# Credit Market Imperfections, Nominal Rigidities, and Business Cycles

by

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# Declaration

**I declare that this thesis is the candidate's own work. I also declare that any material contained in this thesis has not been submitted for a degree to any other university.**

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# Abstract

This thesis is a theoretical study of the role of credit market imperfections in business cycle dynamics. In particular, Chapters 2 to 4 focus on the credit channel of the monetary transmission mechanism, while Chapter 5 studies the role of shocks to credit markets in generating business cycle dynamics. The common framework used throughout the thesis is a New Keynesian (NK) framework characterised by imperfect competition and staggered pricesetting.

The essence of the credit channel of monetary transmission is endogenous movements in the external finance premium, which, in turn, are caused by endogenous movements of agency costs generated in the presence of credit frictions. The credit channel works to complement the interest rate channel inherent to the standard NK model.

Chapter 2 aims to shed light on the workings of the credit channel by presenting an analytical solution for the simplified case where agency costs are modelled acyclical. I show that when acyclical agency costs are incorporated into an otherwise standard NK model, they amplify the real impact of money shocks but reduce the persistence of the real effects. This happens because credit frictions flatten both aggregate supply (AS) and aggregate demand (AD) relations of the model, where the former is essentially the New Keynesian Phillips curve while the latter is derived from the consumption Euler equation and money market equilibrium condition.

Chapter 3 replaces the assumption of economy-wide input markets made in Chapter 2 with the one of segmented input markets. The reason for doing this is twofold. First, the latter assumption seems to capture the reality better. Second, the previous literature shows that the segmented market assumption is a crucial determinant for the degree of the persistence of the real effects of money shocks. I show that for given agency costs, both the real impact of money shocks and the persistence of the real effects are much greater in a model with the segmented input market assumption. This happens because the new assumption greatly flattens the AS curve.

Chapter 4 directly studies the workings of the endogenous agency costs. Focusing on credit frictions in borrowing by firms (entrepreneurs), it compares the different business cycle dynamics generated by two alternative modelling strategies. The first assumes that entrepreneurs make a consumption/saving decision to maximise their intertemporal utility, but have a higher discount rate than households (original lenders). The second assumes that a constant fraction of entrepreneurs die each period and they consume all the accumulated wealth just before their death. These assumptions are widely used in the literature to keep agency costs operative. I show that the choice of the modelling strategies is key to the way the credit channel operates within the NK framework.

Chapter 5 investigates the effect of shocks to credit markets on business cycle dynamics. Using the framework developed in Chapter 2, I show that shocks to credit markets affect agency costs and thus the external finance premium faced by entrepreneurs (borrowers). In turn, this causes a change in output. Then, turning to the framework developed in Chapter 4 with endogenous agency costs, I highlight that there is a feedback effect from macroeconomic conditions to the premium through endogenous developments in entrepreneurs' net worth. The change in the premium caused by the feedback effect leads to the further change in output.

# 1 Chapter One: General Introduction

This thesis is a theoretical study of the role of credit markets in business cycles. In particular, Chapters 2 to 4 of the thesis focus on the credit channel of the monetary transmission mechanism. Chapter 5 studies the role of shocks to credit markets in generating business cycle dynamics.

Chapters 2 to 4 consider the role of credit market imperfections particularly in the output dynamics of the response to monetary shocks. The empirical evidence from the vector autoregression (VAR) analysis, such as the one provided by Christiano, Eichenbaum, and Evans (1999), suggests that exogenous monetary policy shocks have sizable and persistent real effects. One specific feature of the response of output is that it shows a hump-shaped pattern. That is, if the shock is contractionary, a trough is reached only after a lag.

However, it has been found that the sizable and persistent real effects of monetary shocks are unlikely to be explained in a framework with perfect competition and flexible prices and wages. In such a framework, money can have real effects qualitatively, but the effects are rather trivial quantitatively. For example, when money is incorporated into an otherwise conventional real business cycle (RBC) model using the Money in Utility (MIU) approach[1], a persistent increase in money supply growth rate causes a rise in expected inflation, which then induces agents to hold less real money balances. When their utility function is not separable between consumption and real money balances, this, in turn, affects their marginal utility of consumption and thus affects their labour/leisure choice. Thus money can have real effects even in the flexible price environment. However, Walsh (2003, chapter 2), for example, shows that the effect is quantitatively weak.

In parallel to the development of the RBC theory, staggered price/wage setting was studied as a potentially strong propagation mechanism of monetary shocks. However, in early works such as Taylor (1979), parameters in the price/wage setting

[1]Alternatively, one can incorporate money into the standard RBC model using the Cash-in-advance constraint as in Cooley and Hansen (1989).

equations are specified exogenously at the outset, not dependent on microeconomic parameters. Although Blanchard and Fischer (1989) give some microfoundations based on Blanchard and Kiyotaki's (1987) static optimising model with imperfect competition, dynamics in their staggered setting model is still superimposed. In comparison with the RBC models, the microfoundations of those early works are rather weak.

More recently, however, staggering has been studied in a model with proper microfoundations, where price/wage setting equations are derived as a result of the firm/worker's intertemporal optimisation subject to staggering as a constraint. That is, the Keynesian idea of nominal rigidities (specifically, staggered price/wage setting) and imperfect competition has been integrated into the intertemporal optimisation approach from the RBC literature. Galí (2008) call this integrated framework the New Keynesian (NK) framework.[2] The principal channel through which the NK framework exhibits non-neutrality of monetary policy is the interest rate channel. In the presence of nominal rigidities, changes in the nominal interest rate are reflected in the real interest rate. In turn, this affects the consumption, investment and output.

Using a variant of the NK framework, a large number of works have studied the staggered price/wage setting as a propagation mechanism to monetary shocks. On the one hand, Chari, Kehoe and McGrattan (2000) conclude that the staggered price model does not produce the observed degree of persistence in the real effects of money shocks. Their work is thorough in the sense that they draw the conclusion after taking into account a few mechanisms which are known to enhance persistence.[3] On the other hand, Ascari (2000) argues that the staggered wage model does produce strong persistence after money shocks, as long as the model is approximated around the zero inflation environment. In the paper, he shows that the source of the strong persistence in his model comes from the segmentation of the input market implicit in his set up, rather than the staggered wage setting itself. Edge (2002), Ascari

[2] Goodfriend and King (1997) denote this integration as the "New Neoclassical Synthesis".

[3] One of the features considered in their paper is the "convex demand" (p.1169) as suggested by Kimball (1995). When the elasticity of demand becomes larger as price increases, the mark-up charged by monopolistic competitors becomes smaller. Then, after a monetary shock, they charge less for a given change in marginal cost. This results in a more persistent movement in the output.

(2003) and Woodford (2003, chapter 3) confirm his contention, showing that even in the price staggered setting, the segmentation of the input markets results in strong persistence. Realising that Chari et al. (2000) assume the economy wide input markets rather than segmented ones in their framework, the segmentation of the input markets seems to be a crucial factor for the staggered price/wage setting to generate large persistence in the real effects of monetary shocks. Indeed, Woodford (2003, chapter 3) supports this, after comparing the segmented factor with other persistence enhancing features.

Having acknowledged the potential importance of the segmented input markets as a strong propagation enhancer in the NK models, it is noticeable that there is one common feature in the aforementioned models, which is, in fact, shared by the many of the NK models in the literature. That is, they ignore potential imperfections in the credit market such as moral hazard due to asymmetric information. However, ignoring credit frictions might not be an innocuous simplification, especially because a streak of both theoretical and empirical literature has pointed out the potentially important role of credit frictions in macroeconomy. Then, given the fact that the NK framework has become an essential base for monetary policy analysis these days[4], it is important to clarify how credit frictions alter the output dynamics to monetary shocks particularly within the framework.

From an empirical point of view, by pointing out that external capital is not a perfect substitute for internal funds, Fazzari, Hubbard and Petersen (1988) argue that financing constraints are important determinants for investment decisions for many firms. In general, Hubbard (1998) reviews the empirical literature of credit market imperfections on investment and emphasises the importance of firms' net worth in investment decisions. As for the household side, while a number of papers (for example, Hall and Mishkin (1982)) discuss the importance of current income in explaining consumer behaviour thus challenging the Life-Cycle Permanent Income Hypothesis, Jappelli and Pagano (1989) argue that the sensitivity of consumption to current income can be due to capital market imperfections.

[4]See Woodford (1999), Walsh (2003) and Galí (2008) among others.

As for a theoretical development, since the seminal work by Bernanke and Gertler (1989), it has become popular to incorporate credit market frictions into a dynamic general equilibrium (DGE) environment using the Costly state verification (CSV) framework of Townsend (1979) and Gale and Hellwig (1985). Within the framework, entrepreneurs (borrowers) can observe their production outcome costlessly while if financial intermediaries (lenders) want to know the outcome, they need to pay monitoring costs. This informational asymmetry causes a moral hazard problem, because entrepreneurs, absent monitoring, might have an incentive to under-report their own production outcome. After Bernanke and Gertler (1989), a large number of papers such as a series of works by Carlstrom and Fuerst (1997, 98, 2001) incorporate credit market frictions into a DGE framework using the CSV approach.

Yet another popular way to model credit market imperfections is initiated by Kiyotaki and Moore (1997). In their framework, credit constraints arise because it is difficult for lenders to force borrowers to repay their debts unless the debts are secured by collateral. Thus, to the extent that the price of collateralized assets is affected by shocks to the economy, the credit limit is also affected. This, in turn, affects the amount borrowers can spend and invest. What is more, credit limits and asset prices turn out to interact in a dynamic way, which turns relatively small disturbances to the economy into persistent fluctuations in output. As an example of papers based on their approach, Iacoviello (2005) demonstrates the importance of collateral constraints tied to the value of real estate for households as well as for firms in the business cycle dynamics.

Following the CSV approach and focusing on corporate borrowing (rather than household borrowing), Chapters 2 to 4 of this thesis study the credit channels of the monetary transmission mechanism specifically within the NK framework. I first describe how credit channels work under the CSV approach intuitively. As pointed out by Bernanke, Gertler and Gilchrist (1999), the important aspect to look at is the endogenous changes of the external finance premium, i.e. the cost of external funds paid by borrowers minus the opportunity costs of their internal funds. Specifically, in the presence of informational asymmetry between lenders and borrowers, the external finance premium is likely to be inversely related to borrowers' net worth. Intuitively, for a given amount of finance required, the smaller borrowers' net worth is, the larger the premium that is required, because agency costs caused by the informational asymmetry are aggravated when borrowers' financial position is weak (i.e., they have little net worth). Then, to the extent that monetary policy shocks influence borrowers' net worth in a pro-cyclical fashion, the external finance premium is expected to move counter-cyclically.[5] In short, compared to the otherwise conventional NK model, the dynamics in the face of monetary policy shocks are further enriched by the endogenous movement of agency costs.

Then, the natural question is, how do the endogenous movements of agency costs alter the output dynamics intrinsic to the otherwise conventional NK model? Specifically, what happens to amplification/persistence in the real effects of monetary shocks? Further, does the model with credit channels replicate the empirically observed fact of the hump shaped reaction to the shocks? I address these questions in Chapter 4 of this thesis.

However, before considering these questions, I would like to address a more fundamental question. That is, do we know everything about the workings of credit channels within the NK framework? Although the endogenous movements of external finance premium surely seem to be critical in the credit channel, is this really the whole channel through which credit frictions alter the output dynamics? The reason why I have this question is as follows. From a technical point of view, the endogenous agency costs are generated by the addition of borrowers' net worth as a state variable.[6] This state variable works as a source of additional dynamics. However, when the state variable is added to the NK framework, which already incorporates the imperfect competition and staggered price/wage setting into the conventional RBC

[5] While Bernanke et al. (1999) primarily look at the corporate borrowing to see the effect of the endogenous premium on corporate investment, Aoki, Proudman and Vlieghe (2004) focus on household borrowing to see the effect on housing investment and consumption.

[6] Strictly speaking, this is not precise in the model presented in Chapter 4. It shows that what is predetermined is borrowers' capital holding. However, given that their net worth is mainly composed of their capital, net worth is almost predetermined.

framework, it makes it very difficult to solve the system analytically. Thus, the models with endogenous agency costs are usually solved numerically. While convenient, the numerical solution often makes it difficult to obtain clear insights into what lies behind the outcome.

Acknowledging this, Chapter 2 considers a NK model with credit frictions in which agency costs are modelled as time-invariant, i.e. acyclical. Without the time-varying agency costs, the model can be solved analytically. Indeed, Chapter 2 finds that even when agency costs are acyclical, they still alter the output dynamics intrinsic to the otherwise standard NK model. Given that the endogenous movements of agency costs are deliberately set aside in the analysis, the primary purpose of Chapter 2 is to shed more light on the workings of credit channels from a qualitative standpoint.

The model in Chapter 2 nests a standard NK model such as the baseline model of Chari, et.al (2000) as a special case where credit frictions are absent. When frictions are incorporated (so that acyclical agency costs are present), they amplify the impact of an unexpected change in money supply on real output while actually reducing the persistence of the real effects. Analytical solution clarifies that these effects take place because credit frictions work to flatten both the aggregate supply (AS) and aggregate demand (AD) relations, where the former relation is essentially so-called the New Keynesian Phillips curve (NKPC) and the latter is derived from the consumption Euler equation and money market equilibrium condition. Then, for a given upward shift in the AD relation caused by a money supply shock, the impact effect on real output is amplified. As for persistence of real effects, the flatter AS curve works to enhance the persistence while the flatter AD reduces it. Overall, however, the persistence turns out to be smaller. The main purpose of Chapter 2 is to elaborate how (acyclical) agency costs flatten both the AS and AD curves. A supplementary quantitative investigation indicates that the amplification of impact can be significant, but the effect on persistence is rather trivial. Thus, in the light of the VAR analysis which reveals that exogenous monetary shocks have sizable and persistent real effects, the acyclical agency costs, on their own, appear to be a modification in the direction towards greater realism. (As pointed out, however, since the cyclical movements of agency costs are set aside, the quantitative result needs to be interpreted with care.)

Chapter 3 is an extension to Chapter 2. As mentioned above, the literature points out that the crucial factor which enhances the persistence of the real effects of money shocks within the NK framework is the segmentation of input markets. What characterises segmented input markets is that movements of inputs such as labour across segments of the economy are restricted. The lack of free movement of labour can prevent wages from being equalised across the segments such as industries. Within the NK framework, the absence of transmission of pressure on wages is shown to be a decisive factor in enhancing the degree of persistence of real effects of money shocks. Besides, the assumption seems plausible, especially in the short run. It certainly appears difficult for workers to move to other industries freely since it often requires them to acquire different kinds of skills.

Acknowledging its potential relevance for monetary transmission and greater descriptive realism, Chapter 3 replaces Chapter 2's assumption of economy-wide input markets (where input costs are always equalised by the free movement of inputs) with the segmented input markets assumption. The model is again solved analytically. First, I find that the effect of time-invariant agency costs on output dynamics is robust to the different ways of modelling input markets. That is, even with the segmented markets assumption, they amplify the real impact of money shocks while reducing the persistence of the real effect. Second, for a given level of credit frictions, i.e. for the same agency costs, both the real impact of money shocks and the persistence of the effects are much greater with segmented input markets. Essentially, this happens because the segmented markets assumption flattens the AS relation greatly. Then, this indicates that the segmented markets assumption seems to be an important modification in the direction towards greater realism even in an environment where credit markets are imperfect.

Having analysed the workings of credit channel within the NK framework in Chapters 2 and 3, Chapter 4 directly highlights the role of endogenous agency costs. As pointed out by Bernanke et al. (1999) and Carlstrom and Fuerst (2001), for example, when one studies the role of endogenous agency costs in business cycles, there is normally one important modelling issue. That is, the situation can ultimately arise where firms (entrepreneurs) accumulate enough net worth so that external finance is not required and thus agency costs disappear.[7] To avoid this situation, the literature typically offers two alternative modelling strategies whereby the accumulation of entrepreneurial net worth is dampened. The first strategy assumes that entrepreneurs make a consumption/saving decision to maximise their intertemporal utility, but have a higher discount rate than households (original lenders). The second assumes that a constant fraction of entrepreneurs die each period and they consume all the accumulated wealth just before their death. The population is kept constant by the birth of new entrepreneurs. For instance, the former is adopted by Carlstrom and Fuerst (1997, 98) while the latter is by Bernanke et al. (1999). Then, based on Carlstrom and Fuerst (2001), Chapter 4 compares the output dynamics in the face of monetary shocks between the two modelling strategies within the NK framework. In fact, it shows that the dynamics generated are quite different depending on the strategies adopted. The main difference from Carlstrom and Fuerst (2001) is that they compare the dynamics caused by monetary shocks between the two strategies in a flexible price environment.

While Chapters 2 and 3 focus on an exogenous shock to the money supply, Chapter 4 assumes that monetary policy is represented by an interest rate rule and considers a shock to the rule. Given that most central banks today use a nominal interest rate as the instrument for policy implementation, one might argue that this is a more realistic approach. Under this assumption, the output dynamics in response to the interest shock differs between the two strategies as follows. In the former case where entrepreneurs make consumption/saving decision following the Euler equation, the output dynamics are characterised by a hump-shaped reaction. Although the VAR

[7] In the framework of Chapters 2 and 3, this issue does not arise because I there assume that borrowers live only one period and their saving decision is irrelevant. (In fact, this assumption helps me present analytical solutions in those chapters.)

analysis in the literature typically reveals this sort of reaction to the shock, a standard NK model often fails to replicate this reaction. Meanwhile, in the latter case with a constant death ratio, although the real impact is not amplified (compared to a standard NK case), the dynamics are characterised by greater persistence in the real effects. It turns out that what causes these different output dynamics between the two strategies is the different developments of entrepreneurs' net worth. Indeed, how to model entrepreneurs' consumption/saving decision is critical to the way the credit channel operates within the NK framework.

However, I argue that the output dynamics observed under the latter strategy (with a constant death ratio) is more realistic. The reason is as follows. Throughout the chapter, entrepreneurs (borrowers) are assumed to be risk neutral in order to simplify the contracting problem with financial intermediaries. In fact, assuming risk-averse entrepreneurs complicates the problem greatly. Then, under the former strategy (with entrepreneurs whose consumption/saving decision follows the Euler equation), what is implied is the lack of consumption smoothing. In relation to this, entrepreneurs' consumption pattern turns out to show rather extreme volatility. As elaborated below, this volatile consumption is a natural outcome of their rational behaviour to maximise their intertemporal utility. However, it is found that the volatility is unrealistically large. On the other hand, with the latter assumption with a constant death ratio, aggregate entrepreneurial consumption/saving changes rather smoothly after the shock. Thus, to the extent that entrepreneurs have a consumption smoothing motive, I tend to argue that the assumption of a constant death ratio leads to more realistic dynamics. Thus, I conclude that the defining effect of endogenous agency costs in the NK model is to make real effects more persistent. However, since the real impact is not amplified, the endogenous agency costs are not necessarily a modification towards greater realism.

Having studied the credit channels of monetary transmission in Chapters 2 to 4, Chapter 5 looks at the role of shocks to credit markets in generating business cycle dynamics. Although a large number of studies have discussed the role of exogenous shocks such as technology shocks or monetary shocks in business cycle dynamics, since many of the models do not take account of credit market imperfections, shocks happening to credit markets are often ignored. Given this, Chapter 5 studies the potential importance of such shocks as drivers of the business cycles.

As in Chapters 2 to 4, Chapter 5 models credit market imperfections following the CSV approach. As an example of shocks to credit markets, I focus on the shock to the variance of idiosyncratic shocks entrepreneurs (borrowers) are subject to. Based on the framework developed in Chapter 2, I show that an increase in the variance lead to an increase in the external finance premium faced by entrepreneurs. This happens because given that entrepreneurs' production outcome is their own private information, a rise in the variance, implying the aggravation of the informational asymmetry with financial institutions (lenders), increases agency costs. The increase in the premium, in turn, decreases production. Also, I observe that an increase in the variance causes higher inflation. This is because an increase in agency costs (deadweight loss) is reflected in an increase in prices.

Then, I move on to study the effect of the credit market shock in the framework developed in Chapter 4. Using this framework with endogenous net worth, I highlight that there is a feedback effect from macroeconomic conditions to the external finance premium through endogenous movements in entrepreneurs' net worth. In fact, as long as the credit market shock has a recessionary effect, their net worth decreases. Then, as emphasised in Chapter 4, the decrease in net worth brings about an increase in the external finance premium. This feedback effect, in turn, further decreases production. As in Chapter 4, how net worth evolves depends on the assumption made on the entrepreneurs' consumption/saving decisions. Therefore, the choice of the assumption matters when one considers the propagation of credit market shocks into the economy. However, again for the reason specified above (entrepreneurs' consumption shows rather extreme volatility when risk neutral entrepreneurs follow consumption Euler equation), the dynamics under the assumption with a constant death ratio seems more plausible. With this assumption, in the face of an exogenous increase in the variance, the output dynamics is characterised by highly persistent movement. That is, recession persists for long time. Also, high inflation turns out to persist too. In short, an increase in the variance leads to prolonged stagflation.

In the sense that the particular credit market shock is a second moment shock in nature, Chapter 5 is related to the literature which studies the role of uncertainty in the business cycles. For example, Bernanke (1983) shows that under the assumption that investment project is irreversible due to high adjustment costs, events whose long run implications are uncertain gives investors an incentive to wait for undertaking investment. More recently, looking at a wide range of proxies for uncertainty both in micro and macro levels, Bloom, Floetotto and Jaimovich (2009) propose the following stylised fact:[8] both idiosyncratic uncertainty about the evolution of micro level variables and aggregate uncertainty about the evolution of macro level variables are strongly countercyclical. Then, they construct a general equilibrium model and show that a rise in uncertainty at both micro and macro levels leads to a drop in output[9]. In their model, the important feature which causes the second moment shock to have the real effects is the non-convex adjustment costs in both capital and labour. When uncertainty is high, firms become cautious and their investment and hiring slow down thus output falls. Meanwhile, Chapter 5 observes that an increase in uncertainty at micro level can have an adverse macro effect including the fall in output. However, the propagation mechanism is the endogenous developments in credit markets. Thus, the chapter implies that credit market imperfections also might be a factor which explains the stylised fact of countercyclical micro level uncertainty observed by Bloom et al. (2009).

[8]As for micro level, their proxies include the cross-sectional spread of firm-level sales growth rates and also the spread of industry-level sales growth rates, while the proxies for uncertainty at macro level includes an index of stock market volatility.

[9]They also report the overshooting phenomenon: output rebounds and increases beyond the initial level.

# 2 Chapter Two: Credit Market Imperfections, Staggered Pricesetting, and Output Dynamics of the Response to Money Shocks

## 2.1 Introduction

In this chapter, I study the role of credit frictions in the monetary transmission analytically. To solve the model analytically, agency costs due to credit frictions are modelled as acyclical. As emphasised in the general introduction, the baseline framework is the NK framework. I show that when acyclical agency costs are incorporated into an otherwise standard NK model with economy-wide input markets, they amplify the impact of money shocks on output while reducing the persistence of the real effects. Analytical solution clarifies that credit frictions affect monetary transmission by changing the slopes of both aggregate supply (AS) and aggregate demand (AD) curves of the model.

To see the intuition behind the result, I first describe the exact nature of credit frictions in the model. Agents called entrepreneurs have production opportunities, but lack internal funds to employ labour as the input for production. This induces them to seek external funds. Meanwhile, households, separate agents, have some funds to spare, but do not have the production opportunities. Thus, entrepreneurs borrow funds from households through financial intermediaries called banks. Entrepreneurs' production is subject to an idiosyncratic shock. Credit frictions arise because the outcome of production is entrepreneurs' private information and it is costly for banks to monitor it. This informational asymmetry causes a moral hazard problem, since entrepreneurs, in the absence of monitoring, could have an incentive to under-report their own production to increase the profits. In the literature, this sort of framework is known as the costly state verification framework (Townsend, 1979).

Importantly, monitoring costs create a wedge between household consumption,

$c_t$ and output, $y_t$, where $c_t$ and $y_t$ are log deviations from the steady state values[10]. Intuitively, the wedge is created as follows. First, when monitoring costs are absent, entrepreneurs do not make profits because their ability of monitoring their own investment outcome free is virtually shared by lenders and thus it does not earn them economic "rents". The absence of economic rents results in zero entrepreneurial consumption.[11] Then, when output in the model is composed of household and entrepreneurial consumptions, household consumption corresponds to output; $c_t = y_t$.

However, when it is costly for lenders to monitor the outcome, entrepreneurs do enjoy rents because their ability of free monitoring is now special. Thus, entrepreneurial consumption is not trivial any more. Meanwhile, suppose that aggregate output increases due to an unexpected increase in money supply. This bids up the price of goods entrepreneurs produce relative to the price of separate goods they consume. This increase in the relative price makes entrepreneurial consumption increase more than output, i.e. one percent increase in output is accompanied by more than one percent increase in entrepreneurial consumption. Correspondingly, household consumption increases less than proportionally. Thus, in terms of $c_t = dy_t$, monitoring costs decrease the value of $d$ from unity. The wedge is thus created between $c_t$ and $y_t$.

Having observed this, I can show how monitoring costs affect the real effect of money shocks. Imagine the following static money demand equation; $m_t - p_t = c_t$ where $m_t$ is the aggregate money demand, $p_t$ is the aggregate price and $c_t$ is household consumption, all variables in terms of log deviation from the steady state. Incorporating $c_t = dy_t$ discussed above, this becomes $m_t - p_t = dy_t$. Assuming that the money market is in equilibrium, I interpret this as an aggregate demand (AD) relation. Notice that monitoring costs flatten the AD relation by lowering $d$ (in a space where the horizontal axis represents output and the vertical axis price). Now, suppose that there is an unexpected permanent increase in the money supply.

[10] It is below defined as the zero inflation rate steady state.

[11] Strictly speaking, since entrepreneurs' production function exhibits decreasing returns to scale, they obtain some profit and their consumption is not zero even in the absence of monitoring costs. However, I here ignore this profit for the intuitive discussion.

This shifts up the AD curve in the space. Then, the flatter the curve is, the more output increases (given upward sloping aggregate supply (AS) curve). Monitoring costs amplify the real impact of money shock by flattening the AD curve.

The impact effect is further amplified because monitoring costs also flatten the AS curve[12]. Households supply labour in the (economy-wide) labour market. In the model, the equilibrium real wage is subject to an income effect, i.e. the wage is increasing in household consumption. However, since monitoring costs create a wedge between $c_t$ and $y_t$, the wage does not change much for given $y_t$. Meanwhile, there are other agents called retailers, who are monopolistic competitors subject to staggered pricesetting. The smaller change in the wage caused by the weaker income effect leads to a smaller change in retailers' real marginal cost. That is, monitoring costs make the real marginal cost less pro-cyclical. This, in turn, flattens the AS relation. Indeed, the flatter AS curve also amplifies the real impact of money shock. Overall, the flatter AD and AS curves both contribute to the amplification of impact.

Next, given that staggered pricesetting makes the real effect of the shock persist, how do monitoring costs affect the degree of persistence? On the one hand, the flatter AS relation implies a slower adjustment of aggregate price during the transition after the money shock. In turn, this implies the slower change in output. The flatter AS relation actually makes the persistence greater. However, the flatter AD relation is shown to reduce persistence of the shock.[13] Although the effects through AD and AS relations contradict, I theoretically show that the one through the AD relation is dominant so that monitoring costs reduce the persistence overall.

The structure of the model in this chapter is similar to some NK models with credit frictions in the literature, such as Bernanke et al. (1999). As pointed out, however, the crucial difference is that my model limits agency costs to be acyclical while their models endogenise them. The purpose of this chapter is to shed light on the working of credit channel of monetary transmission within the NK framework by

[12] As shown below, this is essentially the New Keynesian Phillips curve.

[13] Although in a different context, Taylor (1979) also reports that a flatter AD relation leads to smaller persistence. In his case, the slope of AD relation is determined by the degree of accommodation of aggregate demand policy to wage changes.

presenting the analytical solutions for the simplified model.

This chapter is organised as follows. Section 2 provides an overview of the model. Section 3 studies how credit frictions, represented by non trivial monitoring costs, make retailers' real marginal cost less pro-cyclical. Then, section 4 shows that credit frictions flatten the AS relation. Section 5 looks at how they also flatten the AD relation. Having seen the effects on AS and AD relations, Section 6 studies how credit frictions alter output dynamics of the response to money shocks. Section 7 conducts quantitative analysis. Section 8 checks the robustness of the results by generalising households' utility function. Section 9 concludes.

## 2.2 The model: overview

Figure 1 shows the overview of the model.

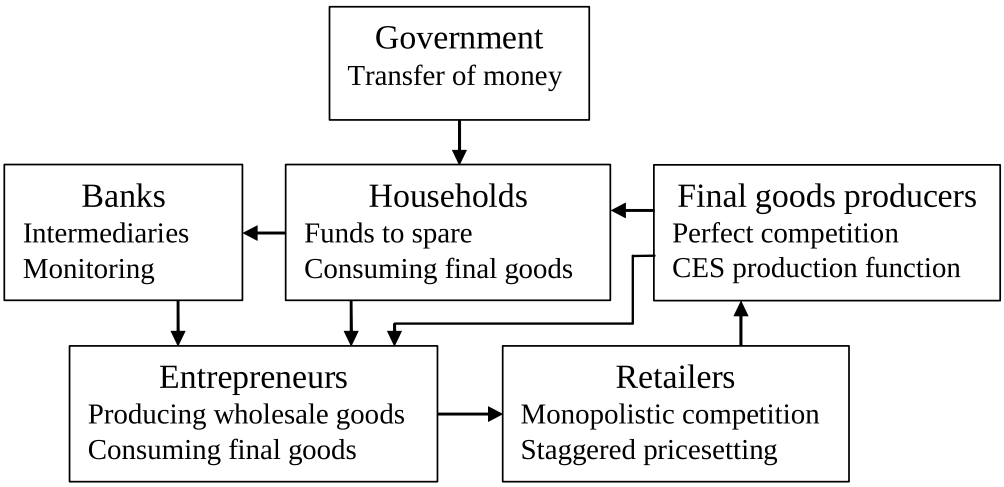


Figure 1: Overview of the model

Entrepreneurs produce homogeneous goods called wholesale goods employing labour supplied by households. Lacking internal funds, they borrow the labour cost from households through banks. Given that the outcome of wholesale goods production is entrepreneurs' private information, banks need to pay monitoring costs to observe it. As mentioned, this asymmetry of information is the source of credit frictions.

Using wholesale goods as the only input, retailers produce differentiated retail goods. Being monopolistic competitors, they are subject to the Calvo style staggered pricesetting (Calvo 1983): in each period, only a fraction of randomly selected retailers are given an opportunity to reset their prices. The reason why I model entrepreneurs and retailers as separate producers is to consider a contracting problem with banks and an intertemporal profit maximisation problem under staggered pricesetting separately. This strategy is often taken when one incorporates credit frictions into the NK framework.[14]

Retailers face a downward sloping demand curve because of final goods producers. Being perfect competitors, final goods producers make composite retail goods called final goods relying on the CES (constant elasticity of substitution) production function.[15] Households and entrepreneurs consume final goods.

As a money shock, this chapter focuses on an unexpected permanent increase in the money supply. Raising revenue through seigniorage, the government makes a direct transfer of money to households. For simplicity, there is no government spending.

## 2.3 Retailers' real marginal cost

To see how monitoring costs alter output dynamics of the response to the money shock, I first study their effect on retailers' real marginal cost. Assuming that retailers' production function exhibits constant returns to scale, the real marginal cost is equal to the real price of wholesale goods. Thus, I now study how the real price of wholesale goods is determined.

[14] Examples are Bernanke et al. (1999), Iacoviello (2005) and Faia and Monacelli (2007).

[15] It should be clear that our "retailers" are not sellers of final goods, contrary to the everyday meaning of the word.

### 2.3.1 Entrepreneurs

Entrepreneur $j$ produces wholesale goods, $Y_t^W(j)$ using household labour, $H_t(j)$. The production function exhibits decreasing returns to scale:

$$Y_t^W(j) = \omega_t(j) H_t(j)^\alpha, \tag{1}$$

where $0 < \alpha < 1$. $\omega_t(j)$ is an iid random variable with an expected value of unity; $E(\omega) = 1$. It represents an idiosyncratic shock to the production. The distribution of the random variable is common across individuals and also time-invariant.

Entrepreneurs, who live only one period, have no initial wealth. I assume that household labour cost, $W_t H_t(j)$, where $W_t$ is nominal wage, has to be paid before the production takes place[16]. Thus, entrepreneurs need to raise the labour cost externally. Meanwhile, households, who do not have an access to this production technology, have funds to spare. Entrepreneurs borrow the input cost from households through banks.

**Costly state verification framework** The realised value of the idiosyncratic shock is entrepreneur $j$'s private information. In order for banks to observe the actual amount of wholesale goods produced, they need to incur monitoring costs. This framework is known as the costly state verification (CSV) framework (Townsend, 1979). I assume that the cost for monitoring entrepreneur $j$'s production is a proportion of the expected amount of production, i.e. $\mu H_t(j)^\alpha$. The monitoring costs parameter, $\mu$ is common across entrepreneurs and time-invariant.[17]

**Standard debt contract** Under the CSV framework, the form of optimal contract is derived as a standard debt contract (Townsend (1979), Gale and Hellwig (1985)).

[16] Carlstrom and Fuerst (1998, 2001) also assume this, although in their models, the input bill contains capital rental fee as well.

[17] I assume that $\mu < 1$, since $\mu$ of as high as 1 implies that monitoring costs amount to the expected wholesale goods production. Monitoring costs should not be that high. For example, a possible range of the parameter, $\mu$ suggested by Carlstrom and Fuerst (2001) is between 0.04 and 0.36.

If the production of wholesale goods happens to exceed the predetermined amount of repayment, entrepreneur $j$ pays this predetermined amount and keeps the rest. However, if it turns out to be less, he defaults and his bank pays monitoring cost and takes all the remaining goods.[18]

The amount of repayment is expressed as $\Psi_t W_t H_t(j)$ in nominal terms, where $\Psi_t$ is the predetermined gross interest rate. Denoting the nominal price of wholesale goods as $P_t^W$, notice that there is a cut-off value of $\omega_t(j)$, $\overline{\omega_t}(j)$ such that

$$\Psi_t W_t H_t(j) = \overline{\omega_t}(j) P_t^W H_t^\alpha.$$

That is, if $\omega_t(j)$ turns out to be below the cut-off value, he defaults.

The contract between entrepreneur $j$ and his bank is made before the idiosyncratic shock is realised. It determines $\overline{\omega_t}(j)$ and the size of the project represented by $H_t(j)$. As mentioned, entrepreneurs consume final goods. Given that entrepreneurs live only one period, they consume all the profits they have made. Then, practically, entrepreneurs aim for maximising their expected consumption of final goods.

Assuming that lending takes place within a period, the net interest rate households obtain from lending to banks is zero, because their opportunity cost of not lending is zero.[19] For simplicity, banks do not employ inputs to operate. Assuming that perfect competition prevails in the banking sector, banks do not make any profits. By lending the input costs, $W_t H_t(j)$ to a large number of entrepreneurs, they recoup the amount they have lent on average.

Denoting the real price of wholesale goods as $\varphi_t \left(= \frac{P_t^W}{P_t} \text{ where } P_t \text{ is the price of final goods}\right)$ and real wage as $w_t \left(= \frac{W_t}{P_t}\right)$, the contracting problem is formally expressed as:

*maximise*

$$\varphi_t f\left(\overline{\omega_t}(j)\right) H_t(j)^\alpha, \tag{2}$$

[18] Given that monitoring takes place only when entrepreneurs go bankrupt, the monitoring costs can be interpreted as bankruptcy costs, which include not only direct costs for auditing but also indirect costs such as costs from asset liquidation and the interruption of business.

[19] Because of this intra period nature of the loan, lending to entrepreneurs does not compete with investing in an interest bearing bond whose return is paid intertemporally.

*with respect to* $\overline{\omega_t}(j)$ and $H_t(j)$, *subject to*

$$\varphi_t g(\overline{\omega_t}(j)) H_t(j)^\alpha = w_t H_t(j). \tag{3}$$

Entrepreneur $j$'s expected real profit is expressed as $\varphi_t f(\overline{\omega_t}(j)) H_t(j)^\alpha$, where $f(\overline{\omega_t}(j))$ is his expected share of the real revenue from wholesale goods production. Under the standard debt contract, the share is expressed as:

$$f(\overline{\omega_t}(j)) = \int_{\overline{\omega_t}(j)}^{\infty} \omega d\Phi(\omega) - (1 - \Phi(\overline{\omega_t}(j))) \overline{\omega_t}(j), \tag{4}$$

where $\Phi$ stands for a cumulative distribution function of $\omega$[20]. In the banks' participation constraint (Eq.3), $g(\overline{\omega_t}(j))$ is the banks' expected share of the revenue. The share is given as

$$g(\overline{\omega_t}(j)) = \int_0^{\overline{\omega_t}(j)} \omega d\Phi(\omega) + (1 - \Phi(\overline{\omega_t}(j))) \overline{\omega_t}(j) - \mu\Phi(\overline{\omega_t}(j)). \tag{5}$$

Notice that there is deadweight loss under this setting. Adding the shares, $f(\overline{\omega_t}(j))$ and $g(\overline{\omega_t}(j))$, I obtain:

$$f(\overline{\omega_t}(j)) + g(\overline{\omega_t}(j)) = 1 - \mu\Phi(\overline{\omega_t}(j)).$$

This clarifies that, on average, the fraction, $\mu\Phi(\overline{\omega_t}(j))$ of the wholesale goods production is lost in the monitoring process. This deadweight loss represents agency costs from the informational asymmetry.

**Real price of wholesale goods,** $\varphi_t$ Solving the contracting problem, I first derive the following implicit labour demand function:

$$w_t = \frac{1}{s(\overline{\omega_t}(j))} \varphi_t \alpha H_t(j)^{\alpha-1},$$

[20] $\phi$ will be used for its probability density function.

where

$$s\left(\overline{\omega_t}\left(j\right)\right) = \frac{1}{1 - \mu\Phi\left(\overline{\omega_t}\left(j\right)\right) + \frac{\mu\phi\left(\overline{\omega_t}(j)\right) f\left(\overline{\omega_t}(j)\right)}{f'\left(\overline{\omega_t}(j)\right)}}.$$

Substituting this into the constraint, $\varphi_t g\left(\overline{\omega_t}\left(j\right)\right) H_t\left(j\right)^{\alpha} = w_t H_t\left(j\right)$, I obtain

$$\alpha = g\left(\overline{\omega_t}\left(j\right)\right) s\left(\overline{\omega_t}\left(j\right)\right). \tag{6}$$

Given that the distribution of $\omega$ is common to all the entrepreneurs and time-invariant, the cut-off value is also common and time-invariant: $\overline{\omega_t}\left(j\right) = \overline{\omega}$. Thus, Eq.6 simplifies to:

$$\alpha = g\left(\overline{\omega}\right) s\left(\overline{\omega}\right), \tag{7}$$

where

$$s\left(\overline{\omega}\right) = \frac{1}{1 - \mu\Phi\left(\overline{\omega}\right) + \frac{\mu\phi(\overline{\omega}) f(\overline{\omega})}{f'(\overline{\omega})}}. \tag{8}$$

This relation indicates that once the distribution of $\omega$ is fully specified, the cut-off value of $\overline{\omega}$ can be obtained for given $\alpha$ and $\mu$.

Furthermore, since $\overline{\omega_t}\left(j\right) = \overline{\omega}$, I know from the labour demand function that the demand is symmetric across entrepreneurs, $H_t\left(j\right) = H_t$. Thus, the function is rewritten as:

$$w_t = \frac{1}{s\left(\overline{\omega}\right)} \varphi_t \alpha H_t^{\alpha - 1}.$$

Rearranging this, I obtain:

$$\varphi_t = s\left(\overline{\omega}\right) \frac{1}{\alpha H_t^{\alpha-1}} w_t. \tag{9}$$

This relation says that the relative price of wholesale goods, $\varphi_t$ is set as the mark up, $s\left(\overline{\omega}\right)$ over wholesalers' real marginal cost, $\frac{1}{\alpha H_t^{\alpha-1}} w_t$. Intuitively, entrepreneurs need to charge the mark up in order to cover the agency costs. Indeed, $s\left(\overline{\omega}\right)$ takes unity in the absence of monitoring costs ($\mu = 0$). However, when $\mu > 0$, $s\left(\overline{\omega}\right)$ is larger than one given that $f'\left(\overline{\omega}\right) \left(= -\left(1 - \Phi\left(\overline{\omega}\right)\right)\right) < 0$.

Finally, notice that the agency costs, $\mu\Phi\left(\overline{\omega}\right)$ (and the mark, $s\left(\overline{\omega}\right)$) is acyclical. Once the time-invariant distribution function of $\omega$ is specified, this can be obtained for given values of $\mu$ and $\alpha$. The acyclical agency costs enable me to obtain below the explicit solution for the output dynamics to the money shock.

### 2.3.2 Households

Having looked at entrepreneurs' behaviour to obtain the pricing relation (Eq.9), I now turn to the representative household's utility maximisation problem. The representative household cares about consumption of final goods, $C$, real money balances, $\frac{M}{P}$ and labour supply $H$. I assume that a rise in real money balances increases utility, given that it facilitates transaction. For simplicity, the utility function is assumed to be separable. The utility function is given as:

$$\sum_{t=0}^{\infty} \beta^t \left( \delta \frac{C_t^{1-\sigma}}{1-\sigma} + (1-\delta) \frac{\left(\frac{M_t}{P_t}\right)^{1-\zeta}}{1-\zeta} - \chi H_t^{\eta} \right), \tag{10}$$

where $\beta$ is the discount rate and it is assumed that $0 < \delta < 1$, $\sigma \geq 0$, $\zeta \geq 0$, $\chi > 0$, and $\eta > 1$. The budget constraint is expressed as:

$$P_t C_t + M_t + B_t = W_t H_t + M_{t-1} + (1 + i_{t-1}) B_{t-1} + \Pi_t + T_t,$$

where $B_t$ is bond holding, $i_{t-1}$ is a nominal interest rate (accrued in period t), $\Pi_t$ is profit share from retailers[21], and $T_t$ is a lump sum tax, all in nominal terms. In the equilibrium, the aggregate supply of bonds is zero.

The government budget constraint is $M_t - M_{t-1} = T_t$. In case $T_t > 0$, it means that the government makes a direct transfer of money to households, raising revenue through seigniorage. In case $T_t < 0$, it is actually a tax collected by the government from households. The government spending is zero in the model.

Solving this utility maximisation problem, I obtain the consumption Euler equation,

$$C_t^{-\sigma} = \beta (1 + r_t) C_{t+1}^{-\sigma}, \tag{11}$$

[21] Households are the shareholders of retailers.

the money demand function,

$$\frac{M_t}{P_t} = \left(\frac{1-\delta}{\delta}\frac{1+i_t}{i_t}C_t^{\sigma}\right)^{\frac{1}{\varsigma}}, \tag{12}$$

and the labour supply function

$$w_t = \frac{\chi\eta}{\delta}\frac{H_t^{\eta-1}}{C_t^{-\sigma}}. \tag{13}$$

In deriving money demand function, use is made of the consumption Euler equation and the Fisher equation: $1 + r_t \equiv (1 + i_t)\frac{P_t}{P_{t+1}}$.

### 2.3.3 Elasticity of retailers' real marginal cost with respect to output

Substituting the labour supply function (Eq.13) into the wholesalers' pricing relation (Eq.9) and rearranging, I can express the real price of wholesale goods, equivalently retailers' real marginal cost, as a function of household consumption $C_t$ and employment, $H_t$:

$$\varphi_t = \frac{\chi\eta}{\delta}\frac{1}{\alpha}s(\varpi)\,C_t^{\sigma}H_t^{\eta-\alpha}\text{[22]}. \tag{14}$$

In order to derive the elasticity of retailers' real marginal cost with respect to output, I now rewrite $\varphi_t$ as a function of output, $Y_t$.

**Elasticity of retailers' real marginal cost with respect to output** Entrepreneur $j$'s production function is given as $Y_t^W(j) = \omega_t H_t^{\alpha}$.[23] Given $E(\omega) = 1$, the "gross" aggregate production of wholesale goods is $H_t^{\alpha}$. Subtracting the aggregate deadweight loss due to monitoring, $\mu\Phi(\varpi)H_t^{\alpha}$, I obtain the "net" aggregate wholesale good production, $Y_t^W$ as follows:

$$Y_t^W = (1-\mu\Phi(\varpi))\,H_t^{\alpha}. \tag{15}$$

[22] In fact, this is a labour market equilibrium condition.

[23] Remember that $H_t(j) = H_t$ for all $j$.

Under the assumption that entrepreneurs spend all the profits on the consumption of final goods, the aggregate entrepreneurial consumption, $C_t^e$ is given as

$$C_t^e = \varphi_t f(\overline{\omega}) H_t^{\alpha}.$$

Using Eq.15, this becomes

$$C_t^e = \varphi_t \frac{f(\overline{\omega})}{1-\mu\Phi(\overline{\omega})} Y_t^W. \tag{16}$$

Meanwhile, the market clearing condition for the final good is:

$$Y_t = C_t + C_t^e, \tag{17}$$

where $Y_t$ is aggregate final good production and $C_t$ is aggregate household consumption. From Eqs.16 and 17, $C_t$ is expressed as:

$$C_t = Y_t - \varphi_t f(\overline{\omega}) \frac{Y_t^W}{1-\mu\Phi(\overline{\omega})}. \tag{18}$$

Finally, using Eqs.15 and 18, retailers' real marginal cost (Eq.14) is rewritten as:

$$\varphi_t = \frac{\chi\eta}{\delta}\frac{1}{\alpha} s(\overline{\omega}) \left( Y_t - \varphi_t \frac{f(\overline{\omega})}{1-\mu\Phi(\overline{\omega})} Y_t^W \right)^{\sigma} \left( \frac{Y_t^W}{1-\mu\Phi(\overline{\omega})} \right)^{\frac{\eta-\alpha}{\alpha}}. \tag{19}$$

Observe that there is $\varphi_t$ in the expression for $C_t$ on the right hand side. This is because $C_t^e$ is a function of $\varphi_t$: the higher the relative price of wholesale goods, the more entrepreneurs consume on aggregate. Generally, $\varphi_t$ can not be factored out apart from the case where $\sigma = 0$ or $\sigma = 1$. $\sigma$ represents the elasticity of intertemporal substitution of consumption. $\sigma = 0$ indicates that the subutility over consumption is linear, $\delta C_t$, while $\sigma = 1$ implies that it is logarithmic, $\delta \ln C_t$. Due to the extremity of the former case, my focus will be on the latter ($\sigma = 1$) from here onwards.

With $\sigma = 1$, the real marginal cost, $\varphi_t$ can be written as a function of $Y_t^W$ and

$Y_t$:

$$\varphi_t\left(Y_t^W, Y_t\right) = \frac{\frac{\chi\eta}{\delta}\frac{1}{\alpha}s\left(\overline{\omega}\right)\left(\frac{Y_t^W}{1-\mu\Phi(\overline{\omega})}\right)^{\frac{\eta-\alpha}{\alpha}} Y_t}{1+\frac{\chi\eta}{\delta}\frac{1}{\alpha}s\left(\overline{\omega}\right)f\left(\overline{\omega}\right)\left(\frac{Y_t^W}{1-\mu\Phi(\overline{\omega})}\right)^{\frac{\eta}{\alpha}}}.^{24} \tag{20}$$

I now log-linearise Eq.20 around the flexible price steady state to obtain the elasticity of the real marginal cost with respect to output. To do this, I first show that aggregate wholesale good production, $Y_t^W$ and aggregate final good production, $Y_t$ are the same in the steady state, i.e. $\overline{Y^W} = \overline{Y}$ (see Appendix 1). The appendix also shows that the log deviation of $Y_t^W$ from $\overline{Y}$, $\widehat{Y_t^W}$ and the deviation of $Y_t$ from $\overline{Y}$, $\widehat{Y}_t$ are the same, i.e. $\widehat{Y_t^W} = \widehat{Y}_t$.

Incorporating these findings, log-linearisation yields:

$$\widehat{\varphi}_t = \xi\widehat{Y}_t, \tag{21}$$

where

$$\xi = \frac{\left(\varphi_{Y^W}\left(\overline{Y},\overline{Y}\right)+\varphi_Y\left(\overline{Y},\overline{Y}\right)\right)\overline{Y}}{\varphi\left(\overline{Y},\overline{Y}\right)}. \tag{22}$$

In Eq.21, $\widehat{\varphi}_t$ is the log deviation of $\varphi_t$ from the steady state, $\overline{\varphi}$. In Eq.22, $\varphi_{Y^W}\left(\overline{Y},\overline{Y}\right)$ is the first order partial derivative of $\varphi$ with respect to $Y^W$ evaluated at the steady state and $\varphi_Y\left(\overline{Y},\overline{Y}\right)$ is defined likewise. Thus, $\xi$ represents the elasticity of retailers' real marginal cost with respect to output. In order to express $\xi$ as a function of deep parameters of the model, I now show how the steady state value of $\overline{Y}$ is determined.

**Flexible price steady state** Retailers, monopolistic competitors, face a downward sloping demand curve. In order to derive the demand curve, I study the behaviour of final good producers. They produce the final good (composite retail good) using all the differentiated retail goods. Their production function is of the CES type:

$$Y_t = \left(\int_0^1 Y_t\left(z\right)^{\frac{\epsilon-1}{\epsilon}}dz\right)^{\frac{\epsilon}{\epsilon-1}}, \tag{23}$$

[24] The cutoff value, $\overline{\omega}$ is a function of neither $Y_t$ nor $Y_t^W$, since it is modelled as acyclical (see Eq.7).

where $Y_t$ is the final good and $Y_t(z)$ is the retail good produced by retailer $z$. Solving their cost minimisation problem, I obtain the following demand function for retail good $z$,

$$Y_t(z) = Y_t \left( \frac{P_t(z)}{P_t} \right)^{-\epsilon}, \tag{24}$$

where

$$P_t = \left( \int_0^1 P_t(z)^{1-\epsilon} dz \right)^{\frac{1}{1-\epsilon}}. \tag{25}$$

Since final good producers act competitively and have a constant returns to scale technology, $P_t$ is the price of the final good as well as the price index of the retail goods.

To define the output in the flexible price steady state, $\overline{Y}$, I study how retailer $z$ behaves in a flexible price setting. Given that there are a large number of retailers, the aggregate price, $P_t$ is treated as given. As mentioned, retailers use wholesale goods as the only input. They transform homogeneous wholesale goods into differentiated retail goods using constant returns to scale technology. Thus, retailer $z$'s production function is simply given as: $Y_t(z) = Y_t^W(z)$. Then, his profit maximisation problem is expressed as:

*maximise*

$$\Pi_t(z) = P_t(z) Y_t(z) - P_t^W Y_t(z)$$

*subject to*

$$Y_t(z) = Y_t \left( \frac{P_t(z)}{P_t} \right)^{-\epsilon}.$$

Solving this yields

$$\frac{P_t(z)}{P_t} = \frac{\epsilon}{\epsilon - 1} \varphi_t, \tag{26}$$

where $\varphi_t = \frac{P_t^W}{P_t}$. The optimal real price is expressed as the mark up $\frac{\epsilon}{\epsilon-1}$ over the real marginal cost.

In a flexible price steady state, every retailer is symmetric and charges the same price. Then, given that $P_t(z) = P_t$, I obtain the following relation:

$$1 = \frac{\epsilon}{\epsilon - 1}\varphi\left(\overline{Y}, \overline{Y}\right). \tag{27}$$

This relation implicitly defines $\overline{Y}$. Incorporating Eq.27 into Eq.22, the expression for the elasticity, $\xi$ is derived as:

$$\xi = \frac{\eta}{\alpha}\left(1 - \frac{\epsilon - 1}{\epsilon}\frac{f(\overline{\omega})}{1 - \mu\Phi(\overline{\omega})}\right). \tag{28}$$

In order to see the effect of monitoring costs parameter, $\mu$ on the elasticity, $\xi$, I now specify the distribution function of $\omega$ and derive the expression for the entrepreneurs' share of net revenue from wholesale goods production, $\frac{f(\overline{\omega})}{1-\mu\Phi(\overline{\omega})}$.

**Entrepreneurs' share of the net revenue, $\frac{f(\overline{\omega})}{1-\mu\Phi(\overline{\omega})}$** I assume that the distribution function of $\omega$ is uniform in the region $[1-\rho, 1+\rho]$ $(0 < \rho \leq 1)$ so that $E(\omega) = 1$ and $Var(\omega) = \frac{1}{3}\rho^2$. With the uniform distribution, it can be obtained that $\phi(\overline{\omega}) = \frac{1}{2\rho}$, $\Phi(\overline{\omega}) = \frac{\overline{\omega}-(1-\rho)}{2\rho}$, $f(\overline{\omega}) = \frac{((1+\rho)-\overline{\omega})^2}{4\rho}$, and $g(\overline{\omega}) = \frac{4\rho - 2\mu(\overline{\omega}-(1-\rho)) - ((1+\rho)-\overline{\omega})^2}{4\rho}$.

Having specified the distribution as uniform, the cut off value, $\overline{\omega}$ is now derived as a function of $\alpha$, $\mu$, and $\rho$ from Eq.7: $\alpha = g(\overline{\omega})\,s(\overline{\omega})$, where $s(\overline{\omega}) = \frac{1}{1-\mu\Phi(\overline{\omega}) + \frac{\mu\phi(\overline{\omega})f(\overline{\omega})}{f'(\overline{\omega})}}$. First, I obtain the following two potential values of $\overline{\omega}$:

$$\overline{\omega} = (1+\rho) - \mu + \frac{1}{2}(\alpha\mu \pm \sqrt{\alpha^2\mu^2 + (1-\alpha)(4\mu^2 + 16\rho - 16\mu\rho)}), \tag{29}$$

where inside the root is larger than $\alpha^2\mu^2$ since $\alpha < 1$ and $\mu < 1$. It can be shown that the larger value of $\overline{\omega}$ in Eq.29 is greater than $1+\rho$. Thus, the only feasible option is the smaller one:

$$\overline{\omega} = (1+\rho) - \mu + \varepsilon, \tag{30}$$

where

$$\varepsilon = \frac{1}{2}(\alpha\mu - \sqrt{\alpha^2\mu^2 + (1-\alpha)(4\mu^2 + 16\rho - 16\mu\rho)}). \tag{31}$$

To see that this value indeed maximises the entrepreneurs' expected profit, I study the second order condition for this problem in Appendix 2. There, I prove that the smaller value is the optimal cut-off value chosen by entrepreneurs.

With the uniform distribution, the entrepreneurs' share of the net revenue, $\frac{f(\overline{\omega})}{1-\mu\Phi(\overline{\omega})}$ is expressed as:

$$\frac{f(\overline{\omega})}{1-\mu\Phi(\overline{\omega})} = \frac{(\overline{\omega}-(1+\rho))^2}{4\rho-2\mu(\overline{\omega}-(1-\rho))}. \tag{32}$$

Substituting Eq.30 into Eq.32, $\frac{f(\overline{\omega})}{1-\mu\Phi(\overline{\omega})}$ is now obtained as a function of $\alpha$, $\mu$, and $\rho$:

$$\frac{f(\overline{\omega})}{1-\mu\Phi(\overline{\omega})} = \frac{(\mu-\varepsilon)^2}{4\rho(1-\mu)+2\mu(\mu-\varepsilon)}. \tag{33}$$

Observe that in the absence of monitoring costs ($\mu = 0$), it simplifies to:

$$\frac{f(\overline{\omega})}{1-\mu\Phi(\overline{\omega})} = 1-\alpha.^{25} \tag{34}$$

**Monitoring costs and the elasticity of retailers' real marginal cost with respect to output** Finally, substituting Eq.33 into Eq.28: $\xi = \frac{\eta}{\alpha}\left(1-\frac{\epsilon-1}{\epsilon}\frac{f(\overline{\omega})}{1-\mu\Phi(\overline{\omega})}\right)$, I obtain the expression of the elasticity as a function of deep variables of the model including the monitoring costs parameter, $\mu$.

It is now clear that monitoring costs affect the elasticity through the entrepreneurs' share of the net revenue, $\frac{f(\overline{\omega})}{1-\mu\Phi(\overline{\omega})}$. I can show that for any values of $\alpha$ ($0<\alpha<1$), $\rho$ ($0<\rho<1$) and $\mu$ ($0<\mu<1$), the entrepreneurs' share, $\frac{f(\overline{\omega})}{1-\mu\Phi(\overline{\omega})}$ is an increase function of monitoring costs parameter, $\mu$:

$$\frac{\partial \frac{f(\overline{\omega})}{1-\mu\Phi(\overline{\omega})}}{\partial\mu} > 0. \tag{35}$$

Intuitively, as it becomes more costly for banks to monitor entrepreneurs' investment outcome, entrepreneurs enjoy a higher economic rent because they have the special ability of monitoring the outcome costlessly. This leads to an increase in the entrepreneurs' share of the net revenue, $\frac{f(\overline{\omega})}{1-\mu\Phi(\overline{\omega})}$.

Overall, I know from Eq.28 that the elasticity of retailers' real marginal cost with

[25] In fact, this is the case regardless of the distribution of $\omega$.

respect to output is a decreasing function of monitoring cost:

$$\frac{\partial \xi}{\partial \mu} < 0. \tag{36}$$

In short, monitoring costs make retailers' real marginal cost less pro-cyclical.

### 2.3.4 Intuition behind the effect of monitoring costs on the elasticity

To see the intuition behind how monitoring costs make retailers' real marginal cost less pro-cyclical, I again look at the wholesalers' pricing relation: $\varphi_t = s(\overline{\omega}) \frac{1}{\alpha H_t^{\alpha-1}} w_t$ (Eq.9). It says that the real price of wholesale goods is determined as a time-invariant mark up, $s(\overline{\omega})$ over the marginal cost, $\frac{1}{\alpha H_t^{\alpha-1}} w_t$. Log linearising this, I have:

$$\widehat{\varphi}_t = (1-\alpha)\widehat{H_t} + \widehat{w}_t.\text{[26]} \tag{37}$$

Given that the CRRA parameter, $\sigma$ is set to be unity in the households' utility function, the equilibrium real wage is obtained as $w_t = \frac{\chi\eta}{\delta} C_t H_t^{\eta-1}$ (see Eq.13). Notice that $w_t$ is subject to an income effect. Log linearising this yields

$$\widehat{w}_t = \widehat{C}_t + (\eta - 1)\widehat{H_t}. \tag{38}$$

Substituting Eq.38 into 37, we obtain

$$\widehat{\varphi}_t = \widehat{C}_t + (\eta - \alpha)\widehat{H_t}. \tag{39}$$

The retailers' real marginal cost is an increasing function of household consumption due to the income effect.

It is the case that $Y_t = (1 - \mu\Phi(\overline{\omega})) H_t^{\alpha}$ around the steady state (see Eq.15).[27] Log linearising this leads to

$$\widehat{H_t} = \frac{1}{\alpha}\widehat{Y}_t. \tag{40}$$

[26] As before, the hat notation indicates the log deviation from the steady state.
[27] Remember that $Y_t^W = Y_t$ around the steady state.

Substituting Eq.40 and $\widehat{\varphi}_t = \xi \widehat{Y}_t$ into Eq.39 and rearranging, I obtain the relation between household consumption $\widehat{C}_t$ and output $\widehat{Y}_t$:

$$\widehat{C}_t = d\widehat{Y}_t, \tag{41}$$

where

$$d = 1 - \frac{\eta}{\alpha}\frac{\epsilon - 1}{\epsilon}\frac{f(\overline{\omega})}{1 - \mu\Phi(\overline{\omega})}. \tag{42}$$

I assume that $0 < d < 1$. Given that $\frac{\partial \frac{f(\overline{\omega})}{1-\mu\Phi(\overline{\omega})}}{\partial \mu} > 0$ (Eq.35), $d$ is a decreasing function in $\mu$:

$$\frac{\partial d}{\partial \mu} < 0. \tag{43}$$

That is, the wedge between $\widehat{C}_t$ and $\widehat{Y}_t$ becomes wider as monitoring costs increase.

It is thus clear that monitoring costs make retailers' real marginal cost less procyclical by weakening the income effect. When monitoring costs are high, household consumption does not increase much for a given change in output. This, in turn, leads to a smaller increase in the real wage and retailers' real marginal cost.

Then, why does an increase in monitoring costs widen the wedge between household consumption and output? To grasp this intuitively, observe first that the market clearing condition of the final good, $Y_t = C_t + C_t^e$ implies the following identity relation:

$$\widehat{Y}_t = \frac{\overline{C}}{\overline{Y}}\widehat{C}_t + \frac{\overline{C^e}}{\overline{Y}}\widehat{C_t^e}, \tag{44}$$

where $\overline{C}$ and $\overline{C^e}$ are the steady state values of each variable and $\widehat{C_t^e}$ is the log deviation of $C_t^e$.

Here I focus on the case where $\alpha$ is close to 1, i.e. entrepreneurs' production technology almost exhibits constant returns to scale. In this case, the profits entrepreneurs obtain in the absence of monitoring costs are negligible. Indeed, when $\mu = 0$, entrepreneurs' ability of observing their own investment outcomes free does not earn them economic rents because the ability is virtually shared by banks. Thus, entrepreneurial consumption is negligible. In Eq.44, $\frac{\overline{C^e}}{\overline{Y}}$ is almost zero when $\mu = 0$, which results in the approximate relation of $\widehat{C}_t = \widehat{Y}_t$.

However, when it is costly for banks to monitor entrepreneurs' investment outcomes and only entrepreneurs can monitor them free, entrepreneurs do enjoy economic rents in the contract. This implies that $\frac{\overline{C^e}}{\overline{Y}}$ is not negligible any more. Next, given that $\widehat{C^e_t}$ is expressed as $(1+\xi)\,\widehat{Y}_t$ (see Eq.16) and $\xi > 0$ (Eq.28), I know that $\widehat{C^e_t} > \widehat{Y}_t$. That is, one percent increase in output from the steady state corresponds to more than one percent increase in entrepreneurial consumption. Intuitively, an increase in aggregate output bids up the relative price of wholesale goods, $\frac{P^W_t}{P_t}$, which brings about the more than proportional increase in entrepreneurial consumption. Then, Eq.44 clarifies that $\widehat{C^e_t} > \widehat{Y}_t$, coupled with the non trivial share of $\overline{C^e}$ in $\overline{Y}$, implies that household consumption increases less than proportionally, i.e. $\widehat{C}_t < \widehat{Y}_t$. Monitoring costs create the wedge.

### 2.3.5 Discussion: The model as a generalisation of a standard NK model

Here, I discuss that this model with credit frictions can be regarded as a generalisation of a standard NK model such as the baseline model by Chari et al. (2000, Section 5). When their model adopts the same utility function as mine (Eq.10), my model nests their baseline model as a special case in which monitoring costs are absent ($\mu = 0$) and the wholesalers' production technology exhibits near constant returns to scale ($\alpha$ close to 1).[28]

Chari et al. (2000) do not consider credit market imperfections in their analysis of the real effect of monetary shocks. In their paper, households provide labour supply to monopolistic competitors called retailers.[29] In their baseline model, capital is ignored and retailers use the labour as the only input for production. When the production technology exhibits constant returns to scale, retailers' real marginal cost is equal to the real wage. Then, if the households' utility function is the same as the one in my model with $\sigma = 1$, the log deviation of real marginal cost is given as:

[28] What I call their baseline model is the model introduced in the section 5 of their paper, where they ignore capital as I do in this chapter.

[29] As in my model, retailers face a downward sloping demand curve due to the final good producer who makes the composite retail good (final good) following a CES production function.

$\widehat{C}_t + (\eta - 1)\widehat{H}_t$. Given that the market clearing condition in their baseline model is given as $Y_t = C_t$, it is the case that $\widehat{C}_t = \widehat{Y}_t$. Also it can be shown that $\widehat{H}_t = \widehat{Y}_t$.[30] Then, the elasticity of retailers' real marginal cost with respect to output is given as $\eta$.

Meanwhile, in the special case of my model in which credit frictions are absent ($\mu = 0$) and $\alpha$ is set close to 1, retailers' real marginal cost, $\varphi_t$ is approximately equal to the real wage, $w_t$ (see Eq.9). As argued, in this special case, entrepreneurial consumption, $C_t^e$ is negligible so that it is approximately case that $\widehat{C}_t = \widehat{Y}_t$ and $\widehat{H}_t = \widehat{Y}_t$. Then, the elasticity of retailers' real marginal cost with respect to output, $\xi$ is given as $\eta$ as in Chari et al's baseline case. In this sense, our model nests the standard NK model as the special case.

## 2.4 Aggregate supply (AS) relation

The previous section saw that monitoring costs render the retailers' real marginal cost less pro-cyclical. This section incorporates this insight into the determination of the AS relation of the model. To do this, I first derive the "New Keynesian Phillips curve (NKPC)" from the retailer's intertemporal profit maximisation problem subject to the Calvo style staggered pricesetting.

### 2.4.1 New Keynesian Phillips curve (NKPC)

Retailers (monopolistic competitors) are subject to Calvo style staggered pricing. In each period, only the proportion "$1 - \theta$" of randomly selected retailers are given an opportunity to adjust their prices. $\theta$ is naturally a measure of the degree of price stickiness, i.e. the higher $\theta$ is, the stickier price is, because the higher value of $\theta$ means the longer expected interval between price changes. When they have a chance to set prices, they set prices in such a way that the expected discounted value of current and

[30] This can be shown as follows. Given that the production function of retailer $z$ is $Y_t(z) = H_t(z)$, aggregate employment is expressed as $H_t = \int_0^1 Y_t(z)\, dz$. Log-linearisation of this yields $\widehat{H}_t = \int_0^1 \widehat{Y_t(z)} dz$. The production function of the final good producer takes the CES type: $Y_t = \left(\int_0^1 Y_t(z)^{\frac{\epsilon-1}{\epsilon}} dz\right)^{\frac{\epsilon}{\epsilon-1}}$. When I log linearising this, I obtain $\widehat{Y}_t = \int_0^1 \widehat{Y_t(z)} dz$. Thus, $\widehat{H}_t = \widehat{Y}_t$.

future profits is maximised. Since this Calvo "lottery" takes place every period and past history does not affect the probability of winning, the winning retailers choose the same new price. I denote this reset price as $P^*_{calvo}$. However, before studying how the reset price is determined, I first look at the relation between a given reset price, $P^*_{calvo}$ and inflation rate. The following derivation of the NKPC is mainly based on Walsh (2003, chapter 5).

**Relation between reset price and inflation rate** The price index (equivalently, the price of final good) is given as (Eq.25); $P_t = \left(\int_0^1 P_t(z)^{1-\epsilon} dz\right)^{\frac{1}{1-\epsilon}}$, where $P_t(z)$ is the price charged by retailer $z$ in period $t$. Under the Calvo price setting, the aggregate price index is determined as an average of the price charged by retailers not having an opportunity to adjust their prices at time $t$ (proportion, $\theta$) and the reset price at time $t$ (proportion, $1-\theta$). Since price adjusters are randomly selected, the average of non adjusters (who adjusted at different times in the past) is equal to the aggregate price in the previous period, $P_{t-1}$. Then, given that the adjusters set the same reset price, $P^*_{calvo,t}$, the price index is expressed as:

$$P_t = \left(\theta P_{t-1}^{1-\epsilon} + (1-\theta) P_{calvo,t}^{*1-\epsilon}\right)^{\frac{1}{1-\epsilon}}. \tag{45}$$

I denote the relative reset price, $\frac{P^*_{calvo,t}}{P_t}$ as $Q_t$. Also, denoting the inflation rate in time $t$, $\frac{P_t - P_{t-1}}{P_{t-1}}$ as $\pi_t$, Eq.45 can be written as:

$$1 = \left(\theta (1+\pi_t)^{\epsilon-1} + (1-\theta) Q_t^{1-\epsilon}\right)^{\frac{1}{1-\epsilon}}. \tag{46}$$

I linearise this Eq.46 around the zero inflation steady state. In terms of price levels, this means that $P_{ss} = P_{t-1} = P_t$. Then, in this steady state, the aggregate price is defined as $P_{ss} = \left(\theta P_{ss}^{1-\epsilon} + (1-\theta) P_{calvo,ss}^{*1-\epsilon}\right)^{\frac{1}{1-\epsilon}}$. This, in turn, implies that the reset price equals the price index, $P_{ss} = P^*_{calvo,ss}$, thus $Q_{ss} = 1$. In the zero inflation steady state, every retailer charges the same price as in the flexible price steady state. Linearising Eq.46 around the steady state yields the following relation between the target price and inflation rate:

$$\widehat{q}_t = \frac{\theta}{1-\theta}\pi_t, \tag{47}$$

where $\widehat{q}_t$ is the percentage deviation from the steady state: $\widehat{q}_t = \frac{Q_t - Q_{ss}}{Q_{ss}}$.

**Determination of reset price** Given the chance to adjust their prices, retailers set the price so that the expected discounted value of current and future profits is maximised. Retailers use the wholesale good as the single input in production and the production exhibits constant returns to scale. Then, given the chance in period $t$, retailer $z$ chooses the price, $P_t(z)$ to maximise

$$\sum_{i=0}^{\infty} \theta^i \Lambda_{t,t+i} \left( \frac{P_t(z)}{P_{t+i}} Y_{t+i}(z) - \frac{P^W_{t+i}}{P_{t+i}} Y_{t+i}(z) \right),$$

where $\Lambda_{t,t+i}$ is the retailer's discount rate for time $t+i$. Incorporating the demand function for retailer $z$ (Eq.24), the above objective can be rewritten as

$$\sum_{i=0}^{\infty} \theta^i \Lambda_{t,t+i} \left( \frac{P_t(z) - P^W_{t+i}}{P_{t+i}} \right) \left( \frac{P_t(z)}{P_{t+i}} \right)^{-\epsilon} Y_{t+i}$$

Differentiating this with respect to $P_t(z)$ and setting it to zero, I obtain

$$\frac{P^*_{calvo,t}}{P_t} = \frac{\epsilon}{\epsilon-1} \frac{\sum_{i=0}^{\infty} \left( \theta^i \Lambda_{t,t+i} \frac{P^W_{t+i}}{P_{t+i}} \left( \frac{P_{t+i}}{P_t} \right)^{\epsilon} Y_{t+i} \right)}{\sum_{i=0}^{\infty} \left( \theta^i \Lambda_{t,t+i} \left( \frac{P_{t+i}}{P_t} \right)^{\epsilon-1} Y_{t+i} \right)}.$$

Using $Q_t = \frac{P^*_{calvo,t}}{P_t}$ and $\varphi_{t+i} = \frac{P^W_{t+i}}{P_{t+i}}$, this is expressed as

$$Q_t = \frac{\epsilon}{\epsilon-1} \frac{\sum_{i=0}^{\infty} \left( \theta^i \Lambda_{t,t+i} \varphi_{t+i} \left( \frac{P_{t+i}}{P_t} \right)^{\epsilon} Y_{t+i} \right)}{\sum_{i=0}^{\infty} \left( \theta^i \Lambda_{t,t+i} \left( \frac{P_{t+i}}{P_t} \right)^{\epsilon-1} Y_{t+i} \right)}. \tag{48}$$

The profits made by retailers are distributed among households (households are the shareholders of retailers). Then, as in Walsh (Chapter 5, 2003), the retailer's discount factor, $\Lambda_{t,t+i}$ is given by the household's discount rate, $\beta^i$ and the marginal utility of their consumption in period $t+i$ relative to the one in period $t$. From the households' utility function (Eq.10), $\Lambda_{t,t+i}$ is derived as:

$$\Lambda_{t,t+i} = \beta^i \left( \frac{C_{t+i}}{C_t} \right)^{-1} . \text{[31][32]} \tag{49}$$

Substituting Eq.49 into Eq.48, I obtain

$$\sum_{i=0}^{\infty} \left( \theta^i \beta^i C_{t+i}^{-1} \left( \frac{P_{t+i}}{P_t} \right)^{\epsilon-1} Y_{t+i} \right) Q_t = \frac{\epsilon}{\epsilon - 1} \sum_{i=0}^{\infty} \left( \theta^i \beta^i C_{t+i}^{-1} \varphi_{t+i} \left( \frac{P_{t+i}}{P_t} \right)^{\epsilon} Y_{t+i} \right) . \tag{50}$$

Log linearising both sides around the steady state and doing some further manipulation (see Appendix 3), what I have in the end is

$$\widehat{q_t} = (1 - \theta\beta) \widehat{\varphi}_t + \theta\beta \left( \widehat{q_{t+1}} + \pi_{t+1} \right) , \tag{51}$$

where $\widehat{\varphi}_t$ denotes the percentage deviation of $\varphi_t$ from the zero inflation steady state value.

**New Keynesian Phillips curve** From Eq.47 and Eq.51, I obtain:

$$\pi_t = \phi \widehat{\varphi}_t + \beta \pi_{t+1}, \tag{52}$$

where

$$\phi = \frac{1 - \theta}{\theta} (1 - \theta\beta) . \tag{53}$$

This is the relation known as the New Keynesian Phillips curve (NKPC). The NKPC says that current inflation is a function of the retailers' real marginal cost expressed as a percentage deviation from the steady state value and the expected future inflation[33].

I now rewrite this as a function of output instead of the marginal cost to derive the aggregate supply (AS) relation.

[31] $\sigma$ is set to be 1.

[32] Looking at the consumption Euler equation (Eq.11), this discount factor can be interpreted more intuitively. This is simply the inverse of the product of the gross real interest rates from period $t$ to $t+i-1$; $\beta^i \left( \frac{C_{t+i}}{C_t} \right)^{-1} = \frac{1}{(1+r_t)(1+r_{t+1})\cdots(1+r_{t+i-1})}$.

[33] Notice that since the model is deterministic, the expected inflation is simply denoted as $\pi_{t+1}$.

### 2.4.2 Monitoring costs and AS relation

Previously, I obtained $\widehat{\varphi}_t = \xi \widehat{Y}_t$, where $\xi = \frac{\eta}{\alpha}\left(1 - \frac{\epsilon-1}{\epsilon}\frac{f(\overline{\omega})}{1-\mu\Phi(\overline{\omega})}\right)$ (Eqs.21 and 28). There, the hat notation was defined as the percentage deviation from the flexible price steady state. However, the flexible price and zero inflation steady states are the same, since every retailer charges the same price in both states. Then, the NKPC is expressed as a function of output:

$$\pi_t = \kappa \widehat{Y}_t + \beta \pi_{t+1}, \tag{54}$$

where

$$\kappa = \phi \xi. \tag{55}$$

The NKPC, the relation between inflation and output can be rewritten into a relation between price and output. Realising that $\pi_t$ is approximated as $\ln P_t - \ln P_{t-1}$, Eq.54 is written as:

$$\ln P_t - \ln P_{t-1} = \kappa \widehat{Y}_t + \beta \left(\ln P_{t+1} - \ln P_t\right). \tag{56}$$

I here specify the nature of money shocks. In this chapter, I focus on an unexpected one-off permanent money increase. Initially, money supply is constant and the economy is in the steady state. Then in Period 0, money supply increases by one percent and it stays at that new level indefinitely. Denote the price level in the initial steady state (that is, the price in the period $-1$) as $\overline{P}$. Also denote the percentage deviation of the price level for the period $k \geq 0$ from $\overline{P}$ as $\widehat{P_k}$, which is approximately $\ln P_k - \ln \overline{P}$. Then, Eq.56 is expressed as: $\widehat{P_k} - \widehat{P_{k-1}} = \kappa \widehat{Y_k} + \beta\left(\widehat{P_{k+1}} - \widehat{P_k}\right)$, where $\widehat{Y_k} = \ln Y_k - \ln \overline{Y}$. Rearranging this yields

$$\widehat{P_k} = \frac{\kappa}{1+\beta}\widehat{Y_k} + \frac{1}{1+\beta}\left(\widehat{P_{k-1}} + \beta \widehat{P_{k+1}}\right). \tag{57}$$

The fact that an increase in monitoring costs, $\mu$ leads to a decrease in the elasticity,

$\xi$ (Eq.36) indicates that

$$\frac{\partial \kappa}{\partial \mu} < 0. \tag{58}$$

It is now clear that monitoring costs make the AS relation flatter.

## 2.5 Aggregate demand (AD) relation

### 2.5.1 Derivation of AD relation

To derive the aggregate demand (AD) relation, I now assume that $\zeta = 1$ in households' utility function (Eq.10). That is, the real balance component of the function takes a logarithmic form. As seen from the money demand function (Eq.12), $\frac{1}{\zeta}$ represents interest elasticity of money demand so that $\zeta = 1$ implies the elasticity of unity.[34] I assume this to derive an interest-insensitive aggregate demand (AD) relation (Fender and Rankin (2003)). That is, there is some sort of the dichotomy between real and nominal sector of the economy. As seen below, this helps me to obtain an explicit solution for the output dynamics to an unexpected permanent money shock.

With $\zeta = 1$, the money demand function is expressed as:

$$\frac{M_t}{P_t} = \frac{1-\delta}{\delta}\frac{1+i_t}{i_t}C_t. \tag{59}$$

Assuming that the money market is in equilibrium, the equilibrium condition represents a money market equilibrium condition. Using the consumption Euler equation (Eq.11) and the Fisher equation, Eq.59 can be rewritten as: $\frac{1-\delta}{M_t} = \frac{\delta}{P_t C_t} - \frac{\beta\delta}{P_{t+1}C_{t+1}}$. Then, by multiplying both sides by $M_{t+1}$, this becomes

$$\gamma_{t+1} = \frac{1}{\beta\Psi_{t+1}}\gamma_t - \frac{1}{\beta\Psi_{t+1}}\frac{1-\delta}{\delta}, \tag{60}$$

where $\gamma_t = \frac{M_t}{P_t C_t}$ and $\Psi_{t+1} = \frac{M_t}{M_{t+1}}$. This is a first order difference equation in the

[34]More specifically, what is meant by the interest elasticity of money demand is the elasticity of money demand with respect to opportunity cost variable, $\frac{i_t}{1+i_t}$.

inverse velocity, $\gamma_t$.

As noted above, I focus on the situation where money supply, $M$ increases unexpectedly at Period 0 and then stays at the high level permanently. Then, the inverse of money supply increase, $\Psi_{k+1}$ always equals 1 for $k \geq 0$. Because the inverse velocity is not a predetermined variable, this difference equation (Eq.60) has to be solved in a forward looking manner. Then, for this equation to exhibit the saddlepoint stability, $\beta\Psi_{k+1}$ needs to be less than 1. This requirement is indeed satisfied given that $\Psi_{k+1}$ equals 1 and the discount factor, $\beta$ is less than 1. This observation now enables me to conclude that the inverse velocity is constant:

$$\gamma = \frac{1}{1-\beta}\frac{1-\delta}{\delta}. \tag{61}$$

Thus, I have the following AD relation:

$$C_k = \frac{1}{\gamma}\frac{M_k}{P_k}. \tag{62}$$

This shows the negative relation between price and consumption for a given level of money supply. With the AD relation, it is now clear that the nominal interest rate is constant despite the change in Money supply; $i = \frac{1-\beta}{\beta}$.

Log linearising the AD relation around the initial steady state (the state before Period 0) yields

$$\widehat{P_k} = \widehat{M_k} - \widehat{C_k}, \tag{63}$$

where $\widehat{C_k} = \ln C_k - \ln \overline{C}$ and $\widehat{M_k} = \ln M_k - \ln \overline{M}$, where $\overline{M}$ represents the level of money supply before the shock.

### 2.5.2 Monitoring costs and AD relation

I now rewrite the above AD relation using output instead of the household consumption. I showed above that $\widehat{C_k} = d\widehat{Y_k}$, where $d = 1 - \frac{\eta}{\alpha}\frac{\epsilon-1}{\epsilon}\frac{f(\varpi)}{1-\mu\Phi(\varpi)}$ (Eqs.41 and 42).[35]

[35] Eq.41 was originally obtained as the deviation from the flexible price steady state, while I am here talking about the deviation from the zero inflation steady state. However, since the two steady

With this relation, Eq.63 is expressed as:

$$\widehat{P_k} = \widehat{M_k} - d\widehat{Y_k}. \tag{64}$$

Assuming that $0 < d < 1$, the AD relation is negatively sloped. Since the wedge between $\widehat{C_k}$ and $\widehat{Y_k}$ becomes wider as $\mu$ increases (Eq.43), monitoring costs make the slope of AD curve flatter.

## 2.6 Output dynamics

Having obtained the AS and AD relations (Eqs.57 and 64), I now show how monitoring costs affect the output dynamics of the response to the unexpected permanent increase in money supply. With one percent increase in money, Eq.64 becomes

$$\widehat{P_k} = 1 - d\widehat{Y_k}.$$

Substituting this into the AS relation, the following second order difference equation is obtained:

$$\widehat{P_{k+1}} - \frac{1+\beta+e}{\beta}\widehat{P_k} + \frac{1}{\beta}\widehat{P_{k-1}} = -\frac{e}{\beta}, \tag{65}$$

where

$$e = \frac{\kappa}{d}. \tag{66}$$

Given that the shock hits in Period 0, the initial condition of the difference equation is given by $\widehat{P_{-1}} = 0$. Since money is neutral in the long run, the terminal condition is given by $\lim_{k\to\infty}\widehat{P_k} = 1$.

As mentioned above, I assume that $0 < d < 1$. The reason for this is that when $d$ takes a negative value and the AD curve is positively sloped, the characteristic equation of the associated homogeneous equation of Eq.65 has no real roots. To avoid this scenario and focus on the case where it does have distinct real roots, I impose this assumption.

states are the same, this relation still holds.

When $d$ is positive, $e$ is also positive (Eqs.55 and 66). Then, the solution for the second order difference equation is obtained as follows (Appendix 4). For any $k \geq 0$,

$$\widehat{P_k} = 1 - b_1^{k+1}, \tag{67}$$

where

$$b_1 = \frac{-a_1 - \sqrt{a_1^2 - 4a_2}}{2}. \tag{68}$$

In Eq.68, $a_1 = -\frac{1+\beta+e}{\beta}$ and $a_2 = \frac{1}{\beta}$. I show in the appendix that $0 < b_1 < 1$. Thus, the terminal condition is satisfied.

Given that $\widehat{P_k} + \widehat{C_k} = 1$, I obtain for any $k \geq 0$,

$$\widehat{C_k} = b_1^{k+1}. \tag{69}$$

Using $\widehat{C_k} = d\widehat{Y_k}$, this is expressed in terms of output:

$$\widehat{Y_k} = \frac{1}{d} b_1^{k+1}. \tag{70}$$

This shows the evolution of output after the money shock.

Notice that the output dynamics is characterised by the "impact" parameter, $\frac{b_1}{d}$, which is output in Period 0, and the "persistence" parameter, $b_1$. Then, how do monitoring costs affect these parameters?

### 2.6.1 Effect of monitoring costs on the impact and persistence of the money shock

Since the impact parameter, $\frac{b_1}{d}$ contains the persistence parameter, $b_1$, I first see the effect of monitoring costs on $b_1$.

**Effect on the persistence parameter,** $b_1$ To see the effect, the key parameter to focus on is $e(= \frac{\kappa}{d})$ (Eq.66). Notice from Eq.68 that the parameter $b_1$ is a decreasing function of $e$:

$$\frac{\partial b_1}{\partial e} < 0. \tag{71}$$

A larger value of $e$ leads to smaller persistence.

Remember that higher monitoring costs make both AS and AD relations flatter. A flatter AS curve is represented by a smaller value of $\kappa$ (Eq.57). A smaller $\kappa$ corresponds to a smaller $e$ (for given $d$). That is, higher monitoring costs lead to greater persistence by flattening the AS curve. Meanwhile, a flatter AD curve corresponds to a smaller value of $d$, which, in turn, implies a larger $e$ (for given $\kappa$). This results in smaller persistence. Indeed, monitoring costs, $\mu$ exert contrasting effects on the persistence through the AS and AD curves.

However, it turns out that the second effect through the AD curve is dominant. In order to see this, notice first that the signs of $\frac{\partial e}{\partial \mu}$ and $\frac{\partial \frac{\xi}{d}}{\partial \mu}$ are the same. Next, substituting $\widehat{C_k} = d\widehat{Y_k}$ (Eq.41) and $\widehat{H_k} = \frac{1}{\alpha}\widehat{Y_k}$ (Eq.40) into $\widehat{\varphi_k} = \widehat{C_k} + (\eta - \alpha)\widehat{H_k}$ (Eq.39), I have

$$\widehat{\varphi_k} = (d + \frac{\eta - \alpha}{\alpha})\widehat{Y_k}.$$

Given that $\widehat{\varphi_k} = \xi\widehat{Y_k}$, I know:

$$\xi = d + \frac{\eta - \alpha}{\alpha}, \tag{72}$$

where $d$ (and $\xi$) is a decreasing function of $\mu$. Since the constant term, $\frac{\eta-\alpha}{\alpha}$ is positive[36], I can say that $\frac{\partial \frac{\xi}{d}}{\partial \mu} > 0$. This, in turn, indicates $\frac{\partial e}{\partial \mu} > 0$. It is thus clear that the effect of monitoring costs through the AD curve dominates the one through the AS curve.

Finally, given that $b_1$ is a decreasing function of $e$ (Eq.71), $\mu$ is negatively related with $b_1$:

$$\frac{\partial b_1}{\partial \mu} < 0. \tag{73}$$

An increase in monitoring costs makes the real effects of money shock less persistent.

[36] The preference parameter, $\eta$ is strictly greater than 1 and the technology parameter $\alpha$ is between 0 and 1.

**Effect on the impact parameter, $\frac{b_1}{d}$** Given that $\frac{\partial d}{\partial \mu} < 0$ and $\frac{\partial b_1}{\partial \mu} < 0$ (Eqs.43 and 73), the overall effect of $\mu$ on the impact parameter, $\frac{b_1}{d}$ might seem ambiguous. However, making use of $\xi = d + \frac{\eta - \alpha}{\alpha}$ (Eq.72), I can unambiguously show that

$$\frac{\partial \frac{b_1}{d}}{\partial \mu} > 0. \tag{74}$$

The higher monitoring costs are, the greater the impact of money shock becomes.

Overall, monitoring costs amplify the impact of money shock on real output while they reduce the persistence of the shock.

## 2.7 Quantitative analysis

### 2.7.1 Parameter values

In order to quantify the effects of monitoring costs on the impact and persistence of the money shock, I now set parameter values. Some parameter values are set previously ($\zeta = 1$ and $\sigma = 1$). For the other parameters, I mainly follow the values used in the literature (see Table 1).

Table 1: Parameter values

| | |
|---|---|
| $\eta$: preference parameter | 4.5 |
| $\alpha$: technology parameter | 0.99 |
| $\epsilon$: elasticity of demand | 10 |
| $\mu$: monitoring costs | 0.20 |
| $\rho$: distribution function parameter | unobserved (to be calibrated) |
| $\beta$: discount factor | 0.99 |
| $\theta$: stickiness parameter | 0.75 |

$\eta - 1$ is the elasticity of marginal disutility with respect to work. As pointed out by Ascari (2000), the value of $\eta$ is difficult to pin down. As a tentative value, I follow Ascari and set $\eta = 4.5$. Concerning $\alpha$, I set $\alpha = 0.99$, which allows me to focus on economic rents from entrepreneurs' monitoring ability as the main source of their profits. As argued above, the model nests the standard NK model as a special case of $\alpha$ close to 1 and $\mu$ equal to 0. Thus, with $\alpha = 0.99$ (close to 1), output dynamics in the case of $\mu = 0$ can be regarded as the one of a standard NK model. The elasticity of demand (for retail goods) is 10, following Chari et al. (2000). The monitoring costs parameter, $\mu$ is difficult to determine. For example, Carlstrom and Fuerst (2001) argue for a possible lower bound of 0.04 and an upper bound of 0.36 after looking at some empirical studies (for the lower bound, Warner (1977), for the upper bound, Alderson and Betker (1995)). Here, I take the intermediate value of 0.20. The distribution function parameter of the idiosyncratic shock, $\rho$ is treated as unobservable as in Fuerst (1995). I calibrate this parameter value below. Discount factor, $\beta$ is set 0.99, a fairly conventional value. Finally, the stickiness parameter in Calvo staggered setting, $\theta$ is set 0.75, following Jeanne (1997).

**Calibration of the distribution function parameter, $\rho$** The distribution function of $\omega$ is assumed to be uniform with a support $[1-\rho, 1+\rho]$. Since $\omega$ does not take a negative value, $\rho$ takes a value between 0 and 1: $0 < \rho < 1$. I tie down this unobservable parameter by looking at the empirical measure of quarterly default rate.

The time unit is set as a quarter. Then, the quarterly default rate is given by $\Phi(\overline{\omega})$ in the model. With Eq.30, the default rate is given as:

$$\Phi(\rho, \mu, \alpha) = \frac{2\rho - \mu + \varepsilon}{2\rho}.$$

Following the work by Fuerst (1995) and series of work by Carlstrom and Fuerst (1997, 1998, 2001), $\Phi$ is here set 0.974% (originally reported by Fisher (1994)). Given that $\alpha = 0.99$ and $\mu = 0.20$, the value of $\rho$ is tied down as 0.11.

### 2.7.2 Output dynamics of the response to the money shock

The qualitative analysis shows that monitoring costs amplify the impact of the money shock while reducing the persistence of the effects (Eqs.73 and 74). I now compare the values of the impact and persistent parameters between $\mu = 0$ and 0.20.

First, knowing that entrepreneurs' share of net revenue from wholesale goods production, $\frac{f(\overline{\omega})}{1-\mu\Phi(\overline{\omega})}$ is expressed as a function of $\alpha$, $\mu$, and $\rho$ (Eq.33), I obtain that $\frac{f(\overline{\omega})}{1-\mu\Phi(\overline{\omega})} = 0.11$ for $\mu = 0.20$. Meanwhile, in case $\mu = 0$, the share takes a trivial value of 0.01 (Eq.34). Next, I calculate the elasticity of retailers' real marginal cost with respect to output, $\xi$ and the wedge between household consumption and output, $d$ for $\mu = 0$ and 0.20 (Table 2). Since $\alpha$ is set 0.99 (close to 1), the case with $\mu = 0$ approximates the standard NK model in which $\xi = \eta$ and $d = 1$. In case $\mu = 0.20$, $\frac{f(\overline{\omega})}{1-\mu\Phi(\overline{\omega})}$ increases to 0.11, which leads to the fall in $\xi$ from 4.50 to 4.10. Also, the monitoring costs widen the wedge so that $d = 0.56$.[37]

Table 2: Entrepreneurs' share, elasticity and wedge for $\mu = 0$ and 0.20

| $\mu$ | Entrepreneurs' share, $\frac{f(\overline{\omega})}{1-\mu\Phi(\overline{\omega})}$ | Elasticity, $\xi$ | Wedge, $d$ |
|---|---|---|---|
| 0 | 0.01 | 4.50 | 0.96 |
| 0.20 | 0.11 | 4.10 | 0.56 |

Finally, being the composite of the parameters discussed, the impact parameter, $\frac{b_1}{d}$ and persistence parameter, $b_1$ are obtained for $\mu = 0$ and 0.20 (Table 3). The parameters for $\mu = 0$ approximate the ones for the standard NK case.

Table 3: Impact parameter and persistence parameter for $\mu = 0$ and 0.20

| Monitoring costs | Impact parameter, $\frac{b_1}{d}$ | Persistence parameter, $b_1$ |
|---|---|---|
| 0 (standard NK case) | 0.56 | 0.54 |
| 0.20 (with credit frictions) | 0.83 | 0.46 |

The dynamics is drawn in Figure 2. As seen there, the impact effect of monitoring costs is significant. With credit frictions present, the impact of the money shock is 48 % larger compared to the standard NK case without credit frictions. Although the effect on persistence is less noticeable, it is not negligible. When $\mu = 0.20$, the parameter takes 0.46, which is 15% less than the case with $\mu = 0$. Thus, although

[37] Note that $0 < d < 1$.

the impact is much greater in a model with credit frictions, the real effect is almost the same as the standard case after a few periods.

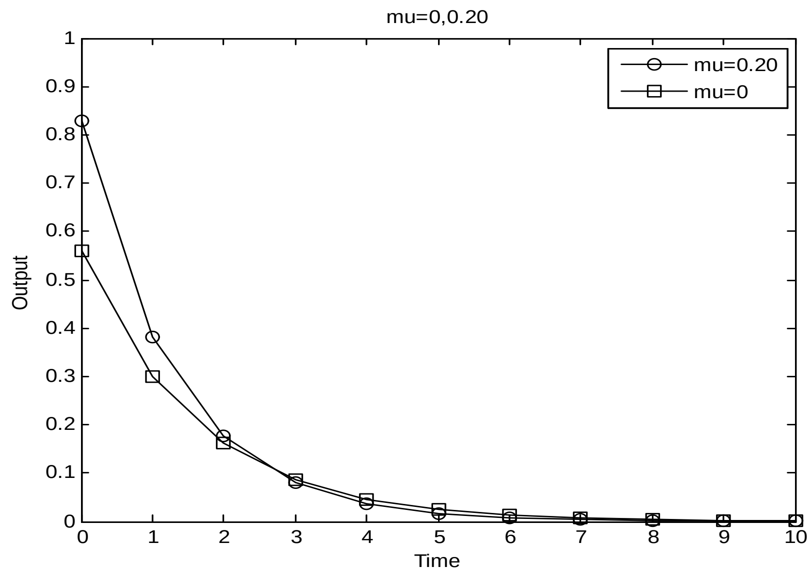


Figure 2: Output dynamics to the unexpected permanent increase in money supply

## 2.8 Robustness

Throughout the qualitative and quantitative analyses conducted above, the subutility over real balances has been assumed to take a log form, i.e. $\zeta = 1$ (Eq.10). Given that the interest elasticity of money demand is given by $\frac{1}{\zeta}$, this implies that the elasticity is unity.[38] This assumption, leading to the dichotomy between real variables and the nominal interest rate, has enabled me to proceed analytically. However, empirical studies tend to show that the interest elasticity of money demand is less than unity, equivalently, $\zeta > 1$. Chari et al. (2000) and Ireland (2001) report the elasticity of 0.39, i.e. $\zeta = 2.56$ and 0.12 i.e. $\zeta = 8.33$ respectively.[39] Given this, I now relax the assumption of $\zeta = 1$ to check the robustness of the results obtained above.

I first study the AS and AD relations for the case of $\zeta > 1$. In fact, the AS relation of the model is not affected by the relaxation of the assumption. For convenience, I

[38] As pointed out above, $\frac{1}{\zeta}$ is the elasticity of money demand with respect to the opportunity cost variable, $\frac{i}{1+i}$. However, I simply call this an interest elasticity of money demand.

[39] The elasticity of 0.12 by Ireland is especially the one for the post-1979 US data.

here replicate the AS relation (Eq.57);

$$\widehat{P_k} = \frac{\kappa}{1+\beta}\widehat{Y_k} + \frac{1}{1+\beta}\left(\widehat{P_{k-1}} + \beta\widehat{P_{k+1}}\right). \tag{75}$$

However, the AD relation is now different when $\zeta \neq 1$. To see this, I first log linearise the consumption Euler equation (Eq.11) to obtain

$$\widehat{C_t} = \widehat{C_{t+1}} - (\widehat{i_t} - \pi_{t+1}), \tag{76}$$

where $\widehat{i_t}$ represents the log deviation of gross interest rate from the steady state value. In terms of price, this is expressed as

$$\widehat{C_t} = \widehat{C_{t+1}} - (\widehat{i_t} - \widehat{P_{t+1}} + \widehat{P_t}). \tag{77}$$

This can be regarded as a dynamic IS relation. Meanwhile, log-linearising the money demand relation (Eq.12) yields

$$\widehat{M_t} - \widehat{P_t} = -\frac{1}{\zeta}\frac{\beta}{1-\beta}\widehat{i_t} + \frac{1}{\zeta}\widehat{C_t}.^{40} \tag{78}$$

With the assumption that money market is in equilibrium, this relation can be seen as a LM relation.

Combining the IS and LM relations (Eqs.77 and 78) and incorporating the relation between household consumption and output (Eq.41): $\widehat{C_t} = d\widehat{Y_t}$ [41], I can obtain a dynamic AD relation. Given that money supply increases by 1% in Period 0 and remains at the new level indefinitely, the AD relation is expressed as follows (for $k \geq 0$):

$$\widehat{P_k} = -\frac{d}{\zeta(1-\beta)+\beta}\widehat{Y_k} + \frac{\beta}{\zeta(1-\beta)+\beta}\left(\widehat{P_{k+1}} + d\widehat{Y_{k+1}} + \frac{\zeta\,(1-\beta)}{\beta}\right). \tag{79}$$

[40] As noted, $\widehat{i_t}$ denotes log deviation of $1 + i_t$ from the steady state value. Then, given that $\ln(1 + i_t) \simeq i_t$, this equation implies that the semi-elasticity of money demand with respect to interest rate, $i_t$ is given by $-\frac{1}{\zeta}\frac{\beta}{1-\beta}$.

[41] This relation is not affected by the relaxation of the assumption of $\zeta = 1$.

From the AS and AD relations (Eq.75 and Eq.79), I obtain the following third order difference equation:

$$\widehat{P_{k+2}} - \frac{2d + d\beta + \kappa}{d\beta}\widehat{P_{k+1}} + \frac{d + 2d\beta + \kappa\zeta(1-\beta) + \beta\kappa}{d\beta^2}\widehat{P_k} - \frac{1}{\beta^2}\widehat{P_{k-1}} = \frac{\kappa\zeta(1-\beta)}{d\beta^2}. \tag{80}$$

Suppose that I set the interest elasticity of money demand to be 0.5, i.e. $\zeta = 2$, while other parameter values are the same as before. This difference equation can be solved numerically for $\mu = 0$ and 0.20 (Appendix 5). It turns out that the characteristic equation of the difference equation gives only one stable root, $b_1$ for each $\mu$. This guarantees the existence of saddle path solution, since there is a single predetermined variable. Using the initial condition of $\widehat{P_{-1}} = 0$ and the steady state value of $\widehat{P_k}$, $\widehat{P} = 1$, the evolution of price is obtained as:

$$\widehat{P_k} = 1 - b_1^{k+1}. \tag{81}$$

Substituting this into the AS relation (Eq.75) yields

$$\widehat{Y_k} = \frac{\beta}{\kappa}\frac{\left(b_1 - \frac{1}{\beta}\right)(b_1 - 1)}{b_1}b_1^{k+1}. \tag{82}$$

This represents the evolution of output for $\zeta \neq 1$. In this case, the impact parameter is given by $\frac{\beta}{\kappa}\left(b_1 - \frac{1}{\beta}\right)(b_1 - 1)$ and the persistence parameter by $b_1$.

Table 4 shows that even when $\zeta = 2$, monitoring costs amplify the impact of the money shock, while reducing the persistence of the effects. The result remains the same with an even lower value of interest elasticity of money demand, 0.125, i.e. $\zeta = 8$. As in $\zeta = 2$, there is only one stable eigenvalue, $b_1$ for each $\mu$ and it is smaller when $\mu = 0.20$. The impact parameter is again greater when credit frictions are present. From a quantitative perspective, the significant effect is mainly seen in the amplification of the impact. The results are robust to the change in households' preferences.

Table 4: Impact and persistence parameter for $\zeta = 1, 2, 8$

| Monitoring costs | Impact parameter | | | Persistence parameter | | |
|---|---|---|---|---|---|---|
| | $\zeta = 1$ | $\zeta = 2$ | $\zeta = 8$ | $\zeta = 1$ | $\zeta = 2$ | $\zeta = 8$ |
| 0 | 0.560 | 0.568 | 0.614 | 0.537 | 0.534 | 0.515 |
| 0.20 | 0.829 | 0.839 | 0.892 | 0.462 | 0.459 | 0.442 |

Incidentally, I find that for given monitoring costs, a decrease in the interest elasticity of money demand (an increase in $\zeta$) leads to an increase in the impact and a decrease in the persistence. The intuition can be given as follows. First, notice that an increase in $\zeta$ flattens the (dynamic) AD relation (Eq.79). This comes from the effect of $\zeta$ on the money demand (Eq.78). In the money demand function, $\frac{1}{\zeta}$ also represents the elasticity of money demand with respect to consumption. In the period of the money increase, if other things are equal, larger $\zeta$ implies a larger increase in consumption therefore output. That is, the impact is amplified. Also, the flattening of the AD curve due to an increase in $\zeta$ results in the reduction in persistence.

## 2.9 Conclusion

In this chapter, I have studied how output dynamics of the response to money shocks are altered when credit market imperfections are incorporated into an otherwise standard NK framework. By modelling agency costs as acyclical, I managed to study the effect of imperfections analytically. I find that monitoring costs flatten both AS and AD curves of the model so that the real impact of the money shock is amplified while persistence of the effects is reduced. The supplementary quantitative analysis indicates that the amplification of the impact can be significant while the effect on persistence is rather small. Thus, in the light of the VAR analysis which indicates that exogenous monetary shocks have sizable and persistent real effects, the acyclical agency costs, on their own, appear to be a modification in the direction towards greater realism.

## 2.10 Appendices to Chapter Two

### 2.10.1 Appendix 1: Proof of $\overline{Y} = \overline{Y^W}$ and $\widehat{Y_t} = \widehat{Y_t^W}$

I first show that aggregate production of the final good and net aggregate production of the wholesale good are the same in the flexible price steady state. That is,

$$\overline{Y} = \overline{Y^W},$$

where $\overline{Y}$ and $\overline{Y^W}$ are the steady state values of $Y_t$ and $Y_t^W$, respectively.

When every retailer sets the same price, $P(z) = P$ (Eq.25). In this case, they also produce the same amount. Then, it is clear from the production function of final goods (Eq.23) that $Y_t = Y_t(z)$ in the steady state. Remember that the production function of retailer $z$ is $Y_t(z) = Y_t^W(z)$. Further, the net aggregate production of the wholesale good, $Y_t^W$ is given by $\int_0^1 Y_t^W(z)\,dz$. Then, when $Y_t^W(z)$ is the same across all the retailers $z$, I know $Y_t^W(z) = Y_t^W$. Therefore, $Y_t = Y_t(z) = Y_t^W(z) = Y_t^W$ in the steady state. Indeed, $\overline{Y} = \overline{Y^W}$.

Also, it can be shown that

$$\widehat{Y_t} = \widehat{Y_t^W},$$

where $\widehat{Y_t}$ is the percentage deviation of $Y_t$ from $\overline{Y}$, while $\widehat{Y_t^W}$ is the percentage deviation of $Y_t^W$ from $\overline{Y^W}$.

The production function of the final good is $Y_t = \left(\int_0^1 Y_t(z)^{\frac{\epsilon-1}{\epsilon}}\,dz\right)^{\frac{\epsilon}{\epsilon-1}}$ (Eq.23). Since the production function of the retail good is $Y_t(z) = Y_t^W(z)$, this can be rewritten as: $Y_t = \left(\int_0^1 Y_t^W(z)^{\frac{\epsilon-1}{\epsilon}}\,dz\right)^{\frac{\epsilon}{\epsilon-1}}$. Log linearising this around the steady state (where $Y_t = Y_t(z) = Y_t^W(z) = Y_t^W$), I have

$$\widehat{Y_t} = \int_0^1 \widehat{Y_t^W(z)}dz, \tag{83}$$

where $\widehat{Y_t^W(z)} = \frac{Y_t^W(z)-\overline{Y}}{\overline{Y}}$. Meanwhile, $Y_t^W = \int_0^1 Y_t^W(z)\,dz$ by definition. Log linearising this, I obtain

$$\widehat{Y_t^W} = \int_0^1 \widehat{Y_t^W(z)} dz. \tag{84}$$

From Eqs.83 and 84, I know $\widehat{Y_t} = \widehat{Y_t^W}$.

### 2.10.2 Appendix 2: Optimal cut-off value of $\omega$, $\overline{\omega}$

I here show that the optimal cut-off value of $\overline{\omega}$ is given by

$$\overline{\omega} = (1 + \rho) - \mu + \varepsilon,$$

where $\varepsilon = \frac{1}{2}(\alpha\mu - \sqrt{\alpha^2\mu^2 + (1-\alpha)(4\mu^2 + 16\rho - 16\mu\rho)})$ (Eqs. 30 and 31). I first confirm that the banks' share of the net production of wholesale good, $g(\overline{\omega})$ is hump shaped.

The general form of $g(\overline{\omega})$ is given as: $g(\overline{\omega}) = \int_0^{\overline{\omega}} \omega d\Phi(\omega) + (1 - \Phi(\overline{\omega}))\overline{\omega} - \mu\Phi(\overline{\omega})$ (see Eq.5). Differentiating once, we have $g'(\overline{\omega}) = (1 - \Phi(\overline{\omega})) - \mu\phi(\overline{\omega})$. Rearranging this yields

$$g'(\overline{\omega}) = (1 - \Phi(\overline{\omega}))\left(1 - \mu\frac{\phi(\overline{\omega})}{1 - \Phi(\overline{\omega})}\right), \tag{85}$$

where $\frac{\phi(\overline{\omega})}{1-\Phi(\overline{\omega})}$ is the hazard rate. For the uniform distribution, it is obtained as: $\frac{\phi(\overline{\omega})}{1-\Phi(\overline{\omega})} = \frac{1}{(1+\rho)-\overline{\omega}}$. Substituting this into Eq.85, I find that $g'(\overline{\omega}) = 0$ at the point where $\overline{\omega} = (1+\rho) - \mu$. Given that the hazard rate is an increasing function in $\overline{\omega}$, I find that $g'(\overline{\omega}) > 0$ $(< 0)$ at the point where $\overline{\omega}$ is less (larger) than $(1+\rho) - \mu$. That is, the function $g(\overline{\omega})$ is hump shaped.

Since $\alpha < 1$ and $\mu < 1$, $\varepsilon$ is negative. Then, $\overline{\omega} = (1+\rho) - \mu + \varepsilon$ is located where $g'(\overline{\omega}) > 0$. Looking at the second order condition of the optimisation problem, this value of $\overline{\omega}$, which corresponds to $g'(\overline{\omega}) > 0$, is proved to be the optimal cut-off value chosen by the entrepreneurs. The profit maximisation problem of entrepreneur $j$ is replicated for convenience (Eqs.2 and 3):

*maximise*

$$\varphi_t f(\overline{\omega_t}(j)) H_t(j)^{\alpha},$$

*with respect to* $\overline{\omega_t}(j)$ and $H_t(j)$, *subject to*

$$\varphi_t g(\overline{\omega_t}(j)) H_t(j)^\alpha = w_t H_t(j).$$

The Lagrangian of this problem can be set up as:

$$L(\overline{\omega_t}(j), H_t(j), \lambda_t(j)) = \varphi_t f(\overline{\omega_t}(j)) H_t(j)^\alpha + \lambda_t(j)(\varphi_t g(\overline{\omega_t}(j)) H_t(j)^\alpha - w_t H_t(j)).$$

Solving this, we obtain Eq.6: $\alpha = g(\overline{\omega_t}(j)) s(\overline{\omega_t}(j))$. Then, as argued above, the cutoff value is found to be common across entrepreneurs and also time invariant, $\overline{\omega_t}(j) = \overline{\omega_t} = \overline{\omega}$. With this insight, we also find that $H_t(j) = H_t$. Now, notice that the value of the multiplier, $\lambda_t(j)$ is also common and time invariant: $\lambda = -\frac{f'(\overline{\omega})}{g'(\overline{\omega})}$.

The bordered Hessian is expressed as:

$$\begin{pmatrix} 0 & \frac{\partial h}{\partial \overline{\omega}} & \frac{\partial h}{\partial H_t} \\ \frac{\partial h}{\partial \overline{\omega}} & \frac{\partial^2 L}{\partial \overline{\omega}^2} & \frac{\partial^2 L}{\partial \overline{\omega} \partial H_t} \\ \frac{\partial h}{\partial H_t} & \frac{\partial^2 L}{\partial H_t \partial \overline{\omega}} & \frac{\partial^2 L}{\partial H_t^2} \end{pmatrix},$$

where $h(\overline{\omega}, H_t) = \varphi_t g(\overline{\omega}) H_t^\alpha - w_t H_t$. Subsequently, the determinant is given as:

$$\begin{aligned} &-(\varphi_t g'(\overline{\omega}) H_t^\alpha)^2 \varphi_t \alpha(\alpha - 1) H_t^{\alpha-2}(f(\overline{\omega}) + \lambda g(\overline{\omega})) \\ &-\varphi_t H_t^\alpha (f''(\overline{\omega}) + \lambda g''(\overline{\omega}))(\varphi_t g(\overline{\omega}) \alpha H_t^{\alpha-1} - w_t)^2. \end{aligned} \tag{86}$$

I know that $g'(\overline{\omega}) > 0$ at $\overline{\omega} = (1+\rho) - \mu + \varepsilon$. Then, I check the sign of this determinant for the case of $g'(\overline{\omega}) > 0$. For the profit to be maximised, the sign of the determinant has to be positive. To find out the sign, I first discuss the signs of "$f(\overline{\omega}) + \lambda g(\overline{\omega})$" and "$f''(\overline{\omega}) + \lambda g''(\overline{\omega})$" in the determinant.

**Sign of** $f(\overline{\omega}) + \lambda g(\overline{\omega})$

Given that $\lambda = -\frac{f'(\overline{\omega})}{g'(\overline{\omega})}$, $f(\overline{\omega}) + \lambda g(\overline{\omega}) = f(\overline{\omega}) - \frac{f'(\overline{\omega})}{g'(\overline{\omega})} g(\overline{\omega})$. Differentiating $f(\overline{\omega})$ $(= \int_{\overline{\omega}}^\infty \omega d\Phi(\omega) - (1 - \Phi(\overline{\omega}))\overline{\omega}$ (Eq.4)), I obtain $f'(\overline{\omega}) = -(1 - \Phi(\overline{\omega}))$, which is negative. Thus, when $g'(\overline{\omega}) > 0$, it is the case that $f(\overline{\omega}) + \lambda g(\overline{\omega}) > 0$.

**Sign of** $f''(\overline{\omega}) + \lambda g''(\overline{\omega})$

$f''(\overline{\omega})$ is given as $f''(\overline{\omega}) = \phi(\overline{\omega})$. Further, I obtain that $g''(\overline{\omega}) = -\left(\mu\frac{\partial\phi(\overline{\omega})}{\partial\overline{\omega}} + \phi(\overline{\omega})\right)$. In the case of the uniform distribution of $\omega$, $f''(\overline{\omega}) + \lambda g''(\overline{\omega})$ can be simplified to: $\frac{-\mu\phi(\overline{\omega})}{g'(\overline{\omega})}$, where $\phi(\overline{\omega}) = \frac{1}{2\rho}$. Then, when $g'(\overline{\omega}) > 0$, $f''(\overline{\omega}) + \lambda g''(\overline{\omega})$ is negative as long as $\mu > 0$.

These observations enables me to conclude that the determinant of the bordered Hessian is positive when $g'(\overline{\omega}) > 0$. Thus, $\overline{\omega} = (1+\rho) - \mu + \varepsilon$ is the optimal cut-off value.

### 2.10.3 Appendix 3: Derivation of Eq.51

As mentioned above, the derivation presented below is largely based on Walsh (2003, Chapter 5).

Solving the retailer's intertemporal profit maximisation problem under Calvo style price stickiness, I obtain the following relation (Eq.50):

$$\sum_{i=0}^{\infty}\left(\theta^i\beta^i C_{t+i}^{-1}\left(\frac{P_{t+i}}{P_t}\right)^{\epsilon-1} Y_{t+i}\right) Q_t = \frac{\epsilon}{\epsilon-1}\sum_{i=0}^{\infty}\left(\theta^i\beta^i C_{t+i}^{-1}\varphi_{t+i}\left(\frac{P_{t+i}}{P_t}\right)^{\epsilon} Y_{t+i}\right).$$

To obtain Eq.51, I first linearise the above relation around the zero inflation steady state. To do this, I need to obtain the steady state value of the marginal cost (for retailers), $\varphi_{ss}$. Remember that in the zero inflation steady state, $Q_t = 1$, that is, the optimal reset price, $P^*_{calvo,t}$ is the same as the price index, $P_t$. Then, I know from Eq.48 that $\varphi_{ss} = \frac{\epsilon-1}{\epsilon}$.

Using this, the linearisation of Eq.50 results in

$$\frac{\widehat{q}_t}{1-\theta\beta} = \sum_{i=0}^{\infty}\theta^i\beta^i\left(\widehat{\varphi_{t+i}} + \widehat{P_{t+i}} - \widehat{P}_t\right).$$

Multiplying the both sides by $1-\theta\beta$ and add $\widehat{P}_t$ to the both, I obtain

$$\widehat{q}_t + \widehat{P}_t = (1-\theta\beta)\sum_{i=0}^{\infty}\theta^i\beta^i\left(\widehat{\varphi_{t+i}} + \widehat{P_{t+i}}\right). \qquad (87)$$

Realising $(1-\theta\beta)\sum_{i=1}^{\infty}\theta^i\beta^i\left(\widehat{\varphi_{t+i}}+\widehat{P_{t+i}}\right)=\theta\beta\left(\widehat{q_{t+1}}+\widehat{P_{t+1}}\right)$, Eq.87 is now rewritten as:

$$\widehat{q_t}=(1-\theta\beta)\,\widehat{\varphi_t}+\theta\beta\left(\widehat{q_{t+1}}+\widehat{P_{t+1}}-\widehat{P_t}\right). \tag{88}$$

Finally, since $\pi_{t+1}=\widehat{P_{t+1}}-\widehat{P_t}$, I obtain Eq.51:

$$\widehat{q_t}=(1-\theta\beta)\,\widehat{\varphi_t}+\theta\beta\left(\widehat{q_{t+1}}+\pi_{t+1}\right).$$

#### 2.10.4 Appendix 4: Solving the 2nd order difference equation, Eq.65 with $e>0$

Denoting $-\frac{1+\beta+e}{\beta}$ as $a_1$ and $\frac{1}{\beta}$ as $a_2$, Eq.65 can be written as: $\widehat{P_{k+1}}+a_1\widehat{P_k}+a_2\widehat{P_{k-1}}=-\frac{e}{\beta}$. When $e$ is positive, it is the case that $a_1^2>4a_2$. Then, as long as $a_1^2>4a_2$, the solution of this difference equation takes the following form:

$$\widehat{P_k}=A_1b_1^k+A_2b_2^k+\widehat{P}, \tag{89}$$

where $A_1$ and $A_2$ are constants to be determined and $b_1$ and $b_2$ are the eigenvalues determined from the characteristic equation:

$$b^2+a_1b+a_2=0.$$

In Eq.89, $\widehat{P}$ is the steady state value of $\widehat{P_k}$. From Eq.65, $\widehat{P}$ is equal to 1. Solving the characteristic equation, I have $b_1=\frac{-a_1-\sqrt{a_1^2-4a_2}}{2}$ and $b_2=\frac{-a_1+\sqrt{a_1^2-4a_2}}{2}$.

I can say that $0<b_1<1$ and $b_2>1$ as follows. The left hand side of the above characteristic equation, $D(b)=b^2+a_1b+a_2$ is a quadratic function and takes a parabola shape exhibiting a valley. Given $D(0)>0$ and $D(1)<0$, it is the case that $0<b_1<1$. Further, since $D\left(\beta^{-1}\right)<0$, $b_2$ is larger than $\beta^{-1}$, i.e. larger than 1. Then, to prevent the system from becoming explosive, the value of $A_2$ is set to be 0. This guarantees the saddle path solution for the model:

$$\widehat{P_k}=A_1b_1^k+1. \tag{90}$$

Using the initial condition of $\widehat{P_{-1}} = 0$, the value of $A_1$ is tied down as $-b_1$. Then, Eq.90 becomes

$$\widehat{P_k} = 1 - b_1^{k+1},$$

as given in Eq.67.

### 2.10.5 Appendix 5: Solving the 3rd order difference equation, Eq.80

Denoting $-\frac{2d+d\beta+\kappa}{d\beta} = a_1$, $\frac{d+2d\beta+\kappa\zeta(1-\beta)+\beta\kappa}{d\beta^2} = a_2$, and $-\frac{1}{\beta^2} = a_3$, Eq.80 becomes

$$\widehat{P_{k+2}} + a_1\widehat{P_{k+1}} + a_2\widehat{P_k} + a_3\widehat{P_{k-1}} = \frac{\kappa\zeta(1-\beta)}{d\beta^2}.$$

The solution to this difference equation takes the following form:

$$\widehat{P_k} = A_1 b_1^k + A_2 b_2^k + A_3 b_3^k + \widehat{P}, \tag{91}$$

where $A_1$, $A_2$ and $A_3$ are constants to be determined and $b_1$, $b_2$ and $b_3$ are the eigenvalues determined from the characteristic equation:

$$b^3 + a_1 b^2 + a_2 b + a_1 = 0.$$

Also, $\widehat{P}$ is the steady state value of $\widehat{P_k}$. I know from Eq.80 that $\widehat{P}$ is equal to 1.

Now, setting $\zeta = 2$, I solve the characteristic equation for each value of $\mu$ ($\mu = 0$ and 0.20). When $\mu = 0.20$, it turns out that $b_1 = 0.46$, $b_2 = 1.02$ and $b_3 = 2.18$. To prevent the explosive path and guarantee the saddle-path stability for $\widehat{P_k}$, $A_2$ and $A_3$ are set to be 0. The coefficient $A_1$ can be tied down by realising that $\widehat{P_{-1}} = 0$. From $\widehat{P_{-1}} = A_1 b_1^{-1} + 1$, I know that $A_1 = -b_1$. Then, I obtain the saddle-path solution for the difference equation (Eq.81):

$$\widehat{P_k} = 1 - b_1^{k+1}.$$

Substituting this into $\widehat{Y_k} = -\frac{\beta}{\kappa}\widehat{P_{k+1}} + \frac{1+\beta}{\kappa}\widehat{P_k} - \frac{1}{\kappa}\widehat{P_{k-1}}$ (the AS relation (Eq.57)) and rearranging, I have (Eq.82):

$$\widehat{Y}_k = \frac{\beta}{\kappa} \frac{\left(b_1 - \frac{1}{\beta}\right)(b_1 - 1)}{b_1} b_1^{k+1}.$$

For $\mu = 0$ as well, I obtain the only one stable eigenvalue. Thus, the output dynamics is given by the same equation.

# 3 Chapter Three: Credit Market Imperfections, Staggered Pricesetting, and Output Dynamics of the Response to Money Shocks -extension to segmented input markets-

## 3.1 Introduction

The previous chapter studied how acyclical agency costs alter the output dynamics of the response to money shocks within the New Keynesian (NK) framework. It revealed that they amplify the impact on real output of money shocks while they reduce the persistence of the real effects. Analytical solution clarified that credit frictions flatten both the aggregate supply (AS) and aggregate demand (AD) relations of the model. Quantitatively, I found that the amplification of the impact can be significant, but the effect on persistence is rather small.

One important assumption of the previous framework is the existence of economy-wide input markets. However, one might argue that the assumption of economy-wide input markets is not innocuous, especially in the short run (for example, Woodford (2003, chapter 3)). In a model with economy-wide input markets, there is always a common input price across the economy. What is assumed is that inputs are perfectly mobile across different segments of the economy such as industries or geographic areas. This free mobility guarantees a common input price in the economy. However, in practice, movement of inputs might be rather restricted especially in the short run.

Suppose that some exogenous shock brings about the situation in which salaries for workers are different in different sectors of the economy. Although workers in a sector with low salaries might have an incentive to move to the one with high salaries, they might find it difficult to move instantaneously if the move requires them to acquire different kind of skills. As for the capital market, although capital should be transferred from the sector with low utilisation rate to the one with high rate, instantaneous transfer is quite unlikely. Then, it might be more plausible to assume that inputs do not move across different sectors, i.e. input markets are segmented across sectors. In particular, this assumption seems more realistic in a short run analysis. Acknowledging this argument, this chapter considers the role of credit market frictions in the NK model with segmented input markets.

Credit frictions temporarily aside, the segmentation of input markets leads to greater persistence in the real effects of money shocks with staggered price/wage setting (Ascari (2000), Edge (2002) and Woodford (2003)). Focusing on staggered prices, intuition can be given as follows. Suppose that there is an increase in aggregate demand due to an unexpected increase in money supply. After the shock, the demand for the price-keeping monopolistic competitors (the ones who keep their prices fixed due to the staggered setting) could go up greatly. To satisfy this increase in demand, they require more inputs, which bids up the price of the inputs. Meanwhile, the demand for the price-resetting producers (the ones who reset their prices) does not go up as much, because they tend to increase their prices. When economy-wide input markets are assumed so that inputs move freely, the bidding from the price keeping producers affects the input prices for price resetting ones as well, i.e. they also go up greatly. However, when the movement of inputs is absent, price-resetting producers do not suffer from the spill-over effect. This leads to rather low reset prices and thus aggregate price. In turn, the slow adjustment of aggregate price implies greater persistence in the real effects of the money shock.

This chapter has two findings. First, when the segmented input market assumption is incorporated in the NK model with credit frictions, the qualitative and quantitative results obtained with the economy wide input markets still hold. That is, credit frictions still amplify the impact effect of the shock while reducing the persistence of the effects. Also, the impact effect can be quantitatively significant, but the effect on persistence is rather small. Second, for a given degree of credit frictions (for given agency costs), both the impact and persistence of the shocks are much greater in a framework with segmented input markets than in one with economy-wide markets. I show that this happens because for given credit frictions, the AS relation is much flatter with segmented markets.

The structure of the paper is parallel to the Chapter 2. Section 2 gives the overview of the model. In section 3, I look at the effect of credit frictions on the monopolistic competitors' (retailers) real marginal costs. Then, section 4 studies the effect of credit frictions on the AS relation. Section 5 turns to their effect on the AD relation. Having seen the effects of credit frictions on the AS and AD relations, section 6 analyses their qualitative effect on the output dynamics to money shocks. Given that the most of the arguments in sections 4-6 overlap with the ones in the corresponding sections in Chapter 2, those sections are kept brief. Section 7 conducts a quantitative analysis Section 8 concludes.

## 3.2 The model: overview

As in Chapter 2, the current framework has three types of producers. First, retailers, who are monopolistic competitors subject to Calvo-style staggered pricesetting (Calvo 1983), produce differentiated retail goods. They face a downward sloping demand curve due to the second type of producers, final good producers. Being perfect competitors, they make the composite retail good (the final good) with the CES production function. Retailers use the wholesale goods as the only input, which are made by entrepreneurs, the third type of producers. Entrepreneurs employ household labour as the input for production. They are the ones who lack internal funds for production and thus borrow funds from banks. Credit frictions are again modelled following the Costly State Verification (CSV) framework (Townsend 1979).

The distinctive feature of this new framework is that the whole production process (apart from the production of final goods) is divided into a large number of industries (Figure 3). Each industry consists of its own set of agents, i.e. households, banks, entrepreneurs and retailers. Importantly, inputs markets (household labour and wholesale goods markets) are segmented across industries. This means that the flow of inputs is cut off across industries. This lack of flow can be justified when the inputs are industry specific and households and entrepreneurs find it too costly to acquire different production skills specific to different industries.

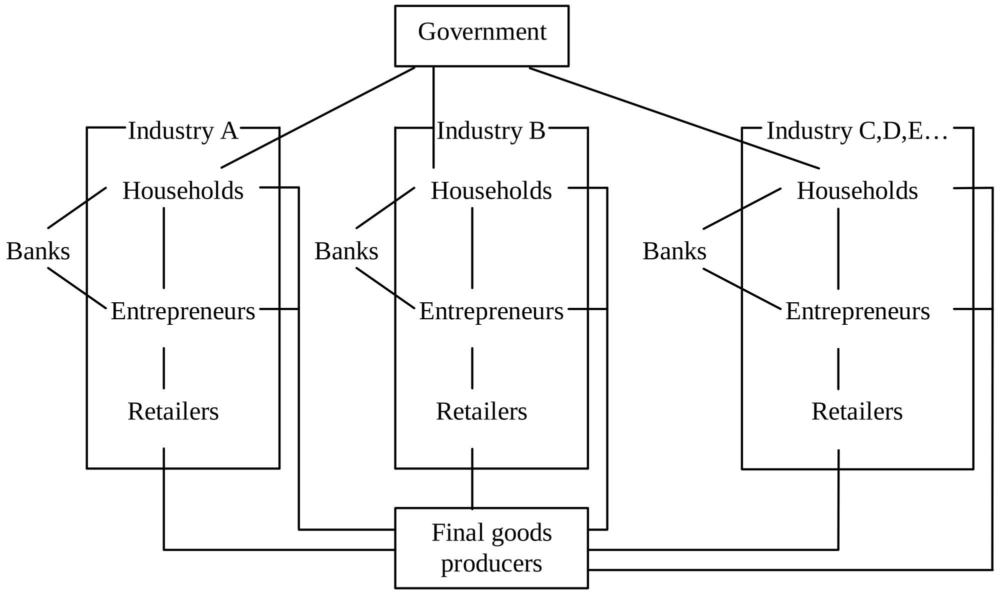


Figure 3: Overview of the model

Following Woodford (2003), I assume that retailers from the same industry always reset their price at the same time, in other words, they draw the Calvo lottery together. Further, although they adjust price simultaneously, they do so in a non-collusive way. Given that they always charge the same price, the demand for their goods is also the same. However, since retailers from different industries draw the lottery separately and can charge different prices, the demand for goods produced in different industries can be different. This leads to different input prices across industries.

Although retailers from the same industry always charge the same price, they actually produce differentiated goods. However, I assume (probably plausibly) that retail goods from the same industry are differentiated rather slightly, while goods from different industries are differentiated largely. In short, there are two stages of product differentiation in the current framework: within industry and across industries. This implies that when all the retail goods are used by perfectly competitive final good producers, the substitutability of the goods within an industry is higher than across industries.

As in Chapter 2, the final goods are consumed by both households and entrepreneurs. The only role of government is to make a direct transfer of money to households in each industry after raising revenue through seigniorage. There is no government spending.

## 3.3 Retailers' real marginal cost in each industry

This section studies the elasticity of retailers' real marginal cost in each industry with respect to aggregate output. I focus on the elasticity in a flexible price environment. Knowing this turns out to be useful to derive the AS relation in the presence of staggered pricesetting. Assuming that retailers' production function exhibits constant returns to scale, retailers' marginal cost in industry $z$ is equivalent to the price of wholesale goods in the same industry. Thus, I first study how the price of wholesale goods is determined.

### 3.3.1 Entrepreneurs

The production function of individual entrepreneur $j$ from industry $z$ is given as:

$$Y_t^W(j,z) = \omega_t(j,z)\, H_t(j,z)^{\alpha}, \tag{92}$$

where $H_t(j,z)$ is household labour employed by the entrepreneur. Assuming that $0 < \alpha < 1$, the production technology exhibits decreasing returns to scale. $\omega_t(j,z)$, being an iid random variable with an expected value of unity, represents an idiosyncratic shock occurring to the entrepreneur.

Entrepreneurs live only one period. At the beginning of each period, they come into the scene with zero initial wealth. Since they pay the household wage before starting the production of wholesale goods, they need to borrow the input cost. They raise funds from households through banks. While households have funds to spare, they do not have technology to produce wholesale goods.

Credit frictions between entrepreneur $j$ from industry $z$ and a perfectly competitive bank in industry $z$ is again modelled following Townsend's costly state verification (CSV) framework. As mentioned in Chapter 2, the optimal contract under the CSV setting is the standard debt contract. Given that the nature of the CSV framework and the standard debt contract were explained in Chapter 2, I do not repeat here.

The contract between entrepreneur $j$ and the bank specifies the threshold value of $\omega$, $\overline{\omega_t}(j,z)$ and the amount of labour employed $H_t(j,z)$. The monitoring costs are a fraction, $\mu$ of the expected wholesale good production, i.e. $\mu H_t(j,z)^\alpha$. The parameter $\mu$ is common to all the retailers across industries and also time-invariant. Entrepreneurs live only one period and consume all the profits they have obtained[42]. While entrepreneur $j$ maximises the expected consumption of final goods, perfectly competitive banks break even. By lending to a large number of entrepreneurs, banks simply recoup the labour cost, $w_t(z)\,H_t(j,z)$ on average. $w_t(z)$ is the real wage prevailing in industry $z$.

Formally, I have the maximisation problem:

*maximise*

$$\varphi_t(z)\,f\left(\overline{\omega_t}(j,z)\right)H_t(j,z)^\alpha \tag{93}$$

*with respect to* $\overline{\omega_t}(j,z)$ and $H_t(j,z)$ *subject to*

$$\varphi_t(z)\,g\left(\overline{\omega_t}(j,z)\right)H_t(z)^\alpha = w_t(z)\,H_t(j,z)\,. \tag{94}$$

Denoting the price of final goods as $P_t$, $\varphi_t(z)\ (=\frac{P_t(z)}{P_t})$ is the real price of the wholesale goods in industry $z$. Since entrepreneurs do not have any market power in industry $z$, $\varphi_t(z)$ is taken as given. The functions $f\left(\overline{\omega_t}(j,z)\right)$ and $g\left(\overline{\omega_t}(j,z)\right)$ represent the expected shares of the wholesale goods production taken by the entrepreneur and the bank, respectively. The distribution of the idiosyncratic shock is time invariant and identical to all the entrepreneurs across industries. Under the standard debt

[42] If they default, they consume nothing.

contract, the expected shares are expressed as:

$$f\left(\overline{\omega_t}\left(j,z\right)\right)=\int_{\overline{\omega_t}(j,z)}^{\infty}\omega d\Phi\left(\omega\right)-\left(1-\Phi\left(\overline{\omega_t}\left(j,z\right)\right)\right)\overline{\omega_t}\left(j,z\right)$$

and

$$g\left(\overline{\omega_t}\left(j,z\right)\right)=\int_{0}^{\overline{\omega_t}(j,z)}\omega d\Phi\left(\omega\right)+\left(1-\Phi\left(\overline{\omega_t}\left(j,z\right)\right)\right)\overline{\omega_t}\left(j,z\right)-\mu\Phi\left(\overline{\omega_t}\left(j,z\right)\right).$$

There, $\Phi$ represents a cumulative distribution function of $\omega$ (and $\phi$ is used for its probability density function).

As in Chapter 2, notice that $f\left(\overline{\omega_t}\left(j,z\right)\right)+g\left(\overline{\omega_t}\left(j,z\right)\right)=1-\mu\Phi\left(\overline{\omega_t}\left(j,z\right)\right)$. This implies that on average, the fraction of $\mu\Phi\left(\overline{\omega_t}\left(j,z\right)\right)$ is lost through monitoring. This deadweight loss represents the agency cost of the model.

Solving the maximisation problem, what we obtain first is the following (implicit) labour demand function:

$$w_t\left(z\right)=\frac{1}{s\left(\overline{\omega_t}\left(j,z\right)\right)}\varphi_t\left(z\right)\alpha H_t\left(j,z\right)^{\alpha-1}, \tag{95}$$

where

$$s\left(\overline{\omega_t}\left(j,z\right)\right)=\frac{1}{1-\mu\Phi\left(\overline{\omega_t}\left(j,z\right)\right)+\frac{\mu\phi\left(\overline{\omega_t}(j,z)\right)f\left(\overline{\omega_t}(j,z)\right)}{f'\left(\overline{\omega_t}(j,z)\right)}}.$$

Substituting this into the constraint (Eq.94), we obtain $\alpha=g\left(\overline{\omega_t}\left(j,z\right)\right)s\left(\overline{\omega_t}\left(j,z\right)\right)$.

Given that both the distribution of idiosyncratic shock and monitoring costs are time invariant and identical across industries, the cut-off value of $\omega$ is also time-invariant and identical to all the entrepreneurs across industries, i.e. $\overline{\omega_t}\left(j,z\right)=\overline{\omega}\left(j,z\right)=\overline{\omega}$. Replacing $\overline{\omega_t}\left(j,z\right)$ with $\overline{\omega}$, I have:

$$\alpha=g\left(\overline{\omega}\right)s\left(\overline{\omega}\right), \tag{96}$$

where

$$s\left(\overline{\omega}\right)=\frac{1}{1-\mu\Phi\left(\overline{\omega}\right)+\frac{\mu\phi(\overline{\omega})f(\overline{\omega})}{f'(\overline{\omega})}}. \tag{97}$$

Thus, once the distribution of idiosyncratic shock, $\omega$ is specified, the cutoff value of $\overline{\omega}$ is obtained for given $\alpha$ and $\mu$.

Likewise, the labour demand function (Eq.95) can be rewritten as: $w_t\left(z\right)=\frac{1}{s(\overline{\omega})}\varphi_t\left(z\right)\alpha H_t\left(j,z\right)^{\alpha-1}$. Then, I know from this relation that $H_t\left(j,z\right)=H_t(z)$. That is, entrepreneurs from the same industry employ the same amount of household labour. Incorporating this, the labour demand function is expressed as

$$w_t\left(z\right)=\frac{1}{s\left(\overline{\omega}\right)}\varphi_t\left(z\right)\alpha H_t\left(z\right)^{\alpha-1}.$$

Finally, notice that this implicit labour demand function also represents entrepreneurs' pricing equation. Rewriting this, we obtain

$$\varphi_t\left(z\right)=s\left(\overline{\omega}\right)\frac{1}{\alpha H_t\left(z\right)^{\alpha-1}}w_t\left(z\right). \tag{98}$$

The real price of wholesale goods in industry $z$, $\varphi_t\left(z\right)$ is determined as a mark up, $s\left(\overline{\omega}\right)$ over the real marginal cost, $\frac{1}{\alpha H_t(z)^{\alpha-1}}w_t\left(z\right)$. In case monitoring costs are present ($\mu>0$), the mark up, $s\left(\overline{\omega}\right)$ is greater than unity.[43] On the other hand, in the absence of monitoring costs ($\mu=0$), $s\left(\overline{\omega}\right)=1$. When monitoring costs are non trivial, the real price of wholesale goods has to be set higher than the real marginal cost to offset the agency costs.[44]

### 3.3.2 Households

I now turn to the utility maximisation problem of households. Households within the same industry are symmetric. The utility maximisation problem of a representative household in industry $z$ is as follows:

[43] This can be seen from the fact that $f'\left(\overline{\omega}\right)\ (=-\left(1-\Phi\left(\overline{\omega}\right)\right))$ is negative in $s\left(\overline{\omega}\right)$ (Eq.97).

[44] Entrepreneurs charge the mark up not because they have the market power but because they need to cover the agency costs.

*maximise*

$$\sum_{t=0}^{\infty}\beta^{t}\left(\delta\ln C_{t}\left(z\right)+\left(1-\delta\right)\frac{\left(\frac{M_{t}(z)}{P_{t}}\right)^{1-\zeta}}{1-\zeta}-\chi H_{t}\left(z\right)^{\eta}\right),^{45} \tag{99}$$

*subject to*

$$P_{t}C_{t}\left(z\right)+M_{t}\left(z\right)+B_{t}\left(z\right)=W_{t}\left(z\right)H_{t}\left(z\right)+M_{t-1}\left(z\right)+(1+i_{t-1})B_{t-1}\left(z\right)+\Pi_{t}\left(z\right)+T_{t}\left(z\right), \tag{100}$$

where $C_t(z)$, $M_t(z)$, $B_t(z)$, $\Pi_t(z)$ and $T_t(z)$ are industry $z$ consumption, money holding, bond holding, profit from retailers[46], and a lump sum tax, all variables in nominal terms. $i_{t-1}$ is an nominal interest rate accrued in period t. Solving this, I obtain the consumption Euler equation:

$$\left(C_{t}\left(z\right)\right)^{-1}=\beta\left(1+r_{t}\right)\left(C_{t+1}\left(z\right)\right)^{-1},$$

the money demand function:

$$\frac{M_{t}\left(z\right)}{P_{t}}=\left(\frac{1-\delta}{\delta}\frac{1+i_{t}}{i_{t}}C_{t}\left(z\right)\right)^{\frac{1}{\zeta}}.^{47}$$

and the labour supply function:

$$w_{t}\left(z\right)=\frac{\chi\eta}{\delta}C_{t}\left(z\right)H_{t}\left(z\right)^{\eta-1}$$

I assume that the profit share from retailers, $\Pi_t(z)$ and the lump sum tax, $T_t(z)$ are common across industries. However, as discussed above, wage and labour supply can be different across industries under the staggered pricesetting. This would mean that households' wealth levels would also be different across industries. Following Ascari (2000) and Woodford (2003), I now assume that some insurance scheme is

[45] In the previous chapter, the subutility over consumption is initially set as the CRRA form and later restricted as the logarithmic form. Here, I focus on the logarithmic form from the outset.

[46] Householders are the shareholders of retailers.

[47] As usual, in deriving the money demand function, use is made of the consumption Euler equation and the Fisher equation: $1+r_t\equiv(1+i_t)\frac{P_t}{P_{t+1}}$.

available in the economy so that the difference in households' wealth levels is offset in every period. This implies that households in different industries always consume exactly the same amount of final goods and holds the same real balances, i.e. $C_t(z) = C_t$ and $\frac{M_t(z)}{P_t} = \frac{M_t}{P_t}$. Thus, the consumption Euler equation and money demand function become

$$C_t^{-1} = \beta (1 + r_t) C_{t+1}^{-1} \tag{101}$$

and

$$\frac{M_t}{P_t} = \left( \frac{1-\delta}{\delta} \frac{1+i_t}{i_t} C_t \right)^{\frac{1}{\varsigma}}, \tag{102}$$

respectively. Also, the labour supply relation can be rewritten as:

$$w_t(z) = \frac{\chi\eta}{\delta} C_t H_t(z)^{\eta-1}. \tag{103}$$

Taking into account that the real price of wholesale goods in industry $z$, $\varphi_t(z)$ is equivalent to retailers' real marginal cost in the same industry, I know from Eqs.98 and 103 that the real marginal cost is expressed as:

$$\varphi_t(z) = \frac{\chi\eta}{\delta} \frac{1}{\alpha} s(\overline{\omega}) C_t H_t(z)^{\eta-\alpha}. \tag{104}$$

Having obtained the expression for the retailers' real marginal cost in each industry, I consider the elasticity of this marginal cost with respect to aggregate output. Specifically, I discuss the elasticity in a flexible price environment.

I now look at how retailers in each industry behave when prices are flexible.

### 3.3.3 Retailer's profit maximisation problem in a flexible price environment

First, the demand function for retail goods in each industry is derived by solving final goods producers' cost minimisation problem. Final goods producers make final goods, using all the differentiated retail goods from all the industries. As pointed out above, there are two stages of product differentiation: within an industry and across industries. Therefore, final goods producers' production function is expressed as follows:

$$Y_t = \left( \int_0^1 Y_t(z)^{\frac{\epsilon-1}{\epsilon}} dz \right)^{\frac{\epsilon}{\epsilon-1}}, \tag{105}$$

where

$$Y_t(z) = \left( \int_0^1 Y_t(j,z)^{\frac{\nu-1}{\nu}} dj \right)^{\frac{\nu}{\nu-1}}. \tag{106}$$

$Y_t(j,z)$ is the amount of retail good produced by retailer $j$ in industry $z$ and $Y_t(z)$ is the composite of all the retail goods from industry $z$. Likewise, $Y_t$ is the composite of all the industry composite goods. It seems natural to assume that the substitutability of retail goods within an industry is higher than composite goods across industries. The difference in substitutability is represented by $\nu \geq \epsilon$.

Given that perfect competition prevails in the final goods sector, the final goods producers minimise the cost of production. In the first stage, they minimise the cost at an industry level. Solving the cost minimisation problem for any given industry $z$, we obtain the demand function for retailer, $j$ in industry $z$ as follows:

$$Y_t(j,z) = Y_t(z) \left( \frac{P_t(j,z)}{P_t(z)} \right)^{-\nu} \tag{107}$$

where

$$P_t(z) = \left( \int_0^1 P_t(j,z)^{1-\nu} dj \right)^{\frac{1}{1-\nu}}. \tag{108}$$

In the second stage, the final good producers minimise the cost at an economy-wide level. The solution leads to the following demand function for an industry-composite retail good:

$$Y_t(z) = Y_t \left( \frac{P_t(z)}{P_t} \right)^{-\epsilon}, \tag{109}$$

where

$$P_t = \left( \int_0^1 P_t(z)^{1-\epsilon} dz \right)^{\frac{1}{1-\epsilon}}. \tag{110}$$

$P_t$ is the price of the final good. I now turn to retailers' profit maximisation problem.

Retailers in industry $z$ use the industry specific wholesale goods as the only input, simply transforming them into differentiated retail goods. Given that the production technology shows constant returns to scale, retailer $j$'s production function is expressed as

$$Y_t(j,z) = Y_t^W(j,z),$$

where $Y_t^W(j,z)$ is the amount of industry $z$'s wholesale goods used by retailer $j$. I study his profit maximisation problem in a flexible price environment.

Given the demand function for an individual retail good (Eq.107), retailer $j$ from industry $z$ solves the following profit maximisation problem:

*maximise*

$$\Pi_t(j,z) = P_t(j,z)\,Y_t(j,z) - P_t^W(z)\,Y_t(j,z),$$

where $P_t^W(z)$ is the price of the industry-specific wholesale good, *subject to*

$$Y_t(j,z) = Y_t(z)\left(\frac{P_t(j,z)}{P_t(z)}\right)^{-\nu}.$$

Since there are a large number of retailers in each industry, an individual retailer does not have any market power in the industry specific wholesale goods market. Therefore, retailer $j$ takes the price of the wholesale good, $P_t^W(z)$ as given. Also, the fact that there are a large number of retailers in each industry implies that the effect of an individual price and output on the industry aggregate price and output, $P_t(z)$ and $Y_t(z)$ is negligible. Thus, $P_t(z)$ and $Y_t(z)$ are given to retailer $j$.

The first order condition for $P_t(j,z)$ leads to the following relation: $\frac{P_t^*(j,z)}{P_t(z)} = \frac{\nu}{\nu-1}\frac{P_t^W(z)}{P_t(z)}$, where $P_t^*(j,z)$ is his optimal price. Multiplying both sides by $\frac{P_t(z)}{P_t}$, this becomes

$$\frac{P_t^*(j,z)}{P_t} = \frac{\nu}{\nu-1}\varphi_t(z), \tag{111}$$

where $\varphi_t(z)$ is the real price of the industry $z$ wholesale goods, $\frac{P_t^W(z)}{P_t}$. The real optimal price under a flexible price setting is a constant mark up, $\frac{\nu}{\nu-1}$ over the real marginal cost, $\varphi_t(z)$, with the size of the mark up determined by the elasticity of demand, $\nu$.

### 3.3.4 Elasticity of retailers' real marginal cost with respect to output

I now derive the elasticity of retailers' real marginal cost in each industry with respect to output in a flexible price environment. First, taking into account the deadweight loss, the net industry aggregate wholesale good production, $Y_t^W(z)$ is given by

$$Y_t^W(z) = (1 - \mu\Phi(\overline{\omega}))\, H_t(z)^\alpha\,. \tag{112}$$

With retailers' constant returns to scale technology, the industry aggregate retail good production, $Y_t(z)$ [48] is equal to $Y_t^W(z)$. Incorporating this, Eq.112 becomes

$$H_t(z) = \left(\frac{Y_t(z)}{1 - \mu\Phi(\overline{\omega})}\right)^{\frac{1}{\alpha}}. \tag{113}$$

Meanwhile, the economy-wide aggregate consumption for entrepreneurs, $C_t^e$ is given as:

$$C_t^e = f(\overline{\omega}) \int \varphi_t(z)\, H_t(z)^\alpha\, dz. \tag{114}$$

Substituting Eq.113 into Eq.114, I obtain

$$C_t^e = \frac{f(\overline{\omega})}{1 - \mu\Phi(\overline{\omega})} \int \varphi_t(z)\, Y_t(z)\, dz. \tag{115}$$

From the market clearing condition, $Y_t = C_t + C_t^e$, aggregate consumption, $C_t$ is expressed as

$$C_t = Y_t - \frac{f(\overline{\omega})}{1 - \mu\Phi(\overline{\omega})} \int \varphi_t(z)\, Y_t(z)\, dz \tag{116}$$

Substituting Eqs.113 and 116 into Eq.104, the real marginal cost in industry $z$,

[48] In principle, aggregate output of retail goods have to be defined by integrating output of each type of good over, since retail goods are not homogeneous. However, under the symmetry in each industry, aggregate output, $Y_t(z)$ is equal to output of a typical retailer, $Y_t(j, z)$.

$\varphi_t(z)$ is expressed as:

$$\varphi_t(z) = \frac{\chi\eta}{\delta}\frac{1}{\alpha}s(\overline{\omega})\left(Y_t - \frac{f(\overline{\omega})}{1-\mu\Phi(\overline{\omega})}\int\varphi_t(z)Y_t(z)\,dz\right)\left(\frac{Y_t(z)}{1-\mu\Phi(\overline{\omega})}\right)^{\frac{\eta-\alpha}{\alpha}}. \quad (117)$$

Multiplying $\varphi_t(z)$ by $Y_t(z)$ and then integrating over industries, I have

$$\begin{aligned}&\int\varphi_t(z)Y_t(z)\,dz\\ =\;& \frac{\chi\eta}{\delta}\frac{1}{\alpha}s(\overline{\omega})\left(Y_t - \frac{f(\overline{\omega})}{1-\mu\Phi(\overline{\omega})}\int\varphi_t(z)Y_t(z)\,dz\right)\left(\frac{1}{1-\mu\Phi(\overline{\omega})}\right)^{\frac{\eta-\alpha}{\alpha}}\int Y_t(z)^{\frac{\eta}{\alpha}}\,dz.\end{aligned}$$

Rearranging this yields

$$\int\varphi_t(z)Y_t(z)\,dz = \frac{\frac{\chi\eta}{\delta}\frac{1}{\alpha}s(\overline{\omega})\left(\frac{1}{1-\mu\Phi(\overline{\omega})}\right)^{\frac{\eta-\alpha}{\alpha}}Y_t\int Y_t(z)^{\frac{\eta}{\alpha}}\,dz}{1+\frac{f(\overline{\omega})}{1-\mu\Phi(\overline{\omega})}\frac{\chi\eta}{\delta}\frac{1}{\alpha}s(\overline{\omega})\left(\frac{1}{1-\mu\Phi(\overline{\omega})}\right)^{\frac{\eta-\alpha}{\alpha}}\int Y_t(z)^{\frac{\eta}{\alpha}}\,dz}. \quad (118)$$

Substituting this back into Eq.117, I obtain

$$\varphi_t(z) = \frac{\frac{\chi\eta}{\delta}\frac{1}{\alpha}s(\overline{\omega})Y_t}{1+\frac{\chi\eta}{\delta}\frac{1}{\alpha}s(\overline{\omega})f(\overline{\omega})\left(\frac{1}{1-\mu\Phi(\overline{\omega})}\right)^{\frac{\eta}{\alpha}}\int Y_t(z)^{\frac{\eta}{\alpha}}\,dz}\left(\frac{Y_t(z)}{1-\mu\Phi(\overline{\omega})}\right)^{\frac{\eta-\alpha}{\alpha}}. \quad (119)$$

Recall that the real optimal price charged by retailer $j$ in industry $z$ under a flexible price setting, $\frac{P_t^*(j,z)}{P_t}$ is $\frac{\nu}{\nu-1}\varphi_t(z)$. Substituting this into the demand function (Eq.109), we obtain

$$Y_t(z) = Y_t\left(\frac{\nu}{\nu-1}\varphi_t(z)\right)^{-\epsilon}.$$

Then, plugging this into Eq.119 and factoring out $\varphi_t(z)$, I have

$$\varphi_t(z) = \left(\frac{\frac{\chi\eta}{\delta}\frac{1}{\alpha}s(\overline{\omega})Y_t}{1+\frac{\chi\eta}{\delta}\frac{1}{\alpha}s(\overline{\omega})f(\overline{\omega})\left(\frac{1}{1-\mu\Phi(\overline{\omega})}\right)^{\frac{\eta}{\alpha}}\int Y_t(z)^{\frac{\eta}{\alpha}}\,dz}\left(\frac{\left(\frac{\nu}{\nu-1}\right)^{-\epsilon}}{1-\mu\Phi(\overline{\omega})}Y_t\right)^{\frac{\eta-\alpha}{\alpha}}\right)^{\frac{1}{1+\epsilon\frac{\eta-\alpha}{\alpha}}}$$

Log linearising this around the flexible price steady state[49], I obtain

$$\widehat{\varphi_t(z)} = \xi \widehat{Y}_t, \tag{120}$$

where

$$\xi = \frac{\frac{\eta}{\alpha}\left(1 - \frac{\nu-1}{\nu}\frac{f(\overline{\omega})}{1-\mu\Phi(\overline{\omega})}\right)}{1+\epsilon\frac{\eta-\alpha}{\alpha}}. \tag{121}$$

The hat notation represents the log deviation from the steady state. Indeed, $\xi$ is the elasticity of retailers' real marginal cost in industry $z$ with respect to output in a flexible price environment.

**Effect of monitoring costs on elasticity** To see the effect of monitoring costs on the elasticity, the entrepreneurs' share of the net revenue from wholesale goods production, $\frac{f(\overline{\omega})}{1-\mu\Phi(\overline{\omega})}$ needs to be specified. As pointed out above, once the distribution of idiosyncratic shock, $\omega$ is specified, the cutoff value of $\overline{\omega}$ is obtained for given $\alpha$ and $\mu$ from $\alpha = g(\overline{\omega})\, s(\overline{\omega})$ (Eq.96). As in Chapter 2, I specify the distribution as uniform in the region $[1-\rho, 1+\rho]$. As shown there, $\overline{\omega}$ is then obtained as a function of $\alpha$, $\rho$, and $\mu$:

$$\overline{\omega} = (1+\rho) - \mu + \varepsilon, \tag{122}$$

where

$$\varepsilon = \frac{1}{2}(\alpha\mu - \sqrt{\alpha^2\mu^2 + (1-\alpha)(4\mu^2 + 16\rho - 16\mu\rho)}).$$

With the uniform distribution, the entrepreneurs' share, $\frac{f(\overline{\omega})}{1-\mu\Phi(\overline{\omega})}$ is obtained as

$$\frac{f(\overline{\omega})}{1-\mu\Phi(\overline{\omega})} = \frac{(\overline{\omega}-(1+\rho))^2}{4\rho - 2\mu(\overline{\omega}-(1-\rho))}. \tag{123}$$

Substituting Eq.122 in Eq.123, $\frac{f(\overline{\omega})}{1-\mu\Phi(\overline{\omega})}$ is expressed as a function of $\alpha$, $\rho$, and $\mu$

$$\frac{f(\overline{\omega})}{1-\mu\Phi(\overline{\omega})} = \frac{(\mu-\varepsilon)^2}{4\rho(1-\mu) + 2\mu(\mu-\varepsilon)}. \tag{124}$$

[49] The steady state value of aggregate output, $\overline{Y}$ is obtained as follows. First, notice from Eq.111 that $\overline{\varphi} = \frac{\upsilon-1}{\upsilon}$, where $\overline{\varphi}$ is the real marginal cost in the steady state. Then, given that $\overline{\varphi}$ can be expressed as a function of $\overline{Y}$ from Eq.119, $\overline{Y}$ can be tied down.

In the absence of monitoring costs ($\mu = 0$), it simplifies to $\frac{f(\varpi)}{1-\mu\Phi(\varpi)} = 1 - \alpha$.

Chapter 2 also shows that $\frac{f(\varpi)}{1-\mu\Phi(\varpi)}$ is an increasing function in monitoring costs, $\mu$:

$$\frac{\partial \frac{f(\varpi)}{1-\mu\Phi(\varpi)}}{\partial \mu} > 0. \tag{125}$$

Again, the intuition is that when monitoring costs become larger, entrepreneurs enjoy the higher economic rent because of their special ability of monitoring the outcome of their projects costlessly. I thus know from Eq.121 that

$$\frac{\partial \xi}{\partial \mu} < 0. \tag{126}$$

In short, monitoring costs make the real marginal cost for retailers in industry $z$ less pro-cyclical.

## 3.4 Aggregate supply (AS) relation

I obtained the elasticity of retailers' real marginal costs with respect to output in a flexible price setting. Given this, I derive the AS relation when prices are set in a staggered way. The derivation is based on Woodford (2003).

As in Chapter 2, I first obtain the relation between the reset price under Calvo staggering, $P^*_{calvo,t}$ and inflation rate, $\pi_t$. Denoting the relative reset price, $\frac{P^*_{calvo,t}}{P_t}$ as $Q_t$, I have

$$\widehat{q_t} = \frac{\theta}{1-\theta}\pi_t, \tag{127}$$

where $\widehat{q_t}$ is the log deviation of $Q_t$ from the zero inflation steady state value (unity) and $\theta$ is the proportion of industries which do not have a chance to reset their prices.

Next, given the chance to reset the price, retailer $j$ from industry $z$ choose $P_t\,(j,z)$ to maximise

$$\sum_{i=0}^{\infty} \theta^i \Lambda_{t,t+i} \left( \frac{P_t\,(j,z)}{P_{t+i}} Y_{t+i}\,(j,z) - \frac{P^W_{t+i}\,(z)}{P_{t+i}} Y_{t+i}\,(j,z) \right),$$

where $\Lambda_{t,t+i}$ is the discount factor.[50] Incorporating the demand function, $Y_t(j,z) = Y_t(z)\left(\frac{P_t(j,z)}{P_t(z)}\right)^{-\nu}$ (Eq.107), this becomes

$$\sum_{i=0}^{\infty}\theta^i\Lambda_{t,t+i}\left(\frac{P_t(j,z)-P_{t+i}^W(z)}{P_{t+i}}\right)\left(\frac{P_t(j,z)}{P_{t+i}(z)}\right)^{-\nu}Y_{t+i}(z). \tag{128}$$

Now, remember that in a flexible price environment, the optimal relative price, $\frac{P_t^*(j,z)}{P_t}$ is proportional to the marginal cost, $\varphi_t(z)$. Then, given that $\widehat{\varphi_t(z)} = \xi\widehat{Y}_t$ (Eq.120), I obtain $\ln\left(\frac{P_t^*(j,z)}{P_t}\right) = \xi\widehat{Y}_t$. Therefore, when I differentiate Eq.128 with respect to $P_t(j,z)$, set it to zero and further log-linearise, I have

$$\sum_{i=0}^{\infty}(\theta\beta)^i\left(\ln\left(\frac{P_{calvo,t}^*}{P_{t+i}}\right)-\xi\widehat{Y_{t+i}}\right)=0.^{51}$$

This can be further rewritten as:

$$\widehat{q}_t = (1-\theta\beta)\,\xi\widehat{Y}_t + \theta\beta\left(\widehat{q_{t+1}}+\pi_{t+1}\right). \tag{129}$$

From Eqs.127 and 129, I obtain the new Keynesian Phillips curve (NKPC):

$$\pi_t = \kappa\widehat{Y}_t + \beta\pi_{t+1}. \tag{130}$$

There, $\kappa = \phi\xi$ where $\phi = \frac{1-\theta}{\theta}(1-\theta\beta)$. In the following, I assume that there is an unexpected permanent increase in money supply in Period 0 and the economy was in its steady state before the shock. Expressing Eq.130 in terms of price for $k \geq 0$, I obtain the following AS relation:

$$\widehat{P_k} = \frac{\kappa}{1+\beta}\widehat{Y}_k + \frac{1}{1+\beta}\left(\widehat{P_{k-1}}+\beta\widehat{P_{k+1}}\right), \tag{131}$$

where $\widehat{P_k}$ represents the log deviation of price from the initial steady state value.

[50] The discount factor $\Lambda_{t,t+i}$ is exactly the same as the previous paper: $\Lambda_{t,t+i} = \beta^i\left(\frac{C_{t+i}}{C_t}\right)^{-1}$, where $\beta$ is the discount rate. In fact, this simply represents the inverse of the product of the gross real interest rates from period $t$ to $t+i-1$.

[51] In the zero inflation steady state, $\Lambda_{t,t+i}$ is simply $\beta^i$.

Given that $\frac{\partial \xi}{\partial \mu} < 0$ (Eq.126), it is the case that

$$\frac{\partial \kappa}{\partial \mu} < 0. \tag{132}$$

That is, monitoring costs make the AS curve flatter.

### 3.4.1 Discussion: comparison of the slope of the AS curve between segmented and economy-wide input markets assumptions

Notice that the difference in the slope comes from the difference in the elasticity of retailers' real marginal cost with respect to output. Denoting the elasticity for economy-wide input markets as $\xi_{econwide}$, Eq.28 in Chapter 2 says

$$\xi_{econwide} = \frac{\eta}{\alpha}\left(1 - \frac{\epsilon - 1}{\epsilon}\frac{f(\overline{\omega})}{1 - \mu\Phi(\overline{\omega})}\right).$$

I denote the elasticity with the segmented market as $\xi_{segmented}$. For simplicity, suppose that $\nu = \epsilon$, i.e. the substitutability of retail goods within an industry is the same as the one of the composite goods across industries. Then, I have

$$\xi_{segmented} = \frac{\frac{\eta}{\alpha}\left(1 - \frac{\epsilon-1}{\epsilon}\frac{f(\overline{\omega})}{1-\mu\Phi(\overline{\omega})}\right)}{1 + \epsilon\frac{\eta-\alpha}{\alpha}}.$$

Given that $0 < \alpha < 1$, $\eta > 1$, and $\epsilon > 1$, the denominator of the expression for $\xi_{segmented}$ is greater than unity. I can thus say that for given parameter values,

$$\xi_{segmented} < \xi_{econwide}. \tag{133}$$

The elasticity is smaller with the assumption of segmented input markets. This is because the spill-over of the upward pressure of input prices across industries is absent when markets are segmented.

Therefore, I can say that for given parameter values, the slope of the AS curve is flatter with the segmented input markets.

## 3.5 Aggregate demand (AD) relation

As in Chapter 2, when $\zeta = 1$, i.e. the subutility over real balance takes a log form in the households' utility function, the consumption Euler equation and money demand function (Eqs.101 and 102) lead to the first order difference equation in the inverse velocity, $\frac{M_t}{P_t C_t}$. Solving this in the case of the unexpected permanent increase in money supply in Period 0, I again obtain the following AD relation:

$$\widehat{P_k} = \widehat{M_k} - \widehat{C_k}, \tag{134}$$

where $k \geq 0$. $\widehat{M_k}$ is the log deviation from the initial steady state.

Now, substituting Eq.118 into Eq.116 and log linearising around the zero inflation steady state, I obtain:

$$\widehat{C_t} = d\widehat{Y_t}, \tag{135}$$

where

$$d = 1 - \frac{\eta}{\alpha}\frac{\nu - 1}{\nu}\frac{f(\overline{\omega})}{1 - \mu\Phi(\overline{\omega})}. \tag{136}$$

With Eq.134, I obtain:

$$\widehat{P_k} = \widehat{M_k} - d\widehat{Y_k}. \tag{137}$$

Assuming that $0 < d < 1$, the relation is negatively sloped. Since $\frac{f(\overline{\omega})}{1-\mu\Phi(\overline{\omega})}$ is increasing in monitoring costs, $\mu$ (Eq.125), higher monitoring costs, $\mu$ flatten the AD relation.

Finally, I compare the slope of AD curve with the one with the economy-wide input markets. In Chapter 2, I obtained $d = 1 - \frac{\eta}{\alpha}\frac{\epsilon-1}{\epsilon}\frac{f(\overline{\omega})}{1-\mu\Phi(\overline{\omega})}$ for the economy-wide markets (Eq.42). Then, I find that as long as $\nu = \epsilon$, the slope is the same between the two cases.

## 3.6 Output dynamics

Suppose that money supply increases permanently by 1% in Period 0. Then, Eq.137 becomes

$$\widehat{P_k} = 1 - d\widehat{Y_k}.$$

Substituting this into the AS relation (Eq.131), I have the same second order difference equation as the one in Chapter 2:

$$\widehat{P_{k+1}} - \frac{1+\beta+e}{\beta}\widehat{P_k} + \frac{1}{\beta}\widehat{P_{k-1}} = -\frac{e}{\beta},$$

where $e = \frac{\kappa}{d}$. Solving this, I have the output dynamics equation:

$$\widehat{Y_k} = \frac{1}{d}b_1^{k+1}, \tag{138}$$

where

$$b_1 = \frac{-a_1 - \sqrt{a_1^2 - 4a_2}}{2}[52].$$

Eq.138 indicates that the output dynamics is characterised by the "impact" parameter, $\frac{b_1}{d}$ and the "persistence" parameter, $b_1$.

### 3.6.1 Effect of monitoring costs on the impact and persistence of the money shock

Following the line of proof given in Chapter 2, it can be again shown that

$$\frac{\partial \frac{b_1}{d}}{\partial \mu} > 0,$$

and

$$\frac{\partial b_1}{\partial \mu} < 0.$$

In words, monitoring costs amplify the impact effect of the money shock while reducing the persistence of the real effects. Thus, I conclude that the qualitative result

[52] $a_1$ and $a_2$ are given as: $a_1 = -\frac{1+\beta+e}{\beta}$ and $a_2 = \frac{1}{\beta}$.

obtained in Chapter 2 is robust to the change in the way of modelling input markets.

## 3.7 Quantitative analysis

In this section, I conduct a simple calibration exercise to see if the assumption of segmented input markets makes a notable quantitative change. As for parameters, the only new element in the current framework is that I now have two parameters for different elasticity of demand, i.e. $\nu$ and $\epsilon$. For simplicity, I set $\nu = \epsilon$. All the other parameter values are set to be the same as Chapter 2. I here replicate the parameters for convenience.

Table 5: Parameter values

| | |
|---|---|
| $\eta$: preference parameter | 4.5 |
| $\alpha$: technology parameter | 0.99 |
| $\nu, \epsilon$: elasticity of demand | 10 |
| $\mu$: monitoring costs | 0.20 |
| $\rho$: distribution function parameter | 0.11[53] |
| $\beta$: discount factor | 0.99 |
| $\theta$: stickiness parameter | 0.75 |

Table 6 shows the values of the elasticity of the retailers' real marginal costs with respect to output, $\xi$ and the wedge between the household consumption and output, $d$ for $\mu = 0$ (no credit frictions) and 0.20. The values of $\xi$ in brackets are the values in the framework with economy-wide input markets.[54] Indeed, $\xi$ is much smaller with segmented input markets for given monitoring costs (as implied by Eq.133).

[53] As explained in Chapter 2, the value of $\rho$ is calibrated to match the quarterly default rate of 0.974% (Fisher (1994)).

[54] The values of $d$ are the same between the two frameworks for each $\mu$.

Table 6: Values of $\xi$ and $d$ for each $\mu$

| $\mu$ | $\xi$ | $d$ |
|---|---|---|
| 0 (no credit frictions) | 0.12 (4.50) | 0.96 |
| 0.20 | 0.11 (4.10) | 0.56 |

The values of the impact and persistence parameters are given in Table 7. Again, the values in the brackets are the ones obtained for the case of economy-wide input markets. Focusing on the impact parameters for the segmented input case, monitoring costs make a huge difference; compared to the case without frictions, the impact with $\mu = 0.20$ is 68 % higher than the one with $\mu = 0$. On the other hand, although monitoring costs reduce the degree of persistence, the effect is small.

Table 7: Impact parameter and persistence parameter for $\mu = 0$ and 0.20

| $\mu$ | Impact parameter, $\frac{b_1}{d}$ | Persistence parameter, $b_1$ |
|---|---|---|
| 0 | 0.94 (0.56) | 0.90 (0.54) |
| 0.20 | 1.58 (0.83) | 0.88 (0.46) |

Having seen the values of the impact and persistence parameters, the output dynamics of the response to the unexpected permanent increase in money supply is drawn in Figure 4.

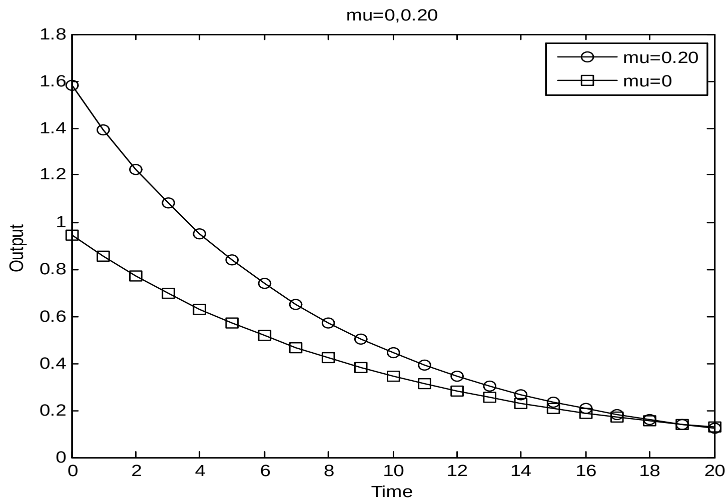


Figure 4: Output dynamics with segmented input markets

Next, I compare the output dynamics between segmented and economy-wide input market cases for $\mu = 0.20$. Table 7 indicates that the assumption of segmented input markets notably enhances both the impact and persistence. The difference in output dynamics is clear (Figure 5). For given agency costs[55], the real effects of the money shock are much more significant with segmented input markets.

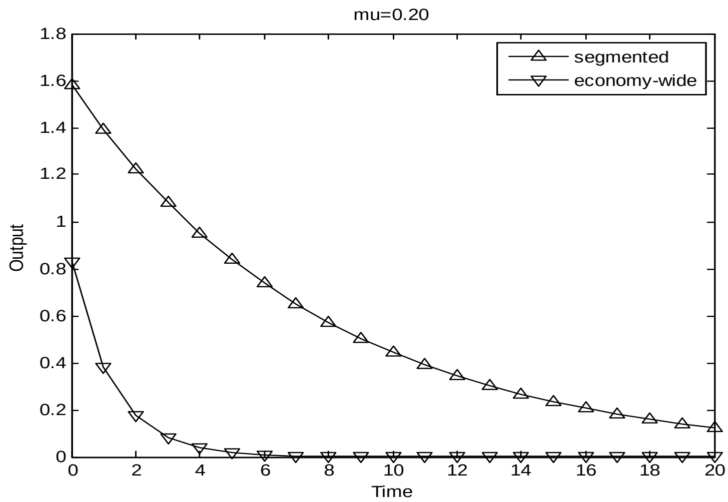


Figure 5: Comparison of output dynamics between different input market structures

## 3.8 Conclusion

This chapter replaced the economy-wide input markets assumption made in Chapter 2 with the segmented input markets assumption. I showed that the effect of credit market imperfections on the output dynamics in response to money shocks is robust to the different ways of modelling input markets. Qualitatively, they still amplify the impact of money shocks on output while reducing the persistence of the real effects. Quantitatively, the amplification of the impact is significant, but the effect on persistence is small. I also showed that, for given agency costs, both the impact of money on output and the persistence of the effects are much greater in a framework with segmented input markets. Analytical solutions clarified that this happens because the segmented input market assumption flattens the AS relation greatly. Thus, in the light of the VAR analysis, the segmented markets assumption seems to be an important modification in the direction towards greater realism even in an environment where credit markets are imperfect.

[55] Indeed, the deadweight loss, $\mu\Phi(\overline{\omega})$ is the same for both frameworks.

# 4 Chapter Four: The Role of Net Worth in Business Cycle Dynamics

## 4.1 Introduction

In the previous two chapters, I studied how agency costs, when modelled as acyclical, affect the real effect of money shocks within the NK framework. By modelling the agency costs as acyclical, I managed to present the analytical solution. Now, I turn my focus directly on the role of endogenous movements of agency costs. What is behind them is the movements of borrowers' (entrepreneurs) net worth. Intuitively, greater net worth corresponds to smaller agency costs. This is because when entrepreneurs' net worth is large, the discrepancy in interests between entrepreneurs and banks is reduced. Thus, agency costs become smaller. Further, if agency costs are smaller, the premium charged on entrepreneurs' borrowing falls. This, in turn, increases their production. Indeed, the way net worth evolves has an important implication on business cycle dynamics through its effect on agency costs.

This chapter compares business cycle dynamics between the two different modelling strategies. The aim of these strategies is to avoid the situation where entrepreneurs ultimately become self-financed.[56] The first strategy assumes that entrepreneurs make a consumption/saving decision to maximise their intertemporal utility, but they have a higher discount rate than households. The second strategy assumes that a constant fraction of entrepreneurs die each period and they consume all the accumulated wealth just before their death. The population is held constant by the birth of new entrepreneurs. This is equivalent to assuming that a constant fraction of entrepreneurs' profit is consumed/saved in each period. For example, the former strategy is adopted by Carlstrom and Fuerst (1997, 98), while the second by Bernanke, Gertler and Gilchrist (1999). For convenience, I call the first type of entrepreneurs as Euler equation entrepreneurs, while the second as overlapping-generations (OLG) entrepreneurs. Based on Carlstrom and Fuerst (2001), I show

[56] If that happened, agency costs would not arise any more.

that the two different strategies lead to different business cycle dynamics in the response to monetary policy shock. The difference from their work is that I make the comparison in the NK framework, while they do so in a flexible price framework.

Focusing on the output dynamics, the dynamics with Euler equation entrepreneurs are characterised by the hump shaped reaction. This type of reaction, although widely observed in the VAR studies, is normally not reproduced in the standard NK model. With OLG entrepreneurs, however, the defining feature of the dynamics is its high persistence. Indeed, for a given shock, the real effect persists much longer than the standard case. I show that the different reactions of net worth to the same monetary shock between the two strategies are the key factor to grasp the different output dynamics.

Suppose that there is an expansionary monetary shock in Period 0. First, note that net worth, which is mainly composed of their capital holding, is practically predetermined. Euler equation entrepreneurs increase savings at the end of Period 0 so that their net worth greatly increases in Period 1. Doing this reduces agency costs and the external finance premium in the subsequent periods, which enables them to consume more and enjoy higher utility in the long run. Because of the large decrease in the external finance premium in Period 1, entrepreneurs produce more in Period 1 than in Period 0. This is what is behind the hump shaped reaction of output with Euler equation entrepreneurs.

Meanwhile, in the case of OLG entrepreneurs, with whom a constant fraction of aggregate profits is consumed and saved, the large increase in net worth in Period 1 is absent. In fact, net worth changes only gradually over time, which in turn causes a gradual change in the external finance premium. With small change in the premium each period, entrepreneurs' production also changes only gradually. This results in the greater persistence in output dynamics than in the standard NK case. Indeed, the way the credit channel works within the NK framework depends on the assumption one makes on entrepreneurs' consumption/saving decisions.

This chapter is organised as follows. Section 2 presents the model with endogenous net worth. Section 3 conducts simulation analysis to compare the impulse responses to monetary policy shock between the two strategies. Section 4 concludes.

## 4.2 The model

There is a continuum of entrepreneurs with measure $1 - b$, who own production technology to produce wholesale goods. In addition to their initial net worth, they seek external finance to maximise the profits from their production. They are risk neutral. Meanwhile, households with a continuum of measure $b$, have funds to spare. However, they do not have an access to the production technology. What happens in equilibrium is that entrepreneurs borrow funds from households through banks.

### 4.2.1 Credit frictions

Unlike the previous chapters, entrepreneurs live long.[57] Since they are subject to idiosyncratic shocks each period, they are not homogenous. In fact, there is a distribution of net worth across entrepreneurs at a given period. For each different level of net worth, there are a large number of entrepreneurs. In principle, entrepreneurs with different levels of net worth face different interest rates and different production choices.

Entrepreneurs produce wholesale goods using labour as well as capital. The labour input is the composite of labour provided by households and entrepreneurs. Homogeneous capital is provided by both households and entrepreneurs. In nominal terms, entrepreneurs with an initial net worth of $N_t$ require the loan of $A_t^N - N_t$, where $A_t^N$ is the whole finance required to undertake the production. As seen below, both entrepreneurs and households consume final goods whose price is $P_t$. Then, in real terms (relative to final goods), they seek for the loan of

$$a_t^n - n_t,$$

[57]As seen below, Euler equation entrepreneurs live indefinitely. Meanwhile, OLG entrepreneurs die some point. However, as long as the death probability each period is not unity, some of them are bound to live for multiple periods. In fact, the expected periods of survival depends on the probability of death.

where $a_t^n$ is the whole real finance required by entrepreneurs with real net worth of $n_t$.

I assume that all the input costs have to be paid before the production takes place.[58] Then, $a_t^n$ is expressed as:

$$a_t^n = w_t H_t^n + w_t^e H_t^{en} + r_t K_t^n. \tag{139}$$

$w_t$, $w_t^e$, and $r_t$ are the household and entrepreneurial real wages and real capital rental rate. $H_t^n$ and $H_t^{en}$ are the household and entrepreneurial labour employed by an entrepreneur with real net worth of $n_t$ and $K_t^n$ is the capital rented by the entrepreneur.[59]

To explain the nature of frictions, I first introduce the entrepreneur's production function:

$$Y_t^{Wn} = \omega_t F(H_t^n, H_t^{en}, K_t^n), \tag{140}$$

where $Y_t^{Wn}$ is the production of goods by entrepreneur with net worth of $n_t$. Specifically, the technology exhibits constant returns to scale:

$$F(H_t^n, H_t^{en}, K_t^n) = \left( (H_t^n)^{\Omega} (H_t^{en})^{1-\Omega} \right)^{\alpha} (K_t^n)^{1-\alpha}. \tag{141}$$

In Eq.140, $\omega_t$ is an iid random variable with an expected value of unity ($E(\omega_t) = 1$), which represents an idiosyncratic shock to the production. Since the distribution of the random variable is common to all the entrepreneurs with different net worth, there is no superscript $n$.

Credit frictions are modelled following the costly state verification (CSV) framework. Since Chapter 2 provided the detailed discussion, I here discuss it briefly. The realised value of the idiosyncratic shock, $\omega_t$ is the private information of the entrepreneur who actually undertakes the production. In order for banks (or any

[58] Carlstrom and Fuerst (1998, 2001) also assume this.

[59] An entrepreneur obtains entrepreneurial labour from a competitive market. Meanwhile, he himself provides labour to the market and obtains the wage. Having this source of income (entrepreneurial wage) prevents their net worth from becoming zero even after he defaults. Aggregation can be done easily when every entrepreneur has positive net worth, however small it is.

other agents) to observe the actual amount of wholesale goods produced, they need to incur monitoring costs. Without monitoring, entrepreneurs have an incentive to underreport the production outcome. This moral hazard problem is the source of agency costs of the model. The cost of monitoring an entrepreneur with net worth of $n_t$ is a proportion of his expected amount of production, i.e. $\mu F(H_t^n, H_t^{en}, K_t^n)$. I assume that the monitoring costs parameter, $\mu$ is time-invariant and common across entrepreneurs with different levels of net worth.

### 4.2.2 Contracting problem

As mentioned in Chapter 2, under the CSV framework, the form of optimal contract is derived as a standard debt contract. Briefly speaking, if the production of wholesale goods happens to exceed the predetermined amount of repayment, an entrepreneur pays this predetermined amount and keeps the rest. However, if it turns out to be less, he defaults and his bank pays monitoring cost and takes all the remaining goods.

The amount of repayment is expressed as $\Psi_t^n \left(a_t^n - n_t\right)$, where $\Psi_t^n$ is the gross interest rate. Denoting the nominal price of wholesale goods as $P_t^W$, the real price of wholesale goods is $\varphi_t \left(= \frac{P_t^W}{P_t}\right)$. Now, notice that there is a cut-off value of $\omega_t$, $\overline{\omega_t}^n$ such that

$$\Psi_t^n \left(a_t^n - n_t\right) = \overline{\omega_t}^n \varphi_t F(H_t^n, H_t^{en}, K_t^n). \tag{142}$$

If $\omega_t$ turns out to be below the cut-off value, he can not repay the debt.

The contract between entrepreneurs and banks is made before the idiosyncratic shock is realised. For an entrepreneur with initial net worth of $n_t$, the contract determines $\overline{\omega_t}^n$ as well as the size of the project, i.e., the amount of each input used, $H_t^n$, $H_t^{en}$ and $K_t^n$. In the contract, entrepreneurs aim to maximise the expected profits. Meanwhile, perfectly competitive banks simply recoup the amount they have lent. They manage to break even by lending to a large number of entrepreneurs. Banks do not incur any cost for their operation. Lending takes place within a period.[60]

[60] Notice that entrepreneurs simply rent capital. Indeed, capital is treated in a similar way to labour as an input.

Then, the profit maximisation problem of an entrepreneur with net worth of $n_t$ is expressed as:

*maximise*

$$\varphi_t f\left(\overline{\omega_t}^n\right) F(H_t^n, H_t^{en}, K_t^n), \tag{143}$$

*with respect to* $\overline{\omega_t}^n$, $H_t^n$, $H_t^{en}$ and $K_t^n$ *subject to*

$$\varphi_t g\left(\overline{\omega_t}^n\right) F(H_t^n, H_t^{en}, K_t^n) = a_t^n - n_t. \tag{144}$$

In the expression for the entrepreneur's expected real profit, $f\left(\overline{\omega_t}^n\right)$ is their expected share of the real revenue from wholesale goods production. Under the standard debt contract, the share is expressed as:

$$f\left(\overline{\omega_t}^n\right) = \int_{\overline{\omega_t}^n}^{\infty} \omega d\Phi\left(\omega\right) - \left(1 - \Phi\left(\overline{\omega_t}^n\right)\right)\overline{\omega_t}^n, \tag{145}$$

where $\Phi$ stands for a cumulative distribution function of $\omega$. In the banks' participation constraint (Eq.144), $g\left(\overline{\omega_t}^n\right)$ is the banks' expected share of the revenue. The share is expressed as

$$g\left(\overline{\omega_t}^n\right) = \int_0^{\overline{\omega_t}^n} \omega d\Phi\left(\omega\right) + \left(1 - \Phi\left(\overline{\omega_t}^n\right)\right)\overline{\omega_t}^n - \mu\Phi\left(\overline{\omega_t}^n\right). \tag{146}$$

Adding the shares, $f\left(\overline{\omega_t}^n\right)$ and $g\left(\overline{\omega_t}^n\right)$, I obtain:

$$f\left(\overline{\omega_t}^n\right) + g\left(\overline{\omega_t}^n\right) = 1 - \mu\Phi\left(\overline{\omega_t}^n\right). \tag{147}$$

As in the previous chapters, there is deadweight loss. On average, the fraction, $\mu\Phi\left(\overline{\omega_t}^n\right)$ of the wholesale goods production is lost in the bankruptcy process. This represents the agency costs.

**Solving the problem** Solving the contracting problem, I first derive the implicit demand functions for each input:

$$w_t = \frac{\varphi_t}{s_t^n} F_H(H_t^n, H_t^{en}, K_t^n), \tag{148}$$

$$w_t^e = \frac{\varphi_t}{s_t^n} F_{H^e}(H_t^n, H_t^{en}, K_t^n), \tag{149}$$

and

$$r_t = \frac{\varphi_t}{s_t^n} F_K(H_t^n, H_t^{en}, K_t^n). \tag{150}$$

In those equations,

$$s_t^n = \frac{1}{1 - \mu\Phi\left(\overline{\omega_t}^n\right) + \frac{\mu\phi(\overline{\omega_t}^n) f(\overline{\omega_t}^n)}{f'(\overline{\omega_t}^n)}}, \tag{151}$$

where $\phi$ represents the probability density function of $\omega$.

Since the production function exhibits constant returns to scale, Euler's theorem indicates that

$$F(H_t^n, H_t^{en}, K_t^n) = H_t F_H(H_t^n, H_t^{en}, K_t^n) + H_t^{en} F_{H^e}(H_t^n, H_t^{en}, K_t^n) + K_t^n F_K(H_t^n, H_t^{en}, K_t^n). \tag{152}$$

I obtain from the implicit demand functions (Eqs.148 to 150) and Eq.152 that

$$s_t^n a_t^n = \varphi_t F(H_t^n, H_t^{en}, K_t^n). \tag{153}$$

Given that $a_t^n$ and $\varphi_t F(H_t^n, H_t^{en}, K_t^n)$ are total costs and revenue, $s_t^n$ can be interpreted as a mark up over the production costs. Notice from Eq.151 that $s_t^n$ is unity when $\mu = 0$ while greater than unity when $\mu > 0$[61]. This mark up is imposed to cover the agency costs in the presence of monitoring costs.

Notice from Eqs.148 to 150 that the marginal rate of technical substitution (MRTS) between any of two inputs is the same regardless of the initial net worth.[62] Since the production function exhibits constant returns to scale, the MRTS is con-

[61] This is because $f'\left(\overline{\omega}\right) \left(= -\left(1 - \Phi\left(\overline{\omega}\right)\right)\right) < 0$.

[62] For example, if it is between household and entrepreneurial labour, Eqs.148 and 149 tell us $\frac{F_H(H_t^n, H_t^{en}, K_t^n)}{F_{H^e}(H_t^n, H_t^{en}, K_t^n)} = \frac{w_t}{w_t^e}$.

stant along the rays from the origin. Then, the ratio of the inputs required by entrepreneurs is the same regardless of the level of net worth. Thus, the marginal product of each input is also the same. This realisation, combined with any of the implicit demand functions, indicates that $s_t^n$ is a common value for any entrepreneur with different net worth, i.e. $s_t^n = s_t$. Given that the distribution of $\omega$ is common, I know from Eq.151 that the cutoff value of $\omega$ is also a common value, i.e. $\overline{\omega_t}^n = \overline{\omega_t}$.[63]

Substituting Eq.153 into the banks' participation constraint (Eq.146) and incorporating $\overline{\omega_t}^n = \overline{\omega_t}$ and $s_t^n = s_t$, I obtain

$$g\left(\overline{\omega_t}\right) s_t = 1 - \frac{n_t}{a_t^n}. \tag{154}$$

It is thus clear that the ratio of net worth, $n_t$ to the whole finance required, $a_t^n$ is also common across entrepreneurs with different net worth.

Further, combining Eqs.153 and 154, I obtain the expression for common expected return on the entrepreneur's net worth (internal funds), $\zeta_t$[64]:

$$\varphi_t f\left(\overline{\omega_t}\right) F\left(H_t^n, H_t^{en}, K_t^n\right) = \zeta_t n_t, \tag{155}$$

where

$$\zeta_t = \frac{s_t f\left(\overline{\omega_t}\right)}{1 - s_t g\left(\overline{\omega_t}\right)}. \tag{156}$$

**External finance premium** From Eq.142, the gross interest rate, $\Psi_t^n$ is expressed as:

$$\Psi_t^n = \frac{1}{a_t^n - n_t} \overline{\omega_t} \varphi_t F(H_t^n, H_t^{en}, K_t^n).$$

Using Eqs.153 and 154, this is simplified as: $\Psi_t^n = \frac{\overline{\omega_t}}{g(\overline{\omega_t})}$. Given that this is a common value, I express it as:

$$\Psi_t\left(\overline{\omega_t}\right) = \frac{\overline{\omega_t}}{g\left(\overline{\omega_t}\right)}. \tag{157}$$

[63] Unlike the previous chapters, the cut-off value and mark-up are time-varying.

[64] Given that entrepreneurs could always receive zero return on the internal fund within a period, $\zeta_t$ has to be larger than unity. This is confirmed later in a quantitative analysis.

Regardless of the level of net worth, entrepreneurs face the same loan interest rate.[65] Intuitively, this is because entrepreneurs' contribution in their projects, $\frac{n_t}{a_t^n}$ is the same regardless of their net worth (see Eq.154)

As in Faia and Monacelli (2007), I define the external finance premium as the gross interest rate minus the safe gross rate of return. Given that the lending takes place within a period, the safe rate of return is unity. Thus, the external finance premium, $\psi_t$ $(= \Psi_t - 1)$ is given as:

$$\psi\left(\overline{\omega_t}\right) = \frac{\overline{\omega_t}}{g\left(\overline{\omega_t}\right)} - 1. \tag{158}$$

### 4.2.3 Households

Having discussed the contracting problem, I now discuss how households (original lenders) and entrepreneurs (borrowers) behave in a dynamic economy.

The representative household gains utility from consumption of the final goods $C_t^h$[66]. They also benefit from holding real money balances, $\frac{M_t}{P_t}$ and enjoying leisure, $1 - H_t$, where $H_t$ is household labour supply. The utility function is separable:

$$\sum_{t=0}^{\infty} \beta^t \left( \frac{\left(C_t^h\right)^{1-\sigma}}{1-\sigma} + \frac{\left(\frac{M_t}{P_t}\right)^{1-\xi}}{1-\xi} - \left(H_t\right)^{\eta} \right), \tag{159}$$

where $\beta$ is the discount rate and $\eta \geq 1$.

The budget constraint in nominal terms is given as:

$$P_t C_t^h + M_t + B_t + P_t K_t^h = W_t H_t + R_t K_{t-1}^h + (1-\delta) P_t K_{t-1}^h + (1+i_{t-1}) B_{t-1} + M_{t-1} + \Pi_t + T_t, \tag{160}$$

where $M_t$ is nominal money, $B_t$ is bond holding[67], $R_t$ is a nominal rental rate of capital[68], $K_t^h$ is households' capital holding, $\delta$ is the depreciation rate, $i_{t-1}$ is a

[65] Even when $\mu = 0$, $\Psi_t$ is larger than unity. This reflects the fact that banks still need to be compensated for the fact that some of the entrepreneurs go bankrupt and fail to repay their debt.

[66] In the previous chapters, it is expressed as $C_t$.

[67] In equilibrium, the aggregate supply of bonds is zero.

[68] Thus, the real rental rate of capital $r_t$ is given by $\frac{R_t}{P_t}$.

nominal interest rate accrued in period $t$, $\Pi_t$ is profit share from retailers[69], and $T_t$ is a lump sum tax. The left hand side of the constraint shows the allocation of his total income in period $t$, which is composed of the expressions in the right hand side.

He has two ways to save: either investing in bonds or capital. I assume that the arbitrage condition ensures that the real returns from each investment are the same, i.e.

$$\frac{1+i_t}{1+\pi_{t+1}} = r_{t+1} + 1 - \delta, \tag{161}$$

where $\pi_{t+1}\left(= \frac{P_{t+1}-P_t}{P_t}\right)$ is the inflation rate and $r_{t+1}$ is the real rental rate of capital.

Unlike investment in bonds or capital, lending to banks takes place within a period. Then, since the opportunity costs of not lending is zero, the net interest rate households obtain from lending to banks is zero in equilibrium. This is the reason why the return from lending to the bank does not appear in the budget constraint.

The government budget constraint is $M_t - M_{t-1} = T_t$. In case $T_t > 0$, it means that the government makes a direct transfer of money to households, raising revenue through seigniorage. In case $T_t < 0$, $T_t$ is actually a tax collected by the government from households. The government spending is zero.

Solving this utility maximisation problem, I obtain the consumption Euler equation,

$$\left(C_t^h\right)^{-\sigma} = \beta \left(C_{t+1}^h\right)^{-\sigma} \frac{1+i_t}{1+\pi_{t+1}}. \tag{162}$$

I also have the money demand function,

$$\frac{M_t}{P_t} = \left(\frac{1+i_t}{i_t}\left(C_t^h\right)^{\sigma}\right)^{\frac{1}{\xi}}, \tag{163}$$

and the labour supply function

$$w_t = \eta \left(C_t^h\right)^{\sigma} H_t^{\eta-1}. \tag{164}$$

In deriving the money demand function, use is made of the consumption Euler

[69] Households are the shareholders of retailers.

equation.

### 4.2.4 Entrepreneurs

Previously in a discussion of the contracting problem, entrepreneurs' initial net worth was taken as given. I first show how it is actually determined. Then, for the case where entrepreneurs do not default, I discuss how they spend the profits after repayment of the debts. Specifically, I study their consumption/saving decisions about final goods. However, if they default, they consume/save nothing. The sequence of events happening to an individual entrepreneur $j$ is summarised in Table 8.

Table 8: Sequence of events happening to entrepreneur $j$ in a given period $t$

| |
|---|
| 1: Aggregate shock (monetary shock) is realised. |
| 2: Entrepreneur $j$ with initial net worth, $n_t$ pays for input costs, $a_t^n$. |
| (The difference is borrowed from the bank.) |
| 3: Using the inputs, he produces wholesale goods. |
| (The outcome of production is subject to an idiosyncratic shock.) |
| 4: If $\omega_t(j) \geq \overline{\omega}$, he repays his debt. If not, he goes bankrupt |
| and the bank pays monitoring costs and takes the remaining. |
| 5: In the former case, he makes a consumption/saving decision. |

**Determination of net worth** Entrepreneurs' net worth is composed of gross return from their capital holding and wage they earn by providing one unit of their labour inelastically. Formally, entrepreneur $j$'s net worth, $n_t$ is expressed as $(r_t + 1 - \delta)K_t^e(j) + w_t^e$, where $\delta$ is the depreciation rate, $K_t^e(j)$ is the capital holding by entrepreneur $j$, and $w_t^e$ is the entrepreneurial real wage. If their capital holding is zero, their net worth is simply $w_t^e$. Aggregating across all the entrepreneurs, I have

$$nw_t = (r_t + 1 - \delta)K_t^e + w_t^e, \tag{165}$$

where $nw_t$ and $K_t^e$ are aggregate net worth and entrepreneurial capital.

**Consumption/saving decision** If entrepreneur $j$ is solvent after the production and still keeps the real profit of $(\omega_t(j) - \overline{\omega_t})\varphi_t F(H_t^n, H_t^{en}, K_t^n)$[70], he then decides how much of the final goods to consume and how much to save for the next period. If saved, its gross return from the capital market forms a part of his period $t+1$ net worth.

Given that the production function of wholesale goods exhibits constant returns to scale, I obtain the following aggregate relation (cf. Eq.143):

$$\varphi_t f(\overline{\omega_t}) F(H_t, H_t^e, K_t) = C_t^e + K_{t+1}^e, \tag{166}$$

where $H_t$, $H_t^e$, and $K_t \left(= bK_t^h + (1-b)K_t^e\right)$ are aggregate household and entrepreneurial labour and aggregate capital. $C_t^e$ and $K_{t+1}^e$ are aggregate entrepreneurial consumption in period $t$ and capital in period $t+1$.

In what follows, I compare two alternative modelling strategies for entrepreneurs' consumption/saving decision. The strategies are useful to avoid the situation where entrepreneurs ultimately become self financed (by accumulating enough net worth) and agency costs become irrelevant. In the first strategy, they make the decision to maximise their intertemporal utility, but they have a higher discount rate than households. Since they make a decision so as to satisfy the Euler equation, I call them Euler equation entrepreneurs. In the second, a constant fraction of entrepreneurs die each period and they consume all the accumulated wealth just before their death. The population is kept constant by the birth of new entrepreneurs.[71] In this case, a constant share of aggregate profit, $\varphi_t f(\overline{\omega_t}) F(H_t, H_t^e, K_t)$ is consumed/saved in each period. I call them overlapping-generations (OLG) entrepreneurs. The first strategy is adopted, for example, by Carlstrom and Fuerst (1997) and the latter adopted by Bernanke, Gertler and Gilchrist (1999).[72] I show below that the effects of credit

[70] Using the external finance premium, the profit can also be expressed as $\omega_t(j)\varphi_t F(H_t^n, H_t^{en}, K_t^n) - (1+\psi_t)(a_t^n - n_t)$.

[71] The initial net worth of the newly born entrepreneurs is $w_t^e$.

[72] As noted in the introduction, the comparison of the strategies was made by Carlstrom and Fuerst (2001). The difference from their work is that I make the comparison in the NK framework, while they do so in a flexible price framework.

frictions on business cycles differ depending on which strategy is taken.

**Euler equation entrepreneurs** I specify Euler equation entrepreneur $j$'s utility function as:

$$E_t \sum_{t=0}^{\infty} (\gamma\beta)^t C_t^e(j). \tag{167}$$

where $C_t^e(j)$ is his consumption in period $t$. Under the assumption that entrepreneurs are risk neutral, the utility function is linear. Since entrepreneurs have the investment technology to produce wholesale goods, their expected return from the investment is greater than households' investment return. To eliminate the scenario in which entrepreneurs keep postponing their consumption to the future and accumulating large wealth, I assume that entrepreneurs discount utility at a higher rate than households, i.e. $0 < \gamma < 1$.

Saving one unit of final good and investing in the capital market in period $t$, he has $r_{t+1} + 1 - \delta$ unit of the good at the beginning of period $t+1$. Then, by investing this into the production of wholesale good, he has an expected gross return of $\zeta_{t+1} = \frac{s_{t+1} f(\overline{\omega_{t+1}})}{1 - s_{t+1} g(\overline{\omega_{t+1}})}$ (Eq.156) within period $t+1$. Then, his expected gross real return across the periods is $\zeta_{t+1}(r_{t+1} + 1 - \delta)$.[73]

Now, given that the utility function takes a linear form, I can tell that his consumption/saving decision is governed by the following Euler equation:

$$1 = \gamma\beta\zeta_{t+1}(r_{t+1} + 1 - \delta). \tag{168}$$

The left hand side is the utility he obtains from consuming one unit of final good in period $t$. The right hand side is the utility obtained in period $t+1$ from consuming the expected return from the saved one unit in period $t$, multiplied by the discount factor of $\gamma\beta$. When they are equal, he is indifferent between consumption and saving, which should be the case when he makes the optimal choice. Observe that this is also an aggregate relation.

[73] However, households' intertemporal return is just $r_{t+1} + 1 - \delta$, since their investment is more limited than entrepreneurs.

**Overlapping-generations (OLG) entrepreneurs** I denote the exogenous probability of death for any entrepreneur as $\Gamma$.[74] As noted, the population is held constant by the birth of new entrepreneurs. Given that OLG entrepreneurs consume all the accumulated wealth just before death, aggregate entrepreneurial consumption, $C_t^e$ is simply given as:

$$C_t^e = \Gamma \varphi_t f\left(\overline{\omega_t}\right) F\left(H_t, H_t^e, K_t\right). \tag{169}$$

$\Gamma$ also represents a constant share of aggregate entrepreneurial profit allocated on consumption each period.

### 4.2.5 Final goods producers

Households and entrepreneurs consume final goods. Entrepreneurs produce wholesale goods, which are, in turn, used by retailers as the input (Figure 6). Retailers are monopolistic competitors who maximise profits intertemporally subject to staggered pricesetting.[75] The final goods are a composite of all the retail goods.

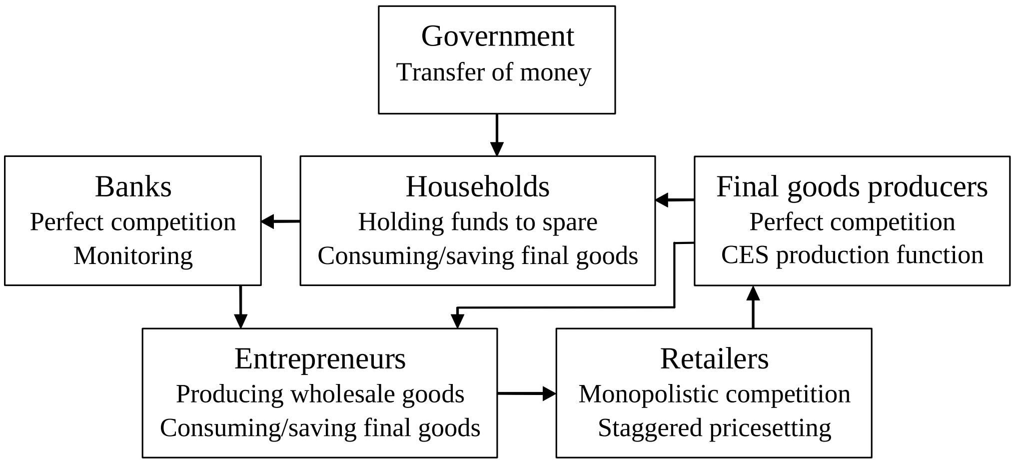


Figure 6: Overview of the model

Final goods producer's problem is identical to the one in Chapter 2. They are perfect competitors and their production function is of the CES (constant elasticity

[74] This implies that their expected survival periods are $\frac{1}{\Gamma}$ periods.

[75] As stated in Chapter 2, the reason why I model two different sets of intermediate goods producers is to separate the contracting problem from the intertemporal profit maximisation problem with staggered pricesetting.

of substitution) type:

$$Y_t = \left( \int_0^1 Y_t(z)^{\frac{\epsilon-1}{\epsilon}} dz \right)^{\frac{\epsilon}{\epsilon-1}}, \tag{170}$$

where $Y_t$ is the production of final goods and $Y_t(z)$ is the retail good produced by retailer $z$. Solving their cost minimisation problem, I obtain the following demand function for retail good $z$,

$$Y_t(z) = Y_t \left( \frac{P_t(z)}{P_t} \right)^{-\epsilon}, \tag{171}$$

where

$$P_t = \left( \int_0^1 P_t(z)^{1-\epsilon} dz \right)^{\frac{1}{1-\epsilon}}. \tag{172}$$

$P_t$ is the price of the final good. $\epsilon$ represents the elasticity of demand.

### 4.2.6 Retailers

Since retailers' problem is also the same as the one in Chapter 2, I describe it only briefly.

Retailers produce retail goods using wholesale goods as the only input. The production function exhibits constant returns to scale. Facing the downward sloping demand curve (Eq.171), retailers act as monopolistic competitors. Retailers are subject to Calvo-style staggered pricesetting. Thus, once they are given a chance to reset their prices in period $t$, retailer $z$ chooses the price, $P_t(z)$ to maximise

$$\sum_{i=0}^{\infty} \theta^i \Lambda_{t,t+i} \left( \frac{P_t(z)}{P_{t+i}} Y_{t+i}(z) - \varphi_{t+i} Y_{t+i}(z) \right), \tag{173}$$

where $\theta$ is the probability of not obtaining the opportunity to change the price, $\Lambda_{t,t+i}$ is the retailer's discount rate for time $t+i$[76], $Y_{t+i}(z)$ is the production by retailer $z$ at time $t+i$, and $\varphi_{t+i}$ is the real price of wholesale good at time $t+i$. Incorporating the demand function for retailer $z$ (Eq.171), differentiating the expression with respect

[76]The discount rate is defined as the inverse of the product of the gross real interest rates from period $t$ to $t+i-1$, i.e., $\frac{1}{(1+r_t)(1+r_{t+1})\cdots(1+r_{t+i-1})}$.

to $P_t(z)$ and setting it to zero, I obtain

$$Q_t = \frac{\epsilon}{\epsilon - 1} \frac{\sum_{i=0}^{\infty} \left( \theta^i \Lambda_{t,t+i} \varphi_{t+i} \left( \frac{P_{t+i}}{P_t} \right)^{\epsilon} Y_{t+i} \right)}{\sum_{i=0}^{\infty} \left( \theta^i \Lambda_{t,t+i} \left( \frac{P_{t+i}}{P_t} \right)^{\epsilon - 1} Y_{t+i} \right)}, \tag{174}$$

where $Q_t$ is the optimal real price chosen by the retailer, i.e., $Q_t = \frac{P^*_{calvo,t}}{P_t}$.

Under Calvo price setting, the price index is determined as an average of the price charged by retailers without an opportunity of adjusting their prices at time $t$ (proportion of $\theta$) and the reset price at time $t$ (proportion of $1 - \theta$). Given that price adjusters are randomly selected, the average price of non adjusters in period $t$ is the aggregate price in the previous period, $P_{t-1}$. Since the adjusters set the same nominal price $P^*_{calvo,t}$, the price index in period $t$ is given as:

$$P_t = \left( \theta P_{t-1}^{1-\epsilon} + (1-\theta) P_{calvo,t}^{*1-\epsilon} \right)^{\frac{1}{1-\epsilon}}.$$

Using inflation rate, this can be rewritten as:

$$1 = \left( \theta (1 + \pi_t)^{\epsilon - 1} + (1-\theta) Q_t^{1-\epsilon} \right)^{\frac{1}{1-\epsilon}}. \tag{175}$$

### 4.2.7 Equilibrium

I now collect the equilibrium conditions for the model. The solution of the contracting problem can be summarised as (c.f. Eq.155)

$$\varphi_t f(\overline{\omega_t}) Y_t^W = \zeta_t n w_t. \tag{176}$$

Given that each entrepreneur provides one unit of labour inelastically, the aggregate production of wholesale goods, $Y_t^W$ is given as:

$$Y_t^W = H_t^{\Omega\alpha} K_t^{1-\alpha}. \tag{177}$$

The aggregate return on the entrepreneur's net worth (internal funds), $\zeta_t$ is given as (Eq.156):

$$\zeta_t = \frac{s_t f\left(\overline{\omega_t}\right)}{1 - s_t g\left(\overline{\omega_t}\right)}, \tag{178}$$

where the mark up, $s_t$ is expressed as (Eq.151):

$$s_t = \frac{1}{1 - \mu\Phi\left(\overline{\omega_t}\right) + \frac{\mu\phi\left(\overline{\omega_t}\right) f\left(\overline{\omega_t}\right)}{f'\left(\overline{\omega_t}\right)}}. \tag{179}$$

Incorporating the demand functions for capital and entrepreneurial labour (Eqs.149 and 150) into Eq.165, aggregate net worth is given as:

$$nw_t = \left(\frac{\varphi_t}{s_t}\frac{Y_t^W}{K_t}\left(1-\alpha\right) + 1 - \delta\right) K_t^e + \frac{\varphi_t}{s_t} Y_t^W \left(1 - \Omega\right)\alpha. \tag{180}$$

Taking into account that retailers' production function shows constant returns to scale and that a fraction of the wholesale goods is lost through monitoring, I have the following relation[77]:

$$\left(1 - \mu\Phi\left(\overline{\omega_t}\right)\right) Y_t^W = Y_t, \tag{181}$$

where the left hand side is the net (after deducting the deadweight loss) aggregate production of wholesale goods and the right hand side is the aggregate production of final goods. With Eq.181, the market clearing condition for the final goods is given as:

$$\left(1 - \mu\Phi\left(\overline{\omega_t}\right)\right) Y_t^W = bC_t^h + (1-b)C_t^e + K_{t+1} - (1-\delta)K_t \tag{182}$$

Eqs.148 and 164 give the household labour market equilibrium condition:

$$\eta\left(C_t^h\right)^{\sigma} H_t^{\eta-1} = \frac{\varphi_t}{s_t}\frac{Y_t^W}{H_t}\Omega\alpha. \tag{183}$$

Households' consumption/saving relation is governed by the Euler equation (Eq.162):

$$\left(C_t^h\right)^{-\sigma} = \beta\left(C_{t+1}^h\right)^{-\sigma}\left(\frac{\varphi_{t+1}}{s_{t+1}}\frac{Y_{t+1}^W}{K_{t+1}}\left(1-\alpha\right) + 1 - \delta\right). \tag{184}$$

[77] Strictly speaking, this has to be proved and this is the case only as an approximation around the zero inflation steady state. The proof is found in Chapter 2.

Also, the arbitrage condition (Eq.161) ensures

$$\frac{1+i_t}{1+\pi_{t+1}} = \frac{\varphi_{t+1}}{s_{t+1}} \frac{Y_{t+1}^W}{K_{t+1}} (1-\alpha) + 1 - \delta, \tag{185}$$

Entrepreneurs' aggregate profit, $\varphi_t f(\overline{\omega_t}) Y_t^W \ (= \zeta_t nw_t)$ is either consumed or saved (Eq.166):

$$\varphi_t f(\overline{\omega_t}) Y_t^W = C_t^e + K_{t+1}^e. \tag{186}$$

In the case of Euler equation entrepreneurs, entrepreneurs' consumption/saving decision is governed by (Eq.168):

$$1 = \gamma\beta\zeta_{t+1} \left( \frac{\varphi_{t+1}}{s_{t+1}} \frac{Y_{t+1}^W}{K_{t+1}} (1-\alpha) + 1 - \delta \right). \tag{187}$$

Meanwhile, with OLG entrepreneurs, I have the following relation (Eq.169):

$$C_t^e = \Gamma \varphi_t f(\overline{\omega_t}) Y_t^W. \tag{188}$$

Finally, the retailers' pricing decision and the evolution of aggregate price are summarised by Eqs.174 and 175:

$$Q_t = \frac{\epsilon}{\epsilon - 1} \frac{\sum_{i=0}^{\infty} \left( \theta^i \Lambda_{t,t+i} \varphi_{t+i} \left( \frac{P_{t+i}}{P_t} \right)^{\epsilon} Y_{t+i} \right)}{\sum_{i=0}^{\infty} \left( \theta^i \Lambda_{t,t+i} \left( \frac{P_{t+i}}{P_t} \right)^{\epsilon-1} Y_{t+i} \right)}, \tag{189}$$

where $Q_t \left( = \frac{P_{calvo,t}^*}{P_t} \right)$ is the retailers' real reset price and

$$1 = \left( \theta (1+\pi_t)^{\epsilon-1} + (1-\theta) Q_t^{1-\epsilon} \right)^{\frac{1}{1-\epsilon}}, \tag{190}$$

where $\pi_t \left( = \frac{P_t - P_{t-1}}{P_{t-1}} \right)$ is the inflation rate of final goods price.

There are 15 variables to be determined: $\overline{\omega_t}$, $s_t$, $\zeta_t$, $nw_t$, $H_t$, $K_t$, $K_t^e$, $\varphi_t$, $Y_t^W$, $Y_t$, $C_t^h$, $C_t^e$, $Q_t$, $\pi_t$ and $i_t$. Meanwhile, there are 14 equations for each case with Euler equations entrepreneurs and OLG entrepreneurs. The model is then closed with an interest rate rule (introduced below).

## 4.3 Simulations

I compare the business cycle dynamics in response to monetary policy shocks for the cases with Euler equations and OLG entrepreneurs. To do this, I log linearise the model around the zero inflation steady state and conduct numerical simulations. The reason why I use simulation is because the model with endogenous agency costs[78] has multiple state variables (as clarified soon) and thus it is difficult to solve analytically. Indeed, the aim of simulations is to explore theoretical possibilities for both cases.

In what follows, I first discuss how to calibrate the model. Then, I explain the solution strategy to obtain the state space form for each case.

### 4.3.1 Calibration

The time unit is a quarter. I assume that the distribution function of idiosyncratic shock to entrepreneurs, $\omega_t$ is uniform in the region $[1-\rho, 1+\rho]$ so that $E(\omega_t) = 1$ and $Var(\omega_t) = \frac{1}{3}\rho^2$. Concerning monitoring costs parameter of $\mu$, I set $\mu = 0.20$ as in Chapter 2. Then, I calibrate the steady state cutoff value and the distribution parameter, $\rho$ such that the quarterly default ratio, $\Phi(\overline{\omega_t})$ is 0.974% and the quarterly external finance premium, $\psi(\overline{\omega_t})$ is 0.5%. The former value is taken from the series of works by Carlstrom and Fuerst (1997, 98 and 2001) and the latter from Faia and Monacelli (2007). With $\mu = 0.20$, I obtain $\rho = 0.609$ and $\overline{\omega} = 0.403$ (the steady state value of cut off value), which in turn implies the steady state mark up, $\overline{s}$ and the entrepreneurs' return on the net worth, $\overline{\zeta}$ of 1.112 and 1.199, respectively.

Euler equation entrepreneurs are more impatient than households. Combining the two Euler equations (Eqs.184 and 187), I have $\gamma = \frac{1}{\zeta}$ in the steady state. This enables me to obtain the patience parameter, $\gamma$ of 0.834.

All the baseline parameter values of the model are summarised in Table 9.

[78] Endogenous agency costs are represented by $\mu\Phi(\overline{\omega_t})$. Unlike the previous chapters, the cut-off value $\overline{\omega_t}$ is not time-invariant.

Table 9: Parameter values

| | |
|---|---|
| $\mu$: bankruptcy cost parameter | 0.20 |
| $\rho$: distribution function parameter | 0.61 |
| $\gamma$: patience parameter (for Euler equation entrepreneurs) | 0.83 |
| $\sigma$: preference parameter (1) | 1 |
| $\eta$: preference parameter (2) | 4.5 |
| $\alpha$: technology parameter (1) | 0.67 |
| $\Omega$: technology parameter (2) | 0.99 |
| $\epsilon$: elasticity of demand | 10 |
| $\beta$: discount factor | 0.99 |
| $b$: households' proportion | 0.7 |
| $\delta$: depreciation rate | 0.025 |
| $\theta$: stickiness parameter | 0.75 |
| $\Gamma$: constant consumption ratio (for ad hoc entrepreneurs) | 0.18 |

The value of household's preference parameter, $\sigma$ (its inverse is the intertemporal elasticity of substitution of consumption) is set to be unity. $\eta - 1$ is the elasticity of marginal disutility with respect to work. Following Ascari (2000), I set $\eta = 4.5$. The production function of wholesale goods is $\left((H_t)^{\Omega}(H_t^e)^{1-\Omega}\right)^{\alpha}(K_t)^{1-\alpha}$, where I set $\alpha = 0.67$ and $\Omega = 0.99$. The elasticity of demand for retail goods is 10, following Chari et al. (2000). Discount factor, $\beta$ is set 0.99. In the continuum of 1, the households' share $b$ is 0.7. The depreciation rate of $\delta = 0.025$ is from Faia and Monacelli (2007). Finally, following Jeanne (1997), the stickiness parameter in Calvo staggered setting, $\theta$ is set 0.75.

Having set the parameter values, I can obtain values of each variable for the Euler equation entrepreneurs in the zero inflation steady state. The only parameter value specific to the case with OLG entrepreneurs (instead of the patience parameter, $\gamma$) is the constant ratio of entrepreneurs' consumption, $\Gamma$. Assuming all the other parameter values are the same as in Table 9, this value can be tied down so that all the steady state values of variables in the OLG entrepreneurs' case are the same as the ones in the Euler equations entrepreneurs' case. The obtained value of $\Gamma$ is 0.184. Given that $\Gamma$ also represents the entrepreneurs' death ratio, it implies that on average, entrepreneurs survive for 5.4 periods.

### 4.3.2 Solution strategy

**Euler equation entrepreneurs** First, realise from Eqs.177 and 183 that the real price of wholesale goods, $\varphi_t$ is expressed as a function of $C_t^h$, $s_t$, $H_t$ and $K_t$. Then, noticing from Eqs.178 and 179 that the cut-off value of $\overline{\omega_t}$ and return on the internal funds, $\zeta_t$ are implicit functions of the mark up, $s_t$, Eqs.176 and 180 lead to aggregate household labour, $H_t$ as a function of $K_t$, $K_t^e$, $C_t^h$ and $s_t$. It is then clear that $Y_t^W$ and $\varphi_t$ are also a function of the four variables. Meanwhile, Eq.186 indicates that entrepreneurial consumption, $C_t^e$ is a function of $K_t$, $K_t^e$, $C_t^h$, $s_t$ and $K_{t+1}^e$.

Now, observe that the market clearing conditions of final goods and two consumption Euler equations for households and entrepreneurs can be expressed as a function of $K_t$, $K_t^e$, $C_t^h$ and $s_t$ and $K_{t+1}$, $K_{t+1}^e$, $C_{t+1}^h$ and $s_{t+1}$. I then log linearise these three equations around the zero inflation steady state to obtain three first order difference equations in $\widehat{K_t}$, $\widehat{K_t^e}$, $\widehat{C_t^h}$ and $\widehat{s_t}$. (In what follows, the term with upper hat indicates the log deviation from the steady state value.) In a matrix form, they are expressed as:

$$\begin{pmatrix} A_{11} & A_{12} & A_{13} & A_{14} \\ A_{21} & A_{22} & A_{23} & A_{24} \\ A_{31} & A_{32} & A_{33} & A_{34} \end{pmatrix} \begin{pmatrix} \widehat{K_{t+1}} \\ \widehat{K_{t+1}^e} \\ \widehat{C_{t+1}^h} \\ \widehat{s_{t+1}} \end{pmatrix} = \begin{pmatrix} B_{11} & B_{12} & B_{13} & B_{14} \\ B_{21} & B_{22} & B_{23} & B_{24} \\ B_{31} & B_{32} & B_{33} & B_{34} \end{pmatrix} \begin{pmatrix} \widehat{K_t} \\ \widehat{K_t^e} \\ \widehat{C_t^h} \\ \widehat{s_t} \end{pmatrix}, \tag{191}$$

where the coefficients can be obtained numerically with parameter values introduced above.

I assume monetary policy is represented by an interest rate rule. Specifically, I define the rule as a Taylor rule (1993):

$$\widehat{i_t} = \theta_1 \pi_t + \theta_2 \widehat{Y_t} + l_t, \tag{192}$$

where $\widehat{i_t}$ is the log deviation of gross nominal interest rate and $l_t$ is a shock term which follows an AR(1) process:

$$l_{t+1} = \theta_3 l_t. \tag{193}$$

Next, I log linearise the arbitrage relation (Eq.185) and combine it with the interest rate rule so that the interest rate, $\widehat{i_t}$ is eliminated. Given that $Y_t$ is also an function of $K_t$, $K_t^e$, $C_t^h$ and $s_t$, I can rewrite the combined relation as a linear difference equation in $\widehat{K_t}$, $\widehat{K_t^e}$, $\widehat{C_t^h}$ and $\widehat{s_t}$ as well as $\pi_t$ and $l_t$. Defining $\widehat{Y_t} = a_1 \widehat{K_t} + a_2 \widehat{K_t^e} + a_3 \widehat{C_t^h} + a_4 \widehat{s_t}$ and $\widehat{H_t} = b_1 \widehat{K_t} + b_2 \widehat{K_t^e} + b_3 \widehat{C_t^h} + b_4 \widehat{s_t}$, I obtain

$$\begin{aligned} & \chi(\eta b_1 - 1) \widehat{K_{t+1}} + \chi \eta b_2 \widehat{K_{t+1}^e} + \pi_{t+1} + \chi(\sigma + \eta b_3) \widehat{C_{t+1}^h} + \chi \eta b_4 \widehat{s_{t+1}} \\ = \; & \theta_2 a_1 \widehat{K_t} + \theta_2 a_2 \widehat{K_t^e} + l_t + \theta_1 \pi_t + \theta_2 a_3 \widehat{C_t^h} + \theta_2 a_4 \widehat{s_t}. \end{aligned} \tag{194}$$

Next, log linearising retailers' reset pricing equation and the equation linking the reset price and aggregate price (Eqs.189 and 190), I obtain the New Keynesian Phillips curve (NKPC):

$$\pi_t = \phi \widehat{\varphi}_t + \beta \pi_{t+1}, \tag{195}$$

where $\phi = \frac{1-\theta}{\theta}(1 - \theta\beta)$. Given that the retailers' real marginal cost can be expressed as a function of $K_t$, $K_t^e$, $C_t^h$ and $s_t$, I obtain yet another first order difference equation in $\widehat{K_t}$, $\widehat{K_t^e}$, $\widehat{C_t^h}$ and $\widehat{s_t}$ as well as $\pi_t$. Defining $\chi_1 = -\phi(\eta - \Omega\alpha)$, I have

$$\beta \pi_{t+1} = (\chi_1 b_1 - \phi(\alpha - 1)) \widehat{K_t} + \chi_1 b_2 \widehat{K_t^e} + \pi_t + (\chi_1 b_3 - \phi\sigma) \widehat{C_t^h} + (\chi_1 b_4 - \phi) \widehat{s_t}. \tag{196}$$

Combining Eqs.191, 193, 194, and 196, I have the following system of first order difference equations in a state space form:

$$\begin{pmatrix} A_{11} & A_{12} & 0 & 0 & A_{13} & A_{14} \\ A_{21} & A_{22} & 0 & 0 & A_{23} & A_{24} \\ A_{31} & A_{32} & 0 & 0 & A_{33} & A_{34} \\ \chi\left(\eta b_1 - 1\right) & \chi\eta b_2 & 0 & 1 & \chi\left(\sigma + \eta b_3\right) & \chi\eta b_4 \\ 0 & 0 & 0 & \beta & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \widehat{K_{t+1}} \\ \widehat{K_{t+1}^e} \\ l_{t+1} \\ \pi_{t+1} \\ \widehat{C_{t+1}^h} \\ \widehat{s_{t+1}} \end{pmatrix} \tag{197}$$

$$= \begin{pmatrix} B_{11} & B_{12} & 0 & 0 & B_{13} & B_{14} \\ B_{21} & B_{22} & 0 & 0 & B_{23} & B_{24} \\ B_{31} & B_{32} & 0 & 0 & B_{33} & B_{34} \\ \theta_2 a_1 & \theta_2 a_2 & 1 & \theta_1 & \theta_2 a_3 & \theta_2 a_4 \\ \chi_1 b_1 - \phi\left(\alpha - 1\right) & \chi_1 b_2 & 0 & 1 & \chi_1 b_3 - \phi\sigma & \chi_1 b_4 - \phi \\ 0 & 0 & \theta_3 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \widehat{K_t} \\ \widehat{K_t^e} \\ l_t \\ \pi_t \\ \widehat{C_t^h} \\ \widehat{s_t} \end{pmatrix}$$

More succinctly, it is described as $Ax_{t+1} = Bx_t$. In the vector $x_t$, $\widehat{K_t}$, $\widehat{K_t^e}$, and $l_t$ are predetermined variables while $\pi_t$, $\widehat{C_t^h}$, and $\widehat{s_t}$ are control variables. It is the existence of $\widehat{K_t^e}$ as a predetermined variable that enriches the dynamics compared to the standard NK model without credit frictions.

Because of the entrepreneurs' Euler equation, the matrix $B$ is not invertible. Thus, I can not use an eigenvalue-eigenvector decomposition proposed by Blanchard and Kahn (1980) to solve the model. Instead, I use generalised Schur decomposition as in Klein (2000) and Söderlind (1999).[79]

**OLG entrepreneurs** The only difference from the Euler equation entrepreneurs case is that the entrepreneurs' consumption/saving allocation is governed by Eq.188 instead of Eq.187. Thus, the solution strategy is similar to the one above. First, I again obtain $H_t$, $Y_t^W$, and $\varphi_t$ as a function of $C_t^h$, $s_t$, $K_t$, and $K_t^e$. Then, I realise from Eq.180 that $nw_t$ is also a function of those variables. Combining Eqs.186 and

[79] I used Matlab to conduct the decomposition.

188, I have the first intertemporal relation as a function of $C_t$, $s_t$, $K_t$, $K_t^e$ and $K_{t+1}^e$:

$$K_{t+1}^e = (1 - \Gamma)\, \zeta_t n w_t.$$

With the market clearing condition (Eq.182) and households' consumption Euler equation (Eq.184), I again obtain three first order difference equations in $\widehat{C_t^h}$, $\widehat{s_t}$, $\widehat{K_t}$ and $\widehat{K_t^e}$.

The rest of the process to obtain a state space form is the same as the previous case. That is, obtaining another three first order difference equations from the interest rate rule and arbitrage condition, NKPC and shock process, I again have the system of equations described succinctly as $Cx_{t+1} = Dx_t$. As in the previous case, the vector $x_t$ is composed of $\pi_t$, $\widehat{C_t^h}$, and $\widehat{s_t}$ as control variables and $\widehat{K_t}$, $\widehat{K_t^e}$, and $l_t$ as predetermined variables. Since $D$ is invertible this time, I use the eigenvalue-eigenvector decomposition to solve the model.[80][81]

### 4.3.3 Impulse responses to interest rate shocks

I now compare business cycle dynamics to interest rate shocks for Euler equation and OLG entrepreneurs. As mentioned, the monetary policy is modelled as a Taylor rule: $\widehat{i_t} = \theta_1 \pi_t + \theta_2 \widehat{Y_t} + l_t$ (Eq.192). Following Taylor (1993), I set $\theta_1 = 1.5$ and $\theta_2 = 0.5$. The shock term, $l_t$, follows the AR(1) process: $l_{t+1} = \theta_3 l_t$ (Eq.193), where I set $\theta_3 = 0.8$.

Before comparing the dynamics for each case, it is convenient first to consider the dynamics in the NK model without credit frictions as a reference point.

**The NK model without credit frictions (standard model)** In order to model credit frictions, I specifically introduced entrepreneurs (borrowers) and banks (financial intermediaries). Indeed, the model without these agents can be regarded as a standard NK model with capital stock. In the standard NK model I consider here,

[80] To conduct the eigenvalue-eigenvector decomposition, I used the Matlab code written by Ryo Kato. His published codes are found in http://ideas.repec.org/e/pka55.html.

[81] Instead, I can again use the generalised Schur decomposition as above. I obtain the same result.

those agents are absent and households provide labour and capital to retailers. Using these inputs, retailers, who are monopolistic competitors subject to Calvo-style staggered pricesetting, produce retail goods. Otherwise, the model structure is the same (see Figure 6). Since the solution strategy to solve this standard NK model is similar to the ones discussed above (in fact, simpler than those), the detail is given in the Appendix.

Figure 7 presents the impulse responses of output, consumption and investment in the face of an unexpected expansionary monetary shock in Period 0, represented by $l_0 = -0.3$.[82] Responding to this expansionary shock, output increases in Period 0 and then decreases gradually. The inflation rate also has the maximum impact in Period 0 then decreases (Figure 8). One seemingly counter-intuitive aspect is that this expansionary shock leads to an increase in the nominal interest rate (Figure 8). This happens because given that the nominal interest rate is determined endogenously by the Taylor rule, although the shock itself is expansionary, the nominal rate rises responding to positive reactions of output and inflation rate. This increase in nominal interest rate also corresponds to an increase in the real interest rate (not shown in the figure). This is what explains the initial fall in consumption through the consumption Euler equation. On the other hand, an investment increases. Finally, Figure 7 shows that this standard NK model fails to replicate the hump-shaped response of output, which is typically observed in the VAR analysis in the literature.

In the following, I see how business cycle dynamics to the same interest rate shock ($l_0 = -0.3$) are altered by the endogenous agency costs. The dynamics turn out to differ dependent on whether Euler equation or OLG entrepreneurs is assumed.

[82]Note that the steady state gross nominal interest rate is given as $\frac{1}{\beta}$ (cf. Eqs.184 and 185). Then, it represents a reduction in the nominal rate of about 0.3% in Period 0.

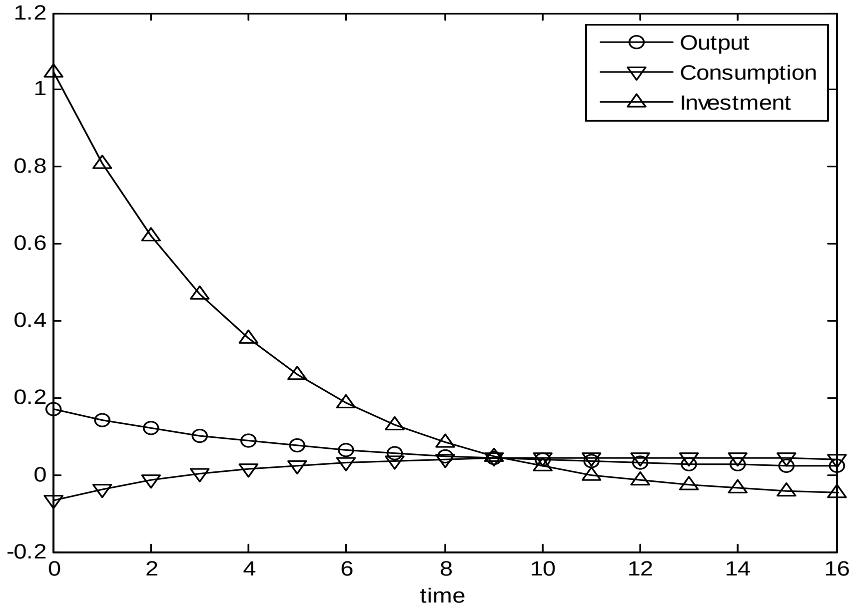


Figure 7: Standard NK model, dynamics 1

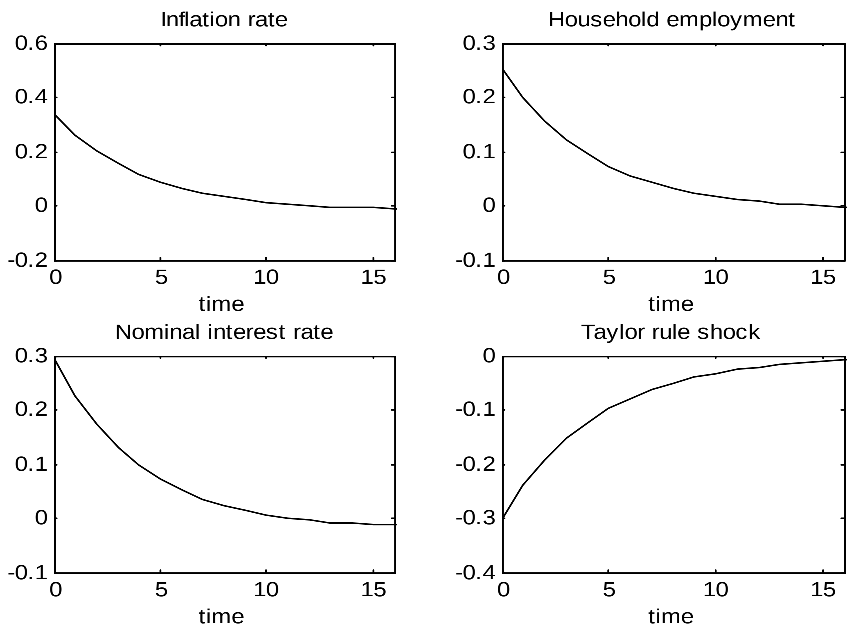


Figure 8: Standard NK model, dynamics 2

**Euler equation entrepreneurs** An obvious advantage of incorporating credit market frictions into a standard NK framework is that I can now study the behaviour of some additional variables in the business cycle dynamics, such as net worth, default ratio, and external finance premium. What is more, I show that the behaviour of these variables are important to understand how other variables, such as output, investment and consumption evolve over time.

Figure 9 shows that in Period 0 when the expansionary interest rate shock hits, the change in entrepreneurs' net worth is rather small. This is because the main component of net worth is their capital saved from the previous period (Eq.180). Though the rental rate of capital and entrepreneurial wage go up, the increase in net worth is still small. Meanwhile, responding to an increase in demand, the whole finance required by entrepreneurs increases greatly. Thus, what happens is that entrepreneurs' contribution in the whole finance becomes smaller.[83] Intuitively, this increases agency costs. In fact, it leads to an increase in the probability of default, $\Phi\left(\overline{\omega_t}\right)$ and deadweight loss, $\mu\Phi\left(\overline{\omega_t}\right)$ in Period 0 (both of which are increasing in the cutoff value, $\overline{\omega_t}$). Then, to offset the increase in agency costs, the mark up entrepreneurs charge over production costs, $s_t$ increases. Also, banks charge higher external finance premium (as reflected in the increase in the loan interest rate).

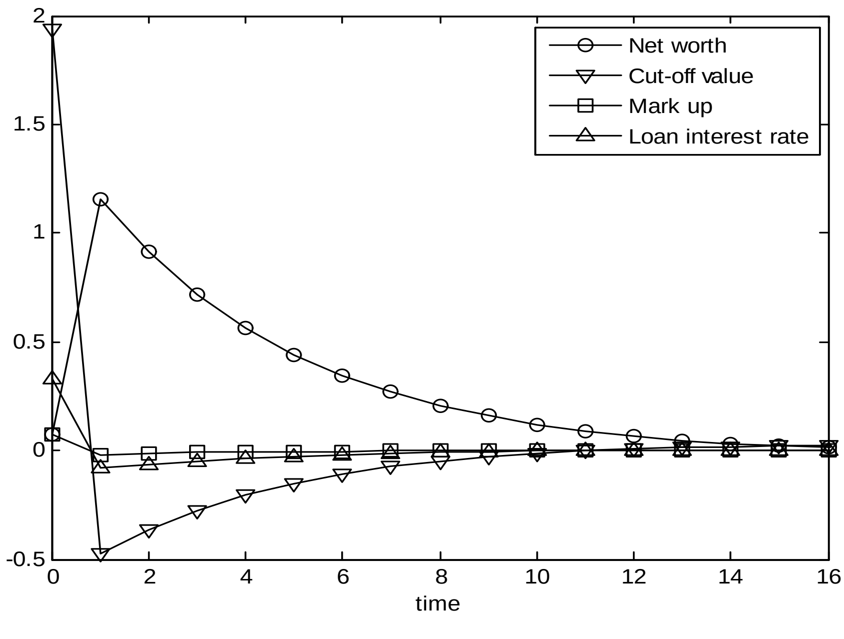


Figure 9: Euler equation entrepreneurs, dynamics 1

At the end of Period 0, entrepreneurs decide to save substantially by giving up their consumption (Figure 10)[84]. Compared to the standard NK case, that is what causes a large amplification in aggregate investment in Period 0. The reason why

[83]In other words, the leverage ratio rises.

[84]Entrepreneurial consumption in Period 0 falls to almost 4% below the steady state level.

entrepreneurs do this becomes clear when I look at what happens to the net worth and external finance premium in the subsequent periods. Reflecting the saving decision, net worth increases greatly in Period 1. By increasing the entrepreneurs' contribution in the whole finance and thus reducing the agency costs, it reduces the external finance premium greatly. In fact, the latter becomes negative. Over the subsequent periods, net worth decreases only gradually. Thus, the external finance premium is kept low. Indeed, by increasing the net worth and decreasing external finance premium, they can keep high level of consumption from Period 1 onwards. For the Euler equation entrepreneurs, this turns out to be the best way to maximise their intertemporal utility.

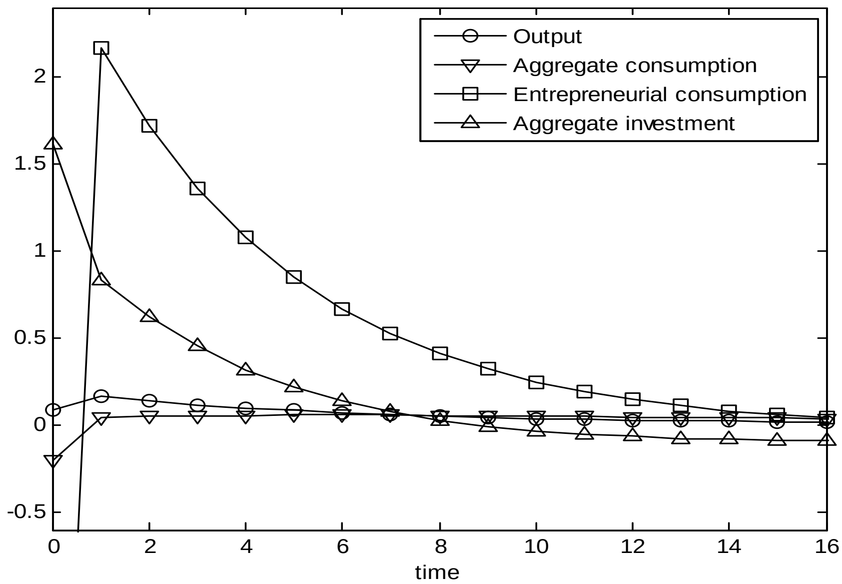


Figure 10: Euler equation entrepreneurs, dynamics 2

Focusing on the output dynamics, notice that output shows a hump shaped reaction to the interest rate shock. This, in fact, is a phenomenon widely confirmed in VAR literature (for example, Christiano, Eichenbaum and Evans (2005)). The mechanism behind this hump shape is again related to the behaviour of net worth. As discussed, given the almost predetermined nature of net worth, agency costs, which are mirrored in the deadweight loss, increase on impact. However, since net worth jumps in Period 1, the external finance premium falls.[85] In turn, this induces entre-

[85]In fact, the premium reacts pro-cyclically in the impact period and then counter-cyclically

preneurs to produce more. Therefore, output increases. Indeed, $\widehat{Y}_0 = 0.092$ (output is higher than the steady state by 9.2% in Period 0) and $\widehat{Y}_1 = 0.171$, while in the standard NK case, $\widehat{Y}_0 = 0.170$ and $\widehat{Y}_1 = 0.142$.[86] The impact effect on output in the Euler equation entrepreneurs' case is dampened relative to the standard case.

The dynamics of other variables such as inflation rate and household employment do not show much difference compared to the standard NK case presented in Figure 8 (thus not shown here).

**OLG entrepreneurs** Given its almost predetermined nature, net worth again does not change much in Period 0. However, the evolution afterwards will differ greatly with OLG entrepreneurs. When a fixed proportion of entrepreneurs' profit is saved (and consumed), net worth increases only gradually (Figure 11). Indeed, OLG entrepreneurs do not increase investment greatly in Period 0 to take advantage of subsequent periods of low external finance premium. Therefore, a fall in the agency costs happens rather gradually. This is reflected in the sluggish fall in the external finance premium (see the loan interest rate). The premium falls below the steady state level only after more than a year (Period 5).

afterwards. Interestingly, the VAR study presented by Bernanke et al. (1999) also shows the same pattern of evolution of the premium in the face of expansionary monetary policy shock. (The premium in their paper is measured as the spread between prime lending rate and T bill rate.)

[86] Note that the steady state level of output is different with and without the frictions: the output with frictions turns out to be 21% lower than the one without.

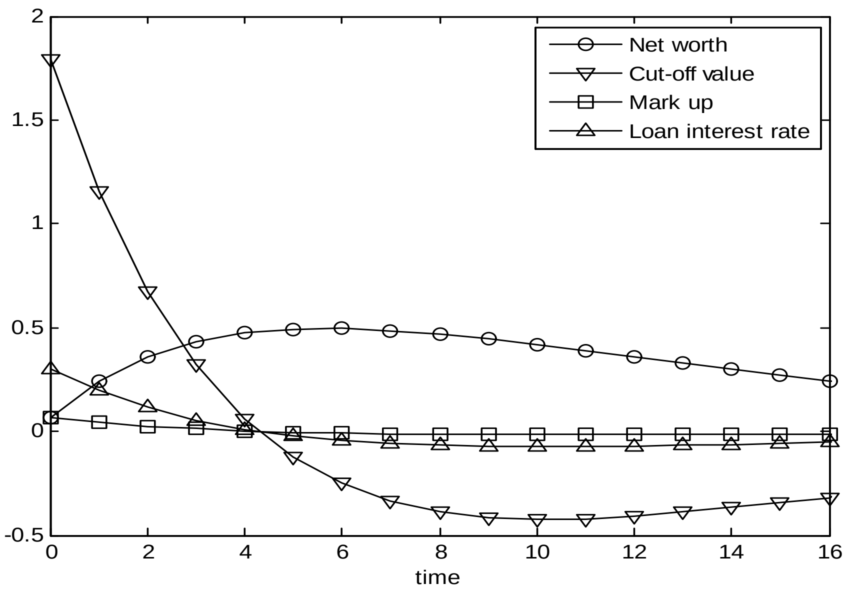


Figure 11: OLG entrepreneurs, dynamics 1

Without the large increase in investment by entrepreneurs in Period 0, the reaction of aggregate investment does not show amplification compared to the standard NK case (Figure 12). Also, with only a gradual decrease in the external finance premium following the shock, the hump shape reaction of output is not as clear as before.[87] More generally, the sluggish adjustment of net worth, implying the sluggish change in the external finance premium, results in the persistent movement of output. Even in Period 8, $\widehat{Y_8} = 0.074$ while $\widehat{Y_2} = 0.094$. In contrast, in the standard NK case, $\widehat{Y_8} = 0.050$ while $\widehat{Y_0} = 0.170$ (and $\widehat{Y_2} = 0.120$).[88] Indeed, output dynamics with OLG entrepreneurs are characterised by more persistent movement compared to the standard case.

As for the dynamics of other variables such as inflation rate, they again do not seem affected much.

[87]Actually, the peak is in Period 2 at 9.4% higher than the steady state (i.e. $\widehat{Y_2} = 0.094$).

[88]In case of Euler equation entrepreneurs, $\widehat{Y_1} = 0.171$ (the peak) while $\widehat{Y_8} = 0.062$.

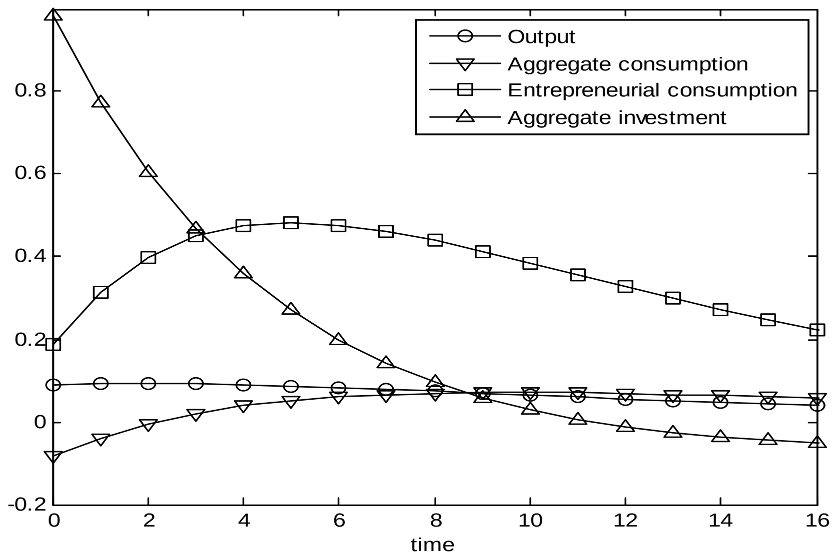


Figure 12: OLG entrepreneurs, dynamics 2

## 4.4 Conclusion

This chapter studied the difference in business cycle dynamics to monetary policy shocks between the two strategies; Euler equation entrepreneurs and OLG entrepreneurs. I showed that the dynamics are indeed different. Output dynamics with Euler equation entrepreneurs is characterised by the hump shaped reaction, while the dynamics with OLG entrepreneurs is by the highly persistent nature. The key factor behind this is the different behaviour of entrepreneurs' net worth. Therefore, the choice of modelling strategies matters when one considers the credit channels within the NK framework.

However, I argue that the output dynamics observed under the OLG entrepreneurs is more realistic. The reason is as follows. Throughout the chapter, entrepreneurs are assumed to be risk neutral in order to simplify the contracting problem with financial intermediaries. In fact, this assumption of risk neutral entrepreneurs is quite common in the literature (for example, Bernanke et al. (1999) and Carlstrom and Fuerst (1997, 98, 2001) among others). Assuming risk-averse entrepreneurs complicates the contracting problem greatly. Then, under the Euler equation entrepreneurs, what is implied is the lack of consumption smoothing. In relation to this, the entrepreneurs' consumption pattern showed rather extreme volatility. On the other hand, with the OLG entrepreneurs, aggregate entrepreneurial consumption/saving evolves rather smoothly after the shock. Thus, to the extent that entrepreneurs have a consumption smoothing motive, I tend to argue that the assumption of OLG entrepreneurs leads to more realistic dynamics. Thus, I conclude that the defining effect of endogenous agency costs in the NK model is to make real effects more persistent. However, in relation to the empirical evidence from the VAR analysis, since the real impact is not amplified due to the increase in the external finance premium, the endogenous agency costs are not necessarily a modification towards greater realism.

## 4.5 Appendix to Chapter Four: Standard New Keynesian model

In the standard NK case, entrepreneurs and banks are absent. Households provide labour and capital to retailers. Retailer $i$'s production function is $Y_t(i) = H_t(i)^{\alpha} K_t(i)^{1-\alpha}$. Using Lagrangian, his cost minimisation problem is described as: $L_t = w_t H_t(i) + r_t K_t(i) + \varphi_t (Y_t(i) - H_t(i)^{\alpha} K_t(i)^{1-\alpha})$. Given that the production function exhibits constant returns to scale, $\varphi_t$ is the real marginal cost as well as real average cost. Differentiating this with respect to $H_t(i)$ and $K_t(i)$ and setting them to be zero, I have the optimisation conditions: $w_t = \varphi_t \alpha \left(\frac{Y_t(i)}{H_t(i)}\right)$ and $r_t = \varphi_t (1-\alpha) \left(\frac{Y_t(i)}{K_t(i)}\right)$.

Since each retailer uses each input in the same proportion, the individual production function can be aggregated as: $\int_0^1 Y_t(i)\, di = H_t^{\alpha} K_t^{1-\alpha}$, where $H_t$ and $K_t$ are aggregate labour and capital. Final good producers produce final goods following the CES production function: $Y_t = \left(\int_0^1 Y_t(i)^{\frac{\epsilon-1}{\epsilon}} di\right)^{\frac{\epsilon}{\epsilon-1}}$. Then, around the zero inflation steady state where $Y_t(i) = Y_t$, the aggregate production function can be expressed as

$$Y_t = H_t^{\alpha} K_t^{1-\alpha}. \tag{198}$$

Likewise, I can express the above mentioned optimisation conditions as

$$w_t = \varphi_t \alpha \left( \frac{Y_t}{H_t} \right)$$

and

$$r_t = \varphi_t (1 - \alpha) \left( \frac{Y_t}{K_t} \right).$$

The market clearing condition for the final goods is:

$$Y_t = C_t + K_{t+1} - (1 - \delta) K_t, \tag{199}$$

where $C_t$ is an aggregate consumption (i.e. household consumption). Solving the same households' utility maximisation problem as in the main text, I have the labour supply relation: $w_t = \eta C_t^{\sigma} H_t^{\eta - 1}$ (cf. Eq.164). With the labour demand function, the labour market equilibrium condition is obtained as:

$$\eta C_t^{\sigma} H_t^{\eta - 1} = \varphi_t \alpha \left( \frac{Y_t}{H_t} \right). \tag{200}$$

Incorporating the demand function for capital, the consumption Euler equation (cf. Eq.184) becomes

$$(C_t)^{-\sigma} = \beta (C_{t+1})^{-\sigma} \left( \varphi_{t+1} (1 - \alpha) \left( \frac{Y_{t+1}}{K_{t+1}} \right) + 1 - \delta \right). \tag{201}$$

Again, since households have a choice of saving either in bonds or capital, the arbitrage condition (cf. Eq.185) is given as:

$$\frac{1 + i_t}{1 + \pi_{t+1}} = \varphi_{t+1} (1 - \alpha) \left( \frac{Y_{t+1}}{K_{t+1}} \right) + 1 - \delta. \tag{202}$$

Retailers maximise profits subject to the Calvo-style staggered pricesetting. As before, their reset real price, $Q_t \left(= \frac{P^*_{calvo,t}}{P_t}\right)$ is given as (cf. Eq.189):

$$Q_t = \frac{\epsilon}{\epsilon - 1} \frac{\sum_{i=0}^{\infty} \left(\theta^i \Lambda_{t,t+i} \varphi_{t+i} \left(\frac{P_{t+i}}{P_t}\right)^{\epsilon} Y_{t+i}\right)}{\sum_{i=0}^{\infty} \left(\theta^i \Lambda_{t,t+i} \left(\frac{P_{t+i}}{P_t}\right)^{\epsilon-1} Y_{t+i}\right)}. \tag{203}$$

Also, the evolution of price index is expressed as (cf. Eq.190):

$$1 = \left(\theta \left(1 + \pi_t\right)^{\epsilon-1} + \left(1 - \theta\right) Q_t^{1-\epsilon}\right)^{\frac{1}{1-\epsilon}}. \tag{204}$$

Thus, I have 8 endogenous variables, $Y_t$, $H_t$, $K_t$, $C_t$, $\varphi_t$, $i_t$, $Q_t$ and $\pi_t$ and 7 equations. The model then can be closed with the interest rate rule (Taylor rule). The solution strategy is similar to the ones in the main text. First I can obtain the steady state values in the zero inflation steady state. Log linearising around the state, Eqs.198,199,200 and 201 lead to two first order linear difference equations in $\widehat{C}_t$, $\widehat{H}_t$ and $\widehat{K}_t$. Next, log linearising the arbitrage condition (Eq.202) and equating it with the Taylor rule, $\widehat{i}_t = \theta_1 \pi_t + \theta_2 \widehat{Y}_t + l_t$ (cf. Eq.192), I have another first order difference equation in $\pi_t$, $\widehat{C}_t$, $\widehat{H}_t$, $\widehat{K}_t$ and $l_t$. I obtain two further difference equations in the form of New Keynesian Phillips curve (obtained from Eqs.203 and 204) and the AR(1) process of the shock term, $l_t$. Thus, I obtain 5 first order difference equations composed of control variables of $\pi_t$, $\widehat{C}_t$, and $\widehat{H}_t$ and predetermined variables of $\widehat{K}_t$ and $l_t$. Having obtained the state space form, the model can be solved using the eigenvalue-eigenvector decomposition as in Blanchard and Kahn (1980).

# 5 Chapter Five: The Effect of Credit Market Shocks on Business Cycle Dynamics

## 5.1 Introduction

Using a variant of DGE frameworks, a large number of models have discussed the role of exogenous shocks such as technology shocks or monetary shocks in business cycle fluctuations. However, since many of the DGE models do not take account of credit market imperfections, shocks happening to credit markets are often ignored. Acknowledging this, I study the potentially important role of credit market shocks as a source of business cycle dynamics.

As in the previous chapters, credit frictions are modelled following the Costly State Verification (CSV) framework. As an example of shocks to credit markets, I focus on the shock to the variance of idiosyncratic shock firms (entrepreneurs) are subject to. An increase in the variance, for example, aggravates the informational asymmetry so that agency costs increase. The increase in agency costs is then reflected in an increase in the external finance premium faced by entrepreneurs. In the sense that the shock has a direct effect on the contractual relationship between entrepreneurs and financial intermediaries, this is indeed a shock to credit markets. This chapter considers the effect of the credit market shock on business cycle dynamics.

First, I consider the effect of the credit market shock using the framework developed in Chapter 2. The framework is convenient to highlight the direct effect of the shock on business cycle dynamics. As noted above, the increase in the variance, implying the worsening of informational asymmetry, leads to an increase in the external finance premium. In turn, this has a recessionary effect by discouraging entrepreneurs' production.[89]

Then, using the model developed in Chapter 4, I demonstrate that the shock (the

[89] This counter-cyclical external finance premium appears to be supported by data. For example, Gomes, Yaron and Zhang (2003) report a negative correlation between total factor productivity and alternative measures of financing premium. Also, Levin, Natalucci and Zakrajsek (2004) show that credit spread on corporate borrowing is particularly low in the late 1990s in the US when the output growth is high.

increase in the variance) has an indirect effect through the endogenous development in entrepreneurs' net worth. What happens is that their net worth decreases as long as the credit market shock has a recessionary effect. Then, as emphasised in Chapter 4, the decrease in net worth leads to an increase in the external finance premium. Indeed, there is a feedback effect on the premium. In turn, this feedback effect decreases output further.

Furthermore, as Chapter 4 implies, the way net worth evolves in the face of the credit market shock turns out to depend on how to model entrepreneurs' consumption/saving decisions. Specifically, I again consider two alternative assumptions: Euler equation and OLG entrepreneurs. Euler equation entrepreneurs make the decision following their consumption Euler equation. Meanwhile, OLG entrepreneurs simply consume all the accumulated profits before dying. As mentioned, the latter arrangement implies that a constant fraction of their aggregate profits is consumed/saved in each period.

Suppose that the credit market shock persists for some periods, i.e. it takes a while for the increase in the variance to die down. In this case, the behaviour of net worth to the shock differs between the two assumptions as follows. Euler equation entrepreneurs choose to decrease their saving greatly soon after the shock happens so that their net worth drops rapidly. After the large fall, their net worth starts bouncing back immediately. However, with OLG entrepreneurs, their saving decreases only gradually so that net worth goes down slowly. It actually takes a while before the net worth starts increasing.

One important reason why the Euler equation entrepreneurs behave that way is better understood if I first discuss how external finance premium evolves with OLG entrepreneurs. With OLG entrepreneurs, while agency costs from aggravated informational asymmetry decrease over time as the shock calms down (as the variance falls), agency costs from gradually lowering net worth actually work to offset the decrease in the agency costs. As a result, the external finance premium decreases only quite slowly. In turn, this persistent premium has a prolonged negative effect on the entrepreneurial consumption. With Euler equation entrepreneurs, however, net worth goes down swiftly and starts bouncing back immediately. Then, although it leads to a higher external finance premium in the short run, it falls rather quickly. Therefore, the negative impact on their consumption dies down quickly. This is what prompts Euler equation entrepreneurs to decrease saving and thus their net worth quickly.

The different ways external finance premium evolves are reflected in the different output dynamics. With Euler equation entrepreneurs, the rapid increase in the premium and the subsequent fall leads to the hump shaped behaviour of output, i.e. decreases in the period after the shock and then starts increasing. Meanwhile, with OLG entrepreneurs, the external finance premium shows high persistence (i.e. takes time to die down). This, in turn, is reflected in persistently low output.

In relation to empirical evidence, Bloom, Floetotto and Jaimovich (2009) shows that a cross-sectional spread of firm- and industry-level growth rates is higher during recessions. For example, the spread of firm-level sales growth rates measured by the quarterly inter quartile range is 23.1% higher during recessions. The variance of idiosyncratic shocks firms are subject to appears volatile in business cycles. This gives the validity of considering the effect of the credit market shocks on business cycle dynamics.

The structure of the chapter is as follows. Based on the model in Chapter 2, Section 2 introduces the basic model to focus on the direct effect of the credit market shock. Then, based on the model in Chapter 4, Section 3 introduces the 'full' model in which the credit market shock has also an indirect effect through the endogenous movement of net worth. As mentioned, two different versions, i.e. Euler equation and OLG entrepreneurs are considered there. Section 4 concludes.

## 5.2 The basic model

### 5.2.1 The model

The basic model is based on the model presented in Chapter 2 (cf. Figure 1). Thus, I here only sketch the model without detailed explanation. The only important difference is that the variance of idiosyncratic shock to entrepreneurs' production is now time-varying. I assume that the distribution function of $\omega$ is uniform in the region $[1-\rho_t, 1+\rho_t]$ $(0 < \rho_t \leq 1)$ so that $E(\omega) = 1$ and $Var(\omega) = \frac{1}{3}\rho_t^2$. An increase in $\rho_t$, which corresponds to the increase in the variance, implies that the lending becomes riskier. Indeed, if $\rho_t = 0$, entrepreneurs' production outcome is public information and no informational asymmetry arises. I am interested in the role of an exogenous shock to $\rho_t$ (i.e., credit market shock) in business cycle dynamics.

Solving the entrepreneurs' optimisation problem as before, I obtain the time-variant cut-off value, $\overline{\omega_t}$:

$$\overline{\omega_t} = (1+\rho_t) - \mu + \varepsilon_t, \tag{205}$$

where

$$\varepsilon_t = \frac{1}{2}(\alpha\mu - \sqrt{\alpha^2\mu^2 + (1-\alpha)(4\mu^2 + 16\rho_t - 16\mu\rho_t)}). \tag{206}$$

Given that $\alpha$ and $\mu$ are still time-invariant, $\overline{\omega_t}$ is simply a function of $\rho_t$: $\overline{\omega_t} = \overline{\omega}(\rho_t)$.

The solution of the problem also leads to the implicit labour demand function:

$$w_t = \frac{1}{s(\overline{\omega_t})}\varphi_t \alpha H_t^{\alpha-1}, \tag{207}$$

where

$$s(\overline{\omega_t}) = \frac{1}{1 - \mu\Phi(\overline{\omega_t}) + \frac{\mu\phi(\overline{\omega_t})f(\overline{\omega_t})}{f'(\overline{\omega_t})}}. \tag{208}$$

In the previous equation, $\Phi(\overline{\omega_t})$ represents the default ratio: $\Phi(\overline{\omega_t}) = \frac{\overline{\omega_t}-(1-\rho_t)}{2\rho_t}$. The probability density function of $\omega_t$: $\phi(\overline{\omega_t})$ is given as $\frac{1}{2\rho_t}$, while the expected share of revenue from wholesale goods going to entrepreneurs, $f(\overline{\omega_t})$ as $\int_{\overline{\omega_t}}^{2\rho_t} \omega d\Phi(\omega) - (1-\Phi(\overline{\omega_t}))\overline{\omega_t}$. $s(\overline{\omega_t})$ can be again interpreted as a mark up entrepreneurs charge over their marginal costs to cover the deadweight loss, $\mu\Phi\left(\overline{\omega_t}\right)$.

The external finance premium, $\psi\left(\overline{\omega_t}\right)$ is defined by the cost of external finance (gross interest rate), $\Psi_t$ minus the safe rate of return, which is unity[90]:

$$\psi\left(\overline{\omega_t}\right) = \Psi\left(\overline{\omega_t}\right) - 1, \tag{209}$$

where

$$\Psi\left(\overline{\omega_t}\right) = \frac{\overline{\omega_t}}{g\left(\overline{\omega_t}\right)}. \tag{210}$$

In the expression of $\Psi\left(\overline{\omega_t}\right)$, $g\left(\overline{\omega_t}\right)$ is the expected share of the revenue going to banks: $g\left(\overline{\omega_t}\right) = \int_0^{\overline{\omega_t}} \omega d\Phi\left(\omega\right) + \left(1 - \Phi\left(\overline{\omega_t}\right)\right)\overline{\omega_t} - \mu\Phi\left(\overline{\omega_t}\right)$.

I now turn to the households' utility maximisation problem. They maximise $\sum_{t=0}^{\infty}\beta^t\left(\ln C_t^h + \frac{\left(\frac{M_t}{P_t}\right)^{1-\zeta}}{1-\zeta} - H_t^\eta\right)$ subject to $P_tC_t + M_t + B_t = W_tH_t + M_{t-1} + (1 + i_{t-1})B_{t-1} + \Pi_t + T_t$.[91] Solving this, I obtain the consumption Euler equation,

$$\left(C_t^h\right)^{-1} = \beta\left(1 + r_t\right)\left(C_{t+1}^h\right)^{-1}, ^{92} \tag{211}$$

the money demand function,

$$\frac{M_t}{P_t} = \left(\frac{1 + i_t}{i_t}C_t^h\right)^{\frac{1}{\zeta}}, \tag{212}$$

and the labour supply function

$$w_t = \eta C_t^h H_t^{\eta - 1}. \tag{213}$$

Given the opportunity to reset the price under Calvo-style staggering, retailers

[90] This is because lending takes place within a period.

[91] While the utility function is slightly simplified compared to the one in Chapter 2, the constraint is exactly the same.

[92] Due to the Fisher equation: $1 + r_t \equiv \left(1 + i_t\right)\frac{P_t}{P_{t+1}}$.

set the real price of $Q_t$:

$$Q_t = \frac{\epsilon}{\epsilon - 1} \frac{\sum_{i=0}^{\infty} \left( \theta^i \Lambda_{t,t+i} \varphi_{t+i} \left( \frac{P_{t+i}}{P_t} \right)^{\epsilon} Y_{t+i} \right)}{\sum_{i=0}^{\infty} \left( \theta^i \Lambda_{t,t+i} \left( \frac{P_{t+i}}{P_t} \right)^{\epsilon - 1} Y_{t+i} \right)}. \tag{214}$$

Inflation rate and the reset price are related as:

$$1 = \left( \theta \left(1 + \pi_t\right)^{\epsilon - 1} + (1 - \theta) Q_t^{1-\epsilon} \right)^{\frac{1}{1-\epsilon}}. \tag{215}$$

**Aggregate relations** I now obtain the aggregate relations. First, taking account of the deadweight loss incurred through the bankruptcy process, $\mu\Phi\left(\overline{\omega_t}\right)$, the net aggregate output of wholesale goods, $Y^W_{t,net}$ is obtained as $Y^W_{t,net} = \left(1 - \mu\Phi\left(\overline{\omega_t}\right)\right) H_t^{\alpha}$.[93] Rewriting this, the aggregate household labour is expressed as

$$H_t = \left( \frac{Y^W_{t,net}}{1 - \mu\Phi\left(\overline{\omega_t}\right)} \right)^{\frac{1}{\alpha}}. \tag{216}$$

Given that $\overline{\omega_t}$ is a function of $\rho_t$, the household labour, $H_t$ is a function of $\rho_t$ and $Y^W_{t,net}$. Since entrepreneurs, who live only one period, spend all the profits on the consumption of final goods, the aggregate entrepreneurial consumption, $C_t^e$ is given as $C_t^e = \varphi_t f\left(\overline{\omega_t}\right) H_t^{\alpha}$.Using Eq.216, it is written as

$$C_t^e = \varphi_t \frac{f\left(\overline{\omega_t}\right)}{1 - \mu\Phi\left(\overline{\omega_t}\right)} Y^W_{t,net}. \tag{217}$$

With the market clearing condition for final goods, $Y_t = C_t^h + C_t^e$, the aggregate household consumption, $C_t^h$ is given as

$$C_t^h = Y_t - \varphi_t \frac{f\left(\overline{\omega_t}\right)}{1 - \mu\Phi\left(\overline{\omega_t}\right)} Y^W_{t,net}. \tag{218}$$

Finally, from the household labour market equilibrium condition (Eqs.207 and 213), the real marginal cost for retailers is obtained as: $\varphi_t = \frac{\eta}{\alpha} s\left(\overline{\omega_t}\right) C_t^h H_t^{\eta - \alpha}$. Substituting

[93] In Chapter 2, the net aggregate output of wholesale goods was denoted as $Y_t^W$.

Eqs.216 and 218 into this and rewriting, I have

$$\varphi_t = \frac{\frac{\eta}{\alpha} s\left(\overline{\omega_t}\right) \left(\frac{Y^W_{t,net}}{1-\mu\Phi(\overline{\omega_t})}\right)^{\frac{\eta-\alpha}{\alpha}} Y_t}{1 + \frac{\eta}{\alpha} s\left(\overline{\omega_t}\right) f\left(\overline{\omega_t}\right) \left(\frac{Y^W_{t,net}}{1-\mu\Phi(\overline{\omega_t})}\right)^{\frac{\eta}{\alpha}}}. \tag{219}$$

Notice that $\varphi_t$ is a function of $\rho_t$, $Y_t$ and $Y^W_{t,net}$. Subsequently, $C^h_t$ and $C^e_t$ are also a function of those variables.

### 5.2.2 Simulations

To consider the effect of credit market shocks on business cycle dynamics, I log linearise the model around the zero inflation steady state and conduct simulation analysis. Specifically, I look at the impulse responses to a shock to the variance parameter, $\rho_t$. As in Chapter 4, the aim of the simulation is to explore theoretical possibilities of the model.

**Steady state** In the zero inflation steady state, every retailer sets the same price every period[94]. Then, Eq.214 implies that the retailers' real marginal cost in the steady state, $\overline{\varphi}$ is given as

$$\overline{\varphi} = \frac{\epsilon - 1}{\epsilon}. \tag{220}$$

I assume that the variance parameter is constant in the steady state, i.e., $\rho_t = \overline{\rho}$. Subsequently, the cutoff value of idiosyncratic shock is also constant in the state, i.e., $\overline{\omega_t} = \overline{\omega}$. Further, the aggregate productions of wholesale goods and final goods are the same in the steady state: $Y_t = Y^W_{t,net} = \overline{Y}$.[95] Then, I can obtain $\overline{Y}$ from Eq.219:

$$\overline{Y} = \left(\frac{\overline{\varphi}\left(1 - \overline{\mu}\Phi\left(\overline{\omega}\right)\right)^{\frac{\eta}{\alpha}}}{s\left(\overline{\omega}\right)\frac{\eta}{\alpha}\left(1 - \overline{\varphi} f\left(\overline{\omega}\right) - \overline{\mu}\Phi\left(\overline{\omega}\right)\right)}\right)^{\frac{\alpha}{\eta}}. \tag{221}$$

[94] In this sense, zero inflation steady state is equivalent to flexible price steady state.
[95] This is proved in Chapter 2.

Making use of Eqs.216, 217 and 218, I have

$$\overline{H} = \left( \frac{\overline{Y}}{1 - \overline{\mu}\Phi\left(\overline{\omega}\right)} \right)^{\frac{1}{\alpha}}, \tag{222}$$

$$\overline{C^e} = \overline{\varphi}\frac{f\left(\overline{\omega}\right)}{1 - \overline{\mu}\Phi\left(\overline{\omega}\right)}\overline{Y}, \tag{223}$$

and

$$\overline{C^h} = \left(1 - \overline{\varphi}\frac{f\left(\overline{\omega}\right)}{1 - \overline{\mu}\Phi\left(\overline{\omega}\right)}\right)\overline{Y}. \tag{224}$$

**Parameter values** To conduct an impulse response analysis, I set parameter values as shown in Table 10.

Table 10: Parameter values in the Basic model

| | |
|---|---|
| $\eta$: preference parameter | 4.5 |
| $\alpha$: technology parameter | 0.99 |
| $\epsilon$: elasticity of demand | 10 |
| $\mu$: monitoring costs | 0.20 |
| $\beta$: discount factor | 0.99 |
| $\theta$: stickiness parameter | 0.75 |

The parameter values apart from the variance parameter are the same as the ones in Chapter 2. Again, the variance parameter is treated as an unobservable parameter. I tie down its steady state value, $\overline{\rho}$ so that an annual external finance premium is 0.02 (200 basis points). This is the steady state value followed by Faia and Monacelli (2007). Given that the time unit is a quarter, I solve the relation: $\psi\left(\overline{\omega}\right) = \frac{\overline{\omega}}{g(\overline{\omega})} - 1 = 0.005$. Given that $\overline{\omega}$ is a function of $\overline{\rho}$, I obtain $\overline{\rho} = 0.11$.

**Log linearisation** In this basic model, the cutoff value, $\overline{\omega_t}$ is a function of $\rho_t$, independent of macroeconomic variables. Thus, other contract related variables such as the default ratio $(dr_t)$, $\Phi\left(\overline{\omega_t}\right)$, mark up, $s\left(\overline{\omega_t}\right)$, and loan interest rate, $\Psi\left(\overline{\omega_t}\right)$ (thus external finance premium, $\psi\left(\overline{\omega_t}\right)$) are also functions of $\rho_t$. Letting a hat notation represent the log deviation from the steady state values, I have $\widehat{\omega_t} = \nu_1 \widehat{\rho_t}$, $\widehat{dr_t} = \nu_2 \widehat{\rho_t}$, $\widehat{s_t} = \nu_3 \widehat{\rho_t}$, and $\widehat{\Psi_t} = \nu_4 \widehat{\rho_t}$. Given the above parameter values, the coefficients, $\nu_1$ to $\nu_4$ can be calculated.

I now turn to macroeconomic relations. As for the aggregate demand side, log linearising the consumption Euler Equation (Eq.211) leads to:

$$\widehat{C_t^h} = \widehat{C_{t+1}^h} - \left(\widehat{i_t} - \pi_{t+1}\right)^{96}. \tag{225}$$

Aggregate supply relation is summarised by the New Keynesian Phillips curve (NKPC), which is obtained by combining log linearised versions of Eqs.214 and 215:

$$\pi_t = \phi \widehat{\varphi_t} + \beta \pi_{t+1}, \tag{226}$$

where $\phi = \frac{1-\theta}{\theta}(1-\theta\beta)$. Further, I assume that monetary policy is represented by the simple Taylor rule (1993):

$$\widehat{i_t} = \delta_1 \pi_t + \delta_2 \widehat{Y_t}. \tag{227}$$

I saw above that $C_t^h$, $C_t^e$ and $\varphi_t$ are a function of $\rho_t$, $Y_t$ and $Y_{t,net}^W$. Then, in a log linearised form, $\widehat{C_t^h}$, $\widehat{C_t^e}$ and $\widehat{\varphi_t}$ are a function of $\widehat{\rho_t}$, $\widehat{Y_t}$ and $\widehat{Y_{t,net}^W}$. Further, recognising that $\widehat{Y_t} = \widehat{Y_{t,net}^W}$[97], they are simply functions of $\widehat{Y_t}$ and $\widehat{\rho_t}$. Likewise, employment, $\widehat{H_t}$ is a function of $\widehat{Y_t}$ and $\widehat{\rho_t}$. Then, I have

$$\widehat{C_t^h} = \gamma_{11} \widehat{Y_t} + \gamma_{12} \widehat{\rho_t}, \tag{228}$$

$$\widehat{C_t^e} = \gamma_{21} \widehat{Y_t} + \gamma_{22} \widehat{\rho_t}, \tag{229}$$

$$\widehat{\varphi_t} = \gamma_{31} \widehat{Y_t} + \gamma_{32} \widehat{\rho_t}, \tag{230}$$

and

$$\widehat{H_t} = \gamma_{41} \widehat{Y_t} + \gamma_{42} \widehat{\rho_t}. \tag{231}$$

[96] $\widehat{i_t}$ is the log deviation of gross nominal interest rate from the steady state value.
[97] This is proved in Chapter 2.

The coefficients are again calculated with the given parameter values.

**State space form** I express the dynamics of $Y_t$, $\pi_t$ and $\rho_t$ in a state space form. First, substituting the expression for $\widehat{\varphi}_t$ (Eq.230) into the NKPC (Eq.226), I obtain

$$\beta \pi_{t+1} = -\gamma_{31} \phi \widehat{Y}_t + \pi_t - \gamma_{32} \phi \widehat{\mu}_t. \tag{232}$$

Second, substituting the expression for $\widehat{C_t^h}$ (Eq.228) and the interest rate rule (Eq.227) into the consumption Euler equation (Eq.225), I have

$$\gamma_{11} \widehat{Y_{t+1}} + \pi_{t+1} + \gamma_{12} \widehat{\mu_{t+1}} = (\gamma_{11} + \delta_2) \widehat{Y}_t + \delta_1 \pi_t + \gamma_{12} \widehat{\mu}_t. \tag{233}$$

Third, I assume that shock process follows an autoregressive process:

$$\widehat{\rho_{t+1}} = \chi \widehat{\rho}_t + \varepsilon_t, \tag{234}$$

where $\varepsilon_t$ is an exogenous shock term. Overall, I have the three linear first order difference equations in the following state space form:

$$\begin{pmatrix} 0 & \beta & 0 \\ \gamma_{11} & 1 & \gamma_{12} \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \widehat{Y_{t+1}} \\ \pi_{t+1} \\ \widehat{\rho_{t+1}} \end{pmatrix} = \begin{pmatrix} -\phi\gamma_{31} & 1 & -\phi\gamma_{32} \\ \gamma_{11} + \delta_2 & \delta_1 & \gamma_{12} \\ 0 & 0 & \chi \end{pmatrix} \begin{pmatrix} \widehat{Y}_t \\ \pi_t \\ \widehat{\rho}_t \end{pmatrix}. \tag{235}$$

This system of equations can be solved using an eigenvalue-eigenvector decomposition (Blanchard and Kahn (1980)).[98]

**Impulse response to credit market shocks** I consider the role of the credit market shock in business cycle dynamics. Specifically, suppose that there is an exogenous shock to $\widehat{\rho}_t$ in Period 0: $\varepsilon_0 = 0.3$ and the AR(1) coefficient, $\chi$ is set to be 0.9 (Eq.234).

[98] As in Chapter 4, to conduct the eigenvalue-eigenvector decomposition, I used the Matlab code written by Ryo Kato.

Figure 13 looks at the effect on contract related variables, cut-off value, default ratio, mark up, and loan interest rate. The horizontal axes represent time. As pointed out, the contract related variables, being functions of only $\rho_t$, are independent of macro variables.

Intuitively, an increase in $\rho_t$ corresponds to the aggravation of informational asymmetry. This implies an increase in the agency costs, which is mirrored in an increase in default ratio (and the deadweight loss). In order to cover the loss, the mark up charged by entrepreneurs need to increase. At the same time, the loan interest rate goes up. Given the safe rate of (intratemporal) return is unity, the increase in the loan rate corresponds to an increase in the external finance premium.

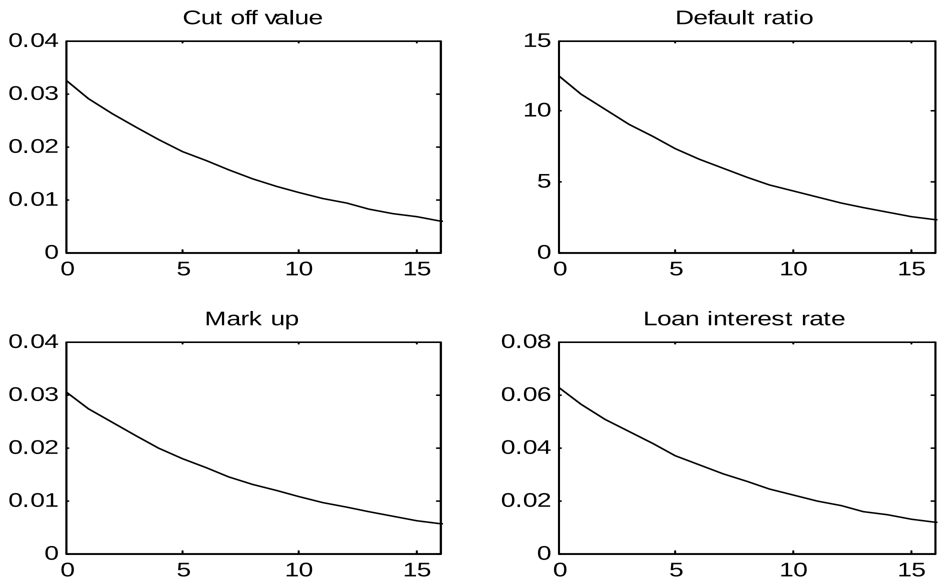


Figure 13: Basic model, dynamics 1

Figure 14 shows the effect on macro variables. Reflecting the increase in the external finance premium, entrepreneurs reduce production. Correspondingly, production of final goods also falls. Observe that the external finance premium moves in a counter-cyclical way. As for the component, $C_t^e$ decreases relatively more because the share going to entrepreneurs, $f(\overline{\omega_t})$, goes down when the cut-off value increases. The inflation rate rises because of the mark up charged to cover the agency costs: if the wholesale good become more expensive, it affects retailers' reset prices and thus aggregate price rises.

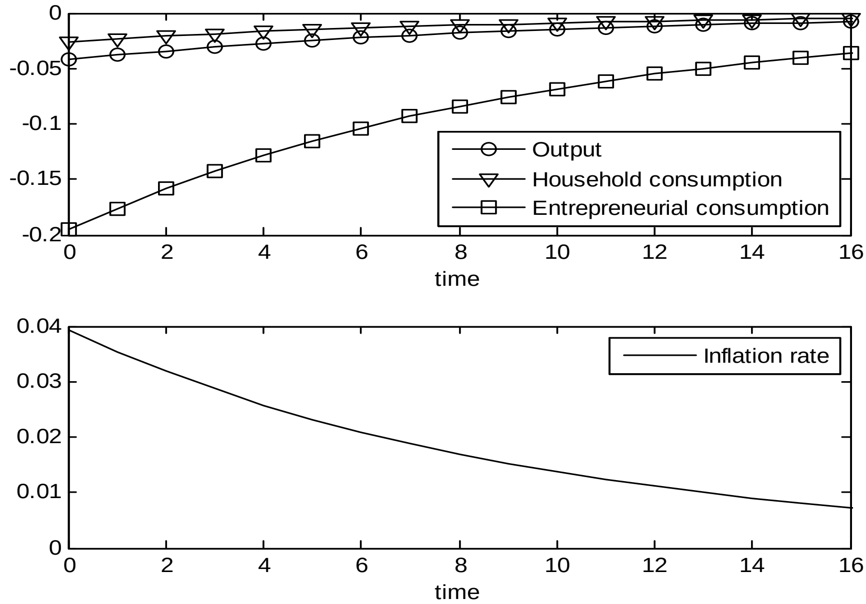


Figure 14: Basic model, dynamics 2

### 5.3 The full model: with endogenous net worth

In the basic model, all the contract related variables, such as the external finance premium, are independent of macroeconomic conditions. Thus, there is no feedback from macroeconomic conditions to the external finance premium. However, in a framework with endogenous entrepreneurs' net worth, the change in macroeconomic conditions does have a feedback effect on the premium through its effect on the net worth. Indeed, Chapter 4 shows that in a boom caused by an expansionary interest rate shock, entrepreneurs' net worth increases. The increase in net worth, in turn, works to decreases the agency costs so that the external finance premium falls.

Having acknowledged this feedback effect, I now study the role of credit market shocks in business cycles using a model with the endogenous net worth. To distinguish from the basic model presented previously, I call it the full model. As in Chapter 4, I consider two different strategies of modelling entrepreneurs' consumption/saving decisions; Euler equation entrepreneurs and overlapping generations (OLG) entrepreneurs. It turns out that the different strategies again lead to different evolution of net worth in response to credit market shocks. Expect that the variance of the idiosyncratic shock is now time-varying, the model presented below is directly based on the one in Chapter 4. Therefore, the detailed explanation is not repeated.

### 5.3.1 The model

First, the solution of the contracting problem between entrepreneurs and banks can be summarised as:

$$\varphi_t f\left(\overline{\omega_t}\right) Y_t^W = \zeta_t n w_t, \tag{236}$$

where $Y_t^W$ is the gross aggregate wholesale goods production (i.e., the output before subtracting the deadweight loss):

$$Y_t^W = H_t^{\Omega\alpha'} K_t^{1-\alpha'} {}^{99}. \tag{237}$$

The expected intratemporal return on the entrepreneur's net worth (internal funds), $\zeta_t$ is given as:

$$\zeta_t = \frac{s_t f\left(\overline{\omega_t}\right)}{1 - s_t g\left(\overline{\omega_t}\right)}, \tag{238}$$

where the mark up, $s_t$ is expressed as:

$$s_t = \frac{1}{1 - \mu\Phi\left(\overline{\omega_t}\right) + \frac{\mu\phi\left(\overline{\omega_t}\right) f\left(\overline{\omega_t}\right)}{f'\left(\overline{\omega_t}\right)}}. \tag{239}$$

The cut-off value, $\overline{\omega_t}$ is an implicit function of $\rho_t$ and $s_t$. The same applies to the return on internal funds, $\zeta_t$.

The aggregate net worth, composed of the gross return from capital holding and entrepreneurial wage, is given as:

$$nw_t = \left(\frac{\varphi_t}{s_t}\frac{Y_t^W}{K_t}\left(1-\alpha'\right) + 1 - \delta\right) K_t^e + \frac{\varphi_t}{s_t} Y_t^W \left(1-\Omega\right)\alpha'. \tag{240}$$

The net aggregate production of wholesale goods (the production after deducting

[99] Given that $\alpha$ was already used in the basic model, I use $\alpha'$ instead of $\alpha$.

the deadweight loss) is equal to the aggregate production of final goods[100]:

$$(1-\mu\Phi(\overline{\omega_t}))\, Y_t^W = Y_t. \tag{241}$$

With Eq.241, the market clearing condition for the final goods is expressed as:

$$(1-\mu\Phi(\overline{\omega_t}))\, Y_t^W = bC_t^h + (1-b)C_t^e + K_{t+1} - (1-\delta)K_t \tag{242}$$

I assume that households' utility function is the same as the one in the basic model[101]. Then, the household labour market equilibrium condition is:

$$\eta C_t^h H_t^{\eta-1} = \frac{\varphi_t}{s_t}\frac{Y_t^W}{H_t}\Omega\alpha'. \tag{243}$$

Households' consumption/saving relation is governed by the Euler equation:

$$\left(C_t^h\right)^{-1} = \beta\left(C_{t+1}^h\right)^{-1}\left(\frac{\varphi_{t+1}}{s_{t+1}}\frac{Y_{t+1}^W}{K_{t+1}}(1-\alpha') + 1 - \delta\right). \tag{244}$$

As for the households' saving decision, the arbitrage condition ensures

$$\frac{1+i_t}{1+\pi_{t+1}} = \frac{\varphi_{t+1}}{s_{t+1}}\frac{Y_{t+1}^W}{K_{t+1}}(1-\alpha') + 1 - \delta, \tag{245}$$

Entrepreneurs' aggregate profit, $\varphi_t f(\overline{\omega_t})\, Y_t^W$ $(=\zeta_t n w_t)$ is either consumed or saved:

$$\varphi_t f(\overline{\omega_t})\, Y_t^W = C_t^e + K_{t+1}^e. \tag{246}$$

Euler equation entrepreneur $j$ makes a consumption/saving decision to maximise his utility function: $E_t \sum_{t=0}^{\infty} (\gamma\beta)^t C_t^e(j)$. Then, the consumption Euler equation

[100] As pointed out in Chapter 4, this is actually an approximate relation which only holds around the zero inflation steady state.

[101] The households' utility function in the Chapter 4 is slightly more general in that the consumption component of the function takes the CRRA form: $\frac{(C_t^h)^{1-\sigma}}{1-\sigma}$. I here set $\sigma = 1$ from the outset.

indicates that the decision is governed by:

$$1 = \gamma\beta\zeta_{t+1}\left(\frac{\varphi_{t+1}}{s_{t+1}}\frac{Y^W_{t+1}}{K_{t+1}}(1-\alpha') + 1 - \delta\right). \tag{247}$$

In the case of OLG entrepreneurs, an aggregate entrepreneurial consumption is simply a constant fraction, $\Gamma$ of the aggregate profit:

$$C^e_t = \Gamma\varphi_t f(\overline{\omega_t})\, Y^W_t. \tag{248}$$

Finally, the retailers' pricing decision and the evolution of aggregate price are the same as the ones in the basic model (Eqs.214 and 215):

$$Q_t = \frac{\epsilon}{\epsilon - 1}\frac{\sum_{i=0}^{\infty}\left(\theta^i \Lambda_{t,t+i}\varphi_{t+i}\left(\frac{P_{t+i}}{P_t}\right)^{\epsilon} Y_{t+i}\right)}{\sum_{i=0}^{\infty}\left(\theta^i \Lambda_{t,t+i}\left(\frac{P_{t+i}}{P_t}\right)^{\epsilon-1} Y_{t+i}\right)}, \tag{249}$$

and

$$1 = \left(\theta(1+\pi_t)^{\epsilon-1} + (1-\theta)\, Q_t^{1-\epsilon}\right)^{\frac{1}{1-\epsilon}}. \tag{250}$$

### 5.3.2 Simulations

Again, the aim of the simulation is to explore theoretical possibilities of the model.

**Parameter values** The parameters which also appear in the Basic model have the same values here; that is, $\eta = 4.5$, $\epsilon = 10$, $\mu = 0.2$, $\beta = 0.99$ and $\theta = 0.75$. The other values specific to the full model are shown in Table 11.

Table 11: Parameter values specific to the full model

| | |
|---|---|
| $\alpha'$: technology parameter (1) | 0.67 |
| $\Omega$: technology parameter (2) | 0.99 |
| $b$: households' proportion | 0.7 |
| $\delta$: depreciation rate | 0.025 |

From Chapter 4, I know that the steady state value of $\rho$ is 0.61. Likewise, the patience parameter, $\gamma$ (specific to Euler equation entrepreneurs) is tied down as 0.83, while the constant consumption ratio, $\Gamma$ (specific to OLG entrepreneurs) as 0.18.

**Solution strategies** The solution strategies for both Euler equation and OLG entrepreneurs are quite similar to the ones presented in Chapter 4. The appendix provides the sketch for both cases.

**Impulse responses to credit market shocks** I now study the impulse responses to the same credit market shock as the basic model. That is, I assume that there is an exogenous shock to $\widehat{\rho_t}$ in Period 0: $\varepsilon_0 = 0.3$ and the shock persists following the AR(1) process; $\widehat{\rho_{t+1}} = 0.9\widehat{\rho_t} + \varepsilon_t$ (Eq.234).

**Euler equation entrepreneurs** First, in Period 0 when the shock hits, the agency costs (reflected in the deadweight loss) increase due to the worsening of informational asymmetry. That is mirrored in the positive mark up and loan interest rate (thus external finance premium) in the period (Figure 15). Meanwhile, net worth, mainly composed of the capital from the last period, barely changes. At the end of Period 0, however, entrepreneurs radically decrease investment as reflected in the large fall in aggregate investment (Figure 16).[102] Thus, in Period 1, the net worth falls greatly. This fall leads to a further increase in the deadweight loss in the period. Thus, there is an increase in the mark up and external finance premium.

[102] On the other hand, entrepreneurial consumption jumps in Period 0 (76% higher than the steady state).

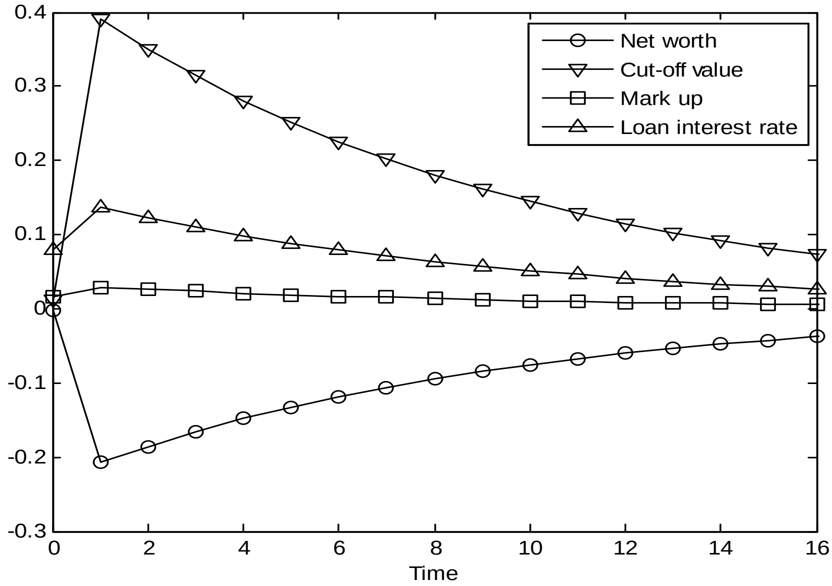


Figure 15: Euler equation entrepreneurs, dynamics 1

The rapid increase in the external finance premium in Period 1 corresponds to the decrease in output in the period; $\widehat{Y_1} = -0.047$ (the output is 4.7% lower than the steady state), while $\widehat{Y_0} = -0.032$ (Figure 16). After Period 1, net worth starts increasing and thus the premium starts falling. Correspondingly, output starts increasing. The hump shaped reaction in output is the notable feature with Euler equation entrepreneurs. The reason why Euler equation entrepreneurs decrease investment in Period 0 is partly because they benefit from the high consumption in Period 0. (Another important reason becomes clear once the behaviour of OLG entrepreneurs is studied.)

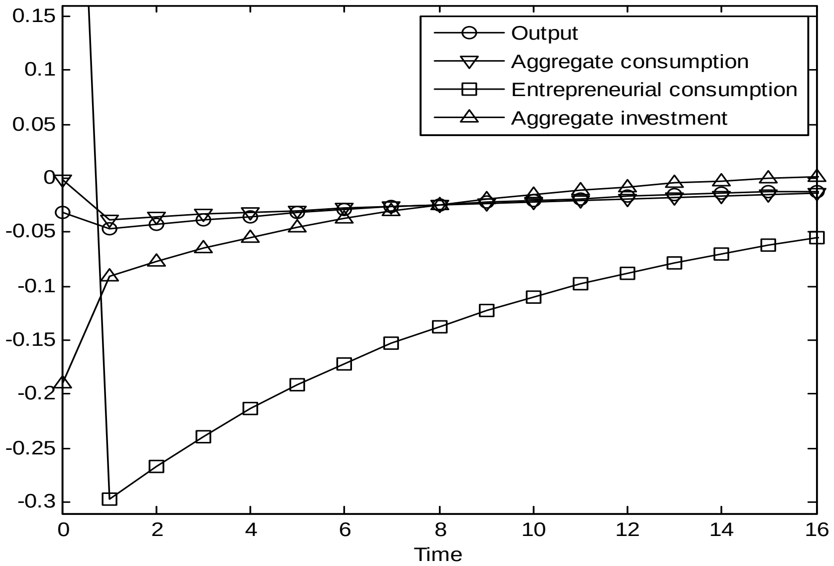


Figure 16: Euler equation entrepreneurs: dynamics 2

After Period 1, output and aggregate consumption starts increasing. The reason why aggregate consumption recovers more slowly than output is because of the high real interest rates (Figure 17). Due to the large drop in the investment at the end of Period 0, capital level tends to be low afterwards. That causes the subsequent high real interest rates. They, in turn, dampen household and entrepreneurial consumption. The inflation rate increases because of the mark up charged to cover the agency costs. Its hump shaped reaction reflects the low mark up in Period 0.

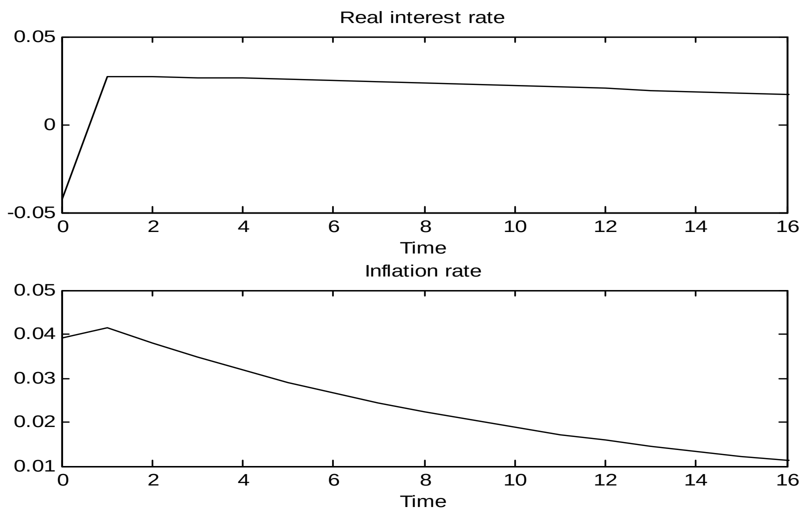


Figure 17: Euler equation entrepreneurs, dynamics 3

**OLG entrepreneurs** When the constant fraction of the aggregate profits is invested each period, entrepreneurs' net worth adjusts only gradually (Figure 18). While agency costs from the uncertainty shock lessens over time because of the AR(1) process of the shock, the decreasing net worth work to slow down the decrease of the agency costs. Further, even after the net worth reaches the bottom after 2 years, its subsequent increase is quite slow. This explains why both mark up and external finance premium are highly persistent. Looking at the loan interest rate, it is 8.8% higher than the steady state in Period 0. However, it still stands 6.2% in two years and 3.6% even in four years. Meanwhile, with Euler equation entrepreneurs, although the loan interest rate is 13.6% higher in Period 1 at its peak, it falls to 6.4% in two years after the shock and 2.7% in four years. This highly persistent external finance premium in case of the constant saving ratio is the important reason why Euler equation entrepreneurs choose to decrease the net worth rapidly in Period 1.

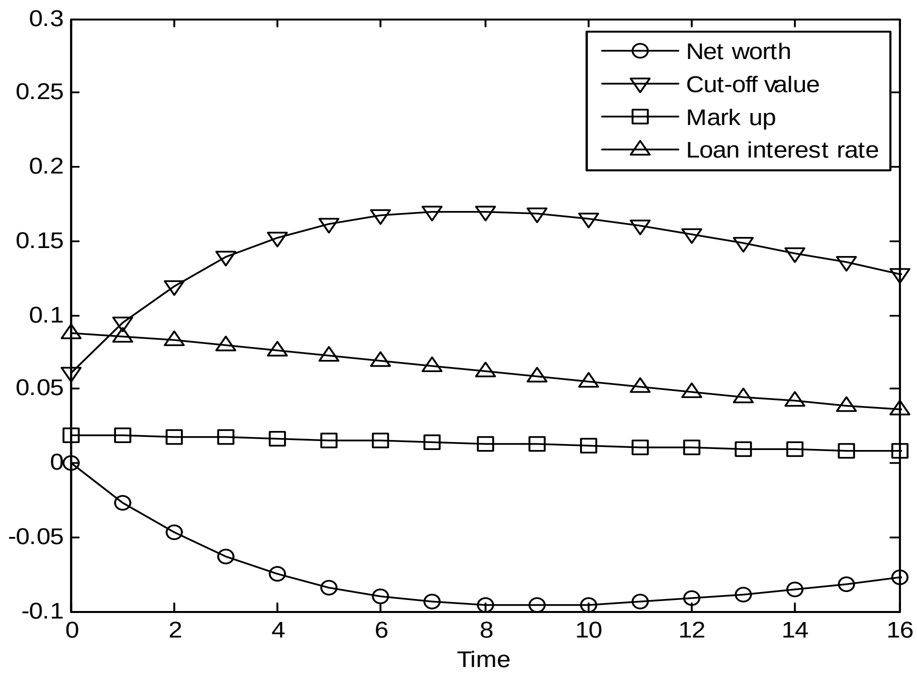


Figure 18: OLG entrepreneurs, dynamics 1

The persistent external finance premium leads to the slow recovery of output (Figure 19). While output is 3.1% lower than the steady state value in Period 0, it is still 2.4% lower in 2 years and 1.5% lower even in 4 years. With Euler equation entrepreneurs, output in Period 1 (bottom) is 4.7% lower than the steady state while 2.5% lower in 2 years after the shock and 1.2% lower in 4 years.[103] Indeed, output dynamics with OLG entrepreneurs are characterised by the highly persistent nature.

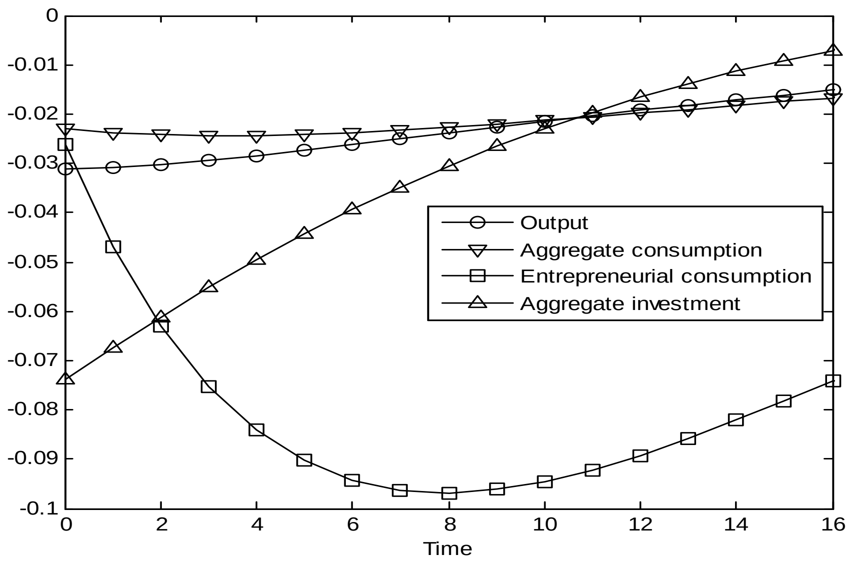


Figure 19: OLG entrepreneurs, dynamics 2

The slow adjustment of entrepreneurs' investment and thus aggregate capital contribute to the sluggish reaction in the real interest rate (Figure 20). It only starts decreasing after 4 years. This further slows the recovery of household consumption and thus aggregate consumption. Inflation rate is also quite persistent: it halves only after 4 years. This again reflects the slow adjustment of agency costs and the mark up.

[103] In the basic model, while the output is 4.2% lower than the steady state in Period 0, it becomes 1.8% lower after 2 years and 0.8% in 4 years.

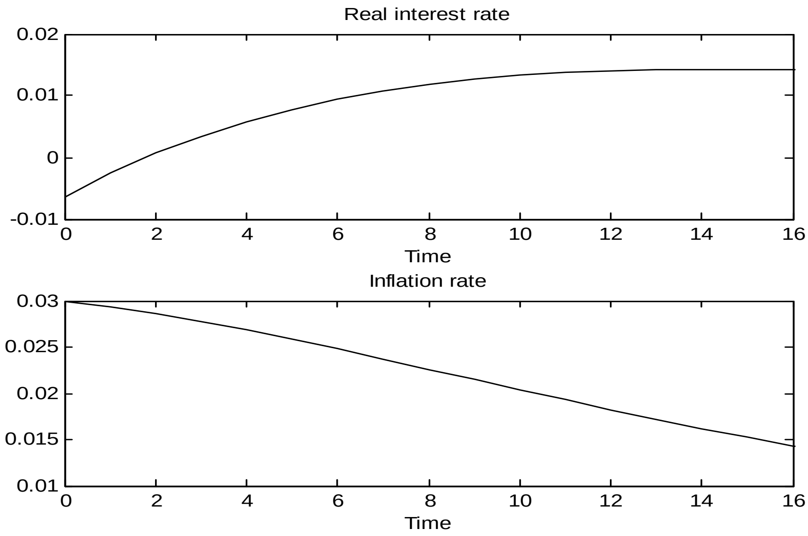


Figure 20: OLG entrepreneurs, dynamics 3

## 5.4 Conclusion

This chapter studied the role of credit market shocks in generating business cycle dynamics. The basic model (based on Chapter 2) demonstrates that an increase in the variance of idiosyncratic shocks entrepreneurs are subject to leads to an increase in the external finance premium. This, in turn, decreases output. Then, the full model (based on Chapter 4) highlights that there is an important feedback effect from macroeconomic conditions to the external finance premium through endogenous net worth. The way net worth evolves depends on whether Euler equation entrepreneurs or OLG entrepreneurs are assumed. Therefore, output dynamics also differ dependent on the assumption.

However, again for the reason specified in the previous chapter (entrepreneurs' consumption shows rather extreme volatility when risk neutral entrepreneurs follow the consumption Euler equation), the dynamics under OLG entrepreneurs seem more plausible. With OLG entrepreneurs, in the face of an exogenous increase in the variance, the output dynamics is characterised by highly persistent movement. That is, recession persists for long time. Also, inflation stays high for long time. To sum up, an increase in uncertainty at micro level brings about prolonged stagflation through the endogenous developments in credit markets.

This result is related to the empirical evidence provided by Bloom, Floetotto and Jaimovich (2009). They show that a cross-sectional spread of firm- and industry-level growth rates is higher during recessions. For example, the spread of firm-level sales growth rates measured by the quarterly inter quartile range is 23.1% higher during recessions. The variance of idiosyncratic shocks firms are subject to appears to be countercyclical. The results of this chapter indicate that credit market frictions can be one of the main factors behind this evidence.

## 5.5 Appendix to Chapter Five: Solution strategy for the full model

### 5.5.1 Euler equation entrepreneurs

The solution strategy is quite similar to the one in Chapter 4. The only difference is that the cut-off value, $\overline{\omega_t}$ and return on internal funds, $\zeta_t$ are now implicit functions of the mark up, $s_t$ as well as the variance parameter of $\rho_t$ (Eqs.238 and 239), while they are the functions of only $s_t$ in Chapter 4.

First, the market clearing condition of final goods (Eq.242) and two consumption Euler equations for households and entrepreneurs (Eqs.244 and 247) can be expressed as a function of $K_t$, $K_t^e$, $\rho_t$, $C_t^h$ and $s_t$ and $K_{t+1}$, $K_{t+1}^e$, $\rho_{t+1}$, $C_{t+1}^h$ and $s_{t+1}$. Log linearising the relations, I obtain the following system of first order difference equations:

$$\begin{pmatrix} A_{11} & A_{12} & A_{13} & A_{14} & A_{15} \\ A_{21} & A_{22} & A_{23} & A_{24} & A_{25} \\ A_{31} & A_{32} & A_{33} & A_{34} & A_{35} \end{pmatrix} \begin{pmatrix} \widehat{K_{t+1}} \\ \widehat{K_{t+1}^e} \\ \widehat{\rho_{t+1}} \\ \widehat{C_{t+1}^h} \\ \widehat{s_{t+1}} \end{pmatrix} = \begin{pmatrix} B_{11} & B_{12} & B_{13} & B_{14} & B_{15} \\ B_{21} & B_{22} & B_{23} & B_{24} & B_{25} \\ B_{31} & B_{32} & B_{33} & B_{34} & B_{35} \end{pmatrix} \begin{pmatrix} \widehat{K_t} \\ \widehat{K_t^e} \\ \widehat{\rho_t} \\ \widehat{C_t^h} \\ \widehat{s_t} \end{pmatrix}.$$

As before, the hat notation indicates the log deviation from the zero inflation steady state.

The second step is again to combine the log linearised arbitrage condition (Eq.245) and the Taylor rule: $\widehat{i_t} = \theta_1 \pi_t + \theta_2 \widehat{Y_t}$ where $\theta_1 = 1.5$ while $\theta_2 = 0.5$. Given that both $Y_t$ and $H_t$ are functions of $K_t$, $K_t^e$, $\rho_t$, $C_t^h$ and $s_t$, I have $\widehat{Y_t} = a_1 \widehat{K_t} + a_2 \widehat{K_t^e} + a_3 \widehat{\rho_t} + a_4 \widehat{C_t^h} + a_5 \widehat{s_t}$ and $\widehat{H_t} = b_1 \widehat{K_t} + b_2 \widehat{K_t^e} + +b_3 \widehat{\rho_t} + b_4 \widehat{C_t^h} + b_5 \widehat{s_t}$. Incorporating these, the arbitrage condition and Taylor rule yields

$$\begin{aligned} & \chi (\eta b_1 - 1) \widehat{K_{t+1}} + \chi \eta b_2 \widehat{K_{t+1}^e} + \chi \eta b_3 \widehat{\rho_{t+1}} + \pi_{t+1} + \chi (1 + \eta b_4) \widehat{C_{t+1}^h} + \chi \eta b_5 \widehat{s_{t+1}} \\ = \; & \theta_2 a_1 \widehat{K_t} + \theta_2 a_2 \widehat{K_t^e} + \theta_2 a_3 \widehat{\rho_t} + \theta_1 \pi_t + \theta_2 a_4 \widehat{C_t^h} + \theta_2 a_5 \widehat{s_t}, \end{aligned}$$

where $\chi = 1 - \beta + \beta \delta$.

Third, with the New Keynesian Phillips curve: $\pi_t = \phi \widehat{\varphi_t} + \beta \pi_{t+1}$ where $\phi = \frac{1-\theta}{\theta} (1 - \theta \beta)$ (derived from Eqs.249 and 250), I have another first order difference equation:

$$\beta \pi_{t+1} = (\chi_1 b_1 - \phi (\alpha - 1)) \widehat{K_t} + \chi_1 b_2 \widehat{K_t^e} + \chi_1 b_3 \widehat{\rho_t} + \pi_t + (\chi_1 b_4 - \phi) \widehat{C_t^h} + (\chi_1 b_5 - \phi) \widehat{s_t},$$

where $\chi_1 = -\phi (\eta - \Omega \alpha)$.

Finally, I assume that the log linearised variance term follows an AR(1) process:

$$\widehat{\rho_{t+1}} = \theta_3 \widehat{\rho_t}.$$

Combining all the linear difference equations, I obtain the following state space form:

$$\begin{pmatrix} A_{11} & A_{12} & A_{13} & 0 & A_{14} & A_{15} \\ A_{21} & A_{22} & A_{23} & 0 & A_{24} & A_{25} \\ A_{31} & A_{32} & A_{33} & 0 & A_{34} & A_{35} \\ \chi\left(\eta b_1 - 1\right) & \chi\eta b_2 & \chi\eta b_3 & 1 & \chi\left(1+\eta b_4\right) & \chi\eta b_5 \\ 0 & 0 & 0 & \beta & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \widehat{K_{t+1}} \\ \widehat{K^e_{t+1}} \\ \widehat{\rho_{t+1}} \\ \pi_{t+1} \\ \widehat{C^h_{t+1}} \\ \widehat{s_{t+1}} \end{pmatrix}$$

$$= \begin{pmatrix} B_{11} & B_{12} & B_{13} & 0 & B_{14} & B_{15} \\ B_{21} & B_{22} & B_{23} & 0 & B_{24} & B_{25} \\ B_{31} & B_{32} & B_{33} & 0 & B_{34} & B_{35} \\ \theta_2 a_1 & \theta_2 a_2 & \theta_2 a_3 & \theta_1 & \theta_2 a_4 & \theta_2 a_5 \\ \chi_1 b_1 - \phi\left(\alpha - 1\right) & \chi_1 b_2 & \chi_1 b_3 & 1 & \chi_1 b_4 - \phi & \chi_1 b_5 - \phi \\ 0 & 0 & \theta_3 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \widehat{K_t} \\ \widehat{K^e_t} \\ \widehat{\rho_t} \\ \pi_t \\ \widehat{C^h_t} \\ \widehat{s_t} \end{pmatrix}.$$

Succinctly, $Ax_{t+1} = Bx_t$. In the vector, $x_t$, $\widehat{K_t}$, $\widehat{K^e_t}$, and $\widehat{\rho_t}$ are predetermined, while $\pi_t$, $\widehat{C^h_t}$, and $\widehat{s_t}$ are control variables. Given that $B$ is not invertible, this is again solved using generalised Schur decomposition (Klein (2000) and Söderlind (1999)).

### 5.5.2 OLG entrepreneurs

Again, the solution strategy for OLG entrepreneurs is quite similar to the one shown in Chapter 4. I obtain the first intertemporal relation: $K^e_{t+1} = (1-\Gamma)\,\zeta_t n w_t$ from Eqs.236,246 and 248. Noticing that $\overline{\omega_t}$ and $\zeta_t$ are now implicit functions of $s_t$ as well as $\rho_t$, $nw_t$ is a function of $C^h_t$, $s_t$, $K_t$, $K^e_t$, and $\rho_t$. Then, the above relation is expressed as a first order difference equation in $C^h_t$, $s_t$, $K_t$, $K^e_t$, and $\rho_t$. Likewise, the market clearing condition (Eq.242) and the consumption Euler equations for households (Eq.244) can also be expressed as first order difference equations in $C^h_t$, $s_t$, $K_t$, $K^e_t$, and $\rho_t$. Log linearising the three difference equations, I have three linear first order equations. Next, I have three additional difference relations in $\pi_t$, $C^h_t$, $s_t$, $K_t$, $K^e_t$, and $\rho_t$ from the interest rate rule and arbitrage condition, the NKPC and the shock process. Without the non-invertibility issue this time, the system of equations is solved using an eigenvalue-eigenvector decomposition (Blanchard and Kahn (1980)). Using the generalised Schur decomposition yields the same result.

# 6 Chapter Six: General Conclusion

Chapters 2-4 of the thesis studied the credit channel of monetary transmission mechanism within the NK framework. Chapter 2 presented an analytical framework to shed more light on the workings of the channel. Although the endogenous developments of agency costs are an undoubtedly critical channel through which credit frictions alter the output dynamics intrinsic to the otherwise standard NK model, Chapter 2 showed that even when agency costs are modelled as acyclical, the output dynamics are still altered. Importantly, by solving the model analytically, the chapter revealed how this happens. It was shown that time invariant agency costs flatten both AS and AD relations by making a wedge between household consumption and output. As a result, compared to the otherwise standard NK model, the real impact of money shocks is amplified, but the persistence is reduced. Given that the cyclical developments of agency costs are deliberately cut off, the chapter is, in nature, qualitatively oriented. However, the supplementary quantitative exercise indicates that the amplification of the impact is sizable while the reduction of persistence is rather negligible. Thus, in the light of the VAR analysis which reveals that exogenous monetary shocks have sizable and persistent real effects, the acyclical agency costs, on their own, seem to be a modification in the direction towards greater realism.

Chapter 3 extended the framework in Chapter 2 by incorporating a segmented input markets structure. This chapter can be seen as a robustness check of the results obtained in Chapter 2. Indeed, the qualitative and also quantitative results obtained in Chapter 2 (with economy wide input markets) still hold. That is, credit frictions still amplify the impact effect of the shock while reducing the persistence of the effects. Also, the impact effect can be quantitatively significant, but the effect on persistence is small. Also, this chapter showed that for a given degree of credit frictions (for given agency costs), both the impact and persistence of the shocks are much greater in a framework with segmented input markets than in one with economy-wide markets. The analytical solution clarified that this happens because the segmented input market assumption flatten the AS curve greatly. In the light of the VAR analysis, this finding indicates that even in the presence of credit market imperfections, the assumption of segmented input markets is a modification in the direction towards greater realism.

Chapter 4, focusing directly on endogenous agency costs, showed that how the credit channel operates within the NK framework depends on the assumptions made on entrepreneurs' consumption/saving behaviour. Judging from the rather extreme fluctuations in entrepreneurs' consumption pattern in the Euler equation entrepreneurs' case, I argued that dynamics in the OLG entrepreneurs' case is more realistic. In the OLG case, although the real effects persist longer than in the standard NK case without credit frictions, the impact is not amplified. In the case of expansionary monetary shock, the smaller increase in output at impact is explained by the initial increase in the external finance premium. This, in turn, is caused by the fact that net worth is practically predetermined while the whole finance required is expanded in the boom. This decrease in entrepreneurs' contribution in the whole finance worsens the agency problem and increases the premium. On the other hand, the greater persistence is due to the slow adjustment process of external finance premium. Overall, in the sense that the real effects are not sizable although persistent, the fully fledged credit channel characterised by endogenous agency costs might not be a modification towards greater realism.

The results obtained for the OLG case are comparable with the results obtained by Carlstrom and Fuerst (2001). As pointed out, the key difference is that they consider the real effect of monetary shocks in a flexible price environment. Their results show that in the case with the OLG entrepreneurs[104], compared to an otherwise standard RBC model with monetary sector, endogenous agency costs lead to less real impact but greater persistence in the effects. My results indicate that the introduction of NK elements, i.e., imperfect competition and staggered price setting, does not alter the way endogenous agency costs exert their effect on output dynamics.

However, Bernanke et al. (1999), which incorporates endogenous agency costs

[104] They rather use the term "permanent income entrepreneurs".

into a NK framework, reports that the real effect of monetary shock is amplified in the case with OLG entrepreneurs.[105][106] However, there are potentially important differences in the assumptions. For example, the timing of financial contract between entrepreneurs and banks is different. In my framework (also in the Carlstrom and Fuerst (2001) framework), the contract is intra-temporal, but in the Bernanke et al., it is inter-temporal. Thus, in their framework, aggregate (monetary) shock is observed after the contract is made and before the repayments are made (if entrepreneurs do not default). Meanwhile, in my setting, the contract is subject to only idiosyncratic shock and not aggregate shock. Carlstrom and Fuerst (2001) casts a doubt on Bernanke et al.'s setting by pointing out that agents would be better off by signing a contract which is indexed to the aggregate shock (Bernanke et al. actually assume a non-indexed contract). Carlstrom and Fuerst argue that this lack of agents' natural action is one important factor behind the amplification result in their setting. I leave rigorous investigation on this amplification/non-amplification issue as a topic for future research.

Although the focus of Chapters 2 to 4 was monetary theory, especially the workings of credit channels within the NK framework, the NK framework is often used to answer normative questions thanks to its solid microfoundations inherited from the RBC literature (see Clarida, Galí, and Gertler (1999) among others). For example, it allows one to consider how monetary policy should be designed in the face of shocks to the economy in such a way that agents' welfare losses are minimised. Indeed, with monetary non-neutrality and solid microfoundations, the NK framework has become an essential basis for monetary policy analysis in many central banks around the world. Given that monetary theory is important when one considers optimal monetary policy, the theoretical understanding of credit channels obtained from Chapters 2 to 4 can be potentially useful for normative analysis.[107]

[105] They do not use the term, "OLG entrepreneurs", but adopts the same assumption of constant death ratio to restrict the accumulation of net worth.

[106] Unlike Chapter 4 of the thesis, they do not make a comparison between the OLG entrepreneurs and Euler equation entrepreneurs.

[107] Some recent works such as Gilchrist and Leahy (2002) and Faia and Monacelli (2007) discuss optimal monetary policy in the NK framework incorporating credit frictions.

Chapter 5 studied the role of credit market shocks in business cycle dynamics. Specifically, the chapter focused on an exogenous increase in the variance of idiosyncratic shocks entrepreneurs are subject to. In the sense that this has a direct influence on the contractual relationship between entrepreneurs and banks, it can be regarded as a credit market shock. Based on the framework developed in Chapter 4, it showed that the shock has important macroeconomic effects. Chapter 5 again compared the dynamics under the Euler equation and OLG entrepreneurs. Following the same logic as Chapter 4 (concerning the entrepreneurs' consumption pattern), I argued that the dynamics under the OLG entrepreneurs is more plausible. Then, what was observed is prolonged stagflation. The output is low because of the increase in agency costs due to the worsening of the information asymmetry. Further, it shows persistence because of the slow adjustments of net worth and agency costs. Also, the persistently high agency costs are reflected in the prolonged inflation. The result indicates that policy makers need to be aware of the macroeconomic effects of this sort of second moment shocks at the micro level. In relation to the stylised fact observed by Bloom et al. (2009), the chapter suggests that the countercyclical uncertainty at the micro level can be caused by endogenous developments in credit markets.

Finally, I state a few ideas for future research. First, although the numerical analysis conducted in Chapter 4 surely indicates the importance of time-varying agency costs in the monetary transmission mechanism, the analytical exercise in Chapter 2 reveals that this is not the only channel through which credit frictions alter the output dynamics intrinsic to the otherwise standard NK model. Indeed, Chapter 2 shows that time invariant agency costs flatten both AS and AD curves by creating a wedge between household consumption and output. Then, what I am interested in doing is to decompose the overall effects observed with endogenous agency costs. It is difficult to do this simply using the frameworks used in those chapters, because the settings are quite different. For example, in Chapter 2, the input for production used by entrepreneurs is only labour, while Chapter 4 also incorporates capital stock whose accumulation also plays a part in dynamics. Further, Chapter 2 considers a change in money supply while Chapter 4 looks at the shock to the interest rate rule. Importantly, the Taylor rule itself affects the dynamics. Thus, I am interested in making a unifying framework which can be used for the decomposition exercise.

Next, I entirely focused on corporate borrowing in the thesis. For example, it totally abstracts away from credit frictions in household borrowing. However, in reality, a large proportion of household borrowing is secured by real estate. In fact, the role of frictions in household borrowing in monetary transmission mechanism seems to be non trivial. Suppose that expansionary monetary shocks lead to an increase in the house price. When a house is used as collateral by households, it can then increase their borrowing capacity. That leads to an increase in household spending and output. It thus appears that the real effect of monetary shocks can be amplified. More generally, Iacoviello (2005) demonstrates the importance of collateral constraints tied to the value of real estate for households (as well as for firms) in the business cycle dynamics. The macro effect through household borrowing constraints is one of the research areas I am interested in.

Finally, the thesis also disregards the potential role played by banks' balance sheets condition. In fact, this appears to be a critical factor which aggravated the financial crisis subsequent to the US subprime crisis of 2007. Before the crisis, many banks were already highly leveraged, that is, they held small capital. Then, as asset prices started falling, their capital became even smaller. Since they found it increasingly difficult to obtain finance in money markets amid the fear of them becoming insolvent, they were often forced to sell their assets at "fire sales prices". In case other banks had the same assets, this reduced their capital. In turn, this could lead to yet another fire sales by those other banks. The contagion effect amplified the initial negative impact and led to rapid deterioration of banks' balance sheet in general.

Furthermore, an increase in uncertainty over the value of asset prices seemed to play a role behind the deterioration of banks' balance sheets. Due to the development of securitisation, it has often become difficult to assess the true value of assets. This implies that the perceived risk of banks going bankrupt increased. Then, it became even more difficult for banks to finance themselves in money markets. This, again, prompted the fire sales of assets, led to the contagion effect and worsened the banks' balance sheets condition further. (These amplification mechanisms introduced here are discussed in Blanchard (2008) in more detail.) Overall, banks had to tighten lending standards and thus credit spreads increased. Consequently, the flow of funds in the economy stagnated and output fell dramatically.

Having seen the above intuitive arguments, there seems little doubt that a disruption of financial intermediation played an important role in the recent economic crisis around the world. However, looking at the theoretical literature on the role of credit market imperfections in the macroeconomy, the emphasis has been rather given on the credit market constraints on non-financial borrowers. In fact, there have been not many formal models on the role of financial intermediaries.[108] Given this state, I would like to work on the role of financial intermediaries in the economy.

[108] One exception is Chen (2001), which formally studies the role of bank net worth and asset prices in macroeconomy.

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