# Sequential Coordination and Input Price Leadership in Bilateral Oligopoly*

Robin Naylor[†] Christian Soegaard[‡]

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**Abstract**

We analyse the leader/follower incentives of upstream suppliers in a bilateral duopoly setting with decentralised bargaining over input prices, showing that upstream suppliers prefer to set prices sequentially rather than simultaneously. We characterise equilibria involving sequential coordination demonstrating that there is a first mover advantage to the upstream supplier with relatively little bargaining power over input price and a second mover advantage to the supplier with relatively greater bargaining power.

*Keywords:* Sequential coordination, bilateral oligopoly, bargaining, first and second mover advantages.

*JEL classification:* D21, D43, J41, L13.

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[†]Department of Economics, University of Warwick, Coventry CV4 7AL, UK. Email: Robin.Naylor@warwick.ac.uk, corresponding author.

[‡]Department of Economics, University of Warwick, Coventry CV4 7AL, UK. Email: C.Soegaard@warwick.ac.uk.

# 1 Introduction

An extensive literature addresses the related questions of whether oligopoly firms will have incentives to act as a leader or as a follower in *horizontal* product markets and why equilibria associated with sequential behaviour might dominate outcomes resulting from the simultaneous behaviour such as Bertrand and Cournot equilibria.[1] We analyse the leader/follower incentives of upstream suppliers in *vertical* markets characterised by bilateral oligopoly in which each upstream supplier bargains with a downstream firm over the input price.[2] We show that all equilibrium outcomes involve situations in which upstream suppliers unambiguously prefer to set input prices sequentially rather than acting simultaneously. We demonstrate how the properties of sequential coordination depend on the distribution of input price bargaining power within and across bargaining pairs, identifying two distinct types of such equilibria. First, there are assured equilibria in which the two upstream suppliers share a common interest in who leads and who follows and, second, there are conflicted equilibria in which the suppliers have conflicting interests over the sequencing. The assured equilibria arise in the presence of sufficient asymmetry in bargaining powers across the bilateral pairs.

Our paper is closely related to Dobson (1994) and Corneo (1995), both of which consider unions as the upstream supplier of labour inputs. In Dobson (1994), a centralised union chooses whether to bargain sequentially or simultaneously with two independent downstream firms. With asymmetry in the bargaining powers of the firms, the union will prefer to bargain first with the *weaker* downstream firm. In contrast, we show that when bargaining is decentralised, equilibria emerge in which the input price leader will be the supplier which faces the *stronger* downstream firm. Corneo (1995) also analyses decentralised bargaining but assumes (i) that the relative bargaining power of upstream over downstream agents is symmetric across the bargaining pairs and (ii) that preferences over sequencing are based on the joint payoffs of upstream and downstream agents. He finds that there is always a second mover advantage and hence that the order of moves will be contested. In contrast, we find that if there is sufficient asymmetry in the bargaining powers of the respective upstream firms, equilibria emerge in which upstream suppliers share a common interest in who leads and who follows.

# 2 The model

We characterise the interactions between two upstream suppliers and their respective downstream firms as a 3-stage game. We allow for asymmetry across the two bargaining pairs in the relative bargaining powers of the bargaining partners. Our objective is to explore how the upstream suppliers' preferences to lead or to follow vary with the distribution of bargaining power within and across bargaining pairs. In Stage 1, upstream suppliers simultaneously and independently select the time at which to negotiate with their respective downstream firms: either 'early' at $T_0$ or 'late' at $T_1$, as in Corneo (1995). Hence, there are two types of outcomes. In one, the two suppliers both choose the same time period, either $T_0$ or $T_1$, in which case at the subsequent Stage 2 input prices are determined by simultaneous and independent Nash bargaining within each bilateral pair.

[1] See, for example, Dixit (1979), Albaek (1990) and Dowrick (1986).

[2] Analysis of equilibrium leader/follower choices in vertical markets is relatively under-researched given that, as Belleflamme and Peitz (2015) observe, inputs are often provided by upstream firms with significant input market power.

In the second type of outcome of the Stage 1 game, one of the two upstream suppliers chooses $T_0$ and the other $T_1$, with input price bargaining in Stage 2 characterised by sequential bargaining and modelled as a game with two substages, $a$ and $b$. In stage $2a$, the upstream supplier acting as the *leader* bargains over the input price with its downstream firm, applying the standard Nash bargaining solution, anticipating the best-response function of the rival follower. In the subsequent Stage $2b$, the upstream *follower* bargains with its downstream firm over their input price, taking as given the input price bargained within the rival bargaining pair in Stage $2a$.

Regardless of whether Stage 2 is characterised by a simultaneous or by a sequential mode of bargaining, in the final stage of the game, Stage 3, the two downstream firms compete à la Cournot in the product market. For each of the two bargaining modes, we solve for Stages 2 and 3 by backward induction, generating payoff values to the upstream suppliers for each mode and these are then compared in order to establish the nature of the final equilibrium outcome. The key questions we address concern the conditions under which bargaining will be simultaneous or sequential and, in the case of the latter, the conditions determining who leads and who follows in the setting of input prices. First, we consider Stage 3.

(i) *Stage 3: Cournot competition in the final product market*
We assume that the final product is a non-differentiated good for which demand is linear, $p = a - b\,(q_1 + q_2)$, where $p$ is the market price and $q_1$ and $q_2$ represent, respectively, outputs of downstream firms $D_1$ and $D_2$. The $i$th downstream firm chooses output so as to maximise profits:

$$\max_{q_i} \pi_i^D = (p - w_i)\, q_i = [a - b(q_i + q_j) - w_i]\, q_i, \;\; i = 1, 2, \;\; j = 1, 2, \;\; j \neq i, \tag{1}$$

where $w_i$ represents the input price paid by $D_i$ to its upstream supplier, $U_i$.[3] The solution to the profit-maximisation problem yields the following output in Cournot equilibrium:

$$q_i = \frac{(a + w_j - 2w_i)}{3b}. \tag{2}$$

The corresponding expressions for firm $j$ can be found by interchanging $i$ and $j$. We now solve for Stage 2 outcomes, first considering the case of sequential bargaining.

(ii) *Stage 2 under sequential bargaining: Stage (2b) The upstream follower*
The profits of upstream supplier $U_i$ can be written as:

$$\pi_i^U = (w_i - \overline{w})\, q_i, \tag{3}$$

where $\overline{w}$ represents a reservation price (assumed common across upstream suppliers). Assume $U_j$ and $D_j$ constitute the follower pair. The maximand is:

$$B_j^F = \left[\pi_j^U\right]^{\beta_j} \left[\pi_j^D\right]^{1-\beta_j} = \left[(w_j - \overline{w})\, q_j\right]^{\beta_j} \left[(p - w_j)\, q_j\right]^{1-\beta_j}, \tag{4}$$

where the second equality uses the expressions for the profits of the upstream supplier in (3) and those of the downstream firm in (1). $\beta_j \in (0; 1]$ and $1 - \beta_j$ are the upstream and downstream agents' respective bargaining powers. The assumption $\beta_j > 0$ ensures that

[3] $w_i$ can be interpreted as the wage if the upstream supplier is a labour union or the wholesale price in the case of an upstream manufacturer.

the upstream suppliers have at least some bargaining power. From substitution of Stage 3 output values in (2), we obtain:

$$\max_{w_j} B_j^F = \frac{1}{3^{2-\beta_j} b} \left[w_j - \overline{w}\right]^{\beta_j} \left[a + w_i - 2w_j\right]^{2-\beta_j} . \tag{5}$$

Solving the maximisation problem, it follows that the input price bargained by the follower pair is given by:

$$w_j = \overline{w} + \frac{(a + w_i)\beta_j - 2\beta_j \overline{w}}{4}, \tag{6}$$

which represents the follower's best-response function to the input price, $w_i$, bargained by the upstream leader. We notice that from the point of view of the follower, the input price is a strategic complement, $\frac{dw_j}{dw_i} > 0$ for $\beta_j > 0$.

(iii) *Stage 2 under sequential bargaining: (2a) The upstream leader*
Input price bargaining between the upstream leader and its downstream bargaining partner solves:

$$\max_{w_i} B_i^L = \frac{1}{3^{2-\beta_i} b} \left[w_i - \overline{w}\right]^{\beta_i} \left[a + w_j - 2w_i\right]^{2-\beta_i} . \tag{7}$$

subject to the follower's best-response function in (6). The superscript $L$ denotes the input price leader and $\beta_i \in (0; 1]$ and $1 - \beta_i$ the bargaining powers of $U_i$ and $D_i$, respectively. Solving (7) using (6), we obtain an expression for the equilibrium value of the leader's input price:

$$w_i^L = \overline{w} + \frac{(a - \overline{w})\beta_i(4 + \beta_j)}{2(8 - \beta_j)}. \tag{8}$$

Now substituting (8) into (6), we get the equilibrium value of the follower's input price:

$$w_j^F = \overline{w} + \frac{(a - \overline{w}) \left[4(4 + \beta_i) - (2 - \beta_i)\beta_j\right] \beta_j}{8(8 - \beta_j)}. \tag{9}$$

Substituting bargained input prices into the expressions for the upstream suppliers' profits, we obtain the following equilibrium profits:

$$\pi_i^{UL} = \frac{(a - \overline{w})^2 \beta_i (2 - \beta_i)(4 + \beta_j)^2}{48b(8 - \beta_j)}; \tag{10}$$

$$\pi_j^{UF} = \frac{(a - \overline{w})^2 \beta_j (2 - \beta_j) \left[16 - \beta_j(2 - \beta_i) + 4\beta_i\right]^2}{96b(8 - \beta_j)^2}. \tag{11}$$

(iv) *Stage 2 under simultaneous bargaining*
In the event that in Stage 1 the upstream suppliers both choose $T_0$ (or $T_1$), we assume that they engage simultaneously in input price bargaining. The maximands of, respectively, $U_i$ and $U_j$, thus become:

$$B_i^B = \frac{1}{3^{2-\beta_i} b} \left[w_i - \overline{w}\right]^{\beta_i} \left[a + w_j - 2w_i\right]^{2-\beta_i} \text{ and } B_j^B = \frac{1}{3^{2-\beta_j} b} \left[w_j - \overline{w}\right]^{\beta_j} \left[a + w_i - 2w_j\right]^{2-\beta_j} . \tag{12}$$

The solution to this maximisation problem involves the following symmetric input prices:

$$w_i^B = \overline{w} + \frac{\beta_i\left[(4+\beta_j)(a-\overline{w})\right]}{16-\beta_i\beta_j} \text{ and } w_j^B = \overline{w} + \frac{\beta_j\left[(4+\beta_i)(a-\overline{w})\right]}{16-\beta_i\beta_j}, \tag{13}$$

and the corresponding upstream profits are:

$$\pi_i^{UB} = \frac{2\beta_i(a-\overline{w})^2(2-\beta_i)(4+\beta_j)^2}{3b(16-\beta_i\beta_j)^2} \text{ and } \pi_j^{UB} = \frac{2\beta_j(a-\overline{w})^2(2-\beta_j)(4+\beta_i)^2}{3b(16-\beta_i\beta_j)^2}. \tag{14}$$

(v) *Stage 1: Upstream Leader/Follower preferences*
We now consider the Stage 1 sub-game, comparing the profits of the upstream suppliers across the two modes of bargaining, sequential and simultaneous. Based on the expressions derived for profits for both the sequential and the simultaneous price setting arrangements as shown in (10), (11) and (14), we establish Proposition 1.

**Proposition 1.** *The profits to $U_i$ (and by symmetry $U_j$) from acting as either leader or follower in the sequential-move game are strictly greater than the profits obtained under a simultaneous-move game irrespective of the values of the relative bargaining power within each pair.*

*Proof.* See the Appendix. □

In the Stage 1 sub-game, each of the upstream suppliers selects whether to move in $T_0$ or in $T_1$, making their decisions independently and simultaneously. We illustrate the Stage 1 game in normal form with the suppliers' continuation payoffs as follows:

| $i, j$ | $T_0$ | $T_1$ |
|---|---|---|
| $T_0$ | $\pi_i^{UB}$,$\pi_j^{UB}$ | $\pi_i^{UL}$,$\pi_j^{UF}$ |
| $T_1$ | $\pi_i^{UF}$,$\pi_j^{UL}$ | $\pi_i^{UB}$,$\pi_j^{UB}$ |

Table 1: The Stage 1 game

Since each of the upstream firms prefer to set input prices sequentially rather than simultaneously, there are two pure-strategy Nash equilibria in the Stage 1 subgame – ($T_0$, $T_1$) and ($T_1$, $T_0$).

# 3 Results

We now address the issue of what determines who might lead (bargain in $T_0$) and who might follow (bargain in $T_1$) and whether there are situations in which the two suppliers have compatible preferences over the sequence of moves. From the expressions for profits, (10) and (11), Figure 1 plots the indifference relation $\overline{U}_i$, that is, combinations of $\beta_i$ and $\beta_j$ consistent with $\pi_i^{UL} = \pi_i^{UF}$, and the corresponding function for $\overline{U}_j$, setting parameter values as $a = 2$, $\overline{w} = 1$, and $b = 1$.[4] The two curves intersect each other at $\beta_i = \beta_j = 0.775$.

[4]Results are not qualitatively sensitive to the choice of parameter values.

Combinations of $\beta_i$ and $\beta_j$ that lie to the left of $\overline{U}_i$ (below $\overline{U}_j$) correspond to points for which $U_i$ ($U_j$) has a first mover advantage. There are four cases to consider. First, with relatively high values of $\beta_i$ and low values of $\beta_j$, $U_i$ has an incentive to move second, and $U_j$ first. The second case is the mirror image of this, with high values of $\beta_j$ and low values of $\beta_i$ in which the incentives of respectively $U_i$ and $U_j$ are reversed. In both of these cases the preferences regarding the sequencing of moves are mutually compatible: one firm prefers to lead and the other to follow. These regions are indicated in the figure by the shaded areas. In the third case, the value of the bargaining parameters are both low and both firms have a preference to lead by bargaining in period $T_0$. In contrast, the fourth case is characterised by high values of $\beta_i$ and $\beta_j$, implying that both firms have a second mover advantage. In these last two cases, the two suppliers wish to sequence their actions but they disagree on the order of moves.

The intuition for our result is as follows. The two upstream suppliers will both benefit from sequential price setting due to strategic complementarity: the first mover will set a higher price than would have been the case under simultaneous price-setting, inducing the follower to do the same. There are two countervailing effects at play: moving first allows a firm to set a price (with a corresponding output level in the final stage) which permits the bilateral pair of which it is a member to move closer to the monopoly solution. However, moving second allows the upstream agent to undercut its rival, thus giving its downstream agent a competitive advantage in the final output market. This second effect is more likely to dominate for the firm with higher bargaining power owing to a higher markup over the input price, facilitating profitable undercutting.

# 4 Conclusions

In this paper, we have identified two types of equilibrium outcomes characterised by sequential coordination: (i) assured equilibria in which the two firms' preferences for the sequence of price setting are mutually compatible and (ii) conflicted equilibria characterised by conflicting preferences over the sequence of moves. Which type prevails depends on (a) how input price bargaining power is distributed within bargaining pairs and (b) how it varies across them. When upstream suppliers are very different from each other in terms of their bargaining power over input prices, the equilibrium is more likely to take the form of a Common Interest Game in which suppliers have mutually compatible preferences. In such an equilibrium, the upstream supplier which is the weaker in terms of input price bargaining power will be the one to act as the input price leader. In contrast, the more similar are the bargaining powers of the upstream suppliers the more likely it is that their interaction will have the characteristics of a game of Battle: either both will prefer to lead or both will prefer to follow.

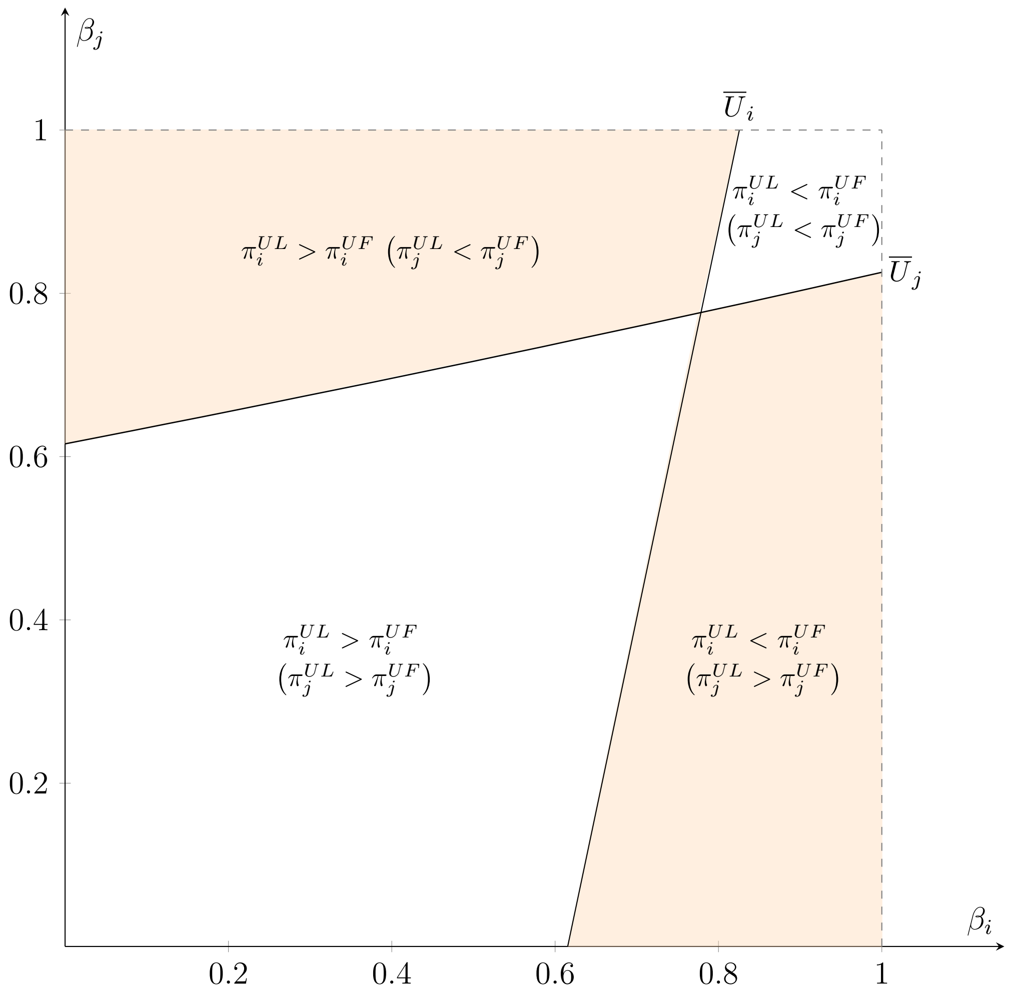


Figure 1: Illustration of iso-profit curves of the upstream suppliers.

# Appendix

*Proof of Proposition 1.* We need to evaluate the difference between $\pi_i^{UL} - \pi_i^B$ and $\pi_i^{UF} - \pi_i^B$. We use the relevant expressions in (10), (11) and (13), and after algebraic rearranging, we obtain:

$$\pi_i^{UL} - \pi_i^{UB} = \frac{\beta_i \beta_j (a - \overline{w})^2 \left[32(1 - \beta_i) + \beta_i^2 \beta_j\right] (2 - \beta_i)(4 + \beta_j)^2}{48b(8 - \beta_j)(16 - \beta_i \beta_j)^2} > 0, \tag{A1}$$

and,

$$\pi_i^{UF} - \pi_i^{UB} =$$

$$\frac{\beta_i \beta_j^2 (a - \overline{w})^2 \left[512 + 128\beta_i - (64 + 8\beta_i + 4\beta_i^2)\beta_j + (2 - \beta_i)\beta_i \beta_j^2\right] (2 - \beta_i)(2 - \beta_j)(4 + \beta_j)}{96b(8 - \beta_j)^2(16 - \beta_i \beta_j)^2} > 0, \tag{A2}$$

The strict inequalities in, respectively, (A1) and (A2) arise due to the parameter restrictions on the $\beta$s. The corresponding values for $U_j$ can be found by interchanging $i$ and $j$. □

# References


Albaek, S. (1990). Stackelberg Leadership as a Natural Solution under Cost Uncertainty. *Journal of Industrial Economics*, 38(3):335–347.

Belleflamme, P. and Peitz, M. (2015). *Industrial Organization: Markets and Strategies*. Cambridge University Press, Cambridge.

Corneo, G. (1995). National wage bargaining in an internationally integrated product market. *European Journal of Political Economy*, 11(3):503–520.

Dixit, A. (1979). A Model of Duopoly Suggesting a Theory of Entry Barriers. *Bell Journal of Economics*, 10(1):20–32.

Dobson, P. W. (1994). Multifirm unions and the incentive to adopt pattern bargaining in oligopoly. *European Economic Review*, 38(1):87–100.

Dowrick, S. (1986). von Stackelberg and Cournot Duopoly: Choosing Roles. *RAND Journal of Economics*, 17(2):251–260.