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Aggregate Skewness and the Business Cycle

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Abstract

We develop a data-rich measure of expected macroeconomic skewness in the US economy. Expected macroeconomic skewness is strongly procyclical, mainly reflects the cyclicality in the skewness of real variables, is highly correlated with the cross-sectional skewness of firm-level employment growth, and is distinct from financial market skewness. Revisions in expected skewness lead to business cycle fluctuations nearly indistinguishable from those induced by the main business cycle shock of Angeletos et al. (2020). This result is robust to controlling for macroeconomic volatility and uncertainty, and alternative macroeconomic shocks. Our findings suggest an important role of higher-order dynamics for business cycle theories.

JEL classification: C22, C38, E32

Keywords: Business cycles, downside risk, skewness
1 Introduction

“FOMC participants (Board members and Reserve Bank presidents) indicated that considerable uncertainty surrounded the outlook for economic growth and that they saw the risks around that outlook as skewed to the downside.”

Monetary Policy Report to Congress, Federal Reserve Board, Feb. 2008 (p.2)

“The outlook for the UK and global economies remains unusually uncertain. [...] The risks are skewed to the downside.”


Assessing macroeconomic risks and analysing their potential impact on the economy is a key focus of economic policy institutions. Such risks are often not balanced around the baseline outlook, and the concept of skewness has been a device for policymakers to communicate their beliefs about the evolution of risks. The academic literature has also used skewness to characterize the asymmetric effects of economic shocks due to, for instance, non-linearities (e.g. Petrosky-Nadeau et al., 2018; Jensen et al., 2020; Mumtaz and Theodoridis, 2020) or particular adverse events (e.g. Barro, 2009; Gourio, 2012; Jordà et al., 2020). Yet it remains unclear to what extent business cycles exhibit large swings in the asymmetry of risk and, most importantly, whether those matter for our understanding of the macroeconomy. In this paper, we develop a new measure of expected macroeconomic skewness for the US economy, reflecting variations in the balance of risks of a large number of macroeconomic and financial indicators. We contrast this measure with alternative measures of macro and micro skewness, and show that revisions in expected skewness lead to sizeable business cycle fluctuations.

A long-standing literature has argued that macroeconomic fluctuations are asymmetric, highlighting that recessions tend to be relatively deeper and more pronounced than expansions (Neftci, 1984; Hamilton, 1989; Sichel, 1993; Morley and Piger, 2012).
More recent work has studied the asymmetry of the conditional distribution of GDP growth, documenting the presence of procyclical GDP growth skewness related to the state of macro-financial conditions (e.g. Adrian et al., 2019; Loria et al., 2020; Delle Monache et al., 2022; Forni et al., 2021). These studies focus on measuring (expected) asymmetry of a single macroeconomic variable, namely GDP growth. While GDP is one of the most representative measures of the business cycle, it is unclear to what extent conditional skewness in GDP growth summarizes unbalanced risk in the broader macroeconomy. We derive a new measure of aggregate expected skewness, which represents a common factor driving the individual conditional skewness series of a large number of macroeconomic and financial indicators. Individual measures of skewness are computed using robust asymmetry measures (Kelley, 1947), where time-varying asymmetry derives from the relative movements of the conditional quantiles (Koenker and Bassett, 1978; Engle and Manganelli, 2004a). This procedure allows us to i) derive summary measures for different subgroups (e.g. prices, labor market indicators and financial variables) and ii) understand which variables contribute most to overall skewness.

The common skewness factor is strongly procyclical and explains only a limited

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1Theoretical and empirical contributions highlighting the role of time-varying skewness include, for example, Colacito et al. (2016), Dew-Becker et al. (2019), Jensen et al. (2020) and Fève et al. (2021) at the macro level, and Busch et al. (2022), Salgado et al. (2019), and Dew-Becker (2022) at the micro level.

2The simple and transparent derivation of our expected skewness factor allows to update it seamlessly and monthly updates can be downloaded from the authors’ websites.
part of the dynamics in expected skewness for most of the indicators. It explains more of the skewness variation of the real economy variables (including income, labor markets, orders and sales, and production indicators) compared to, for example, prices. Moreover, the factor accounts for a non-negligible fraction of the conditional asymmetry in some financial indicators, in particular non-household balance sheet and stock market indicators. Our measure of skewness is far from perfectly correlated with the conditional skewness of GDP growth, meaning that the latter may not always capture economy-wide risks. Aggregate skewness also comoves with the GDP growth skewness that conditions on financial conditions (Adrian et al., 2019). This is despite the fact that our measure captures common movements in conditional asymmetry across many indicators, where the skewness of each variable is not forced to move together with financial conditions and is derived using only information contained in past observations of the variable itself. Our expected skewness factor relates closely to a summary measure of Fed economists’ perception of risks to the economic outlook distilled from verbal information contained in the “Greenbook” documents prepared for Federal Open Market Committee (FOMC) meetings (Aruoba and Drechsel, 2022). In addition, aggregate skewness also highly correlates with the cross-sectional skewness of employment growth computed at the firm level by Salgado et al. (2019). Both findings are remarkable since the data and methodologies used to construct these indicators are extremely different. By contrast, our measure displays limited correlation with indicators of financial market skewness, including stock return skewness, either computed at the market level (Dew-Becker, 2022) or the firm level (Salgado et al., 2019).

Our second contribution relates to investigating the role of our skewness factor in the US business cycle. In recent studies, Salgado et al. (2019) and Forni et al. (2021)
demonstrate that shocks to the cross-sectional skewness of firm-level stock returns and the predictive GDP growth distribution, respectively, can produce contractionary movements in macroeconomic and financial indicators. We show that revisions in expected skewness, which are associated with an increase in perceived downside risk, lead to a substantial contraction in output, consumption, and investment, while leaving prices and TFP broadly unaffected. Remarkably, revisions in expected skewness largely overlap with the main business cycle (MBC) shock identified in Angeletos et al. (2020) and give rise to nearly identical impulse response functions (IRFs). This finding is robust to various sensitivity exercises. Specifically, revisions in expected skewness are distinct from movements in aggregate volatility and uncertainty, and appear unrelated to alternative shocks capturing investors’ risk appetite, productivity, fiscal policy, and monetary policy.

Our empirical results suggest that models striving to explain the main force of macroeconomic fluctuations may benefit from allowing for higher-order dynamics and possibly relate those to economic agents’ varying perception of downside risk. In this regard, within theories that suggest that a single shock is driving the business cycle, this key driver of macroeconomic fluctuations should also account for the bulk of the variation in revisions of perceived macroeconomic risk. Theories allowing for i) confidence or sentiment shocks (Angeletos and La’O, 2013; Angeletos et al., 2018); ii) the possibility of rare disasters (Rietz, 1988; Barro, 2006; Barro and Ursúa, 2008; Gabaix, 2008; Barro, 2009; Gourio, 2012; Wachter, 2013; Petrosky-Nadeau et al., 2018; Jordà et al., 2020); iii) informational frictions and learning asymmetries (Veldkamp, _______________)

3Basu et al. (2023) find that an equity risk premium shock also gives rise to similar cyclical dynamics.
2005; Ordonez, 2013); or iv) left-skewed uncertainty of households or firms (Salgado et al., 2019), could provide promising avenues.

2 A data-rich skewness measure for the US economy

This section presents a new measure of expected asymmetry based on a large dataset of macroeconomic and financial variables. We use the quarterly version of the McCracken and Ng (2016) dataset (FRED-QD) that contains 246 time series starting from 1959 and categorized into 14 groups. All variables are transformed to make them stationary by using the transformations suggested by the authors. We remove those series that have missing observations over our sample period 1960:Q1–2022:Q3, which reduces the number of variables to \( N = 210 \). Next, we estimate for each (de-meaned) variable \( y_i \) and each quantile level \( p = \{10\%, 50\%, 90\%\} \), the following autoregressive quantile regression as developed in Engle and Manganelli (2004a)

\[
Q^p_{i,t} = \beta^p_0 + \beta^p_1 Q^p_{i,t-1} + \beta^p_2 y_{i,t-1} \mathbb{I}(y_{i,t-1} > 0) + \beta^p_3 y_{i,t-1} \mathbb{I}(y_{i,t-1} < 0),
\]

(1)

where \( i = 1, \ldots, N \) and \( t = 2, \ldots, T \). This asymmetric slope model (Engle and Manganelli, 2004a) allows for a different impact of past observations on the respective quantiles, depending on whether they lie above or below the unconditional mean of the series. This permits an asymmetric impact of contractions and expansions in each variable,

These are national income and product accounts (NIPA); industrial production; employment and unemployment; housing; inventories, orders, and sales; prices; earnings and productivity; interest rates; money and credit; household balance sheets; non-household balance sheets; stock markets; exchange rates; and other.
so that, for instance, a recession can affect downside risk without necessarily affecting upside risk. In addition, the model allows the quantiles to be persistent, which seems appropriate given the well-documented persistence of the first two moments of many macroeconomic series (see, e.g., Antolin-Diaz et al., 2017). Engle and Manganelli (2004b) highlight the ability of this model to recover the correct time-varying quantiles in a detailed simulation study.

The conditional quantile autoregressive model belongs to the class of observation-driven models, for which the trajectories of the time-varying parameters are perfectly predictable one-step-ahead given past information (Cox, 1981). Using the estimated model parameters from these quantile regressions, and assuming that agents’ use Equation (1) to form their expectations, we compute for each variable the one-step-ahead expected, or predicted, Kelley skewness (Kelley, 1947)

\[
\mathbb{E}_t[Skew_{i,t+1}] = \frac{\mathbb{E}_t[Q_{i,t+1}^{0.9}] + \mathbb{E}_t[Q_{i,t+1}^{0.1}] - 2\mathbb{E}_t[Q_{i,t+1}^{0.5}]}{\mathbb{E}_t[Q_{i,t+1}^{0.9}] - \mathbb{E}_t[Q_{i,t+1}^{0.1}]}.
\] (2)

Intuitively, the Kelley skewness quantifies asymmetry by comparing the spread of a (conditional) distribution to the right of the median with the spread to the left. Since each quantile estimate is computed as a (variable-specific) moving average of a non-

\footnote{The coefficients are estimated by regression quantiles (Koenker and Bassett, 1978) and further details can be found in Engle and Manganelli (2004a). Since we are interested in capturing cyclical movements in skewness rather than slow-moving trends, we restrict the degree of persistence, i.e. \(0 < \beta_1^p < 0.8\).}

\footnote{Moreover, Taylor (2005) shows that using intervals of symmetric quantiles provides volatility forecasts that outperform those obtained from standard volatility models.}
linear function of the variable itself, there is no reason ex-ante to expect that the skewness of any series displays a particular cyclical behaviour or comoves across indicators. Our overall measure of expected asymmetry is then constructed as the first principal component obtained from the set of series-specific skewness measures, where each measure is first standardized by subtracting the series-specific mean and dividing by its standard deviation (see, e.g., Stock and Watson, 2002). Since the skewness factor is based on PCA, its sign is not identified. We identify the sign by assuming a positive correlation between the skewness factor and the skewness of GDP growth. The factor reflects common movements of skewness across many macroeconomic and financial indicators and does not necessarily overlap with the skewness of any specific indicator, e.g. the skewness of GDP growth. Moreover, the common factor should be relatively immune to idiosyncrasies and noise in the measurement of expected skewness for each of the individual series arising, for instance, from the estimation of the time-varying quantiles. In fact, our measure is also robust to large variation in the data, such as those observed during 2020, when many of the underlying skewness indicators exhibit instabilities.\footnote{Figure D-3 in Appendix D shows our skewness factor estimated with different data vintages, indicating that data revisions and the re-estimation of the model only have a very limited impact.} One should be concerned if our procedure was to predict large variation in aggregate asymmetry when in fact this is not a feature of the data. Appendix A presents a simulation exercise showing that our two-step approach to construct the skewness factor does not yield spurious results, i.e. the factor collapses to zero if the DGP does not feature conditional skewness.

The skewness factor explains around 12% of the variation across the individual
skewness series, reflecting the presence of many series with little asymmetry that load only weakly on the common factor.\textsuperscript{8} Table 1 shows the share of variation explained by the skewness factor for each group of variables. The skewness factor tends to explain more of the skewness variation of the real economy variables including NIPA, labor markets, and production indicators compared to, for example, prices. Moreover, the factor accounts for a non-negligible fraction of the conditional asymmetry in some financial indicators such as non-household balance sheets and stock markets. The last column of Table 1 highlights that our expected skewness factor is robust to the data composition. Specifically, the factor remains largely unaffected by the omission of any of the groups of variables.\textsuperscript{9,10}

\textsuperscript{8}For comparison, the first principal component of the actual data accounts for around 24\% of the variation, while a common (GARCH) volatility factor accounts for around 29\% of the variation in dispersion. Lastly, a common factor of a quantile-based dispersion measure (expected interquartile range), accounts for around 26\% of the variation.

\textsuperscript{9}Figure D-1(b) in Appendix D shows alternative factors when omitting groups of variables. We have also computed an alternative skewness factor based on a subset of 101 variables, that largely match those used in\textsuperscript{} Stock and Watson (2012). Figure D-1(a) compares the two skewness factors which are highly correlated.

\textsuperscript{10}We further investigated the extent to which the time variation in the left and right dispersion of the conditional distribution for each indicator influences the common variation in skewness. Our findings indicate that the two dispersions: (a) do not provide substantial additional information regarding underlying risks beyond what is already captured by the skewness measures, and (b) contribute approximately equally to the skewness factor.
Existing studies have largely focussed on the conditional asymmetry of a single variable, i.e. GDP growth (see, for example, Adrian et al., 2019; Jensen et al., 2020; Loria et al., 2020; Forni et al., 2021; Castelnuovo and Mori, 2022). This is different from our data-rich approach where the skewness factor reflects variation in risks across numerous macroeconomic and financial indicators. The top left panel of Figure 1 compares the expected skewness factor with the individual (de-meaned) skewness series of GDP growth obtained from different conditional quantile models. Aggregate expected skewness is highly procyclical: it drops strongly during recessions and increases/stabilises during the expansionary phases of the cycle. Our skewness factor is positively correlated with the skewness series of GDP growth retrieved using the autoregressive quantile model. Despite their similarities, there are also differences between our skewness factor and the expected skewness of GDP growth. The latter features a distinct downward trend in the last part of the sample, which is in line with the findings of Delle Monache et al. (2022) and appears to be a feature not shared by other indicators.

Quantile regressions that include financial conditions imply a more asymmetric conditional growth distribution and a longer left tail during recessions (Adrian et al., 2019). We document a correlation of around 0.4 between our expected skewness factor and the (Kelley) skewness of GDP growth which conditions on financial conditions. This highlights that elevated asymmetry during downturns is a feature shared by a number of economic indicators and not necessarily related to fluctuations in financial conditions. Note that we report the comparison with GDP growth skewness with the latter estimated over a sample ending before the Covid-19 pandemic. When including data for the pandemic period, both estimates of GDP skewness reported in Figure
change substantially, with especially the Adrian et al. (2019) measure becoming unstable. By contrast, our skewness factor is not affected by this issue.\footnote{Figure D-4 in Appendix D shows the two GDP growth skewness measures estimated over the full sample.} Moreover, the VAR analysis in Section 3 shows that unexpected changes in aggregate skewness and measures of GDP growth skewness can in some cases exert similar, but in other cases very different effects on the macroeconomy.

Documents of economic policy institutions often contain a verbal assessment of risks to the economy. In case of the US, the language with which Fed economists describe the subtleties around the economic outlook reflects such an informal economic risk assessment and provides valuable information beyond what is contained in purely numerical predictions (see, for example, Aruoba and Drechsel, 2022; Cieslak et al., 2022). Figure 1(b) highlights the close relationship between the skewness factor and Fed economists’ perception of risks to the economic outlook. The latter is constructed as the first principal component of more than 250 sentiment indicators extracted using natural language processing techniques from the Fed “Greenbook” documents by Aruoba and Drechsel (2022).\footnote{To maintain close comparability with Aruoba and Drechsel (2022), we also constructed a monthly version of our indicator using the FRED-MD dataset (McCracken and Ng, 2016). The quarterly average of monthly skewness is consistent with the skewness factor extracted from quarterly data. Different from our measure, the sentiment measures of Aruoba and Drechsel (2022) are only available with a five-year lag given the publication delay of the “Greenbook” documents.}

We also compare our measure of macro skewness with micro-level and financial
market measures of asymmetry. Figure 1(c) compares the cross-sectional (Kelley) skewness of firms’ employment growth (Salgado et al., 2019) with our expected skewness factor. Both series move together closely and share a correlation of around 0.8. Given the different underlying methodologies, we interpret this result as i) potential evidence that the same shocks or mechanisms drive both firm-level and aggregate skewness and ii) an affirmation of our interpretation of the skewness factor as an economy-wide skewness measure. Figure 1(d) contrasts our expected skewness factor with two measures of financial market skewness. Specifically, we show the option-implied skewness of the S&P 500 index computed at the market level by Dew-Becker (2022), and the cross-sectional firm-level series of stock return skewness of Salgado et al. (2019). The correlation between the skewness factor and these two series is relatively low. This further supports the interpretation of the aggregate skewness factor as a measure of macroeconomic skewness which is distinct from financial market skewness.

Lastly, our skewness measure correlates with – but is still quite distinct from –

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13To preserve the forward-looking character of the skewness factor, we compute the annual average for each year $t$ over the period Q4 ($t$) to Q3 ($t + 1$). However, this implies that for the annual series, expectations about skewness in $t + 1$ are no longer formed conditional on information in year $t$ only. The firm-level skewness series was taken from the replication files provided by Salgado et al. (2019) who compute this based on the US Census Bureau’s Longitudinal Business Database.

14Ludvigson et al. (2021) highlight a similar disconnect between macro and financial market uncertainty.
aggregate volatility and uncertainty.\textsuperscript{15} Table D-1 in Appendix D shows a correlation matrix including the expected skewness factor, the first principal component of the actual data ($X$) and squared data ($X^2$) akin to Gorodnichenko and Ng (2017), a common factor of the expected interquartile ranges derived from Equation (1), an expected volatility (GARCH) factor, and two popular measures of uncertainty (Jurado et al., 2015; Ludvigson et al., 2021).\textsuperscript{16} Given the procyclicality of the skewness factor, it is not surprising to find negative comovement with uncertainty, which moves countercyclically (see, e.g., Jurado et al., 2015).

Before proceeding, it is worth highlighting two criticalities regarding the construction of our skewness factor. First, calculating the Kelley skewness requires picking a specific level of risk. Our choice of the standard 10\% and 90\% quantiles considers implicit (effective) sample limitations in quantile regressions (see, e.g., Chernozhukov and Umantsev, 2001). However, Figure D-2(a) in Appendix D shows that our skewness factor remains very similar when using the 5\% (2.5\%) and 95\% (97.5\%) quantiles. Second, we also computed a factor from an alternative (score driven) time-varying skewness model, which models each variable’s conditional distribution as a skew-t distribution with time-varying moments (Delle Monache et al., 2022). Such a substantially different model retrieves an aggregate skewness factor which closely resembles

\textsuperscript{15}Orlik and Veldkamp (2014) highlight how within a Bayesian learning framework, where agents attempt to learn the evolving distribution of GDP growth, uncertainty, skewness and therefore downside risk, are naturally related to one another.

\textsuperscript{16}The fact that the quantile-based volatility measure is strongly correlated with the GARCH factor ($>0.9$) and macroeconomic uncertainty ($>0.8$) provides reassurance that our procedure also reliably measures skewness.
our baseline factor, with a correlation of over 0.8 (see Figure D-2(b)). These exercises highlight that the presence of common variation in skewness remains robust to the exact measurement approach.

3 Macroeconomic effects of shifts in aggregate skewness

In this section we investigate the dynamic relationship between expected skewness and the macroeconomy by adding our skewness factor to an otherwise standard VAR model. The empirical specification, the variables included, as well as the estimation approach largely follow Angeletos et al. (2020). Within this set up, we study the relationship between revisions in expected skewness and the main business cycle shock of these authors.

The baseline VAR contains the following variables: the expected skewness factor, real GDP per capita, real investment per capita, real consumption per capita, hours worked per person, unemployment rate, labor share, effective federal funds rate, inflation, labor productivity (non-farm business sector), and a measure of TFP.\textsuperscript{17} The analysis is conducted over the period 1960:Q1–2019:Q4.\textsuperscript{18} Details on the variables can

\textsuperscript{17}As in Uhlig (2005), we do not include a constant in the VAR (see also Uhlig, 1994). The results remain virtually unchanged when including a constant.

\textsuperscript{18}We end the sample in 2019:Q4 to avoid that the results are affected by the Covid-19 pandemic (see, for example, Lenza and Primiceri, 2022). For the VAR analysis, we also extract the skewness factor from this shorter sample (see Figure D-5(a) in Appendix D) to ensure consistency, in particular with the GDP growth skewness measures (see Figure 1(a)). However, this skewness factor shares a correlation of 0.95
be found in Appendix B. The VAR model has the following representation:

\[ y_t = \sum_{p=1}^{P} \Theta_p y_{t-p} + u_t, \quad u_t \sim N(0, \Sigma) \]  

where \( \Theta_p \forall p = 1, \ldots, P \) are coefficient matrices, and \( u_t \) is a vector of reduced-form disturbances, which are linear combinations of the underlying structural (orthogonal) shocks \( u_t = A_0 \varepsilon_t \). \( A_0 \) is the matrix containing the contemporaneous responses, where \( A_0 A_0' = \Sigma \). Due to the relatively large dimension of the VAR, we adopt a Bayesian estimation approach and employ a Minnesota-type prior. The parameter controlling the tightness of this prior is set to \( \lambda = 2 \) and Section 4 shows that the results hold even for looser configurations. Appendix C contains details on the prior specification and the estimation approach. We choose a lag length of \( P = 2 \) and demonstrate robustness with respect to this choice in Section 4.

Identifying exogenous variation in expected skewness is challenging, with theory providing little guidance. Our baseline approach imposes zero restrictions on the matrix containing the contemporaneous responses. Specifically, \( A_0 \) is identified as the lower triangular matrix obtained from a Cholesky decomposition of \( \Sigma \). Ordering our skewness measure first, this simple identification scheme provides an intuitive interpretation of the identified shock as the revision, i.e. the ‘unexpected change’, in expected skewness: \( \mathbb{E}_t[Skew_{t+1}] - \mathbb{E}_{t-1}[\mathbb{E}_t[Skew_{t+1}]] \) where the expectation \( \mathbb{E}_{t-1} \) is conditional on the information set spanned by the VAR. We loosely refer to this as a “skewness shock”. However, this should not be interpreted as a structural shock, but is better

with the ‘full sample’ factor shown in Figure 1. Finally, all key results hold when excluding the Great Recession, i.e. ending the sample in 2007:Q2.
ter understood as the (fixed) linear combination of (structural) shocks, i.e. the *skewness anatomy* following the lexicon of Angeletos et al. (2020), which explains unexpected changes in aggregate skewness. Section 4 shows that an alternative approach which relaxes the zero restrictions and identifies the shock that explains the largest share of unexpected variation in skewness over a given horizon based on Uhlig (2003) yields very similar results.

Revisions to expected skewness are ‘small’ compared to the overall variation of aggregate skewness, highlighting a certain sluggishness of underlying risks in the macroeconomy. Figures 2 and 3 show the impulse response functions following a (one-standard deviation) downward revision of expected skewness, and the corresponding forecast error variance contributions, together with those of the MBC shock of Angeletos et al. (2020). The latter is identified as the shock that explains the bulk of the variation of unemployment using the max-share approach of Uhlig (2003), targeting four quarters in the time domain. Both shocks are identified within the same VAR specification. A revision in expected skewness generates business cycle dynamics that are very similar, even quantitatively, to the *business cycle anatomy* documented in Angeletos et al. (2020). These dynamics reflect a sizeable, but relatively short-lived, comovement between GDP, investment, consumption, hours worked, and unemployment, without meaningful movements in inflation and TFP. Table 2 shows that the (unconditional) correlation between the MBC shock and our skewness shock is above

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19 Figure D-5(a) in Appendix D contrasts the skewness factor and its revisions.

20 For example, GDP falls by around 0.6%-0.7% within one year, with the negative effect vanishing after slightly more than three years.
Angeletos et al. (2020) use the business cycle anatomy to shed light on the transmission of macro shocks and, in particular, on the drivers of the business cycle. Our evidence underlines that the key source of business cycle fluctuations also accounts for short-term revisions in expected macroeconomic asymmetries. Put differently, while the MBC and the skewness shock are likely no structural shocks – but rather a combination of such shocks – our results suggest that the same combination of structural shocks explains both revisions in expected skewness and business cycle fluctuations.

In Section 2 we show that our skewness factor is correlated with alternative measures of macroeconomic skewness. It is therefore natural to ask whether their revisions also display similarities with the business cycle anatomy or whether introducing a broader (PCA-based) measure of skewness is crucial to obtaining this result. As a first exercise, we replace the expected skewness factor with the individual expected skewness series of GDP growth. The results are shown in Figures D-6 and D-7 in Appendix D. Despite the sizeable correlation between aggregate macro skewness and the skewness of GDP growth, revisions in the latter do only generate a smaller amount of comovement among the key macroeconomic variables. As a result, the correlation between revisions in GDP growth skewness and the MBC shock is small and even flips sign (Table 2). When comparing our baseline results with the impact of revisions in expected GDP growth skewness computed based on the approach of Adrian et al. (2019), we find larger similarities. While revisions in this measure of growth skewness, largely reflecting revisions related to financial conditions, have a much more short-lived impact on macroeconomic asymmetry (Figures D-8 and D-9), they produce sizeable co-movement among all key macroeconomic quantities. However, several quantitative

\[ \text{Figure D-5(b) in Appendix D contrasts revisions in skewness and the MBC shock.} \]
differences compared to the impact of revisions in aggregate expected skewness remain. The correlation with the MBC shock is clearly positive, but stays significantly below the baseline result (Table 2). This is evidence that our broader skewness factor contains additional information which matters when analysing the impact of changing risks.

We also investigate whether revisions in financial market skewness produce dynamics consistent with the ones reported above. To this end, we replace the skewness factor with the option-implied market skewness series of Dew-Becker (2022) and the cross-sectional stock return series of Salgado et al. (2019), both shown in Figure 1(d). First, Table 2 shows that revisions to the S&P 500 skewness series are negatively correlated with the MBC shock. A downward revision in this skewness measure is associated with an expansionary response of the main business cycle indicators, and non-negligible positive inflation (Figures D-10 and D-11). This result is in line with Dew-Becker (2022), who finds financial market skewness to move countercyclically. Second, when including the cross-sectional firm-level measure of stock return skewness, we only find a minor correlation between its revisions and the MBC shock (see Table 2, and Figures D-12 and D-13).

To conclude this section, we explore the impact of revisions in expected skewness beyond the baseline set of macroeconomic variables through augmented specifications, including selected financial variables (see Appendix E). We consider three augmented models that in addition include either i) excess returns and the term premium (Figures E-1 and E-2); ii) real house prices and real stock prices (Figures E-3 and E-4); or iii) yields of 10-year government bonds (Figures E-5 and E-6). A downward revision of expected skewness is associated with lower stock prices, excess returns and
government bond yields while the term premium, and to a lesser extent house prices, increase. Moreover, revisions in skewness contribute to a non-negligible share of the variation in government bond yields, the term premium and stock prices. Yet in line with the original evidence in Angeletos et al. (2020), a revision in expected macroeconomic skewness appears to matter somewhat more for macroeconomic than financial variables.

4 Robustness checks

We check the robustness of our baseline results along different dimensions. Detailed results can be found in Appendix F. First, the results are robust to a change in the identification scheme. In particular, to be closer to Angeletos et al. (2020), we also identify skewness shocks using the Uhlig (2003) approach which maximizes the explained share of skewness variation over four quarters in the time domain. The results (Figure F-1) are very similar to those based on the recursive identification.

Second, we augment our baseline specification with measures of macroeconomic volatility, uncertainty and geopolitical risk. Figure F-2 presents the effects of a revision in expected skewness when controlling for aggregate expected volatility, achieved by ordering this measure first in the Cholesky identification. This isolates the contribu-

22For these robustness checks we only report the IRFs.

23The volatility measure is also based on a data-rich approach. Specifically, we estimate a GARCH(1,1) model on each (de-meaned) series of the McCracken and Ng (2020) dataset and obtain the first principal component of all standardized expected volatility (conditional standard deviation) series.
tion associated with the revision in expected macroeconomic skewness that is orthogonal to variation in overall volatility. The IRFs remain very similar and the correlation between revisions in expected skewness and the MBC shock remains quite strong (Table 2). In a related exercise, we control for macro and financial uncertainty (Jurado et al., 2015; Ludvigson et al., 2021). While the IRFs (Figure F-3) change somewhat more in this case, they still remain similar to the baseline results. The positive comovement between output and uncertainty after a downward revision in expected skewness implies that the transmission of skewness revisions is distinct from the transmission of an uncertainty shock, which is generally characterized by a negative comovement between output and uncertainty. Table 2 shows that the correlation between the skewness shock and the MBC shock remains sizeable. These results are largely consistent with those in Forni et al. (2021), who show that the transmission of downside uncertainty and skewness shocks is distinct from that of a standard (symmetric) uncertainty shock, with a widening of the left tail causing economic contractions (see also Segal et al., 2015). Moreover, to test whether revisions in expected skewness relate to geopolitical risk, we augment our baseline specification with the Geopolitical Risk Index of Caldara and Iacoviello (2022). Here, we find that the IRFs (Figure F-4) as well as the correlation with the MBC shock, remain nearly unchanged.

Third, we show that revisions in expected skewness are unrelated to other standard shocks. We control for: i) shocks to risk appetite measured as the exogenous variation in the Gilchrist and Zakrajšek (2012) excess bond premium (Figure F-5); ii) productivity shocks measured as the exogenous variation in the growth rate of the Fernald (2014) TFP series (Figure F-6); iii) shocks to government expenditure as identified in Ramey and Zubairy (2018) (Figure F-7); and iv) monetary policy shocks measured by
the surprise series of Jarociński and Karadi (2020), which is purged of the central bank information component (Figure F-8). In all cases the IRFs are similar to the baseline model and range from being nearly identical (TFP and fiscal policy) to showing some differences (EBP and monetary policy). The skewness shock continues to be highly correlated with the MBC shock across specifications (Table 2), highlighting that revisions in expected skewness are roughly orthogonal to these shocks.

Finally, we change the lag order in the VAR and the Minnesota prior. Figure F-9 presents the results using a lag order of \( P = 4 \), which remain very similar compared to the baseline model. Figure F-10 shows that applying an even looser configuration of the Minnesota prior (\( \lambda = 10 \)) leaves the baseline results essentially unchanged.

5 Conclusion and direction for future research

We construct a factor that summarizes expected macroeconomic skewness. This factor is the first principal component of the time-varying expected skewness indicators of a large number of macroeconomic series. Aggregate macroeconomic skewness is strongly procyclical, comoves with, but is quite distinct from, the expected GDP growth skewness series based on the approach of Adrian et al. (2019), and is highly correlated with the cross-sectional skewness of firm-level employment growth (Salgado et al., 2019). In addition, our skewness factor comoves with the economic risks perceived by Fed staff economists (Aruoba and Drechsel, 2022). We then document that the impulse responses of a set of macroeconomic variables associated with a revision in expected macroeconomic skewness, and the corresponding variance contributions, closely match the business cycle anatomy of Angeletos et al. (2020). In fact, expected skewness revisions largely overlap with the main business cycle shock identi-
fied in Angeletos et al. (2020). The results are robust to changes in the identification scheme, controlling for macroeconomic volatility, uncertainty, and frequently considered alternative shocks.

Our results highlight the importance of accounting for a procyclical variation in conditional skewness of macroeconomic data. Variation in conditional skewness requires the presence of non-linearities in the transmission of Gaussian shocks (see, e.g., Fernández-Villaverde and Guerrón-Quintana, 2020), or can directly derive from skewed shocks hitting the economy (Bekaert and Engstrom, 2017; Salgado et al., 2019). Angeletos and La’O (2013) and Angeletos et al. (2018) highlight how waves of optimism and pessimism regarding both firms’ expected employment and production decisions as well as consumers’ beliefs about future employment opportunities and income generate dynamics of output, employment, spending and prices akin to the business cycle patterns observed in the data. The former could potentially arise from learning asymmetries in the presence of informational frictions as in Veldkamp (2005). To the extent that fluctuations in sentiment or confidence are associated with a reassessment of upside and downside risk over the cycle, and hence shifts in expected skewness, our results help addressing the problem that “a direct, empirical counterpart to the confidence shock is hard, if possible at all, to obtain” (Angeletos et al., 2018, p. 1692). Our results are also consistent with a relevant role for expectations of rare disasters in explaining economic fluctuations (Rietz, 1988; Barro, 2006, 2009; Gabaix, 2008; Gourio, 2012; Wachter, 2013; Petrosky-Nadeau et al., 2018; Jordà et al., 2020). In particular, our results highlight the importance of allowing for time variation in the severity (Gabaix, 2008) and/or probability of such rare disasters (see, e.g., Gourio, 2012; Wachter, 2013; Giglio et al., 2021), which could generate sizeable variation in ex-
pected skewness. Lastly, our results provide insights for macroeconomic theories that search for shocks and propagation mechanisms behind macroeconomic fluctuations. Any such theory should be able to reproduce variation in aggregate skewness whose revisions are strongly affected by the main source of business cycle fluctuations.
References


Table 1: Descriptive statistics of skewness variation explained by first principal component (in %)

<table>
<thead>
<tr>
<th>Group</th>
<th>No.</th>
<th>Mean</th>
<th>Median</th>
<th>Max.</th>
<th>Min.</th>
<th>Corr. w/o</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inventories, orders, and sales</td>
<td>6</td>
<td>21.0</td>
<td>23.5</td>
<td>38.3</td>
<td>0.6</td>
<td>0.99</td>
</tr>
<tr>
<td>National inc. and product accounts</td>
<td>22</td>
<td>17.9</td>
<td>13.5</td>
<td>50.3</td>
<td>0.0</td>
<td>0.98</td>
</tr>
<tr>
<td>Employment and unemployment</td>
<td>44</td>
<td>15.7</td>
<td>12.6</td>
<td>43.3</td>
<td>0.1</td>
<td>0.95</td>
</tr>
<tr>
<td>Industrial production</td>
<td>15</td>
<td>14.8</td>
<td>8.9</td>
<td>59.7</td>
<td>1.1</td>
<td>0.99</td>
</tr>
<tr>
<td>Stock markets</td>
<td>5</td>
<td>13.5</td>
<td>9.0</td>
<td>31.5</td>
<td>2.0</td>
<td>1.00</td>
</tr>
<tr>
<td>Non-household balance sheets</td>
<td>11</td>
<td>13.3</td>
<td>12.1</td>
<td>30.2</td>
<td>0.1</td>
<td>0.99</td>
</tr>
<tr>
<td>Housing</td>
<td>6</td>
<td>8.1</td>
<td>4.2</td>
<td>19.8</td>
<td>0.0</td>
<td>1.00</td>
</tr>
<tr>
<td>Interest rates</td>
<td>18</td>
<td>7.8</td>
<td>5.7</td>
<td>42.4</td>
<td>0.0</td>
<td>0.99</td>
</tr>
<tr>
<td>Prices</td>
<td>46</td>
<td>7.6</td>
<td>2.7</td>
<td>51.5</td>
<td>0.0</td>
<td>0.98</td>
</tr>
<tr>
<td>Earnings and productivity</td>
<td>10</td>
<td>7.1</td>
<td>2.3</td>
<td>26.4</td>
<td>0.1</td>
<td>1.00</td>
</tr>
<tr>
<td>Exchange rates</td>
<td>4</td>
<td>6.4</td>
<td>6.7</td>
<td>11.1</td>
<td>1.2</td>
<td>0.99</td>
</tr>
<tr>
<td>Household balance sheets</td>
<td>9</td>
<td>6.0</td>
<td>3.9</td>
<td>26.3</td>
<td>0.0</td>
<td>1.00</td>
</tr>
<tr>
<td>Money and credit</td>
<td>13</td>
<td>6.0</td>
<td>3.9</td>
<td>21.4</td>
<td>0.1</td>
<td>0.99</td>
</tr>
</tbody>
</table>

Note: This table presents descriptive statistics for the shares of variation of the individual skewness series explained by the skewness factor (in %). The last column contains the correlation between the skewness factor and an alternative skewness factor obtained from the original dataset but where the variables of the respective group were omitted. The grouping follows McCracken and Ng (2020). The group Other is dropped from this table as it only contains one variable.
Figure 1: Skewness factor vs. other (skewness) measures

(a) Exp. GDP skew and exp. skewness factor

(b) Fed staff sentiment and exp. skewness factor

(c) Firm-level skew and exp. skewness factor

(d) Fin. market skew and exp. skewness factor

Note: Figure 1(a) shows the expected skewness factor (right axis) together with the individual (Kelley) skewness series of quarter-over-quarter real GDP growth (left axis) derived based on the quantile specification in Equations (1)-(2) and the approach of Adrian et al. (2019) (ABG), respectively. The latter series is based on quantile regressions of real GDP growth on lagged growth and the lagged National Financial Conditions Index (NFCI) computed by the Chicago Fed. Figure 1(b) shows the monthly version of the skewness factor together with the first principal component of all sentiment indicators (Oct. 1982–Dec. 2016) computed in Aruoba and Drechsel (2022) (AD).
Figure 1(c) shows the skewness factor (annual avg., right axis) together with the (employment-weighted) cross-sectional Kelley skewness of firms’ log employment growth (1976–2014) obtained from Salgado et al. (2019) (SGB, left axis). Figure 1(d) shows the skewness factor (right axis) together with i) the monthly option-implied measure of market skewness for the S&P 500 developed in Dew-Becker (2022) (DB, quarterly avg., 1983:Q2–2021:Q4, left axis), and ii) the cross-sectional (Kelley) skewness of firms’ daily stock returns within a month computed in Salgado et al. (2019) (SGB, quarterly avg., 1964:Q1–2015:Q1, left axis). All alternative skewness series are de-meaned and the scale of the SGB financial skewness measure is adjusted for comparability with the DB measure. The blue shaded areas are the bootstrapped confidence bands (90%) around the skewness factor based on Gonçalves and Perron (2020). Gray areas are NBER recessions.
Note: The blue lines are the posterior mean responses to a negative one S.D. shock to expected skewness along with the 68% highest density interval. The skewness shock is identified through a Cholesky decomposition. The black lines are the responses to a one S.D. shock to the MBC shock of Angeletos et al. (2020). This shock is identified using the approach of Uhlig (2003). The IRFs represent percentage deviations from the steady state, except for inflation, the policy rate, and unemployment, which are reported in percentage points. Sample period: 1960:Q1–2019:Q4.
Figure 3: Baseline model: Forecast error variance contributions

Note: Posterior mean of the forecast error variance contributions along with the 68% highest density interval for a shock to expected skewness (blue) and the MBC (unemployment) shock (black).
<table>
<thead>
<tr>
<th>Baseline model</th>
<th>Other skewness measures</th>
<th>Robustness checks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Skew. shock</td>
<td>MBC shock</td>
<td>MBC shock</td>
</tr>
<tr>
<td>Med.     0.87</td>
<td>(1960:Q1–2019:Q4)</td>
<td>0.82</td>
</tr>
<tr>
<td>95% HDI -0.21</td>
<td>(1960:Q1–2019:Q4)</td>
<td>0.47</td>
</tr>
<tr>
<td>0.31</td>
<td>(1963:Q2–2019:Q4)</td>
<td>0.04</td>
</tr>
<tr>
<td>0.70</td>
<td>(1960:Q1–2019:Q4)</td>
<td>0.60</td>
</tr>
<tr>
<td>0.66</td>
<td>(1960:Q3–2019:Q4)</td>
<td>0.55</td>
</tr>
<tr>
<td>0.87</td>
<td>(1960:Q1–2019:Q4)</td>
<td>0.82</td>
</tr>
<tr>
<td>0.78</td>
<td>(1973:Q1–2019:Q4)</td>
<td>0.69</td>
</tr>
<tr>
<td>0.88</td>
<td>(1960:Q1–2019:Q4)</td>
<td>0.82</td>
</tr>
<tr>
<td>0.86</td>
<td>(1960:Q1–2015:Q1)</td>
<td>0.80</td>
</tr>
<tr>
<td>0.86</td>
<td>(1990:Q1–2016:Q4)</td>
<td>0.79</td>
</tr>
</tbody>
</table>
Note: Each row corresponds to a VAR specification and shows the correlation between downward revisions in (expected) skewness and the (contractionary) MBC shock (Angeletos et al., 2020). We report the median correlation across MCMC draws along with the 95% highest density interval (HDI). Revisions in (expected) skewness are identified through a Cholesky decomposition by ordering skewness first if no alternative shock/variable is included and second/third otherwise. Specification a) is our baseline model whereas in b), c), d) and e) the skewness factor is replaced with the exp. skewness of GDP growth, the exp. skewness of GDP growth based on the approach of Adrian et al. (2019), the option-implied skewness of the S&P 500 (quarterly avg.) computed by Dew-Becker (2022), and the cross-sectional firm-level skewness of stock returns (quarterly avg.) computed by Salgado et al. (2019), respectively. The alternative variables/shocks are: f) a data-rich measure of expected volatility based on a GARCH(1,1); g) the macroeconomic and financial uncertainty indices (quarterly avg.) of Jurado et al. (2015) and Ludvigson et al. (2021); h) the (historical) geopolitical risk index (quarterly avg.) of Caldara and Iacoviello (2022); i) the excess bond premium (EBP, quarterly avg.) (Gilchrist and Zakrajšek, 2012); j) the (annualized) growth rate of the utilization-adjusted TFP measure of Fernald (2014); k) the government spending shock of Ramey and Zubairy (2018); and l) the monetary policy surprises (quarterly avg.) of Jarociński and Karadi (2020).
Appendix A Monte Carlo exercise

This section addresses the concern that our two-step approach to constructing an aggregate skewness factor could yield spurious results, i.e. indicate time-varying conditional skewness in cases, where in fact there is none. For this, we conduct a Monte Carlo exercise and generate 500 datasets of size $N = 70$ and $T = 250$ from two different data generating processes (DGP), both of which do not feature conditional skewness. The first DGP has a time-varying mean and volatility, which both have a factor structure. Specifically, DGP 1 is defined as

\begin{align*}
y_{i,t} &= \mu_{i,t} + e^{h_{i,t}/2} \varepsilon_{i,t}, \quad \varepsilon_{i,t} \sim \mathcal{N}(0, 1), \\
\mu_{i,t} &= \lambda^f_i f_t + \omega_{i,t}, \\
h_{i,t} &= \lambda^h_i \tilde{h}_t + \nu_{i,t}, \\
f_t &= \rho^f f_{t-1} + z_t, \quad z_t \sim \mathcal{N}(0, \sigma_z^2), \\
\tilde{h}_t &= \rho^h \tilde{h}_{t-1} + u_t, \quad u_t \sim \mathcal{N}(0, \sigma_u^2), \\
\omega_{i,t} &= \rho^\omega_i \omega_{i,t-1} + \epsilon_{i,t}, \quad \epsilon_{i,t} \sim \mathcal{N}(0, \sigma_{\epsilon,i}^2), \\
\nu_{i,t} &= \rho^\nu_i \nu_{i,t-1} + \kappa_{i,t}, \quad \kappa_{i,t} \sim \mathcal{N}(0, \sigma_{\kappa,i}^2).
\end{align*}

The parameters of the DGP are set to: $\rho^f = 0.9, \rho^h = 0.98, \rho^\omega_i = 0.9, \rho^\nu_i = 0.98, \sigma_z^2 = 1, \sigma_u^2 = 0.1, \sigma_{\epsilon,i}^2 = 1, \sigma_{\kappa,i}^2 = 0.1 \forall i = 1, ..., N$. The factor loadings in the mean and log-volatility equation, $\lambda^f_i$ and $\lambda^h_i$, are drawn from independent normal distributions with the
moments chosen such that the average variation explained of the mean and log-volatility of the variables is 20% and 25%, respectively.

DGP 2 is similar to DGP 1 but includes the so-called leverage effect, i.e. a negative contemporaneous correlation between the innovations to the mean and volatility factors, as well as the innovations to the idiosyncratic mean and volatility components. Under this assumption, it is well-known that the model remains conditionally Gaussian, but features unconditional left-skewness (e.g. Omori et al., 2007). In particular, in this case, we assume that $z_t$ and $u_t$ follow a multivariate normal distribution with correlation $\rho_{z_t,u_t} = -0.9$. A similar assumption is introduced for the correlation between $\epsilon_{i,t}$ and $\kappa_{i,t}$ is chosen randomly from a uniform distribution $[0, -0.9]$ for each variable $i = 1, \ldots, N$.

For each simulated dataset and both DGPs, we estimate the skewness factor as outlined in Section 2. Since the scale of the skewness factor is not identified, and since in this case we are interested in assessing how far the retrieved factor is from the zero line, we normalize the factor so that its standard deviation matches the mean value of the standard deviation of the individual skewness series in the dataset. Figure A-1 presents the distribution of the estimated skewness factors across the Monte Carlo samples. The results provide evidence for the strong performance of the model and show that our two-step approach to construct the skewness factor does not capture “spurious skewness”. In particular, since both DGPs do not feature conditional skewness, the distribution of the estimated factors across Monte Carlo samples is centred around the zero line with only limited dispersion.

**Figure A-1: Results of Monte Carlo simulation**

![Graphs showing the distribution of estimated skewness factors for DGP 1 and DGP 2](image)

Note: The largest shaded area corresponds to the 90% confidence interval, with shades corresponding to increasing probability ranges of 10%, 20%, ..., 90%.
Appendix B Data

### Dataset used to derive the skewness factor

McCracken and Ng (2016) / McCracken and Ng (2020) datasets: [https://research.stlouisfed.org/econ/mcclacken/fred-databases/](https://research.stlouisfed.org/econ/mcclacken/fred-databases/)

Vintage: December 2022.

### Variables in baseline VAR

<table>
<thead>
<tr>
<th>Name</th>
<th>Description</th>
<th>Transformation</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>GDP</td>
<td>Real GDP per capita</td>
<td>100*\log(X)</td>
<td>(AL)FRED</td>
</tr>
<tr>
<td>Investment</td>
<td>Real investment per capita</td>
<td>100*\log(X)</td>
<td>(AL)FRED</td>
</tr>
<tr>
<td>Consumption</td>
<td>Real consumption per capita</td>
<td>100*\log(X)</td>
<td>(AL)FRED</td>
</tr>
<tr>
<td>Hours worked</td>
<td>Hours worked per person</td>
<td>100*\log(X)</td>
<td>(AL)FRED</td>
</tr>
<tr>
<td>Unemployment</td>
<td>Civilian unemployment rate</td>
<td>–</td>
<td>(AL)FRED</td>
</tr>
<tr>
<td>Labor share</td>
<td>Labor share in the non-farm business sector</td>
<td>100*\log(X)</td>
<td>(AL)FRED</td>
</tr>
<tr>
<td>Policy rate</td>
<td>Effective federal funds rate</td>
<td>(X/4)</td>
<td>(AL)FRED</td>
</tr>
<tr>
<td>Inflation</td>
<td>Percentage change in GDP deflator</td>
<td>100*\log(\frac{X_t}{X_{t-1}})</td>
<td>(AL)FRED</td>
</tr>
<tr>
<td>Labor productivity</td>
<td>Real (non-farm) output per hours of all persons</td>
<td>100*\log(X)</td>
<td>(AL)FRED</td>
</tr>
<tr>
<td>TFP</td>
<td>Level of total factor productivity</td>
<td>100*\log(X)</td>
<td>Femald (2014)</td>
</tr>
</tbody>
</table>

### Variables in augmented VARs

<table>
<thead>
<tr>
<th>Name</th>
<th>Description</th>
<th>Transformation</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Excess returns</td>
<td>Change of S&amp;P 500 minus 3-month treasury yield</td>
<td>–</td>
<td>FRED</td>
</tr>
<tr>
<td>Term premium</td>
<td>Treasury term premium for 10-year gov. bonds</td>
<td>–</td>
<td>FRBSF/ Adrian et al. (2013)</td>
</tr>
<tr>
<td>House prices</td>
<td>Real house prices (nominal HPI divided by CPI)</td>
<td>100*\log(X)</td>
<td>FRED</td>
</tr>
<tr>
<td>S&amp;P 500</td>
<td>Real stock prices (S&amp;P 500 index divided by CPI)</td>
<td>100*\log(X)</td>
<td>FRED</td>
</tr>
<tr>
<td>Government bond yield</td>
<td>10-year government bond yield</td>
<td>–</td>
<td>FRED</td>
</tr>
</tbody>
</table>

Note: The variables in the baseline VAR match those included by Angeletos et al. (2020) and more details can be found in that reference.
Appendix C  VAR model and prior choice

This appendix contains details on the VAR model and the prior specification. Since we do not outline every step of the Bayesian treatment of a VAR model, we refer to standard references for further details (e.g. Koop and Korobilis, 2010; Chan, 2020). The starting point of our empirical analysis is a vector autoregressive model of order \( P \) denoted as VAR(\( P \))

\[
y_t = \sum_{p=1}^{P} \Theta_p y_{t-p} + u_t, \quad u_t \sim \mathcal{N}(0, \Sigma),
\]

where \( u_t \) is a \( N \times 1 \) vector of reduced-form errors that is normally distributed with zero mean and covariance matrix \( \Sigma \). The regression-equation representation of this system is

\[
Y = X\Theta + U,
\]

where \( Y = [y_{h+1}, ..., y_T] \) is a \( N \times T \) matrix, \( X = Y_{-h} \) is a \( (NP) \times T \) matrix containing the \( h \)-th lag of \( Y \), \( \Theta = [\Theta_1, ..., \Theta_P] \) is a \( N \times (NP) \) matrix, and \( U = [u_{h+1}, ..., u_T] \) is a \( N \times T \) matrix of disturbances.

The Bayesian estimation of VAR models has become standard in empirical macroeconomics. We use a Minnesota-type prior (Doan et al., 1984; Litterman, 1986). It is assumed that the prior distribution of the VAR parameters has a Normal-Wishart conjugate form

\[
\theta | \Sigma \sim \mathcal{N}(\theta_0, \Sigma \otimes \Omega_0), \quad \Sigma \sim \mathcal{IW}(v_0, S_0),
\]

where \( \theta \) is obtained by stacking the columns of \( \Theta \). In contrast to Litterman (1986), the covariance matrix \( \Sigma \) is not replaced by an estimated and thus known (diagonal) counterpart. Therefore, sampling from the conditional posterior distributions described below requires Gibbs sampling (see also Mumtaz and Zanetti, 2012). Our results are based on 25,000 draws and we discard the initial 5,000 draws as burn-in. The prior moments of \( \theta \) are given by

\[
E[(\Theta_p), i, j] = \begin{cases} 
\delta_i & i = j, p = 1 \\
0 & \text{otherwise}
\end{cases}, \quad Var[(\Theta_p), i, j] = \lambda \sigma_i^2 / \sigma_j^2,
\]

and, as outlined in Bańbura et al. (2010), they can be constructed using the following \( T_D \) dummy observations

\[
Y_D = \left( \frac{\text{diag}(\delta_1 \sigma_1, ..., \delta_N \sigma_N)}{\lambda}, 0_{N \times (P-1)N}, ..., \text{diag}(\sigma_1, ..., \sigma_N)' \right)' \quad \text{and} \quad (C-5)
\]

\[
X_D = \left( J_P \otimes \frac{\text{diag}(\sigma_1, ..., \sigma_N)}{\lambda}, ..., 0_{N \times NP, ...} \right)', \quad (C-6)
\]
where $J_P = \text{diag}(1, 2, ..., P)$ and diag denotes the diagonal matrix. The prior moments in Equation (C-3) are functions of $Y_D$ and $X_D$, $\Theta_0 = Y_DX_D'(X_DX_D')^{-1}$, $\Omega_0 = (X_DX_D')^{-1}$, $S_0 = (Y_D - \Theta_0X_D)(Y_D - \Theta_0X_D)'$ and $v_0 = T_D - NP$. Finally, the hyper-parameter $\lambda$ controls the tightness of the prior and our baseline choice is $\lambda = 2$.

Since the normal-inverse Wishart prior is conjugate, the conditional posterior distribution of this model is also normal-inverse Wishart (Kadiyala and Karlsson, 1997)

$$\theta|\Sigma, Y \sim N(\bar{\theta}, \Sigma \otimes \bar{\Omega}), \ \Sigma|Y \sim \text{IW}(\bar{v}, \bar{S}),$$

where variables with a bar denote the parameters of the posterior distribution. Defining $\hat{\Theta}$ and $\hat{U}$ as the OLS estimates from Equation (C-2), the parameters of the conditional posterior distribution can be computed as $\bar{\Theta} = (\Omega_0^{-1}S_0 + YX')(\Omega_0^{-1} + X'X)^{-1}$, $\bar{\Omega} = (\Omega_0^{-1} + X'X)^{-1}$, $\bar{v} = v_0 + T$, and $\bar{S} = \hat{\Theta}X'\hat{\Theta} + \Theta_0\Omega_0^{-1}\Theta_0 + S_0 + \hat{U}\hat{U}' - \hat{\Theta}\hat{\Omega}^{-1}\hat{\Theta}'$. Lastly, as in Mumtaz and Zanetti (2012), the values of the persistence parameter $\delta_i$ and the error standard deviation $\sigma_i$ of the AR(1) model are obtained from its OLS estimation.

**Appendix D** Additional results

**Figure D-1**: Skewness factor based on different datasets

(a) Smaller subset of variables

(b) Excluding groups of variables

Note: Figure D-1(a) shows the skewness factor based on the full McCracken and Ng (2020) dataset and an alternative skewness factor based on a subset of variables similar to those used in Stock and Watson (2012). The blue shaded areas are the bootstrapped confidence bands (90%) around the skewness factor based on Gonçalves and Perron (2020). Figure D-1(b) shows the skewness factor based on the full dataset together with alternative skewness factors obtained from the original dataset where one group of variables is omitted at a time. In both figures, the scale of the alternative skewness factors is adjusted to match the one of the original factor.
**Figure D-2:** Additional robustness checks for skewness factor

(a) Kelley skewness based on other quantiles

(b) Score-driven modelling approach

Note: Figure D-2(a) contrasts the baseline skewness factor with alternative skewness factors that use the 5%/95% and 2.5%/97.5% conditional quantiles (instead of 10% and 90%), respectively, to compute the expected Kelley skewness for each variable. Figure D-2(b) compares the skewness factor with an alternative skewness factor based on a simplified version of the score-driven model developed in Delle Monache et al. (2022).

**Figure D-3:** Real-time skewness factor

**Figure D-4:** GDP growth skewness until 2022:Q3

Note: Figure D-3 shows the expected skewness factor estimated over the full sample (using the FRED-QD “2022-12” vintage (McCracken and Ng, 2020)) together with an alternative skewness factor that is estimated in real-time using FRED-QD vintages since May 2018 (earliest available vintage). Specifically, the latter starts by using the “2018-05” vintage to estimate the skewness factor over the period 1960:Q1–2018:Q1. After that, the real-time estimation uses four vintages per year (January, April, July, and October), re-estimating the model repeatedly using the latest vintage, and adding one observation at a time. The last re-estimation is done using the “2022-10” vintage. Figure D-4 shows the expected skewness factor together with measures of GDP growth skewness when, also for the latter, using the full sample. See also Figure 1.
Table D-1: Correlation of skewness factor and different volatility/uncertainty measures

<table>
<thead>
<tr>
<th></th>
<th>PC (skew)</th>
<th>PC (X)</th>
<th>PC (X²)</th>
<th>PC (P_{75}-P_{25})</th>
<th>PC (GARCH)</th>
<th>Macro unc.</th>
<th>Fin. unc.</th>
</tr>
</thead>
<tbody>
<tr>
<td>PC (skew)</td>
<td>1.00</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>PC (X)</td>
<td>0.43</td>
<td>1.00</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>PC (X²)</td>
<td>-0.37</td>
<td>-0.51</td>
<td>1.00</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>PC (P_{75}-P_{25})</td>
<td>-0.72</td>
<td>-0.69</td>
<td>0.82</td>
<td>1.00</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>PC (GARCH)</td>
<td>-0.64</td>
<td>-0.47</td>
<td>0.91</td>
<td>0.93</td>
<td>1.00</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Macro unc.</td>
<td>-0.70</td>
<td>-0.44</td>
<td>0.56</td>
<td>0.83</td>
<td>0.76</td>
<td>1.00</td>
<td>-</td>
</tr>
<tr>
<td>Fin. unc.</td>
<td>-0.48</td>
<td>-0.34</td>
<td>0.29</td>
<td>0.54</td>
<td>0.45</td>
<td>0.61</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Note: This table contains correlations of the exp. skewness factor PC (skew) and different measures of volatility and uncertainty. PC (X), PC (X²), PC (P_{75}-P_{25}), and PC (GARCH) are, respectively, the first principal component of the McCracken and Ng (2020) dataset, the first principal component of the squared observations (Gorodnichenko and Ng, 2017), the first principal component of the expected interquartile ranges, and the first principal component of the expected individual GARCH standard deviations. Macro unc. and Fin. unc. are the macroeconomic and financial uncertainty indices (quarterly averages) developed in Jurado et al. (2015) and Ludvigson et al. (2021).

Figure D-5: Revisions in expected skewness

(a) Exp. skewness factor and its revisions

(b) Revisions in exp. skewness and MBC shock

Note: The left panel shows the expected skewness factor and its revisions obtained from the VAR analysis. The blue shaded areas are the bootstrapped confidence bands (90%) around the skewness factor based on Goncalves and Perron (2020). The dark gray shaded areas are the 90% HDI of the skewness revisions across all MCMC draws. Light gray areas are NBER recessions. The right panel contrasts revisions in expected skewness with the MBC shock obtained following Angeletos et al. (2020). The reported correlation reflects the one of the median shocks computed across all MCMC draws.
Results of baseline model with GDP skewness

**Figure D-6: Impulse response functions**

Note: Posterior mean responses to a negative one S.D. shock to expected GDP skewness along with the 68% highest density interval. Identification through Cholesky decomposition. Sample period: 1960:Q1–2019:Q4.

**Figure D-7: Forecast error variance contributions**

Note: Posterior mean of the forecast error variance contributions along with the 68% highest density interval.
Results of baseline model with GDP skewness (Adrian et al., 2019)

**Figure D-8: Impulse response functions**

Note: Posterior mean responses to a negative one S.D. shock to expected GDP skewness (Adrian et al., 2019) (blue lines) along with the 68% highest density interval. Identification through Cholesky decomposition. Sample period: 1971:Q1–2019:Q4. The gray lines are the posterior mean responses to a negative one S.D. shock to the expected skewness factor and the green dotted lines are the responses to the MBC shock of Angeletos et al. (2020) (see Figure 2).

**Figure D-9: Forecast error variance contributions**

Note: Posterior mean of the forecast error variance contributions for a shock to expected GDP skewness (Adrian et al., 2019) (blue) along with the 68% highest density interval, for a shock to the expected skewness factor (gray), and for the MBC shock (green) of Angeletos et al. (2020) (see Figure 3).
Results of baseline model with S&P 500 skewness (Dew-Becker, 2022)

**Figure D-10: Impulse response functions**

![Impulse response functions for various macroeconomic variables](image)


**Figure D-11: Forecast error variance contributions**

![Forecast error variance contributions](image)

Note: Posterior mean of the forecast error variance contributions along with the 68% highest density interval.
Results of baseline model with firm-level return skewness (Salgado et al., 2019)

Figure D-12: Impulse response functions

Note: Posterior mean responses to a negative one S.D. shock to the cross-sectional firm-level skewness of stock returns (Salgado et al., 2019) along with the 68% highest density interval. Identification through Cholesky decomposition. Sample period: 1964:Q1–2015:Q1.

Figure D-13: Forecast error variance contributions

Note: Posterior mean of the forecast error variance contributions along with the 68% highest density interval.
Appendix E  Augmented models including financial variables

Results of model augmented with excess returns and term premium

Figure E-1: Impulse response functions

Note: Posterior mean responses to a negative one S.D. shock to expected skewness along with the 68% highest density interval. Identification through Cholesky decomposition. Sample period: 1961:Q3–2019:Q4.

Figure E-2: Forecast error variance contributions

Note: Posterior mean of the forecast error variance contributions along with the 68% highest density interval.
Results of model augmented with house prices and stock prices

**Figure E-3: Impulse response functions**

Note: Posterior mean responses to a negative one S.D. shock to expected skewness along with the 68% highest density interval. Identification through Cholesky decomposition. Sample period: 1975:Q1–2019:Q4.

**Figure E-4: Forecast error variance contributions**

Note: Posterior mean of the forecast error variance contributions along with the 68% highest density interval.
Results of model augmented with government bond yields

**Figure E-5: Impulse response functions**

![Impulse response functions](image_url)

Note: Posterior mean responses to a negative one S.D. shock to expected skewness along with the 68% highest density interval. Identification through Cholesky decomposition. Sample period: 1960:Q1–2019:Q4.

**Figure E-6: Forecast error variance contributions**

![Forecast error variance contributions](image_url)

Note: Posterior mean of the forecast error variance contributions along with the 68% highest density interval.
Appendix F  Robustness checks

Results of baseline model with max-share identification approach

Figure F-1: Impulse response functions

![Figure F-1](image)

Note: Posterior mean responses to a negative one S.D. shock to expected skewness along with the 68% highest density interval. Identification through max-share approach (Uhlig, 2003). Sample period: 1960:Q1–2019:Q4.

Model results controlling for (GARCH) volatility

Figure F-2: Impulse response functions

![Figure F-2](image)

Note: Posterior mean responses to a negative one S.D. shock to expected skewness along with the 68% highest density interval. Identification through Cholesky decomposition. Sample period: 1960:Q1–2019:Q4.
Model results controlling for macroeconomic and financial uncertainty

**Figure F-3: Impulse response functions**

Note: Posterior mean responses to a negative one S.D. shock to expected skewness along with the 68% highest density interval. Identification through Cholesky decomposition. Sample period: 1960:Q3–2019:Q4.

Model results controlling for geopolitical risk

**Figure F-4: Impulse response functions**

Note: Posterior mean responses to a negative one S.D. shock to expected skewness along with the 68% highest density interval. Identification through Cholesky decomposition. Sample period: 1960:Q1–2019:Q4.
Model results controlling for excess bond premium

Figure F-5: Impulse response functions

Note: Posterior mean responses to a negative one S.D. shock to expected skewness along with the 68% highest density interval. Identification through Cholesky decomposition. Sample period: 1973:Q1–2019:Q4.

Model results controlling for TFP growth

Figure F-6: Impulse response functions

Note: Posterior mean responses to a negative one S.D. shock to expected skewness along with the 68% highest density interval. Identification through Cholesky decomposition. Sample period: 1960:Q1–2019:Q4.
Model results controlling for fiscal policy shocks

Figure F-7: Impulse response functions

Note: Posterior mean responses to a negative one S.D. shock to expected skewness along with the 68% highest density interval. Identification through Cholesky decomposition. Sample period: 1960:Q1–2015:Q4.

Model results controlling for monetary policy shocks

Figure F-8: Impulse response functions

Note: Posterior mean responses to a negative one S.D. shock to expected skewness along with the 68% highest density interval. Identification through Cholesky decomposition. Sample period: 1990:Q1–2016:Q4.
Results of baseline model with lag order $P = 4$

Figure F-9: Impulse response functions

![Graphs showing the response of various economic indicators to a negative one S.D. shock to expected skewness. Each graph represents the posterior mean responses along with the 68% highest density interval. Identification through Cholesky decomposition. Sample period: 1960:Q1–2019:Q4.]

Note: Posterior mean responses to a negative one S.D. shock to expected skewness along with the 68% highest density interval. Identification through Cholesky decomposition. Sample period: 1960:Q1–2019:Q4.

Results of baseline model with looser prior configuration in VAR ($\lambda = 10$)

Figure F-10: Impulse response functions

![Graphs showing the response of various economic indicators to a negative one S.D. shock to expected skewness. Each graph represents the posterior mean responses along with the 68% highest density interval. Identification through Cholesky decomposition. Sample period: 1960:Q1–2019:Q4.]

Note: Posterior mean responses to a negative one S.D. shock to expected skewness along with the 68% highest density interval. Identification through Cholesky decomposition. Sample period: 1960:Q1–2019:Q4.
References


