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Yahong Li, Kexian Li, Xiangwei Zeng, Yuegang Fu, Peter J. Bryanston-Cross, "Three-dimensional polarization ray-tracing Mueller algorithm for an optical system with arbitrary surface type," Proc. SPIE 12935, Fourteenth International Conference on Information Optics and Photonics (CIOP 2023), 1293553 (24 November 2023); doi: 10.1117/12.3008229

Event: Fourteenth International Conference on Information Optics and Photonics (CIOP 2023), 2023, Xi'an, China

Three-dimensional polarization ray-tracing Mueller algorithm for an optical system with arbitrary surface type

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ABSTRACT

In order to investigate the relationship between the interface parameters of an optical interface/system and its polarization characteristics, a three-dimensional (3D) polarization ray-tracing Mueller algorithm is proposed in this paper. Firstly, using the optical design simulation software CODE V or ZEMAX, the vector modeling of the optical system and the pupil sampling or field of view sampling of the incident light are carried out. Secondly, according to the surface type of each optical interface in the optical system and whether the optical coating is plated, the 3D polarization ray-tracing of each optical interface is carried out, and the 3D Mueller matrix $\mathbf{M}_l$ (9×9 order) of each optical interface under the respective local coordinate system is calculated. Then, a 3×3 order rotation transformation matrix $\mathbf{R}$ is introduced by using the rotation transformation of the global coordinate system, and the 3D Mueller matrix $\mathbf{M}_g$ (9×9 order) of each optical interface under the global coordinate system is obtained. Based on the 3D polarization algorithm proposed in our published paper [1], the 3D Mueller matrix $\mathbf{M}$ of each sampled ray through whole optical system is calculated. Finally, if the polarization state of the incident light of the optical system is known, the polarization state of the emitted light can be accurately calculated. Especially, the 3D polarization ray-tracing Mueller algorithm is not only suitable for handle the totally, partial and unpolarized light through the optical system, but also suitable for quantitative calculation of the polarization properties of an arbitrary surface, including spherical/aspherical/free-form surface.

Keywords: 3D polarization, polarization ray-tracing, Mueller algorithm, optical system, surface type

1. INTRODUCTION

How to describe the polarization characteristics of light waves and accurately deal with the interaction between polarized light and an optical interface/system, several polarization algorithms have been proposed, such as the three-dimensional (3D) polarization ray-tracing calculus proposed by Russell A. Chipman based on the 3D extension of Jones vector[2]. The 3D Jones vector (3×1 order) represents the polarization characteristics of the incident and emergent light waves, and the 3D Jones matrix (order 3×3) represents the retardance [3] and diattenuation [4] of an optical system. G. Yun proposed the 3D polarization ray-tracing tensor (3×3×3×3 order) by using the 3D coherence matrix (3×3 order). Compared with the 3D polarization ray-tracing calculus proposed by Russell A. Chipman, the 3D polarization ray-tracing tensor can be used to calculate all the polarization characteristics of an optical system, including the de-polarization [5]. In addition, Colin J. R. Sheppard derived the 3D generalized form of Stokes vector based on Chandrasekhar phase basis matrix (3×3 order), and proposed the mathematical model of polarization action of 3D Mueller matrix (9×9 order) [6]. Meanwhile, based on 3 × 3 coherency matrix, a 3D polarization ray tracing calculus (3×3 order) for partially polarized light has been introduced by H. Zhang et al [7]. However, these existing 3D polarization algorithms have not explored the quantitative relationship between the 3D polarization matrix (i.e., Jones matrix, Mueller matrix) or 3D polarization tensor and the interface parameters in an optical system.

In general, an optical system contains multiple optical interfaces. In addition to optimizing the design of each optical interface parameter from the perspective of imaging performance, it is often necessary to comprehensively consider multiple factors to meet the transmission or reflection energy requirements. It will inevitably cause the changes of the polarization properties for the entire optical system to affect the imaging quality of the system. Especially for some polarization-sensitive optical systems, such as high NA microscope objective [8-10], lithographic projection objective [11] or large aperture astronomical telescope [12], the polarization properties must be strictly controlled during the

Fourteenth International Conference on Information Optics and Photonics (CIOP 2023),
edited by Yue Yang, Proc. of SPIE Vol. 12935, 1293553
© 2023 SPIE · 0277-786X · doi: 10.1117/12.3008229

optical design of the system. Therefore, how to quantitatively trace the influence of each optical interface parameter on the polarization properties of the optical system, which provides a strong theoretical basis for high-performance optical imaging to guide the optimization design.

2. VECTOR MODELING OF AN OPTICAL SYSTEM

Vector modeling of the optical interface/system under study and the incident light is required. There are two main steps: 1) the definition of the global coordinate system of an optical interface/system: is a right-handed coordinate system {XYZ} with the Z-axis along the optical axis, and the direction of light propagation is always Z-axis forward. 2) the definition of the local coordinate system of each sampled ray incident on the optical system: is a right-handed coordinate system $\{x_i y_i z_i\}$ with the ray propagation vector $\boldsymbol{k}_i$ as the local coordinate system Z-axis. Taking a single lens as an example, the definitions of global and local coordinate systems are shown in Fig. 1(a).

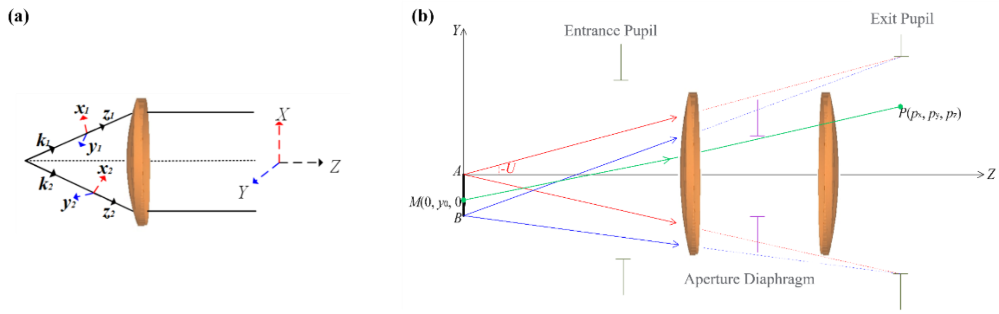

Figure 1 Definitions of the local coordinate system for an optical system and global coordinate system for each ray

Firstly, before vector ray tracing is performed on an optical interface/system, the propagation vector of incident light must be modeled. The linear field of view (i.e., object height h) is taken as an example, as shown in Fig. 1(b).
The origin of the global coordinate system {XYZ} of the optical system is located at the central object point, the known object height AB is $\pm h/2$, the entry pupil diameter is D_{ent}, the entrance pupil distance is P_z, and the incident light is sampled by m×m at the entrance pupil. The sampling range in the x and y directions is -D/2 to D/2. Similarly, the linear field of view (FoV) is sampled by 1×n, and the sampling range is -h/2 to h/2. For any object point M, its coordinates in the global coordinate system {XYZ} is M (0, y_0, 0). Once the object point coordinates are fixed, the FoV of the optical system is also fixed. Different incident rays only depend on pupil sampling location in the entrance pupil plane. When the x and y coordinates of the sampling point P are correspondingly Px and Py, the P coordinates in the global coordinate system {XYZ} is P=(Px,Py,Pz). Thus, the propagation vector of an incident ray $\boldsymbol{k}_i$ can be uniquely determined as

$$\boldsymbol{k}_i = MP/\left|MP\right| = \pm \frac{(p_x, p_y - h_0, p_z)}{\sqrt{p_x^2 + (p_x - h_0)^2 + p_z^2}} \tag{1}$$

where the ± sign depends on the distance between the entrance pupil position P_z and the global coordinate origin. When the entrance pupil distance is positive P_z>0, the expression of the propagation vector of incident ray is + sign, otherwise, it is - sign.

Secondly, on the premise that the propagation vector $\boldsymbol{k}_i$ of the incident ray is known, combined with the surface equation of the incident optical interface, such as plane, sphere, aspherical or free-form surface, the global coordinates of the incident point of the incident ray arriving at the incident optical interface can be uniquely determined, that is, the incident point P_1. According to the definition of normal vector, the vector direction connecting the center of incident optical interface C1 and the incident point P_1 is the normal vector N_1. In addition, it is noted in particular that the normal direction of any incident ray always points from the exiting medium n2 to the incident medium n1, and the normal vector is directly related to the surface shape of the optical interface.

Finally, under the global coordinate system {XYZ}, based on the vector Snell law [13, 14], the vector ray tracing of each optical interface in the optical system is completed. It is obtained that the propagation vectors of the incident and emitted rays on each optical interface, namely $\boldsymbol{k}_{in}$ and $\boldsymbol{k}_{out}$. Furthermore, the local coordinate system $\{x_i y_i z_i\}$ corresponding to each optical interface is uniquely determined by,

$$\begin{cases} \mathbf{z}_{in} = \boldsymbol{k}_{in} & \mathbf{z}_{out} = \boldsymbol{k}_{out} \\ \mathbf{y}_{out} = \dfrac{\mathbf{z}_{in} \times \mathbf{z}_{out}}{\left|\mathbf{z}_{in} \times \mathbf{z}_{out}\right|} & \mathbf{x}_{out} = \mathbf{z}_{in} \times \mathbf{y}_{out} \\ \mathbf{y}_{out}{}' = \mathbf{y}_{out} & \mathbf{x}_{out}{}' = \mathbf{z}_{out} \times \mathbf{y}_{out} \end{cases} \tag{2}$$

where the bold x, y and z represent the direction of the three axes of the local coordinate system, x_{out}'and y_{out}' represent the local coordinate system of the reflected rays, x_{out} and y_{out} represent the local coordinate system of transmitted rays.

3. 3D POLARIZATION RAY-TRACING MUELLER ALGORITHM

The local coordinate system$\{x_i y_i z_i\}$ is defined in the z-axis positive always along the ray propagation vector $\boldsymbol{k}$. Since the electric field vector of a light wave is a transverse wave, the z-electric field component in the local coordinate system is always 0. When the propagated ray incident on the optical interface is reflected or transmitted, its effect on the polarization of the incident ray in the local coordinate system $\{x_i y_i z_i\}$ can be expressed by,

$$\begin{pmatrix} E_{t\_x} \\ E_{t\_y} \\ 0 \end{pmatrix} = \begin{pmatrix} t_s & 0 & 0 \\ 0 & t_p & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} E_x \\ E_y \\ 0 \end{pmatrix} \quad \begin{pmatrix} E_{r\_x} \\ E_{r\_y} \\ 0 \end{pmatrix} = \begin{pmatrix} r_s & 0 & 0 \\ 0 & r_p & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} E_x \\ E_y \\ 0 \end{pmatrix} \tag{3}$$

where the symbol t and r mean the reflected or transmitted ray, E_x and E_y are the x-field component and y-field component of incident ray, t_s and t_p is the Fresnel coefficient of transmitted ray, r_s and r_p is the Fresnel coefficient of reflected ray.

Obviously, the polarization calculation of the above optical interface concluded in Eq. (3) is obtained in the corresponding local coordinate system. In order to maintain the consistency of the polarization calculation results of each optical interface, the polarization calculation result of each optical interface must be converted to the same coordinate system, that is, the global coordinate system {XYZ}. To this end, the rotation transformation matrix between the local coordinate system $\{x_i y_i z_i\}$and the global coordinate system {XYZ} is introduced, that is, a 3×3 order matrix $\boldsymbol{R}$,

$$\boldsymbol{R} = \begin{pmatrix} cos\alpha & 0 & sin\alpha \\ 0 & 1 & 0 \\ -sin\alpha & 0 & cos\alpha \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & cos\beta & sin\beta \\ 0 & -sin\beta & cos\beta \end{pmatrix} \tag{4}$$

where α and β are the two rotation angles, and only depends on the propagation vector of the light,

$$\boldsymbol{k} = (a,b,c)^T \quad \alpha = arctan(b/c) \quad \beta = arcsin(a/\sqrt{a^2+b^2+c^2}) \tag{5}$$

It is noticed that the polarization calculation of an optical interface in a local coordinate system can be written to 2D Jones matrix and 2D Mueller matrix by reduced dimension, and the inherent relationship between them can be derived by,

$$\boldsymbol{M} = \boldsymbol{A} \cdot < \boldsymbol{J} \otimes \boldsymbol{J}^{\dagger} > \cdot \boldsymbol{A} \quad \boldsymbol{A} = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & -1 \\ 0 & 1 & 1 & 0 \\ 0 & i & -i & 0 \end{pmatrix} \tag{6}$$

Refer to the mathematical representation of 3D polarized light in our previous publication [1], the relationship between 3D coherence vectors and 9×1 order Stokes vectors is derived. Meanwhile, according to the definition of the

3D coherence vector/matrix [15], we can finally deduce the polarization effect of the optical interface on the incident ray according to Eqs.(3)-(6),

$$\mathbf{M} = \boldsymbol{Q}_{9\times9} \cdot \boldsymbol{N}_g \cdot \boldsymbol{Q}_{9\times9}^{-1} \tag{7}$$

$$\boldsymbol{Q}_{9\times9} = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2} & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & \sqrt{3} & 0 & \sqrt{3} & 0 & 0 & 0 & 0 & 0 \\ 0 & i\sqrt{3} & 0 & -i\sqrt{3} & 0 & 0 & 0 & 0 & 0 \\ \sqrt{3} & 0 & 0 & 0 & -\sqrt{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & \sqrt{3} & 0 & 0 & 0 & \sqrt{3} & 0 & 0 \\ 0 & 0 & i\sqrt{3} & 0 & 0 & 0 & -i\sqrt{3} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \sqrt{3} & 0 & \sqrt{3} & 0 \\ 0 & 0 & 0 & 0 & 0 & i\sqrt{3} & 0 & -i\sqrt{3} & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 2\sqrt{2} \end{pmatrix} \tag{8}$$

where ***N***g and **M** are the 3D coherency transformation matrix (9×9 order) and 3D Mueller matrix (9×9 order) of an optical interface in global coordinate system {XYZ}.

The polarization effect of the entire optical system on the incident ray is equal to the sequential left multiplication of the 3D coherency transformation matrix or 3D Mueller matrix corresponding to each optical interface, that is

$$\mathbf{M}_{Total} = \prod_{q=m,-1}^{1} \mathbf{M}_q = \mathbf{M}_m \cdot \ldots \cdot \mathbf{M}_q \cdot \ldots \cdot \mathbf{M}_1 \tag{9}$$

where m is the total number of optical interfaces contained in the optical system, and q is the sequence in which the incident ray passes through each optical interface.

4. CONCLUSIONS

A new 3D polarization ray-tracing Mueller algorithm for an optical system with arbitrary surface type is proposed to explore the quantitative relationship between the surface parameters and the polarization characteristics. It can be used to reverse guide the optimal design of high-performance imaging optical systems. Based on the proposed 3D polarization algorithm, the 3D Mueller matrix of each optical interface/system is calculated, which includes all the polarization characteristics, i.e., retardance, diattenuation and depolarization.

FUNDINGS

National Natural Science Foundations of China (NSFC) (11904044, 62105136).

DISCLOSURES

We declare no conflicts of interest.

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