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Parameter Learning in Production Economies*

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Abstract

We examine how parameter learning amplifies the impact of macroeconomic shocks on equity prices and quantities in a standard production economy where a representative agent has Epstein-Zin preferences. An investor observes technology shocks that follow a regime-switching process but does not know the underlying model parameters governing the short-term and long-run perspectives of economic growth. We show that rational belief updating endogenously generates long-run risks that help explain various asset pricing facts, most prominently, dividend yield variance decomposition. The asset pricing implications of endogenous long-run risks depend crucially on the introduction of a procyclical dividend process.

Keywords: Parameter learning, dividend yield variance decomposition, return predictability, business cycles, Markov switching
JEL: D83, E13, E32, G12

1. Introduction

The key premise of the real business cycle model is to explain aggregate asset prices and macroeconomic dynamics jointly. The literature in macro-finance has proposed several models with various amplification mechanisms that range from parsimonious approaches such as behavioural assumptions (extrapolative beliefs, ambiguity aversion) to

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more complex models with heterogeneous agents. Within the purely rational paradigm, rational pricing of parameter uncertainty has recently been developed to successfully explain standard asset pricing moments in endowment models (Collin-Dufresne, Johannes and Lochstoer, 2016; Johannes, Lochstoer and Mou, 2016; Andrei, Carlin and Hasler, 2019a; Andrei, Hasler and Jeanneret, 2019b). It is unclear, however, what effect it has on asset prices, macroeconomic variables, and predictability patterns in a real business cycle model with an endogenous dividend process.[1] The main objective of this paper is to examine the effect of parameter learning on the economy and asset prices in a production setting where parameter uncertainty is rationally priced in equilibrium.

This paper examines a production economy that features (1) a two-state Markov-switching process for productivity growth, (2) Bayesian learning about unknown parameters in the technology process, (3) asymmetric quadratic adjustment costs, (4) financial leverage, and (5) recursive preferences. We follow Collin-Dufresne et al. (2016) and analyze the implications of fully rational pricing of parameter uncertainty in the technology process for asset prices and quantities. Our analysis focuses on uncertainty about parameters governing the persistence and magnitude of economic growth. Specifically, we study learning about transition probabilities and mean growth rates in the two-state process for productivity growth with known and constant volatility across states.

Incorporating rational parameter learning into a parsimonious real business cycle model can simultaneously generate several important stylized facts in the data. First, it helps capture key moments of financial variables. Intuitively, rational pricing of belief updating gives rise to subjective, long-lasting macroeconomic risks. Coupled with endogenous long-run consumption risks due to consumption smoothing (Kaltenbrunner and Lochstoer, 2010), these risks are priced under the investor's preference for early resolution of uncertainty. Quantitatively, this yields a two-fold increase in the risk premium and Sharpe ratios on a levered dividends claim, a two-fold decline (increase) in the mean

[1] The reason is that some mechanisms successfully studied in exchange economies do not extend well to production economies. See, for example, Rouwenhorst (1995), Jermann (1998), Lettau and Uhlig (2000), Boldrin, Christiano and Fisher (2001), Cochrane (2007), and Kaltenbrunner and Lochstoer (2010) among others for a discussion why it is more challenging to reconcile aggregate quantities and asset prices jointly.

(volatility) of interest rates, and a significant decline in the average price-dividend ratio relative to the known parameter economy. Meanwhile, parameter learning has quantitatively minimal effects on macroeconomic moments. This finding echoes Tallarini (2000), Campanale, Castro and Clementi (2010), and Liu and Miao (2015), who find no effect of increasing agents' sensitivity to risk on macroeconomic variables.

Second, a model with rational parameter learning generates a realistic dividend yield variance decomposition (Cochrane, 2011). The lower interest rates reduce the sensitivity of dividends to changes in capital through financial leverage, decreasing the volatility of dividend growth and the price-dividend ratio. A combination of more volatile equity returns due to parameter learning permanent risks and less volatile dividend yields and cash flows leads to a higher fraction of dividend yield variation explained by discount rates. In the model with known parameters, however, all price-dividend ratio volatility is driven by the variation in dividend growth. This demonstrates that rational parameter learning not only serves as risk magnification, which helps generate realistic asset prices with empirically consistent aggregate quantities, but it attributes the variability of dividend yield primarily to expected returns and less to dividend growth, a novel implication for drivers of endogenous prices and cash flows.

Third, time-varying posterior beliefs and their rational pricing are crucial for reproducing the long-term forecastability of excess returns by macroeconomic and financial ratios. The time variation in beliefs leads to fluctuations in the equity risk premium. Fully rational pricing of parameter uncertainty further magnifies the impact of belief revisions on the conditional equity premium, generating stronger predictability in the model with parameter uncertainty relative to the known parameter framework. Interestingly, unlike long-run risk models, we show that the endogenous long-run risks implied by parameter learning are consistent with weak consumption growth predictability.

We compare priced parameter uncertainty (PPU) with a common approach to dealing with uncertain parameters in general equilibrium models called anticipated utility (AU) (Kreps, 1998; Cogley and Sargent, 2008; Piazzesi, Schneider et al., 2009). With PPU, the investor accounts for future changes in posterior beliefs when calculating current utility and prices. In contrast, the AU agent learns about unknown parameters over time but treats current beliefs as true and fixed parameter values in the decision-making. Consequently, fluctuations in parameter beliefs are not priced in equilibrium with AU. This model generates virtually the same results as the full information framework. Despite time-varying beliefs, the AU model cannot capture the long-horizon predictability of excess returns and predicts that cash flows mainly drive dividend yields. Intuitively, the firm's optimal investment allows the household to easily smooth consumption and eliminate consumption risks generated by myopic AU pricing.

We perform several checks to understand better the key elements leading to our conclusions. First, asymmetric adjustment costs and financial leverage enable the model to generate procyclical dividends: the former reduces the decline in investment during the recession, and the latter ensures that dividends are affected by the procyclical net balance of the long-term debt. We consider the models with symmetric quadratic and convex adjustment costs to evaluate the role of procyclical dividends. In both cases, the resulting countercyclical dividends substantially reduce the amplification effect of parameter uncertainty. Thus, it is critical to generate procyclical payoffs to obtain a significant impact of rational learning. Second, we consider alternative mechanisms — increasing risk aversion and leverage in the economy — to amplify financial moments. If parameter uncertainty is turned off, then a much higher risk aversion and counterfactually high leverage of more than 200% are required to match the equity premium. Yet, the fraction of discount rates in dividend yield variance decomposition is too small in both cases. Finally, we study the model with unknown transition probability and document that asset pricing implications of PPU diminish marginally. Thus, hidden persistence has the most significant asset pricing effect, extending the results in endowment economies.

Methodologically, our paper is the closest to Collin-Dufresne et al. (2016) studying PPU in the endowment economy. Our paper is the first to apply this learning-based theory to a real business cycle model. Unlike the endowment model, the production economy needs to generate procyclical dividends to amplify asset pricing moments significantly. Moreover, rather than exploring learning about rare events (Rietz, 1988; Barro, 2006), we consider learning about parameters governing business cycle dynamics. We show that long-run consumption risks generated by consumption smoothing magnify the impact of endogenous belief revisions on asset prices; therefore, less is needed in terms of the speed of parameter learning in the production economy.

Our paper speaks to production-based asset pricing under imperfect information. This includes models with subjective expectations (Cagetti, Hansen, Sargent and Williams, 2002), state uncertainty and ambiguity (Jahan-Parvar and Liu, 2014), and extrapolative expectations (Hirshleifer, Li and Yu, 2015). We complement their results by showing that salient features of asset prices can be explained in the model with Bayesian learning without resorting to behavioral biases or preferences, provided that PPU is applied. Ai (2010), Liu and Miao (2015), and Liu and Zhang (2022) examine a model with a production sector and state learning but price exogenous dividends to explain asset pricing phenomena, in contrast to a production economy with parameter uncertainty and endogenous cash flows in our case. Winkler (2020) analyzes learning about endogenous prices and shows a strong amplification of macroeconomic moments and no effect on risk premiums by price learning, whereas PPU generates pronounced asset pricing results. Finally, none of these other models consider dividend yield variance decomposition.

Davis and Segal (2023) analyze a production economy in which the agent learns about trend and business cycle shocks. The agent's inability to distinguish between the two shocks implies that each shock inherits the properties of its counterpart. This predicts that transitory shocks are the primary driver of long-run risk. Further, learning flips the cyclicality of the equity premium, investment, and valuation ratios. The learning problem in our model is distinct from the one in Davis and Segal (2023) because they abstract from parameter uncertainty and its rational pricing, a key focus of our paper. In our model, rational parameter learning becomes instrumental in generating permanent risks, helping capture the long-horizon stock return predictability patterns and variance decomposition of dividend yields.

Our paper also connects to production-based asset pricing models under full information with long-run risks (Kaltenbrunner and Lochstoer, 2010; Croce, 2014), rare disasters (Gourio, 2012), disappointment aversion (Campanale, Castro and Clementi, 2010), external habit (Chen, 2017), search frictions (Bai, 2021; Bai and Zhang, 2022) and richer production economies with heterogeneous agents (Ai, Croce and Li, 2013; Favilukis and

Lin, 2016; Ai, Croce, Diercks and Li, 2018). We differ from these papers by considering an alternative channel — rational parameter learning — for understanding asset returns, predictability patterns, and variance decomposition of dividend yields.

The paper proceeds as follows. Section 2 presents the model. Section 3 provides the calibration. Section 4 reports quantitative analysis. Section 5 concludes.

2. Model

The model is a standard real business-cycle framework (Kydland and Prescott, 1982; Long and Plosser, 1983) populated by a representative household with Epstein-Zin preferences and a representative firm with Cobb-Douglas production technology. The firm produces a consumption-investment good using labor and capital as inputs subject to productivity shocks and capital adjustment costs. The household participates in the production process by working for the firm and providing investment for capital. Additionally, the household trades firm shares and risk-free bonds to maximize the lifetime utility of a consumption stream subject to a sequential budget constraint. The firm maximizes its value by choosing labor and investment demand.

2.1. Household

A household has recursive preferences of Epstein and Zin (1989):

$$U_t = \left\{ (1-\beta) V_t^{1-1/\psi} + \beta \left(E_t \left[U_{t+1}^{1-\gamma} \right] \right)^{\frac{1-1/\psi}{1-\gamma}} \right\}^{\frac{1}{1-1/\psi}}. \tag{1}$$

Here the utility index V_t depends on both consumption C_t and hours worked N_t through a standard Cobb-Douglas form:

$$V_t = C_t (1 - N_t)^{\nu}, \tag{2}$$

where $\nu > 0$ is the leisure preference. We include leisure in a household's utility function following Gourio (2012). $E_t[\cdot]$ is the expectation operator, $\beta \in (0, 1)$ is the discount factor, $\gamma > 0$ is the parameter controlling risk aversion, and $\psi > 0$ is the elasticity of intertemporal substitution (EIS). Note that endogenous labor allows the household to absorb asset return shocks through labor hours, altering the household's attitude towards risk. The labor margin has a significant effect in the specification with additively separable utility (Swanson, 2012) or recursive preferences (Swanson, 2018). We refer to those studies for a detailed discussion of endogenous-labor measures of risk aversion.

2.2. Firm

The representative firm produces the consumption good using a constant return to scale Cobb-Douglas production function:

$$Y_t = K_t^{\alpha}(A_t N_t)^{1-\alpha},$$

where Y_t is the output, K_t is the capital stock, N_t is labor hours, and A_t is an exogenous, labor-enhancing technology level (which we also refer to as productivity). The firm's capital accumulation equation incorporates capital adjustment costs and is given by:

$$K_{t+1} = (1-\delta)K_t + \varphi(I_t/K_t)K_t,$$

where $\delta \in (0,1)$ is the capital depreciation rate, $I_t = Y_t - C_t$ is a gross investment, and $\varphi(\cdot)$ is the capital adjustment cost function. We adopt an asymmetric capital adjustment cost function (Abel and Eberly, 1996; Zhang, 2005), which takes a quadratic form:

$$\varphi(x_t) = x_t - \frac{\theta_t}{2}\cdot(x_t - x_0)^2,\ \theta_t = \theta^+ \cdot \mathbb{I}(x_t \geq x_0) + \theta^- \cdot \mathbb{I}(x_t < x_0),$$

where $\mathbb{I}(\cdot)$ denotes the indicator operator that equals 1 if the condition is satisfied and 0 otherwise. We choose the constant x_0 such that there are no adjustment costs in the non-stochastic steady state, which implies $x_0 = \exp(\bar{\mu}) - 1 + \delta$. The remaining two parameters θ^+ and θ^- satisfy the condition $0 < \theta^+ < \theta^-$ to ensure that the representative firm faces higher capital adjustment costs for low investments.

2.3. Technology

Productivity growth follows a two-state Markov switching model:

$$\Delta a_t = \mu_{s_t} + \sigma\varepsilon_t, \quad \varepsilon_t \overset{\text{iid}}{\sim} N(0,1), \tag{3}$$

where Δa_t is the log productivity growth, s_t is a Markov chain with a transition matrix $\Pi = [\pi_{ij}]_{i,j=1,2}$, with $\pi_{ij} \in (0,1)$. We label $s_t = 1$ the "good" regime with high productivity growth and $s_t = 2$ the "bad" regime with low productivity growth.

2.4. Asset Prices

Following Gourio (2012), the representative household's stochastic discount factor (SDF) for recursive preferences is:

$$M_{t+1} = \beta \left(\frac{C_{t+1}}{C_t}\right)^{-\frac{1}{\psi}} \left(\frac{1-N_{t+1}}{1-N_t}\right)^{(1-\frac{1}{\psi})\nu} \left(\frac{U_{t+1}}{\left(E_t\left[U_{t+1}^{1-\gamma}\right]\right)^{\frac{1}{1-\gamma}}}\right)^{\frac{1}{\psi}-\gamma}. \tag{4}$$

A calibration with $1/\psi \neq \gamma$ implies the utility function is not time-additive. The last multiplicative term in the SDF reflects the household's preferences for an earlier resolution of uncertainty in the calibration with $\gamma > \frac{1}{\psi}$. In this case, when the household's continuation utility U_{t+1} is below the certainty equivalent, the last multiplier in the pricing kernel increases, raising a premium for future low utility states.

In equilibrium, the following condition for a gross return $R_{j,t+1}$ holds:

$$E_t\left[M_{t+1}R_{j,t+1}\right] = 1. \tag{5}$$

In particular, the equation above holds for the investment return $R_{I,t+1}$:

$$R_{I,t+1} = \frac{1}{Q_t}\left[Q_{t+1}\left(1-\delta+\varphi\left(\frac{I_{t+1}}{K_{t+1}}\right)\right) + \frac{\alpha Y_{t+1} - I_{t+1}}{K_{t+1}}\right], \tag{6}$$

where Q_t is Tobin's marginal Q defined as $Q_t = \frac{1}{\varphi'\left(\frac{I_t}{K_t}\right)}$. The return on investment can be interpreted as the return of an equity claim to the unlevered firm's payouts (Restoy and Rockinger, 1994). As the firm behaves competitively, the labor input is chosen at a level equal to its marginal product:

$$w_t = \partial Y_t/\partial N_t = (1-\alpha)A_t^{1-\alpha}K_t^{\alpha}N_t^{-\alpha} = (1-\alpha)Y_t/N_t. \tag{7}$$

The unlevered firm value FV_t is given by $FV_t = Q_t K_{t+1}$, and the firm's unlevered dividends D_t^u are defined by:

$$D_t^u = Y_t - w_t N_t - I_t \overset{(7)}{=} \alpha Y_t - I_t. \tag{8}$$

The equity prices observed on the market are for levered corporations in contrast to unlevered dividend payments of production companies in the model. Since the observed aggregate stock market dividends are not directly comparable to the endogenous payouts, we consider pricing levered equity claims. We introduce financial leverage in the spirit of Jermann (1998). The Modigliani and Miller conditions hold; hence, the financial leverage does not change the equilibrium allocations. It only influences the dynamics of a firm's payouts and how we report the returns on a claim to its dividends. In particular, financial leverage increases the volatility of dividends and makes equity returns riskier.

Following Liu and Miao (2015), we assume that the firm issues n-period discount bonds and pays back its outstanding debt of n-period maturity in each period. The fraction ω of the firm's capital K_t at time t is financed by long-term bonds. Let $B_{t,n}$ denote the price of the n-period discount bonds in period t and assume the payoff of this bond is equal to one at all states n periods ahead. Define the levered dividends as:

$$D_t = \alpha Y_t - I_t + \omega \left(K_t - K_{t-n} / B_{t-n,n} \right), \tag{9}$$

where $\alpha Y_t - I_t$ is the operating cash flow of an unlevered claim, ωK_t denotes proceeds from newly issued bonds at time t, and $\omega K_{t-n} / B_{t-n,n}$ represents repayments of bonds purchased at time $t - n$ and price $B_{t-n,n}$.

The prices of the n-period bonds are defined recursively by:

$$B_{t,n} = E_t \left[M_{t+1} B_{t+1,n-1} \right], \tag{10}$$

with the boundary condition $B_{t,0} = 1$ for any t. We denote the price of the levered equity claim by P_t and the levered equity return by $R_{t+1} = (P_{t+1} + D_{t+1}) / P_t$. As is well known, one can readily compute the equity price as $P_t = FV_t - DV_t$, where FV_t is a firm's value and DV_t is a net balance of the long-term debt issued over the period from $t - n + 1$ to t. The quantities FV_t and DV_t satisfy the conditions:

$$FV_t = Q_t K_{t+1} \quad \text{and} \quad DV_t = \sum_{j=1}^{n} \frac{B_{t,j} \omega K_{t-n+j}}{B_{t-n+j,n}}. \tag{11}$$

2.5. Equilibrium Characterization and Model Solution

This section presents the equilibrium conditions in recursive form. In each period, the social planner chooses consumption C_t, investment I_t, and labor supply N_t to maximize the household's utility subject to the resource constraint and the law of motion of the capital accumulation.[2] Due to the homogeneity of recursive preferences and the common trend A_t of quantities, we can reformulate the problem in terms of stationary variables $\{\tilde{C}_t, \tilde{I}_t, \tilde{Y}_t, \tilde{K}_t, \tilde{U}_t\} = \left\{\frac{C_t}{A_t}, \frac{I_t}{A_t}, \frac{Y_t}{A_t}, \frac{K_t}{A_t}, \frac{U_t}{A_t}\right\}$. Formally, the social planner's maximization problem is defined by the Bellman equation:

$$\tilde{U}_t = \max_{\tilde{C}_t, \tilde{I}_t, N_t} \left\{ (1-\beta)\tilde{V}_t^{1-\frac{1}{\psi}} + \beta \left(E_t \left[\tilde{U}_{t+1}^{1-\gamma} \cdot e^{(1-\gamma)\Delta a_{t+1}} \right] \right)^{\frac{1-\frac{1}{\psi}}{1-\gamma}} \right\}^{\frac{1}{1-\psi}} \tag{12}$$

subject to the constraints:

$$\tilde{V}_t = \tilde{C}_t (1 - N_t)^{\nu} \tag{13}$$

$$\tilde{C}_t + \tilde{I}_t = \tilde{K}_t^{\alpha} N_t^{1-\alpha} \tag{14}$$

$$e^{\Delta a_{t+1}} \tilde{K}_{t+1} = (1-\delta)\tilde{K}_t + \varphi\left(\frac{\tilde{I}_t}{\tilde{K}_t}\right)\tilde{K}_t \tag{15}$$

$$\Delta a_t = \mu_{s_t} + \sigma\varepsilon_t, \quad \varepsilon_t \sim N(0,1). \tag{16}$$

The main aim of this paper is to adopt PPU developed in the consumption-based setting by Collin-Dufresne et al. (2016) to the production-based setting and explore its implications for the economy and asset prices. In doing so, we compare the models with (1) full information (FI), (2) unknown parameters using anticipated utility pricing (AU) (Kreps, 1998; Cogley and Sargent, 2008; Piazzesi et al., 2009), and (3) unknown parameters using fully rational parameter uncertainty pricing (PPU) (Collin-Dufresne et al., 2016). The remainder of this section briefly discusses solution methods for the three cases (the detailed description of methodologies is in Internet Appendices A-C).

First, the solution method for the model with known parameters is standard. One needs to solve the Bellman equation through value function iteration and to find the equilibrium utility as a function of the productivity growth regime and capital. Second, AU assumes investors learn about unknown parameters over time, but in each period they treat their current mean beliefs as true values. In each period, the current and future

[2]The economy can also be decentralized following a standard approach: the household works for the firm and trades its shares and risk-free bonds to maximize the lifetime utility over a consumption stream, while the firm chooses labor and capital to maximize its value, the present value of future cash flows.

utilities depend only on the state of the economy and capital because the true parameter values are set to the current mean beliefs, which are assumed to remain unchanged in the future. In this case, the numerical solution for the model with AU pricing of parameter uncertainty simplifies to solving the Bellman equation when all parameters in productivity growth are known and equal to the current mean beliefs.

Third, Collin-Dufresne et al. (2016) has developed solution methods for priced parameter uncertainty when agents rationally acknowledge future changes in beliefs in equilibrium. Figure 1 illustrates their approach, which consists of two steps. First, the household will have learned the true parameters when a long data history becomes available. Thus, the household begins solving the boundary economy with all known parameters. Second, the household applies the recursive equilibrium conditions to go backward and compute the utility starting from boundary conditions. This procedure generates the parameter learning-generated, permanent risks, which are priced with recursive utility. This is because posterior beliefs of parameters are martingales and hence Bayesian learning produces permanent shocks to agents' expectations.

[Insert Figure 1 about here]

Here, we sketch the backward recursion used in the second step. Having found the limiting boundary economies from the first step, we then perform a backward recursion using the following state variables: the regime of the economy s_t, capital $\tilde{K}_t$, and the vector X_t of sufficient statistics for the priors. We employ standard, conjugate priors distributions for the unknown parameters in the productivity growth process (e.g., beta, normal, and inverse gamma distributions for the transition probabilities, mean parameters, and variance parameters, respectively). Since the state s_t is observable, we can update $X_{t+1} = g(s_{t+1}, \Delta a_{t+1}, s_t, X_t)$ via standard Bayes' rule. Further, the law of motion of capital implies that $\tilde{K}_{t+1} = f(\Delta a_{t+1}, s_t, \tilde{K}_t, X_t)$ is the function of the observed realized productivity growth, the state, capital, and sufficient statistics.

Given this, we can now write the equilibrium utility as:

$$\tilde{U}_{t+1}(s_{t+1}, \tilde{K}_{t+1}, X_{t+1}) = \tilde{U}_{t+1}(s_{t+1}, \Delta a_{t+1}, s_t, \tilde{K}_t, X_t)$$

to indicate that the utility evolution is the function of the two observable variables, s_{t+1} and Δa_{t+1}. Using these notations, we can rewrite the Bellman equation as follows:

$$\tilde{U}_t(s_t, \tilde{K}_t, X_t) = \max_{\tilde{C}_t, \tilde{I}_t, N_t} \left\{ (1-\beta)\tilde{V}_t^{1-\frac{1}{\psi}} + \beta \left(E_t \left[\tilde{U}_{t+1}^{1-\gamma}(s_{t+1}, \Delta a_{t+1}, s_t, \tilde{K}_t, X_t) \cdot e^{(1-\gamma)\Delta a_{t+1}} \middle| s_t, \tilde{K}_t, X_t \right] \right)^{\frac{1-\frac{1}{\psi}}{1-\gamma}} \right\}^{\frac{1}{1-\psi}} \tag{17}$$

where the expectation on the right-hand side is equivalent to:

$$E_t \left[\tilde{U}_{t+1}^{1-\gamma} \left(s_{t+1}, \Delta a_{t+1}, s_t, \tilde{K}_t, X_t \right) \cdot e^{(1-\gamma)\Delta a_{t+1}} \middle| s_t, \tilde{K}_t, X_t \right] \tag{18}$$
$$= \sum_{s_{t+1}=1}^{2} E_t(\pi_{s_{t+1}, s_t} | s_t, X_t) E_t \left[\tilde{U}_{t+1}^{1-\gamma} \left(s_{t+1}, \Delta a_{t+1}, s_t, \tilde{K}_t, X_t \right) \cdot e^{(1-\gamma)\Delta a_{t+1}} \middle| s_{t+1}, s_t, \tilde{K}_t, X_t \right].$$

We have an analytical expression for the conditional expectation of transition probabilities and can use quadrature-type numerical methods to evaluate the second expectation in Eq. (18) (see Internet Appendix C). In sum, the backward recursion is completely defined by the constraints (13)-(16), the evolution equation $X_t = g(s_{t+1}, s_t, \Delta a_{t+1}, X_t)$ given by standard Bayes' rule, and the recursive Bellman equation (17)-(18).

3. Calibration

This section calibrates parameter values in a production economy, discusses the importance of uncertainty about different parameters in a productivity growth process, and then describes the set of unknown parameters and the evolution of beliefs.

3.1. Parameter Values

Panel A in Table 1 reports the parameter values of an investor's preferences, production and capital adjustment cost functions. The coefficient that controls risk aversion (γ) is equal to 10.[3] The subjective discount factor (β) is set to 0.995. This value allows the

[3]We need to carefully interpret this number due to the presence of leisure in our model. If the period utility is given by the Cobb-Douglas function with constant returns to scale (see, e.g, Van Binsbergen et al., 2012; Gourio, 2013), relative risk aversion is indeed equal to γ. It is worth noting that our specification is an amplified version of Gourio (2012). In this case, Swanson (2018) shows that the coefficient of relative risk

benchmark calibration to match the low risk-free rate in the data. There is no consensus in the literature about the value of the elasticity of intertemporal substitution. We follow the disaster risk literature (Gourio, 2012) and long-run risks models (Bansal and Yaron, 2004; Ai, Croce and Li, 2013; Bansal, Kiku, Shaliastovich and Yaron, 2014) by setting EIS (ψ) to 2. Consistent with the real business-cycle literature, the constant capital share in a Cobb-Douglas production function (α) is 0.36. We set the quarterly depreciation rate (δ) to 0.025, which implies an annual rate of 10% (Favilukis and Lin, 2016).

[Insert Table 1 about here]

To calibrate the adjustment cost coefficient (θ^+) and the degree of asymmetry (θ^-/θ^+), we jointly set values of these parameters by using the estimates from prior studies and by matching the volatility of investment and the correlation between consumption and levered dividends. The empirical estimates of θ^+ vary from 2 to 8 in quarterly frequency. We choose a middle point of this range and set $\theta^+ = 5$ as in Zhang (2005). There is limited empirical evidence on the degree of asymmetry. We set $\theta^-/\theta^+ = 20$ in the benchmark calibration. These parameter choices allow the model to match the large volatility of investment and to generate procyclical levered dividends of the firm.

Following Stock and Watson (1999), we use the macroeconomic data to construct the cumulative Solow residuals and scale these residuals by the labor share $(1-\alpha)$ to interpret them as labor-augmenting technology. We estimate a two-state Markov switching process of quarterly productivity growth rates by applying the expectation-maximization algorithm developed by Hamilton (1990). Panel B in Table 1 reports the maximum likelihood estimates for the transition probabilities (π_{ii}), productivity growth rates (μ_i) as well as the constant volatility (σ). Productivity is estimated to grow quarterly at about 0.52 percent in expansions and about -1.86 percent in recessions. The productivity volatility comes out to around 1.47 percent. The transition probability to the expansion (recession) conditional on being in the expansion (recession) is estimated

aversion is roughly $(1+\nu)\gamma$. This implies a risk aversion of 30 in our calibration. Even though this value exceeds an upper bound of the interval considered plausible by Mehra and Prescott (1985), a production model estimated with US data by Van Binsbergen et al. (2012) reveals a higher risk aversion of 65.78.

at around 0.961 (0.625). These numbers imply the average duration of the high-growth expansion state of about 25.64 quarters and the average duration of the low-growth recession of about 2.67 quarters. Our maximum likelihood estimates are consistent with the values reported by Hamilton (1989) and Cagetti, Hansen, Sargent and Williams (2002).

Regarding financial leverage, the firm issues long-term bonds with a maturity of $n = 60$ quarters. For each model, we choose the leverage parameter (ω) to match the average debt-to-equity ratio of around 1:1, similar to Gourio (2012) and Jahan-Parvar and Liu (2014). The calibrated values of ω in different models are in the interval $[0.75\%, 1.00\%]$. The model-implied leverage ratio is consistent with an empirical estimate of the average debt-to-capital ratio of 50% (Rauh and Sufi, 2010; Jermann and Quadrini, 2012).

3.2. Which Parameters Are Important and Why?

We consider a two-state Markov switching process for log productivity growth as in Eq. (3). The household observes the state of the economy. There are five remaining parameters in the productivity growth process. If all parameters are assumed to be unknown, using conjugate priors would result in many state variables associated with parameter uncertainty. Coupled with additional state variables, s_t and $\tilde{K}_t$, and the need to solve the optimization problem for the endogenous investment and labor hours, the problem quickly becomes computationally costly. To reduce computational costs, we identify the parameters for which accounting for parameter uncertainty is particularly important in utility and focus on uncertainty about those most important parameters while assuming others are known.

We identify these parameters by adopting the metric of Collin-Dufresne et al. (2016) used to evaluate the importance of uncertainty about different parameters in the endowment economy. Specifically, we compute the fraction of wealth an agent would pay to resolve all uncertainty by setting parameters equal to their prior mean.[4] Let U_t^{AU} denote the utility in the parameter learning model with AU and U_t^{One} denote the utility in the model where parameters are uncertain but revealed in one period. The *parameter*

[4]Internet Appendix D presents a formal definition. This metric is also related to Lucas (1987) studying the economic importance of business cycle risk and Epstein et al. (2014) focusing on long-run risk.

uncertainty premium is defined as:

$$\pi_t = 1 - \frac{U_t^{One}}{U_t^{AU}}. \tag{19}$$

Panel C in Table 1 demonstrates the parameter uncertainty premiums for various degrees of prior information. Several results are noteworthy. First, uncertainty about π_{22} has the most significant utility effects. The reason is that the persistence of the bad state has particularly adverse effects on equilibrium utilities in the model with known parameters. Second, although the welfare loss of uncertainty about π_{22} is the largest, the decline in the welfare loss over time is comparable across different parameters. For instance, the welfare loss drops by about one-third for all parameters over a 50-year sample period. Intuitively, the regime switches in productivity growth happen at the frequency of the business cycle. Therefore, there is relatively frequent updating about model parameters, and the speed of learning about different parameters is comparable. Third, the welfare loss of uncertainty about the volatility parameter is negligible. The reason is that the productivity growth volatility is the second moment and has a little effect on the utility function. Further, the volatility of productivity growth does not switch between the two states. Thus, belief updating about a volatility parameter happens each period, and it is relatively easier to learn the productivity growth volatility.

This significant premium for the uncertain persistence of economic growth aligns well with established notions about learning in the asset pricing context. The extant literature has shown that learning about persistence risk can amplify asset prices in endowment economies (Collin-Dufresne et al., 2016; Andrei et al., 2019b,a) and rationalize the market patterns of recent major crises (Gillman et al., 2015; Ghaderi et al., 2022). Our work is a solid first step to understanding the effects of rationally priced hidden persistence in the production-based setting. Analyzing the additional implications for the real economy in richer models with heterogeneous agents and multiple sectors would be fruitful. There are other financial settings where a strong amplification mechanism of priced parameter uncertainty might play a significant role. For instance, forward-looking pricing of hidden persistence contains information about future economic growth and, hence, can be used to analyze the impact of macroeconomic announcements. The amplification effect of rational learning might explain a higher risk premium during announcement days. Finally, investors might disagree about persistence and, therefore, rational pricing of their belief dynamics might generate additional implications in the time-series (e.g., explaining the momentum and reversal effects) and cross-sectional (e.g., connecting underreaction and overreaction to news shocks with future returns) asset pricing.

3.3. Choice of Unknown Parameters and Initial Beliefs

Here, we describe a set of parameters assumed to be uncertain and then discuss the choice of priors. The household observes the state of the economy (s_t). The assumption of an observable state is required to solve the model with PPU. Motivated by the results of the welfare loss of uncertainty about various parameters, the household is assumed to know the true volatility parameter (σ) and is uncertain about the true transition probabilities (π_{11}, π_{22}) and mean growth rates in each regime (μ_1, μ_2). The household starts with beta priors $\mathcal{B}(a_{i,0}, b_{i,0})$ for π_{ii}, normal priors $\mathcal{N}(\mu_{i,0}, \sigma_{i,0})$ for μ_i, and updates beliefs from observed states and productivity growth rates using Bayes' rule. Given priors, the hyperparameters of posteriors are

$$
\begin{aligned}
a_{i,t} &= a_{i,0} + \#(\text{state } i \text{ has been followed by state } i), \\
b_{i,t} &= b_{i,0} + \#(\text{state } i \text{ has been followed by state } j), \\
\mu_{i,t} &= \mu_{i,t-1} + \mathbf{1}_{s_t=i} \frac{\sigma^2_{i,t-1}}{\sigma^2 + \sigma^2_{i,t-1}} \left(\Delta a_t - \mu_{i,t-1}\right), \\
\sigma^{-2}_{i,t} &= \mathbf{1}_{s_t=i} \cdot \sigma^{-2} + \sigma^{-2}_{i,t-1}.
\end{aligned}
$$

Let $\tau_{i,t}$ and $\lambda_{i,t}$ denote the time spent in regime i and the mean belief of π_{ii}. Then, $\tau_{i,t}$ and $\lambda_{i,t}$ are defined as follows:

$$
\tau_{i,t} = a_{i,t} + b_{i,t} \quad \text{and} \quad \lambda_{i,t} = E_t[\pi_{ii}] = \frac{a_{i,t}}{a_{i,t} + b_{i,t}}.
$$

Since $\sigma^{-2}_{i,t}$ is determined by $\tau_{i,t}$, the vector $X_t = \{\tau_{1,t}, \lambda_{1,t}, \tau_{2,t}, \lambda_{2,t}, \mu_{1,t}, \mu_{2,t}\}$ provides sufficient statistics for posterior beliefs. Note that s_t, K_t, and X_t are state variables of the economy with rational parameter learning.

We set the initial values of the vector of sufficient statistics with the following aims. First, we evaluate the persistence of the impact of parameter learning. In doing so, we inject priors encoding different training samples (e.g., 100, 150, 200, and 10000 years of prior learning). Second, we center initial beliefs on the true values of unknown parameters estimated from the post-war sample to illustrate that our results do not require pessimistic initial beliefs and are robust to using look-ahead information. Note that calibrating priors based on historical data is a more realistic procedure and would likely improve the model's performance due to pessimism induced by the Great Depression and World Wars. Following these guidelines, we formally define the initial values for a given training sample length of T quarters as:

$$\tau_{i,0} = T \times \frac{1-\pi_{jj}}{2-\pi_{11}-\pi_{22}}, \quad \lambda_{i,0} = \pi_{ii} \quad \text{and} \quad \mu_{i,0} = \mu_i.$$

In sum, we evaluate the performance of the production economy assuming different information settings. For incomplete investor knowledge, we distinguish between rational parameter learning and more commonly assumed AU. We use standard, conjugate priors for unknown parameters: beta and normal distributions for transition probabilities and mean growth rates. We employ look-ahead unbiased initial beliefs and fine-tune prior hyperparameters to embody various samples of prior learning. We numerically solve the models using the methodologies outlined in Section 2.5.

4. Quantitative Analysis

This section compares the performance of different models. Then, it conducts sensitivity analysis and considers a simpler model with unknown transition probabilities to examine the impact of hidden persistence.

4.1. Unconditional Moments

We now assess the model performance by looking at key moments of quantities, dividends, and returns. Panel A in Table 2 presents the quarterly moments of macroeconomic variables from different models and the data. The data column shows that output is more volatile than consumption and hours worked but less volatile than investment. Also, there is a positive but not perfect correlation between the series. Comparing the empirical moments with the model-generated statistics, the models with PPU, AU, and

FI explain the empirical moments reasonably well. Parameter learning increases the correlation of changes in hours with investment and output growth and has quantitatively negligible effects on other macro moments.

[Insert Table 2 about here]

Panel B in Table 2 shows that PPU significantly improves the asset pricing performance. The last two columns demonstrate that the production economies with known parameters, or with unknown parameters but AU pricing, generate a too-high average risk-free rate and price-dividend ratio as well as a too-low mean and volatility of excess equity returns compared to the data.[5] Columns 3 to 6 show that rational pricing of parameter uncertainty in the productivity growth process leads to a lower risk-free rate and price-dividend ratio. The risk premium, the Sharpe ratio of excess equity returns, and the volatility of the risk-free rate are more than two times higher with rational parameter learning. The equity volatility also increases substantially. Overall, the production economy with PPU and 100 years of prior learning can match the first and second moments of interest rates, generating a large equity premium and around three-quarters of its volatility while generating large equity Sharpe and low price-dividend ratios.

Furthermore, the financial moments remain amplified in the model with rational parameter learning and compare well with the data even after 200 years of prior learning. This impact is long-lasting despite the calibration's conservative amount of parameter uncertainty. Indeed, applying a 200-year prior in 1947 effectively implies that households had access to productivity data from the beginning of the Industrial Revolution in the 1750s. In reality, however, one would expect a higher degree of parameter uncertainty due to a shorter sample of the productivity data. Also, there is a considerable amount of uncertainty faced by investors when calibrating prior beliefs, which are set at the true parameter values in our simulations. Finally, the household in our model faces uncertainty about parameters governing business-cycle fluctuations, which are relatively

[5]For the AU case, we report the results for a prior period of 100 years because the results remain similar across different lengths of prior samples.

frequent. Augmenting the productivity growth process with a rare state is likely to amplify the magnitude and persistence of the impact of parameter learning.

Before advancing further, it is essential to address an important aspect of a frictionless production economy that fails to capture the firm's procyclical dividends, resulting in a low equity premium and equity volatility.[6] This countercyclicality is entirely because investment is more volatile than output and hence capital's share of output. A shock that raises capital will also cause higher investment. Since dividends net out investment expenditure from capital's share, dividends are countercyclical unless the model is modified.

Armed with this understanding, we explore how our model uniquely approaches this challenge. Specifically, we show that a combination of financial leverage and asymmetric adjustment cost can match the observed procyclical dividends.[7] The bottom of Panel B in Table 2 shows key moments of levered dividends from the data and different models. While all specifications reasonably capture the positive correlation between consumption and dividends, the PPU specification better captures the volatility of dividends. Intuitively, the impact of investment frictions on levered dividends works as follows. In bad times, it is more difficult for a representative firm to reduce investment due to higher costs, which would lead to a smaller drop in investment compared to the symmetric capital adjustment cost. Thus, net profits after deducting investment appear less countercyclical. With financial leverage, a firm's dividends are the sum of a firm's profits and the net balance of the long-term debt. The latter is proportional to capital and, therefore, declines in the recession. The sum of less countercyclical profits and strongly procyclical net issuance of long-term debt results in procyclical dividends.

Our findings extend the previously documented results by Collin-Dufresne et al. (2016) in the consumption-based setting to the production-based one. However, it is

[6]Section 3.6 of Kaltenbrunner and Lochstoer (2010) provides a comprehensive discussion of the countercyclical firm payouts and procyclical aggregate stock market dividends.

[7]Uhlig (2007), Belo, Lin and Bazdresch (2014), and Favilukis and Lin (2016) introduce wage rigidity to generate more volatile and procyclical dividends. This extension can further improve our results and possibly magnify the effect of parameter learning.

worth emphasizing the differences in the mechanisms due to endogenous dividends in the production economy. Rational parameter learning amplifies the impact of shocks on marginal utility (especially during bad times), decreasing the interest rates in both settings. In the production model, however, the interest rates have additional implications for dividends, equity returns, and the price-dividend ratio through financial leverage. Intuitively, the lower interest rates reduce the dividend sensitivity to changes in capital through financial leverage. Consequently, dividend growth and the price-dividend ratio become less volatile, while parameter learning permanent risks amplify the volatility of equity returns. More volatile returns coupled with less volatile dividend yields and cash flows lead to a higher fraction of dividend yield variation explained by discount rates.

This mechanism within the production economy works as follows. First, the reduction in long-term interest rates reduces the interest rate payments and hence increases the level of dividends (Eq. (9)) in the PPU specification. In turn, this reduces the sensitivity of dividends to changes in capital and decreases their volatility. Second, long-term rates exhibit a more significant decline compared to short-term rates because the effect of parameter uncertainty on marginal utility is accumulated over multiple periods (Eq. (10)). This decreases the level of debt and increases its sensitivity to fluctuations in capital (Eq. (11), debt levels are affected by the ratios of the n-period interest rate to interest rates with maturities ranging from 1 to n periods). In contrast, the firm's value remains largely unaffected because capital and the investment-to-capital ratio are less affected by parameter uncertainty. As a result, the equity price increases and becomes more sensitive to changes in capital due to a debt component. Coupled with endogenous long-run risk due to consumption smoothing, this amplified sensitivity leads to more volatile returns. Third, the increase in equity prices is offset by a significant boost in dividends; therefore, the price-dividend ratio declines. Although the equity price is somewhat more sensitive to capital fluctuations due to a debt component, the sizable reduction in dividend sensitivity overweights this effect and decreases the volatility of the price-dividend ratio.

In sum, combining a smoother price-dividend ratio, less volatile dividends, and more volatile equity returns leads to a higher importance of discount rates in explaining dividend yield variation in the PPU model. To explore this relationship more formally, the following section presents the dividend yield variance decomposition results.

4.2. Dividend Yield Variance Decomposition

A key finding of empirical finance is that variation in the price-dividend ratio is primarily due to variation in discount rates (Campbell and Ammer, 1993; Cochrane, 2011). A production economy offers a particularly suitable framework for understanding this variation because both stock returns and cash flows are endogenously determined in the model. Following Cochrane (2011), we examine whether our model with PPU passes the test on drivers of stock price variation.

We first employ the Campbell and Shiller (1988) approximate present value to obtain

$$dp_t \approx \sum_{j=1}^{k} \rho^{j-1} r_{t+j} - \sum_{j=1}^{k} \rho^{j-1} \Delta d_{t+j} + \rho^k dp_{t+k}, \tag{20}$$

where $dp_t = \ln(D_t/P_t)$ is the log dividend yield, $r_t = \ln(R_t)$ is the log stock market return, and $\rho = 1/(1 + E[dp])$ is an approximation constant. We consider univariate regressions of k-quarter ex-post returns, dividend growth, and dividend yield on the lagged dividend yield:

$$\sum_{j=1}^{k} \rho^{j-1} r_{t+j} = a_r + b_r^k dp_t + \varepsilon_{t+k}^r, \tag{21}$$

$$\sum_{j=1}^{k} \rho^{j-1} \Delta d_{t+j} = a_{\Delta d} + b_{\Delta d}^k dp_t + \varepsilon_{t+k}^d, \tag{22}$$

$$dp_{t+k} = a_{dp} + b_{dp}^k dp_t + \varepsilon_{t+k}^{dp}, \tag{23}$$

Regressing both sides of Eq. (20) on the dividend yield and using Eq. (21)-(23), the following approximation should hold:

$$1 \approx b_r^k + \left[-b_{\Delta d}^k\right] + \rho^k b_{dp}^k. \tag{24}$$

Alternatively, we can rewrite Eq. (24) as follows:

$$1 \approx \frac{cov\left(dp_t, \sum_{j=1}^{k} \rho^{j-1} r_{t+j}\right)}{var(dp_t)} + \left[-\frac{cov\left(dp_t, \sum_{j=1}^{k} \rho^{j-1} \Delta d_{t+j}\right)}{var(dp_t)}\right] + \rho^k \frac{cov\left(dp_t, dp_{t+k}\right)}{var(dp_t)}. \tag{25}$$

Thus, we can also interpret the terms in Eq. (24) as the fractions of dividend yield variation attributed to discount rates, cash flows, and future dividend yield.

Table 3 reports the dividend yield variance decomposition results in the data. The empirical estimates based on direct regressions indicate that two-thirds of the variation in dividend yield is mainly attributed to variation in discount rates. In contrast, only one-fifth and one-tenth of the variation are attributed to dividend growth and past dividend yield. Interestingly, the VAR-based estimates show that only one-third of dividend yield variation is due to the variability of discount rates and two-thirds to dividend growth, whereas the past dividend yield has a negligible effect. Our estimates are consistent with Schorfheide, Song and Yaron (2018) who document that around 70% of the dividend yield variation is attributed to variation in discount rates in the case of the direct estimation. The VAR attributes slightly less (more) than half of the variation to discount rates (dividend growth). Overall, the variability of dividend yield is driven by both returns and cash flows, though the exact weights depend on the estimation procedure.

[Insert Table 3 about here]

Table 3 also reports the results of the models with PPU, AU, and FI. Several observations are noteworthy. First, the model with FI specification yields a dividend growth forecast coefficient close to 1, while other estimates are essentially zero. Thus, all price-dividend ratio volatility in the FI case corresponds to variation in dividend growth, inconsistent with the empirical evidence. Second, the framework with AU pricing marginally improves the direct estimation results by increasing (decreasing) the portion of the dividend yield variability due to future returns (cash flows). However, the coefficients are far away from the empirical point estimates. Also, the VAR-based results remain unchanged relative to the FI case. Third, introducing fully rational parameter learning substantially improves the model fit with the data. In the direct estimation, the portion of dividend yield variability explained by expected returns (cash flows) increases (declines) slightly above (below) half, bringing the model performance closer to the data. In the VAR estimation, the fractions closely match the data point estimates.[8]

To better understand the sources of model improvement, we can iterate the Campbell and Shiller (1988) approximate present value and impose a no-bubble condition $\lim_{k\to+\infty} \rho^k dp_{t+k}$ to decompose the dividend yield into two main components:

$$dp_t \approx \sum_{j=1}^{+\infty} \rho^{j-1} r_{t+j} - \sum_{j=1}^{+\infty} \rho^{j-1} \Delta d_{t+j}. \tag{26}$$

Rearranging the terms, taking variances of both sides and dividing by $var(dp_t)$ leads to

$$\frac{var\left(\sum_{j=1}^{+\infty} \rho^{j-1} r_{t+j}\right)}{var(dp_t)} \approx 1 + \frac{var\left(\sum_{j=1}^{+\infty} \rho^{j-1} \Delta d_{t+j}\right)}{var(dp_t)} - 2\left[-\frac{cov\left(dp_t, \sum_{j=1}^{+\infty} \rho^{j-1} \Delta d_{t+j}\right)}{var(dp_t)}\right], \tag{27}$$

in which the expression in square brackets corresponds to a fraction of variation in dividend yield due to variation in dividends $-b_{\Delta d}^{(\infty)}$.

The magnitude of $-b_{\Delta d}^{(\infty)}$ can be explained by key moments reported in Table 2 and the relationship given by Eq. (27). The FI specification fails to capture the empirical dividend yield variance decomposition because it generates little variation in stock returns and too much variability in dividends, pushing $var\left(\sum_{j=1}^{+\infty} \rho^{j-1} r_{t+j}\right) \Big/ var(dp_t)$ toward zero and $var\left(\sum_{j=1}^{+\infty} \rho^{j-1} \Delta d_{t+j}\right) \Big/ var(dp_t)$ toward one. This implies that $-b_{\Delta d}^{(\infty)}$ approaches one, inconsistent with the empirical evidence. Thus, a model aiming to match the contribution of dividends to dividend yield variation should substantially alter the second moments of key variables.

The model with AU marginally increases (decreases) the volatility of equity returns (dividend growth) relative to the FI case, which leads to a minimal improvement in the direct regression results. In contrast, rational parameter learning generates substantially smoother dividend growth, less variability of the price-dividend ratio, and the amplification of the return volatility (see Table 2 for the results and Section 4.1 for the discussion).

[8] In unreported results, we compute the 5th and 95th percentiles for simulated statistics in the three models. For AU and FI cases, the 90% confidence intervals fail to contain the data point estimates of $b_r^{(k)}$ and $-b_{\Delta d}^{(k)}$. For PPU, the confidence intervals contain the data estimates.

The combination of a smoother price-dividend ratio and more volatile returns increases the left part of Eq. (27). Further, since the volatility of dividends becomes twice smaller and the volatility of the price-dividend ratio declines only by a quarter, as shown in Table 2, the second term in the right part of Eq. (27) declines. As a result, the last term should become smaller, leading to the decreasing importance of dividends in explaining dividend yield variation and, therefore, increasing the importance of discount rates.

4.3. Excess Return and Consumption Predictability

The empirical literature has documented that excess returns at an aggregate level can be predicted by the investment-capital (Cochrane, 1991; Bansal and Yaron, 2004), Tobin's Q (Pontiff and Schall, 1998; Lewellen, 2004), dividend-price (Campbell and Shiller, 1988; Fama and French, 1989), and consumption-wealth (Lettau and Ludvigson, 2001) ratios. High dividend yields and high book-to-market and consumption-wealth ratios predict high future excess returns, whereas high investment rates forecast low future excess returns. The predictive regressions also suggest that the slope coefficients (in absolute terms) and R^2s are relatively large and tend to increase over the forecast horizon. These regularities pose a challenge for the real business-cycle model. This section compares the long-term predictability patterns generated by production economies with parameter uncertainty (PPU and AU pricing with a 100-year training sample and unbiased priors) and known parameters to the predictability observed in the post-war data.

Tobin's Q, the investment-capital and consumption-wealth ratios are endogenously determined in the model. Defining wealth as the present discounted value of a household's consumption, wealth can be recursively specified as $W_t = C_t + E_t\left[M_{t+1}W_{t+1}\right]$. We guess wealth of the form $W_t = U_t / \frac{\partial U_t}{\partial C_t}$ and verify that it satisfies the Bellman equation. Consequently, the wealth-consumption ratio in our model with endogenous labor is

$$\frac{W_t}{C_t} = \frac{1}{1-\beta}\left(\frac{U_t}{C_t(1-N_t)^{\nu}}\right)^{1-1/\psi},$$

where the agent's utility and consumption are endogenously determined. We run the predictive regressions and report results in Table 4 using these model-generated quantities. All models can generate monotonic patterns in the slope coefficients and R^2s over the forecast horizon, but the magnitudes differ across different frameworks.

[Insert Table 4 about here]

Panels A and B show that in the regressions with investment rates and Tobin's Q, PPU generates larger R^2s in both cases, while there is no noticeable difference between the slope coefficients across the three models. Panel C shows that in the regression with dividend yields, the FI model generates too small slope coefficients and R^2s at around 0.1 and 5%, respectively. Surprisingly, the AU model leads to slightly worse results. In contrast, the model with PPU displays significant return predictability, with the magnitudes of coefficient estimates and R^2s being comparable to the empirical results. Panel D shows that in the regression with consumption-wealth ratios, both PPU and AU frameworks dominate the model with FI in terms of R^2s. At the same time, PPU better captures the coefficient estimates by producing the lowest slopes among the three models.

[Insert Table 5 about here]

One may wonder whether subjective long-run risks implied by parameter learning would lead to counterfactually strong predictability of future consumption growth, a common problem of the models featuring long-run risks. To test this implication, we run forecasting regressions of future consumption growth on the lagged log price-dividend ratio and log Tobin's Q. Table 5 presents the results. In the data, future consumption is unconnected to valuation ratios as measured by small R^2s, especially at forecasting horizons longer than two years. Like in the data and FI model, expected consumption growth is modestly predictable in the models with parameter learning.

In sum, rational pricing of risks generated by Bayesian learning helps explain excess return predictability observed in the data. Intuitively, dynamic updating of beliefs about unknown parameters generates the time-variation in the equity risk premium, leading to stronger return predictability in parameter learning models. Furthermore, rational pricing of parameter uncertainty amplifies the impact of belief revisions on equilibrium quantities and the risk premium. Hence, it increases (in absolute terms) the slope coefficients and R^2s compared to AU pricing. PPU has a stronger effect on financial variables than on macroeconomic quantities. Therefore, there is a more significant improvement in the regression results using dividend yields and consumption-wealth ratios as predictors. Finally, subjective long-run risks generated by parameter learning are consistent with modest predictability of future consumption growth.

4.4. Sensitivity Analysis

The sensitivity analysis below evaluates the impact of procyclical dividends and tests alternative channels used to improve asset pricing performance. We conduct a two-step comparative statistics exercise using alternative adjustment cost functions and examining the economy with complete information under alternative risk aversion and leverage parameters calibrations. First, we consider the parameter-learning models with symmetric quadratic or more common convex adjustment costs (Jermann, 1998):

$$\varphi(x) = a_1 + \frac{a_2}{1 - 1/\xi} x^{1-1/\xi}, \tag{28}$$

in which ξ is the elasticity of the investment rate to Tobin's Q.[9] To put the models considered on a comparable footing, the adjustment cost parameters are chosen $\theta^+ = \theta^- = 15$ and $\xi = 2.5$ to deliver similar investment volatility. Second, Tallarini (2000) shows that risk aversion strongly impacts asset pricing predictions. Thus, it is a natural candidate to improve the model's performance. We increase the risk aversion to the point where the specification with known parameters matches the observed equity premium. The required parameter is $\gamma = 16.5$. Yet another alternative channel is the financial leverage controlling the riskiness of dividends. Thus, as an additional exercise, we consider increasing the model-implied leverage ratio to match the empirical equity premium. The required leverage of the economy is more than 200%.

Table 6 shows the sensitivity analysis results. We report asset pricing moments to save space, whereas the remaining statistics are presented in Internet Appendix F. Panel A shows that shutting off asymmetry in adjustment costs leads to a modest amplification of equity moments by PPU. However, rational parameter learning still has a significant effect on marginal utility. Although we observe a sizable decline in the mean and a

[9]Following Boldrin et al. (2001), we set $a_1 = \frac{1}{\xi - 1}(1 - \delta - \exp(\bar{\mu}))$ and $a_2 = (\exp(\bar{\mu}) - 1 + \delta)$ with $\bar{\mu}$ being unconditional mean of μ_{s_t} so that there are no adjustment costs in the non-stochastic steady state.

two-fold increase in the volatility of interest rates, the equity premium and its volatility are marginally amplified by rational learning, and the price-dividend ratio becomes only slightly lower. Since equity moments are not significantly affected, Panel B shows that the specification with rational parameter learning yields the same variance decomposition predictions as the model with known parameters: dividends explain the whole variation in dividend yields. The results of the predictive regression using the dividend-price ratio as a predictor become substantially weaker too, as shown in Panel B. Overall, this deterioration suggests that the asset pricing implications of PPU in the production economy depend crucially on introducing a procyclical dividend process.

[Insert Table 6 about here]

This conclusion is not merely a technical consequence of using a convex (or symmetric) adjustment cost function. We show this by solving the model with Jermann adjustment costs and pricing exogenous procyclical dividends. Internet Appendix F demonstrates that the amplification mechanism provided by PPU is strong in the economy with Jermann adjustment costs once a firm's dividends become procyclical.

Table 6 shows the impact of increasing the risk aversion and leverage parameters in the FI setting. The model with higher risk aversion does a good job of matching the key asset pricing moments of financial variables. It also improves the variance decomposition results and yields stronger stock return predictability, though it cannot fully capture the empirical estimates. Increasing the leverage ratio also improves asset pricing moments, particularly equity return volatility, but at the expense of risky dividends. Hence, this specification still predicts that dividend yields are primarily driven by dividends. In sum, the benchmark calibration with known parameters and extreme risk aversion and leverage levels falls short of replicating the empirical variance decomposition of the price-dividend ratio despite matching the equity premium.

4.5. Learning About Hidden Persistence

The benchmark model with parameter uncertainty considers joint learning about expected productivity growth and persistence of the two regimes. Having multiple sources of unknown, however, does not allow us to inspect individual contributions of various parameters. Motivated by the research on hidden persistence (Cogley and Sargent (2008), Gillman et al. (2015), Andrei et al. (2019b), and Andrei et al. (2019a)), we consider the model with learning about unknown transition probabilities.

Table 7 presents asset pricing moments, whereas the remaining results are shown in Internet Appendix F. Panel A demonstrates that shutting down uncertainty about expected productivity growth leads to lower uncertainty in the economy. Consequently, compared to the benchmark moments in Table 2, the equity risk premium, its volatility, and Sharpe ratios become lower, while the risk-free rate and the price-dividend ratio are on average higher. Nevertheless, the amplification of equity moments by rational learning about transition probabilities is comparable to the benchmark specification. Panel B shows that learning about mean productivity growth plays a non-negligible role in the variance decomposition results. The model with priced uncertainty about the hidden persistence now predicts that, based on direct estimation, dividends explain the largest fraction of the price-dividend variability, counter to what we observe empirically. The reason is that dividends become more volatile without learning about permanent shocks. Coupled with a weaker amplification of return volatility, this strengthens the role of cash flow variability in explaining fluctuations of dividend yield. Panel C illustrates that the predictive regression results remain virtually unchanged compared to the benchmark.

[Insert Table 7 about here]

In sum, the uncertainty premium for hidden persistence is quantitatively larger than for expected growth rates, consistent with parameter uncertainty importance in Section 3.2. A multidimensional learning problem amplifies the impact of shocks on the marginal utility (Johannes et al., 2016), improving the variance decomposition results.

5. Conclusion

Introducing parameter uncertainty into a parsimonious real business cycle framework improves the model's ability to reproduce salient moments of macroeconomic and asset return data. Combined with investment frictions in the form of asymmetric costs, parameter uncertainty gives rise to additional macroeconomic risks that help capture the key asset pricing moments (e.g., the mean and volatility of the risk-free rate, the large equity premium, and around three-quarters of excess return volatility, the large equity Sharpe ratio and the level of the price-dividend ratio), while respecting stylized moments of macroeconomic quantities. Furthermore, parameter learning implies a relatively higher importance of discount rates in explaining dividend yield variation, helping the model generate a realistic dividend yield variance decomposition. Finally, time-varying posterior beliefs about unknown parameters reproduce the long-horizon predictability of excess returns by macroeconomic and valuation variables as observed in the data. The asset pricing implications of subjective long-run risks crucially depend on the procyclicality of dividends consistent with the data.

Future research may consider extending our mechanism to a richer model with sticky prices and financial frictions. In particular, modeling wage rigidity in the spirit of Favilukis and Lin (2016) can help endogenously generate procyclical dividend growth in the model. The interaction between sticky prices and learning effects may have additional interesting implications for the labor market. Motivated by a large strand of the literature on time-varying macroeconomic uncertainty, it is interesting and straightforward to extend our methodology to learning about volatility risks. This might have additional asset pricing implications, especially for volatility-sensitive assets. Finally, the multi-sector and multi-agent models with rationally priced parameter uncertainty will likely have additional implications. We leave this important avenue for future research.

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Tables and Figures

Table 1. Benchmark Calibration

Parameter	Description	Value
Panel A: Preferences, Production and Capital Adjustment Cost Functions		
β	Discount factor	0.995
γ	Parameter controlling risk aversion	10
ψ	EIS	2
ν	Leisure preference	2
α	Capital share	0.36
δ	Depreciation rate	0.025
θ^{+}	Adjustment cost coefficient	5
θ^{-}/θ^{+}	Asymmetry of adjustment costs	20
Panel B: Markov-switching Model of Productivity Growth		
π_{11}	Transition probability from expansion to expansion	0.961
π_{22}	Transition probability from recession to recession	0.625
μ_1	Productivity growth in expansion	0.52
μ_2	Productivity growth in recession	-1.86
σ	Productivity volatility	1.47

Panel C: Parameter Importance

	100 yrs	150 yrs	200 yrs	10000 yrs
π_{11}	7.52	4.73	3.45	0.00
π_{22}	27.90	18.08	12.85	0.24
μ_1	5.03	3.19	2.33	0.05
μ_2	6.93	4.46	3.29	0.05
σ	0.50	0.30	0.10	0.00

Note: This table reports the parameter values in the benchmark calibration and the parameter uncertainty premium in the two-state Markov-switching model. Panel A presents preference parameters and values in the production and adjustment costs functions. Panel B shows the maximum likelihood estimates of parameters in a two-state Markov-switching model for productivity growth. We obtain these estimates by applying the expectation-maximization algorithm (Hamilton, 1990) to quarterly total factor productivity growth rates from 1947:Q2 to 2016:Q4. Panel C shows the parameter uncertainty premium for various prior distributions based on 100, 150, 200, and 10000 years of initial learning.

Table 2. Sample Moments

	Data	PPU				AU	FI
		100 yrs	150 yrs	200 yrs	10000 yrs	100 yrs	
Panel A: Macroeconomic Quantities							
$E(\Delta c)$	0.47	0.30	0.30	0.30	0.29	0.28	0.29
$\sigma(\Delta c)$	0.72	0.97	0.98	0.98	0.98	0.98	0.99
$ar1(\Delta c)$	0.21	0.14	0.14	0.14	0.14	0.14	0.14
$\sigma(\Delta i)$	2.39	2.53	2.51	2.50	2.50	2.37	2.40
$\sigma(\Delta y)$	1.28	1.22	1.21	1.22	1.21	1.19	1.19
$\sigma(\Delta n)$	0.67	0.53	0.52	0.54	0.52	0.48	0.50
$\rho(\Delta i, \Delta y)$	0.62	0.86	0.86	0.86	0.85	0.86	0.85
$\rho(\Delta c, \Delta y)$	0.45	0.76	0.76	0.75	0.76	0.80	0.77
$\rho(\Delta c, \Delta i)$	0.35	0.34	0.33	0.33	0.33	0.39	0.36
$\rho(\Delta n, \Delta i)$	0.62	0.88	0.87	0.86	0.83	0.86	0.79
$\rho(\Delta n, \Delta y)$	0.68	0.54	0.52	0.51	0.47	0.51	0.43
Panel B: Financial Variables							
Debt/Equity	1.00	1.01	1.01	0.99	0.99	1.00	1.01
$E(R_f) - 1$	0.23	0.20	0.27	0.31	0.42	0.45	0.46
$\sigma(R_f)$	0.40	0.44	0.36	0.32	0.22	0.20	0.20
$E(R - R_f)$	1.59	1.83	1.56	1.35	0.88	0.79	0.76
$\sigma(R - R_f)$	7.75	5.47	5.12	4.71	3.93	4.07	3.92
$SR(R - R_f)$	0.21	0.35	0.31	0.30	0.24	0.20	0.21
$E(pd)$	4.38	4.13	4.27	4.39	4.71	4.78	4.81
$\sigma(pd)$	0.34	0.22	0.23	0.24	0.28	0.28	0.29
$ar1(pd)$	0.96	0.88	0.86	0.85	0.82	0.82	0.82
$E(\Delta d)$	0.49	0.23	0.25	0.27	0.31	0.30	0.31
$\sigma(\Delta d)$	5.25	7.85	8.98	10.65	14.99	15.23	16.02
$ar1(\Delta d)$	0.01	−0.10	−0.10	−0.10	−0.11	−0.12	−0.11
$\rho(\Delta c, \Delta d)$	0.09	0.15	0.16	0.18	0.21	0.15	0.23

Note: This table reports moments from the data and the production economies with unknown transition probabilities and mean growth rates, as well as known parameters. The empirical statistics are reported in the "Data" column and correspond to the U.S. data from 1947:Q2 to 2016:Q4. The models with parameter learning assume priced parameter uncertainty or anticipated utility ("PPU" and "AU" columns) with unbiased initial mean beliefs. For fully rational pricing of parameter uncertainty, we present the results for the models with 100, 150, 200, and 10 000 years of prior learning. For anticipated utility pricing of parameter uncertainty, we consider the model with a 100-year prior because employing alternative lengths generates virtually the same results. The "FI" column presents the results of the full information case where the parameters are known. The model-based moments are means of statistics based on 1000 simulations. Simulated statistics are calculated for the length of time corresponding to a full sample size and are expressed in quarterly terms. $E(x), \sigma(x), SR(x), ar1(x)$, and $\rho(x, y)$ denote the sample mean, standard deviation, Sharpe ratio, the autocorrelation of x, and correlation between x and y, respectively.

Table 3. Dividend Yield Variance Decomposition

	Data			PPU			AU			FI		
	$b_r^{(k)}$	$-b_{\Delta d}^{(k)}$	$\rho^k b_{dp}^{(k)}$	$b_r^{(k)}$	$-b_{\Delta d}^{(k)}$	$\rho^k b_{dp}^{(k)}$	$b_r^{(k)}$	$-b_{\Delta d}^{(k)}$	$\rho^k b_{dp}^{(k)}$	$b_r^{(k)}$	$-b_{\Delta d}^{(k)}$	$\rho^k b_{dp}^{(k)}$
Direct	0.69	0.20	0.08	0.53	0.45	0.04	0.16	0.85	−0.02	0.02	1.01	−0.04
VAR(60)	0.33	0.66	0.01	0.36	0.63	0.00	0.03	0.96	0.00	0.05	0.94	0.00
VAR(∞)	0.33	0.67	0.00	0.36	0.63	0.00	0.03	0.96	0.00	0.05	0.95	0.00

Note: This table reports the results of dividend yield variance decomposition based on the direct and VAR-based estimation from the data and the production economies with unknown transition probabilities and mean growth rates, as well as known parameters. For the direct estimation (the first row), we run a set of univariate regressions of 60-quarter ex-post returns, dividend growth, and dividend yield ($k = 60$) on a constant term and the dividend yield. For the VAR-based estimation, we consider the first-order VAR specification (the second row) and also infer long-run coefficients ($k \to \infty$) from one-quarter VAR (the third row). The empirical statistics are reported in the "Data" columns and correspond to the U.S. data from 1947:Q2 to 2016:Q4. The models with parameter learning assume priced parameter uncertainty or anticipated utility ("PPU" and "AU" columns) with unbiased initial mean beliefs and a 100-year prior. The "FI" columns present the results of the full information case where the parameters are known. For each model, we simulate 1000 economies at a quarterly frequency with a sample size equal to the empirical counterpart. We obtain regression coefficients for each simulation and report average sample statistics over all 1000 artificial series.

Table 4. Return Predictability

	Data		PPU		AU		FI	
h	β	R^2	β	R^2	β	R^2	β	R^2
	Panel A: Investment-capital ratio							
1Y	−0.59	0.12	−0.34	0.07	−0.24	0.04	−0.25	0.03
2Y	−0.99	0.17	−0.57	0.12	−0.45	0.08	−0.45	0.06
3Y	−1.46	0.27	−0.76	0.16	−0.62	0.11	−0.62	0.09
4Y	−1.88	0.35	−0.93	0.20	−0.79	0.13	−0.78	0.11
5Y	−2.28	0.39	−1.08	0.24	−0.93	0.16	−0.92	0.13
	Panel B: Tobin's Q							
1Y	−0.45	0.08	−0.43	0.09	−0.29	0.05	−0.28	0.05
2Y	−1.57	0.13	−0.69	0.15	−0.51	0.10	−0.49	0.09
3Y	−1.95	0.16	−0.91	0.21	−0.70	0.13	−0.67	0.13
4Y	−2.47	0.18	−1.10	0.26	−0.87	0.16	−0.84	0.16
5Y	−2.75	0.25	−1.28	0.30	−1.03	0.19	−0.99	0.19
	Panel C: Dividend-price ratio							
1Y	0.07	0.03	0.13	0.09	0.03	0.01	0.01	0.01
2Y	0.16	0.09	0.20	0.14	0.05	0.03	0.01	0.02
3Y	0.24	0.14	0.25	0.17	0.06	0.03	0.00	0.03
4Y	0.31	0.19	0.29	0.20	0.07	0.04	−0.00	0.04
5Y	0.40	0.24	0.33	0.22	0.08	0.05	−0.01	0.04
	Panel D: Consumption-wealth ratio							
1Y	2.18	0.08	2.86	0.09	2.90	0.08	5.01	0.04
2Y	3.78	0.12	4.37	0.13	5.21	0.14	8.50	0.06
3Y	5.11	0.16	5.54	0.16	7.19	0.19	11.60	0.08
4Y	6.49	0.21	6.62	0.21	8.99	0.24	14.44	0.10
5Y	7.44	0.23	8.02	0.25	10.57	0.28	16.95	0.12

Note: This table reports return predictability statistics from the data and the production economies with unknown transition probabilities and mean growth rates, as well as known parameters. We consider univariate regressions of cumulative excess log equity returns on several valuation and macroeconomic variables over various forecasting horizons (h years; 1 to 5). We use the investment-capital ratio, Tobin's Q, log dividend-price, and log consumption-wealth ratios as the right-hand side variable (x_t) in the linear projection:

$$r^{ex}_{t+1\to t+h} = \text{Intercept} + \beta(h) \times x_t + \varepsilon_{t+h},$$

where $r^{ex}_{t+1\to t+h}$ are h-year future excess log equity returns. The empirical statistics are reported in the "Data" columns and correspond to the U.S. data from 1947:Q2 to 2016:Q4. The models with parameter learning assume priced parameter uncertainty or anticipated utility ("PPU" and "AU" columns) with unbiased initial mean beliefs and a 100-year prior. The "FI" columns present the results of the full information case where the parameters are known. For each model, we simulate 1000 economies at a quarterly frequency with a sample size equal to the empirical counterpart. We obtain the slope coefficients and R^2s for each simulation and report average sample statistics over all 1000 artificial series.

Table 5. Consumption Predictability

h	Data	PPU	AU	FI	Data	PPU	AU	FI
	Panel A: Price-dividend ratio				*Panel B: Tobin's Q*			
1Y	0.06	0.08	0.05	0.06	0.05	0.03	0.03	0.03
2Y	0.04	0.07	0.04	0.05	0.03	0.03	0.03	0.03
3Y	0.01	0.07	0.04	0.05	0.01	0.04	0.04	0.04
4Y	0.01	0.06	0.04	0.05	0.01	0.05	0.05	0.05
5Y	0.00	0.06	0.04	0.05	0.00	0.05	0.06	0.06

Note: This table reports consumption predictability statistics from the data and the production economies with unknown transition probabilities and mean growth rates, as well as known parameters. We consider univariate regressions of cumulative consumption growth rates on several valuation and macroeconomic variables over various forecasting horizons (h years; 1 to 5). We use log price-dividend ratio and log Tobin's Q (the latter is normalized to have a standard deviation of one) as the right-hand side variable (x_t) in the linear projection:

$$\Delta c_{t+1\to t+h} = \text{Intercept} + \beta(h) \times x_t + \varepsilon_{t+h},$$

where $\Delta c_{t+1\to t+h}$ are h-year future consumption growth. The empirical statistics are reported in the "Data" column and correspond to the U.S. data from 1947:Q2 to 2016:Q4. The models with parameter learning assume priced parameter uncertainty or anticipated utility ("PPU" and "AU" columns) with unbiased initial mean beliefs and a 100-year prior. The "FI" column presents the results of the full information case where the parameters are known. For each model, we simulate 1000 economies at a quarterly frequency with a sample size equal to the empirical counterpart. We obtain R^2s for each simulation and report average sample statistics over all 1000 artificial series.

Table 6. Sensitivity Analysis

	Data	Symmetric		Convex		FI	
		PPU	AU	PPU	AU	$\gamma\uparrow$	$\omega\uparrow$
Panel A: Financial Variables							
Debt/Equity	1.00	1.01	1.00	0.99	1.00	1.00	2.22
$E(R_f)-1$	0.23	0.27	0.46	0.26	0.45	0.28	0.46
$\sigma(R_f)$	0.40	0.38	0.21	0.38	0.21	0.22	0.19
$E(R-R_f)$	1.59	0.68	0.43	0.76	0.41	1.61	1.58
$\sigma(R-R_f)$	7.75	2.17	2.09	2.38	2.14	4.71	9.24
$SR(R-R_f)$	0.21	0.32	0.21	0.33	0.19	0.35	0.20
$E(pd)$	4.38	5.01	5.21	4.93	5.27	4.25	4.41
$\sigma(pd)$	0.34	0.31	0.29	0.32	0.33	0.21	0.35
$ar1(pd)$	0.96	0.92	0.95	0.94	0.94	0.84	0.81
$E(\Delta d)$	0.49	0.20	0.29	0.20	0.28	0.31	0.32
$\sigma(\Delta d)$	5.25	11.04	7.48	10.09	9.15	8.95	17.11
$ar1(\Delta d)$	0.01	−0.12	0.00	−0.08	−0.01	−0.09	−0.14
$\rho(\Delta c,\Delta d)$	0.09	−0.12	−0.72	−0.31	−0.72	0.27	0.24

Panel B: Dividend Yield Variance Decomposition

	Data			Symmetric (PPU)			Convex (PPU)		
	$b_r^{(k)}$	$-b_{\Delta d}^{(k)}$	$\rho^k b_{dp}^{(k)}$	$b_r^{(k)}$	$-b_{\Delta d}^{(k)}$	$\rho^k b_{dp}^{(k)}$	$b_r^{(k)}$	$-b_{\Delta d}^{(k)}$	$\rho^k b_{dp}^{(k)}$
Direct	0.69	0.20	0.08	0.09	1.05	−0.13	0.04	1.08	−0.12
VAR(60)	0.33	0.66	0.01	0.04	0.95	0.02	−0.01	0.97	0.03
VAR(∞)	0.33	0.67	0.00	0.04	0.95	0.00	−0.01	1.00	0.00

	Data			$\gamma\uparrow$ (FI)			$\omega\uparrow$ (FI)		
	$b_r^{(k)}$	$-b_{\Delta d}^{(k)}$	$\rho^k b_{dp}^{(k)}$	$b_r^{(k)}$	$-b_{\Delta d}^{(k)}$	$\rho^k b_{dp}^{(k)}$	$b_r^{(k)}$	$-b_{\Delta d}^{(k)}$	$\rho^k b_{dp}^{(k)}$
Direct	0.69	0.20	0.08	0.44	0.57	−0.02	0.34	0.69	−0.04
VAR(60)	0.33	0.66	0.01	0.18	0.80	0.00	0.14	0.85	0.00
VAR(∞)	0.33	0.67	0.00	0.18	0.81	0.00	0.16	0.83	0.00

Panel C: Return Predictability (Dividend-Price Ratio)

	Data		Sym. (PPU)		Conv. (PPU)		$\gamma\uparrow$ (FI)		$\omega\uparrow$ (FI)	
	β	R^2	β	R^2	β	R^2	β	R^2	β	R^2
1Y	0.07	0.03	0.01	0.02	0.02	0.03	0.11	0.05	0.09	0.04
2Y	0.16	0.09	0.03	0.04	0.04	0.06	0.17	0.08	0.14	0.05
3Y	0.24	0.14	0.04	0.05	0.05	0.08	0.22	0.10	0.17	0.06
4Y	0.31	0.19	0.05	0.07	0.06	0.10	0.26	0.11	0.20	0.07
5Y	0.40	0.24	0.05	0.08	0.07	0.12	0.30	0.13	0.23	0.09

Note: This table reports moments from the data and the production economies with (1) unknown transition probabilities and mean growth rates and alternative adjustment costs and (2) known parameters and different parameters controlling risk aversion (γ) and financial leverage (ω). In all calibrations, other parameters are set at the original values. The empirical statistics are reported in the "Data" column and correspond to the U.S. data from 1947:Q2 to 2016:Q4. We solve parameter-learning models with symmetric or convex adjustment costs assuming either priced parameter uncertainty or anticipated utility ("PPU" and "AU" columns). We solve known-parameter models with higher values of γ or ω. The model-based moments are means of statistics based on 1000 simulations.

Table 7. Unknown Transition Probabilities

	Data	PPU				AU
		100 yrs	150 yrs	200 yrs	10000 yrs	100 yrs
Panel A: Financial Variables						
Debt/Equity	1.00	0.98	1.01	0.99	0.99	0.98
$E(R_f) - 1$	0.23	0.27	0.32	0.35	0.43	0.45
$\sigma(R_f)$	0.40	0.37	0.32	0.29	0.21	0.19
$E(R - R_f)$	1.59	1.59	1.43	1.26	0.93	0.82
$\sigma(R - R_f)$	7.75	5.14	5.01	4.65	4.49	4.25
$SR(R - R_f)$	0.21	0.32	0.29	0.28	0.21	0.20
$E(pd)$	4.38	4.26	4.34	4.45	4.69	4.77
$\sigma(pd)$	0.34	0.22	0.23	0.24	0.25	0.28
$ar1(pd)$	0.96	0.84	0.83	0.82	0.78	0.79
$E(\Delta d)$	0.49	0.25	0.26	0.27	0.30	0.30
$\sigma(\Delta d)$	5.25	9.44	10.33	11.46	14.24	15.86
$ar1(\Delta d)$	0.01	−0.09	−0.10	−0.10	−0.11	−0.11
$\rho(\Delta c, \Delta d)$	0.09	0.23	0.23	0.24	0.24	0.22

Panel B: Dividend Yield Variance Decomposition

	Data			PPU			AU		
Direct	0.69	0.20	0.08	0.41	0.58	0.00	0.16	0.86	−0.03
VAR(60)	0.33	0.66	0.01	0.23	0.75	0.00	0.04	0.95	0.00
VAR(∞)	0.33	0.67	0.00	0.23	0.76	0.00	0.03	0.95	0.00

Panel C: Return Predictability (Dividend-Price Ratio)

	Data		PPU		AU	
h	β	R^2	β	R^2	β	R^2
1Y	0.07	0.03	0.12	0.08	0.04	0.02
2Y	0.16	0.09	0.18	0.11	0.06	0.03
3Y	0.24	0.14	0.23	0.14	0.07	0.04
4Y	0.31	0.19	0.27	0.16	0.08	0.05
5Y	0.40	0.24	0.31	0.18	0.09	0.06

Note: This table reports moments from the data and the production economies with unknown transition probabilities. The empirical statistics are reported in the "Data" column and correspond to the U.S. data from 1947:Q2 to 2016:Q4. The models with parameter learning assume priced parameter uncertainty or anticipated utility ("PPU" and "AU" columns) with unbiased initial mean beliefs. For fully rational pricing of parameter uncertainty, we present the results for the models with 100, 150, 200, and 10000 years of prior learning. For anticipated utility pricing of parameter uncertainty, we consider the model with a 100-year prior because employing alternative lengths generates virtually the same results. The model-based moments are means of statistics based on 1000 simulations. Simulated statistics are calculated for the length of time corresponding to a full sample size and are expressed in quarterly terms. $E(x), \sigma(x), SR(x), ar1(x)$, and $\rho(x, y)$ denote the sample mean, standard deviation, Sharpe ratio, the autocorrelation of x, and correlation between x and y, respectively.

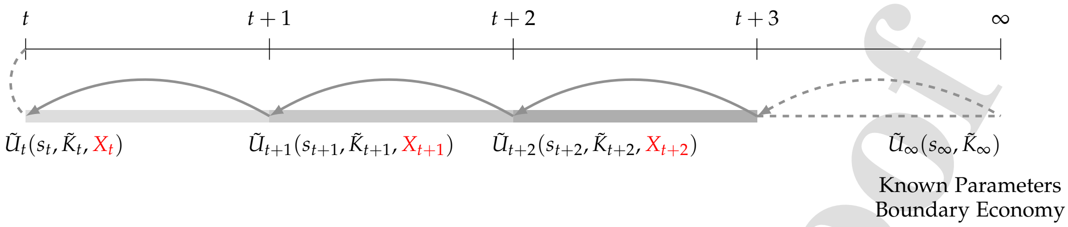

Figure 1. Priced Parameter Uncertainty.

Note: The figure illustrates the agent's continuation utility in the production economy with priced parameter uncertainty. The equilibrium utility is the function of the state s_t, capital $\tilde{K}_t$, and sufficient statistics for unknown parameters X_t. To find $\tilde{U}_t = \tilde{U}_t(s_t, \tilde{K}_t, X_t)$ at time t, the agent uses the backward recursion starting from the known parameters boundary as shown by arrows in the diagram. The boundary economy, in turn, is solved assuming the agent knows the true parameters in the productivity growth process.

Highlights

Rational parameter learning in a real business cycle model amplifies asset prices

The magnitude of the amplification depends crucially on procyclicality of dividends

Rational parameter learning generates realistic dividend yield variance decomposition

Time-varying beliefs reproduce the long-term forecastability of equity returns