A Study of the Relationship Between the Mathematical Beliefs and Teaching Practices of Home-Educating Parents in the Context of their Children's Perceptions and Knowledge of Mathematics

By

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Abstract

Home-education, also known as home-schooling, is an educational choice made by families to facilitate learning at home rather than in school. Research by Rothermel (2002) and Rudner (1999) shows that, on average, home-educated children far outperform school-educated children on standard mathematics tests. But at present, no study has yet investigated the key reasons behind this phenomenon – indeed, no research has taken an in-depth look into the ways in which parents facilitate the learning of mathematics at home and the resultant effects on their children's mathematical development. Therefore, in this study, we will consider the nature of mathematics education through the eyes of the home-educating parent and their children.

Through questionnaires, this research examines the relationship between the educational and mathematical beliefs of home-educating parents. Parental views are compared with the children's perceptions of the home learning environment, their mathematical beliefs and their mathematical understanding. Furthermore, the children's mathematical understanding is addressed through consideration of their responses to a series of mathematical questions set within the context of Key Stages 1-3 of the National Curriculum. To obtain the research sample, home-educating families from across the United Kingdom were contacted via the Internet, and information was collected through both email and postal response. From the parental data, three categories of home-educator were highlighted: (1) Structured, (2) Semi-Formal and (3) Informal (as described by Lowe and Thomas, 2002). The children's questionnaire responses were then analysed, using illustrative case studies to demonstrate how different home-educating approaches of their parents could result in

different perceptions of mathematics and mathematical learning in the children. For example, children learning via a 'structured' approach were less likely to be able to measure their own level of mathematical ability than children from the other families; they also mentioned limited resources and less independence when learning mathematics.

When examining the children's assessed work, selective case studies, together with detailed analysis, revealed a strong link between the home-educating approach and the problem-solving strategies of the children. Children from structured families were often competent when solving more routine, 'calculation-type' problems, but less able to adapt their knowledge to problems that required a 'deeper' understanding of the concept. Children from families where the parent themselves had a mathematical background (e.g. mathematician or mathematics teacher) typically used formal mathematical reasoning in their work. On the other hand, children learning from 'informal' families (where emphasis was placed on 'child-directed' learning) seldom used 'standard procedural' type approaches to solve problems, but instead displayed a range of creative strategies.

The findings suggested that a home-educating parent's conception of mathematics not only influenced the way in which they attempt to teach mathematics but also their children's mathematical beliefs and learning style. Furthermore, there was evidence to suggest that certain home-educating approaches encouraged a 'type' of mathematical understanding that could be applied in a range of situations, whereas other approaches, particularly where both the learning materials and interaction with others was restricted, resulted in a more limited level of mathematical understanding.

Chapter 1: Introduction

In the United Kingdom, state education is free and available to all children within the local educational area. Parents also have the option of sending their children to private schools - including boarding schools, religious schools, and those that promote particular teaching philosophies, such as the Montessori schools. Yet, there is a third choice available to parents, which is to *educate their children at home*. Up until the 1970's, home education was virtually non-existent in the UK, however, it is now estimated that there are up to 100, 000 home-educating families in the UK (BBC, 2006). While a number of families remain 'invisible' to their local education department (as they prefer not to disclose their home-educating status) it is clear is that the number of families choosing to educate their children at home is increasing. Not surprisingly, the research community has tried to determine the effects of this somewhat 'unorthodox' educational approach on the home-educated children's academic learning.

Within both the US and UK, studies have shown that home-educated children appear to perform at a higher academic level than their school educated peers – for example, Rudner (1999) tested 11,930 American home-educating families, finding that a quarter of home-educated students were working at **one or more grades above** their age-level peers in public and private schools. In the UK, Rothermel's study (2002) of 419 families showed that home-educated children **largely outperformed their schooled counterparts** on a general mathematics test, achieving an average mark of 81%, compared to the school educated pupils average mark of 45%.

As a mathematics education researcher, I was particularly interested in the effects that home-education could have on the children's mathematical thinking and development. Given that home-educating parents could come from a range of different educational and employment backgrounds, perhaps with no prior knowledge of teaching, was there any evidence that the parents' previous experiences influenced the ways in which they taught mathematics? Furthermore, what types of mathematical understanding were developed when a child was no longer 'formally' educated within the school environment?

Although Rothermel (2002) and Rudner (1999) have shown that home-educated children generally outperform their schooled peers on mathematics tests, no studies have yet focused on the 'mathematical thinking' of the children. In both of the previous studies, the children were given a series of standard tests on a number of subjects, and attained marks were measured for comparison purposes. But there was no 'in-depth' examination of the *mathematical reasoning* of home-educated children, nor any consideration of the *children's attitudes* toward their mathematical learning. Therefore, in this research, I seek to explore the key educational factors that could influence a child's mathematical development. In order to construct a suitable study, I use my experiences and perspectives as: (1) A mathematics education researcher, (2) An individual who was educated both at home and at school, and (3) A teacher and learner of mathematics.

1.1. Construction of the Research Questions

This study aims to identify: (1) The different approaches to home-education, (2) How the selected approach affects the learning of mathematics and (3) The resultant influence on the children's mathematical thinking.

To establish the ways in which a parent may approach the teaching of mathematics at home, initial consideration is given to why the parents chose to home-educate in the first place, i.e. *"What made the parents feel that home was a better option than school?"* Both American (Romanowski, 2001) and British (Rothermel, 2002) studies show that reasons for home-education can be varied, including an unhappiness with the schools' methods of teaching, social factors and religious reasons. Whilst this study examines the parental reasons for choosing home-education, unlike other studies, it also seeks to measure the level of influence the *children* have on this educational decision. Thus we seek to answer the questions:

- Why do parents choose home education?
- Do the children have any influence on the parents' decision to home educate?

Mathematics Education research has shown that the teaching approach followed in school can have a significant effect on a student's understanding of the subject (Boaler, 1998). Having been home-educated for a number of years prior to entering university and through my interactions with the general home-educating communities, I am aware that there are a number of different approaches to home-education. Some parents may follow an approach similar to that of school, closely following the National Curriculum, whilst at the other extreme, there are parents who have abandoned all forms of 'formal teaching', stressing that their children can learn ‘through life experiences’. In order to understand what makes a parent choose a particular home-educating approach, we ask the question:

• How do the **mathematical background** and **experiences** of the parent affect their approach to home-education with regards to the learning of mathematics?

This leads us to a consideration of *mathematical background, mathematical belief*, and *teaching practice*. Mathematics education researchers (Thompson, 1984; Askew, Brown, Rhodes, Johnson and William, 1997) claim a **teacher’s** mathematical beliefs, stemming from their personal schooling experiences, have an influence on their teaching practice. But the **home-educating parent’s** mathematical beliefs may not only come from their ‘school experiences’ of mathematics – for example they may have held previous employment that involved mathematical applications, which also contributes to their perceptions of the subject. So in order to better understand *why* a parent chooses a particular approach with regards to their children’s mathematical learning, we first have to understand what a parent *believes* about mathematics. Parental beliefs on mathematics could provide the basis behind their educational philosophies when teaching the subject, and hence guide their choice of curriculum, methods of imparting mathematical knowledge, and the ways in which they interact with their children. We therefore ask:

- What are the core **beliefs** held by the home-educating parents?
- How might their beliefs influence the **teaching of mathematics**?

An individual’s perceptions of *mathematics* and the *learning of mathematics* may indicate how they think about, and hence understand the subject. Thus, the next stage of the study examines the effects that the home environment has on the children’s perceptions and understanding of mathematics, i.e.:

• What are the **children's feelings** towards both their learning environment (i.e. learning mathematics at home) and mathematics, as a subject?

The responses identified from this question related to those clarifying parental belief may then give some insight into the question:

• Are the parental beliefs likely to **influence** their children's views of home-education/mathematics?

The final aim of the study is to examine the effects of home-education on the children's *mathematical understanding*. We examine the main theoretical arguments relating to mathematical understanding (including Skemp, 1976; Hiebert and Carpenter, 1992 and Sfard, 1991), with a view towards establishing a way of adequately classifying and/or measuring the understanding of the children. An important issue arising here are the responses to the question "When is a mathematical idea understood?" Since the study is considering both parents and children, there are two forms of question associated with this notion:

• When does the **parent** believe their child **understands** elements of mathematics?

• When does the **child** believe s(he) **understands** mathematics?

Associated with these questions we may identify issues from the ways in which parents seek to measure their children's levels of mathematical understanding, the identification of whether or not children have *the different forms of understanding* and if relationships between these forms and the home-educating approach can be established.

1.2 Overview of Research

In order to answer the main research questions, it was clear that I would require information from both home-educating parents and their children. To establish a suitable research methodology, I noted that the majority of the identified research questions were open, that is, as a researcher I sought to *draw and infer relationships and meanings from the data,* rather than establish a pre-conceived hypothesis. It thus seemed appropriate to follow a *phenomenographical approach,* where one would attempt to define the different ways in which the parents' experience, interpret and perceived particular phenomena relating to mathematical learning at home, in line with the approach suggested by Marton (1981). Thus it was imperative that during the construction of the research instruments (Chapter 3), and also when conducting data analysis (Chapters 4, 5 and 6), I chose a method that would allow each participant to *give meaning* to the topic in question.

The specific research questions indicated that I would require qualitative data, in order to investigate areas such as belief, and also quantitative data – for example, I needed a way of measuring the children's mathematical abilities. A mixed methodology appeared to be the most suitable approach, and the process through which the research instruments were developed is described in Chapter 3. In particular, the study used *questionnaires* for both parents and children, followed by *three groups of mathematics questions* that were to be attempted by the children, depending on their level of mathematical ability.

Whilst it is impossible to avoid bias in any study, a sample of 28 home-educating families from the United Kingdom was chosen through *use of the Internet* so that I would not be restricted by travel constraints, and to enable a quick and efficient form of communication with the participants. It is also, however, understood that this approach can lead to some bias (Dillman, Tortora, and Bowker, 1998b), and so the advantages and disadvantages of the use of the Internet during data collection will be evaluated in Chapter 3 (Section 3.8). At the same time, writing ‘mathematical script’ can be particularly hard, and thus a postal approach was taken when obtaining the children’s answers to mathematics questions.

Once the responses were collated, the data analysis was conducted with two key aims in mind, namely to: (1) Establish the main ‘themes or phenomena’ related to the research questions, and (2) Use the identified themes from the questionnaire and mathematics questions to answer the specific research questions. Therefore the data analysis process was structured as follows:

1. Using the data from the parental questionnaires, Chapter 4 first identifies themes relating to the parents’ mathematical beliefs, their approaches to teaching mathematics, and their notions of understanding. Relationships between the various themes are established, where a number of beliefs relating to the importance of mathematics, and the reasons for adopting a particular approach will be highlighted.

2. Chapter 5 then focuses on the influence of the *overall home-educating approach* on the children’s mathematical beliefs and understanding. Three key *types* of home-educating family are identified and examined in detail – namely, the structured/formal, semi-formal and informal approaches. Illustrative case studies are used to demonstrate how these different

approaches impact on the children's mathematical learning. The children's assessed work from the three groups is also analysed, and consideration is given to the specific areas of mathematics that were covered (such as Arithmetic).

3. Once the main findings have been established in Chapters 5, Chapter 6 draws together the key results from Chapters 4 and 6, with the aim of answering the question: "*How did each of the three main home-educating approaches affect the children's perceptions and understanding of mathematics?*"

Chapter 7 concludes the study, returning to answer the original research questions. It is found that the parent's mathematical teaching beliefs can be influenced by a number of factors – namely their personal beliefs towards mathematics, their previous experiences of mathematics in everyday life and work, and their children's previous academic experiences in school. The chosen home-educating approach is often a reflection of their mathematical and teaching beliefs, but it is also observed that the majority are open to changing the way in which they teach mathematics if their children lose interest or fail to understand a concept. The children's mathematical beliefs, whilst sometimes reflective of the parental mathematical beliefs, are also strongly influenced by other factors, such as: (1) The mathematical problems that they engage in, (2) The different ways in which the mathematics is related to them, and finally (3) The beliefs and attitudes of the parents.

When considering the effect of the home-educating approaches on the children's mathematical understanding, one can identify a relationship between the 'ways in which the child learns mathematics at home' and 'the problem solving strategies used in the assessed work. That is, there is evidence to suggest that certain home-educating approaches result in a type of mathematical understanding that can be used in a range of situations whilst other home-educating styles may result in a more limited understanding of mathematics.

Chapter 2: Literature Review

This chapter aims to examine relevant literature to support further development of the research questions. Where appropriate, this chapter will also establish a number of sub-questions, associated with the main research aims that were identified in Section 1.1.

Since this study is focused on two specific areas; that is, *home-education* (Section 2.1) and *mathematics education* (Section 2.2), the literature review is divided accordingly.

2.1 Home-Education

We begin with a historical overview of home-education, followed by an in-depth look at the home-educating population in the UK (Section 2.1.2). Next, Section 2.1.3 focuses upon the main reasons for families to choose home-education, followed by consideration of the different approaches to home-education (Section 2.1.4). Finally, recent research concerning the mathematical 'abilities' of home-educated children will be summarised and analysed in Section 2.1.5, with the intention of highlighting a number of issues that have not yet been considered in previous research.

2.1.1 History of Home-Education in the United Kingdom

Home-education, also known as 'home-schooling' in North America, is a growing form of educational diversity, where parents provide their children with an education at home, rather than sending them to school. Although it is now considered 'unusual' for children to be taught at home, historically it was common for prosperous Victorian families to employ a nursery governess to teach their sons and daughters at home until the age of eight (Menzo and Whitaker, 2001). After the age of eight, girls continued being educated at home by a governess, learning household duties and aesthetic skills, such as music and dancing, until the ages of seventeen or eighteen, when they would generally marry. Boys ceased being taught at home after the age of eight, in order to enter preparatory school. This educational approach stemmed from the Victorian belief that the education of boys was of vital importance — they would eventually become the maintainers of their own families. Girls were valued more for their personal fortunes, appearance and manners, as these qualities would affect their future prospects for marriage (Menzo and Whitaker, 2002).

In the early 20th Century, compulsory education laws, beginning with the 1870 Forster Education Act, along with the introduction of a basic network of mainstream schools, resulted in a shift from 'home-learning' to 'school-learning', with the vast majority of children attending either a state school or 'private school'. At the same time, the education of children was no longer considered primarily the parents' responsibility — the government desired that all children received an education of some form, regardless of their financial background. Consequently, until fairly recently, the majority of children who were educated at home were Travellers (e.g. Romany Gypsies) or from geographically isolated families (Lowe and Thomas, 2002). The decision to teach their children at home was generally a result of the 'unorthodox' lifestyles led by such families, where the option of sending the children to mainstream schools was either impossible or unacceptable. For example, Travelling Families often felt their children would lose their cultural identity in mainstream schools.

Yet over the past thirty years, home-education has become increasingly popular in the UK, the US, and in Australasia. In the UK, when the home-education support group

Education Otherwise first started in 1977, it consisted of a handful of members. These days, Arora (2003) notes that Home Education UK quotes a figure of 1% of the UK population being educated at home (around 85,000 children) rising to an estimate of around 140,000 (Furedi, 2002). Whatever the true figure, all the evidence indicates that the number of UK families choosing to educate their children at home is rising.

2.1.2 Home-Education in the United Kingdom

The UK home-educating community encompasses families from a diverse range of religious, philosophical and political backgrounds, and economic levels. Families vary in size, from single parent families teaching an only child, to parents educating several children at home. Meighan (1997) estimated that there were around 50,000 children 'home-educated' throughout the UK, whilst a more recent estimate (Lowe and Thomas, 2002) puts the figure somewhere between 10,000 and 150,000. Such uncertainty about the figures may be answered by the fact that these 'invisible' families include parents who have chosen to home-educate their children from birth, thus their children will never have been registered with their local education department. It is not compulsory for one to register as a home-educator or seek official permission (Arora, 2003) and indeed, Rothermel (2002) found in her study of 419 home-educating families that between 31 to 65% of home-educators were unknown to their local education department.

As can be inferred from previous studies on home-education, some families may make the choice to home-educate before their child reaches school age, whilst others withdraw their children from school due to dissatisfaction with the educational arrangements. Therefore, one of the key features of this study will be to determine how the family's circumstances influence their decision to choose home-education.

2.1.3 Reasons for Choosing Home-Education

Arora (2003) notes that there has been an increase in UK home-educating families, and suggests a number of factors, including a growing dissatisfaction with the education provided by schools, the introduction of National Testing (especially in primary schools), the widespread availability of educational materials (e.g. Internet resources), and strong support from other home-educating families. For example, Education Otherwise (EO) is a registered charity based in England for families whose children are being educated outside of school. Support is provided through online resources, by telephone and via newsletters.

In the US, Romanowski (2001) noted that there are two main categories of home-educators: *ideologues* and *pedagogues*. Ideologues home-educate because ‘they object to what they believe is being taught in private and public schools and they seek to strengthen their relationship with their children’ (Van Galen and Pitman, 1991, p.66-67). Ideologues aim to pass their values, beliefs and skills onto their children, and claim that the home-environment is the best place to do so. Pedagogues take a different view of home-education, believing that schools are unable to provide a suitable level of teaching to cater for their children’s individual educational needs. Pedagogues may observe that schools have a negative effect on their children’s academic and emotional behaviour, and hence argue that: “breaking from the traditional formal mode of teaching will lead to improved understanding and learning in their children.” (Marchant and MacDonald, 1994, p.66).

In the UK, Rothermel (2002) felt parents tend towards pedagogical reasons for home-educating:

> "...in the US reasons given were; because parents considered it their responsibility to educate the children; to avoid negative peer influences (parents and children) and to control the instructional materials used. In the UK, the main motivations were **freedom and flexibility** so that children could learn **in their own style** and the family could maintain **a close relationship** with time together." (Rothermel 2002, p.345)

Arora (2003) notes that the introduction of National Testing, especially in primary schools, has led to an 'instructive' teaching approach through which teachers tend to be the 'givers of all information', rather than 'encouragers' who enable children to discover ideas for themselves. Indeed, in 2003, delegates at the annual conference of the Association of Teachers and Lecturers in Blackpool argued that young children were highly constrained and pressurised by target-setting and the requirements of the curriculum (BBC 2003, Accessed June 2, 2006). The speakers noted that there needed to be a greater recognition of fun and creativity in the teaching approaches and also indicated that, within primary and secondary schools, the 'instructive' approach led to an increase in health and behavioural problems. Perhaps not surprisingly then, a number of home-educators particularly dislike the emphasis on National Tests in mainstream schools, and the associated effects such tests have on the schools' teaching philosophies and their children.

It appears that both ideologues (most prevalent within the US) and pedagogues (most prevalent within the UK) believe that home-education will strengthen the parent-child relationship, and help avoid possible negative influences from the school environment. Rothermel (2002) indicates that only 13.14% of UK parents home-educate for 'moral reasons', and just 4.17% due to religion. In her small scale study, Yusof (2003) noted that UK home-educators cited the 'inflexibility of schools' and there was a perception that they could provide a 'better' learning environment at home, although she also noted that bullying, religious/moral differences, and children with special needs were also factors that influenced their decision.

A sector of the UK home-educating community that is quite different from those described above are members of the Travelling and Gypsy community. A study conducted by Lancashire County Council (2005) involving over 50% of Local Authorities showed that within the Gypsy/Roma community, 26% elected to be home-educated at the point of transfer to secondary school. Clearly this sector of the home-education community is unique in a number of ways. For instance, since only 10-11% of the families were recorded to be living in housing, the children may have found it hard to attend school regularly if they did not have a fixed address. Additional factors that influenced Gypsy/Roma and Traveller families opting to take their children out of school included a fear of cultural erosion, a supposed lack of relevance within the school curriculum and the fear of racist bullying (Dyer, Anders and Dean, 2004/5).

It can be seen that Gypsy/Roma and Traveller families are a ‘special case’ within the home-educating community as their primary reasons for home-educating are centred on their unique cultural status. For this reason, such families will not be considered in this study, as it would be wiser to consider the particular attributes of this group as part of a separate study, where their special circumstances can be taken into account. It is also important to be aware of children who ‘do not attend school’, but are not given any form of education at home (e.g. those playing truant or suspended/expelled from school). In the context of this research, the term ‘home-educators’ will be used to describe *a family who seeks to make the home environment an educational* *alternative to school* – where some form of learning takes place, rather than any child who is 'not attending school'.

As can be seen from the various literature on home-education, there are a number of different reasons for parents to educate their children at home. However, whilst previous studies all focus on the *parents'* reasons for choosing a home-education, there has been no consideration of the children's level of influence on the decision to stay at home. This is something the current study attempts to correct by eliciting whether or not children contribute towards the final decision. Two interconnected questions thus formed part of a parental questionnaire (See Appendix 1):

- *What were the main reasons behind your decision to home-educate?*
- *Was this decision based mainly on your own personal educational beliefs or did your child express his/her feelings to be educated at home?*

Once the reasons for home-educating have been established, the next step is to examine the various ways in which parents chose to provide an education for their children within the home-environment. This would allow the 'reasons for choosing home-education' to be analysed in the context of a parent's home-educating style, perhaps indicating a relationship between the two.

2.1.4 Different Approaches to Home-Education within the UK

> "...it should be borne in mind that the home educating community is a broad and diverse community of educational philosophies." Mike Fortune-Wood (2005, p.9)

Home-education allows for a range of creative learning opportunities with regards to a number of different educational variables; for example, *where and when* learning takes place, the *learning materials* used, the *rate of learning*, and the *children's level* *of influence* on their learning. Bearing these four variables in mind, Lowe and Thomas (2002) suggest there are three main approaches to home-education: a structured approach, an informal approach and a semi-formal approach.

2.1.4.1 A Structured/Formal Approach

Structured approaches to home education include the more formal methods of working to a strict timetable and adhering to a set curriculum (such as the National Curriculum or a commercial scheme). These parents may need reassurance that they are covering all compulsory subjects available at school:

> "We (the Joubert family) have opted for a learning programme **based on workbooks** at the moment for various reasons. Our two eldest are at an age where they need to start looking at **qualifications** i.e. **GCSE's** or similar...
>
> ...I feel less concerned about things like "are they learning enough?"... Lastly, this method requires the least input from me and yet I feel confident that the children are learning **relevant material**." (School at Home Website, Accessed 2006)

This approach is also frequently used in preparation for formal exams — a structured curriculum enables home-educated children to cover the required syllabus – however, Lowe and Thomas (2002) note that younger children often grow bored with a curriculum that is too predictable and rigid.

2.1.4.2 An Informal Approach

When following an *informal approach* (also known as the *autonomous* approach), the parents generally hold the principle that their children know best as to what suits his/her learning. Accordingly, learning follows the child's questions, and is based on the child's particular interests and problems that he/she is currently addressing. Parents often stress the importance of learning skills to equip their children for the 'real-world', and their teaching approach frequently reflects this belief:

> "M is always encouraged to take part in decisions regarding her education and her own future. She is also encouraged **to use her own initiative** and to make her own judgements...we have found from our previous experience with our son that children are very good at learning all that they need to know if trusted to do so.
>
> ...We usually tend to look at topics in the form of a theme or project, usually taken from some **interest expressed** by M. **Discussion** plays a great part in our approach.
>
> One of our main aims is to fit M for **life in the real world.** We encourage basic skills such as reading and writing, use of computer and calculator, house and garden maintenance, personal safety, self discipline, respect and care for others, for animals, for the immediate community and environment and **for the world as a whole**."
>
> Home-Education.org [Online]

2.1.4.3 A Semi-Formal Approach

> "Time limits **– they're up to you**! Either my partner, depending on who's at home that day, or myself would **encourage the children** to get on with something. That something is pretty well **up to them**, (telly watching not included here, but that's our personal choice!) it could be writing a letter, playing, drawing, using a CD Rom, making something, reading, constructing with Lego, taking an old video to pieces, cooking, even cleaning out the bedroom.
>
> ...We find **workbooks valuable to cover any short fall** we feel there might be in their skills, but we find ourselves turning to them less and less. There are so many beautifully illustrated books and computer programmes that the children love to sit and use, or explore virtual worlds... Or how many metres such and such is, (from playing in the garden)."
>
> Education Otherwise [Online]

The above extract describes a typical implementation of the *semi-formal approach*, where the learner has considerable influence on the educational arrangements, while parents are able to use their experience and organisational skills to help facilitate their children's learning through a range of activities. For example, note that the family above does not adhere to a strict timetable, and the parents do not necessarily choose the day's activities. A variety of resources are used as required, including workbooks, computer programmes, and real-life activities. When adopting a semi-formal approach, learning typically involves much discussion and interaction between child and parent. At other times, the children may work independently, with the opportunity to explore areas of interest without adult intervention or time limits.

2.1.4.4 Summary of Approaches

Structured	Semi Formal	Informal
• Children have little input towards the choice of material – it is the parent who determines what is to be studied, and when	• Both parents and children have input into the choice of learning activity, as children have the opportunity to follow their own interests when learning	• All learning is child-directed, and based on the child's current interests. Parents' role is to provide an 'educationally stimulating' environment
• Majority of the teaching is from a set curriculum, usually consisting of workbooks, followed in a sequential order	• Parent plays the role of a facilitator, with an aim of encouraging the children towards educational activities. Workbooks are generally used only when required	• No set curriculum or work books are used
• A timetable of learning at set times during the day	• No strict 'timetable' of when learning should take place.	• No timetable used
• The need to cover materials based on the National Curriculum	• Evidence of a range of learning activities, such as real-life, computer programs and workbooks	• There may be an emphasis on 'learning to cope in the real-world'
• Emphasis on "knowing" - little discussion.	• Discussion is encouraged, with parents providing additional support when the child has difficulty grasping a concept	• Discussion based activities are very common

Table 2.1: Comparison of Different Approaches to Home Education within the UK

As can be seen from the above approaches to home-education, the main variations are associated with the range of activities used, timetables of learning, use of schemes and workbooks, degree of influence of the children, and the aims and goals of the parent/children. Lowe and Thomas (2002) felt that the majority of home-educating families favoured a semi-formal approach to home-education since this allowed some form of structure when required, but maintained the flexibility to adopt informal learning approaches. Rothermel (2002) also found the majority of families adopting both formal and child-centred learning activities in their daily routines, with 3.4% of parents taking a formal or structured approach to learning, 59.3% following both structured and informal learning during the day, and 37% adopting the informal ‘child-directed’ approach. Considering each of these approaches in the context of teaching and learning mathematics, Yusof (2003) reported that of the 30 families in her study, 60% were ‘completely flexible’ as to when their children studied mathematics. Though home-educators averaged 1-2 hours of mathematical learning per day this varied greatly amongst families. Teaching was frequently one-to-one for 96% of the families, regardless of the number of children, and the choice of mathematical activities suggested four main priorities:

- The child’s interests
- The real-life application of the mathematical concept
- The child’s personal learning style, and whether it would help the child’s understanding,
- Personal preferences of the parents and available resources etc.

Thus, there was little evidence that many families in Yusof’s study followed a structured approach. Instead, they generally devised a mathematics curriculum according to the needs of their child; formal textbooks were largely used as a guide or to provide suitable questions when focusing on a particular mathematical concept. Therefore, giving support to Lowe and Thomas (2002) and Rothermel’s observations, Yusof (2003) found that the majority of families adopted a semi-formal approach with regards to the teaching of mathematics. Yusof’s (2003) study also suggested that within the home-educating community, different philosophies towards learning mathematics existed, with evidence to indicate that the *educational* and *mathematical beliefs* of the parents had an effect on the ways in which their children were taught mathematics. For example, a number of home educators felt school would jeopardise their child’s education, believing that the curriculum was inflexible. These parents gave their children relative freedom as to ‘where and when’ they studied. Other home-educators tried to incorporate mathematical activities into their everyday lives whenever possible, which was consistent with their belief that ‘mathematics is essential for everyday life’. As mentioned earlier, one aim of this study is to gain a better understanding of how these various approaches to teaching mathematics can affect a child’s perceptions and understanding of mathematics. But before considering literature associated with the learning of mathematics, we first review previous research to gain an insight into the possible effects of home-education on mathematical achievement.

2.1.5 Home-Education and Mathematical Achievement

Although no previous study has considered the relationship between home-education and mathematical understanding in any depth, past research has measured home-educated children’s performance on standard mathematics tests. Two earlier studies, the first conducted in the US (Rudner, 1999) and the other in the UK (Rothermel, 2002) will be discussed.

2.1.5.1 Demographics of Rudner’s Home-Educating Families

Rudner (1999) considered 11,930 home-educating American families in his study on the ‘*Scholastic Achievement and Demographic Characteristics of Home School Students in 1998’*. He found that the home-educating parents had **more formal education** than the general population of the US, i.e. 88% of these parents continued their education beyond high school, compared to 50% for the US as a whole.

2.1.5.2 Performance on Standard Tests

The home-educated children in Rudner's study were given the Iowa Tests of Basic Skills (ITBS) or the Tests of Achievement and Proficiency (TAP) across a range of subjects, including mathematics. Relationships were found between student achievement, parental educational background and the resources used. It was found that **25% of home-educated students were working at one or more grades above their age-level peers** in public and private schools, where the median scores were generally in the 70th to 80th percentile, and far exceeded those of public and private school students. Moreover, Rudner indicated that children who had been home-educated throughout their life achieved higher test scores than students who had also attended other education programs, but no significant differences in achievement were noticed with regards to gender, or formal teaching experiences.

2.1.5.3 Demographics of Rothermel's Home-Educating Families

Rothermel's (2002) study considered the aims and practices of 419 home-educating families from diverse socio-economic backgrounds. 196 assessments were used to evaluate the psychosocial and academic development of home-educated children aged eleven years and under, and according to Rothermel, it was the first UK study involving home-educated children and their families to use a range of methodologies and a large sample. Approximately half the home-educating families in Rothermel's (2002) study mentioned their poor experiences with schools, whilst the remaining families were influenced by their choice of lifestyle. Nevertheless, although a number of parents cited a dissatisfaction with school as the main motivating factor, once they began home-educating their children, many other benefits of home-education were noted. For example, one advantage was having the space to develop non-academic intelligences. Furthermore, there was also greater opportunity for family activities, discussion and spontaneity within the learning. A feature that was common to all families was the 'flexible approach' to education and the high level of parental attention received by the children.

Rothermel (2002) found that around 50% of the home-educated children had been home-educated from birth and 50% had been withdrawn from school, where families tended to withdraw one child from school and then subsequent children would be home-educated from the beginning. Parents who were more confident generally avoided following the National Curriculum, whereas those who were less confident tended to base their teaching around the National Curriculum.

2.1.5.4. Performance on Assessed Work

The PIPS (Performance Indicators in Primary Schools) assessments were developed by the Centre for Evaluation and Monitoring (CEM) at Durham University to track a number of aspects of schooling as pupils moved through the primary sector in England and Wales (PIPS Assessments, Online reference). They are generally used by schools to gather data on a range of variables that are grouped into measures of academic attainment, developed ability and attitude, which are then used to calculate measures of relative progress. For example, in Year 2 the assessment is made up of three sections, each taking about half an hour to complete. The first two sections assess mathematics and reading, an example of a mathematics question (Figure 2.1) shown below:

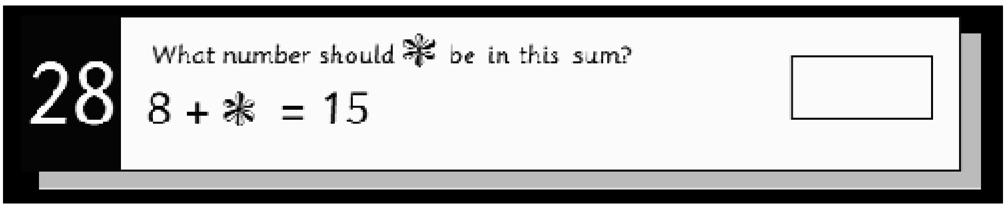


Figure 2.1: Example of Mathematics Problem from the PIPs Assessments (Online)

Rothermel (2002) tested each home-educated child at three different stages: **Start of Reception Mathematics** (4-5 year olds), **End of Reception Maths** (6 year olds), and **Year 2 Maths** (7 year olds) using the PIPS Baseline Assessments. This allowed her to measure the children's performance at each stage of the assessment as well as their progress over the years.

The PIPS Baseline assessment data revealed that 64% of the home-educated children scored over 75% on the assessment, whereas, nationally, just 5.1% of children scored over 75%. Socio-economic class was not an indicator of achievement – in fact, the home-educated children from lower socio-economic groups performed better than those from the middle class. Furthermore, at least 14% of the home-educating parents were employed in manual and unskilled occupations. The children's level of attainment was not limited in any way by their parent's level of education, where approximately 38% of parents were educated at comprehensive schools, and 21% had no post-school qualifications. Although 47.5% of the home-educators in Rothermel's study had attended university, at least 27.7% had not. Ray (1997) believes that because the majority of home-educating parents taught their children on a one-to-one basis, the parent's educational background had less influence on their children's academic performance.

2.1.5.6 Summary

Rudner (1999) and Rothermel's (2002) studies both support the fact that, in the UK and US, home-educated children are outperforming their schooled counterparts.

Lubienski (2003) argues that while Rudner's (1999) study showed that home-educated children outperformed school children on standardised tests, these findings do not prove that home-education is a more effective form of education than school. He believes that there are additional factors that may affect the results of such comparative studies – e.g. the higher income and educational levels of the home-educating parents compared to the parents of the schooled children, as was noted by Rudner (1999). The findings of Rothermel's (2002) study, however, suggest that the income and educational levels of the parent are not indicators of achievement. In fact, it was the children from lower income groups who outperformed those from higher groups. Rothermel believes that the key factors in determining the children's academic development and progress was the level of parental input, along with a flexible approach to education. However, Lubienski (2003) suggests that:

> "Without knowing how many people are home educating, for what reasons, in what ways and to what effect, we cannot draw compelling conclusions about the degree to which the act of homeschooling boosts academic performance, especially relative to other forms of education..."
> Lubienski (2003, p.172)

Although prior studies have measured and compared the children's marks on standard mathematics tests, there is no consideration of the ways in which the parents teach their children mathematics, the particular ways in which home-education benefits mathematical learning, and the consequent effect on their children's mathematical understanding.

> "…whilst the academic assessments showed how well these children could perform, they gave no indication of the type and breadth or depth of education these children were engaged in."
> (Rothermel, 2002, BERA Working Paper)

Unlike previous studies, this research seeks to determine the range of understanding exhibited by home-educated children through their problem solving approaches rather than an evaluation of their scores on a standardised mathematics test.

But before turning our attention to the area of mathematical understanding, it is necessary to consider the relevant factors that can influence a child's mathematical knowledge. We begin with review of the literature on mathematics teaching in mainstream schools, with a focus on 'the main factors that could affect a teacher's approach when teaching mathematics in a classroom'.

2.2 Mathematics Education

Earlier studies on home-education (Rothermel, 2002 and Lowe and Thomas, 2000) identify three main categories of home-educating approaches (i.e. structured, semi-formal and informal) but **no attempt** has been made to identify the relationship between parental reasons for home-educating and the chosen approach to mathematical learning. Consequently, in the context of mathematics learning and as an extension of the author's previous study (Yusof, 2003) the current study addresses the following areas:

(1) Parent's beliefs about mathematics and the effects that these beliefs may have on their home-educating approach.

(2) The effect of the parent's mathematical/teaching background on their home-educating approach.

(3) The effects of a particular style of home-education on the children's perceptions of home-education and their learning environment.

(4) The effects of the parents' home-educating approach on their children's perceptions of themselves as the learners of mathematics, and their mathematical understanding.

In an attempt to address these issues, we first review the literature on mathematics teaching in mainstream schools. This will begin with a consideration of *mathematical belief* (Section 2.2.1) and then *mathematical knowledge* (2.2.3), with an aim of identifying the main factors that could influence the approach used when teaching mathematics.

2.2.1 Mathematical Beliefs

Beliefs underpin an individual's personal thoughts and behaviour thus influencing their disposition to adopt certain practices but not others (Swan, 2006 and Schoenfeld, 1992). In other words, an examination of the home-educating parents' beliefs could enable us to form a picture of the environment in which the mathematical learning takes place. Underhill (1988) summarises research on mathematical beliefs into the following four areas:

1. *Beliefs about mathematics as a subject*, in particular, the nature of mathematics. For example, one commonly held belief is that 'mathematics is about following a set of rules to solve arithmetical problems'.

2. *Beliefs about the learning of mathematics* – e.g. "What is helpful/unhelpful when learning mathematics?"

3. *Beliefs about teaching* – e.g. "What is effective teaching?"

4. *Beliefs about the social context* – "How are the students influenced by the behavioural norms of the learning environment?"

Ernest (1991a, 1991b) divides a mathematics teacher's belief system into three components: (1) Their *conception of mathematics* as a subject for study, (2) The *nature of mathematics teaching* and (3) The *process of learning* mathematics. Note that both Ernest and Underhill describe beliefs concerning the *nature of mathematics as a subject*, beliefs about *teaching*, and beliefs related to the *process of learning mathematics*. However, Underhill's fourth point is also highly relevant to this study since a key difference between the 'teachers' of mathematics in this study (i.e. the parents) and school teachers is that the social context of the mathematical learning environment is *at home* rather than *at school*. For this reason, the current study will examine beliefs associated with the parents':

(1) Conceptions of mathematics as a subject

(2) Beliefs associated with the teaching and learning of mathematics and

(3) Beliefs about the social context of the home-educating environment in relation to learning mathematics – "What behaviours and practices are encouraged in a home-educating environment?"

Additionally, the *children's beliefs* regarding each of the above components will also be addressed since this will identify some degree of relationship or difference between the beliefs of the child and parent.

Given that an individual holds beliefs from each of the above components, we next ask the question: *"If a parent holds certain mathematical beliefs, what effect will this have on the family's approach to learning mathematics?"*

2.2.2 The Influence of Beliefs on Teaching Practice

It can be argued that beliefs are manifested through classroom practice associated with conceptions (Ernest, 1989a) and orientation (Askew, Brown, Rhodes, Johnson and William, 1997). In their study of effective teachers of Numeracy, Askew et al (1997) identified three types of teaching orientation that that could be associated with beliefs concerning the nature of mathematics:

- **Transmission –** projecting a view that mathematics is a series of *rules and truths*, which must be conveyed to the student through an instructional approach until fluency is attained.

- **Discovery –** where mathematics is viewed as a human creation and students are encouraged to learn through individual exploration and reflection. Teachers are seen as facilitators.

- **Connectionist –** mathematics is seen as network of ideas that the student and teacher construct through joint discussion. In addition, the teacher aims to challenge the students' thinking.

To observe a teacher's *beliefs,* and determine their particular teaching orientation, with regards to mathematical learning, Ernest (1989a) suggested that the teacher's main *aims,* together with their *behaviour during the lessons* should be considered. Therefore, in this study, questions will be asked regarding the home-educating parents' perceptions of themselves as teachers, their aims during mathematics lessons, and the mathematical goals and targets set for their children. This may help us to identify the key teaching beliefs of the parents and relate this to their mathematical beliefs and their teaching practice.

2.2.3 The Influence of Subject Knowledge on Mathematics Teaching

The mathematical beliefs of a teacher are not the only influence on the teaching approach – the *mathematical knowledge* of the teacher can also play a major role (Ball (1991). Whilst the mathematical knowledge of parents who home-educate has received limited (if any) consideration, within this study it will be considered in the context of the parents' actions in developing the mathematical understanding of their children. In contrast, a number of studies have considered the relationship between a school teacher's knowledge and belief systems, and the influence of these elements on the teacher's approach towards teaching mathematics. This knowledge will be considered through two main areas, namely: Section 2.2.4 - Subject matter knowledge and Section 2.2.5 - Pedagogical knowledge.

2.2.4 Subject Matter Knowledge

Shulman (1986, p.9) describes *subject matter knowledge*, as 'the amount and organisation of knowledge per se in the mind of the teacher'. Ball (1991) writes that a teacher's knowledge about mathematics includes their views on what it means to 'do and know' the subject, and their philosophical opinions on mathematical learning. He stresses that *teachers need to have strong subject-matter understanding to be effective teachers of mathematics.*

Within the current study, a number of factors associated with a home-educating parent's mathematical knowledge will be investigated to examine the effect of 'parental mathematical knowledge' on their home-educating approach. Yusof's study (2003) showed that in some families, one (or both) of the parents had higher educational qualifications. On the other hand, some were only educated up to GCSE's/O-Levels. This was also noticed in Rothermel's (2002) study on home-education, which revealed that approximately half of the parents had received no further education after leaving school. Therefore, it is conjectured that there may be three main differences between the knowledge base of a home-educator and a mathematics teacher:

(1) The parent may not have had any formal teaching experience

(2) Their formal qualifications and knowledge will vary considerably – some may have been educated up to GSCE level, others could have reached postgraduate level

(3) The parents' mathematical experiences could be derived from a number of varied situations – the workplace, their former schooling, or the home environment

Taking these points into consideration, the study will focus on the parents':

- Highest level of mathematical qualification – GCSE, A-level, degree/university level, etc.

- Use of mathematics in current or former employment – either by the respondent, or other close family members (husband and so on)

- Effect of parent's prior mathematical experiences on their home-education approach – were they an advantage or disadvantage?

Once their mathematical background has been established, it remains to be seen if the home-educator's level of mathematical knowledge will have an influence on the way mathematics is taught within the family home.

2.2.5 Pedagogical Knowledge

Although it is indicated that there is **no simple relationship** between a teacher's formal qualifications, their understanding of mathematics and their teaching practice, this does not imply that these factors have little influence on their students' understanding of mathematics. Rather, there may be other issues that could affect the teaching; for example, the content delivery style (e.g. rote learning) and the mathematical activities used to illustrate a concept, may both affect a student's perception and knowledge of mathematics. These notions are associated with the idea of *Pedagogical Knowledge* which, according to Shulman (1986), consists of knowledge of the curriculum, teaching and management, students, and the evaluation of students' progress. Key factors that influence the mathematics lessons **in school** (Leinhardt, Putnam, Stein and Baxter, 1991) include knowledge of:

1. The mathematical goals to be accomplished, and the steps needed to attain those goals

2. A curriculum that consists of the concepts to be taught, activities used and any concept-related problems that could arise in the future. This contributes to the running of the class and is strongly influenced by the exam system, where teachers aim to cover the syllabus required for the forthcoming exams, such as SATs, GCSE's and A-levels

3. The types of experiences and activities that are used during the course of instruction, which could facilitate student learning

4. Activities used during explanation of a concept, including workbooks, verbal discussion and pictorial images, which are used to promote understanding, and an awareness of the strengths and weaknesses of each approach

Accordingly, in this study, we ask: *"What pedagogical knowledge do home-educating parents possess?"* Although Rothermel's (2002) UK sample showed that 29% of the parents were teacher-trained, there is no evidence that this sample is representative of all UK home-educators, as it is generally understood that families who choose to participate in home-education research tend to have a strong interest in educational issues (Arora, 2003) and it is not surprising that a large percentage would have a teaching background of some form. On other the hand, even if parents possess a level of teacher-training, this does not necessarily imply that they will home-educate their children according to methods derived from their teacher training experiences. A home-educating family in the UK is under no obligation to have any plan with regards to the mathematical content cover, as the legal requirements of the Education Act 1996 indicate that:

> "The 1996 Act makes no attempt to define 'suitable education', and disputes over educational provision rarely come to court, so there is little case law to help with this. However, in the case of *Harrison & Harrison v Stevenson* (appeal to Worcester Crown Court 1981), education was held to be 'suitable' if it was such as to prepare children for life in modern civilised society; and to allow them to achieve their full potential.
>
> This definition is a very general one and can encompass a variety of educational styles and methods. Families are entitled to choose whatever they feel to be the most suitable approach to learning at home for their child." Education Otherwise (Online)

Consequently, as well as considering possible previous teaching experiences of the parents, we ask if these experiences have influenced their approach to home-education.

Now that we have considered mathematics education literature in the context of the 'teachers of mathematics', the next section will examine the possible effects of different teaching approaches on the children's mathematical understanding.

2.2.6 Children's Mathematical Understanding

> "One of the most widely accepted ideas within the mathematics education community is that students should understand mathematics. The goal of many research and implementation efforts in mathematics education has been to promote learning with understanding. But achieving this goal has been like searching for the Holy Grail. There is a persistent belief in the merits of the goal, but designing school learning environments that successfully promote school understanding has been difficult."
>
> Hiebert and Carpenter (1992, p.65)

The topic of mathematical understanding has been an area of much discussion and debate over the years, where the idea of 'learning with understanding' has been examined in areas far beyond the boundaries of mathematics education. For example, within the cognitive science sector, much emphasis has been placed on modelling internal representations with considerable precision (Gardner, 1985). Researchers from the anthropology and social science disciplines favoured theories based on situated knowledge and social influences in attempts to explain the understanding associated with learning in everyday situations and the lack of such understanding that resulted from formal school learning environments (Lave, 1988; Brown, Collins, and Duguid, 1989; Nunes, Schliemann and Caraher, 1993). Clearly understanding is a fundamental part of learning; hence it is also clear that many researchers believe a model of learning should include aspects of understanding - regardless of the importance given to social and environmental factors.

Yet despite extensive study no school education system has been designed that can promote understanding in the mathematics classroom with any reasonable degree of certainty (Hiebert and Carpenter, 1992). It is also the case that although the mathematical achievements of home-educated children have been measured through standard tests, there has been no consideration of the children's mathematical understanding. Moreover, there is no data on home-educated children's strategies used when solving mathematical problems, nor of their perceptions of what it means to 'understand' something in mathematics.

Hence, in this section, some of the key issues related to mathematical understanding will be addressed. Theories concerning the various types of understanding associated with mathematical thinking are considered together with the consequences of understanding (or not understanding) a mathematical concept. The first question to be addressed is *"What do we mean by the term 'understanding'?"* In other words, when is it possible to say that someone has successfully understood an idea?

2.2.7 What Does It Mean To Understand Something in Mathematics?

The dictionary definition of 'understand' is "to grasp the meaning of… to have thorough technical acquaintance with… to be thoroughly familiar with the character and propensities of." (Merriam-Webster's Online Dictionary). It is easy to say "I understand" or "I do not understand" in specific situations. But when learning mathematics, one cannot simply rely on intuitive feelings to measure depth of understanding. Nickerson (1985, p.216) writes:

> "Consider the relatively straightforward question of what it means to understand a word or term. One might take the position that one understands a word if and only if one can define it correctly. But this answer will not do. Clearly, it is possible to understand a word well enough to use it appropriately in specific contexts without being able to produce a definition that is unambiguous in all contexts".

At the same time, being able to define a word accurately does not imply that someone fully understands the meaning of the word, as understanding depends on the context.

Nevertheless, although it may be impossible to say whether or not one fully understands something, Nickerson (1985) believes that if understanding is to form a primary part of education, then one is obliged to make an effort to understand, even though this is only a partly successful endeavour.

Some studies appear to associate understanding with the ability to ‘do’ or ‘have’ a set of skills - for example, Gagné and Driscoll (1988) focus on instructional methods of teaching. They classify intellectual skills into **rules** (which allow a student to *do* something) and **higher order rules -** a set of simple rules combined to form a more complex rule, stressing that, “The statement of a rule is merely the representation of it – the rule itself is a learned capacity of the learner” (Gagné and Driscoll, 1988, p.51). In other words, a learner has only learnt the rule when he can use it in a variety of situations. But one might ask whether knowing how to correctly *apply* a rule means that the learner *understands* the rule.

Within mathematics and mathematics education, recommendations such as those of Skemp (1976), Hiebert and Lefevre (1986) and Sfard (1991) emphasise the importance of ‘knowing more than just the rules’. They conjecture that this additional knowledge can lead to a deeper form of understanding. Though they describe a higher form of understanding based on ‘knowing why the rules are used’ (Skemp, 1976), the connections between pieces of mathematical knowledge (Hiebert and Lefevre, 1986; Duffin and Simpson, 2000) and the structural properties of the mathematical concept (Sfard, 1991), a common assumption made by these researchers is that understanding can vary in degree or completeness.

2.2.8 Relational and Instrumental Understanding

According to Skemp (1976), there are two meanings of the word 'understanding', which are distinguished by the terms:

- **Relational Understanding** 'knowing both what to do and why it is done'
- **Instrumental Understanding** 'applying rules to the problem without justification'

Skemp provides an example of a newcomer to a city when describing the differences between these types of understanding. A person may learn their way from their house to their place of work, and then learn the route from their house to a friend's house. These *isolated bits* of knowledge are compared to the knowledge gained by going out and *exploring* the city. Skemp suggests that a person who only knows some isolated facts is more likely to get lost than someone who builds a mental map. In the same way, a person who only has an instrumental understanding of a mathematical concept may have difficulty when facing a problem that does not 'fit the rule'.

Skemp argues that when these differing concepts of understanding are applied to the teaching of mathematics, they result in such different kinds of knowledge it suggests that 'there are two effectively different subjects being taught under the same name, 'mathematics' ' (Skemp, 1976, p. 22). It may be the case that teaching 'rules' is easier than teaching 'why we use those rules' because less cognitive effort is required from both the teacher and the pupils. Moreover, Skemp suggests if a student's primary mathematical goals are exam grades then this can incline them towards a very instrumental approach, regardless of how their teacher presents the material. Thus the student's attitude towards mathematics learning can also affect their understanding but perhaps what may be even more damaging is a student's inclination to learn relationally while the teacher does not have the time, the resources or possibly even the subject and pedagogic knowledge to teach in this way.

2.2.9 Procedural and Conceptual Understanding

Hiebert and Carpenter (1992) distinguish two kinds of mathematical knowledge, namely *procedural* and *conceptual* knowledge; although interestingly they make no reference to Skemp's notions of understanding. They identify procedural knowledge as the knowledge necessary to carry out a sequence of actions. Such knowledge is often manifested through the manipulation of symbols in a step-by-step process; for example, applying the quadratic formula to obtain the solutions to a quadratic equation, without fully comprehending the derivation of the formula.

In contrast, conceptual knowledge is seen to be knowledge that is rich in relationships and consequently it possesses two key features — it is part of a network and it is essential for expertise through its relationship with procedural knowledge. Their suggestion that a mathematical concept is understood if it is part of an internal network of representations implies that the more connections within the network to a particular mathematical idea, the more strongly the idea will be understood. An awareness of these mathematical structures and their appropriate connections is the essence of understanding (Duffin and Simpson, 2000), and thus a failure to understand derives from a limited perception of possible connections with the consequence that the learner tries to implement a set of procedures without knowing 'why' these procedures are being applied.

Duffin and Simpson (2000) use the term *reconstructing* to indicate that a learner is able to develop understanding by reconstructing knowledge through use of appropriate connections — such an individual does not need to remember all of the detail. In contrast, they suggest that the individual who does not fully understand a mathematical concept is merely *reproducing* the idea. There are many similarities between Hiebert et al. (1992) and Duffin and Simpson's theoretical ideas, as both are based on a framework of internal connections, with the strength of understanding associated with the number of connections. Though they refer to understanding (Skemp, 1976) and knowledge (Hiebert and Lefevre, 1986), the similarities between the two ideas draw attention to different perceptions of mathematics and its application. From the perspective of definitions, both notions could lead to the perception that the different forms of understanding/ knowledge are at opposite ends of a spectrum. At one end, the understanding and knowledge of mathematics reflects strong networks and connections, whereas at the other end of the spectrum, the understanding and knowledge is isolated and reflects limited connections. On the other hand, perhaps an individual may view some areas of mathematics conceptually, but other areas are understood procedurally. Of interest within this study is the *'perception of mathematics that is encouraged through the home educating parents' teaching'* and the *'result as perceived by the child and reflected through their learning'*.

Furthermore, we ask, *"What types of mathematical understanding are encouraged by the parents, and how is this reflected within the child's articulation and application of 'what it means to understand/not understand' a mathematical concept?"*

2.2.10 How Can a Student's Level of Understanding Be Measured?

We shall first consider the ways of detecting when a student *does not* possess a complete understanding of a mathematical concept. One way of noticing a lack of understanding is in the application of rules, particularly in their overgeneralisation. Kuhn and Phelps (1982) observed that students tended to persist in using procedures once the rules were well-rehearsed, but unfortunately they frequently applied them with little reflection or examination of the calculation process, leading to overgeneralisations of a particular rule. Hiebert and Carpenter (1992) indicate that many misconceptions in mathematics arise from such attempts. Unfortunately, this kind of learning is hard to detect until the student is unable to solve a problem, as Skemp (1987, p.33) pointed out:

> "Learning to manipulate symbols in such a way as to obtain the approved answer may be very hard to distinguish from conceptual learning. The learner cannot distinguish between the two if he has no experience of genuinely understanding mathematics…
>
> The amount a bright child can memorise is remarkable, and the appearance of learning mathematics may be maintained until a level is reached at which only true conceptual learning is adequate to the situation."

He suggested that teachers can overcome this problem by testing the adaptability of the learner to new, though mathematically related situations. Within this study, in order to determine whether or not a home-educated child is 'blindly' applying a rule, the children will be given a number of problems that involve the same mathematical concept (e.g. fractions), but in a variety of situations (see Section 3.4 in methodology).

Nickerson (1985) claims that, in general, we expect our understanding of something to increase over time; nevertheless, there are many factors, other than time, that can affect it. Even if a concept is understood the first time a student uses it, practice and discussion is also needed to ensure the idea 'sinks in'. Hiebert and Carpenter (1992) claim that a failure to receive sufficient reinforcement immediately after a new concept is introduced provides the strong possibility that essential points will be forgotten. If initially learned with sufficient depth, the concept may, more appropriately, be stored within ‘long-term’ memory. The incorrect use of a mathematical concept over a long period of time is unlikely to increase understanding. In reality, it is more likely to *decrease* understanding, as misconceptions are formed and reinforced. These misconceptions can be deep-rooted and extremely hard to remove (Fischbein and Schnarch, 1997).

In this study, the home-educated children will be asked what they do if they have difficulty understanding a particular concept. The purpose will be to identify whether they seek and receive clarification from their parent (or tutor). Furthermore, the home-educators will be asked to identify the ways in which they measure their children’s understanding, which may reveal the different ‘signs’ they look for in order to be reassured that their children have understood the concept, and in particular whether the emphasis is upon instrumental or relational understanding.

2.2.11 Social and Cultural Factors Affecting Understanding

Resnick (1987b), Lave (1988) and Greeno (1989) suggest that a complete description of understanding should include analysis of situated and social activity. In this study, the mathematical learning takes place within the home environment, but since there has been no consideration of the different types of mathematical understanding found in home-educating families, we turn to research from mainstream schools in the UK in order to clarify the possible factors that could be of influence.

Boaler (1998) examined the influence of two quite different teaching approaches on a number of Year 9 to Year 11 (pre-GCSE) school-educated students. She considered the teaching practices of two schools, where one school strictly followed a ‘closed’ traditional textbook approach, with little room for discussion or exploration. The other school predominantly used open-ended ‘project based’ activities when teaching mathematics. It was found that students who followed the traditional approach developed a procedural, rule-based understanding of mathematics that was of limited use in mathematics problems that were ‘not typical of their textbook questions’ (i.e. ‘real-life’ situations or an unusually worded assessment) even though the students possessed the necessary skills to satisfy the requirements of their exams.

Students in the open, project-based learning environment developed a conceptual understanding that gave them the ability to apply their mathematical knowledge in a range of assessments and situations. Although their mathematical learning was somewhat unstructured they were more successful than the ‘traditional students’ when applying mathematics to real-life tasks, and performed equally well in their GSCE mathematics exams. In other words, Boaler’s research revealed that *different approaches to learning led to different forms of mathematical* understanding, giving support to Skemp (1976), who proposed that the way in which mathematics was taught could determine whether the student learned a concept relationally or instrumentally.

Boaler (1998) believed that a flexible, open approach to mathematics gave students a deeper level of mathematical understanding that was transferable to situations outside the classroom environment. Students appeared to develop the ability to explore unfamiliar situations and then choose a suitable method of solution. But unfortunately, a major disadvantage was that **20% of the students** attending the progressive ‘project-oriented’ school **intensely disliked the unstructured approach.** Some students could not work in an unsupervised setting without guidance – they would have preferred following formal exercises in addition to the projects. As a result, only 11% of the ‘open, project-based’ school students, and the same percentage of the ‘traditional’ students, achieved an A-C grade in their GCSE mathematics exams, although the former showed a superior performance on real-life problems. This suggests that **neither a predominantly ‘rule-based’** nor an **entirely ‘open’ project based approach** enables students to perform well in *both* real-life and exam situations, especially if the students are unhappy with the approach. Therefore, in this research, we investigate if the home-educators’ approaches to teaching mathematics allows the children to develop an understanding of mathematics that allows them to use mathematics in a variety of situations. At the same time, we see if this approach gives their children the opportunity to learn mathematics in the way that they feel most comfortable with.

2.3 Summary of Literature Review

In summary:

- Section 2.1.1 of the literature review considered the history of home-education in the UK, before examining the nature of this educational choice as it is today (Section 2.1.2). Parents chose home-education for a number of reasons, the two main reasons being: (a) **Academic –** parents were dissatisfied with the teaching in school and (b) **Social/ideological** – parents were not happy with the ‘values’ and social behaviours within the school environment. This study will consider the reasons parents chose home-education, as well as whether the opinions and views of their children influenced the decision in any way.

- Section 2.1.4 examined the different ways that parents could implement home-education, with the three main approaches being: (1) Structured, (2) Informal and (3) Semi-Formal. These approaches mainly differed with regards to the level of influence the child has on the learning, the use of curriculum and other learning activities and the regularity of teaching. An important aim of this research will be to identify various approaches to teaching mathematics, and the resultant effects on the learning of mathematics.

- Section 2.2 focused on the key areas of mathematics education that will feature in the study – (1) Mathematical and teaching beliefs (Sections 2.2.1-2.2.2), (2) Mathematical and teaching knowledge (Sections 2.2.3-2.2.5), and (3) The effects of (1) and (2) on the teaching and learning of mathematics. All three areas will be addressed when considering the mathematical background of the parent.

- The latter sections (Sections 2.2.6 to 2.2.11) focus on mathematical understanding, where two main meanings of the word *understanding* are distinguished: (1) An understanding that is rule-based, where one is able to follow a sequence of procedures, and (2) An understanding that consists of a number of connections, as well as knowing the relationships between these connections. We seek to identify the types of understanding within the home-educating environment, as well as the particular features of the children's learning environment that could encourage a certain 'type' of understanding, a focus of Section 2.2.11, where the influence of social cultural aspects of the environment on mathematical knowledge was examined.

Now that the research questions have been drawn out from a review of the literature the next chapter describes the approach that was used in order to collect the necessary data.

Chapter 3: Methodology

3.1 Introduction

This chapter covers the development of the research methodology that was constructed for the purpose of data collection. To select the most appropriate approach, we first refer to the specific aims of this research, which were highlighted in Section 1.1, and summarised as follows:

1. Understand the main reasons for the families to choose home-education.
2. Examine the influence that parental mathematical belief/background has on the home-educating approach.
3. Investigate the different types of mathematical belief and understanding that exist amongst home-educated children.
4. Formulate relationships between parental teaching approaches and their children's mathematical beliefs/understanding.

As the study aims to address a number of different areas of education - such as the *parents' perceptions of themselves as teachers* and the possible *types of mathematical understanding exhibited by the childre*n - it is clear that a number of factors may need to be measured. To clarify the ways in which the methodology will be developed to address these areas, Figure 3.1 illustrates the main research issues associated with the parents, with Figure 3.2 illustrating the issues associated with the children.

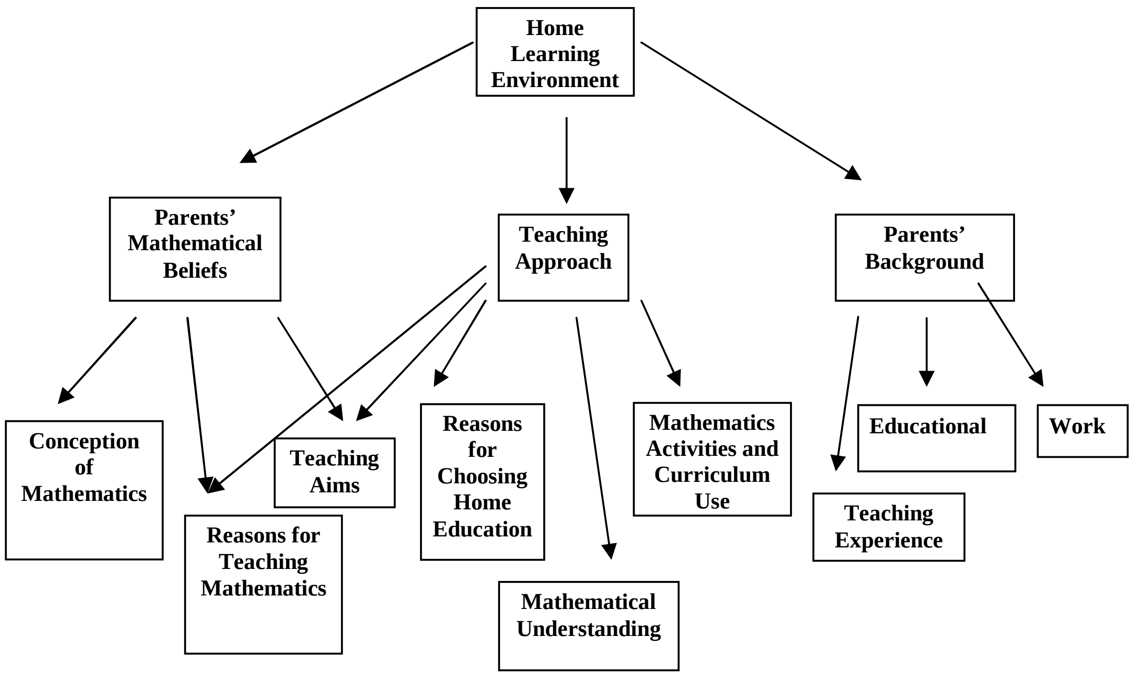


Figure 3.1: Issues Considered in the Investigation of Home-Educating Parents Teaching of Mathematics.

For the parents, we focus on the: (1) Teaching approach, including the reasons for choosing home-education, (2) Mathematical beliefs, and how this could influence the ways in which mathematics is taught, and (3) Background of parents. At the same time, relationships are sought between all three areas. Yusof (2003) used questionnaires to gather information associated with home-educating parents and their approaches to mathematics education. Information on areas such as the 'parental perceptions of mathematics learning' and 'the range of mathematical activities used at different stages of the child's development' was obtained, leading to outcomes that suggested home-educating parents generally adapted their teaching according to the needs of their child. Hence, for the current study, it was considered that for the **first phase** of the data collection process, which would *only consider the parents*, it would be appropriate to follow a similar approach. That is, *using questionnaires to obtain information from the parents* for the issues highlighted in Figure 3.1.

At the same time, due to the limitations of a small-scale dissertation, Yusof's study did not consider the effects of home-education on a child's perceptions of mathematics. Nor did it examine the 'types' of *mathematical understanding* that could result from a home learning environment. These additional areas of study were also noted by Rothermel (2004), who wrote, regarding the use of standard tests:

> "Whilst on the one hand the assessment provided data useful to the researcher, on the other hand it told us **very little about the children's knowledge and experiences**, that is, the hard to describe components which many parents reported as part of the quintessential appeal of home education."

Thus, the **second phase** of this current study will take these areas into account, as illustrated in Figure 3.2:

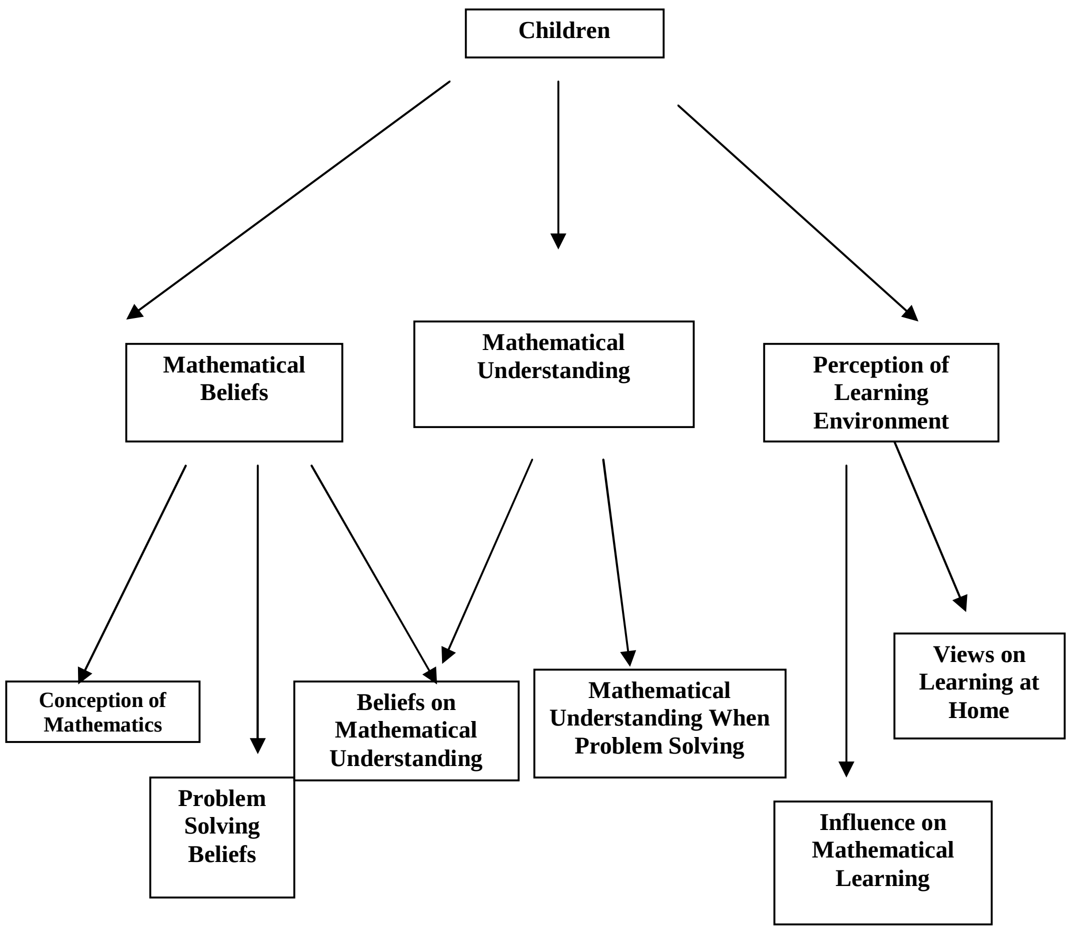


Figure 3.2: Issues Considered in the Investigation of Home-Educating Children's Learning of Mathematics

Like the data collection method used for the first phase of the study for the parents, *a questionnaire will be given to the children*, in an attempt to draw out the children's attitudes and beliefs towards the learning of mathematics. Secondly, a specifically designed *set of mathematics questions* will be used to identify the children's understanding of the mathematics they have engaged in through the education received at home. Finally, data from both phases of the research will be re-examined to help establish possible relationships between the children's mathematical understanding and the parental teaching approach.

3.2 Framework of the Methodology

When considering research into the development of mathematical thinking, it can be seen that a variety of indirect methods have been used to make inferences about mental processes and their relationships with other variables connected with mathematical thinking. Koshy (2001, p.56) uses questionnaires to investigate students' perceptions of mathematics and the relationship with mathematical thinking:

> "Asking pupils and parents to complete separate **questionnaires** about children's work habits can also provide valuable insights into the ways of their thinking and the nature of the strategies used."

Clinical interviews and discourse analysis (Rowland, 1999b; Gray, 1991) have also been used to analyse students' cognitive structuring of mathematics within the classroom, which may be of a general kind (e.g. concept formation, abstraction etc.) or related to knowledge about the construction of knowledge of specific topic areas, such as fractions. But this approach was not considered suitable for the study, due to a desire to draw upon as widely based a sample as possible, without needing the expenses required for 'face-to-face' contact with each participating family. Thus an alternative approach was sought. Moreover, as well as considering methodologies used within the mathematics education field, it was also essential to investigate data collection methods appropriate for home-education research.

Within the home-education research arena, Rudner (1999) considered 11,930 American families in his study on the *'Scholastic Achievement and Demographic Characteristics of Home School Students in 1998'*, where the children were given tests of skills/proficiencies to measure academic achievement, whilst their parents responded to a questionnaire requesting background and demographic information. The data provided information on the children's academic performance across a range of subjects, including mathematics, and relationships were established between student achievement, parental educational background and the resources used. Although this study is designed to be on a much smaller scale than that of Rudner, given the possible regional distribution of the sample it would seem that questionnaires, supported by other mechanisms of data collection, would fit the requirements of the study, and from Rudner's experience at least, provide appropriate data for further consideration. An added dimension to the study, and following the approach of Rudner, will be the use of mathematics questions to establish the processes children employ to solve a series of mathematics problems. In the loosest sense, the mathematics problems could be considered as 'structured interviews', used to illuminate the information obtained from the questionnaires. Consequentially, in this study, the data collection process will utilise: (1) Questionnaires – the first designed for parents and the second for their children, and (2) A set of mathematics problems, for the children.

With regards to obtaining a suitable sample of home-educating families, a number of factors had to be considered – namely, the expenses involved, and the location of the possible participants. Home-educating families are located throughout the UK, and it was felt that a sample should reflect the views of this distribution, rather than only considering families within the researcher's locality. As a result, the Internet was considered a valuable means of contacting possible participants. For example, contact with online home-education support groups was *predominantly over the Internet*. In these instances, email was frequently the primary contact for parents, where questionnaires were mainly sent (and responded to) via email. But by choosing to obtain data primarily over the Internet, it was noted that home-educating families who did not have access to this resource were eliminated from the study, and this bias will be discussed in Section 3.8.

Other methods of data collection were also used if deemed appropriate. For example, the vast majority of children posted their answers to the mathematics questions during the second stage of the study, since it was clearly much easier to 'write out' mathematical statements by hand than to type them out on a computer. In order to justify the chosen methods of data collection, we shall now discuss the associated advantages and disadvantages of each approach.

3.3 Questionnaires

In large scale studies, questionnaires have traditionally been used to gather information concerning areas of interest, and require some introspection on the part of the respondent. According to Gay (1987), questionnaires require less time and expense than interviews since they generally take the form of a set of questions to which the respondents make written (or typed out) responses. At the same time, they allow the researcher to pose a variety of questions; for example, one can set questions that are open, closed or use some form of scaling. A wide range of information can be collected, as respondents can be asked about their attitudes, values, beliefs and past behaviours.

Within the field of mathematics education, questionnaires can be a useful way of investigating mathematical belief, as was noted through Boaler's (2004) longitudinal study. Over a four year period, she monitored approximately 700 students from three different high schools, where the aim was to monitor the impact of different teaching approaches upon students' understanding of mathematics, using questionnaires to determine their perceptions of both mathematics and their learning environment. Her research is indicative of how questionnaires, with a variety of 'question types', can be used to investigate mathematical belief:

> "The **questionnaires** combined closed, Likert response questions with more open questions that we analyzed and coded. The questionnaires asked students about their **experiences** in class, their enjoyment of mathematics, their **perceptions** about the **nature of mathematics**, learning, and students. The observations, interviews and questionnaires combined to give us information on the teaching and learning practices in the different approaches and students' responses to them.
>
> Teachers from each approach were also interviewed at various points in the study although the teachers' perspectives on their teaching were not a major part of our analyses."
>
> (Boaler, 2004, p.3)

Consequently, for this study, it was felt that questionnaires provided a suitable data collection method when considering mathematical belief and various teaching approaches. One strength of this approach is that questionnaires allow the researcher to use standardised questions to focus on areas of interest (e.g. home-education), and additional expenses, such as time and money, are not wasted on irrelevant questions.

If certain questions require greater reflection by the participants, a written form of response allows extra time to think about the answer. This was particularly relevant for this study, as a home-educating parent may not have time to answer all the questions in one sitting, since they often have to combine full-time roles as both parent and teacher. Thus the use of questionnaires gave families the chance to respond in their own time, since they could ponder and address the questions when it best suited their convenience.

Unlike interviews, through questionnaires, information can be obtained from participants who live in otherwise inaccessible locations for the researcher — the researcher does not have to travel to each respondent. They are also less intrusive than interviews, as the respondent can choose to remain anonymous. Furthermore, if certain questions are considered personal in nature, questionnaires allow the respondent to answer more freely than if they were in a one-to-one interview situation, thus eliminating interviewer bias.

At the same time, questionnaires rely on the respondents' motivation, honesty, and memory to respond. For example, a respondent may not be inclined to give accurate answers if they want to present themselves in a favourable light. Additionally, the people who choose to complete a questionnaire for the research may be different from those who do not respond, thus, as noted above, biasing the study. To reduce the likelihood of this bias, it was made clear to the participants that there were 'no right or wrong answers'. They were informed that the main interest was in investigating the different 'types' of home-educating approaches that existed, rather than pinpointing the most 'successful' approach. In other words, parents were encouraged to honestly describe the methods of teaching that worked (or did not work!) for their children. Furthermore, since I myself came from a home-educating family, the families possibly believed that the aim of my research was not to criticise any shortfalls in their educational practices but rather to ‘provide an arena’ for them to share their feelings on their home-educating experiences. As many home-educated families are somewhat isolated from the larger educational community, it would also allow their children to demonstrate their mathematical abilities to an ‘outsider’.

It was hoped that these factors would help overcome some of the previously mentioned biases that could occur in educational research and in this research in particular. In addition, other elements of bias can be reduced by careful questionnaire design; hence, we shall now discuss the construction of the questionnaires.

3.3.1 Questionnaire Design

Two questionnaires were needed in this study, the first of which was for the parents, designed to obtain information on the backgrounds and home-educating approaches of each family. The second questionnaire aimed to address the children’s views on mathematics and their mathematical learning environment. Thus, the questionnaire design for this PhD research was crucial, since both questionnaires formed the key instrument when considering *child and parental mathematical belief*.

3.3.2 Critique of Questionnaire Used in Earlier Study

Each question had to be relevant to the key research questions identified in Chapter 2, and yet be written in a language that was accessible to every parent and child, as no assumptions could be made concerning their educational background. Consequently, before constructing the questionnaires, a critical analysis of an earlier questionnaire used by Yusof (2003) was undertaken. The questions in this initial study were formulated through a literature review of both home-education and mathematics education research (e.g., Rothermel, 2002; Thomas, 1998; Lowe et al. 2002). They also drew on knowledge gained through my personal home-education experiences. Although this approach could lead to bias, I attempted to ensure any questions based on personal experiences could be generalised to a wide range of home-educating families, rather than those from a particular background. To summarise, the parental questionnaire used by Yusof was divided into three main sections that focused on:

1. *Parental reasons for home-educating their children*, aiming to identify the main reasons behind the parents' decision to home-educate.

2. *Home-educators' views on mathematics.* This section examined the parent's conceptions of mathematics, and their teaching beliefs.

3. *The nature of mathematical activities through which mathematics could be developed.*

The questionnaire provided insight into the ways in which certain activities were used to develop mathematical thinking via parental descriptions of various learning activities. The responses also demonstrated the diverse methods of teaching different children within the same family, as well as identifying ways in which the home-educating parents measured their children's level of mathematical understanding. Comments from the examiners for the dissertation indicated that the questionnaire:

> "...contains both open and closed questions. The analysis uses a mixed methodology that presents data in a descriptive way supported by qualitative statement...indicates the author's ability to establish a mechanism to obtain answers, make a comprehensive assessment of the responses and attempt to establish theory..."
>
> (Examiner's comments for MSc Dissertation, Yusof, 2003)

At the same time, the MSc **did not** provide enough in-depth information on the home-educators' approaches to mathematical learning. For example, it was important to know whether a parent had formal teaching experience, and if this affected their home-educating approach - as we observed earlier, *pedagogic knowledge* could have a variety of effects on an individual's teaching approach (Section 2.2.5). Parental educational level may also be a relevant factor. Therefore, as well as the questions from the MSc questionnaire, additional questions were designed and included to cover areas that were not addressed in the initial MSc study. For these questions, there is a greater focus on the various approaches to learning mathematics. In particular, they consider the home-educators' views relating to their:

1. Perceptions of themselves as teachers ('Q.8, Mathematical Activities' and 'Q.8, Why Teach Mathematics' in the Questionnaire For Parents, Appendix 1)

2. Personal experiences of mathematics, and the resulting influence on their approach to mathematics education ('Why Teach Mathematics', Final Section, Questionnaire for Parents, Appendix 1)

3. Aims of questions/discussion when their child is learning mathematics ('Mathematical Activities, Q.8' in the Questionnaire for Parents)

4. Advantages/disadvantages of the home as a mathematical learning environment ('Why Teach Mathematics, Q.10 and 11 in the Questionnaire for Parents)

The specific features of the questions formulated to address these issues shall now be explained.

3.3.3 Types of Questions Used in the Questionnaires

In this study, personal, detailed and descriptive information was needed from the home-educating population:

> "Where a site-specific case study is required, then **qualitative**, less structured, word-based and **opened** ended questionnaires may be more appropriate as they can capture the specificity of a particular program."   Cohen, Manion and Morrison (2000, p. 247)

Hence, the focus was on data of a qualitative nature, but it was also acknowledged that a range of different question types could aid triangulation during data analysis. For this reason, it was felt that a semi-structured questionnaire, consisting of both open and closed questions, would allow me to set the agenda, but not presume the response. In particular, the various types of question that were included in the questionnaires are detailed and justified below (note that the full questionnaire can be seen in Appendix 1).

3.3.4 Open Questions

Open questions were used to obtain specific information regarding certain educational aspects. For example, the question, *"What signs do you look for in your child's thinking to show that he or she understands the mathematics that you've just taught them?"* gives parents the chance to personalise their answers according to their particular style of home-education. Therefore, open questions were frequently asked to allow for a more complete and detailed response.

3.3.5 Closed Questions

Whilst being a source of detailed information, open questions can be difficult to code and may require too much work for the respondent. Gay (1987) recommends that structured (or closed form) questions should be used whenever possible, including both multiple choice and questions requiring scaling, as this would allow respondents to prioritise their answers. Closed questions also facilitate data analysis, since they are quick to complete and straightforward to code (Cohen et al., 2000), and do not rely on the respondent's level of articulation (Wilson and McLean, 1994). However, closed questions do not allow respondents to include any additional information in the form of remarks, qualifications and explanations, and there is a risk that the categories might not be exhaustive and may have bias in them (Oppenheim, 1992). Therefore, closed questions were only used in situations when either: (a) Straightforward information was required (e.g. the number of children in the family, children's ages etc.), or (b) I wanted the parents or children to prioritise their answers. In particular, I made extensive use of *Likert-type questions.*

3.3.6 Likert Scales

Likert scales describe a multi-item scale, designed to measure attitudes, which adhere to certain formal requirements. According to Uebersax (2006), Likert scaling presumes the existence of an underlying continuous variable, whose value characterises the respondents' attitudes and opinions, where typically:

(1) The scale contains a number of items

(2) The response levels are arranged horizontally, using consecutive integers or verbal labels to represent 'evenly-spaced' intervals

(3) Verbal symbols are symmetrical about a neutral middle ground

At the same time, Uebersax (2006) writes that a researcher is reasonably justified to use an even number of response levels provided that both criteria (1) and (2) are maintained, and hence, there need not be an exact middle or neutral category. Thus, some questions in the questionnaires did not have a 'middle' category, for example, Q.8 (Table 3.1).

When you ask your child a series of questions, your aim is to:	Never	Rarely	Often	Always
See if they know the correct answer				
Get them to justify and explain their reasoning				
To allow them to gain confidence				
To solve a problem in an everyday situation				
Find out if they are paying attention				
Give them the opportunity to direct the lesson				
Discover their ideas and opinions				
Help you to understand something better as well as your child				
Find out what is interesting about the mathematical topic				

Table 3.1: Question 8, P.6, Likert Type Question in Questionnaire for Parents

Question 8 examines parental aims when questioning their child and this question had no ‘neutral’ category, as it was felt that ‘ambivalent’ responses would be of little value. The question was structured so that the parent’s strength of response would be indicative of their preferred teaching approach. For example, the options highlighted in green can be related to a ‘child-led’ approach to home-education – if a parent gave the option *“Give them the opportunity to direct the lesson”* a ranking of “Often” or “Always”, this could be evidence that their children had a strong influence on the mathematics learning. The options highlighted in red, e.g. *“Get them to justify and explain their reasoning”* may indicate the parent’s attitude towards the understanding of the mathematical concept.

One disadvantage of Likert scale questions is that they can lead to unclear data sets - they are relative only to a personal abstract notion concerning "strength of choice".

For instance the choice "moderately disagree" may mean different things to different respondents and to the researcher interpreting the data. But Goldstein and Hersen (1984, p.52) argue that a:

> “...level of scaling obtained from the procedure is rather difficult to determine, the scale is clearly at least ordinal. Those persons with the higher level properties in the natural variable are expected to get higher scores than those persons from lower properties”.

In other words, one may be able to gain a perspective of how *strongly* a respondent feels about a particular issue, especially when considering very ‘high’ or ‘low’ rankings. For example, Question 8 in ‘Mathematical Activities’ of the parents’ questionnaire (see Appendix 1 for complete questionnaire) asks the respondent to rank a number of statements that describe different approaches when teaching mathematics, e.g. *“Children won’t really learn mathematics unless I cover it in a structured way”*. The higher the ranking, the better the description fits the parent. If a home-educator gives the statement a ranking of 1, this means that the description describes them well, and we would expect them to have a much stronger emphasis on structured learning than a parent who gives a ranking of 5. In other words, this is the level of analysis that will be employed in the study when interpreting responses from Likert-type questions.

3.3.7 Case Studies

The questionnaire also addresses a number of criteria highlighted by Koshy (2001) when selecting appropriate learning strategies for ‘mathematically promising’ pupils (page 9 and 10 of the Questionnaire for Parents). These criteria were re-written from the perspective of a home-educating parent rather than a teacher, in order to determine the philosophy that underlies each parent’s approach to mathematics education. For example, do the home-educators believe that their children’s mathematical abilities are innate, and will their mathematical knowledge develop without the need for parental guidance? Or does every child need support from an adult? If a child has major shortfalls in one area (e.g. writing) would this affect their learning of mathematics?

Instead of asking these questions directly, parents are given the fictional cases studies of two home-educated students, based on the case studies in Koshy (2001, p.6-7 and p.10). The case studies aim to provoke reflection - some home-educators may even recognise their own children within these descriptions.

3.3.8 Layout of Questions

Cohen and Manion (1994) stress that the *appearance of the questionnaire* is very important, and emphasise the use of suitable spacing between questions to avoid a ‘compressed look’. Researchers are also advised to arrange the questions in such a way as to maximise participation. Therefore, the layout of both the parents’ and children’s questionnaires was deliberately varied, with more personal, attitude questions interspersed with ‘scaling questions’ to ensure that each respondent remained interested. For example, if four ‘Likert-type’ questions appeared consecutively, the respondent may be inclined to give a rating of ‘5’ to every question as an automatic response, without giving serious reflection to their answers. Consequently, care was taken to avoid a long series of ‘repetitive’ questions.

The previous sections describe the types of questions that were used for both the parents and children’s questionnaires, since the justification for their inclusion in this research applies to both. However, whilst the focus has been on the questions that were devised for the parents' questionnaire, we will now describe the construction of the data collection instruments used when considering the children.

3.4 Questionnaires and Assessed Work for Children

Although the parents' questionnaire may provide relevant information on the ways in which mathematical learning is undertaken at home, it does not consider the children's views, which is, perhaps, as important (if not more important) if we are to investigate the development of mathematical thinking at home.

We also need to *see the children's mathematical work* to determine if the parents' home-educating approaches influence their children's mathematical understanding in any way. Therefore, in this study, these areas will be investigated through the use of a *questionnaire*, to address the children's *perceptions* of their learning, and *assessed work*, which will examine the types of mathematical *understanding* through their problem solving strategies.

3.4.1 Exploratory Study for Children's Questionnaire

During the first quarter of 2004, two exploratory studies were used to gain an insight into the nature of children's mathematics beliefs, using a mixture of interviews and questionnaires. This was mainly to help formulate appropriate questions for the children's questionnaires. Four clinical interviews were conducted in January of 2004. The sample, largely an opportunity sample, involved children between seven and eight years of age; six of whom attended local primary schools, and one who was home-educated. Three of the interviews were one-to-one and one consisted of a group interview with three girls.

The interviews lasted from 20-30 minutes and were unstructured as the questions were simply formulated according to the children's previous responses. However there were some common elements to each interview; in particular, the initial questions were used to get a sense of the child's conceptions of mathematics as a subject. Secondly, each child was asked whether they could describe a fraction, and then tested on simple questions such as 1/3 + 4/3, to see if they were capable of applying basic arithmetical rules to the fractions. In each interview, some attempt was made to discuss algebra (what is $2a$ plus $3a$ and so on) and this area was 'explored' depending on the child's ability to engage in a constructive dialogue. As this was my first experience of conducting interviews the exploratory study highlighted the importance of 'good' questioning technique, in particular; I learned how to avoid 'leading' questions. Due to the young age of the participants, it was initially felt that algebra might prove to be too difficult a topic for discussion. Nevertheless, most of the children were able to discuss and answer simple problems such as $2a + 3a$. In summary, it was found that:

(1) Questions like 'What is mathematics?' could be difficult for children to answer, as this was a very broad question. It was easier for them to describe their 'feelings when they understood something in mathematics' and most were capable of relating their views on how they would indicate 'someone who was good at mathematics' and 'someone who was bad' at mathematics.

(2) Fractions were often described through the use of a 'circle' or a pizza. It was clear that some children used this representation as a basis for all their calculations – as soon as they were asked a question on fractions, they began drawing a circle.

(3) The age of the child was not necessarily an indicator of their mathematical ability.

After this initial exploratory study, the questionnaire for home-educated children was designed, taking into account some of the issues identified in the pilot study.

3.4.2 Questionnaire Design for Home-Educated Children

In the parents' questionnaire, I sought to obtain information regarding the parents' *mathematical beliefs, teaching approaches,* views on *mathematical understanding*, and their *educational background*. Likewise, three key areas of focus in the children's questionnaire were: the children's mathematical *beliefs*, their notions of mathematical *understanding*, and their views on *home-education* (in relation to the learning of mathematics). However, one additional area of consideration in the children's questionnaire was the children's *problem solving beliefs*. Each of these areas will now be discussed.

3.4.3 Beliefs on Mathematics as a Subject

It has been shown that the failure in solving problems is not only due to a **lack of knowledge**, but also due to the **incorrect use of knowledge** which is often inhibited by both **general and specific** beliefs about mathematics (Schoenfeld, 1983, McLeod, 1992). Some common beliefs associated with mathematics are (Schoenfeld, 1985, p.43):

- Mathematics problems are always solved in less than ten minutes, if they are solved at all
- Only geniuses are capable of discovering or creating mathematics.

Pehkonen and Törner (1996) write that mathematical beliefs can indicate an individual's experiences of learning mathematics, which can be otherwise difficult to observe. They also are a force of inertia, as beliefs form part of a student's understanding and feelings of mathematics and can thus shape the way the individual engages in mathematical behaviour. Consequently, in this questionnaire, it was important to discover the children's mathematical beliefs, as this could help us understand the way they *engaged in a mathematical problem*.

Two main areas of mathematical belief were examined in this study, namely the children's beliefs regarding 'mathematics as a subject' and their beliefs on 'mathematical problem solving'. For example, Q.1 (see Appendix 2, fourth page of Questionnaire for Children) asks the child to write down the words that they would use to describe mathematics, and to indicate their feelings towards the subject. With regards to their notions of 'problem solving', Q.1 on the first page of Questionnaire for Children (see Appendix 2) is based on Schoenfeld's (1985) hypothesis that students who believe "*Mathematics problems are always solved in less than ten minutes"* tend to give up if they have not reached a solution after 10 minutes. Other questions are based on Zan and Poli's (1995) 'Beliefs about Mathematical Problems Questionnaire', where according to Zan and Poli (1995, p.103), it was found that:

> "…the good solvers and the poor solvers have a significantly different concept of a mathematical problem."

For example, Question 6 (p.2) of the children's questionnaire is based on Zan and Poli's question: *"In a problem is it worse to make a calculation error or choose the wrong operations?"* In Zan and Poli's research, the majority of good problem solvers believed that it was worse to choose the wrong operation, and the poor problem solvers tended to believe that it was worse to make a calculation error.

3.4.4 Mathematical Understanding

One of the main aims of the children's questionnaire was to investigate their perceptions of mathematical understanding. Krutetski (1976) writes that those who are mathematically 'able' display characteristics including a swiftness of reasoning, ability to generalise, flexible thinking and use of mathematical structure. Sheffield (1994) looked at criteria of a behavioural nature, mentioning curiosity, awareness, creativity and a high level of energy and persistence.

Questions 7 and 8 (second and third pages of the Questionnaire for Children, Appendix 2) focus on the ways in which the children 'choose to identify understanding/lack of understanding' when doing mathematics. They ask the children to rate the importance of a number of characteristics related to both the behavioural aspects of understanding and those identified by Krutetski (1976). The children's responses can also be compared against their parents' approaches when 'measuring their children's level of mathematical understanding'. Question 4 (p.5) considers the respondent's source of help when they fail to understand a mathematical concept.

3.4.5 Perceptions of Mathematical Learning

As well as considering how the children *view mathematics as a subject*, we also wish to find out *how they view themselves as learners of mathematics and the environment in which they learn mathematics.* Therefore, the questions on page 5 of the questionnaire mainly address the children's perceptions of their own mathematical ability. We also investigate the ways in which the children measure mathematical ability; for example, they are asked *"How can you tell when someone is bad at mathematics?"* Responses to questions on mathematical ability may be related to the child's notions of mathematical understanding.

Moreover, a set of questions is posed to allow the child to describe the learning environment at home, where Q.8 (p.6) addresses the factors that influence the child to choose a particular mathematical topic to learn – is this choice based on their own interest, parent-driven, or guided by other means? The question corresponds to Question 3 (page 3) of the parents' questionnaire, where the parent is asked, *"What guides your choice of a particular activity to teach mathematics?"*, which may enable us to determine possible relationships between child and parental views on the factors that influence the mathematical learning.

Questions 5 – 7 (p.5 and 6) concern the child's current mathematical activities and any mathematical targets that the respondent may be aiming for. Question 9 (p.6) asks the child to list the main benefits of learning mathematics at home (similar to Question 10, p.7 of the parents' questionnaire), while Question 10 (p.7) addresses any improvements that could be made to the home-educating environment. The final set of questions on page 7 relates to more general aspects – e.g. the children's ages and the length of time they have been learning at home.

3.5 Pilot Study of Questionnaires

In December 2004 and January 2005 both the parents and children's questionnaires were piloted to assess the suitability of the research instruments. The majority of the questionnaires were sent and responded to via email, supporting a quick and efficient method of collecting feedback. Six families took part – some providing very useful information towards improving the questionnaire design. For example, it was noticed that certain questions needed to be made more specific. Respondents also wrote that it was important to note the age at which each child initially began home-education in the family. For those who used an informal or ‘autonomous’ approach, the word ‘teaching’ sometimes proved to be a problem, as some parents felt that their children mainly learned mathematics independently, and they did not actually ‘teach’ their children. The parents suggested that the questions should be formulated to take these issues into account; e.g. “How many hours do you spend teaching your children?” followed by “How many hours does your child spend learning mathematics independently?” As a result of the pilot study feedback, necessary changes to the questionnaire were made prior to distribution.

3.6 Distribution of Home-Education Questionnaires

One of the biggest problems faced by home educators in the UK is their ongoing conflict with Local Authority officials. A number of parents have faced court proceedings, intervention from the Social Services and interference from other relatives - simply due to their decision to home-educate their children. Without going into a long debate on the issue, home-educating families are often very wary of any ‘official educational authority’. A number of online egroups (Education Otherwise, etc.) have been set up so that parents can offer and exchange advice on various home-educating issues, such as the legalities of home-education or different home-educating styles. I felt that the members of these support groups could provide an ideal sample for my research, although inevitably there would be some bias, as will be discussed later in Section 3.8. However, in order to join the egroups, one has to give reasons for wanting to become a member, as there have been a number of incidents where anti-home educators joined the support groups, only to cause disruption by posting numerous ‘negative’ comments. Permission from the site moderators was therefore essential before any contact with the home-educating parents could be initiated. But I had two key advantages, which were:

1. I came from a home educating family and could therefore empathise with the families’ situations.

2. The research focussed on home-education, and thus might be of benefit to the parents.

Previously, during the course of my MSc research, I had made contact with a number of home-educating families throughout the United Kingdom from the ‘Education Otherwise’ Yahoo egroup, which is the largest home-education organisation in the United Kingdom. I emailed the moderator of the Education Otherwise egroup, and asked permission to use the egroups for my MSc study. The moderator was extremely helpful, giving me access to the email lists and suggesting other home-education egroups. The moderators of these egroups were also contacted, and in total, I obtained twenty questionnaires during the course of my MSc study. A further eight questionnaires were obtained via other means (local home-education contacts). Utilising my MSc experiences, for my PhD study, these groups were again used to obtain a sample of home-educators, following a similar approach to my MSc research. Both the parents and children’s questionnaires were sent (through email or post) to each family from the sample, so that if their child(ren) wished to answer the children’s questionnaire, the families could first scan the questionnaire to assess its contents. After completion, the families had the option of either emailing or posting the completed questionnaires.

The letter sent out to the parents via email can be seen in Appendix 3. The majority of the questionnaire data were obtained through this process, although data were also received from home-educators who were 'not members of any egroups', since a number of parents forwarded the request for information to other home-educators within their locality, and so the sample was not just restricted to members of the support groups. If follow up questions were needed, an email was sent requesting more information, which parents were generally happy to provide; although for the majority of the completed questionnaires, there was more than enough detail in the initial answers.

3.7 Assessed Work

Whilst the questionnaires covered issues relating to parental and child's perceptions of mathematical learning within the home-environment, a key focus of this study was to investigate the mathematical thinking of home-educated children through their problem-solving approaches. Consequently, a third research instrument was designed for this purpose, where a number of mathematics questions were constructed to allow the researcher to identify various 'types of mathematical thinking' from the children's problem-solving approaches - without the need for face-to-face contact with the participants.

Mislevy (2003) believes that educational assessment is "reasoning from observations of what students do or make in a handful of particular circumstances, to what they know or can do more broadly" (p. 237). According to the National Research Council (2001b), there are three broad purposes of assessment (NRC, 2001b), geared towards the following aims:

(1) Formative: To assist student learning

(2) Summative: The assessment of student achievement, and

(3) Evaluative: Evaluate existing programs or new interventions.

In the current study, however, the focus is on the identification of *different types of mathematical thinking* through the children's problem solving approaches, rather than simply measuring 'achievement'. We also wish to evaluate the effects of the 'home-education' learning environment on mathematical understanding, and hence questions should be constructed to enable one to determine a link between the parental approach towards mathematics education and their children's solution strategies. With these aims in mind, the first step was to formulate an appropriate set of mathematics questions. These could then be trialled in an exploratory study to assess their suitability, with regards to (a) the degree of difficulty, (b) the clarity of the questions and (c) the 'types' of mathematical response elicited from the children.

3.7.1 Exploratory Study for Assessed Work

The first set of questions in the exploratory study was selected to assess children's approaches to 'real-life' problems. The questions typically required the application of relatively simple mathematics procedures to 'real-life' problems, and were taken from examples suggested by the National Numeracy Strategy for use in the Daily Mathematics Lessons for Year 5 to Year 6 children (see Appendix 4). These questions were chosen as they involved relatively simple arithmetical skills that the children should be familiar with, but they also required a correct interpretation of the 'real-life' context of the text in order to perform the necessary arithmetic. It was felt that such questions could be appropriate for the main study, given that I wished to examine both the children's mathematical skills, and the influence that the home-environment may have on their thinking.

An opportunity sample of nine children (aged from 10-14 years) took the test in January/February, 2004. The children's mathematics tutor informed me that from her assessment of the children's work over the previous months, the mathematical abilities of the children ranged from Level 3 – Level 5 Key Stage 2, up to Foundation Level GCSE (the 14 year-old student). It was thus felt that the majority of the students should have the prerequisite mathematical skills needed to complete a range, if not all, of items within the test. The children were given unlimited time to attempt the problems, as the aim was to observed their thinking during the tests, rather than measure speed of completion. The results of the test are briefly summarised as follows:

1. Most found the questions to be too difficult; in particular, they often found it hard to correctly identify an appropriate mathematical approach. Furthermore, some made simple calculation errors after identifying the correct approach. The average mark across all students was 46%.

2. The two highest marks came from the youngest children; a nine-year old home-educated student obtained a mark of 89% and a ten-year old student achieved 79%. Both appeared confident when applying arithmetical procedures, and rarely made calculation errors. From their workings, I observed that these children were quick to find an appropriate approach, whilst for the other children, many 'failed attempts' were noticed. The findings from this exploratory study highlighted the importance of using test questions that

required some ‘working’, since it was perhaps the only way to identify the range of solution strategies.

3. The results showed that age did not appear to be a significant factor with regards to the level of performance on the test.

Building on the feedback gathered from this initial exploration, I then conducted a more structured pilot assessment, choosing National Key Stage tests as a way of exploring the children’s mathematical understanding in a few selected areas of mathematics. Three sets of sample Key Stage Test questions (taken from the Qualifications and Curriculum Authority Website) at the Key Stage 1, Key Stage 2 and Key Stage 3 levels were given to 16 children, aged from 7 to 16 years. Note that 20 sets of questions were attempted as four children who took the KS2 questions also attempted those from KS3. This would permit the observation of any differences by age-group. The children came from a variety of backgrounds, some from independent schools whilst others attended local state schools. All were school-educated (except for one, who was home-educated on a part-time basis). As the main objective of the pilot study was to test the suitability of the sample Key Stage questions as a measure of ‘understanding’ with regards to certain mathematical concepts, the fact that the children came from mainstream schools was not felt to be an issue. Note that not all of the questions used in the pilot study were used in the actual set of questions used in the main study. Some were replaced or deleted as they proved too difficult/easy, as will be indicated in the table of questions for each of the pilot studies.

3.7.2 KS1 Pilot Study

We first address the questions that were given to four KS1 students, Children U, Z, T and D (see Appendix 5 for a breakdown of the marks across all questions). This set consisted of a range of questions covering: (1) Basic Arithmetic, (2) Shape, (3) Arithmetical Word Problems, and (3) 'Real-life' problems. Essentially, it was hoped that the questions would help to measure the children's understanding of arithmetic through its application in various situations, their knowledge of shape, and also the 'real-world' applications of mathematics. Table 3.2 summarises the questions and provides an indication of the level of responses achieved from the children who participated in the KS1 test.

Basic Arithmetic		**No. of correct responses (out of 4)**	**Comment**	**Discriminator**	**Used in Main Study**
Q4	Two Digit Addition	2	Range of strategies	Yes	Yes
Q5	Three Digit Subtraction	3	Range of strategies	Yes	Yes
Q6	Simple Division	4		No	No
Shape					
Q1	Symmetrical Properties	0	Triangle /Parallelogram difficulties	Yes	Yes
Q2	Transposition of Perimeter	1	One correct response	Yes	Yes
Arithmetical Word Problems					
Q3	Money	2	Calculation errors	Yes	No
Q9	Establishing multiple cost	2	Range Strategies	Yes	Yes
Q13	Finding numbers	1	Some with help	Yes	Yes
Real Life Situations					
Q7	Interpretation of a time table	0	None answered both parts	No	No
Q8	Coin Recognition	4	Easy!	No	No
Q10	Reading weight scale	4	All correct	No	No
Q11	Reading Analogue Clock	4	All Correct	No	No
Q12	Reading measuring jug	1	Only one correct response	No	Yes

Table 3.2: Summary of Questions used in the KS1 Pilot Study

Within Table 3.2 the questions are grouped to reflect the main issues under consideration; for example the children's ability to demonstrate their basic arithmetical skills, their knowledge of shape or their interpretation of real life situations. Comments provide a brief indication of the impressions left after the response were considered, these leading to indications of whether or not each question discriminated between children, either in the way it was responded to, or in the success rate achieved. It was as a consequence that questions were then chosen for use in the main study. The average mark across all questions was 59%, with Child U achieving the highest mark of 85%, Child D and Z both obtaining 54%, and Child T achieving 42%. However, as the aim of the exploratory study was not to simply measure the overall marks of the children, but to examine the various 'types of strategies' elicited from the questions, we will now briefly focus on some of the *solution strategies* elicited from the problems.

3.7.2.1 <u>Shape</u>

When considering suitable 'Shape' questions, Question 1 revealed that whilst all 4 children could draw lines of symmetry for the hexagon, **none** realised that the triangle had more than one line of reflectional symmetry:

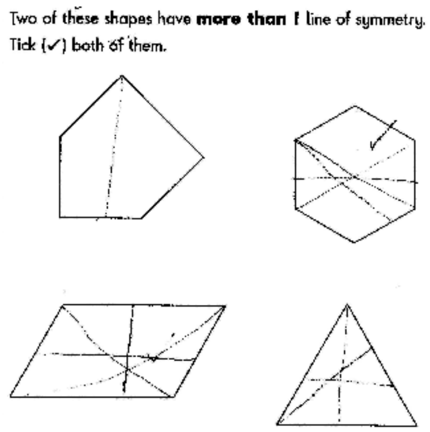


Figure 3.3: Child Z, Q.1, Trial of KS1 Questions

Similarly, for Question 2, only one child was able to correctly answer the question - the solution strategies indicated that most were able to determine the necessary perimeter but unable to draw the correct shape:

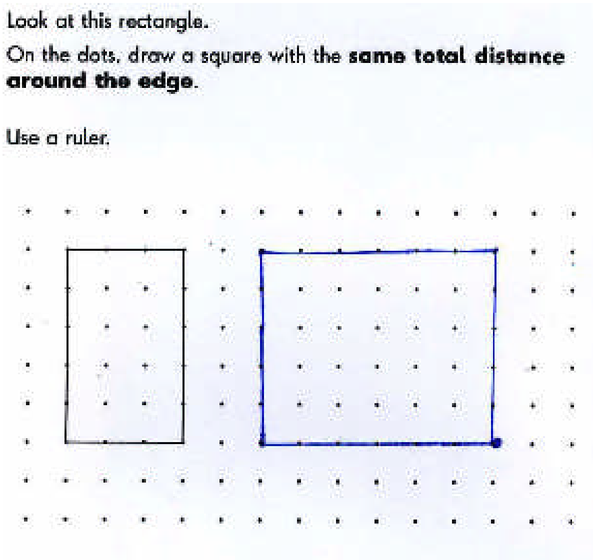


Figure 3.4: Child D, Q.2, Trial of KS1 Questions

Both Questions 1 and 2 were therefore included in the main study.

3.7.2.2 <u>Arithmetic</u>

With regards to the arithmetic questions, it was felt that the most valuable problems would be those that generated a range of solution strategies, such as Question 4, where the use of both partitioning (Child Z) and following a formal procedure (Child D) were observed:

Child D **Child Z**

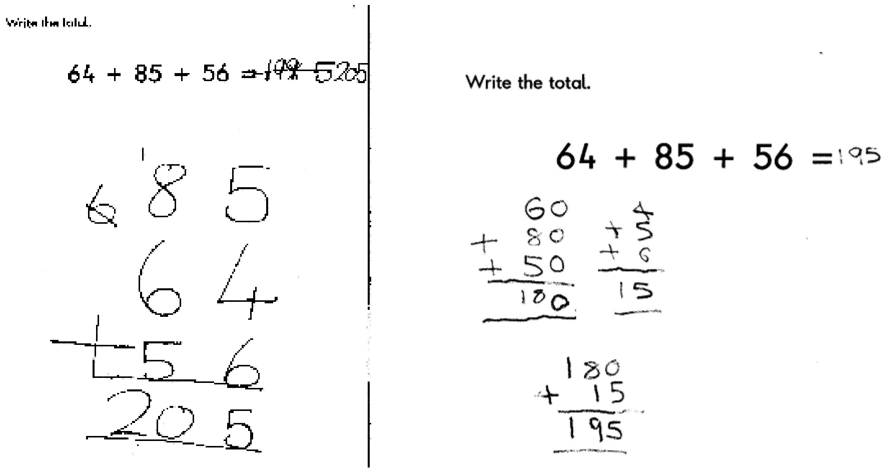


Figure 3.5: Children D and Z, Q.2, Trial of KS1 Questions

For some questions, the children were unable to fully comprehend the text of the problem. Question 7, for instance, proved to be particularly difficult, as no child was able to successfully answer both parts. Therefore, this question was replaced by an alternative 'real-life question', namely Question 3, Group 1 Questions (See Appendix 6).

3.7.2.3 Summary of Results from KS1 Pilot Study

This pilot test indicated that with slightly more 'complex' arithmetical questions, such as Questions 4 and 5, a range of different solution strategies could be observed. It was therefore felt that these questions should be included within the set of problems given to the home-educated children. Nearly all the KS1 children had problems with 'shape' questions, therefore Questions 1 and 2 were selected for use in the research instrument. Furthermore, two 'word problems' and a question on 'reading scales' were set in order to determine the home-educated children's abilities to answer 'everyday life' mathematics questions. The children were also set Question 13, as this question, whilst proving somewhat difficult for the KS1 students, does not necessarily require more advanced mathematical knowledge, but rather it depends on whether the child can find an appropriate strategy. The full set of questions that were posed at the KS1 level can be found in Appendix 6.

3.7.3 KS2 Pilot Study

Table 3.3 is constructed in the same way as Table 3.2 and indicates the questions used in the KS2 Pilot Study, and if they were used in the main study:

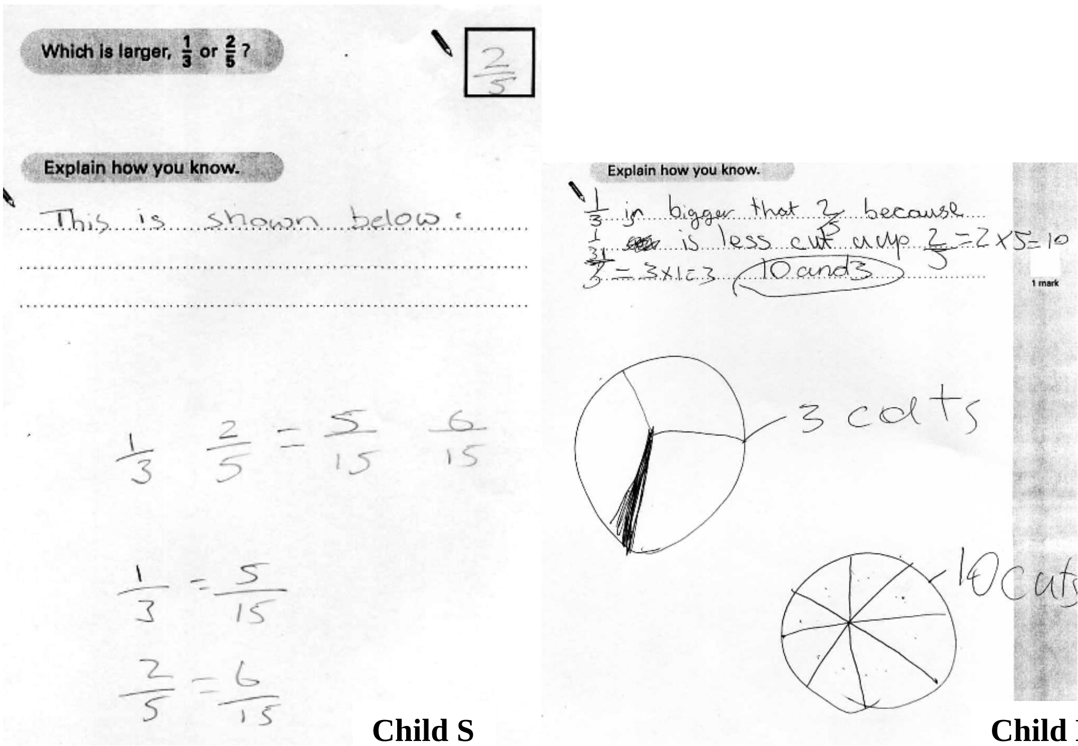
Arithmetic		**No of Correct Answers**		**Discriminator**	**Used in Main Study**
Q1	Fractions, order, size	4 out of 7	Justifying larger fraction	Yes	Yes
Q3	Fractions of surface	2 out of 7	Reliance on procedure	Yes	Yes
Q9	Proportion	2 out of 7	Multiple procedures proved difficult.	No	No
Q12	Multiplication	3 out of 7	Straightforward	No	
Shape					
Q2	Finding Angles	3 out of 7	Facts and application	Yes	Yes
Q7	Lines of symmetry	1 out of 7	Interesting approaches	Yes	Yes
Q8	Symmetry	5 out of 7		No	No
Q10	Area and Shape	2 out of 3	Too easy!!	No	No
Q11	Area	3 out of 7	Difficult	Yes	Yes
Q13	Area	0	Range of strategies	Yes	Yes
Word Problem (Real Life)					
Q4	Interpretation	0	Variety of strategies	N/A	Yes
Q5	Find a number	5 out of 7		Yes	Yes
Q14	Area of Paving.	1 out of 7	Few attempted	No	Yes
Logical thinking					
Q6	Finding number	7 out of 7	Showed a range of strategies	No	Yes
"Algebraic"					
Q6	Value of symbols	7 out of 7	Very easy	No	Yes

Table 3.3: Summary of Questions Used in KS2 Pilot Study

The questions were categorised according to type (e.g. Arithmetic) and the responses were examined in order to distinguish if the questions would provide an indication of the children's thought processes. Seven children were set questions at the KS2 level, where the average mark for this test was 48% (see Appendix 5 for a breakdown of the marks across all questions).

3.7.3.1 <u>Arithmetic</u>

Questions 1 and 3 demonstrated that those who incorrectly calculated problems involving fractions were usually unsuccessful due to the inappropriate use of visual representations, as can be observed in Child K's solution to Question 1 below. On the other hand, Child S has successfully applied her knowledge of equivalent fractions. It was felt that these questions would provide a useful discriminator when considering the home-educated children's understanding of fractions and associated visual imagery.



Child S **Child K**

Figure 3.6: Child S and Child K, Question 1, Trial of KS2 Questions

3.7.3.2 Shape

Question's 7, 10, 11 and 13 all required knowledge of Area and Shape. As was previously noticed in the KS1 pilot test, Question 7 showed that most children struggled with questions on symmetry. Some of their drawings had more than one line of symmetry. Most of their drawings had no lines of symmetry! Only one child gave the correct answer to both parts. Many also struggled to find a method of solution for Question 11, when the length/width had to be determined from the given information, as can be observed in Child Sb's[1] solution below. This question was included in the main study as a guide to the home-educated children's application of area facts:

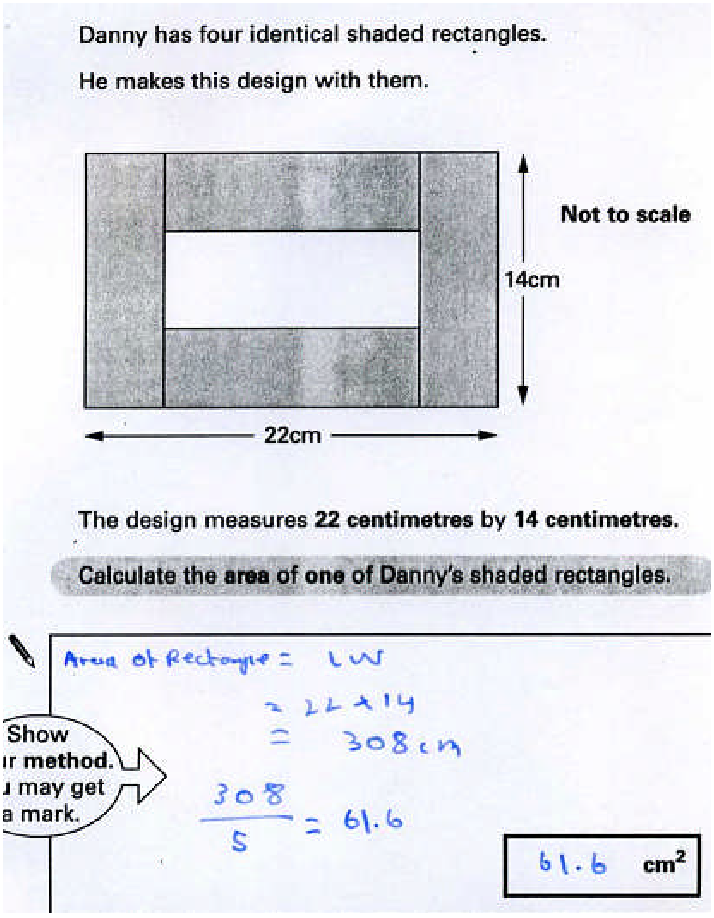


Figure 3.7: Child Sb, Question 11, Trial of KS2 Questions

[1] Note that two children had names beginning with 'S', so the second child whose name began with 'S' was called 'Child Sb' to distinguish her from 'Child S'. This was also done for other children who shared the same initial letter for their first name.

Question 2 involved angle computation for different shapes, and the responses demonstrated a clear distinction between those who ‘knew the facts and could apply them’ and those who ‘knew some of the facts, but not the method of solution’. Only three children were able to complete the problem successfully. Child Al’s answer shows his numerous attempts to find the correct approach:

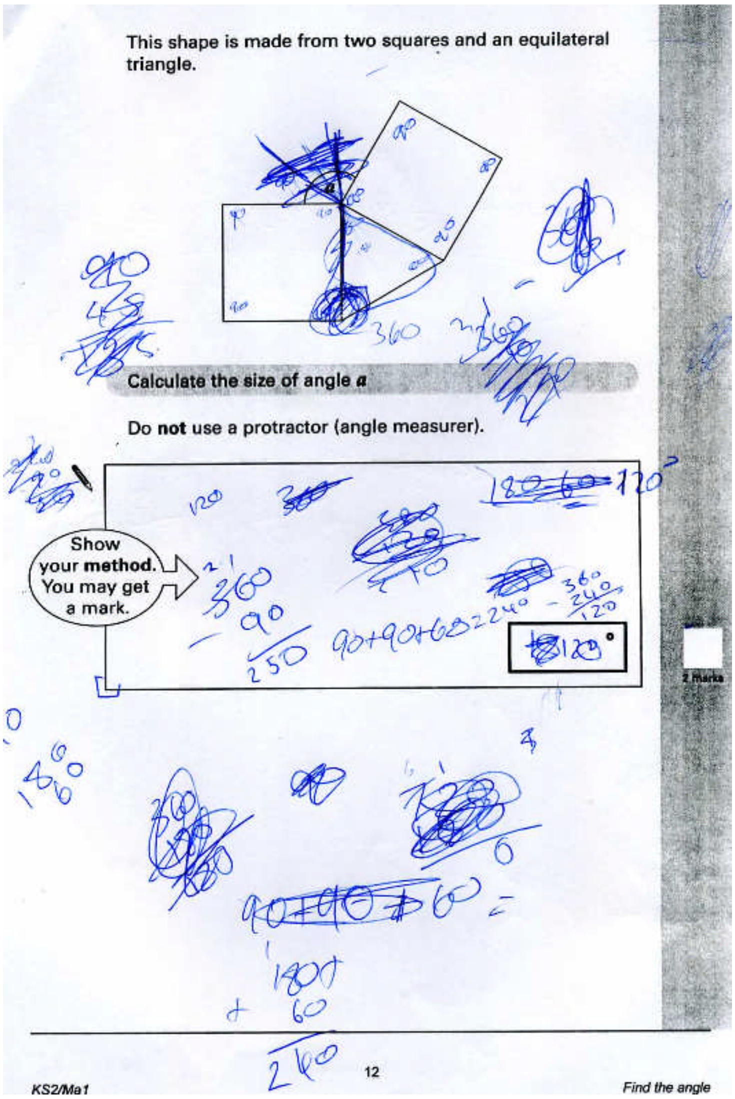


Figure 3.8: Child Al, Question 2, Trial of KS2 Questions

Only three children attempted Question 10 (all obtaining correct answers) and this question was not used in the main study as it appeared too simple.

3.7.3.3 Word Problems

Questions 4, 5 and 14 involved the interpretation of word problems. None of the children who were given Question 4 were able to complete it, but those who did showed a number of interesting methods of solution, e.g. writing out the multiples of 5 that fell between 30 and 15, and then adding one. All the 'word' problems resulted in a variety of solutions, and were hence included in the home-educated children's test.

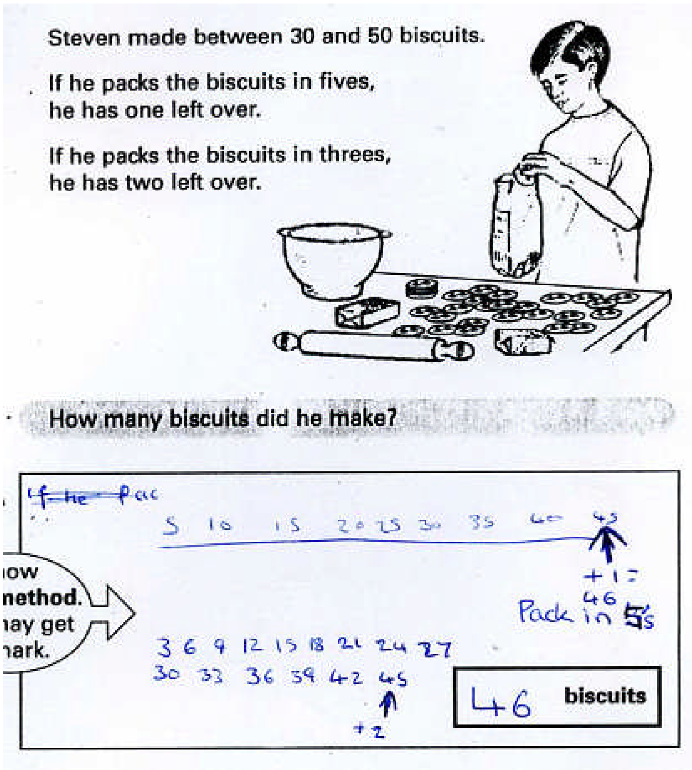


Figure 3.9: Child N, Question 4, Trial of KS2 Questions

3.7.3.4 Summary of Results from KS2 Pilot Study

The responses showed two main types of error. For some questions, the children appeared to know the correct solution strategy, but did not have the arithmetical skills necessary to complete their answer (e.g. Question 5). On the other hand, for problems such as Question 2, the majority of children could not find an appropriate strategy to begin with, despite the relative simplicity of the arithmetical calculations involved.

At the same time, the pilot test showed that when it came to problems involving shape, such as symmetry and area, children who appeared confident with arithmetical procedures did not seem to be any better at obtaining a method of solution than those without such skills. The full set of questions selected for the main study can be seen in Appendix 7.

3.7.4 Key Stage 3 Pilot Study

Table 3.4 indicates the outcomes of KS3 questions given to nine children.

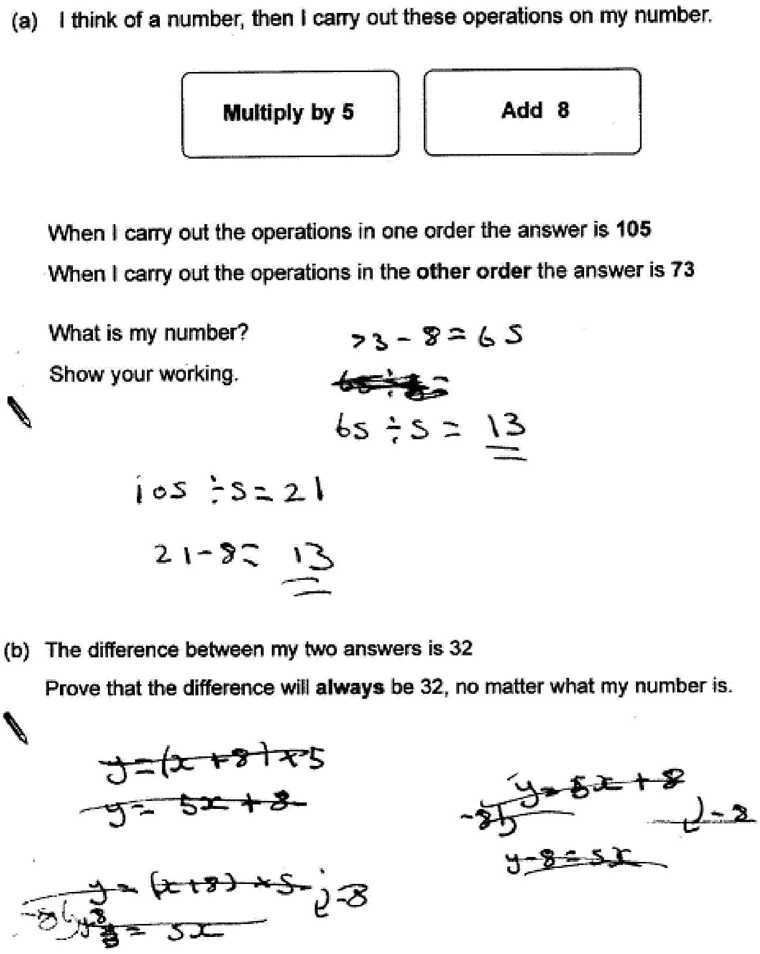
Arithmetic		**No of Correct Answers**		**Discriminator**	**Used in Main Study**
Q1	Multiplication facts	9 out of 9	Found to be relatively simple	No	Yes
Q4	Fractions	4 out of 9	Range of solutions	Yes	no
Q7	Basic Arithmetic facts	7 out of 9	Not very difficult	No	No
Q9	Proportion	6 out of 9	Straightforward	No	No
Shape					
Q6	Finding dimensions of shape	0 out of 9	Facts and interpretation	Yes	Yes
Q10	Angles	1 out of 9	Interesting approaches	Yes	Yes
Q11	Area and shape	0 out of 9	Many guesses at correct answer, but no solution offered	Yes	Yes
Logical thinking					
Q3	Ratio	2 out of 9	Most could not even make an attempt	No	No
"Algebraic"					
Q2	Value of unknowns using algebraic manipulation	1 out of 9	Too difficult	Yes	No
Q5	Algebraic puzzle	0 out of 9	Too difficult	No	No
Q8	Translating text into algebraic statements	0 out of 9	Showed interesting attempts	Yes	Yes

Table 3.4: Summary of Questions Used in KS3 Pilot Study

The average (mean) mark was 41%, with marks ranging from 75% to 21% (see Appendix 5 for more details). Some of the students' answers are summarised below, including a brief description of the types of understanding observed in their working.

3.7.4.1 Algebra

The children appeared to have considerable difficulty when manipulating algebraic expressions. For example, for Question 8, six children were able to solve the first part, as was Child Sb. But none were able to set up an expression to prove the more general result. Question 5 also showed the universal lack of ability to manipulate algebraic expressions, where most found it hard to follow instructions such as ‘divide the algebraic expression by two’.


(a) I think of a number, then I carry out these operations on my number.

Multiply by 5	Add 8

When I carry out the operations in one order the answer is **105**

When I carry out the operations in the **other order** the answer is 73

What is my number?

Show your working.

73 − 8 = 65

65 ÷ 5 = 13

105 ÷ 5 = 21

21 − 8 = 13

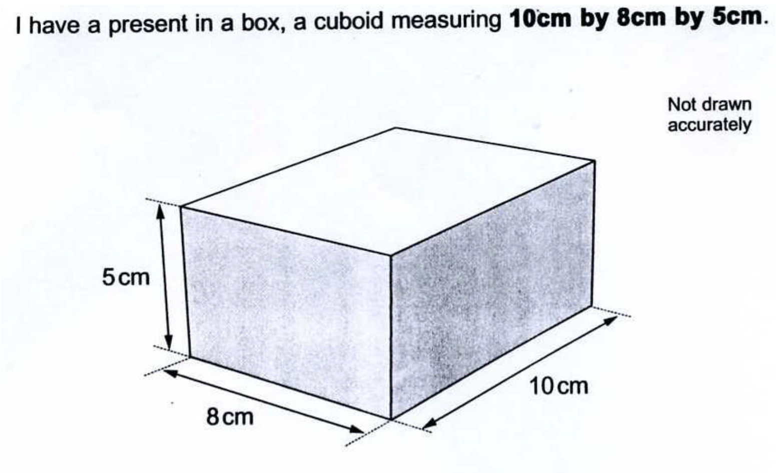
(b) The difference between my two answers is 32

Prove that the difference will **always** be 32, no matter what my number is.

Figure 3.10: Child Sb, Question 8, Trial of KS3 Questions

3.7.4.2 Shape

Question 6, involving aspects of measurement (surface area) and the correct interpretation of the text, was not solved correctly by any of the children in this pilot study. It appeared that this question was confusing to some children, as can be observed in Child Al's answer:



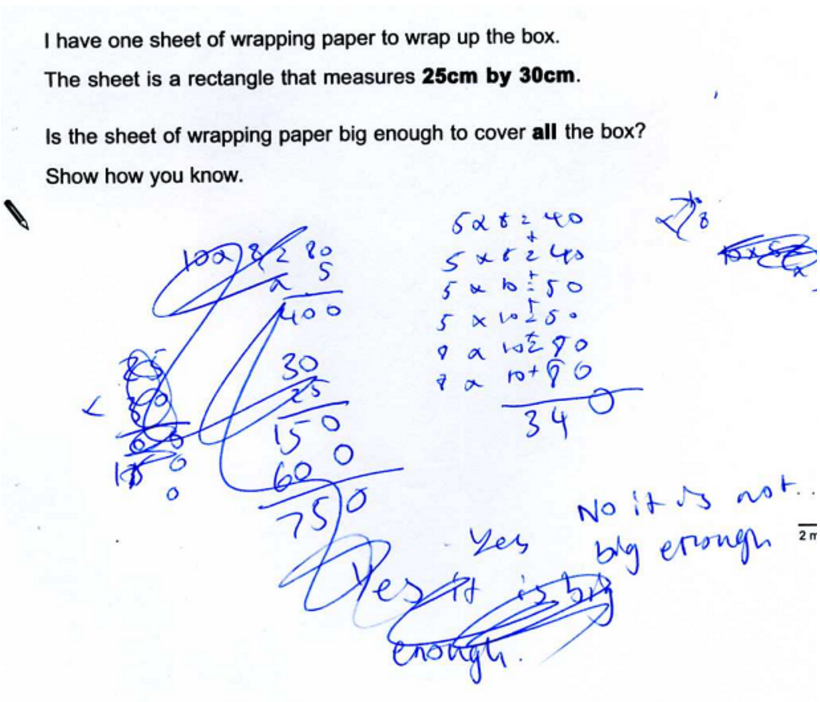


Figure 3.12: Child Al, Question 5, Trial of KS3 Questions

However it was decided that this question would be included in the main study to see if any of the home-educated children would be able to progress further towards the correct solution. With regards to the question on angles (Question 11), two were able to find the angle for the first part, which had a 'labelled triangle' – however, only Child Sb was able to complete the second part, which had no labelled angles.

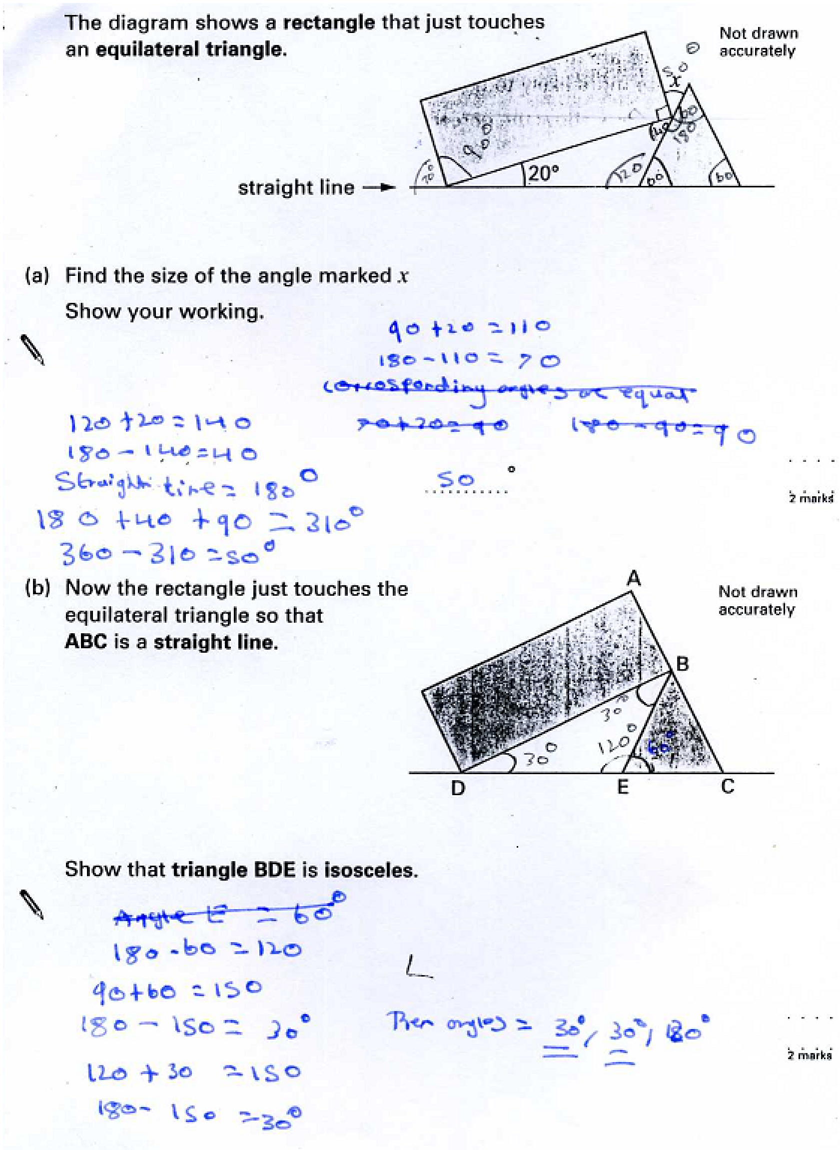


Figure 3.13: Child Sb, Question 10, Trial of KS3 Questions

3.7.4.3 Arithmetic

Questions 1 and 7 proved relatively simple for the children in the test but Question 4 showed that over half the children were unable to solve relatively simple questions involving fractions. For Question 3, on ratio, only two children were able to obtain a correct answer. The majority of the children made no attempt at the problem, and a few just guessed at the answer, e.g. a third etc.

3.7.4.4 Summary of Results from KS3 Pilot Study

It was noticed that questions on algebra generally proved difficult. Although most could substitute a given value of x into an expression, it was felt that many were unable to manipulate equations in order to make x the subject of an expression. Hence, it was felt that the assessed work posed at the KS3 level should include questions that require a degree of algebraic knowledge. The children in this pilot test had difficulty applying their knowledge to a 'real-life problems' that required them to first interpret the text of the question (e.g. Questions 3 and 6). Thus, Question 6 was chosen as part of the research instrument, to assess whether the home-educated children were able to make progress on such questions.

On the whole, most were confident with arithmetical operations – but it is clear that some did not fully understand fractions, with a number of errors appearing in basic calculations (addition of fractions, etc.). Rather than asking a number of straightforward arithmetical questions in the assessed work, the questions used in the main study included questions on trigonometry, algebra and shape, all of which required arithmetical manipulations as well as an understanding of the specific topic. This would enable one to determine how the home-educated children *applied* their arithmetical knowledge in a variety of different scenarios. The full set of questions can be found in Appendix 8.

3.7.5 Distribution of Assessed Work

The first stage of the main study involved the distribution of the questionnaires to both children and parents, which took place in the spring of 2005. A few months later, these parents were contacted to ask if their children wished to participate in the second stage of the study, where their children would be given the opportunity to attempt one (or more) of the assessed work tests. At the same time, around five 'new' families asked to participate in both the first and second stages of the study. This was not an issue, since I had not yet begun analysing data from the earlier questionnaires. An email was sent out to each family explaining that the questions were divided into three age-groups, which roughly corresponded to the equivalent Key Stage ages for each level, as defined by the National Curriculum:

- Questions, Group 1: 5-8 year-olds
- Questions, Group 2: 9-12 year-olds
- Question, Group 3: 13-16 year-olds

The parents were asked to select the most suitable set of questions for their children, with the opportunity to ask for a more appropriate set if the initial choice proved too easy/difficult for their children. Families were informed that there was no time limit for the questions, as the aim of the assessed work was to consider the various methods the children used to solve the problems, rather than their overall mark. The families were also given the choice of being sent the questions via email or post. However, I encouraged the participants to post the children's answers to the assessed work rather than sending them through email, as I was aware that it was very difficult and time-consuming to type out mathematical symbols. So whilst the main means of communication was via email, all of the children returned their answers to the assessed work via post. In total, 13 children took part in this stage of the study, with 20 sets of questions completed, as some children answered questions from 2 or more groups. We now discuss the advantages and disadvantages of the data collection method, which was primarily conducted over the Internet.

3.8 Use of the Internet in Educational Research

In order to gain access to the target population of home-educating families in the UK, it was felt that the Internet, as a primary method of data collection, would be invaluable, as it would allow contact and communication with families who would otherwise be inaccessible.

Internet-based surveys, while sharing many commonalities with paper-based surveys, have their own distinguishing features. They have the advantage of quicker response rates than those received via post. Especially with home-educating parents, there may be a long interval between the time when the questionnaires are initially posted to the respondent and when the completed documents are returned, simply due to the inconvenience of having to complete the questionnaire by hand, then going out to post the letter (that is, assuming a self-addressed envelope has been provided by the researcher).

On the other hand, with emailed questionnaires, the researcher does not have to worry about the questionnaire going missing; as email is generally considered to be a secure means of sending and receiving information. Hence, it is unlikely the questionnaire will be ‘lost’ either before or after completion. Most parents are likely to find it relatively simple to complete a questionnaire at the computer, before sending it as an emailed attachment. Whilst there is a possibility that recipients may not open their mail to access the questionnaire, it can be assumed that families who take an interest in participating in an online study are most likely to check their email at least a few times a week unless there are extenuating circumstances, such as holidays or illness. If there is no response after the initial questionnaire has been sent, the researcher can email a polite reminder to encourage a response.

3.8.1 Technical Considerations of Internet Research

When considering the technical features of the questionnaire, I experimented with both *'web-based' surveys*, where one could both design and post the questionnaire online and then forward the link to respondents to be completed online, and simpler *'Microsoft Word'* questionnaires that could be sent to participants via email.

Whilst it was found that the web-based surveys certainly had a clearer and more attractive layout than the Word questionnaires, there are a number of associated disadvantages with these surveys, which are highlighted by Dillman, Tortora and Bowker (1998b) and Dillman, Tortora, Conradt and Bowker (1998a). The respondents' Internet service might be too slow to download the pages that host the questionnaire. If some families did not have Internet access, it may prove difficult and time-consuming to convert the Internet page into a suitable format for printing. Moreover, some parents could find the 'click and display' method of answering questions via the web survey complicated, due to a lack of experience in answering online surveys. It was also felt that a web-based survey would not offer the same flexibility as a Word survey, since previously it was noticed that a number of parents would write clarifying comments concerning their responses to closed questions, and this would not be possible with a web-based survey. Therefore, the decision was made to design and distribute the questionnaire as a Word document, to allow for ease of completion both online, and as a paper-based document. Since this form of questionnaire had already been used to collect data for my MSc research, I was confident that the majority of parents would be able to express their perspectives on home-education and mathematics through this medium.

3.8.2 Possible Bias

One of the main issues associated with Internet-based research is the possibility of bias, where some sub-group of the target population may be under-represented amongst the respondents. In this study, I was aware that families without Internet access were likely to be under represented in this study, even though some families took part exclusively through postal questionnaires. At the same time, I felt that the vast majority of home-educating families would make use of the Internet as an educational resource, and so the percentage of ‘under-represented’ families would be relatively small. Furthermore, Eysenbach and Wyatt (2002) write that, especially for studies that are mainly qualitative in nature, it is not necessary to obtain an ‘average view’ of the parent population but rather an in-depth understanding of particular groups within the parent population. Another point to consider is that Smith and Leigh (1997) found there was no difference in income, education or ethnicity between users and non-users of the Internet.

Consequently it is recognised that there may be a strong bias in the data, however, in seeking to establish a relationship between the parents teaching approach, their background and their children’s mathematical understanding it was felt that even with bias the outcome would be informative. In this instance, the study may be seen as an exploratory study with additional developments considered within the final chapter.

3.8.3 Ethical Considerations

During an Internet-based study, the email addresses can immediately identify the individuals, especially since most home-educating parents use their real names. To ensure that such information remained private, all families (both children and parents) were kept anonymous during the study, and parental consent was always obtained before receiving information from the children. In fact, it was the parents who distributed the children's questionnaire and assessed work to their children; therefore, I did not have direct contact with the children at any time during the study. In the children's questionnaire, I also made it clear that the children should ask permission from their parents before giving out any additional contact details. All participants were provided with information regarding my personal education and background to reassure them that I would conduct the research in a considerate manner, including the name of my university, and the department to which I belonged.

3.9 Validity, Reliability and Generalisabilty

In this section, we will address issues of validity and reliability in relation to the chosen methodology. There are a number of different types of validity and reliability and whilst it may not be possible to achieve a perfect level of these measures, it is possible to reduce the risk of producing invalid or unreliable findings. In addition, issues of generalisation will be discussed in Section 3.9.5.

3.9.1 Validity

> "Validity is an important key to effective research. If a piece of research is invalid then it is worthless."
> (Cohen, Manion and Morrison, 2000, p.105)

Generally speaking, validity is a demonstration of how well a research instrument measures 'what it is supposed to measure'. In *qualitative research*, validity may be addressed by focusing on the honesty, depth, richness and scope of the data achieved, the participants approached, the extent of triangulation, and the objectivity of the researcher (Winter, 2000). This study therefore attempts to establish the following types of ‘qualitative’ validity (Maxwell, 1992), as explained below:

(1) **Descriptive Validity –** that the participants are honest in their accounts. To encourage an honest participation in this research, parents were informed that the study sought to discover the range of home-educating approaches used. There was no suggestion that any particular approach would be considered ‘better’ than the others, as my aim was to discover ‘what approach best suited their family, and why’. Thus, I hoped the home-educators would not feel the need to fabricate descriptions of home-education in order to meet any expectations of the researcher. Furthermore, a number of questions in the parents’ questionnaire were centred on the same topic (e.g. preferred method of teaching mathematics), and similar questions were posed for the children. Thus any contradictory answers could be addressed.

(2) **Interpretative/Theoretical Validity –** research catches the meaning of the situation, and is able to explain the phenomena. During the data analysis process, the meaning and interpretations of the data will be taken from the observed phenomena within the parental/child response. As I will not use pre-conceived categories of description, it is hoped that the observed phenomena will, to a large extent, capture the ‘meanings’ of the data.

I also aimed to increase the credibility of my research via the following means, as suggested by Lincoln and Guba (1985):

1. **Prolonged engagement** in the field – my research into home-education prior to the data collection took place over a three-year period. I was also familiar with the nature of the research population, as a former home-educated child.

2. **Persistent observation –** repeated requests for data were made, and follow-up questions were used if any points needed further clarification

3. **Triangulation –** where two or more methods of data collection are used on the same object of study. This was important when considering the two main areas of this research – approaches to mathematics education, and mathematical understanding, hence, all three research instruments were designed to consider these areas of focus.

4. **Peer comparison –** I had the opportunity to discuss my work with other home-education researchers (both in the UK and internationally) through our research email group. A home-education symposium at the British Educational Research Association Conference also allowed other academics within the home-education field to examine my study (Yusof, 2004), and provide feedback. Within the area of mathematics education, I presented my research findings during our mathematics education research department SUMINARS to gain a critical perspective from my peers in the mathematics education arena.

5. **Participant validation** - giving the participant the opportunity to add further validation. If any answers to the questions were unclear, the parents/children were given the chance to clarify their answers.

3.4.5 Reliability

Reliability refers to the dependability, consistency and replicability over time, of the research instruments and participants. Bodgan and Bilken (1992, p. 149) argue that, 'in qualitative research, reliability can be regarded as a fit between what the researchers record as data, and what has actually taken place in the original setting'. To achieve greater reliability in this study, the aim was to minimise bias in areas such as: (1) the attitudes, opinions and expectations of the researcher, (2) pre-conceived notions, and seeking answers to support these views, and (3) misunderstandings between researcher and respondent. This was done by:

(1) A prolonged exposure in the field, where the correspondence with the home-educators took place over at least six months up to a year. This enabled me to gain a better understanding of the research population and counter the effects of any pre-conceived notions.

(2) Triangulation – asking the same question through a variety of questions reduced the chances of possible misunderstandings from researcher or respondent

(3) Auditing – all emails and written correspondence with the home-educators was kept to ensure that the data could be verified, if necessary.

We now briefly discuss how issues of validity and reliability affect the particular research instruments that were used in this study.

3.9.3 Validity and Reliability of Questionnaires

Questionnaires tend to be more reliable than interviews, as participants may be more honest when they can respond without the pressure of 'face-to-face' contact – they can also remain anonymous, if desired. On the other hand, with questionnaires, there is little opportunity for clarification of questions, as the same questions could have different meanings for different people. Likewise, questionnaires could pose problems to those of limited literacy. To help overcome such issues, the language and structure of the questions were tested through the use of a pilot test (Section 3.5), where the participants were encouraged to highlight any areas that appeared confusing or required greater clarification.

3.9.4 Validity and Reliability of Tests

Issues that affect the reliability of tests include the perceived importance of the test, level of formality of the test, ways in which the test is administered, and the marking process (Cohen, Manion and Morrison, 2004). As I, as the researcher, did not intend to personally administrate the test, it was essential that the parents did not feel tempted to help their children with the questions, as this would obviously invalidate the results. Consequently, in my correspondence with the participants, it was emphasised that the study was mainly focused on 'how the children attempted the questions', rather than their overall performance on the tests. I did not set a time-limit in which the test could be completed, and the children were free to select the set of questions that they felt most confident to make an attempt at. The use of pilot testing and relying on the parents' choice of appropriate 'question sets' also ensured that there was little risk of the children being unable to comprehend the text of problems.

One advantage of letting the parents administrate the assessed work was that I was of no direct influence during the testing process. Since it could be reasonably assumed that most of the children will have attempted the assessed work questions at home, this eliminated the possible bias that can result from taking tests in unfamiliar settings.

When marking the questions during data analysis, I adopted an approach that was in-line with the phenomenographical nature of the study, where I classified the responses according to the 'solution strategy' used and tried to remain consistent in the classification process once a particular category had been defined.

3.9.5 Generalisabilty

The generalisabilty of a study is the view that the theory generated from the research may be useful in understanding other similar situations within the specific groups, communities or circumstances (Maxwell, 1992). To address the generalisabilty of the study, the results will be compared to previous home/mathematics education research, which may demonstrate the extent to which this study supports (or contradicts) the findings from similar communities. At the same time, it is recognised that this study is the first to consider the mathematical thinking of home-educated children in the United Kingdom, and hence the opportunities for comparison may be somewhat limited at present. However, as recommended by Lincoln and Guba (1985), I aimed to obtain sufficiently rich data so that readers and users of this research could determine its applicability and comparability to other situations.

3.10 Summary of Methodology Chapter

To summarise:

- Section 3.2 examined various methodologies used in previous studies on home-education. An approach utilising questionnaires was felt to provide a means of obtaining data related to the parental teaching approach, and the mathematical beliefs of both child and parent.

- Sections 3.3 and 3.4 then discussed the particular features of questionnaires, and types of questions (open and closed questions, use of Likert-type questions etc.) that could be used to investigate the different approaches to mathematics education. Furthermore, Section 3.4 described how exploratory studies and an examination of literature relating to *mathematical belief, mathematical understanding* and *problem solving* aided design of the children's questionnaire.

- Section 3.5 mentioned how pilot studies were used to gain feedback on the parents and children's questionnaires and make necessary changes, with Section 3.6 detailing the distribution process of these research instruments.

- Section 3.7 examined the use of assessed work to determine children's mathematical thinking. Two exploratory studies were undertaken in order to identify appropriate sets of questions – with the different 'types of problem solving approach' observed in the responses taken to be the determining factor as to whether a question was included in the final study. Three groups of questions were constructed, corresponding to different levels of ability (Key Stage 1, Key Stage 2 and Key Stage 3).

- Section 3.8 critiqued the use of the Internet when conducting research, and Section 3.9 covered issues of validity, reliability and the generalisation of the study.

Now that the appropriate methods of data collection have been discussed, Chapter 4 analyses the data obtained from home-educating families in the United Kingdom via the various research instruments.

Chapter 4: Data Analysis for the Parents' Questionnaires

From data obtained through the parental and child questionnaires and the children's answers for the assessed work, the data analysis chapters (Chapters 4 and 5, and 6) aim to give an insight into the ways in which the parent's approach to mathematical learning influences their children's perceptions and understanding of mathematics.

One of the key aspects of the phenomenographical approach is the identification of qualitatively distinct categories from which the researcher can then try to understand and establish relationships (Marton, 1994). In Chapter 4, we shall therefore consider the responses from home-educating parents, with the intention of identifying the range of approaches used to teach mathematics, the mathematical beliefs of the parents and their perceptions of themselves as teachers. The data used to generate these findings was collected from the parental questionnaires (see Appendix 1), where 28 families participated in this stage of the study.

Particular areas of investigation will initially focus on the *reasons for choosing home-education* (Section 4.1) and the *background* of the parents (4.2). Next, Section 4.3 considers the mathematical *beliefs of the parents*, and beliefs on the 'importance of learning mathematics' (4.4). Section 4.5 examines how parents *view themselves as 'teachers of mathematics'* and the main influences on their *teaching activities* (4.6). Section 4.7 then takes a closer look at the *use of textbooks* and other activities that were beneficial to aid the learning of mathematics (e.g. computers, games, everyday life activities and so on), with Section 4.8 asking parents to detail the current mathematical topic that their child(ren) was learning. Once the activities used to learn mathematics have been established, Section 4.9 examines the use of learning timetables when teaching mathematics, with Section 4.10 then considering the parents' views on how home-education has helped (or hindered) their children's mathematical development.

The above analysis aims to enable us to form a picture of the different approaches to teaching mathematics within a home-education family, and with this in mind, Section 4.11 considers the ways in which parents observe their *children's mathematical understanding*. Finally, Sections 4.12 and 4.13 examine the incentives given to the children and the 'long-term' goals of the parent, with regards to their children taking formal mathematics exams.

4.1 The Home-Educating Parents

28 parents completed the questionnaires. All 28 were observed to be the mother in the family, although in some instances, the fathers' teaching approaches were referred to within the questionnaire responses. Each respondent will be identified by the family that they belong to, e.g. Family 28 will refer to the parent of the family and Child 28 will refer to the child.

Eight families had chosen to home-educate from birth, while the remaining twenty parents had initially sent their children to school before the decision was made to home-educate all school-aged children. For the latter families, any subsequent children were home-educated from birth. The median number of children per family was two – where seven families had an only child and at the other extreme, Family 20 had seven children, all educated at home. No questions concerning the income-levels of the parents were asked – note Rothermel (2002) found that socio-economic background was not a significant factor with regards to the children's performance on mathematical tests. Secondly, it was felt such questions may be considered too personal. However, the families' educational backgrounds (particularly with regards to mathematics) and prior teaching experiences were felt to be important, and these will be examined in Section 4.2.

In an attempt to identify the various approaches to home-education, initial consideration will be given to the parents' chief motivations for choosing to home-educate their children.

4.1.1 Main Reasons for Choosing Home-Education

Before presenting the findings for this section, a brief explanation of the process of categorisation will be given. For questions that resulted in qualitative data, the following steps were used to establish the categories:

1. All data pertaining to the specific question were collated and tabulated.

2. A number of possible categories were identified by common words or phrases that appeared in the parental quotes, e.g. 'everyday life', 'bullying', 'happiness'.

3. Once possible categories had been identified, each quote was re-examined with respect to each of the categories of description – colour coding was used to aid this process. An example of this when establishing the 'main reasons for home-educating' can be seen in Appendix 11. This step was repeated a number of times until appropriate categories were identified, both in terms of

the size of each category, and the extent to which the categories accurately described the families.

4. Once the categories were deemed suitable, numerical coding was used to assist the formation of relationships and aid triangulation.

It was found that the most common reason to choose home-education stemmed from the belief that the schooling system was too restrictive, with **20 out of the 28** home-educators believing that the ‘inflexible’ school learning environment resulted in unhappiness and limited personal development, thus restricting their child’s learning potential:

> “I feel that early formal education is often **harmful** to children’s personal and academic development and makes many young children **unhappy**. In institutionalised education, **timetables, standardised teaching methods and content cannot meet the emotional or intellectual needs of each specific child**.” Family 4 (daughter aged 5)

Words associated with emotions of happiness were cited by **13** parents:

> “He **hated** school! (had to stop chatting and couldn’t sit for hours on one task)”
> Family 9 (children taken out of school when eldest was 7 years old)

Whilst a concern with the development of the child influenced the decision of 20 of the parents, there were also instances where home-educators drew upon their own experiences of the education system to justify their decision:

> “Mother (me) has **experience of working in education** system and hated the lack of individual care and respect for each child. My husband was **educated privately** and did not feel that the **state system** would be beneficial as she [the daughter] is so lovely.”
> Family 27 (daughter aged 3 years, never been to school)

There was also an explicit belief that children would learn more if they had greater influence on the nature of their learning environment:

"I feel that children are more likely to retain their natural love of learning if they are allowed to **control the content and pace** of what they learn, and if they can **choose to learn things** at a time when they are personally relevant."

Family 22 (children aged 5, 3 and 1 years, never been to school)

A quarter (7 families) identified their children as having 'special needs' or requiring additional help as their children were labelled 'slow'. They frequently drew upon the notion of 'individual need' and a 'lack of support' as their justification for educating their children at home:

"I have several children with Asperger's syndrome and two with autism and feel that their **needs were not met in school** and the youngest, who are autistic, have never been as I do not think schools are able to give children **individual learning programmes** they need with SEN [Special Educational Needs]

Family 20 (7 children: eldest left school aged 11)

Six chose home-education because they felt that their children required extra time or additional help to learn certain concepts. They believed that if their children were at school, they were likely to be labelled as 'slow':

"My first daughter **was slow** to reach all her developmental milestones in comparison with her peers (although she got to them all eventually) and I just didn't want her to be **labelled as "slow"** by some teacher when she was just 4 as I knew this label would stick with her forever.

Family 3 (children aged 5 and 2 years, never been to school)

Social reasons were also a factor for some families, as **8** parents indicated that their children had been bullied at school, either by other pupils or, as in some isolated instances, by a particular teacher:

"Our oldest Tim, was **being bullied**. The school refused to accept our complaints, and they believed our son was not sociable and therefore [he] became a target!"

Family 16 (children taken out of school at 6 and 5 years of age)

"Went to small local school – only 7 in class. Was **bullied by teacher** for wanting to do maths!!!! Lasted 5 weeks!"

Family 21 (eldest taken out of school at 4 years, younger sibling never attended school)

It is perhaps not surprising that 7 out of the 8 families who cited bullying as their main reason for home-education also noticed their children were unhappy in school. Family 18's comments illustrate how pressure from teachers and bullying in the playground appeared to make home-education one of their few remaining options:

> "Daniel is brilliant at all subjects except numbers. After two difficult years at school Daniel 'shut down' in maths – the teachers put pressure on him to move on with the rest of the class before he was ready. Doing times table tests before he could understand number bonds to 10 they made no allowances for his weakness. They were dismissive of our request for advice & help. He was also bullied in the playground over two years, he seemed to draw attention to himself, he was **angry with school** which he saw as unjust.
>
> Daniel's feelings were of paramount importance to us – he cried a lot and couldn't cope with the thought of doing maths lessons although he enjoyed other lessons. We decided after researching home education for a year that it had to be better than school – that **education came second to our child's well being**. It soon appeared that Daniel was learning far more at home then he had ever learnt at school." Family 18 (youngest child taken out of school at 8)

4.1.2 Summary

The belief that the *'school environment would be detrimental to the children's social or academic development'* was a key influence on the decision to choose home-education over school. With regards to their children's academic needs, parents of children labelled by school as 'too slow' felt that their children were not given enough time to develop their mathematical abilities, and an inflexible educational setting would result in their children holding a negative view of learning.

On the other hand, parents of children felt to be 'ahead of the class' mentioned their child's feelings of 'boredom' at school – they believed that the teachers were unable or unwilling to cater for their children's individual abilities. For these parents, the schooling system did not cater for children who, it was perceived, fell outside of the 'normal' range of ability – home-education would give their children greater control over 'what they are learning, and when they are learning'.

Similarly, families with ‘special needs’ children also cited the lack of individual support within mainstream schools, and thus believed that their children would receive greater attention at home. This finding is similar to Rothermel’s (2000) claim that UK home-educators seek to provide their children with a flexible learning environment to cater for their individual needs. It was also clear from the responses that the majority of home-educators in this study could be considered pedagogues, as described by Romanowski (2001), since the parental decision to home-educate was generally motivated by a desire to provide a better education than available at school. Negative *social aspects of the school environment* such as bullying, an inability to associate with peers, or perceived persecution from the teachers, were other key reasons for home-education. These circumstances led parents to believe that, within the school environment, their children had high levels of stress, were unwilling to attend classes, and generally felt unhappy. Whilst there are signs that parents tended to attempt to resolve these issues with the school authorities, a lack of progress resulted in the conclusion that home-education was the only remaining option.

Although it was mainly the parents who made the final decision as to whether or not their children should be educated at home, in some cases the *children* also had an influence on the decision.

4.1.3 Influence of Children When Choosing a Home-Education

The level of influence of the children when making the decision for home-education appeared to be *age dependent*. For **8 of the 14** families who began home-educating when their **children were under the age of seven years**, the parents generally made a sole decision - they felt their children were unable to make an informed choice at this early age:

> “It was **my decision**, as my child was, and is, **too young to understand**. Though my child says she prefers home education, this is largely the result of the pro-HE propaganda I have given her, rather than a reasoned opinion. She has **never been to school** and so cannot know whether she would prefer it to home education.” Family 4 (daughter aged 5)

On the other hand, for nine families in this study, it was the *children who requested* to learn at home. This was usually after the children had experienced problems in school:

> “The main reason was the **children’s repeated and firm requests** to be home educated. My son was originally taken out due to bullying and lack of support, then tried school in year 1 and couldn’t cope so requested to be home educated again. My daughter entered school at reception but **asked to be removed** as soon as she went into year 1. My personal beliefs have influenced us too and I will not be putting my youngest child in school when she reaches that age. Family 7 (eldest child left school aged 4, the rest were home-educated)

In some instances, it was the child’s refusal to attend school that led to this decision:

> “My child made it clear he was not going to school anymore. We found home education was a possible only option left for us. Family 10 (youngest child was taken out of school at 10)

In those instances where the decision was a joint parent/child decision (12 families) there appeared to be an awareness of the possibilities of home educating:

> “A bit of both really. I had always been **interested in HE** [Home Education], and she had mentioned it but it was not feasible when I was working, so we are now happily HE and totally broke!” Family 1 (daughter taken out of school at 12)

Thus, the evidence suggests that families with younger children relied mainly on the parents’ preference for home-education, whereas families with children of ‘school going age’ were more likely to give their children a greater say in the decision making process.

After having identified the primary reasons for choosing home-education, we next focus on the particular areas that may help distinguish the families’ home-educating approaches, beginning with an analysis of the parents’ mathematical backgrounds

(Section 4.2.1), and their previous teaching experiences (Section 4.2.2). This may provide some insight into the parent's educational choices to be discussed later in the chapter (Section 4.5 onwards).

4.2 Qualification and Experience of Parents

In this section, the parental experiences of mathematics will be examined.

4.2.1 Mathematical Background of Parents

The majority of parents (**18 out of 28**) studied mathematics in formal education no higher than the GCSE/O-Level stage. A further three parents had obtained A-levels in mathematics, whilst five claimed to have used mathematics as an important aspect of their degree **–** e.g. in engineering or business studies at university:

> "Warwick MBA (statistics, accounting etc.) DH is a fund manager." Family 12

Three families wrote 'other' when describing their highest mathematical qualification, or level of achievement:

> "I have a degree module in maths from the Open University but have not finished my degree [Engineering]." Family 18

Asked to indicate whether they or any other close family member (e.g. mother, father, grandparents, or children) had worked in a job that is mathematical or numerical in nature, **20 parents** wrote that they had at least one family member within 'mathematically dependent' employment. For example, **8 out of the 28** home-educators had a close family member who worked in banking or other financially oriented work:

"Husband moves large pieces of freight and has **constant maths problems** to solve to fit cargo in and pricing. I was a bank clerk. I worked in a bank and used a lot of maths that developed my knowledge." Family 10

"Grandfather **maths teacher**, father **bank manager**, brother **accountant**." Family 5

Six parents mentioned engineering or electronics:

"James – computer engineer

Marissa – aeronautical engineer apprentice

Victoria – bank clerk." Family 18

Interestingly, four wrote that either they or a family member had **taught mathematics** at some stage in their working lives (Families 5, 8, 9 and 17):

"My husband **taught maths** in secondary school." Family 8

"I'm an **ex-maths teacher**!" Family 9

An almost equal number (5 families) worked with computers or within the Information Technology industry:

"I used to work in **computers** as a project manager/systems designer. My husband is a computer project manager." Family 16

18% of families (5 of the 28) had had employment where some mathematics was used:

"Father has worked in data analysis and statistics." Family 19

"I worked in a lab and used maths a lot." Family 24

4.2.2 Teaching Experience

Parents were asked if they had previous teaching experience, and asked to detail the subsequent effect on the teaching at home. Eleven of the parents had some experience of teaching, and of these parents, five held recognised teaching qualifications.

Of the eleven who had taught previously, six felt that their previous teaching experiences were *useful* when home-educating their children. Some noted an awareness of different teaching techniques, leading to the formulation of their own teaching approaches when home-educating. This was observed for those with formal teaching qualifications, e.g. Family 3, as well as parents who had taught informally for a number of years, like the mother in Family 4:

> "PGCE. It keeps me **aware that things** like the National Curriculum exist and that I can take as much or little from it as I like." Family 3

> "No qualifications, but I have done teaching for a total of about five years. I have noticed during my teaching that **learning is more effective in small groups** than large ones, that fear greatly inhibits learning, and that motivation and interest are critical for learners. Focussing on exams restricts learning, whereas having time and freedom to explore ideas is liberating and leads to greater understanding in the long run. This has encouraged me to adopt an autonomous approach to my daughter's home education." Family 4

Note, however, that the teaching experiences were not necessarily within the same subject area as the topics taught at home. Sometimes parents would draw upon the *ways in which the topics could be taught* from their past teaching employment. For example, the mother in Family 28 found some of the learning theories that were covered during her course on Office Studies could also be implemented in her family's home-educating pedagogy:

> "Yes, I have the RSA teachers' certificate but only to teach office studies. However, I did have to study **two learning theories** (Gestalt and Pavlov) and these have helped me see, and change, methods of teaching as and when the need arises. For instance, repetition is good for spelling and punctuation, but for history we might watch the TV or visit an historical place. Flexibility is all important."

At the same time, three parents felt that whilst many of the ideas from their previous teaching experiences were useful, the approaches that they had learnt for classroom teaching could not be always be fully implemented when home-educating:

> "At first when I studied Montessori, I realised how much I needed to know to make the maths fun yet educational, despite the four years I had spent in university and actual teaching time in schools. I think I was **too formal to start with as this is how I was taught to teach in school**, but have slowly mellowed as the years have passed."
>
> Family 2 (Parent has B.Ed to teach Primary and Special Needs children)

> "I have a certificate for Montessori 3-6 classroom assistant and **tried to use Montessori methods with her but many of them did not suit her**. It did give me many ideas for maths games such as the "Bank game" which we have used a little and will use more as time goes on."
>
> Family 25

Only one home-educator believed that her teaching experiences were detrimental to her home-educating:

> "Was a **hindrance t**o begin with until I ignored all we had learned except for some John Holt reading!"
>
> Family 9

Rather surprisingly, it was noted that this parent was formally a mathematics teacher!

4.2.3 Summary

To summarise, while the **majority of parents had studied mathematics no further than the GCSE stage**, nearly three-quarters had a family member who was working in a job that required mathematical skills. Eleven had teaching experience, most finding that they had used **some elements** of their previous teaching experiences when implementing their home-education approach, generally by identifying 'what worked/didn't work' in the classroom and making adjustments to suit their child.

Associating the *parents' mathematical beliefs* with their educational background or employment may indicate the resultant effect on their perceptions of the subject. It is to these that we now turn since they provide an additional influence to the chosen approach to home-education.

4.3 Parents' Mathematical Beliefs

This section aims to give insight into the nature of the parents' mathematical beliefs through an examination of their responses to the questionnaire item: "*What does mathematics, as a subject, mean to you?*" (Question 1, p.4 of the parents' Questionnaire, Appendix 1). A close examination of all responses suggests that eight specific beliefs associated with mathematics can be identified (Table 4.1):

Belief 1	Important to everyday life
Belief 2	Fun/enjoyable activity
Belief 3	Logical/mental or abstract exercise
Belief 4	Numbers
Belief 5	Important for scientific areas or other careers
Belief 6	Dislike mathematics
Belief 7	Mathematics is hard
Belief 8	I do not think that it is that important

Table 4.1: Specific Beliefs Associated With Mathematics

A large proportion of the sample (19 out of 28) identified mathematics as an *important skill for everyday life*:

> "…I do think maths and **day to day living** go hand in hand – from baking a cake, paying the bill, working out the area of tiles needed in the kitchen etc." Family 6

At one extreme Family 22 perceived 'basic maths/arithmetic' as a key component of this skill:

> "Mathematics in the form of **basic arithmetic** is essential in **many areas of life** and initially I aim to give my children a solid foundation in those areas." Family 22

At the other extreme, Family 7 believed that knowledge of 'more complex' mathematical concepts was useful in many different areas of life:

> "It is **essential to daily life**, my family were builders and used **trigonometry**, calculation of **mass etc** daily so I grew up being very comfortable with **all forms of mathematics**. Also maths is fun, we all really enjoy it and it is needed for other subjects, particularly science. We are surrounded by **numbers in life** even looking at proportion in art is mathematics."
>
> Family 7

Two believed mathematics was *not particularly important* though they acknowledged its basic utilitarian benefit:

> "**I don't think mathematics is as important** as we would like to think. To many people it is irrelevant other than being able to do day-to-day calculations. Some people have different strengths and shouldn't "HAVE" to do mathematics." Family 24

Almost one quarter of the sample (six families), included a reference to *'numbers'* in their perception of mathematics:

> "I see it as a means to an end, **an ability to manipulate numbers** to help with everyday life situations" Family 14

But nine believed mathematics was more than 'just operations with numbers', and stressed the *logical or abstract nature* of mathematics:

> "**It means logic**. Those who are good at maths are generally well organised and methodical with an inquisitive mind. They want to **problem solve** and are good at it. They see **patterns**. They also have a high earning potential." Family 28

The explicit contribution that mathematics can make towards the *later careers* of their children was identified by five families:

> "EVERYTHING. I believe it encourages LEARNING in the broadest sense and should be the essence of learning. People who are mathematically astute are capable of understanding the sciences, and even the arts. It is **no wonder that graduates of mathematics** are in great demand worldwide." Family 16

Eleven families held the view that mathematics was *a fun and enjoyable activity*:

> "It is an indispensable tool for life, but also an **enjoyable activity** and a great brain stretcher, there's always something new to learn." Family 5

On the other hand, three parents, whilst acknowledging its value, *did not favor mathematics* as a subject. In fact, it was in response to her own dislike of mathematics that the respondent from Family 13 indicated her intention to make mathematical learning enjoyable for her children:

> "I want to **make maths fun** and show how it is used and important in everyday life**. I disliked maths** and the way it was taught to me as a child and want it to be different for my children."
>
> (Family 13)

In order to help identify possible relationships between the different perceptions of mathematics, Table 4.2 indicates the percentage of families that subscribe to each belief, listing the families belonging to each 'category of belief'.

Perception of mathematics	**Percentage of parents with this belief**	**Parents with this belief**
1: Important to everyday life	68%	1, 2, 3, 4, 5, 6, 7, 10, 13, 14, 15, 16, 18, 20, 22, 23, 25, 26, 27
2: Fun/enjoyable activity	39%	1, 4, 5, 7, 8, 10, 11, **13**, 18, 19, 21
3: Logical/mental or abstract exercise	32%	2, 5, 6, **8**, 17, 19, 23, 27, 28
4: Numbers	25%	7, **8**, 9, 10, 14, 18, 26
5: Important for scientific areas or other careers	18%	4, 7, 16, 27, 28
6: Dislike mathematics	11%	12, **13**, 20
7: Mathematics is hard	7%	6, 15
8: I do not think that it is that important	7%	18, 24

Table 4.2: Mathematical Beliefs of Parents

Notice in Table 4.2 that the percentages in the second column **do not** add up to 100% as a parent could view mathematics in more than one way. For example, the mother in Family 1 believes mathematics is important to everyday life but also finds the subject 'fun'. Therefore, whilst the categories are discrete, the parental responses may include a multiple set of qualities – i.e. responses that fall into more than one category.

On inspection of Table 4.2 it can be seen that the **most commonly** held belief is: **(1)** *Mathematics is important to everyday life*. Conversely, the **least common** belief, with only two parents falling into this category, is **(8)** *Mathematics is not that important.* Such views could be identified as the extremes of a continuum and similarly belief **(2)** *Mathematics is a fun and enjoyable activity* and beliefs **(6)** and **(7)** *Mathematics is hard/I dislike Mathematics'* also appear to be extremes in a continuum. The mother from Family 13 appears to be an exception, holding beliefs from **both categories (2) and (6),** as highlighted in red in Table 4.2. Clarification of such a view is obtained from the mother's comments who, though she disliked mathematics at school, attempts to encourage different mathematical beliefs in her children by making the subject 'fun' when teaching.

We see that families who believe mathematics is a subject that predominantly involves *numbers* (Belief 4) **do not** generally hold a *'logical/abstract'* (Belief 3) notion of the subject. An exception is Family 8 (highlighted in blue), who holds beliefs belonging to both categories. Whilst the respondent does not appear to be strongly inclined towards mathematics, she emphasises its numerical applications to everyday life. Her husband is a mathematics teacher, whilst their son, a self-motivated learner of mathematics, is happy to explore mathematical concepts in his own time.

Perhaps through her family's mathematical experiences, the mother in Family 8 has come to appreciate both the numerical and the 'more abstract' applications of mathematics. Now that parents' core beliefs associated with mathematics have been identified, we next examine the relationship between their views of mathematics, and their approach to teaching mathematics. It is to this that we now turn by considering responses to the question: *"Why do you teach your children mathematics?"*

4.4 Parent's Reasons for Teaching Mathematics

In this section, the level of importance that the home-educators give to the various 'reasons for teaching mathematics' will be discussed. Parental reasons for teaching mathematics (Q.4 in Questionnaire for Parents, Appendix 1) were identified through ranking a seven point scale ranging from **"Most important" (1) to "Least important" (7)**.

	Degree of importance ←				→ Degree of Unimportance				
I want my child to learn mathematics because:	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**Modal ranking**	**Median ranking**
(a) Mathematics is an interesting subject	19%	**22%**	19%	19%	4%	4%	15%	2	3
(b) We all need to know some mathematics to deal with everyday situations	**82%**	14%	0%	0%	0%	0%	4%	1	1
(c) It helps children think in a logical way	25%	**36%**	18%	11%	0%	4%	7%	2	2
(d) I don't want them to be afraid of the subject as they grow older	**32%**	14%	11%	7%	14%	11%	11%	1	3
(e) Most other scientific disciplines require mathematics	7%	**29%**	11%	25%	18%	4%	7%	2	4
(f) They need to pass exams	4%	7%	0%	7%	**29%**	25%	**29%**	5 and 7	6
(g) It is a subject covered in every school curriculum	0%	0%	0%	4%	7%	18%	**71%**	7	7

Table 4.3: % of Parents Who Gave a Particular Ranking to Each Option, and the Associated Mode and Median Ranking

Table 4.3 indicates the percentage of home-educators who assigned a particular ranking to each statement (note that each parent approximately equals 4% of the sample), and summarises the modal and median rank for the statement. The figures in red indicate the percentage of home-educators for each modal class.

Perhaps the most interesting feature to emerge from the table is the fact that, of 49 possible responses, only two give a clear indication that there is a consensus amongst parents. That is, **82%** (23 parents) place a high degree of importance on learning mathematics because their children will need to deal with *everyday situations* whilst **71%** (20 parents) place a low degree on learning mathematics because it is *in the school curriculum*. Each of the reasons will now be discussed in the order from most strongly held opinion to least strongly held opinion, that is in the order: **(b), (g), (c), (d), (e), (a), and (f).**

Clearly parents' indications that their children "… *need to know some mathematics to deal with everyday situations"* stands out as the important reason for learning mathematics. Indeed only one parent (4% of the total) regards it as unimportant. Family 12's comments on the issue of learning mathematics illustrate the more general perception of its importance to every-day life:

> "It is making sense of our **everyday lives**: how we record abstract as well as concrete thoughts. It helps us understand the world we live in and see its use daily. Maths is everywhere and we are part of it." Family 2 (children aged 7and 3)

16 out of the 19 families who mention the everyday aspects of mathematics in their **'perception of mathematics as a subject'** (Table 4.2) also rank the 'application of mathematics to everyday life' as the most important reason for **teaching mathematics**

to their children. In other words, parents who view mathematics as a subject necessary for everyday life strongly believe their children should learn mathematics ‘so they can deal with everyday life situations’.

No home-educator ranked reason (g) = *“It is a subject covered in every school curriculum”* as important, with 20 families ranking (g) as 7 – the **least important reason for teaching mathematics**. This is reflected in Family 20’s reasons for choosing a home-education:

> “I was never, ever happy with the school system, the **limited learning opportunities the N.C** [National Curriculum] gives and the way children behave in schools – especially secondary schools.” Family 20, children aged 15, 14, 13, 12, 10, 7 and 7

79% (17 parents) suggest that an important reason for learning mathematics is because it ‘helps children to think in a logical way’. It was perhaps no surprise that the three families (Families 17, 19 and 28) in this study who perceived mathematics as a logical subject gave statement (c) a ranking of 1:

> “**Logical thinking** and reasoning” Family 17

We might therefore infer that parents who believe mathematics is ‘logical’ tend to teach mathematics because they assume that mathematics will ‘help their children think logically’. Interestingly, the two families who *did not* regard logical thinking as an important reason for teaching mathematics both followed an *autonomous/informal* approach to home-education. Both parents claimed that the **only reason** their children were taught mathematics was for its application to everyday life tasks, and consequently, ranked **every reason as ‘unimportant’**, apart from those associated with everyday life.

Apart from item (b), the item that evoked the highest ranking of 1 was item (d) concerning *the development of a fear of mathematics*. However, this was identified as the most important reason for learning mathematics by only one in three parents. In contrast, **just over a third** of parents regarded it as **relatively unimportant** in their desire that their children learn mathematics. These figures suggest that parents who felt most strongly about 'the fear of mathematics' may possess a particular characteristic that distinguishes them from those who feel less strongly. If we consider those parents who ranked (d) as most important, we see that **6 out of the 9** who ranked reason (d) as 'most important' mention that their children 'disliked' or 'hated' school:

> "1st child was unhappy and not doing as well as we knew she could at school. 2nd/3rd – travelling for a year, no. 3 decided not to go back – doesn't like school and is bored there."
>
> Family 19

Perhaps these parents sought to counteract the 'negative' experiences of school by formulating a positive experience of mathematical learning at home. From Table 4.3, it can be seen that the majority (60%, i.e. 16 out of the 27 who gave a ranking for this item) of home-educators have given (a) a ranking of 1, 2 or 3. This can be noticed in Family 19's view on the subject:

> "Useful way of understanding some things, can be **fun and interesting**. Good to develop logical thinking." Family 19, children aged 15 and 14

But conversely, four families (Families 8, 20, 21 and 24) consider the 'interesting aspects' of mathematics to be the least important reason. These families all claim to be autonomous or take an informal home-educating approach (see Section 2.1.4), and as mentioned earlier, such parents tend to give little importance to 'teaching' their children any topic, apart from those skills necessary for everyday life.

For (e) = *'Most other scientific disciplines require mathematics'*, Table 4.3 shows a modal ranking of 2, but a median of 4. This indicates that there may be a 'particular type' of home-educating family who place more importance on the application of mathematics to scientific areas than the 'average family' in this sample. To help discover the reasons for this, we consider the parents' backgrounds. It was noticed that 20 out of the 28 families in this study have family members who work (or have worked) in jobs that are numerical in nature. On further inspection, it was somewhat surprising to find that **none of the families with a parent who had taught mathematics** in school give (e) much importance, all ranking in the range of 4-7 (i.e. relatively unimportant). For example, the parents from Family 8 and Family 9 give rankings of 7 and 4 respectively, where the father in Family 8 and the mother in Family 9 both taught mathematics in secondary school. In other words, the ex-mathematics teacher parents appear to place little importance on the utilitarian aspect of mathematics in the scientific context. But by *excluding the four families* where a parent is (or was) a mathematics teacher, it can be seen that 12 of the 16 home-educating parents with a close family member in a 'mathematically related' job believe the application of mathematics to the sciences is an **important reason** for teaching mathematics. Perhaps the parents' mathematical experiences within the workforce resulted in them valuing the utilitarian scientific applications of mathematics.

The majority of families (83%, i.e. 23 out of 28) feel that the *'need to pass exams'* is **a relatively unimportant reason** for teaching their children mathematics. Family 2 gives this reason a ranking of 7; in fact, her dislike of tests and assessments was one of the motivating factors behind her decision to home-educate:

> “**Hate assessments** of young children, tests etc.” Family 2, children aged 7 and 6

However, while the findings suggest that most consider ‘exams’ to be an insignificant motivating factor, three families (Families 11, 14 and 28) give the ‘need to pass exams’ relatively high rankings from 1-2. Families 11 and 28 both had children undertaking IGCSE/GCSE Mathematics exam – in fact, these were the only families with home-educated children who were taking exams at the time of this study. Thus, the parents’ beliefs towards exams could have been influenced by their children’s current exam involvement.

On the other hand, although her children were not currently taking exams, the parent in Family 14 also gave this feature for teaching mathematics a high ranking of 2. Family 14 had only taken their children out of school within the past year, and had initially followed a structured teaching approach, making use of standard textbooks with a focus on exam orientated study. However, after the first three months of home-education, their approach changed to a more flexible, child-centred and interest based approach. It would be interesting to see whether the parent’s views towards exams change after a longer period of time spent following a less exam-focussed approach.

Half of the respondents (14 families in total) provided **additional reasons** for teaching mathematics. Ten families indicated that they taught mathematics because their children enjoyed the subject:

> “It can be **great fun**. My daughter seems interested and inclined to seeing patterns.” Family 27

The notion of confidence was mentioned by four home-educators, with one mother drawing upon her own mathematical experiences to exemplify the benefits of such confidence:

> "Total self-confidence is a wonderful by-product of mathematics, and never to be "blinded by science". When I left university it was remarkably easy for me, a black girl, to be employed by a large corporate computer organisation, at a time when not many women were in the industry, let alone ethnic minorities. I have had a wonderful career and now I hope to assist our children down a similar path." Family 16

Like the mother of Family 16, **3 out of the 14** parents who provided additional reasons believed mathematical knowledge would be important to their children's *later careers*:

> "It's important in life and general understanding. It's great exercise for the brain, and opens up new ways of thought. My nine year old loves **physics** and **chemistry**, and will need math to understand or work further in that." Family 23

Two parents taught mathematics because they themselves *personally found the subject enjoyable*:

> "There are so many **fun things** to do and it's wonderful to watch my children work at and grasp ideas that then become part of their thinking and working out skills." Family 2

4.4.1 Summary

In this study, the findings show that the most important reason for home-educators to teach their children mathematics stems from a belief that *mathematics is an essential aspect of everyday life*. With regards to this particular belief, it is also evident that the **parental perceptions of mathematics** influence their **beliefs on teaching mathematics**, as 84% of the parents who **believe** 'mathematics is a part of everyday life' also feel that learning 'mathematics in order to deal with everyday life situations' is the **most important reason** for their children to learn mathematics.

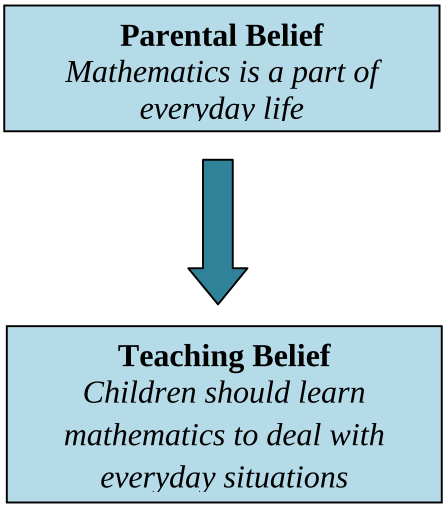


Figure 4.1: Beliefs on the Everyday Applications of Mathematics

Families who consider themselves autonomous write that this is the main, and perhaps the only, reason they encourage their children to learn mathematics.

Judging from the median rankings within Table 4.3, the second most important reason to teach mathematics stems from the belief that *mathematics helps children to think logically*. Once again, there is also evidence that the parents' mathematical beliefs affected their teaching beliefs, as all three of the parents who explicitly mentioned the word 'logic' in their description of mathematics as a subject gave the **most importance** to *teaching mathematics to help improve their children's logical thinking*.

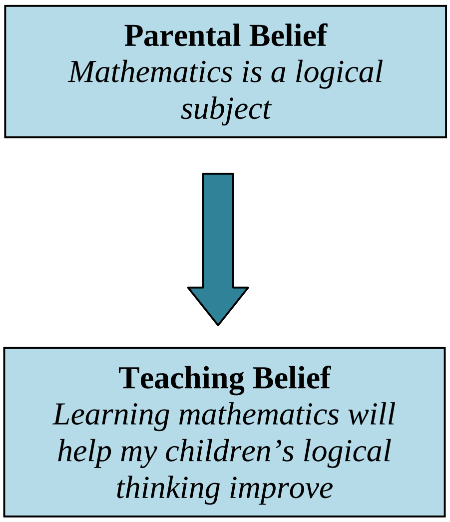


Figure 4.2: Beliefs Associated With the Logical Aspect of Mathematics

For those who teach their children so that they *will not be afraid of mathematics in later life* there is evidence that their mathematical teaching is governed by the need to prevent a negative perception of the subject - most had children who did not have favourable experiences at school.

The majority of parents (60%) find mathematics ‘interesting’, and this appears to motivate their involvement with regards to their children’s mathematical learning. Four parents write that the ‘interesting aspect’ of mathematics is the **least important reason** for teaching the subject their children – but as three of these parents considered themselves autonomous, perhaps the issue of whether the parents personally find mathematics interesting is of no relevance to the children’s learning.

When considering the *application of mathematics to the other sciences*, the responses indicate that **12 of the 16** home-educators who have families working, or who have worked, within employment involving mathematics (excluding mathematics teachers) believe the scientific application of mathematics is a ‘relatively important’ reason.

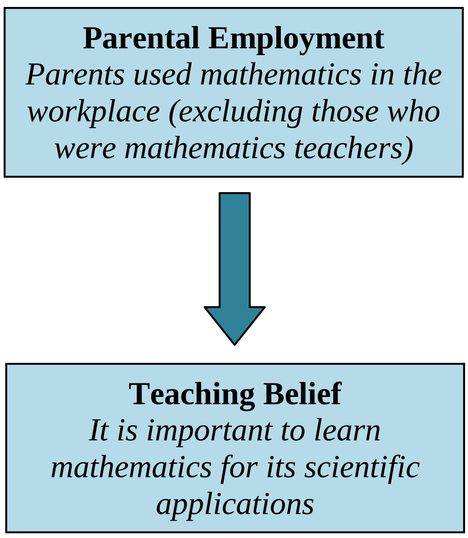


Figure 4.3: Beliefs Associated With the Scientific Applications of Mathematics

On the other hand, in all four instances where one of the parents had taught mathematics in school, it was noticed that **very little importance** was given to the scientific application of mathematics. One could hypothesise that when a home-educating parent has used mathematics extensively in the workplace **outside of the school environment** they are more likely to **emphasise the scientific applications of mathematics** as an aspect of their teaching pedagogy than the average home-educator. As for why it appears that home-educators who taught mathematics in school were **less likely** to consider the scientific applications as a relevant reason for teaching mathematics, one would need a greater number of ex-mathematics teachers in order to conduct a more detailed analysis.

The majority of the families believe *'the need to pass exams'* is a relatively unimportant reason for teaching mathematics; indeed, some explicitly state that it was their dislike of standardised tests and exams that led to a disillusionment with the school system. Moreover, with regards to home-educators teaching mathematics because *'mathematics is part of the school curriculum'*, 20 of the 28 parents consider this to be the **least important reason** for learning the subject. Those who felt that exams were important were relatively structured in their teaching approaches, using workbooks and 'standard textbooks' to ensure that the syllabus for each exam is completely covered – furthermore, two had children studying for formal mathematics exams at the time of this study, and this may have influenced their beliefs.

Additional reasons for teaching mathematics included: parental/child enjoyment of the subject, the benefits of mathematical knowledge to children's future careers, and the perceived levels of confidence for those who are 'mathematically able'. The issue now is to consider the way that the parental beliefs on mathematics and mathematics teaching can affect the teaching of mathematics at home. Initially, we shall consider how the parents view themselves as 'teachers of mathematics' within the home-environment.

4.5 Parent's Views of Themselves as Teachers

The statements within Table 4.4 are drawn from the parents' questionnaire (Q.8, fourth page of Questionnaire for Parents, Appendix 1) whilst the rankings illustrate the quality of their response on a five point scale: **1 =** *'Very much like me'***, 2 = '***Often like me'***, 3 =** *'Sometimes like me'***, 4 =** *'Rarely like me'* and **5 =** *'Never like me'*. In this table, only the **strongly felt opinions are included** - parents who believed that they only 'sometimes' followed a particular approach are not included.

Statement	Ranking of 1 or 2	Ranking of 4 or 5
A = I try to provide mathematical 'learning opportunities' or resources for my child to discover or construct mathematical ideas for themselves	2, 3, 4, 5, 6, 7, 12, 16, 18, 19, 20, 21, 25, 27	8, 10, 15, 23, 24, 28
B = Children won't really learn the material unless I cover it in a structured way	11, 12, 14, 16, 17	1, 2, 3, 4, 7, 8, 9, 10, 13, 19, 20, 21, 22, 24, 25, 26, 27, 28
C = It is my aim to demonstrate the mathematics to my child	7, 12, 14, 16, 17, 20, 23, 27	1, 2, 5, 8, 9, 11, 22, 24, 28
D = The most important part of the lesson is the content of the curriculum	12, 24	1, 3, 5, 7, 8, 9, 10, 11, 14, 15, 17, 18, 20, 21, 22, 25, 26, 27, 28
E = I aim to provide mathematical learning experiences through everyday experiences	1, 2, 3, 4, 5, 6, 7, 12, 13, 14, 15, 16, 19, 20, 21, 22, 24, 25, 26, 27, 28	18, 23
F = I allow my child to learn mathematics by themselves, independently of me	1, 3, 4, 5, 6, 8, 10, 13, 15, 19, 20, 21, 24, 25, 26, 27, 28	14, 17, 18
G = Children should always understand what they are learning, i.e. it should 'make sense' and encourage thinking	1, 2, 3, 4, 5, 6, 7, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 28	8, 26
H = It is useful for students to become familiar with many different areas of mathematics even if their understanding for now is limited.	2, 4, 6, 10, 11, 12, 13, 14, 15, 16, 17, 19, 25, 27, 28	1, 3, 8, 18, 20, 21, 22

Table 4.4: Parent's Aims When Teaching Mathematics

Table 4.5 below presents a more detailed analysis of these parent statements and highlights (in red) the highest percentage response to each item. Note that in some instances the total number of families answering the question is less than 28 – for example, for statement D, the total was 25. This is because some parents did not assign a ranking to that particular statement.

Description	1 = Very much like me		2 = Often like me		3 = Sometimes like me		4 = Rarely like me		5 = Never like me	
	No.	%	No.	%	No.	%	No.	%	No.	%
A	9	**32%**	5	18%	8	29%	4	14%	2	7%
B	1	4%	4	14%	5	18%	10	**36%**	8	29%
C	3	11%	5	18%	10	**36%**	7	25%	3	11%
D	1	4%	1	4%	4	16%	7	28%	12	**48%**
E	14	**50%**	7	25%	5	18%	1	4%	1	4%
F	11	**39%**	6	21%	8	29%	3	11%	0	0%
G	14	**40%**	10	36%	2	7%	1	4%	1	4%
H	11	**39%**	4	14%	6	21%	4	14%	3	11%

Table 4.5: % Breakdown of Parental Aims When Teaching

4.5.1 Everyday Experiences

From Table 4.5 we observe that the majority of parents *'aim to provide mathematical experiences through everyday experiences'*; with three-quarters frequently incorporating their children's mathematical learning into their daily activities. It can also be seen that **15 out of the 20** families who believe mathematics is 'important to everyday life', and teach mathematics so their children can cope with everyday situations, also *describe themselves as teachers* who 'always' or 'often' use real-life experiences to teach mathematics**:**

> "Maths is **a daily part of life** that is not contrived. Children go shopping, cook and sew as a part of their lives and so maths is a natural part of their lives."
>
> Family 20 (children aged 15, 14, 13, 12, 10, 7 and 7)

This indicates that the home-educators' **mathematical beliefs** influenced their **reasons for teaching mathematics**, and both these factors affected the way the parents **perceived themselves as teachers**.

Of the two families who believed it was important to learn mathematics to deal with everyday life situations but very **rarely taught mathematics through everyday** activities, it was noted that both made extensive use of workbooks and followed a rather structured approach. For example, whilst valuing the use of mathematics in everyday life, the mother from Family 23 believed a structured approach was the most appropriate way to learn mathematics:

> "I would ultimately like my children to learn, at least, all the math they would learn at school. I think that for math, more than for any other "subject", this requires **some structure** and some sort of aids to set out the topics to cover and exercises for the kids to do."
>
> Family 23 (children aged 9, 8 and 5)

She believes workbooks offer some continuity to her teaching:

> "I started working with workbooks regularly when my daughter was 6 and a half and my son 8. Of course they knew much of the math in the workbooks by then. I just keep building on what we've already done. Usually, we go through the workbooks, sometimes breaking to make up our own problems. Or we've done that to focus on addition, subtraction, multiplication."
> Family 23 (children aged 9, 8 and 5)

So whilst the parent believes **'mathematics is useful for everyday life'**, she also holds the belief that '**mathematics is best taught'** through a structured approach, in order to follow a typical school approach to mathematics. In her case, the teaching activities appear more strongly influenced by her teaching beliefs rather than her mathematical beliefs. In a different scenario, the mother in Family 18 stresses the importance of learning mathematics through real-life activities, but writes her eight-year old son has had difficulty learning 'basic mathematics'. This has led a heavy dependence on guidance presented within the Kumon curriculum:

> "He's eager to try new things but put off quickly if counting numbers are involved. With the help of Kumon maths now his 6th month he can do some mental maths with simple number bonds under 20 – he must feel his achievement at being able to add in his head but won't give any credit to himself or Kumon. We would like a professional to tell us if they think David has dyscalcular [sic] or has he just 'shut down' - we would then know if we should stop Kumon

maths which he dislikes but has helped him OR should we take things even slower and accept that he has a problem with numbers." Family 18 (child aged 8)

4.5.2 Understanding

Over three-quarters of the home-educators aim to ensure that their children **fully understand** each concept they are learning (Item G, Tables 4.4 and 4.5). Only two 'autonomous' families (Families 8 and 26) gave little importance to 'mathematical understanding', but they claim that they do not actually 'teach' their children. Notice from Family 26's comment, however, that although the respondent does not make 'mathematical understanding' a key aim of her teaching, she believes she is *aware* of her son's level of understanding.

> "It is **obvious if a child understands**. They will be looking happy and feeling relaxed. My son often tells me an answer before I can work it out myself - sometimes I can't work it out at all and he has to help me! This is happening with the maths book we are working on at the moment. Other than that I ask my son if he understands and he says yes or no."
>
> Family 26 (child aged 9

There appears to be some contradiction in the views expressed by parents to item (G) referring to 'always understanding' and the notion of wide experience vs. 'limited understanding' (Item H, Tables 4.4 and 4.5). Though the majority of families (76%) indicate that children should always understand what they are learning, they are happy to put this as a secondary issue to the need to provide a wide set of experiences (H). When constructing this question, it was conjectured that parents would see these statements as 'opposite' ranking e.g. if they rank Description G as 1 = "Very much like me", they are expected to rank Description H as 4 = "Rarely like me" or 5 = "Never like me". However, notice in Table 4.6 that only 7 out of 28 families (25%) rank Descriptions G and H in this way, as has been highlighted in red.

Statement	Ranking of 1 or 2	Ranking of 4 or 5
G = Children should always understand what they are learning, i.e. it should 'make sense' and encourage thinking	**1, 2, 3, 4, 5, 6, 7, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 28**	**8, 26**
H = It is useful for students to become familiar with many different areas of mathematics even if their understanding for now is limited.	**2, 4, 6, 10, 11, 12, 13, 14, 15, 16, 17, 19, 25, 27, 28**	**1, 3, 8, 18, 20, 21, 22**

Table 4.6: Comparison of Parental Beliefs on Understanding

It therefore appears that nearly half the home-educators in this study not only want their children to *understand* the mathematics, but also be *exposed to a range of mathematical concepts*. However, problems could arise if the child fails to understand a concept – do the parents then 'move on to introduce new concepts', or wait until the child has achieve a sufficient level of understanding?

4.5.3 Independent Learning

All the home-educators in this study encourage the *independent learning* of mathematics (Item F, Tables 4.4. and 4.5), with 60% (17 parents) writing that their children learn independently most (if not all) of the time. For example, the mother in Family 3 is a qualified secondary school teacher, well aware of school educational standards. Yet she does not feel that a formal teaching structure is necessary at home, and claims that, most of the time, her daughters are learning mathematics independently:

> "DD1 [Eldest daughter] surprised us by **suddenly deciding to add up low numbers** (up to 10) a few months ago so we practice that. I saw the reception class maths targets and I think DD1 can do most of them so no need to really do anything further. Think things will just develop at their own pace. **I haven't really taught them**. They just demonstrate that they understand something by teaching me!" Family 3

It is noticed that home-educators with ‘special needs’ children are the **least likely of all the home-educators** to encourage independent learning – for instance, both Families 17 and 18 have ‘special needs’ children who require much support from their parents, and perhaps it is for this reason they do not encourage independent learning:

> “My youngest is dyslexic and will never remember his tables or any sequences. We have to cover and recover topics in various ways to help him find a key to remember things.”
>
> Family 17

4.5.4 Mathematical Learning Opportunities

Half of the home-educators regularly provide learning opportunities for their children to discover and construct mathematical ideas for themselves (Item A, Tables 4.4. and 4.5):

> “Always look for a fun way to teach a concept… often find interesting things at nearly new sales or charity shops. Am aware of the sorts of things they need to know (i.e. National Curriculum) but try to work around with as many diversifying activities rather than bore them with ‘the same old thing’.” Family 2 (children age 7 and 6)

A third occasionally provide such opportunities for their children, which suggests that such families take a mixed approach of both ‘structured learning’ and ‘discovery-based’ learning. This approach is typified by Family 1:

> “Sometimes we use workbooks, sometimes it is verbal etc, just depends on where we are at the time. We don’t go by topic at all. A mixture of various methods are used.”
>
> Family 1 (child age 15)

Of the six families who rarely or never provide such experiences, three are ‘structured’ in their teaching approach, making extensive use of set curricula and workbooks when teaching:

> “Usually do topics. Oldest child now has a curriculum, other two we muddle along with. Try to keep lots of variety and a balance between hands on games etc. and more traditional workbook type learning. Usually will adjust with mine/child’s mood.”
>
> Family 15 (children aged 12, 7 and 4)

The other three are ‘autonomous’, writing that, since they never actually teach their children, there is no need to ‘provide them’ with learning experiences. Only if their children instigate a need to discuss a particular mathematical concept will the parents provide assistance, as can be observed in Family 10’s approach:

> “**No one routinely ‘teaches’ him** but when a situation occurs where a calculation is required we will show him how to do it.” Family 10 (child aged 14)

At the same time, it is important to note that while a home-educating family may claim to be autonomous, the evidence may suggest otherwise. For example, the mother in Family 24 writes:

> “My child learns autonomously and isn’t learning any particular topic. But she has been doing a lot of algebra games online.” Family 24 (children aged 10 and under a year)

> “We don’t ‘teach’ maths. She plays mathematics games online about 4 times a week and one lesson with our neighbour a week.” Family 24 (children aged 10 and under a year)

Although she states that her daughter learns autonomously and is ‘not learning any particular topic’ her daughter’s weekly mathematics lesson suggests otherwise. It should therefore be noted that some home-educators may describe their approach as ‘autonomous’ but in reality the children’s mathematical learning is influenced and guided by the parents to a much greater extent that would be expected from an entirely ‘child-led’ family.

4.5.5 Demonstrating Mathematics

It appeared that the sample was somewhat equally split between those who ‘frequently’, those who ‘sometimes’, and those who rarely/never demonstrated concepts to their children (Item C, tables 4.5 and 4.5). Ten parents ‘sometimes’ taught in this manner, whilst eight felt that they regularly demonstrated mathematics concepts:

“Our daughter acts as our primary guide. We don’t teach as such but if she shows interest we respond and **give her information** or show her e.g. in cooking **I show her on the scale** and then say that we need a bit more flour.” Family 27 (child aged 3)

On the other hand, 10 parents (36%) rarely or never aimed to demonstrate the mathematics to their children.

4.5.6 Structured Work/Use of Curriculum

64% of the sample rarely felt that a structured approach (Item B, Tables 4.4. and 4.5) was an essential aspect of mathematical learning, implying that most home-educators do not consider ‘structure’ to be a key feature of their teaching. Furthermore, the vast majority (76% i.e. 19 out of 28) of home-educators seldom consider ‘curriculum adherence’ (Item D, Table 4.4 and 4.5) to be an important factor when teaching mathematics. This is not surprising when we recall that, for some parents, it was their child’s dislike of the structured learning in school that resulted in a move to home-education:

“Disappointment with school situation. Son wasn’t happy or learning well, and found that the **structure and the system didn’t suit him**. Soon realised that I don’t agree with much that the school system does and do not think that it is an efficient way of learning.

Family 26 (child aged 9)

Whilst the parent in Family 26 occasionally uses books when home-educating, the family do not ‘rigidly’ adhere to a particular mathematics book or curriculum:

We are currently working on mental maths. We have a book called “shortcut to fractions success” which has a number of tests in the book which the child can complete when they are in a “maths mood”. The tests are short and they introduce more complicated fraction ideas as they go through the book. It is **quite unusual for us to complete maths books** - we usually learn via computer games or games we play together. It just so happens that at the moment we are working on this - in a very relaxed and informal way though.”

Family 26 (child aged 9)

68% of families who consider ‘the National Curriculum’ to be an irrelevant reason for teaching mathematics rarely followed a set ‘curriculum’ when teaching. But of the two home-educators who emphasised the use of a curriculum when teaching, it was noticed that Family 24 used both Letts revision workbooks [Online Reference] and BBC Revisewise [Online Reference] textbooks to guide their daughter's learning:

> "They make sure she knows in general what her peers are learning and give her simple explanations of certain concepts." Family 24 (children aged 10 and under a year)

These findings indicate that the parental 'attitude towards the National Curriculum' could influence the 'ways in which the parent teaches mathematics'.

4.5.7 Summary of Results

Three-quarters of the home-educators encouraged their children to learn mathematics through real-life experiences, and the findings suggested that parental beliefs on the relevance of mathematics in everyday life activities had a positive influence on this teaching approach.

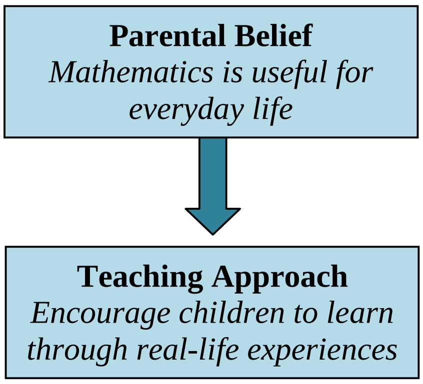


Figure 4.4: Beliefs on the Everyday Applications of Mathematics Affect Teaching Approach

The same percentage of parents (76%) focused on their children obtaining a 'deep' understanding of each concept. It was also observed that 40% would like their children to become familiar with *many different areas* of mathematics even if their present level of understanding was limited - they emphasised **breadth** of learning, rather than **depth** of learning.

Structured learning and the use of curriculum were rarely important aspects of mathematical teaching amongst the home-educators in this sample. This supports the earlier finding in Table 4.3, where it was found that 20 parents believed the **least important reason** for teaching their children mathematics was because ‘mathematics is found in the school curriculum’. On the other hand, parents who followed curricula tended to hold the belief that their children ‘should learn the same areas of mathematics that are covered in school’. The use of curriculum gave these home-educators the confidence that their children were at least receiving a mathematical education that was comparable to that of their schooled peers. Thus, with regards to the use of a mathematics curriculum, the home-educator’s **teaching approach is** generally a reflection of their **teaching beliefs** (Figure 4.5)**.**

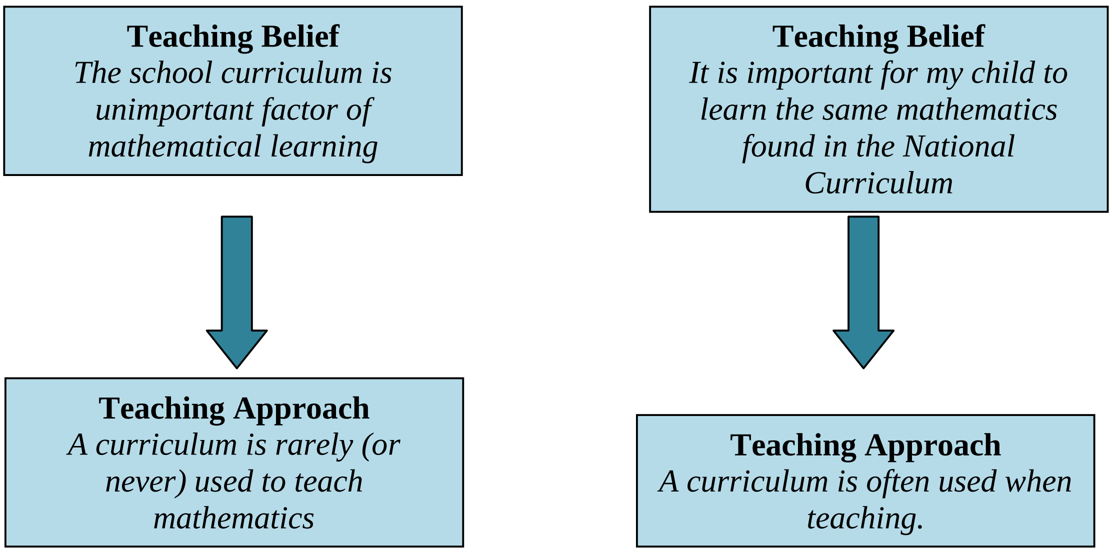


Figure 4.5: Influence of Parental Beliefs on Curriculum Use on the Teaching Approach

Independent learning was encouraged by the majority of the parents (17 out of 28) but those with special needs children tended to give their children extra support when teaching mathematics, and were consequently less inclined to promote this form of learning. Half the families in this study regularly provided ‘mathematical activities’ to enable their children to discover concepts for themselves. Those parents who rarely did so were evenly split between ‘autonomous’ and ‘structured’ families – two groups at the ‘opposite’ end of the home-educating spectrum. It is hypothesised that because autonomous families believe their children’s mathematical learning should entirely be determined by the child themselves, there is no need for the parents to ‘provide’ learning activities, unless asked to do so by the child. On the other hand, structured families may follow textbooks throughout most of their teaching, and hence there is little opportunity for their children to explore concepts via alternative activities. The distribution of parents ‘teaching a concept through demonstration’ appeared to be equally split amongst the home-educators. A third of the parents frequently taught via this approach, with the same percentage of families ‘sometimes’ demonstrating concepts to their children. Now that we have examined the main parental beliefs regarding mathematics and mathematics teaching, we give consideration to the different types of activities used to facilitate the learning of mathematics at home.

4.6 Factors that Guided the Home-Educator’s Choice of Activity

As mentioned in the literature review (Section 2.2.5) home-educators in the United Kingdom are not obliged to follow a particular curriculum, and therefore the choice of activities used to learn mathematics is entirely up to the family. This section will examine the main justifications for the teaching activities, as well as the circumstances that could result in a change of activity.

4.6.1 Interest-Based Learning

> "As the child gets older **his or her interests change** with maturity and knowledge. Life changes and opportunities change. What works at 6 won't at 12."
>
> Family 20 (children aged 15, 14, 13, 12, 10, 7 and 7 – all home-educated)

16 parents chose topics according to their children's interests. This teaching approach was particularly evident in those parents who identified themselves as autonomous, as three-quarters of these families teach mathematics according to their children's specific interests.

> "If my son becomes **bored or disinterested** with the way we currently do things."
>
> Family 26 (son aged 9 years)

One third of the home educators indicated that as well as being interesting, an activity should also be fun for **both** child and parent:

> "It has to be fun, have a point and be **something I can stand repeating often**."
>
> Family 25 (daughter aged 4 years)

> "Overwhelmingly, my choice is governed by **my own interest**. If I think it is fun and interesting, I introduce it to my child. If and when she shows an interest as well, we proceed. Occasionally, she will spontaneously show an interest in something I have not mentioned or shown to her, in which case we pursue it." Family 4 (daughter aged 5)

4.6.2 Workbooks and Curriculum-Based Learning

Approximately one quarter of the participating families mention **workbooks** or a **curriculum** as being the main guide for their choice of mathematical activity:

> "We have been **following the Edexcel IGCSE syllabus** and using text books."
>
> Family 11 (children aged 16 and 14)

> "I just keep building on what we've already done. **Usually, we go through the workbooks**, sometimes breaking to make up our own problems. Family 23 (children aged 9, 8 and 5)

Furthermore, a quarter state that while they are not heavily dependent on workbooks at present, they aim to make greater use of workbooks (or a defined curriculum) as their children grow older. Family 22 writes that if her children were to be enrolled for a GCSE mathematics exam, this could lead to a change from their current 'child-led' approach to a more structured mathematics curriculum:

> "At the moment our home ed. is very much child led so if the children express an interest in doing something I try to go along with it. We also sometimes watch BBC school TV (numbertime etc.) and pick up on an activity we see on there. I suppose that **in the longer term if the girls were to take GCSE maths** I would need to ensure that they had **covered the curriculum**, so we would probably need to introduce a **more structured plan** at that point."
>
> Family 22 (children aged 5, 3, and 1)

This form of change, from 'informal learning' to a 'more structured, workbook based' approach, is also commented upon by other families with younger children:

> "At the moment we are taking a **very informal** and playful approach. We don't demand her attention. When she is **older** (nearer to ten) then I would think we will use a **more formal approach** (possibly) so that she can then choose to go into scientific fields if that is her interest. It does depend on what she wants." Family 27 (child aged 3)

4.6.3 Everyday Life Activities

Five home-educators in this study claimed their choice of activity is primarily governed by everyday tasks:

> "I will usually relate to **a mathematical example** whenever an opportunity arises. For instance, whilst **baking a cake**, we can convert quantities to metric, we can halve quantities given in the recipe, we can also work out how long it will take to bake, and precisely when it will finish baking. Our children **always expect a question** from us, be it mathematical or not, to be directed towards them at anytime, anywhere." Family 16 (children aged 8 and 6)

Some try to mix formal workbooks with 'real-life' activities depending on their children's preferences:

> "**At first I bought a curriculum guideline book** from WHS with class plans in it and thought we would follow that**. Now** (3 and a half months into home ed.) **I am more led by the children** as I am already beginning to see that they **learn better when it's something they**

want to learn. So, now I might give them a choice of topics – money, measuring etc. or just follow on something that happens as part of the day i.e. telling time or weighing."

Family 14 (children aged 8 and 6)

Notice that Family 14 initially used a curriculum, but gradually switched to a more informal mode of learning. The parent feels the children learn best through a less restrictive approach. On the other hand, Family 25's daughter appears to prefer more structured work, despite the parent 'not really liking workbooks':

"Finding something that really catches Beth's attention (like the Singapore workbooks seem to be doing just now) is what makes me change how we do things. I don't really like workbooks but because she likes doing them I let her run with them."

Family 25 (child aged 4)

One can observe that the daughter is allowed to study according to her preferred learning style, (which just happens to be Singaporean workbooks) as the mother puts her daughter's interests **above** her own teaching preferences.

4.6.4 Changes in Approach to Learning Mathematics

The main influences resulting in a change of mathematical teaching approach are the children's age, ability and interests, as can be inferred from the previous section. However, notwithstanding a preferred teaching approach, ten respondents indicate that their approach will change if their child had difficulty *understanding* an idea:

"If a **concept was not working**. For example, my younger son is having a hard time learning how to read '11' and '13'. I tried using flashcards but to no avail. We are now using dot-to-dot pictures to naturally introduce recognition of the next number."

Family 12(children aged 8 and 5)

But five parents in this sample feel it is unlikely that they will need to change their teaching approach at any point. Of these, three consider themselves 'autonomous', and see no need to change their activities. The remaining two families have children studying for GCSE/IGCSE mathematics exams (Family 11 and 28) thus are unlikely to consider changing their teaching activities at this point.

4.6.5 Different Approaches for Different Children

Of those families who are home-educating more than one child (21 families in total), three-quarters adjusted the approaches according to the particular child in question. 13 parents emphasised that *different children may have different learning styles*:

> "Some of my children are **very visual**. They like concepts being shown with manipulatives and watching maths videos. Others like **hands on maths** with cooking, playing shops and computer games. One likes workbooks!!!!!!!!!!" Family 20 (children aged from 15 to 7)

Five parents feel that their children's *mathematical ability* is the main influence on the teaching approach:

> "M now 19 was exceptional at maths and passed his computer science A' level when he was 14 years old, he didn't need games to understand maths.
>
> C now 17 passed GCSE maths with a 'B' & seems to have coped well although she liked playing games especially UNO.
>
> D nearly 9 needs lots of help and forgets how to do things overnight – times tables have been difficult for him to remember." Family 18 (child age 8)

Only Family 6 mentions age being a relevant factor, nevertheless it is clear that the children's personal learning styles are the main influencing factor on this family's teaching approach:

> "Age and ability. Child no 3 follows a maths course online on the pc. Child No. 4 refuses to be 'taught' anything and does not use workbooks/sheets/text books at present. Instead we provide a range of 'hands on' activities for him. Mathematical linking cubes, triangles & quadrants. Measuring equipment – rulers, tape measures, spring balances, scales, wooden geometric shapes, board games, bingo, construction toys, using money to buy things, pc games that require maths skills etc." Family 6 (children aged 13 and 8)

4.6.8 Summary

The home-educators in this study tend to take their children's **interest** and **enjoyment** as the main criteria when selecting mathematical activities. Those from 'child-led' families appear to allow their children to choose **all** the mathematical activities according to their particular interests. Lack of interest is also the main stimulus for these home-educators to change the teaching activity.

These parents believe that when their child is *emotionally* willing to engage in an activity (that is, they find the activity interesting) this is the ideal time to use the activity to teach mathematics.

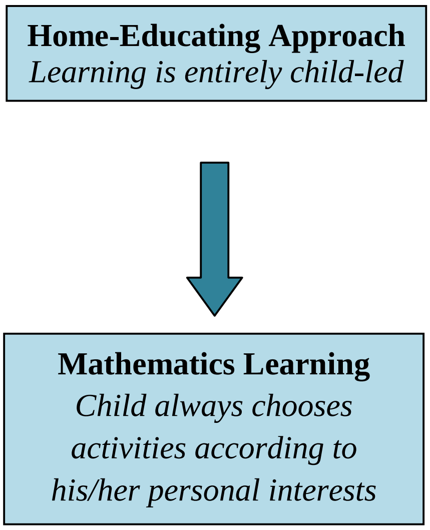


Figure 4.6: The Influence of the 'Child-led' Approach on Mathematical Learning

When considering families with more than one child, it is noticed that the **majority adapt their teaching approaches** according to each child's particular 'mathematical personality'. This is largely governed by the children's preferred learning styles and their particular interests. On the other hand, over a quarter of the home-educators use a mathematics **curriculum** to guide activities, with the same number of 'interest-based' home-educators considering a switch to a more 'formal', structured approach when their children are older. This change could also be necessary for families with children wishing to study for exams such as GCSE’s or A-levels.

The findings show that home-educating families use a variety of approaches to teach mathematics, and as the next step in the analysis, we examine the **learning resources** that helped them implement their chosen approach to mathematics education. As there is no requirement for home-educating parents to follow a curriculum when teaching mathematics, initial consideration is given to the books used when teaching mathematics, in order to determine what guided their choice of textbook.

4.7 Learning Resources

Q.9 of the parental questionnaire (Appendix 1) asked the parents to list the published books used to learn mathematics, and to explain the ways in which these books aided their children’s mathematical learning. They were also asked to provide examples of activities used when teaching a particular mathematical concept to their children.

4.7.1 Books Used When Learning Mathematics at Home

The home-educators in this study used a wide variety of books, which sometimes made it difficult to categorise responses. For example, Family 4 provides the following list of books, where most are of general interest - many of which show applications of mathematical concepts:

> “(These are not textbooks as such.)
> Adler, David A & Nancy Tobin. Shape Up!
> Anno, Masaichiro & Mitsumasa Anno. Anno’s Mysterious Multiplying Jar
> Boynton, Sandra. Hippos Go Berserk!
> Enzensberger, Hans Magnus. The Number Devil, illus Rotraut Susanne Berger, translated by Michael Henry Heim
> Leedy, Lorraine. Fraction Action
> Neuschwander, Cindy & Marilyn Burns. Amanda Bean’s Amazing Dream: A Mathematical Story, illus Liza Woodruff

Nozaki, Akihiro & Mitsumasa Anno. Anno's Hat Tricks
Pinczes, Elinor J. One Hundred Hungry Ants, illus Bonnie MacKain
Tang, Greg. The Grapes of Math, illus Heather Cahoon
Tang, Greg. Math Fables, illus Heather Cahoon
Tang, Greg. Math for All Seasons: Mind-Stretching Math Riddles illus Harry Briggs
Tang, Greg. Math-Terpieces, illus Greg Paprocki
Times Tables 1 – 12
Usborne First Book of Mathematics
Vorderman, Carol. How Mathematics Works (Eyewitness Science Guides)."

The mother's selection is based on her daughter's enjoyment and interest, and also to aid pattern recognition:

"Most of these books introduce **interesting** concepts. Some provide **practice** at computation. Some (specifically the times tables book) also provide information which can be used to **spot patterns** and provoke thought – the products 11 to 99 in the eleven-times table never fail to delight, though she is perplexed by the fact that 121 and 132 look less attractive than the earlier products!" Family 4 (child aged 4)

The above example for Family 4 highlights two points of consideration: (1) Parents may use *many different books* to teach their children, and (2) There may be a *number of different reasons* for using each book. Table 4.7 below was constructed by examining these factors, with the figures in blue indicating the parents who used a particular 'type of book', and the subsequent columns showing their 'reasons for choosing this type of book'.

	Parents	Structure	Practice or introduce concepts	Interest fun	Exams	Visual/ enactive
CGP	**1, 6, 7, 8, 16, 18, 23**	18, 23	7, 18			7
Letts	**1, 5, 7, 10, 19, 24**	10, 24	7, 24	5, 10, 19		7
Other series: MEP, Kent, BBC, Ladybird, Oxford Maths	**2, 5, 6, 13, 14, 15, 16, 17, 20, 24, 26**	13, 14, 24	2, 5, 13, 17, 24, 26	16		
Singapore	**12, 15, 20, 25**	12, 15		20, 25		

‘Standard’ GCSE/IGCSE books based on syllabus	**8, 11, 28**		8, 11	11	8, 11	
Tarquin/Miquon	**9, 12, 25**	12		25		9
Non-textbooks	**4, 15, 21, 27**		4, 15	4, 27		27

Table 4.7: Books Used By Parents

As can be observed in Family 4’s list of books, over half (54%) the families in this study mention the use of **two or more** different textbooks when teaching mathematics, which suggests that each book was chosen for a specific purpose. Hence, a brief description of the *different types* of books that were most commonly used by the home-educators will be given, as well as the *reasons for using these books*.

20 out of the 28 families in this study made use of mathematical textbooks that formed part of a **series** or **curriculum** (i.e. Letts, Oxford Maths, Singapore maths etc.). But, as we will now observe, the books were not always used *as a curriculum*, that is, the children did not necessarily complete each book in the series in a sequential way. Three particular mathematics series were quite popular - namely, Coordination Group Publications (CGP), Singapore Maths and the Letts Series. **One quarter** of families made use of CGP books, and as Family 18 indicates:

> “CGP key stages 2 www.cgpbooks.co.uk. CGP – matched to the **National Curriculum** with deliberate use of humour. Giving clear instruction using examples.” Family 18 (child aged 8)

Family 18’s description typifies the main feature of this particular mathematics series, as according to the CGP website, all of the books are geared towards the National

Curriculum. Each book caters for the various stages of learning, from Key Stage 1 up to A-Level mathematics. However, whilst CGP specialises in National Curriculum orientated textbooks, **only two families** in this study (Families 18 and 23) use the books in a *structured way*:

> "We use CGP, found them the most useful textbook series. I would ultimately like my children to learn, at least, all the math they would learn at school. I think that for math, more than for any other "subject", this requires some structure and some sort of aids to set out the topics to cover and exercises for the kids to do." Family 23 (children aged 9, 8 and 5)

A further two parents use the CGP books to provide practice, or introduce new concepts, Family 7 noting that concepts were often explained visually:

> "They enabled them to see the subject explained in visual format both numerically and graphically and gave them the opportunity to practice." Family 7 (children aged 7, 5 and 2)

The format of the books, their use of humour and colourful pictures when explaining concepts, seemed to appeal to both parents and children. On examination of the home-educators' use of the Lett's mathematics range of books, a similar pattern is observed – once again, these books adhere to topics covered by the National Curriculum, but they are written in a 'creative and fun' style:

> "*Monstrous Maths* covers essential topics from the National Curriculum and is based around the popular theme of magic. Wizard Whimstaff's blend of fun with curriculum-based activities enchants and educates young learners. Children work towards attaining a Wizard's Trophy of Excellence at the end of the book."
>
> Lett's Website, Monstrous Maths [Online]

Two families use these books to provide some *form of structure* to their teaching, and the same number use them to be introduced to, or practice, concepts. The Lett's range of textbooks is more extensive than that of CGP – although they start from Pre-school mathematics up to A-level, they also include 'less standard' material. For example, 'The World of Maths' books provides problems in a variety of 'real-life' settings, and their 'Star Maths' series caters for students who are 'gifted in mathematics' (Letts

Website, Star Maths). Perhaps not surprisingly, three parents mention that their *children* find the Letts books appealing:

> "**They are fun to dip** into when looking for some structure or ideas of what to do."
>
> Family 10 (child aged 14)

Moreover, the Lett's textbooks are sometimes used for the parent's own mathematical knowledge:

> "Letts KS3 (mostly for **own reference** though son enjoys reading it too). He also reads the Letts book to learn new concepts."
> Family 5 (child aged 7)

In summary, home-educators find that CGP and Letts texts cover most of the mathematics that would be found at school (or the National Curriculum), whilst explaining the concepts in an 'interesting' way. While 22 parents use books that form part of a series or curriculum, **only 8** of these families mentioned the 'structured' aspects of the texts when justifying their choice of texts. This lack of emphasis on the structured nature of the text may be due to a number of factors. For example, Family 16 (children aged 8 and 6 years) suggest that it was the inflexible nature of CGP books that led to a switch to computer-based learning:

> "We started using the CGP books (KS2/KS3). However, we find using PC products much more beneficial and more flexible."

Furthermore **12 parents** felt the main benefit of textbooks was to *introduce and practice* new concepts:

> "Oxford Maths Zone. Hodder Home Learning. Gold Stars. They have colourful pages to reinforce practical learning with stars as their rewards. They go over the basics in a similar format, some even have problems to solve. We use them to review ideas and concepts."
>
> Family 2 (children aged 7 and 6)

The books may also have been chosen to suit the children's *particular interests* or learning style, as was noticed for eight families in this sample:

> "Singapore and Horizon. One child **enjoys workbooks**. The Singapore ones are brightly coloured and have stickers initially. She enjoyed this. The books are based on everyday situations that we do anyway." Family 20 (children aged 15, 14, 13, 12, 10, 7 and 7)

Notice that Family 20 believes the workbooks reflect their 'everyday-life' approach to learning mathematics. On the other hand, of the eight parents who explicitly referred to the **notion of structure** in choosing texts, the reference to structure was frequently associated with notions of *teaching* rather than *learning*:

> "We use the Singapore Primary Maths curriculum, supplemented with the Miquon curriculum for the early years. They give structure, allow me to **teach easily** and are an excellent mathematical education." Family 12 (children aged 8 and 5)

The mother in Family 15 lacks confidence, and finds that the Singapore mathematics books help her child daughter learn independently:

> "Singapore maths is set out, as **I am not a confident mathematician**, it means a 12 yr old can work through it alone - just coming to me when stuck. Family 15 (children aged 12, 7 and 4)

Three families based their choice of books on the *exam syllabus* for their children's GCSE/IGCSE exams. Although the parent in Family 8 is unable to remember the titles of the GSCE books used by her son (who took the exam at 13 years of age, obtaining a B), the books were primarily used for exam preparation:

> "I have thrown away the textbooks my son was using, sorry. I can't remember the names but they were standard GCSE textbooks. He read through them and worked on the exercises. Anything he couldn't work out he asked his father to explain."

Two families wrote that they **do not use any books**. In Family 21's case, the parent personally does not like textbooks, while Family 27's 3-year old daughter is using posters to learn mathematics informally – perhaps this is because she has not yet learnt to read:

> “Exeter Maths Centre for Mathematical Excellence posters which are aimed at kindergarten age. It’s very informal and we play at counting with questions e.g. “how many trees? We like looking at the pictures and counting.” Family 27 (child aged 3)

4.7.2 Summary

The home-educators use a wide range of books for a variety of reasons, the most common being that the books provided both an *introduction*, as well as additional *practice* to mathematical concepts. Books also gave *structure* to the learning, and supported less ‘mathematically confident’ parents. A number of home-educators based their selection of textbook on their children’s interests and learning styles, and some chose textbooks based on the way that the mathematical material was presented to their children, for example, the use of visual imagery.

4.7.3 Examples of Activities Used to Teach a Concept

Recognising that textbooks may not be the only teaching resource used to teach mathematics, the families were also asked to provide an example of an activity that had been used to teach their children a mathematical concept. The varied examples included:

1 Verbal activities (e.g. singing songs, having a conversation about a topic)

2 Arithmetic manipulation

3 Pictorial representation

4 Real life activity

5 Invented game

6 Hands-on activity

7 Counting numbers sequentially

Since the number of parents belonging to each of the above categories is relatively small, no in-depth analysis of the results could be done. However, each category will be summarised briefly in order to illustrate the ways in which the activities were used to teach a particular concept. The most common example of a mathematical activity involved *arithmetic*, with **approximately half** giving such an example:

> "Playing shops – **addition**, **subtraction**, **multiplication** – what can I afford to buy – how can I make up the total with the coins I possess – how much does it cost to buy two of the same item – how much money do I have left (these activities at present require a lot of support from me!)." Family 22

11 home-educators mentioned *verbal activities*, such as 'counting songs':

> "Through **songs/nursery rhymes** when very young such as 1, 2 buckle my shoe or 10 green bottles. My youngest has learnt to count through singing such songs." Family 13

The same number **(11 parents)** provided an example involving *counting/number sequencing*:

> "I taught them both to **count down** from ten by pretending they were rockets blasting off. They learned the days of the week and the months of the year from songs I made up." Family 12

Almost one-fifth of home-educators gave an example of an *invented game*:

> "Here is an example of the "child-led" approach in action:
>
> One of my daughter's favourite games is an **arithmetic quiz she invented** which she calls "**Ask Numbers**." She has many variations on the rules; for example she may specify "You ask me subtraction questions where the answer is an odd number," or "I'll tell you an answer and you think of a question to match it. Then I'll say whether your question is right or wrong for my answer." Family 4 (daughter aged 5)

> "Car number plates were a great way to teach the fastest way to add/multiply numbers. We would all add up the number of the car in front and yell "done" when we had finished. Then we would compare the order we did it in. For example L952BNP. The easiest way to multiply the numbers is 2X5=10 and then 10X9=90. It is much more time consuming to work out 9X5=45 and then multiply by 2." Family 17

The above examples show how parents utilised creative approaches to teach arithmetic. **Two parents** provided examples of games that were more '*conventional'* (board games, computer games etc.):

> "Board games like **Monopoly**, **Snakes and Ladders**, and chess were introduced to appreciate money, counting (adding and subtracting). We would recommend the game of Monopoly to all families as it encourages every child to read, work out difficult problems and of course simple arithmetic, not to mention an enjoyable family activity." Family 16

These games all involve an 'hands-on' element, and indeed 21% of home-educators named activities that made use of *'physical objects'* :

> "Used **wooden numbers** to develop recognition of numbers, they had pegs to put in – a corresponding amount for each number and colour coded. We would hide them underneath a scarf and try to guess which number they were. We would count **the pegs** as they were put in and talk about why we couldn't put this colour in that number (introducing concepts like too many, not enough, more, less, the same)." Family 2

Two give examples that involve *pictorial representations*:

> "Using **diagrams** or **pictures** to show division, multiplication etc. (i.e. putting circles round groups of objects)." Family 5

Ten families gave an example from a *real-life activity*:

> "Giving a four year old the money to buy some **pick and mix** sweets that cost 1p or 2p each so he had to add up how many he could buy." Family 10

This study also examined the mathematical concepts their children were currently studying, and the resources and activities used to learn this concept.

4.8 Mathematical Concept Child is Currently Learning

It was noticed that families could be classified according to their 'reason for choosing a mathematical concept'. For example some taught according to *"Whatever came next in the workbooks/computer course"*, others chose **"***Topics that appeared in everyday* *life tasks"* whilst some parents wrote that they were ***"****Not going through anything in particular"*. These three categories will thus be considered first, before focusing on the mathematical concepts that were being learnt.

4.8.1 No Particular Topic Was Being Learned

Families who define themselves as autonomous generally write that their children tend to learn mathematics whenever they feel interested. As a result, **one quarter** of respondents indicate that their children are *not going through any topic in particular*, with three families in this category defining themselves as **autonomous**. But although these parents write that their children are not learning any specific topic, some mention the occasional use of online resources:

> "My dd [dear daughter] is **not currently learning any mathematical topic**, and is not using any activities, however she did recently have a go on www.educationcity.com when we were offered a free trial and whizzed through the maths exercises up to year 7 (age 11-12) getting top marks on all of them. She found it boring however so we didn't subscribe."
>
> Family 8 (daughter, aged 9, 19 yr old son at university was also home-educated)

4.8.2 Textbook Learning

For four families in this study, the children learnt *via textbooks*. However, the evidence also suggests that in some families only *one particular* child adopted this method of studying, whereas their sibling(s) chose to adopt a different approach, with less reliance on formal workbooks. Other families may be working through a textbook only temporarily, as in the case below:

> "We are **currently working on mental maths**. We have a book called "shortcut to fractions success" which has a number of tests in the book, which the child can complete when they are in a "maths mood". The tests are short and they introduce more complicated fraction ideas as they go through the book. It is quite unusual for us to complete maths books - we usually learn via computer games or games we play together. It just so happens that at the moment we are working on this - in a very relaxed and informal way though." Family 26 (child aged 9)

Family 28's 14 year old son learns through a GCSE textbook and is supported by a tutor, but is not going through the topics in any strict order:

> "My son is working towards a higher maths GCSE paper. I think that he **works through a textbook**, but not in any particular order. Whatever the teacher thinks best."

On the other hand, Family 23's children are both working through Year 5 and Year 4 textbooks in a structured way:

> "Various, **through workbooks**. 9 year old is halfway through year 5 series. 8 year old nearly halfway through year 4." (children aged 9, 8 and 5)

4.8.3 Mathematics Through Daily Activities

Two families centre their mathematical teaching on various opportunities that arise through their *daily activities*, as illustrated through the examples below:

> "No set topic - just as **events occur that need maths, e.g. checking shares**, pocket money, making things etc." Family 10 (child aged 14)

> "We don't use topics. We **use our everyday life and a very holistic approach** because we believe maths is all around e.g. wheels on cars, measuring ingredients, water volume play. She can count to 20 and recognises shapes (e.g. 2d triangle, circle and 3d sphere, cube) – we look at pictures and have some blocks too." Family 27 (child aged 3)

Due to the fact that the children's ages ranged from 5 to 15 years of age, a number of different areas of mathematics were being covered at the time of the study. We now briefly discuss some of the main topics that were mentioned by the parents, and the types of activities used to teach these concepts.

4.8.4 Arithmetic

Just under half the children in this study were studying arithmetic, employing a *variety of methods*, including oral/verbal discussion, 'made-up' games, visual representations, workbooks and online activities. **Twelve** of the parents (all with children under the age of 8) indicated that their children were studying *basic arithmetic or 'working with numbers'*, although the approach could vary from family to family:

> "My youngest has **learnt to count through songs –** I am not making any effort to teach him at his age. My 7 year old enjoys working with numbers and likes to add and is learning to subtract. She enjoys counting her pocket money." Family 13 (children aged 7 and 3)
>
> "**She learns many topics at the same time**, but seems most interested in place value at the moment. (Having had a strong preference for oral over written work, she is only now beginning to look at numbers.) She looks at numbers around her and asks for confirmation of what they are "what's a one followed by a five?"
>
> She **especially likes trying to read the very large numbers** which indicate her score in a computer game, sometimes asking for help with this and then exclaiming: "I have five million, two hundred thirteen thousand, six hundred seventy points!"" Family 4 (child aged 5)

Notice that Family 4 is teaching through an oral approach since the daughter appears to prefer learning through dialogue. On the other hand, Family 7 takes a more varied approach, using visual imagery to serve as a 'reminder', as well as workbooks for practice. The response below suggests that the use of computer software helps the children from Family 7 experience the concept in a different medium:

> "They are both **learning basic times tables** (2,3,5,10) and place values. They are using visual reminders (for the tables) and base 10 (place value). They are both using mathematics workbooks to practice addition and subtraction (5 year old) and multiplication and division (7 year old). I act as scribe for my son when he does this. They are still using software and games to expand on these topics." Family 7 (children aged 7, 5, and 2)

In fact, **eleven parents** mentioned *computing resources* as a learning aid:

> "Y (aged 6) is working on subtractions and learning her 2x, 5x, and 10x tables. My husband wrote a **Visual Basic program** which helps them learn any tables very effectively."
> Family 16 (children aged 8 and 6)
>
> "Algebra – my child is learning by a game and **with a computer programme**."
> Family 20 (children aged 15, 14, 13, 12, 10 and twins aged 7)

4.8.6 Other Mathematical Topics

Other topics that were being covered by the children at the time of this study will now be mentioned briefly. Four (aged from 7 to 14 years) were learning *algebra*:

> "Various – we are looking at geometry, **some algebra**. We use story problems to practice various topics, we use polydrons, and we are using a book called groovy geometry which introduces use of protractors etc." Family 5 (child aged 7)

Four children (aged from 3 to 16 years) were covering geometry/shape:

> "The latest topics we covered were Differentiation and Geometry. Elder daughter is much more visual and wanted to produce everything in graph or diagrammatical form and this helped her to grasp the concepts. Younger daughter just has an innate ability in maths."
>
> Family 11(children aged 16 and 14)

It is important to note from the comments below that the set of those learning a particular concept is not discrete. For example, the child in Family 25 was covering a range of topics:

> "Counting to 20 – counting anything, everything
>
> Time – just pointing out what the time is, etc
>
> Days, weeks and months – looking at the calendar every day, counting down days and weeks to special events. Graphs – simple bar charts of how many of an item there are in a pictures (using Singapore books). Family 25 (child aged 4)

In fact eight families mentioned that their children were studying *two or more concepts* at the time of this study. This indicates that home-educated children may frequently study **more than one mathematical concept** within the 'same' period of time, perhaps signifying a tendency towards 'breadth of learning'.

4.8.7 Summary

To review the parents' use of activities:

- In four of the families in this study, the children study whichever mathematical concept comes next in their textbook. However, the evidence suggests that in some families *one particular* child may adopt this method of studying, whereas their sibling(s) may choose to adopt a different approach, with less reliance on formal workbooks

- *Verbal activities* were commonly associated with *counting/number* sequencing. *'Hands-on' activities* were used by a fifth of the parents - this could be through the use of physical objects to aid number recognition, or using board games to develop arithmetical skills.

- Just over a third made use of *computer-based* mathematical activities, including use of GCSE mathematics CD-ROMs, using Excel to draw graphs and so on.

- Over a third provided examples of *real-life activities* (such as cooking and shopping).

- Four children were learning *graphs*, and four were learning geometry, using a range of resources, including computer software, enactive activities (such as paper folding) and workbooks.

Now that the range of resources used has been identified, we next examine the amount of time parents spent teaching their children. As home-educators are not obliged to follow a set timetable, the learning routines can vary.

4.9 Time Spent Learning Mathematics With Others

Whilst a flexible approach to the learning of mathematics was mentioned by 61% (17 of the 28 families), the word 'flexible' could mean different things to different parents.

4.9.1 Highly Flexible

Three home-educators only 'teach' mathematics when asked – the rest of the time the child is left to study independently. It was noticed that all such families classify themselves as 'autonomous' or 'child-led':

> "My child is taught mathematics **only if they have asked to be taught**. Then they are taught by whoever is the most appropriate person. When my son decided at 13 he needed a GCSE maths we bought some text books and he worked his way through them with help from his father and took the exam 8 months later, gaining a B grade. They learn their mathematics through daily events in their lives and using maths in real life situations."
>
> Family 8 (child aged 9)

This approach can imply that for a period of time mathematics may be done intensively but then this intensity may be followed by a period of no mathematics whatsoever:

> "Sometimes every day, sometimes not for a week or so. It depends what else is happening in our lives at that time. We **do not follow a timetable** nor would we ever consider it as we do not feel it is appropriate in HE." Family 1 (child aged 15)

4.9.2 Claim to be Flexible But Comments Indicate Otherwise

Twelve parents (43%) teach an **average of four times** a week, although many still describe themselves as 'flexible':

> "I am totally flexible. Our children have a lot of playtime. In fact, I would only need to teach 1 hour or a maximum of 2 hours a day. I do teach maths every day, and usually after breakfast."
>
> Family 16 (children aged 8 and 6)

Two aim to teach regularly, but find it hard to adhere to a timetable and as a result have felt that a flexible timetable suits their learning:

> "It turns out to be, on average an hour a day, two or three days a week. By me. **Very flexible** timetable – I aim to do about 4 days a week, almost never do." Family 23 (children aged 9 and 8)

> "**Flexible,** about 3 times a week when we are on plan, about once a month when not!"
>
> Family 9 (children aged from 11 to 6)

4.9.3 Never Flexible

Thirteen families teach mathematics on a daily basis, and therefore from a perspective of regularity may be *less flexible* than those mentioned previously:

> "Kumon - 10 minute booklet everyday including weekends & holidays – he hates it but it has helped him. Games - 20 minutes most days." Family 18 (child aged 8)

4.9.4 Initially Flexible But Change

Families who are new to home-education sometimes do little formal work initially, but gradually introduce a timetable in order to make greater progress:

> "Very flexible, we now try and do at least three half hour sessions a week with the books we're using, but probably do more – we often do maths story problems in the bath! Until about a year ago there was little or no formal learning but his need to progress has meant we now do a little more 'formal' maths work." Family 5 (child aged 7)

Indeed, three families note that as their children get older, the amount of formal teaching increases and consequently, the learning may be less flexible:

> "Oldest 20 minutes, 4 days a week, plus everyday use, such as cooking chips etc.
> Middle one, 1 task a day for 4 days plus everyday use.
> Youngest –maths when he asks to do it plus everyday use."
> Family 15 (children aged 12, 7 and 4)

4.9.5 Neither Flexible Nor Inflexible

One family claims to never teach mathematics:

> "**Never** unless you include my husband or random people about the house who just happen to be in the right place at the right time." Family 3 (children aged 5 and 3)

In the pilot study, it was mentioned that one should also take into account the mathematical learning that took place 'outside of formal teaching sessions', and the next section considers the area of 'informal learning'.

4.9.6 Informal Learning

Parents were also asked to write down the amount of time their children spent learning mathematics *informally*, through activities outside the 'teaching' periods. Ten parents (39%) found it hard to quantify the number of hours:

> "Very difficult to say as this would not only vary from day to day but you don't always realise (as their parent) that they are learning or informally doing maths. The information is taken in independently. If I had to give a rough figure I would say about half an hour a day maybe."
>
> Family 26 (child aged 9)

Overall, 17 of the 28 parents indicated that that their children learned mathematics informally *at least once a day*, with a quarter noting that their children were expected to engage with mathematics on a daily basis through *everyday activities*:

> "Pretty much **all the time**. She counts objects, compares sizes, likes pretending to measure with rulers, talks about shapes, weighs cooking ingredients with me and we sing songs with numbers in (1,2 Buckle my shoe)." Family 27 (child aged 3)

> "Difficult to answer. A bit **every day I** suppose. He's 14 so out and about, checking money, credit, bidding on Ebay, etc. He doesn't really work out area, diameter, etc, unless he has a specific task." Family 28 (child aged 14)

Only the parent from Family 11 did not believe that her children (aged 15 and 16) were learning mathematics outside of their 'formal lessons' – perhaps because they were revising for their IGCSE Mathematics exam.

4.9.7 Summary

To summarise, the majority of families followed a flexible timetable, with just under a half learning mathematics for approximately four days a week. The pattern of study tended to vary amongst families, with some willing to allow periods where relatively little mathematics was done, whereas others tried to enforce daily study. Of those who classified themselves as *autonomous,* teaching only took place when requested by the child. However, nearly all of the families in the study believed their children learned mathematics through various informal activities that took place during the day, such as shopping, cooking, and bidding on Ebay.

Now that the parental mathematical backgrounds, perceptions of mathematics/mathematics teaching and the different teaching resources have been considered, the next question to ask is *"How may these factors have affected their children's mathematical learning?"* We first ask the parents to describe their feelings on how their approach to home-education has benefited their children's mathematical learning, and also to outline any perceived disadvantages.

4.10 Advantages/Disadvantages of Learning Mathematics at Home

Parents were asked to specify 'how the home-environment helped their children to learn mathematics' through an open question that would allow them to articulate the main benefits of this educational choice. The fact that a home-education gave children the *flexibility to study mathematics at their own pace, according to ability, and whenever they wanted* was the most citied advantage (43%, i.e. 12 parents):

> "My child has one-to-one attention from a parent who is more interested in her happiness than her SATs grade, and who is enthusiastic about maths. She has **time to pursue ideas** whenever they take her fancy rather than following someone else's timetable, so she can spend an entire morning on maths when she wants to and leave it alone for weeks when she wants to. She is well-rested and unstressed.
>
> She can investigate mathematical topics in any order she likes instead of the order specified by a curriculum, and can **work at her own level**." Family 4

> "They are with me all day, so if a mathematical topic comes up it can be discussed and related to their formal work. In addition, we are not tied to classroom periods, so if they are interested, we can keep going. We can **go at the child's pace**: DS did very little maths at school and was very inattentive, because the level was well below his capabilities. His teacher had no idea that he was good at maths". Family 12

A flexible approach seemed to give parents a belief that they could determine their child's level of *understanding* before going further:

> "It is far quieter than a classroom and they have a lot more support. We have more time to be able to cover things they find difficult and are able to be much more flexible, not holding back a child who is 'too far ahead' or pushing one who is 'too behind' into **completing something they don't fully understand**." Family 7

Ten parents feel that the frequent opportunity to learn mathematics through informal situations is a key advantage of home-education:

> "There are a wide range of **everyday life situations** from putting things away in cupboards (size, 3d etc) to counting birds that are in the garden. There is water for volume, ingredients to cook. **It's very real**. We have lots of paper, crayons to draw shapes with. There are pouring items in the bathroom for water play. We talk about shapes in the home and in the garden."
>
> Family 27

> "We cook, measure and weigh ingredients. We go shopping. We work out budgets. We work out how long it will take to save for something. We work out how many tiles we need for the bathroom, how much glue and paint etc. We play games such as monopoly, which have calculations as part of the game. My children play a lot on the computer, including lots of maths games." Family 8

Just over one third of the parents (10 parents) note their children are learning in a *less pressured, more relaxing environment* than at school:

> "**No pressure, no negative comments** from anyone else, they see no reason why they shouldn't be able to do it which is not the case in school." Family 15

The same proportion highlights the opportunities for *exploring mathematics* according to their specific interests:

> "Time not limited,
>
> Materials available
>
> Time to **play/explore unlimited**." Family 9

> "I can adapt the learning styles to suit my children. Maths is **fun & not pressured**. Also with our day-to-day activities they are learning about maths in 'the real world' not just in textbooks." Family 6

The benefits of *one-to-one attention*, and the *resources available* in the home-environment, are other advantages of home-education:

> "We can tackle things on a **one-to-one** basis and we can spend as much time on things as they need." Family 14

> "Because most of the tools we use at the moment are **Internet based**, being at home means they **can access them at any time**. As I am a mathematician my children have an advantage over many school children since in my experience, primary schools are rarely equipped with teachers with a mathematical background." Family 16

The parents were also asked to outline the disadvantages of teaching mathematics at home. It was observed that almost two thirds of parents (18 out of 28) felt that there were no real disadvantages:

> “**None providing you make arrangements for them to see friends** – 3 times a week and to do outside activities. Daniel has more friends now than he had before and we also have a wider range of activities including lake sailing, football, piano, lots of outings with other home educated children.” Family 18

> “**I can’t think of any**. I think maths in the classroom is boring and very book orientated except in reception. Children at home have endless opportunities to play, use sand, water, games, computers and shop etc. which the class has to artificially make up.” Family 20

However, several used this opportunity to express the notion of disadvantage by highlighting a concern associated with their own ability and mathematical knowledge.

> “My main disadvantage is that I am **not very good at maths** myself and so have to learn alongside my children (but that could actually be thought of as a positive as well.” Family 6

> “Once we get into serious algebra, I will have to use an on-line course or a tutor, as **I don’t think I am qualified to teach at that level**.” Family 12

A number of other concerns given by parents are quoted below:

> “I am not sure if I have **missed any important areas** of knowledge but hope if it is relevant it will all pop up at some time.” Family 10

> “I sometimes **wonder what concepts to introduce** at what age – however just by looking at a basic workbook can solve this problem.” Family 13

Families 28 and 4 feel that home-educating an only child can be difficult:

> “Lack of specialised teaching. Lack of **teamwork if no siblings**.” Family 28

> “It can be fun for children to learn from each other; in the home of an **only child** this rarely happens. It can also be helpful if children see that learning is not always effortless, by seeing other kids work to master ideas: children who learn primarily from their own parents may become discouraged at the discrepancy between the child’s knowledge and the adult’s knowledge. Adults’ basic computations may appear effortless, and therefore out of reach to the child.” Family 4

4.10.1 Summary

Parents believed that the main advantage of home-education was the opportunity for their children to learn at their own pace, with one-to-one attention and support. This flexibility also allowed parents to devote more time to concepts that they or their child perceived to need greater attention. Around 40% mentioned the frequency of learning mathematics in informal situations at home, through everyday activities. Some parents noted that this learning was quite different from the mathematics taught in school, which they believed was somewhat abstracted from reality, in artificially construed situations. A similar number of parents felt that home-education created a more relaxing and less pressured learning environment than school.

Over 60% believed there were no real disadvantages to learning mathematics at home, although parents who lacked confidence at mathematics occasionally had difficulty teaching concepts that they themselves did not understand well. The same number of parents believed their children would eventually ‘overtake’ them with their level of mathematical knowledge. Other disadvantages included a lack of certainty that their children were covering the ‘necessary’ mathematical concepts, and an inability to ‘compare’ learning with others in the case of families with an only child.

The evidence from these findings has shown that home-educating parents teach mathematics in a variety of ways; some are ‘more structured’ using a variety of workbooks, others teach through ‘real-life activities’ whereas others claim that they never teach their children! We have noted that home-educated children do not have to complete a set amount of mathematics work within a certain timeframe, study via a particular curriculum, nor are they obliged to take formal mathematics exams, such as SATs or GCSEs. Therefore, we next ask the question: *“How does the parent identify when their child has understood a concept?”*

4.11 Mathematical Understanding

The parents were asked: "What signs do you look for in your child's thinking to show that he or she understands the mathematics that you've just taught them?" Through their responses, it was discovered that two thirds use a variety of methods to measure their child's understanding:

> "**Consistency** e.g. she knows a triangle is a triangle and does not forget (now she is three – when she was younger she seemed to forget what shapes were and got in a muddle both verbally and with her fingers in terms of counting). She also shows **signs of enjoyment** when she understands something. She often **talks to her dolls** about concepts **or re-enacts** them."
>
> Family 27

In the example above, we can observe that independent use, the demonstration of the concept, as well as the child's behaviour, all play a role in helping the parents measure the level of understanding. The most common measure was through the child's *application of the concept*, with 17 of the 28 families observing understanding through activities such as their child 'playing games' or *using the mathematics in a real-life situation*:

> "Child 4 [aged 6] demonstrates his skills by **playing more complex games**, **building** ever more complex construction models, **adding up his pocket money** & **telling me** how much more he needs to buy 'x' etc."
> Family 6

Or, as seen in Family 11's response below, the children *complete examples* in order to see if they obtain the *correct answer*. The parent believes her approach is different to the sometimes repetitive work in school, in that they can move on as soon as it is apparent that the concept is 'understood':

> "We do a **few examples**, if they have grasped it we move on. I don't expect them to do pages of the same thing as they would in school. We work through the examples together, I give pointers if they need help, so that we get to the **correct answer together**, rather than them being disillusioned by getting them wrong."
> Family 11 (daughters aged 16 and 14)

The second most common way of identifying mathematical understanding **(15 of the 28 parents)** was *through discussion* or the child's *explanation of the concept*:

> "Can they **explain or show** me what they have done and or **why that** happened. **Listen** to their games and how they **talk to each other**." Family 2 (children aged 7 and 6)

In fact, two families exclusively used the children's explanation and comments to measure understanding:

> "**I don't teach her**; she initiates everything. I can sometimes tell what she understands by the questions she asks and the **comments and observations** she makes. I often do not know the extent of her understanding, which is fine: my style of home education does not require constant assessment." Family 4 (child aged 5)

Notice that in Family 4's case, there may be instances where the parent is unable to measure the daughter's level of understanding, yet this is not considered to be a problem. Indeed, *independent work* by the child is a common measure of understanding for eight of the home-educators:

> "I look for the ability to reproduce **the procedure independently**. Usually when she gains a new skill she wants to practice it independently without any encouragement from me, which is a pretty good indicator." Family 22 (children aged 5, 3 and under a year)

Finally, the child's *emotions and behaviour* are taken to be an indication of understanding by **seven** of home-educators:

> "You can tell, when you're sitting there with them. For instance, they get the answer. Their **facial expression** shows if they get it or are **frustrated** or c**o**nfused. We talk through the problems." Family 24 (children aged 10 and under a year)

4.11.1 Summary

It can be observed that the home-educators measure their children's mathematical understanding in a number of ways, the most frequent being whether the child can apply the concept being learned; this could be by answering questions from a textbook, or by applying the concept in a real-life situation. The children's ability to explain their knowledge of the concept, or independently reproduce/use the mathematics, are also common ways of determining the children's levels of understanding. The findings indicate that many home-educators use an interactive process of discussion and questioning, thus, it could be beneficial to investigate the overall aims of such discussions, and this will form the basis for the next section.

4.12 Parental Aims When Asking Mathematics Questions

Though questioning appears to be an issue raised by many parents when considering their child's knowledge of a concept, it was also featured as part of the questionnaire (Appendix 1, Q.8). The purpose was to see more specifically the role it played within teaching, and to perhaps triangulate the quantitative data against the parental comments regarding their chosen teaching method. Table 4.8 illustrates the frequency with which parents used questioning with particular intentions in mind. The table is constructed to illustrate the items associated with reasons for questions and the frequency and percentage of responses associated with the frequency of response on a four point scale:

	4 = Always		3 = Often		2 = Rarely		1 = Never	
Aim when asking questions	**No.**	**%**	**No.**	**%**	**No.**	**%**	**No.**	**%**
A: See if they know the correct answer	6	**21%**	11	**39%**	9	**32%**	2	**7%**
B: Get them to justify and explain their reasoning	4	**14%**	12	**43%**	11	**39%**	1	**4%**
C: To allow them to gain confidence	9	**33%**	16	**59%**	0	**0%**	2	**7%**
D: To solve a problem in an everyday situation	3	**11%**	23	**82%**	1	**4%**	1	**4%**
E: Find out if they are paying attention	2	**7%**	4	**14%**	15	**54%**	7	**25%**
F: Give them the opportunity to direct the lesson	3	**12%**	14	**54%**	6	**23%**	3	**12%**
G: Discover their ideas and opinions	14	**52%**	13	**48%**	0	**0%**	0	**0%**
H: Help you to understand something better as well as your child	6	**22%**	14	**52%**	7	**26%**	0	**0%**
I: Find out what is interesting about the mathematical topic	4	**15%**	11	**41%**	10	**37%**	1	**4%**

Table 4.8: Parental Aims When Asking Questions

Each of the aims will be addressed in the frequency with which they were used by the home-educators.

4.12.1 Discover Children's Ideas and Opinions

Although the parents in this study may possess a range of mathematical beliefs and adopt different ways of teaching mathematics, **all consider the children's ideas and opinions** to be an important element during their mathematical learning, with over half claiming to always use such an approach:

> "We **discuss what we've been doing** and think up problems for each other to solve."
>
> Family 13 (children aged 7 and 3)

4.12.2 Increase Confidence

92% frequently question their children with the aim of increasing confidence. This was especially noticeable in families with 'special needs' children, where **5 out of the 8** families with special needs children *always* aimed to improve their children's confidence. One example is Family 27, whose youngest child has dyslexia:

> "My **youngest is dyslexic** and will never remember his tables or any sequences. We have to cover and recover topics in various **ways to help him find a key** to remember things."
>
> Family 17 (children aged 18, 15, and 12)

Only two families never seek to improve their child's confidence, but both write that their children are home-educated 'autonomously':

> "Neither of my children have asked me to teach them – ever. They have asked for my help when they don't understand something they are learning. If I turn into teacher mode on them they very soon lose interest. Family 8 (children aged 19 – at university and 9)

4.12.3 Everyday Situations

There is again a strong emphasis on the 'real-life' applications of mathematics, as Table 4.8 shows the vast majority of parents frequently initiate questions in order to solve a mathematical problem from an everyday situation. The results also suggest a relationship between the parental teaching beliefs and their teaching approaches. 87% of parents who believe *'the most important reasons for teaching mathematics is to deal with everyday situations'* often/always question their children on the real-life applications of the subject, helping to establish validity through triangulation:

> "I want to make maths fun and **show how it is used and important in everyday life**. I disliked maths and the way it was taught to me as a child and want it to be different for my children. I try to make maths fun and use **real life maths** rather than sit down written work. We are very flexible – learning **may take place while out shopping or baking a cake**. We use workbooks sparingly.
>
> It is real maths that needs to be used every day. It isn't a subject that is studied on its own away from the world. My children **see how useful it is** and how it relates to their lives (e.g. saving up pocket money to buy a toy, learning to tell the time so they know when their swimming lesson is, making grandma a birthday cake.)"
>
> Family 13 (children aged 7 and 3)

Only two families rarely or never initiate such discussions:

> "I cannot really think of a situation where I would ask my child a series of questions, unless I didn't understand something and they were explaining it to me."
>
> Family 8 (children aged 19 – at university, and 9)

4.12.4 Improve Parents Understanding

As was also noticed in Family 8's response above, three quarters of the parents in this study regularly ask their children mathematics questions because they themselves have trouble understanding a mathematical concept. For six of the parents, this is their main motivation for initiating a mathematical discussion with their child:

> "My son **often tells me an answer before I can work it out myself** - sometimes I can't work it out at all and he has to help me! This is happening with the maths book we are working on at the moment."
>
> Family 26 (child aged 9)

4.12.5 Give Child the Opportunity to Direct the Lesson

66% of the home-educators write that asking questions is a way of giving their children the opportunity to direct the mathematics lesson. This is most likely to occur in families where the choice of mathematical activity is predominantly based on their children's interests:

> "The **interest of the child**. E.g. my youngest is very train oriented so I use a train and wagons with blocks on to teach base 10." Family 20 (children aged 15, 14, 13, 12, 10, 7 and 7)

Conversely, with regards to the nine families who 'rarely/never' give their children the chance to direct the learning, five adhere to a formal curriculum, and mainly use workbooks. Thus, there is little opportunity for their children to determine the course of the lessons, as their parents have a set teaching routine:

> "We have been following the Edexcel IGCSE syllabus and using text books."
>
> Family 11 (children aged 16 and 14)

4.12.6 Correct Answers

The majority of families (60%) in this study wrote that checking 'correct answers' was a frequent occurrence during their children's mathematical learning. However, there appears to be a relationship between the emphasis placed on 'checking children's answers' and the 'ways of measuring mathematical understanding'. Of the eleven parents who seldom emphasise 'checking answers' when teaching, nine do not use 'correct' answers to measure their children's level of understanding. For example, although the mother in Family 25 has taught her 4-year old daughter a number of different mathematical concepts, including graphs, time and counting, she does not list 'correct answers' as a measurement of understanding.

> "Repeating ideas back to me or relating them to another situation."
>
> Family 25 (child aged 4)

4.12.7 Justify and Explain Their Mathematical Reasoning

Just over half of the parents frequently ask their children to justify and explain their answers, but 12 parents rarely or never do so. Family 9 is one parent who often questions her children in this way:

> "Can say verbally what the answer is and can **give an explanation** of why it's right."
>
> Family 9 (children aged 11, 7, and 6)

5.12.8 Interest Generated From Mathematical Topic

Around half regularly initiate mathematical discussions in order to discover the interesting aspects of the concept, and 6 out of 10 parents who *personally find mathematics fun or interesting* belong to this group:

> "I **liked maths** at school and took it to A Level, I have enjoyed going back to subjects which I haven't needed or used since school. I now understand calculus much better than the first time around."
>
> Family 11 (children aged 16 and 14)

But surprisingly, **2 out of the 4 home-educators** who did **not** enjoy mathematics at school also frequently question their children out of interest, as was the case for the mother from Family 20:

> "I believe maths is an important life skill but **I am not a great fan** of it as a subject though my children in the main appear to be. I am happy for them to learn the basics and anything else is a bonus. So far they have all surpassed this themselves."
>
> Family 2 (children aged 7 and 6)

Conversely, 10 parents rarely or never question their children to discover the interesting aspects of a mathematical concept – one being the parent from Family 24, who does not appear to enjoy the subject herself:

> "I don't think mathematics is as important as we would like to think. To many people it is irrelevant other than being able to do day-to-day calculations. Some people have different strengths and shouldn't "HAVE" to do mathematics."
>
> Family 24 (children aged 10 and under a year)

4.12.9 Paying Attention

The majority (79%) of parents in this study rarely ask questions to find out if their children are paying attention. In particular, it was noticed that families who used a flexible timetable (17 in total) seldom (82%) checked if their children are paying attention but half of the home-educators who have a **structured** timetable frequently checked their children's concentration levels.

4.12.10 Summary of Results

Figure 4.7 summarises the frequency of response for each item discussed in section 4.12.9, the most common reasons situated at the top of the list. Thus, 'discovering the child's opinions' is a frequent reason for initiating discussion, whilst 'checking the child's attention' is rarely given as a reason for doing so.

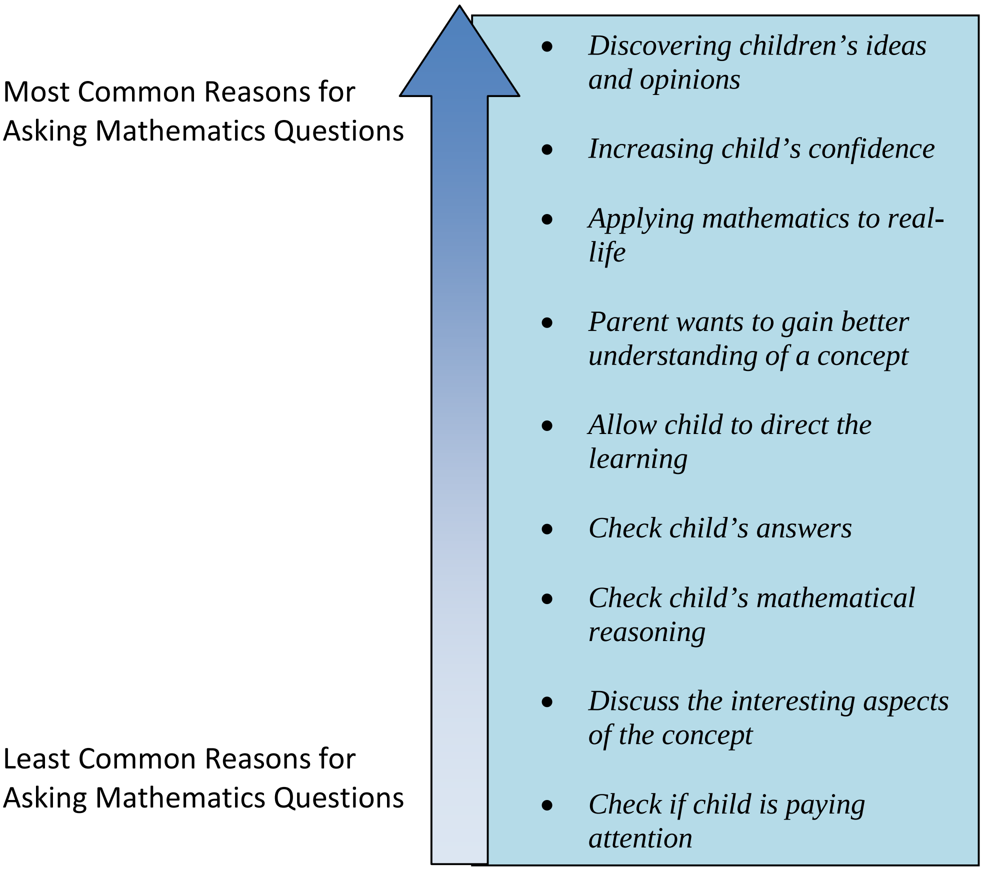


Figure 4.7: Frequency of Response for Parents' Motivations For Asking Their Child Mathematics Questions

From Figure 4.7, we see that the options for discussion, real-life and application of concepts are the main reasons for parents to question their children, which reflect the most common ways of measuring mathematical understanding, as identified in Section 4.11.1. Issues such as interest may be influenced by the personal attitudes of the parents' towards mathematics, whilst it appears that home-educators rarely need to check the attention levels of their children.

Now that the various approaches to learning mathematics at home have been considered, we next examine the long-term goals of the families, and consider the incentives parents give their children to motivate their mathematical learning.

4.13 Incentives and Future Goals

The responses showed that the majority of the families **did not** give their children any incentives, but of those who did, the goals/incentives tended to consist of verbal praise or achieving good exam results rather than material items. Three of the ten families who provided a goal or target gave their children *verbal praise*:

> "**Verbal praise** when they work at understanding a concept, or completing a task. Their workbooks often have stickers (stars) in them and the children like to receive them." Family 2

Three families felt that 'receiving a sticker or star' was an adequate reward:

> "Kumon - sticker and treats, pocket money .30p – 50p for each completed Kumon booklet - D says he doesn't want the money (so it doesn't work)!" Family 18 (child aged 8 years)

For the families below, *external rewards* such as a place at university, or good exam grades were suitable incentives:

"Tim wants to go to **Cambridge University**, where my brother read medicine and his grandfather (my husband's father) is a retired master of Hughes Hall and a renowned neuro-radiologist at Addenbrookes Hospital. He knows what is required of him and should he want to achieve all these things there is really no substitute for hard work."

Family 16 (children aged 8 and 6)

Others give their children 'time-off' from studying mathematics rather than a material reward:

"No incentives but must complete same everyday Monday → Thursday. Friday then a free day unless they didn't do the work in the week – then they have to finish it on Friday **before they can do anything else**."

Family 15 (children aged 12, 7 and 4)

Family 4 does not give her daughter (aged five) a material incentive as her reward is *greater attention*:

"Actually, I suppose I do give her an incentive: **my attention**. She knows that I am far likelier to agree to engage in a mathematical discussion than a craft activity with her, because that is where my interest lies. If she wants me to stop washing up and interact with her, asking me to read her a maths book never fails!"

On the other hand, **three** families believe that their children are motivated by material incentives:

"Child 4 has incentives in that he has new 'hands on' stuff to use which we buy." Family 7

Family 28 writes that although they do not give their son (aged 14) any incentives, they hope his tutor does:

"I don't, but I hope the tutor does. You have jogged me into doing something about this!"

Unlike school, there are no compulsory exams, nor competition from peers apart from perhaps their siblings (if any). 13 out of 24 parents write that they were neutral as to when the exams were taken:

> "It really depends on why the exam is being taken. I hope my children will take the exams they need to progress in their chosen lives/careers. **I don't mind when they take them** as long as they feel ready and are not too unduly stressed." Family 2

Although most would leave the final decision to the child, a third encouraged taking exams early as it was felt there were some benefits, such as spacing out exams, and helping the children with their future careers:

> "**Yes. I think it is a good idea to take exams whenever the child is interested**. Also, it may be useful to focus on one or two subjects at a time, which would mean some must be taken earlier than others." Family 4

> "**Yes – if the student wants to and is able to** and if it doesn't stop them from enjoying life. My eldest son passed his A'level computer science when he was 14 years old – on an accelerated course run by Ryde College Watford – it looked good on his C.V. and he now works for an excellent company and earns his own money at 19." Family 18

The families below write that their home-educated children have already taken exams early, but again stress that it *depends on the child*:

> "I don't know if it is a good idea for every child, it worked out fine for my son, who took his **Maths GCSE at 13 and got a B grade**." Family 8

> "My **11 yr old did the GCSE** but only intermediate level. He doesn't seem interested in going further right now, so can't say?!" Family 9

On the other hand, a small number of parents would not encourage their children to take exams early, believing that children needed to be emotionally ready for exams, and that exam study could affect the child's learning.

> "If the child wants to, but **I feel the more time the child** can consolidate their learning the better so **later exams better**." Family 10

> "Personally **I don't think so**, would see it mostly as a need to prove something to yourself. Though **if my kids want to, that would be fine with me**. I wouldn't ultimately want to send them to university early, because of emotional development and social issues." Family 23

Only two parents felt children should not take exams at all.

> "We **don't believe in exams**." Family 1

> "GCSE useless – my six year old could probably do it next year!!! 'A' level not much better! **Not doing exams** – waste of time!" Family 21

An equal number feel that their children *should take their exams early*:

> **"Yes, why not?"** Family 28

In summary, whilst the majority did not give their children any incentives, a few motivated their children through verbal praise and offer academic targets, such as good exam grades – only three parents would give material incentives. With regards to taking exams, the home-educators mainly felt it was the child's decision.

4.12 Summary of Chapter 4: Data Analysis for Parents

Before considering the children's views on the home-educating situation, we briefly summarise the main findings from this chapter:

- Sections 4.1.2 - 4.1.3 considered the reasons that parents and children chose home-education. Key reasons included the flexibility of learning, and the happiness of the child. Older children generally had a greater influence on the decision to learn at home.

- Section 4.2 investigated the mathematical background and teaching experiences of the parents – most had not studied mathematics beyond GSCE level, but nearly three-quarters had a close family member working in an area that required extensive mathematical knowledge. While 40% of the parents had some experience of teaching, opinion was divided as to whether these experiences were beneficial to the home-educating approach – most feeling that only certain aspects were applicable.

- Sections 4.4 and 4.5 looked at the mathematical and mathematical teaching beliefs of the home-educators. One prominent belief was that ‘mathematics is important for everyday life’ and this generally led to the teaching belief that ‘one should learn mathematics to deal with everyday life situations’. Other common beliefs included: ‘mathematics is a logical subject’, ‘it is enjoyable’ and, where the parent regularly used mathematics in the workplace - ‘mathematics is important for its scientific applications’. Parents whose children were unhappy at school were often more worried about their children developing a fear of mathematics. Exams and school curriculum were generally considered unimportant.

- Section 4.5 showed that the key aims of the home-educators when teaching were to: (1) Prepare their children for everyday life and (2) Enable their children to develop a strong understanding of each concept. Parental teaching beliefs also appear to influence their teaching approaches – for example, if they did not consider the school curriculum an important reason for learning mathematics then they rarely or never used a curriculum when teaching. However, if they did believe the school curriculum to be an important guidance for mathematical learning, then aspects of curriculum use featured heavily in their teaching approach.

- Section 4.6 showed that interest and enjoyment were the main criteria when choosing an appropriate activity to learn mathematics. Just over a quarter found that a curriculum could also be a useful guide when teaching. Parents often changed their teaching approach when it was evident that the child could not understand the concept, or if boredom (in either child or parent!) set in.

- The use of textbooks was discussed in Section 4.7, and a range of different series of books was identified – key benefits of textbooks including: practice, enabling independent learning, interest, and giving structure and support to the parents. Section 4.8 showed that home-educators also used a range of other activities when teaching, including visual aids (graphs, computer-based activities), everyday life tasks (shopping, cooking, bidding on Ebay), games (both invented and conventional, such as Monopoly) and formal activities (e.g. textbooks).

- Section 4.9 showed that the majority of home-educators used a flexible timetable of learning, with many incorporating informal learning into their daily routines. Some of the key advantages of home-education, highlighted in Section 4.10, were: (1) The opportunity for the child to learn at their own pace, (2) One-to-one support from the parent, and (3) Learning took place in a 'relaxing' environment. Most felt there were few real disadvantages, apart from the fact that an 'only child' could miss out on learning with others, and some parents worried that their child's mathematical knowledge would advance their own.

- In Section 4.11, it was revealed that parents frequently determined their children's levels of mathematical understanding through their child's applications of the concept (perhaps independently of the parent), and the child's explanations of 'what they thought the concept was about'. Indeed, Section 4.12 showed that most felt that their children should gain a 'good' understanding of each concept before moving on to a new area. Parents also regularly questioned their children to discover their ideas on a mathematical topic, and were often happy for the child to direct the learning. Very few parents felt the need to check if their child was paying attention, nor did they place much emphasis on constantly checking the child's mathematical reasoning.

- Section 4.13 showed that incentives and goals were not common amongst the home-educators – and whilst the parents were generally supportive of their children taking formal mathematics exams, the majority stressed that the child should choose the appropriate time as to when such exams were taken.

Chapter 4 has given an insight into the parental beliefs with regards to their home-educating approach, and the ways in which this approach is implemented when teaching mathematics. In Chapter 5, we will consider the children's views of mathematics, and the environment in which they learn the subject.

Chapter 5: Influence of the Parents' Home-Educating Approach on Their Children's Mathematical Learning

In order to identify the general home-educating approaches of the 28 home-educating parents in this study, Chapter 4 examined questionnaire responses from *all participating parents*. This enabled the identification of relevant themes within the areas of focus, including the mathematical background of the parents, the widely-held beliefs on mathematics teaching and parental notions of mathematical understanding.

As mentioned in Section 2.2.6, this study also aims to establish relationships between the home-educating approach of the parent and their children's mathematical understanding. Accordingly, a number of illustrative case-studies from each category of the following three 'types of home-educating approach', namely: *Structured* families, *Semi-formal* families, and finally *Informal* families, will be provided to show the effects of the particular home-educating approach on the children's mathematical beliefs and understanding. This will be followed by a consideration of the children's perceptions of their learning environment (Sections 5.4-5.5), and their views on mathematics (Section 5.6), problem solving (5.7) and mathematical understanding (Section 5.9).

Finally, Sections 5.10 to 5.13 will focus on identifying the different types of solution strategies observed in the children's answers to the assessed work, in particular, to investigate whether the way in which the children *do mathematics* is a result of how they *learn mathematics at home*.

We begin with a case study of a structured family (as described in Chapter 2, Section 2.1.4), aiming to identify the ways in which a structured home-educating approach could affect the children's perceptions of learning mathematics and their mathematical beliefs.

5.1 Structured Families

The main characteristics of a structured family are: **(1)** The families make extensive use of a *curriculum and textbooks***,** and **(2)** Learning often takes place at *regular intervals* during the week. This group includes families where the participating child chooses to learn mathematics via a structured approach, even if their siblings followed a semi-formal/informal approach. Families 6, 11, 15, 23 and 28 all fell into the category of structured families. We now have a closer look at Family 23 in order to examine how the structured home-educating approach may affect children's perceptions of mathematics.

5.1.1 Case Study of Family 23

Apart from the eldest spending a short time at nursery school, Family 23 have been home-educating their three children (son aged nine, daughter aged eight and a five year old) since birth:

> "I started thinking they start school too young. I thought my son was happier and learning more at home than in nursery. The longer I was in it, the more I came to think they could learn more, be happier, have higher self-esteem and individualism if they continued to be home-educated."
> Family 23

The mother felt that her personal experiences of mathematics had a neutral effect on her home-educating approach, because she preferred her children's learning to be "less abstract" and more grounded in reality than that of a 'typical school approach to mathematical learning". She describes mathematics in the following way:

> "It's one of the tools you need to make sense of the world, and get along in life. It's great mental exercise."

The mother believed their home-educating approach allowed their children to learn in a secure environment, and at their own pace, writing: "Once the child has understood a concept they can progress onto the next". However, she also comments, regarding her teaching:

> "Well, I have to <u>do</u> it, and come up with a plan."

The children's learning is predominantly through workbooks (Coordination Group Publications, Year 3 and Year 1), as the mother believed that this approach would allow her children to cover all the mathematics they would learn at school, in a structured way. She aimed to build upon the material her children had previously covered by working through topics and exercises from the workbooks, and occasionally asked the children to make up their own problems. Teaching took place three to four days a week, but this programme was relatively flexible. The children's facial expressions and verbal discussion were used to measure the level of mathematical understanding.

5.1.2 Influence of Family 23's Approach on Child's Perceptions of Mathematics

Upon examination of the children's responses to Q.8 of the children's questionnaire *"How do you choose which mathematics topic to study?"* (see Appendix 2), we see that Child 23a (aged 9) believed his learning was mainly governed by parental choice and everyday activities. His sister, Child 23b (aged 8), felt that her activities are entirely based on *'whatever comes next in the textbook'*, and wrote that she never had the chance to study concepts that were personally interesting to her. More relevant is the fact that Child 23a could not list the current area of mathematics (or activity) that he was learning at the time of the study - despite the fact that he was taught mathematics three to four times per week, he wrote:

> "Don't have one."

His sister was currently learning from her Year 3 books but stated that she had not used any additional learning activities. Although the mother listed a number of advantages of home-education (e.g. her children's happiness), neither of her children could list a single aspect of home-learning that they found beneficial. Only Child 23b (aged 8) expressed an opinion of her mathematical learning, writing:

> "I just do it".

When asked to give an indication of their mathematical abilities, both children indicated, "I do not know."

The mother's beliefs about mathematics appeared to have influenced her son's beliefs regarding the 'uses of mathematics', as both the parent and the children highlighted its relevance to everyday life, to other subjects and to the passing of exams. At the same time, the second child, a girl, did not find mathematics interesting, and felt that *'most people do not like mathematics'*. Indeed, both siblings expressed the belief that mathematics is **useful but boring** – suggesting that the mother's structured approach, which largely consists of textbook exercises, led to a lack of interest in mathematics by the children.

However, the mother may not be aware of her children's views, as indicated in her comments on the fictional case study of Richard (see Appendix 10):

> "One of the beauties of home ed. is that you can go at the speed of what they're good at. This keeps it all interesting for them. (In fact my nine-year-old sounds a lot like Richard!)".

In fact, Family 23 was the only instance where the parent was interested in mathematics but the children were not.

Note that the children's perceptions of mathematics as a subject are not the only beliefs affected by their parents' structured approach. When considering the children's problem solving beliefs, Child 23a stated: *"A mathematics problem is numbers with some words and a question"* and his sister wrote: *"A mathematics problem is an exercise during a mathematics lesson"*. Not surprisingly, Child 23a believed that 'every mathematics problem should involve numbers' and both siblings expressed the belief that 'it is the same to make a calculation error as it is to choose the wrong method or operation'.

Whilst in general terms Child 23a identified *confidence, parent approval*, *explaining* the concept to another party, finding *patterns*, and *applying* the concept to real-life situations as important signs of mathematical understanding, the latter two suggest a tendency to focus on the 'relational' aspects of understanding. His sister also appears to value the application of concepts to a real-life situation as a key sign of understanding. Child 23a then contrasts this with what may be a more 'instrumental' perception by identifying *wrong answers* and a fear of *making mistakes* as the main signs of lacking mathematical understanding. However, it appears that both children rely on their feelings and the 'real-life applications of the concept' as important signs of mathematical understanding, despite their structured textbook approach to learning.

When *doing mathematics* both children aimed to understand each concept, although it is also observed that Child 23a values **quick completion** of work and **correct answers above the application of the concept**.

We next consider a family at the opposite end of the spectrum, that is, an informal family.

5.2 Informal Families

Families following the informal, or ‘autonomous’ home-educating approach claim that *learning is entirely child-led*. The parents often believe their children are the best judge of ‘how they should learn mathematics’, and frequently comment that their children are never actually ‘taught’. Families 4, 8, 9, 10, 20, 21 and 26 could all be classified as ‘informal home-educating’ families, since their children’s learning was centred on a ‘child knows best’ educational philosophy — many only taught their children mathematics if help was requested from the child.

5.2.1 Case Study of Family 26

The nine-year-old son of family 26 had been home-educated for the past three years:

> “Son wasn’t happy or learning well and found that the structure and the system didn’t suit him. Soon realised that I don’t agree with much that the school system does and do not think that it is an efficient way of learning.

The parent viewed mathematics as a subject that involved an understanding of numbers, which was necessary for everyday life. She noted the home environment allowed her son to learn and explore mathematics whenever he wanted to, for as long as he liked, and this philosophy, she believed, helped prevent boredom.

Her views on teaching can also be observed in her responses to the fictional case studies of Joe and Richard (see Appendix 10):

> You shouldn't make a child do anything but if he can apply his maths to the real world and enjoys this then it would be helpful to him. If he is not ready for this then leave him to work in his own way again.

From her comments on the case study of Richard (Appendix 10), it can be seen the mother believes that if children already appear to have a 'good' understanding of mathematics, then this understanding will continue to develop 'naturally' without the need for much support.

When teaching her own child, she tried to create many opportunities for him to learn mathematics in informal and relaxed settings, for example, when cooking. Learning activities, usually through computer games and other mathematically orientated games were, she believed, generally 'fun and engaging' – they were only changed if the son became bored or disinterested. As a result, she felt that their son enjoyed and was 'good' at mathematics. Books were sometimes used to cover certain topics and to provide practice; for example, the son was currently learning and practicing fractions from a textbook. The parent measured her son's mathematical understanding in the following way:

> "They will be looking happy and feeling relaxed. My son often tells me an answer before I can work it out myself - sometimes I can't work it out at all and he has to help me! This is happening with the maths book we are working on at the moment. Other than that I ask my son if he understands and he says yes or no."

5.2.2 Influences of Parental Approach on Child's Perceptions of Mathematics

The family felt that home education promoted their underlying philosophy of the home-educating approach — their son was able to direct his mathematical learning — and his perception of his mathematical learning reflected their beliefs:

> "Nobody pressurises me. Can finish when I am bored."
>
> Child 26 (aged 9, at home for 3 years)

His mother believed home-education has given him the opportunity to learn mathematics in a relaxed way, through 'useful activities', and through his responses we see that Child 26 also appreciated the **everyday applications** of mathematics.

The mother of family 26 taught mathematics because she believed it was a *useful everyday life skill*, and she suggested her son enjoyed the subject. She felt her son was *good at mathematics* and this gave him confidence. Her views appear to have **influenced** Child 26's *mathematical beliefs*, as he found the subject interesting and useful for everyday life, and writes:

> "I enjoy maths and compared to English it's a breeze. I don't find maths as complicated as this questionnaire."
>
> Child 26

With regards to his problem solving beliefs, although Child 26 generally learns mathematics through everyday activities, he seems to have a relatively restrictive view of 'the attributes of a mathematics problem', holding the belief that **all mathematical problems are numerical**, and that such problems are simply *'numbers with some words and a question'*, which can only have one correct answer. These **problem solving beliefs** may be *'number oriented'* because of his mother's perception of mathematics**:**

> "It means to have **an understanding of numbers** which are useful and used in everyday life."

Child 26's notions of *mathematical understanding* also appear to be **influenced** by his mother's teaching. When teaching, the mother's main aims are to: (1) See if her son knows the *correct answer,* (2) Increase *confidence,* and (3) To solve a *real-life* problem. Similarly, Child 26 believes that the **top three important signs of understanding** are: (1) *Correct answers,* (2) Using the mathematics in a *real-life situation*, and (3) *Feeling confident.* When doing mathematics, Child 26's two main priorities are to: (1) Finish the work *quickly* and (2) Get *correct* answers. Such an attitude to mathematical work could be at the expense of his understanding (which he rated as less important). However, he does not feel that it is possible to judge whether someone is good (or bad) at mathematics.

5.2.3 Comparison of the Family 23 and 26

As can be seen, the different approaches to teaching mathematics at home have led to quite different perceptions of mathematical learning, with the children from Family 23 generally believing that mathematics is boring, and having no notion of their own mathematical abilities. Perhaps this is because the majority of their learning is through textbooks, where it is observed that the parent herself expresses little enthusiasm for the teaching. On the other hand, Child 26 is relatively confident at mathematics, and enjoys the subject – his mathematical learning reflects his family's philosophy that learning should be child-directed and through everyday life activities. Interestingly, both the children Family 23 and 26 hold similar beliefs on mathematical understanding and problem solving – having a numerical view of mathematics problems, and valuing correct answers, speed of calculation, confidence and application to real-life situations when identifying mathematical understanding.

Given that ‘extreme’ parental approaches can give rise to extreme attitudes to mathematics in their children but at the same time strong similarities in the children’s perceptions of problem solving and understanding, what is the outcome when a family adopts a ‘mixed’ home-educating approach using a range of teaching activities, guided by the children’s particular interests?

5.3 Example of a Semi-Formal Family

In semi-formal home-educating families, the children have significant influence on their mathematical learning — they frequently determine the resources used and amount of time spent learning. At the same time, the *parent acts as a mentor*, suggesting areas of improvement, facilitating learning and perhaps initiating change if their current learning approach is unsuccessful. Families 5, 7, 16, 17, 18, 19, 22 and 24 all belonged to the category of ‘semi-formal’ families, where the children were typically provided with a range of learning activities (often based on their personal interests) with guidance from their parents.

5.3.1 Example of a Semi-Formal Family

Family 7 had been home-educating their three children aged seven, five and two years, for the past three years. The eldest had special needs and was bullied at school, and they also felt their five year old daughter received an inadequate level of education. As a result, both children requested to be taken out of school. Although the mother had no formal teaching experience she had previously trained adults as part of her previous employment, and a number of family members held jobs that involved mathematical applications (e.g. builders, who often used trigonometry and so on). The mother personally enjoyed mathematics, finding it interesting and fun - she believed her love of mathematics had been passed to her children.

The mathematical learning followed a flexible timetable although the parent drew on her knowledge of the National Curriculum, aiming to cover each concept via a creative approach. Consequently, this family's teaching was organised according to the children's needs, using a range of activities such as workbooks, visual reinforcement (Cuisenaire rods and computer software), with adjustments made if the children did not appear to be learning. For example, as can be seen from her comments on Richard's case study (Appendix 10), we note that the parent believes that if a child is struggling with a particular area of learning (in this case writing), then the parent should intervene:

> "If he struggles with writing then he may also struggle with mathematics in a literary format rather than numerical, so this would need to be looked at to see if it can be helped."

Her comments on the case study of Joe also indicate an emphasis on adjusting the teaching activities according to the needs of the child, a key aspect of the semi-formal approach:

> "Give him whatever is suitable. Work for children above his age gap may be suitable only in certain areas of mathematics, things could easily be 'tailored' to suit him individually."

The mother measured her children's understanding through successful real-life applications of the concept, and if the children demonstrated 'different ways of working out problems'.

5.3.2 Influences of Parental Approach on Child's Perceptions of Mathematics

Child 7a (aged 7) who answered the questionnaire, and his mother, noted that the home-environment was more conducive to studying mathematics than school, with Child 7a writing:

> "I can ask lots of times if I don't understand and it's nice and quiet and I can have a rest whenever I want one."

Both also mentioned that when learning mathematics, the priority was to cover concepts that required *greater understanding* and those that had *applications to everyday life,* with Child 7a mentioning that interest and textbooks occasionally influenced his choice of topic. The semi-formal approach was also evidenced by Child 7a's current learning activities, where he used times tables, workbooks, money and the computer, to learn multiplication and division.

The mother stressed that *mathematics is important to everyday life*, which appears to have influenced her son's perceptions of the subject. Child 7a believed *mathematics is about 'finding out things'* and a mathematics problem is **'***a situation that you can solve using mathematics'*. Like his mother, Child 7a finds the subject *interesting* and *enjoyable*. The parent mentions that their immediate and extended family members are generally 'comfortable with mathematics' and so, perhaps not surprisingly, Child 7a feels that he is *good at mathematics*, explaining that those who are *'good at mathematics'* know how to *'work things out'*. On the other hand, according to him, those who are *'bad at mathematics'* try to *avoid the subject.* All but one of Child 7a's problem solving beliefs fell into the category of a 'good problem solver' (see Section 3.4.3). The mother measured her children's understanding via the following approach:

> "When they **apply it to real life** or when they show me a **different way of working something out**, for e.g.: when adding 10+5 my son immediately answered 15 then went on to explain that it was the same as 3x5 because 10 is made up of 2 groups of 5. It showed how much he had understood the concept of grouping and the way in which addition and multiplication are linked."
> Family 7

For Child 7a, the most *important signs of understanding* were: finding patterns, correct answers, explaining the concept to others, independent work without help, real-life applications, and making connections with existing knowledge. So unlike Families 23 and 26, there is evidence to suggest that Child 7a's notions on problem solving and mathematical understanding are influenced by his mother's home-educating approach because his *notions of understanding* are *very indicative* of his mother's approach towards mathematics education.

The parental home-educating philosophy towards mathematics and the influence this has on their children's attitudes as exemplified within the three case studies above may be seen as illustrative of the wider picture within the sample considered. We now turn to consider the home-educated children's perception of their mathematics learning environment, and highlight any links between the children's views and the parental teaching approach.

5.4 Children's Perceptions of the Home-Educating Environment

From the sample of 28 home-educating families, 21 children (11 girls and 10 boys) completed the questionnaire. The children's ages ranged from 6 to 18 years of age, where the mean age was 11 years. On average, the children had been home-educated for 5 years (median of three years). Three children were relatively new to home-education, having been out of school for less than a year, whilst others ranged from those who had never attended school, to those who had been home-educated for a number of years.

The children were asked to detail both the advantages and disadvantages of learning mathematics at home (Q.9, p.6 and Q.10, p.7 of the Children's Questionnaire, Appendix 2). Regardless of the home-educating approach of the parent, just under half the children (8 out of 21) felt that the flexibility of learning was one of the most important aspects of home-education:

> “I can work at my **own pace** and if I don’t understand anything I can **spend as long** as I want learning it.” Child 17a (aged 18, was at home for 11 years)

The results highlighted in Section 4.1.1 show that 70% of parents chose to home-educate since they believe ‘school is too restrictive’, suggesting that both children and parents emphasise *flexibility of learning* as a key advantage of home-education. *‘Receiving help when needed’* is the next most common response, with a third of the children believing assistance can easily be found at home, and a similar fraction noting that the home environment was conducive to study:

> “I can work at my own pace, and my mum who used to be a math’s teacher can **help me** when I’m stuck.” Child 9 (aged 11, at home for 5 years)

> “It is an **easy environment** to work in.” Child 17c (aged 12, always taught at home)

However, a quarter felt that their concentration could be improved when studying mathematics:

> “**Concentration** skills. Learning to apply social skills outside of busy environments. Learning to find the will to learn when you can be doing other things. Time management.”
> Child 28 (aged 15, at home for 1 year)

Two children would have liked more resources:

> “More **copies of the textbook** so more than one person can see diagrams/questions.”
> Child 11 (aged 15, at home for 1 year)

Child 18’s comments list a number of possible improvements:

> “1. Not to get cross with my head when it won’t add up.
> 2. Play more games
> 3. Play the piano more because I’m good at it.
> 4. Throw the Kumon into a rubbish lorry.” Child 18 (aged 8, at home for 1 year)

Child 18 mentions a strong dislike of the Kumon mathematics program, which he works through on a daily basis. It is noticed that he is the only child in this study to express an aversion towards his parents' chosen teaching approach, which his mother describes in the following way:

> "Kumon - 10 minute booklet everyday including weekends & holidays – he hates it but it has helped him. Games - 20 minutes most days. He's eager to try new things but put off quickly if counting numbers are involved.
>
> With the help of Kumon maths now his 6th month he can do some mental maths with simple number bonds under 20 – he must feel his achievement at being able to add in his head but won't give any credit to himself or Kumon. We would like a professional to tell us if they think David has dyscalcular or has he just 'shut down' — we would then know if we should stop Kumon maths which he dislikes but has helped him OR should we take things even slower and accept that he has a problem with numbers.
>
> We don't know how to get this help."

From these comments we observe that Child 18 has difficulty learning basic arithmetic, which was one of the reasons he was home-educated, due to a lack of support from his school teachers. But although the parent believes the Kumon program is resulting in some improvement, Child 18 does not favour this approach.

5.5 Choice of Mathematical Activity According to the Children

Table 5.1 shows the results from Q.8, p.6 of the questionnaire, which asked the children to indicate how frequently each of the listed factors affected their choice of activity. The highest percentages for each activity are highlighted in red.

Choice of Activities	Always	Sometimes	Never
A = My parent/teacher chooses it for me	5%	52%	43%
B = I choose something that I'm interested in	33%	61%	5%
C = I study whatever comes next in the textbook	10%	71%	19%
D = It is important to work on the areas I don't understand	38%	57%	5%
E = We find mathematics in everyday life (e.g. shopping etc.)	67%	29%	5%
F = I work on the areas that are needed for my exams	24%	48%	29%

Table 5.1: Children's Perspective on Choice of Activity When Learning Mathematics

The highest percentage observed in Table 5.1 indicates the *occasional use of textbooks*, with around three quarters of the children studying whatever concept came next in their textbooks. Child 17b used the books for *practice and explanation*:

> "I use a textbook for explanations and exercises."
>
> Child 17b (aged 15, at home for 9 years)

Textbooks were sometimes used in addition to other activities, as in the case of Child 7:

> "Times tables, **workbooks**, money, computer."
>
> Child 7a (aged 7, at home for 2 years)

But Child 15 (aged 12, at home for 3 years) believed that she did not learn from any activities apart from workbooks, exclusively using a 'maths scheme' to learn mathematics:

> "I **don't do any [other activities]** really, I work from my **maths scheme**."

Child 15 followed a structured approach to learning mathematics, with the parent writing:

> "Usually do topics. Oldest child now has a curriculum...must complete same everyday Monday → Thursday. Friday then a free day unless they didn't do the work in the week – then they have to finish it on Friday before they can do anything else."

Similarly, Child 28 (aged 15, at home for one year) is also using an exclusive activity through a structured approach, learning from a GCSE textbook, with support from a tutor. So it can be seen that those children whose parents adopt a structured approach felt that most of their mathematical learning was through textbooks.

In contrast, four children (from Families 8, 20, 21 and 24), all from informal families, wrote that they **never used a textbook** to direct their learning, which indicates that children from 'child-led' informal families were less likely to use textbooks than those from structured/semi-formal families. It was also observed that the three children from Families 8, 9 and 26 were not studying any mathematics at the time of this study. This suggests that as well as the 'informal' children choosing *what they learned*, there was also the flexibility to choose *when they learned*.

Similar to the parents (Section 4.5.1), we see from Table 5.1 that the children also emphasise learning through *everyday life activities*, suggesting that the home-educating environment tends to encourage children to apply their mathematical skills in both formal and informal learning situations. A parental emphasis on 'catering for their children's interests' also appears to be an influencing factor — of the eight parents whose choice of activities was primarily governed by their *children's interests*, seven have children who believe they can 'always' choose mathematical activities based on their personal interests.

In fact, the only children who wrote that they *never had the opportunity* to study topics according to their interests were Child 23b and Child 18. As we saw earlier in Section 5.1.1, the children in Family 23 learned through a highly structured, textbook based learning that was entirely parent-directed. Child 18 specifies that his parents always choose his mathematical activities – but this may be a result of the fact that his parents believe he requires a great deal of support when learning mathematics.

38% of the children claimed that they 'always' focused on improving their *understanding*, and a quarter chose to focus on areas that were relevant to their exams:

> "**Practise** exam questions on the topic, and **textbooks** that **explain** how it works."
> Child 11 (aged 15, at home for 1 year)

Now that we have considered the children's perceptions of their learning environment, we next ask, *"What effect does such an environment have on their views of mathematics?"*

5.6 Children's Perceptions of Mathematics as a Subject

This section examines the children's notions of mathematics as a subject, and also considers possible relationships between child and parental mathematical beliefs. Question 2 (p.4) of the children's questionnaire (Appendix 2) measured the strength of agreement given to a series of statements, the results of which are shown in Table 5.2 below:

Statement	Strongly Agree	Agree	Neutral	Disagree	Strongly Disagree
A = Mathematics is just about numbers	0%	0%	**47%**	33%	19%
B = Mathematics is interesting	29%	**33%**	19%	10%	10%
C = We need mathematics for everyday life	33%	**48%**	19%	0%	0%
D = Mathematics is useful for other subjects	25%	**60%**	15%	0%	0%
E = Most people do not like mathematics	10%	25%	**35%**	25%	5%
F = I do not like mathematics	14%	0%	19%	14%	**52%**
G = It is important to learn mathematics to pass exams	24%	**38%**	14%	19%	5%
H = I enjoy mathematics	**33%**	**33%**	19%	5%	10%

Table 5.2 Percentage of Children's Level of Agreement for Each Statement

The majority (85%) believed that mathematics was useful for other subjects and for everyday life (81%), with Child 9, aged 11 years, providing an indication of its worth:

"...many forms of mathematics are **in all subjects**." Child 9

Of the five children who ‘strongly agreed’ that mathematics was useful for other subjects, three had parents who rated the ‘application of the mathematics to other sciences’ as the most or (second most) important reason for teaching mathematics. This suggests that the parent’s *motives for teaching the subject* influenced their children’s perceptions of mathematics. On the other hand, in Family 28, whilst the mother did not believe that the scientific applications of mathematics were important motivations, her son, Child 28 (aged 15), valued the application of mathematics to other subjects. Interestingly, Child 28 was taught mathematics by a tutor, not his parents, which could explain why the mother’s mathematical beliefs had little influence on the son’s views. Indeed, the vast majority of children ‘agreed’ with this statement, irrespective of the level of importance given by their parents.

Over two-thirds (66%) of the children disagreed with the statement “I do not like mathematics”, with half reporting strong disagreement. Perhaps not surprisingly then, the majority of children (62%) also found the subject interesting:

> “Maths has **interesting** concepts and you can play around with it in many different ways.”
> Child 20 (aged 15)

Furthermore, both enjoyment and interest appeared to be influenced by parental belief, as of the nine parents in this study who liked mathematics, seven had children who also enjoyed the subject. Similarly, of the nine home-educators who taught their children mathematics because they personally believed ‘mathematics is an interesting subject’, eight had children who found the subject interesting. However, while it appears that *parents who liked mathematics tended to pass their positive views onto their children*, there was evidence to suggest that *negative perceptions of mathematics by parents did not necessarily lead to a similar belief in their offspring*. For example, of the two parents who did not like mathematics, Family 20 writes that all her children enjoy the subject, with her son verifying this fact, whereas Child 24 holds the same dislike of mathematics as her mother. In fact, only three children in this study (Children 8, 18 and 24) expressed a dislike towards the subject:

> "I find it kind of **boring**. I **don't like it much**" Child 8 (aged 9, never attended school)

Most of the children who expressed negative or neutral views towards mathematics came from rather 'structured' families, as was the case for the children from Families 6, 14, 23 and 24, whose parents mainly used workbooks. This relationship between a 'very structured home-educating approach' and a negative view of mathematics is illustrated in Family 18. Child 18's view of mathematics, however, appears to depend on the *area of mathematics*, and how this area is taught.

> "Shape = **enjoyable**
>
> Symmetry = I like it
>
> Adding = it's **O.K.** sometimes
>
> Take away = **takes ages** and I **hate** it
>
> Division = I don't know much yet" Child 18 (aged 8, at home for 1 year)

His mother appears to use a very structured, systematic approach to teach addition and subtraction, mainly through the daily use of Kumon Maths worksheets, which Child 18 clearly dislikes:

> "He's eager to try new things but put off quickly if counting numbers are involved. With the help of Kumon maths now his 6th month he can do some mental maths with simple number bonds under 20 – he must feel his achievement at being able to add in his head but won't give any credit to himself or Kumon." Family 18

Child 19 makes the observation that one's perception of mathematics primarily depends on the person:

> "A mess of recurring and unrecurring **logic**, that can be made, **enjoyable**, unenjoyable, **boring** or **exciting**, dependent on the individual." Child 19 (age 14, at home for 1 year)

Around two thirds of the children held the belief that it was *important to learn mathematics to pass exams*. Indeed, out of the 13 children who expressed this view, 3 mention exams as one of their future targets:

> "I would like a GCSE in maths and maybe an A-level."
> Child 15 (aged 12, at home for 3 years)

However, of the 5 children who disagreed/strongly disagreed with the statement, 2 mention taking mathematics exams as a future goal:

> "I want to take an A level eventually." Child 20 (aged 15)

These findings cautiously suggest that there is **no relationship** between the level of importance given to 'learning mathematics for exams' and the child's future aims of taking an exam.

Other viewpoints of mathematics that could be determined from the responses were *challenging* and *logical***:**

> "**Challenging**, interesting, **logical** puzzles" Child 10 (aged 14, at home for 2 years)

The evidence from the parents' questionnaires suggested children who perceive mathematics as a 'logical subject' tend to have parents who *believe that learning mathematics will improve their children's logical thinking*. Of the six children who explicitly mention that 'mathematics is logical', five had parents who believed an important reason for teaching their children mathematics was because *'Mathematics helps children think in a logical way'* (see Section 4.4). This was the case for the mother in Family 9, and we can see that her son shares this belief:

> "I would say that **mathematics is simple and straightforward** when you can understand what formula to use. Sometimes it can be considered mathematics but is actually **simple logic**. It's sort of like a map if you don't know where you're going you can't get there."
> Child 9 (aged 11, at home for 5 years)

5.7 The Usefulness of Mathematics

All but one child believed that the mathematics they learnt at home would be useful when they were older, and although they were not asked to give a specific reason, those who did emphasised its use in everyday life tasks (4 out of 21), jobs (4 out of 21) and as a useful skill for other academic subjects:

> "The mathematics that I am learning will be of help to me **in my job**, for writing statistical reports. Maths is also useful for **working out VAT and tax**, both of these are already useful, but they will become more useful as I grow older."
>
> Child 17b (aged 15, at home for 9 years)

> "When I **want a job** and have lots of money to spend – I will need to do sums."
>
> Child 18 (aged 8, at home for 1 year)

> "Yes. Because I want to go to Cambridge University **to study sciences**."
>
> Child 16 (aged 8, at home for 2 years)

Just under 40% did not have a specific mathematical goal or target. However, a third were targeting an A-Level/GCSE in mathematics:

> "To get an **A/A* in GCSE** and maybe **A level**." Child 19 (aged 14, at home for 1 year)

> "I want to take an **A level** eventually." Child 20 (aged 15, at home for 4 years)

Three wrote that their target is to learn a particular mathematical concept:

> "To **know all my times tables** and to be good at division."
>
> Child 14 (aged 8, at home for 1 year)

> "**I want to be as good at sums as my friends** – When I went to school two of my friends Clare & Adam tried to help me with sums but they got told off by the teacher and she didn't ever help me – she just told me to do sums not how to do sums and kept me in at playtime to do them but didn't tell me how." Child 18 (aged 8, at home for 1 year)

5.8 Beliefs on Problem Solving

In addition to considering the children's perceptions of mathematics, their problem solving beliefs were also examined. We also seek to gain an insight into the question: "*What are the main factors that could affect a child's problem-solving beliefs?*" Family 17's home-educating approach demonstrates how the semi-formal teaching approach caters for the different learning styles of their three children (aged eighteen, fifteen and twelve years). In this extract, the findings suggest that whilst all the siblings share the same teacher, a range of different problem-solving beliefs are evident.

5.8.1 Example of the Range of Beliefs in Family 17

The eldest, Child 17a was accelerated at school, which resulted in her 'being away from her friends'. She asked her parents to consider teaching her at home, and eventually all three children were taken out of school, where they have been home-educated for the past eleven years. Although the mother previously taught GCSE mathematics at school, she felt her teaching experiences had no significant influence on her home-educating approach; her personal belief being that mathematics means 'logical thinking and reasoning'.

She was aware that there were many different ways of teaching mathematics, but personally emphasised 'mathematical understanding', and was guided by each child's learning style:

> "My eldest has a near photographic memory and knew all her times tables up to 12 by 5 1/2years. My youngest is dyslexic and will never remember his tables or any sequences. We have to cover and recover topics in various ways to help him find a key to remember things."

Teaching took place four to five times a week, and they also made use of everyday tasks that involved mental mathematics. From Table 5.3 below that was constructed from Children 17a, 17b and 17c's questionnaire responses, one can observe that there are more differences than similarities in the siblings' perceptions of mathematics, despite sharing the same teacher (i.e. their mother) for a number of years.

Similarities	Differences
All three children list **'understanding'** as being their main objective when learning mathematics, and have similar beliefs on mathematical understanding.	The older two, **Children 17a (aged 18) and 17b (aged 15),** both try to focus on the **everyday applications** of mathematics, with **17b** also concentrating on areas of **interest** and those needed for exams, writing that her parents **'never'** influence her learning activities. But **Child 17c (aged 12)** is **generally guided by textbooks,** and his parents occasionally choose the mathematical topics.
The mother indicates that mathematics is *interesting* and *useful* for everyday life. **All her children enjoy mathematics,** find it interesting, and express the view that it is important for everyday life.	The **older two** feel that they are **good at mathematics** most of the time, but **Child 17c** writes that he is good at **mathematics only occasionally**. Perhaps this is because Child 17c had dyslexia, and sometimes had difficulty remembering simple concepts. Therefore, he required extra support and this may have resulted in a lower perception of his own mathematical abilities.
	Children **17a and 17c** feel that a mathematics problem is **'an exercise when you decide which operations to be done and then perform them correctly'.** Both **Children 17a and 17c** believe that 'being able to perform calculations easily' is a measure of mathematical ability.
	On the other hand, **Child 17b** sees mathematics **'as a set of formulae used to calculate the effects of actions'** and believes a mathematics problem is 'a situation you can solve using mathematics'. **Child 17b** believes that a sign of being 'good at mathematics' is the ability to **apply the concepts** to everyday life, and an **understanding** of the formulae and methods.

Table 5.3: Similarities and Differences in the Mathematical Perceptions of the Children from Family 17

In particular, Table 5.3 shows that while *all of the children* appear to be influenced by the parental **emphasis on understanding**, the *particular learning styles* of the children are the strongest influence on their problem-solving beliefs and their perceptions of themselves as mathematicians. For example, Child 17b focuses on learning the applications of mathematics and topics that interest her, seldom relying on her parents for guidance. She has a different set of problem solving beliefs to her younger brother, Child 17c, who mainly learns from a curriculum. There may be age and maturity issues associated with these differences, something that will now be explored through the full samples responses to the nature of a mathematics problem and the general beliefs about problem solving.

5.8.2 "What is a Mathematics Problem?"

As well as providing examples of mathematics problems, the children were asked to identify the best description of a mathematics problem from the following statements (Question 3, p.1 of the Children's Questionnaire):

***Statement 1**: "A mathematics problem is numbers with some words and a question"*

***Statement 2**: "A mathematics problem is a situation you can solve using mathematics"*

***Statement 3**: "A mathematics problem is an exercise where you decide which operations to be done, and then perform them correctly"*

***Statement 4**: "A mathematics problem is an exercise during a mathematics lesson"*

Age appeared to be a determining factor when considering the distribution of response. Just under half the sample (nine children) chose Statement 2 but of the nine who possessed this belief, **seven were aged 10 years or over**. Furthermore, of the seven children (35% of the total sample) who felt that *"A mathematics problem is an exercise where you decide which operations to be done, and then perform them* *correctly"*, five are also aged 10 years or older. Indeed it was observed that **all of the children who were 10 years or above** chose either Statement 2 or 3.

The younger children (under 10 years of age) showed **no such pattern of response**, with three believing *"A mathematics problem is numbers with some words and a question"*. Two (Children 16 and 18) believe a mathematics problem is an exercise where operations need to be performed, and Child 23b believed *"A mathematics problem is an exercise during a mathematics lesson"*.

5.8.3 Specific Problem Solving Beliefs

To obtain clarification to the children's best descriptions of a mathematics problem **four specific problem solving scenarios** were presented to them.

They were first asked how they viewed the statement: *"Mathematics problems are always solved in less than ten minutes"* (Q.1, p.1, Children's Questionnaire). The majority (71%) **did not agree with this statement** and in this instance, the age of the children did not appear to have an influence on their view. Secondly, when asked if there *"Does there exist a mathematics problem without numbers?"* thirteen children (65%) believed that such a problem **could exist -** once again, the age of the child did not appear to influence this problem solving belief. The majority of the sample (68%) also disagreed with the statement *"All mathematics problems only have one correct answer*" (Q.5, p.1).

Finally, the children were asked to indicate what they believed to be the 'worst' of the following three problem solving errors: (1) A calculation error, (2) Choosing the wrong method or operation, or (3) It's the same, there is no difference. The children’s responses were almost equally distributed across each option, where eight children indicated that both errors were equivalent, seven suggested that ‘the worst error is to choose the wrong operation or method’, and six believed ‘the worst error is a calculation mistake’. Again, the age of the children did not appear to be a factor.

5.8.4 Relationships between the Problem Solving Beliefs

Parental mathematical beliefs did not appear to have any influence on the children’s problem solving beliefs. But the following pattern of response was noticed when considering each of the problem solving beliefs discussed above. Of the **nine children** who believed “A mathematics problem is a situation you can solve using mathematics”, **seven felt** ‘there exist mathematics problems without numbers’. But of the 10 children who believed “A mathematics problem is an exercise, or numbers with some words and a question” **over half** held the belief that ‘every mathematics problem *should involve numbers’* – i.e. children with a ‘less holistic view’ of a mathematics problem tended to adopt a numerical conception of a mathematics problem.

5.8.5 Summary

Briefly summarising the main problem solving beliefs, we note the following:

- The majority of home-educated children perceived a mathematics problem to be *‘any situation where mathematics can be used’* or *‘an exercise where mathematical operations need to be performed’*, where the overall aim is to find a solution using mathematics.

- Age is rarely an influence on the problem-solving beliefs, nor does there appear to be any relationship with parental mathematical beliefs.

- Most believed that it could take longer than 10 minutes to solve a mathematics problem, and that ‘problems without numbers’ existed**.** Those who viewed a mathematics problem as ‘any situation where mathematics could be used’ were more likely to perceive the existence of a mathematics problem ‘without numbers’ than children who held an ‘operational’ or ‘numerical’ perception of mathematics problems.

As we saw earlier in Section 5.2.3, both children from Family 23, which was structured and Child 26, from an informal family, had similar beliefs on understanding, even though their family’s approaches to learning mathematics were quite different. In order to help clarify a relationship between understanding and the home-educating approach we now turn to the *children’s perceptions of understanding*.

5.9 Children’s Views on Mathematical Understanding

To begin with, the children’s views on ‘what it means to be good/bad’ at mathematics may help to illustrate their perceptions of understanding. Therefore, we first consider the responses to Questions 1 and 2 (Appendix 2, p.5 of the Children’s Questionnaire), which asked children to describe characteristics that they felt would indicate when someone was good (or bad) at mathematics.

5.9.1 Children’s Perceptions of Mathematical Ability

The children identified the speed of completion of mathematics questions, especially performing mental calculations correctly, as a key measure of ability, with 9 of the 21 children mentioning that those who are good at mathematics can *‘do mathematics problems quickly’*:

> "They solve their problems in a **short amount of time**, they can do it **within their head**, they rarely need help from others and can memorise things quickly."
> Child 8 (aged 9, always educated at home)

Thus, those who are deemed 'bad at mathematics' are expected to be slower:

> "If you give them a worksheet they'll **take ages on it** and usually groan and **want to give up** on a really hard sum. Also they are not bothered about answering questions."
> Child 14 (aged 8, at home for 1 year)

Seven claim that being able to *solve problems correctly* indicates that one is good at mathematics:

> "They usually get **most of their questions** right and most of the time quite quick at working out mental maths." Child 24 (aged 10, at home for 9 years)

The same number feel *'getting wrong answers'*, and 'constantly requiring extra help' are characteristic of those who are bad at mathematics:

> "They get their answers **wrong sometimes**, find it extremely difficult and takes them ages to complete a question." Child 6 (aged 13, at home for 1 year)

Indeed, six out of eight children who cited 'correct answers' as a sign of being *'good at mathematics'* also mention 'incorrect answers' as a sign of being *'bad at mathematics'*. As well as getting the answers correct, **five** mention *'mental calculations'* as a key characteristic of a good mathematician:

> "The type of questions they answer. How quick they are at adding, multiplying etc. **in their head.**" Child 28 (aged 15, at home for 1 year)

Four write that *'understanding the mathematical concept'* is a relevant factor when someone is good at mathematics, and **three** believe a lack of understanding will be apparent when someone is 'bad' at mathematics. This belief is illustrated by Child 17b's (aged 15) answers:

> "They can apply maths to everyday life and they can **understand the formulae** and methods."
> [Good at mathematics]

"They often do not like maths and have a lack of confidence. They tend to be **unable to understand methods** and formulae, they also are not able to relate maths to everyday life." [Bad at mathematics]

Child 18 (aged 8, at home for 1 year) gives an interesting response to the questions, mentioning the particular careers of each individual ability level:

"They are **happy and quick at sums** and get a good job like an **accountant** or a **bank manager** or become **the manager of a football team**." [Good at mathematics]

"They are **slow at sums** but they still get a good job like an **artist, sports person, pop star** or **gardener**. They just don't do any." [Bad at mathematics]

Having illustrated the ways in which the children measure mathematical ability, we now consider the options they take when they have difficulty understanding a concept.

5.9.2 Child's Strategy When Unable to Understand a Concept

When asked to write down what they would do if they were unable to understand a mathematical topic, only two out of the twenty one would stop working on the concept:

"**Stop doing it**, or ask my mum." Child 8 (aged 9, always educated at home)

On the other hand, nine children wrote that they would think through the problem again before asking for external help:

"I would **re-read and explore the topic** for anything familiar. I would look at a question in the topic and look at the answer then try to find out how the answer came about and see if it works, then I would see if it worked with other questions. If I had no idea I would ask."

Child 9 (aged 11, home-educated for 5 years)

A similar percentage **(12 out of 21)** asked a parent or family member for help:

"**Ask** my parents or my brother." Child 6 (aged 13, home-educated for 1 year)

Four would turn to alternative resources, such as textbooks or the Internet:

"Find **a different source** that covers the same information in a different way."

Child 11 (aged 15, home-educated for 1 year)

"I look in text books or on the internet to **find a different method** or a different explanation. This should help me to understand it." Child 17b (aged 15, home-educated for 9 years)

Two wrote that they would feel upset if they could not understand a concept – however, Child 18's response appears to depend on the topic:

"Sums & telling the time = **get cross & sad** and don't do any work

Shape & symmetry = try harder because I want to learn this

Handling data = think hard."

Child 18 (aged 8, home-educated for 1 year)

5.9.3 Important Signs of Mathematical Understanding

To consider the children's perceptions of mathematical understanding we focus on the signs that indicate whether (or not) they have understood a particular mathematical concept. Q.7 (p.2) and Q.8 (p.3) of the children's questionnaire (Appendix 2) asked the children to rank the importance of each of the following 'signs', from **1 = Not important at all**, to **5 = Always Important**, as a measure of whether they had/had not understood a particular concept. Table 5.4 indicates the options through which the children indicated the degree of importance they attached to statements denoting measures of understanding or otherwise. Each statement is identified by letters A to J (Understanding) and A to I (Not Understanding).

Signs of Understanding	Signs of 'Not Understanding'
A. Parent/teacher says I understand the mathematics	A. My parent/teacher says that I don't understand the mathematics
B. Can see a 'pattern' in the mathematics	B. Most of my answers are wrong
C. Answers are all correct	C. It is hard to explain the mathematics
D. I can do the questions without help from my parent/teacher	D. Cannot see how the mathematics is used in real-life
E. Have memorised the formula or method	E. I can't see how the mathematics is connected to any other mathematical idea
F. I know how each part of the formula or method works	F. I am afraid that I will make a mistake
G. Can explain the mathematics to another person	G. The formula/method is too hard to remember
H. The mathematics can be used in a real-life situation	H. I can't explain how the formula/method works
I. There is a connection to some mathematics I know already	I. I get stuck all the time without help from my parent/teacher
J. I feel confident	

Table 5.4: Signs of Mathematical Understanding

Table 5.5 (below) indicates the frequency with which the importance of each item was considered. Note the responses of 20 children are recorded – Child 21 (aged 6) did not answer this question.

Signs of Understanding	1 = Not important at all	2 = Rarely important	3 = Sometimes important	4=Important most of the time	5 = Always important
A	4	2	6	2	6
B	0	3	3	6	8
C	1	0	5	8	6
D	0	1	7	6	6
E	1	0	6	12	1
F	1	1	4	5	9
G	2	2	2	8	6
H	0	2	5	4	9
I	1	4	7	5	3
J	0	0	2	2	12
Signs of 'Not Understanding'	**1 = Not important at all**	**2 = Rarely important**	**3 = Sometimes important**	**4=Important most of the time**	**5 = Always important**
A	4	4	4	1	7
B	0	1	4	6	8
C	0	2	7	8	3
D	2	2	7	7	2
E	2	0	8	6	4
F	4	4	4	3	5
G	0	2	7	8	3
H	0	1	5	10	4
I	2	2	1	5	10

Table 5.5: Distribution of Responses for Signs of Understanding

5.9.4 Confidence

According to the children, the **most important** sign of understanding was the **'***level of confidence'* (Item J) felt when working through a concept (**90%)**. Interestingly, when considering the importance given to 'the fear of making a mistake' (Item F) as an indication that they had **not understood** a concept, Table 5.5 shows an almost equal number of children for each ranking of importance. This indicates that there is less relevance given to *fear* as an indicator of 'not understanding' than *confidence* as a sign of 'understanding'.

5.9.5 Explaining to Others

70% of the children felt the ability to explain the concept to another person was an important sign of understanding (Item G). It was noticed that **three out of the four** children (Children 18, 23b and 24) who did not think that this was important were taught mathematics via a structured home-educating approach.

5.9.6 Knowledge of the Formula or Method

Just under three quarters (70%) felt *knowledge of each part of the formula/method* (Item F) was a clear sign of understanding, and as would be expected, three-quarters of the sample gave **similar levels of importance** to Item H (not being able to explain the formula/method) being a sign of lack of understanding.

On the other hand, five children gave **quite different levels of importance** to each statement, with the most extreme difference noted in Child 23b's response. That is, Child 23b did not feel it necessary to know each part of the formula/method in order fully understand a concept, but at the same time, she believed an inability to explain 'how the formula/method works' clearly indicated a lack of understanding. It was noticed that Child 23b's family are **structured**, as were the backgrounds of all five children whose responses regarding this statement followed a similar pattern to Child 23b's. However, this is an area that would require greater investigation in order to fully comprehend the reasons for the disparity in response for children from structured families.

5.9.7 Memorisation

The majority of children (65%) felt that *memorisation of a formula/method* (E) was an important sign of understanding, and 85% of these gave the same (or one level more/less) importance to both memorisation statements. But three gave responses that demonstrated quite differing levels of importance, with both Child 6 (aged 13) and Child 14 (aged 8) believing that while memorisation is not a good sign of understanding, an 'inability to memorise' demonstrates a lack of understanding. On the other hand, Child 17a (aged 18) takes the opposite view, claiming that memorisation is 'always' a sign of understanding, but lack of memorisation seldom indicates a lack of understanding.

5.9.8 Real-life Situations

Nine of the children felt that a successful application of a concept to a real-life situation was a clear sign of understanding (Item H), and it was noted that eight of these had parents who mentioned the application of mathematics to real-life, either within their **mathematical beliefs,** their **teaching activities,** or their **ways of measuring their children's understanding:**

> "Maths means being able to use numbers in a useful way when needed, such as balancing a bank statement, adding up a bowling score or working out how much carpet to buy." Family 8

On the other hand, of the two children (Child 6 and Child 11) who did not rate real-life applications as important signs of understanding, it was observed that their mathematical learning was rather structured:

> "Child 3 has to do some of her PC [personal computer] maths each day and her incentive is to do it so she can then do other things!!!!" Family 6

> "To pass their IGCSE with a decent grade. I believe GCSE's in Maths and English are the minimum requirements for many jobs. In the few weeks before the exams we were doing about 1 hr per day." Family 11

5.9.9 Patterns

70% of the children in this study believed that the ability to *see patterns in the mathematical concept* (B) was an important sign of understanding most/all of the time. Again it was noticed that the only two children who did not consider this to be a sign of understanding were both from structured families (Children 6 and 23b).

5.9.10 Correct Answers

Of the sample, only Child 20 (aged 15, at home for 4 years) believed correct answers (Item C) were 'never' an indication of understanding; however, he also felt that if 'most of his answers were wrong', it was a very clear indication that the concept had not been understood. On the other hand, Child 14 (aged 8, at home for 1 year) believed that while correct answers were a good sign of understanding, incorrect answers rarely implied a lack of understanding.

5.9.11 Parent/Teacher's Influence

60% of the children believed that once they could do questions *without assistance from their parent or teacher* (Item D), this was a strong indication that the concept had been understood. But parental/teacher *acknowledgement of understanding* (Item A) was of varying levels of importance to the children, with six stating that Item A was rarely an important indication of understanding, the same number feeling it was 'sometimes' important and eight believing it was 'often' an indication of understanding. Furthermore, one could observe that children from both autonomous (e.g. Child 8, from an informal/autonomous family) and structured families (e.g. Child 23a) felt parental/teacher acknowledgement was an important sign of understanding, but similar diversity was found within those who felt this was of little importance. It could be inferred that this particular 'sign of understanding' is dependent on a child's individual need to seek assurance from another individual, rather than their family's educational approach.

The general observation from Section 5.9 is that, the majority of the home-educated children rated most of the ‘signs’ of understanding/not understanding, as important. However, there is also evidence to suggest that children from structured families do not appear to utilise as many ‘indicators’ of understanding as the children from semi-formal and informal families. It may be that since their learning is more restricted to textbooks, these children have narrower perceptions of mathematics from which to ‘measure’ understanding.

To establish links between the teaching approach followed by the parent, the mathematical beliefs of the child and parent and the children’s mathematical understanding we now consider case studies associated with four families and the qualitative relationship between the parental home-educating philosophy and their children’s understanding, as evidenced in the children’s assessed work.

5.10 Parental Philosophy and Children’s Mathematical Understanding: Four Case Studies

Of the four families to be considered within this section, Family 11 follows a ‘structured’ approach, Family 4 follows an informal approach whilst two, Families 7 and 16, follow a semi-formal approach.

5.10.1 Philosophy and Understanding within a Structured Family

Family 11 had been home-educating their two daughters (aged seventeen and fifteen) for the past two years.

> “Eldest daughter refused to go to school, became school phobic. Younger daughter decided to join her ‘because lessons are boring and the people are horrible’” Family 11

The mother worked for British Telecom and personally liked mathematics. She enjoyed covering the concepts again with her children as this also improved her own understanding of mathematics. The father studied electronics and now worked in the field of Information Technology. From her responses to the fictional case study of Richard (see Appendix 10), we get a sense that the mother believes that all children, regardless of natural ability, need guidance, and that mathematical learning should not take precedence over learning other subjects:

> "He shouldn't do maths to the exclusion of everything else."

With regards to her own home-educating approach, the mother believed that the family could make mathematical learning more interesting than it appeared to be in school - her children would not have to carry out repetitive work once a concept was understood. However, she recognised one disadvantage associated with mathematics teaching and learning — her younger daughter often comprehended concepts faster than she did. Both daughters followed the IGCSE Edexcel syllabus and textbooks for around one hour per day, although they had different learning styles – the elder being more visual, preferring to use graphs and diagrams to aid understanding, while the younger daughter appeared to understand concepts very easily, from a symbolic perspective. The daughters' mathematical understanding was measured by the family going through examples together, whilst the mother provided assistance if there was hesitation on her daughters' part.

5.10.1.1 Influence of Parental Approach on Child's Perceptions of Mathematics

Only Child 11b's (aged 15) questionnaire answers could be compared with feedback from the mother, as Child 11a (aged 17) did not complete a questionnaire.

The mother believed that home-education allowed her children to avoid unnecessary repetitive work when learning mathematics, and this belief was also expressed by Child 11b, who wrote that she was:

> "Free to move at my own pace and choose what area to study."
>
> Child 11b (aged 15, at home for 1 year)

Surprisingly, although the children followed a structured approach, making exclusive use of textbooks, Child 11b felt that her mother **never dictated** the choice of topics to be learned. Instead, her learning was centred on topics that required **greater understanding** and those necessary for forthcoming **exams**. Thus it appears that it was the child, rather than the parent, who chose the particular mathematical concepts to concentrate on. Indeed, Child 11b believed that this freedom of choice was one of the main advantages of home-education.

It can be noted, however, that the mother's perceptions of mathematics did have some influence on the daughter's **mathematical beliefs**. Both child and parent viewed mathematics as a subject that **is important for everyday life**, and believed that gaining a **'thorough' understanding** of each concept was an essential aspect of the family's mathematical learning. Both also perceived mathematics to be a **'logical subject'**, and found mathematics **enjoyable and interesting**:

> "Logical, numerical, interesting, useful." Child 11b (aged 15)

The mother's positive attitude towards mathematics may have also influenced the daughter's perception of herself as a 'learner of mathematics', where Child 11b believed that she is **'good at mathematics most of the time'**.

All of Child 11b's problem solving beliefs were characteristic of those of a **'good problem solver'** (see Section 3.4.3 for a definition), for example, she believed a mathematics problem was 'any situation that can be solved using mathematics' and that 'not all mathematical problems need involve numbers'. Child 11b's responses suggested that 'levels of confidence when applying a concept' were an indication of mathematical ability. Her mother also appeared to consider **'confidence'** to be an important aspect of the learning process — for example, she did not highlight her daughter's incorrect answers in order to avoid feelings of disappointment.

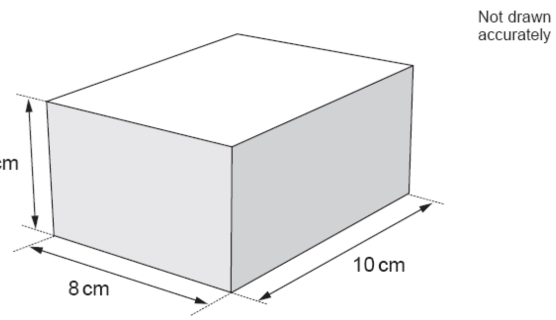
Child 11b **did not consider** the 'application of concepts to real-life situations' to be an important indication of understanding, perhaps because the majority of her learning was through textbooks. For her, the key signs of understanding were: finding *patterns*, *correct answers*, *confidence*, both an *understanding* as well as the *memorisation* of the formula/method, and working *independently* without help. Though the real life applications of mathematics did not figure as an important aspect of the subject, the key features of Child 11b's notions of understanding suggest a leaning towards a relational or conceptual understanding of mathematics. Furthermore, her aims when 'doing mathematics' also revealed a tendency towards the **application and understanding** of each concept (i.e. relational/conceptual understanding) rather than memorisation, correct answers, and quick completion (i.e. procedural/instrumental understanding).

5.10.1.2 Child 11a and Child 11b’s Assessed Work (Group 3)

Both Child 11b (aged 15) and her sister Child 11a (aged 17) completed the Group 3 assessed work (see Appendix 8 for the complete set of questions). By considering both children’s work, we can compare the different problem solving strategies of each sister. Child 11a achieved 78% on the test, whilst Child 11b, attained 94%, which was the highest obtained mark for the Group 3 questions amongst the home-educated children. Both sisters answered Questions 1 to 4 correctly, the majority of which were ‘calculation type questions’.

However, neither sister was able to answer Question 5.

I have a present in a box, a cuboid measuring **10cm by 8cm by 5cm**.



I have one sheet of wrapping paper to wrap up the box.

The sheet is a rectangle that measures **25cm by 30cm**.

Is the sheet of wrapping paper big enough to cover **all** the box?

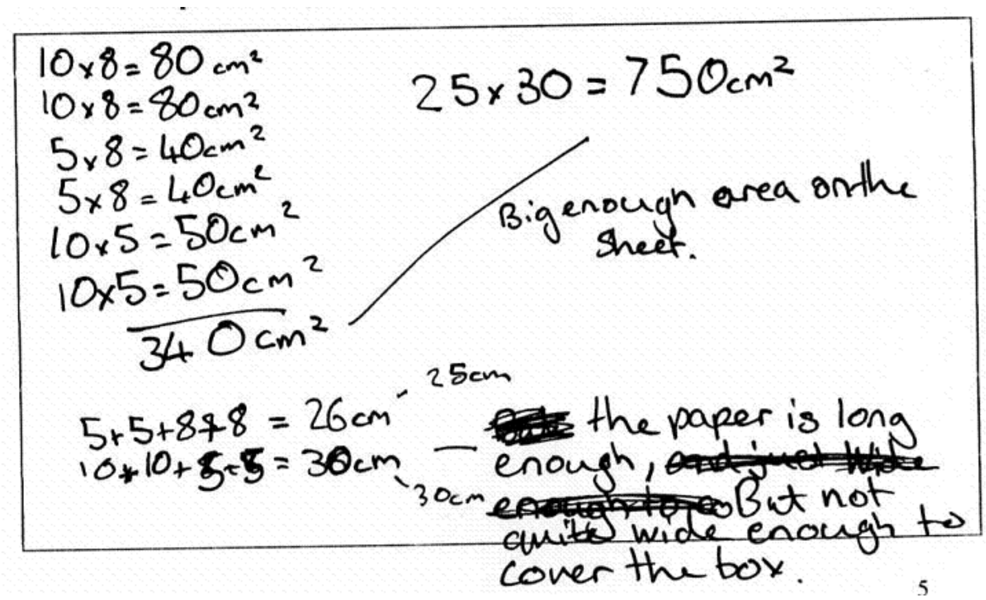
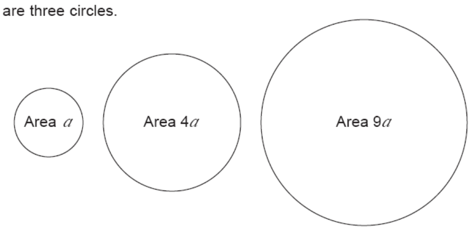


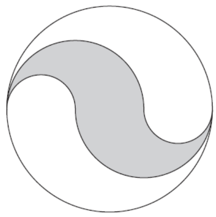
Figure 5.1: Q.5, G.3, Child 11b, Aged 15

In this question, Child 11b correctly calculates the surface area of the box, showing that the area of the sheet is greater than the surface area of the box, and initially appears to believe that the paper is big enough to cover the box. But when calculating the length and width of the paper, she calculates the 'width of net of the box' as 26cm with a corresponding 'length of the net of the box' of 30cm. She crosses out her answer, and concludes that the paper is **not big enough to cover the box.** Drawing the net of the box would have led her to the required width, as her method was correct. In Question 9, however, Child 11b does make use of a diagram to justify her solution:

Here are three circles.



Parts of the circles are used to make the design below.



What **fraction** of the design is **shaded**?

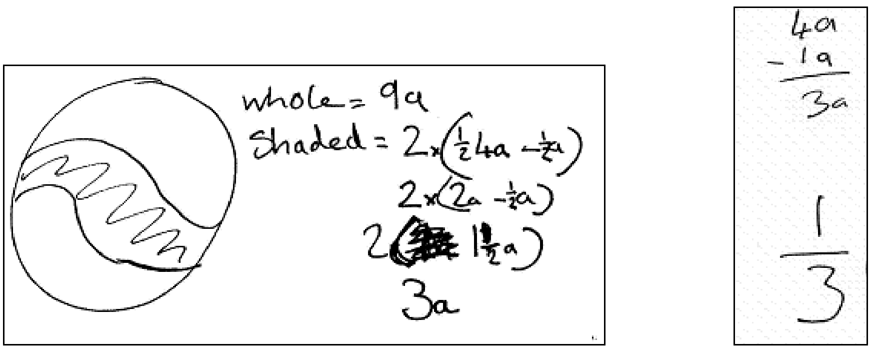


Figure 5.2: Q.9, G.3, Child 11b, Aged 15 and Child 11a, Aged 17

Child 11b's solution demonstrates her algebraic reasoning, linking the areas of the circles, the shaded region, and her algebraic calculations. On the other hand, Child 11a's understanding of the situation appeared to be more pictorial, in that she can 'see that the area of the shaded region is a third', and therefore uses less justification than her sister. However, for Q.5, like her sister, Child 11a did not make use of any imagery, and made an identical mistake, writing "The sheet is not big enough to cover the box because the circumference of the box is 26cm and the paper is only 25cm wide".

The following question, which has two parts, indicates that Child 11b may have a ‘better’ understanding of algebra than her older sister. The first part was given as follows:

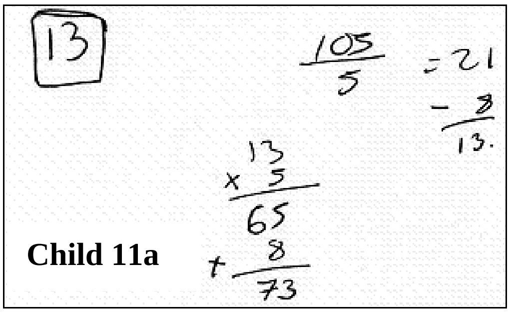
I think of a number, then I carry out these operations on my number.

Multiply by 5	**Add 8**

When I carry out the operations in one order the answer is **105**

When I carry out the operations in the **other order** the answer is **73**

What is my number?



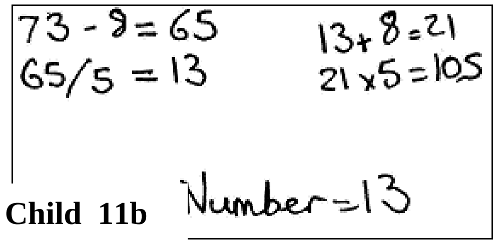


Figure 5.3: Q.8, G.3, Child 11a, Aged 17 and Child 11b, Aged 15

Observe that both sisters are able to find the correct solution to the first part of Question 8 using similar approaches, but different starting points. However, for the second part of Question 8, Child 11b is able to generalise her answer, whilst although Child 11a can see how 32 *could be obtained as a difference,* she is unable to generalise the solution for all numbers.

The difference between my two answers is 32. Can you prove that the answers will always be 32, no
matter what my number is?

$$(5 \times 8) - 8 = 32$$

Child 11a

$$\rightarrow = n \times 5 + 8 \quad — \quad (5n+8) - 5n = 8$$
$$\leftarrow = (n+8) \times 5 \quad — \quad (5(n+8)) - 5n = 5 \times 8 = 40$$
$$40 - 8 = 32.$$

Child 11b

Figure 5.4: Q.8, G.3, Child 11a, Aged 17 and Child 11b, Aged 15

5.10.1.3. The Structured Family — Parental Philosophy and Child's Understanding

Through their responses, we can observe that Children 11a and b found questions that were calculation-based relatively straightforward. This suggests that both were confident with arithmetic type problems and had an instrumental understanding of these areas. Furthermore, Child 11b (aged 15) demonstrated algebraic reasoning for both questions 8 and 9 – which also required proof. Both sisters, despite being the oldest children of the sample, were unable to correctly answer Q.5 (area of a box) – although, to be fair, none of the children in this study were able to do so. A correct solution to Q.5 would have been more easily obtained if the net of the box had been drawn out. Even though the parent wrote that Child 11a was a 'visual learner', perhaps the children's reliance on learning through textbooks, where they seldom explored applying their mathematical knowledge in situations outside of their routine exam preparation, made them less eager to experiment by drawing the dimensions of the paper.

5.10.2. Philosophy and Understanding within an Informal Family

Family 4 has been identified as a family using an informal approach. Their only child, a daughter (aged six years old), had never attended school, as her parent held the belief that *'home is better than school'*. The mother, who had previously taught mathematics for five years, had experienced the benefits of small class sizes and teaching based on motivation and interest. From her comments on the case studies (Appendix 10), we can see that she favours an approach where children have the chance to control their own learning:

> "Making him do anything is likely to turn him off what seems to be an enjoyable activity for him, which he is good at."
>
> "Such a decision should be at the child's own initiative, and he probably isn't old enough to make such a decision himself yet."

At the same time, she believes children should be given encouragement and support when learning mathematics:

> "I wouldn't be concerned about him, but I would want to provide him with opportunities and encouragement to continue learning (rather than just 'leaving him alone')"

Her beliefs are reflected when teaching her own child, where a one-to-one approach is used, selecting topics based on her own personal interests as well as her daughter's. Child 4 was encouraged to initiate the teaching, and mathematical learning often took place through discussion and games rather than bookwork:

> "Here is an example of the "child-led" approach in action:
>
> One of my daughter's favourite games is an **arithmetic quiz she invented** which she calls "**Ask Numbers**." She has many variations on the rules; for example she may specify "You ask me subtraction questions where the answer is an odd number," or "I'll tell you an answer and you think of a question to match it. Then I'll say whether your question is right or wrong for my answer."
>
> Family 4 (daughter aged 5)

Books were mainly used to supplement learning and provoke thought, as well as to aid pattern recognition. Mathematical learning was expected to take place whenever the opportunity was there to 'discuss' a concept, but the intention was to ensure her child learned the 'basics of mathematics'. Mathematical understanding was then assessed by listening to the comments made by her daughter as she talked through a problem. Although the mother frequently felt that she did not precisely know how much her daughter understood, she felt that this was not considered to be an issue.

5.10.2.1. Influence of Parental Approach on Child's Perceptions of Mathematics

Child 4 was aged 5/6 years at the time of this study, and was deemed too young to answer the questions for the Children's Questionnaire. However, a set of simple questions (see Appendix 9 for the questions and Child 4's answers) were constructed in order to assess her feelings towards mathematics.

Both child and parent appeared to enjoy mathematics, and though only five, the daughter's knowledge of numbers appeared to be advanced — she was able to give examples of 'big' numbers described as 'googol', and 'small' numbers, such as 'negative googol'. The mother often taught mathematics through everyday activities, and indeed Child 4 wrote, in answer to the question "Where can you find numbers?":

> "Everywhere! I can count the drawers on the chest and the panes of glass in the desk and the handles on the drawers. There are numbers on a chessboard and there are numbers of squares there too. There are numbers of weights on a scale and there is the number of chairs in our house."

She was familiar with the notions of 'heavier' and 'longer' and mentioned that she enjoyed drawing a variety of shapes, such as triangles, squares etc. and constructing 3D objects.

5.10.2.2 Child 4's Assessed Work (Group 1)

Child 4 (aged 6) answered many of the questions **verbally**, so her mother wrote down a transcript of the discussions that took place as her daughter articulated the problems. Although she only answered 56% of the questions correctly, Child 4's solutions to the

Group 1 questions (which were set at the Key Stage 1 level) showed a very comprehensive understanding of arithmetic:

Write the total.

$$64 + 85 + 56 =$$

C: 60+80, what's that? How much is 6+8?
S: 14.
C. Then it's 140. Plus 4 plus 5. 9. 149. Then the 6.
149+6... 149+1 is 150, 149+2 is 151, 149+3 is 152,
149+4 is 153, 149+5 is 154, 149+6 is 155. Then the
big 50. 155+50 is 205! That's it, it's 205.

Figure 5.5: Q.2, G.1, Child 4, Aged 6

For Question 2, Child 4 (identified as C) realises that she can get 60 plus 80 by calculating 6 plus 8. She next adds the units 4 and 5 to 140 to make 149. She then adopts a *different strategy* to add the 6 (the unit in '56'), *counting on 6* from 149 to get to 155. Finally she adds the 'big 50' to the 155 in one step to obtain the correct answer.

Manjit buys **6** stamps.
Each stamp costs **19p**.

How much does he pay?

Show how you work it out in the box.

C: If they were 20p it would be 120p. But they're only 19p, so it's six less. 120p minus 6p is 114p. (Writes 141) Is that 114?

S: No, you've written 141.

6

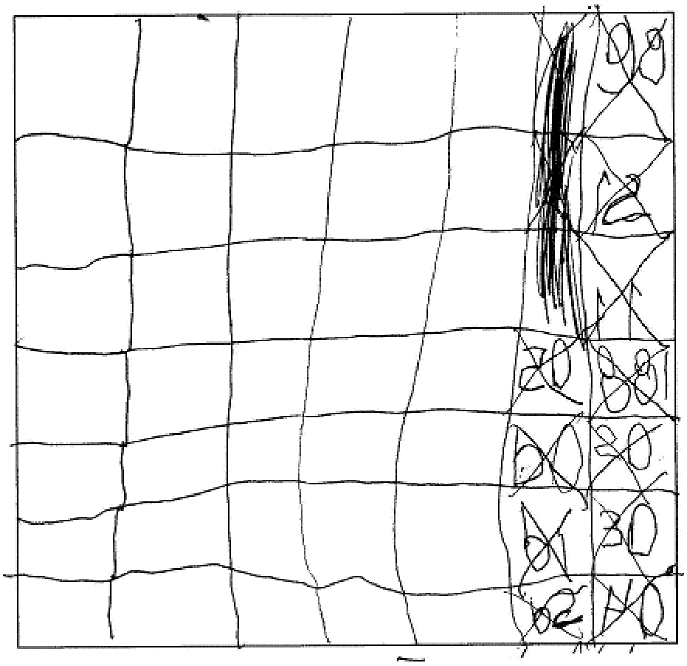
C: Oh, it's 1-1-4 (writes 114).

Figure 5.6: Q.6, G.1, Child 4, Aged 6

In Question 6, it is interesting to note how Child 4's *'mental and oral'* abilities are **ahead** of her *'written'* abilities. She is able to deduce that '6 stamps at 19p each' is just 6 less than '6 stamps at 20p each', showing a *relational* understanding of multiplication when making the connection between multiplication and addition/subtraction. At the same time, she is unable to write down the number '114' correctly **–** indeed, the mother mentions that Child 4 is still learning to write numbers according to their 'place value'.

In her questionnaire responses, the mother mentions that they often play numerical games where the daughter tries to 'find the correct number' according to a set of 'arithmetical rules' (e.g. "What is the biggest number?") and similar reasoning skills are demonstrated in Question 7:

$205 - 143 =$



C: I'll put a guess in each box. That should be plenty
of boxes. 99? 99 + 143 is too big. Try 12.
12+ 143 is too small. 41. 41 + 143 is too small.
Try a bigger number. . . (PTO)

So. 50+143 is 193, that's close.
60 then. 60 + 143 is 203. Almost!
61. 61 +143 is 204. We only need one
more! 62! It must be 62!

Figure 5.7: Q.7, G.1, Child 4, Aged 6

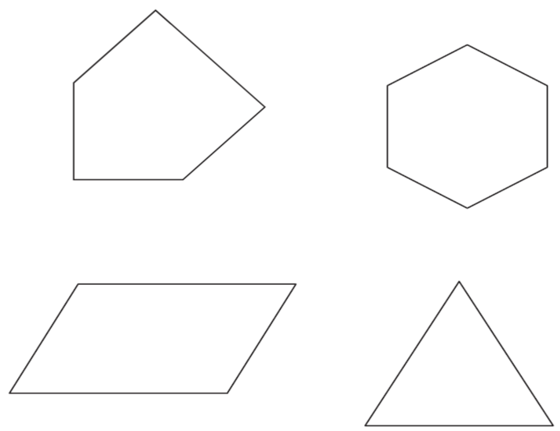
Here we can see that Child 4 uses an iterative process, combining ‘guessing’ with extra calculations to arrive at the answer. She is also attempting the subtraction by adding – to her, the notion of subtraction and addition are easily related**.** The numerical reasoning during this process reveals an ability to perform numerous subtractions to ‘check’ each guess, and she also uses the result from each subtraction to make an ‘informed’ guess, in order to get closer to the correct solution.

From her comments, Child 4 also appears to be familiar with various 2 dimensional and 3 dimensional shapes:

> "I like making pyramids out of rods and balls, and I like making cubes and triangles and squares and I like making very long things with my rods and balls."

Her 'hands-on' approach to geometrical objects is demonstrated in her solution to Question 8:

Two of these shapes have **more than 1** line of symmetry.
Tick (✓) both of them.



Transcript of Child 4's approach:

C: "Let's cut them out and fold them. Parallelograms no good."

C's mother: "Not this one, but a rectangle would be."

C: "But of the shapes we have…

C's mother: "Which ones have two or more lines of symmetry?"

C: "Triangle and hexagon. Three ways for triangle. Lots and lots of ways for hexagon. Lots!"

Draws the four shapes (as originals have been cut out!) and ticks the hexagon and triangle.

Figure 5.8: Q.8, G.1, Child 4, Aged 6

Notice that although the mother mentions that a rectangle would fit the criteria, Child 4 ignores this comment to focus on the problem.

The mother believes that the *'usage of mathematics in everyday life'* is a fundamental reason for her daughter to learn mathematics and encourages her daughter to investigate problems that occurred in 'everyday situations'. In the dialogue below, after the mother describes the dimensions of 'ml', child and parent go off to investigate what a millimetre looks like in 'real-life'.

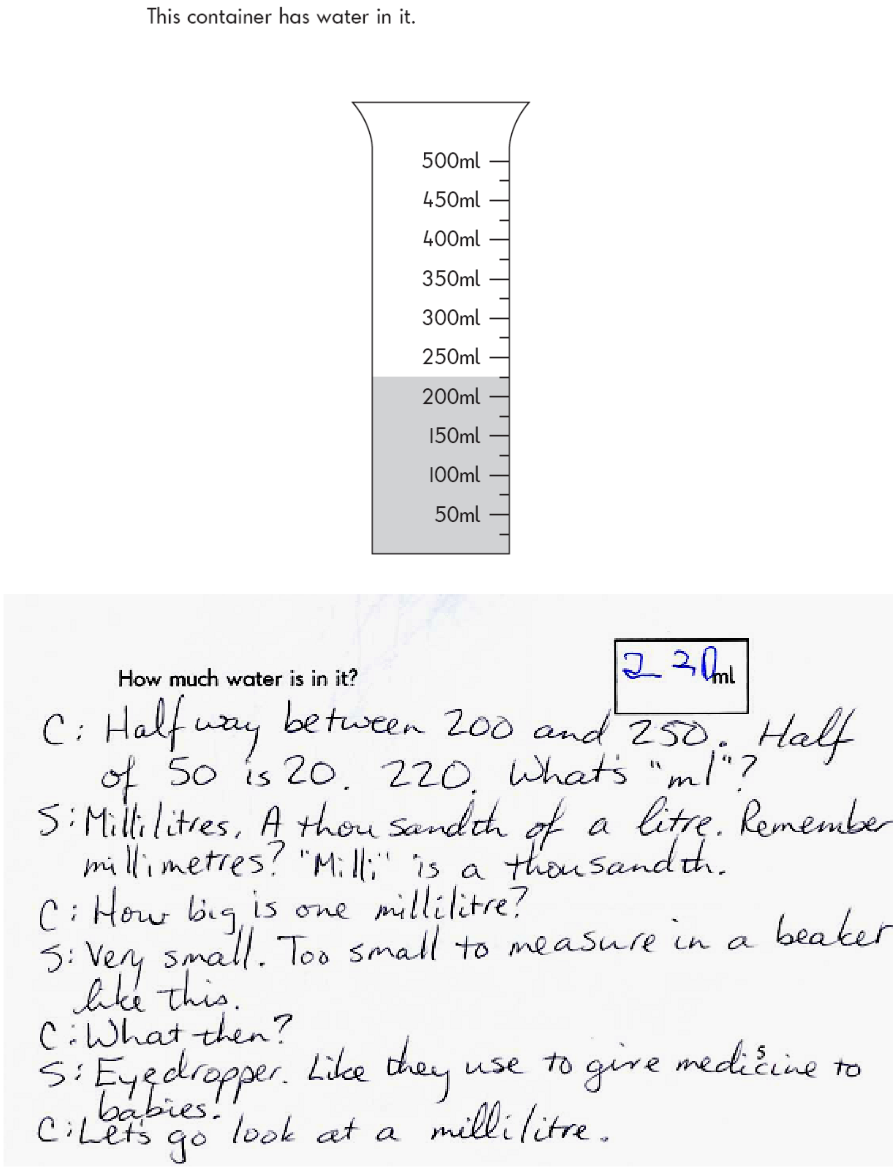


Figure 5.9: Q.5, G.1, Child 4, Aged 6

However, in Question 3 below, we see that Child 4 is concentrating on the 'real-life' aspect of the problem to the extent that it hinders her mathematical reasoning:

The shop sells candles in boxes of **25**

Emma needs **69** candles.
How many boxes of candles does she need to buy?

C: She can't buy boxes of 25 to get 69.
S: It's OK if she buys more. She could have extras[3].
C: Why won't the shopkeeper let her buy the number she needs?
(PTO)

S: Probably it isn't convenient for them.
C: It's wasteful.
S: Maybe. If she can't use the extra candles they might get wasted.
C: She should complain. What's her name?
S: Emma.
C: Emma should complain.
S: Maybe she should. But if ~~the~~ shopkeeper says no, and she has to buy boxes of 25?
C: Two isn't enough.
S: Isn't it?
C: No. 50 isn't enough. She wants 69.
S: How many boxes then?
C: Three.
S: Why three?
C: Because she only needs 19 more. So one more box.
S: Oh, OK.
C: I'll write 3 in the box.

Figure 5.10: Q.3, G.1, Child 4, Aged 6

Notice how the conflict between the 'mathematical solution' and the real-life situation of 'wasted candles' causes confusion, as Child 4 sought a 'real-life' resolution to avoid 'wastage'. It is only after her mother emphasises the fact that the shopkeeper is restricted to selling boxes of 25 that Child 4 is able to solve the problem.

5.10.2.3. The Informal Family — Parental Philosophy and Child's Understanding

From her answers, we see that Child 4 took a number of creative approaches when solving arithmetical questions, never following a procedural strategy. Note how in Q.7 (Figure 5.7), Child 4 attempted the subtraction by adding. For her the sense of addition seems to be so intertwined with subtraction that subtraction appears to be easily related with addition (Gray and Tall, 1994). Her understanding of Shape was evidenced through her responses to the questions detailed in Appendix 9, and demonstrated in her solution to Q.8 where she adopted a hands-on approach to determine the symmetrical properties of the shapes, and also revealed knowledge of the shape 'parallelogram'. She was also very interested in the 'real-life' elements of the problems (e.g. Q.3 and 5) – the 'real-life' aspect of mathematics being a key feature of the informal home-educating style taken by her mother.

We have seen examples of both structured and informal families, and will now consider two examples of semi-formal families, seeing as they were the most represented group amongst the home-educators. We first look at Family 7, whose background was described earlier in Section 5.3.1.

5.10.3.2 Child 7a and Child 7b's Assessed Work (Group 1)

Although Child 7b (aged 6) did not complete the children's questionnaire, she attempted all of the assessed work questions from Group 1, as did her older brother, Child 7a (aged 7). The children's mother mentioned that her teaching approach is adjusted according to each child's personal preference, as her children work in different ways:

> "If we can teach mathematics in a very clear graphic way, or in a hands-on activity it is always better absorbed and understood than just being verbally explained. We find maths manipulatives like Cuisenaire rods, base 10, abacus etc essential and maths games and software excellent for practice."
>
> "My daughter is happy to do written work and enjoys having her old books to look back on, likes rewards such as stickers. My son hates reward systems and avoids written work as he has dyspraxia and dyslexia, so he prefers to work via computer, games, or discussion."
>
> Family 7, children aged 7, 5 and 2

The **difference in the children’s mathematical problem** solving is evident in their assessed work from Group 1, as illustrated in Table 5.6 below. For the majority of questions (6 out of 9), the children have **not chosen the same method of solution**, despite having the same teacher (their mother) and being only a year apart in age.

	Q.1	Q.2	Q.3	Q.4	Q.5	Q.6	Q.7	Q.8	Q.9
7a	Worked strategy	Column procedure	Writes down answer	Correct drawing	Writes down answer	Column procedure	Writes down answer	Writes down answer	Worked strategy
7b	Writes down answer	Worked strategy	Writes down answer	Correct drawing & writes 16	Worked strategy	Worked strategy	Worked strategy	Writes down answer	Worked strategy

Table 5.6: Comparison of Children’s Solution Strategies From Family 7

In Table 5.6, the red ‘worked strategy’ labels describe a solution that was reached through an understanding of the relationship between different mathematical facts/concepts, as can be noted in Child 7b’s approach to Questions 2 and 6 (see Figures 5.11 and 5.12 below). However, for these questions, Child 7a twice uses a **‘column procedure’** (e.g. a vertical decomposition method, highlighted in pink in Table 5.6) - a comparison of the siblings’ different approaches shown below:

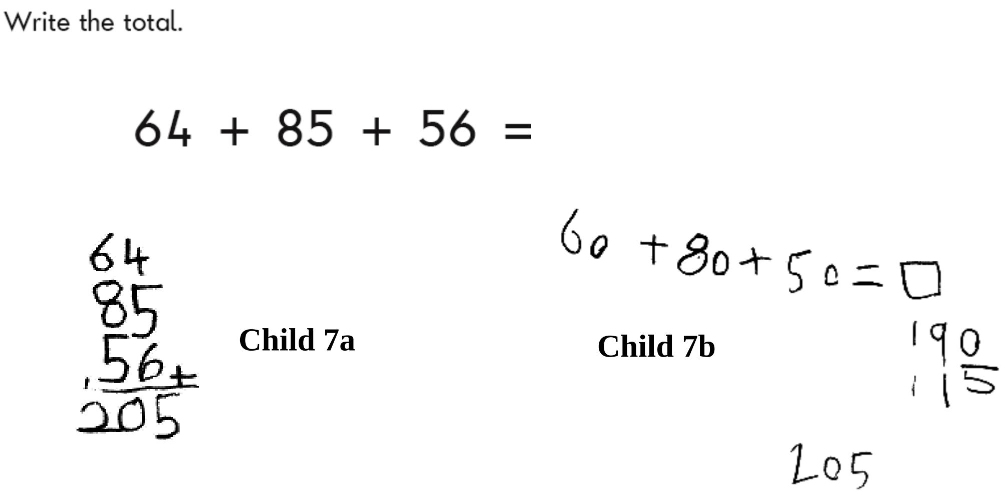


Figure 5.11: Q.2. G.1, Child 7a, Aged 7 **Figure 5.12: Q.2, G.1 Child 7b, Aged 6**

Here, Child 7a writes out the numbers in column format and carries over the ‘one’ to the tens column, whilst his sister demonstrates an understanding of how this process works in her calculations, by first adding the ‘tens’ and then carrying over the one after adding the units. A similar scenario occurs in Question 6 (see Appendix 6), where the siblings use their arithmetical knowledge of addition and multiplication in two quite different ways. Although Child 7a successfully applies a procedural method for addition, adding the number 19 six times, his sister demonstrates a ‘working understanding’ of relationships between multiplication, addition and subtraction (see Figure 5.31) - evidence that she is making connections between her existing knowledge of multiplication and addition, and perhaps demonstrating a *relational understanding of arithmetic*. This could also suggest that her problem-solving approach is influenced by her mother’s teaching, as according to the mother, ‘making connections’ is indication of mathematical understanding. Further evidence of Child 7b’s mathematical knowledge can be seen in Question 7, where she demonstrates an understanding of complementary addition (i.e. adding numbers to 143 in order to reach 205):

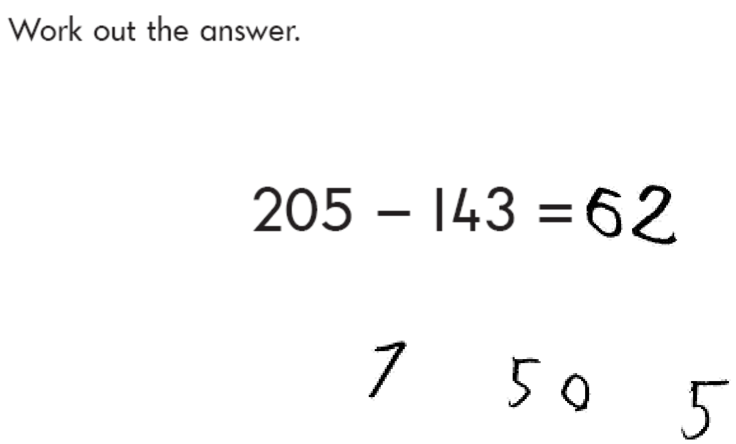


Figure 5.13: Q.7, G.1, Child 7b, Aged 6

One can hypothesise that she has used the following strategy to reach her solution:

(1) 143 + 7=150
(2) 150 + 50=200
(3) 200 + 5=205

She then totalled the numbers in red to arrive at the answer of 62.

Both children from Family 7 answered **all of the Group 1 questions correctly**, apart from Question 8, where they made identical mistakes, incorrectly identifying the parallelogram as having more than one line of symmetry, when in fact it has none:

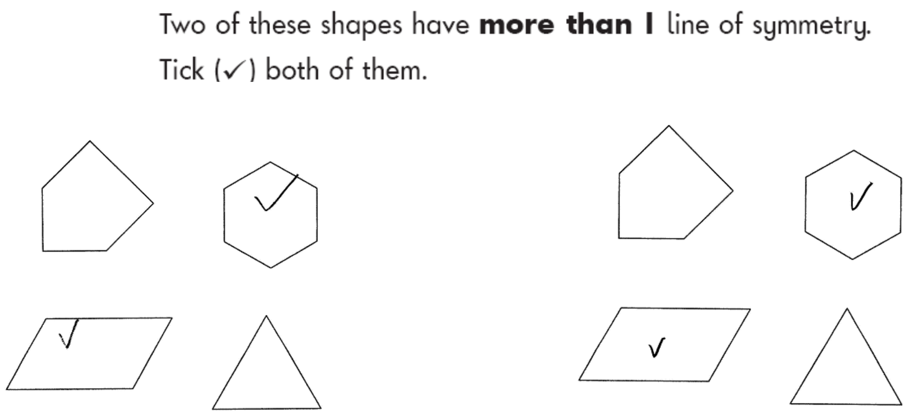


Figure 5.14: Q.8, G.1, Child 7a, Aged 7 and Child 7b, Aged 6

5.10.3.3 The Semi-Formal Family – Parental Philosophy and Child's Understanding

It was interesting to note that although both sets of siblings from Families 7 and 11 had the same teacher (i.e. their mother), siblings from within the same family often took quite different problem solving approaches for the same group of questions. This was particularly evident in Family 7, where the younger sibling, Child 7b, often took a quicker, less procedural approach to her brother when solving arithmetic problems. Perhaps Child 7a, who at seven years of age, was a year older than his sister, was beginning to follow a more formal learning approach with regards to arithmetic, where he had obtained knowledge of standard procedures. Furthermore, their mother mentioned that Child 7a had dyslexia, which may have influenced his problem solving approach – perhaps he preferred adopting ‘standard’ methods of solutions rather than those that could require more writing.

Our final case study is of another semi-formal family, where in this instance the mother holds a mathematics degree.

5.10.4 Parental Philosophy and Child’s Understanding—A Second Semi-Formal Family

Family 16 had had been home-educating their two children (aged eight and six) for the past two years. Their son was bullied at school, and it was felt that this was due to his mixed-race background. The parents also stated that their son was ‘academically ahead’ and his school could not cater for his individual needs. Indeed her comments on the case study of Joe (Appendix 10) show that she feels children with ‘above average’ mathematical abilities should be given challenging work in order to maintain an interest in the subject:

> “Boredom must not be allowed to set in. Joe needs challenges...”
>
> “This will help widen his scope and offer greater challenges for him.”

The mother held a degree in mathematics and had worked with computers as a project manager, as did her husband. She believes mathematics can be found everywhere, and that an advanced (university level) knowledge of mathematics was an asset for future employment. In addition, the parent believed that ‘being good at mathematics generally gives children confidence’. Her experiences of mathematics were cited as an advantage when teaching:

> “It is a great advantage having a home-educating parent who has a mathematics background. I can imagine it is not easy to impart such knowledge to young children without some adequate

standards in mathematics. For instance, I wanted Tom to be able to do more challenging problems than KS2 level, but he needed to know his tables well.

So having mastered his 2,5 and 10 times tables, if he was asked to multiply 9 x 8, he was taught to get to the easiest table and add/subtract to get the answer, e.g. 10 x 8 = 80, 80 – 8 = 72. He would also realise the numbers in the 9 x table add up to 9 (7 + 2). Just little fun tricks children appreciate."

She also noted that at home, the children could make significant use of Internet resources compared to school, and did not feel that there were any disadvantages in their home-educating approach.

The family made extensive use of board games (e.g. Monopoly) to encourage the children to appreciate money, counting and arithmetic skills, and they also tried to relate to mathematical examples throughout the day; for example, when cooking. The parent was open to changing the teaching approach if the children did not appear to understand the current concept; in fact, the parents often discussed alternative strategies for teaching between themselves. The children were taught for one hour every day, and their understanding was measured through an application of the concept to a number of examples, to ensure that their child 'felt comfortable' with the concept. Each concept was covered regularly to aid memorisation.

5.10.4.1 Influence of Parental Approach on Child's Perceptions of Mathematics

Child 16 (aged 8) appeared to enjoy learning mathematics at home but mentioned that he 'hated' making mistakes. In the context of learning activities, his focus was on the **everyday applications** of mathematics, and indeed, his parents tried to **relate to mathematical examples throughout their daily activities**. As an example of a mathematics problem, Child 16 gave a **real-life example**:

"If 5 mugs cost £25.00 how much will 6 mugs cost?

1 mug will cost 25/5 = £5.00

Therefore 6 mugs will cost 6 x 5 = £30.00"

Textbooks, understanding, parental influence and interest occasionally guided his learning activities, thereby demonstrating the 'semi-formal' approach to mathematics, where a wide range of learning resources is used alongside parental guidance. An example of this approach is illustrated through Child 16's current learning activity, where he learned problem solving through questions devised by his mother:

> "Mummy gives me problems using the characters in the Lord of the Rings which are my favourite books."

His mother strongly emphasised the relevance of mathematics to the workplace as well as other scientific careers, mentioning how her personal mathematical abilities were a significant asset in her later career. This may have influenced her son's mathematical goals, where he aimed to 'get the highest grades possible to study sciences at Cambridge University'. The mother's belief that **'mathematics helps children think logically'** appeared to have an effect on Child 16's belief that mathematics:

> "... makes your brain work properly and logically." Child 16 (aged 8)

Like his mother, Child 16 also believed mathematics is **enjoyable**, **interesting** and **useful** both in everyday life and other subjects. However, he also commented that 'most people do not like mathematics'. Interestingly, despite the family's focus on the 'everyday life' learning of mathematics, Child 16 believes *'a mathematics problem is an exercise when you decide which operations to be done and then perform them correctly'*. He feels that he is **good at mathematics**, and that those who are good at mathematics can perform mental calculations quickly. It was also noticed that Child 16 'hates getting wrong answers, and is determined to get correct answers'. Perhaps then, his notion of a mathematics problem is influenced by the need to find 'correct answers'.

However, he notes that a mathematics problem can have more than one correct answer and also feels that it is 'worse to choose the wrong operation or method than to make a calculation error'. As we will observe later, Child 16 is familiar with algebra, yet his questionnaire responses indicate a belief that 'all mathematics problems must have numbers'. Child 16's notions of mathematical understanding are reflective of his parents' teaching approach with nearly every 'sign of understanding' given an equal level of importance. This is also noticeable in his priorities when *doing mathematics* (Q.9, p.3 of Children's Questionnaire, Appendix 2), as **all aims are given equal priority**, apart from 'finishing the work quickly', which he gives the least importance. We now consider his solutions to the assessed work problem, where he attempted questions for all three groups.

5.10.4.2 Child 16's Assessed Work (Group 1, 2 and 3)

Child 16 (aged 8) answered **all of the questions from Group 1 correctly**, a summary of his approaches shown below:

Q.1	**Q.2**	**Q.3**	**Q.4**	**Q.5**	**Q.6**	**Q.7**	**Q.8**	**Q.9**
Column procedure	Column procedure/ Worked strategy	Worked strategy	Worked strategy	**Writes down answer**	Column procedure/ Worked strategy	Column procedure	**Writes down answer**	Worked strategy using algebra

Table 5.7: Summary of Child 16's Solution Strategies

Table 5.7 shows that Child 16 adopted a number of different arithmetical skills. For example, his solution to Question 2 shows use of the column layout for addition, but he also 'breaks down' the addition into tens and units as an alternative approach. Note that his mother encourages her children to appreciate the range of '**different' strategies for arithmetic problems**, so conceivably Child 16's two answers for the question (Figure 5.15) are influenced by this teaching aim.

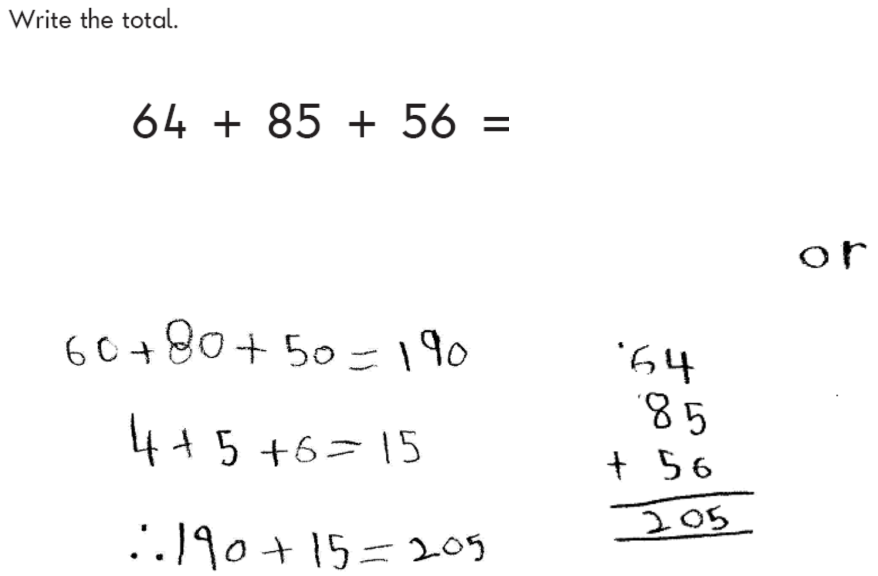


Figure 5.15: Q.2, G.1, Child 16, Aged 8

For Question 3, he reasons that dividing 69 by 25 would provide a 'mathematically' correct solution (Figure 5.16), but taking the real-life situation into consideration he notices that it is impossible to have 2.76 boxes and concludes that 3 boxes are needed.

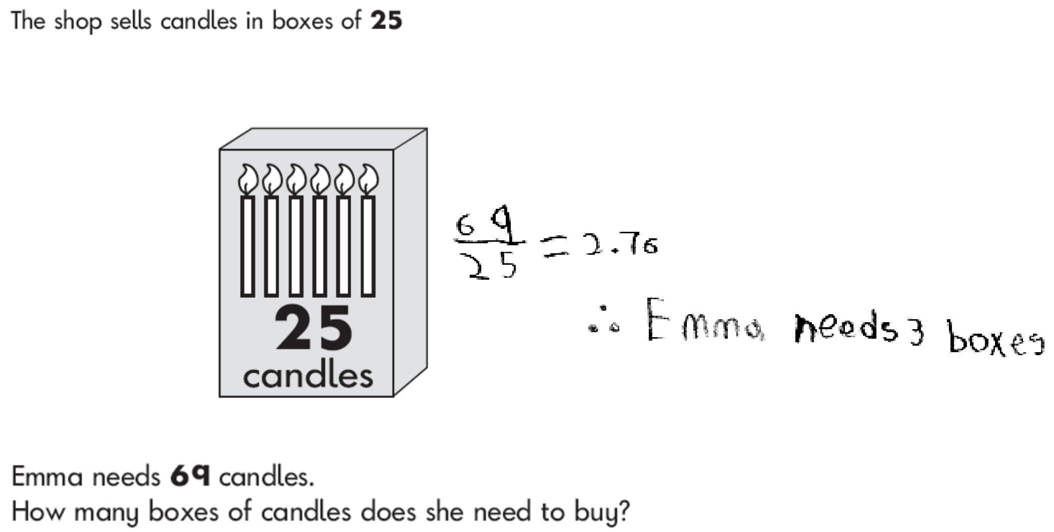


Figure 5.16: Q.3, G.1, Child 16, Aged 8

With regards to the questions from Group 2 (see Appendix 7), Child 16 only made an error in one question (Q.1), where he appeared to forget to write the final answer down. However, although his work showed a high level of calculation accuracy, the solution strategies were not always the quickest, as can be seen in Question 2 (Figure 5.17), below:

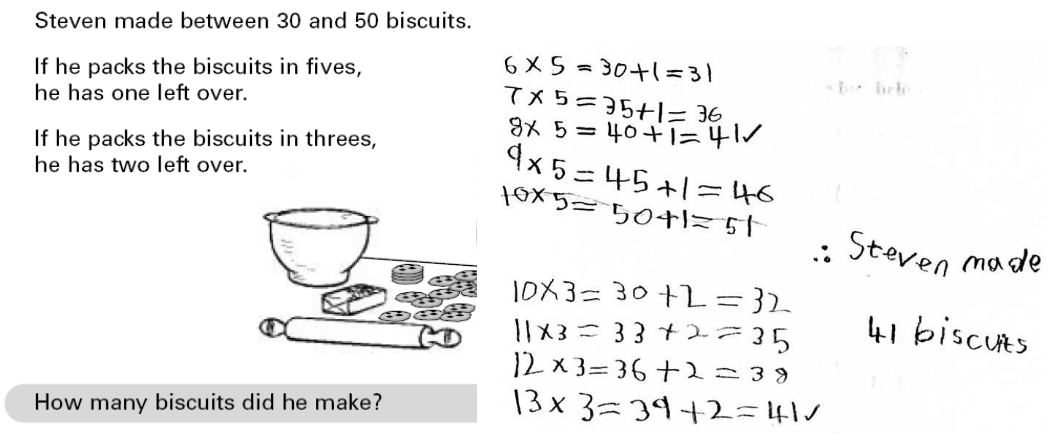


Figure 5.17: Q.2, G.2, Child 16, Aged 8

Notice that his work is **very systematic**, perhaps indicative of Child 16's strong desire to **obtain correct answers** - indeed, he wrote that he 'hates it' when he gets questions wrong.

For Question 4 (see Appendix 7), Child 16 demonstrates his knowledge of fraction to decimal conversion (see Figure 5.36 later on) and in Question 7 he establishes the fraction of the diagram by correctly interpreting the pictorial information (Figure 5.16):

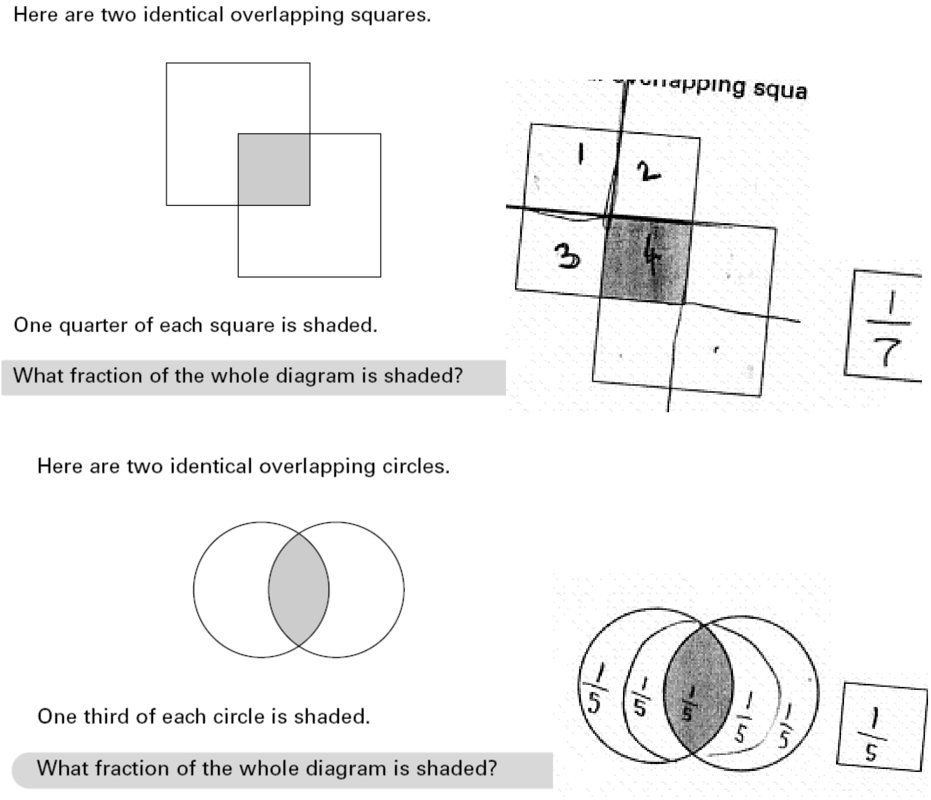


Figure 5.18: Q.7, G.2, Child 16, Aged 8

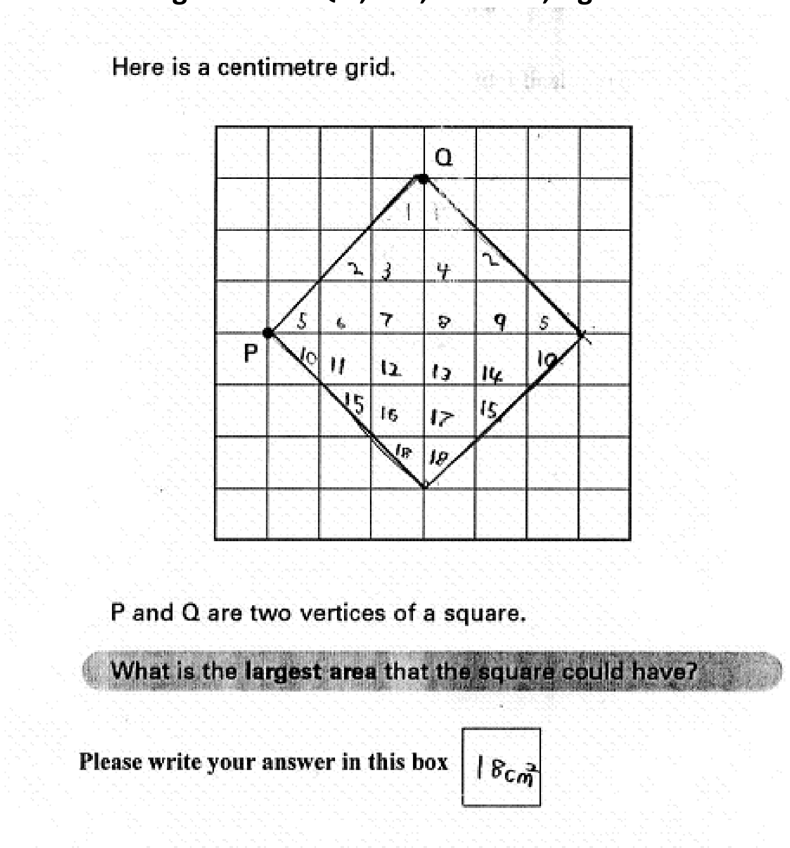


Figure 5.19: Q.5, G.2, Child 16, Aged 8

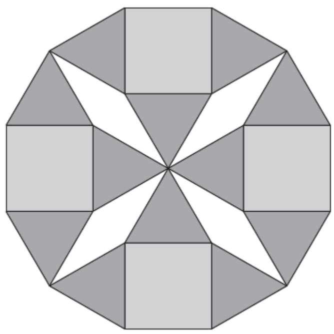
Child 16's use of visual information is also observed in Question 5 (and Question 3, Group 3, see Appendix 8), where he has labelled each area covered by the square in order to calculate the largest possible area of the required square (Figure 5.19).

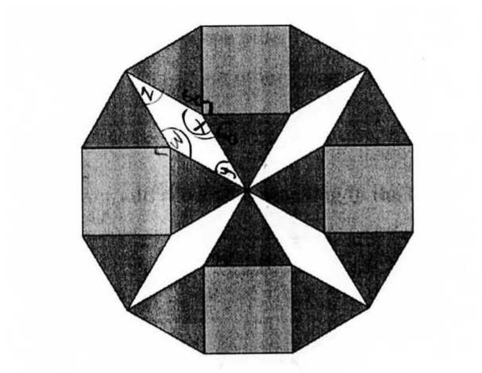
Child 16's answers to the Group 2 questions showed an in-depth understanding of fractions and area. Whilst some of his problem-solving strategies were occasionally laborious for a child who appeared to have a good understanding of arithmetic, he obtained the correct answers to practically all of the questions. We next examine Child 16's answers to the Group 3 questions, which were aimed at Key Stage 3 students, for Levels 3-8. He was able to correctly answer the first four 'calculation' type questions, but like the other children, was unable to fully justify his answer for Question 5.

I have some tiles that are squares and some tiles that are equilateral triangles.

The side lengths of the tiles are all the same.

I arrange the tiles like this.





I want to fill the gaps by making four tiles that are rhombuses.

What should the angles in each rhombus be?

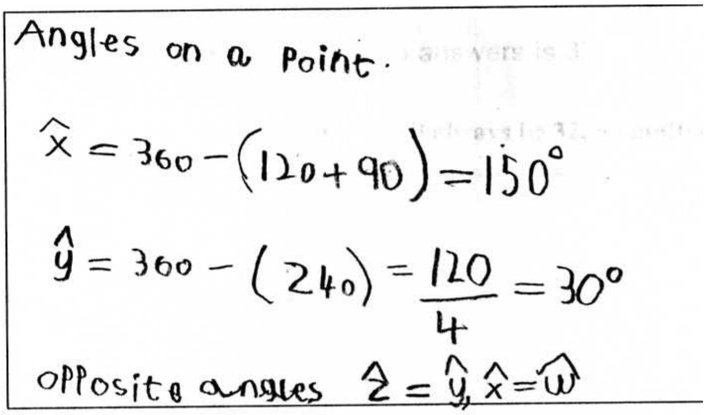


Figure 5.20: Q.7, G.3, Child 16, Aged 8

Child 16’s familiarity with formal mathematical notation is evident in Question 7, where he uses the ‘standard mathematical’ notation to label the angles of the rhombus, and lays out his argument in an organised way. As his mother is a mathematician, it appears that not only has she taught him the mathematical concepts, he has also been taught the **‘formal’ aspects of mathematical notation and argument**. Question 9, Group 1 (see Figure 5.21) and Question 8, Group 2 (see Figure 5.22) demonstrate Child 16’s understanding of algebraic concepts, where one can observe knowledge of simultaneous equations and finding ‘unknowns’:

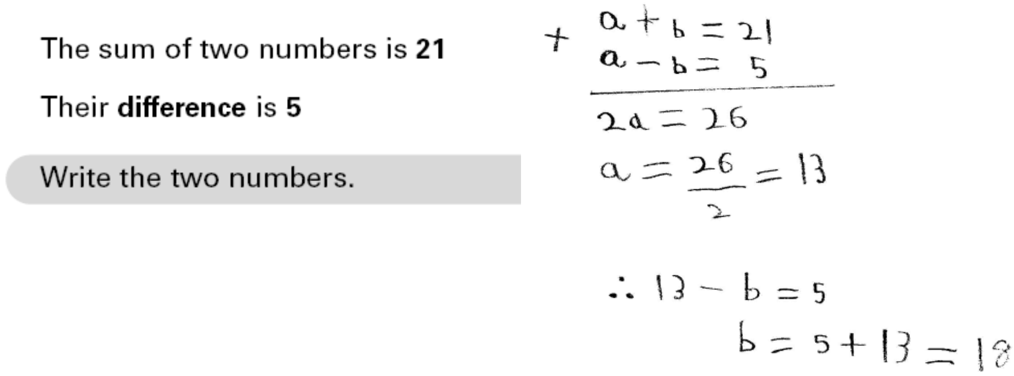


Figure 5.21: Q.9, G.1, Child 16, Aged 8

In Question 8, Group 3, for the first part, he begins by defining *x* as the ‘unknown’ and successfully sets up two algebraic equations to solve for *x*.

I think of a number, then I carry out these operations on my number.

Multiply by 5 **Add 8**

When I carry out the operations in one order the answer is **105**

When I carry out the operations in the **other order** the answer is **73**

What is my number?

Let x = number

$(x+8)5 = 105$

$5x+8=73$

$x = \frac{73-8}{5} = 13$

$x+8 = \frac{105}{5} = 21$

$\therefore x = 21-8=13$

Figure 5.22: Q.8, G.3, Child 16, Aged 8

However, he does not appear to be able to begin the second part, which requires a proof that ‘the answers will always be 32, no matter what the number’. Thus although Child 16 shows an understanding of algebra when solving for particular values of *x*, he does not yet appear to be able to generalise the result. At the same time, for Question 9, Group 3 (Figure 5.23), he is able to use algebra to correctly prove the fraction of the shaded shape is a third.

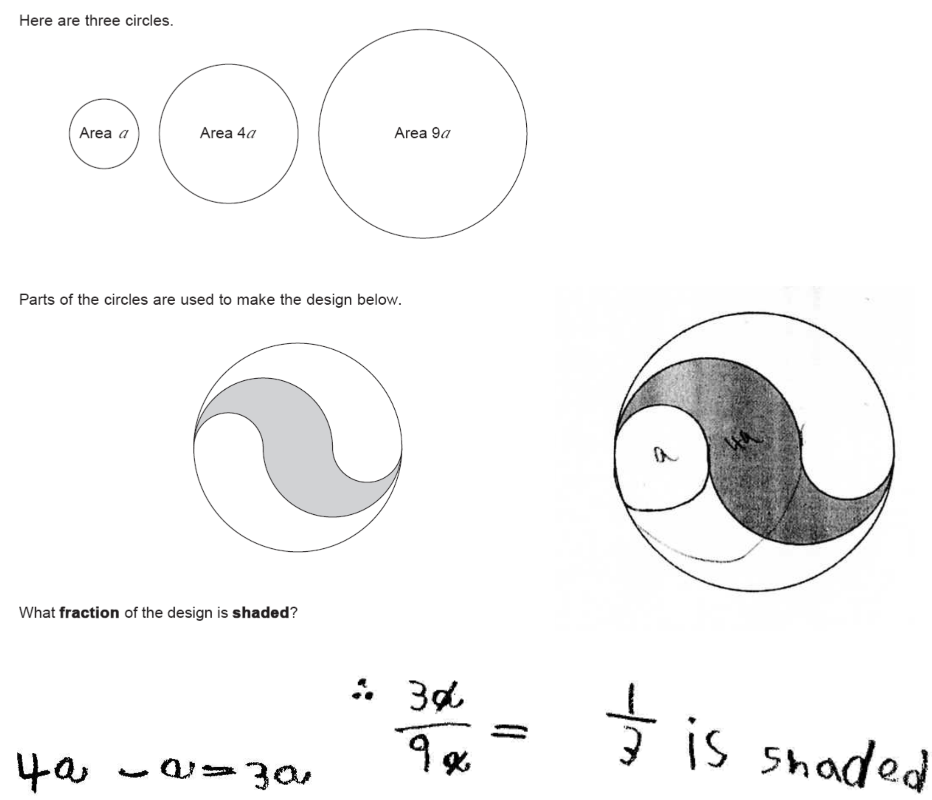


Figure 5.23: Q.9, G.3, Child 16, Aged 8

These findings indicate that Child 16 is able to construct simple proofs when there is a ***visual image*** to help him 'begin' his solution, as was noticed in a number of questions where Child 16 **utilised the pictures** to help calculate area, or set up algebraic expressions. But when no visual image was available (e.g. Question 8, Group 3), he was sometimes unable to proceed and at other times, his calculations were somewhat time-consuming. We can thus hypothesise that Child 16 could be a **'visual' learner**. At the same time his mother has facilitated Child 16's development of a level mathematical understanding that is very advanced for an eight year old, in both the **range of topics** understood (e.g. algebra, shape, simple proofs, trigonometry) and in his **'formal' mathematical** reasoning.

5.10.5 Comparison of Understanding Observed from the Three Different Home-Educating Approaches

As was noted earlier, both the siblings from the structured family (Family 11) were confident with arithmetic type problems, and appeared to possess an instrumental understanding of these areas. Children from the informal/semi-formal families (4, 7 and 16) also demonstrated an aptitude for arithmetical problems, however, for certain questions, they also displayed a level of relational understanding within their solution strategies. In particular, the two youngest children, Child 4 and Child 7b (both aged 6) used a number of creative approaches when solving arithmetical problems – they never applied a formal procedural method. Note how Child 4, from an informal family, was drawn into the real-life aspect of the problems at times, perhaps due to influence of the ‘learning through everyday life’ approach of such families.

For Child 11b (aged 15), there was evidence of algebraic reasoning and an understanding of proof. Child 16 (aged 8) also appeared confident with algebraic strategies and his solutions for the questions from Groups 2 and 3 demonstrated the influence of his mother’s mathematical background.

Now we have had a look at how the various home-educating approaches could affect the children’s problem solving strategies, we examine *all the assessed work answers* in order investigate the main relationships between the learning environment and the children’s mathematical understanding. We first begin with an analysis of the Group 1 questions, with the questions grouped according to concept type, e.g. Arithmetic.

5.11 Analysis of Group 1 Questions

The Group 1 questions were attempted by eight children, aged from six to nine years. Table 5.8 shows that the average mark across all questions was 86%, with Children 9b, 16 and 26 getting every question correct. These children were all also slightly 'older' (see Table 5.8), suggesting that age may be a factor with regards to accuracy of solution. However, as the findings below will demonstrate, the age of the child did not necessarily imply that their method of solution was quicker.

Group 1											
Child	**Age**	**Q.1**	**Q.2**	**Q.3**	**Q.4**	**Q.5**	**Q.6**	**Q.7**	**Q.8**	**Q.9**	**Mark**
4	6	0	1	1	0	0	1	1	1	0	**56%**
5	8	1	1	1	1	1	0	1	1	1	**89%**
7a	7	1	1	1	1	1	1	1	0	1	**89%**
7b	6	1	1	1	1	1	1	1	0	1	**89%**
9b	8	1	1	1	1	1	1	1	1	1	**100%**
16	8	1	1	1	1	1	1	1	1	1	**100%**
22	5	1	1	1	1	0	0.5	0.5	1	0	**67%**
26	9	1	1	1	1	1	1	1	1	1	**100%**
Mean	**7.1 years**	**87.5%**	**100%**	**100%**	**88%**	**75%**	**81.3%**	**93.8%**	**75%**	**75%**	**86%**

Table 5.8: Marks for the Group 1 Questions

As well as considering the particular solution strategies used for each type of question (e.g. 'Shape'), the following analysis will also seek to determine if there is evidence that the home-educating background of the child plays a role in the children's choice of solution strategy. Table 5.9 provides an indication of how each of the children responded to the Group 1 questions. Subdivided into four sections: Basic Arithmetic, Shape, Arithmetical Word Problems, and problems associated with Real Life Situations, the table indicates children who obtained correct or incorrect solutions for each problem. Where a child did not attempt the problem this is also indicated. Each child is also identified in accordance with the general teaching philosophy of the parents. Note that no children from structured families attempted the Group 1 questions, perhaps suggesting that this approach is not commonly used for younger home-educated children.

Basic Arithmetic		**Correct**	**Wrong**	**Did Not Attempt**
Q1	Three Digit Subtraction	**5, 7a, 7b, 16, 22, 9b, 26**	**4**	
Q2	Two Digit Addition	**5, 7a, 7b, 16, 22, 4, 9b, 26**		
Q7	Three Digit Subtraction	**5, 7a, 7b, 16, 4, 9b, 26**	**22**	
Shape				
Q4	Transposition of Perimeter	**5, 7a, 7b, 16, 22, 9b, 26**	**4**	
Q8	Symmetrical Properties	**5, 16, 22, 4, 9b, 26**	**7a, 7b**	
Arithmetical Word Problems				
Q6	Establishing multiple cost	**7a, 7b, 16, 4, 9b, 26**	**5, 22**	
Q9	Finding numbers	**5, 7a, 7b, 16, 9b, 26**		**4, 22**
Real Life Situations				
Q3	Calculating Cost	**5, 7a, 7b, 16, 22, 4, 9b, 26**		
Q5	Reading measuring jug	**5, 7a, 7b, 16, 9b, 26**	**4**	**22**

Children highlighted in green were from informal families

Children highlighted in blue were from semi-formal families

Table 5.9: Children's Results for the Group 1 Questions, by Area of Mathematics

5.11.1 Arithmetic Questions, Group 1

Questions 1, 2 and 7 all involved straightforward arithmetical operations. Overall, only two mistakes were made – as Table 5.9 shows, the majority of the children appeared very comfortable with this type of mathematics problem. In general, three different solution strategies were evident, as was observed in Question 1. For this question, three out of the eight children (5, 7b and 26) just wrote down the answer – Child 5's answer illustrative of this approach:

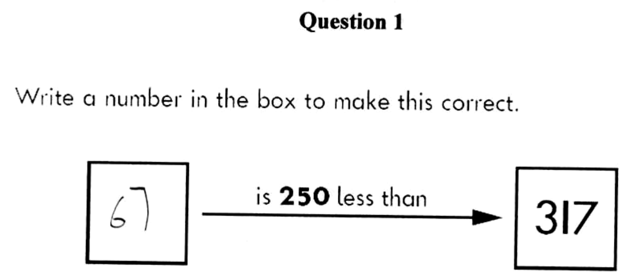


Figure 5.24: Q.1, G.1, Child 5, Aged 8

Children 16 and 22 both wrote down their working in a ‘column format’, and ‘take away’ from the 3 hundreds:

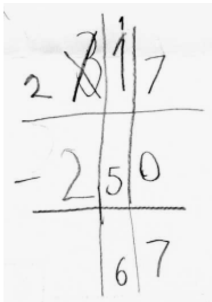


Figure 5.25: Q.1 G.1, Child 22, Aged 5

On the other hand, Children 4, 7a and 9b used their understanding of ‘less than’ to subtract 250 from 317, as illustrated by Child 4’s approach (Figure 5.26). Although she does not get the correct answer, Child 4 realises that she can go back to 317 by adding a number to 250, as this is equivalent to the statement, *“What number is 250 less than 317?”:*

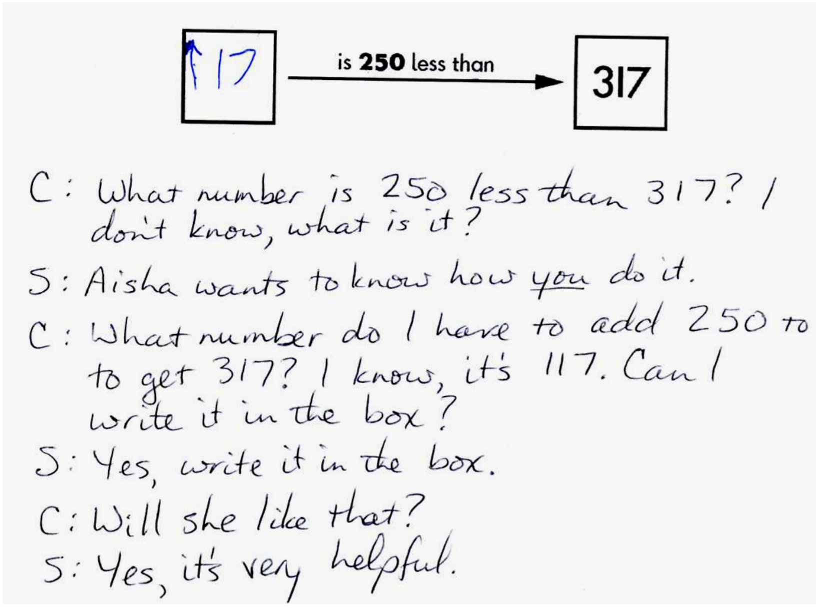


Figure 5.26: Q.1, G.1, Child 4, Aged 5

For Question 2, all solutions were obtained via a worked strategy or a 'column procedure'. Children 5, 7a, 16, 22 and 26 used a column format, with carrying over.

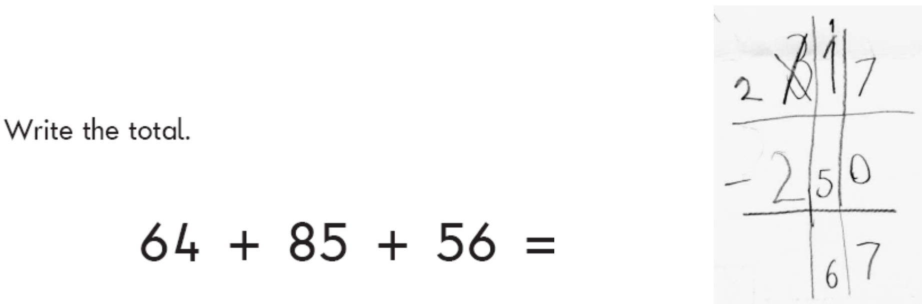


Figure 5.27: Q.2, G.1, Child 22, Aged 5

In fact, it was noticed that Child 22 adopted a 'column procedure' for **all questions** involving arithmetic. On the other hand, Children 9b and 16's strategies demonstrated use of partitioning, where they undertook separate calculations for the tens and units, before summing the results at the end:

$60+80+50=190$

$4+5+6=15$

$\therefore 190+15=205$

Figure 5.28: Q.2, G.1, Child 16, Aged 8

5.11.2 Shape Questions, Group 1

Questions 4 and 8 involved an understanding of ‘shape’. Question 4, which required knowledge of ‘perimeter’, was correctly answered by all children, except Child 4. Three out of the eight children (5, 7a, and 26) just wrote down the answer without any working. On the other hand, Child 22 wrote the number 16 next to the square, and then used trial and error.

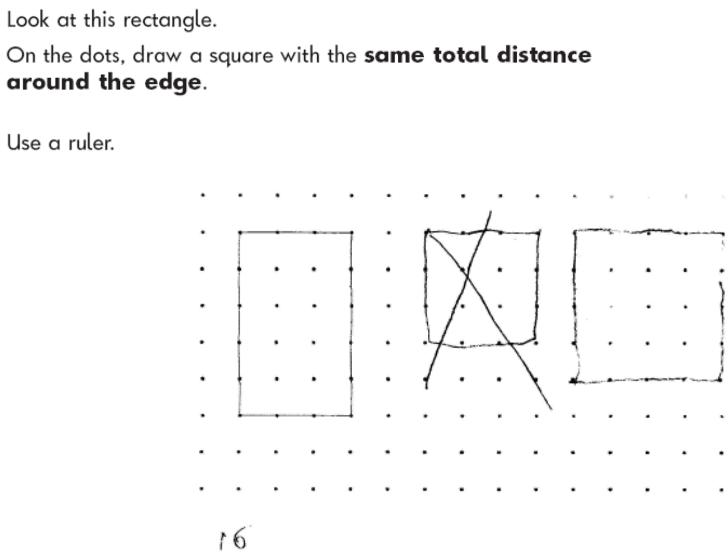


Figure 5.29: Q.4, G.1, Child 22, Aged 5

Child 9b takes an unusual approach to this problem, as the parent writes: *“He went off to look at the bath floor – it’s those stick on mosaic tiles and said he knew from that!”* Perhaps there is a similar pattern of squares/rectangles on his bathroom floor!

Question 8 required knowledge of reflective symmetry and an understanding of the condition ‘more than one line of symmetry’. Children 9b, 22 and 26 correctly chose both the hexagon and triangle without any working, with the parent of Child 22 writing: “*F did this really quickly just by looking at the shapes”*. But Children 7a and 7b both correctly identified the hexagon but incorrectly chose the parallelogram. Children 5 and 16 draw lines of symmetry, whilst Child 4 actually cut out the shapes in order to visualise the lines of symmetry, and then eliminated the shapes that ‘didn’t work’!

Two of these shapes have **more than 1** line of symmetry.
Tick (✓) both of them.

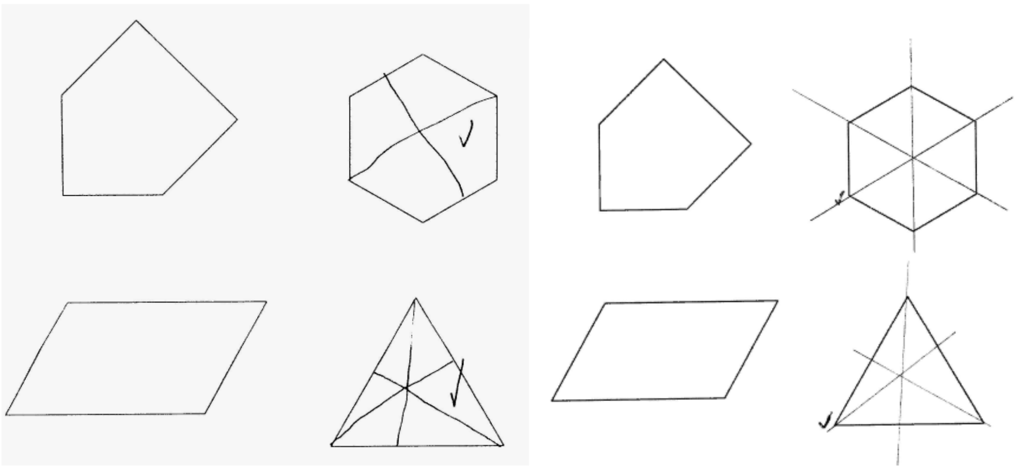


Figure 5.30: Q.8, G.1, Child 5, Aged 8 and Child 16, Aged 8

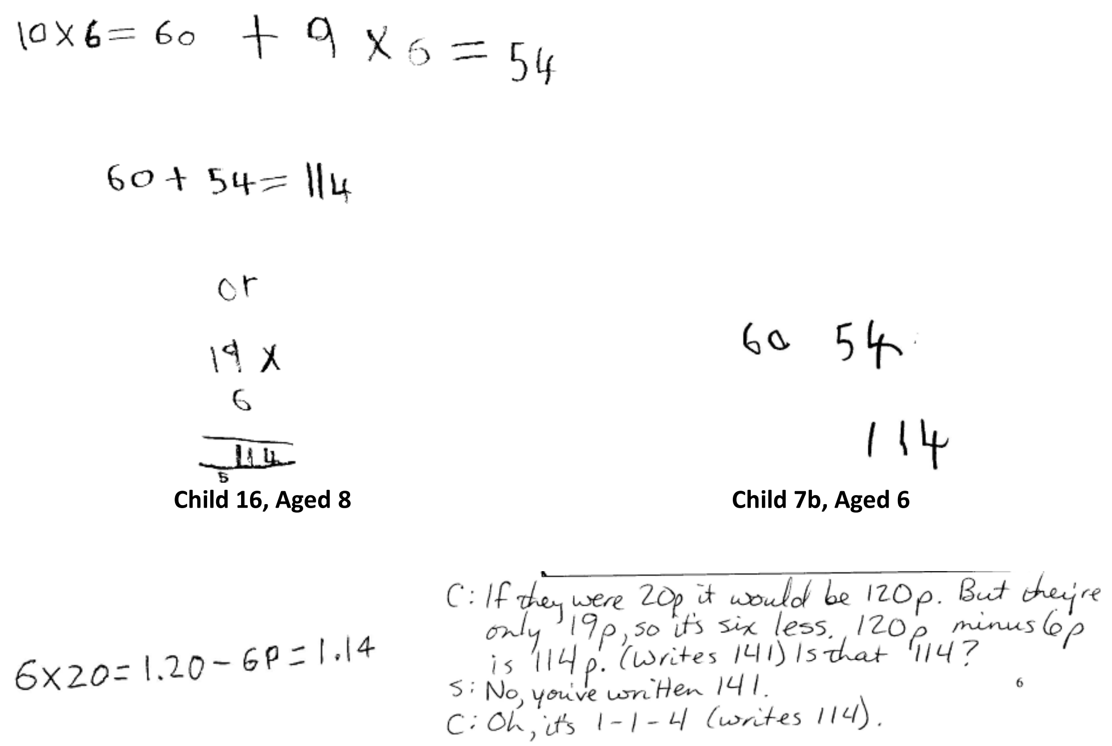
5.11.3 Arithmetical Word Problems, Group 1

For Question 6, both procedural answers and worked strategies were evident:

Manjit buys **6** stamps.
Each stamp costs **19p**.

How much does he pay?

Show how you work it out in the box.



Child 16, Aged 8 **Child 7b, Aged 6**

Child 26, Aged 9 **Child 4, Aged 6**

Figure 5.31: Range of Strategies Used to Answer Question 6, Group 1

Figure 5.31 shows that only Child 16 demonstrates the more formal procedure of multiplication. Children 7b and 26 use a strategy that is based on rounding to 20, multiplying by 6 and then subtracting 6, with Child 4's answer demonstrating the thought processes behind her answer. Question 9, which was in fact the only question within Group 1 set at the KS2 level, proved too difficult for Children 4 and 22, the youngest children within this study. On the other hand, Children 5 and 26 were able to just write down the answer.

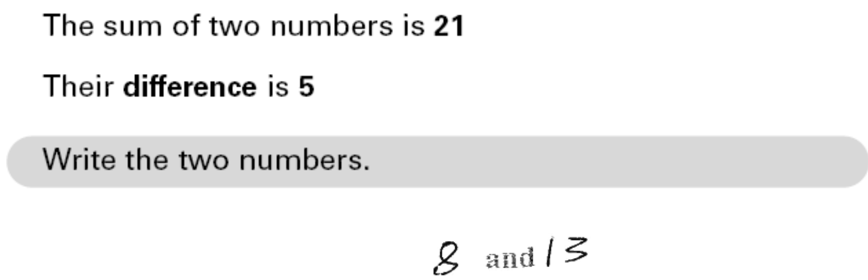


Figure 5.32: Q.9, G.1, Child 26, Aged 9

The most common approach however (used by Children 9b, 7a and b) was to list pairs of numbers that sum to make 21 and then consider the difference, as illustrated by Child 9b's (aged 8) strategy:

Parent writes*: "He was pretty orderly on this*".
1. Started 10 and 11 and said, "Didn't work".
2. 9 and 12 - didn't work
3. Missed the 8 and 13
4. 7 and 14 – didn't work
5. 6 and 15 – didn't work
Realised he had missed one and got 8 and 13

One participant, Child 16, takes a unique approach to this question, by using algebra and solving two simultaneous equations (as detailed earlier in Section 5.10.4.2, Figure 5.21).

5.11.4 Real Life Situations, Group 1

Questions 3 and 5 both involved a 'real-life' element. For Question 3, three out of the eight children (7a, 7b, and 26) were able to write down the answer without any working.

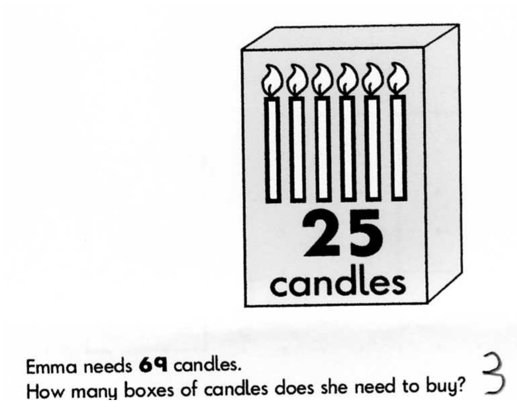


Figure 5.33: Q.3, G.1, Child 7a, Aged 7

Only Child 22 used a column procedure:

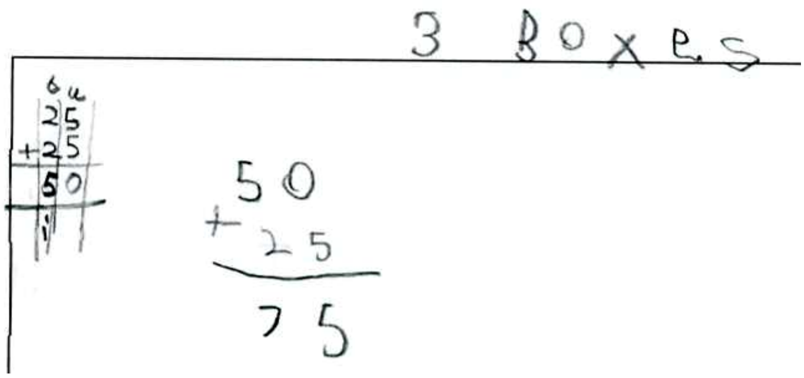


Figure 5.34: Q.3, G.1, Child 22, Aged 5

The mother writes *"We've not really looked at multiplication much yet – it took a while for her to think how to do this"*, and Figure 5.34 demonstrates how Child 22 had to employ her knowledge of addition to reach the correct multiple of 25. On the other hand, Children 5 and 9b are able to list multiples of 25 until they reach a number greater (or equal to) 69, where Child 9b then includes the additional information that there are 6 candles to spare. Child 4 was caught up in the real-life element of the situation, where a conflict between the mathematical solution and the 'reality' of having to buy 6 unnecessary candles hindered her progress (see Section 5.10.2.2, Figure 5.10).

For Question 5 (Figure 5.35), four out of the eight children (5, 7a, 16 and 26) just wrote down the answer. Child 9b realises that half of 50 is 25 and thus the answer is 225 – with Child 4 taking a similar approach, but again getting caught up in the real-life aspect of the problem, where she is very curious to discover what a 'ml' represents (see Section 5.10.2.2, Figure 5.9).

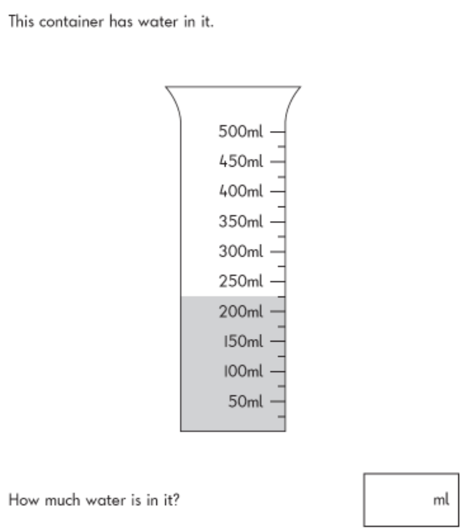


Figure 5.35: Q.5, G.1

5.11.5 Summary of Group 1 Questions

Drawing upon the basic characteristics of Table 5.9, Table 5.10 illustrates the methods used by each child to solve the Group 1 questions.

Basic Arithmetic		**Worked strategy**	**Procedure**	**Just wrote down answer**
Q1	Three Digit Subtraction	7a, 4, 9b	16, 22	7b, 5, 26
Q2	Two Digit Addition	7b, 16, 4, 9b	5, 7a, 16, 22, 26	
Q7	Three Digit Subtraction	5, 7b, 4, 9b	16, 22, 26	7a
Shape				
Q4	Transposition of Perimeter	7b, 16, 22, 4, 9b (used an analogy from 'real-life')		5, 7a, 26
Q8	Symmetrical Properties	4		5, 7a, 7b, 16, 22, 9b, 26
Arithmetical Word Problems				
Q6	Establishing multiple cost	5, 7b, 16, 4, 9b, 26	7a, 16, 22	
Q9	Finding	7a, 7b, 16, 9b		5, 26

Children highlighted in green were from informal families

Children highlighted in blue were from semi-formal families

	numbers			
Real Life Situations				
Q3	Calculating Cost	**5, 16, 4 (also initiated real-life discussion with parent), 9b**	**22**	**7a, 7b, 26**
Q5	Reading measuring jug	**7b, 4 (also initiated real-life discussion with parent), 9b**		**5, 7a, 16, 26**

Table 5.10: Method of Solution for Group 1 Questions

Notice from Table 5.10 that children from informal families were generally less likely to use 'procedural' answers to their questions than those from semi-formal families – in fact, Child 26 was the only 'informal child' to apply this approach when solving Q.1 and 7. Furthermore, Table 5.10 shows that Children 4 and 9b, both from informal families, also considered the real-life context of the problems during their attempts to reach a solution.

We can also see that Child 26 was able to 'write down the answer' for 6 out of the 8 questions without the need for working – conversely, Child 16 appeared to justify his results as much as possible, even if the additional working was somewhat unnecessary. We now summarise the particular methods used for each type of question:

- **Arithmetic and Arithmetic Word Problems –** The home-educated children appeared relatively confident with this type of problem, with only 6 mistakes made out of a total of 40 questions across all eight children. This also was verified by the fact that apart from the two youngest children (Children 4 and 22) all were able to correctly answer the KS2 level question, Question 9. From

Table 5.10 a variety of approaches is evident – only Child 22 used the same method for each question, i.e. a procedural 'column-format'.

- **Shape –** Apart from Child 4, all of the children were able to answer the question on perimeter (Q.4) correctly. Only the children from Family 7 answered the question on symmetry incorrectly – and every child, except Child 4, wrote the answer down without the need for working.

- **Real-Life –** None of the children had any difficulty calculating the cost when given a 'shopping' situation in Q.3 – although it was observed that Child 4, from an informal family, tended to dwell on the real-life context of the problem. Her interest in the context and 'meaning' of the situation was again evident in Q.5, where she expressed great interest in discovering the properties of a 'ml'. There did not appear to be any connection between the approach taken by the children and their home-educating background, apart from the fact that Child 22 again favoured a procedural 'vertical composition' approach for Q.3.

It is important to note that no child from a 'structured' home-educating family attempted the Group 1 questions. What the results from this analysis appear to suggest is that there are no major differences between the problem-solving approaches of children from semi-formal and those from informal home-educating families for this set of questions, apart from the fact that semi-formal children are slightly more likely to employ procedural approaches. This may be because informal families generally only use textbooks for mathematics questions when their child has a particular interest in a concept, whereas children from semi-formal families adopt elements of a textbook, structured approach alongside activities based around the children's preferred learning style. We next consider the Group 2 questions.

5.12 Analysis of Group 2 Questions

Table 5.11 shows that the average mark for the Group 2 questions, which were attempted by Children 5, 9a, 16 and 26, was 86%. Interestingly, the oldest child who attempted this set of questions (Child 9a, aged 11) made the most errors, with mistakes made on Q.5 and Q.7.

Group 2											
Family	**Age**	**Q.1**	**Q.2**	**Q.3**	**Q.4**	**Q.5**	**Q.6**	**Q.7**	**Q.8**	**Q.9**	**Mark**
5	8	1	1	1	1	0.5	1	1	1	1	**94%**
9a	11	1	1	1	1	0	1	0	1	1	**78%**
16	8	0.5	1	1	1	1	1	1	1	1	**94%**
26	9	1	1	0	1	1	1	0	1	1	**78%**
Mean	**9 years**	**87.5%**	**100%**	**75%**	**100%**	**62.5%**	**100%**	**50%**	**100%**	**100%**	**86%**

Table 5.11: Marks for the Group 2 Questions

Table 5.12 above shows that, similar to the Group 1 questions, no child from a structured family attempted this set of questions. It can also be observed that whilst all four children made an error of some kind (or did not attempt the question), those from semi-formal families (Families 5 and 16) only had one such error each, while the children from informal families (Families 9a and 26) had two each.

Arithmetic		Correct	Wrong	Did Not Attempt
Q4	Fractions, order, size	5, 16, 9a, 26		
Q7	Fractions of shape	5, 16	9a, 26	
Shape				
Q1	Area	5, 9a, 26	16 (correct method but did not finish)	
Q3	Finding Angles	5, 16, 9a		26
Q5	Area	16, 26	5 (correct square), 9a	
Q6	Symmetry	5, 16, 9a, 26		
Word Problem (Real Life)				
Q2	Interpretation + understanding of remainder	5, 16, 9a, 26		
Logical thinking				
Q9	Find a number	5, 16, 9a, 26		
Algebraic				
Q8	Value of symbols	5, 16, 9a, 26		

Children highlighted in blue were from semi-formal families

Children highlighted in green were from informal families

Table 5.12: Children's Results for the Group 2 Questions, by Area of Mathematics

5.12.1 Arithmetic Questions, Group 2

Questions 4 and 7 both focussed on the children's knowledge of fractions. For Q.4, the children had to compare two fractions to find the larger and provide justification for this choice, and all did this correctly. While Child 26 did not explain the reasoning behind his choice, Child 16 expressed both fractions as decimals to justify his answer, whereas Child 9a used percentages:

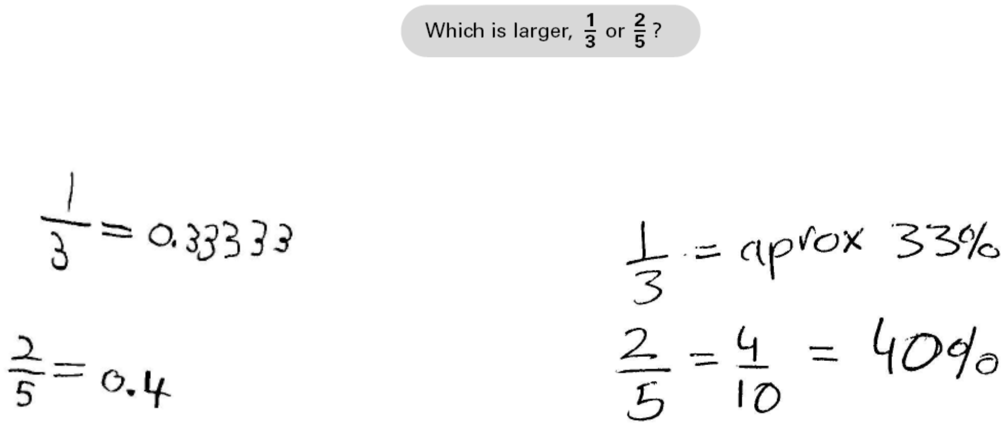


Figure 5.36: Q.4, G.2, Child 16, Aged 8 Child 9a, Aged 11

Child 5 took a different approach, arguing that since 2/5 requires multiplication by a smaller number than 1/3 to make 1, 2/5 is the larger of the two:

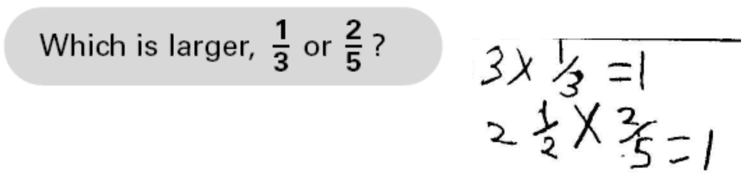


Figure 5.37: Q.4, G.2, Child 5, Aged 8

For Q.7, Children 26 and 9b do not show any working, and got both parts wrong:

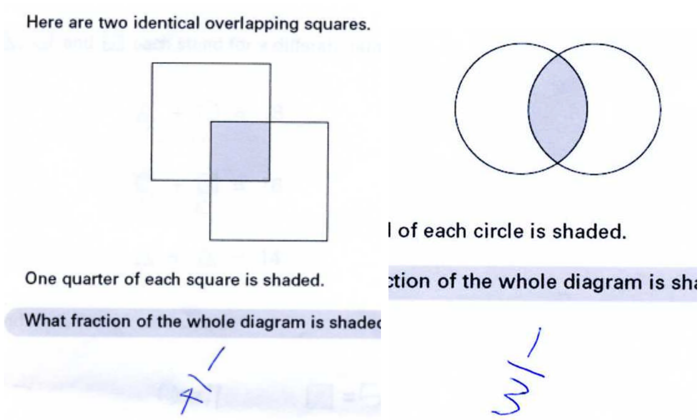


Figure 5.38: Q.7, G.2, Child 26, Aged 9

On the other hand, Child 5 gets both parts correct without showing any working. Child 16 divided the first square into four quarters and realised that '7 equal squares' form the entire shape. He also appears to 'observe' that the circle can be divided into 5 equal parts:

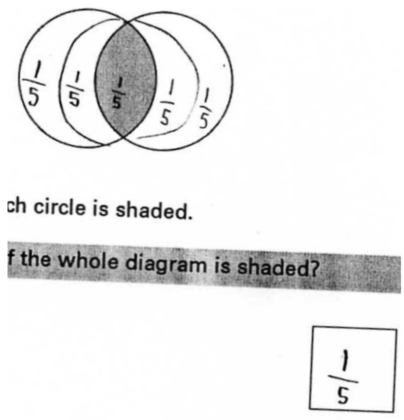
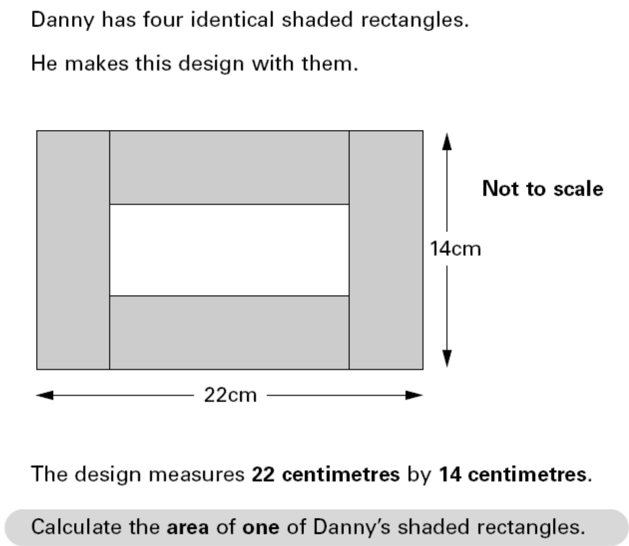


Figure 5.39: Q.7, G.2, Child 16, Aged 8

5.12.2 Shape Questions, Group 2

Four questions from Group 2 involved Shape, with Questions 1 and 5 focusing on Area, and Q.3 and Q.6 looking at Angles and Symmetry, respectively. All children appeared to be confident solving Q.1. Children 5 and 9a adopted an approach within which they initially calculated the width of the shaded rectangle before determining the area:



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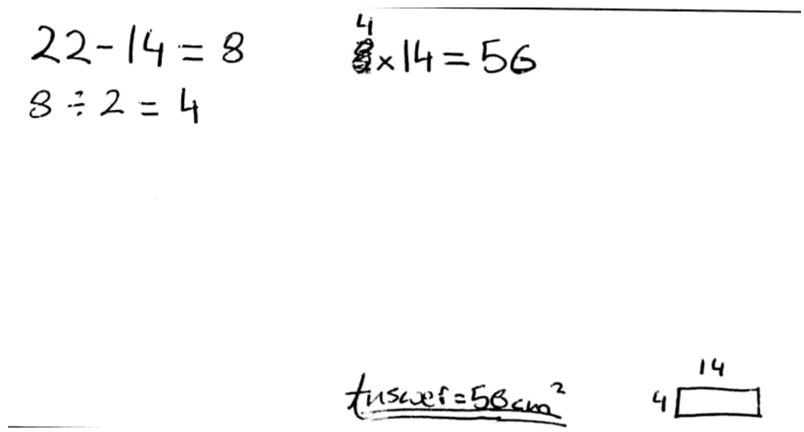
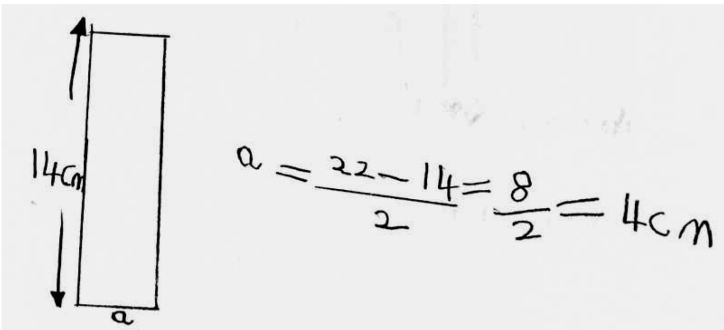


Figure 5.40: Q.1, G.2, Child 9a, Aged 11



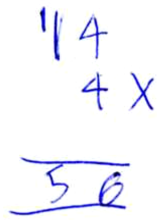


Figure 5.41: Q.1, G.2, Child 16, Aged 8 **Child 26, Aged 9**

It can be seen that the approach used by Child 9a is also used by Child 16 but with the additional feature of labelling the unknown side *'a'* and then recording a more inclusive approach to calculating the width. However, Child 16 then omits to calculate the area! On the other hand, Child 26 appears to perform most of his calculations mentally and only shows working during his calculation of the area for which he uses a standard approach to the multiplication of 4 x 14.

Question 5 required the children to interpret the text appropriately, where an understanding of the term 'vertices' is needed. Secondly, they had to visualise the correct position for the square so that the area is maximised. Finally, they needed to choose a method for calculating the square's area.

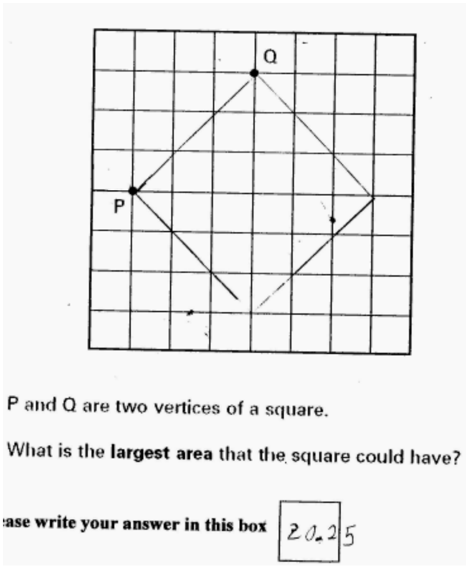
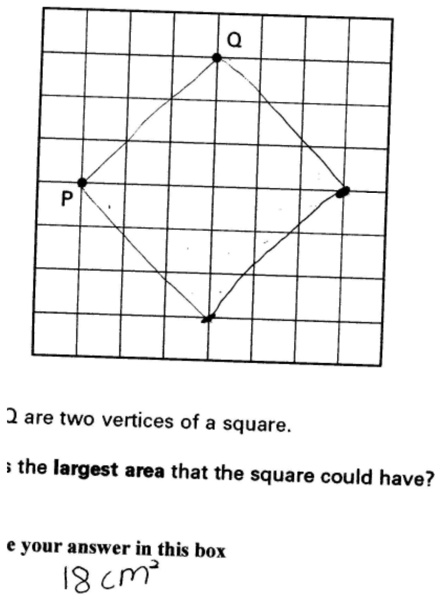


Figure 5.42: Q.5, G.2, Child 5, Aged 8

In Figure 5.42, we see that Child 5 drew the correct square but was unable to work out the area - he appears to have calculated the multiple of two 'estimated' lengths to get an area of 20.25 cm squared. Child 9a did not draw the correct square to begin with.



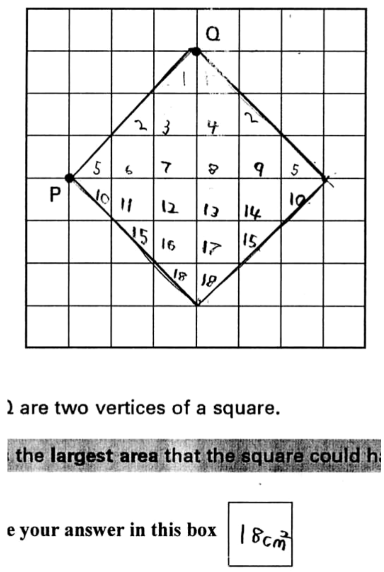


Figure 5.43: Q.5, G.2, Child 26, Aged 9 Child 16, Aged 8

In Figure 5.43 we can again observe how Child 26 has obtained a correct answer without any working. Child 16 also gets the correct answer, but he methodically labels each 'cm squared', pairing half squares, to get a total of 18cm squared.

Question 3 required knowledge of angle properties of equilateral triangles and squares, as well as the ability to perform the appropriate calculations when finding the missing angle *'a'*. Child 26 did not attempt this question, but the other three all successfully found the correct angle, with Child 9a and 16 labelling the angles on the diagram before using their knowledge of the angle at a point (360 degrees) to calculate their answer:

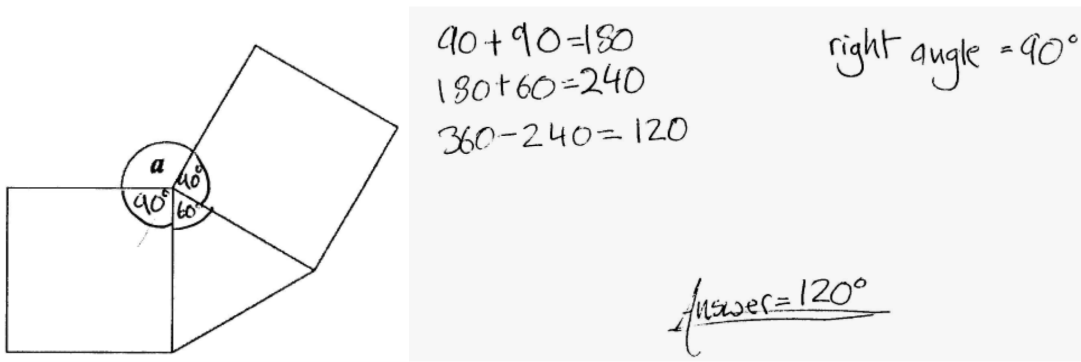


Figure 5.44: Q.3, G.2, Child 9a, Aged 11

Child 5's approach (see Figure 5.45) is somewhat different to the above, where he writes down the basic information with regards to the angles of a square and an equilateral triangle. He deduces that 90 degrees is a quarter of 360 degrees. Next he works out '90 minus 60' to give him the 'amount taken away from one quarter to make *a*'. He then adds this back to the 'other quarter' i.e. '90 plus 30'. This reasoning is quite unusual, but it works!

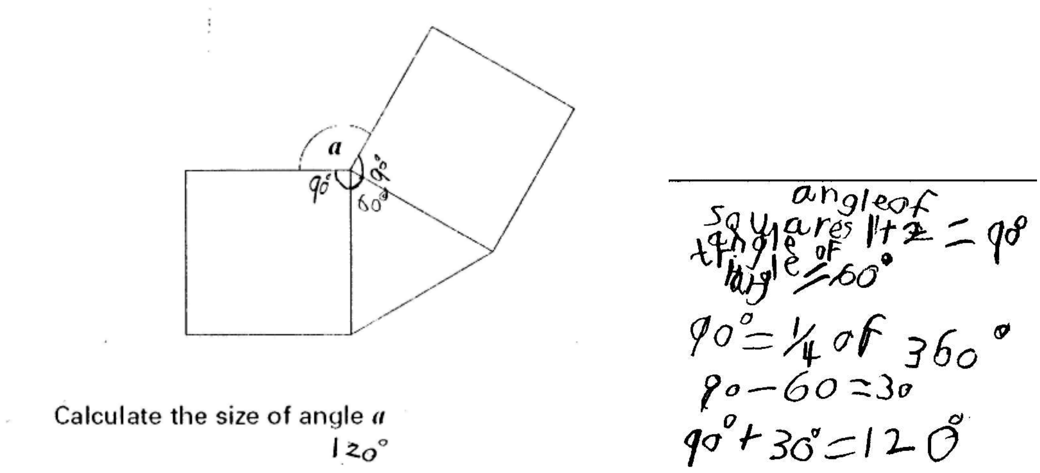


Figure 5.45: Q.3, G.2, Child 5, Aged 8

None of the four children had any difficulty with Q.6, which was on symmetry. In his answer below we see that Child 26 included sketches of two additional shapes that did not satisfy the given criteria, indicating a process where each shape was 'tested' against the criteria.

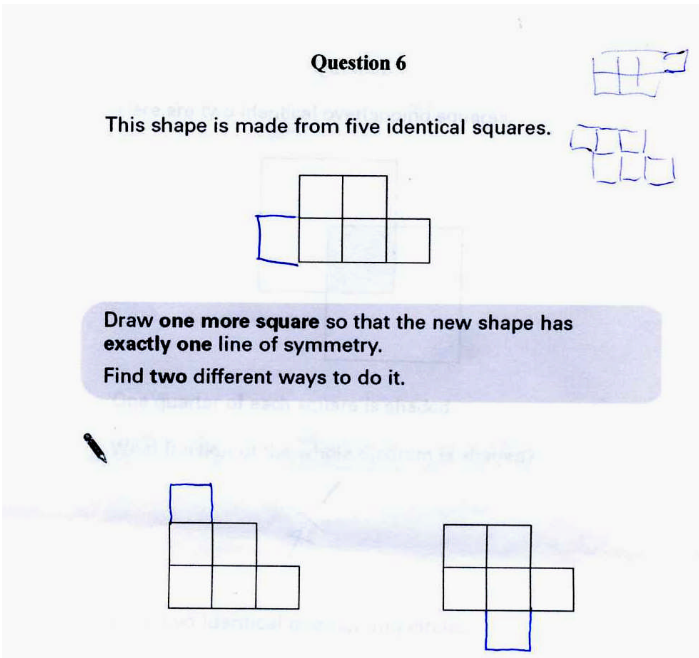


Figure 5.46: Q.6, G.3, Child 26, Aged 9

5.12.3 Word Problems (Real Life), Group 2

For Question 4 (see below), Child 16 systematically considered multiples of five and three in order to find a number that met both criteria. On the other hand Child 5 and 9a's approach was more direct – they mentally calculated the numbers that fit the 'biscuits in five, one left over' rule, and checked these numbers for the 'biscuits in three, two left over' criteria.

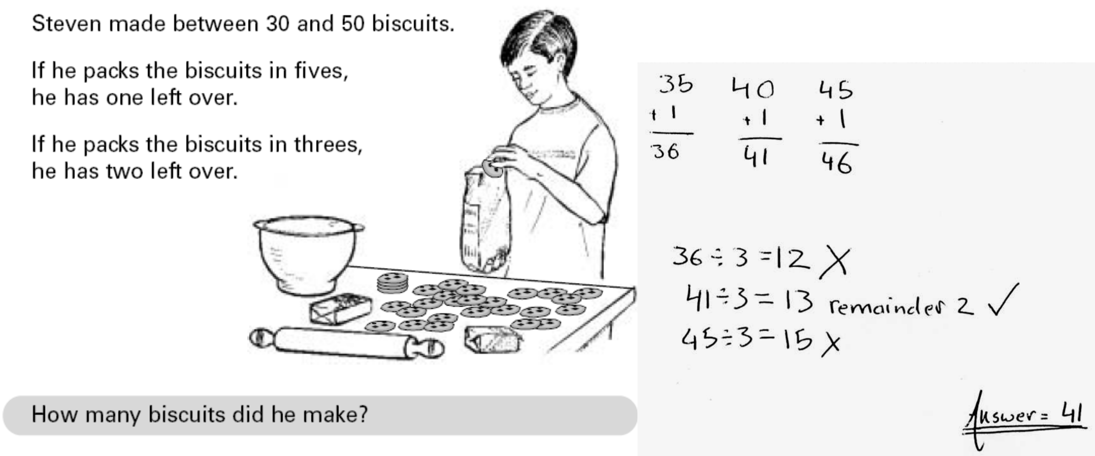


Figure 5.47: Q.2, G.2, Child 9a, Aged 11

Child 26 understands that to satisfy the 'multiple of 5, one left over' criteria, the number must end in 1 or 6. He therefore only checks numbers ending in one or six in order to satisfy the second criteria:

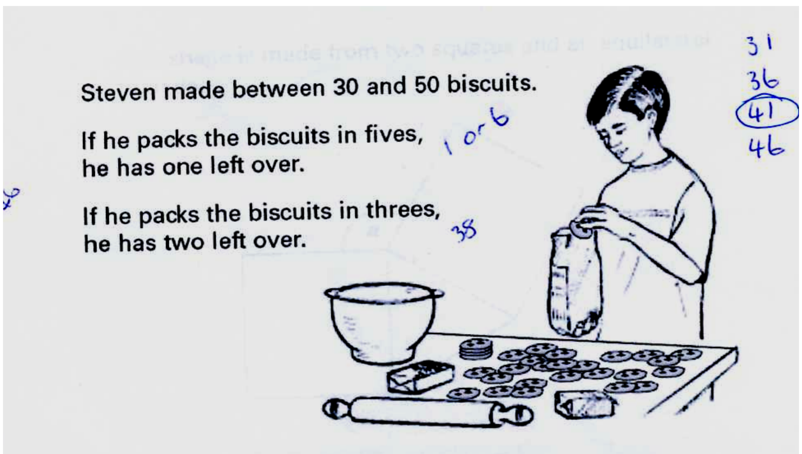


Figure 5.48: Q.2, G.2, Child 26, Aged 9

5.12.4 Logical Thinking, Group 2

Question 9 is similar to Question 2 (see above), where an understanding of the requirements of the three criteria is needed, as well an appropriate strategy for determining the numbers. All of the children were able to successfully find the possible numbers, and their methods demonstrated a diversity of strategies. Child 16 again demonstrated a systematic approach, checking each number against the criteria and eliminating those that did not meet the conditions:

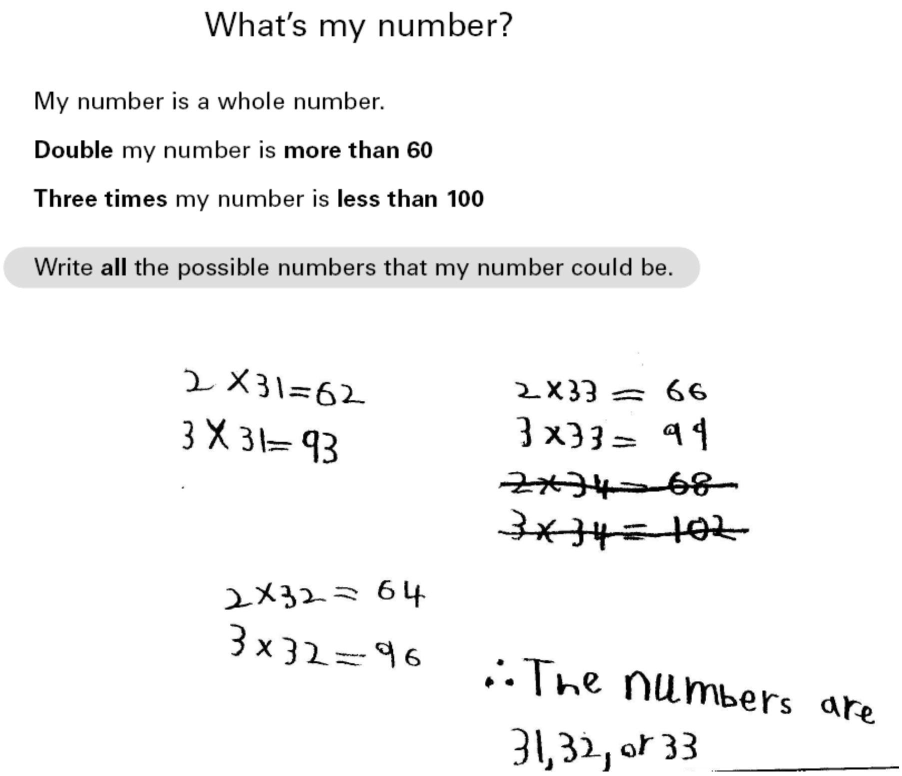


Figure 5.49: Q.9, G.2, Child 16, Aged 8

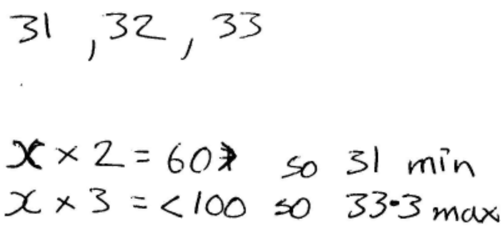


Figure 5.50: Q.9, G.2, Child 9a, Aged 11

Child 9a expressed the numbers as an unknown ‘*x*’. He then set up two inequalities to determine possible values of *x* (Figure 5.50).

On the other hand, Children 5 and 26 just wrote down the correct answer.

5.12.5 Algebraic Questions, Group 2

All of the children in this study were able to find the three unknowns successfully for Q.8 (see below). Child 5 simply wrote down the answer, as did Child 9a, while Child 26 wrote down the value below each shape, suggesting that he may have substituted each known value to find the remaining shapes during his working. In fact, Child 16 is the only participant to detail his method of solution, where he uses an algebraic method of substitution:

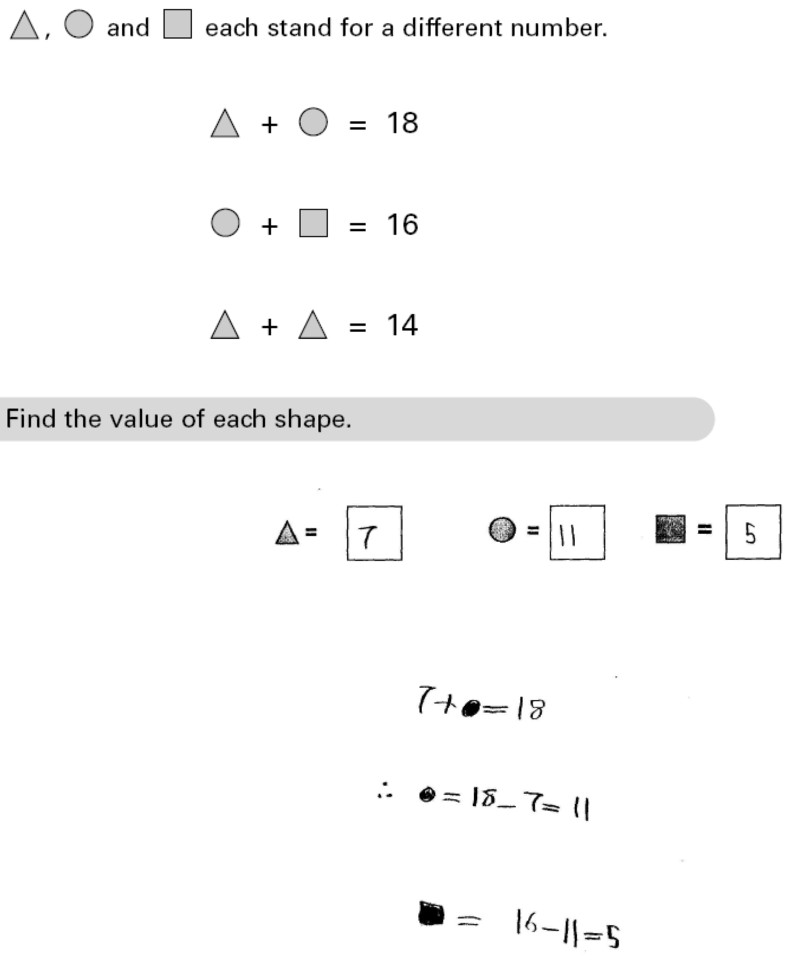


Figure 5.51: Q.8, G.2, Child 16, Aged 8

5.12.6 Summary of Group 2 Questions

Arithmetic		Worked Strategy	Procedure	Just wrote down answer
Q4	Fractions, order, size	5, 16, 9a		26
Q7	Fractions of shape	9a, 16		5, 26
Shape				
Q1	Area	5, 16, 9a, 26		
Q3	Finding Angles	5, 16, 9a		
Q5	Area	16		5, 9a, 26
Q6	Symmetry	26		5, 16, 9a
Word Problem (Real Life)				
Q2	Interpretation + understanding of remainder	5, 9a, 26	16	
Logical Thinking				
Q9	Find a number	9a, 16		5, 26
Algebraic				
Q8	Value of symbols	16, 26		5, 9a

Table 5.13: Method of Solution for Group 2 Questions

Constructed from the basic format of Table 5.12, Table 5.13 illustrates the methods used by each child to solve the Group 2 questions. One observation that can be made from Table 5.13 is that in only a single instance was a 'procedural' approach identified (for Child 16), and as was also noticed in the Group 1 set of questions, the children from the semi-formal and informal families were relatively similar in their problem solving approaches. To summarise the various approaches:

- **Arithmetic –** Both arithmetic questions involved fractions, and all correctly answered Q.4, which involved determination of the 'larger fraction'. However, two (Children 9a and 26) were unable to solve Q.7, which required a consideration of visual representations to determine the size of the shaded region in its fractional form. There did not appear to be any connection between the home-educating background of the children and the children's approach to these questions, although it was noted that Child 26 again appeared to favour ‘just writing down the answer’, similar to his general approach for the Group 1 questions.

- **Shape –** All were able to solve the question on symmetry, and were equally able when faced with a question that required the calculation of area with a ‘to-be-determined’ length (Q.1). But for Q.5, where they had to first draw a square with the largest area, and then determine this area, without any given lengths or measurements, this proved too difficult for Children 9a and 5.

- **Word Problems/Logical Thinking –** All four children answer these questions correctly, with only Child 16 taking a procedural approach for Q.2.

- **Algebraic Questions –** Again, the children found this question relatively simple – Child 16 (algebraic approach) and Child 26 (substitution) using a worked strategy, whilst the other two were able to write down the answers immediately.

The final set of questions to be considered were the Group 3 questions, which were set at the Key Stage 3 level.

5.13 Analysis of Group 3 Questions

The average mark for the Group 3 questions, which were attempted by eight children, was 64%. It was noticeable that apart from Child 16 (aged 8), all of the children under the age of 10 (Children 9b and 26) struggled with this set of questions.

Group 3											
Family	**Age**	**Q.1**	**Q.2**	**Q.3**	**Q.4**	**Q.5**	**Q.6**	**Q.7**	**Q.8**	**Q.9**	**Mark**
9b	8	1	1	0	1						**33%**
9a	11	1	1	1	1	0.5	1	0	0.5	0	**67%**
11a	17	1	1	1	1	0	1	1	0.5	0.5	**78%**
11b	15	1	1	1	1	0.5	0.5	1	1	1	**89%**
16	8	1	1	1	1	0.5	0.5	1	0.5	1	**83%**
19	14	1	1	1	1	0.5	0	1	0.5	0	**67%**
26	9	0.5	0.5		1						**22%**
28	14	1	1	1	1	0.5	0.5	1	1	0	**72%**
Mean	**12 years**	**93.8 %**	**93.8 %**	**75 %**	**100 %**	**31.3 %**	**43.8 %**	**62.5 %**	**44 %**	**31.3 %**	**64%**

Table 5.14: Marks for the Group 3 Questions

The Group 3 questions were structured in a way that allowed the children to initially attempt a set of arithmetical questions, which assessed their knowledge of basic multiplication, decimals etc. followed by a range of problems that required the application of concepts such as Area, Angles and Algebra in a number of different situational contexts (real-life, pictorial etc.).

Arithmetic		**Correct**	**Wrong**	**Did Not Attempt**
Q1	Multiplication facts	**9b, 9a, 26, 11a, 11b, 16, 19, 28**		
Shape				
Q3	Area of shape	**9a, 11a, 11b, 28, 16,19**		**9b, 26**
Q5	Finding dimensions of shape		**9b, 9a, 11a, 11b, 16, 19, 28**	**26**
Q7	Angles	**11a, 11b, 28, 16, 19**	**9a**	**9b, 26**
Q9	Area and shape	**11a, 11b,16**	**9a, 28**	**19, 9b, 26**
Logical thinking				
Q2	Arithmetic puzzles	**9b, 9a, 26 (correct, but no justification given), 11a, 11b, 28, 16, 19**		
Q4	Puzzle involving decimals	**9b, 9a, 26, 11a, 11b, 28, 16, 19**		
Q6	Square numbers justification	**9a, 11a, 11b, 28, 16**		**19, 9b, 26**
"Algebraic"				
Q8	Translating text into algebraic statements	**11b**	**9a, 11a, 28, 16, 19**	**9b, 26**

Children highlighted in red were from structured families

Children highlighted in blue were from semi-formal families

Children highlighted in green were from informal families

Table 5.15: Children's Results for the Group 3 Questions, by Area of Mathematics

5.13.1 Arithmetic, Group 3

Q.1 required relatively simple arithmetical knowledge, and all eight children were able to obtain the correct answer:

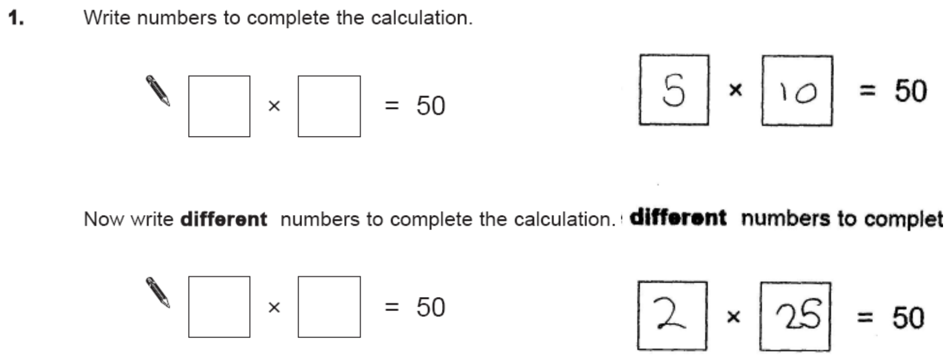


Figure 5.52: Q.1, G3, Child 19, Aged 14

5.13.2 Logical Thinking, Group 3

While the questions within this group involved the use of arithmetic, the children were also required to justify their mathematical thinking. For example in Q.2, although Child 26 was able to identify diagram B as the ‘odd one out’, he failed to explain the reason for his choice.

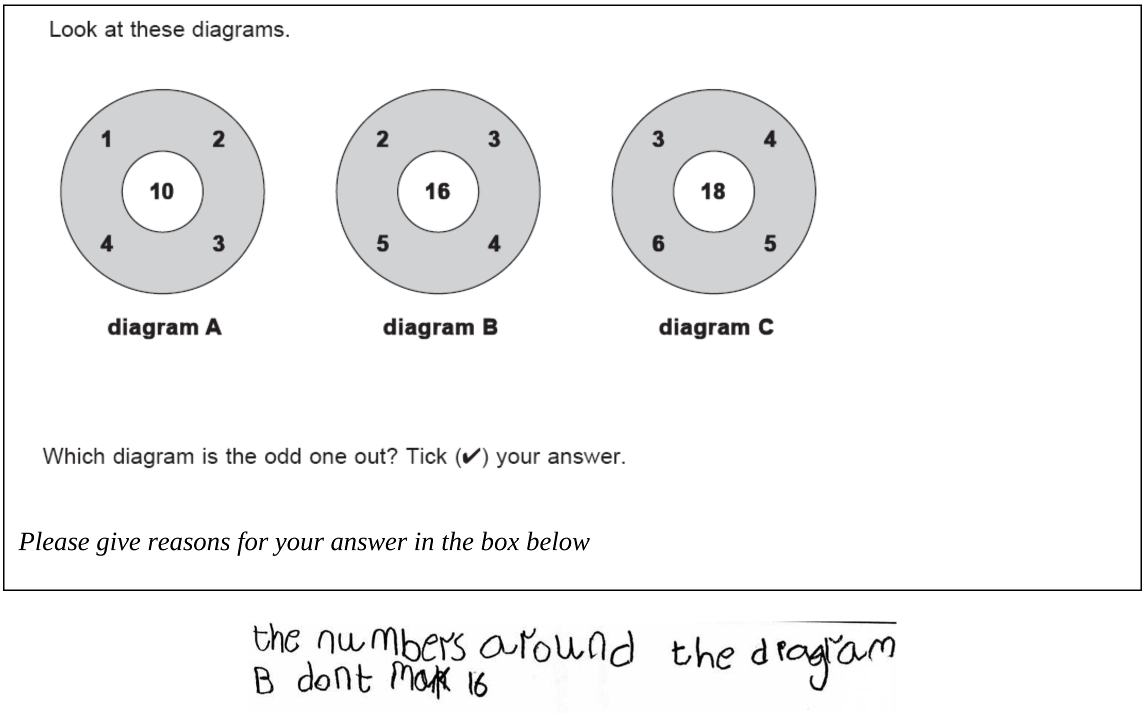


Figure 5.53: Q.2, G.3, Child 9b, Aged 8

Child 9b, on the other hand, attempted to explain his answer, as did the majority of the children (6 out of 8), who made reference to the fact that the numbers in the outer circle of diagram B do not ‘add up’ to make 16. All of the children showed a similar competence in solving Q.4, with none demonstrating any working. Question 6 (see below) proved more challenging - Children 9b, 19 and 26 did not attempt a solution. The most common response was to claim that the two possible values for the last digit of the square root are 4 and 6, but neither of these numbers is prime. Children 9a, 11a and 11b all used this approach:

I am thinking of a six-digit **square number** with a units digit of 6

_ _ _ _ _ 6

Could its square root be a **prime number**?

Tick (✔) Yes or No.

☐ Yes ☐ No

Please explain why you ticked Yes/No in the box below

I ticked no because 4x4=16 it ends in 6 and 4isnt prime. The sam with 6x6=36 and so on.

Figure 5.54: Q.6, G.3, Child 9a, Aged 11

Children 16 and 28 focus more on the nature of prime numbers, arguing that since the only even prime number is two, the square route cannot be prime, as it will never equal two:

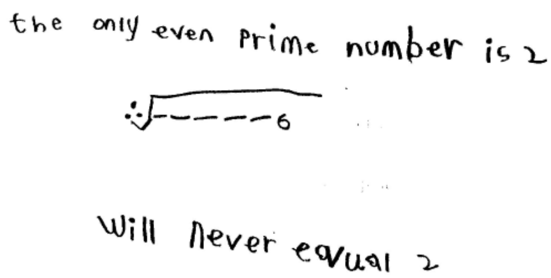


Figure 5.55: Q.6, G.3, Child 16, Aged 8

5.13.3 Shape, Group 3

Four questions addressed the children’s knowledge of shape. Note that Children 9b and 26 did not attempt Questions 3, 7 and 9. Question 3 involved area and Children 9a, 11a, 11b, 19 and 28 all drew the correct rectangle without showing any working. The most common orientation of the rectangle was ‘horizontal’, as in Child 28’s answer:

The grid shown below is made of regular hexagons.

On the grid, draw a rectangle with an area **6 times** as big as the area of one hexagon.

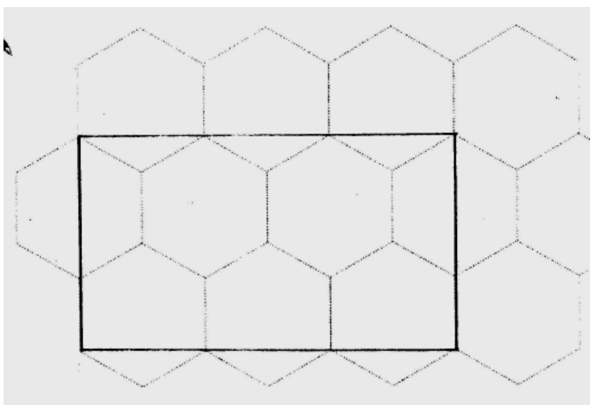


Figure 5.56: Q.3, G.3, Child 28, Aged 14

Unlike the others, however, Child 11b drew a ‘vertical’ rectangle:

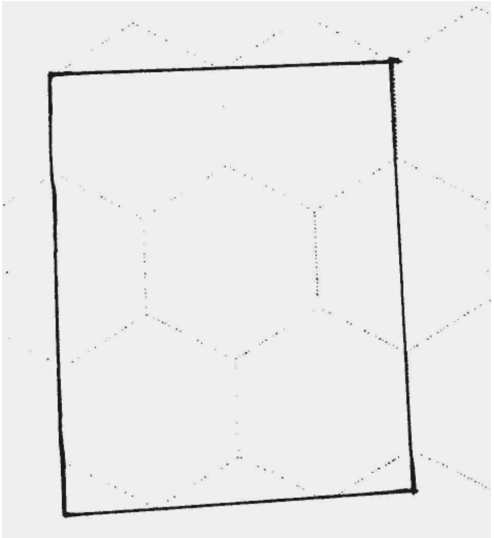


Figure 5.57: Q.3, G.3, Child 11b, Aged 15

Child 16 was the only participant who showed working for this question. Notice how he has ‘numbered’ each of the enclosed hexagons so that the rectangle can clearly be seen to cover six hexagons:

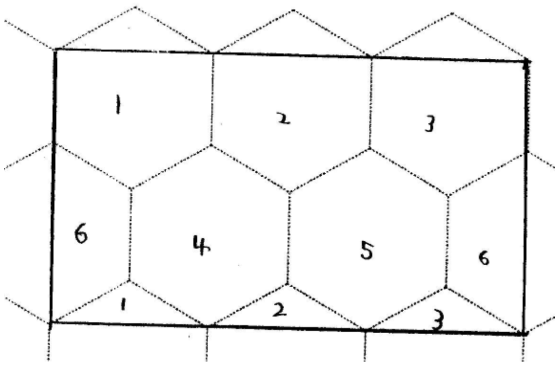
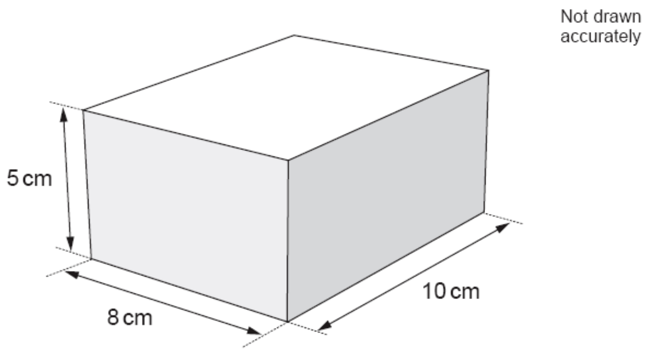


Figure 5.58: Q.3, G.3, Child 16, Aged 8

Question 5 required the children to be able to convince themselves that the wrapping paper is large enough to cover the box, and to give a mathematical explanation that justifies their conclusion. A full justification should consider whether the wrapping paper is large enough to cover the box in one piece – it is not sufficient to show that the area of the paper is larger than the surface area of the box. None of the children were able to give a complete justification of their conclusion. However, although

Child 9b did not show his calculations, he concluded that the paper was large enough, and wrote down his thoughts on how the answer could be justified:

I have a present in a box, a cuboid measuring **10cm by 8cm by 5cm**.



I have one sheet of wrapping paper to wrap up the box.

The sheet is a rectangle that measures **25cm by 30cm**.

Is the sheet of wrapping paper big enough to cover **all** the box?

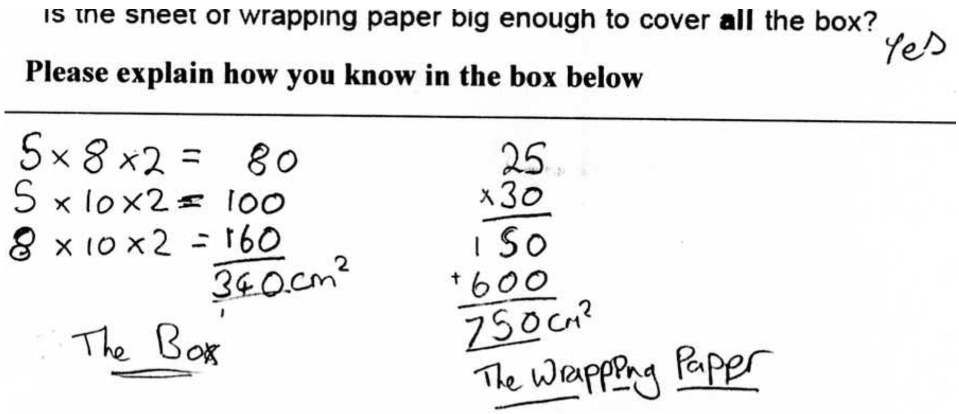
Is the sheet of wrapping paper big enough to cover **all** the box? yes

Please explain how you know in the box below

add up the nummbers
on the piece of paper
if its less than the
you have ~~te~~ then
it will cover it

Figure 5.59: Q.5, G.3, Child 9b, Aged 8

Children 9a, 16, 19 and 28 all felt that the paper was large enough but they only compared the area of the wrapping paper to the surface area of the box:



Is the sheet of wrapping paper big enough to cover **all** the box? Yes

Please explain how you know in the box below

$5 \times 8 \times 2 = 80$
$5 \times 10 \times 2 = 100$
$8 \times 10 \times 2 = 160$
$340 cm^2$
The Box

25
$\times 30$
150
$+600$
$750 cm^2$
The Wrapping Paper

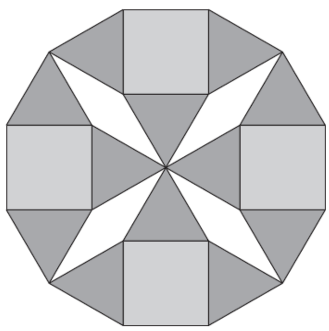
Figure 5.60: Q.5, G.3, Child 19, Aged 14

Question 7 addressed the children's knowledge of angle properties. Child 9a attempted Q.9, but his incorrect assumption of the properties of a rhombus lead to the wrong solution:

I have some tiles that are squares and some tiles that are equilateral triangles.

The side lengths of the tiles are all the same.

I arrange the tiles like this.



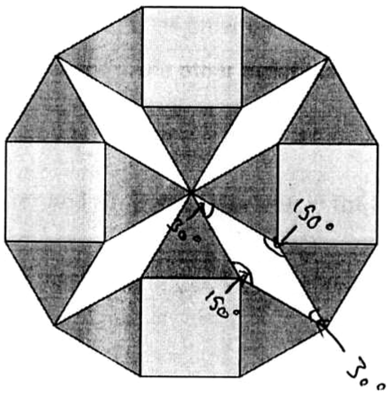
I want to fill the gaps by making four tiles that are rhombuses.

What should the angles in each rhombus be?

rhombus corner is $\frac{1}{2}$ of 90° so
2 corners should be 45° and the
other 2 should be 135. ~~Bea~~ Because
a rhombus is a tilted square and a square
~~equal~~ has 360° in side and 135×2=270
45+45=90
+ 360°
so i am right

Figure 5.61: Q.7, G.3, Child 9a, Aged 11

On the other hand, Children 11a, 11b, 16, 19 and 28 were able to find the correct angles, with Child 28 writing down some of the angle properties to justify his answer:



I want to fill the gaps by making four tiles that are rhombuses.

What should the angles in each rhombus be?
SEE ABOVE

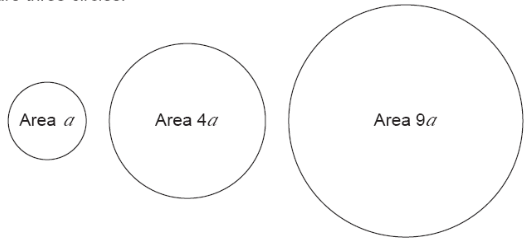
Please show your calculations in the box below

EACH ANGLE IN EQUALATERAL TRIANGLE IS 60°. THE ACUTE ANGLES IN THE RHOMBUS MUST BE 30°. THE REST MUST BE 150°.

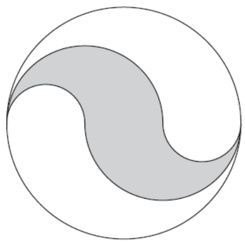
Figure 5.62: Q.7, G.3, Child 28, Aged 14

The majority of the children found it hard to justify their solution to Question 9:

Here are three circles.



Parts of the circles are used to make the design below.



What **fraction** of the design is **shaded**?

Please show your answer and working on the next page

Children 9b, 19 and 26 did not attempt this question, and while Children 9a and 28 were able to identify the fraction as a third, they could not justify their answer:

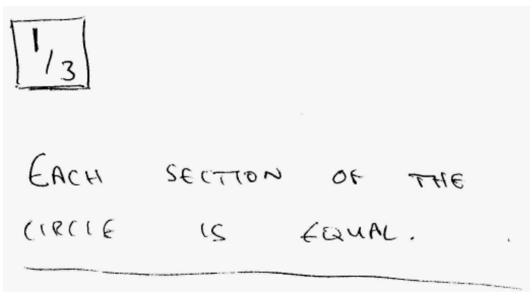


Figure 5.63: Q.9, G.3, Child 28, Aged 14

Child 11a wrote a brief but adequate proof, using the relevant fractions of the shape to justify her answer. Her sister, Child 11b, wrote a more detailed proof, as did Child 16:

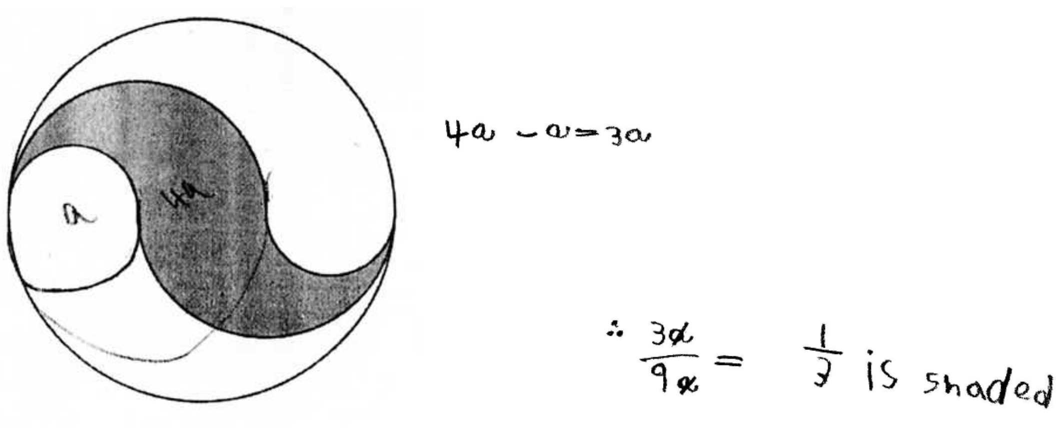


Figure 5.64: Q.9, G.3, Child 16, Aged 8

5.13.4 "Algebraic", Group 3

I think of a number, then I carry out these operations on my number.

Multiply by 5	Add 8

When I carry out the operations in one order the answer is **105**

When I carry out the operations in the **other order** the answer is **73**

What is my number?

Please give your answer and show your working in the box below

Figure 5.65: Q.8, G.3

Children 9b and 26 did not attempt this question, but the remaining children were able to solve the first part to find the unknown number. Child 9a made an attempt to solve the second part, showing his ability to prove the statement for a particular number (i.e. 2) but not in the general sense:

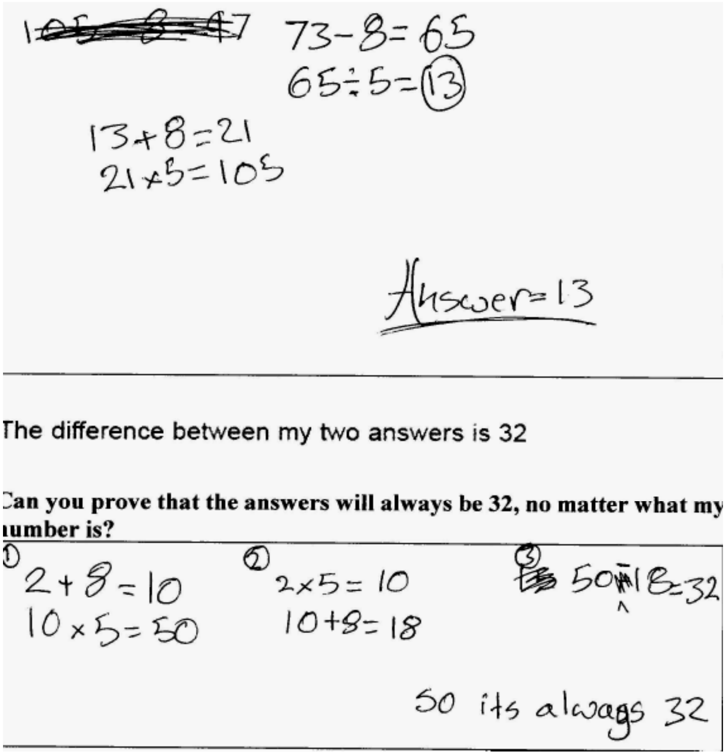


Figure 5.66: Q.8, G.3, Child 9a, Aged 11

Children 16, Child 28 and Child 11b all attempted to use algebra to justify their answers; for example, Child 28 wrote down what could be considered the beginnings of a proof but ended up using a particular example:

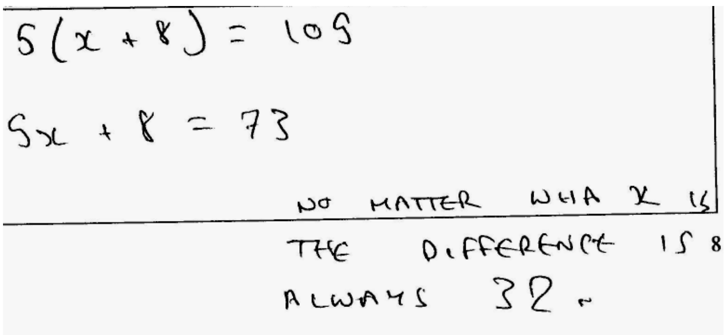


Figure 5.67: Q.8, G.3, Child 28, Aged 14

Only Child 11b was the able to prove the second part.

$\rightarrow = n \times 5 + 8$ — $(5n+8) - 5n = 8$

$\leftarrow = (n+8) \times 5$ — $(5(n+8)) - 5n = 5 \times 8 = 40$

$40 - 8 = 32.$

Figure 5.68: Q.8, G.3, Child 11b, Aged 15

5.13.5 Summary of Group 3 Questions

Note that the particular problem solving strategies (e.g. procedural) used for the Group 3 questions are not summarised in a table. This is because the majority of the questions required use of a worked strategy and hence did not have a method of solution via a particular arithmetical approach. In summary, the findings revealed:

- **Arithmetic –** For Q.1, no child had any difficulty in finding two pairs of numbers.

- **Logical Thinking –** Q.2 and Q.4 again demonstrated the children's strengths with arithmetical-type problems – even the two youngest children, aged eight (Child 9b and 16), could use their numerical reasoning to solve the problems. Child 16 was also able to successfully apply his knowledge of prime numbers in Q.6, as did five of the other children – on the other hand, three were unable to make a start.

- **Shape –** When given Q.3, which involved drawing a shape that had a 'given area', all but two of the youngest children (Child 9b and 26) were able to obtain the correct answer. However, none were able to fully justify their solutions for Q.5 – whilst Child 11b took the correct approach she made a mistake when finding the dimensions. Q.9, which linked fractions with area, was only solved by Children 11a, b and 16; noticeably those who appeared to have developed a sense of *'formal' mathematics reasoning* – Child 16's mother was a mathematician, and the sisters Child 11a and b were both studying for their IGCSE's (which is at a slightly 'higher' standard than GCSE mathematics). Apart from two of the younger children who did not attempt Q.7 and Child 9a who made a wrong assumption, all were able to apply their knowledge of angle properties to find the correct angle.

- **Algebraic –** Q.8 was a useful indicator of those who were able to prove results in the general sense, and those who could only give a 'particular' solution. Half of the children could only solve the first part. Only Child 11b could give a proof – Child 28 made an attempt, but then reverted to the use of particular solutions – note that both of these children are from structured families. This indicates that, as in Q.9, which also required a justification of the results, having a sense of 'formal mathematical' reasoning appeared to be key when constructing proof.

The next chapter will bring together the results obtained from both Chapters 4 and 5 in an attempt to link together the key ways in which the home-educating approaches of the families affected their children's mathematical beliefs and thinking.

Chapter 6: Discussion of the Home-Education Approaches and Their Effect on the Children's Mathematical Development

This section brings together the analysis from Chapters 4 and 5 in an attempt to discuss the ways in which the three main home-educating approaches: (1) Structured, (2) Semi-Formal and (3) Informal, influenced the children's mathematical beliefs and understanding.

6.1 Structured families

We first begin with a consideration of the *Structured* (or Formal) families. Note that in the literature review (Section 2.1.4.1), this category was also described as 'Formal', however, it was felt that the word 'Structured' provided a better description of the home-educating approach, and hence this term was used throughout the study.

6.1.1 Home-Educating Approach and Children's Learning of Mathematics

In structured families, the children used *textbooks* as the main mathematical learning resource:

> "We have been following the Edexcel IGCSE syllabus and using text books."
> Family 11, children aged 16 and 14

> "Various, through workbooks. 9 year old is halfway through year 5 series. 8 year old nearly halfway through year 4." Family 23, children aged 9 and 8

Learning also took place at *regular intervals* during the week:

> "Oldest 20 minutes, 4 days a week, plus everyday use, such as cooking chips etc. Middle one, 1 task a day for 4 days plus everyday use. Youngest –maths when he asks to do it plus everyday use."
> Family 15

> "Child 3 [aged 13 years] has to do some of her pc maths each day and her incentive is to do it so she can then do other things!!!!"
> Family 6, children aged 13 and 8

It is noticed that the parental ‘reasons for following a structured approach’ were the **main factors** in determining the children’s choice of mathematical topics for Families 6, 11, 15, 23 and 28 (Sections 4.6.3 and 5.5). Children within Families 11 and 28 were both studying for examinations at the time of this study, and their mathematical learning was focused on covering the syllabus in order to attain the best possible marks, i.e., their *‘mathematical goals’* required a structured learning approach.

Family 23 is structured by *parental choice*, and it can be observed that neither Child 23a nor 23b have the option of choosing the mathematical content; textbooks identify the core of the curriculum and as a consequence both children find mathematics “boring” (Section 5.1.2). Child 15 learns mathematics through a structured curriculum because her mother lacks confidence, and the curriculum is influenced by areas that reflect her mother’s philosophy — ‘greater understanding’ and the ‘everyday learning’ of mathematics (Section 5.5.). At the same time Child 15, aimed to ‘get correct answers and finish the work quickly when learning’ — reflecting her mother’s teaching priorities.

It appeared that the structured families’ *use of textbooks* influenced their children’s views of mathematics. For example, Child 28 was encouraged to apply his mathematical knowledge to real-life situations and found mathematics both interesting and enjoyable. But his learning was predominantly **focused on the areas needed for his exams**, and only occasionally guided by concepts that interested him and the everyday applications of mathematics. As was also noticed for Family 11, *a focus on exams appeared to require more structure to the learning*.

From the above evidence, the following relationship between a structured approach and their child’s learning is proposed, particularly in instances where it is the parent who has chosen the home-educating approach:

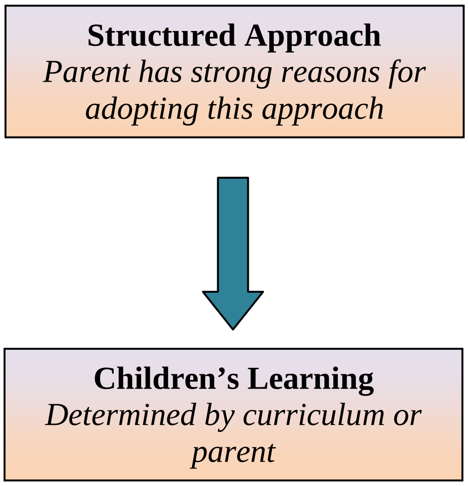


Figure 6.1: Effects of the Structured Approach on the Children’s Learning of Mathematics

On the other hand, Child 6 **chose** to learn from a structured curriculum, and writes that her parents had **no influence** on the topics that she learned – she instead prioritised **understanding** when learning**.**

Although the children appear to have different ways of implementing the structured approach into their mathematical learning, **mathematical understanding** appears to have high priority amongst the ‘structured families’. However, only two out of the six children made the ‘application of concepts’ a priority – two other children felt that obtaining correct answers was most important, whilst the remaining two judged ‘finishing the work quickly’ more important than the ‘application of the concept’.

6.1.2 Home-Educating Approach and Children's Mathematics Beliefs

The children from the structured families demonstrate a range of mathematical beliefs. Family 23's children found mathematics 'boring and useful', but Child 11b, perhaps reflecting her mother's views, 'enjoys mathematics and finds it interesting', as does Child 28. Child 15's father uses mathematics as a key aspect of his job, and his daughter writes that mathematics is 'interesting and enjoyable', but Child 6 is neutral towards the subject — and her parent found mathematics very difficult at school. It seems that a **parental confidence and interest** in mathematics is the strongest indicator of whether or not their children will like/dislike the subject. Regardless of their personal feelings towards the subject, all of the children considered mathematics to be useful in many areas of life, including everyday activities, exams and their future work.

With regards to their problem solving beliefs, Children 23a, 23b perceived a mathematics problem as 'part of a mathematics lesson' or 'questions with numbers and words', and it is conjectured that this attitude was established from their use of textbook. Out of the five questions related to problem solving beliefs, the children from Family 23 had only one belief that was identified by Zan and Poli (1995) as belonging to 'good problem solvers'. Child 15's operational view of a mathematics problem and her problem solving beliefs also reflected her structured learning approach - she **perceived mathematics** to be a subject *'where you work something out and try to comply'*, and believed 'a *mathematics problem is an exercise when you decide which operations to be done and then perform them correctly'*. Although Children 11 and 28 both currently learn through textbooks (for their exams) there is evidence that the children have used mathematics in 'everyday' situations (e.g. bidding on Ebay), and they both view a mathematics problem as *'a situation where* *one can use mathematics'*, a perception shared by Child 6, whose mother encouraged her children to observe the real-life applications of mathematics (Section 4.6). To conclude, it appears that the children's problem solving beliefs are influenced by the quality of problems they are exposed to. Those exposed to a variety of 'mathematical situations' (e.g. real-life mathematics), have a less restrictive view of problem solving than those who predominantly learn through textbooks — therefore, the mathematics they engage in influences the **types of mathematics problems they perceive to exist.**

6.1.3 Relationship Between the Structured Approach and Mathematical Understanding

Interestingly Children 23a, 23b and 6 **could not say** what their 'standard of mathematics' was – thus **half the children from structured families were unable to measure their own level of mathematical ability**.

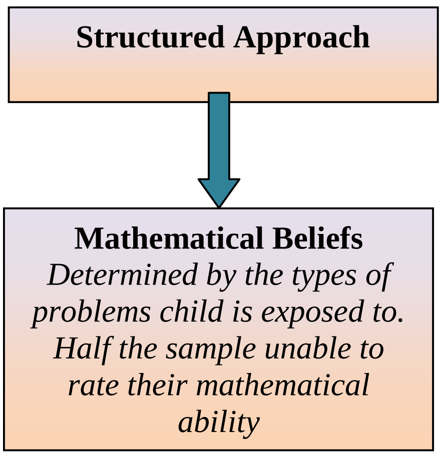


Figure 6.2: Effects of the Structured Approach on the Children's Mathematical Beliefs

On the other hand, both Child 11, 28 and 15 are able rate their personal mathematical abilities, with the former two believing they are 'good at mathematics' and the latter feeling she is 'sometimes good at mathematics'.

The children's indications of 'the sign of someone who is good/bad at mathematics' suggests that their mainly 'textbook/curriculum' learning approach is a major influence. For example, the majority listed correct answers or 'speed of calculation' as a sign that someone is good at mathematics. Child 28 also mentions accuracy for a 'range of mathematical' questions, perhaps as a result of his extra-curricular use of mathematics.

The important signs when measuring their *own levels of mathematical understanding* showed that despite the sometimes restricted learning approaches, the children from structured families sought a range of signs – e.g. the application of the concept to real-life concepts, confidence and making connections with existing knowledge. However, on average, the children from structured families rated a fewer number of indicators of understanding as 'important' than those from semi-formal/informal families (Section 5.9.3).

Only three children from the structured families attempted the assessed work questions, and all attempted the Group 3 set. Child 11a (aged 17) obtained 78%, having little difficulty with the 'calculation type' questions, or those that required the application of trigonometric results. However, she struggled with the questions where proof was required, as did Child 28 (aged 14), who also obtained 78%, and had difficulty with precisely the same questions as Child 11a. As could be observed from their answers to Question 9 (Section 5.13.3), both children had an understanding of 'how to begin their proofs', but they either failed to generalise the result, or were unable to follow through with their argument. It could be suggested that Children 11a and 28 had an operational or procedural understanding of the concept (as defined by

Hiebert and Carpenter, 1992; Skemp, 1976), but had yet to establish enough connections to move beyond this step.

On the other hand, Child 11b (aged 15) displayed a different level of mathematical understanding in her *formal understanding* of mathematics (the presentation and layout of her solutions) and in her ability to prove the results conclusively. For example, her proof to Q.8 (Section 5.13.4) showed her ability to generalise the result, and the solution to Q.9 (Section 5.10.1.2) linked her algebraic justification to the designated areas of the circle. Furthermore, in Q.5 (Section 5.10.1.2) she was the only individual to compare the length and width of wrapping paper with the box (the other children only compared areas). Her understanding of the mathematical problems showed a 'knowledge of each concept that is rich in relationships' – thus, as well as a procedural understanding of the situations, there was also evidence that she understood the situation *conceptually*. Interestingly **all of her problem solving beliefs** fell into the category of a 'good problem solver', as identified by Zan and Poli (1995).

The assessed work showed that although Child 11b was a year younger than her sister, and had the same teacher (their mother), using the same syllabus, Child 11b appears to have developed a 'more relational' understanding of the mathematical concepts required for Group 3 than her sister. Indeed, the mother indicated that Child 11b's was often quicker than her when learning new concepts.

6.2 Semi-Formal Families

6.2.1 Home-Educating Approach and Children's Learning of Mathematics

It was noticed that in these families, the parents were generally opened-minded with regards to their teaching and were willing to adapt the learning activities according to the needs of their child, as noticed in Family 5's comments:

> "More or less time available, introducing a concept which requires a different approach, or if I feel the **current approach is not working** for my child, discovering new resources."
>
> Family 5, son aged 7

Eight families (5, 7, 16, 17, 18, 19, 22 and 24) belonged to this category. Although the children used a range of learning activities, it was clear from the children's responses that their **personal interest** guided the choice of activity. Other factors such as mathematical understanding, everyday life applications and parents occasionally influenced their choice. The range of activities used to learn mathematics (Internet learning, songs, worksheets etc.) also demonstrated the flexibility of this approach.

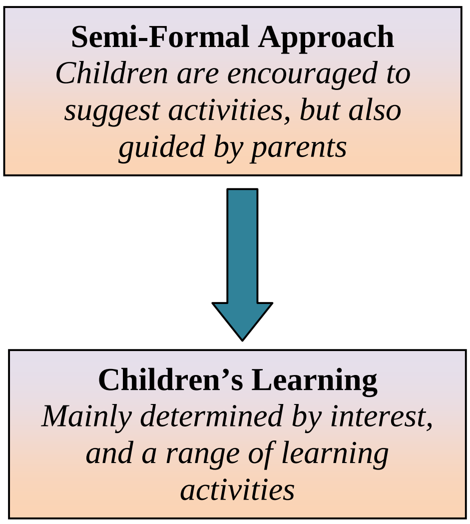


Figure 6.3: Effects of the Semi-Formal Approach on the Children's Learning of Mathematics

When *doing mathematics*, **six out of the nine children** wrote that their top two priorities were **applying** the concept and **understanding** the mathematics. Of the remaining three children, Children 17a and 18 wrote that **understanding** and **correct answers** were their main aims. Interestingly, both had special needs, where they had trouble remembering simple mathematical concepts and perhaps this is why they give high importance to correct answers — they tend to learn the 'hard to remember' concepts via a structured, memory-based approach. Thus it appeared that parents whose children had special needs were more inclined to teach *procedurally,* and this may have resulted in the children's prioritising correct answers over the ability to apply their knowledge to a range of situations (i.e. an indicator of relational understanding as defined by Skemp (1976)).

The only 'semi-formal child' who felt that neither understanding nor applications were priorities was Child 14. She aimed to obtain correct answers and finish her work quickly. On further investigation, it was noted that her family has recently changed from a structured approach to a semi-formal home-educating style, and she had previously learned from a set curriculum. Therefore, it seems that the children from families who were 'strictly semi-formal' gave a higher priority to the **application** of concepts than the children who were 'more structured' in some aspects of their learning – these children placed a higher priority on **correct answers.** However, both groups of children valued the *'understanding of each concept'* when learning.

6.2.2 Home-Educating Approach and Children's Mathematics Beliefs

Six out of the nine children (7a, 16, 17a, 17b, 17c and 19) found mathematics **enjoyable** and **interesting**, with Child 18 enjoying *some* areas of mathematics, but not through his Kumon workbooks (Section 5.6). Note that all six of these children had parents who personally enjoyed mathematics. On the other hand, Child 24 did not currently like mathematics, but her aim was to *learn to enjoy mathematics.* Thus the **majority of the children** from semi-formal families had a **positive attitude** towards mathematics which could be associated to their parents' perceptions of mathematics. As was also noticed in the children from structured families, all of the children considered mathematics to be useful in many areas of life, including everyday activities, exams and their future work.

Overall, seven out of the nine children had the majority (three or more) of their beliefs associated with those identified as 'good problem solvers'. Only Child 14, who until recently had learned mathematics through a structured approach, and 17c, who has dyslexia, had three or more beliefs belonging to the category of 'bad problem solvers' (Zan and Poli, 1995).

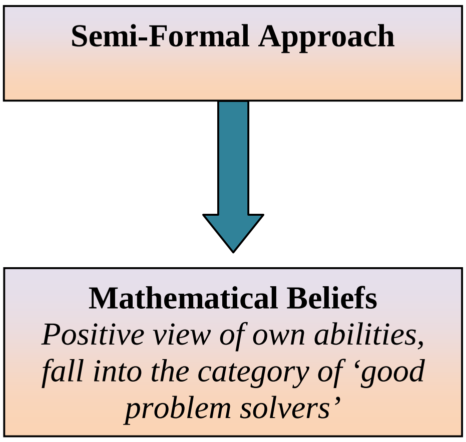


Figure 6.4: Effects of the Structured Approach on the Children's Mathematical Beliefs

6.2.3 Relationship between the Semi-Formal Approach and Mathematical Understanding

All of the children from the semi-formal families had an opinion of their personal mathematical ability, with five claiming they were 'good at mathematics most of the time' and four stating they were good at mathematics 'some of the time'.

The children's beliefs on 'the signs of someone who is good/bad at mathematics' were extremely varied, the indications included signs of confidence, accuracy, speed, like/dislike of mathematics, applications to other areas and even 'the jobs held by the individuals'. However, it was noted that the children's beliefs associated with the qualities of a 'good mathematician' were usually a reflection of their 'aims when doing mathematics' – for example, if their priority when *doing mathematics* was to apply the concept, to them, this was often a *sign of someone who is good at mathematics.*

The children's important indicators when measuring their *own levels of mathematical understanding* were similar to those seen within the structured families. However, where the children sought a range of signs, these appeared to depend more on the *individual child* rather than the home-education approach. For example, in Family 17, Child 17a and 17c both felt that the 'parent/tutor saying they understood the concept' was **often** a sign of understanding, whereas their sister, Child 17b **rarely** asked for parental advice. She preferred to look for alternative sources of information, such as the Internet or textbooks when faced with a concept that was hard to understand, rather than asking her parents. This suggests that the views on mathematical understanding can vary amongst siblings from the same semi-formal family.

Children 5 (aged 7), 7a (aged 7), 7b (aged 6), 16 (aged 8), 19 (aged 14) and 22 (aged 5) from the semi-formal families all attempted the assessed work, although Child 5, 7b and 22 did not complete the children's questionnaires. From the solution strategies, it could be observed that Child 5, 16 and 7b all showed signs of *conceptual understanding* (Hiebert and Carpenter, 1992), where their answers often demonstrated knowledge of the relationships between various arithmetical operations, and in particular, Children 5 and 16 used very creative approaches for a number of questions (e.g. Q.7 Group 2, Figure 5.39).

Child 5 comes from a family where his mathematical understanding is measured by the ability to develop ideas independently, and this ability to 'see concepts differently' is evident in a number of Child 5's solution strategies (see Q.4, Group 2, Figure 5.37). Child 16's mother is a mathematician, and for an eight year old, he appears to have developed a *formal understanding* of concepts within algebra and trigonometry, using the appropriate notation and layout to express his mathematical arguments. At the same time, his answers were sometimes laborious (see for example Section 5.10.4.2, Figure 5.15) which may indicate that this familiarity with a range of procedures may perhaps have inhibited his ability to seek quick method of solution. It was felt that Child 16's priority when solving problems often appeared to indicate an eagerness to demonstrate his range of mathematical knowledge rather than to obtain a quick solution. His mother wrote that her son's work was never viewed by anyone outside the family, and mentioned that Child 16 was very happy to receive feedback from the study – so perhaps this is why he chose to show 'as much justification as possible'. On the other hand, Child 16's various solution strategies to the problems indicated that he had developed a conceptual understanding of arithmetic, and was able to apply his algebraic knowledge to a range of problems (e.g. Q.8, Group 2, Figure 5.51).

Child 19 appeared comfortable with questions that required calculations and procedural knowledge, but struggled with those that asked for a proof. She learned mathematics independently through a GCSE textbook and CD and rarely explained her mathematical reasoning to others, which could have made it harder for her justify her answers in the assessed work.

Child 22 also tended to be procedurally inclined, using a ‘column layout’ for every question, whenever possible. When unable to calculate the answers via a column procedure, she did not appear to have any alternative strategy. However, she displayed an ‘intuitive’ understanding of the visual/pictorial questions, where her mother noted that she explored such problems by trial and error, or ‘saw’ the answer very quickly. In other words, one could postulate that Child 22 had developed a ‘procedural understanding of arithmetical concepts’ and a conceptual understanding of ‘mathematical concepts that can be represented visually’, such as area or symmetry.

6.3 Informal Families

6.3.1 Home-Educating Approach and Children’s Learning of Mathematics

Seven families (4, 8, 9, 10, 20, 21 and 26) belonged to this category. Child 4 did not complete a questionnaire, but attempted the assessed work from Group 1. From the children’s responses it was noticed that the main influence on their choice of mathematical topic was ‘the concepts that occurred in everyday life’, with five out of the six children writing that they *always learned* through everyday life activities. Secondly, four out of the six children noted that their **personal interest** was a key factor in the choice of concept to learn. The factors that *never* influenced their learning activities were **parents** (five of the children) and **textbooks** (noticed in four out of the six families). Thus a child from an informal family typically based their mathematical learning on everyday activities, especially those of interest, and almost never referred to parents or textbooks as guidance. When *doing mathematics***,** four out of the six children mentioned that **understanding** was one of their main priorities, along with **correct answers**. Only Child 20 felt that the ‘application of the mathematical concepts’ was a key aim.

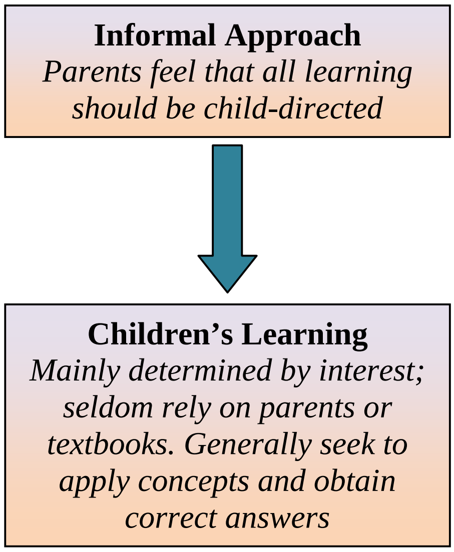


Figure 6.5: Effects of the Informal Approach on the Children’s Learning of Mathematics

6.3.2 Home-Educating Approach and Children’s Mathematics Beliefs

Six out of the seven children from informal families found mathematics **enjoyable** and **interesting**. Only Child 8 did not like mathematics - she found the subject ‘boring’. Therefore, practically all of the **children** from informal families had a **positive attitude** towards mathematics. Again, all of the children considered mathematics to be useful in many areas of life, including everyday activities, exams and their future work. Interestingly, although the parent from Family 8 did not give any value to formal mathematics exams, her daughter, Child 8, believed that it was important to learn mathematics to pass exams.

With regards to the children’s **problem solving beliefs** the responses were very mixed. Two out of the five children who answered the question viewed a mathematics problem as ‘a situation you can solve using mathematics’ (Children 10 and 20). Two (Children 8 and 26) believed they were ‘numbers with some words and a question’, and Child 9a believed a mathematics problem was ‘an exercise where you decide which operations to be done, and then perform them correctly’. With regards to their overall problem solving beliefs, Children 8 and 20 had three out of five beliefs belonging to the category of ‘good problem solvers’, but the other children only had one or two beliefs that fell into this category. Overall, the children from the informal families did not appear to have many of the characteristics of ‘good problem solvers’ as identified by Zan and Poli (1995).

6.3.3 Relationship between the Informal Approach and Mathematical Understanding

All but one of the children from the informal families felt they were good at mathematics *most of the time*. Child 8 wrote she was good at mathematics ‘some of the time’, moreover, she was the only child who did not like the subject. This indicated that the children from the child-led families were generally confident with their own mathematical abilities.

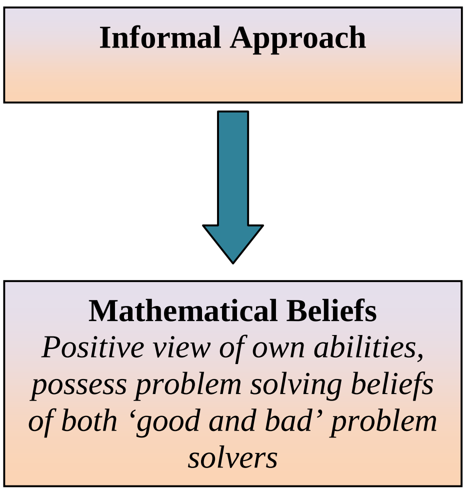


Figure 6.6: Effects of the Informal Approach on the Children’s Mathematical Beliefs

When considering ‘the signs of someone who is good/bad at mathematics’, none of the children mentioned ‘correct answers’ – instead they considered the individual’s ability to perform mental calculations and explain the concepts, the speed at which they worked and a sense that they ‘knew what they were doing’.

When measuring their *own levels of mathematical understanding*, like the children from the structured/semi-formal families, they sought a range of indications. However, out of all the children from the informal families, only Child 8 felt that the ‘parent/tutor saying they understood the concept’ was **often** a sign of understanding – none of the others regarded their parents’ ‘clarification of understanding’ as important.

Children 4 (aged 6), 9a (aged 11), 9b (aged 7) and 26 (aged 9) attempted the assessed work, although Children 4 and 9b did not complete the children’s questionnaires. From the solution strategies (Section 5.10.6), it could be observed that Child 4 showed signs of *conceptual understanding*, as **all of her solutions** used ‘worked strategies’, where she frequently identified the ‘type of problem’, then constructed a solution using a range of arithmetical skills. Child 26 demonstrated the ability to use both ‘column layouts’ and ‘worked strategies’ although he had a tendency to write as little as possible, and this made it difficult to determine the approach used.

Child 9b had little difficulty with the Group 1 questions, and as mentioned in Section 5.11.2 he used the flooring in his bathroom to help solve Q.4, Group 1, which was on shape. Similarly, Child 4 related a number of problems from Group 1 to ‘real-life’ issues, signifying that these children appreciated the applications of the mathematical concepts to everyday situations.

Child 9b also attempted a few questions from Group 3, showing an understanding of the approach needed to solve Q.5, even though he was unable to calculate the required areas. His older brother, Child 9a (aged 11), had recently taken his GCSE intermediate exam, obtaining a Grade B. Child 9a demonstrated a confidence with numerical questions, which was observed in both ‘calculation’ type questions, and when proofs were required. He appeared less familiar with algebraic proofs, and incorrectly answered three questions (Q.5, G.2, Q.7, G.2 and Q.9, G.3) that involved pictorial elements (e.g. areas, fractions etc.). His mother used to be a mathematics teacher, and as was also noticed in Child 16’s work, Child 9a’s ‘formal understanding’ of concepts was relatively advanced, using the correct mathematical notation and expressions when constructing his answers.

Now that the relationships between the three main home-educating approaches and the children’s perceptions and understanding of mathematics have been discussed, Chapter 7 returns to address the original research questions that were identified in Chapter 2 and associate them with the outcomes of data analysis within the study.

Chapter 7: Conclusion

7.1. Introduction

The key focus of this research was to gain a better understanding of how mathematical knowledge is developed through a home-educating environment. In particular, it sought to identify how different approaches to the teaching of mathematics at home could affect the children's perceptions and understanding of the subject. This chapter brings together the results from the data analysis in an attempt to address the four aims of the research, as summarised in Chapter 3:

1. Understand the main reasons for the families to choose home-education.
2. Examine the influence that parental mathematical belief/background has on the home-educating approach.
3. Investigate the different types of mathematical belief and understanding that exist amongst home-educated children.
4. Formulate relationships between parental teaching approaches and their children's mathematical beliefs/understanding.

To address the first two aims, Section 7.2 considers the main reasons for choosing home-education before examining parental background and teaching approach. Next, to address the third aim, we consider the children's perceptions of mathematics and their learning environment (Section 7.3), before discussing the topic of *mathematical understanding* (Section 7.4). Section 7.5 then addresses aim 4 by describing possible relationships between the parental home-educating approach and the children's mathematical learning. Finally, issues regarding the methodological approach (Section 7.7) and possible areas for further research (Section 7.8) are briefly discussed.

7.2 Approaches to Teaching Mathematics within the Home Educating Environment

This section covers the core issues of parental reasons for choosing home-education (Section 7.2.1) and examines the mathematical background and the philosophy of parents towards the teaching and learning of mathematics (Section 7.2.2). It goes on to examine the ways through which parents believe that home-education may help children's mathematical learning (Section 7.2.3), the parents' perceptions of themselves as teachers (Section 7.2.4) and the relationship between their beliefs and approaches in the teaching of mathematics (Section 7.2.5)

7.2.1 Reasons for choosing Home-Education

As identified in Section 1.1, two research questions informed the reasons for parents choosing home education:

- *What were the main reasons behind the parent's decision to home-educate?*
- *Was this decision based mainly on the parent's personal educational beliefs or did their children express his/her feelings to be educated at home?*

Section 4.1 indicated that the main reason for parents to choose home-education was a belief that schools could not cater for the particular academic (or social needs) of their children; that is, a home-education would offer *greater flexibility* for their children's learning. This finding supported the research of Arora (2003) and Rothermel (2000) where it was noted that 'a dissatisfaction with the state schooling system' was the chief reason for parents in the UK to choose home-education over school. Especially in instances where the children were felt to be 'ahead' of their peers or for children with Special Education Needs, parents believed home-education gave children the opportunity to learn at their own pace. It could therefore be hypothesised that parents with children who do not fall into the 'average' range of academic ability may consider home-education when they find that their child is not being taught at a suitable level in school. These parents believe that they will be able to provide an educational environment that is more conducive to their child's academic abilities.

It was also noted that parents with children who were unhappy at school due to social issues (student/teacher bullying) indicated that their children's happiness took precedent over the advantages of receiving a mainstream school education, and hence their children were taken out of school with the hope that they, as parents, would be able to provide a suitable level of education at home.

One aspect of this study that had not been addressed in previous studies of home-education in the UK was the level of influence that the *children* had in the decision to choose home-education. Section 4.3.1 showed that *age* appeared to be a determining factor with regards to the level of influence of the children, with children under the age of seven generally having little say in the matter, often being taught at home since birth. On the other hand, parents of children of school-going age (e.g. aged seven and above) were more likely to consider their children's requests to remain at home as an important factor when making the decision to take them out of school.

7.2.2 Mathematical Backgrounds of the Families and the Effect on Their Mathematical Beliefs and their Teaching

To establish how qualified parents felt they were to teach mathematics in the home situation and what their philosophy towards the teaching and learning of mathematics may be, two areas were considered:

(1) *The effect of the parent's mathematical/teaching background on their home-educating approach*

(2) *Parents' beliefs about mathematics and the effect that these beliefs may have on their home-educating approach*

7.2.2.1 Mathematical and Teaching Background of Parents

Whilst Section 4.2.1 showed that the majority of the families had formal mathematics qualifications no higher than the GCSE/O Level stage, nearly three-quarters of the sample had at least one close family member who was employed (or had been employed) in a job that required extensive applications of mathematics, for example, some had worked in accounting, engineering, and four mentioned that they had a family member who had taught mathematics in school.

Rothermel's (2002) UK sample showed that 29% of the 419 participating parents were teacher-trained (Section 2.2.5), and in this study it was found that, out of the 28 participating families, eleven parents had *formal teaching experience* of some kind (e.g. 39% of the sample), but perhaps this is due to the much smaller sample size of this research. Of these eleven parents, it was interesting to observe that only six felt the knowledge gained during their teaching experiences was useful when home-educating their own children. The others claimed that whilst *some of the methods* learnt during their teaching jobs were beneficial when home-educating, many of the techniques used to teach in school were too formal and structured to suit their own children. Consequently these parents would selectively adopt or reject elements of the pedagogical knowledge gained through 'school-teaching experiences' depending on the nature of their child's particular learning style. For example, in Section 4.2.2, it was noted that some parents who were 'teacher trained' felt that the formal methods that they were taught to implement in school were not really appropriate when home-educating, whilst others 'picked up' certain aspects of particular educational approaches, such as the use of mathematical games from Montessori, especially when these methods appeared to appeal to their children. We could hypothesise that *'the teaching at home adapted to the child rather than the child adapting to the teaching'*.

7.2.2 Mathematical Beliefs of Parents

Ernest (1991a, 1991b) and Underhill (1998) divided a mathematics teacher's beliefs into three components: (1) Their conception of mathematics as a subject for study, (2) The nature of mathematics teaching and (3) The process of learning mathematics, with a fourth component described by Underhill (1988) as (4) Beliefs about the social context of the learning environment.

Section 4.3 showed that parental *personal beliefs about the nature of mathematics* (with regards to its everyday life applications/logical aspects) appeared to be the strongest influence on their *teaching beliefs*.

The main perception of mathematics as a subject held by the home-educators was that 'mathematics is a part of everyday life', with an associated key teaching belief being that their children should learn mathematics 'in order to deal with everyday life situations'. In fact, those parents who claimed to adopt an informal/autonomous approach sometimes stressed that this was the **only reason** they felt it necessary for their children to learn mathematics. The 'logical' aspect of mathematics was also treated as important by nearly 80% of the families, indeed, it was noted that **all of the**

parents who explicitly mentioned ‘mathematics as logical’ in their descriptions of the subject gave the highest level of importance to *‘teaching mathematics as it would help their children to think in a logical way’*.

Parents whose children had had negative (particularly in the academic sense) experiences at school tended to emphasise ‘teaching mathematics in order to reduce possible fears of the subject’. For these parents, perhaps their *children’s prior experiences at school* affected their motivations for teaching the subject. Another factor that appeared to drive parental teaching beliefs, in particular, the scientific applications of mathematics, was the *employment background* of the family. Parents from families who themselves had made extensive applications of mathematics *in the workplace* (e.g. engineering) gave more importance to ‘teaching mathematics to their children due to its scientific applications’ than parents without such family backgrounds. However, it was somewhat a surprise to note that those with *mathematics teachers* in the family gave little importance to the scientific applications of mathematics. More research would be needed in order to identify the reasons for this phenomenon. *Exams* and the *school curriculum*, both identified as typical influencing factors on mathematics lessons at **school** by Leinhardt et al. (1991), were given **very little importance** by the home-educators.

The results from this study highlighted four possible categories of mathematical beliefs that most strongly affected the home-educators’ reasons for teaching mathematics:

1. Their personal beliefs on the nature of mathematics (e.g. logical, interesting etc.)

2. The areas in which the family used mathematics (e.g. employment, everyday life)

3. Previous academic experiences of their children, generally at school

4. A dislike of the school curriculum/exams

From this, it is suggested that as well as their beliefs on the *nature of mathematics*, a parent's teaching beliefs could be composed of: (1) Their perceptions on the applications of mathematics from their previous life experiences (both in everyday life and their previous employment), and (2) Their perceptions of their children's previous 'mathematical history'. Unlike a 'typical' primary/secondary school mathematics teacher, the home-educating parent may establish their mathematical beliefs from many different areas of employment over a number of years, with a consequential influence on their teaching practice. Unlike a school teacher, a home-educating parent may have (or aim to acquire) an in-depth knowledge of their child's preferred learning style, simply due to the fact that they can build up a learning relationship over a number of years.

It is hypothesised that the core *teaching beliefs* of the parents could influence the ways in which they teach mathematics. We first consider the ways in which parents felt a home-education aided their children's mathematical learning.

7.2.3 How might Home-Education Help Mathematical Learning?

Underhill (1988) wrote that a teacher's conceptions of 'what is helpful/unhelpful when learning mathematics' forms one of the key components of mathematical belief. In Section 4.10 it was found that the key cited benefit of home-education was that it offered children the flexibility to learn mathematics at their own pace. In addition, parents felt that their children had the chance to understand each topic before moving onto something new, and could apply their mathematical knowledge to everyday life situations, as and when they occurred – for example, when cooking, shopping or helping their parents' business transactions. Other advantages included one-to-one attention, less pressure, and the utilisation of a range of learning activities.

Whilst the disadvantages cited by the parents were few in comparison, some mentioned that 'learning from other children' was important but this was hard when they had an only child. Others worried that their level of mathematical knowledge was only just ahead of their child's, and that they might accidentally 'miss' teaching an important concept, due to their own lack of mathematical knowledge.

Leinhardt et al. (1991) wrote that one of the key factors that influenced mathematics lessons in schools was the teacher's knowledge of *'the mathematical goals to be accomplished'*. In order to establish the main criteria that guide their particular teaching approach, we shall next discuss the aims of the parents when teaching.

7.2.4 Parental Aims, and Their Perceptions of Themselves as Teachers

Section 4.5 showed that the majority of parents frequently aimed to *'provide mathematical learning experiences through everyday experiences'*, an aim that was also related to the perception that mathematical knowledge is 'needed for everyday life'. In terms of frequency, the next most common aim was to ensure that their children *fully understood* each concept. But it was also interesting to note that nearly half of the parents who emphasised the 'full understanding' of each concept were also prepared for a limited level of understanding in certain areas if this meant that their children could be exposed to a range of different mathematical concepts. All of the 28 parents in this study encouraged their children to *learn independently* – however, this was less common amongst families where their children had Special Educational Needs. Perhaps these parents believed their children required additional support when learning mathematics.

Going back to the benefits of home-education to their child's mathematical learning, it can be seen that both the aims of the parents, and the 'cited benefits of home-education' highlight the importance of mathematical understanding and the application of mathematics in everyday situations. One might also postulate that a degree of flexibility is offered when the parents encourage their children to learn independently – *'when you learn mathematics by yourself, you set the pace at which you cover the material'*.

In order to get a clearer picture of how the parents might translate their teaching beliefs into teaching practice, we now examine the parental teaching approaches with respect to Askew et al.'s (1997) three types of teaching orientation, beginning with 'Transmission':

(a) Transmission: 'Projecting the belief that mathematics is a series of rules and truths, where mathematics is conveyed through an instructional approach until fluency is attained.'

It was observed that while a third of the parents regularly demonstrated concepts, the same fraction ‘sometimes’ did this, and the rest never did so. The majority believed that following a curriculum (or the use of a structured teaching method) was rarely/never important when teaching, suggesting that wholly instructional approaches were seldom used. The tendency amongst all home-educators to encourage independent learning also gives support to the conclusion that although a small proportion of parents may possess some elements of the ‘transmission’ orientation, there was little evidence to suggest that any of the parents in this study followed such an approach in its entirety.

(b) Discovery – ‘Mathematics is viewed as a human creation, where students are encouraged to learn through individual exploration and reflection. Teachers are seen as facilitators.’

Section 4.5.4 showed that half of the 28 parents in this study generally saw themselves as the ‘facilitators of learning, where they provided experiences for the children to construct ideas for themselves’. Given that all 28 parents also encouraged their children to learn independently, this suggests that at least half of the families in this study could be described as having a ‘discovery’ orientation. Of the six parents who rarely or never viewed themselves as facilitators of learning, half were structured in that they generally adopted a workbook/curriculum based approach. This could indicate that parents following a structured approach were less likely to have a discovery orientation. The remaining three parents were completely autonomous/informal, hence *all of their children’s learning was child-directed* – and so these parents did not see the need to facilitate mathematical learning or guide their children in any way.

(c) Connectionist – 'Mathematics is seen as a network of ideas that the students and teachers construct together through joint discussion, where the teacher also aims to challenge the student's thinking'.

It was observed that approximately half the parents frequently asked their children to justify and explain their reasoning and all 28 parents encouraged their children to share their ideas and opinions of a concept. Moreover, three-quarters of the parents frequently initiated mathematical discussion with their child when they themselves had difficulty understanding a concept, signifying a relationship of 'mutual learning'.

In summary, the results indicate that the majority of home-educating parents in this study adopted a teaching approach that suggests elements of *both the discovery and connectionist orientation*, whilst there was little evidence to support prevalence of the transmission orientation.

7.2.5 Relationship between Teaching Beliefs and the Parent's Approach to Teaching Mathematics

The results discussed in Sections 4.5.7 and 4.6.8 identify four possible relationships:

(1) Parents who believed 'mathematics is useful for everyday life' tried to follow a teaching approach where their children were encouraged to learn mathematics through everyday activities.

(2) Those with the belief that their children should learn ‘the mathematics that is covered in school’ (i.e. the National Curriculum) would often use a curriculum when teaching.

(3) In contrast, the majority of the parents did not believe the school curriculum was an important influence when teaching, so these parents rarely (or never) adhered to a curriculum.

(4) Those who identified themselves as informal/autonomous stressed that all of their children’s mathematical learning was child-directed, and hence the teaching approach followed their child’s interests as the primary, and perhaps only, driver. Such parents were often entirely flexible as to when their children learnt mathematics.

Whilst there was evidence to support the above relationships, it was noted that of the 21 families who were home-educating more than one child, three-quarters *adjusted their teaching approach according to the child that they were teaching*. For example, some parents had one child who was a visual learner, whilst their sibling preferred hands-on activities, and another child favoured workbooks. These parents displayed a willingness to alter their teaching activities according to the particular learning styles and interests of each child, irrespective of their perceptions of mathematics or personal teaching beliefs. Other factors that could result in a change of teaching approach included: the age of children, levels of understanding reached or the future aims of the child (e.g. formal exams or ambitions to study at university). Through these results we can again observe the ‘flexibility of the home-educating approach’.

The parents' mathematical backgrounds, their beliefs and the chosen teaching approach may well have an effect on their children's perceptions of mathematics and their learning environment and it is to these issues that we now turn.

7.3 Children's Perceptions of Mathematics and Their Learning Environment

Two of the research questions, identified in Section 1.1 (p.13), considered the relationship between the parents' philosophy towards mathematics and their children's beliefs:

- *Was there a relationship between the parental mathematical beliefs and their children's perceptions of mathematics?*
- *Was there a relationship between the parental perceptions of home-education and their children's perceptions of home-education?*

Section 7.3.1 draws conclusions from the evidence identified to address the first of these two questions whilst Section 7.3.2 considers the second. Consideration is then given to the way in which parents' and children's beliefs and perceptions guided the nature of the children's mathematical activities (Section 7.3.3).

7.3.1 Relationships between the Parent and Children's Mathematical Beliefs

Section 5.6 showed that the prevalent perception of mathematics held by the children was that *'mathematics is useful for everyday life, and in its application to other subjects'* – which was the same as the parents' primary reason for teaching mathematics - so that their children *'could learn to deal with everyday life situations'*.

It was noticed that two-thirds of the children in the study *enjoyed learning mathematics*, and again parental beliefs were a possible influence, since nearly every parent who enjoyed mathematics had children who shared these beliefs. On the other hand, no such relationship was evident when considering those parents who had a negative view of mathematics – so perhaps additional factors, such as other close family member's mathematical beliefs or the teaching approach should be taken into account. Other parental mathematical beliefs, including an interest in the subject, and the belief that mathematics is 'logical' also appeared to result in similar beliefs expressed by their offspring.

7.3.2 Children's Perceptions of Their Learning Environment

In Section 7.3.1 we examined the parents' views on how home-education benefited their children's mathematical learning. From the children's perspective, the main advantage of home-education was the chance to learn mathematics at their own pace (see Section 5.4) – a view also expressed by the parents. Another common view held by both children and parents was that the mathematical learning was more relaxing at home.

At the same time it was interesting to observe that while around two-thirds of the parents in this study felt there were no real disadvantages to their children's mathematical learning at home, three-quarters of the children indicated specific areas where they felt they could improve their learning. Thus even though the majority of parents were generally happy with their children's mathematical learning at home, the children were still able to identify a number of areas where they felt they could improve their learning.

7.3.3 What Guided the Children's Mathematical Activities?

Now that the children's perceptions of mathematics and their learning environment have been discussed we attempt to answer the question:

- *What were the main influences on the children's learning activities at home?*

Section 4.6.8 showed that for the majority of the parents, their children's interests and levels of enjoyment were the key criteria when choosing appropriate learning activities, and these parents tended to have children who also believed that their mathematical learning was primarily guided by their personal interest.

A quarter of the parents mentioned textbooks, and three-quarters of the children also quoted the occasional use of textbooks, with a third currently using textbooks at the time of this research. Whilst the majority did not strictly adhere to textbooks from a curriculum, it appeared that most parents found textbooks to be a valuable resource for introducing new concepts, to provide practice questions, and giving some structure when necessary – for example, when their children were studying for formal exams.

From the children's perspectives, other common factors that influenced their choice of activities included:

- A focus on areas that required greater understanding
- Learning mathematics through everyday activities

Note how both of the above criteria reflect the *aims of the parents* that were highlighted in 7.3.2. Overall, there is strong evidence to show that the main elements that guided the parental teaching approach were also demonstrated in their children's perceptions of their learning activities.

7.4 Parents' and Children's Perceptions of Mathematical Understanding

We now return to final key area of this research - the area of *mathematical understanding.* Responding to this issue, this section is subdivided into three main sections, beginning with the parents' notion of understanding (Section 7.4.1), their children's notions of understanding (Section 7.4.2) and the types of mathematical understanding that could be observed through the children's problem solving approaches (Section 7.4.3). This latter subsection is sub-divided into four sections, the children's notions of problem solving (Section 7.4.3.1), the understanding observed from the children's solutions to Group 1 questions (Section 7.4.3.2), Group 2 questions (Section 7.4.3.3) and finally the Group 3 questions (Section 7.4.3.4).

7.4.1 Parents' Notions of Understanding

In Section 4.3, it was noted that a primary aim of the home-educators was to ensure that their children reached a 'sufficient' level of understanding for each concept that they were learning. The question is:

- *How do parents determine the level of understanding reached by their child?*

Section 4.11 showed that two-thirds of the parents used *more than one measure* to determine their children's level of mathematical understanding, the most common measure being their child's *application of the concept*, often in a variety of situations (e.g. everyday life). Whilst one might argue that the repeated application of a concept to the same situation may only indicate an instrumental level of understanding (Skemp, 1976), it was noticed that some home-educators mentioned a range of situations in which they expected their child to apply the concept, indicating that these parents may have been seeking the development of a relational level of understanding. To support this notion, it was also observed that 58% of the parents asked their children to *justify and explain their reasoning*, i.e. ‘the parents wanted the children to explain *why* they had taken a particular approach’. In fact, all 28 parents would regularly discuss mathematical concepts with their children in order to obtain their child’s viewpoint (Section 4.12). For three-quarters of the parents, these discussions were a way of improving their own knowledge of a concept, thus a two-way learning exchange was taking place. Such interactions may encourage their children to ‘construct’ their own notions of a concept, because rather than the parent being seen as ‘the source of all knowledge’, the parent acts as a fellow learner; there to challenge, discuss and learn the concepts alongside their child.

A third of the parents believed that the *independent application* of the mathematics would indicate understanding – that is, once their child could answer the questions without support, it was deduced that they had understood the concept. However, one could argue that a child could successfully work through a number of questions with only an instrumental level of understanding, so to gain a greater insight into the mathematical understanding of the children, we asked:

- *How did the children measure their levels of understanding?*

7.4.2 Children’s Notions of Understanding

For the home-educated children in this study, the **most important measures of understanding** were a feeling of confidence and ‘*knowledge* of each part of the formula or method'. The majority also believed that an inability to explain the method used was a strong indication that they had not understood a concept. The **least important sign of understanding** was 'the parent saying that they (the children) understood the concept'. Indeed, Section 5.9.2 showed that when unable to understand a concept, nearly half of the children in this study would reconsider the concept *by themselves* before seeking parental aid, while a fifth would turn to alternative resources, such as a book or the Internet. Nevertheless, one should view these results with caution, as Table 5.5 showed that the vast majority of the children in this study felt *all the mentioned criteria* were important signs of understanding. Only children from structured families consistently identified fewer important signs of understanding.

In an attempt to determine the different types of mathematical understanding that arose through the various home-educating styles, we now turn to the children's notions of problem solving and their solutions to the assessed work.

7.4.3 Types of Mathematical Understanding Observed through the Children's Problem Solving Strategies

This section draws conclusion to the following questions:

- *What were the children's notions of problem solving?*
- *What types of mathematical understanding could be observed through their answers to the assessed work questions?*

7.4.3.1 Children's Notions of Problem Solving

Section 5.8 showed that most children felt that it could take longer than 10 minutes to solve a mathematics problem and that there could be more than one correct answer.

The majority perceived a mathematics problem to be ‘any situation that can be solved using mathematics’. This belief appeared to be age-related, as most were aged 10 years or older; furthermore, none of the children from this age group believed that ‘a mathematics problem is numbers with some words and a question’. On the other hand no patterns of response could be observed for children under ten years of age, and age did not appear to be a factor for the other problem solving beliefs.

Apart from these observations, no evident relationships between the children’s mathematical beliefs, their parent’s beliefs, and the children’s problem solving beliefs were observed. In an attempt to distinguish the different types of understanding that could be identified through their problem solving strategies we consider the conclusions that may be drawn from the children’s responses to the three groups of questions.

7.4.3.2 Types of Understanding Observed Through the Children’s Problem Solving Strategies for the Group 1 Questions

From the Group 1 questions, which were set at the KS1 level and attempted by eight children (with an average age of 7 years), it was noted that the average mark across all questions was 86%. The three oldest children who attempted this set of questions (aged from 8-9 years) all achieved 100% on the test, but the level of accuracy achieved on the test did not necessarily imply the ‘quickest’ method of solution.

For the *arithmetic/arithmetical word problems* all eight children appeared relatively competent, with only 6 mistakes made across a total of 40 questions. Table 5.10 shows that a variety of solution strategies were used by all except Child 22, indicating that most had developed an understanding of arithmetic that could be applied in a range of situations, i.e. they had, or were developing, a relational understanding of arithmetic. Only one child, Child 22 (aged 5) always adopted a procedural 'step-by-step approach' for these questions, so perhaps she had only reached an instrumental/procedural understanding of basic arithmetic, as described by Skemp (1976) and Hiebert and Carpenter (1992) and perhaps, as Sfard (1991) describes, an indication towards an operational understanding of arithmetic. However, it is suggested that she is operating at a more sophisticated stage with the variety of questions than many of her peers would be at the same age.

It was hard to determine the different 'types' of understanding for the *shape* questions, since nearly every child (apart from Child 4) wrote the answer down without showing any working. However, across all 16 questions (in total, as there were two questions on shape for each of the eight children) only 3 mistakes were made. Since the questions required knowledge of distance (Q.4) or symmetry (Q.8) (see Appendix 6), where it was necessary to apply each concept in a situation that required more than simply identifying lengths/identifying lines of symmetry, it might be proposed that the children had achieved a relational understanding of such concepts.

For the questions that involved the application of arithmetic to a real-life situation, once again, only Child 22 used a procedural approach. Further investigation into this type of problem and the thought processes that the children went through when selecting an appropriate strategy would be required before a statement could be made regarding the types of understanding used – because as noticed in Child 4's narrative for Question 3, Group 1 (Figure 6.31), one can only really 'see' how the child reaches a solution strategy by engaging in a discussion with the child.

7.4.3.3 Types of Understanding Observed Through the Children's Problem Solving Strategies for the Group 2 Questions

Four children aged from 8 to 11 years of age attempted the Group 2 questions, obtaining an average mark of 86%. For this set of questions, there **did not** appear to be a relationship between accuracy and age – it was the 11 year old who made the most mistakes, getting two questions wrong (see Table 5.12).

For the arithmetic questions, which both involved fractions, whilst all of the children were able to determine the larger fraction (for Question 4, Group 2), two made mistakes on Q.7, where they had to use the visual representations to calculate the size of the shaded area. One might suggest that they had a relational understanding of fractions when using symbolic representations, as illustrated in Child 9a's answer (Figure 5.36). However, the children did not appear to have a strong multi-representational view of fractions in that they had not yet reached this level of understanding for an associated pictorial image. A further investigation into this area would be needed to determine if this is the case.

The children all appeared competent with the questions on symmetry and angles (apart from Child 26 who did not attempt this question). However, two were unable to solve Question 5, which required a level of understanding of area that encompasses the fact that 'area is the space enclosed by the shape', rather than simply the product of two lengths.

All four children were able to solve the questions on Logical Thinking and Algebra with little difficulty, suggesting that they, like the children in Group 1, were confident in applying their arithmetical knowledge to a range of situations and held a relational understanding of this area of mathematics.

7.4.3.4 Types of Understanding Observed Through the Children's Problem Solving Strategies for the Group 3 Questions

Eight children (aged from 8 to 17 years) attempted the questions from Group 3 (set at the KS3 Level), obtaining an average mark of 64%.

Apart from Child 16 (aged 8) those under the age of 10 years struggled with most of these questions (see Section 5.13). However, for the arithmetical and logical thinking questions, even the youngest children were successful in answering these questions, again showing the home-educated children's arithmetical strengths. Age was a discriminator for the question on angles – all of the children (apart from Child 16, aged 8) who were able to successfully answer Question 7 were aged 14 or older. The questions on algebra identified those children who were able to generalise a proof, and those who could only provide justification using a particular solution – only Child 11b was able to successfully construct a proof.

In conclusion, the children's answers to the three groups of assessed work showed that the majority possessed relational understanding in many fundamental arithmetical concepts, whilst they had varied levels of understanding across the other areas of mathematics. To distinguish how the parental home-educating approach could play a role in determining the methods used to solve the problems, we turn to the final research question, which was to address the links between the home-educating approach, the children's mathematical beliefs, and the ways in which they do mathematics.

7.5 Influence of the Parental Home-Educating Approach on the Children's Mathematical Learning

In Chapter 2 (Section 2.2.11), we saw how Boaler (1998) considered the teaching practices of two schools, where one school followed a 'closed' traditional textbook approach, whilst the other predominantly used open-ended 'project based' activities. Boaler observed that students who followed the traditional approach developed a procedural, rule-based understanding of mathematics that was of limited use in mathematics problems that were 'not typical of their textbook questions'. Students in the open, project-based learning environment developed a conceptual understanding that gave them the ability to comprehend and apply their mathematical knowledge to a range of assessments and situations. Similar findings may be associated with children from home-educating families, evidenced as follows.

In my research, three different categories of home-educating were established, and the particular families belonging to each category were identified in Chapter 5. This chapter also contained a detailed, case-study analysis of the relationships between the families' approach and the children's perceptions and understanding of mathematics. In line with the phenomenographical nature of this study, Chapter 6 then discussed how the particular features of each home-educating approach resulted in different perceptions and 'types of understanding' in the children. Summarising the results from Chapter 6, the following observations can be made:

- **Relationship Between Home-Educating Approach and Choice of Activities:** Children with parents who followed a *Structured Approach* tended to learn via activities that were determined by either the chosen curriculum or their parents. Families who adopted a *Semi-Formal* home-educating approach generally encouraged their children to suggest mathematical activities according to their personal preferences, with guidance from the parents. But children from *Informal/Autonomous* families seldom relied on textbooks or their parents to determine the mathematical learning activities – it was the children who had full control over what they learnt, and when.

- **Relationship Between Home-Educating Approach and the Children's Mathematical Beliefs:** For the children from *Structured* families, their mathematical beliefs tended to be determined by the range of problems that they encountered, with half of those from this group being unable to rate their own level of mathematical ability. On the other hand, children from *Semi-Formal* families tended to hold a positive view of their mathematical abilities and many of their beliefs fell into the category of a 'good problem-solver', as identified by Zan and Poli (1995). Children from the Informal/Autonomous families also tended to have a positive view of their level of mathematical ability, but possessed problem solving beliefs of both good and bad problem solvers.

- **Relationship Between Home-Educating Approach and the Children's Mathematical Understanding:** As noted earlier, Boaler (1998) observed that students following the traditional approach developed a procedural, rule-based understanding of mathematics. Of the three children from the

> *Structured/Formal* families, one child displayed signs of both instrumental and relational understanding in her work whilst the others showed a competency in solving questions that required procedural answers, but there were signs that they still had a procedural level of understanding for areas such as algebra.
>
> Boaler (1998) noted that students in the open, project-based learning environment developed an understanding that gave them the ability to comprehend and apply their mathematical knowledge to a range of assessments and situations. For the six children from the *Semi-Formal* families, three showed clear signs of conceptual understanding, especially in their arithmetical solutions. On the other hand, two children showed a preference for procedural solution strategies. Children from the *Informal/Autonomous* families almost never used procedural methods – they either wrote down the answer directly or constructed their own solution strategy using their knowledge of the concept. Two related their solution strategies to real-life experiences.

Boaler's (1998) study showed that the way children learned mathematics resulted in the development of different forms of understanding. The results of this study support her research, as there is evidence to suggest that the majority of the children learning in a Structured/Formal home-educating family have developed a procedural understanding of mathematics, those from Semi-Formal families tended to use both procedural and conceptual methods of solution and hence possessed both types of knowledge, whilst children from Informal/Autonomous families demonstrated a conceptual knowledge of mathematics, almost never adopting procedural methods of solution. There were also instances when the children from Informal families related their answers to everyday situations, suggesting that their mathematical understanding may have been developed by learning through real-life situations.

It was difficult to compare the general levels of achievement for the tests because children from informal families tended to be ten years or under, whilst all the children from structured families were 14 years or older, and hence only attempted the Group 3 questions. However, a rudimentary analysis of the results showed that for the Group 1 questions, the average mark for those from informal families was 85%, whilst children from semi-formal families obtained a similar average of 86%. However, the Group 2 questions showed an average mark of 78% for children from informal families compared with an average of 94% for those from semi-formal families – indicating that children from semi-formal families were, on average, outperforming those from informal families when the level of difficulty was increased.

For the Group 3 questions, the average age of the children from informal families who attempted this set was 9 years, so perhaps not surprisingly, they found these questions rather difficult, only obtaining an average mark of 41%. On the other hand, the average age of those from structured families was 15 years, and their average mark was 80%. But the children from semi-formal families obtained an average mark of 75%, even though the average age was only 11 years.

7.6 The Children's Future Aims

Practically all of the children believed that mathematics would be useful in their future, some emphasising its use in everyday life, jobs, and its application to other subjects. Whilst only two of the parents mentioned a specific mathematical target for their children, 42% of the children had a goal or target that they wished to attain, giving support to the parents' general view that 'it was up to the children if they wished to take exams'. In fact, nearly a third of the children mentioned that they were targeting a good grade in their GCSE/IGCSE/A-level's. Two children had already taken their exams at an early age, where both had obtained 'B' grades in their GCSE mathematics exams, one at the age of 13 years, and the other at 11 years of age.

7.7 Methodological Issues

This thesis used data from questionnaires and mathematics tests, both of which were distributed over the Internet and in the case of the questionnaires, returned via email. The use of the Internet to contact families and collect data, whilst an advantage in many ways (being cost effective and the ease of communication) introduced bias as the sample was restricted to those with Internet access. Whilst some might argue that home-educators with such access are not fundamentally different to families without the Internet (Smith and Leigh, 1997), a more representative sample could be obtained by contacting families via other means, such as local home-education groups.

With regards to the parents' questionnaires, it was noted that some families who were home-educating more than one child adopted a range of different teaching approaches depending on the child in question – for example, some parents made a point of mentioning that their aims when teaching mathematics to Child X were quite different to their aims when teaching their sibling, Child Y. Thus a possible improvement to the questionnaire could encompass:

- General questions which are applicable for all children in the family, e.g. reasons for home-educating, mathematical beliefs and background of the parent.

- Specific questions dependent on the child in questions, e.g. “Do you adopt the same/different approaches for the children in your family?”, “If so, please complete each set of questions for the particular child in question” – in this case, extra pages would be included for each child in the family.

One disadvantage however is that it could require a lot of extra writing from the parent – one family in this study had seven children who all appeared to have quite different learning preferences!

When attributing particular descriptive categories to the parents and children (e.g. the description of semi-formal etc.), it should be noted that extent to which each participant belonged to the category could vary. For example, suppose the mother in Family X teaches mathematics at the same time every day, uses a set mathematics curriculum for all her teaching (through textbooks), and allows very little input from her children. The mother in Family Y also follows a curriculum, uses textbooks for most of the teaching, and sometimes follows a timetable, with occasional input from her children. In this study both Family X and Family Y would have been classified as ‘structured’, although it is clear that the mother from Family Y is more flexible in her use of this approach. Thus when analysing the data, rather than simply categorising the parents or children as belonging to a particular type of home-educating family, it may be useful to measure how strongly they exhibit a particular characteristic. E.g. for the categorisation of home-educating approaches one might construct the following diagram, where the mother in Family X would favour the 'Very Structured' category more so than the mother from Family Y, who demonstrates some semi-formal tendencies:

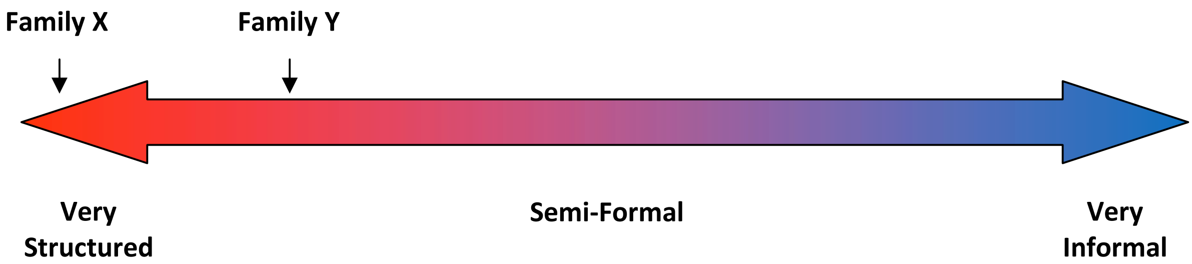


Figure 7.1: Diagram to Illustrate Range of Home-Educating Approaches

Those families who exhibited a number of 'structured' traits, such as timetables, curriculum use and instructional work would tend towards the red side of the diagram, while the more 'informal' families would be situated in the blue area. Semi-formal families, with a quite 'balanced' mix of structured and informal teaching would lie in the central, purple region.

It was noticed that it would have been useful for: (a) All of the children whose parents had answered the parental questionnaire to answer the children's questionnaires, and (b) For each child to attempt both the questionnaire and the assessed work. This would have made it easier to identify possible relationships between the parents and children, and between each sibling within the family.

Due to time constraints, it was not possible to do more than a rudimentary analysis of the responses to the fictional case studies (see Section 3.3.7).

However, it was felt that the results could provide a useful indicator of the parents' main home-educating philosophies and so a selection of illustrative comments were used throughout the case studies in Chapter 5, in order to identify the parents' general views towards mathematical learning.

When considering the responses to the assessed work, it was noticed that some children attempted questions from both Groups 1 and 3, but not Group 2! Others only attempted questions from Group 1, although it was felt that they could have made an attempt at the Group 2 questions. Upon seeing the initial solutions to the first group of attempted questions, perhaps one could have encouraged the children to try the second group of questions to allow for a more comprehensive measure of their level of understanding.

7.8 Further Research

With regards to the generalisation of this study, Strauss and Corbin (1990) believe that in order to truly examine if a theory developed in one situation will also apply to another, it is necessary to conduct further research. Possible areas for extending this research include:

- Identifying families on the basis of their different approaches to home-education (e.g. structured, semi-formal, informal), and then conducting a detailed analysis of their mathematical understanding through both clinical interviews and assessed work, as this would enable the researcher to examine the thought processes behind the children's mathematical reasoning.

- Constructing a series of tasks involving: (a) Questions that require a straightforward ‘procedural’ application of mathematical concepts, (b) Questions that require a conceptual understanding of these concepts, and finally (c) A practical ‘real-life’ problem that requires an application of the concept in an everyday situation. Whilst the assessed work in this study contained elements of all three question types, no child from a structured family attempted questions from Groups 1 or 2, so their answers could not be compared against those from informal/semi-formal families. Asking the same set of questions to children of a similar age/ability from different home-educating backgrounds would enable a better comparison to be made.

- Undertaking a longitudinal study of families from each of the three different home-educating types (structured/semi-formal/informal) in order to investigate how the children’s understanding develops over the years, and the ways in which the home-educating style could affect this development. One could also consider whether the parents’ approaches change over the years, and the overall mathematical understanding attained at the end of the home-education period.

This study has shown that whilst many home-educating parents may share similar beliefs regarding home-education and the nature of mathematics as a subject, their different teaching beliefs, and the range of teaching approaches, can lead to varied perceptions of mathematics and different ‘types’ of mathematical understanding in their children.

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Appendix 1: Questionnaire for Parents

General Home-Education Questions

The following questions are just general questions on home education, and it is not essential that you answer them. However they are useful in giving me an overview of the home-educating environment, and so I would be grateful if you could complete this section.

1. How many children do you have?

Please write down their ages in the boxes below, and indicate whether they are being home-educated:

	Age of Child	Home educated	Age that they began home ed.	Not Home educated
1st Child				
2nd Child				
3rd Child				
4th Child				
5th Child				

2. What were the main reasons behind your decision to home-educate?

3. Was this decision based mainly on your own personal educational beliefs or did your child express his/her feelings to be educated at home?

Mathematical Activities

1. How old were your children when you first introduced them to mathematics?

2. Please indicate in the table below, the approximate ages at which you used a particular representational form.

	0 - 4	5 - 7	8 - 11	12+
Songs				
Games				
Stories (telling a story and asking your child to relate the content to you, i.e. getting your child to remember key facts and figures)				
Cooking				
Car Journeys, bus journeys, train journeys, learn about time, cost, etc.				
Objects (e.g. money, dice, counters etc.)				
Computer Programs				
Television Programs				
Pictures				
Drawings				
Diagrams				
Graphs				
Numbers				
Algebraic statements				
Other (Please specify here)				

Please give an example of the way in which you used one of the activities in the table above to teach a mathematical idea.

3. What guides your choice of a particular activity to teach mathematics?

4. What circumstances might encourage your teaching activities to change?

5. Are the same approaches used for all of the children in your household?

YES ☐ **NO** ☐ **(Please check)**

How might they differ?

6. What mathematics topic is your child currently learning, and which activities are they using during this time?

7. What signs do you look for in your child's thinking to show that he or she understands the mathematics that you've just taught them?

8. Which of these statements describes you best?

Use the numbers below to give a ranking to the following statements:

- **1 = Very much like me**
- **2 = Often like me**
- **3 = Sometimes like me**
- **4 = Rarely like me**
- **5 = Never like me**

Statement	Ranking
I try to provide mathematical ‘learning opportunities’ or resources for my child to discover or construct mathematical ideas for themselves	
Children won’t really learn the material unless I cover it in a structured way	
It is my aim to demonstrate the mathematics to my child	
The most important part of the lesson is the content of the curriculum	
I aim to provide mathematical learning experiences through everyday experiences	
I allow my child to learn mathematics by themselves, independently of me	
Children should always understand what they are learning, i.e. it should ‘make sense’ and encourage thinking	
It is useful for students to become familiar with many different areas of mathematics even if their understanding for now is limited.	

Why Teach Mathematics?

1. What does mathematics, as a subject, mean to you?

2. Do you give your children any motivational incentives or targets with regards to their mathematics?

Yes ☐ No ☐

3. What are these incentives? (Please write them in the space below).

4. Below are some reasons that you might have for teaching your children mathematics. Please give a number from 1-7 in terms of their importance to you as your child's teacher. 1 = MOST IMPORTANT REASON 7 = LEAST IMPORTANT REASON. You may use the same number twice if they have equal importance

"My child should learn mathematics because…"

Mathematics is an interesting subject ☐

We all need to know some mathematics to deal with everyday situations ☐

It helps children to think in a logical way ☐

I don't want them to be afraid of the subject, as they grow older ☐

Most other scientific disciplines require mathematics ☐

They need to pass exams ☐

It is a subject that is covered in every school curriculum ☐

5. Are there any other reasons for which you teach your children mathematics?

6. How often is your child taught mathematics by you or another person? Please indicate if you use a flexible timetable.

7. How often does your child spend learning mathematics 'informally' through everyday activities?

8. When you ask your child a series of questions, your aim is to:	**Never**	**Rarely**	**Often**	**Always**
See if they know the correct answer				
Get them to justify and explain their reasoning				
To allow them to gain confidence				
To solve a problem in an everyday situation				
Find out if they are paying attention				
Give them the opportunity to direct the lesson				
Discover their ideas and opinions				
Help you to understand something better as well as your child				
Find out what is interesting about the mathematical topic				

9. In the table below, please indicate the curriculum that you are following (tick all that apply):

Curriculum	Never	Rarely	Sometimes	Often	Always
Your own					
Alternative curriculum					

Please write down the name of any mathematics textbooks that you have found useful:

How did these books help your child to learn at home?

10. In what ways do you think the home environment helps your children to learn mathematics?

11. Are there any disadvantages when teaching your child mathematics at home, and if so, what are they?

12. Is it a good idea to take mathematics exams (A-level/GCSE's) early?

Please indicate your highest level of mathematics qualification by ticking the appropriate box.

Up to O-Level/GCSE ☐ **Advanced Level** ☐

Degree Level (i.e. have used a high-level of mathematics in university or college degree e.g. engineering degree etc.)

Other (please write in box provided):

Does any one in your family work (or formerly worked) in a job that is mathematical or numerical in nature?

Do you think that your experiences of mathematics were an advantage/disadvantage when home educating?

Have you any formal teaching qualifications? If so, what effect has this had on your approach to home-education?

"Joe is six years old, rarely interacts with other children, but asks very interesting questions. He would like to talk to his parents the whole time if possible, and to share his ideas with them. His interests are very different and at a more sophisticated level to his peers, although he is dreamy and solitary most of the time.

Joe is very able in most subjects, especially mathematics. He can predict number sequences, cope with complex rules, and understands time (24 hour clock). Joe enjoys problem-solving activities, but during these sessions he lives in a world of his own without any interaction."

Which of these options would you suggest for Joe?

Option	**Yes/No**	**Reason for choice**
Teach Joe mathematics that is for children one or two years above his age group		
Get a mathematics tutor for Joe		
Make arrangements to prepare Joe to take his GCSE Mathematics exam when he is nine years old.		
Give him extra mathematics problems for him to solve in his own time		
Make him apply his mathematics to real-world situations		
Join the local mathematics club		
		Other suggestions? Please write here:

"Richard is nine years old, and is very good at mathematics. You may be surprised at that because you will see very little writing in his mathematics book, indeed he doesn't like writing in general. He tends to do sums in his head and just writes the answers down. The answers are nearly always correct, but he writes no method of solution, and rarely explains his reasoning.

On the other hand, he loves problem solving – he can often think of three or four different approaches to the same problem."

Which of these options would you suggest for Richard?

Option	Yes/No	Please give a reason for your choice
Richard very good at mathematics and can take care of himself. Leave him alone to fulfil his talent		
You need to assess his knowledge and skills to see if he is able to undertake calculations efficiently.		
Encourage him to undertake more problem solving tasks		
Richard needs to improve his written and language skills before he can be given extra work in mathematics		
His written work could be developed through his mathematics		
His mathematical ability will be underdeveloped if it is not nurtured right away		
		Other suggestions? Please write here

Appendix 2: Questionnaire for Children

Please try and answer all of the questions, hope you have fun and remember that for most questions, there is no "right or wrong answer"! [2]

1. Mathematics problems are always solved in less than ten minutes. Is this true or false?

True ☐ **False** ☐

2. Please give an example of a mathematics problem (you may write as much in the box below):

☐

3. Tick <u>one statement</u> that you believe is the best description of a mathematics problem:

A mathematics problem is numbers with some words and a question ☐

A mathematics problem is a situation you can solve using mathematics ☐

A mathematics problem is an exercise where you decide which operations to be done, and then perform them correctly ☐

A mathematics problem is an exercise during a mathematics lesson ☐

4. Does there exist a mathematics problem without numbers?

Yes ☐ **No** ☐

5. All mathematics problems only have one correct answer

True ☐ **False** ☐

[2] Page 1 of Questionnaire

6. Tick one statement that you agree with most. [3]

When solving a mathematics problem, is it worse to:

Make a calculation error ☐

Choose the wrong method or operation ☐

It's the same, there is no difference ☐

7. Which of the reasons below are important when you understand something in mathematics? Give the reasons a number from 1 to 5, where

1 = Not important at all
2= Rarely important
3= Sometimes important
4= Important most of the time
5 =Always important

Reason	Number
Parent/teacher says I understand the mathematics	
Can see a 'pattern' in the mathematics	
Answers are all correct	
I can do the questions without help from my parent/teacher	
Have memorised the formula or method	
I know how each part of the formula or method works	
Can explain the mathematics to another person	
The mathematics can be used in a real-life situation	
There is a connection to some mathematics I know already	
I feel confident	

[3] Page 2 of Questionnaire

8. Which of the reasons below are important signs when you <u>cannot understand something in mathematics?</u> Give the reasons a number from 1 to 5, where[4]

1 = Not important at all
2= Rarely important
3= Sometimes important
4= Important most of the time
5 =Always important

Reason	Number
The formula is hard to remember	
My parent/teachers says that I don't understand the mathematics	
Most of my answers are wrong	
It is hard to explain the mathematics	
Cannot see how the mathematics is used in real-life	
I can't see how the mathematics is connected to any other mathematical idea	
I am afraid that I will make a mistake	
The formula/method is too hard to remember	
I can't explain how the formula/method works	
I get stuck all the time without help from my parent/teacher	

9. Which of the reasons listed below is the most important when doing mathematics? 1 = The most important, 2 = The second most important, 3 = The third most important, 4 = The fourth most important, 5 = The least important. Put a number in the box next to each of the reasons.

Finishing my work quickly ☐

Getting all of my questions correct ☐

Memorising the formula or method ☐

Understanding how the mathematics works ☐

Being able to apply the mathematics and 'see how it works' ☐

[4] Page 3 of Questionnaire

1. What words would you use to describe mathematics? Please write your thoughts in the box below.[5]

2. What do you think of the following statements?

Statement	Strongly Agree	Agree	Neutral	Disagree	Strongly Disagree
Mathematics is just about numbers					
Mathematics is interesting					
We need mathematics for everyday life					
Mathematics is useful for other subjects					
Most people do not like mathematics					
I do not like mathematics					
It is important to learn mathematics to pass exams					
I enjoy mathematics					

3. Do you think that the mathematics you learn will be useful to you when you are older?

[5] Page 4 of Questionnaire

1. How can you tell when someone is good at mathematics?[6]

2. How can you tell when someone is bad at mathematics?

3. Do you think you are good at mathematics? Tick one box

Yes, I am good at mathematics most of the time ☐

Sometimes I am good at mathematics ☐

I am not good at mathematics ☐

I don't know ☐

4. What do you do if you can't understand a mathematical topic?

[6] Page 5 of Questionnaire

5. Do you have a mathematical goal or target? If so, please write it here:[7]

6. What area of mathematics are you studying now?

7. Please write down the activities that you use to help you learn this topic.

8. How do you choose which mathematics topic to study?

	Always	**Sometimes**	**Never**
My parent/teacher chooses it for me			
I choose something that I'm interested in			
I study whatever comes next in the textbook			
It is important to work on the areas I don't understand			
We find mathematics in everyday life (e.g. shopping etc.)			
I work on the areas that are needed for my exams			

[7] Page 6 of Questionnaire

9. What do you like about studying mathematics at home?[8]

10.What do you think you could improve when learning at home?

A few details

How old are you?

Are you a boy or a girl? **Boy** **Girl**

How long have you been studying at home?

Please write your parents/guardians' name here:

Please write their email/telephone number/address here (ask permission first ☺). I might need to ask them a few questions and you may be invited to take part in the next stage of the study.

[8] Page 7 of Questionnaire

Appendix 3: Email Sent to Parents

Dear Home-Educators,

I hope that everyone is enjoying the warm weather right now - hopefully it will last!

I am a researcher from the mathematics education research centre, University of Warwick, and my doctorate will be on 'home-education and mathematics'. Although this may sound like an unusual choice of thesis, since I was home-educated for 15 years before I began my mathematics degree, home-education is an area that seems 'natural' to study.

Much research has been conducted on children's mathematical education in the school environment, focusing on the relationship between teacher's beliefs, teaching styles, and children's levels of understanding. However, the home environment is a very different setting to that of school and this is an area that should be investigated, given the increasing number of home educating families in the UK.

The first part of my study focused on the parents - the home-educating environment, the use of different activities, and the reasons for home-education.

The second stage of my study will hopefully identify:

* 'Different types of home-educators'

* Home-educated children's perceptions of mathematics and learning at home

* Children's methods of learning (what helps them to learn, what do they find interesting about mathematics etc,)

* The effects of a home-education on children's mathematical understanding

I am looking for around 50 families to take part in the first stage of this study. It would be great if I could get responses from parents from a variety of backgrounds, so please take part if you have the time. The only conditions that the parents and children need to satisfy are:

(1) The children are home-educated (i.e. the parents have chosen the home-environment as an alternative to school).

(2) This stage of the study involves the parent and child completing a questionnaire. It does not involve 'doing any mathematics' - it just asks children for their views/opinions etc. so hopefully it should take too long to complete. The questionnaire is in Word format, so as long as you have Word and can open attachments, you can just email any responses back to me.
If you and your children would like to take part, please reply to this email at:

kuching48@yahoo.co.uk and I will send you a copy of both the children's/parents questionnaires (either through email, or post if you'd prefer a paper copy).

I am aware that many families are going to HesFes so this isn't the best time to collect data, but I will just send the email again next week. If you are a member of any other home-education groups, please could you forward this email on to them :-) it would be a great help.

Many thanks in advance and best wishes to all,

Aisha
Mathematics Education Research Centre,
University of Warwick

Appendix 4: Exploratory Study Questions

Taken from the 'National Numeracy Strategy for use in the Daily Mathematics Lessons for Year 5 to Year 6 children' [Online] Available at: http://www.mathsyear2000.org/resources/numeracy/pdfs/y456str3.pdf [Accessed 14th of January 2005]

Problems Set at the Year 5 Level

Single-step operations

• Three children play Tiddlywinks.
What was each child's score?
Yasmin 258 + 103
Steven 177 + 92
Micky 304 + 121

• I think of a number, then divide it by 15.
The answer is 20.
What was my number?

• There are 12 eggs in a box.
How many eggs in 9 boxes?
How many boxes will 192 eggs fill?

• A bus seats 52 people. No standing is allowed. 17 people got off a full bus. How many were left on?
How many seats for two people are there?
How many people can sit on 6 buses?
How many buses are needed to seat 327 people?

Multi-step operations

• I have read 134 of the 512 pages of my book.
How many more pages must I read to reach the middle?

• There are 8 shelves of books.
6 of the shelves hold 25 books each.
2 of the shelves have 35 books each.
How many books altogether are on the shelves?

• I think of a number, subtract 17, and divide by 6.
The answer is 20. What was my number?

• You start to read a book on Thursday.
On Friday you read 10 more pages than on Thursday. You reach page 60.
How many pages did you read on Thursday?

• Ravi bought a pack of 30 biscuits.
He ate one fifth of them on Thursday. He ate one eighth of the remaining biscuits on Friday.
How many biscuits did he have left?

Problems Set at the Year 6 Level

Single-step operations

• 12 500 people visited the museum this year.
This is 2568 more than last year.
How many people visited the museum last year?

• There are 35 rows of chairs.
There are 28 chairs in each row.
How many chairs are there altogether?
How many rows of chairs do 420 people need?

• A school has 486 pupils and 15 classes.
What is the average class size?

• Gwen has a box of 250 staples to make kites.
She uses 16 staples to make each kite.
How many complete kites can she make?

• Use a calculator or a written method.
A full box has 180 pins.
How many full boxes can be made
from 100 000 pins?

Multi-step operations

• There is space in the multi-storey car park for
17 rows of 30 cars on each of 4 floors.
How many cars can park?

• 196 children and 15 adults went on a school trip.
Buses seat 57 people.
How many buses were needed?

• 960 marbles are put into 16 bags.
There is the same number of marbles in each bag.
How many marbles are there in 3 of these bags?

• In a dance there are 3 boys and 2 girls in every
line. 42 boys take part in the dance.
How many girls take part?

• I think of a number, add 3.7 and multiply by 5.
The answer is 22.5. What was my number?

• Of the 96 children in Y6, three quarters have pets.
45 children have a dog. 21 children have a cat.
How many Y6 children have other kinds of pets?

Appendix 5: Marks on the Key Stage Pilot Study Questions

KS1 Test	Question	D	U	Z	T	% correct
1	Symmetry	1 out of 2	1 out of 2	1 out of 2	1 out of 2	50%
2	Area	0	0	1	0	25%
3	Money	0	1	0	1	50%
4	Addition	1	1	0	0	50%
5	Subtraction	1	1	0	1	75%
6	Arithmetic	2 out of 2	2 out of 2	2 out of 2	2 out of 2	100%
7	Time	1 out of 2	1 out of 2	1 out of 2	0 out of 2	38%
8	Money	2 out of 2	2 out of 2	2 out of 2	2 out of 2	100%
9	Arithmetic	0	1	1	0	50%
10	Measure	1	1	1	1	100%
11	Time	1	1	1	1	100%
12	Measure	0	1	0	0	25%
13	Arithmetic	0	1	0	0	25%
Mark (%)		**54%**	**85%**	**54%**	**42%**	

Table to show marks achieved on the KS1 pilot test.

In the table above, the letters D, U, Z and T represent the names of each child.

KS2 Test	Question	N	K	Ad	Al	Sb	Sh	H	Correct
1	Fractions	0	0	1	1	1	1	0.5	64%
2	Angles	0	0	0	1	1	1	0	42%
3	Fractions	0 out of 2	0 out of 2	2 out of 2	0 out of 2	2 out of 2	0 out of 2	0 out of 2	29%
4	Arithmetic	0	0	0	void	void	void	void	0%
5	Arithmetic	0	1	1	1	1	1	0	71%
6	Algebra	1	1	1	1	1	1	1	100%
7	Symmetry	2 out of 2	1 out of 2	1 out of 2	0 out of 2	0 out of 2	0 out of 2	1 out of 2	36%
8	Symmetry	1	1	1	1	0	1	0	71%
9	Arithmetic	1 out of 2	0 out of 2	2 out of 2	1 out of 2	2 out of 2	1 out of 2	0 out of 2	50%
10	Area	1	0	1	void	void	void	void	67%
11	Area	1	0	1	0	0	1	0	42%
12	Measure	2 out of 2	0 out of 2	2 out of 2	1 out of 2	1 out of 2	2 out of 2	1 out of 2	64%
13	Area	0	0	0	0	0	0	0	0%
14	Arithmetic/ Area	0	0	1 out of 2	0 out of 2	2 out of 2	1 out of 2	0 out of 2	29%
Mark (%)		**43%**	**25%**	**71%**	**50%**	**62%**	**67%**	**21%**	

Table to show marks achieved on the KS2 pilot test

KS3 Test	Question	U	Sz	Az	Hl	Al	Hz	Sb	H	Sh	Correct
1	Arithmetic	1	1	1	1	1	1	1	1	1	100%
2	Algebra	0 out of 3	1 out of 3	1 out of 3	2 out of 3	2 out of 3	2 out of 3	3 out of 3	2 out of 2	2 out of 3	55%
3	Ratio	0	0	0	0	0	0	1	0	1	22%
4	Fraction	0 out of 2	0 out of 2	0 out of 2	2 out of 2	2 out of 2	2 out of 2	2 out of 2	0 out of 2	0 out of 2	44%
5	Algebra	0	0	0	0	0	0	0.5	0	0	5%
6	Measure	0	0	0	0	0	0	0	0	0	void
7	Arithmetic	1 out of 3	3 out of 3	3 out of 3	2 out of 3	3 out of 3	3 out of 3	3 out of 3	3 out of 3	3 out of 3	88%
8	Algebra	1 out of 2	0 out of 2	0 out of 2	0 out of 2	1 out of 2	1 out of 2	1 out of 2	1 out of 2	1 out of 2	33%
9	Arithmetic	1	0	0	1	1	1	1	0	1	67%
10	Angles	0 out of 2	0 out of 2	0 out of 2	1 out of 2	0 out of 2	0 out of 2	2 out of 2	0 out of 2	0 out of 2	17%
11	Area	0	0	0	0	0.25	0.25	0.25	0	0.25	11%
Mark (%)		**26%**	**21%**	**21%**	**44%**	**49%**	**49%**	**75%**	**31%**	**49%**	

Table to show marks achieved on the KS3 pilot test

Appendix 6: Group 1 Questions (Key Stage 1)

Question 1

Write a number in the box to make this correct.

Question 2

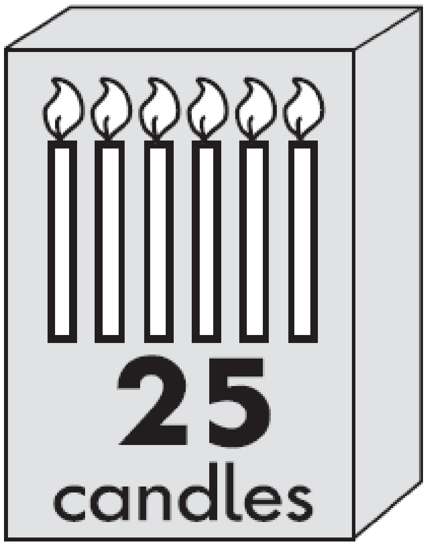
Write the total.

$$64 + 85 + 56 = \boxed{}$$

You can do your working in the box below

Question 3

The shop sells candles in boxes of **25**



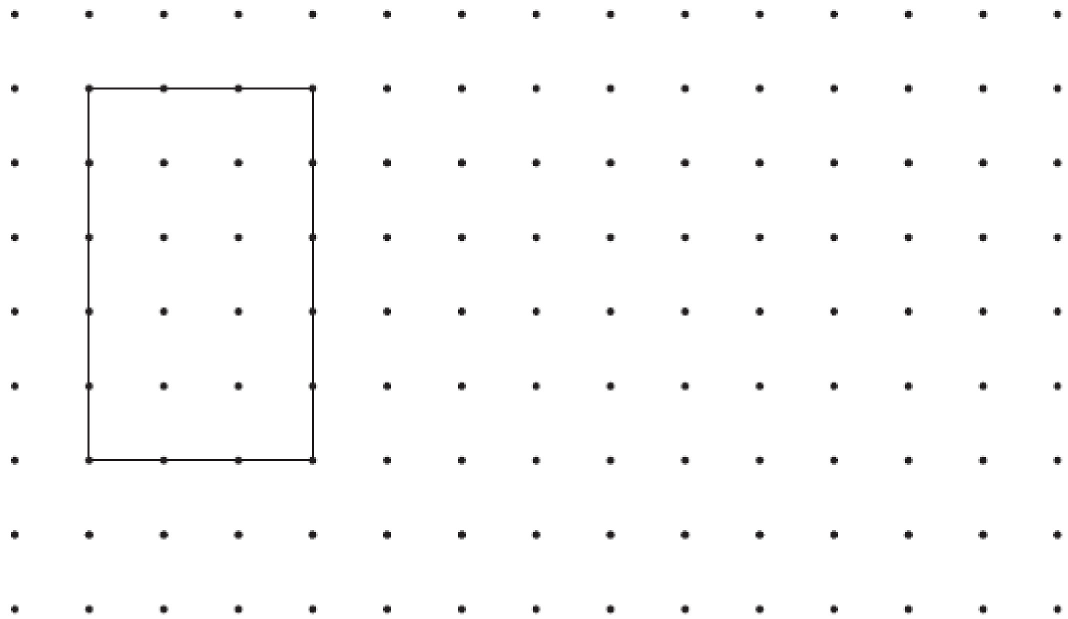
Emma needs **69** candles.
How many boxes of candles does she need to buy?

Question 4

Look at this rectangle.

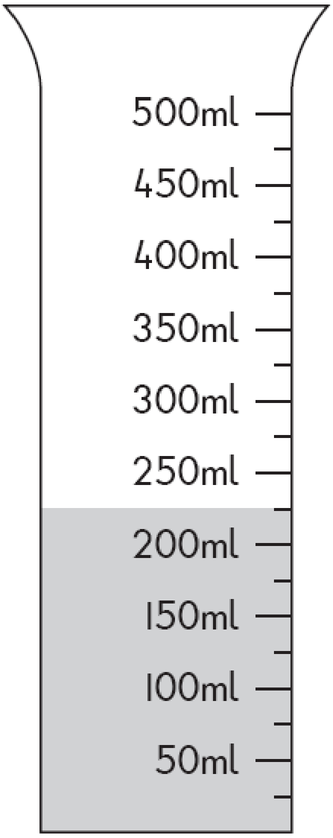
On the dots, draw a square with the **same total distance around the edge**.

Use a ruler.



Question 5

This container has water in it.



How much water is in it?

ml

Question 6

Manjit buys **6** stamps.

Each stamp costs **19p**.

How much does he pay?

Show how you work it out in the box.

Question 7

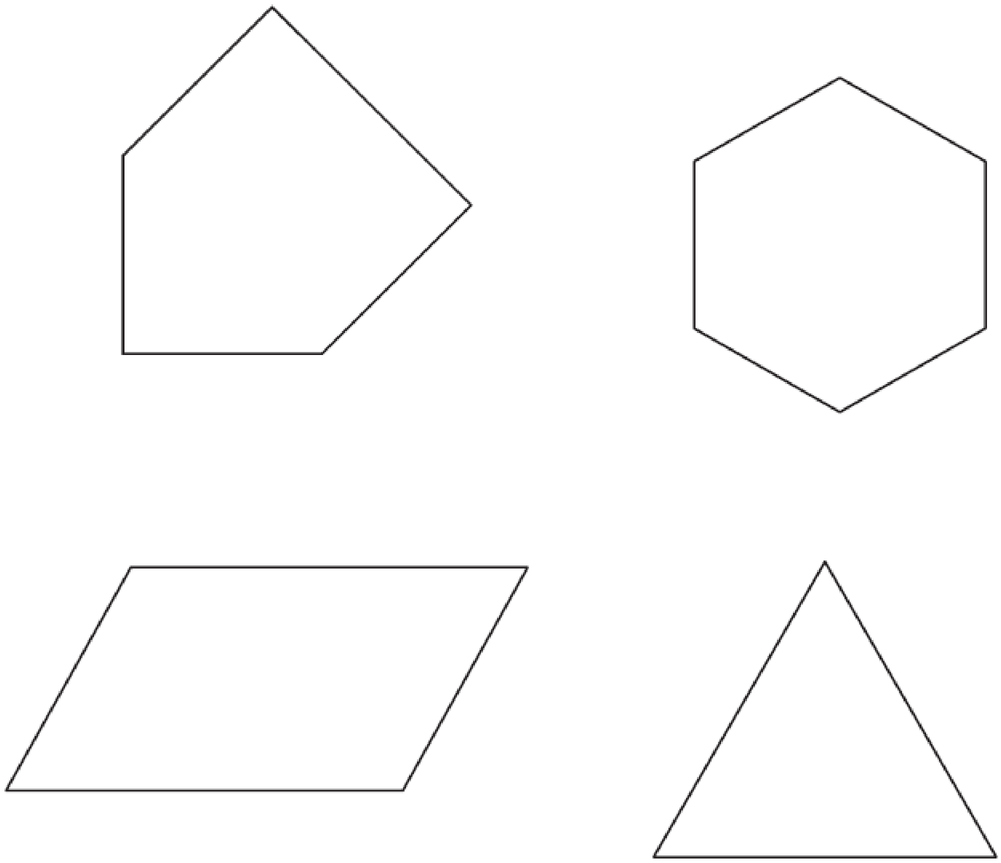
Work out the answer.

$$205 - 143 = \boxed{}$$

You can do your working in the box below

Question 8

Two of these shapes have **more than 1** line of symmetry.
Tick (✓) both of them.



Question 9

The sum of two numbers is **21**

Their **difference** is **5**

Write the two numbers.

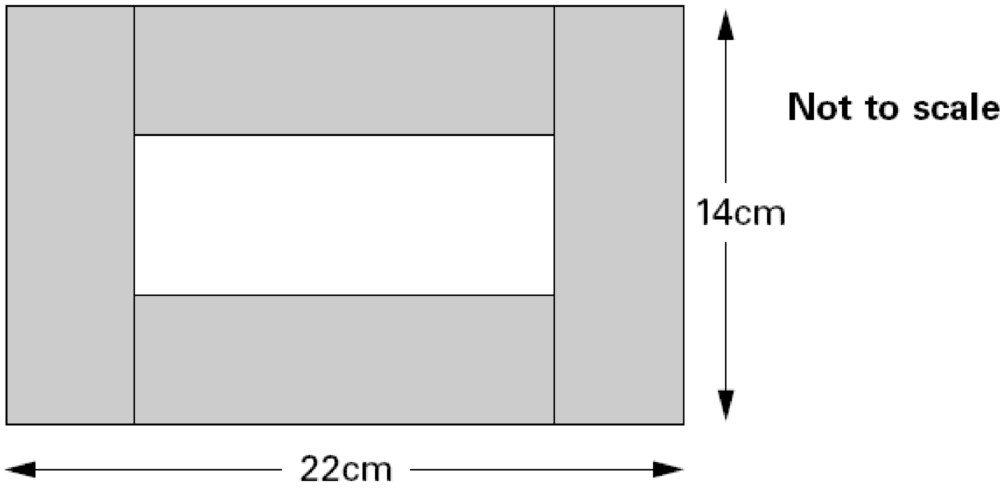
	and	

Appendix 7: Group 2 Questions (Key Stage 2)

Question 1

Danny has four identical shaded rectangles.

He makes this design with them.



The design measures **22 centimetres** by **14 centimetres**.

Calculate the **area** of **one** of Danny's shaded rectangles.

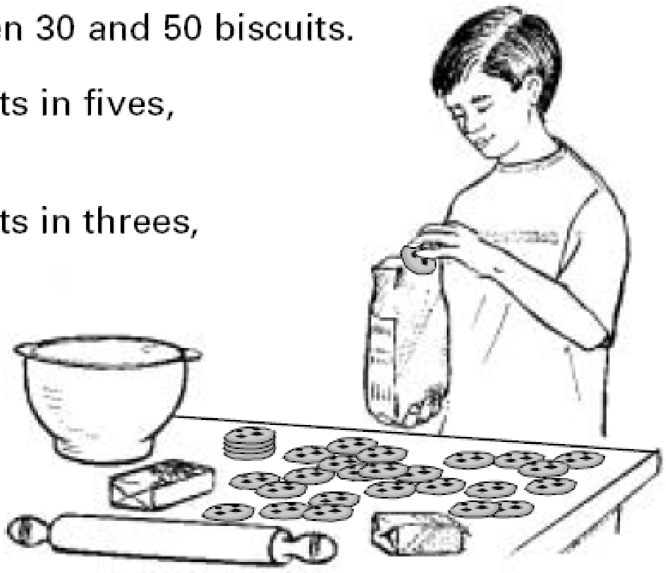
Please show your working in the box below

Question 2

Steven made between 30 and 50 biscuits.

If he packs the biscuits in fives,
he has one left over.

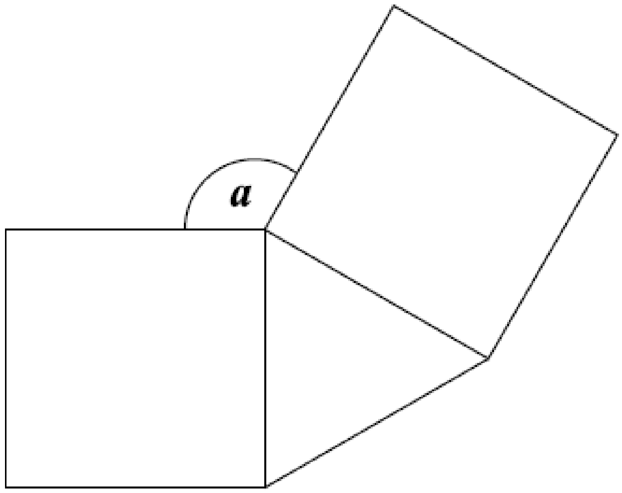
If he packs the biscuits in threes,
he has two left over.



How many biscuits did he make?

Question 3

This shape is made from two squares and an equilateral triangle.



Calculate the size of angle a

Please do not use a protractor

Write your answer and working in the box below

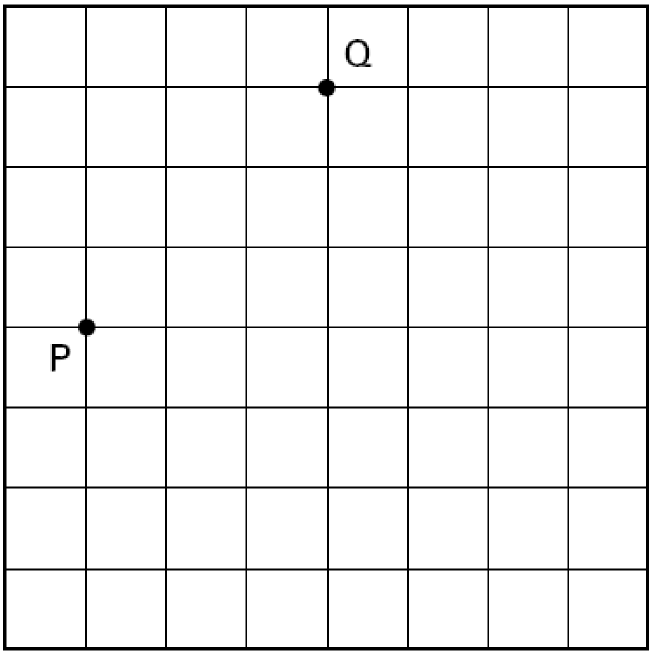
Question 4

Which is larger, $\frac{1}{3}$ or $\frac{2}{5}$?

Please write how you found out your answer in the box below

Question 5

Here is a centimetre grid.



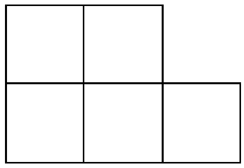
P and Q are two vertices of a square.

What is the **largest area** that the square could have?

Please write your answer in this box

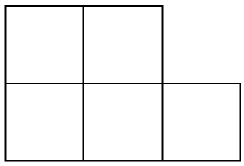
Question 6

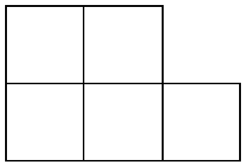
This shape is made from five identical squares.



Draw **one more square** so that the new shape has **exactly one** line of symmetry.

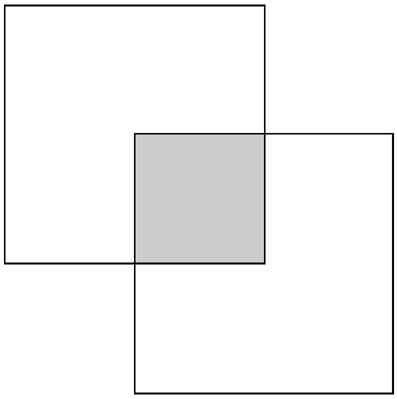
Find **two** different ways to do it.





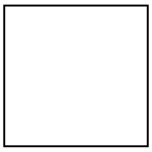
Question 7

Here are two identical overlapping squares.

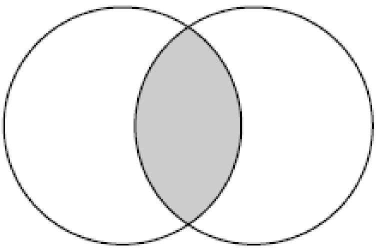


One quarter of each square is shaded.

What fraction of the whole diagram is shaded?

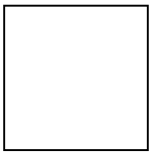


Here are two identical overlapping circles.



One third of each circle is shaded.

What fraction of the whole diagram is shaded?



Question 8

$\triangle$, $\bigcirc$ and $\square$ each stand for a different number.

$$\triangle + \bigcirc = 18$$

$$\bigcirc + \square = 16$$

$$\triangle + \triangle = 14$$

Find the value of each shape.

$\triangle = \boxed{}$ $\qquad$ $\bigcirc = \boxed{}$ $\qquad$ $\square = \boxed{}$

Question 9

What's my number?

My number is a whole number.

Double my number is **more than 60**

Three times my number is **less than 100**

Write **all** the possible numbers that my number could be.

Appendix 8: Group 3 Questions (Key Stage 3)

Question 1

1. Write numbers to complete the calculation.

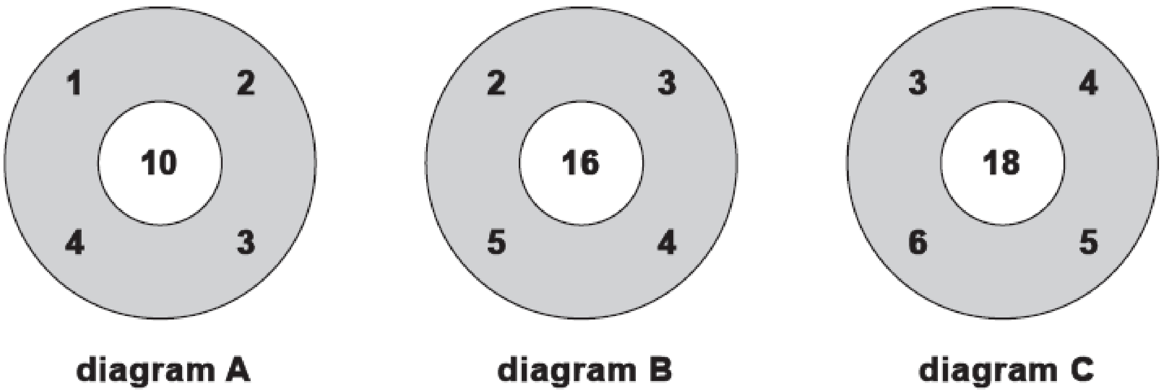
$$\square \times \square = 50$$

Now write **different** numbers to complete the calculation.

$$\square \times \square = 50$$

Question 2

Look at these diagrams.



Which diagram is the odd one out? Tick (✔) your answer.

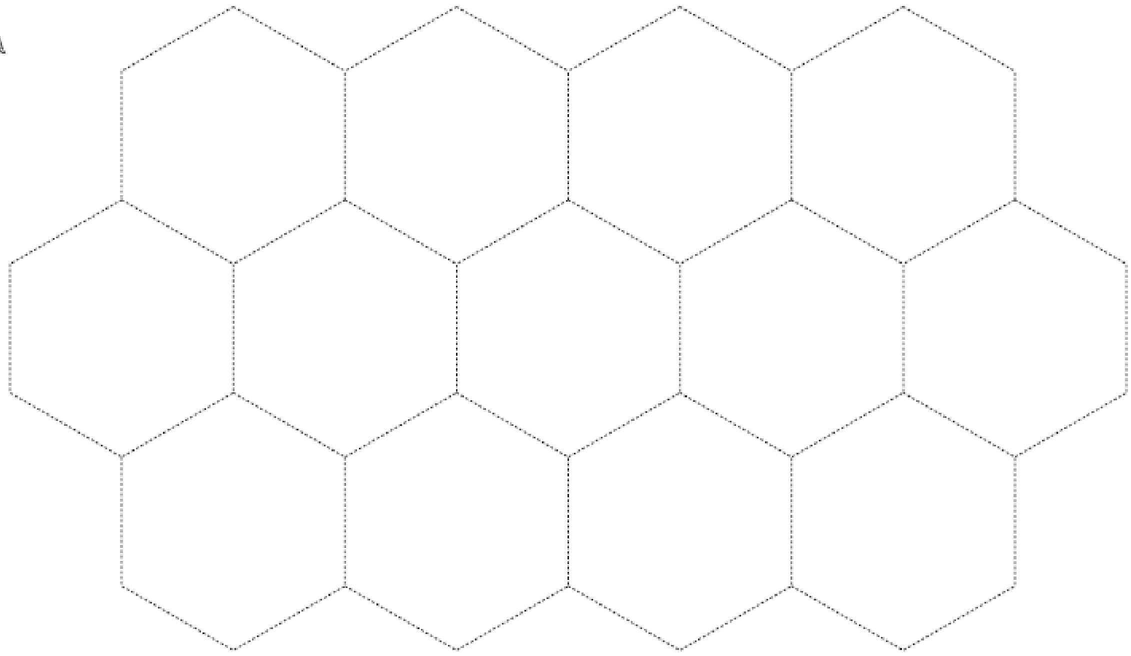
diagram A ☐ **diagram B** ☐ **diagram C** ☐

Please give a reason for your answer in the box below

Question 3

The grid shown below is made of regular hexagons.

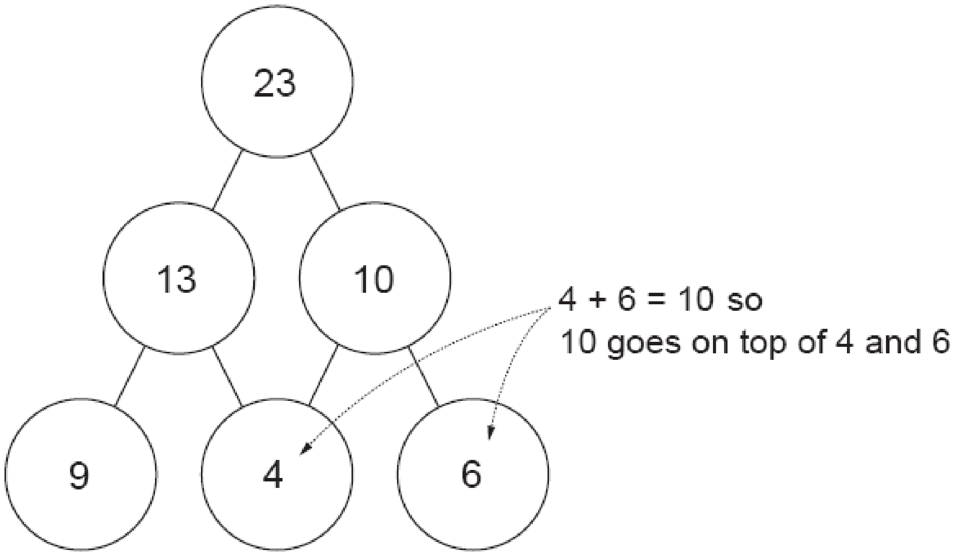
On the grid, draw a rectangle with an area **6 times** as big as the area of one hexagon.



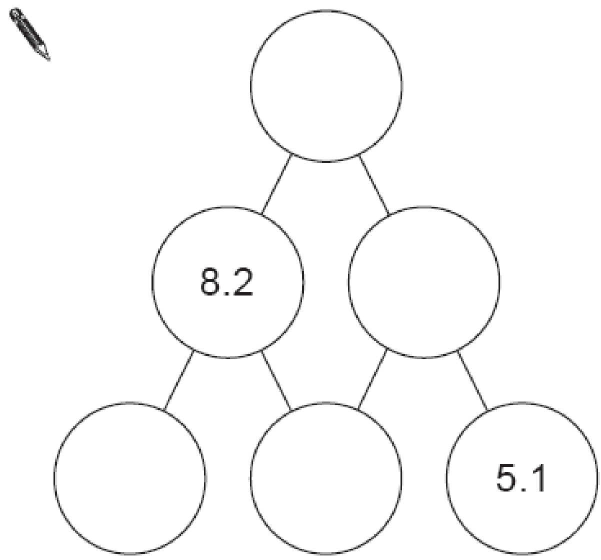
Question 4

Look at this number puzzle.

You **add two numbers** to work out the number that goes on top of them.

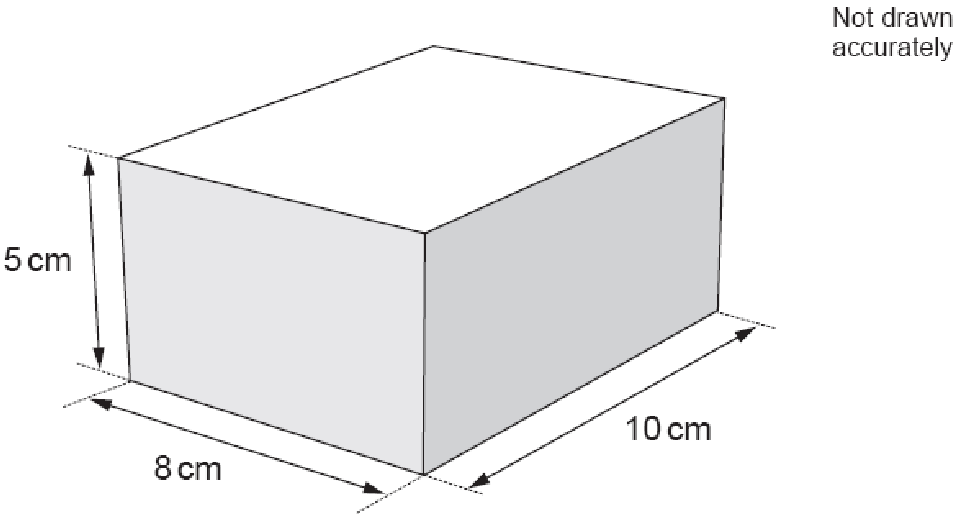


Complete the number puzzle below.



Question 5

I have a present in a box, a cuboid measuring **10cm by 8cm by 5cm**.



I have one sheet of wrapping paper to wrap up the box.

The sheet is a rectangle that measures **25cm by 30cm**.

Is the sheet of wrapping paper big enough to cover **all** the box?

Please explain how you know in the box below

Question 6

I am thinking of a six-digit **square number** with a units digit of 6

_ _ _ _ _ 6

Could its square root be a **prime number**?

Tick (✔) Yes or No.

☐ Yes ☐ No

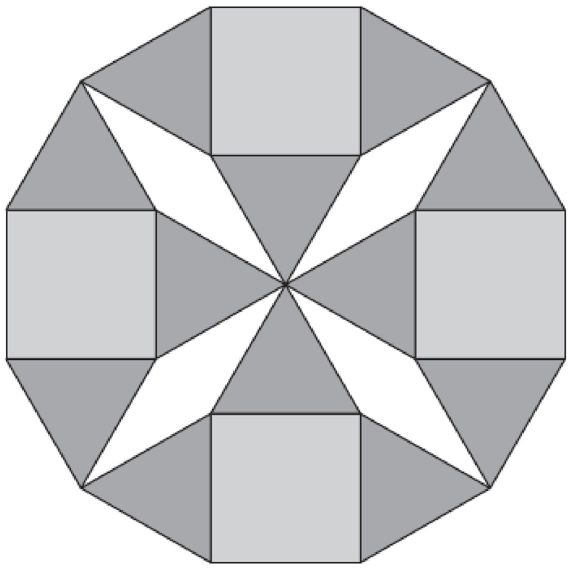
Please explain why you ticked Yes/No in the box below

Question 7

I have some tiles that are squares and some tiles that are equilateral triangles.

The side lengths of the tiles are all the same.

I arrange the tiles like this.



I want to fill the gaps by making four tiles that are rhombuses.

What should the angles in each rhombus be?

Please show your calculations in the box below

Question 8

I think of a number, then I carry out these operations on my number.

Multiply by 5	**Add 8**

When I carry out the operations in one order the answer is **105**

When I carry out the operations in the **other order** the answer is **73**

What is my number?

Please give your answer and show your working in the box below

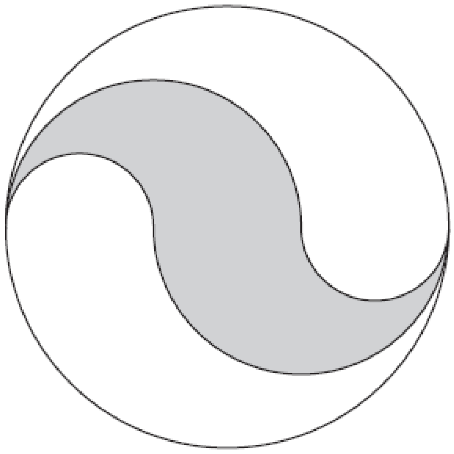
The difference between my two answers is 32

Can you prove that the answers will always be 32, no matter what my number is?

Question 9

Here are three circles.

Parts of the circles are used to make the design below.



What **fraction** of the design is **shaded**?

Please show your answer and working on the next page:

Answer and working for Question 9:

Appendix 9: Questions for Child 4

1. Can you count? **Yes** [X] **No** [ ]

2. What is the biggest number you know? Write it in this box.

A googol

3. What is the smallest number you know? Write it in this box.

Negative googol

4. How do you feel when you see numbers? Tick the box to show how you feel.

I feel happy ☺ [X]

I don't feel happy ☹ [ ]

5. Where can you find numbers? Write down all the places in the box.

Everywhere! I can count the drawers on the chest and the panes of glass in the desk and the handles on the drawers. There are numbers on a chessboard and there are numbers of squares there too. There are numbers of weights on a scale and there is the number of chairs in our house.

6. Where can you find shapes? Write down all the places in the box below.

There are shapes everywhere. I don't want to say where.

7. Which shapes can you draw?

Square	**Yes**	X	**No**	
Circle	**Yes**	X	**No**	
Triangle	**Yes**	X	**No**	
Rectangle	**Yes**	X	**No**	

8. Are there any other shapes that you like to draw? Please write them here.

Yes. I like to draw circles and squares and triangles and rectangles.

9. Which games do you like to play?

I like making pyramids out of rods and balls, and I like making cubes and triangles and squares and I like making very long things with my rods and balls.

10. Do you know any counting songs? Please write them here:

One two three four, Mary at the cottage door, five six seven eight, eating cherries off a plate.
One two three four, who's that knocking at my door? Five six seven eight, who's that tapping at my gate?

11. Which one is longer?

My fingers ☐ **My arms** X

12. Which one is heavier?

An elephant X **A cat** ☐

Appendix 10: Responses to Case Study Questions

Joe's Story:

"Joe is six years old, rarely interacts with other children, but asks very interesting questions. He would like to talk to his parents the whole time if possible, and to share his ideas with them. His interests are very different and at a more sophisticated level to his peers, although he is dreamy and solitary most of the time.

Joe is very able in most subjects, especially mathematics. He can predict number sequences, cope with complex rules, and understands time (24 hour clock). Joe enjoys problem-solving activities, but during these sessions he lives in a world of his own without any interaction."

Richard's Story:

"Richard is nine years old, and is very good at mathematics. You may be surprised at that because you will see very little writing in his mathematics book, indeed he doesn't like writing in general. He tends to do sums in his head and just writes the answers down. The answers are nearly always correct, but he writes no method of solution, and rarely explains his reasoning.

On the other hand, he loves problem solving – he can often think of three or four different approaches to the same problem."

What would you suggest for these home-educated children?

Family 26' s responses:

Option	**Yes/ No**	**Reason for choice**
Teach Joe mathematics that is for children one or two years above his age group	Yes	If Joe is able then to provide him with maths above his age range should not be a problem. This should only be done if he isn't going to struggle as this would be bad for confidence.
Get a mathematics tutor for Joe	No	Joe seems to work well on his own and becomes absorbed in his work. If this works for him I think he should be left to this way of working.

Make arrangements to prepare Joe to take his GCSE Mathematics exam when he is nine years old.	Yes/ No	If Joe wants to then yes, if not, then no.
Give him extra mathematics problems for him to solve in his own time	Yes	This would be good if he wants to do this.
Make him apply his mathematics to real-world situations	Yes	You shouldn't make a child do anything but if he can apply his maths to the real world and enjoys this then it would be helpful to him. If he is not ready for this then leave him to work in his own way again.
Join the local mathematics club	Yes/ No	If Joes wants to - then yes. He seems to like his own company though so may not want to. If this is that case he should be left alone to do things his way.
		Other suggestions? Please write here:

Which of these options would you suggest for Richard?

Option	**Yes/ No**	**Please give a reason for your choice**
Richard very good at mathematics and can take care of himself. Leave him alone to fulfil his talent	Yes	Seems as though Richard has a very good brain. No interference needed.
You need to assess his knowledge and skills to see if he is able to undertake calculations efficiently.	No	He can obviously do this as he comes to the correct answers.
Encourage him to undertake more problem solving tasks	Yes	If Richards wants to then this would be good fun for him.

Richard needs to improve his written and language skills before he can be given extra work in mathematics	No	Richards written and language skills will develop at their own rate. No need to worry about trying to improve them.
His written work could be developed through his mathematics	Yes/ No	If Richard is happy for written work to be included in the maths then yes. Other than that the answer is no. Making him include writing in his maths when he doesn't want to will probably stop him doing maths all together.
His mathematical ability will be underdeveloped if it is not nurtured right away	No	His mathematical ability will be just fine. He can work with numbers, calculate in his mind and has a good understanding already. He will continue to develop naturally.
		Other suggestions? Please write here

Family 7's responses:

Which of these options would you suggest for Joe?

Option	*Yes/No*	*Reason for choice*
Teach Joe mathematics that is for children one or two years above his age group	no	Give him whatever is suitable. Work for children above his age gap may be suitable only in certain areas of mathematics, things could easily be 'tailored' to suit him individually.
Get a mathematics tutor for Joe	no	He is already very able, he needs opportunities to learn himself, but why would he need a tutor for a subject he has a strong affinity for?
Make arrangements to prepare Joe to take his GCSE Mathematics exam when he is nine years old.	no	It would be of no benefit to him and is unnecessary at this stage. He should be allowed to continue exploring and enjoying his favourite subject and only face examinations when more emotionally mature.
Give him extra mathematics problems for him to solve in his own time	yes	He obviously enjoys the subject, so giving him more to work with in a relaxed way would seem sensible.

Make him apply his mathematics to real-world situations	yes	It may help him interact more with the world around him and will help him in his adult life.
Join the local mathematics club	yes	Again this gives more opportunity and encouragement to interact with others and the world around him.
		Other suggestions? Please write here:

Which of these options would you suggest for Richard?

Option	*Yes/No*	*Please give a reason for your choice*
Richard very good at mathematics and can take care of himself. Leave him alone to fulfil his talent	no	If he struggles with writing then he may also struggle with mathematics in a literary format rather than numerical, so this would need to be looked at to see if it can be helped.
You need to assess his knowledge and skills to see if he is able to undertake calculations efficiently.	yes	If he has writing difficulties then it would be fairer to assess his abilities verbally or kinaesthetically to find out what he can do and what he understands.
Encourage him to undertake more problem solving tasks	no	Not necessarily, it's hard to know if he actually has a problem working them out or just explaining how he did it. Extra practice is not likely to change things.
Richard needs to improve his written and language skills before he can be given extra work in mathematics	no	He does need help with written and language skills but this does not mean that his mathematics needs to be held back.
His written work could be developed through his mathematics	yes	If this is a subject he enjoys then there is more incentive to try and improve written work.
His mathematical ability will be underdeveloped if it is not nurtured right away	no	Not necessarily but finding out what he can do and what he struggles with can only help him to develop and learn more.

		Other suggestions? Please write here

Family 11's responses:

Which of these options would you suggest for Richard?

Option	*Yes /No*	*Please give a reason for your choice*
Richard very good at mathematics and can take care of himself. Leave him alone to fulfil his talent	Yes	I wouldn't be concerned about him, but I would want to provide him with opportunities and encouragement to continue learning (rather than just "leaving him alone")
You need to assess his knowledge and skills to see if he is able to undertake calculations efficiently.	No	If he consistently gets the right answer to a variety of types of problem, he's calculating efficiently.
Encourage him to undertake more problem solving tasks	Yes	He should be encouraged to do whatever he loves.
Richard needs to improve his written and language skills before he can be given extra work in mathematics	No	If his written and language skills are holding him back (it doesn't sound like they are, at least not yet), then they can be developed at the same time as maths. Withholding appropriate-level maths until he catches up in other areas sounds like a cruel punishment. It will be hard for him to remain motivated if he is made to work exclusively on skills (such as writing) which are difficult and unpleasant for him.
His written work could be developed through his mathematics	Yes	
His mathematical ability will be underdeveloped if it is not nurtured right away	Yes	I disagree with the urgency implied by this statement, but he'll be happier and more mathematically competent if his abilities are nurtured now.
		Other suggestions? Please write here Try using computers more, or have someone else do his writing for him, to encourage him to communicate. Try giving him much more difficult problems to work on.

Family 4's responses:

Which of these options would you suggest for Joe?

Option	***Yes/No***	***Reason for choice***
Teach Joe mathematics that is for children one or two years above his age group	Yes	Or higher – Joe may lose interest if his maths is not at the correct level for him. He may need to work on "the basics" (eg arithmetic facts) concurrently if he hasn't already mastered them.
Get a mathematics tutor for Joe	No	I suppose this might be a good idea if Joe's parents aren't very confident of their own maths skills and Joe likes to talk about maths and cannot learn on his own. But I would guess that this isn't the case, or he wouldn't be so good at maths already.
Make arrangements to prepare Joe to take his GCSE Mathematics exam when he is nine years old.	No	Such a decision should be at the child's own initiative, and he probably isn't old enough to make such a decision himself yet.
Give him extra mathematics problems for him to solve in his own time	Yes	If he likes them
Make him apply his mathematics to real-world situations	No	Making him do anything is likely to turn him off what seems to be an enjoyable activity for him, which he is good at.
Join the local mathematics club	Yes	If Joe finds other children who share his interests and abilities, he may enjoy interacting with them and develop better self-esteem. Though he may be a true loner, it is also possible that the reason he doesn't interact with other children is that he hasn't found any who are sufficiently like him.

Which of these options would you suggest for Richard?

Option	*Yes /No*	*Please give a reason for your choice*
Richard very good at mathematics and can take care of himself. Leave him alone to fulfil his talent	Yes	I wouldn't be concerned about him, but I would want to provide him with opportunities and encouragement to continue learning (rather than just "leaving him alone")
You need to assess his knowledge and skills to see if he is able to undertake calculations efficiently.	No	If he consistently gets the right answer to a variety of types of problem, he's calculating efficiently.
Encourage him to undertake more problem solving tasks	Yes	He should be encouraged to do whatever he loves.
Richard needs to improve his written and language skills before he can be given extra work in mathematics	No	If his written and language skills are holding him back (it doesn't sound like they are, at least not yet), then they can be developed at the same time as maths. Withholding appropriate-level maths until he catches up in other areas sounds like a cruel punishment. It will be hard for him to remain motivated if he is made to work exclusively on skills (such as writing) which are difficult and unpleasant for him.
His written work could be developed through his mathematics	Yes	
His mathematical ability will be underdeveloped if it is not nurtured right away	Yes	I disagree with the urgency implied by this statement, but he'll be happier and more mathematically competent if his abilities are nurtured now.
		Other suggestions? Please write here Try using computers more, or have someone else do his writing for him, to encourage him to communicate. Try giving him much more difficult problems to work on.

Family 16's responses:

Which of these options would you suggest for Joe?

Option	*Yes/ No*	*Reason for choice*
Teach Joe mathematics that is for children one or two years above his age group	Yes	Boredom must not be allowed to set in. Joe needs challenges.
Get a mathematics tutor for Joe	Yes	If that is possible, although finding a good tutor is very difficult. As statistics show, very few children gain from hired tutors.
Make arrangements to prepare Joe to take his GCSE Mathematics exam when he is nine years old.	Yes	Only if he wants to. Not just to gratify the parents.
Give him extra mathematics problems for him to solve in his own time	Yes	Problems that could involve interaction with other people. How about a game of monopoly or chess?
Make him apply his mathematics to real-world situations	Yes	This will help widen his scope and offer greater challenges for him.
Join the local mathematics club	Yes	Not many places offer such facilities, but if they cannot find one locally, how about finding groups on the internet?
		Other suggestions? Please write here: Parents should spend a great deal more time with him, as this would appear is what he enjoys best. He is only 6, confidence will come later.

Appendix 11: Example of Coding

- Bullying and social problems
- Parents were not happy with the school system. E.g. felt that school was inflexible and wanted children to learn at their own pace, according to their individual capabilities.
- Parents felt that their children would be happier (prefer) to be at home
- Children had special needs (Asperger's etc.) or needed extra help

Family	What were the main reasons behind your decision to home-educate?
1	Bullying leading to school phobia and associated problems with health including narcolepsy, sleep paralysis, abdominal migraines. Also 4 broken wrists in 6 months
2	Hate assessments of young children, tests etc. Had most of the stuff needed (as a teacher) so they could learn in their own time and pace. Love their individuality.
3	I am a teacher (just keeping my hand in 1 day a week at the moment) and as a language teacher in secondary school I get to ask kids their birthday quite a lot. Allowing for the odd exception, the kids who were born Sept-Nov were all in Set one (at least for languages) and Dec-Feb in Set 2 and so it went on. Many with SEN on a low level (such as behavioural difficulties or just could try harder types were July-August born. I tried 3 times for September babies to try and beat the system. The first one was born the following May. 2nd time we struck lucky on our first try and had a September due date. She kicked our plans into touch by bring born 2 weeks early on 27th August despite me trying to keep my legs closed! 3rd time again conception was curiously evaded until a June due date was appropriate. So this got us thinking. And also my husband is Japanese and we have lived in Japan where they start 2 years later and literacy rates are about 10% higher than the UK. My first daughter was slow to reach all her developmental milestones in comparison with her peers (although she got to them all eventually) and I just didn't want her to be labelled as "slow" by some teacher when she was just 4 as I knew this label would stick with her for ever.
4	I feel that early formal education is often harmful to children's personal and academic development and makes many young children unhappy. In institutionalised education, timetables, standardised teaching methods and content cannot meet the emotional or intellectual needs of each specific child.
5	Child was bright for his age and schools seemed unable to cater for this due to the current lack of flexibility for teachers, also social influences at local schools were undesirable, and pressure on young children was too high.
6	Child 3 had clinical depression and school was making her very unhappy Child 4 has Aspergers Syndrome & ADHD and was getting no support at school.
7	In my son's case (7) due to lack of support for SEN and bullying. In my daughter's case (5) it was because she was receiving very little education and found the peer pressure too much to cope with. Also she found the school day just too long for her at 4 and 5.
8	I wanted to give them the freedom to choose their own education.
9	He hated school! (had to stop chatting and couldn't sit for hours on one task) And problems with religion and their perception of it!!
10	My son was very unhappy at school and the environment (with bullying/disrespect/lack of learning/lack of good teaching) was not what I consider is good for a child.
11	Eldest daughter refused to go to school, became school phobic. Younger daughter decided to join her "because lessons are boring and the people are horrible"
12	My older son is profoundly gifted but his motor skills are delayed. This was a very difficult fit in our British-style school (in Hong Kong) : they gave him help with writing/PE but no extension activities. I had to fight for six months even to get suitable readers for him (he was reading Harry Potter at five). We then moved to China, where we didn't want to inflict the local, high pressure, education system on our kids. Home educating seemed an ideal solution.
13	My oldest was struggling at school.

14	To spend more time with the children We thought there was lots of wasted time at school
15	Eldest child bored and not challenged at school. Second child not ready for such a big move when it was his time to go to school. Now they are out of school I realise the impact school had on their personalities and decision making – I wouldn't put them back. Also felt their faith would suffer in a school environment.
16	Our oldest T, was being bullied. The school refused to accept our complaints, and they believed our son was not sociable and therefore became a target! Thomas is a highly intelligent boy, speaks his mind freely and eloquently but also very kind and fair. This is the way he and his sister have been raised by us. We are a mixed-marriage family. My husband is quintessentially English, and I come from African, Lebanese and Iranian roots; and I am also a Muslim. Therefore I was never comfortable with the school in so far as my son's background could have created the unspeakable reaction from a predominantly white school.
17	Eldest was accelerated at school until she was away from her friends and was not too happy
18	D is brilliant at all subjects except numbers. After two difficult years at school D 'shut down' in maths – the teachers put pressure on him to move on with the rest of the class before he was ready. Doing times table tests before he could understand number bonds to 10 they made no allowances for his weakness. They were dismissive of our request for advice & help. He was also bullied in the playground over two years, he seemed to draw attention to himself he was angry with school which he saw as unjust.
19	1st child was unhappy and not doing as well as we knew she could at school. 2nd/3rd – travelling for a year, no. 3 decided not to go back – doesn't like school and is bored there.
20	I was never, ever happy with the school system, the limited learning opportunities the N.C gives and the way children behave in schools – especially secondary schools. I have several children with Asperger's syndrome and two with autism and feel that their needs were not met in school and the youngest, who are autistic have never been as I do not think schools are able to give children individual learning programmes they needs with SEN.
21	We always thought we would - No.1: Very good at math, could talk in sentences at 2, other kids at 7! Went to small local school – only 7 in class. Was bullied by teacher for wanting to do maths!!!! Lasted 5 weeks!
22	I feel that children are more likely to retain their natural love of learning if they are allowed to control the content and pace of what they learn, and if they can choose to learn things at a time when they are personally relevant.
23	I started thinking they start school too young. I thought my son was happier and learning more at home than in nursery. The longer I was in it, the more I came to think they could learn more, be happier, have higher self-esteem and individualism if they continued to be home-educated.
24	My child was bored and bullied. The curriculum was not varied enough. School was putting a great stress on the family with their insistence of being 'on time' and never absent.
25	R did not cope well with nursery and does much better in calmer, less chaotic environment
26	Disappointment with school situation. Son wasn't happy or learning well and found that the structure and the system didn't suit him. Soon realised that I don't agree with much that the school system does and do not think that it is an efficient way of learning.
27	Philosophical: we homebirthed, still breastfeed and didn't like the idea of sending her off into the hands of complete strangers. Home ed is a natural extension of our parenting – we feel we are best suited to facilitate her learning and it is done with love and we want her to have a happy childhood. Mother (me) has experience of working in education system and hated the lack of individual care and respect for each child. My husband was educated privately and did not feel that the state system would be beneficial as she is so lovely.
28	Tendency to truant. Too much stress/work/homework/time out of the office. Always in trouble at school. School not providing for our needs. Lots of disruption in classes. Silly set homework. Poor teaching. Unhappy child!