruptcy causes ex ante firm borrowing costs to go down, which in turn leads to higher investment and firm value.

We emphasize that our analysis is limited to two types of CDS intervention: A hedged lender who is a tough negotiator in an out-of-court debt restructuring and a protection seller who can avoid a credit event by infusing capital into the firm. We do not claim that other kinds of CDS interventions, such as manufactured defaults, create firm value. Also, our analysis is limited by several simplifying assumptions that make the model tractable. For example, we only have a single lender, we assume symmetric information, we do not endogenize the number of protection sellers, and we ignore the role of liquidity in the CDS market and the bond market (see Oehmke and Zawadowski (2015)).

Keeping these caveats in mind, our results have important policy implications. Future regulation that prohibits intervention by protection sellers might reduce firm value instead of increasing it.

Figure 1: Timeline of events in the active CDS buyer model

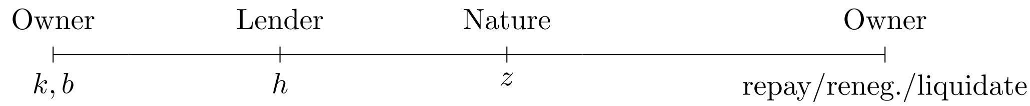


Figure 2: Optimal default decision in the active CDS buyer model

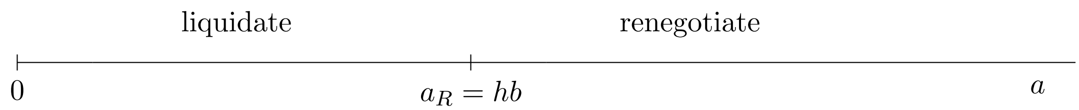


Figure 3: Timeline of events in the extended model

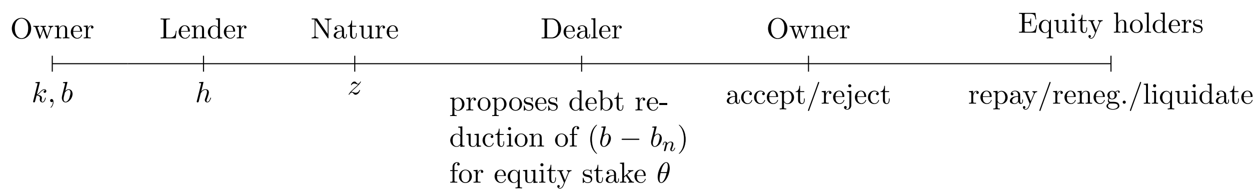
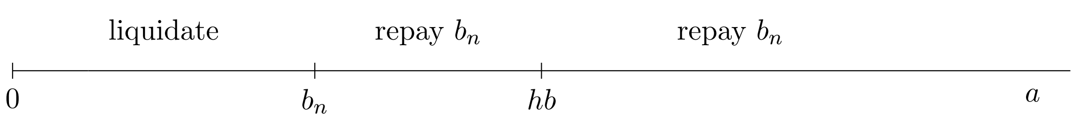
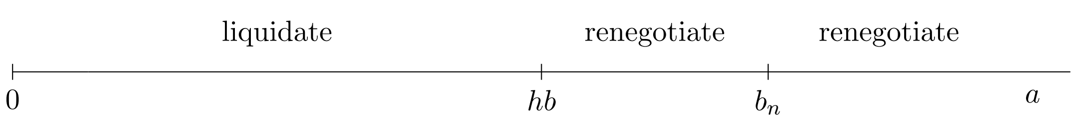


Figure 4: Optimal default decision – extended model

If $b_n \leq hb$:



If $b_n > hb$:



Firm	Year	Summary	Source
Codere	2013	Protection buyer (Blackstone GSO) offers financing to Codere in return for technical default. The losing party is the protection seller (unknown).	Bloomberg
Caesars Ent.	2014	Protection buyer (Elliott) pushes for early bankruptcy. Protection seller (Blackstone GSO) is against bankruptcy. Elliott is also a major bondholder. Blackstone GSO is a major loan holder. Eventually, Caesars files for bankruptcy before CDS maturity, Elliott profits.	WSJ
Forest Oil	2014	Financially distressed Forest Oil wants to merge with Sabine Oil & Gas, a competitor, to avoid default. Protection buyers (unknown) purchase stocks in order to vote against the deal.	WSJ
RadioShack	2014	BlueCrest Capital Management, DW Investment Management and Saba Capital Management sell CDS protection on RadioShack. When RadioShack becomes financially distressed, the protection sellers offer new loans to avoid default.	WSJ
Norske Skog	2016	Protection seller (Blackstone GSO) keeps financially distressed firm alive and collects CDS spread.	Reuters
iHeartMedia	2016	iHeartMedia misses a payment on a bond owed to its subsidiary, which triggers a CDS credit event. This eliminates the CDS contracts of bondholders, which might make a future debt restructuring easier for the firm.	WSJ
Matalan	2017	Protection sellers (unknown group of hedge funds) offer financing to keep financially distressed firm alive. They request that the new debt is issued by a new legal entity.	FT
Hovnanian	2018	Protection buyer (Blackstone GSO) offers financing in return for default of a subsidiary of Hovnanian.	WSJ
McClatchy	2018	Protection seller (Chatham) provides financing to McClatchy, asks to move debt to subsidiary. Protection seller collects CDS spread on parent company with zero default risk. Deal is cancelled a few months later.	Bloomberg

(*continued*)

Table 1: **Summary of recent cases of CDS investor involvement**.

Firm	Year	Summary	Source
Supervalu	2018	United Natural Foods wants to acquire Supervalu and needs to issue new debt to finance the deal. Protection buyers (unknown) for Supervalu would lose money in the deal, because their CDS protection would become worthless. They convince debt underwriter (Goldman Sachs) to make Supervalu a co-borrower of the new debt.	Bloomberg
Windstream	2019	Protection buyer (Aurelius Capital Management) sues Windstream for violating bond covenant. Windstream files for bankruptcy.	WSJ
Neiman Marcus	2019	Protection buyer (Aurelius Capital Management) pushes Neiman Marcus to link more of its debt to CDS contracts.	WSJ
Thomas Cook	2019	Bondholders hedged with CDSs threaten to block a debt restructuring because they want a credit event to be triggered. Additionally, a protection seller offers financing to the firm.	FT

Table 1: **Continued**

	Empty creditor	Cash injection by protection seller	Manufactured default	Orphaned CDS
Description	Bondholder with CDS protection demands better terms in an out-of-court debt restructuring	Protection seller injects debt or equity into a distressed firm	Protection buyer provides financing to distressed firm, demands a technical default	Protection seller injects cash into a distressed firm, demands that the debt is moved to a subsidiary
Intervening party	Protection buyer	Protection seller	Protection buyer	Protection seller
Example	YRC Worldwide	Norske Skog, Radioshack	Codere, Hovnanian	McClatchy

Table 2: **Types of CDS intervention.** Most of the cases mentioned in the last row are explained in detail in Table 1. For a description of the case of YRC Worldwide, see Bolton and Oehmke (2011).

	Owner	Dealer	Lender
Debt repayment	$(1-\theta)(a-b_n)$	$\theta(a-b_n)$	b_n
Debt renegotiation	$(1-\theta)(a-b_r)$	$\theta(a-b_r)$	b_r
Liquidation	0	$-hb$	hb

Table 3: **Extended model – terminal payoffs for each player.** The symbol a denotes the asset value of the firm; b is the face value of debt; b_n is the new face value of debt after a possible equity injection; b_r is the face value of debt that is agreed on in a debt renegotiation; h is the lender's CDS hedge ratio; θ is the fraction of the firm owned by the CDS dealer after a possible equity injection.

	Active buyer & seller	Active buyer	Δ	Comments
Investment k	$k^* = \left(\frac{\alpha Z}{2}\right)^{\frac{1}{1-\alpha}}$	$k^* = \left(\frac{\alpha Z}{2(1+\tau)}\right)^{\frac{1}{1-\alpha}}$	> 0	Difference larger for high τ.
Face value of debt $b(k)$	Zk^α	$\frac{Zk^\alpha \tau}{1+\tau}$	> 0	For fixed value of k.
Market value of debt M	$M(k, b, h)$	$M(k, b, h)$	> 0	For fixed k, b, and h.
Tobin's Q (V/k)	$1/\alpha - 1$	$1/\alpha - 1$	0	
Firm value V	Qk^*	Qk^*	> 0	Difference larger for high τ.
Book leverage b/k	$2/\alpha$	$\tau 2/\alpha$	> 0	
Market leverage b/V	$2/(\alpha Q)$	$\tau 2/(\alpha Q)$	> 0	
Prob. of liquidation	0	> 0	< 0	
$\partial k/\partial \tau$	0	< 0	> 0	
$\partial V/\partial \tau$	0	< 0	> 0	
$\partial b/\partial \tau$	0	> 0	< 0	For fixed value of k.
$\partial M/\partial h$	> 0	> 0	> 0	For $h \in [1/2, 1]$.

Table 4: **Comparative statics – Active CDS buyer and seller vs. active CDS buyer.** We compare the predictions of the model with an active CDS buyer and seller (Section 2) to the predictions of the model with only an active CDS buyer (part of Section 1). In the columns, we show (i) the predictions of the model with an active buyer and seller, (ii) the predictions of the model with an active buyer, and (iii) the difference between the two, i.e., the first column minus the second column. In the rows, we show the optimal level of investment k; the optimal face value of debt as a function of investment $b(k)$; the market value of debt M for fixed values of (k, b, h); Tobin's Q (V/k) at the optimal (k, b, h); firm value V at the optimal (k, b, h); book leverage at the optimal investment and debt b/k; quasi-market leverage at the optimal investment and debt b/V; the probability of liquidation in equilibrium; the effect of corporate taxes on optimal investment $\frac{\partial k}{\partial \tau}$; the effect of taxes on optimal firm value $\frac{\partial V}{\partial \tau}$; the effect of taxes on the face value of debt for fixed values of investment, $\frac{\partial b}{\partial \tau}$; and the effect on an exogenous change in the hedge ratio on the market value of debt, $\frac{\partial M}{\partial h}$.

	Active buyer & seller	Firms without CDS	Δ	Comments
Investment k	$k^* = \left(\frac{\alpha Z}{2}\right)^{\frac{1}{1-\alpha}}$	$k^*_{\text{no-CDS}} = \left(\frac{(1-\tau)\alpha Z}{2}\right)^{\frac{1}{1-\alpha}}$	> 0	
Face value of debt $b(k)$	Zk^α	0	> 0	For fixed value of k.
Market value of debt M	$M(k, b, h)$	0	> 0	Implies opposite predictions for yield.
Tobin's Q (V/k)	$1/\alpha - 1$	$1/\alpha - 1$	0	
Firm value V	Qk^*	Qk^*	> 0	
Book leverage b/k	$2/\alpha$	0	> 0	
Market leverage b/V	$2/(\alpha Q)$	0	> 0	
Prob. of liquidation	0	0	0	
$\partial k/\partial \tau$	0	< 0	> 0	
$\partial V/\partial \tau$	0	< 0	> 0	
$\partial b/\partial \tau$	0	0	0	For fixed value of k.
$\partial M/\partial h$	> 0	n/a	n/a	For $h \in [1/2, 1]$.

Table 5: **Comparative statics – Firms with CDSs vs. firms without CDSs.** We compare the predictions of the model with an active CDS buyer and seller (Section 2) to the predictions of the model without a CDS market (part of Section 1). In the columns, we show (i) the predictions of the model with an active CDS buyer and seller, (ii) the predictions of the model without a CDS market, and (iii) the difference between the two, i.e., the first column minus the second column. In the rows, we show the optimal level of investment k; the optimal face value of debt as a function of investment $b(k)$; the market value of debt M for fixed values of (k, b, h); Tobin's Q (V/k) at the optimal (k, b, h); firm value V at the optimal (k, b, h); book leverage at the optimal investment and debt b/k; quasi-market leverage at the optimal investment and debt b/V; the probability of liquidation in equilibrium; the effect of corporate taxes on optimal investment $\frac{\partial k}{\partial \tau}$; the effect of taxes on optimal firm value $\frac{\partial V}{\partial \tau}$; the effect of taxes on the face value of debt for fixed values of investment, $\frac{\partial b}{\partial \tau}$; and the effect on an exogenous change in the hedge ratio on the market value of debt, $\frac{\partial M}{\partial h}$.

References

Aghion, Philippe, and Patrick Bolton, 1992, An Incomplete Contracts Approach to Financial Contracting, *Review of Economic Studies* 59, 473–494.

Arping, Stefan, 2014, Credit protection and lending relationships, *Journal of Financial Stability* 10, 7–19.

Bartram, Söhnke M, Jennifer Conrad, Jongsub Lee, and Marti G Subrahmanyam, 2021, Credit Default Swaps around the World, *Review of Financial Studies* forthcoming.

Batta, George E, and Fan Yu, 2018, Credit Derivatives and Firm Investment, *SSRN*.

Bolton, Patrick, and Martin Oehmke, 2011, Credit Default Swaps and the Empty Creditor Problem, *Review of Financial Studies* 24, 2617–2655.

Buccola, Vincent SJ, Jameson K Mah, and Tai Zhang, 2020, The myth of creditor sabotage, *U. Chi. L. Rev.* 87, 2029.

Caglio, Cecilia, R Matthew Darst, and Eric Parolin, 2019, Half-full or Half-empty? Financial Institutions, CDS Use, and Corporate Credit Risk, *Journal of Financial Intermediation*.

Campello, Murillo, and Rafael Matta, 2020, Investment risk, CDS insurance, and firm financing, *European Economic Review* 125, 1–10.

Chang, Xin Simba, Yangyang Chen, Sarah Qian Wang, Kuo Zhang, and Wenrui Zhang, 2019, Credit Default Swaps and Corporate Innovation, *Journal of Financial Economics* 134, 474–500.

Colonnello, Stefano, Matthias Efing, and Francesca Zucchi, 2019, Shareholder Bargaining Power and the Emergence of Empty Creditors, *Journal of Financial Economics* 134, 297–317.

Danis, Andras, 2017, Do Empty Creditors Matter? Evidence from Distressed Exchange Offers, *Management Science* 63, 1285–1301.

Danis, Andras, and Andrea Gamba, 2018, The Real Effects of Credit Default Swaps, *Journal of Financial Economics* 127, 51–76.

Degryse, Hans, Yalin Gündüz, Kuchulain O'Flynn, and Steven Ongena, 2020, Identifying Empty Creditors with a Shock and Micro-Data, *SSRN*.

Diamond, Douglas W., and Philip H. Dybvig, 1983, Bank Runs, Deposit Insurance, and Liquidity, *Journal of Political Economy* 91, 401–419.

Donaldson, Jason Roderick, Edward R. Morrison, Giorgia Piacentino, and Xiaobo Yu, 2021, Restructuring Vs. Bankruptcy, Columbia University.

Fletcher, Gina-Gail S, 2019, Engineered Credit Default Swaps: Innovative Or Manipulative, *NYUL Rev.* 94, 1073.

Gertner, Robert, and David Scharfstein, 1991, A Theory of Workouts and the Effects of Reorganization Law, *Journal of Finance* 46, 1189–1222.

Goldstein, Itay, and Ady Pauzner, 2005, Demand–Deposit Contracts and the Probability of Bank Runs, *Journal of Finance* 60, 1293–1327.

Guest, Paul M, Nikolaos Karampatsas, Dimitris Petmezas, and Nickolaos G Travlos, 2017, Credit Default Swaps, Financial Distress and Corporate Acquisitions, *SSRN*.

Hart, Oliver, and John Moore, 1988, Incomplete Contracts and Renegotiation, *Econometrica* 56, 755–785.

Hu, Henry T C, and Bernard Black, 2008, Equity and debt decoupling and empty voting II: Importance and extensions, *University of Pennsylvania Law Review* 156, 625–739.

Kim, Gi H, 2016, Credit derivatives as a commitment device: Evidence from the cost of corporate debt, *Journal of Banking and Finance* 73, 67–83.

Lubben, Stephen J, 2007, Credit derivatives and the future of Chapter 11, *American Bankruptcy Law Journal* 81, 405.

Morris, Stephen, and Hyun Song Shin, 2010, Global Games: Theory and Applications, in Mathias Dewatripont, Lars Peter Hansen, and Stephen J. Turnovsky, eds.: *Advances in Economics and Econometrics* (Cambridge University Press,).

Narayanan, Rajesh, and Cihan Uzmanoglu, 2018, Credit Default Swaps and Firm Value, *Journal of Financial and Quantitative Analysis* 53, 1227–1259.

Noldeke, Georg, and Klaus M Schmidt, 1995, Option Contracts and Renegotiation: A Solution to the Hold-up Problem, *The RAND Journal of Economics* 26, 163.

Oehmke, Martin, and Adam Zawadowski, 2015, Synthetic or Real? The Equilibrium Effects of Credit Default Swaps on Bond Markets, *Review of Financial Studies* 28, 3303–3337.

Subrahmanyam, Marti G, Dragon Yongjun Tang, and Sarah Qian Wang, 2014, Does the Tail Wag the Dog?: The Effect of Credit Default Swaps on Credit Risk, *Review of Financial Studies* 27, 2927–2960.

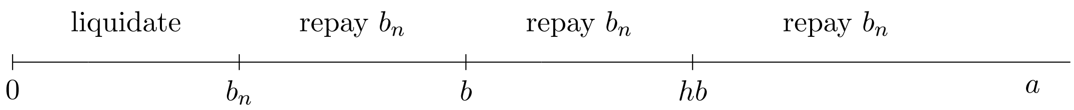
Wong, Tak-Yuen, and Jin Yu, 2021, Credit Default Swaps and Debt Overhang, *Management Science* forthcoming.

Appendices

A. Proof that the optimal hedge ratio is $h \leq 1$ in the extended model in Section 2

We solve the model by backwards induction under the assumption $h > 1$, and show that the resulting debt value cannot be optimal from the debt holder's perspective.

From Figure 4, because for $h > 1$ we have necessarily $b < hb$, considering the case $b_n \leq b < hb$, the firm's default decision is



In order to determine the outside option of the initial owner in case she rejects a proposal from the protection seller, we also derive the optimal default decision of the owner without an equity injection (this figure is analogous to Figure IA.1 in the Internet Appendix).

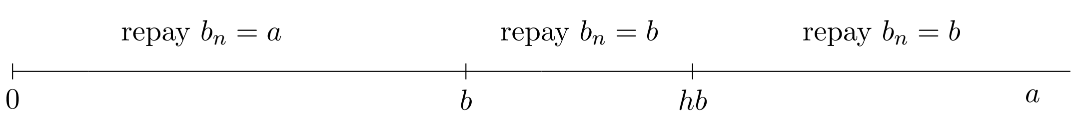
Using the same logic as in the proof of Lemma 2, we can derive the optimal θ in the case $h > 1$:

$$\theta = \begin{cases} 1 - \frac{a-b}{a-b_n} & \text{if } a \geq b, \\ 1 & \text{if } a < b. \end{cases}$$

Analogously to the proof of Lemma 3, one can derive the optimal b_n, which becomes:

$$b_n = \begin{cases} b & \text{if } a \geq b, \\ a & \text{if } a < b. \end{cases}$$

Both cases, $a \geq b$ and $a < b$, are followed by repayment of debt. Having the optimal θ and b_n, we can summarize the solution graphically as follows:



Similarly to Equation (18), we can now derive the market value of debt,

$$M = \int_0^{b/k^\alpha} [\underbrace{(b-a)}_{\text{early repaym.}} + \underbrace{a}_{\text{remaining repaym.}}] d\Gamma(z) + \int_{b/k^\alpha}^{Z} \underbrace{b}_{\text{full repayment}} d\Gamma(z) - \underbrace{\int_0^{b/k^\alpha} (b-a) d\Gamma(z)}_{\text{upfront CDS premium}},$$

which can be simplified to

$$M = \int_0^{b/k^\alpha} a d\Gamma(z) + \int_{b/k^\alpha}^{Z} b d\Gamma(z).$$

Because this expression does not depend on h, the debt holder cannot do better (nor worse) by increasing his hedge ratio beyond 1.

B. Proof of Lemma 1

From Equation (12), debt renegotiation is only feasible if $a \geq hb$. Also, we know that if renegotiation is feasible, the equity holders prefer repayment to renegotiation if $b_n \leq hb$. If renegotiation is infeasible, the equity holders prefer repayment to liquidation if $a \geq b_n$. Combining these three inequalities produces the stated result.

C. Proof of Lemma 2

We consider two possible cases: $b_n \leq hb$ and $b_n > hb$. As for the first case,

- if $a \geq hb$, the owner's payoff if the offer is accepted is $(1-\theta)(a-b_n)$, corresponding to debt repayment, and if the offer is rejected it is $a - b_r(b) = a - hb$, corresponding to renegotiation.[25] From equating the two payoffs we have

$$\theta = 1 - \frac{a - hb}{a - b_n}.$$

  Because $a \geq hb \geq b_n$, then $\theta \leq 1$, with strict inequality if either of the two inequalities $a \geq hb$ and $hb \geq b_n$ is strict.

- if $b_n \leq a < hb$, the payoff if the offer is accepted is $(1-\theta)(a-b_n)$ from debt repayment, and 0 from liquidation if the offer is rejected. Equating the two payoffs yields $\theta = 1$;

[25] Notice that the $b_r(b)$ is the b_r under rejection, i.e., without a debt reduction, while we use $b_r(b_n)$ for the renegotiated face value after a debt successful debt reduction. Under our assumption of high liquidation costs, the two expressions for b_r are the same, but in general they are not. This is because here the outside option of the bondholder is hb, which depends on b but not on b_n. But with an arbitrary liquidation cost $\xi \in [0,1]$, the outside option becomes $(1-\xi)a + hb - hb(1-\xi)a/b_n$, which depends on b_n as well.

- if $a < b_n$, the payoff is 0 from liquidation regardless if the offer is accepted or rejected. The dealer can choose any $\theta \in [0, 1]$. We assume that in equilibrium, $\theta = 1$.

As for the case $b_n > hb$,

- if $a \geq b_n$, the owner's payoff if the offer is accepted is $(1-\theta)(a - b_r(b_n)) = (1-\theta)(a - hb)$, deriving from renegotiation of the debt b_n, and if the offer is rejected it is $a - b_r(b) = a - hb$, corresponding to renegotiation of the debt b. Equating the two payoffs yields $\theta = 0$;
- if $hb \leq a < b_n$, the payoff if the offer is accepted is $(1-\theta)(a - b_r(b_n))$, from renegotiation of b_n, and if the offer is rejected it is $a - b_r(b)$, from renegotiation of b. Hence, $\theta = 0$;
- if $a < hb$, the payoff is 0 from liquidation regardless if the offer is accepted or rejected. As before, we assume that in equilibrium, $\theta = 1$.

D. Proof of Lemma 3

Motivated by Lemma 2, and the expression of the optimal θ, we consider two possible cases: $a > hb$ and $a < hb$. As for the first case:

- If the dealer injects a lot of capital, i.e., chooses a sufficiently low b_n such that $b_n \leq hb$, this would eventually lead to repayment of b_n (see Lemma 1), so his payoff is

$$\max_{b_n} \left\{ -(b - b_n) + \theta(a - b_n) \right\}.$$

  After substituting for $\theta = 1 - \frac{a-hb}{a-b_n}$ from Lemma 2, this simplifies to $\max_{b_n}(-b + hb)$, which is independent of b_n and negative.

- If the dealer injects little capital, i.e., chooses a sufficiently high b_n such that $b_n > hb$, this eventually leads to renegotiation (see Lemma 1), so his payoff is

$$\max_{b_n} \left\{ -(b - b_n) + \theta(a - b_r(b_n)) \right\}.$$

  Based on Lemma 2, the optimal θ is 0 in this case. Hence, the payoff simplifies to $\max_{b_n}(-(b - b_n))$, which is maximized at $b_n = b$. This implies that no recapitalization takes place, and the dealer's payoff is zero, which is higher than in the case considered above.

Therefore, if $a > hb$, it is optimal for the dealer not to reduce the debt.

As for the second case, $a < hb$:

- suppose the dealer chooses a sufficiently low b_n such that $b_n \leq hb$.

  – If he chooses a sufficiently low b_n such that $b_n \leq a < hb$, because this would be a case of repayment, he gets
  $$\max_{b_n} \{-(b - b_n) + \theta(a - b_n)\}.$$
  After substituting for $\theta = 1$ from Lemma 2, this simplifies to $\max_{b_n}(a - b)$, which is independent of b_n. Because $a < hb$, then $a < b$, and therefore the payoff is negative.

  – Otherwise, if he chooses a value of b_n such that $a < b_n \leq hb$, then he gets
  $$\max_{b_n} \{-(b - b_n) - hb\},$$
  because this would be a case of liquidation.

  We now show that the dealer is better off by choosing b_n such that $b_n \leq a < hb$ rather than $a < b_n \leq hb$. This is the case because, for the dealer's payoff in the two cases, it is true that $a - b > \max_{b_n} \{-(b - b_n) - hb\}$, and this inequality is equivalent to $a > \max_{b_n} \{b_n - hb\}$. In the last inequality, the left-hand side is always positive, and the right-hand side is negative or zero, since $b_n \leq hb$ by assumption. We conclude that $b_n \leq a$.

- If the dealer chooses a sufficiently high b_n such that $b_n > hb$, then this would be a case of liquidation with payoff
  $$\max_{b_n} \{-(b - b_n) - hb\}.$$
  The maximum payoff is $-hb$, which is attained at $b_n = b$.

Comparing the optimal payoffs in the cases $b_n \leq hb$ and $b_n > hb$ reveals that there is no globally optimal b_n. If $-hb > a - b$, it is optimal to choose $b_n = b$, which is followed by liquidation. Otherwise, the optimal new debt is $b_n = a$, which is followed by repayment.

We conclude the case $a < hb$ as follows: If $-hb > a - b$, or $a < b - hb$, it is optimal to choose $b_n = b$, which is followed by liquidation. Otherwise, the optimal new debt is $b_n = a$, which is followed by repayment.

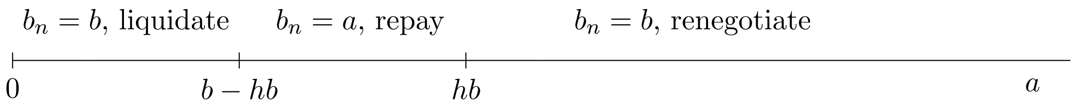
To describe the optimal b_n across all possible cases, we have to take into account two thresholds, hb and $b - hb$. It is easy to show that $hb < b - hb$ is equivalent to $h < 1/2$. Hence, we have Figure 5.

E. Proof of Lemma 4

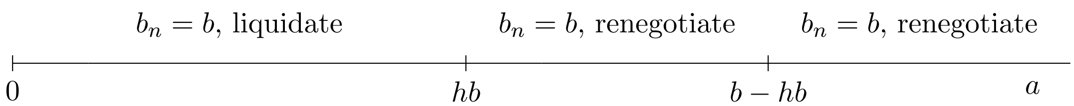
From the proof of Lemma 3, if $a \geq hb$, there is renegotiation of the debt, no intervention by the dealer, and no payment from the CDS contract. On the other hand, if $a < hb$ the equilibrium

Figure 5: Optimal intervention and optimal default decision – extended model

If $b - hb < hb$ (i.e., $h > 1/2$):



If $b - hb \geq hb$ (i.e., $h \leq 1/2$):



strategy depends on whether h is higher or lower than $1/2$, see Figure 5. If $h \leq 1/2$, there is no equity injection, or $b_n = b$. This is followed by liquidation and a payment from the protection seller. If $h > 1/2$, there is an equity injection and debt is reduced to $b_n = a$ if $b - hb \leq a < hb$, and if $a < b - hb$ there is no intervention followed by liquidation, with the compensation from the CDS contract. Hence, if $b - hb \geq hb$, the expected total payoff to the debt is

$$M = \int_0^{z_R} [\underbrace{0}_{\text{bond payoff}} + \underbrace{hb}_{\text{CDS payoff}}] d\Gamma(z) + \int_{z_R}^{Z} \underbrace{hb}_{\text{renegotiation}} d\Gamma(z) - \underbrace{\int_0^{z_R} hb d\Gamma(z)}_{\text{CDS premium}}, \tag{18}$$

where $z_R = hb/k^\alpha$. This expression simplifies to

$$M(k, b, h) = \int_{z_R}^{Z} hb d\Gamma(z). \tag{19}$$

If $b - hb < hb$, then the total expected payoff to the debt is

$$M = \int_0^{z_L} (\underbrace{0}_{\text{bond payoff}} + \underbrace{hb}_{\text{CDS payoff}}) d\Gamma(z) + \int_{z_L}^{z_R} [\underbrace{(b-a)}_{\text{recapitalization}} + \underbrace{a}_{\text{debt paym.}}] d\Gamma(z)$$
$$+ \int_{z_R}^{Z} \underbrace{hb}_{\text{renegotiation}} d\Gamma(z) - \underbrace{\left[\int_0^{z_L} hb d\Gamma(z) + \int_{z_L}^{z_R} (b-a) d\Gamma(z)\right]}_{\text{CDS premium}},$$

where $z_L = (b - hb)/k^\alpha$ is the liquidation threshold for this case.