Addressing Mathematics Anxiety: A Case Study in a High School in Brazil

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Abstract
Mathematics Anxiety is debilitating and causes negative impacts on students’ academic lives. In this case study, we use Participatory Action Research methodology to address Mathematics Anxiety in a public high school in Rio de Janeiro (Brazil). An intervention was performed using tools such as the Growth Zone Model and the hand model of the brain in order to address anxiety and build Mathematical Resilience. The levels of Mathematics Anxiety were measured before and after the intervention using the revised MAS scale (MAS-R), and narrative records were made. The data indicate positive evidence in performing this kind of intervention, adding to the literature on building Mathematical Resilience and leading us to continue developing our practice as anxiety-informed teachers, and to recommend future interventions with bigger and more diversified samples.

Keywords
Mathematics Anxiety, Mathematical Resilience, Participatory Action Research, Anxiety-Informed Mathematics Teaching, Growth Zone Model

1. Introduction
Mathematics Anxiety (MA) has been defined as a “feeling of tension and anxiety that interferes with the manipulation of numbers and the solving of mathematical problems in ordinary life and academic situations” (Richardson & Suinn, 1972: p. 551). It has been studied for over 60 years by many teachers, researchers in mathematics education, psychologists, and neuroscientists (Richardson & Suinn, 1972; Ashcraft, 2002; Johnston-Wilder et al., 2014; Finlayson, 2014; Dowker et al., 2016). MA is disabling and, if not addressed, it interferes with the teaching and learning of mathematics (Ashcraft, 2002). It is characterized by
“fear about performing mathematics and is associated with delayed acquisition of core mathematics and number concepts and poor math competence” (Richardson & Suinn, 1972: p. 553). Many researchers have reported the consequences of being anxious about mathematics, including the inability to do mathematics, the decline in mathematics achievement, the avoidance of mathematics courses, the limitation in selecting college majors and future careers, and negative feelings of guilt and shame (Richardson & Suinn, 1972; Betz, 1978; Dowker et al., 2016).

Many actions and interventions have been tried to address mathematics anxiety, including mindfulness, managed breathing and writing (Carmo, et al., 2020). In essence, these actions have as their main goal the development of what is known as Mathematical Resilience in students and apprentices. Mathematical Resilience (MR) is defined as “a learner’s stance towards mathematics that enables pupils to continue learning despite finding setbacks and challenges in their mathematical learning journey” (Johnston-Wilder & Lee, 2010: p. 38). It is known that all learning requires resilience to an extent, but the authors claim that the resilience required for learning mathematics (Mathematical Resilience) is “a particular construct because of the difficulties presented when learning mathematics” (Johnston-Wilder & Lee, 2010: p. 38). This is, in part, because prevalent mathematics pedagogy often leaves learners feeling that learning mathematics is tedious, isolated, rote, elitist, and depersonalized (Nardi & Steward, 2003) and in part because of pervasive fixed mindsets in relation to mathematics.

In this paper, we also introduce the concept of anxiety-informed mathematics teaching, based on the Scottish report (Homes & Grandison, 2021), which considers the necessity to adopt practice that raises awareness of anxiety, recognizes Mathematics Anxiety in learners and seeks to resist causing further harm. This is an important step for the establishment of a safer teaching environment that can promote mathematical resilience and enhance achievement in mathematics.

The relationship between anxiety toward mathematics and achievement in mathematics is difficult to establish; higher attaining students are often found to have high Mathematics Anxiety (Carey et al., 2016), but, in general, higher-level anxiety is associated with lower-level achievement. This negative relationship has been displayed across age populations (Ma, 1999); Mathematics Anxiety is high in Brazil (OECD, 2012) and the mean score for achievement in mathematics of 15-year-old Brazilian students of 377 points is significantly lower than the average (490) of students of other country members of the Organization for Economic Co-operation and Development (OECD, 2017).

Numerous studies have shown that emotional factors may play a large part in mathematical performance, with MA playing a particularly large role. McLeod’s studies (1992; 1994) reveal value given to the affective dimension in knowledge acquisition, especially when it comes to mathematics learning. The author emphasizes that beliefs, attitudes, and emotions all play important roles in the learning of mathematics. Students’ beliefs about mathematics and about them-
selves form an important part of the context for their cognitive evaluations of the interruptions and blockages that are an inherent part of solving mathematical problems (Bandura, 1988). Ashcraft (2002) states that one possible reason for the negative association between MA and actual performance is that students who have higher levels of MA are more likely to avoid activities and situations that involve mathematics, leading them to have less practice which may reduce their fluency and their future mathematical learning. Another aspect that might influence performance is overloading working memory (Ashcraft & Kirk, 2001). This problem affects anxious people whose thoughts that they are doing badly in mathematics cause distraction, distress and loss of focus on the task or problem they have to solve.

Some authors (Fuerst & Hitch, 2000; Caviola et al., 2012) suggest that anxiety affects working memory and has a particularly strong effect on arithmetic, “as working memory has been found in many studies to be strongly associated with arithmetical performance, especially in tasks that involve multi-digit arithmetic and/or involve carrying” (Dowker et al., 2016: p. 4).

Estimates of the prevalence of Mathematics Anxiety vary widely and are dependent on the populations being sampled, on the measures used (although many studies involve similar measures), and, especially, on what criteria are used to categorize people as “mathematics anxious”. The percentage of people currently excluded from mathematics by anxiety is unclear, but a useful estimate is about 30% in the UK (Dowker et al., 2016), rising to 49% in Brazil (OECD, 2013). Mathematics Anxiety arises from adverse prior experiences, whether direct or vicarious. Mathematics Anxiety is debilitating and results in reduced progress in Mathematics.

2. Anxiety-Informed Mathematics Teaching

The need to include emotions in mathematics and science education has been emphasized by many authors (Hart, 1989; Damasio, 1994; McLeod, 1992, 1994; Gómez Chacón, 2000; Hidalgo et al., 2013) and underpins the necessity of introducing strategies that help to address mathematics anxiety.

Frequently, there is a mismatch between the pedagogy of mathematics teachers and what anxious learners need to thrive when learning mathematics. In this paper, we draw upon a recent report from Scotland (Homes & Grandison, 2021) into teachers and other professionals working with traumatized clients and apply it to addressing mathematics anxiety as teachers. We propose that there is a need to develop anxiety-informed mathematics teachers who can adopt a practice that raises awareness of anxiety, recognizes Mathematics Anxiety in learners, responds appropriately and resists causing further harm.

The literature offers tools that can help teachers and students to become more aware of Mathematics Anxiety and to understand how to reduce its impact and build resilience (Johnston-Wilder et al., 2020). One of these tools is the Growth Zone Model (GZM) (Lugalia et al., 2013), which is a framework for learners to name and communicate their current, otherwise hidden, feelings when learning
mathematics. The GZM has three “zones”, as depicted in Figure 1, which represent ways individual learners might experience a situation.

The green zone, or comfort zone, represents feeling safe and confident while learning mathematics, being able to use existing knowledge alone if needed and not experiencing stress. The red zone symbolizes danger or threat to well-being. In this zone, students might experience a “fight, flight or freeze” impulse to be angry, or to run, or to cry, or a sudden inability to think clearly. The amber or growth zone represents feeling challenged, which is when students can learn most effectively. Students who have a growth mindset (Dweck, 2006) are prepared to undertake challenges, believing they can improve. They may feel nervous but motivated and engaged, prepared to develop new skills by taking managed risks, exploring strategies, making mistakes, persisting, using coping strategies, feeling supported by their learning community, encouraging, asking questions and accessing support.

The GZM is a tool for enabling awareness of emotions to develop in the mathematics classroom with a shared language, thus enabling learners to develop Mathematical Resilience (Johnston-Wilder et al., 2020). The GZM gives a framework for learners to name and communicate their feelings.

The Growth Zone of the GZM has four elements: value, struggle, growth mindset and relationships (Cousins et al., 2019). It is important that students understand the value to them of the mathematics they are studying, why they are learning this topic. Value is related to Ryan and Deci’s (2017) notion of autonomous motivation. Struggle summarizes the need for persistence and perseverance when meeting challenges; perseverance includes recruiting support when needed (Williams, 2014). The Growth mindset (Dweck, 2006) is included because of the prevalence of a fixed mindset in mathematics teaching and learning. Relatedness is included because of its importance (Ryan & Deci, 2017) in psychological safety and being part of a community (Wenger, 1998) in learning mathematics. Group work is one way to increase relatedness and community in teaching and learning mathematics (Koçak et al., 2009).

![Figure 1. Growth zone model (GZM) (Lugalia et al., 2013).](image)
Also, the hand model of the brain (HMB) (Siegel, 2010) and the relaxation response (Benson, 1983) are useful tools in developing anxiety-informed mathematics teaching and building Mathematical Resilience (Johnston-Wilder et al., 2021). The former is a tool to recognize when emotional distress is impacting learning and the latter is to manage emotional distress. Siegel’s HMB (see Figure 2) can be introduced to help to explain to the students, in an easy and accessible way based on neuroscience, why MA may lead to temporary feelings of stupidity due to panic, distress or self-defense response when facing math tests or math challenges (Johnston-Wilder et al., 2021). It is suggested that these tools be used once the students are familiar with the concept of the GZM. HMB is a tool that helps learners to recognize when their brain has “flipped” into the red zone, and to take appropriate action. These actions include telling the teacher by folding and lifting their fingers, using a calming technique to trigger the relaxation response, or removing themselves from the situation.

Benson’s “relaxation response” is described as a self-induced state of consciousness and is believed to be the counterpart of the emergency response (Benson, 1983). According to the author “the regular elicitation of the relaxation response reduces the peripheral responsiveness to secreted noradrenaline: more noradrenaline is required to bring about increases in blood pressure and heart rate” (Benson, 1983: p. 284). The relaxation response is a contrasting state to “fight and flight” known as “rest and digest”. Some important elements to elicit the relaxation response can be the repetition of a sound, phrase, word or thought.

### 3. Study

X is a state network of technical schools in the city that was created in 1997 and stands as the only state technical school network in the country. Currently, the network serves about 300,000 students per year in more than 130 educational institutions.
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In X, students have enormous difficulty with mathematics and abstract thinking. Most of them come from Social Classes C and D (IBGE, 2017; 2018). Students suffer from low self-esteem, lack of confidence, and school-related anxiety. According to OECD (2013), in Brazil, 49% of students reported very nervous doing mathematics problems. In addition, 81% of students reported that even if they are well prepared for a test, they feel very anxious, and 56% related that they get very anxious when they study mathematics (OECD, 2017).

In view of this scenario, there is an urgent necessity to perform interventions aiming to change the reality of students who suffer from MA and to implement strategies in order to develop MR and improve mathematics learning. According to Lyons and Beilock (2012), “educational interventions emphasizing control of negative emotional responses to math stimuli (rather than merely additional math training) will be most effective in revealing a population of mathematically competent individuals, who might otherwise go undiscovered” (Lyons & Beilock, 2012: p. 2102).

In this study, we performed an intervention using the GZM and related tools to promote MR in a public high school in Brazil, where an initial survey had revealed that from 2018 to 2019, 63% of the chosen class had decreased their score in mathematics, 6% remained the same and only 31% improved their performance. Teachers may have many explanations for this such as boredom, distraction or laziness; however, to us, as anxiety-aware teachers, these results were taken as a strong indicator of the need to test for high Mathematics Anxiety in the class. The students were in their second year of high school, studying an ICT-based course in Statistics. In this paper, statistics is considered by both teacher and students to be a subset of mathematics.

When teachers are conducting classroom research related to improving MA, it is informative to gather qualitative data, such as field notes, students’ writing, interviews; it can also be helpful for teachers to find ways of assessing and measuring MA (Bai et al., 2009). Most measures for assessing MA involve extensive questionnaires and rating scales and are predominantly used in research with adolescents and adults. The first measure of MA using rating scales was made by Dreger and Aiken (1957); other examples include the Mathematics Anxiety Research Scale or MARS (Richardson & Suinn, 1972) and the Fennema-Sherman Mathematics Attitude Scales (MAS) (Fennema & Sherman, 1976).

Bai et al. (2009) emphasize the importance of measuring Mathematics Anxiety not only to understand its nature but also for planning future interventions and instructional delivery. In their work on the measurement of mathematics anxie-
ty, the authors compiled and described several unidimensional psychometric scales, and presented the Betz scale (1978) which is intended to measure Mathematics Anxiety from positive and negative affect. According to Watson (1988), positive affect is a degree of pleasurable interaction with the environment (sense of ease, lack of discomfort, and the absence of fear); and negative affect reflects an adverse response to the environment (feelings of discomfort, restlessness, uneasiness, and confusion). As previous research suggested that some MAS items needed to be revised for wording (Pajares & Urdan, 1996), some improvements were proposed by Bai et al. (2009) and these changes were made to establish the MAS-R scale. Many teachers conducting classroom-based research used the MAS-R scale (see for example: Baker, 2021; Nyama, 2021; Gokhool et al., 2022). It is important to note here that self-report scales only offer a partial picture and the qualitative data collected in this study adds enormously to our understanding of maths anxiety and developing mathematical resilience.

The field interventions were conducted by the first author who is a researcher and high school teacher, with the supervision and discussion of the second author.

4. Methodology

In this paper, a critical paradigm is adopted (Bohman et al., 2021); we seek to challenge conventional pedagogy and to make a change to the experience of learners with adverse prior experiences of mathematics. The work is rooted in the mismatch between what teachers of mathematics know and do, and what anxious mathematical learners need. Our paper is informed by the Mathematical Resilience Framework previously described and notions of trauma-informed practice (Homes & Grandison, 2021) which we adapt to anxiety-informed practice in the context of teaching mathematics.

The methodology used in this study is named PAR (Participatory Action Research). PAR is a version of action research (Savin-Baden & Wimpenny, 2007; Young, 2013) in which participants collaborate with the action researcher (Berggold & Thomas, 2012). Participants in PAR work to generate new knowledge to address a specific issue or problem (Young, 2013). Participatory Action Research is defined as a process which develops practical knowledge while pursuing worthwhile human purposes. It combines theory and practice, action and reflection with the participation of stakeholders who seek practical solutions to concerns and issues, allowing the flourishing of those stakeholders and their communities as a result of the research process (Jacobs, 2016). The qualitative data were analyzed using deductive latent thematic analysis (Clarke & Braun, 2017) because we were working with subjective experiences and using a pre-existing framework: the Mathematical Resilience Framework.

The specific objectives of the study were to:

1) Assess the level of Mathematics Anxiety of students who had previously regressed in mathematics, trying to identify those students who need more attention and follow-up;
2) Implement tools and strategies to bring psychological safety, collaboration, relatedness, growth mindset and autonomous motivation towards learning mathematics;

3) Evaluate qualitative and quantitative data BEFORE and AFTER the intervention;

4) Discuss the results with the student participants, giving students experience of statistics in the real world.

The intervention was divided into four cyclic phases (reflecting, planning, acting and observing, evaluating) as depicted in Figure 3 and discussed below. The duration of the intervention was 16 weeks. Field notes were collected during the intervention to record the perceived and stated reactions of the students. According to Sammons et al. (2014), qualitative field notes provide one view of effective teaching practice, recording the climate for learning as the participants perceive it. In addition, some notes were written by students at the end of the questionnaires.

The sample in our study was small (N = 32). For this purpose, a paper-based MAS-R questionnaire was applied BEFORE and AFTER the intervention (see Figure 4). Asterisked items are reverse scored (items 1, 3, 5, 8, 10, 12 and 13), so that the opposite is true (i.e., 1 = Strongly agree and 5 = Strongly disagree). Thus, a high score on a question indicates higher anxiety or negativity towards mathematics.

4.1. Planning Phase

In the Planning phase, the level of anxiety was measured in one class using the
revised MAS-R scale (Bai et al., 2009). The BEFORE-measure revealed for 19 students a total Mathematics Anxiety score equal to or greater than 50 points which means that 59% of the class had a high level of Mathematics Anxiety and negativity. Another 28% of the students showed totals from 45 to 49 which is still high. Only 4 students (13%) scored less than 45, that is, 28 students were suffering from high or very high Mathematics Anxiety. When analyzing the responses to positive questions, for example, two students (MV and ESP) each presented high levels of negativity in positive questions such as: “I find math interesting”; “Math relates to my life”; “I would like to take more math classes”; “Math is one of my favorite subjects”, and “I enjoy learning mathematics”.

After assessing the level of anxiety of students using the BEFORE-measure, a class was allocated so that students could bring their questions and solve some mathematics problems. It was interesting that students felt free to bring their questions from other fields that involve mathematics such as physics and chemistry.

Based on the results of this BEFORE measure, plans were made to introduce the Growth Zone Model to the students in the second week, starting by using an exercise that was chosen to be relevant to the students. This exercise consisted of listing on the whiteboard students’ responses to the question “What does Statistics mean to you?” Then the students were introduced to the GZM model described in section 1. To describe it, handouts describing the tools were given out (Johnston-Wilder & Moreton, 2018). Learners were invited to describe how they felt in each zone represented by scenarios: relaxing on the beach (green); trying something new (amber), and a tiger coming into the room (red). The idea was to enable the students to identify which zone they were in, red, amber or green, at
any moment. Strategies were then developed to deal with different student experiences; a set of activities and exercises was established to support the learners in different zones outside of class time. In this intervention, students were taught how to use long out-breaths to trigger the relaxation response. Sometimes, “a permission to have a break” was used.

4.2. Acting and Observing Phase

The GZM was introduced to the class in the second week (see Figure 5) and the students were asked to write their names on a small rectangle of colored paper and put the rectangles on the area they felt closer to at that time. Students were very enthusiastic with the act of positioning themselves on board. Some put themselves in the border between the GROWTH-ANXIETY zones, the majority positioned themselves in the ANXIETY zone, and none in the COMFORT zone. This phase was very important as a step to build communication as the GZM schema was available to students all the time in the classroom. Some of them recalled that they were in the red zone from time to time.

The hand model of the brain was easy to explain; the students used it whenever they felt “stuck” or in a difficult situation. One student (MCR) used to lift her fingers in many situations in class. Her problems related to not understanding what was asked and many times the relaxation response was used with her. Also, it was sometimes difficult for the teacher to attend to so many students and so students were instructed to use the relaxation response such as breathing using a longer out-breath or going to have some water (making a pause). This created time for the teacher to attend to students individually.

The new methods to deal with their mathematics anxiety had good impact on students as a whole and permitted the development of empathy in the classroom. Also, in class, understanding of topics was prioritized, and problems proposed by all students were explored (trying to leave no one behind). The pressure to “cover the syllabus”, even if it involves knowingly leaving learners behind (Alderton & Gifford, 2018), was resisted.

When the BEFORE-measurement was completed, mathematics situations and exercises from daily life were taught using Excel. For example, in one activity, students were presented with a research table with one product brand that

Figure 5. Application of GZM in class (February 2019; Photos taken by the authors).
people use the most (toothpaste) and were asked to study the absolute and relative frequencies. For the data to become meaningful, they were asked to calculate the relative frequency \( (Fr) \) of the data defined as the ratio between the absolute frequency \( (Fa) \) and the total number of observations \( (n) \). They used the formula:

\[
Fr = \frac{Fa}{n}
\]

As the course is ICT-based, they built Table 1 with Excel and then they built a pie-chart to represent the results.

Discussion was promoted with students and teacher (participatory) to understand the new dynamics of the class. Before the intervention, most students were afraid of speaking their responses aloud. Since the students were very anxious in mathematics, as demonstrated by the BEFORE-measure, these reactions were newly interpreted by the anxiety-informed teacher as self-defense responses. The teacher also observed that students really did not want to think for themselves; they asked to be given answers. In the beginning, it was hard for them because answers were not given, and they had to think for themselves to find solutions to the proposed problems. Students had difficulty solving the problems and gave up. However, after four classes, the class members were observed trying to find the solution by themselves, forming informal discussion groups to find the best solutions; this revealed increased resilience in the Growth Zone because their previous response to activities had been to wait passively for the teacher. After six classes, students were more participative and they felt safer asking questions, as demonstrated by observations and comments recorded in the field notes.

In addition, some activities in Statistics were given, with opportunities for the students to talk about how they felt about them. These activities were based on Graham (2006). The activities were selected using the criteria of the Growth Zone, in particular those activities that were perceived as valuable and amenable for collaborative group work.

4.3. Evaluating Phase

In the Evaluating phase (sixteenth week), the MAS-R scale (Bai et al., 2009) was applied again. Data were collected after the application of the paper-based questionnaire. Therefore, it became possible to discuss the statistical BEFORE and AFTER results with the students. This discussion was a key activity to reinforce

<table>
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<th>Percentage (%)</th>
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<tr>
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and give feedback to the next planning phase, where the cycle may start again. Also, field notes were collected in this Evaluating phase.

5. Results

A sample of 32 high-school students from X took part in the research project. The sample consisted of 9 males and 23 females aged from 15 to 17; four of these collaborated with the teacher with the Statistical Analysis. The research protocol was approved by the Ethics Committee from X. The quantitative results are presented first, then the qualitative.

5.1. Quantitative Results

Only 4 students (13%) scored less than 45 on the BEFORE measure. The AFTER-measure indicated that 7 students (22%) scored less than 45. Thus, a few more students showed a low level of Mathematics Anxiety after the intervention.

When comparing BEFORE and AFTER measures, evidence of Mathematics Anxiety was observed in both positive and negative questions. There was some improvement in positive questions: 1. I find math interesting; 10. I would like to take more math classes; and 13. I enjoy learning mathematics. These improvements indicated increased motivation within mathematics. Also, in negative questions: 9. Math makes me feel nervous, and 14. Math makes me feel confused, there was a decrease, indicating a small reduction in anxiety level (less nervous and less confused).

These small-scale quantitative results indicate the potential for this kind of intervention. The fact that the sample is small makes it difficult to obtain p-values lower than 0.05 even if the intervention is effective. However, since this sample gave some indications that the intervention made a difference, it is recommended to carry out a study with a larger and more diversified sample and more adequate conditions to assess the impact of the tools. In this paper, the focus is on the impact on the teacher as Action Researcher.

5.2. Qualitative Results

The qualitative data were analyzed using deductive latent thematic analysis because we were working with subjective experiences and using a pre-existing framework: the Mathematical Resilience Framework. Thus, in our latent thematic analysis, we looked for evidence of anxiety, psychological safety, growth mindset motivation and value, struggle, and relatedness.

Mathematics Anxiety: There was evidence of anxiety in the qualitative data, and also, at times, evidence of underlying experience that could be described as causal, such as feeling lost.

GAS described being completely lost in mathematics prior to the intervention.

“I was good at math before; however I have always had difficulty with simple calculus in a way that I always had mistakes in questions for not knowing how to make a sum or a subtraction. From the 6th grade on I see myself
completely lost, and I had to study day and night to keep the average. Since then, I have difficulty with sine rules and geometry.”

JR described the state of anxiety very vividly:

“I always had a good relationship with math, and I had some teachers who made jokes in class until my 9th grade. When I began high school, I felt great difficulty as I was used to another methodology. Now, the same subject that I used to like because I thought it was easy and funny makes me terrified and in a crisis of anxiety, panicking and panting. I have tried to improve but it has been difficult.” He also declared that: “I have anxiety crisis”.

VMS described feeling: “lost and confused in math classes” and that “I always do math tests as fast as I can to get rid of it”.

Student WN revealed that he was very nervous when facing numbers. Student ESP said that: “I get so nervous in math tests that I always forget to sign my name on the test”.

Students MV and JR declared that they: “get lost in math explanations because it is too much information”.

Student AAM declared that: “when I arrived to attend classes I felt lost”. These students had been expected to study mathematics in their previous year without the teacher having been anxiety-aware, and for many, their mathematics had regressed rather than progressed. Given the level of MA revealed in this study, it seems very important that their teacher be anxiety-aware.

Psychological Safety: After a few classes, students felt safer compared to the initial classes: they asked more questions; they sometimes asked for a certain topic to be explained again or an explanation to be repeated. They also expressed positive comments about the rhythm of the class. As a consequence of the safer environment established, for example, student LS said: “I feel myself capable of doing these classes, however I am not capable in normal math classes”.

One interesting observation was made during classes with student MCR, who always asked the other students for help in activities and exercises during class. She used to behave like that on many occasions, even if the activity was intended to be very simple. During the intervention, the teacher tried to tell her not to ask for the help of her peers immediately and to try to think by herself, saying “I’ve done this path and now it is important that you do it by yourself”. It was very interesting that all class members tried to convince her to do the same, and she did not feel discriminated against or isolated for this; she felt psychologically safe (from the point of view of the teacher, the student felt more relaxed to talk about her difficulties during the activity). She was encouraged to think for herself, and she succeeded in doing the exercise.

Growth mindset: In their prior experience learning mathematics, students demonstrated a fixed mindset. Student BS said that: “I was not born to deal with numbers”. Student MCR revealed her reaction by saying: “… before I thought
the subject of math was not made for me”. Student WN revealed that he was very nervous when facing numbers and that: “in general, I do not know what to do with them”.

After the study, some students experienced growth in their study of mathematics. Student MCR said “I feel less dumb in your class”. Student AAM said that “… today I have less difficulty in the subject”. Student BR declared that: “I am still confused however there is some progress”. Student AOR declared that: “classes helped me to progress in math”. Student CSO declared that: “classes helped me to improve my performance in math”. Student LS said that: “I feel myself capable of doing these classes, however I am not capable in normal math classes”. Student EV said: “I feel relieved for being able to solve”.

The qualitative data show the changes the students perceived in the extent to which they had developed a growth mindset mathematics despite starting the class with high Mathematics Anxiety and fixed mindset.

**Motivation and value:** We also identified data which we categorized as related to motivation to study mathematics and perceived value. Before the intervention, most students did not find mathematics motivating or see the value in studying it. Student GF declared that: “I do not see the reason for studying math”. Meanwhile, student SB told that: “I hope I will never have to study math again”.

Another student, EV, said that: “when I had my first math class, I liked it and it was easy to learn. However, since the 7th grade, I began to find it very difficult and now I don’t like mathematics.”

A few students had previously experienced mathematics as motivating. For example, JM positioned himself in the amber area of GZM because: “I find math intriguing”.

During and after the intervention, PHD declared that “time passes so quickly here”, indicating substantial motivation. Student GLGT said that: “after classes, I got more interested in the subject”.

Student MCR said that: “I like your classes, sometimes I feel it a bit complicated, but I find it interesting”. Student CR declared that: “I feel classes are more interesting and more useful”.

An indication of change in motivation also arose in the quantitative data. In answer to the question, I find math interesting, the proportion of students disagreeing dropped from 38% BEFORE to 9% AFTER. These data combined indicate changes in how students perceived value, interest and usefulness in mathematics, having started from a low level of motivation.

**Relatedness and Struggle:** Relatedness was built into the intervention design in that students were encouraged to collaborate in groups and communicate if they experienced being in their red zone. After four classes of intervention, the teacher observed the learners forming informal discussion groups to find solutions to tasks they were struggling with, having previously waited passively for the teacher. After six classes, students were more participative and asked more questions.
6. Discussion

We use the Mathematical Resilience Framework to structure our discussion. We found resonance with the Scottish model for trauma-informed practice (Homes & Grandinson, 2021) and we adopt the use of the term anxiety-informed mathematics teaching in this discussion. That means that we are interested in mathematics anxiety related to students learning mathematics and, for teachers, the need for realization, recognition, and response to mathematics anxiety and avoidance of (and resistance to) re-harm.

Hence this section has six parts: in the first part, we consider the evidence available to the teacher about mathematics anxiety, which relates to adverse prior experiences. In the second part, we discuss psychological safety. In part three, we discuss the growth mindset element of the MR framework. In part four, we discuss autonomous motivation and relate it to value in the MR model. In part five, we consider the importance of relatedness and relate it to group work. In part six we discuss struggle and collaboration from the MR model. In Figure 6, we present an enhanced Growth Zone Model.

6.1. Mathematics Anxiety

Many teachers of mathematics are not aware that a significant proportion of the students in front of them are suffering from mathematics anxiety, including Author 1 at the start of the study. Teachers do not necessarily need to know the specific adversities previously faced by their own students as the common adversities are well-explored in the literature and can be built into training. In this study, students initially were experienced by the teacher-researcher as: passive, apathetic, unmotivated, avoidant, and unwilling to try. From the perspective of
an anxiety-informed teacher, the students are now perceived as suffering from mathematics anxiety. They previously experienced feeling lost, confused, helpless, and powerless, being left behind, being unable to ask questions and find appropriate help. They felt unsafe. This echoes findings in the literature (Cousins et al., 2019; Carey et al., 2016) and lays the foundation for teachers of mathematics to realize the extent of MA, learn to recognize avoidance as a symptom of MA, learn the importance of exploring how to respond effectively and how to resist and avoid causing any further harm.

6.2. Psychological Safety

The teacher-researcher adopted the GZM, the HMB and the RR tools explicitly to promote understanding of the role of psychological safety in learning mathematics in her class. Psychological safety is relatively well-established in work environments (Edmondson, 1999) but not yet in education and is a vital and effective concept in avoiding further harm to students who have developed mathematics anxiety. The tools have been previously shown to promote understanding and communication (see for example: Baker, 2021; Johnston-Wilder et al., 2021) when learners are feeling unsafe in mathematics classes and this study adds to this evidence.

6.3. Growth Mindset and Learning from Mistakes

The teacher-researcher gave the students tools to change their state from panic or comfort to growth. Also, the knowledge of these tools helped students to develop confidence with activities in their Growth zone. Students became aware that they maybe could not do an activity yet, but they would be able to do it later. The work of Dweck (2006) is foundational in terms of growth and fixed mindsets. From the teacher-researcher point of view, the GZM was found to be empowering, permitting the students to visualize their anxiety in terms of zones (challenge or threat) and offering a language for communicating feelings of threat. Johnston-Wilder et al. (2020) found that “the GZM empowered students by giving them both a vocabulary for, and an understanding of, the emotions they were experiencing while learning, without judgment” (p. 1340). Students felt encouraged to ask questions, to take the risk of showing that they didn’t know the answer, able to ask questions, an essential part of learning. Whether to ask questions, or whether to participate, these were not options previously experienced by the students as they were anxious (uncomfortable, very lost, confused, or having an anxiety crisis). The intervention empowered students, helping them to understand ‘I am not stupid; I am panicking’; panic is temporary, unlike stupidity, and can be changed. "Even me, I can do it!!!" (Field notes).

6.4. Value and Autonomous Motivation

Having created a safer environment, and introduced the growth mindset, Author 1 also presented tasks that were perceived as more relevant to the students.
The tasks were more aligned with the students’ goals and values, and this created autonomous motivation (Ryan & Deci, 2017). This is aligned with the principles of andragogy which brings attention to the importance of value, motivation, and relevance to adult learners (Knowles, 1984). Favoring students’ autonomy may include communicating expectations, establishing goals, introducing strategies and procedures, providing directions, and giving constructive feedback.

Finally, some interesting quantitative data were obtained. The percentage disagreement with the question “I find mathematics interesting” went down from 38% to 9%. These results indicate increases in motivation.

6.5. Relatedness and Peers

A different relationship was built between learners and the teacher. Using the tools (GZM, hand model of the brain, and relaxation response) improved communication about the hidden emotional barriers to learning and allowed students to spend more time thinking about mathematics as they had learned ways to address perceived threats and their ‘fight and flight’ response; the students learned to recover and go back to mathematics when they felt safer. The GZM tool in particular allowed learners to understand that all effort is constructive, mistakes can be learned from and that they were not alone, as other learners in the class shared similar experiences. Also, it brought respect and honesty to both student-student and student-teacher relationships; the students felt listened to and the teacher was able to resist causing further harm. Students experienced opportunities to communicate, participate, ask questions, and create and work in groups to solve problems; in other words, they experienced relatedness, a basic psychological need (Ryan & Deci, 2017), without which learning can become unsafe.

Behaviour also significantly improved. Before the intervention, any request by the teacher for silence had to be repeated during the explanation of a topic or a difficult issue. Another positive improvement was that some absence was very common at the beginning of classes; as the intervention progressed, attendance improved considerably. It was not surprising that some anxiety continued to be present in this group after such a brief intervention, but improvements were noted.

6.6. Struggle and Collaboration

Effective struggle with mathematics challenges involves both persistence and perseverance (Williams, 2014). Persistence involves keeping trying, which arises out of a growth mindset and autonomous motivation. Perseverance involves recruiting more support as needed to tackle challenges. The support can come from various sources including peers, the course textbook, ICT and the teacher. We have illustrated that transforming pedagogy from experiences of isolation to ones of collaboration, and developing strategies for working together with peers, can be an effective part of addressing mathematics anxiety. Being included as
part of a supportive learning community, collaborating on something of perceived value, meets basic psychological needs of relatedness, competence and autonomous motivation (Ryan & Deci, 2017).

7. Conclusion

Our main goals were to assess the level of MA in a class of students that had previously regressed in mathematics; to explore some strategies to respond to that anxiety, to resist re-harm, and to give the students an opportunity to engage with real-life statistics.

Pre-intervention, the students avoided and procrastinated, being passive and non-responsive in class. The teacher-researcher had read about Mathematics Anxiety and realized that this was a possible explanation.

A high level of Mathematics Anxiety was detected in the initial survey. In order to create an environment of psychological safety, the teacher introduced the tools from the Mathematical Resilience Framework: the Growth Zone Model, the hand model of the brain and the relaxation response. The teacher introduced collaborative activities both between students and with the teacher to increase relatedness. From the point of view of the teacher-researcher, it was easier to be empathetic and respond appropriately to students disclosing they were in the red zone than when they previously did not take part or were disruptive. Students found the activities valuable. Also, sharing with students the results of the intervention played an essential role as they could see statistics being used in a real problem that involved them.

The intervention developed an environment of reduced anxiety, and increased empathy and motivation in class. It led to more positive attitudes towards mathematics. The data presented in this study indicate that this kind of intervention can make a difference in the lives of students of mathematics in public schools in Brazil. The intervention enabled the creation of a safe, collaborative, empowering and autonomy-promoting learning space for the students involved. The sample was not big enough for robust statistical evidence of impact, and, in future, large-scale studies are desirable. However, the intervention was perceived to be successful based on qualitative analysis of field notes showing learners’ reports of anxiety and suffering towards mathematics before the intervention and reduced anxiety and suffering after. The intervention also enabled the development of activities using tools to reinforce safety, relatedness and motivation towards mathematics classes; this intervention promoted Mathematical Resilience and helped the students to overcome adverse prior experiences with learning mathematics.

Also, the intervention brought a new and different framework to the teacher, enabling the teacher to become an anxiety-informed practitioner. The teacher will continue to develop her practice, explicitly creating a safer environment, using the GZM, hand model of brain, and relaxation response so that students can communicate and regulate their feelings about mathematics and feel emplo-
wered to tackle challenging questions, encouraging collaboration within groups in the class, and using PAR to bring increased communication and mutual respect.

Every mathematics teacher in Brazil, and around the world, is likely to work with a significant proportion of students with mathematics anxiety in each class. In Brazil, the proportion is estimated at 49% (OECD, 2013). This action research adds evidence to the growing number of teacher-researchers exploring the MR tools and their impact on teacher practice (see for example: Baker, 2021; Nyama, 2021; Gokhool et al., 2022). We have shown that to build an anxiety-informed mathematics teaching profession, “small changes can make a big impact” (Homes & Grandinson, 2021).

There are implications from this study for teacher training and for National Policy. The tools that were used reinforced psychological safety and autonomous motivation in learning mathematics and the development of Mathematical Resilience, thus helping students to overcome adverse prior experiences and current difficulties in mathematics classes. We argue that these tools should be embedded in teacher practice to help achieve, in the context of mathematics, the sustainable development goals SDG 4, good health and well-being, and SDG 5, quality education, defined by the UN (2015) in the 2030 Agenda for the sustainable development of nations: ensuring inclusive and equitable quality mathematics education and promoting lifelong learning opportunities for all stakeholders.

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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