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# Essays on Financial Systems 

by

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A thesis submitted in fulfilment of the requirements for the degree of Doctor of Philosophy in Economics

University of Warwick, Department of Economics December 2008
to my family

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## Declaration

An earlier version of Chapter Two appeared as "Self-Enforcing Multilateral Relational Contracts", presented at the Centre for the Study of Globalisation and Regionalisation, Hitotsubashi University, University of Warwick and Royal Economic Society Easter School. An earlier version of Part II appeared as "Soft Budget as a Credit Cycle: a Mechanism of Zombie Firms", presented at the Centre for the Study of Globalisation and Regionalisation.

## Summary

It is said among historians, that there are two remarkable innovations in modern finance: deposit banking in southern Europe and negotiable bills in northern Europe, especially Antwerp. Although negotiable bills are as important as deposit banking (because they became a foundation of modern commercial banking and stock markets), they are not often studied. Part I of the thesis studies indirect loan contracts which do not rely on either bank-specific technologies or legal protection. It focuses on the concept of negotiability and explains its characteristics, including the substitutability of deposit banking and negotiable bills. Negotiable bills, or resaleable bills, can be interpreted as an indirect loan contract. The buyer of the bill, i.e. the initial lender, can re-sell the bill to a third party to satisfy his liquidity needs. So the initial issuer of the bill borrows from a third party, through the initial lender (acting as an intermediary). Previous studies have focused on direct loan contracts: between banks and borrowers, depositors and banks, or suppliers and buyers. There are few papers studying the incentive problems faced by all three players. To fill this gap, in Chapter Two, we study indirect loan contracts that a lender and a borrower can make only through an intermediary agent, where the borrower and the lender cannot observe any transaction between the other two. Under this severe information asymmetry, the existence of loan contracts as a sequential equilibrium is proved, although they are less efficient compared with direct loan contracts. In Chapter Three, we consider role of collateral in improving efficiency. Chapter Four concludes, summarising the characteristics of these contracts: only less risky borrowers can issue negotiable bills and riskier borrowers need to seek a direct relationship with lenders (or, they are rationed).

In the 1990s, the Japanese economy experienced a prolonged recession, the so-called 'lost decade'. It is discussed that a cause of the problem was the "zombie lending" problem: chronic loss-making firms (zombies) still obtained finance from their banks. Part II of the thesis aims to address the following issues with a microeconomic model. Firstly, why did banks not liquidate bankrupt borrowers? Secondly, how did it affect macroeconomic productivity? And thirdly, how did it affect the procyclicality of land prices as in Kiyotaki and Moore(1997)'s credit cycle? A bank, in this model, has an incentive not to liquidate insolvent borrowers: the liquidation of collateral asset (land) will invite the collapse of land market and the bank has to bear a large loss. The loss may make the bank under-capitalised and force it to close its business. The bank, to avoid the forced closure, does not liquidate insolvent borrowers. This "zombie borrowers" occupy their land unused, and the bank can squeeze land supply to push up land price: the bank's own capital is then kept higher than it should be. In the final chapter, based on this model, optimal post-crisis policies are discussed by comparing two options; public capital injection and toxic asset purchasing scheme.

## Part I

## Self-Enforcing Trade Credit

## Chapter 1

## Introduction

We study multilateral self-enforcing financial contracts among non-financial institutions, where players can make contracts only through an intermediary. In particular, we first explain when and how firms can make indirect loan contracts without any help of bank-specific technologies or legal enforcement. Then we discuss their inefficiency and possible solutions.

There have been two remarkable innovations in the history of finance ${ }^{1}$. One was the emergence of deposit banking in southern Europe, and the other was the resaleability or negotiability of bills in northern Europe. The success of deposit banking is widely known: in the thirteenth century, large deposit banks in Italy became a centre of international settlements and their bank-notes were circulated as a convertible money (so-called 'inside money'). Since coinage was always scarce in medieval Europe and it was costly and risky to bring metal coins to remote merchants, this enabled the rapid expansion of international trade in Europe.

On the other hand, the Low Countries (Netherlands, Belgium and Luxembourg) innovated a different style of finance, based on Antwerp, another predominant financial centre in medieval Europe (Van der Wee(1963), Kohn(1999a)). The remarkable feature of Antwerp was that businesses were run without banks: in contrast to southern Europe, the rulers of the city had banned deposit banking since the 1480s. This caused a serious settlement problem in the city, since bank deposits and bank-notes were the main instruments for settlement. Merchants in

[^0]Antwerp solved the problem by making their debts (especially bills) assignable, transferable or negotiable ${ }^{2}$. This meant that current holders of the bills (or other debts, like bonds) could make payment by giving the bills they possessed, instead of money, when they needed to pay. This innovation made bills (bills obligatory and bills of exchange) a convertible money with a certain limitation compared with cash. Thus merchants in Antwerp succeeded in developing a completely different type of inside money that did not rely on deposit banks. Later on, this second innovation, negotiability of bills, was mainly utilised for international trade credit; while the first innovation, deposit banking, provided a clearing service for local transactions, as we now explain.

In the seventeenth century, after Antwerp had declined due to the malfunctioning of its port, Amsterdam took up the position of financial centre of northern Europe, with two remarkable institutional innovations: the Bank of Amsterdam and the Dutch East India Company. The Bank of Amsterdam, established in 1609, was the first exchange bank in northern Europe and succeeded in combining these two financial systems: deposit banking and negotiable bills. According to Quinn and Roberds(2006), the municipality of Amsterdam required the settlement of bills of exchange through the bank: so merchants in the city had to open a deposit account with the bank. This was the first link between deposit banking and credits using negotiable bills. The negotiable bills then transferred to London and became even more sophisticated, with the innovation of the discounting of bills. Bills of exchange had been popular as an instrument of short-term loans in London until the First World War, when they were replaced by the commercial banking system in the form of bank advances (Day(1957)).

Even though their importance has declined compared with the medieval era, negotiable bills still play an important role in modern financial markets, especially in trade credit market. Even in Japan, known for the dominance of deposit banking, Kagabayashi(2004) points out that the sales credit (trade receivable) of small-medium sized manufacturing firms occupies more than

[^1]$25 \%^{3}$ of their total assets. Emery(1984) reports similar figures for US manufacturing firms: he points out that $26.5 \%$ of the total assets of manufacturing firms was accounts receivable created by trade credit sales.

The other remarkable innovation in Amsterdam was the first stock issuance in history - made by the Dutch East India Company. As Neal(1990) writes, the permanent capital fund that the company issued in 1609 was irredeemable, and shareholders were entitled to receive annual dividends and to transfer their shares to other investors. He points out that this first issuance of shares was a successful blend of 'early financial innovations - safe and rapid international movement of funds in the bill of exchange, and safe and liquid long-term investment in the perpetual annuity'.

Considering these two factors, it may not be an exaggeration to say, as some historians do, that the negotiability initiated in Antwerp in the sixteenth century became the foundation of modern financial markets.

There are two critical features in this transaction involving negotiable bills: no bank-specific technologies or informality. First, it does not rely on any bank-specific technologies, since it was innovated in Antwerp, where deposit banking was banned. This feature allows both nonfinancial firms and banks to use this instrument for their credit transactions. This feature was especially helpful for northern European countries, where deposit banking was weak (e.g. London), to establish efficient financial markets without banks.

As a modern illustration of this non-banking characteristic, consider what happened in Ireland in 1970 when all offices and branches of major clearing banks were closed for six months due to industrial disputes and it took a year for the banking business to go back to normal. Bank deposits, which accounted for more than $80 \%$ of the money supply, were unavailable for a year; the economy lost a major instrument of settlement, or finality, for all transactions. Murphy(1978) shows how the Irish economy sorted this problem out. Businesses issued their

[^2]own IOUs, instead of bank-notes or bills of exchange drawn on the bank, for their settlements. Sometimes the privately issued IOUs were re-sold for further settlements, and acted as alternative settlement instruments instead of bank deposit until the clearing banks restarted their businesses.

The second characteristic of negotiable bills was informality. As Kohn(1999a) points out, bills in Antwerp were basically informal documents ${ }^{4}$. As we will see below, courts ratified the merchants' custom (implicit contract) of negotiability; and finally the custom was enacted into law. Thus, the informal contracts in Antwerp became the archetype of modern negotiability. This implies the importance if studying negotiable bills as informal loan contracts. Furthermore, it is not unrealistic to assume that inter-firm credit has less legal protection than bank credit, considering its business nature. It would be less verifiable, negotiation costs are relatively larger for small firms, and there are too many firms to audit. Some empirical studies actually show that trade credit tends to be used in areas with less legal protection (see below).

In the following chapters, we will see what kind of contract could re-create these features. As both history and some empirical research (see below) show, these two characteristics coexist in many cases. This implies the importance of studying self-enforcing debt contracts without bank-specific technologies. Furthermore, to re-create the negotiability (or resaleability) of bills in a game-theoretic framework, we focus on indirect debt contracts, where borrowers and lenders can make loan contracts only through an intermediary agent.

In this chapter, we will proceed as follows. In discussing previous studies of trade credit and its characteristics, it is shown that trade credit studies tend to assume some "supplier-specific" technologies, instead of bank-specific technologies; but we cannot take this approach because negotiable bills have been used among pure financial institutions ${ }^{5}$ as well (c.f. negotiable $\mathrm{CDs}^{6}$ ). Instead, we focus on relationship lending (using reputation as the enforcement mechanism), which does not have to rely on bank-specific or supplier-specific technologies. At the same

[^3]time, it is shown the reason why negotiability or resaleability is difficult to describe in this relationship-lending approach. The significance of the informal contract is emphasised, although legal protection easily allows the negotiability. Last of all, several past studies relating to indirect loan contracts are reviewed, followed by a summary of the core model of this part, to be explained in Chapters Two and Three.

### 1.1 Literature Review

### 1.1.1 Trade Credit

Trade credit, non-banking inter-firm credit transactions that resemble the financial system in Antwerp without deposit banking, has been studied in the last several decades, mainly to tackle the question of why interfirm credit is used while bank credit is available. As many empirical research show, implicit interest rates on trade credit are usually higher than that of bank credits. For instance, Cuñat(2006) shows that a common trade credit, " $2-10$ net $30 "$ contract, implicitly imposes about $44 \%$ annually ${ }^{7}$. This is obviously higher than the bank loan rate, and it is natural to expect that bank loans should dominate trade credit.

Yet, as Emery(1984), Kagabayashi(2004) and other papers show, trade credit plays an important role in today's financial markets. Most of the studies of trade credit try to deal with this paradox. According to Petersen and Rajan $(1997)^{8}$, there are two major categories of trade credit studies. The first emphasises its industrial aspect. For instance, Brennan, Miksimovic and Zechner(1988) argue that trade credit is used as a tool of price discrimination among buyers, by considering the case in which a credit customer has lower reservation price than a cash customer. Lee and Stowe(1993) argue that trade credit (delayed payment) rather gives an opportunity for the buyer to check the quality of a purchased good. Thus, for the sellers, offering delayed payment acts as a signal of product quality.

[^4]The second answer was in terms of the imperfect credit circumstance of trading firms. Petersen and Rajan(1997) further categorise this into three, and the following taxonomy is its modification. The first one regards trade credit as a result of ex-ante information asymmetry. For instance, $\operatorname{Smith}(1987)$ argues that trade credit is used as a screening device. If buyers prefer trade credit to cash transactions at discount, it implies the buyers' credit deterioration. Or, Biais and Gollier(1997) regard trade credit as a signalling device. Their signalling mechanism is similar to Holmstrom and Tirole(1997): if a supplier has private information of a buyer's creditworthiness, the supplier's behaviour (of lending / not lending) acts as a signal for a third party, a bank. Burkart and Ellingsen(2004) suppose that suppliers have an information advantage about their borrowers' transaction, since traded intermediate goods are firm-specific, in contrast to cash. This creates a relative advantage over banks who deal with cash, which is anonymous.

The second assumes that a supplier can control a buyer's behaviour to some extent, to solve a borrower's moral hazard problems. Typical assumptions are that the supplier has some relationship with the buyer and it is costly to reconstruct the relationship again. With the relationship, the supplier can enforce the buyer's repayment by threatening them with the cutoff of the relationship. At the same time, the supplier has an incentive to help the buyer who is suffering from temporary liquidity shortage; or, the buyer goes bankrupt and the supplier loses the valuable relationship with the buyer. The relationship enables the supplier to lend to the buyer who is credit-rationed by a bank due to information asymmetry. Smith(1987) was the first one who mentioned such a relationship and its role, and Cuñat(2006) models it fully. He argues that such a kind of relationship is achieved when a buyer produces final goods with a specific intermediate good that is produced by a monopolistic supplier of the good. It is costly for both the buyer and the supplier if the relationship is broken, since the buyer has to use generic goods which give lower productivity, and the supplier loses the customer. Kim and Shin(2007) can also be categorised into this "trade credit as a result of moral hazard problems". They consider a vertical production chain: a supplier produces an intermediate good for another supplier, who produce another intermediate good for another, downstream supplier. The chain ends when it reaches a supplier of the final good. They argue that the longer production chain means there is a longer time lag between the production of an intermediate good and the receipt of its revenue.

It means that each supplier has a claim on his downstream supplier, and the claim acts as a "stake" of the relationship (the production chain itself). The final good supplier can force the suppliers of the intermediate goods to stay in the chain by borrowing from upstream suppliers and promising to repay appropriately (the suppliers would lose their revenues (repayments from downstream suppliers) if they stop participating into the chain).

The third one assumes that suppliers have an advantage in liquidating collateral seized from default buyers. This skill, better than banks, allows trade credit to be advantageous (see Mian and Smith(1992)). Frank and Maksimovic(2004) make a similar model, with many players. In the theories of banking, Diamond and Rajan(2001) use a similar approach.

### 1.1.2 Characteristics of Trade Credit Studies

The common characteristics of these trade credit studies is that they assume some specialities of non-bank suppliers; the necessity of signalling or screening, the collateral collection skill of suppliers, and so on. Of course it is natural to assume this, to show the preference of trade credit to bank credit. Yet instruments or technologies used in trade credit transactions are actually common among bank credit transactions, too. As we see in the history of Antwerp (see Appendix), trade credit emerged through utilizing bills obligatory and bills of exchange in medieval Europe. Some historians point out that these instruments began to be used for pure financial transactions (e.g. exchange rate arbitrage) as well, soon after these have been used for trade credit. The boundary of trade credit and bank credit would be, that is to say, more ambiguous than was thought previously. The ambiguity can be observed in the case of Ireland (see Appendix for further information). Trade credits with cheques drawn on the closed bank substituted bank credit during the period of bank closure. It is essentially not bank credit, because people might not expect that the cheques would be repaid ${ }^{9}$. Thus, it should be regarded as non-bank credit. The cheques became banks' credit again when the banks restarted business.

[^5]Considering this, it would be valuable to concentrate on the characteristics and mechanisms of the instruments used in trade credit, instead of focusing on the characteristics of its users by distinguishing banks and non-banks as previous researchers have done. This means that we need to make a model of credit transactions using bills, which relies neither on special banking skills (i.e. monitoring, collateral collection, commitment to a demand-deposit contract etc.) nor on the suppliers' special skills (i.e. enforcement ability using intermediate goods, collateral collection skill, signalling needs etc.). Through the model, we may see why trade with bills has become the foundation of the modern financial system, with its flexibility that can be used by any players.

Considering the characteristics of bills transactions, we do not have to limit our attention to the literature about trade credit. Although it is rarely referred to in trade credit studies, theories of banking have similar approaches to the arguments in trade credit. For example, the signalling model in the information acquisition approach mentioned above is similar to Holmstrom and Tirole(1997). In their model, the bank is not a signal receiver, but a signal maker. The third parties who receive the signal are uninformed investors, not banks, in Holmstrom and Tirole(1997). It is also common in banking theories to assume a certain ability to enforce repayment as well. Diamond(1984) assumes that a bank can enforce a borrower's repayment directly and a depositor can enforce a bank's repayment by non-pecuniary penalty. Calomiris and Kahn(1991) assume that banks can commit to a demand-deposit contract and this gives the incentive of monitoring to depositors. Or, Diamond(1991), Rajan(1992) and Boot, Greenbaum and Thakor(1993) study how relationship and reputation work in financial contracts. The advantage in collateral liquidation skill is studied by Diamond and Rajan(2001), for example.

### 1.1.3 Relationship Lending and Resaleability

Unless we assume the existence of supplier-specific or bank-specific skills, debt contracts are enforceable only through relationship (reputation). Such relational contracts have been studied both in trade credit and banking theories. In the theories of trade credit, an exclusive relationship is assumed in $\operatorname{Smith}(1987)$, Wilner(2000), Cuñat(2006) and others. Smith(1987) and

Wilner(2000) argue that a relationship between lender and borrower gives the lender an incentive to help a borrower in trouble, because the relationship is assumed to be valuable for the lender and it will be lost if the borrower goes bankrupt. This can be regarded as a Soft Budget Constraint of Kornai(1979) or Dewatripont and Maskin(1995), as Boot(2000) points out in the context of banking studies. Cuñat(2006) uses the relationship to enforce borrowers to repay as well. Smith(1987) and Wilner(2000) assume that making such relationships needs a certain sunk cost that is exogenously given, while Cuñat(2006) uses user-specific intermediate goods to make an exclusive relationship between a lender and a borrower. A positive correlation between trade credit and firms' relationship is empirically shown by McMillan and Woodruff(1999) and Uchida, Udell and Watanabe(2006).

Similar issues are discussed in the theories of banking. One of the most notable topics is competition between relationship-based banking and transaction-oriented banking, which Boot(2000) surveyed. According to Boot(2000)'s definition, relationship-based banking relies on private information obtained by bank-specific monitoring or screening and also on multiple interaction that enables intertemporal information reusability. The transaction-oriented banking is, in contrast, mainly done by "less-informative" investors like bond-holders; arm's length lending, proprietary trading and other debts frequently traded in secondary markets are included in this category. Many papers study the reasons why relationship-based banking is dominating, or coexisting, with transaction-oriented banking. For instance, Diamond(1991) argues that high-credit firms choose transaction-oriented banking because their reputation is sufficient to bind themselves to behave honestly towards less-informed investors. By contrast, lower credit firms need banks' costly monitoring technology. Sharpe(1990) and Rajan's(1992) arguments are similar to this point. Holmstrom and Tirole(1997) describe how banks provide credible signals to other uninformed investors.

One common characteristic of relationship lending, regardless of trade credit or banking, is that it is basically focusing on the bilateral relationship between lender and borrower. Although some banking studies look like games with three players (borrowers, banks and depositors), these studies concentrate on the relationship between banks and depositors, by assuming that there is no incentive problem between borrowers and banks. For instance, Diamond(1984) assumes
banks' skills of monitoring their borrowers. Diamond and Rajan(2001) assume that borrowers' loans are perfectly secured by collateral. Neither models describe borrowers' decision-making. In this sense, these banking studies are still essentially bilateral loan contracts.

The impossibility of the three players' multilateral loan contract is explained by Diamond(1984). He points out that bank loans are not resaleable, because banks lose the incentive to enforce their borrowers' repayments once the banks sell their loan contracts to third parties. As Boot and Thakor(2000) argue, most banking studies, including the consumption insurance approach as in Diamond and Dybvig(1983), are part of relationship-based banking. Accordingly, loan contracts cannot be enforceable once the relationship has broken down. If the bank sells its loan to a third party who does not have a relationship with the initial borrower, neither the bank nor the third party can enforce repayment since the initial relationship is lost (even if the bank retains the relationship, it has no incentive to help the third party, since there is no relationship between the bank and the third party). This is the reason why loan sale is not allowed in the theories of relationship-based banking.

This restriction would be too strong, considering the modern financial system from the sixteenth century onward. As explained in the history of financial development in medieval Europe, bills obligatory and bills of exchange, forms of loan contracts, have been re-sold for settlement. At the present time, Boyd and Gertler(1994) and Gorton and Pennacchi(1995) point out the rapid growth of securitisation and the sale of bank loans. In that sense, as Boot and Thakor(2000) point out, the boundary of relationship-based banking and transaction-oriented lending is ambiguous. These facts imply the importance of studying resaleable loan contracts, in contrast to the previous studies both of banking and trade credit.

### 1.1.4 Informality of Loan Contracts

One would think that these re-sales of loans is enabled by legal enforcement, and that the relationship is not necessary in the system; i.e., observed resaleability of loans does not contradict the claim of relationship-based banking theories. In fact, this is partly true. Many historians, e.g. Van der Wee(1964) and $\operatorname{Kohn}(1999 a)$, argue that legal enforcement played a certain role
in diffusing resaleable financial instruments, i.e. negotiable bills. At the present time, bond issuers are legally enforced to repay their debts to any holders, regardless of the existence of a relationship.

Yet, as Kohn(1999a) points out, bills of exchange and bills obligatory initially emerged as an informal instrument. Legal enforcement was made afterwards, in accordance with merchants' custom (see Guest(1998), for instance) ${ }^{10}$. The custom was initially ratified by court in 1437 , in the case of Burton v. Davy. The court confirmed the mercantile customs at that time and gave legal guarantee. This famous case is regarded as a milestone of financial laws among jurists (see Beutel(1938) or Munro(2003)). See Greif(1989,1993) and others as well for historical studies of informal relationship.

Even at the present time, legal enforcement mechanism is still far from perfect, especially in developing countries. For instance, McMillan and Woodruff(1999) surveyed private firms in Vietnum and found that 91 percent of the firms did not think courts could enforce a contract with a customer. Johnson, McMillan and Woodruff(1999) surveyed transition countries ${ }^{11}$ as well in 1997, and found that more than $30 \%$ of the firms had negative view regarding reliability of courts' enforceability ${ }^{12}$. Furthermore, less than 40 percent of the firms used courts when they had the latest dispute. Such kind of informal finance is empirically reported by Ghate(1992) and Fisman and Love(2003) as well.

### 1.1.5 Multilateral Loan Contracts

These issues stated above lead us to consider informal multilateral loan contracts, relying on neither bank-specific technologies nor supplier-specific technologies. There are a few studies of multilateral loan contracts with more than three players. Kiyotaki and Moore(1997) study a kind of production chain. Each player produces an intermediate good that is specific to the other

[^6]player. This is slightly different from normal loan contracts, because suppliers (lenders) give their products in exchange for payment (in a sense, it is perfectly collateralised). They study the propagation of liquidity shock in this chain. Kim and Shin(2007) study a different type of multilateral contracts, a production chain. In their model, each intermediate good's supplier has to bear a time lag from his/her production until payment of the production as the production chain becomes longer. This is because it takes time to transfer their intermediate goods to downstream firms. This loan asset makes the suppliers stake-holders, and loan contracts giving sufficient rent to every suppliers are enforceable. Shin(2005) studies a circle-shaped production chain and obtains the unique equilibrium asset price determined in the circle. Mobius and Szeidl(2006) study a loan contract from the network approach. They assume a valuable network (friendship) as an exogenously given condition and consider loan contracts in the network.

These studies assume symmetric information, although some of them deal with the (recursive) moral hazard problem (Kim and $\operatorname{Shin}(2007)$ ). This is the reason why they can avoid the impossibility noted by Diamond(1984). Symmetric information is a reasonable simplification, but it would be worth removing this assumption because information asymmetry is the heart of relationship-based lending, as Boot(2000) argues. As far as I know, there are two studies tackling multilateral loan contracts with information asymmetry. One is Frank and Maksimovic(2004). There are three players, bank, seller and buyer. The bank can lend to the buyer directly, but the seller has better collateral collection skill; accordingly, it would be better for the bank to lend to the buyer through the seller. Yet, in this indirect finance, the bank cannot observe the buyer's creditworthiness, which is the seller's private information. The authors study when and how the bank chooses indirect lending through the seller (trade credit).

Another one is Kiyotaki and Moore(2003), studying the resaleability and possibility of resaleable papers being used as inside money. In their model, the initial lender and final borrower have to trade through an intermediary, due to the intertemporal lack of double coincidence. The repayments from the borrower to the intermediary and the intermediary to the lender are guaranteed by collateral (and equivalent collateral collection skill). Yet there is an adverse selection problem. The initial creditor can learn about credit quality before the creditor re-sells the claims she possesses, so she has an incentive to pick bad claims to sell these to third parties.

This adverse selection problem prevents third parties from purchasing second-hand claims and the claims become non-resaleable. The authors showed that this adverse selection problem can be solved by sufficiently low credit rationing in bilateral contracts, or by a banking technology to bundle the claims that prevents the initial creditor from picking.

### 1.2 Summary of the Model

The model explained below is another version of multilateral loan contracts. The critical difference of this model from previous studies is its self-enforcing characterization. It does not assume any legal enforcement, any collateral, or any special technologies of banks or suppliers. Additionally, it assumes severer information asymmetry: the lender cannot observe any transaction between the borrower and intermediary, and the borrower cannot observe any transaction between the lender and intermediary. This assumption makes players' beliefs complicated, compared with Frank and Maksimovic(2004), which gives players' beliefs exogenously ${ }^{13}$.

Instead, I have simply assumed that the relationships among players are given and fixed: a lender can lend resources to a borrower only through an intermediary agent. The borrower can repay the loan only through the intermediary agent. This assumption contrasts with previous trade credit studies that try to explain the reason why an exclusive relationships are sustained between players (player-specific intermediate goods achieve the relationship in many papers). This ex-ante relationship is basically for simplification, but can be justified with McMillan and Woodruff(1999)'s survey. They used location of firms as a proxy of relationship and find positive correlation to the amount of trade credit. This means that relationships among firms are given by geographical conditions - namely, exogenously given. The Central Bank of Ireland(1971) also mentioned that personal relationships made in pubs played an important role in the financial

[^7]transactions among non-banking firms. In the next chapter, I assume that the lender and the borrower live in different countries and only the intermediary agent, à large merchant, has branches in both countries. This is close to the situation in medieval Europe, where cross-border transactions became popular.

We will see in the next chapter that such kinds of multilateral loan contracts are possible, but less efficient, because a severer penalty strategy is necessary and it is inevitable for a borrower to be punished even when the borrower behaves honestly. The penalty strategy needs to be exponentially more severe as the borrower's investment project is riskier; and more severe penalty inevitably lowers the efficiency of the economy. We will see how we can ease the penalty in Chapter Three, by discussing collateral or mediator. If the borrower has sufficient amount of collateral, we can achieve the first best equilibrium (although this requires the borrowers to be wealthy to some extent).

This part continues as follows. The core model is discussed in Chapter Two. Chapter Three modifies the model with collateral. Chapter 4 concludes, and we will see that the characteristics of the model show a good fit to the case of Antwerp, Ireland and others.

### 1.3 Appendix to Chapter One

### 1.3.1 Emergence of Transaction with Bills in Antwerp

It is said that there are two remarkable innovations in modern finance. One is the emergence of deposit banking in southern Europe and the other is the resaleability or negotiability of bills in northern Europe. These two innovations arose from the same need: scarcity of money (coinage). Many historians point out that metal money was scarce in medieval Europe, even during periods of economic downturn. Kohn(1999c) points out that there was a serious shortage of gold coin in Bruges around $1400^{14}$, for example.

[^8]
## Shortage of Coinage and Emergence of Banking

There are three main reasons for this shortage. The first one is its bad quality. According to Kohn(1999c), after the collapse of the Roman empire, minting policy became very loose. Each manor minted to their own standard, and minting was taken over by private goldsmiths and silversmiths. Due to the crude minting technology, it was easy to counterfeit coins. Clipping and culling were common among money-changers and others. It lowered the amount of coinage available for transactions. Second, quantity of gold minted was stagnant by the middle of the fifteenth century. During the fourteenth century, silver, which had been minted badly, was largely replaced by gold, which was generally of good quality. Yet European economies relied on Africa for the production of gold and it was scarce historically, due to the prolonged interruption in Mediterranean trade during the Dark Ages. The scarcity continued until new mines were discovered in central Europe and then America. Third, as de Roover(1948) points out, long-distance trades, which became common in medieval Europe, lowered the velocity of these coins significantly. It took a long time to ship gold bullion or other metal currencies, and merchants had to prepare à large amount of commodity money. Fourth, these precious metals had been used not only for transaction purposes, but also for non-monetary uses, such as hoarding and decoration, as $\operatorname{Kohn}(1999 \mathrm{c})$ points out.

Van der Wee(1963), Kohn(1999d) and other historians point out that money-changers, goldsmiths and silversmiths began to act as a sort of bankers in this era. Since coinage varied in quality, merchants needed to ask money-changers to check and certify this quality in every transaction. According to Kohn(1999d), it was more efficient for the money-changers to count and check coins only once and then store them, rather than check the same coin in every transaction. When merchants needed to pay or receive money, they ordered it to the money-changers and the money-changers settled by transferring the money on their books. Kohn(1999d) points out that this is a primitive book-entry system. Goldsmiths and silversmiths, who had to keep precious metals for the time being, issued a kind of securities in trust, and these acted as a primitive form of deposit certificates. These primitive bankers made it possible to trade without coinage.

## Frequent Bank Failure and Bans of Banking

Yet, as de Roover(1948, p331) wrote, 'In the Middle Ages, commercial banking was not a conservative business but a risky game': their business was unstable due to several reasons. First, they did not have safe borrowers: sovereign debt was one of the most risky loans for the bankers, since they needed to rely on their personal relationships with royal families to collect the repayments. They lent to high-risk business ventures directly, which was of course illiquid. The size of their balance-sheets was still too small to be a buffer from liquidity shocks. In addition, central banks, which prevent liquidity shocks from propagating, did not exist at that time. De Roover(1948) mentioned an episode in which 96 banks out of 103, which had existed in Venice at one time or another, came to a bad end. Frequent bankruptcy in the other cities are also reported; Bruges by de Roover(1948) and Antwerp by Van der Wee(1963).

These bank failures, accompanied by the ruin of "all classes of people" (de Roover(1948, p340)), made rulers of Antwerp prohibit banking business itself. The Dukes of Burgundy declared bans on banking business in 1433, 1467 and 1480; Van der Wee(1963) showed that only one money-changer was still reliable in 1480 and the last money-changer finally run in 1484. Also, in 1486 there was a series of bankruptcies of bankers, and deposit and clearance banking businesses in Antwerp were completely ruined.

De Roover(1948) points out that the rulers had other reasons to prohibit the banking business. Since the banking business was risky, as mentioned above, the bankers needed to hold sufficient amounts of gold specie as their own capital, a buffer from liquidity shock. This means that currency debasement was favourable for them: the value of gold bars became relatively higher than coin and the bankers' capital was reinforced. In a sense, currency devaluation policy was not only a monetary policy, but also a policy of prudence to rescue ailing banks. In addition, the bankers' picking and culling also debased coinage and had a great benefit to the bankers. The Dukes of Burgundy or Maximilian of Austria intended to improve the quality of the coinage by forcing the bankers to bring their species back into circulation. This monetary stability policy and the following deflation (currency appreciation) was critical for the banking
business ${ }^{15}$.

## Emergence of Bills as a Financial Revolution

Merchants in Antwerp, then, had to seek another measure of settlement other than banking, since coinage was still scarce. This prompted the development of another financial technology, bills of exchange and bills obligatory. Merchants paid by bills, and offset the liability by another receivable. Bills were circulated instead of coins. Yet trades with bills had significant drawbacks compared with coinage; that is, the resaleability of bills was very limited, while coins could be handed over to anybody. That is, a seller who received a bill might face difficulty when he/she had a liquidity need, since he/she could not pay by the bill received (that is to say, they could not re-sell the bill received). This was because the third party, who is offered the bill by the seller, did not know about the creditworthiness of the issuer of the bill. The seller would correctly expect this and refused to receive the bill. Such a kind of problem did not occur when they traded with coins, or through banks; since every transaction was concentrated on banks, the banking business did not face such a resaleability problem.

Accordingly, we can say that the history of bills in medieval Europe is the history of making the bills resaleable. Van der Wee(1963) categorises the developmental progress of transactions with bills into three stages. The first one was normal assignment, which had been diffused in the fifteenth century. Van der Wee(1963) points out that the assignment was not a transfer of the rights of a creditor. The seller of a bill remained the debtor, until the initial creditor's receivable was satisfied by the issuer of the bill. The second stage was the assignment of bills that was payable to bearer, which was established in Antwerp by 1510. Compared with the bills that was payable to order, bills that was payable to the bearer were convenient for creditors, since anybody, not only the creditors themselves or their proxies, could receive the repayment from their debtor, if they presented the bills. Yet it was risky system at the same time, since

[^9]even robbers of the bills could obtain the right to receive the repayment. The other problem was the anonymity caused by assignment. If bills that was payable to the bearer were assigned several times, it became difficult for creditors (holders of the bills) to specify the initial debtor. This problem was solved in the third stage, endorsement. Endorsement required every seller to sign their names when they re-sold their bills. Important point is that signers undertook an obligation to repay the debt if the initial debtor failed to repay. If the holder of the bill could not receive repayment from the initial borrower, the holder could claim the repayment against the last endorser(signer). The endorser could then claim the repayment against the previous endorser, and this continues until it reaches the initial debtor. That is to say, endorsement was, and is, a sequential guarantee mechanism. This mechanism could solve the problem of anonymity cited above, and the alternative financial system reached completion. They could use bills for their settlement instead of coinage, without the help of deposit or clearance banking.

Many historians regard this negotiability of bills as a financial revolution in medieval Europe. This technology was introduced in London in the seventeenth century, and became the foundation of the modern financial system.

### 1.3.2 Informal "Money" in Ireland in 1970

Antwerp was an interesting case showing how the economy worked without banks in medieval Europe. Now we look at the other economy without banks in 1970. From May 1st to November 17th $1970^{16}$, Ireland's major clearing banks (Associated Banks, Chase, and Bank of Ireland International) were closed for business due to an industrial dispute. Since the Associated Banks (main clearing banks) accounted for $75 \%$ of total bank credits (Central Bank of Ireland(1971)), this closure meant that people in Ireland lost most of their instruments for settlement. (Murphy (1978) points out that current and deposit accounts with the banks made up $85 \%$ of M2 in 1970.) The decline in money supply was partly offset by increases in currency in circulation and by the current accounts of the other operating banks. But Murphy(1978) estimated that the increase constituted less than $1 / 12$ of the Associate Banks' current and deposit accounts.

[^10]A notable characteristics of the banks' closure in Ireland is that the malfunctioning bank credit had been replaced by cheques drawn against the closed banks and other trade credits. Usually, the cheque drawn by an issuer would be settled in a few days. Yet, during the bank closure, an acceptor of the bill could not expect immediate repayment of the bill; furthermore, the acceptor could not be sure when the banks would restart the business and he/she would receive the repayment. As Murphy(1978) argues, essentially the cheques or bills were drawn not against the current accounts of the closed banks, but against cheques and bills the issuer had received (but, of course, not settled) or the acceptor's view as to the issuer's creditworthiness. That is to say, during the bank closure, issuers and acceptors of the cheques or bills made implicit debt contracts without the banks' support or legal support. These debt contracts, which should be regarded as trade credits, functioned as an alternative money in Ireland.

This alternative settlement mechanism had several distinct features compared with the banking system. First, acceptors (lenders) had to consider not only their direct borrowers' creditworthiness, but the borrowers' payers. In many cases, one borrower's repayment was contingent on another's, who was indebted to the borrower (this was the case when the borrower had drawn the bills against his/her unpaid cheque or bill received). Second, there was limited negotiability. If we trade with money, we do not have to consider how many times the note has been used, since perfect resaleability (or negotiability) of money is assured. Yet, the "alternative money", cheques or bills did not have perfect negotiability. Murphy(1978) writes that negotiated bills or cheques were accepted only when acceptors had sufficient information about initial issuers' creditworthiness. Third, this alternative money was accepted mainly within Ireland; foreign firms accepted it less.

The main question is why the alternative finance system worked without banks' support ${ }^{17}$. Both Central Bank of Ireland(1971) and Murphy(1978) point out the close-knit social networks in Ireland. The population in Ireland was only three million in 1970, which enabled "a highdegree of personal contact amongst members of the community" (Murphy (1978) $)^{18}$. This would

[^11]also explain why the alternative money was rarely accepted by foreign firms.

Thanks to the system, Ireland's rate of growth was still positive in 1970 ( $4 \%$ in 1969 and $2 \%$ in 1970, according to the Central Bank of Ireland(1971)), although the money supply declined drastically at that time.

Another notable fact was a seamless linkage of bank credits and non-bank credits. The alternative money, trade credit, mainly used cheques drawn on the closed banks as an instrument, although they did not rely on bank credit directly. After the bank closure, however, the trade credits with cheques drawn on the banks became a part of bank credit, although these were not permitted by the banks ex ante. As a result, bank credit was increased after the closure. According to the Central Bank of Ireland(1971), the overall increase in credit was $20 \%$ (year on year basis) in April 1971, which was 8\% points higher than expected.
system. The firms used cheques drawn on the closed banks, knowing that they did not settle. It should be interpreted that issuers and acceptors of the cheques made implicit debt contracts, with a stochastic guarantee that the closed banks would settle the cheques some day. According to the Central Bank of Ireland(1971), some market participants expected that the bank closure would not last long and they would not accept these cheques if they correctly anticipated the length of the closure.

## Chapter 2

## Trade Credit as a Multilateral Relational Contract

### 2.1 Introduction

This chapter studies multilateral self-enforcing financial contracts among non-financial institutions, which players can make contracts only through an intermediary. In particular, this chapter explains when and how firms can make an indirect loan contract with their counterparts through an intermediary, without any help of bank-specific technologies or legal enforcement. We will then discuss its inefficiency: possible solutions are discussed in the next chapter.

As explained in the last chapter, the phenomenon we are going to examine is "how a lender and a borrower who cannot make contact directly make a loan contract via an intermediary agent, without assuming any bank-specific technologies or legal enforcement". The clearest example was observed in Antwerp in the fifteenth and sixteenth century. At that time, the rulers in Antwerp banned the deposit banking business due to frequent bank failures and bankers' immoral activities. This made bank-notes as a major settlement instrument unavailable, and merchants in the city needed to innovate alternative measure of settlement. The system of bills of exchange and bills obligatory emerged in such circumstances. Initially, bills were issued in bilateral relationship, but gradually merchants tended to pay by bills issued by the other
merchants. The resaleability of bills was initially informal contracts; later on, courts accepted the merchants' custom and ratified them. Such kind of resale of IOUs without legal enforcement can be observed in many cases, as we have seen in the last chapter. Many historians and economists point out in the studies of these cases that relationships among agents (firms) played an important role in such kinds of financial contracts.

This implies that we can use relational contracts to describe these phenomena. The previous studies regarding relational contracts have, however, focused on bilateral contracts (Thomas and Worrall(1988), MacLeod and Malcomson(1989) or Levin(2003)). In this chapter, by contrast, I will consider multilateral relational contract, which lender and borrower cannot have contact directly.

I assume that there are three players in the economy $(A, B \text { and } C)^{1} . A$ is a borrower, and he has a profitable project but without input goods to start. C is a lender: she is endowed with the input good but has no project. It is assumed that A and C cannot trade directly for some reason (e.g. they are living in different countries). Only B, who has neither a project nor endowment, has a relationship with both A and C ; C can lend to A only through B (i.e. B is à large merchant and has branches in both countries). C lends to $\mathrm{B}, \mathrm{B}$ then lends to A ; then A starts the project and receives its stochastic outcome; A repays to B based on the outcome, and B transfers A's repayment to $\mathrm{C}^{2}$. This stage game is repeated infinitely. See Figure 2 for an overview of the whole transaction.

With the information asymmetry, C faces several incentive problems. She is not sure whether A will honestly repay, and whether B will honestly transfer A's repayment; in addition, A is not sure whether C will really lend to B or B will steal C's loan. In order to convince C that she will be repaid appropriately, B has to bear some penalty in case A fails to repay.

[^12]By showing that default is costly for $B, C$ is sure that $B$ will repay appropriately to $C$, and she lends her endowment to B who transfers it to A . Considering that this indirect finance resembles the transaction of negotiable (resaleable) bills (see footnote 2 in Chapter One), this B's commitment to C can be interpreted as an endorsement (backing) of the bill. Endorsement, a critical mechanism that has enabled the negotiability of bills, is a declaration of the seller that he is responsible for the debt he is selling (the buyer of the bill can claim the repayment against the seller in the case of the initial borrower's (issuer's) default).

There are many papers studying more than one principal and more than one agent in the relational contracts approach, but such multilateral contracts, in which principal and agent do not meet directly, have never been studied as far as I know ${ }^{3}$.

This structure has something in common with Kiyotaki and Moore(2000), who also assume three players and that lenders cannot lend directly, due to an intertemporal lack of doublecoincidence. But the self-enforcing nature of this model makes two critical differences. First, they assume exogenously that a lender ( C , in this chapter) can extract only a certain fraction of debt service from her indirect borrower (A) (the so-called resaleability constraint). In this model, whether the lender can extract repayment from the indirect borrower or not is endogenously determined. Second, they assume that all loans are collateralised and all players have the same collateral collection skills. This assumption helps them to define inside money (circulated bills) without banking. In this model, there is no collateral; we see that bills can be traded only with reputation as a bind ${ }^{4}$.

Yet this self-enforcing indirect finance is less efficient compared to direct finance, as we will see. The indirect finance mechanism needs more severe penalties and safer projects (risky projects cannot obtain finance even though they are profitable). Some inefficiency was reported in Antwerp as well, before legal enforcement system was established(Kohn(1999a)). We see

[^13]that such inefficiency can be eased, or solved, by several measures such as legal enforcement mechanisms or collateral.

I believe that such multilateral relational contracts are widely observed phenomena and are worth studying. Other than the financial contracts we are going to study, it is common that suppliers of intermediate goods trade with assembly firms through trading agencies, or labor force is supplied to companies through staffing agencies. In particular, we cannot expect strong legal supports for international contracts, contracts among small to medium size firms or contracts in emerging markets. I hope that the self-enforcing contracts studied here shed light on these fields as well.

In this chapter, we will proceed as follows. First, we study bilateral loan contracts, where a borrower and a lender can make a loan contract directly. Secondly, the loan contract is extended to the three players' case, namely, multilateral contracts: conditions for the existence of equilibrium and the efficiency of the equilibrium obtained there are then discussed. Section Four concludes.

### 2.2 The Bilateral Game

### 2.2.1 The Nature of the Game

## Stage Game

The basic structure of the game is analogous to the traditional debt contract studies with limited liability and without costly state verification, like Sappington(1983) or Innes(1990) (we will assume ex-post information asymmetry, so it is closer to Sappington(1983) in this sense). As we will focus on self-enforcing relational contracts, however, we cannot apply their models directly; they have assumed that principals (lenders) can commit their state-contingent payoff function to their borrowers.

In addition, we will restrict the principals' penalty measures further, by assuming that principals cannot control repayment from borrowers. The principal's action is assumed to be
only the choice of lending. Therefore, it is trivial that we cannot construct the stage-game loan contract and we have the unique autarky equilibrium that the borrower cheats and the lender does not lend. Yet, in the repeated game context, it is still available for principals to punish borrowers in case of default, by a termination threat, and trading equilibrium may be retained. In this sense, the following argument about bilateral debt contract is close to Bolton and Scharfstein(1990), although our game is repeated infinitely ${ }^{5}$.

Now we will set up the model in detail. There are two risk-neutral infinitely-lived players A and C (we assume linear utility to eliminate consumption smoothing matters, which are not relevant in this argument). Player A has a profitable investment project, which requires one unit of endowment to start. Only player C is endowed, while she does not have the project. The investment project yields stochastic output $y \in\{0, Y\}$, with a prior probability distribution $\{\pi, 1-\pi\}$ where $\pi>0$. Its expected value is

$$
\begin{equation*}
E[y]=\pi Y+(1-\pi) \cdot 0 \geq 1 \tag{2.1}
\end{equation*}
$$

and its distribution is i.i.d. over time.

At the beginning of the period t , C receives a unit of endowment and chooses her action, lending amount to A, $a_{t}^{C} \in\{0,1\}$, based on her history $h_{t}^{C}=\left\{\tau_{0}, \ldots, \tau_{t-1}, a_{0}^{C}, \ldots, a_{t-1}^{C}\right\} \in H^{C}$, thus $\tilde{a}^{C}: H^{C} \rightarrow\{0,1\}$ (the endowment is assumed to be indivisible for simplicity). Player A immediately starts his investment project only when he receives $a_{t}^{C}=1$ (We ignore A's choice of whether he invests the borrowed resource or not for simplicity).

The project matures in the same period and nature picks $y \in\{0, Y\}$, which is not observable for C. Based on his type $y$ and his history $h_{t}^{A}=\left\{\tau_{0}, \ldots \ldots, \tau_{t-1}, a_{0}^{C}, \ldots . ., a_{t-1}^{C}\right\} \in H_{t}^{A}$, A chooses his repayment amount $\tau \in[0, Y]$ to C. Stage game payoffs are $g^{A}=a_{t}^{C}(y-\tau)$ and $g^{C}=-a_{t}^{C}+a_{t}^{C} \cdot \tau$. Since we assume a linear utility function for both players, the payoffs are at the same time players' utility. This stage game is repeated infinitely. The discount factor $\delta \in(0,1)$ is common among players, and both $\pi$ and $Y$ are common knowledge among players.

[^14]

Figure 2-1: Stage Game Transaction in Bilateral Contracts
Since the players' utility is obviously bounded, and the probability distribution over the outcome $y$ is i.i.d. and $\delta<1$, we can restrict our attention to a stationary structure without any loss of generality, from the principle of optimality. In addition, we assume no negative wealth constraint:

$$
\begin{equation*}
0 \leq \tau \leq y \tag{2.2}
\end{equation*}
$$

The Nash Equilibrium of the stage game, after any history, is $\left\{\tilde{a}^{A *}, \tilde{a}^{C *}\right\}=\left\{\tau=0, a^{C}=0\right\}$, as explained above.

## Repeated Game

Consider now the repeated game. The stage game shown above is repeated infinitely, given the common discount factor $\delta$. One of the repeated game equilibria is obvious: infinite repetition of stage-game Nash equilibrium $\left\{\tilde{a}^{A *}, \tilde{a}^{C *}\right\}=\left\{\tau=0, a^{C}=0\right\}$. Under the equilibrium, all financial transactions are halted. The payoffs, denoted as autarky payoffs, are $U_{\text {autarky }}^{A}=0$ and $U_{\text {autarky }}^{C}=(1-\delta) \sum_{i=0}^{\infty} \delta^{i} \cdot 1=1$.

We will show that A and C have better equilibria, which player A honestly repays to C. A repeated game payoff is defined as $U_{t}^{i}=(1-\delta) \sum_{j=t}^{+\infty} \delta^{i-t} g_{j}^{i}(\cdot)$, where $g^{i}(\cdot)$ is player $i$ 's stage game payoff. A surplus from autarky payoffs is defined as $u^{i}=U^{i}-U_{\text {autarky }}^{i}$ for $\forall U^{i}$ and $\forall i=\{A, C\}$. We are going to specify sequential equilibria with a positive surplus of the game.

Now we can define a Bayesian extensive game ( $\Gamma,\{\Theta\},\{p\},\{u\}) ; \Gamma$ is a two-players extensive game with perfect information defined above, a set of the agent's type is $\Theta=\{0, Y\}, p$ is a probability measure on $\Theta$ and $p(Y)=\pi$ and $p(0)=1-\pi$, and $u$ is a utility function such that $u^{i}: \Theta \times H \rightarrow \Re_{+}$and $u_{t}^{i}=\sum_{j=t}^{\infty} \delta^{j-t} g^{i}\left(y_{j}, a_{j}^{A}, a_{j}^{C}\right)$ for $i \in\{A, C\}$.

Since we can limit our attention to the stationary structure, we can describe the payoff function $u^{i}$ in the functional form. The continuation payoff function $\mathbf{x} \in X \subseteq \Re_{+}^{2}$ is defined as follows; $\mathbf{x}=\left(x^{A}, x^{C}\right): \Re_{+} \times \Re_{++} \rightarrow \Re_{+}^{2}$, mapping from C's repayment amount $\tau$ and discount factor $\delta$ to continuation payoff vector $\mathbf{x}$.

Players' utility in the functional form can be defined as follows:

$$
\begin{align*}
u^{A} & =E\left[(1-\delta) a^{C}(y-\tau)+\delta x^{A}(\tau, \delta)\right]  \tag{2.3}\\
u^{C} & =E\left[(1-\delta)\left(-a^{C}+a^{C} \cdot \tau\right)+\delta x^{C}(\tau, \delta)\right] \tag{2.4}
\end{align*}
$$

The continuation payoffs $x^{i}(\cdot)$ should be designed to punish the cheater, or honest behaviour cannot be enforced. Continuation payoffs are:

$$
\begin{align*}
x^{i}(\tau) & =u^{i} & & \text { for } \forall \tau \geq \hat{\tau}  \tag{2.5}\\
x^{i}(\tau) & =\delta^{l} \cdot u^{i} & & \text { for } \forall \tau<\hat{\tau} \tag{2.6}
\end{align*}
$$

for $i \in\{A, C\}$. This means that when C finds $\tau$ is smaller than the threshold $\hat{\tau}$, he will choose $a^{C}=0$ for $l$ periods then come back to $a^{C}=1$ at the period $l+1$, regardless of A's choices during the penalty phase ${ }^{6}$. A's best response for this strategy is, only when $\tau$ is smaller than the threshold $\hat{\tau}, \tilde{a}^{A}(Y)=0$ for $l$ periods and then come back to $\tilde{a}^{A}(Y)=\hat{\tau}$ as well, since he cannot expect any reward by choosing honest behaviour during the phase. This strategy is a Nash Equilibrium since C's best response to A's response is identical to her original strategy (this strategy is not stationary but the corresponding payoff is stationary).

[^15]If A's repayment is not smaller than $\hat{\tau}$, he will keep lending to A. Under the definition, we can simplify A's strategy space as $a^{A} \in\{0, \hat{\tau}\}$, where $\hat{\tau} \leq Y$, since A's utility function is monotonically decreasing w.r.t. $\tau$. The level of $\hat{\tau}$ depends on the relative bargaining power between A and C ; it is given and fixed throughout the chapter, as mentioned before.

In order to attain its enforceability, the following constraints must hold:

$$
\begin{align*}
(1-\delta) a^{C}(Y-\hat{\tau})+\delta x^{A}(\hat{\tau}, \delta) & \geq(1-\delta) a^{C}\left(Y-\tau_{0}\right)+\delta x^{A}\left(\tau_{0}, \delta\right)  \tag{2.7}\\
(1-\delta) a^{C}\left(0-\tau_{0}\right)+\delta x^{A}\left(\tau_{0}, \delta\right) & \geq(1-\delta) a^{C}(0-\hat{\tau})+\delta x^{A}(\hat{\tau}, \delta) \tag{2.8}
\end{align*}
$$

The latter violates the non-negative wealth constraint and can be ignored. In addition, we have two participation constraints,

$$
\begin{equation*}
u^{i} \geq u_{\text {autarky }}^{i}=0 \text { for } i \in\{A, C\} \tag{2.9}
\end{equation*}
$$

The first constraint automatically holds from the non-negative wealth constraint. By substituting (2.4), (2.5) and (2.6) into (2.9), A's participation constraint (2.9) can be rewritten as

$$
\begin{equation*}
\pi \hat{\tau} \geq 1 \tag{2.10}
\end{equation*}
$$

on the assumption that the enforcing constraints hold. If (2.10) holds, (2.9) also holds for any long period of autarky.

The equilibrium strategies of this game are combinations of A's choice $\left(a^{A} \in\{0, \hat{\tau}\}\right)$, C's loan $\left(a^{C} \in\{0,1\}\right)$ and C's choice of continuation function $\left(x^{A, C}\right)$, which satisfy the participation constraint (2.10), and the enforcing constraint (2.7). Here we skip to define the corresponding posterior beliefs, since the state of the economy changes every period. We will show that this combination of strategies and beliefs is sequential equilibria.

### 2.2.2 Model Solution

For (2.7), substitute continuation payoffs (2.5) and (2.6) into (2.7) and we have the following enforcing constraint:

$$
(1-\delta) a^{C}(Y-\hat{\tau})+\delta u^{A} \geq(1-\delta) a^{C}\left(Y-\tau_{0}\right)+\delta^{l+1} u^{A}
$$

If $a^{C}=0$ then the above inequality obviously holds, so we assume $a^{C}=1$ and solve for $u^{C}$ :

$$
\begin{equation*}
\left(1-\delta^{l}\right) u^{A} \geq \frac{1-\delta}{\delta} \hat{\tau} \geq \frac{1-\delta}{\delta} \frac{1}{\pi} \tag{2.11}
\end{equation*}
$$

The right inequality comes from (2.10).

Now we will check the one-stage deviation. Define A's expected surplus if he behaves honestly as follows;

$$
\begin{equation*}
u^{A}=(1-\delta) \pi(Y-\hat{\tau})+\delta\left\{\pi \cdot u^{A}+(1-\pi) \delta^{l} \cdot u^{A}\right\} \tag{2.12}
\end{equation*}
$$

and solve this for $u^{A}$, and substitute it into the enforcing constraint:

$$
\begin{equation*}
\ln \left(\frac{\delta \pi Y-\hat{\tau}}{\pi Y-\hat{\tau}}\right) \geq(l+1) \ln \delta \tag{2.13}
\end{equation*}
$$

Since $\frac{\delta \pi Y-\hat{\tau}}{\pi Y-\hat{\tau}}<1$ and $\delta<1$, these logarithms are negative.

$$
\begin{equation*}
l \geq \underline{1}=\left|\frac{\ln \left(\frac{\delta \pi Y-\hat{\tilde{\imath}}}{\pi Y-\tilde{\tau}}\right)}{\ln \delta}\right|-1 \tag{2.14}
\end{equation*}
$$

This inequality holds only when $\delta \pi Y-\hat{\tau}>0$. This lower bound is strictly bigger than zero, since $\delta>\frac{\delta \pi Y-\hat{\tau}}{\pi Y-\hat{\tau}}$. Thus, we have the following lemma:

Lemma 1 The necessary condition that the finite period autarky penalty can enforce $A$ to repay honestly is $\delta \pi Y>\hat{\tau}$.

Combined with the participation constraints, we have $\pi Y>\frac{\hat{\tau}}{\delta} \geq \frac{1}{\pi \delta}$. Or, $E[y]=\pi Y>\frac{1}{\pi \delta}$. At the same time, A can enjoy strictly positive benefit $\pi Y-\frac{\hat{\tau}}{\delta}$ if he succeeds to repay. This fee is necessary for A to have an incentive to repay honestly.

Proposition 2 if $E[y]>\frac{1}{\pi \delta}$, then $\exists \hat{\delta}$ s.t. for $\forall \delta>\hat{\delta}$ and $\forall l \geq l(\hat{\delta}), \tilde{a}^{A}(y=Y)=\hat{\tau}$, which $A$ repays $C$ honestly, is enforceable.

Proposition 3 Autarky length $l$ has an uniform lower bound for any $\pi, Y, \hat{\tau}$ and $\delta$ in any trading Sequential Equilibrium.

We have two equilibria: one is trivial autarky equilibrium, $\left(\tilde{a}^{A}(y)=0, a^{C}=0, x^{A}(\cdot)=\right.$ $\left.x^{C}(\cdot)=\delta^{\infty} u^{A, C}\right)$ for $\forall y$. The other equilibria are $\left(\tilde{a}^{A}(Y)=\hat{\tau}, \tilde{a}^{A}(0)=0, a^{C}=1, x^{i}(\hat{\tau})=\right.$ $\left.\delta u^{i}, x^{i}(0)=\delta^{l} u^{i}\right)$ for $\forall i \in\{A, C\}, \forall l \geq 1$, satisfying $\delta \pi Y>\hat{\tau} \geq \frac{1}{\pi}$. Corresponding beliefs can be calculated with the Bayesian rule but it is skipped here, since the state is updated every period. Thus, C has to punish A even with a firm belief of A's honesty. If it trembles, $\mu^{C}\left(y_{t}=Y \mid \tau_{t}=0\right)=\varepsilon$ for some small $\varepsilon>0$, C's optimal strategy still does not change since C has taken sufficiently severe punishment from the beginning. All the other information set is singleton, on the assumption that players do not refer to past history in the trading phase (in the punishment phase, they only care about the length of the punishment); the Nash Equilibria are, therefore, consistent. Sequential Rationality has been proven above, so the strategy and corresponding beliefs are Sequential Equilibria. This study, however, has not specified whole equilibrium set of the game. An aim of the study is proving that the existence of some interesting equilibria and that the equilibria are sequential equilibria.

### 2.3 The Multilateral Game

### 2.3.1 Nature of The Game

There are three risk-neutral infinitely-lived players A, B and C. Player A has a profitable investment project, which is equal to that of previous section, yielding $y \in\{0, Y\}$ with the
constraint (2.1). Only player C is endowed with one unit of good to start the project, while he does not have the project. Player B has neither project nor endowment.

There is an additional assumption here. C cannot lend to A directly while C can lend to B and B can lend to A. In other words, only B is endowed with an asset of "relationship". This assumption can be justified in several ways. First is simply assuming that the network structure has been restricted somehow: i.e. A and C speak a different language and cannot make a contract without B, who can speak both languages. Second, some specific time structures à la Kiyotaki and Moore(2000) with intertemporal lack of double coincidence can justify a similar situation. Third, if B has better person-specific monitoring skills of A than C has, it is better for C to lend his endowment to B and ask him to lend to A with monitoring. This argument is common in banking research(i.e. Diamond and Rajan(2001)).

Here, we adopt the first justification for simplicity. This exogenously-assumed relationship can be justified empirically. For example, Mcmillan and Woodruff(2000) study positive correlation between location of firms and amount of trade credit. Or, the Central Bank of Ireland(1971) and $\operatorname{Murphy}(1978)$ point out that personal relationships (in pubs) help loan contracts among people or firms. With this assumption, each player's information set becomes slightly complicated. The three players' histories are, respectively:

$$
\begin{aligned}
h_{t}^{A} & =\left\{a_{0}^{B}, \ldots, a_{t}^{B}, \tau_{0}, \ldots, \tau_{t-1}, y_{0}, \ldots, y_{t}\right\} \\
h_{t}^{B} & =\left\{a_{0}^{C}, \ldots, a_{t}^{C}, a_{0}^{B}, \ldots, a_{t-1}^{B}, \tau_{0}, \ldots, \tau_{t-1}, \omega_{0}, \ldots, \omega_{t-1}\right\} \\
h_{t}^{C} & =\left\{a_{0}^{C}, \ldots, a_{t-1}^{C}, \omega_{0}, \ldots \omega_{t-1}\right\}
\end{aligned}
$$

A's output $y$ is private information. C cannot observe any trade between A and B ( $\tau$ and $\left.a^{B}\right)$. A cannot observe any trade between B and C ( $a^{C}$ and $\omega$ ) either. See Figure 2 for an overview of this.

## Stage Game and Participation Constraints

At the beginning of the period t , C receives an endowment and decides $a_{t}^{C} \in\{0,1\}$, amount of lending to B, based on his history $h_{t}^{C}=\left\{a_{0}^{C}, \ldots, a_{t-1}^{C}, \omega_{0}, \ldots \omega_{t-1}\right\}$. If B receives a unit of endowment from C and he chooses his action, amount of lending to $\mathrm{A}, a_{t}^{B} \in\{0,1\}^{7}$, based on his history $h_{t}^{B}=\left\{\tau_{0}, \ldots, \tau_{t-1}, \omega_{0}, \ldots, \omega_{t-1}, a_{0}^{B}, \ldots, a_{t-1}^{B}\right\} \in H^{B}$, thus $\tilde{a}^{B}: H^{B} \rightarrow\{0,1\}$. Player A immediately starts his investment project if he receives $a_{t}^{B}=1$ (We ignore A's choice of whether he invests the borrowed resource or not for simplicity). A cannot start his project if he receives $a_{t}^{B}=0$.

The project matures in the same period and nature picks $y \in\{0, Y\}$, then A receives $a_{t}^{B} \cdot y$. A observes this and repays $\tau$ to $\mathrm{B}\left(\tilde{a}^{A}(y)=\tau\right)$. B observes $\tau$ and then determines his transfer amount $\omega\left(\tilde{a}^{B 2}(\tau)=\omega\right)$. C receives $\omega$ and the stage game ends.

This stage game is repeated infinitely. Discount factor $\delta \in(0,1)$ is common among players. We limit our attention to the stationary structure as well, and each player's surplus from autarky is

$$
\begin{align*}
u^{A} & =E\left[(1-\delta) a^{C} \cdot a^{B}(y-\tau)+\delta x^{A}(\tau)\right]  \tag{2.15}\\
u^{B} & =E\left[(1-\delta) a^{C} \cdot a^{B}(\tau-\omega)+\delta x^{B}(\tau, \omega)\right]  \tag{2.16}\\
u^{C} & =E\left[(1-\delta) a^{C} \cdot a^{B} \cdot \omega+\delta x^{C}(\omega)\right] \tag{2.17}
\end{align*}
$$

and their participation constraints are $u^{i} \geq 0$ for $i \in\{A, B, C\}$.

We will specify the continuation payoff functions later, but it is clear that B and C have to impose a trigger strategy to enforce honest behaviour on their counterparts, based on their received repayments $\tau$ and $\omega$. The thresholds of the trigger strategy, $\hat{\tau}$ and $\hat{\omega}$, are given and fixed throughout the argument, as well as the bilateral game. The restriction for $\hat{\tau}$ and $\hat{\omega}$ will be given later, but the following arguments apply to any $\hat{\tau}$ and $\hat{\omega}$ which suffice for the restrictions.

[^16]

After Project Ends (Repaying)

Figure 2-2: Stage Game Transaction in Multilateral Contracts

It is assumed that $\hat{\tau}$ and $\hat{\omega}$ is common knowledge among players, which implies that the players know their relative bargaining power.

## Basic Structure of the Game

Since there is no public signal among the players, it is not trivial to construct the equilibria of the game. First, we will specify A and B's possible strategy (and corresponding belief) sets, and define C's best response for each of the strategies and beliefs. Then we will specify B and C's possible strategy and belief sets, and consider A's best response. Finally, we will specify A and C's possible strategy and belief sets, and find B's best response. The fixed points of the best responses and corresponding consistent beliefs are the sequential equilibria of the game. The purpose of the section is to reveal the characterization of the equilibria. We will find that there are several additional incentive problems and that these are solved in the equilibria.

Since the rule of the game is common knowledge, each player can delete their counterparts' strictly dominated choices from the possible strategy set. This enables us to simplify the possible strategy set for each combination of the players.

### 2.3.2 Equilibria in the Repeated Game

## A's Possible Strategies

A has to decide $\tilde{a}^{A}(Y)$ and his continuation payoff function $x^{A}(\cdot)$ relying on his beliefs about $a^{C}$, $\tilde{a}^{B}(\cdot), \tilde{a}^{B 2}(\hat{\tau}), x^{B}(\cdot)$ and $x^{C}(\cdot)$. He can choose $\tilde{a}^{A}(Y)=\hat{\tau}$ only when the following conditions hold:

$$
\begin{align*}
\tilde{a}_{t}^{B 2}(\hat{\tau}) & =\hat{\omega}  \tag{2.18}\\
\tilde{a}_{t+1}^{C}\left(\omega_{t}\right. & =\hat{\omega})=1  \tag{2.19}\\
\tilde{a}_{t}^{B}\left(a_{t}^{C}\right. & =1)=1  \tag{2.20}\\
\delta \pi Y & >\hat{\tau}  \tag{2.21}\\
l & \geq \underline{1}(\delta, \pi, Y, \hat{\tau}) \tag{2.22}
\end{align*}
$$

The latter two conditions are equivalent to the bilateral case; $1(\delta, \pi, Y, \hat{\tau})$ is defined at (2.14). If these conditions are violated then A finds it profitable to cheat $\tilde{a}^{A}(Y)=0$. The first three conditions are additional conditions in the multilateral contracts; which are necessary to ensure $a_{t+1}^{B}=1$ after $\tau_{t}=\hat{\tau}$. In the bilateral game, A can be sure that C always has an incentive to lend to A as long as A successfully repays. In the multilateral contracts, by contrast, A cannot be sure that A can borrow C's endowment after the successful repayment, since (1) B might steal A's repayment and report to C that A did not repay, (2) C might not lend her endowment to B even after C receives repayment $\hat{\omega}$, or (3) B might steal C's endowment and report to C that A did not repay.

Here A only believes that the first three conditions hold; he will find out that his belief is correct in the trading equilibria. (2.21) and (2.22) hold when B adopts the punishment strategy defined in the proposition 2. From the fifth condition, A's continuation payoff function becomes $x^{A}(\tau=\hat{\tau})=u^{A}$ and $x^{A}(\tau=0)=\delta^{l} u^{A}$.

If any of the conditions is violated, A chooses a cheat strategy, $\tilde{a}^{A}(Y)=0$ (B and C are never repaid).

## B's Possible Strategies

B has to decide $\tilde{a}^{B}\left(a^{C}\right), \tilde{a}^{B 2}(\tau)$ and the continuation payoff function $x^{A}(\cdot)$ based on his beliefs about $\tilde{a}^{A}(Y), a^{C}, x^{A}(\cdot)$ and $x^{C}(\cdot)$. From the non-negative wealth constraint, we can assert that $\tilde{a}_{t}^{B}\left(a_{t}^{C}=0\right)=0$ and $\tilde{a}_{t}^{B 2}\left(\tau_{t}=0\right)=0$.

B's Incentive Problem ( $\tilde{a}^{B}$ ): Firstly, B will choose $\tilde{a}^{B}\left(a^{C}=1\right)=1$ only when the condition $\tilde{a}^{A}(Y)=\hat{\tau}$ holds. As we have seen in the bilateral contracts (or in the previous paragraph), this condition is equivalent to the following two constraints:

$$
\begin{align*}
\delta \pi Y & >\hat{\tau}  \tag{2.23}\\
l & \geq \mathfrak{l}(\delta, \pi, Y, \hat{\tau}) \tag{2.24}
\end{align*}
$$

B's Incentive Problem ( $\tilde{a}^{B 2}$ ): Secondly, we will think the condition of when B chooses $\tilde{a}^{B 2}(\hat{\tau})=\hat{\omega}$. B chooses to repay honestly only when the future reward is sufficiently large compared with the current sacrifice $\hat{\omega}$. So B and C need to construct the following penalty strategy: B and C enter the penalty phase $\left(a^{C}=0, a^{B}=0\right.$ and $\left.\tilde{a}^{B 2}(\hat{\tau})=\hat{\omega}\right)$ if they observe $\omega=0$. The penalty phase continues for n periods; both B and C continue the punishment strategy for any outcome. They restart the trading strategy at $n+1$ periods, regardless of what has happened in the penalty phase. This penalty strategy is obviously a Nash Equilibrium (a similar argument to the bilateral contract applies). We will formulate this enforcing constraint below.

Since the players' payoff function has a stationary structure, as we have seen in the bilateral contracts, we use a one-stage deviation principle. B's honest payoff in functional form is described as follows:

$$
\begin{equation*}
u_{t}^{B}=(1-\delta) \pi(\hat{\tau}-\hat{\omega})+\delta\left\{\pi u^{B}+(1-\pi) \delta^{n} \cdot u^{B}\right\} \tag{2.25}
\end{equation*}
$$

The continuation payoff is defined as follows: $x^{i}(\omega)=u^{i}$ if $\omega \geq \hat{\omega}$ and $x^{i}(\omega)=\delta^{n} u^{i}$, otherwise (for $i \in\{B, C\}$ ). This continuation payoff function is imposed by C, as we will see later. The participation constraints are $u^{B} \geq 0$. Then, the enforcing constraint is:

$$
(1-\delta)(\hat{\tau}-\hat{\omega})+\delta u^{B} \geq(1-\delta)(\hat{\tau}-0)+\delta^{n+1} u^{B}
$$

Substitute (2.25) into the inequality above, we have

$$
\begin{equation*}
\left|\frac{\ln \left(\frac{\delta \pi \hat{\hat{\jmath}}-\hat{\omega}}{\pi \hat{\tilde{\jmath}}-\hat{\omega}}\right)}{\ln \delta}\right|-1=\underline{\underline{n}} \leq n \tag{2.26}
\end{equation*}
$$

This inequality holds only when $\delta \pi \hat{\tau}-\hat{\omega}>0$. We can show the LHS of the inequality $\underline{n}>0$ using the same procedure as in the previous section. Therefore, we have the following lemma:

Lemma 4 The necessary condition that the finite period autarky penalty can enforce $B$ to transfer $A$ 's repayment $(\tau)$ honestly is $\delta \pi \hat{\tau}>\hat{\omega}$.

Therefore, $n$ has a uniform lower bound $\underline{n}$ for any $\pi, \hat{\tau}, \hat{\omega}$ and $\delta$ if $\delta \pi \hat{\tau}>\hat{\omega}$. At the same time, we have the following relationships from the lemma 4 and the participation constraints:

$$
\begin{aligned}
\pi Y & >\frac{\hat{\tau}}{\delta}>\frac{\hat{\omega}}{\delta^{2} \pi} \geq \frac{1}{\delta^{2} \pi^{2}} \\
E[y] & >\frac{1}{\delta^{2} \pi^{2}}
\end{aligned}
$$

In addition, this lower bound can be effective only when B and C have a belief $\tilde{a}^{A}(Y)=\hat{\tau}$. Thus, we have the following proposition.

Proposition 5 if $E[y]>\frac{1}{\delta^{2} \pi^{2}}$ and $B$ and $C$ have a belief $\tilde{a}^{A}(Y)=\hat{\tau}$, then $\exists \hat{\delta}$ s.t. for $\forall \delta>$ $\hat{\delta}$ and $\forall n \geq n(\hat{\delta}, \pi, \hat{\tau}, \hat{\omega}), \tilde{a}^{B 2}(\tau=\hat{\tau})=\hat{\omega}$, which $B$ transfer $A$ 's repayment to $C$ honestly, is enforceable. Autarky length $n$ has a uniform lower bound for any $\pi, \hat{\tau}, \hat{\omega}$ and $\delta$.

Subsequently, we can now formulate B's conditions to be honest about $\tilde{a}^{B 2}(\hat{\tau})$.

$$
\begin{align*}
\delta \pi \hat{\tau} & >\hat{\omega}  \tag{2.27}\\
n & \geq \underline{n}(\delta, \pi, \hat{\tau}, \hat{\omega}) \tag{2.28}
\end{align*}
$$

If these conditions do not hold, B finds out that $\tilde{a}^{B 2}(\hat{\tau})=0$ is profitable.

If either of the first two conditions (2.23) and (2.24) is violated, the latter conditions (2.27) and (2.28) become meaningless and B chooses cheating strategy $a^{B}=0$ and $\tilde{a}^{B 2}(\hat{\tau})=0$ eternally (if C lends to B , she will never be repaid). If the first two conditions hold (thus $\tilde{a}^{A}(Y)=\hat{\tau}$ ) but either of the latter two is violated, $B$ will choose the other cheating strategy $a_{t}^{B}=1$ and $\tilde{a}_{t}^{B 2}(\hat{\tau})=0$ for any $t$. If all four conditions hold, B chooses trading strategy $a^{B}=1$ and $\tilde{a}^{B 2}(\hat{\tau})=\hat{\omega}$.

## C's Possible Strategies

C has to decide $a^{C}$ and continuation payoff function $x^{C}(\cdot)$ based on her beliefs about $\tilde{a}^{A}(Y)$, $\tilde{a}^{B}\left(a^{C}=1\right), \tilde{a}^{B 2}(\hat{\tau}), x^{A}(\cdot)$ and $x^{B}(\cdot) . \mathrm{C}$ can choose $a^{C}=1$ only when she can expect A's honest repayment and B's honest transfer; thus, the following four conditions are necessary:

- $\hat{\omega} \geq \frac{1}{\pi}$
- $\tilde{a}^{B}\left(a^{C}=1\right)=1$
- $\tilde{a}^{A}(Y)=\hat{\tau}$
- $\tilde{a}^{B 2}(\hat{\tau})=\hat{\omega}$

The first one is C's participation constraint. The fourth condition is equivalent to the following two conditions, as we have seen:

- $\delta \pi \hat{\tau}>\hat{\omega}$
- $n \geq \underline{n}(\delta, \pi, \hat{\tau}, \hat{\omega})$

However, C knows that these two conditions require $\tilde{a}^{A}(Y)=\hat{\tau}$. So C has to be assured that A chooses trading strategy. This is not an easy task for C , since C cannot observe the flows between A and B , so that C cannot confirm $l \geq \mathrm{l}(\delta, \pi, Y, \hat{\tau})$ directly(since we have assumed that C can observe the threshold $\hat{\tau}$ and that $\{\pi, Y\}$ are common knowledge, she can check directly whether $\delta \pi Y>\hat{\tau}$ or not).

We can overcome this problem, under the assumption that only C has an endowment to start the project; from the non-negative wealth constraint, C knows that $l \geq n$. C can calculate $\underline{\mathrm{l}}(\delta, \pi, Y, \hat{\tau})$ by herself, so C can enforce A's honest behaviour indirectly by setting $n \geq \mathrm{l}(\delta, \pi, Y, \hat{\tau})$. C also knows that the second condition is always satisfied when the third constraint holds. Accordingly, C's conditions for a trading strategy $\left(a^{C}=1\right)$ can be rewritten as follows:

- $\delta \pi Y>\hat{\tau}$
- $\delta \pi \hat{\tau}>\hat{\omega}$
- $n \geq \underline{l}(\delta, \pi, Y, \hat{\tau})$
- $n \geq \underline{\mathrm{n}}(\delta, \pi, \hat{\tau}, \hat{\omega})$
- $\hat{\omega} \geq \frac{1}{\pi}$

The corresponding continuation payoff function is $x^{C}(\omega=\hat{\omega})=u^{C}$ and $x^{C}(\omega=0)=\delta^{n} u^{C}$. If any of the conditions does not hold, C chooses an autarky strategy, $a^{C}=0$, eternally.

## Fixed Points of the Best Responses

From the argument above, each player chooses the trading strategy only when all the following constraints are satisfied (the consistency of these beliefs are shown in the appendix):

- $l=n \geq \max (\underline{l}, \underline{n})$
- $\delta \pi Y>\hat{\tau}$
- $\delta \pi \hat{\tau}>\hat{\omega}$
- $\hat{\omega} \geq \frac{1}{\pi}$

Equilibrium strategies should satisfy the condition $l=n$. If $l>n(l \nless n$ because of the nonnegative wealth condition), B and C restart the trading phase; nevertheless, A is still choosing autarky. Of course B cannot lend to A, and he cannot repay $\hat{\omega}>1$ (from C's participation constraint). C regards this as a deviation and starts another penalty phase. When A restarts the trading phase, A has already entered another penalty phase and B cannot lend to A. This triggers another penalty between A and B. This is Pareto inefficient, and all players have incentive to choose $l=n$. In addition, the trading strategies are supported by the following beliefs:

- $\mu^{i}\left(a_{t}^{B 2}=\hat{\omega} \mid \tau_{t}=\hat{\tau}\right)=1 \quad i \in\{A, C\}$
- $\mu^{i}\left(a_{t+1}^{C}=1 \mid \tau_{t}=\hat{\tau}, \omega_{t}=\hat{\omega}\right)=1 \quad i \in\{A, B\}$
- $\mu^{i}\left(a_{t}^{B}=1 \mid a_{t}^{C}=1\right)=1 \quad i \in\{A, C\}$
- $\mu^{i}\left(\tilde{a}_{t}^{A}\left(y_{t}=Y\right)=\hat{\tau}\right)=1 \quad i \in\{B, C\}$

The conditional beliefs of all information sets can be calculated using these. Obviously, these beliefs are consistent with the corresponding strategy. Now we have the trading equilibria. Of course, we have another trivial equilibrium, autarky, such that $\left\{\tilde{a}^{A}(y=Y)=0, \tilde{a}^{B}\left(a^{C}=1\right)=\right.$ $\left.0, a^{C}=0, \tilde{a}^{B 2}(\tau=\hat{\tau})=0, x^{A}(\cdot)=\delta^{\infty} u^{A}, x_{t}^{B}(\cdot)=\delta^{\infty} u^{B}, x_{t}^{C}(\cdot)=\delta^{\infty} u^{C}\right\}$.

Now we have the following proposition:

Proposition 6 If $\delta \pi Y>\hat{\tau}$ and if $\delta \pi \hat{\tau}>\hat{\omega} \geq \frac{1}{\pi}, \exists \tilde{\delta}$ s.t. for $\forall \delta>\tilde{\delta}$ and $\forall q \geq \max (l, n)$, there are trading equilibria that each player's actions are appropriately enforced with penalty length $q$.

Proposition 7 The penalty length $l=n$ has a uniform lower bound $\max (l, n)>0$ for any $\pi, Y, \hat{\tau}, \hat{\omega}$ and $\delta$ which satisfies the conditions above.

Lemma 8 The equilibria specified here are sequential equilibria.

See Appendix for the proof. As mentioned in the bilateral game, the equilibria we specified here may not be the whole set of equilibria of the game. This study aims to show the existence of some interesting equilibria, and specifying the whole set of equilibria is left for future research.

### 2.3.3 Efficiency of the Self-Enforcing Trade Credit

## Penalty Length and the Second Best Economy

From the definition of the lower bounds for the penalty lengths $\underline{1}$ and $\underline{n},(2.14)$ and (2.26) respectively, we characterise the following properties. First, $\underline{l}$ is strictly increasing against $\hat{\tau}$ while $\underline{n}$ is strictly decreasing against $\hat{\tau}$. This is very intuitive, because a high $\tau$ reduces A's $\operatorname{margin}(Y-\tau)$ while it increases B's margin $(\tau-\omega)$. A higher margin raises the surplus to participate in the contract honestly. Similarly, $\underline{n}$ is increasing against $\hat{\omega}$ for a given $\hat{\tau}$.

Since the penalty is costly for all players, we have some $\hat{\tau}^{*}$ and $\hat{\omega}^{*}$ such that the lower bound of the penalty length $\max (\underline{l}, \underline{n})$ is minimised, namely, the economy is the second-best efficient (the first-best is $l=n=0$, which is impossible in this model). Solving (2.14) and (2.26), we have the following relationship:

$$
\left|\frac{\ln \left(\frac{\delta \pi Y-\hat{\tau}}{\pi Y-\hat{\tau}}\right)}{\ln \delta}\right|-1 \geq\left|\frac{\ln \left(\frac{\delta \pi \hat{\tau}-\hat{\omega}}{\pi \hat{\tau}-\hat{\omega}}\right)}{\ln \delta}\right|-1 \quad \text { if } \underline{1} \geq \underline{n}
$$

Since logarithm is monotonic and all $\ln (\cdot)$ is negative, it is equivalent to

$$
\begin{aligned}
\frac{\delta \pi Y-\hat{\tau}}{\pi Y-\hat{\tau}} & \leq \frac{\delta \pi \hat{\tau}-\hat{\omega}}{\pi \hat{\tau}-\hat{\omega}} \\
Y \cdot \hat{\omega} & \leq \hat{\tau}^{2}
\end{aligned}
$$

For any given $\hat{\omega}, \max (\underline{l}, \underline{\underline{n}})$ is minimised when we set $\hat{\tau}$ such that $Y \cdot \hat{\omega}=\hat{\tau}^{2}$. This is because, if $\underline{l} \geq \underline{n}$ then we can reduce $\underline{l}$ and increase $\underline{n}$ by lowering $\hat{\tau}$; thus $\max (\underline{l}, \underline{n})=\underline{l}$ is lowered. If $\underline{l} \leq \underline{n}$
then we can reduce $\underline{n}$ and increase $\underline{l}$ by increasing $\hat{\tau}$; thus $\max (\underline{l}, \underline{n})=\underline{n}$ is lowered. Such kind of improvement becomes impossible only when $\underline{\underline{l}}=\underline{\underline{n}}$, namely, $Y \cdot \hat{\omega}=\hat{\tau}^{2}$. $\underline{n}$ is minimised for given $\hat{\tau}$ when we set $\hat{\omega}=\frac{1}{\pi}$. Thus, if we can pick $\hat{\tau}$ and $\hat{\omega}$ to achieve the second best economy, $\hat{\tau}^{*}=\sqrt{\frac{Y}{\pi}}$ and $\hat{\omega}^{*}=\frac{1}{\pi}$.

## Optimization and Endogenised Thresholds

Here we will endogenise $\hat{\tau}$ and $\hat{\omega}$, which have been assumed to be exogenously given and fixed, by assuming that B writes all contracts. Then $\hat{\tau}$ and $\hat{\omega}$ can be obtained as maximisers of the following optimization problem:

$$
\max _{\hat{\tau}, \hat{\omega}} u^{B}=(1-\delta) \pi(\hat{\tau}-\hat{\omega})+\delta\left\{\pi u^{B}+(1-\pi) \delta^{\max (\underline{l}, \underline{\underline{n}})} u^{B}\right\}
$$

subject to

- $l=n=\max (\underline{l}, \underline{n})$
- $\delta \pi Y>\hat{\tau}$
- $\delta \pi \hat{\tau}>\hat{\omega}$
- $\hat{\omega} \geq \frac{1}{\pi}$

Players' enforcing constraints and participation constraints are summarised in the four conditions. It is obvious that we have a corner solution for $\hat{\omega}, \hat{\omega}=\frac{1}{\pi}$ since lowering $\hat{\omega}$ raises B's surplus and shortens the lower bounds of the penalty length $\underline{n}$. But $\hat{\tau}$ must be an interior solution, because the higher $\hat{\tau}$ increases B's current payoff but lowers the continuation payoff through longer penalty length $\left(\tau \geq \sqrt{\frac{Y}{\pi}}\right)$. This prevents the economy from achieving the second-best equilibrium. Therefore, the self-enforcing contract can achieve at best a third-best equilibrium.

## Only Less Risky Projects Can Be "Resaleable"

That the multilateral contract requires $E[y]>\frac{1}{\delta^{2} \pi^{2}}$ implies that it is difficult for A to issue liquid (resalable) IOUs if A's project is risky ( $\pi$ is small). If $\pi$ decreases, the required $E[y]$ would increase exponentially; the borrowers with low $\pi$ find it difficult to obtain finance even if the expected profit of their project is the same as the borrowers with a high $\pi$.

This happens because the lender C has to share a certain amount of her surplus with both A and B , as a compensation for punishment on the equilibrium path ( A and B are punished even if they behave honestly, because C has no way to distinguish their deviation from the unlucky event that $y=0$. C has to punish A and B, nevertheless C is sure that A and B do not deviate on the equilibrium path, to give $A$ and $B$ an appropriate incentive to behave honestly). This mechanism is slightly similar to the recursive moral hazard by Kim and $\operatorname{Shin}(2007)^{8}$.

If the project is riskier, compensation to both A and B will increase. This is the reason why the requirement of profitability becomes more severe exponentially, as the risk of the project increases ( $\pi$ decreases). This implies that if we have a longer credit chain, with more than four players, the requirement to $\mathrm{E}[\mathrm{y}]$ accelerates further as $\pi$ decreases. Thus, only the borrowers with a very high $\pi$ can issue "very liquid" IOUs that can be re-sold more than twice, although I do not provide formal proof here. In contrast, borrowers with high risk have to rely on bilateral contracts, because the requirement is looser, $E[y]>\frac{1}{\delta \pi}$, as proposition 2 shows. Their loan contracts are not resaleable.

It would be possible to regard the bilateral contracts as a kind of bank loan, which is also not resaleable. Or, it would be better to regard it as 'angels', who invest in venture businesses with their own money. Less risky borrowers do not have to rely on these banks or angels, because they can issue resaleable IOUs like bonds. This division of roles in the financial market would be a plausible hypothesis and convenient to explain several phenomena, such as the emergence

[^17]of banking in northern Italy in the thirteenth century or banking in developing countries. Why did big merchants become bankers? Why do banks in the countries with undeveloped property right systems behave as financial intermediaries, even though undeveloped property rights mean that collateral collection, the transfer of property right, is very difficult? With this definition, we can answer such the questions, as an essential feature of banking is neither monitoring ability nor collateral collection skills but bilateral relationships with many borrowers. We will discuss this again in Chapter Four, after we discuss collateral and other efficiency-improving mechanisms.

### 2.3.4 Relationship to the past literature

## One-Sided Private Monitoring

Compared with normal public monitoring games using the framework of Abreu, Pearce and Stacchetti $(1986,1990)$ or Fudenberg, Levine and Maskin(1994), the model discussed here is complicated. This complexity comes from its characteristics which resembles private-monitoring. In the normal public monitoring games, where players share common signals, players can shift to a different phase (e.g. penalty phase) at the same time, according to the public signal. Abreu, Pearce and Stacchetti(1990) show that we can construct a simple recursive structure to have coordination, under certain assumptions. In the private monitoring game, however, we cannot use this approach since players do not share any common information and they cannot be sure when punishment is "triggered". That is to say, we cannot coordinate players of any game to punish a deviator from equilibrium strategies: trigger strategies, commonly used in public monitoring games, are not available, or at least very difficult to utilise.

This characteristic of private monitoring requires players to construct a conditional belief of other players' private histories, to infer their actions that is a function of their own private histories, as Kandori(1992) surveyed. At the same time, a recursive structure is not available because they cannot have a simple public strategy that relies only on any current public signal. These factors introduce considerable complexity into the model. Due to the above reasons, as far as I know, a coordination equilibrium is found only in extreme cases; for example, almost
perfect and almost public monitoring by Mailath and Morris(2002); almost perfect monitoring by Ely and Välimäki(2002) and Piccione(2002); or the other extreme case, assuming the signal is completely non-informative by Matsushima(2004). Yet, the intermediate case remains an open question.

The model discussed here has similar characteristics. The borrower A and the lender C do not share any common signals (A cannot observe $\omega$, the amount C receives, and C cannot observe $\tau$, the amount A pays). In this sense, this model can be categorised as a private monitoring game. Moreover, $\omega$ contains sufficiently large noise, so it is impossible to apply the almost-perfect monitoring approach. Yet we are still able to construct a non-trivial equilibrium, because C is sure that when C starts her penalty phase, A must start the penalty phase as well. This assurance comes from the non-negative wealth constraint and from the assumption that only C has an endowment to start A's project. If C stops lending, A has no way to obtain an endowment to start his project. That is to say, this game is not the full private monitoring discussed by previous studies, but a "one-sided" private monitoring game, in which C is assured of A's private histories ${ }^{9}$ but A needs to infer C's private histories.

This one-sided assumption simplifies the problem drastically, although it is still complex compared with public monitoring. In the "double-sided" private monitoring games discussed in the above papers, we have a higher-order belief problem. When one of the players of the "double-sided" private monitoring game chooses his action, he needs to make his conditional belief about other players' private histories to determine their best responses. At the same time, he knows that the other players also infer his private histories; he has to infer their conditional beliefs as well. This inference continues infinitely.

Such a kind of higher order problem does not occur in public monitoring games. Every player knows that the other players' actions only depend on public signals (and their history), so no inference is needed. In the "one-sided" private monitoring, A has to infer C's private history about $\omega$. By contrast, C does not have to infer $a^{B}$ (to be precise, C needs to infer $y$, but C does not use this inference for coordinated punishment). The inference stops at the

[^18]first order, since A knows that C can correctly infer $a^{B}$ and he does not have to infer C's conditional belief about $a^{B}$ and its history. Thus, "one-sided" private monitoring is relatively more tractable, as shown below and in the Appendix.

Compared with public monitoring games, A has a difficulty to find the timing of the penalty phase off the equilibrium path. If he finds $a_{t+1}^{B}=0$ after he repays $\tau_{t}=\hat{\tau}$, he understands that B or C deviates equilibrium strategy, but he cannot distinguish which of them deviates. If B deviates by stealing $\tau$ and choosing $\omega_{t}=0$, this is the first period of the penalty phase. If C deviates by choosing $a_{t+1}^{C}=0$ after receiving $\omega_{t}=\hat{\omega}$, or B deviates by choosing $a_{t+1}^{B}=0$ after $a_{t+1}^{C}=1$, this is the last period of the trading phase and the next period is the first period of the penalty phase. Although A has no clue to distinguish when the penalty phase starts at this time, he can find that the penalty phase is finished when he observes $a_{t+l+1}^{B}=1$. Otherwise, the penalty phase finished at $t+l+1$ period and the trading phase restarts from the following period. Thus, A can overcome the coordinated punishment problem of the private monitoring game (see Appendix for a detailed explanation).

The one-sided private monitoring gives certain limitations to the model at the same time. Since C can infer correctly what A observes is critically important, and this is sustained by the assumption that C's endowment is only the source of A's investment, any other endowments to the other players are not allowed. For instance, if B also receives an endowment for the initiation of A's project, the equilibrium discussed here would collapse since this assumption leads to double-sided private monitoring. Even if C chooses a sufficiently long penalty phase to enforce A's repayment, there is no guarantee that B cooperates with the penalty. If C chooses too long period of the penalty, B would find that bilateral trade with A is more profitable than multilateral trade with A and C, even though C's endowment is somehow preferable to B's endowment. C also rationally expects the possibility that B does not cooperate with the penalty strategy; this prevents players from coordinative equilibrium.

## Impossibility of Statistical Inference

Some point out that the trigger strategy is not efficient (Radner(1981), Abreu, Milgrom and Pearce (1991), Stocken(2000), and so on), regardless of the length of the penalty phase. In the case that the principal cannot distinguish an agent's deviation from stochastic bad events, it is inevitable to punish occasionally the agent who behaves honestly. Since the wrong punishment normally leads to a Pareto inefficient outcome, as in the model discussed here, strategies with the trigger strategy cannot be efficient (some penalty strategies do not incur Pareto inefficiency, especially in renegotiation-proof contracts like Kletzer and Wright(2000). In their model, penalty means transfer of excess surplus of the contracts and it does not incur inefficiency).

One possible solution to the problem is statistical inference. If the principal can track credit history and calculate its average repayment, the principal can review her agent's behaviour more precisely. This type of strategy, studied by Radner(1981,1985,1986), Abreu, Milgrom and Pearce(1991), Stocken(2000) and Matsushima(2004), is called a review strategy. Its basic mechanism is as follows. In repeated games with infinite repetition, the principal fixes his action for $T$ periods: A set of the $T$ periods is called a review phase. She receives an outcome every period, which is a function of the agent's action and stochastic circumstances (both variables are the agent's private information, although the principal knows the probability distribution of the stochastic circumstances). The principal collects the outcomes in the review phase and calculates the success ratio. If the success ratio is sufficiently low compared with the expected ratio when her agent behaves honestly, the principal switches to the penalty phase, a finite repetition of stage game Nash equilibrium. After the penalty phase finishes, the economy restarts the review phase again.

The review strategy has an advantage over trigger strategies in terms of precision of signal interpretation, by making the length of the review phase $T$ sufficiently large. Yet it has serious drawbacks at the same time. The largest drawback is that the principal has to keep the information unused for the time being, until the review phase ends. Generally speaking, such kind of delay cannot be efficiency-improving. Matsushima(2004) points out that the delay finally breaks down coordination between principal and agent. If the agent fortunately succeeds
certain times and he is sure that he will pass the review, or if the agent fails certain times and he is sure that he cannot pass the review at the end of the phase, the agent has no incentive to act honestly during the rest of the review phase. Based on the fact that the agent will choose cheating action by the end of the phase, keeping the same (cooperative) action by the end of the phase cannot be the best response for principal either. The principal should stop the review phase immediately and start a new phase. Yet this principal's response must be correctly anticipated by the agent. The agent's best response to the interruption of the phase is to choose a cheating action one period earlier than when he is sure that he can cheat safely. Of course, this is correctly inferred by the principal and the review phase ends one period earlier. Matsushima(2004) points out that this repetition continues and finally breaks down all coordination during the review phase ${ }^{10}$.

This problem has been solved (or avoided) in various ways. Radner $(1981,1986)$ assumes the discount factor to be unity. By assuming this, any one-shot deviation profit becomes negligible in infinitely repeated games. Accordingly, the agent is still indifferent between an honest and a cheating action even after he is sure that he has passed (or that he will never pass) the review; a cooperative strategy is still weakly supported as an equilibrium. Abreu, Milgrom and Pearce(1991) simply assume that actions during the phase are fixed. Abreu, Milgrom and Pearce's(1991) assumption is non-generic and hard to apply. Radner's(1981,1986) nodiscounting assumption is criticised by Matsushima(1994). He argues that Radner's(1981,1986) arguments strongly rely on the characteristics of the time-average criterion that $x / T \rightarrow 0$ as $T \rightarrow$ $\infty$ for any large $x$; this assumption is extreme and not plausible, he argues. Matsushima(2004), instead, assumes the non-informative property, that neither principal nor agent can infer their counterpart's history of private information at all. With this property, the agent does not have any incentive to deviate in the middle of review phase. The non-informative property is, however, still non-generic for application purposes.

Stocken(2000) studies the review strategy from a different viewpoint. The papers cited

[^19]above assume private monitoring, that players do not share any common signals. Stocken(2000) focuses on public monitoring (cheap talk game). Since it is public monitoring, now we can shorten the review phase when players are sure that the agent passes or fails the review. Under this assumption, we just need to ensure that the agent has no incentive to deviate when he expects the phase is closed in the next period. Stocken(2000) shows that the review strategy can achieve coordination between principal and agent if the discount factor is sufficiently close to one the and the probability of success is sufficiently close to $100 \%$ (the stochastic circumstance is almost non-stochastic). A problem of this research is that the trigger strategy will be almost efficient when the discount factor and success probability are close to one (for instance, penalty length converges to zero, as discount factor $\delta$ and probability of success $\pi$ converge to one. This is obviously efficient). If the review strategy is possible, trigger strategy becomes almost efficient and the review strategy is unnecessary.

The impossibility of review strategy does not change even when we allow mixed strategy. We can randomise players' actions at two different points. The first one is at the end of each phase. When the principal finds that the agent passes the test, she restarts the review phase with certain probability. The principal can randomise her action when she shifts from the penalty phase to the review phase. By setting appropriate probability, we can make the choice of the review phase and penalty phase indifferent for the players. This simplifies the definition of equilibria drastically, since both choosing the review phase and penalty phase are the best responses to any history given. This approach, Interchangeability, is initially adopted by Ely and Välimäki(2002) and Piccione(2002), although they study normal grim trigger strategies.

The other possible randomisation is using mixed strategy equilibrium in every stage game. In this case, of course, we do not have the incentive problem cited above. Yet this is obviously inefficient because the principal now has to set the threshold much lower than pure strategy equilibria. In this case, there is no guarantee that the review strategy is more efficient than trigger strategies. Moreover, mixed strategy equilibrium is possible only when the players are indifferent among some elements of their choice sets. It is easier to find such a kind of set in the case of the simultaneous move game, but the stage-game in the model we have discussed is a sequential move game and one choice (deviation) strictly dominates the other. Consequently,
mixed strategy equilibrium does not help us to solve the impossibility of statistical inference ${ }^{11}$.

### 2.4 Conclusion

The multilateral relational contracts discussed in this chapter show several interesting characteristics. First, it is possible to construct indirect loan contracts with reputational bind, although the degree of information asymmetry is more severe than bilateral contracts. A, the borrower, does not know how B and C trade, and C , the lender, does not know how A and B trade (B is the intermediary agent). This is a difficult problem setting, because Yet, the penalty strategy that punishes A and B at the same time, can sort this problem out.

The penalty strategy, which punishes both A and B at the same time, incurs inefficiency as well. Even if A fails to repay and B does not do anything wrong, B would be punished as well. This is because C cannot distinguish whether A or B deviates from the equilibrium strategy, and C does not have any measures to punish A or B specifically. This means that B needs a compensation for the unreasonable punishment. The lender C has to share a certain amount of her profit with B , to give B an incentive to behave as an honest intermediary. Similar compensation is necessary for A as well. Since A's investment outcome is stochastic, A also would be punished even though he behaves honestly, when the investment outcome turns out to be zero with some positive probability. A requires compensation for the unreasonable punishment.

The punishment needs to be more severe when A's investment project is riskier. Accordingly, the compensation must be larger if A's investment project is risky. The compensation increases exponentially as the project is riskier, because C has to pay the compensations to both A and B. This means that if the project is risky, C's participation constraint would be violated unless the project is very profitable. Since the requirement of the expected profitability is

[^20]exponentially more severe as the project is riskier, only less risky borrowers can obtain finance in this multilateral relational contract.

In bilateral contracts, C only has to pay compensation to her lender (A) directly, and the requirement to the expected profitability is relaxed. In this sense, multilateral contracts are less efficient than bilateral contracts; of course, they are still better than autarky, if the indirect relationship is given exogenously.

The implications of the model for the real economy are discussed in Chapter Four.

### 2.5 Appendix

### 2.5.1 Multilateral Contracts as Sequential Equilibria

In order to show that the trading equilibria shown in the body text are sequential equilibria, we need to show their sequential rationality and consistency. It is not trivial that the equilibria are consistent. First, we see that, on the equilibrium path, the equilibrium strategies are sequentially rational w.r.t. consistent beliefs. Each player has one non-singleton information set on the equilibrium path. A's information set when A chooses $\tau_{t}$ has two elements: one is $\left(\tau_{t-1}=\hat{\tau}, \omega_{t-1}=\hat{\omega}, a_{t}^{C}=1, a_{t}^{B}=1\right)$ and the other is $\left(\tau_{t-1}=\hat{\tau}, \omega_{t-1}=0, a_{t}^{C}=1, a_{t}^{B}=1\right)$. If the former is true, A should choose $\tilde{a}^{A}(Y)=\hat{\tau}$, but if A is at the latter node, A should choose $\tilde{a}_{t}^{A}(Y)=0$, because it is the first period of the penalty phase and he does not expect $a_{t+1}^{B}=1$ in the following period, if B and C follow an equilibrium strategy. B also has an information set with two elements when it receives $\tau=0: y=0$ and $y=Y$. C's non-singleton information set, after receiving $\omega=0$, has four elements: $(y=0, \tau=0),(y=Y, \tau=0),(y=Y, \tau=\hat{\tau})$ and $a^{B}=0$.

## An Additional Assumption and the Flavour of Private Monitoring

The penalty strategy is slightly complicated because of this model's private-monitoring-like characteristics. In the public monitoring argument, where every players observes the same
signal which triggers the penalty, each player knows the current status(phase) of the economy: I.e. everybody knows when the economy entered the penalty phase and when it will come back to the trading phase. In this model, by contrast, player A cannot realise when the economy shifts to the penalty phase, if B deviates in the previous period. And if C deviates from her own punishment and chooses $a^{C}=1$, A has no clue to realise he is in the penalty phase, and may choose $\tilde{a}^{A}(Y)=\hat{\tau}$, or, this equilibrium strategy itself cannot be sustainable in the worst case. We will see that the players can infer when the penalty is triggered even if they do not share the common public signal.

The detailed strategies are as follows:

$$
\begin{aligned}
\sigma^{A}\left(a^{A}\right. & =\hat{\tau} \mid y=Y)=1 \text { in the trading phase } \\
\sigma^{A}\left(a^{A}\right. & =0 \mid \forall y)=1 \text { in the penalty phase } \\
\sigma^{B}\left(a^{B}\right. & \left.=1 \mid a^{C}=1\right)=1 \text { in the trading phase } \\
\sigma^{B}\left(a^{B}\right. & \left.=0 \mid \forall a^{C}\right)=1 \text { in the penalty phase } \\
\sigma^{B}\left(a^{B 2}\right. & =\hat{\omega} \mid \tau=\hat{\tau})=1 \text { in the trading phase } \\
\sigma^{B}\left(a^{B 2}\right. & =0 \mid \forall \tau)=1 \text { in the penalty phase } \\
\sigma^{C}\left(a^{C}\right. & =1 \mid \omega=\hat{\omega})=1 \text { in the trading phase } \\
\sigma^{C}\left(a^{C}\right. & =0 \mid \forall \omega)=1 \text { in the penalty phase }
\end{aligned}
$$

## Consistency of A's On-Path Information Set

Since we have limited our attention to the pure strategy equilibria, A can choose $\tilde{a}^{A}(Y)=\hat{\tau}$ without doubt. But if we introduce trembles, A chooses $\tilde{a}^{A}(Y)=\hat{\tau}$ only when the following inequality holds:

$$
\begin{equation*}
\mu^{A}\left\{(1-\delta)(Y-\hat{\tau})+\delta u^{A}\right\}+\left(1-\mu^{A}\right)\left\{(1-\delta)(Y-\hat{\tau})+\delta^{n} u^{A}\right\} \tag{2.29}
\end{equation*}
$$

$$
\geq \mu^{A}\left\{(1-\delta) Y+\delta^{l+1} u^{A}\right\}+\left(1-\mu^{A}\right)\left\{(1-\delta) Y+\delta^{n} u^{A}\right\}
$$

where $\mu^{A}$ is A's belief that he is at an on-path node. $1-\mu^{A}$ is the probability that B deviates from the trading phase and C deviates from the penalty phase. This is based on the equilibrium strategy of B and C, $\sigma^{B}(\omega=\hat{\omega} \mid \tau=\hat{\tau})=1-\varepsilon_{1}, \sigma^{C}\left(a^{C}=1 \mid \omega=\hat{\omega}\right)=1-\varepsilon_{2}$, $\sigma^{C}\left(a^{C}=0 \mid \omega=0\right)=1-\varepsilon_{3}, \sigma^{B}\left(a^{B}=1 \mid a^{C}=1, \omega_{-1}=\hat{\omega}\right)=1-\varepsilon_{4}$ and $\sigma^{B}\left(a^{B}=1 \mid a^{C}=\right.$ $\left.1, \omega_{-1}=0\right)=1-\varepsilon_{5}$. From the Bayesian rule, if B deviates with a small probability,

$$
\mu^{A}=\frac{1-\varepsilon_{2}}{\left(1-\varepsilon_{2}\right)+\varepsilon_{3}}
$$

And if C deviates (with a small probability $\varepsilon$ ),

$$
\mu^{A}=\frac{\left(1-\varepsilon_{1}\right)\left(1-\varepsilon_{4}\right)}{\left(1-\varepsilon_{1}\right)\left(1-\varepsilon_{4}\right)+\varepsilon_{1}\left(1-\varepsilon_{5}\right)}
$$

From the proposition 5, 6 and 7 , for any $l \geq 1$ and $\delta \geq \delta$, in the inequality (2.29) are sufficiently large such that the first term in the LHS is larger than the first term of the RHS. Since $\delta^{l+1}$ is monotonically decreasing with respect to $l$ and $\delta$, for any $l>\underline{1}$ and $\delta>\delta$, the first term in the LHS is strictly rather than the one in the RHS. For these $l$ and $\delta$, the inequality (2.29) holds with $\mu^{A}$ that is sufficiently close to 1 . Namely, the inequality (2.29) holds for sufficiently small $\varepsilon_{3}>0$ or $\varepsilon_{1}>0$ (other $\varepsilon_{i}$ for $i=2,4,5$ can take any value).

## Consistency of B's On-Path Information Set

B has a non-singleton information set when B receives $\tau=0 . \mu^{B}$ denotes B's ex-post belief about A's decision $a^{A}$. As we have seen above, B will punish A even when A behaves honestly $\left(\tilde{a}^{A}(Y)=\hat{\tau}\right)$. B's optimal strategy is, therefore, sequentially rational with respect to any tremble of A's action.

## Consistency of C's On-Path Information Set

C has a non-singleton information set with four elements: $(y=0, \tau=0, \omega=0),(y=Y, \tau=$ $0, \omega=0),(y=Y, \tau=\hat{\tau}, \omega=0)$ and $a^{B}=0$. For any node, C's optimal strategy is punishing A and B: C's penalty strategy (n periods autarky) is sequentially rational for any perturbation of A and B .

## Consistency of A's Off-Path Information Set

When A observes $a_{t}^{B}=0$ after $\tau_{t-1}=\hat{\tau}$, there are two possibilities. One is that $\omega_{t=1}=0$, and that this period is the first period of the penalty phase (Case 1 hereafter). The other is that $a_{t}^{B}=0$ or $a_{t}^{C}=0$ after $\omega_{t=1}=\hat{\omega}$; its penalty phase starts at the next period (Case 2 hereafter). In addition, in the case that $a_{t}^{B}=1$ and $a_{t}^{C}=1$ after $\omega_{t=1}=0, \mathrm{~B}$ and C restart the penalty phase from the next period; this is also included in Case 2. At this moment, A has no clue to distinguish Cases 1 and 2. This causes a problem at the end of the penalty phase; at $t+n+1$ period, A might be in the first period of the restarted trading phase, or the last period of the penalty phase. But A gets a clue at this period. If $a_{t+n+1}^{B}=1$, it implies that the penalty phase is over. Instead, if $a_{t+n+1}^{B}=0$ then it is the last period of the penalty phase. A should choose $\tilde{a}^{A}(Y)=\hat{\tau}$ in the former case and $\tilde{a}^{A}(Y)=0$ in the latter case ${ }^{12}$.

Assume that $\sigma^{B}(\omega=\hat{\omega} \mid \tau=\hat{\tau})=1-\varepsilon_{1}, \sigma^{C}\left(a^{C}=1 \mid \omega=\hat{\omega}\right)=1-\varepsilon_{2}, \sigma^{C}\left(a^{C}=0 \mid \omega=\right.$ $0)=1-\varepsilon_{3}, \sigma^{B}\left(a^{B}=1 \mid a^{C}=1, \omega_{-1}=\hat{\omega}\right)=1-\varepsilon_{4}$ and $\sigma^{B}\left(a^{B}=1 \mid a^{C}=1, \omega_{-1}=0\right)=1-\varepsilon_{5}$. The conditional belief of A in the information set $a^{B}=0$ after $\tau=\hat{\tau}$, denoted as $P$, is obtained from the Bayes rule as follows:

$$
P=\frac{\varepsilon_{1}\left(1-\varepsilon_{3}\right)}{\varepsilon_{1}\left(1-\varepsilon_{3}\right)+\left\{\left(1-\varepsilon_{1}\right)\left(1-\varepsilon_{2}\right) \varepsilon_{4}+\left(1-\varepsilon_{1}\right) \varepsilon_{2}+\varepsilon_{1} \varepsilon_{3} \varepsilon_{5}\right\}}
$$

Therefore, the penalty phase ends at period $t+n$ with probability P , and ends at period $t+n+1$ with probability $1-P$.

[^21]By focusing on single player's deviation, P becomes:

$$
P=\frac{\varepsilon_{1}}{\varepsilon_{1}+\left(1-\varepsilon_{1}\right) \varepsilon_{4}} \quad \text { when only B perturbs its actions }
$$

$$
P=0 \quad \text { when only } \mathrm{C} \text { perturbs its actions }
$$

In the former case, however, B cannot choose its action when $a^{C}=0$. Accordingly, the probability that A observes $a_{t}^{B}=0$ after $\tau_{t-1}=\hat{\tau}$ is zero with any arbitrary perturbation of a single player.

## The Other Information Sets

All A's information sets in the penalty phase on the equilibrium path are singleton. B can observe almost all information except for $y$, so only the non-singleton information set is $y=$ $\{0, Y\}$ when he observes $\tau=0$. However, he has no choice in this information set so we can ignore this. C's non-singleton information set is the one when he chooses $a^{C}$. The element of the set is $\left\{a^{B}=0, a^{B}=1\right.$ but $y=0, a^{B}=1$ and $y=Y$ but $\tilde{a}^{A}(Y)=0, a^{A}=\hat{\tau}$ but $\left.a^{B 2}(\tau)=0\right\}$. In the trading phase, based on A and B's equilibrium strategies, the probability assigned for the elements is $\{0,1,0,0\}$. Since C chooses to enter the penalty strategy even though she is sure that both A and B do not cheat, any tremble cannot change her choice. Therefore, the consistency problem does not arise here.

Now we have seen that the trading equilibria satisfy both sequential rationality and consistency. The trading equilibria are therefore sequential equilibria. The autarky equilibrium is obviously a sequential equilibrium as well.

## Chapter 3

## Collateral, Money and the Efficiency of the Economy

### 3.1 Introduction

In the arguments above, we have assumed that any loans are not secured by collateral. This assumption may not be plausible. As Coco(2000) reports, $85 \%$ of small business loans in the UK are more than fully-secured by collateral (Black, de Meza and Jeffreys(1996)). In the US, 'nearly $70 \%$ of all commercial and industrial loans are currently made on secured basis'(Berger and Udell(1990)). In this chapter, we will see how collateral coexists with the relationship lending discussed in the last chapter, and how it improves the efficiency of the economy.

### 3.1.1 Literature Review

The role of collateral has been discussed in the last several decades and it can be categorised as follows. The first one assumes ex-ante information asymmetry (hidden information, adverse selection) and uses collateral to reveal the hidden information. For instance, the lender would offer two different loan contracts: one imposes low interest rates but requires collateral and the other has a high interest rate but is not secured by collateral. Low-risk borrowers would
prefer former one, since they know their own default ratio, which is private information, and they can infer that collateral is likely to be back on their hands; i.e. for low risk borrowers, giving collateral does not incur a large cost. In contrast, high-risk borrowers would prefer a non-collateralised loan, because they know that the probability of collateral seizure is high. This is a common argument that uses collateral as a screening device. Such a kind of argument is made by Bester(1985), Besanko and Thakor(1987) and others. Chan and Kanatas(1985) uses collateral as a signalling device.

The second one assumes intermediate information asymmetry (hidden action or moral hazard). If the lender cannot observe her borrower's effort level (say), the threat of seizing collateral in the case of project failure can discipline the borrower and the borrower would make his maximum effort to raise the probability of success. Such kind of argument has been made by Aghion and Bolton(1992) ${ }^{1}$, Boot and Thakor(1994) ${ }^{2}$, and others.

These two approaches are, however, not supported by empirical studies. A prediction of these models is that a collateralised loan is safer than a non-collateralised one, because only safer borrowers want to pledge more collateral in the adverse selection model and because borrowers would make more effort when they pledge their collateral. Yet, as Berger and Udell(1990), Jiménez and Saurina(2003) and other papers show, a positive correlation between the amount of pledged collateral and the risk premium of the borrowers (or default probability) is observed. This contradicts with the previous literatures.

There are several studies which attempt to solve the contradiction. For example, Inderst and Mueller(2007) explain the positive correlation with a incomplete competition model, not with asymmetric information. The model explained in this chapter also has the positive correlation between riskiness and collateral, using asymmetric information. As shown in the previous chapter, it is assumed in the model that there is ex-post information asymmetry. We will see that the positive correlation can be naturally observed in this setting ${ }^{3}$.

[^22]
### 3.1.2 Informal Collateralised Loan

A collateralised loan is a debt contract that gives lenders a legal right to seize collateralised assets in the case that their borrowers fail to repay. With collateral, lenders can secure their loan and can lend to borrowers easily; or lenders can discipline (or screen) their borrowers. It is an important mechanism to avoid inefficient credit rationing, for example.

A possible problem that might occur when we introduce collateral into the model discussed in the last chapter is inconsistency of assumption. The self-enforcing structure assumed there postulates a lawless economy. If we assume pledgeability of collateral (exogenous commitment technology of collateral in case of default), it might be inconsistent with this lawlessness. Yet, historically, collateralised loans have been made with poor legal protection. For instance, mortgage loans were popular in the eleventh or twelfth century, mainly to finance the Crusades, according to Kohn(1999e). Land was used as a collateral, but courts were sympathetic to the borrowers, i.e. the Crusaders, and did not give sufficient legal protection. Moreover, Kohn(1999e) also points out that land-pawning has a long history, dating back 5000 years to ancient times. It is hard to suppose that the collateral was secured by an exogenous legal system.

Accordingly, here I try to model two different informal collateralised loans, which do not need exogenous legal guarantees. This means that borrowers need to commit their collateral delivery. One way of doing this is through pawnbroking. This requires borrowers to deliver their collateral ex ante, when they receive the loan. According to De Roover(1948), many goods were brought to pawnbrokers, such as jewelry, plate, clothing, kitchenware, or furniture. An advantage of the pawnbroking business is that there is no commitment problem. Until the borrower's repayment, lenders can control the collateral. The other is the rather normal collateral. Borrowers are required to deliver their collateral when they fail to repay. If they did not deliver the pledged collateral, they would be punished. Such kind of collateralised loan were common historically as well. For example, according to Kohn(1999d), loans to peasants took the form of the forward-purchase of their crops in Florence around 1500. Essentially This was collateralised loan, but lenders avoided using a loan contract because this would be regarded as usury, which was prohibited. So, I will compare these two contracts.

In the following argument, we adopt a simple approach: at the beginning of each period, the borrower A receives some endowment of output good, $d$. He can consume it freely, but this is not an input good; he still has to borrow C's endowment (input good) for his production. He can also use this output good as a collateral ${ }^{4}$.

The borrower can give his collateral in the different timing. First, he can give the output good before the project starts, when he receives C's endowment. It means that he "bought" C's endowment partially. Namely, the output good acts as commodity money in this case. Second, he can give the output good after the project is finished. If he finds out that $y=0$, then he gives his collateral to the lender instead. Namely, the output good here acts as collateral under the normal definition. We will study the both cases in this section, because both definition might be plausible under specific situation. For example, we can think a situation that delivery of output good is costly. In this case, the former definition might be less useful (suppose that C ships her endowment to A. the ship waits at A until the project finishes, and bring A's output good back to C. In this case, A has to hire another ship to deliver his commodity-money to C at the beginning of the project). Then we will compare these efficiency and conclude the section.

### 3.2 Bilateral Contracts

### 3.2.1 Settings

First, we will discuss the bilateral contract. The extensive form game is almost the same as the one without collateral, but now A receives a certain amount of collateral $d \leq \hat{\tau}$. A has a chance to receive $d$ at the beginning of every period, and C knows it. We will add a new action $\tilde{a}^{A 3}: H^{A} \rightarrow\{0, d\}$. Indivisibility of $d$ is assumed for simplicity. If the loan contract is pawnbroking, namely, borrowers give their collateral when they borrow, $\tilde{a}^{A 3}$ is chosen at the

[^23]beginning of each period. If it is a normal collateralised loan, $\tilde{a}^{A 3}$ is chosen after the choices of $y$ and $\tau$.

### 3.2.2 Collateralised Loan (ex post Delivery)

First, we will assume that A will give his collateral when he finds out that he cannot repay. Then the collateral is seized, or A gives it to C voluntarily. It is common in the moralhazard arguments to assume that a lender has some collateral collection skill (equivalently, assuming that collateral is somewhat verifiable in terms of Hart and Moore(1994) as Kiyotaki and Moore(2000) or Diamond and Rajan(2001)). By contrast, here we assume that the borrower voluntarily gives his collateral to avoid the longer penalty. Lenders cannot seize their borrowers' collateral by compulsion, due to an imperfect legal system.

Now the enforcing constraints imposed to the borrower are as follows:

$$
\begin{align*}
(1-\delta)(Y+d-\hat{\tau})+\delta u^{A} & \geq(1-\delta)(Y+d-d)+\delta^{k+1} u^{A}  \tag{3.1}\\
(1-\delta)(Y+d-d)+\delta^{k+1} u^{A} & \geq(1-\delta)(Y+d-0)+\delta^{l+1} u^{A} \\
(1-\delta)(0+d-d)+\delta^{k+1} u^{A} & \geq(1-\delta)(0+d-0)+\delta^{l+1} u^{A} \tag{3.2}
\end{align*}
$$

The first one guarantees that the borrower wants to repay $\tau$ rather than give $d$ when he succeeds to obtain $Y$. The second and third one compel the borrower to give collateral in the event that he does not repay $\tau$. The penalty length $k$ is imposed when the borrower fails to repay $\hat{\tau}$ but gives his collateral $d$ to C . If he does not give the collateral, C would impose penalty length $l$ (this $l$ would be different from non-collateralised case in the previous chapter). It is obvious that the second and third are equivalent ${ }^{5}$.

Then we can rewrite the equation (2.12), A's expected surplus when he behaves honestly, as:

[^24]$$
u^{A}=\frac{1-\delta}{1-\delta \pi-(1-\pi) \delta^{k+1}}\{\pi(Y-\hat{\tau})+d\}
$$

Substituting this into (3.1) and (3.2) :

$$
\begin{aligned}
& \frac{\delta\left(1-\delta^{k}\right)}{1-\delta \pi-(1-\pi) \delta^{k+1}}\{\pi(Y-\hat{\tau})+d\} \geq \hat{\tau}-d \\
& \frac{\delta\left(\delta^{k}-\delta^{l}\right)}{1-\delta \pi-(1-\pi) \delta^{k+1}}\{\pi(Y-\hat{\tau})+d\} \geq d
\end{aligned}
$$

The first one is arranged as follows:

$$
\begin{align*}
\frac{\delta \pi Y-\hat{\tau}+d(1+\delta-\delta \pi)}{\pi Y-\hat{\tau}+d(2-\pi)} & \geq \delta^{k+1}  \tag{3.3}\\
\left|\frac{\ln \left(\frac{\delta \pi Y-\hat{\tau}+d(1+\delta-\delta \pi)}{\pi Y-\hat{\tau}+d(2-\pi)}\right)}{\ln \delta}\right|-1 & =\mathrm{k} \leq k
\end{align*}
$$

The last inequality works as long as $\delta \pi Y-\hat{\tau}+d(1+\delta-\delta \pi)>0$ and it obviously holds $\delta \pi Y-\hat{\tau}$, the necessary condition of lemma 1 . Note that this lower bound $\underline{\underline{k}}$ is smaller than $\underline{\underline{l}}$, which is defined in the bilateral contracts without collateral.

Lemma 9 The necessary condition for the enforceability is eased, $\delta \pi Y-\hat{\tau}+d(1+\delta-\delta \pi)>0$, when the loan contract is collateralised.

The second inequality is arranged in a similar way and we have:

$$
\begin{equation*}
\frac{\delta^{l+1}\{\pi(Y-\hat{\tau})+d\}+d(1-\delta \pi)}{\pi(Y-\hat{\tau})+d(2-\pi)} \leq \delta^{k+1} \tag{3.4}
\end{equation*}
$$

While the first inequality gives a lower bound of k , the second one gives its upper bound. The $k$ satisfying both constraints exists only when the following inequality exists:

$$
\frac{\delta \pi Y-\hat{\tau}+d(1+\delta-\delta \pi)}{\pi Y-\hat{\tau}+d(2-\pi)}>\frac{\delta^{l+1}\{\pi(Y-\hat{\tau})+d\}+d(1-\delta \pi)}{\pi(Y-\hat{\tau})+d(2-\pi)}
$$

The inequality holds, because the denominator of RHS is greater than the one of LHS and the numerator of LHS is greater than the one of RHS, if $l$ is sufficiently large such that $\delta \pi Y-\hat{\tau}+\delta d>\delta^{l+1}\{\pi(Y-\hat{\tau})+d\}$. Therefore, there exists $k$ that satisfies the two enforcing constraints.

### 3.2.3 The Mechanism of Pawnbroking

Second, we will examine the different usage of the collateral. In the previous argument, we have assumed that the collateral is given when the borrower fails to repay. This is a normal assumption, but we have the other usage of the collateral: ex-ante delivery of the collateral. The borrower has received $d$ at the beginning of each period. Accordingly, he can give the collateral to C when he contracts with C . If he successfully repays, the collateral comes back to A (in this model, the collateral and repaid output good are the same, so it would be balanced out). This type of contract can be regarded as a pawnshop business, or, A "purchases" C's input good by paying with an output good. The latter definition implies that the collateral can act as a commodity money.

This is obviously a better strategy for both A and C compared with the non-collateralised case, because now A does not have to commit to give collateral in the case of default and he just needs to commit $\hat{\tau}-d$ when he succeeds in the project. Now C has only to impose one constraint:

$$
(1-\delta)\{Y-(\hat{\tau}-d)\}+\delta u^{A} \geq(1-\delta) Y+\delta^{l+1} u^{B}
$$

This is equivalent to the constraint (3.1) in the previous ex-post collateral delivery. (Since A does not have to commit the collateral delivery, we do not need (3.2)). The penalty length $l$ is, accordingly, the same as the penalty $k$ of the ex-post delivery case. In this sense, the exante delivery (pawnbroking) and the ex-post delivery (collateral) are equivalent in this bilateral
contract. Of course, it is the same as the case in which lenders can seize borrowers' collateral by compulsion.

Proposition 10 Efficiency of the ex-ante collateral delivery and that of the ex-post delivery are equivalent in the bilateral contracts, in terms of the penalty length.

### 3.3 Multilateral Contracts

Now we are ready to discuss the role of collateral in the multilateral relationship. The general setting is the same as the non-collateralised case discussed above. Additionally, A will receive indivisible collateral $d$ at the beginning of each period, and this is a common knowledge among players.

In the multilateral relationship. the role of the collateral becomes more important. The collateral is delivered from A to B , in order to shorten the penalty length $l$, then B gives the collateral to C , to shorten the length $n$. Now it may not be appropriate to regard the ex-ante delivery of the collateral as pawnbroking. Rather, it is a variation of commodity money. B purchases C's input good by paying the money, and A purchases the good from B by paying the money.

In the case of the ex-post delivery, we will face a similar difficulty to that, which is argued by Diamond and Rajan(2001). C needs the collateral in case of default, but C has to force B to transfer the collateral, not only A. This needs an additional enforcing constraint; we will see that the additional constraint makes the problem very difficult and possibly causes considerable inefficiency.

### 3.3.1 The Mechanism of Pawnbroking (Commodity Money)

First, we start by considering a simpler case. I assume that A delivers his collateral ex-ante, when they contract and that A receives C's input good. This needs only a minor modification
to the non-collateralised case. A gives his collateral d to B ex ante; thus A has to commit to repay $\hat{\tau}-d$ to B at the end of the project. B transfers A 's collateral to C ex ante; thus B has to commit the transfer $\hat{\omega}-d$ to C at the end of the project. Accordingly, now we have the following modified enforcing constraints:

$$
\begin{aligned}
(1-\delta)\{Y-(\hat{\tau}-d)\}+\delta u^{A} & \geq(1-\delta) Y+\delta^{k+1} u^{A} \\
(1-\delta)\{(\hat{\tau}-d)-(\hat{\omega}-d)\}+\delta u^{B} & \geq(1-\delta)(\hat{\tau}-d)+\delta^{n+1} u^{B}
\end{aligned}
$$

The first one is the same as the constraint of the bilateral contracts. The expected payoff $u^{B}$ is the same as in the non-collateralised case (2.25), because B merely transfers the indivisible collateral $d$ from A to C. Substituting (2.25) into the second constraint and rearranging it, we have:

$$
\frac{\delta\left(1-\delta^{n}\right)}{1-\delta \pi-(1-\pi) \delta^{n+1}} \pi(\hat{\tau}-\hat{\omega}) \geq \hat{\omega}-d
$$

Solving this for $\delta^{n+1}$,

$$
\frac{\delta \pi \hat{\tau}-\hat{\omega}+(1-\delta \pi) d}{\pi \hat{\tau}-\hat{\omega}+(1-\pi) d} \geq \delta^{n+1}
$$

The necessary condition for the existence of such $n$ is $\delta \pi \hat{\tau}-\hat{\omega}+(1-\delta \pi) d>0$. This is easier than the non-collateralised one, $\delta \pi \hat{\tau}-\hat{\omega}>0$; namely, some riskier (but profitable) borrowers are now able to obtain finance. If its LHS is bigger than the LHS in the non-collateralised case (2.26), the lower bound of the penalty length becomes shorter in the contract with commodity money. Thus, we need to prove the following:

$$
\begin{equation*}
\frac{\delta \pi \hat{\tau}-\hat{\omega}+(1-\delta \pi) d}{\pi \hat{\tau}-\hat{\omega}+(1-\pi) d}-\frac{\delta \pi \hat{\tau}-\hat{\omega}}{\pi \hat{\tau}-\hat{\omega}}>0 \tag{3.5}
\end{equation*}
$$

Rearrange it,

$$
(1-\delta) \pi \hat{\tau}-(1-\delta) \pi \hat{\omega}>0
$$

This obviously holds as long as $\hat{\tau}>\hat{\omega}$, which is the non-negative wealth constraint we have assumed at the beginning. To be precise, the constraint allows $\hat{\tau}=\hat{\omega}$, but it is possible only when the contract is fully secured, $d=\hat{\omega}^{6}$. Substitute this into (3.5), and we see the inequality holds.

We have already seen that ex-ante delivery of the collateral would shorten the penalty length $l$ between A and B. Now we have proved the following:

Proposition $11 \max (l, n)$ under the ex-ante collateral delivery (pawnbroking) is strictly shorter than the one in the non-collateralised case. Pawnbroking is strictly more efficient than in the non-collateralised case.

The equilibrium strategy profile and the corresponding beliefs are exactly the same as the one in the previous chapter, since the pawnbroking simply reduces the required repayment amounts. Defining these is, therefore, skipped in this section.

### 3.3.2 The Mechanism of collateralised Loan (ex post Delivery)

We then assume that A delivers his collateral ex-post, after he fails to repay. In this case, as we have seen, B and C have to enforce not only their borrower's repayment but also collateral deliveries. A's enforcing constraints are the same as (3.1) and (3.2). B's enforcing constraints are designed as follows:

$$
\begin{align*}
(1-\delta)(\hat{\tau}-\hat{\omega})+\delta u^{B} & \geq(1-\delta) \hat{\tau}+\delta^{n+1} u^{B}  \tag{3.6}\\
(1-\delta)(d-d)+\delta^{p+1} u^{B} & \geq(1-\delta) d+\delta^{n+1} u^{B} \tag{3.7}
\end{align*}
$$

[^25]where $p$ is penalty length if B fails to transfer $\hat{\omega}$, but succeeds to transfer $d$. The penalty length $n$ is imposed when B does not give anything to C. B's expected payoff when he behaves honestly is:
$$
u^{B}=(1-\delta) \pi(\hat{\tau}-\hat{\omega})+\pi \delta u^{B}+(1-\pi) \delta^{p+1} u^{B}
$$

If A gives his collateral when he fails to repay, he has to bear penalty $p$. As long as he behaves honestly, A does not have to bear a penalty for $n$ periods. Solving this for $u^{B}$, we have:

$$
\begin{equation*}
u^{B}=\frac{(1-\delta) \pi(\hat{\tau}-\hat{\omega})}{1-\pi \delta-(1-\pi) \delta^{p+1}} \tag{3.8}
\end{equation*}
$$

Substitute this into (3.6), we have:

$$
\begin{equation*}
\frac{(1-\delta \pi) d+\delta^{n+1} \pi(\hat{\tau}-\hat{\omega})}{\pi(\hat{\tau}-\hat{\omega})+(1-\pi) d} \leq \delta^{p+1} \tag{3.9}
\end{equation*}
$$

The inequality shows $p$ has an upper bound. The necessary condition to hold the inequality is that LHS is strictly greater than zero. This is satisfied if $\delta \pi \hat{\tau}>\hat{\omega}$, the condition of lemma 4.

Likewise, substitute (3.8) into the second constraint (3.7), we have:

$$
\frac{\left(\delta^{p+1}-\delta^{n+1}\right) \pi(\hat{\tau}-\hat{\omega})}{1-\delta \pi-(1-\pi) \delta^{p+1}} \geq d
$$

Solving for $\delta^{p+1}$,

$$
\begin{equation*}
\frac{(1-\delta \pi) d+\delta^{n+1} \pi(\hat{\tau}-\hat{\omega})}{\pi(\hat{\tau}-\hat{\omega})+(1-\pi) d} \leq \delta^{p+1} \tag{3.10}
\end{equation*}
$$

Both (3.9) and (3.10) give the same lower bound of $p$, but no lower bound for $p$ is defined. Additionally, $n$ is increasing w.r.t $p^{7}$. There is no upper bound for $n$ so we can take any arbitrary $n$, because it does not affect the players' expected payoff when they behave honestly.

[^26]Now we have four penalties in the ex-post collateral delivery: $k, l, p$ and $n . k$ is decreasing w.r.t $l$ and $n$ is increasing w.r.t $p$. Players want to lower $p$ as much as possible, but $p$ cannot be lowered below $k$, as we have seen in the non-collateralised case: C will impose $\max (k, p)$ and $\max (l, n)$, in order to avoid the penalty-length mismatch problem. Likewise, now we cannot assume $l=n=+\infty$, because the penalty is no longer an off-path punishment.

Accordingly, we have $k=p<l=n<\infty$. These figures, therefore, have to satisfy the constraints shown above, (3.3), (3.4) and (3.9). Since we have seen that there exists a penalty length $k$ that satisfies the first two constraints, now we need to check if there exists any penalty length $p$ between the upper bound (3.9) and the lower bound (3.3). Namely, we need to check if the following inequality holds or not $(k=p$ and $l=n$ are substituted already):

$$
\frac{\delta \pi \hat{\tau}-\hat{\omega}+(1-\delta \pi) d}{\pi \hat{\tau}-\hat{\omega}+(1-\pi) d}-\frac{(1-\delta \pi) d+\delta^{n+1} \pi(\hat{\tau}-\hat{\omega})}{\pi(\hat{\tau}-\hat{\omega})+(1-\pi) d}>0
$$

Rearranging this:

$$
\begin{aligned}
& d \delta \pi(\pi \hat{\omega}+Y-\pi \hat{\tau}-\hat{\omega})+d \pi(2 \hat{\tau}-Y-\hat{\omega}) \\
& +\delta \pi^{2} Y \hat{\tau}-\delta \pi^{2} Y \hat{\omega}-\pi \hat{\tau}^{2}+\pi \hat{\tau} \hat{\omega}-d^{2}(1-\delta)-\delta^{n+1} \pi(\hat{\tau}-\hat{\omega}) \\
= & \delta \pi\{\hat{\tau}(2-\delta \pi)-\hat{\omega}(1+\delta-\delta \pi)-(1-\delta) Y\}+(\delta \pi Y-\hat{\tau})(\hat{\tau}-\hat{\omega})-\delta^{n+1} \pi(\hat{\tau}-\hat{\omega}) \\
= & \{\delta \pi Y-\hat{\tau}+d(1+\delta-\delta \pi)\} \pi(\hat{\tau}-\hat{\omega})-(1-\delta)(Y-d \pi \hat{\tau})-\delta^{n+1} \pi(\hat{\tau}-\hat{\omega})
\end{aligned}
$$

The first term of the last equation is the same as the necessary condition. Since the latter two terms are negative ${ }^{8}$, the condition that guarantees the existence of appropriate penalty length $p$ is more severe than the necessary condition. Namely, compared with the bilateral loan contract or multilateral pawnbroking, the collateralised multilateral loan contract with ex-post collateral delivery is less efficient.

Proposition 12 In the multilateral relationship, the ex-ante collateral delivery (using collateral

[^27]as a commodity money) is favourable compared with the ex-post collateral delivery, because the feasible enforceable set is smaller.

The equilibrium strategies specified here are sequential equilibria. See the appendix for the proof.

### 3.4 Conclusion

In this chapter, we see how collateral can improve the efficiency of loan contracts in the bilateral and multilateral relationship. We have checked two different collateral systems. The first one is pawnbroking: a borrower gives his collateral ex ante when he borrows. The benefit of pawnbroking is that the borrower does not have to commit his collateral delivery in the case of default. The other one is normal collateral delivery: a borrower gives his collateral only when he fails to repay. If a legal system is established, the borrower still does not have to commit this delivery. Yet historically, such ex-post collateral delivery can be observed without appropriate legal support. In this chapter, therefore, I compare between pawnbroking and ex-post collateral delivery without legal enforcement.

In bilateral contracts, we see that both collateral deliveries (ex ante and ex post) are equivalent and efficiency-improving. Essentially, ex-post delivery is less efficient because there is an extra incentive problem that the borrower may not give his collateral when he fails to repay Yet the lender can enforce the borrower's honest behaviour by threatening him with a very long penalty in the case that the borrower does not give his collateral when he defaults ( $l$ would be close to positive infinity). Since there is no uncertainty for this behaviour (collateral delivery) and the behaviour is observable, a sufficiently long penalty can enforce borrowers' honest behaviours without efficiency loss. In both collateral deliveries, the necessary penalty length is shorter than in the non-collateralised case.

In contrast, in multilateral contracts, ex-post collateral delivery is less efficient. Lender C has two penalty strategies. One punishes B if B does not repay $\hat{\omega}$ (but gives collateral $d$ ) and
the other punishes B if B does not give collateral $d$ when B cannot repay $\hat{\omega}$. C can use different penalty lengths ( $p$ and $n$ respectively), and a sufficiently long penalty can solve the problems. However, if C punishes B by penalty for $p$ periods when B fails to repay $\hat{\omega}$ but gives $d$ (it is necessary for C to force B to repay $\hat{\omega}$ honestly), B would lose the incentive to give $d$ when B has nothing to repay. Furthermore, this penalty length $p$ is automatically applied to $\mathrm{A} ; p$ periods of penalty must be appropriate to force A to repay honestly at the same time. We have seen that there is no guarantee to have such $p$. Accordingly, a feasible set of parameters is smaller in multilateral loan contracts with ex-post collateral delivery, compared with ex-ante delivery (pawnbroking).

The reason why the collateral in this chapter improves efficiency is that the value of the collateral is constant (and therefore, common knowledge among agents). If the value of the collateral is stochastic, as Barro(1976) assumes, players' penalty functions become costly, just as the penalty strategies in the last chapter. In this case, collateral is not necessarily efficiencyimproving. In other words, if the economy has something common among players, it would improve efficiency. For instance, if this economy has a mediator who can reveal the true state of a default borrower with positive probability and announce the revealed information to the public, the information acts as a public signal of the economy. This public signal allows players to coordinate with the public monitoring approach à la Abreu, Pearce and Stacchetti(1990). I have argued this mediator system in another paper, $\mathrm{Ota}(2006)$.

The efficiency of ex-ante delivery in a multilateral loan contract would support the circulation of commodity money or primitive money in less developed financial markets. We will look at this issue in the next chapter.

### 3.5 Appendix: Sequential Equilibria with Ex-post Collateral Delivery

In the case of the ex-ante delivery, pawnbroking, we do not have to change the structure of the game, because the delivery of collateral is done when the contract is made. The ex-ante
delivery is therefore common knowledge among players, and there is no additional action in the game.

In the case of ex-post delivery, on the other hand, the game's action space and the players' beliefs change. First, A has another action $a^{A 2} \in\{0, d\}$, whether A gives its collateral to B when A fails to repay $\tau$ or not. A does not have any incentive to give the collateral when A repays $\tau$, because it violates the participation constraint that the loan contract should provide better payoff than autarky. In the following argument, it is assumed that A does not have the choice of $a^{A 2}$ if $\tau=\hat{\tau}$ for simplicity. B now has three information set: a singleton set $\tau=\hat{\tau}$; $\tau=0$ and $a^{A 2}=d$; and $\tau=0$ and $a^{A 2}=0$. The second set has two elements $y=Y$ and $y=0$. The third one also has two, $y=Y$ and $y=0$. B has another action as well, $a^{B 3} \in\{0, d\}$, whether B deliver A's collateral to C or not.

C also has three information sets. The first one is singleton, $\omega=\hat{\omega}$. The second set is $\omega=0$ and $a^{B 3}=d$. This set has two elements, $\left(y=Y, a^{A 2}=a^{B 3}=d\right)$ and $\left(y=0, a^{A 2}=a^{B 3}=d\right)$. The third set is $\omega=0$ and $a^{B 3}=0$. This set has five elements, $\left(y=Y, \tau=\hat{\tau}, a^{A 2}=a^{B 3}=0\right)$, $\left(y=Y, \tau=0, a^{A 2}=d, a^{B 3}=0\right),\left(y=Y, \tau=0, a^{A 2}=0, a^{B 3}=0\right),\left(y=0, \tau=0, a^{A 2}=d, a^{B 3}=\right.$ $0)$ and $\left(y=0, \tau=0, a^{A 2}=a^{B 3}=0\right)$. The penalty strategies defined above enforce $a^{A 2}=d$ only when $y=0$ and $a^{B 3}=d$ only when $a^{A 2}=d$. The other equilibrium strategies are identical to the previous chapter.

The consistent beliefs are derived in a similar way to the previous chapter. In the previous chapter, when A arrives the informant set $\left(\tau_{t-1}=\hat{\tau}, a^{B}=1\right)$, A has some off the equilibrium nodes. A' ex-post belief on this information set is $\mu^{A}\left(\omega_{t-1}=\hat{\omega}, a_{t-1}^{A 2}=a_{t-1}^{B 3}=0, a_{t}^{C}=1, a_{t}^{B}=\right.$ $1)=1$, even with B or C's trembles (see the previous chapter for the proof).

B has, after receiving $d$, the following ex-post belief:

$$
\mu^{B}\left(y=0 \mid a^{A 2}=d\right)=\frac{(1-\pi) \cdot \tilde{\sigma}^{A}\left(a^{A 2}=d \mid y=0\right)}{\pi \cdot \tilde{\sigma}^{A}\left(a^{A 2}=d \mid y=Y\right)+(1-\pi) \cdot \tilde{\sigma}^{A}\left(a^{A 2}=d \mid y=0\right)}
$$

The incentive compatibility constraint (3.7) ensures that $\tilde{a}^{B 3}\left(a^{A 2}=d\right)=d$ is optimal for any belief $\mu^{B}\left(y=0 \mid a^{A 2}=d\right)$; thus, any belief is consistent to the strategy. Likewise, C's ex-post belief is defined as follows:

$$
\mu^{C}\left(y=0 \mid \omega=0, a^{B 3}=d\right)=\frac{(1-\pi) \cdot \tilde{\sigma}^{A}\left(a^{A 2}=d \mid y=0\right) \cdot \tilde{\sigma}^{A}\left(a^{B 3}=d \mid a^{A 2}=d\right)}{Y}
$$

where $Y=\pi \cdot \tilde{\sigma}^{A}\left(a^{A 2}=d \mid y=Y\right) \cdot \tilde{\sigma}^{B}\left(a^{B 3}=d \mid a^{A 2}=d\right)+(1-\pi) \cdot \tilde{\sigma}^{A}\left(a^{A 2}=d \mid y=\right.$ $0) \cdot \tilde{\sigma}^{A}\left(a^{B 3}=d \mid a^{A 2}=d\right)$. Again, the penalty strategy imposing $p(=k)$ periods autarky is optimal for any belief. Another ex-post belief is defined as follows. $\tilde{\sigma}^{A}(\tau=\hat{\tau} \mid y=Y)=1-\varepsilon^{A 1}$, $\tilde{\sigma}^{A}\left(a^{A 2}=d \mid \tau=0\right)=1-\varepsilon^{A 2}, \tilde{\sigma}^{B}(\omega=\hat{\omega} \mid \tau=\hat{\tau})=1-\varepsilon^{B 1}, \tilde{\sigma}^{B}\left(a^{B 3}=d \mid a^{A 2}=d\right)=1-\varepsilon^{B 2}$,

$$
\begin{aligned}
& \mu^{C}\left(\tau=\hat{\tau} \mid \omega=0, a^{B 3}=0\right) \\
& \quad=\frac{\pi \cdot\left(1-\varepsilon^{A 1}\right) \cdot \varepsilon^{B 1}}{\pi\left(1-\varepsilon^{A 1}\right) \varepsilon^{B 1}+\pi \varepsilon^{A 1}\left(1-\varepsilon^{A 2}\right)\left(1-\varepsilon^{B 2}\right)+\pi \varepsilon^{A 1} \varepsilon^{A 2}+(1-\pi)\left(1-\varepsilon^{A 2}\right) \varepsilon^{B 2}+(1-\pi) \varepsilon^{A 2}} \\
& \quad \mu^{C}\left(y=Y, \tau=0, a^{A 2}=d \mid \omega=0, a^{B 3}=0\right), \mu^{C}\left(y=Y, \tau=0, a^{A 2}=0 \mid \omega=0, a^{B 3}=0\right), \\
& \mu^{C}\left(y=0, \tau=0, a^{A 2}=d \mid \omega=0, a^{B 3}=0\right), \text { and } \mu^{C}\left(y=0, \tau=0, a^{A 2}=0 \mid \omega=0, a^{B 3}=0\right)
\end{aligned}
$$ are also defined in the same way. The optimal penalty (autarky) lengths for each belief are $n$, $k, l, k$, and $l$. The optimal penalty length is $\max (n, l)>k$ : thus, the penalty strategy defined above is optimal for any ex-post beliefs (any ex-post belief with tremble is consistent).

There exist, therefore, two sequential equilibrium strategy profiles. One is, $\tilde{a}^{A}(y=Y)=\hat{\tau}$, $\tilde{a}^{A 2}(\tau=0)=d, \tilde{a}^{B}\left(a^{C}=1\right)=1, \tilde{a}^{B 2}(\tau=\hat{\tau})=\hat{\omega}, \tilde{a}^{B 3}\left(a^{A 2}=d\right)=d, \mathrm{C}$ takes max $(k, p)$ periods autarky if B gives $\omega=0$ and $a^{B 3}=d$, and $\max (n, l)$ periods autarky if B gives $\omega=0$ and $a^{B 3}=0$. Corresponding beliefs are shown above. The other one is autarky: all the above actions becomes zero.

## Chapter 4

## Conclusion: A Financial Revolution

Money is normally categorised into two types: outside money and inside money. Outside money includes fiat money and commodity money, which is guaranteed exogenously. Inside money can be further categorised into two, bank deposits (bank-notes) and negotiable bills. Although we tend to limit our attention to bank deposits when we consider inside money, as money supply statistics show, the history shows that negotiable bills can be a substitute for bank deposit as an inside money(see Chapter One).

From this viewpoint, the study in the previous chapter can be thought as a piece of research into an alternative inside money, which has been rarely studied ${ }^{1}$. In this chapter, we will look at its characteristics and then compare the other studies of inside money with a similar framework, Kiyotaki and Moore(2003).

[^28]
### 4.1 Costs and Benefits of Multilateral Contracts

The arguments shown above give several simple characterizations of the self-enforcing ${ }^{2}$ contract of indirect finance, or Credit Chains, using Kiyotaki and Moore(1997)'s term. First, with reputation, the borrower's IOU can be resaleable only when the intermediary is collectively responsible to A's and his own honest behaviour. This mechanism is equivalent to endorsement, which guarantees a negotiability of the bills, initiated at Antwerp in the sixteenth century.

Endorsement is one of the critical features of modern finance. When a holder of bills sells a bill purchased from somewhere else, she has to sign her name on the back of the bill. As Day(1957) points out, endorsement is the declaration of the seller to be responsible for repayment of the bill; i.e. the buyer of the bill is endowed with a right to claim the debt service in the case that the issuer of the bill fails to repay. In other words, the seller has collective responsibility for the repayment. This joint responsibility mechanism is the same as the model we examined in Chapter Two.

Kohn(1999a) argues that Antwerp had to introduce this because deposit banks were banned and they could not use bank-notes for their settlements. In other words, they used resaleable bills as an alternative instrument for settlement. bank-notes are normally regarded as a form of money because banks' credit is exogenously guaranteed (by government), or banks have certain special technologies that makes bank-notes credible for others. But in our model, as in Antwerp, traders are able to create another form of money (although its function is limited compared with bank-notes) without any special technologies of banks. Such kind of substitutability of trade credits in economics without banking is currently observed as well. For example, Petersen and Rajan(1997) and Demirgüc-Kunt and Maksimovic(2001) empirically show that trade credit tends to grow if the legal system and banking system are less established. The case of Ireland in 1970 has been explained in Chapter One.

[^29]Second, the role of intermediary B is limited compared with other theories of banking. As Boot and Thakor(2000) summarise them, theories of banking have two different definitions of banking: relationship banking and transaction banking. Relationship banking is close to commercial banking; the banks have certain relationships with borrowers and monitor or enforce them with their own technologies. Transaction banking is close to investment banking or securities broking. They underwrite stocks or bonds without relationships with borrowers or technologies. This is possible because information asymmetry is negligible in their markets. The shift to transaction banking has been discussed because of declining information asymmetry, due to publicly observable credit scoring, for instance.

B's function in this model is simple: merely guaranteeing A's repayment by bearing joint liability (endorsement). No special activities such as costly monitoring or person-specific collateral collection skills are required (this resembles British accepting houses). In this sense, this intermediary business is close to transaction banking; nevertheless, this economy is under severe information asymmetry and the model is essentially categorised as relationship banking. There are many papers studying the difference between relationship banking and transaction banking. Yet the model discussed here implies that the boundary of these two banking systems is not so clear, as Boot and Thakor(2000) point out. In this model, A and C do not have a direct relationship and they cannot obtain the other's private information directly, whereas the indirect loan contract between A and C (through B ) requires exclusive multiple interaction between them. Normally, relationship banking presumes that the lender can obtain proprietary information about her borrower through screening, monitoring and so on. Transaction banking tends to assume symmetric information or public signals about borrower's quality, such as credit scoring. The model discussed here is different from both. A and C cannot observe the other's action directly, and do not share any public signal. In this sense, the model in Chapter Two is different from these two major categories.

This feature, different from transaction banking and relationship banking, is useful when considering securitisation as well. As Gorton and Pennacchi(1995) point out, it is difficult to explain securitisation of bank loan with the traditional theory of banking, as in Diamond and Dybvig(1983) or Diamond(1984). On the other hand, it would be difficult to assume that there
is no information asymmetry, as transaction lending tends to assume. The model discussed here is able to give some explanation relating to this issue. The intermediary B acts as an underwriter of the securitised loan, by taking collective responsibility ${ }^{3}$.

Third, a longer credit chain needs a longer period of penalty. As shown above, C needs to impose $\max (l, n)$ to enforce the borrower C's honest behaviour. If we assume that players' relative bargaining power is exogenous, $l$ in the multilateral game must be lower than in the bilateral one. This is because the borrower can enjoy a smaller allocation of the project surplus in the multilateral game and he loses the incentive to behave honestly; his honest repayment should now be enforced by a more severe penalty. Namely, a bilateral loan contract is always weakly preferable to a multilateral contract. The penalty phase can be interpreted as a form of "financial disruption", since no financial transaction occurs during the penalty phase. In this sense, this non-banking informal indirect finance is fragile compared with direct finance, or other financial systems with legal support or with collateral ${ }^{4}$. This implication is supported by some empirical works which argue that financial panics frequently occurred in past, or occur in emerging countries rather than developed countries (see Allen and Gale(1998) or Lindgren, Garcia and Saal(1996)). Also, in medieval Europe several conflicts are reported by Kohn(1999a, 1999e) before legal support for negotiability was established.

Fourth, only borrowers with relatively safe projects can issue resaleable bills. The bilateral and multilateral contracts need $E[y]>\frac{1}{\delta \pi}$ and $E[y]>\frac{1}{\delta^{2} \pi^{2}}$ respectively, or any penalty cannot enforce the honest behaviour of borrowers. This condition is necessary to give sufficient margin to both A and B , to compensate for the utility loss incurred when they behave honestly (due to the information asymmetry, they are punished when the project fails even though they behave honestly). Accordingly, the lender should give a greater margin to all involved players if $\pi$ is low. The restriction is exponentially more severe in the multilateral contract, because now the

[^30]lender should give the bigger margin to both A and B . As a result, longer credit chains need safer projects.

See Figure 4-1, which describes what kind of project can obtain finance assuming that the discount factor $\delta$ is close to 1 . The vertical axis represents risk of the project $\pi$ (higher is safer). The horizontal axis is the expected profit of the project $E[y]$ (à large number means profitable). The dark shaded area is a set of projects that cannot obtain finance under bilateral contracts even though these are profitable. Under the multilateral contracts, projects in the light and dark shaded area are not able to borrow. Only the project with high $\pi$ (blank area) can issue their resaleable bills and obtain finance without any exogenous support. In other words, the fact that the bills have been re-sold acts as a signal of credit quality of the issuers ${ }^{5}$. This explains the reason why the Bank of England discounted commercial bills or took these as collateral only when the bills were 'signed' (endorsed) twice ${ }^{6}$, according to Day(1957). Our model shows the reason why the Bank of England could regard the bills as eligible; only bills with safe investment projects could be traded(re-sold) and endorsed.

### 4.2 Collateralised Loan and Primitive Money

We have seen, in Chapter Three, that collateral can improve the efficiency of the economy, which means shorter penalty length and a wider set of parameters than is feasible for uncollateralised loan contracts (both bilateral and multilateral: see Figure 4-1). One conclusion of the chapter is that ex-ante collateral delivery, which resembles pawnbroking, is better than ex-post delivery, which is close to normal collateralised agreement, because lenders do not have to impose an additional penalty strategy to enforce collateral delivery. Collateral plays a more important role in multilateral contracts, because one collateral can alleviate the commitment problem twice. Firstly A now has to commit a fraction of his repayment $\tau$ (the rest of the repayment has been

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Figure 4-1: Risk, Profitability and Funding Opportunity
covered by the collateral delivered). Secondly, B has to commit a fraction of his transfer $\omega$. (Essentially, ex-ante collateral delivery is equivalent to ex-post collateral delivery with legal collateral collection without additional cost.)

The alleviated incentive problem described in Chapter Three explains why pawnbroking has had a long history of 5000 years (Kohn(1999e)). De Roover(1948) point out that various goods had been used as collateral in pawnbroking in the sixteenth century: jewelry, plate, clothing, kitchenware, kettles and furniture. Pawnbrokers kept these goods until repayment. It is interesting that similar goods were used as primitive money as well. It is famous that metal money such as gold, silver and copper were used as commodity money in the past. Yet, according to Braudel(1967), a wide variety of goods were used as "money" (Braudel(1967) regards these as primitive money). For instance, salt in Upper Senegal (17th century), cotton cloth in the Gulf of Guinea, dried fish in Iceland (15th century), furs in Alaska and Russia, tobacco, sugar and cocoa in colonial America, were all used as kinds of money. He points out that these rudimentary forms of money appeared 'as soon as commodities are exchanged'. As Bernhardt(1986) points out, both money and loans can support intertemporal exchange when there is an absence of a temporal coincidence of wants. Commodity money or primitive money emerged to alleviate possible problems occurring in intertemporal barter transactions. Collateral emerged to alleviate problems occurring in intertemporal financial transactions. In this sense, ex-ante delivery of collateral and primitive money played a very similar role. If collateral is sufficiently large to cover everything (i.e. $d \geq \hat{\tau}$ ) in Chapter Three, borrower A "pays" his collateral to B and purchases C's intermediate good instead. B also pays the collateral to C, to lay in a stock of the good. In this sense, the argument in Chapter Three can be regarded as an explanation of primitive money as well.

### 4.3 Inside Money and Financial Expansion

Figure 4-1 also implies the transition of financial systems as the financial market expands. If the market is small, borrowers would have direct relationships with all possible lenders and it is not difficult for them to make bilateral loan contracts, without an intermediary. As the
market expands, however, the network in the market becomes sparse and it becomes difficult for borrowers to find lenders who have a direct relationship with the borrowers. Figure 4-1 shows that only safer borrowers can obtain finance if the financial market expands and its network becomes sparse, because a sparse network requires negotiable bills to be issued and only less risky borrowers can do this.

It is interesting to compare this result with Kiyotaki and Moore(2003)'s study of inside money. They argue that economies rely on both outside money (e.g. fiat money) and inside money (e.g. bank-notes) initially, but outside money is not required as financial deepening proceeds, and even inside money is not necessary when financial deepening proceeds further. They assume financial deepening to be a higher negotiability of IOUs and a higher degree of mortgageability (high leverage): both are given exogenously in their model (in the model discussed here, by contrast, negotiability is endogenously determined and mortgageability does not exist because leverage through collateral is impossible in our set-up).

We can say that the model discussed here studies the relationship between inside money and financial expansion, while Kiyotaki and Moore(2003) study the relationship between inside money and financial deepening, although their definition of inside money is slightly different ${ }^{7}$. A financially deepened market (with lower banking costs) does not have to rely on outside money and can use inside money (bank-notes) instead. If the market deepens further (higher leverage), then the economy does not need inside money as well. On the other hand, a financially sparse market needs inside money and safer borrowers who can issue inside money (negotiable bills). If its network is dense, the economy does not need inside money because direct loan contracts are available.

Outside money can be defined in this model; as we have seen above, collateral can act as a form of outside money (to be precise, commodity money or primitive money), while Kiyotaki and Moore(2003) assume outside money to be fiat money. With collateral (commodity money), the two curves in Figure 4-1 shift south-west and the rationed area vanishes as collateral increases.

[^32]Accordingly, we can say that the economy has to rely on outside money (collateral) as the financial market expands (as its network becomes sparse).

This implication gives some justifications to historical facts in Antwerp or Ireland and to the empirical finding by Petersen and Rajan(1997) and Demirgüc-Kunt and Maksimovic(2001): trade credit grows when the banking system is less established or unavailable. An established banking system can be defined in this context as a situation in which every player (or, at least majority of the players) has a direct relationship with their bank (the network is dense). If borrowers have direct access to lenders, there is no reason to choose less-efficient multilateral loan contracts. "Less developed banking system" in these empirical studies means that banks do not have a tight-knit network with borrowers. If the economy does not have such a banking network, borrowers have to rely on multilateral loan contracts, which are equivalent to negotiable bills or inside money in the model discussed here, and borrowers in the light-shaded area of Figure $4-1$ are rationed.

### 4.4 Other Efficiency-Improving Mechanisms

Throughout the above chapters I have limited my attention to à lawless economy ${ }^{8}$. By finding the equilibria of informal contracts, the model shows that a legal enforcement mechanism is not essential in these financial systems However, at the same time, the informal contracts are less efficient. Some profitable projects are rationed (even though their expected profitability is very high) and it is inevitable to punish borrowers who behave honestly. Collateral can alleviate the problem, as we have seen. Yet borrowers need to have initial wealth to make collateralised loan contracts. If borrowers do not have any wealth (assets or goods), they will be rationed again.

In order to improve efficiency further, we need to have exogenous legal enforcement. We can, for instance, assume the existence of a mediator, who will audit a defaulting borrower's credit status and reveal whether the borrower behaved honestly or not. If the mediator's announcement is common knowledge among players, players can coordinate based on the public

[^33]signal, as Abreu, Pearce and Stacchetti(1990) argue. The importance of these legal enforcement mechanisms has been observed historically. Kohn(1999e) points out that, without an appropriate legal system, the mortgaged loan in the eleventh century finally declined, because of frequent conflicts about collateral delivery and repayment. The endorsement became popular everywhere in the eighteenth century, after the courts ratified its legal validity. Ghate(1992) also points out that the negotiability of bills normally becomes widespread after certain legal supports are established.

The capability of the legal system does not impair the importance of the informal contracts discussed here. As explained in Chapter One, the legal system (for instance, the Law of Bills of Exchange in the UK) tended to be made in accordance with merchants' customs. The essential features of the informal contracts are preserved, as we have seen in the endorsement mechanism. Moreover, legal enforcement mechanisms cannot be perfect as well, and still partly rely on an informal incentive mechanism, even in modern financial markets.

## Part II

## Optimal Post-Crisis Policies: From Japanese Experience

## Chapter 5

## Introduction: A Lost Decade

### 5.1 A Lost Decade and Zombie Lending

This part tries to answer three questions that arise from the so-called "zombie lending" problem in Japan in the1990s. This was a phenomenon in which default firms were not liquidated by their banks and their productive capital was left unused. This raises the first question: "Why did banks not liquidate defaulting firms, although they knew the firms were loss-making?". The other questions are macroeconomic: "Why did the Japanese economy continue to be stagnant for more than a decade, after the burst of the bubble in the early 1990s?" and "Why did land prices in the last decade not act as a leading indicator of economic growth, as Kiyotaki and Moore(1997) predict?" (see Figure 5-1 ${ }^{1}$ ). By answering to these questions, we can obtain several interesting implications for optimal policies to resolve other banking crisis.

We will find the answer to these questions by combining two different models: one is the soft budget constraint model (SBC hereafter), proposed by Kornai(1979) and Dewatripont and Maskin(1995); and the other is the credit cycles model, proposed by Kiyotaki and Moore(1997).

It has been commonly argued that unhealthy Japanese banks were one of the major contributors to this recession of the 1990s. One hypothesis argued that Japanese banks who suffered

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Figure 5-1: GDP Growth and Land Price
from non-performing loans could not lend to productive borrowers and this caused the socalled "credit crunch", and the consequent under-investment caused the long stagnation of the Japanese economy. This credit crunch hypothesis has, however, been challenged by several authors. For example, Motonishi and Yoshikawa(1999) argued that the credit crunch was not empirically observed during the 1990s - except for the short period from 1997 to 1998. Financng constraints only existed for small firms, but not for large firms. Thus, the Japanese prolonged stagnation cannot be explained by the credit crunch hypothesis. Hayashi and Prescott(2002) also queried the credit crunch hypothesis and argued that shortened working hours and a lowered Total Factor Productivity (TFP hereafter) were the main contributors to the recession, through a calibration of a Real Business Cycle model. They also point out that some empirical observations are inconsistent with the credit crunch hypothesis.

After the challenges to the credit crunch hypothesis, the "zombie firm" hypothesis gripped economists' attention. The definition of zombie lending differs in detail depending on authors, but basically it can be defined as a situation in which banks leave non-performing borrowers in business, by giving additional loans for their repayments (Economically, this is equivalent to


Figure 5-2: Bank Loan by Industries
a rescheduling of the debts). As a result, the inefficient and insolvent borrowers are preserved and continue using human and physical capital; so, aggregate productivity of the economy, or TFP, declines. As Figure 5-2 shows, outstanding loans to poor-performing industries like real estate or construction continued to increase after the bursting of the bubble (as pointed out by Hoshi(2000) $)^{2}$. However, lending to the poor-performing industries did stop after the bank crisis in 1998; note that this occurred despite the injection of public funds to re-capitalise the suffering banking system from 1998 onwards.

Is there any empirical evidence for the existence of such zombie-lending, or Oigashi in Japanese (which literally means 'additional lending')? Kobayashi, Saita and Sekine(2002), for example, tested a hypothesis of zombie lending based on the soft budget constraint (SBC

[^35]hereafter) and show that zombie lending was empirically observed in the construction or the real estate industry. Mihira(2005) estimated bank-loan supply function and observed that bank loans tended to increase to the over-borrowing firms, especially in the construction and real estate industries. Peek and Rosengren(2005) also found a tendency for troubled Japanese banks to lend to impaired borrowers, in order to avoid realizing losses on their balance sheets. Since the troubled banks' capital adequacy ratio was low and close to the threshold under the Basel Accord ( $4 \%$ or $8 \%$ ), any realization of loss would cause a painful suspension of their business.

A negative externality of the zombie lending, especially its influence on the macro economy has also been empirically tested. Caballero, Hoshi and Kashyap(2006) argue that zombie lending prevented the creative destruction of industries in distress. Zombie lending also had severe effects in distorting market conditions, they claim. Since the zombie firms received subsidised loans, they could produce in more severe condition; their inefficient production would lower products' prices and raise wages of the industry. This prevented entry of productive entrepreneurs and investment of non-subsidised incumbents. Thus, zombie lending lowered the efficiency of non-zombie firms as well. They tested this hypothesis empirically and obtained affirmative results. Similarly, Mihira(2006) tested the negative externality of zombie lending to non-zombie firms, using micro data. He argued that the productivity growth in the economy could have fallen by a third on account of the negative externality of zombie lending.

While empirical research into zombie lending has accumulated in the last five years, its theoretical explanation is, by contrast, still relatively undeveloped. For example, Hayashi and Prescott(2002) argue that negative change in TFP is a main contributor; but the change in TFP is taken to be exogenous. Caballero, Hoshi and Kashyap(2006) show a theoretical framework of how the zombie lending affected the aggregate economy, but do not show why banks chose zombie lending. There has of course been some theoretical analysis. Berglof and Roland(1997) modify Dewatripont and Maskin(1995)'s soft budget constraint model and build a model that is similar to the zombie lending situation. Their paper is closely related to my model shown in the next chapter, but some facts still contradict their model.

Before going into the details of the model, we will undertake fact-finding. In the rest of


Figure 5-3: Total Debt Amount of Bankrupt Firms
this chapter, I will summarise what happened in the Japanese economy and the banks after the burst of the bubble, and the kind of measures that were taken. This provides some stylised facts to introduce the theoretical model developed in the next chapter.

### 5.2 Facts of the Lost Decade

After the bubble burst in the early 1990s, the number of bankruptcies surged (Figure 5-3). The banks' balance sheets were severely damaged by the bankruptcies and the following plunges in asset prices. The amount of less-performing loans of major banks increased in 1992 and 1995, and then peaked in 2001 (see Figure 5-4 ${ }^{3}$ ).

The bad-loan problem initially hit so-called "Jusen", housing loan companies initially established for the personal housing loan business through the joint capital investment of banks ${ }^{4}$.

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Figure 5-4: Transition of Low-Performing Bank Loans

These companies obtained finance from commercial banks and arranged housing loans to individuals. During the bubble from the late 1980s to the early 1990s, however, these Jusen drastically increased loans to real estate agencies, which became critical for the Jusen when the bubble burst. These companies finally went bankrupt in 1995, and their debts (JPY 6.4 trillion $\simeq$ USD 60 billion) were mainly written off by the banks and partly covered by government (JPY 0.7 trillion). The public capital injection caused severe political turmoil and delayed further public capital injections to bad banks in the following years.

In 1998, during the bank crisis triggered by the bankruptcies of several major financial institutions in November 1997 (Hokkaido Takushoku Bank and Yamaichi Securities), two major banks (Long-Term Credit Bank and Nippon Credit Bank) went bankrupt, but it took until 1999 for the government to inject public capital into banks on a massive scale. It took several more years to stabilise banks' capital adequacy ratio, through the capital injection and M\&A of the banks (see Figure $5-5^{5}$ ). As Figure $5-4$ shows, the bad loan problem was finally solved by 2005.

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Figure 5-5: Capital Adequacy Ratio and Capital Injections

In the decade from 1995 to 2005, we can find several salient characteristics. First, corporate bankruptcy lagged for several years behind the lowest level of GDP growth, which was in fact negative in 1998 (see Figure 5-3). Corporate default peaked in 2000, but GDP growth had then improved $(2.9 \%)$. A similar lag can be found in the volume of low-performing loans. The bad loan amount peaked in 2001 (Figure 5-4). These lags are consistent with the existence of zombie lending: by giving additional finance to suffering borrowers, banks could disguise these suffering borrowers as less risky borrowers. This could lower corporate bankruptcy and less-performing loans at least for the time being.

Second, bank loan amounts were not correlated with land prices (see Figure 5-6). According to Hart and Moore(1994)'s argument, and to the literature relating to collateralised loans, including Kiyotaki and Moore(1997), land prices and loan amounts would have a positive correlation since land is used as collateral, not only as productive capital. Land prices had kept declining since 1992; however, bank loan growth had been positive until 1997. Bank loan growth bottomed out in 2005, when the bad loan problem was solved (see Figure 5-4), or when land prices bottomed out. In other words, bank loan growth was inconsistent with land prices from the burst of the bubble until the bank crisis in 1998.


Figure 5-6: Land Prices and Bank Loan

Third, GDP growth and the evolution of land prices were inconsistent with the logic of Kiyotaki and Moore(1997). Their macroeconomic model predicted a multiplier effect of land prices and economic growth: a higher land price raises collateral value and thus it increases investment. Higher investment stimulates the economy and raises land prices further. When land price is lowered, we have the opposite vicious cycle. Such a relationship could be observed until the burst of the bubble, as Figure 5-7 shows. But the relationship seemed to collapse after the burst of the bubble. From 1992 to 1998, GDP growth was positive while land price declined by $5 \%$ every year. After the bank crisis in 1998, GDP growth started to improve, while land price declined further (see Figure 5-7).


Figure 5-7: GDP Growth and Land Prices

### 5.3 Literature Review

### 5.3.1 Soft Budget Constraint Models

Several studies examine some of the above factors. But there is little research explaining all of these phenomena: the existence of zombie lending, its negative effect on the macroeconomy, and some of the "anomalies" shown above. A useful approach to tackle the problems is the soft budget constraint model (SBC hereafter). Kornai(1979) highlighted the fact that chronic loss-makers still obtain finance in socialist economies. Dewatripont and Maskin(1995) described the phenomenon as a result of dynamic inconsistency. It is optimal for the bank to threaten its borrower, by saying that the bank will liquidate its borrower when the borrower's performance is not satisfactory. This liquidation is painful for the borrowers and gives him an incentive to work honestly. However, once the borrower's poor performance is revealed, the bank does not have an incentive to liquidate the borrower, because it is costly for the bank as well. Liquidation is, therefore, ex ante optimal but suboptimal ex post. Thus, chronic loss-makers can avoid liquidation and keep obtaining finance.

A related argument for the zombie phenomenon in SBC is Berglof and Roland(1997) ${ }^{6}$. They showed that a bank chooses to refinance bad firms when expected profitability of alternative firms is low or the liquidity value of the bad firm is low. They also showed the reason why the zombie firm phenomenon occurs with credit crunch; refinancing a bad firm leads to crowding out; it means that the bank stops lending to new firms.

Berglof and Roland(1997) is useful to explain some of the zombie phenomenon, but we can still find several problems when applying their model to the Japanese case. First, it is difficult to think that the zombie phenomenon occurred due to low liquidation value, which is exogenously given. Seemed as Caballero, Hoshi and Kashyap(2006) show, the number of zombie firms peaked in 1996; however, as Figure 5-3 shows, Japanese land prices had moderately declined until 1997 and then declined further from 1998 onwards. Figure 5-3 also shows that the Japanese economy was sluggish when the number of zombie firms stopped increasing ${ }^{7}$. These observations are opposite to Berglof and Roland(1997)'s prediction.

Note that they rely on the assumption that refinancing to the bad firm is still more profitable than liquidation ${ }^{8}$, i.e., the going concern value is higher than its liquidation value. This has been a common assumption in SBC since Dewatripont and Maskin(1995)'s seminal paper was published. For Dewatripont and Maskin(1995), this makes the threat of liquidation non-credible and leads a borrower to choose a moral hazard action. Yet if the borrower is truly a "chronic loss maker", as Kornai(1979) wrote, his going concern value might be lower than liquidation value. This is because, before the bad firms become insolvent, they sell their trade rights, patents or the other sources of benefits, so as to avoid bankruptcy as long as possible. After they become insolvent, talented managers and employees leave the firm. Consequently, assuming that the bad firm's profitability is still higher than the liquidation value is not plausible, at least in this Japanese case.

[^38]These factors motivate us to study a different type of SBC. Our model explains the fact that banks had an incentive to make zombies even though they knew it was costly compared with liquidation. They had to make zombies because the liquidation of bad firms (selling their collateral, land) inevitably lowers equilibrium land price (as well as the liquidation value) and the banks had to realise capital loss. This capital loss lowers their own capital and may trigger a bank closure if their own capital falls below a certain threshold, i.e. the BIS accord. In order to avoid the huge utility loss of bank closure, the banks choose to make zombies even though it is loss-making in the long run. In other words, the zombies were made to keep the land price higher than its intrinsic value. This is of course costly for the whole economy, because a higher land price prevents profitable firms from joining the market and because some productive capital (land) possessed by zombies is left unused.

This alteration of the SBC model can explain the other facts of the Japanese lost decade. Much empirical research points out that under-investment and corresponding lower funding demand were observed during the decade. Some papers (e.g. Hayashi and Prescott(2002)) explain these phenomena as a result of exogenous productivity shock. Caballero, Hoshi and Kashyap(2006) argue that these were the results of zombie lending that prevented the constructive destruction: since less productive firms were subsidised and kept in the market, total supply of goods and the labour demand of the industry would increase unnecessarily and lower the profit of healthy firms. Caballero et.al.(2006) made a linkage between zombie lending and low productivity in this way. Alternatively, Peek and Rosengren(2005) argue that banks tend to lend to affiliated borrowers and do not lend to the most appropriate borrowers. This is close to the argument of the crowding-out effect: healthy firms cannot obtain finance because zombie firms obtain their funds instead. In this model, instead, zombie lending raises land prices higher than a fair value that is consistent with their current productivity. It lowers healthy firms' investment in land, which is assumed to be an essential productive capital.

In order to feature these characteristics, we need to create a model with the following characteristics. First, we endogenise liquidation value. The bank makes zombies not because liquidation value is low, but because the bank needs to limit plunges in land prices to avoid
the bank closure required by the BIS accord ${ }^{9}$. Since this intervention or closure is assumed to impose huge utility loss on bankers, they try to raise land prices as much as possible, by stopping the liquidation of default firms' collateral and squeezing land supply ${ }^{10}$.

Second, we do not assume that bad firms can be profitable even ex-post. Once firms default, they produce nothing until the game ends. In other words, we study the phenomenon that bad firms keep obtaining finance even when their going concern value is lower than their liquidation value. As Hart(2000) mentions, such kinds of situation have rarely been discussed in the previous studies, compared with the papers studying how to prevent inefficient fire-selling of debts by creditors, in the context of the Chapter 11 or debt equity swaps. this chapter studies its opposite: how to liquidate bad firms.

### 5.3.2 Credit Cycles and Microeconomics

We will endogenise the liquidation value using the ideas of Kiyotaki and Moore(1997)'s Credit Cycles (KM hereafter). KM's seminal paper studies an amplification mechanism of economic growth through land price in a macroeconomic framework (they show the procyclicality of land price, at the same time). Their macroeconomic model, however, is sometimes too complicated to deal with some microeconomic issues.

Krishnamurthy(2003) solves this complexity problem by re-modelling KM's model into a simple microeconomic model. He simplifies KM into a three-period model with only one competitive market. The banks, deep pockets, lend to firms if they are collateralised with land

[^39](human inalienability as in Hart and Moore(1994) is assumed). The firms purchase land, borrowing money from the bank by pledging the purchased land as collateral , and purchase land further (so it is a leveraged loan). Then, the firms produce output goods using their land as a productive capital. One of the critical differences of Krishnamurthy(2003)'s model from KM is that the economy ends at the third period. The economy vanishes at this period and land becomes worthless because there is no production opportunity. It means that no loan contract is made in the second period, because the banks know that collateral becomes worthless. Thus, the leveraged loan occurs only in the first period, and thus Krishnamurthy(2003)'s amplification effect is limited compared with KM. The other critical difference from KM's model is that only the land market is opened. This simplification allows us to consider some microeconomic issues, such as the introduction of insurance contract into the credit cycles model. Krishnamurthy (2003) shows that the amplification mechanism of KM vanishes if insurance is available; but if the banks, who act as insurance providers at the same time, are also collateral-constrained, then the amplification mechanism is revived.

Gai, Kondor and Vause(2006) modify this Krishnamurthy(2003)'s model further. Krishnamurthy(2003) assumes that the firms are homogeneous, but Gai, Kondor and Vause(2006) assume there are two different firms with negatively correlated productivity shocks. This negative correlation of productivity shock allows them mutual insurance, without using the banks as insurance providers. With this modification, they show the amplification mechanism without assuming collateral-constrained banks: since the insurance contracts here are made among collateral-constrained firms, they do not have to assume an additional constraint to the banks. In addition, they introduce exogenous "liquidation constraint" as in Kiyotaki and Moore(2001) ${ }^{11}$ or Tomura(2006), which gives an upper bound to the fraction of the future value of land holding that a firm can credibly commit to a bank when the firm borrows from the bank. They study how this liquidation constraint affects the firms' equilibrium land holding or land price volatility.

The model discussed in the next chapter is the other variation of Krishnamurthy(2003). Since the purpose of the model is to describe zombie lending, or the soft budget constraint, in-

[^40]surance contracts are omitted and the amplification mechanism as in Kiyotaki and Moore(1997) is not focused on. There are two types of firms (borrowers), good firms and bad firms. Bad firms fail their production and go into default. This assumption is similar to Gai, Kondor and Vause(2006), but nobody knows their own types (the banks cannot observe this either) in this model. At the end of the first period, bad firms go bankrupt and their collateral (land) is seized by the banks. This gives additional risk exposure of the banks against land price. If the equilibrium land price falls due to a negative productivity shock in the second period, the banks' profit would be impaired and their own capital would decline. Here we assume the balance-sheet constraint, which restricts the banks' amount of its own capital (it is similar to the Basel Accord). If a bank's own capital falls below the threshold, the bank is closed and the managers of the bank would suffer from huge utility loss. The other critical difference from the models above is that we do not assume a competitive market. The bank is monopolistic or oligopolistic: this assumption allows the bank to manipulate the equilibrium price. If the bank does not liquidate seized land collateralised by bad firms and hides from the market, aggregate land supply is squeezed. This would raise the equilibrium land price and would improve the bank's own capital, and the bank may be able to pass the balance-sheet constraint. Yet this behaviour of the bank makes zombie firms, because bad firms are not liquidated and their land is left unused in the second period. Lorenzoni(2008) also uses Krishnamurthy(2003)'s framework to discuss inefficient credit boom and following firesale of assets.

Miller and Stiglitz(2008) extend KM in a different way. They show that an asset price bubble and its burst triggers the fire sale of collateral asset (land). Since the fire sale may lead the economy to a complete collapse, they argue the necessity of public capital injection or similar schemes.

In short, the critical differences from the two previous studies are the introduction of the soft budget constraint and monopolistic (or oligopolistic) pricing. Compared with Gai, Kondor and Vause(2006), who limit their attention to the firms'(borrowers') behaviour (in their model, banks virtually do nothing), this model focuses on the behaviour of banks and ignores the firms' behaviours instead. We will look at the details of the model in the next chapter. The policy implications of the model will be discussed in Chapter 7.

## Chapter 6

## Soft Budget Constraint in a Credit Cycle

### 6.1 Introduction

In this chapter, I will create a simple microeconomic model to discuss the zombie lending problem and the behaviours of macroeconomics parameters such as land price or TFP. The basic story is as follows. There are many firms purchasing land for their production, although they do not have sufficient funds to purchase land. They can borrow the funds from a bank by pledging their land as collateral. The firms borrow funds, purchase land, and then pledge the land to borrow more: i.e. the firms are allowed to leverage their debts.

In the next period, some of the firms go bankrupt - this is an unanticipated shock for the bank. The bank can liquidate the default firms and sell their collateralised land in the land market. But this is not always the optimal choice for the bank. The bank is assumed to be a monopolistic deep pocket, but its own capital is restricted by a public mentor: if the bank suffers a huge loss and its own capital declines lower than a predetermined threshold, then the bank is forced to close. If the bank's own capital is close to its threshold, liquidating the default firms is not always a good idea: the liquidation forces the bank to write off its loan, and there is no guarantee that the value of collateralised land is sufficiently high to cover the loss. If land
price in this period is too low, or, if the liquidation invites the plunge of the land market, then the bank find that the loss of liquidating insolvent firms is too costly (because it may trigger the closure of the bank) and it can be optimal for the bank not to liquidate the insolent firms.

If the bank finds the liquidation suboptimal, the bank can leave the default firm untouched. The bank lends additional money to the default firms for their repayments (this is equivalent to indulgence or reschedule of debts). The default firms are assumed to be non-productive and cannot produce anything even after indulgence, so the land held by the firms are left unused. By this "zombie lending", the bank can hide its borrowers' bankruptcy, and can squeeze the land supply to raise its equilibrium price. The zombie lending allows the bank not to write off default firms, and to avoid the plunge of land market: the bank can thus raise its own capital for the time being.

This zombie lending is costly for the bank. Even if the bank reschedules the default firms' debts, the default firms' performance is less likely to improve. Furthermore, since some land, the production capital of the economy, was left unused, the average productivity of the economy decreases after zombie lending. Accordingly, future land price is less likely to increase even if the bank postpone to sell the collateral land. In this chapter, the cost of zombie lending is drastically simplified: the economy closes at the end of the third period and the land price drops to zero regardless of the bank's choice (i.e., we assumes the largest possible cost of zombie lending). In addition, the default firms' performance does not improve in any case. With these two assumptions, the bank's incentive of zombie lending is minimised: the model shows that the bank prefers zombie lending even in this cost structure (the bank prefers zombie lending just to survive another period). These assumptions are made to simplify the story, and to focus on other interesting issues: when and how zombie lending occurs, how it affects macro economy, and what kind of public policy works for this sort of situations.

Although the soft budget constraint (SBC), describing why chronic loss makers can obtain finance, is a useful framework for studying this problem, SBC has an inconvenient assumption for the study: the chronic loss makers' liquidation value is lower than their going-concern value and these values are exogenously fixed. The zombie lending occurred in Japan was, as seen in the previous chapter, leaving insolvent borrowers unliquidated; i.e. liquidation value is higher
than going-concern value. Furthermore, to study the anomalies observed in Japan after the burst of the bubble, we need to endogenise the liquidation value of the borrowers (land price) and the aggregate productivity of the economy in this chapter to study the anomalies. This requires us to modify the SBC model, to consider not only the microeconomic problem in finance, but also the macroeconomic issues like the aggregate productivity of an economy.

In order to discuss both microeconomic (zombie lending) and macroeconomic (land price and aggregate productivity) issues, we use a new approach to macroeconomics that studies macroeconomic issues using microeconomic frameworks. There are several forerunners in this area. Suarez and Sussman(1997) make an overlapping generation model of optimal debt contracts and show an endogenous business-cycle-like reversion, with an amplification effect of credit imperfection. Their "cycle" is still simple and artificial, compared with related studies in macroeconomics, e.g. Carlstrom and Fuerst(1997), but we can obtain clear implications instead. Krishnamurthy(2003) makes a simplified microeconomic model of Kiyotaki and Moore's (1997) credit cycle, in an economy with only three periods. He succeeds in showing how insurance contracts work in the Credit Cycle model analytically. Tirole(2006) adopts the same approach as Suarez and Sussman(1997). He makes a dynamic model with the overlapping generation of Holmström and Tirole(1997), and shows the existence of stationary equilibrium.

The model shown in this chapter is based on Krishnamurthy(2003)'s model. He assumes an economy with three periods and two categories of players, banks and firms. Both players are a continuum of unit measure. The firms are entrepreneurs, and borrowers. They have better investment opportunities than banks, but they may not have sufficient amount of funds to start their projects. The banks are deep pocket, instead, but their profitability of investment is lower than the firms' profitability. The banks would do better, therefore, to lend their funds to the firms and enjoy the return. Yet human inalienability, as in Hart and Moore(1994), is assumed and all loans must be collateralised. The firms can leverage their downpayment by purchasing collateralisable land, which is productive capital at the same time. After production occurs, the debt is repaid and the next period starts. This is the same as Kiyotaki and Moore(1997)'s mechanism, and their amplification mechanism can be found in this Krishnamurthy(2003) model as well.

To simplify the analysis, only the land market is opened in each period in Krishnamurthy(2003)'s model; labour or any other physical capital is not required in the model (each firm's production technology is necessary and a manager of each firm is 'inalienable'). Furthermore, the economy ends at the third period: all players will die at the end of the period, thus land price must be zero in this period as well. There are no loan contracts from the second to the third period, because land price in the third period is zero, thus land is not collateralisable. Since the amplification through collateralised land only occurs once in the first period, Krishnamurthy(2003)'s amplification effect is much smaller than the original credit cycles model by Kiyotaki and Moore(1997). There are no loan contracts from the second to the third period, because land price in the third period must be zero and land is not collateralisable. With these simplifications, Krishnamurthy(2003) can study how insurance contracts works in the credit cycles model analytically. Gai, Kondor and Vause(2006) modify this model and discuss the relationship between procyclicality and insurance in a different way. Banks give insurance contracts in Krishnamurthy(2003), but Gai et.al.(2006) consider mutual insurance between two different borrowers.

These features of Krishnamurthy(2003)'s model are convenient for studying SBC, because both Dewatripont and Maskin(1995) and Krishnamurthy(2003) are three periods models. In the model discussed in this chapter, Krishnamurthy(2003)'s model is merged with Dewatripont and Maskin(1995) or Berglof and Roland(1997)'s SBC model, to study the zombie lending problem. The bank lends to borrowers in the first period and then determines whether the bank "helps" default borrowers or not in the second period. These choices are made in Krishnamurthy(2003)'s credit-cycle-like framework.

The model shown here has several significant differences from Krishnamurthy(2003) or Gai, Kondor and Vause(2006)'s model, in order to discuss the zombie lending problem. First, Krishnamurthy(2003)'s model is competitive equilibrium, but the model in this chapter is monopolistic (the competitive equilibria are also studied for comparison). This alteration enables the bank to decide whether it will liquidate its borrowers or not. (If a borrower has many lenders, e.g. the loan is securitised, the bank cannot determine zombie lending by itself.) The monopoly also enables the bank to squeeze the land supply. If the bank is relatively small
and its liquidation does not invite the collapse of the land market, zombie lending cannot be optimal in any case. In other words, monopoly or oligopoly in both loan contract and land market is essential in zombie lending. We will come back to this in Section 4.

Second, this model does not have insurance contracts, which are focused on in Krishnamurthy(2003) and Gai, Kondor and Vause(2006), because here we limit our attention to the zombie lending problem.

Third, this model has two different shocks, macroeconomic productivity shock and idiosyncratic default shock. The firms are ex-ante identical, but a fraction of the firms go bankrupt (this is unanticipated shock) to consider the zombie lending problem.

At the same time, it is also different from the SBC as in Dewatripont and Maskin(1995). In their model, or in Berglof and Roland(1997), liquidation value of the bad firms is exogenously given; but land price, the liquidation value of bad firms in this model, is endogenously determined. In addition, the SBC models assume that even "chronic loss-makers" are profitable to some extent: in other words, additional lending to bad firms to keep them alive is assumed to be profitable. In the model discussed here, bad firms do not make any profit at all.

To be precise, land in this model is merely an production capital, so we can consider the "land" can include other collateralisable production capitals; Machinery, patents, and factory equipments. In the following argument, these production capitals are denoted as "land" for notational simplicity. But sometimes it is easier for readers to consider other capitals, especially when we think about the bank's monopoly in asset markets.

By solving the model, we will see that the zombie firm phenomenon occurs when the following conditions hold. First, the bank's own capital is not too high and not too low. If the bank has sufficient own capital, the bank can afford liquidating bad borrowers. If the own capital is too low, the bank is forced to close even after zombie lending - the bank has no incentive of zombie lending in this case as well. Second, if the bank lends more to the firms, zombie lending is likely to occur. If the bank lends more, it will seise more collateral after the firms' bankruptcy, and is exposed to the larger risk of land price. The larger exposure to land price, and that the large amount of collateralised land allows the bank to squeeze the land market
further. Third, if larger fraction of the firms go bankrupt, zombie lending is likely to occur as well, due to the same reason of the overlending. These show a good fit to what happened in Japan.

If these conditions are satisfied, the zombie lending will cause several outcomes. The decline in land price becomes moderate because zombie lending raises land price by squeezing land supply. The number of bankruptcy becomes smaller than its real figure: the bank gives additional funding for borrowers' repayment so these firms will not be counted as insolvent. Aggregate productivity decreases, because some production capital is left unused.

This gives a different explanation for the Japanese "lost decade": the period of economic stagnation after the bubble in the early 1990s. In our model, the stagnation of the productivity occurred due to the banks' postponement of collateral-liquidation (or, postponement of the bank crisis). The banks, with their own capital lower ${ }^{1}$, tried to postpone bank closure by making zombies. This had supported the land price, but lowered the productivity of the Japanese economy, until the bank crisis in 1997-98 ${ }^{2}$. After the crisis, banks were nationalised and they could liquidate zombies without anxiety of closure. This improved the productivity, and lowered land price further. In a sense, this is a model of "delayed" credit cycles.
the chapter continues as follows. We will define a three-periods model of the reduced-form credit cycles, mainly following Krishnamurthy(2003) in the next section. Its equilibria and characteristics are discussed in Section 3. Section 4 studies equilibria when there are many banks and the land market becomes competitive. Section 5 concludes. Interpretations and possible solutions to the zombie lending problem are discussed in the next chapter.

[^41]
### 6.2 Setup of the Game

### 6.2.1 Stage Game and General Setup

This economy has three periods, $t=0,1,2$. The economy has continuum firms with measure one who have profitable production opportunities. The economy has their counterparts, $M$ identical banks. Since we do not consider their depositors throughout the argument, they can be named as creditors or investors. In the following argument we assume that $M=1$ for simplicity; we will consider the case of many banks later. The infinite firms (borrowers) are not players of the game. It is a game of the banker, and firms are just devices of the game. The firms mechanically try to maximise their short-term profits, but there is no strategic interaction between the bank and the firms, for the sake of simplicity.

The firms have profitable investment opportunities but no input to start a project. The bank has productive capital, land, but it has less profitable projects only. It is also assumed that the bank is deep pocket, but due to the human inalienability à la Hart and Moore(1994), there is an upper bound for the loan (the bank cannot lend more than the value of the land the firms have).

The objective of the bank is to maximise the following linear utility function:

$$
U=E\left[c_{0}^{b}+c_{1}^{b}+c_{2}^{b}\right]
$$

where superscript $f$ denotes firms, $b$ denotes the bank. Since the discount factor is assumed to be one and the bank has a profitable investment project and a profitable lending opportunity (as we will assume below, the bank anticipates no bankruptcy of borrowers; thus, lending is always profit making opportunity for the bank ex-ante), the bank tries to put their wealth into their projects as much as possible. Therefore, we can assume that they will maximise their next period's wealth in each period without loss of generality. The firms are assumed to be myopic devices in the game. They try to maximise their current production based on their productivity, own wealth and land price offered by the monopolist bank (if the market is competitive, offered by a central auctioneer).

The firms' identical and strictly concave production function at date $t$ is $f\left(z_{t}, k_{t}^{f}\right)$, where $z_{t}$ is stochastic productivity (macro shock) and $k_{t}^{f}$ is firms' land holding. For simplicity, we assume $^{3} f\left(z_{t}, k_{t}^{f}\right)=k_{t}^{f}\left(z_{t}-k_{t}^{f}\right)$. The bank's production function is $f\left(A, k_{t}^{b}\right)=k_{t}^{b}\left(A-k_{t}^{b}\right)$, where $k_{t}^{b}$ is the bank's land holding and $E\left[z_{t}\right] \geq A$, which ensures that the firms are more profitable. $K$ is constant land supply and $K=k_{0}^{f}+k_{0}^{b}$. A is constant, and $A \geq 2 K$ is assumed to ensure monotonically increasing production function globally. We further assume that the output good is not storable.

The firms purchase land with their wealth $w_{0}^{f}$ and bring it to the bank, and borrow money. So, they can leverage their wealth (but they have to pay its interest beforehand as a downpayment, thus $w_{0}^{f}$ provides an upper bound on their loan). These production functions with the collateral constraint specify the demand functions of land, and the marginal revenue of the monopolist bank, which possesses all land at the beginning of the game. Then the production is carried out and $w_{t+1}^{f}$ is updated.

We will find subgame perfect equilibria on the extensive game defined below. We assume firms' behaviour is given, and omit their strategies from the equilibrium strategy set. Thus, equilibrium land price is determined in a monopolistic market. The land market opens twice in the period o and period 1 .

### 6.2.2 How the game proceeds

## Firms at date 0

At the beginning of date 0 , firms' productivity $z_{0}$ and their initial identical wealth $w_{0}^{f}$ is given exogenously. Each firm chooses $k_{0}^{f}$, which maximises his next day wealth $w_{1}^{f}$. The choice of $k_{0}^{f}$ is restricted by the following two constraints. The first one is budget constraint:

[^42]\[

$$
\begin{equation*}
q_{0} k_{0}^{f} \leq D+w_{0}^{f} \tag{6.1}
\end{equation*}
$$

\]

The firms cannot purchase $k_{0}^{f}$ more than their borrowing plus their own wealth. And the borrowing $D$ is restricted by the collateral constraint:

$$
\begin{equation*}
(1+r) D \leq q_{0} k_{0}^{f} \tag{6.2}
\end{equation*}
$$

$r$ is the interest rate from date 0 to date 1 , and they have to repay all the amount in the next period, $R=(1+r) D$ if they can survive until the next period. To be precise, this assumption is not perfectly coherent: we should define $(1+r) D \leq E_{0}\left[q_{1}\right] k_{0}^{f}$ or they cannot cover their risk. Here we assume this just for simplicity.

The expected value of firms' aggregate wealth, $w_{1}^{f}$, is defined as follows:

$$
\begin{align*}
E\left[w_{1}^{f}\right] & =w_{0}^{f}+D-q_{0} k_{0}^{f}+f\left(z_{0}, k_{0}^{f}\right)+E\left[q_{1}\right] k_{0}^{f}-R  \tag{6.3}\\
w_{1}^{f} & =(1-\gamma)\left\{w_{0}^{f}+D-q_{0} k_{0}^{f}+f\left(z_{0}, k_{0}^{f}\right)+E\left[q_{1}\right] k_{0}^{f}-R\right\}
\end{align*}
$$

where $\gamma$ is default ratio, which is unanticipated by the firms and the bank and is stochastically independent from any other factors. This is the second shock to the economy, with the macro shock $z_{0,1}$. The firms are ex ante identical, but different ex post; fraction $\gamma$ of the firms go bankrupt, and the rest survive. Default firms are assumed to lose everything $\left(w_{1}^{f}=0\right)^{4}$, so we just need to multiply $1-\gamma$ (the default firms' collateral is seized by the bank). The firms maximise their $w_{1}^{f}$ by setting $k_{0}^{f}$ appropriately, with the budget constraint (6.1). If the two constraints are not binding, we have their inverted demand function by maximizing (6.3) with respect to $k_{0}^{f}$ :

$$
q_{0}=E\left[q_{1}\right]+f^{\prime}\left(z_{0}, k_{0}^{f}\right)
$$

[^43]$$
=E\left[q_{1}\right]+z_{0}-2 k_{0}^{f}
$$

The marginal revenue of the bank selling $k_{0}^{f}$ to the firms is:

$$
M R=\frac{\partial q_{0}\left(k_{0}^{f}\right) \cdot k_{0}^{f}}{\partial k_{0}^{f}}=E\left[q_{1}\right]+z_{0}-4 k_{0}^{f}
$$

If the budget and collateral constraints ((6.1) and (6.2)) are binding, we have:

$$
\begin{equation*}
k_{0}^{f}=\frac{w_{0}^{f}}{\frac{r}{1+r} q_{0}} \tag{6.4}
\end{equation*}
$$

This budget constraint is strictly downward sloping. The revenue function with the constraint is:

$$
q_{0} k_{0}^{f}=w_{0}^{f} \frac{1+r}{r}
$$

Therefore, marginal revenue with respect to $k_{0}^{f}$ is equal to zero, if the constraints are binding. That is to say, the marginal revenue function is bent at the crossing point of the inverted demand function and the budget constraint, as Figure 6-1 shows. The marginal revenue line bends down and is flat at zero as long as the budget constraint is binding, and again bends upward to go back to the unconstrained marginal revenue function at the second crossing point of the demand function and the budget constraint.

## Bank at date 0

There exists one monopolistic bank here, which is a deep pocket and has less-profitable concave production opportunity. The bank's production function is $f\left(A, k_{0}^{b}\right)$, where $f^{\prime}(\cdot)>0$ and $f^{\prime \prime}(\cdot)<0$. If it does not lend to the firms, it tries to maximise its profit, which consists of her own investment profit $f\left(A, k_{0}^{b}\right)$ and revenue from selling land to the firms $q_{0} k_{0}^{f}=w_{0}^{f}$ :

$$
\begin{equation*}
q_{0} k_{0}^{f}+f\left(A, K-k_{0}^{f}\right)+q_{0}\left(K-k_{0}^{f}\right)+\left(E\left[q_{1}\right]-q_{0}\right)\left(K-k_{0}^{f}\right) \tag{6.5}
\end{equation*}
$$

The first term is the bank's revenue of selling its land to the firms. The second one is its own production. The third one is a part of its endowment holding on its own, and the fourth one is the capital gain by holding $k_{0}^{b}=K-k_{0}^{f}$. and the latter three terms are her cost function (derivatives of the latter three with respect to $k_{0}^{f}$ are negative. The bank's opportunity cost increases as $k_{0}^{f}$ increases). Differentiate the latter three with respect to $k_{0}^{f}$, and we have:

$$
\begin{equation*}
M C=2 k_{0}^{f}-(2 K-A)+E\left[q_{1}\right] \tag{6.6}
\end{equation*}
$$

If the bank makes collateralised loan to the firms, the bank's expected profit is defined as:

$$
q_{0} k_{0}^{f}+f\left(A, K-k_{0}^{f}\right)+q_{0}\left(K-k_{0}^{f}\right)+\left(E\left[q_{1}\right]-q_{0}\right)\left(K-k_{0}^{f}\right)-D+(1-\gamma)(1+r) D+\gamma E\left[q_{1}\right] k_{0}^{f}
$$

The first four terms are the same as above. The fourth term $D$ is loan amount, and the fifth is successful repayment amount. The last term is collateral value seized from default firms. By assuming that the firms are budget-constrained for simplicity, we have $D=\frac{q_{0}}{1+r} k_{0}^{f}$. Substitute this into the above and differentiate with respect to $k_{0}^{f}$, we have:

$$
\begin{equation*}
M C^{\prime}=\frac{2}{1-\frac{\gamma}{1+r}} k_{0}^{f}-\frac{1}{1-\frac{\gamma}{1+r}}(2 K-A)+\frac{1-\gamma}{1-\frac{\gamma}{1+r}} E\left[q_{1}\right] \tag{6.7}
\end{equation*}
$$

Since $0<\frac{\gamma}{1+r}<1$, the sign of each coefficient is the same as the no-loan case (6.6), but the slope is steeper.

## Market Clearing at date 0

Date 0 equilibrium $k_{0}^{f}$ makes $M R$ and $M C$ equal. If the budget constraint is not binding, obtaining equilibrium price $q_{0}$ is straightforward, as Figure 6-1 shows. If the constraint is binding, equilibrium $k_{0}^{f}$ is determined at $\hat{k}_{0}^{f}$. Since the MR of any $k_{0}^{f}>\hat{k}_{0}^{f}$ is zero but the MC is non-zero, the bank has no incentive to sell her land any more. This means that $\hat{k}_{0}^{f}$ is an upper-bound of $k_{0}^{f}$.

To be precise, we need the following lemma to asset that $\hat{k}_{0}^{f}$ is an upper-bound of $k_{0}^{f}$. Since the budget constraint is convex, the constraint becomes unbinding again if $k_{0}^{f}$ is sufficiently


Figure 6-1: Land Market Equilibrium (macro schock is given)


Figure 6-2: Possible unconstrained MR
large; if MR and MC intersect at such a large $k_{0}^{f}$, $\hat{k}_{0}^{f}$ cannot be the upper bound. Figure $6-2$ shows that there are two discontinued unconstrained marginal revenues. Yet the following lemma excludes this possibility.

Lemma 13 There exists an upper bound of $k_{0}^{f}$ and $k_{1}^{f}, \hat{k}_{0}^{f}$ and $\hat{k}_{1}^{f}$ respectively, which is the intersection of the demand function and the budget constraint. If the budget constraint is binding, $k_{i}^{f}=\hat{k}_{i}^{f}$ for $i=\{0,1\}$.

See Appendix for the proof.

We can obtain equilibrium $q_{0}$ from the intersection of MR and MC, and the inverted demand function.

## Firms at date 1

At the end of the date 0 , some of the firms go bankrupt and cannot extract any profit from the investment. The fraction of the default firms is $\gamma$ : this is an unanticipated shock (e.g. negative productivity shock), so the expected $\gamma$ is zero although its actual value is strictly positive. The surviving firms obtain the return, $f\left(z_{0}, k_{0}^{f}\right)$.

The surviving firms (fraction $1-\gamma$ ) sell all their land to the bank to repay the loan. The default firms' wealth becomes zero, and their land is seised by the bank. The default firms' productivity at date 1 is fixed at zero.

Now the surviving firms' objective is to maximize $w_{2}^{f}$, given $w_{1}^{f}$. Thus a similar procedure is repeated at date 1 , but the critical difference is now they cannot borrow money from the bank. Since the economy finishes at date 2 , everything becomes worthless by then; accordingly, the value of land becomes zero at date $2\left(E\left[q_{2}\right]=0\right)$. This means that the bank cannot secure the loan by the collateral land. There is no other collateralisable asset, so no financial transaction would be carried out at date 1. Therefore, the firms' objective function becomes:

$$
w_{2}^{f}=w_{1}^{f}-q_{1} k_{1}^{f}+f\left(z_{1}, k_{1}^{f}\right)
$$

subject to:

$$
w_{1}^{f} \leq q_{1} k_{1}^{f}
$$

where $z_{1} \in\left\{z^{H}, z^{L}\right\}$ is the firms' updated productivity, chosen by nature. The bank's marginal revenue with non-binding budget constraint is:

$$
M R=z_{1}-4 k_{1}^{f}
$$

If binding, we have

$$
\begin{aligned}
k_{1}^{f} & =\frac{w_{1}^{f}}{q_{1}} \\
M R & =0
\end{aligned}
$$

## Bank at date 1

As assumed above, the bank is a deep pocket and there is no budget constraint, as KM assume. However, here we add a restriction to the bank, which resembles the BIS accord regulating banks' capital adequacy ratio. This is that if the bank makes a loss, which reduces its own capital, and if the capital falls below a predetermined threshold, then the bank is forced to be closed ${ }^{5}$. The bank closure incurs a huge utility loss to the bank's manager (infinite loss is assumed for simplicity). This balance sheet constraint ( $\mathrm{B} / \mathrm{S}$ constraint) is described as:

$$
\begin{equation*}
W_{1} \geq \bar{W} \tag{6.8}
\end{equation*}
$$

Denote $W_{1}$ as the bank's own capital at date 1 and $\bar{W}$ as a threshold that is required by an external auditor. If $W_{1}<\bar{W}$, the auditor forces the bank to close its business. $W_{0}$ is endowed ex ante, and $W_{1}$ is updated as follows:

$$
\begin{equation*}
W_{1}=W_{0}+f\left(A, k_{0}^{b}\right)+q_{0} k_{0}^{f}+q_{0} k_{0}^{b}+\left(E\left[q_{1}\right]-q_{0}\right) k_{0}^{b}-D+(1-\gamma)(1+r) D+\gamma(1-\theta) q_{1} k_{0}^{f}+\gamma \theta q_{1} k_{0}^{f} \tag{6.9}
\end{equation*}
$$

The second term of the RHS is the profit coming from the bank's own investment. The third to fifth terms are the profits by selling the bank's land to the firms, and by holding the land for its own. The sixth to ninth are profit coming from loans to the firms.

If the bank expects $W_{1}<\bar{W}$, after observing updated productivity $z_{1}$, it tries to squeeze land supply to raise equilibrium land price $q_{1}^{*}$ and then $W_{1}$ (higher land price is always beneficial

[^44]for the bank, because it can enjoy capital gain from her land: this is proved below). The bank can squeeze land supply by making zombies: i.e., by giving up selling failed firms' land and leaving it on the failed firms' hands, whose productivity is fixed at zero. Since the land held by zombie firms is excluded from the market, aggregate land supply declines by $\gamma \theta k_{0}^{f}$, where $\theta \in[0,1]$ is a fraction of zombies created out of default firms' land. Since the demand function of the firms does not change, as we will see, this squeeze raises $q_{1}^{*}$ under certain conditions. The bank has the incentive to do so anytime when she can raise $W_{1}$, such that $W_{1} \geq \bar{W}$ by making zombies, because it is assumed that the bank manager has to bear infinite utility loss.

After deciding $\theta$, the bank enters the land market at date 1. The bank's concave production function is $f\left(A, k_{1}^{b}\right)$ and FOC is

$$
M C=2 k_{1}^{f}-\left\{2\left(K-\gamma \theta k_{0}^{f}\right)-A\right\}
$$

Since land price will be zero in the next period, no lending occurs and $E\left[q_{2}\right]=0$; this simplifies MC in date 1.

Equilibrium land price $q_{1}^{*}$ is calculated with the MR and MC . Since sufficiently low equilibrium $q_{1}^{*}$ causes $W_{1}<\bar{W}$, (i.e. violates the $\mathrm{B} / \mathrm{S}$ constraint (6.8)) and the bank has to close the business, it needs to infer $q_{1}^{*}$ ex ante and manipulate $\theta$ before the market is cleared, in order to ensure $W_{1} \geq \bar{W}$.

### 6.3 Equilibria (monopolistic bank)

Subgame Perfect Equilibrium strategy defined in the game is a configuration of the following actions of the bank: $k_{0}^{f} \in[0, K], \theta \in[0,1]$ and $k_{1}^{f} \in\left[0, K-\theta \gamma k_{0}^{f}\right]$. The bank may choose $k_{0,1}^{b}$ rather than $k_{0,1}^{f}$, but we use this for notational simplicity (without any loss of generality). As explained above, the bank is assumed to be only able to choose lending or not lending: once the bank decides to start loan business, it has to accept all borrowers with sufficient collateral.


Figure 6-3: Normal Equilibrium and Zombie Equilibrium ( $\hat{q}$ is the land price that achieves $W_{1}=\bar{W}$. If land price is lower than the threshold, the bank is forced to close. See appendix for the definision.)

In the following argument, we will assume that the firms' budget constraint at date 0 is always binding, for computational simplicity. Thus loan amount $D$ becomes endogenous and is omitted from the strategy set. We just assume that $r \in(0, \infty)$ is exogenously given. This assumption is to simplify the equilibrium strategies. All the other parameters are exogenously given.

### 6.3.1 Time Line

The game consists of the following stages:
date 0 The borrowers' identical wealth $w_{0}^{f}$, the bank's initial own capital $W_{0}$, and macro shock $z_{0}$ are given (before the game starts).
date 0 Based on the demand functions of the firms, the bank chooses $k_{0}^{b}$, and $q_{0}$ is determined (bank loan $D$ is automatically determined).
date 0 The outputs $f\left(z_{0}, k_{0}^{f}\right)$ and $f\left(A, k_{0}^{b}\right)$ are realised. The fraction $\gamma \leq 1$ of the firms go bankrupt.
date 1 The successful firms sell their land to the bank and repay $(1+r) D$. The default firms give their property right of their land to the bank.
date 1 Nature chooses $z_{1} \in\left\{z_{1}^{L}, z_{1}^{H}\right\}^{6}$. Its distribution $\mu=\{\pi, 1-\pi\}$ is known ex ante.
date 1 The bank estimates the firms' demand functions based on $z_{1}$ and derives $q_{1}$ and $W_{1}$. If $W_{1}<\bar{W}$, she sets $\theta>0$ (making zombies).
date 1 Given $\theta$ and players' demand functions, $k_{1}^{b}$ and $q_{1}$ are determined.
date 1 The mediator checks $W_{1}$ and shuts the bank down if $W_{1}<\bar{W}$.
date 2 The output $f\left(z_{1}, k_{1}^{f}\right)$ and $f\left(A, k_{1}^{b}\right)$ are realised.

[^45]date 2 The firms and the bank consume everything obtained.
date 2 The economy is closed.

### 6.3.2 Date 1 Land Market

We will construct the equilibrium by backward induction. Since the economy closes at date 2 and no action occurs then, $k_{1}^{f}$ is the last action chosen by the bank, given $z_{1}, w_{1}^{f}$ and $\theta$ : the equilibrium $k_{1}^{b, f}$ and corresponding $q_{1}$ is the first action we need to consider. The budget constraint $\frac{w_{1}^{f}}{q_{1}}$ may or may not be binding. Actually, their actions are trivial at this moment. $q_{1}$ is determined subject to the following conditions:

$$
\begin{aligned}
M R & =z_{1}-4 k_{1}^{f} \text { if the budget constraint is not binding. } \\
M R & =0 \text { if the budget constraint is binding. } \\
M C & =2 k_{1}^{f}-\left\{2\left(K-\gamma \theta k_{0}^{f}\right)-A\right\} \\
k_{1}^{f}+k_{1}^{b} & =K-\gamma \theta k_{0}^{f}
\end{aligned}
$$

Thus the equilibrium action $k_{1}^{b}$ and equilibrium price $q_{1}$ are determined if $\theta, z_{1}$ and $w_{1}^{f}$ are specified beforehand. The existence of a unique equilibrium is guaranteed by the lemma 16 and other discussions above.

### 6.3.3 Decision of Zombie Lending

Based on the estimation relating to the land market at date 1, which is shown above, the bank has to decide $\theta$. At this moment she has observed $z_{1}$ and $w_{1}^{f}$; thus she can calculate correctly whether her wealth $W_{1}$ will fall below $\bar{W}$ or not, from the equation (6.9). If $W_{1}<\bar{W}$, the bank need to raise $W_{1}$ up to $\bar{W}$ or the bank is forced to close. But measures to raise $W_{1}$ are limited. Most components of $W_{1}$ are fixed at date 0 , except for $q_{1}$ and $\theta$, and it is obvious from (6.9) that change in $\theta$ does not have any direct effect to $W_{1}$. Thus the path of the zombie lending
effect should be that higher $\theta$ raises $q_{1}$, and then higher $q_{1}$ raises $W_{1}{ }^{7}$. The latter one, $\frac{\partial W_{1}}{\partial q_{1}}>0$, is obvious from the equation (6.9), but the former one needs to be proved. And we have the following proposition:

Proposition 14 There exist $W_{0}, z_{1}^{L}$ and $r$ such that the bank finds the zombie lending optimal.

See appendix for the proof. From the proof, we have the following implications:

Remark 15 Zombie lending is prone to be chosen when r is small (bank loan was highly leveraged in the previous period, i.e. debt overhang), $\gamma$ is large (the bankruptcy of the firms in a large scale) and $W_{0}$ is small. But if $r$ and $W_{0}$ are too small, then zombie lending cannot improve $W_{1}$ enough, and the bank is forced to close.

Low $r$ allows the firms to borrow more (i.e. high leverage), and their bankruptcy exposes the bank to the larger risk of land price because the bank seises collateralised land from default firms. If $\gamma$ is large, the bank has a larger room to make zombie firms. Low $W_{0}$ is obvious: If the bank does not have sufficient cash buffer, they need to make zombie firms to make up their loss. But if $W_{0}$ or $r$ is too small, the bank gives up to hide the loss and closes the business.

The following lemma assures the uniqueness of optimal $\theta$ :

Lemma 16 If there exists $\theta \leq 1$ such that the bank can raise its own capital $W_{1} \geq \bar{W}$, the bank chooses minimum $\theta$ that ensures $W_{1} \geq \bar{W}$. I.e. the bank's optimal $\theta$ is the one achieving $W_{1}=\bar{W}$.

[^46]See Appendix for the proof. Intuitively, the bank chooses the smallest $\theta$ because zombie lending is costly for the bank. The bank has to keep non-productive land $\left(\gamma \theta k_{0}^{f}\right)$, which becomes zero value for sure in the next period. To minimise the cost of zombie lending, the bank chooses minimum feasible $\theta$.

To be precise, if $W_{1}<\bar{W}$ even when $\theta=1$, any $\theta \in[0,1]$ can be an equilibrium action since the cost of the closure is negative infinite and it dominates all the other factors. In the following argument, I assume that equilibrium $\theta$ is equal to zero in this case to eliminate this non-interesting multiple equilibria.

### 6.3.4 Decision of Date-0 Investment $\left(k_{0}^{f}\right)$

We will proceed one step backward, action at date 0 . At date 0 , the bank, which does not anticipate any firm's bankruptcy $\left(E[\gamma]=0\right.$ ), chooses $k_{0}^{b}$ (or, equivalently, $k_{0}^{f}$ ) at the intersection of the firms' demand function and their budget constraint, where MR of the bank plunges to zero.
based on $r, w_{0}^{f}$ and $z_{0}$ and expectation of $z_{1}, \theta$ and $k_{1}^{f}$. Since we have assumed that the budget constraint for the firms are binding, the bank's optimal action is:

$$
k_{0}^{f}=\frac{\left(E\left[q_{1}\right]+z_{0}\right)+\sqrt{\left(E\left[q_{1}\right]+z_{0}\right)^{2}-8 \frac{1+r}{r} w_{0}^{f}}}{4}
$$

from the proof of Lemma 13. Since $\gamma$ is expected to be zero, $E\left[q_{1}\right]$ is simply defined as:

$$
E\left[q_{1}\right]=\pi \cdot q_{1}\left(z_{1}^{H}, k_{1}^{b, H}\right)+(1-\pi) \cdot q_{1}\left(z_{1}^{L}, k_{1}^{b, L}\right)
$$

where $q_{1}(\cdot)$ are the equilibrium land prices at date 1 without zombie lending (any zombie lending is not anticipated). Therefore, $E\left[q_{1}\right]$ is determined independently from the actions chosen.

### 6.3.5 Equilibrium Strategies

The arguments above proves the existence of the following equilibrium strategies. The strategy set may be a subset of the whole equilibrium set of the game. The aim of the chapter is showing the existence of the zombie lending equilibrium and finding the conditions for the equilibrium; not finding the whole set of equilibria.

The equilibrium strategy of the game, $\left(k_{0}^{f}, \theta, k_{1}^{f}\right)$, is defined as follows:

$$
k_{0}^{f}=\frac{\left(E\left[q_{1}\right]+z_{0}\right)+\sqrt{\left(E\left[q_{1}\right]+z_{0}\right)^{2}-8 \frac{1+r}{r} w_{0}^{f}}}{4}
$$

$$
\begin{aligned}
{ }^{\exists 1} \theta & \in[0,1] \text { s.t. } W_{1}=\bar{W} \text { if } W_{1}(\theta=1) \geq W \text { and } W_{1}(\theta=0)<W \\
\theta & =0 \text { otherwise }
\end{aligned}
$$

$$
{ }^{\exists 1} k_{1}^{f} \text { with given } \gamma, \theta, w_{1}^{f} \text { and } z_{1}
$$

### 6.4 Equilibria (Many Banks)

If there are many banks, we have competitive equilibria with some different features from the monopolistic banking we saw above. The timeline of the game is the same as the one of the monopolistic economy. The date 0 equilibrium, however, is determined at the intersection of the firms' demand function and the banks' MC, because now the banks are also price taker and cannot sell their land at the point where $\mathrm{MR}=\mathrm{MC}$.

The banks' supply function is firstly obtained. The banks' profit function at date 0 is ( $\gamma$ is expected to be zero):

$$
q_{0} k_{0}^{f}+f\left(A, K-k_{0}^{f}\right)+q_{0}\left(K-k_{0}^{f}\right)+\left(E\left[q_{1}\right]-q_{0}\right)\left(K-k_{0}^{f}\right)-D+(1+r) D
$$



Figure 6-4: Zombie lending in competitive banking (infeasible case)

Differentiate this with respect to $k_{0}^{f}$, we have the supply function:

$$
\begin{aligned}
& q_{0}-(A-2 K)-2 k_{0}^{f}-E\left[q_{1}\right]+r \cdot q_{0} \\
q_{0}= & \frac{1}{1+r}\left\{2 k_{0}^{f}+E\left[q_{1}\right]+(A-2 K)\right\}
\end{aligned}
$$

Since it is assumed that the firms are bound by the budget constraint, the date 0 demand function is defined as follows:

$$
q_{0}=\frac{w_{0}^{f}}{\frac{r}{1+r} k_{0}^{f}}
$$

The equilibrium price $q_{0}^{*}$ and the allocation $k_{0}^{f *}$ are obtained at the intersection of the demand and supply functions. It is obvious that the demand and supply functions have unique intersection.

The choice of $\theta$, the fraction of zombie lending out of unanticipated bankruptcy of the firms, is remarkably different in the economy with many banks. The monopolist bank can hide some land $\gamma \theta k_{0}^{f}$ from the market to push up the land price $q_{1}$. But if there are many banks and the size of each bank is infinitesimally small, single bank's zombie lending cannot raise $q_{1}$. Equivalently, when all the other banks choose zombie lending, it is optimal for each single bank to liquidate all insolvent borrowers (deviation from the zombie lending strategy) because each one of the banks have an opportunity of free lunch. The banks' optimal choice is, therefore, $\theta=0$ for any parameters, even though a bank (or banks) is under-capitalised ( $W_{1}<\bar{W}$ ).

The choice of date 1 land allocation $k_{1}^{f}$ is determined in a similar way to the date 0 land market. Since $E\left[q_{2}\right]=0$ and $D=0$, the demand and supply function becomes as follows:

$$
\begin{aligned}
& q_{1}=z_{1}-2 k_{1}^{f} \text { if the budget constraint is not binding } \\
& q_{1}=\frac{w_{1}^{f}}{k_{1}^{f}} \text { if the budget constraint is binding }
\end{aligned}
$$

$$
q_{1}=2 k_{1}^{f}+(A-2 K)
$$

Since the demand function and the budget constraint are strictly decreasing and the supply function is strictly increasing, the uniqueness of the equilibrium land allocation at date 1 is guaranteed.

### 6.5 Conclusion: Zombie Lending and its Rationale

The model explored here is designed to answer the three questions raised in the previous chapter: The mechanism of zombie lending, the reason why Japanese economy continued to be stagnant for more than a decade, and the reason why the land price did not positively correlated with economic growth after the bubble burst. The model provides answers to the three.

First, a bank has an incentive to make zombie firms to make up their flawed own capital. When the bank finds that some of its borrowers go bankrupt, the bank has two choices. One is seising the collateralised land and liquidating it (the loan itself will be written off). The other is behaving as if the insolvent borrowers are solvent: In practice, the bank lends some money to its insolvent borrowers to enable them to repay the interest of their debts (to the bank). The land held by the insolvent borrowers (zombies) are left unused, and then the bank can squeeze aggregate supply of land to raise $q_{1}$. Higher $q_{1}$ raises the value of the bank's asset $\left(k_{0}^{b}+\gamma \theta k_{0}^{f}\right)$ and then its own capital $\left(W_{1}\right)$ higher than the threshold $\bar{W}$. This is costly for the bank as well, since the bank also has to buy expensive land and it has to keep some land $\left(\gamma \theta k_{0}^{f}\right)$ unused, but the cost of forced closure triggered by $W_{1}<\bar{W}$ is assumed to dominate the costs of making zombies.

Second, zombie lending retains non-productive firms and hinders new entrants ${ }^{8}$ from starting new business using cheaper production capital. This lowers the aggregate productivity of the

[^47]economy. This part of the story is similar to Caballero, Hoshi and Kashyap(2006) arguing that zombie lending prevented creative destruction of distressed industries. In this chapter, by contrast, zombie lending is not necessarily the bad thing, because the economy can avoid the cost of banking crisis. If the social cost of bank closure (this is not defined in this chapter) is huge, then zombie lending can be socially efficient even though it prevents the economy from quick recovery. During the Japanese "lost decade", many banks were forced to restructure their business drastically (including M\&A), but there was neither the bankruptcy of major banks, nor the panic liquidation of borrowing firms ${ }^{9}$. The banks had several years to resolve their problems in their balance sheets.

Third, zombie lending brakes the positive correlation between economic growth and land price, as in Kiyotaki and Moore (1997). In a normal credit cycle, negative productivity shock has a negative effect to land price. But if we assume that there is a bank on the propagation path, and if the bank suffers from insufficient capital (cash), the bank has an incentive to prevent the propagation to avoid forced closure. We also can conjecture from the model that land price goes down when the economy experiences a positive productivity shock after zombie lending, because the positive shock provides some cash to the bank to secure their own capital, which allows the bank to liquidate some insolvent borrowers and write off the loan ${ }^{10}$.

The model also tells when we have zombie lending and when we don't have. Zombie lending is likely to occur when the following four conditions are satisfied. First, unanticipated bankruptcy shock $\gamma$ is large. Second, the bank's leverage is high (equivalently, interest rate $r$ is low). Third, the bank's own capital $W_{0}$ is small. Four, $z_{1}^{L}$, lower aggregate productivity, is low (or, $q_{1}$ becomes sufficiently high and the bank does not worry about the capital loss coming from seised land $\gamma \theta k_{0}^{f}$ ). And there is one important necessary condition - the market is not competitive. If there are many banks, each small bank cannot raise market price $q_{1}$ and their optimal $\theta$ is always zero.

The first four conditions imply that zombie lending is prone to occur after the burst of a

[^48]bubble. When a bank experiences unanticipated borrowers' bankruptcy, after making highly leveraged lending, the bank cannot bear the cost of de-leveraging, even if the bank knows it is an efficient choice in the long run. This is the zombie lending.

The last necessary condition implies that this zombie lending may occur only when loan market and land (or, any other collateralisable assets) market are monopolistic or oligopolistic. In Japan, bank loans are rarely securitised (there was virtually no securitised loan market a decade ago) and each firm use only one bank to finance (so-called main bank system ${ }^{11}$ ). Furthermore, the size of bad loan held by major Japanese banks was large compared with annual trading amount of land in Japan (c.f. Footnote 10 of chapter 5). I.e., since the land market was illiquid and Japanese major banks have been huge, monopoly (or oligopoly) in land market is not an unrealistic assumption in Japan.

On the other hand, zombie lending may not occur in the US or the UK, where loans are commonly securitised. If a bank suffering from low capital adequacy ratio aims at zombie lending, the bank has to coordinate with all the other banks and investors holding the ABS (asset-backed security) of the loan. The coordination is at least difficult, since each lender has an incentive to deviate from the coalition and liquidate its share at higher price. In the worst case, the bank may not be able to find identity of all the lenders (coordination is of course impossible in this case).

The arguments above implies that the post-crisis economic growth and asset price in the US or the UK may be different from the one in Japan. In Japan, zombie lending prevented the economy from quick recovery, and also from falling into severe financial panic. The prolonged stagnation, the so-called lost decade, is the result of this. In the US or the UK, by contrast, a post-crisis economy may experience rapid de-leverage (the liquidation of insolvent borrowers) in a large scale and a banking crisis as a result, because zombie lending is difficult. But the crisis may be followed by a quick economic recovery, because bad firms will be promptly eliminated.

This implies that optimal policy for crisis resolution in the US or the UK may be different from the one Japanese government took. We will compare two policies Japanese government

[^49]took, public capital injection and bad assets purchasing schemes, to clarify what we can learn from the Japanese experience, in the next chapter.

### 6.6 Appendix

### 6.6.1 Proof of Lemma 13

As shown in Figure 6-1, there are two disconnected intervals of the MR line. Here we see that the lower, right-hand side interval does not intersect with MC in any conditions allowed by the assumptions made above. First, we will find the intersections of the demand function and the budget constraint. The demand function and the budget constraint are respectively defined as:

$$
\begin{aligned}
q_{0} & =E\left[q_{1}\right]+z_{0}-2 k_{0}^{f} \\
q_{0} & =\frac{w_{0}^{f}}{k_{0}^{f}} \cdot \frac{1+r}{r}
\end{aligned}
$$

Solving these two equations, we have the following:

$$
\begin{array}{r}
\left(E\left[q_{1}\right]+z_{0}-2 k_{0}^{f}\right) k_{0}^{f}=\frac{1+r}{r} w_{0}^{f} \\
2\left(k_{0}^{f}\right)^{2}-\left(E\left[q_{1}\right]+z_{0}\right) k_{0}^{f}+\frac{1+r}{r} w_{0}^{f}=0 \\
k_{0}^{f}=\frac{\left(E\left[q_{1}\right]+z_{0}\right) \pm \sqrt{\left(E\left[q_{1}\right]+z_{0}\right)^{2}-8 \frac{1+r}{r} w_{0}^{f}}}{4}
\end{array}
$$

The bigger solution is the second intersection of the demand function and the budget constraint. Next, the intersection of unconstrained MR and MC is derived. We have complicated MC if the bank lends, but any MC with bank loans is steeper than the MC without bank loans. That is, if intersection of the unconstrained MR and MC without bank loans is smaller than the second intersection of the demand function and the budget constraint, then the same result
can be obtained with any other steeper MC with bank loan. Accordingly, we can limit our attention to the non-lending MC:

$$
\begin{aligned}
M R & =E\left[q_{1}\right]+z_{0}-4 k_{0}^{f} \\
M C & =2 k_{0}^{f}-(2 K-A)+E\left[q_{1}\right]
\end{aligned}
$$

Solving this with $\mathrm{MR}=\mathrm{MC}$, we have:

$$
k_{0}^{f}=\frac{z_{0}+(2 K-A)}{6}
$$

Since $2 K-A<0$ by assumption, $E\left[q_{1}\right]>0$ and $\sqrt{\left(E\left[q_{1}\right]+z_{0}\right)^{2}-8 \frac{1+r}{r} w_{0}^{f}}>0$, we can see

$$
\frac{\left(E\left[q_{1}\right]+z_{0}\right)+\sqrt{\left(E\left[q_{1}\right]+z_{0}\right)^{2}-8 \frac{1+r}{r} w_{0}^{f}}}{4}>\frac{z_{0}+(2 K-A)}{6}
$$

Consequently, for any $z_{0}, E\left[q_{1}\right], w_{0}^{f}, K$ and $A$, the second intersection of the demand function and the budget constraint is (horizontally) larger than the intersection of unconstrained MR and MC. This is sufficient to prove that the first intersection of demand function and budget constraint is the upper bound of $k_{0}^{f}$. We can prove that date- 1 equilibrium has the same upper bound, using a similar (and simpler) derivation.

### 6.6.2 Proof of Proposition 14

The bank has an incentive to make zombie borrowers only when the bank can raise its own capital $W_{1}$ by stop liquidating default borrowers' collateral (land). Namely, the conditions for zombie lending are $W_{1}(\theta=0)<\bar{W}$ and $W_{1}(\theta=1) \geq \bar{W}$. We will rearrange the conditions with respect to $q_{1}$ first. Since $W_{1}$ is strictly increasing w.r.t $q_{1}$, we have a unique threshold $\hat{q}_{1}$ such that $W_{1}\left(\hat{q}_{1}\right)=\bar{W} . W_{1}\left(\hat{q}_{1}\right)=\bar{W}$ is defined as follows:

$$
\begin{equation*}
f\left(A, k_{0}^{b}\right)+q_{0} k_{0}^{f}+\hat{q}_{1} k_{0}^{b}-D+(1-\gamma)(1+r) D+\gamma(1-\theta) \hat{q}_{1} k_{0}^{f}+\gamma \theta \hat{q}_{1} k_{0}^{f}+W_{0}=\bar{W} \tag{6.10}
\end{equation*}
$$

Thus the threshold $\hat{q}_{1}$ is:

$$
\begin{equation*}
\hat{q}_{1}=\frac{D-(1-\gamma)(1+r) D}{k_{0}^{b}+\gamma k_{0}^{f}}-\frac{f\left(A, k_{0}^{b}\right)+q_{0} k_{0}^{f}}{k_{0}^{b}+\gamma k_{0}^{f}}+\frac{\bar{W}-W_{0}}{k_{0}^{b}+\gamma k_{0}^{f}} \tag{6.11}
\end{equation*}
$$

If $q_{1}$ without zombie lending is lower than the threshold, i.e. $q_{1}\left(z_{1}^{L}, \theta=0\right)<\hat{q}_{1}$, and if $q_{1}$ with zombie lending is higher than the threshold, i.e. $q_{1}\left(z_{1}^{L}, \theta=1\right) \geq \hat{q}_{1}$, then the bank has the incentive of zombie lending. These two conditions are satisfied only when $q_{1}\left(z_{1}^{L}, \theta=\right.$ $0)<q_{1}\left(z_{1}^{L}, \theta=1\right)$. The condition is satisfied by choosing appropriate $z_{1}^{L}$ : sufficiently small $z_{1}^{L}$ ensures that the firms' budget constraint does not bind when $\theta=1$.

If the zombie lending can raise $q_{1}$ (if $z_{1}^{L}$ is sufficiently small), we can prove the existence of zombie lending by showing that $\hat{q}_{1}$ can take arbitrary number by choosing appropriate parameters in the equation (6.11). First, $\hat{q}_{1}$ can take any small number (possibly negative) by choosing sufficiently large $W_{0}$. As $W_{0}$ goes to infinity, the threshold $\hat{q}_{1}$ diverges to negative infinity (see the equation 6.11). On the other hand, $W_{0}$ has a lower bound: The bank's initial cash holding $W_{0}$ plus the revenue of land sales to the firms $q_{0} k_{0}^{f}$ must be equal or larger than the regulation level $\bar{W}$ (or, the bank is forced to close at date 0 ). Accordingly, $\hat{q}_{1}$ cannot take any arbitrary large number only by manipulating $W_{0}$.

Suppose that $W_{0}$ is the lowest number it can take. Accordingly, $\bar{W}-W_{0}=q_{0} k_{0}^{f}$. Substitute this into (6.11) and differentiate $\hat{q}_{1}$ with respect to $r$, we have $\frac{\partial \hat{q}_{1}}{\partial r}<0$. The proof is below:

$$
\hat{q}_{1}=\frac{D-(1-\gamma)(1+r) D}{k_{0}^{b}+\gamma k_{0}^{f}}-\frac{f\left(A, k_{0}^{b}\right)+q_{0} k_{0}^{f}}{k_{0}^{b}+\gamma k_{0}^{f}}+\frac{q_{0} k_{0}^{f}}{k_{0}^{b}+\gamma k_{0}^{f}}
$$

Substitute $D=q_{0} k_{0}^{f}-w_{0}^{f}, q_{0} k_{0}^{f}=\frac{w_{0}^{f}}{r /(1+r)}$ (because it is assumed that the firms' budget constraint is binding at date 0 for simplicity) and $k_{0}^{b}=K-k_{0}^{f}$ into the equation above. Then we have:

$$
=\frac{1}{K-(1-\gamma) k_{0}^{f}}\left[\{1-(1-\gamma)(1+r)\}\left(\frac{w_{0}^{f}}{r /(1+r)}-w_{0}^{f}\right)-\left(K-k_{0}^{f}\right)\left(A-K+k_{0}^{f}\right)\right]
$$

As long as the budget constraint is binding at date $0, \frac{\partial k_{0}^{f}}{\partial r}<0$ because higher $r$ tightens the
firms' budget constraint. Further rearranging the equation, we have:

$$
=\frac{1}{K-(1-\gamma) k_{0}^{f}}\left[\left(\gamma+\frac{\gamma}{r}-1\right) w_{0}^{f}-A K+K^{2}+(A-2 K) k_{0}^{f}+\left(k_{0}^{f}\right)^{2}\right]
$$

$\frac{\partial \hat{q}_{1}}{\partial r}<0$ from the equation above. As $r \rightarrow 0, \hat{q}_{1} \rightarrow+\infty$. $\hat{q}_{1}$ can take, therefore, any value by choosing appropriate $z_{1}^{L}, W_{0}$ and $r^{12}$.

### 6.6.3 Proof of Lemma 16

We need to show that the bank's profit is strictly decreasing w.r.t. $\theta$. Thus, the bank tries to minimise $\theta$ and we can ensure that she chooses $\theta$ such that $W_{1}=\bar{W}$.

The final wealth of the bank at date $2, W_{2}$, is defined as follows:

$$
W_{2}=W_{1}+f\left(A, k_{1}^{b}(\theta)\right)+q_{1}(\theta)\left\{k_{0}^{b}-k_{1}^{b}+\gamma(1-\theta) k_{0}^{f}\right\}
$$

Differentiating this w.r.t. $\theta$, we have:

$$
\begin{aligned}
\frac{\partial W_{2}}{\partial \theta} & =\frac{\partial f\left(A, k_{1}^{b}(\theta)\right)}{\partial \theta}-\left\{q_{1}^{\prime}(\theta) k_{1}^{b}(\theta)+q_{1}(\theta) k_{1}^{\prime b}(\theta)\right\}-\left\{\gamma k_{0}^{f} q_{1}(\theta)+\gamma \theta k_{0}^{f} q_{1}^{\prime}(\theta)\right\} \\
& =\frac{\partial f\left(A, k_{1}^{b}(\theta)\right)}{\partial \theta}-q_{1}^{\prime}(\theta)\left\{k_{1}^{b}(\theta)+\gamma \theta k_{0}^{f}\right\}-q_{1}(\theta)\left\{k_{1}^{\prime b}(\theta)+\gamma k_{0}^{f}\right\}
\end{aligned}
$$

In the above equation, all terms but $\frac{\partial f\left(A, k_{1}^{b}(\theta)\right)}{\partial \theta}$ and $k_{1}^{\prime b}(\theta)$ are strictly positive. $\frac{\partial f\left(A, k_{1}^{b}(\theta)\right)}{\partial \theta}$ is strictly negative. $k_{1}^{\prime b}(\theta)=0$ if $\theta \leq \tilde{\theta}$, or, if $k_{1}^{\prime b}(\theta)+\gamma k_{0}^{f} \geq 0$ even though $\theta>\tilde{\theta}$; otherwise, $k_{1}^{\prime b}(\theta)<0$. Here, $\tilde{\theta}$ denotes $\theta$ such that MC intersects with MR at its kinked point (if the budget constraint is not binding at $\theta=0, \tilde{\theta}$ would be negative). Therefore, $\frac{\partial W_{2}}{\partial \theta}<0$ if $\theta \leq \tilde{\theta}$, or, if $k_{1}^{\prime b}(\theta)+\gamma k_{0}^{f} \geq 0$ even though $\theta>\tilde{\theta}$. If we can show that $k_{1}^{\prime b}(\theta)+\gamma k_{0}^{f} \geq 0$ when $\theta>\tilde{\theta}$, $\frac{\partial W_{2}}{\partial \theta}<0$ is globally ensured and the lemma is proved.

[^50]At date 1 , the slope of the MC is 2 and the slope of the MR is -4 . We know that if $\theta$ changes by $+\Delta \theta$ then the MC curve horizontally shifts leftward by $\gamma k_{0}^{f} \cdot \Delta \theta$. As a result, the intersection of MC and MR shifts leftward as well, by $\frac{1}{3} \gamma k_{0}^{f} \cdot \Delta \theta$ (since $\theta>\tilde{\theta}$, the budget constraint is not binding). That is to say, $k_{1}^{f}$ decreases by $\frac{1}{3} \gamma k_{0}^{f} \cdot \Delta \theta$ and $k_{1}^{b}$ also decreases by $\frac{2}{3} \gamma k_{0}^{f} \cdot \Delta \theta$ (since aggregate land supply decreases by $\gamma k_{0}^{f} \cdot \Delta \theta$ ); $k_{1}^{\prime b}(\theta)=-\frac{2}{3} \gamma k_{0}^{f}$. Now we see that $k_{1}^{\prime b}(\theta)+\gamma k_{0}^{f} \geq 0$ when $\theta>\tilde{\theta}$ and $\frac{\partial W_{2}}{\partial \theta}<0$ is globally ensured. The bank has an incentive to minimise her loss by choosing the smallest $\theta$ that ensures $W_{1} \geq \bar{W}$, and the $\mathrm{B} / \mathrm{S}$ constraint holds. Therefore, the $\theta$ such that $W_{1}=\bar{W}$ is the equilibrium action for the bank.

## Chapter 7

## Policy Implications for Post-Crisis Economies

### 7.1 Introduction: Optimal Policies for Post-Crisis Economies

We saw, in the previous chapter, that a post-crisis economic growth is affected by banks' behaviour, and the path of the growth depends on the characteristics of banking industry, the degree of leverage before the crisis, and the size of unanticipated bankruptcy shock that borrowers experience. Especially, the model shows that a recovery path after crises may be different with oligopolistic banking (Japan) and securitised finance (the US or the UK).

We will study, in this chapter, optimal policy for post-crisis economies. The conclusions in the previous chapter imply that the optimal policy may vary relying on some parameters e.g. the features of banking industries, the features of the bubble they experienced, and so on. Particularly, we will focus on two policies Japanese government took 10 years ago: public capital injection to banks that suffered from low capital adequacy ratio, and a toxic asset purchasing scheme, which allowed banks to eliminate illiquid risky assets from their balance sheets.

The most relevant study to this chapter is Miller and Stiglitz(2008), modelling the burst of an asset price bubble, which is caused by a scheduled technology shock and its cancellation. The bubble burst triggers a margin call by lenders, which forces borrowers to make a fire-sale
of their assets. The fire sale lowers land price further, and the price decline triggers another margin call. The vicious cycle may finally make the economy completely collapse. Miller and Stiglitz (2008) argue that some public bail-out schemes must be necessary in these cases, to stop the vicious cycle. They suggest three possible options; debt-equity swap, public capital injection, and loan write-down.

This chapter, instead, focuses on public capital injection and toxic asset purchasing scheme. A question we tackle here is not whether these two policies can help banks or not. In this model, banks can be bailed out if the government injects sufficient amount of capital. Which policy is less costly for taxpayers? What kind of indirect effects do they have? These questions crystallising the costs and benefits of these two policies are what we focus on in this chapter.

Section 2 briefly explains the features of the Japanese policies aiming at resolving its banking problems. Section 3 discusses the costs and benefits of the policies. Section 4 concludes.

### 7.2 Two Crisis Resolution Policies: How These Worked

After the Japanese banking crisis in late 1997, when a major security company and several banks went bankrupt, Japanese government mainly took two different policies to resolve the problem. First one is public capital injection. In February 1998, the government stipulated two bills ${ }^{1}$ for public capital injection. One bill prepared 17 trillion yen ( 170 billion US dollars ${ }^{2}$ ) allowing the Resolution and Collection Corporation (RCC, which is equivalent to US Resolution Trust Corporation, RTC) to purchase defaulted banks' assets and then to liquidate the assets.

The other bill, stipulated in February 1998, secured fund (13 trillion yen) for Deposit Insurance Corporation of Japan (DIC) to inject public capital to risky but still solvent banks. The capital injection was done by mainly purchasing banks' perpetual subordinated debts. The interest rate of debts were 6 months LIBOR plus some spreads (from 100 bps to 300 bps ), and the rates were increased by (approximately) 150 bps if the banks could not repay the debt

[^51]within 5 years (this step-up interest was prepared to give an incentive for banks to repay the public capital as soon as possible). In the following March, approximately 2 trillion yen was injected into 21 banks.

Since the performance of these bills was not very satisfactory, these bills were revised further in October 1998. A notable feature of the revision is allowing DIC to purchase toxic assets from solvent (but possibly risky) banks. Overall funds secured for these policies were doubled to 60 trillion yen. In March 1999, 7.5 trillion yen was additionally injected to major banks, based on the revised bills. The injection was done by mainly purchasing preferred stocks with an exchangeable clause. The preferred stocks did not have voting write, but the holders of the stock have a seniority over normal stock holders in claiming issuers' assets. The exchangeable clause gave the government to exchange the preferred stocks to normal stocks. The clause became selectable 3-5 years after the issuance: Namely, issuing banks that fails to buy-back the preferred stocks within 5 years could be literally "nationalised" by the government exchanging its preferred stocks to normal stocks with voting rights.

A reason why the government changed its injection scheme from subordinated debts to preferred stock is the high interest rate of subordinated debts. Since obtaining taxpayers' support for the public capital injection schemes was very difficult, the government had to design the scheme giving top priority to the security of taxpayers' money. In subordinated debt, a fixed income asset, only the source of profit is its interest payments. The high interest, however, conflicted with the scheme's main purpose, liquidity provision to stricken banks. The banks had to lose their precious cash every 6 months for interest payment, and the burden was not negligible for the banks. To avoid the conflict, the government determined to rely on capital gain, rather than income gain (interest) - preferred stock is a convenient measure for this. The government's decision was literally paid: By selling the stocks, the government could earn some profit.

The toxic asset purchasing scheme from solvent banks, however, did not work well for the time being. DIC purchased toxic assets for only 1.3 trillion yen from 1999 to $2002^{3}$. This is

[^52]mainly because DIC's bids were too low. Since the security of taxpayers' money was the first priority, DIC had to purchase toxic assets only at the price that DIC was not likely to make any loss. As a result, during the three years, DIC's average purchasing price of the assets was $4.5 \%$ of the assets' face value. The bids were remarkably lower than the market prices of these assets, so no banks preferred to sell their toxic assets to DIC. The government amended the bill obliging DIC to secure taxpayers' money as the first priority in January 2002: DIC was then allowed to purchase toxic assets at current price, estimated from discounted cash flow from the assets. The amendment raised DIC's purchasing price to $9.8 \%$ in 2002 ( $15.8 \%$ in 2003 and $13.2 \%$ in 2004) and the purchased amount of toxic assets surged to 2.1 trillion yen in 2002.

By 2008, most public capital injected to banks have been repaid. Total capital injection was 12.4 trillion yen, and the current outstanding is 3.2 trillion yen at the end of September 2008. The debt-collection ratio of the toxic assets purchased from banks is $118.5 \%$ (accumulated basis: total collection / total purchased) in 2007.

### 7.3 Capital Injection and Toxic Asset Purchase

### 7.3.1 Zombie Lending and Toxic Asset Purchase Scheme

Toxic asset purchasing scheme has a similar effect to zombie lending. Zombie lending is banks' squeeze of land supply to raise land price so that the banks can make up their own capital. The banks squeeze the land supply by leaving insolvent borrowers un-liquidated. Toxic asset purchasing scheme has the same effect. The government, on the other hand, purchases nonperforming loans from the banks (to prevent the banks from fire-selling their collateralised land) or their collateral assets (land) directly, to raise land price and secure the banking industry. As Figure 7-1 shows ${ }^{4}$, toxic asset purchasing scheme is described in the exactly same way as zombie lending.

[^53]

Figure 7-1: Two policies: How these work

Toxic asset purchasing scheme, therefore, has the same negative effect to the economy. If the government purchases insolvent borrowers' debts (with their collateralised land) and leave these untouched, the land, a production capital, cannot contribute to production on the period. In other word, toxic asset purchasing scheme prevents new entrants (or solvent and productive incumbents) from buying the land to produce. This interference of creative destruction lowers the aggregate productivity of the economy, as we have seen in the case of Japanese "lost decade".

As long as the government can purchase only toxic assets, the government cannot choose $\theta>1$. Accordingly, the conditions that toxic asset purchase scheme works are identical to the one of zombie lending. This implies the risk of a government adopting toxic asset purchasing scheme: the scheme cannot always help banks. If the banks' own capital is too small, buying all toxic assets still might not be able to raise land price higher than the threshold level $\hat{q}$ and the banks would be forced to close. Or, if the government cannot observe the shape of the demand function of land (or any other collateralisable assets), the government cannot correctly predict how much it can raise land price by purchasing toxic assets. Especially the latter problem should be emphasized in the case of toxic asset purchase scheme. In the case of monopolistic bank, the bank may have sufficient information about the market. But it is obviously unrealistic to assume that the government has better knowledge about the market.

The problems can be eased if the government can purchase toxic assets at inflated price. The difference between the inflated price and its market price is an income transfer from the government to banks. In the model explained in the previous chapter, it is equivalent to public capital injection as well. Of course, if the market price itself was hard to observe, even purchasing toxic assets at inflated price could not secure the banking industry. Adopting toxic asset purchase scheme should, therefore, require careful feasible study beforehand. Or, as Japanese government did, the scheme should be combined with public capital injection policy.

### 7.3.2 Public Capital Injection

In the model shown in the previous chapter, public capital injection raises $W_{1}$ of the bank (or banks) directly, to ensure $W_{1} \geq \bar{W}$ (banking industry is secured). Since $W_{1}$ and $\bar{W}$ are
obviously observable for the government, the information asymmetry problem in the toxic asset purchase scheme does not occur. In Figure 7-1, public capital injection is described as decline in $\hat{q}$, which is defined in the equation (6.11). If the government can lower $\hat{q}$ below the market price $q_{1}\left(z_{1}, \theta=0\right)$, the banks can be secured. This is always feasible for the government, because $\hat{q} \rightarrow-\infty$ as $W_{1} \rightarrow+\infty$.

A benefit of the public capital injection policy is that this policy does not hinder creative destruction, i.e. the liquidation of non-productive insolvent borrowers and replacing them with productive new entrants. This must be efficient in the long run. Since the model in the previous chapter assumes that the insolvent borrowers cannot produce anything, liquidating these insolvents is economically costless in the model.

But, it may not be unrealistic to assume that there is a social (non-pecuniary) externality of liquidating firms, e.g. the surge of unemployment followed by a higher crime rate. In this case, the economy that injects its public capital to the banking sector will experience a recession (or social instability) coming from the liquidation of insolvents, in the short run. The economy then starts to recover since non-productive firms had been eliminated and replaced by productive firms. From this viewpoint, public capital injection is the policy of "hard landing and quick recovery". By contrast, toxic assets purchase scheme or zombie lending can be categorised as "soft landing and slow recovery". The economy that chooses toxic asset purchase scheme (or zombie lending) does not have to rapidly liquidate insolvents. But the economy has to bear long stagnation, because the economy allows the non-productive firms to occupy some production capital for nothing, as Japanese economy experienced after the burst of the bubble.

In the following section, we will compare the performance of these two policies using the model in the previous chapter. Since the model cannot capture the whole costs and benefits of de-leveraging (e.g. the social and economic costs of liquidating borrowing firms, which are assumed to be zero in the model since insolvent firms' productivity is zero), any conclusion obtained in the model cannot be very realistic. But at least the model provides some criterion to consider the optimal policy. Studying the issue further is left for future research.

### 7.3.3 Costs and Benefits of Two Policies

In this section, we will compare the performance of these two policies, public capital injection and toxic assets purchase scheme, using the model in the previous chapter. Since the model cannot capture the social costs of liquidating insolvent borrowers mentioned above, any conclusion obtained in the model cannot be very realistic. But at least the model provides some criterion to consider the optimal policy. Studying the issue further is left for future research. In the following arguments, we assume competitive banking industry, which is closer to the US or the UK.

The costs of public capital injection are $\bar{W}-W_{1}$, the amount of capital injection, and the social and economic cost of de-leveraging. Since the model economy closes at date- 2 , the government cannot receive any repayment from the banks; thus whole injected capital becomes the cost. The latter one, the cost of de-leveraging cannot be studied in this model since the model assumes that insolvent borrowers are value zero in any criteria (the value of the borrowing firms is measured by their productivity $z$ alone). The benefits of capital injection are eliminating the cost of banking crisis and the benefit of de-leveraging, i.e. solvent new comers' production using liquidated land.

The costs of toxic assets purchasing scheme are the purchasing amount $\theta \gamma q_{1} k_{0}^{f}$, which achieves $\bar{W}=W_{1}$, and lower production by making zombie borrowers who do not produce anything. Since the value of the land in the next period (date 2) is zero for sure, the whole purchasing amount becomes cost. $\theta \gamma$ may not be bounded above by 1 in this case, because the government can purchase any amount of land. The benefits of the scheme are no banking crisis and no severe de-leveraging.

We will focus on the direct cost of the policies, $\bar{W}-W_{1}$ and $\theta \gamma q_{1} k_{0}^{f}$ for the sake of simplicity. The cost and benefit of de-leveraging are hard to capture in this model and left for future research. Since $\theta$ is chosen by the government to achieve $\bar{W}=W_{1}$, we have the following equation (since $q_{1}$ is not a linear function of $\theta$, the following equation is a linear approximation):

$$
\frac{\partial W_{1}}{\partial q_{1}} \frac{\partial q_{1}}{\partial \theta} \cdot \hat{\theta}=\bar{W}-W_{1}
$$

where $\frac{\partial W_{1}}{\partial q_{1}}=k_{0}^{b}+\gamma k_{0}^{f}$. Solving this for $\hat{\theta}$, we have:

$$
\hat{\theta}=\frac{\bar{W}-W_{1}}{\left(k_{0}^{b}+\gamma k_{0}^{f}\right) \cdot \frac{\partial q_{1}}{\partial \theta}}
$$

And the cost of toxic asset purchasing scheme is:

$$
\frac{\bar{W}-W_{1}}{\left(k_{0}^{b}+\gamma k_{0}^{f}\right) \cdot \frac{\partial q_{1}}{\partial \theta}} \gamma k_{0}^{f}
$$

Since the cost of capital injection is $\bar{W}-W_{1}$, the cost of capital injection is larger if the following is positive:

$$
\begin{aligned}
& \left(\bar{W}-W_{1}\right)-\frac{\bar{W}-W_{1}}{\left(k_{0}^{b}+\gamma k_{0}^{f}\right) \cdot \frac{\partial q_{1}}{\partial \theta}} \gamma k_{0}^{f} \\
= & \left(\bar{W}-W_{1}\right)\left\{1-\frac{\gamma k_{0}^{f}}{\left(k_{0}^{b}+\gamma k_{0}^{f}\right) \cdot \frac{\partial q_{1}}{\partial \theta}}\right\}
\end{aligned}
$$

Rearranging this, we have the following condition:

$$
\begin{aligned}
\text { Toxic assets purchasing scheme is cheaper if } 1 & <\left(\frac{K / k_{0}^{f}-1}{\gamma}+1\right) \frac{\partial q_{1}}{\partial \theta} \\
\text { Public capital injection is cheaper if } 1 & >\left(\frac{K / k_{0}^{f}-1}{\gamma}+1\right) \frac{\partial q_{1}}{\partial \theta}
\end{aligned}
$$

We have the following proposition from the above:

Proposition 17 Toxic asset purchasing scheme is cheaper than public capital injection if firms' demand function (or their budget constraint) is steep ( $\frac{\partial q_{1}}{\partial \theta}$ is large), and the banks are exposed to the risk of land ( $k_{0}^{b}$ is large).

If the market is very illiquid and its market price is highly sensitive to new bids, toxic asset purchasing scheme can be cheaper for taxpayers than public capital injection. If banks' loan
is highly leveraged (high $k_{0}^{f}$ ), public capital injection is cheaper. The proposition implies that the optimal post-crisis policy depends on the market conditions, and there is no trivial answer to the question which policy is more efficient.

### 7.4 Empirical Evidences

The results obtained in this and the previous chapters are consistent with empirical findings in Japan. The existence of zombie lending itself has been empirically shown by some researchers (see Kobayashi, Saita and Sekine(2002), Mihira(2005) and Peek and Rosengren(2005)) and no counter-argument has been made to the author's knowledge. The results obtained here are consistent with some another details of these empirical founding.

First, zombie lending lowers aggregate productivity. In the model, aggregate productivity declines by $\gamma \theta z_{1}$. Mihira(2006) estimated Japanese zombie lending lowered Japanese TFP (total factor productivity) by $1.1-1.7 \%$ maximum. Caballero et.al.(2006) also find an increase in zombie firms significantly reduces investment and employment growth.

Second, the model is consistent with Mihara's (2006) another finding: the negative impact of zombie lending has a spill-over effect to other industries. The model in the previous chapter does not explicitly model inter-industry externality, but the impact of zombie lending can be transferred to another industry via common collateral asset (possibly land). Zombie lending in an industry raises the price of a collateral asset (land), and the high asset price hinders firms in other industries (using the same production capital) from investing.

Third, the correlation between GDP growth and land price, expected in the KM's model, is broken with zombie lending. As Figure 5-7 shows, the land price (major cities) declined drastically when the bubble burst, and then only slowly decreased until the banking crisis
in 1997. After the crisis and the following public capital injection, while the GDP growth started picking up, the land price decline was in fact accelerated. The negative correlation had continued until 2005, until the banks' balance sheet problem was almost resolved.

The positive correlation between production growth and land price growth was broken in this model, because of zombie lending and the following public capital injection. Banks tried to minimise their liquidation of bad borrowers as much as possible to avoid forced closure (bankruptcy), but they start liquidating bad borrowers when they receive sufficient public capital (excess public capital allows the banks to liquidate bad borrowers and write-off their non-performing debts). Accordingly, the model implies, and Figure 5-7 shows, public capital injection may accelerate the plunge of asset price (land price). Asset price starts picking up only after the banks finish liquidating the "inventory" of their bad borrowers (in 2005: see Figure 5-5 for the resolution of non-performing loan problem in Japan).

Fourth, the lagged bankruptcy of Japanese firms can be also explained in this model. The bankruptcy of Japanese firms reached a peak (value basis) in 2000 (Figure 5-3), while the GDP growth had started to improve (Figure 5-7) in 1999. The banks could afford liquidating their bad borrowers in 2000, because they received ample public capital in 1999. Public capital injection can raise the number of bankruptcy in the short run.

Fifth, Mihira(2005) found out that bank loans were prone to increase to the over-borrowing firms. This is similar to a condition for zombie lending in this model - bank loan should be leveraged (low $r$ ). A bank with over lending (high leverage) is exposed to the larger credit risk of its borrowers and the larger risk of the borrowers' collateral value. The liquidation of large amount of collateral lowers asset price, and then deteriorates the bank's balance sheet further. The bank with highly leveraged loan, therefore, has a strong incentive of zombie lending.

### 7.5 Conclusion

Japanese banking crisis and the following so-called lost decade, and many policies tried during the recession provide many valuable implications. First, there is a trade-off between severe liquidation of borrowers and long economic stagnation. If a bank finds its borrowers insolvent, the bank has two choices: liquidating them, or leaving them un-touched. In other words, they are de-leveraging or zombie lending. This choice is essentially the same as choosing hard landing or soft landing. (Efficient middle-point can be found, although it is left for future study.)

Second, a government facing a post-crisis recession also has to choose the same option; hard landing or soft landing. Public capital injection helps banks from bankruptcy, but accelerates the banks' de-leveraging in the short run. This can be the reason why a government injecting public capital to banks tends to request (or force) the banks to keep lending their borrowers. But in the long run, public capital injection is more efficient because non-productive insolvent firms are replaced with productive one. Toxic asset purchasing scheme also helps banks from bankruptcy by improving the banks' value of their assets. The scheme, by contrast, decelerates de-leveraging because the scheme is economically equivalent to zombie lending. This scheme, on the other hand, would invote the long stagnation of the economy, because the scheme preserves insolvent (and thus non-productive, or at least less productive) firms in the economy. The "zombie" firms hold their production capital (land) for nothing, and lower the aggregate productivity of the economy.

Third, toxic assets purchase scheme can be operationally difficult, if the government does not have sufficient information about the shape of the land market's demand function. If the price is sensitive to new bids, toxic asset purchase scheme can efficiently improve banks' own capital by purchasing the small amount of land (or bad loans). But if the price is less sensitive, the banks may be forced to close even after the scheme. By contrast, public capital injection scheme does not have any information asymmetry problem and the government can secure its banking industry for sure.

Since the model is highly simplified, many possible extensions are left for future research. For instance, if the game continues for many (or possibly infinite) periods, we might be able to see a different business cycle from Kiyotaki and Moore(1997). Removing some assumptions made for simplicity (that firms' bankruptcy is unanticipated by banks, that firms' budget constraint is always binding at date 0 , and so on) can be another possible extension. Whether the implications above can be econometrically confirmed or not is also an interesting question.

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[^0]:    ${ }^{1}$ Van Der Wee(1967), cited and translated by Youji(2004).

[^1]:    ${ }^{2}$ To be precise, these three terms have different definitions. Assignability means that holder of bills can resell them to third parties before its maturity. Transferability makes the bills more impersonal. Negotiability allows the third party to claim a repayment of the default debt not only against the initial borrower, but also against the seller (initial holder) of the bill. What we study is a mixture of assignability and negotiability. To avoid confusion, we will use the word 'resaleability', as in Kiyotaki and Moore(2003), hereafter.

[^2]:    ${ }^{3}$ The ratio was more than $35 \%$ in the 1980s and steadily decreasing (in the large manufacturing firms, the number was around $20 \%$ in 2000). Kagabayashi(2004) points out that the decline was due to the simultaneous increase in the firms' assets and their own capital (not due to large declines in trade credit among non-financial firms).

[^3]:    ${ }^{4}$ He also points out that this does not mean they had no legal protection. We will come back to the point in Chapter Four.
    ${ }^{5}$ De Roover(1948) points out as follows: 'Not all medieval bills of exchange were commercial bills. A large proportion of bills were what might be called finance bills.' It implies that these bills relied neither on bankspecific technologies nor on supplier-specific technologies.
    ${ }^{6}$ Certificates of Deposit: mainly traded in interbank markets.

[^4]:    ${ }^{7}$ Wilner(2000) mentions that trade credit interest rates commonly exceed $18 \%$.
    ${ }^{8}$ Many other papers make similar taxonomies.

[^5]:    ${ }^{9}$ Considering the accumulated backlog of unsettled cheques during the dispute, it was not difficult for people in Ireland to expect that the bank cheques would not be settled on time, at maturity date, according to the Central Bank of Ireland(1971).

[^6]:    ${ }^{10}$ For example, negotiability (resaleability) of bills that are payable to the bearer was left informal for a long time in the UK, as Megrah and Ryder(1983) point out.
    ${ }^{11}$ Poland, Slovakia, Romania, Russia and Ukraine.
    ${ }^{12}$ In particular, both Russian and Ukrainian firms were negative (more than 40 percent of the firms thought courts could not enforce contracts).

[^7]:    ${ }^{13}$ Frank and Maksimovic(2004) need several assumptions to make their model workable. The first one is the independent default probability of the buyer(borrower) and seller(intermediary), which simplifies players' beliefs. The second is that the buyer (borrower) gives his repayment to the initial lender directly in case the seller (intermediary) is bankrupt. This exogenous commitment power given to the borrower helps their model work more easily.

    In this model presented here, default ratios are strongly correlated because the borrower's repayment is only the source of repayment for both borrower and intermediary. This assumption would be plausible, and convenient to descrive chains of defaults (contagion), which are sometimes observed in reality.

[^8]:    ${ }^{14} \operatorname{Kohn}(1999 \mathrm{~d})$ also points out that the scarcity was aggravated by public policy that enforced the use of gold coin, not bills of exchange.

[^9]:    ${ }^{15}$ Van der Wee(1963) countered that the public prohibition was not the critical reason for the ruin of Antwerp banking, by pointing out that after the ordinances of 1433,1467 and 1480 the crafts of money-changers and bankers were flourishing in Antwerp. He argues that a monetary crisis of the last quarter of the 15 th century was fatal for banks.

[^10]:    ${ }^{16}$ Central Bank of Ireland(1971) and Murphy(1978) point out that we can consider that the closure effectively continued for a year, since these banks' clearing business was stuck before and after the closure.

[^11]:    ${ }^{17}$ To be precise, many transactions were done with bills drawn on the closed banks' accounts and it was expected that the banks would settle these after they restarted the business. In that sense, the system was still partly supported by the banks. It must be pointed out, however, that many transactions were actually cleared and cancelled during the closure, because the closure was prolonged for a year.
    ${ }^{18}$ The Central Bank of Ireland(1971) points out that the potential credit of the closed banks also helped the

[^12]:    ${ }^{1}$ To be precise, bills of exchange and bills obligatory have different transaction processes and transaction of bills of exchange is more complicated than the model shown here (bills of exchange need three players: an issuer orders a drawee to pay to a payee). Yet in transactions with bills of exchange, two players out of three participants in contracts belong to the same firm (but different branches) and no incentive problem would occur, as many historians point out. Thus, here we do not assume bills-of-exchange-like transaction, to concentrate on the regotiability of the bills.
    ${ }^{2}$ In terms of bills of exchange, A is an accepter / underwriter, B is a drawer and C is a payee. In terms of bills obligatory, $A$ is a drawer and $B$ is initial payee; then $B$ sells the bill to $C$.

[^13]:    ${ }^{3}$ There are several papers studying financial contracts as relational contracts; i.e. Boot, Greenbaum and Thakor(1993) and Rajan(1992). However, both of them study bilateral contracts, not indirect multilateral contracts.
    ${ }^{4}$ Kiyotaki and Moore(1997) and Cunat(2005) also study trade credits. The difference of the model is that we focus on pure financial transactions. Kiyotaki and Moore(1997) and Cunat(2005) assume that all financial transactions are backed by underlying physical trades. We do not take their idea here, since Kohn(1999a) points out that the bills of exchange have been used for pure financial transactions as exchange rate arbitrage.

[^14]:    ${ }^{5}$ In the studies of trade credits, Cunat(2006) adopts a similar approach.

[^15]:    ${ }^{6}$ This penalty is common in the real economy. In the case that we fail to repay, we cannot borrow money for the time being (in Japan, this takes approximately 2-7 years).

[^16]:    ${ }^{7}$ As in the previous section, we wil limit the action space to $a^{B, C} \in\{0,1\}$ for simplicity.

[^17]:    ${ }^{8}$ Compared with the model here, Kim and Shin's(2007) model is simpler. In their model, studying the moral hazard problem of many players with a vertical relationship that is similar to the model discussed here, it is sufficient to give a certain amount of surplus to each of suppliers to prevent deviation, because there is no uncertainty. In addition, no private-monitoring-like problem occurs, because information is symmetric in their model. This shows that information asymmetry is a critial problem in these multilateral contract models, like credit chain (a la Kiyotaki and Moore(1997)) or production chain problems.

[^18]:    ${ }^{9}$ To be precise, C can be assured of history of $a^{B}$ only. The history of y is still A's private information and C has no way to be assured.

[^19]:    ${ }^{10}$ To be precise, the agent gives up acting honestly immediately after he infers from his private signal that the principal receives a bad signal even though the agent acts honestly. This fragility comes from the severity of the penalty strategy. In order to enforce the best outcome, the principal needs to expand the length of the phase $T$ and to raise the threshold of the success ratio as high as she dares to. Accordingly, any wrong signal immediately breaks down cooperation in the review phase.

[^20]:    ${ }^{11}$ Bhaskar and van Damme(2002) also study mixed strategies under private monitoring, although their set-up is far different from the model here. Their two periods game assumes almost perfect monitoring and that there are two pure Nash Equilibria in the stage-game.

[^21]:    ${ }^{12}$ Even with the strategy, C does not have any incentive to choose $a^{C}=1$ at the end of the penalty phase to shorten the phase, because the given B's strategy is $a^{B 2}=0$ for any outcome at the end of the penalty phase.

[^22]:    ${ }^{1}$ Aghion and Bolton(1992) do not use the word "collateral" explicitly, but they discuss the same thing.
    ${ }^{2}$ Collateral is used in their stage-game equilibrium. In dynamic equilibrium, collateral is not used.
    ${ }^{3}$ Barro(1976) also assumes ex post information asymmetry, but he assumes constant output and stochastic collateral value; there is no concept of "riskiness of borrowers" in his model.

[^23]:    ${ }^{4}$ The assumption that the borrower receives collateral every period as an endowment, is clearly not plausible. However, some degree of simplification is inevitable here, for simplicity. It is not impossible to endogenize the amount of collateral, by assuming the output good is storable. Then we can construct the stochastic dynamic programming problem, with the next period's wealth(collateral) as an action choice and the outstanding collateral as the state variable. But the value function is far from its regular form, and analytically almost intractable because the normal principle of optimality is not available.

[^24]:    ${ }^{5}$ To be precise, it is optimal for C and inefficient for the economy enforcing A to give $\tau+d$ in the case of success, by imposing a longer penalty. We will ignore the possibility here.

[^25]:    ${ }^{6}$ Otherwise, B is indifferent between trading strategy and autarky penalty, and C's punishment never works.

[^26]:    ${ }^{7}$ To be precise, we need to check that the LHS of (3.10) is smaller than 1 and bigger than zero. It is not difficult to show it is smaller than 1 , but for the sufficiently large $p$, it must be negative and any $n$ cannot enforce B's honest behaviour. Even if $p=0, \delta \pi \hat{\tau}-\hat{\omega}-\delta(1-\pi) \eta \hat{\omega}>0$ must hold, which is more strict than the non-collateralized case.

[^27]:    ${ }^{8}$ To be precise, the second term can be positive if $Y-d \pi \hat{\tau}<0$. Yet there exists some $d$ such that $Y-d \pi \hat{\tau}>0$.

[^28]:    ${ }^{1}$ As a study of money, this study is slightly oversimplified compared with other studies. Generally we assume random matching models with finitely many players or something similar in order to study money, as in Bernhardt(1986) or Kiyotaki and Wright(1989). In this thesis, by contrast, we assume only three players and there is no random matching aspect. Yet this assumption still gives some important implications, and Kiyotaki and Moore(2003) also adopt a similar framework to discuss inside money and outside money.

[^29]:    ${ }^{2}$ The contracts discussed here are not renegotiation proof, in the spirit of Farrel and Maskin(1990)'s weak renegotiation proofness. This is because we have to rely on the autarky penalty, which is costly for both. There are several ways to overcome the problems suggested in the related literature (i.e. Levine(2003) or Kletzer and Wright(2000), but, neither are available here. Instead, we can use a different definition of renegotiation proofness, proposed by Pearce(1987). In the relational contracts literature, Macleod and Malcomson(1989) adopt the same definition. See $\mathrm{Ota}(2006)$ for detailed discussion.

[^30]:    ${ }^{3}$ Another explanation of the securitization is Kiyotaki and Moore(2003)'s bundling. In their study, a bank will bundle many borrowers' debts and securitizes their averaged repayment. As long as the borrowers' risk is idiosyncratic, buyers of the securitized loan can presume that repayment is constant. In contrast to the model here, they assume that bundling technology is exogenously given and no picking is allowed (ruled out exogenously) there.
    ${ }^{4}$ Ghosal and Miller(2003) argue that there is a similar fragility in sovreign debt crises. They point out that 'excessive project termination' is required to give an appropriate incentive to sovereign debtors.

[^31]:    ${ }^{5}$ This signalling mechanism has not been discussed in this thesis, because we limit our attention to the three players in charge.
    ${ }^{6}$ One of the signers must be an accepting house.

[^32]:    ${ }^{7}$ In tKiyotaki and Moore(2003)'s argument, inside money is a kind of bank note exogenously given with fixed cost. In the model discussed here, inside money is defined as negotiable bills that are endogenously configurated.

[^33]:    ${ }^{8}$ See Dixit(2004) for lawlessness and economics.

[^34]:    ${ }^{1}$ GDP statisticswere discontinued in 1995.

[^35]:    ${ }^{2}$ One might feel that bank loans to manufacturing industries did not decline after the burst of the bubble in the early 1990s. This would be because the bank loans to manufacturing industries jumped in 1993 in Figure 5-2. It was, however, a technical jump and did not reflect any macroeconomic incident. It was due to a relaxation of a law of commerce and deregulation of lease contracts in 1993. By excluding this jump, we can find (almost) monotonic decline of bank loans in the manufacturing sectors.

[^36]:    ${ }^{3}$ The statistics were discontinued in 1999.
    ${ }^{4}$ Jusen had several similar characteristics to S\&L in US, excluding that Jusen did not borrow from private sector like $\mathrm{S} \& \mathrm{~L}$, but borrowed from banks.

[^37]:    ${ }^{5}$ The line from 1997 to 1999 is the capital adequacy ratio of major banks in Japan, estimated by Okuda(2000). The time series of capital adequacy ratios is hard to plot since its definition changed frequently and banks were merged and absorbed from 1999 to 2002. The capital adequacy ratio of Risona bank soared in 2003 , just because of the public capital injection in 2003 , as shown in the figure.

[^38]:    ${ }^{6}$ They studied European transitional economies, not the Japanese case. Kobayashi, Saita and Sekine(2002) undertook empirical research of the Japanese case based on this study.
    ${ }^{7}$ The other possible explanation of the peaking out of the zombie phenomenon is flaws in the legislation. Tomura(2006) argues that Japanese bankruptcy law prevents banks from liquidating collaterals of default firms. But it is also difficult to explain the peak-out, since the new bankruptcy law (like chapter-11 in US) was enacted in 2000, several years after the peak-out.
    ${ }^{8}$ After the refinancing the bank receive a payoff $R_{p}$. This is bigger than refinancing cost 1 , but smaller than total financing cost 2 (the bank lends twice; lending at the beginning and then refinancing). Thus, the bad firm is ex-ante loss-making but ex-post profitable.

[^39]:    ${ }^{9}$ The Basel Capital Accord requires banks to keep their capital adequacy ratio at a minimum level of $8 \%$ and banks with insufficient capital adequacy ration are not allowed to operate international services. If the ratio falls below $4 \%$, the government will intervene in the bank's operation; the govenment can decide to close the bank as well, if the ratio is lowered further.
    ${ }^{10}$ One possible objection is that each bank may not be large enough to affect land price. Yet we can see that the Japanese major banks were sufficiently large to affect the price. Six major banks in Japan accounted for approximately $50 \%$ of total bank loans ( $\$ 1.9$ trillion out of $\$ 4.2$ trillion) in 2003 ; so each major bank has $5-10 \%$ of the total bank loans ( $\$ 0.3-0.4$ trillion). The aggregate value of Japanese land was $\$ 18.6$ trillion in 1993 (now it must be much smaller).

    Caballero et.al. estimate that $15 \%$ of the firms had received the "subsidized" loans, which make zombies, so the size of each bank's zombies is $\$ 60$ million. I think this is sufficiently large compared to the aggregate value of traded land in 2004, $\$ 400$ million.

[^40]:    ${ }^{11}$ Kiyotaki and Moore(2001) name this as 'resaleability constraint'.

[^41]:    ${ }^{1}$ To be precise, we should use capital adequacy ratio for the balance-sheet constraint, rather than own capital. We use absolute value of own capital due to technical reasons; see below.
    ${ }^{2}$ Practically, it is more realistic to think that banks chose to make zombies with optimistic expectations of future land prices. They hesitated to sell their land because they expected recovery of land prices in the near future. This mechanism is not studied directly in this model, since land price goes to zero for sure when the economy closes; but it is easy to show that an optimistic view of future land price encourages the banks to make zombies. See conclusion for further discussion.

[^42]:    ${ }^{3}$ With this assumption, its first derivative becomes linear and change in productivity $z$ is described as a parallel shift of the derivative.

[^43]:    ${ }^{4}$ This comes from the assumption that the budget constraint is binding. They spend all available funds at date 0 , the production becomes zero, and their land will be seized by the bank in the case of default. Thus we have $w_{1}^{f}=0$.

[^44]:    ${ }^{5}$ The BIS accord restricts capital adequacy ratio, not absolute value of capital. Here we need to modify the restriction because no lending occurs from date 1 to date 2 .

[^45]:    ${ }^{6} z_{1}^{H}$ does not play any important role in this model. This parameter becomes important when we extend the model allowing anticipated bankruptcy, which may discourage the bank to lend (the participation constraint of bank loan may bind). This is left for future research.

[^46]:    ${ }^{7}$ To be precise, the bank can raise $q_{1}$ without making zombie borrowers. The monopolist bank can make a offer with a very high $q_{1}$, and the firms will purchase only the small amount of land $k_{1}^{f}$. Theoretically there is no reason to exclude this opportunity; but we do not consider the case since "purchasing most of the land to raise land price" is not a realistic scenario to describe what happened in Japan. The exclusion of the possibility simply limits our attention to the subset of equilibrium outcome, and thus it does not affect the robustness of the study.

[^47]:    ${ }^{8}$ Or survived productive firms; these are equivalent since we assume atomless firms.

[^48]:    ${ }^{9}$ Of course, some famous banks (including LTCB, NCB and Yamaichi Securities) went bankrupt, and some borrowers were liquidated by banks: But these did not lead to severe financial panics.
    ${ }^{10}$ This has not yet been studied. Proving this is left for future research.

[^49]:    ${ }^{11}$ See, for example, Hoshi, Kashyap and Scherfstein(2003).

[^50]:    ${ }^{12}$ To be precise, lower r may increase $w_{1}^{f}$ and affects $q_{1}\left(z_{1}^{L}, \theta\right)$ (not the threshold $\hat{q}_{1}$ ) if the budget constraint is binding. But high $w_{1}^{f}$ will ease the budget constrant and $q_{1}\left(z_{1}^{L}, \theta\right)$ reaches its upper bound when $r$ goes down to a threshold. Accordingly, there exists an $r$ such that $q_{1}$ satisfies the two conditions.

[^51]:    ${ }^{1}$ For legislative details, see Kamakura (2005) for example.
    ${ }^{2}$ calculated using USD1=JPY100.

[^52]:    ${ }^{3}$ from DIC's webpage ((http://www.dic.go.jp/katsudou/katsudou4-1.html).

[^53]:    ${ }^{4}$ Competitive banking industry is assumed here for notational simplicity.

