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Article Title: Dynamic modelling of the fidelity of random surface measurement by the stylus method

Year of publication: 2009

Link to published article:

<http://dx.doi.org/10.1016/j.wear.2008.04.061>

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# Dynamic Modelling of the Fidelity of Random Surface Measurement by the Stylus Method

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## Abstract

A numerical simulation has been used to systematically investigate the characteristics of a stylus surface profiling instrument. The model bridges together previously used concepts to incorporate both tip size and tip dynamics. The dynamics incorporate a tip flight phenomenon, inferring detachment from zero value of the reaction force and the trajectory from a free vibration solution to a second order system. Tip shape is handled by a rigid kinematic model. Following brief discussions of the modelling and the process for generating test profiles with defined correlation lengths, the influences of the inherent parameters of the stylus instrument and the testing method on the measurement fidelity are examined. The tip shape effect of the stylus instrument is also considered in the dynamic measurement. It is noted that the finite tip size can increase the critical scanning speed to avoid tip flight, but decreases the signal fidelity of the measurement due to the bridging effect.

*Keywords:* Random surface, Dynamic modelling, Tip flight, Stylus method

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## 1. Introduction

The stylus instrument remains an important technique for measuring surface topography and micro-geometry. It has a number of advantages such as convenient utilization, insensitivity to surface contaminants and vibration. Nevertheless, there are some disadvantages that significantly affect the fidelity of surface measurement. The finite size of the stylus tip introduces a lateral smoothing of the measured profile: the recorded signal is a convolution-like non-linear interaction of the profile and the tip shape. Tip flight is another major factor reducing the surface measurement fidelity. Due to the dynamic effects of the stylus instrument, the trajectory of the stylus tip cannot always follow the profile of the measured surface. Thus tip flight phenomena are especially common in stylus measurements involving high scanning speed and/or large profile slope angle.

Much previous work has been done on the fidelity of surface measurement. Damir [1] studied the effects of stylus kinematics for several deterministic surfaces without consideration of the stylus tip radius. The separation point of the stylus tip from the measured surface, the maximum lift and the path of the stylus after separation were established. McCool [2] assessed the combined effects of stylus tip radius and flight on the surface topography measurement. The simulation showed the magnitude of the distortion and the effects of the sample length and frequency on traced profiles. Song and Vorburger [3] described the theoretical and experimental work on stylus flight and introduced different models of stylus flight in the profiling of sinusoidal, rectangular, triangular, and random surfaces. Pawlus [4] recently introduced a simulation model for predicting the flight distortion on the measured surface topography. Whitehouse [5, 6] has made a great contribution to the study of dynamic aspects of scanning surface instruments and microscopes. In an effort to improve measurement fidelity, he proposed that the optimal damping ratio for the mechanical pick-up of a stylus instrument should be  $\zeta=0.59$ . This has been confirmed experimentally by Liu [7] by using a modified stylus instrument to show that the minimal distortion on surface measurement can be achieved with a damping ratio between 0.5 and 0.7.

In this paper, the dynamic model of the stylus instrument has been developed to study the effects of the tip flight on the signal fidelity in the surface measurement. The method and procedure for generating a random surface profile are briefly described and the cubic spline interpolation technique is used to smooth the generated profiles. The influences of the inherent parameters of a stylus instrument and the parameters of the measurement process on the signal fidelity are investigated. The effects of finite size are also discussed by using triangular section tip shapes with different truncation lengths.

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## 2. Dynamic model

The mechanics of a stylus instrument can be modelled as a second order damped mass-spring system as shown in Fig.1. For small angular motion of the lever, the displacement of the stylus tip is given by

$$\ddot{y}(t) + 2\xi\omega_n\dot{y}(t) + \omega_n^2(y(t) + \delta) = Q(t) \quad (1)$$

where  $y(t)$  is the vertical displacement of the stylus tip,  $\delta$  is the initial static displacement,  $\xi = c/2\sqrt{km}$  is the damping ratio,  $\omega_n = \sqrt{k/m}$  is the natural frequency,  $Q(t) = R(t)/m$  is the vertical reaction force, and the time derivatives of  $y(t)$  can be expressed in terms of surface spatial derivatives as

$$\dot{y}(t) = vy'(x), \quad \ddot{y}(t) = v^2 y''(x)$$

providing that the stylus traversing velocity,  $v$ , remains constant.

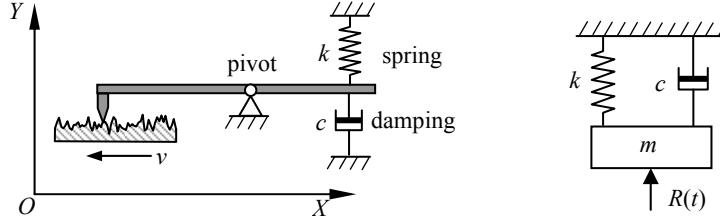


Fig.1. Schematic diagram of the stylus pick-up system

The adhesive forces at the contact between the stylus tip and the surface are normally negligible, and thus the reaction force  $R(t)$  can never be negative. During the scanning process, tip flight will occur at the point  $S(x_s, y_s)$  when the reaction force  $R(t)$  reduces to zero. After the stylus tip separates from the measured profile, the governing equation of the trajectory of the stylus instrument is determined by setting  $Q(t)=0$  in Eq. (1). Solving this, the trajectory of the stylus tip is given as

$$y(t) = (y_s + \delta)e^{-\xi\omega_d t} \left\{ \cos \omega_d t + \frac{\xi}{\sqrt{1-\xi^2}} \sin \omega_d t \right\} + \frac{\dot{y}_s}{\omega_d} e^{-\xi\omega_d t} \sin \omega_d t - \delta \quad (2)$$

where  $\omega_d = \omega_n \sqrt{1-\xi^2}$  is the damped natural frequency,  $y_s$  and  $\dot{y}_s$  are the vertical displacement and velocity at the separation point, respectively. The free flight continues until it drops back into positive contact with the surface. The maximum overshoot can be determined when the vertical velocity of the stylus approaches zero.

## 3. Generation of random profile and stylus tip

Numerical generation is widely used to simulate the natural and engineered surfaces with different bandwidth, which can avoid errors in measurements. There are many methods for generating different surfaces [8, 9]. In this paper, a digital filter and Johnson transfer system are used (in MATLAB®) to generate (pseudo-)random roughness profiles with given skewness and kurtosis. In the dynamic simulations, such sampled profiles must be interpolated in order to determine the real tip trajectory for detecting and modelling stylus flight effects. Instead of piecewise finite difference approximations to the second derivative in evaluating the dynamic response of the stylus, it is preferable, for reasons of numerical stability, to use a smoothed analytical approximation to the discrete profile and then calculate its derivatives analytically. The cubic spline function approximations are chosen as they pass exactly through the discrete points without the ripple of a comparable polynomial fit.

The finite size effects of the stylus tip on surface measurement have been studied by using a triangular section with vertex angle of  $90^\circ$  as the perfect tip, which has a base of  $20 \mu\text{m}$  and a height of  $10 \mu\text{m}$ . The other tips are generated by truncating the perfect tip with a rounded shape to simulate the practical worn effect. The parameters used in this study are based on a practical stylus system with  $\xi=0.02$ ,  $\omega_n=86 \text{ rad s}^{-1}$  and  $m=1.5 \text{ g}$  [7].

## 4. Numerical Simulation

Using the above parameters of the stylus system and the generated random profiles, the simulated scans were carried out based on the established dynamic model. In the simulations, 140 point random profiles with sample interval of  $1 \mu\text{m}$  were generated and interpolated with 20 points between the adjacent original ones. The generated profile with autocorrelation length of  $10 \mu\text{m}$  and the dynamic reaction forces under scanning speeds of 0.01, 0.05 and 0.2 mm/s are shown in Fig.2. Under the two low scanning speeds, 0.01 and 0.05 mm/s, the minimum values of the reaction force  $R(t)$  are positive, indicating that there is no tip flight during the measurement processes. The reaction force  $R(t)$  (Fig.2b) at the scanning speed of 0.01 mm/s is approximately proportional to the measured random profile, which indicates that the spring force is the dominant component. At the scanning speed of 0.05 m/s, the reaction force  $R(t)$  (Fig.2c) fluctuates more than the measured profile (Fig.2a), indicating that the damping and the inertial forces play important roles. With increased scanning speed, for instance  $v=0.2 \text{ m/s}$ ,

the reaction force  $R(t)$  is negative at some peaks with steep slopes as shown in Fig.2d. The stylus tip will then separate from the measured profile. The stylus tip loci with different scanning speeds of 0.2 and 0.35 mm/s are shown in Fig.3. It can be seen that the overshoot of the stylus tip occurs at the steep peaks of the measured profiles and the overshoot of the stylus tip becomes significantly large with increased scanning speed.

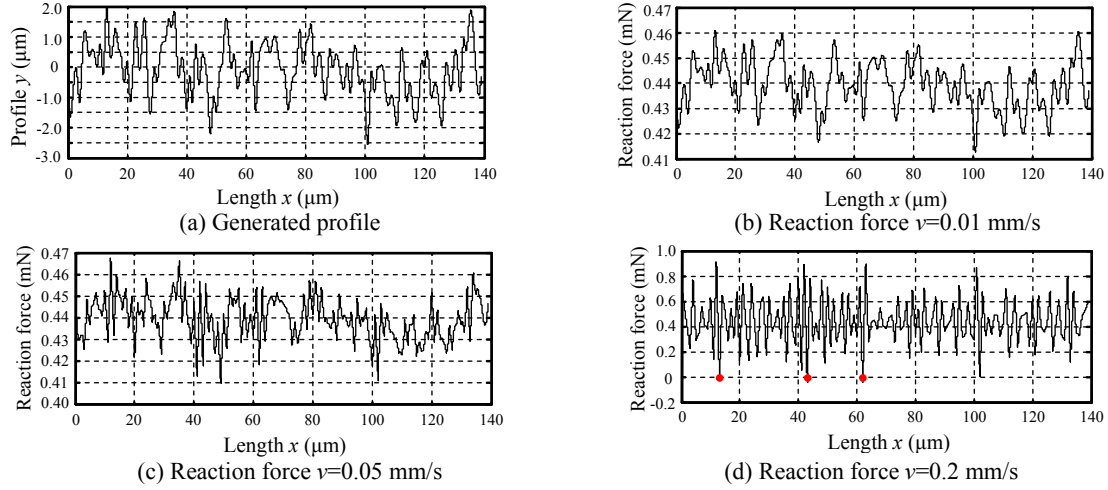


Fig.2. Generated random profile and the dynamic reaction force

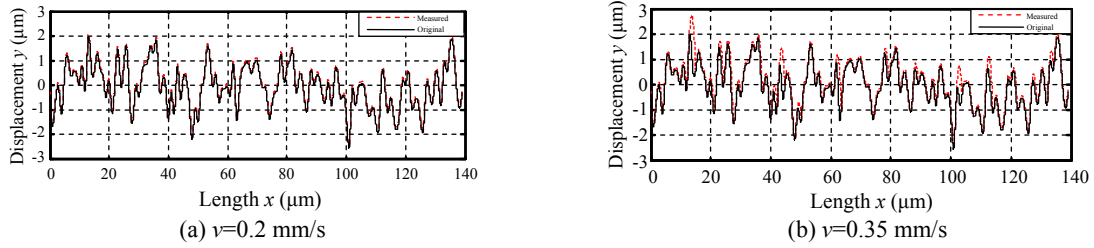


Fig.3. Stylus tip loci with different scanning speed

The effect of the scanning speed on the signal fidelity is also evaluated by means of statistical parameters: the magnitude parameters  $R_a$  and  $R_q$ , RMS slope  $\Delta_q$  and Skewness  $S_k$ . The variations of the statistical parameters over the range of the scanning speeds are shown in Fig.4 where the values are normalized to those at the speed of 0.15 mm/s. It is noted that the measured normalized parameters are constant with the scanning speed below 0.2 mm/s, when the stylus tip can exactly follow the profile and no separation occurs. When the scanning speed exceeds 0.2 mm/s, the measured parameters vary from those obtained at low scanning speed. The deviations of the measured values from the original profile become larger with the increase of the scanning speed because more tip flight occurs. In order to improve the signal fidelity of the stylus instrument, the scanning speed must not exceed the value of 0.2 mm/s which is defined as the critical scanning speed for the current stylus instrument and measurement parameters (such as preload displacement).

The effect of the preload displacement on the critical scanning speed under the current configuration is shown in Fig.5. It can be seen that the critical scanning speed increases with the preload displacement. At a preload displacement less than 3  $\mu\text{m}$ , the critical scanning speed is zero. The reason is that the preload displacement is less than the maximum value of the valley of the measured profile, and the stylus tip cannot follow the profile at the valley even at very low scanning speed. The preload displacement must be large enough to keep the stylus tip in contact with the measured profile during measurement process. On the other hand, a large preload displacement can lead to large static force applied on the measured profile, which may be sufficient to cause scratching and tip wear.

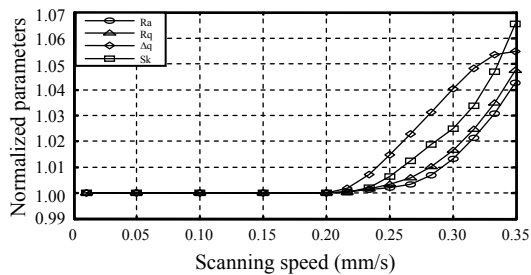


Fig.4. Effects of scanning speed on statistical parameters

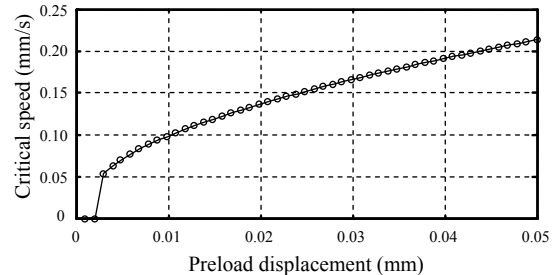


Fig.5. Effect of preload on critical scanning speed

It is intuitively obvious that the higher natural frequencies are likely to improve the fidelity of stylus measurement. The natural frequency has a square root relationship, increasing with stiffness and decreasing with increasing mass. The actual behaviour of the present model was investigated for a preload displacement of 10  $\mu\text{m}$ . Fig.6 shows that a softer spring would lead to a lower critical scanning speed, indicating that the tip flight would occur at a correspondingly low traversing speed. On the contrary, the stronger spring would keep the contact condition between the stylus tip and the profile at a relative high scanning speed. Similarly, the reduction of the equivalent mass of the moving parts of the stylus instrument can improve the critical scanning speed for the given spring stiffness and preload displacement, as shown in Fig.7.

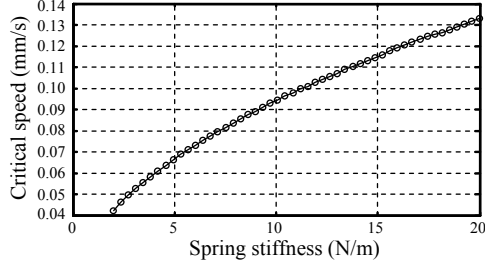


Fig.6. Effect of spring stiffness on critical scanning speed

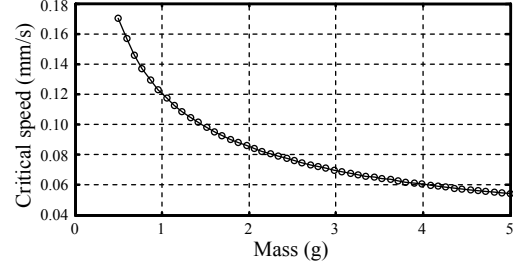
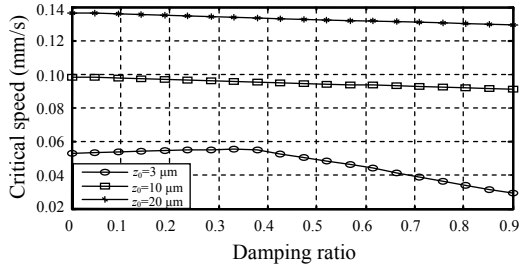
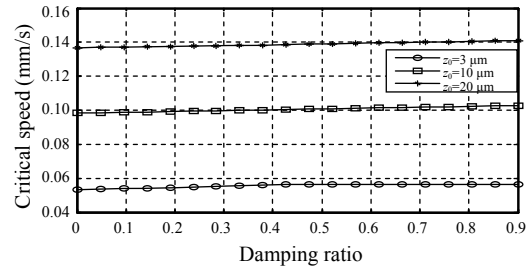


Fig.7. Effect of equivalent mass on critical scanning speed

Although the performance of the stylus instrument can be improved by increasing the natural frequency of the system, there are some disadvantages. The measurement force induced by increasing stiffness can be great and might damage the measured surface. Meanwhile, the reduction of the equivalent mass of the moving part is limited by the minimum stiffness and some functional requirements. Thus, active damping control is an effective method to improve the performance of the stylus instrument [7]. The effect of the damping ratio on the critical scanning speed is shown in Fig.8. From the simulation results, it is noted that the critical scanning speed decreases with the increasing damping ratio (Fig.8a). This phenomenon seems to disagree with the conclusion given in papers [6, 7]. By further investigation, it is clear that the position of the tip separation from the profile occurs easily at the downward sides of the peaks of the profile, where the direction of the damping force is upward, opposing motion. Such separation easily occurs when the damping ratio increases. For the upward side of the peaks of the profile, the separation has little effect on the fidelity of stylus measurement. The critical scanning speed is approximately linear with the damping ratio during the upward separation (Fig.8b). These simulation results still differ somewhat from those of Whitehouse or Liu [6, 7]. The damping ratio has only slight effect on critical speed and its choice is not so important. However, the best value seems to be around 0.4 rather than the 0.5 to 0.7 suggested previously.



(a) Downward separation



(b) Upward separation

Fig.8. Effect of damping ratio on critical scanning speed

Generally, surfaces with larger correlation lengths will have fewer high spatial frequencies and might be expected to have higher critical speeds before fidelity is compromised. Fig.9 shows the effects of the autocorrelation length on the critical scanning speed and the minimum required preload displacement. The larger autocorrelation length of the measured profile will increase the critical scanning speed and decrease the minimum preload displacement to avoid tip flight for a given measurement. Because the majority of functional surfaces in current engineering applications tend to have correlation lengths above 20  $\mu\text{m}$ , the practical variation perhaps seems not so important. However, this may not be the case with modern high-performance structured surfaces.

The finite size effect of the stylus tip on the surface measurement has to be considered, especially in the short wavelength surface metrology, where the wavelength is approximately equal to the radius of the stylus tip. To carry out rather more practical simulations, with consideration of tip wear effects, the perfect triangular tips were truncated and then fitted with an appropriate sized circular quadrant. The effect of the truncation length of the tip on the critical scanning speed is shown in Fig.10. The critical scanning speed gradually increases with the increment of the truncation length of the tip as a whole, but the experimental curve is not monotonic. The critical speed reflects the non-linear filtering effects of stylus size on the profile.

The statistical evaluation of the effect of the tip truncation length on the signal fidelity is shown in Fig.11, where the scanning speed is set to 0.2 mm/s and the autocorrelation length of the random profile is 10  $\mu\text{m}$ . The measured parameters are normalized to those at truncation length of 0.3  $\mu\text{m}$ . It is noted that the statistical parameters will decrease with the increasing

tip truncation length, especially the average slope. This is commonly more important than dynamic effects. The key reason is that the valleys of the stylus tip locus become shallow due to the tip size effect.

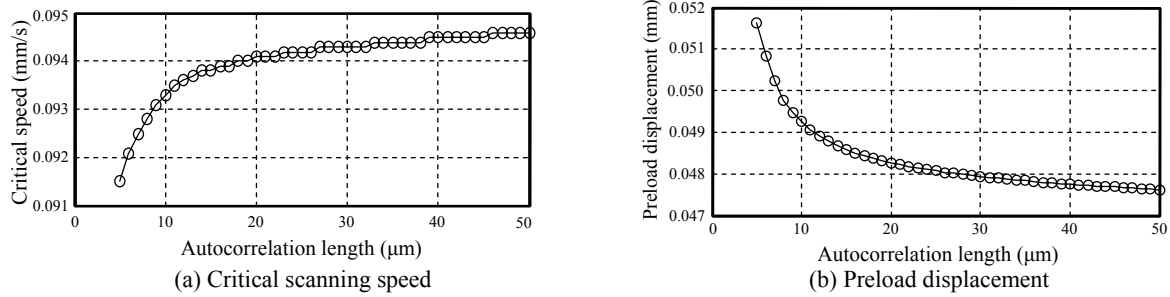


Fig.9. Effect of autocorrelation length on the stylus testing parameters

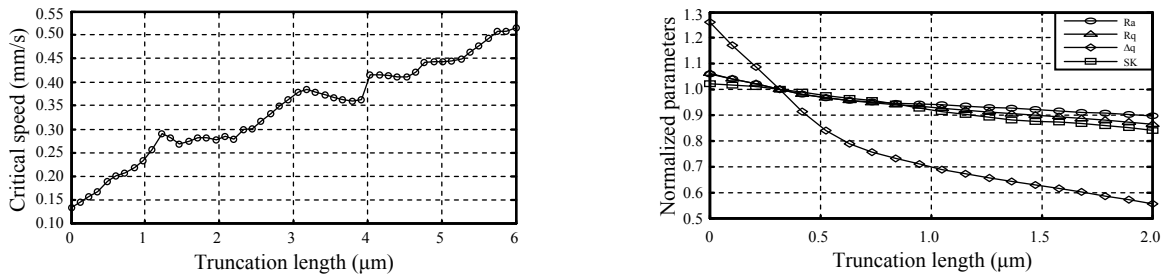


Fig.10. Effect of truncation length on the critical speed

Fig.11. Effects of the tip size on the statistical parameters

## Conclusions

To improve the measurement efficiency, a fast scanning speed is usually preferred. Due to the Hertzian contact between the stylus tip and the measured surface, there is then a possibility that the contact cannot be maintained and the tip will separate from the profile. After tip separation, the stylus instrument will follow a free trajectory different to the surface profile (tip flight), which will affect the measurement accuracy.

We have demonstrated in simulation that fidelity of stylus measurement can be maintained if the stylus is traversed at a speed lower than its critical scanning speed. This critical speed can be determined for a given stylus system and surface. The model shows that the critical scanning speed increases with the increasing spring stiffness, but decreases with the increasing mass. Surfaces with large autocorrelation lengths have faster critical scanning speeds. The model again shows that the damping ratio is best set at a higher value than on most commercial stylus instruments, but slightly lower than previous proposals, at around 0.4. However, from the present results, the choice is rarely likely to be a major factor in the overall measurement fidelity. The dynamic parameter values influence fidelity quite gently for many engineering surfaces generated by conventional manufacturing processes, but will be more critical for the growing range of nano-structured surfaces.

The finite tip size has important effects on the fidelity of stylus measurement but in many cases the geometric smoothing will dominate the dynamic effect. Considering a typical triangular section tip with circular truncation, the tip wear seems to increase the critical scanning speed. However, the information within steep and narrow valleys will be lost from the stylus tip loci. The magnitude of the statistical parameters will generally be reduced with increases in the tip truncation length.

## Acknowledgements

This project is supported in part by the National Natural Science Foundation of China (No. 50705064) and in part by Natural Science Foundation of Tianjin.

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