# Alternative models of

# security price equilibrium

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# Table of contents

# List of figures

# List of tables

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# Abstract

The major determinant of the performance of financial markets is the nature of the information available, both in terms of the overall quality of the information held by investors, and the distribution of information amongst investors. The nature of this issue makes it difficult to model realistically. This thesis marks an attempt to gain insights to the behaviour of securities markets by investigating the consequences of relaxing, in a realistic way, some of the restrictions on information in existing models.

The core of the thesis consists of formal models of the stock market. The first of these is a development of the information aggregation literature, and in particular the model of Hellwig (1980). It looks at the ability of prices to aggregate information that is dispersed among agents who specialise in acquiring information about particular components of the factors that determine the future stock value. We find that narrowing the extent of specialisation beyond a certain point will inevitably lead to a reduction in the informativeness of the market price.

The second model is also a development of the information aggregation literature, and looks at the implications of investors obtaining private information about the extent of liquidity trading. We find that such a framework gives rise to the possibility of multiple equilibria and price 'crashes.' The third model is an extension of the second in which the acquisition of information is made endogenous, and shows that the main results of that model are retained. It also shows up the dominance of the cost of information about value, rather than about liquidity trading, in determining the overall informativeness of the price.

After investigating the possible consequences of, and providing evidence for, the existence of positive feedback trading we investigate the behaviour of a market in which investors exhibiting such behaviour are combined with investors who trade on the basis of value in a form as in De Long, Shleifer, Summers & Waldmann (1989, 1990a). We then apply results from Hart (1977) to determine the conditions under which manipulation can be possible. A number of model characteristics are shown to be possible depending on the specific form of the feedback trading. We finish by adding shocks to the system, as in De Long et. al., and look at the effect of both competitive and monopolistic speculation. We find that competitive speculation may be more destabilising than monopolistic speculation, and that positive feedback trading is more destabilising when it acts after a delay.

# Chapter 1

# Introduction

## 1.1 Overview

Models of the stock market often contain assumptions about the information held by the investing agents that are so restrictive that they are unable to reflect certain aspects of the behaviour of real-world markets, such as crashes. The work contained within this thesis attempts to revise informational assumptions in a realistic way, in order to obtain richer results. Revisions in the informational assumptions can take place in two directions: firstly, in the direction of attempting to reflect the high degree of uncertainty about the future; and secondly, in the direction of better reflecting the informational asymmetries between investors. The latter may best be served by relaxing the strong informational assumptions implied by full rational expectations.

In this chapter, after briefly looking at the role of formal models in the understanding of reality, we look at ways in which models can be constructed, basing this on the development of the literature. We look at the implications for modelling of the assumption of rational expectations, and the ways in which particular streams of the literature, such as those focusing on market microstructure and information aggregation, have developed. We look at issues such as: the importance of the assumptions of common priors and common knowledge; the possibility of 'sunspots' and 'bubbles'; the way in which information is acquired and supplied; the implications of the presence of liquidity traders; and the implications of the presence of investors who are not 'rational' in the sense in which this term is used in the rational expectations literature. At the appropriate points within this chapter, outlines of the work in the remainder of this thesis will be given, in order to highlight the points of departure from the existing literature.

The remainder of the thesis follows a similar path to this introductory chapter, with the initial adherence to rational expectations principles being relaxed as the thesis progresses. The models contained within chapters two, three and four are (noisy) rational expectations models of the information aggregation school: in chapter two we relax the assumption that investors receive information about the full liquidation value by endowing them with partial information only, and investigate the ability of the price to aggregate such 'specialised' pieces of information; in chapter three we allow the investors to receive information about the amount of liquidity trading taking place, and thus look at a situation in which prices aggregate two types of information; and in chapter four we allow the acquisition of information in chapter three to become endogenous. From chapter five onwards we begin to look at the effect on market prices of the existence of investor demand that fluctuates predictably through time or with price changes. In chapter six we argue the case for the existence of some investors whose stock demand is positively related to price changes; in chapter seven we look at the price behaviour and the potential for stock market manipulation in the presence of such investors; and in chapter eight, in the light of this, we investigate the potential for competitive speculation to be destabilising.

## 1.2 The rhetoric of financial modelling

The appropriate method for research depends on the nature of the issues to be investigated, and the environment in which the research takes place. Perlman (1978) made the point that:

> "The essential methodological question is what does it take to convince oneself or others of the validity of an idea? Or, to put it otherwise, what system of proof works - a model, empirical evidence, moral revelation, or what?" (p. 582).

We believe that the issues addressed in this thesis are ones that the construction of formal models can illuminate, and we have attempted to provide such models that can persuade the reader that the results produced can contribute to our understanding of how markets operate.

All models in economics and finance must involve some degree of abstraction from reality in the form of simplifying assumptions. Since our aim is to produce models that exhibit certain aspects of real-world behaviour, the ideal way to assess the acceptability of a model is to compare its results to reality. Unfortunately, the reason we need to construct models is that we do not have a perfect understanding of reality, and so the assessment of models cannot be perfect: often we know certain stylised facts, which determine certain minimum requirements for model performance, but beyond this any assessment becomes subjective. Just as with predicting the future, as we argue below, the paucity of true information prevents a fully objective assessment of competing models. The necessity of subjective assessment implies that the same 'hard' facts can be interpreted differently by different people, and that the generally-accepted interpretation will depend on the prevailing environment.

This reliance on subjectivity in assessing the acceptability of a model inevitably leads to an important role for rhetoric in the process of persuasion.[1] In fact, every argument put forward to promote a theory can be labelled as rhetoric, be it data analysis, model-building, or an accompanying 'story.' In order to be persuasive, models need all three of these types of rhetoric to some degree, even if the data analysis component is simply that the results produced are not demonstrably incompatible with observed market behaviour.

In constructing models built on pure theory we must therefore continually bear in mind their rhetorical context, and so our task is arguably:

[1] See McCloskey (1986) for a good discussion of this issue.

> "to invent marvels [in the sense of stories that clearly violate the laws of nature] that have a point, the way *Animal Farm* has a point. The plots and characters of pure theory have the same relation to truth as those in *Gulliver's Travels* or *Midsummer Night's Dream*. Pure theory confronts reality by disputing whether this or that assumption drives the results, and whether the assumption is realistic" (McCloskey, 1992, pp 30-31).

The astute reader will recognise that this introduction is itself part of the overall rhetoric of the thesis: and the success of the thesis will be judged by whether or not the combination of its rhetorical components serves to persuade its initial readers that it merits the award of a Doctor of Philosophy degree.

# 1.3 Modelling markets and investors

## 1.3.1 Key assumptions

When considering the most appropriate way to model the stock market, the prime considerations must be the structure of the market and the nature of the information held by the agents. We can assume that the market consists of investors alone or with a Walrasian auctioneer, or, as in a more recent development of the literature, that a specialist sets prices in an attempt to break even; we can assume that speculators act competitively or monopolistically or somewhere in-between; and we can assume that all the agents have full knowledge of the structure of the market, in terms of the preferences of the other investors and the quality of their information, or that some (or all) are not so informed.

If we assume that each agent uses the full extent of her knowledge to form beliefs and optimise her behaviour, and constantly updates these beliefs as new information becomes available, we are making the assumption of 'rational expectations' (R.E.) in the sense used in the literature, and first described by Muth (1961). It is possible, however, to make a wide variety of different specific assumptions while remaining under the rational expectations umbrella.

Two of the key assumptions that determine the performance of R.E.-based asset pricing models concern the *prior beliefs* held by the investors about the underlying structure of the world, and the extent to which these are *common knowledge* amongst investors. The prior beliefs held by investors about the state of the world could relate, in specific models, to beliefs about the particular distribution from which a future payoff is taken. Common knowledge, in rough terms, is that knowledge known by all investors to be commonly shared by them all. More specifically:

> "Two people, 1 and 2, are said to have *common knowledge* of an event *E* if both know it, 1 knows that 2 knows it, 2 knows that 1 knows it, 1 knows that 2 knows that 1 knows it, and so on" (Aumann, 1976, p. 1236).

Relevant common knowledge relates to the structure of the system in which the agents are operating, in terms of specifics such as the number of agents, their utility functions and risk aversion, prior beliefs and the precision of their information. It is standard practice in asset pricing models to assume that agents have common priors - although they may subsequently receive private information that causes their posterior expectations to diverge - and that a large part of the structure of the market is common knowledge.

### 1.3.2 The implications of common priors and common knowledge

Aumann (1976) showed that if two people have the same prior beliefs, and their posterior beliefs are common knowledge, then these posterior beliefs must be the same. This ensures that with identical prior beliefs agents cannot agree to disagree. Sebenius & Geanakoplos (1983) showed that this implies that two agents with common priors will not be able to agree on a bet that is acceptable to both parties if a dialogue takes place between them, since the dialogue will reveal the posteriors, which will converge. Milgrom & Stokey (1982) showed likewise that when investors have common prior beliefs, differences in information will not motivate trading activity in a situation where it is common knowledge that the other agents are acting rationally in offering the trade, and where the initial asset allocation is Pareto optimal with respect to the prior beliefs. The importance of the assumption that the initial allocation is Pareto optimal is that in such a situation there can be no motive for trading other than betting - or speculating - on differences of opinion. These results indicate that asymmetric information, rather than leading to trade between agents, is likely to stifle it.

In order for agents with common priors to be willing to trade in the presence of asymmetric information, it must not be common knowledge that there are no overall *gains from trade*. Asymmetric information itself does not create such gains, which is why agents are not willing to trade in the Milgrom & Stokey framework. Gains from trade can result from a desire for portfolio rebalancing due to aversion to risk or preferences regarding immediate rather than delayed consumption. Risk aversion may provide insurance motives for trade, make risk-sharing beneficial, or lead to desired portfolio revision as a result of new information if the initial allocation was not Pareto efficient. Rational investors will therefore be willing to trade either when they have reasons to trade that make up for the possible losses from trading with better-informed investors; or when there is a possibility of trading with other investors who are either less-informed or who are trading for insurance or liquidity motives.

### 1.3.3 Sunspots and bubbles

There has been much investigation into the consequences of common priors and common knowledge about beliefs for the possibility of 'sunspots' and 'bubbles' in a market with rational investors.

'Sunspots' are events that affect prices even though informed investors know they have no implications for value. The origin of the application of the sunspot terminology to the asset-pricing literature follows the work of Stanley Jevons (W.S. Jevons, 1909) who claimed to have discovered a relationship between real-life sunspots and the business-cycle. Although real sunspots could theoretically have influenced the economy through their effect on agricultural activity, the asset-pricing literature has come to use the term sunspot to refer to events which are observable, but for which there is no theoretical connection with economic activity. 'Bubbles' are generally regarded as scenarios where the price increases rapidly above its 'true' value however defined, before suddenly crashing as the bubble bursts.

A simple backward induction argument shows that investor rationality will prevent the price of an asset from exceeding its fundamental value when investors' beliefs are common knowledge (Tirole, 1989). However, Allen, Morris & Postlewaite (1993) have shown that when short sales are restricted and investors have private information, implying that beliefs are not common knowledge, investors may be willing to purchase stock in the belief that they will be able to pass it on to others before the bubble bursts.

Tirole (1982) showed that bubbles will not occur in a market for an infinitely-lived asset in which there are a finite number of rational investors with common priors, unless there is a source of gains from trade. This is because investors will be unwilling to pay more for the asset than they would if forced to hold it forever, since they know that if a seller does not re-enter the market the remaining traders must share a loss. This again relies on the common knowledge of beliefs, but this itself is not enough: Bhattacharyya & Lipman (1995) showed that bubbles can still exist in a market with an infinitely-lived asset, and a finite number of rational investors, in which the fundamental asset value is common knowledge, provided that the initial wealth of each investor is not common knowledge.

The overlapping generations framework first formulated by Samuelson (1958) allows a situation to be modelled in which there is a continual influx of new investors. It has often been shown in this framework that sunspots and bubbles can exist in such an environment when investors have common priors and all relevant variables are common knowledge (see, for example Azariadis (1981), Cass & Shell (1983), Tirole (1985), Jackson & Peck (1991)). A general requirement is that the growth rate of the economy exceeds the interest rate in a bubbleless situation (Tirole, 1989). Investors trade as there are gains that result from the different preferences regarding immediate and delayed consumption of the investors in the different age-groups. The standard use of two periods to represent investors' life-spans, combined with the growth requirement, detracts somewhat from the conclusions, but nevertheless this approach does highlight a plausible consideration, which is that the relationship between the time-horizon of investors and the expected duration of price patterns may be important.

The above indicates the importance of assumptions about common knowledge, priors, and gains from trade. It also shows that, even without the existence of ill-informed investors (in a non-rational sense) bubbles and sunspots are not ruled out. It is therefore likely that bubbles and sunspots are possible when ill-informed investors are present. Indeed, when there are non-rational investors present, even the simple backward-induction argument, and the common priors and common knowledge argument of Tirole (1982) break down. Unfortunately, the above does not indicate much about the effect of the interaction of informed and ill-informed investors, and cannot tell us the effect of the presence of the rational investors, since these are the only investors present, and prices in their absence are undefined. We investigate these issues later in the thesis.

# 1.4 Model types

In this section we begin by dissecting two types of rational-expectations-based stock market models in the literature - information aggregation models and market microstructure models - and highlighting the roles of the underlying assumptions in facilitating the isolation of equilibrium prices. We then go on to look at the assumptions made about future payoffs, information about the extent of liquidity trading, and the acquisition and supply of information. Along the way we discuss the models found in chapters two, three and four.

## 1.4.1 Information aggregation models

The information aggregation literature was founded by Grossman (1976) to show how prices could come to reflect private pieces of information when investors attempt to use the price to learn about the private information of others. The idea of agents using endogenous variables to learn about exogenous ones had previously been developed by Lucas (1972). Equilibrium prices in information aggregation models are determined by a Walrasian auctioneer who equates the demand for and supply of stock.

Grossman's model contains one riskless and one risky asset, both of which make their only payouts one period later. Each investor is endowed with a piece of information revealing the payoff to the risky asset in the following period with a certain degree of noise. The noise in the information of each investor is assumed to be independent of the noise for all other investors, and so the 'average' value of the individual signals accurately reflects the expected value of the payout for large numbers of investors. The supply of stock is fixed and positive, which ensures that there are gains from trade, since the two possible scenarios are that: either other investors currently own the stock and wish to liquidate at any price; or the investors are endowed with stock, and their risk aversion produces a desire for portfolio rebalancing. The investors' utility functions and coefficients of risk aversion, the stock supply, the relationship between the exogenous variables and the price, and the distribution from which the payoff is drawn are all common knowledge. It follows from this that agents have common priors. Since the price contains information about the payoff, it is determined simultaneously with investors' individual stock demands. Grossman showed that the model equilibrium tends towards full revelation of the asset payoff, ensuring that the private information becomes redundant. This, however, cannot be a legitimate equilibrium, since if investors fail to take their private information into account the price cannot reflect it. If prices do not fully reflect the private information, investors will benefit from using it, which will lead back towards full revelation. There is therefore no equilibrium in which investors use their information, and no equilibrium where they do not.

Grossman's model shows that gains from trade are not enough to guarantee an equilibrium in an asset pricing model. Grossman pointed out that adding a source of extrinsic uncertainty to the model would allow the problem of full revelation of information through the price to be overcome, and suggested that this could be done by adding uncertainty about the supply of stock. Diamond & Verrecchia (1981) produced a model that introduced such extrinsic uncertainty. In their model each agent is endowed with stock, and the total stock supply is the sum of all the endowments. The stock endowment of each agent is taken from a known distribution, and agents cannot observe endowments other than their own. These assumptions ensure that each agent has some information about the total stock supply, derived from their priors and their own endowment, but they do not know its precise level. This in turn ensures that prices are only partially-revealing in equilibrium, since the investors are unable to disentangle the price effects of the private signals and the stock supply. One problem with this approach is that as the number of investors becomes large, the supply noise disappears, and the prices tend towards full revelation. In addition, the fact that individual endowments give investors some information about the stock supply makes the model slightly less tractable than the alternative Hellwig (1980) formulation, which explicitly includes noise traders and is discussed below.

It is possible to adapt the stock endowment assumption to ensure that uncertainty in the stock supply remains as the number of investors becomes large. This is the approach taken by Verrecchia (1982), a model we will become more familiar with subsequently. Making this assumption, however, produces results identical to those obtained by including noise traders, and retains disadvantages in terms of tractability. The inclusion of noise in supply can therefore be justified either on the grounds of endowment uncertainty or liquidity trading. The liquidity trading assumption is arguably the more realistic.

Hellwig (1980) was the first to explicitly add liquidity trading to a model of information aggregation. Under his formulation, subsequently adopted by the major part of the literature, the extent of liquidity trading is assumed to be drawn from a (known) normal distribution. Once again, since the effects of the supply outcome and the private information cannot be disentangled by the investors, the prices are only partially revealing, and private information is not completely dominated by the information contained within the price.

The behaviour of the agents and prices in information aggregation models has been replicated experimentally by Sunder (1992). Huberman & Schwert (1985) found evidence supporting noisy rational expectations equilibria for the Israeli bond market. Hellwig (1982) suggests that the root cause of the lack of equilibrium in Grossman (1976) (and other features of information aggregation models) is the simultaneous determination of price and demand. Hellwig develops a dynamic example in which agents base their demand on past prices, and shows that, for short gaps between periods, the returns to becoming informed can be bounded above zero even when prices approach full-revelation arbitrarily closely.

### 1.4.2 Market microstructure models

In market microstructure models, trading centres around a relatively ill-informed specialist who is assumed to aim to break even by setting 'fair' prices. In such a setting the need to incorporate gains from trade is a central issue, as the specialist is involved in each trade, and has no portfolio rebalancing motives for trading, and so will only trade if at least some of the other investors have reasons to trade other than the exploitation of private information. To this end, it is generally assumed, as in the seminal works of Glosten & Milgrom (1985) and Kyle (1985) that some trades occur as a result of liquidity considerations, and that these are equally likely to be sales or purchases. The trades of the informed traders are also equally likely to be sales or purchases, which ensures that there is a strong symmetry in the trading process. The specialist is unable to observe the trading of the liquidity traders.

In Glosten & Milgrom (1985), the specialist sets bid and ask prices on the basis of the expected value of the stock if the next trader is a seller or a buyer. The specialist trades only one stock at a time, and takes account of the possible information contained within the trade when setting the bid-ask spread. The trades of the ill-informed investors provide 'noise' that prevent the specialist from discerning the information of the informed trader, and allows the informed traders to profit at the expense of the others.[2] It can be shown that in this framework uninformed traders cannot profit from manipulating prices purely on the basis of trading strategies, even though the specialist will not know whether or not their trading partner at any given time is acting on information.

In Kyle (1985) the traders can submit demands of any quantity to the specialist, who sets a price after observing the combined demand of the informed and liquidity traders. Unlike Glosten & Milgrom, Kyle allows for continuous trading, under which the incremental stock demand of the liquidity traders is assumed to follow Brownian

[2] Note that, strictly speaking, the others do not lose out in trading; it is just that the informed traders are able to commandeer the lion's share of the overall gains from trade resulting from the divergent preferences.

motion. This allows us to gain insights about into how information comes to be incorporated into prices over time. Kyle termed the liquidity traders 'noise' traders, due to the noise their demand adds to the aggregate demand signal received by the specialist.

The noise traders in market microstructure models therefore tend to play two roles: they provide gains from trade, ensuring trading can take place; and they also add noise to the information set of the rational investors, as they also do in information aggregation models, ensuring that the information of the informed investors is not fully revealed.

### 1.4.3 Focus on information about payoffs

In both the market microstructure and information aggregation literature we have examined so far, the private information is generally a noisy observation of a single future liquidating payoff. In reality, of course, observations must relate to the underlying determinants of the future price, rather than the future price itself; and these observations must be 'converted' into a value estimate. One such conversion process is provided by the linear factor model that underlies the arbitrage pricing theory (A.P.T.) formulated by Ross (1976). Under the A.P.T., the return on an asset is the sum of a constant term, an asset-specific term, and terms consisting of each factor multiplied by a coefficient representing the sensitivity of the asset to the factor. Handa & Linn (1991) use the *n*-asset framework of Admati (1985), which is an extension of Hellwig (1980), to show how the A.P.T. can be set in an information aggregation framework.

Handa & Linn did not investigate the effect on the ability of prices to aggregate information of investors specialising in acquiring information about one factor only. In **chapter two** we use a single-asset model based on Hellwig (1980) to investigate this issue. Although not capturing the difficulty in producing a value estimate from raw data not specifically relating to the payoff itself, this formulation does allow us to assess the ability of the price to aggregate information in a broader sense than has been attempted previously. We show that the potential for specialisation in information acquisition to improve the efficiency with which prices aggregate information is strictly limited.

### 1.4.4 Information about liquidity trading

Up to this point we have assumed that the liquidity trading is unpredictable and unobservable. It is likely, however, that the root causes of the liquidity requirements can be observed to some degree, and so investors will have information about the actual net amount of liquidity trades. Prices in this situation would have to aggregate information about two independent variables: the payoff, and the liquidity demand. In **chapter three** we construct a model in which the investors receive private information about the supply in the same form as their private information about the payoff. Since the investors have some private information about the stock supply that tells them something about the total supply, this model has some similarities with Diamond & Verrecchia (1981): but, unlike that model, the supply uncertainty remains, and private information about the supply retains some value, when the number of investors becomes large. We demonstrate that this may lead to multiple equilibria, with each possible equilibrium representing a different way in which the information can be aggregated, and corresponding discontinuities in the price function.

### 1.4.5 Information acquisition

So far the models we have examined have not dealt explicitly with the process by which the investors obtain their information, but have instead taken the quality of each investor's private information as given. Since, however, the equilibrium price function determines the value of information, the quality of information should be treated as endogenous.

Grossman & Stiglitz (1980) looked at the situation in which agents can choose to purchase a given information signal at a given cost. Since the signal is identical for all who acquire it, this is not a model of information aggregation. Supply uncertainty once again prevents the information from being fully revealed in the price. Grossman & Stiglitz show that in equilibrium the number of investors choosing to observe the information signal is such that the benefits of doing so will exactly offset the cost. They also show that the informativeness of market prices is bounded away from zero, even as the noise in supply becomes small, which contrasts with the results of Hellwig (1982) we revealed earlier (sub-section 1.4.1).

Verrecchia (1982) looks at the issue of information acquisition in the information aggregation framework of Hellwig (1980). The stock supply issue is set up along similar lines to Diamond & Verrecchia (1981), with investors being endowed with stock, and each agent's endowment being taken from the same distribution. The difference here. though, is that the variance of this distribution depends on the number of investors present such that the overall variance of the stock supply does not. This ensures that the stock supply still provides noise as the number of agents becomes large, and under this scenario the situation is identical to that in the large market case of Hellwig. In the Verrecchia model it is assumed that the investors face fixed linear cost functions that relate the cost of acquiring the information to the precision of the information acquired. The key comparative static results are that the informativeness of the price tends to increase with decreases in the level of supply noise, the cost of acquiring information, and the overall risk-aversion of the investors.

Verrecchia looks only at the acquisition of information about the future payoff. In **chapter four** we generalise this to the acquisition of information about both the payoff and the amount of liquidity trading, thus extending chapter three similarly to the way that Verrecchia extended Hellwig (1980). This allows us to see the way in which the acquisition of the two information types impinge upon each other, most notably when the cost of acquiring information changes.

The main analytical result of chapter four is that, when the agents receive no free endowment of value-information, the cost of supply-information does not affect the total quality of information (in other words the conditional variance of the liquidation value) obtained by the agents: the acquisition of supply information merely serves as a way for the agents to obtain the same quality of information more cheaply. A corollary of this is that price discontinuities cannot occur without a positive free endowment of value-information.

### 1.4.6 Information supply

Diamond (1985) utilised a model similar to Verrecchia (1982) to assess the optimal amount of public information a firm should release. Admati & Pfleiderer (1986), still in an information aggregation framework, looked at the strategies a monopolistic seller of information may use when selling information directly to traders. Admati & Pfleiderer (1990) later compared direct selling strategies to indirect ones in which the monopolist sells shares in a portfolio constructed using his private information. In both of these models, the monopolist is prevented from trading in the market on his own account, and is assumed to be always honest in his dealings. Admati & Pfleiderer (1988) show that the possibility of an information seller also trading on his own account can be dealt with fairly easily in a framework based on Kyle (1985), since it is always optimal for the information owner to sell the information signal as he receives it (and not to add noise as Admati & Pfleiderer (1986) show is optimal in an information aggregation framework), and so if he decides to trade, he does so under the same conditions as the other investors who have purchased the information. It is demonstrated that it is the degree of risk aversion of the information owner and that of the other investors that determines how the owner chooses between the three possibilities of: trading and not selling; selling and committing himself not to trade; and both selling and trading. The Kyle framework proves more tractable for the analysis of this issue since the investors are assumed to submit their demands before observing the price, which ensures that the value of information is not dispersed by the partial revelation of this information to uninformed agents via the price. In fact, in this framework rational agents who are not informed and have no liquidity requirements have no reason to trade, and so the only active investors are the noise traders and the investors who have purchased information. If we are interested in the willingness to acquire information when it can partially be revealed to non-acquirers through the price, the information aggregation framework is more appropriate.

# 1.5 'Ill-informed' investors and liquidity traders

## 1.5.1 Focus on liquidity traders

In both the market microstructure and information aggregation models we have seen, liquidity traders have been included to introduce gains from trade and add noise to the system, and their trades have been assumed to come from a distribution that is symmetric. However, as Allen & Gorton (1990) have argued, on closer inspection this assumption of symmetry becomes difficult to justify, since it is difficult to conceive of situations where investors are forced to buy stock in the same way that they are sometimes forced to sell.

Allen & Gorton also noted that, while anyone can take advantage of positive information by buying stock, the exploitation of negative information would involve short-selling unless the stock was already held, and this may not be pursued as vigorously. The combination of this with the asymmetry of liquidity-motivated trades will ensure that a sale is less likely than a purchase to be motivated by information, and so purchases involve more information on average than do sales. Allen & Gorton show that if this feature is introduced into the Glosten & Milgrom (1985) framework, an uninformed manipulator can profit from simple trading strategies if the specialist is not aware of her presence in the market: the manipulator can buy several units of stock at rapidly rising prices, and then sell out at prices that fall at a slower rate, with the average sale price being higher than the average purchase price. As Allen & Gorton point out, however: for the model to conform to the requirements of rational expectations, the specialist should recognise the possibility of speculative trades, and the next step would be to discover an 'equilibrium level of manipulation.' This work shows that the specific assumptions made about the behaviour of noise traders are extremely important in determining a model's character, and so attention should be paid to determining the appropriate characteristics.

Allen & Gale (1992) provide a model that exhibits an equilibrium level of manipulation of the sort we required from Allen & Gorton, albeit in a different framework. They show how an uninformed trader can successfully manipulate a stock price even when the other investors are fully aware of the strategy that is being followed. Allen & Gale's model contains three types of traders: price-taking investors, who originally own the stock; an informed trader, who receives information about the future stock value; and a manipulator, who has no such information. The price-taking investors can tell when one of the two 'large' traders are present in the market, but cannot identify which type of large trader it is. The investors know the correct *a priori* probabilities that they will be trading with the manipulator and informed trader respectively, and it is these that determine the prices at which the investors are willing to trade. The effect of the manipulator's potential presence is not to make the investors worse off, since their rationality protects them from this, but to worsen the terms on which the informed investor can trade. It is only by assuming that the risk-aversion of the large traders is lower than that of the investors, and so the trading of large traders involves a welfare-enhancing transfer of risk from the other investors, that Allen & Gale prevent the manipulator's profit from being mirrored by a loss for the informed trader, and his associated withdrawal from the market. This once again highlights a need in R.E. models for there to be a source of overall gains from trade.

### 1.5.2 Portfolio insurance

Portfolio insurance strategies can provide a motivation for liquidity trading. A dynamic portfolio insurance strategy involving stock and bond market trading to replicate the payoff of a buy-and-hold strategy plus a put option leads to investors buying more stock when prices rise, and selling stock when prices fall. Portfolio insurers therefore engage in *positive feedback trading,* which occurs when net stock demand bears a positive relationship to the current price or past price changes, and also results from trend following and price-induced fads, as we shall see later.

Brennan & Schwartz (1989) constructed a model containing portfolio insurers and a representative (non-portfolio-insuring) investor, and found that the effect of portfolio insurers on the volatility of prices was relatively small. However, Gennotte & Leland (1990) included portfolio insurers in an otherwise standard information aggregation framework, and showed that their behaviour can lead to price discontinuities of the form found in chapter four; although the effect in the absence of the discontinuities is again relatively small.

In Gennotte & Leland, the stock demand from the portfolio insurers is assumed to be a particular function of the market price. It is found that under this specification there may be two possible equilibria, since the total demand curve is of a reverse-s-shape (and supply is fixed). Over a range, the demand curve is therefore of a perverse nature, which is due to the increased desire of the 'rational' investors to hold stocks that results from a lower price being more than offset by the higher stock sales of the portfolio insurers. Over the range of possible levels of the liquidation value, there may therefore be a region with two potential equilibrium prices, instead of just one as elsewhere. Plotting the path of prices as the liquidation value signals fall or rise shows up a discontinuity which occurs at the point where the second potential equilibrium disappears. Gennotte & Leland construct a story to accompany this, in which the discontinuity in the price function resulting from the model is translated into a crash in continuous time, even though all the equilibria result from one-off model runs. In chapter three we follow the precedent set by Gennotte & Leland and use the comparative statics of a single-period model to try to explain a phenomenon that is by its very nature dynamic.

One aspect of the Gennotte & Leland model that arguably violates its internal consistency, is the assumption that the portfolio insurance corresponds to a put-option replication strategy in world in which prices are distributed log-normally, even though in this model prices are distributed normally. Of course, in such a one-period framework it would be difficult to justify any particular form of portfolio insurance: but the fact still remains that it is the particular form of portfolio insurance specified that drives the results.

# 1.6 The effect of uncertainty

## 1.6.1 Uncertainty and the common prior assumption

The presence of uncertainty can undermine the common prior assumption. The three reasons commonly used to justify the assumption of common priors are given by Morris (1995), and are: that it is implied by rationality; that it would otherwise make theorising too easy; and that bounded rationality should form the starting point for looking at differences in beliefs, rather than different priors.

That care must be taken to ensure that any assumption of different priors is reasonable, in order to prevent its use in justifying any result or explaining any event, is uncontroversial. The key issue here, though, as in the other two objections, is whether or not the assumption is appropriate: clearly, if an event is caused by differences in priors, then any explanation should reflect this. In order to assess the rationality of different prior beliefs, it is necessary to investigate the foundations of the theory of predictive probability.

Savage (1972) gives the three possible interpretations of the probabilities applied to future events. These are the *frequentist* view, that probabilities can be determined by observing the outcomes of repeated events; the *personalistic* view, that probabilities are determined subjectively by individuals; and the *logical* view, that probabilities are objectively determined by the nature of the situation. Both the frequentist and logical views point to probabilities that are independent of any subjective opinion, and so provide possible bases for the Common Prior Assumption (C.P.A.). The personalistic view is that differences of opinion are fundamental. In fact:

> "Most holders of personalistic views... envisage the possibility that a person may consider one event more probable than another without having any compelling argument for his attitude. Viewed practically... the holder of a personalistic view typically supposes that the person is under the influence of experience, and possibly even biologically determined inheritance, that expresses itself in his opinions, though not necessarily through compelling argument" (Savage, 1972, p. 65).

As a consequence:

> "The criteria incorporated in the personalistic view do not guarantee agreement on all questions among all honest and freely communicating people, even in principle. That incompleteness... does not distress me, for I think that at least some of the disagreement we see around us is due neither to dishonesty, to errors in reasoning, nor to friction in communication, though the harmful effects of the latter are almost incapable of exaggeration" (Savage, 1972, p. 67-68).

As Morris points out, "The logical view of probability has largely been discredited in the philosophical literature" (*ibid.* p. 234) and that "At best, logic tells us how to update a prior given new information, but not how to choose a prior" (*ibid.* p. 235). The problem is especially acute for beliefs concerning events that are endogenously determined.

The difficulty with applying the frequentist view to future events is that it may be the case that learning is not yet complete; and if this is the case, it is almost inevitable that there will be differences in beliefs. Perhaps the key question is whether or not a pooling of all information and opinions will lead everyone to the same conclusions; or to put it another way, whether or not the 'Harsanyi doctrine,' that all differences in beliefs can be explained by differences in information, holds. If the latter holds, taking information to include all that can be gleaned from the opinions of others, then it would perhaps be appropriate to embrace the C.P.A. and explicitly model the learning process. As we have already seen, however, those who hold the personalistic view would argue that after all such pooling of information, there is still likely to be differences in beliefs. Some of these could be due to information processing errors, and in such cases explicit consideration of this should be taken, and the C.P.A. maintained. However:

> "there are some situations where we have a very clear idea that there exist heterogeneous prior beliefs that have nothing to do with information processing errors in anything other than a tautological sense. In those cases, it will be more insightful to take the heterogeneous prior beliefs as primitive and not attempt to reduce them to information processing errors" (Morris, 1995, p. 242).

Knight (1921) argued that the situation at each point in time is to a large degree unique, and so it is impossible to derive an objective probability distribution for future events. Perfect knowledge of the available information is insufficient to ensure a unique rational estimate. Thus there is a difference between *risk*, where the probabilities relating to the possible outcomes are objectively known; and *uncertainty*, where the probabilities are not known. Morris (1995) suggests that risky situations can be thought of as ones where learning is complete; and uncertain situations as ones where learning is still taking place. The key point is that, until learning is complete, it is not clear what lessons can be learned from the available evidence; and since, in relation to economic issues, the underlying situation is always shifting, uncertainty is ever-present. Keynes also recognised this. In the General Theory he wrote:

> "We are assuming, in effect, that the existing market valuation, however arrived at, is uniquely *correct* in relation to our existing knowledge of the facts which will influence the yield of the investment, and that it will only change in proportion to changes in this knowledge; though, philosophically speaking, it cannot be uniquely correct, since our existing knowledge does not provide a sufficient basis for a calculated mathematical expectation" (1936, IV, p. 152).

As a consequence:

> "human decisions affecting the future, whether personal or political or economic, cannot depend on strict mathematical expectation, since the basis for making such calculations does not exist; .... it is our innate urge to activity which makes the wheels go round, our rational selves choosing between the alternatives as best we are able, calculating where we can, but often falling back for our motive on whim or sentiment or chance" (*ibid.*, VII, p. 162-3).

Uncertainty exists as the future economic situation depends on the outcomes of a huge number of personal decisions, which we cannot predict with certainty. We simply do not have enough information to be able to judge the relative merits of a wide range of predictions, and must rely on subjective judgement to choose between them, thus allowing scope for divergence of opinion.

Evidence of the importance of recognising the existence of true - or 'Knightian' - uncertainty has been unearthed by Dow & Werlang (1992), who show that this can explain the high volatility of stock prices when it is treated as qualitatively different from riskiness. Indeed:

> "Since the future profitability of companies depends heavily on many long-term factors, including political factors, which are extremely difficult to predict, it is natural to think that the stock market is characterized by a high degree of Knightian uncertainty" (Dow & Werlang, 1992, p. 631-632).

Models containing rational investors with different prior beliefs are rather thin on the ground. Varian (1989) argued that the models of heterogeneous beliefs prevailing before the appearance of the rational expectations literature, such as Lintner (1969), can be interpreted as representing those belief differences remaining after all learning has taken place. It would be preferable, however, to model such learning explicitly. Harrison & Kreps (1978) provided a rare example where this occurs. The model is built around two sets of risk-neutral investors who are prevented from selling short. They receive identical information but interpret the implications of this in different ways, and it is assumed that the knowledge of the others' beliefs does not lead to the investors revising their own expectations, but rather to them taking them into account when determining how much they are prepared to pay for stock. Since they know the value the other investors will place on the stock in different circumstances, and so realise that there exists the possibility of selling the stock in the future for more than they believe it is worth, each agent is prepared to pay more in this scenario than they would if prevented from reselling and instead forced to hold forever. This willingness to pay more for something if allowed to sell than if forced to hold forever can form the basis for a definition of speculative behaviour, as we shall see later.

It is clearly possible to argue that differences in priors can exist, and that bounded rationality is therefore not necessarily the appropriate way to model the causes of differences in beliefs. The difficulty comes in judging what differences in prior beliefs are appropriate, and in assessing how agents' beliefs change with exposure to the beliefs of others and new information. This difficulty in determining the correct specification, combined with ease of modelling and a legitimate desire to investigate how markets would behave in the absence of differences in prior beliefs, has led to the almost universal adoption of the C.P.A.

### 1.6.2 The extent of common knowledge

Uncertainty about the structure of the market and the beliefs and nature of its participants, as opposed to uncertainty about the future, is reflected in the extent of common knowledge. It is clear that:

> "When payoffs in a game are not common knowledge, the outcome depends not only on players' beliefs about payoffs, but also on their beliefs about others' beliefs about payoffs, on their beliefs about others' beliefs about their own beliefs, and so on ad infinitum" (Morris, Rob & Shin, 1995).

Such a progression concerning the beliefs of others is known as *higher order uncertainty*, and there is increasing evidence that this can significantly affect model performance (see, for example, Morris, Postlewaite & Shin (1995) and Morris, Rob & Shin (1995)). Making an assumption of a high degree of common knowledge about the structure of the system in which the agents operate does, however, lead to a much higher degree of tractability than would be the case otherwise, and so this continues to be done. As with the possibility of different priors, however, it should always be borne in mind that common knowledge may be narrower in scope than is hypothesised in specific models.

### 1.6.3 Uncertainty and belief formation

When there is uncertainty about the quality of the information on which other agents are basing their trades, there is scope for prices to follow a trend before the uncertainty is removed. For example, an increase in demand from uninformed investors may be interpreted as having informational content by other investors, and lead them to revise upwards their estimate of the underlying stock value, thus leading to a rise in price in excess of that justified by the demand increase itself. A trend in uninformed investor demand can therefore lead to a trend in prices which is reversed when the uncertainty is resolved. The price movement will therefore resemble that of a bubble; but it is not a bubble in the strictest sense, since the price at all times reflects the information of the investors. The greater the degree of uncertainty about the fundamental determinants of stock value, the greater will be the scope for investors to misread the information content of price movements.

Uncertainty concerning the amount of information reflected in the price is also important. If investors do not know whether or not a particular piece of information is already incorporated into the price, they will not know whether or not they should act on it. This could lead to investors trading on information that has already reflected in the price, which Black (1986) has termed noise trading. Such noise trading will be positively related to past price movements, since they will be affected in the same direction by information.

There is no guarantee that any information will be fully incorporated into prices, since no investor will know when this process of incorporation has been completed: it is possible that there is a rapid initial price adjustment, after which investors believe that prices have moved to reflect it, and a subsequent slow price adjustment as the initial pricing errors are eliminated. This is fundamentally different to initial pricing errors caused by uncertainty about the implications of the information, since it implies that the price does not move solely to reflect the implications of the information, but that a component of the price change does not reflect the information, but results simply from the inability of prices to aggregate information successfully. This has implications for event studies. If price changes are equally likely to overestimate the implications of the new information as underestimate them, and the errors have a mean of zero, this effect will not be observable in aggregate empirical data.

As well as leading to trading on noise, a lack of information can also allow investors' demand to be affected more by fads and fashions, and psychological factors. This will even be the case if investors believe in the efficient markets hypothesis, since this will prevent investors from consciously counteracting their whims. In such a scenario, the demand of these investors will be determined independently of the price, and this is arguably a better justification for the symmetric behaviour of noise traders in some of the models we have seen than that of liquidity trading.

The degree of uncertainty influences the lack of informedness of both the ill-informed and relatively well-informed investors; and the latter effect allows the behaviour of the ill-informed investors to exert a greater effect on the price, since the well-informed investors will be less willing to trade strongly against them.

Since fluctuations in such demand is likely to affect the market price, it should be monitored and predicted by investors. Shiller (1984) gives a simple model in which 'smart' investors take the fluctuating demand of other investors into account when determining the level of their own investment, without fully neutralising their effects.

### 1.6.4 Consequences for model-building

We have seen that it is likely that investors have different prior beliefs, and are sufficiently ill-informed to suffer from bounded rationality and to share relatively little common knowledge. There are two approaches that can be taken to modelling under these conditions: either the shortcomings of the investors can explicitly be taken into account; or simple mechanistic rules can be used as a proxy for their effect.

There is much that can be achieved by modelling ill-informed investors explicitly. As we have already argued, however, such modelling may be sufficiently intractable as to be unable to capture investor behaviour realistically. In addition, the assumptions that need to be made in terms of the nature of the prior beliefs, information sources, common knowledge and knowledge concerning the structure of the market may need to be sufficiently *ad hoc* that they offer no advantage over more mechanistic behavioural rules; although as with most things, the relative advantages of the techniques will depend on the particular issue being addressed.

The aim in incorporating mechanistic behavioural patterns for ill-informed investors should be to use reasonable and realistic patterns, and to apply the model to suitable situations, not letting it produce absurd results. We should be aware that there are limits to the extent to which investors can be exploited, and not place too great an emphasis on model conclusions that depend on such scenarios.

Investor strategies such as portfolio insurance and chartism, as well as trading on noise, investor contagion and other psychological effects, indicate that positive feedback trading, which involves a demand function for stock that bears a positive relationship to price movements, may be an appropriate pattern of investor behaviour to model. This conclusion is reinforced by anecdotal evidence of stock price bubbles and episodes of manipulation, as we shall see in **chapter six**.

### 1.6.5 The effect of price-related ill-informed liquidity trader demand

The implications of the existence of ill-informed liquidity traders clearly depend on the nature of the other agents operating in the market. The models we have seen so far add liquidity traders to a market with competitive rational investors, and possibly a manipulator as well. If it is thought that the general investor behaviour roughly follows mechanistic rules - albeit perhaps with a knowledge of the fundamental stock value - and so fits into our category of liquidity traders, then we can legitimately examine the behaviour of a market in which they appear alongside a manipulator alone. Models such as this have been developed to investigate the stabilising effect or otherwise of speculation. Hart (1977) provides a fairly general characterisation of the behavioural characteristics of the mechanistic traders that can allow a manipulator to profitably disturb an initial steady state, and aids the understanding of previous models in the literature, as we shall see in **chapter five**. The possibility of profitable destabilising manipulation is shown to be dependent on a feedback between past prices and mechanistic trader demand. In **chapter seven** we analyse a simple model in which such feedback trading takes place, continuing this in **chapter eight** with the addition of exogenous shocks.

Competitive (informed) speculation is generally thought to be stabilising, and would be in most modelling frameworks. However, De Long, Shleifer, Summers & Waldmann (1989, 1990a) have produced a model that shows that competitive speculation may destabilise prices around a demand or value shock in the presence of positive feedback trading. Even here, though, an increase in the market power of the speculators relative to the feedback traders will have a stabilising effect. We analyse the claims of De Long et. al. in chapter eight.

## 1.7 Agency issues

The management of funds is also subject to the complexities of a principal-agent relationship: the managers may have less incentive than individual investors to extract the best possible value from their portfolios, since what matters to them is predominantly their career progression, which tends to depend on their short-term performance relative to other fund managers. A strategy based on underlying value, that is likely to pay off in the long-term, is an extremely risky strategy for a manager who might be out of a job before it can bear fruit: it is more sensible to act similarly to others.[3] Even if success is achieved in this manner, it may not be recognised:

> "it is the long-term investor, he who most promotes the public interest, who will in practice come in for most criticism, wherever investment funds are managed by committees or boards or banks. For it is in the essence of his behaviour that he should be eccentric, unconventional and rash in the eyes of average opinion.... Worldly wisdom teaches that it is better for reputation to fail conventionally than to succeed unconventionally" (Keynes, 1936, V, p. 157-8).

In addition to this, Allen & Gorton (1993) demonstrate that asymmetric information between investors and portfolio managers can provide managers with an incentive to churn, which can result in bubbles.

## 1.8 Concluding comments

We have seen that rational expectations models often require liquidity traders to provide gains from trade and informational noise. The specific form of liquidity - or 'ill-informed' - trader behaviour is also important, even when there are competitive rational investors present in the market, and is especially so when the bulk of investors follow mechanistic rules. As we stated at the start of this chapter, the thesis as a whole follows a similar structure to this introduction: we can think of it as charting the implications of the presence of liquidity traders as their demand alters

[3] This idea provides the motivation for Scharfstein & Stein (1990).

from being purely random as in the information aggregation models of chapters two, three and four, to varying through time or with price changes as in the later chapters.

The thesis can also be viewed more explicitly in terms of the informational assumptions made. Chapter two looks at the ability of stock prices to aggregate information when this information is dispersed throughout the economy, in the sense that any given investor receives only part of the information required to form an estimate of the future stock value. Chapter three looks at the ability of stock prices to aggregate two different types of information, relating to both the future stock value directly and the amount of liquidity trading. Chapter four looks at the incentives for investors to acquire information of the two types, within the framework constructed in chapter three. We then begin to consider the possibility that some investors may not have sufficient information to be able to conform to the requirements of rational expectations. In chapter five we look at the implications of the presence of such investors, focusing particularly on the fads model of Shiller (1984) and the possibility of profitable destabilising speculation. In chapter six we provide evidence that such 'naive' or 'ill-informed' investor behaviour can take the form of positive feedback trading, under which the demand from these investors bears a positive relationship to current and / or past prices. In chapter seven we investigate the performance of a system containing positive feedback traders along with investors who trade on the basis of underlying value, and, following Hart (1977), assess the potential for manipulation in such a situation. We finish in chapter eight by assessing, in the light of chapter seven, the conclusion of De Long, Shleifer, Summers and Waldmann (1990a) that competitive speculation can be destabilising.

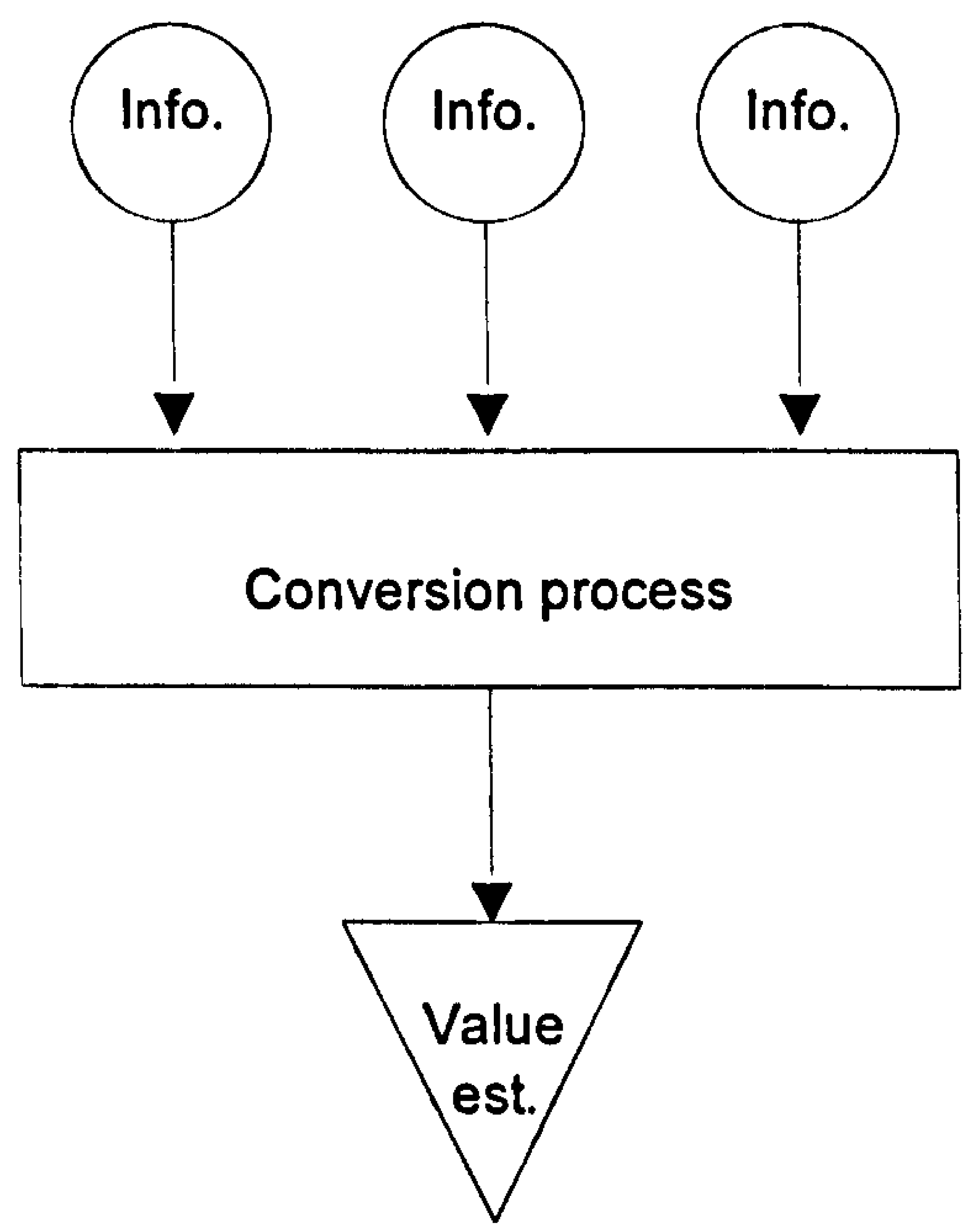
# Chapter 2

# On the aggregation of dispersed information

## 2.1 Introduction

This chapter contains an example of a model in which the noise traders play a facilitating role in allowing an equilibrium to exist, but are not central to the model performance beyond this. The motivation behind this specific formulation is the fact that, in reality, information about future value comes in many different forms, relating to different aspects of the determinants of the future value; and these different pieces of information, which do not correspond directly to estimates of future value, must be aggregated and converted into such an estimate. The assumption used in the information aggregation models we considered previously was that the information obtained by the agents concerning the liquidation value related to the entire liquidation value, and so represents an extreme simplification. Figure 2.1 illustrates the link between information and the value estimate.

*Fig. 2.1: The information conversion process.*




It is possible to think of the 'conversion process' as a model that translates relevant data into a value estimate in the same way that a macroeconomic model uses data about certain underlying economic variables to produce estimates for the future values of key macroeconomic variables.

In addition to this feature of information, the overall stock of information is dispersed among agents, with different agents receiving information about different factors. This raises questions about how stock prices can aggregate information that is dispersed among agents, an issue that has so far not been addressed by the information aggregation literature.

The information aggregation literature, beginning with Grossman (1976), and developing with work such as that by Hellwig (1980) and Diamond and Verrecchia (1981), Admati (1985), Kim and Verrecchia (1991a, b) and Brennan and Cao (1996), has tended to assume that investors receive information signals relating to the entire future value itself, and so has avoided the need to investigate the issues of conversion processes and the dispersion of information.

One possible conversion process is the linear factor model that underlies the arbitrage pricing theory (A.P.T.) of Ross (1976), under which the return on an asset is a linear function of a number of fundamental factors. Handa & Linn (1991) show how the A.P.T. can be embedded in the *n*-asset information aggregation framework of Admati (1985), itself an extension of Hellwig (1980). Handa & Linn made the assumption that each investor receives information about all the factors. In this chapter we use a single-asset model based on Hellwig (1980) to investigate the ability of prices to aggregate information when the investors specialise in acquiring information about one factor only.[1] Although this in a sense circumvents the problem we highlighted earlier concerning the difficulty of converting raw data into value estimates, it does test in a new way the ability of prices to aggregate information across investors.

We will be working in a 'large market,' as first described by Hellwig (1980), in which individual investors do not influence the price. As well as being tractable, this is consistent with the price-taking assumption, and so avoids the problem of

[1] When the following chapter was first written, the author was unfortunately not aware of the work of Handa & Linn (1991).

schizophrenic investor behaviour present in finite-agent models, as pointed out by Hellwig, whereby investors take into account the covariance between their signal and the price, but still act as price-takers. This problem cannot be avoided in the Diamond & Verrecchia formulation, since when the number of agents becomes 'large', the noise in the stock supply disappears and the Grossman (1976) problem of the non-existence of an equilibrium recurs. It is possible to overcome the problem in finite-investor models by assuming that each investor represents a continuum of agents with identical information. Kyle (1989) proposed an alternative method of dealing with the schizophrenia problem, which was to allow the agents to take account of their influence on the price. Kyle showed that each investor trades less aggressively in this case than when perfectly competitive, which reduces the amount of information that is reflected in the price. The results obtained working in a 'large market' framework are, however, more tractable, which is why this has become the standard approach in the literature, and why we will use it in the models contained in the following three chapters.

Grinblatt & Ross (1985) show that in the standard (Hellwig-type) information aggregation model structure it can be profitable for a non price-taking investor to behave strategically to alter the equilibrium price function, even when the strategy is both linear and committed to in advance. We will not consider the possibility of strategic behaviour in the information aggregation work that follows.

The structure of this chapter is as follows. In the following section we derive the equilibrium price function for the general form of the model, which can be thought of as building on the foundations of the large-market case of Hellwig (1980). We then assume full homogeneity across the liquidation-value components, and describe this more tractable case, giving two numerical examples of how the informativeness of the price falls with the number of components. We finish with a look at the possible implications of informational gains from specialisation, and conclude that a broad information base is vital.

## 2.2 Liquidation Value With *N* Additive Components

Consider an economy in which there are two assets, one risk-free and one risky, both of which pay out (the only) units of the single consumption good in the following period. The risk-free asset pays out a single unit of the consumption good with certainty, while the risky asset pays out a random quantity of consumption goods. The payoff to the risky asset - $\tilde{u}$ , the liquidation value - is drawn from a normal distribution with mean and variance as follows:

$$\tilde{u} \sim N\left(\bar{u}\,, 1/h_o\right) \tag{2.1}$$

Assume that the liquidation value of the stock is the sum of a number of different components; in particular:

$$\tilde{u} = \tilde{u}_1 + \tilde{u}_2 + \tilde{u}_3 + \ldots\ldots + \tilde{u}_N \tag{2.2}$$

For the sake of simplicity also assume that these components are distributed independently of each other. Each component can be thought of as a coefficient multiplied by a factor that partly determines the payoff.

The investors are aware of the structure of the economy - specifically the value of the coefficients in the price function - and can also observe the equilibrium price itself before trading. They also receive heterogeneous pieces of private information. There are a 'large' number of these rational agents, and so individually they cannot influence the market price. We assume that there are now $N$ types of competitive agent, present in the market in measure $\mu_n$, with:

$$\sum_{n=1}^{N} \mu_n = 1 \tag{2.3}$$

Following Kim & Verrecchia (1991a), we assume for convenience that the agents of each type form a [0, 1] continuum ($i_n \in [0, 1]$).

Agents' utility depends solely on their consumption of the consumption good in the final period. It is assumed that the agents' preferences can be characterised by utility functions that exhibit constant absolute risk aversion. The importance of this assumption is that it ensures that the price does not feed back to agent demand via wealth, and hence greatly contributes to model tractability. Specifically, agent $i$ has a negative-exponential utility function that takes the following form:

$$U_i(W_i) = -\exp(-W_i / r_i) \tag{2.4}$$

where $W$ represents terminal wealth and $r$ the risk-tolerance coefficient.

The stock supply available to the rational investors ($\tilde{L}$) can be thought of as the sum of the amount of stock outstanding and the net supply from liquidity traders.

We assume that agent *i* of type *n* receives private information $\tilde{y}_{ni}$ regarding one of the components of the liquidation value, where:

$$\tilde{y}_{ni} = \tilde{u}_n + \tilde{\varepsilon}_{ni} \tag{2.5}$$

The random vector $\left(\tilde{u}_n, \tilde{L}, \tilde{\varepsilon}_{ni}\right)$ is assumed to be distributed normally with mean $\left(\bar{u}_n, \bar{L}, 0\right)$ and variance $\left(\frac{1}{h_n}, \frac{1}{p_L}, \frac{1}{s_n}\right)$. It is also assumed that these random variables are distributed independently, and so all covariances are zero.

The agents now hypothesise a price function of the following form:

$$\tilde{P} = \alpha + \beta_1 \tilde{u}_1 + \beta_2 \tilde{u}_2 + \ldots + \beta_N \tilde{u}_N - \gamma \tilde{L} \tag{2.6}$$

where $\tilde{L}$ is the per-capita stock supply, and:

$$\alpha = \alpha_{u_1} \bar{u}_1 + \alpha_{u_2} \bar{u}_2 + \ldots + \alpha_{u_N} \bar{u}_N + \alpha_L \bar{L}$$

We show in Appendix A (section A1) that the coefficients can be expressed as follows:

$$\begin{aligned}
\gamma &= \frac{1}{\sum_{n=1}^{N} \mu_n r_n K_n \left(1 - \alpha_{2n}\right)} \\
\beta_n &= \mu_n r_n K_n \gamma \alpha_{1n} \\
\alpha &= \gamma \sum_{n=1}^{N} \mu_n r_n K_n \alpha_{0n}
\end{aligned} \tag{2.7}$$

where:

$$\alpha_{1n} = \frac{1}{b_n h_n}\left[\left(\sum_{n=1}^{N} \frac{\beta_n^{\,2}}{h_n}\right) - \beta_n \left(\sum_{n=1}^{N} \frac{\beta_n}{h_n}\right) + \frac{\gamma^2}{p_L}\right]$$

$$\alpha_{2n} = \frac{1}{b_n}\left[\left(\frac{1}{h_n}+\frac{1}{s_n}\right)\left(\sum_{n=1}^{N}\frac{\beta_n}{h_n}\right)-\frac{\beta_n}{h_n^2}\right]$$

$$\alpha_{0n} = \left(\sum_{n=1}^{N}\bar{u}_n\right)-\alpha_{1n}\bar{u}_n-\alpha_{2n}\left[\alpha+\left(\sum_{n=1}^{N}\beta_n\bar{u}_n\right)-\gamma\bar{L}\right]$$

$$\begin{aligned}\frac{1}{K_n} = \frac{1}{b_n}\Bigg\{&\left(\sum_{n=1}^{N}\frac{\beta_n^2}{h_n}\right)\left[\left(\frac{1}{h_n}+\frac{1}{s_n}\right)\left(\sum_{n=1}^{N}\frac{1}{h_n}\right)-\frac{1}{h_n^2}\right]\\ &-\left(\sum_{n=1}^{N}\frac{\beta_n}{h_n}\right)\left[\left(\frac{1}{h_n}+\frac{1}{s_n}\right)\left(\sum_{n=1}^{N}\frac{\beta_n}{h_n}\right)-2\frac{\beta_n}{h_n^2}\right]\\ &-\frac{\beta_n^2}{h_n^2}\left(\sum_{n=1}^{N}\frac{1}{h_n}\right)+\frac{\gamma^2}{p_L}\left[\left(\frac{1}{h_n}+\frac{1}{s_n}\right)\left(\sum_{n=1}^{N}\frac{1}{h_n}\right)-\frac{1}{h_n^2}\right]\Bigg\}\end{aligned}$$

$$b_i = \left(\frac{1}{h_n}+\frac{1}{s_n}\right)\left[\left(\sum_{n=1}^{N}\frac{\beta_n^2}{h_n}\right)+\frac{\gamma^2}{p_L}\right]-\frac{\beta_n^2}{h_n^2}$$

The equation for $\gamma$ can be re-written as:

$$\begin{aligned}\frac{1}{\gamma} = &\left(\sum_{n=1}^{N}\frac{\beta_n^2}{h_n}-\sum_{n=1}^{N}\frac{\beta_n}{h_n}+\frac{\gamma^2}{p_L}\right)\left[\sum_{n=1}^{N}\frac{\mu_n r_n K_n}{b_n}\left(\frac{1}{h_n}+\frac{1}{s_n}\right)\right]\\ &+\sum_{n=1}^{N}\frac{\mu_n r_n K_n}{b_n h_n^2}\beta_n(1-\beta_n)\end{aligned} \tag{2.8}$$

As with the original Hellwig (1980) case, the equation for beta can be analysed independently of the others: defining a new variable $Q_n$ as $\beta_n/\gamma$, it can be re-written as:

$$Q_n = \mu_n r_n K_{An}\alpha_{A1n} \tag{2.9}$$

where:

$$\alpha_{An} \equiv \alpha_{1n}\frac{b_n}{\gamma^2} = \frac{1}{h_n}\left[\left(\sum_{n=1}^{N}\frac{Q_n^2}{h_n}\right)-Q_n\left(\sum_{n=1}^{N}\frac{Q_n}{h_n}\right)+\frac{1}{p_L}\right]$$

and:

$$\frac{1}{K_{An}} \equiv \frac{b_n}{K_n}$$

$$= \left\{ \left( \sum_{n=1}^{N} \frac{Q_n^2}{h_n} \right) \left[ \left( \frac{1}{h_n} + \frac{1}{s_n} \right) \left( \sum_{n=1}^{N} \frac{1}{h_n} \right) - \frac{1}{h_n^2} \right] \right.$$

$$- \left( \sum_{n=1}^{N} \frac{Q_n}{h_n} \right) \left[ \left( \frac{1}{h_n} + \frac{1}{s_n} \right) \left( \sum_{n=1}^{N} \frac{Q_n}{h_n} \right) - \frac{2Q_n}{h_n^2} \right]$$

$$\left. - \frac{Q_n^2}{h_n^2} \left( \sum_{n=1}^{N} \frac{1}{h_n} \right) + \frac{1}{p_L} \left[ \left( \frac{1}{h_n} + \frac{1}{s_n} \right) \left( \sum_{n=1}^{N} \frac{1}{h_n} \right) - \frac{1}{h_n^2} \right] \right\}$$

These coefficients denote the 'structure' of market prices. When the agents believe the price function takes the linear form previously assumed, with the coefficients taking the above values, the price function will indeed take a linear form with coefficients taking these values. The agents' behaviour is therefore consistent with rational expectations. Although these models determine prices in one period only, we can perhaps think of each new 'game' as the latest in a long series, during which the agents have learned the structure of the price function. This assumes, of course, that the price function has remained constant over this period. Blume, Bray and Easley (1982) provides a survey of the literature relating to the stability of rational expectations equilibria in general.

# 2.3 Full homogeneity

In this section we will assume full homogeneity, across both agents and components, in order to facilitate the study of the effects of changing the number of components ($N$). In particular, we assume the following:

$$\begin{aligned} &r_1 = r_2 = \ldots\ldots = r_N = r \\ &s_1 = s_2 = \ldots\ldots = s_N = Ns \\ &h_1 = h_2 = \ldots\ldots = h_N = Nh \\ &\mu_1 = \mu_2 = \ldots\ldots = \mu_N = \frac{1}{N} \\ &\bar{u}_1 = \bar{u}_2 = \ldots\ldots = \bar{u}_N = \frac{\bar{u}}{N} \end{aligned} \tag{2.10}$$

These imply the following:

$$\begin{aligned} &K_1 = K_2 = \ldots\ldots = K_N = K \\ &\alpha_{u_1} = \alpha_{u_2} = \ldots\ldots = \alpha_{u_N} = \alpha_u \\ &\beta_1 = \beta_2 = \ldots\ldots = \beta_N = \beta \end{aligned} \tag{2.11}$$

The price function now takes the following form:

$$\tilde{P} = \alpha + (\tilde{u}_1 + \tilde{u}_2 + \ldots\ldots + \tilde{u}_N)\beta - \gamma\tilde{L} \tag{2.12}$$

where:

$$\begin{aligned} \alpha &= (\bar{u}_1 + \bar{u}_2 + \ldots\ldots \bar{u}_N)\alpha_u + \alpha_L \bar{L} \\ &= \alpha_u \bar{u} + \alpha_L \bar{L} \end{aligned}$$

Using the results from the general case, we can find the values for the coefficients, which are as follows:

$$\alpha_u = \frac{h}{K}$$

$$\alpha_L = \frac{1}{K}\frac{rhsp_L}{\left[(h+s)N-s\right]}$$

$$\beta = \frac{1}{K}\frac{h}{\left[(h+s)N-s\right]}\left\{s+\frac{r^2hs^2p_L}{\left[(h+s)N-s\right]}\right\} \qquad (2.13)$$

$$= 1-\alpha_u$$

$$\gamma = \frac{1}{rK}\left\{1+\frac{r^2hsp_L}{(h+s)N-s}\right\}$$

and:

$$K = \frac{h}{(h+s)N-s}\left\{(h+s)N+\frac{r^2hs^2p_L}{\left[(h+s)N-s\right]}\right\}$$

As with previous information aggregation models,[2] the mean price level is as follows:

$$\begin{aligned}\overline{P} &= (\alpha_u+\beta)\overline{u}-(\gamma-\alpha_L)\overline{L}\\ &= \overline{u}-\frac{1}{rK}\overline{L}\end{aligned} \qquad (2.14)$$

The beta-coefficient reveals the extent to which prices reflect the liquidation value, and so gives an indication of the informativeness of the price. We show in the appendix (A2) that beta decreases with the number of liquidation-value components ($N$) (in fact, as this number becomes 'large,' beta approaches zero), and so the smaller the proportion of the liquidation value about which the agents have information, the less informative prices become. The rapidity of the fall-off in beta can be illustrated with the aid of some numerical examples.

[2] See, for example, the following two chapters of this thesis.

# 2.4 Numerical examples

## 2.4.1 Example #1: 'Simple' parameter values

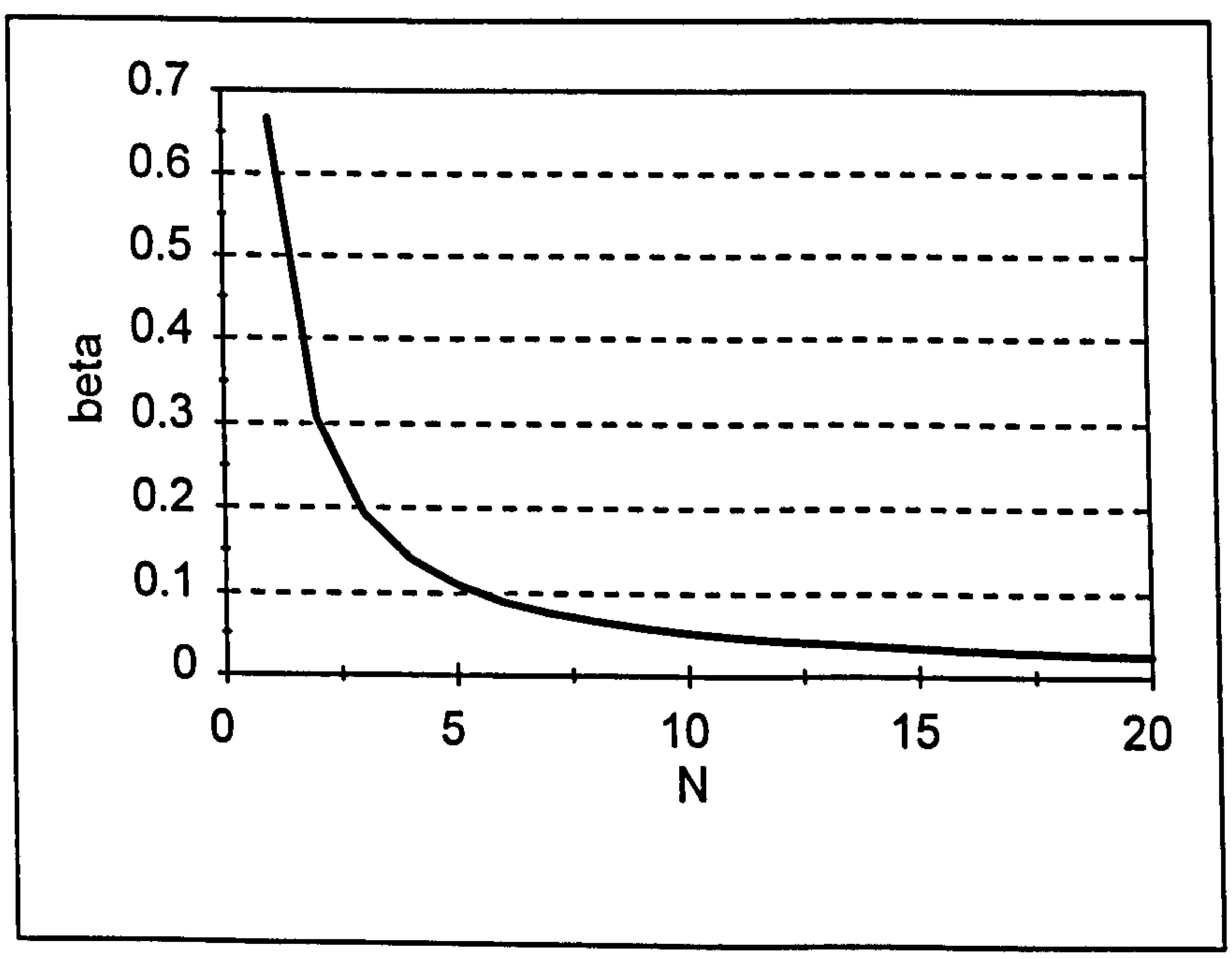
Assume that the parameter values are as given in table 2.1.

*Table 2.1: Simple parameter values.*

| $r$ | $h$ | $s$ | $p_L$ |
|---|---|---|---|
| 1 | 1 | 1 | 1 |

Given these values, the relationship between beta and the number of components making up the liquidation value is as shown in figure 2.2.

*Fig. 2.2: Beta & N with 'simple' parameter values.*




This shows that for these parameter values the informativeness of the price drops off very quickly as the number of components rises above one.

### 2.4.2 Example #2: Gennotte & Leland parameter values

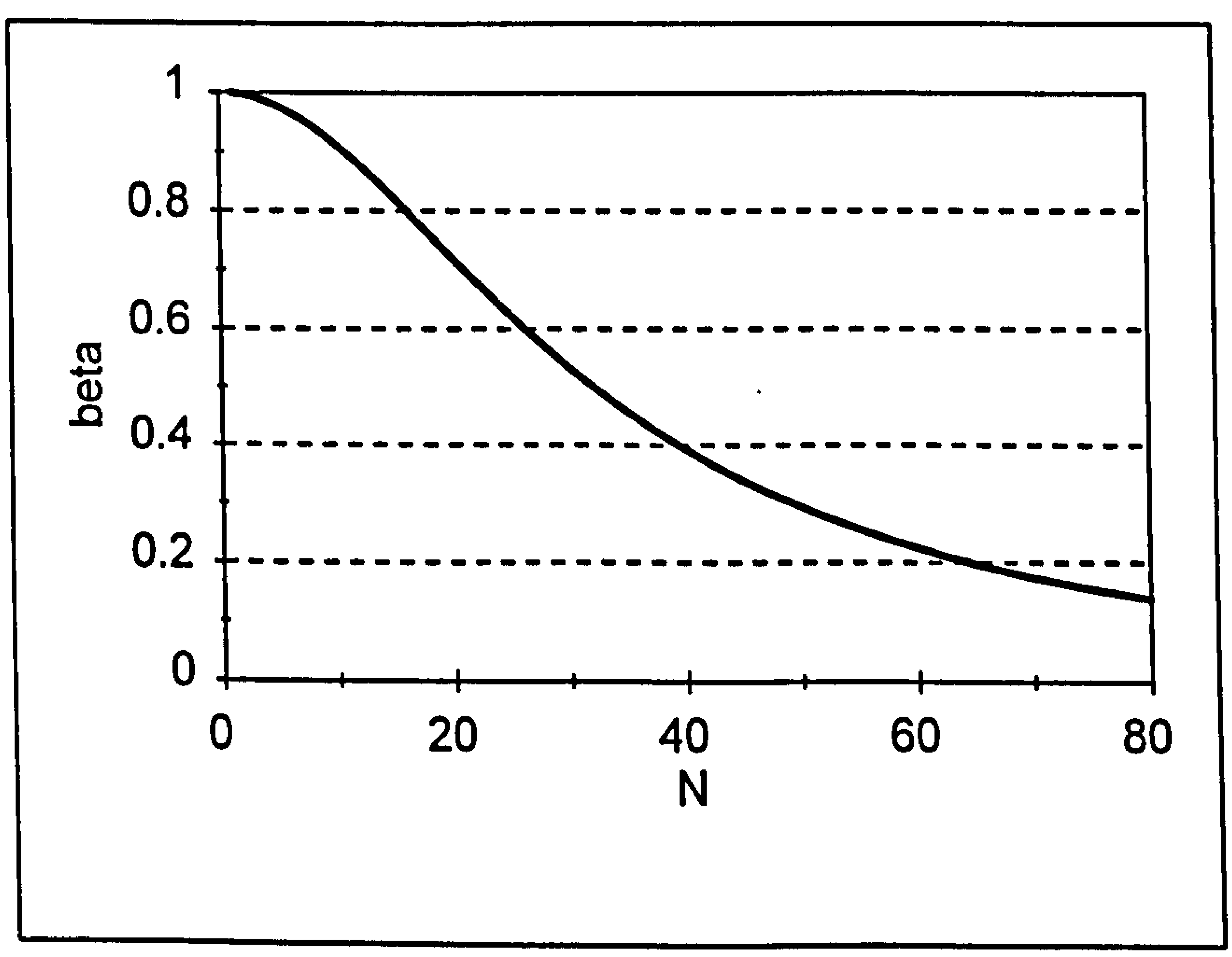
In order to provide an alternative example to the one given above, and one that perhaps better reflects reality, we can use the parameter values given in Gennotte & Leland (1990). These are given in table 2.2.

*Table 2.2: Gennotte & Leland parameter values.*

| $r$ | $h$ | $s$ | $p_L$ |
|---|---|---|---|
| 1 | 12.5 | 2.5 | 2942 |

Given these values, the relationship between $N$ and beta is as given in figure 2.3.

*Fig. 2.3: Beta & N with Gennotte & Leland parameter values.*




It is clear that the value of beta, and hence the informativeness of the price, drops away far more slowly in this example than in the previous one using relatively unsophisticated parameter values; indeed, the number of components must rise to thirty-three before beta is halved. This slower reduction in beta is a consequence of the rate of decrease being in large measure determined by the relative size of the risk tolerance ($r$) and the precision of the stock supply ($p_L$) parameters - as is revealed more clearly by the expression for the reciprocal of beta given in the appendix (A2) - and the fact that in this example the supply precision term is large.

With the liquidation value comprising only one component, as in traditional information aggregation models, the informativeness of the price is extremely high ($\beta = 0.999321$). This would make the incentive to acquire information very weak, as has previously been recognised.[3] By allowing the agents to observe information relating to only one of several components of the liquidation value, the informativeness is reduced to more realistic levels: for example, if we define informativeness as the divergence of beta from one, and changes in informativeness as the new level expressed as a proportion of the base-line level, we can say that increasing the number of components from one to two reduces the level of informativeness by almost 80% ($\beta = 0.996721$); and increasing the number of components to nine reduces it by 99% ($\beta = 0.929132$). This illustration shows that, even though the fall-off in beta may not appear to be as dramatic in the latter example as in the former, it is still enough to have an enormous effect on the level of informativeness.

[3] See, for example, Kyle (1989).

# 2.5 Specialisation versus diversification

In the above we have assumed that, even though we have restricted the agents to the acquisition of information relating to only one of several components, there have been no informational gains from the specialisation: the pooling of information signals by one member of each group will produce a 'super-signal' of merely the same precision as in the single-component case. If we assume that the enforced specialisation will lead to a higher quality of information, and a theoretical super-signal of a higher precision than the single-component case, the effect on price informativeness of increasing the number of components will not be so pronounced.

Instead of constraining the precision of the information for each component to be $N$ times the value of the single-component precision ($s_0$), we can represent it as $N$ times a theoretical super-signal ($s_s$) that is a function of the number of components and the single-component precision, and can exceed this single-component precision:

$$s_i(N) = Ns_s(s_0, N) \geq Ns_0 \tag{2.15}$$

The coefficients of the price function under this formulation can be found simply by substituting in the super-signal precision ($s_s$) for the single-component precision ($s_0$). The beta coefficient can therefore be expressed as follows:

$$\beta = \frac{s_s\left[(h+s_s)N - s_s\right] + r^2 h s_s^2 p_L}{(h+s_s)N\left[(h+s_s)N - s_s\right] + r^2 h s_s^2 p_L} \tag{2.16}$$

where $s_s = s_s(s_0, N)$. Equating this expression with that for the single-component case, as given in the appendix (A3), allows us to discover the functional form for the specialisation function that maintains beta at its initial level as the number of components increases. The form of the specialisation function that maintains a constant level of beta is as follows:[4]

$$s_s(\beta(i = N) = \beta(i = 1)) = \frac{Nhs_0}{h - (N - 1)s_0} \tag{2.17}$$

Since the super-signal precision cannot become negative, there is an upper limit to the value of $N$ ($N_{limit}$) for which it can continue to change to maintain the constant beta. This upper limit is given by the following:

$$N_{limit} = 1 + \frac{h}{s_0} \tag{2.18}$$

Once the number of components rises above this level, no amount of gains from specialisation can prevent the informativeness of the price from falling. This shows that, although specialisation may increase price informativeness as the number of components rises initially, there comes a point where there is no substitute for a wider information base.

Returning to the two examples used previously, we can see that for those cases specialisation would no longer be able to maintain the informativeness of prices as the number of components moves beyond 2 and 6 respectively.

[4] This is equivalent to maintaining a constant level of the conditional precision for each agent ($K_i$.).

## 2.6 Conclusion

We began this chapter by highlighting the necessity of a 'conversion process' between raw data and value estimates. The additive conversion process used subsequently goes only a very small way to recognising this, since the information is still assumed to relate to future value; and so much work is still needed in this area. Using the additive conversion process, and assuming that agents can receive information about one component only, we showed that increasing the number of components necessarily reduces the informativeness of the price when a 'super-signal' consisting of one signal for each of the components would have the same precision as in the single-component case. Once we admitted the possibility of informational gains from specialisation, it became clear that increasing the number of components may initially increase informativeness; but even in this case, informativeness must by necessity begin to fall as the number of components continues to increase beyond a threshold determined by the unconditional precision of the liquidation value and the precision of the information signal in the single-component case. This demonstrates that there is a balance to be struck between specialisation and a broader-based approach to information gathering.

The logical next step would be to relax the assumption that agents can receive information relating to one component alone, in order to study the distribution of information that is likely to occur in equilibrium when information acquisition is made endogenous. This, however, is beyond the scope of this thesis, and will be left as an issue to be addressed in future work.

# Appendix A: For Chapter 2

## A1 General case

Given the postulated price function, $(\tilde{u}, \tilde{y}_{ni}, \tilde{p})$ is distributed normally with mean $M$, where:

$$M = \left[\bar{u}_1 + \bar{u}_2 + \ldots\ldots\ldots + \bar{u}_N, \bar{u}_n,\right.$$
$$\left.(\alpha_{u_1} + \beta_1)\bar{\mu}_1 + (\alpha_{u_2} + \beta_2)\bar{\mu}_2 + \ldots\ldots + (\alpha_{u_N} + \beta_N)\bar{\mu}_N - (\gamma - \alpha_L)\bar{L}\right] \tag{A1}$$

and variance-covariance matrix $V_n$, where:

$$V_n = \begin{pmatrix} \sum_{n=1}^{N} 1/h_n & 1/h_n & \sum_{n=1}^{N} \beta_n/h_n \\ 1/h_n & 1/h_n + 1/s_n & \beta_n/h_n \\ \sum_{n=1}^{N} \beta_n/h_n & \beta_n/h_n & \gamma^2/p_L + \sum_{n=1}^{N} \beta_n^2/h_n \end{pmatrix} \tag{A2}$$

The method for finding the posterior distribution of the liquidation value given the private information and price can be found in texts such as Mood, Graybill & Boes (1973). To begin, partition the vectors $Y$ and $M$, and the matrices $V$ and $R$ as follows:

$$Y = \begin{pmatrix} Y_1^* \\ Y_2^* \end{pmatrix}, \quad M = \begin{pmatrix} M_1 \\ M_2 \end{pmatrix}, \quad V = \begin{pmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{pmatrix}, \quad R = \begin{pmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{pmatrix}. \tag{A3}$$

where $R$ represents the conditional variance-covariance matrix, and $Y_2^* = (\tilde{y}_{ni}, \tilde{P})$.

The conditional mean $(M_1^*)$ and variance $(R_{11}^{-1})$ are as follows:

$$M_1^* = \mu = M_1 + V_{12}V_{22}^{-1}(Y_2^* - M_2)$$
$$R_{11}^{-1} = K^{-1} = \sigma^2 = V_{11} - V_{12}V_{22}^{-1}V_{21} \tag{A4}$$

Using this result we find that, the posterior distribution of $\tilde{u}$ is:

$$E\left(\tilde{X}\middle|\tilde{y}_{ni},\tilde{P}\right)=\alpha_{0n}+\alpha_{1n}\tilde{y}_{ni}+\alpha_{2n}\tilde{P}$$
$$Var^{-1}\left(\tilde{X}\middle|\tilde{y}_{ni},\tilde{P}\right)=K_n \tag{A5}$$

where the values of the coefficients are as given in the main text.

The relationship between the posterior distribution and agent $i$'s demand for stock is as follows:

$$\begin{aligned}D_{ni}\left(\tilde{P},\tilde{y}_{ni};\alpha,\beta_n,\gamma\right)&=r_n\frac{E\left(\tilde{X}\middle|\tilde{y}_{ni},\tilde{P}\right)-\tilde{P}}{Var\left(\tilde{X}\middle|\tilde{y}_{ni},\tilde{P}\right)}\\&=r_nK_n\left[\alpha_{0n}+\alpha_{1n}\tilde{y}_{ni}+\left(\alpha_{2n}-1\right)\tilde{P}\right]\end{aligned} \tag{A6}$$

The total demand can be written as follows:

$$D=\sum_{n=1}^{N}\mu_nD_n \tag{A7}$$

where $\mu_n$ represents the measure of agent type $n$ in the market; we can think of this loosely as the proportion of the total number of agents that receive private information of type $n$. Total demand must equal total supply; this gives the following relationship:

$$\begin{aligned}\tilde{L}&=\sum_{n=1}^{N}\mu_nr_nK_n\left[\alpha_{0n}+\alpha_{1n}\tilde{u}_n+\left(\alpha_{2n}-1\right)\tilde{P}\right]\\&=\sum_{n=1}^{N}\mu_nr_nK_n\left(\alpha_{0n}+\alpha_{1n}\tilde{u}_n\right)+\tilde{P}\sum_{n=1}^{N}\mu_nr_nK_n\left(\alpha_{2n}-1\right)\end{aligned} \tag{A8}$$

Re-arranging reveals:

$$\tilde{P}=\frac{\sum_{n=1}^{N}\mu_nr_nK_n\left(\alpha_{0n}+\alpha_{1n}\tilde{u}_n\right)-\tilde{L}}{\sum_{n=1}^{N}\mu_nr_nK_n\left(1-\alpha_{2n}\right)} \tag{A9}$$

Setting this equal to the hypothesised price function gives the expressions found in the main text.

## A2 Full homogeneity

The expressions for the coefficients can also be expressed as follows:

$$\alpha_u = \frac{[(h+s)N-s]^2}{(h+s)N[(h+s)N-s]+r^2hs^2p_L}$$

$$\alpha_L = \frac{rhsp_L[(h+s)N-s]}{h\{(h+s)N[(h+s)N-s]+r^2hs^2p_L\}}$$

$$\beta = \frac{s[(h+s)N-s]+r^2hs^2p_L}{(h+s)N[(h+s)N-s]+r^2hs^2p_L} \tag{A10}$$

$$\gamma = \frac{1}{rh}\frac{[(h+s)N-s][(h+s)N-s+r^2hs^2p_L]}{\{(h+s)N[(h+s)N-s]+r^2hs^2p_L\}}$$

The reciprocal of the beta coefficient can be written as follows:

$$\frac{1}{\beta} = 1+\frac{[hN-(N-1)s][(h+s)N-s]}{s[(h+s)N-s]+r^2hs^2p_L} \tag{A11}$$

The first derivative of beta with respect to the number of components is as follows:

$$\frac{\partial\beta}{\partial N} = -\frac{(h+s)s[(h+s)N-s]\{(h+s)N-s+2r^2hsp_L\}}{\{(h+s)N[(h+s)N-s]+r^2hs^2p_L\}^2} < 0 \tag{A12}$$

# A3 Single component case

When there is only one component of the liquidation value, the situation becomes equivalent to a special case of the basic Hellwig (1980) model with homogeneous investors. The coefficients are as follows:

$$\begin{aligned} \alpha_u &= \frac{h}{K} \\ \alpha_L &= \frac{rsp_L}{K} \\ \beta &= \frac{s + r^2 s^2 p_L}{K} \\ \gamma &= \frac{1/r + rsp_L}{K} \end{aligned} \tag{A13}$$

and:

$$K = h + s + r^2 s^2 p_L$$

# Chapter 3

# The non-uniqueness of informational equilibria in seasoned-equity markets

## 3.1 Introduction

Stock prices must aggregate information of many different types. They should reflect factors such as future interest rates, exchange rates, technological developments, and the motivations for investors' behaviour. It is not clear *a priori* that there will be a unique way in which such aggregation will occur; and if this is the case, movements between equilibria are likely to lead to prices that are more volatile than the determinants of the underlying value, and may cause price crashes.

The possibility of multiple equilibria has been discovered previously resulting from: adverse selection in the context of a market for a product that can vary in quality

(Wilson, 1980) or similarly in a market for unseasoned equity issues or venture capital projects (Karki and Hodges, 1986); and in a market for seasoned equity in the presence of portfolio insurers (Gennotte and Leland, 1990). To the author's best knowledge, this chapter represents the first example of multiple informational equilibria in a market for seasoned equity in which portfolio insurers are not present.

Standard models in the information aggregation literature, from Grossman (1976) through Hellwig (1980), Diamond and Verrecchia (1981), Admati (1985), Grinblatt and Ross (1985), Gennotte and Leland (1990), Kim and Verrecchia (1991a, 1991b), Naik (1993) and Brennan and Cao (1996), all look at a situation in which the price aggregates information of one type only, which relates to the future liquidation value of the stock. In all these models, except for Grossman (1976), in which there is no equilibrium price, and Gennotte and Leland (1990), this assumption ensures that there is a unique informational equilibrium. In this chapter we look at a situation in which the price aggregates information about both the liquidation value and the stock demand from noise traders, and show that even in the absence of portfolio insurers there may exist multiple potential equilibria for the price. Since the model in this chapter looks at the aggregation of more than one distinct type of information, it differs in approach from the model in the previous chapter, which looks at the aggregation of multiple components of the same type of information.

This work takes as its point of departure the model of Hellwig (1980), to which is added heterogeneous information signals relating to the level of noise trading activity of the same form as the signals relating to the liquidation value. In the section below, the model structure is described. In section three the nature of the equilibria is analysed, and this is followed in section four by a numerical example in which there exist multiple price equilibria.

## 3.2. Model structure

As in the previous chapter, we assume that there are two assets, one risk-free and one risky, both of which pay out (the only) units of the single consumption good in the following period. The risk-free asset pays out a single unit of the consumption good with certainty, while the risky asset pays out a random quantity of consumption goods. As before the liquidation value ( $\tilde{u}$ ) is drawn from a normal distribution with mean and variance as follows:

$$\tilde{u} \sim N\left(\bar{u}, 1/h_o\right) \tag{3.1}$$

As in the previous chapter we work in a 'large market,' and assume for convenience that the agents form a [0, 1] continuum ($i \in [0, 1]$). Agents' utility depend solely on their consumption of the consumption good in the final period. As before we will assume that each agent $i$ has a negative-exponential utility function that takes the following form:

$$U_i(W_i) = -\exp(-W_i/r_i) \tag{3.2}$$

where $W$ represents terminal wealth and $r$ the risk-tolerance coefficient.

The stock supply can be thought of as the sum of the amount of stock outstanding and the net stock supplied by liquidity traders, who do not appear explicitly in the model. Since if agents received information about the total stock supply from liquidity traders the price would become fully revealing, we will assume that the total liquidity supply is the sum of two parts: one part about which agents can have information ($\tilde{S}$); and one part about which they cannot ($\tilde{L}$). It is possible to justify this assumption in many ways; for example by arguing that the portfolio balancing transactions of some institutions are more transparent than of others. The total supply available per rational investor ($\tilde{x}$) can therefore be expressed as follows:

$$\tilde{x} \equiv \tilde{L} + \tilde{S} \tag{3.3}$$

where $\tilde{S}$ and $\tilde{L}$ are assumed to be independently normally distributed.

Each agent receives, before trading, two pieces of information ($\tilde{y}_i$ and $\tilde{w}_i$), in the form of noisy estimates of the liquidation value and the observable part of the stock supply, consisting of the actual values plus noise, as follows:

$$\tilde{y}_i = \tilde{u} + \tilde{\varepsilon}_i \tag{3.4}$$

$$\tilde{w}_i = \tilde{S} + \tilde{\theta}_i \tag{3.5}$$

The random vector $\left(\tilde{u}, \tilde{L}, \tilde{S}, \tilde{\theta}_i, \tilde{\theta}_j, \tilde{\varepsilon}_i, \tilde{\varepsilon}_j\right)$ is distributed normally with mean $\left(\bar{u}, \bar{L}, 0,0,0,0,0\right)$ and variance $\left(\frac{1}{h_o}, \frac{1}{p_L}, \frac{1}{p_S}, \frac{1}{t_i}, \frac{1}{t_j}, \frac{1}{s_i}, \frac{1}{s_j}\right)$. We assume that these random variables are distributed independently, and so all covariances are zero. It will be shown below that there is an equilibrium price function of the following linear form:

$$\tilde{P} = \alpha_u \bar{u} + \alpha_L \bar{L} + \beta \tilde{u} - \gamma_S \tilde{S} - \gamma_L \tilde{L} \tag{3.6}$$

This means that if the traders hypothesise initially that the price function will be of this form, this will indeed be the case, and the value of the coefficients will be as they expected. We show in Appendix B (section B1) that the price function can also be expressed as follows:

$$\begin{aligned}\tilde{P} = \frac{1}{rK}\Bigg\{ & rh_o\bar{u} + \left(\int_0^1 \frac{\beta\gamma_L p_L (p_S + t_i) r_i}{\gamma_L^2 (p_S + t_i) + \gamma_S^2 p_L} di\right)\bar{L} \\ & + \left(rs + \int_0^1 \frac{\beta^2 p_L (p_S + t_i) r_i}{\gamma_L^2 (p_S + t_i) + \gamma_S^2 p_L} di\right)\tilde{u} \\ & - \left(1 + \int_0^1 \frac{\beta\gamma_S p_S p_L r_i}{\gamma_L^2 (p_S + t_i) + \gamma_S^2 p_L} di\right)\tilde{S} \\ & - \left(1 + \int_0^1 \frac{\beta\gamma_L p_L (p_S + t_i) r_i}{\gamma_L^2 (p_S + t_i) + \gamma_S^2 p_L} di\right)\tilde{L}\Bigg\}\end{aligned} \tag{3.7}$$

where $r \equiv \int_0^1 r_i \, di$ , $s \equiv (1/r)\int_0^1 r_i s_i \, di$ , and:

$$K \equiv h_o + s + \frac{1}{r}\int_0^1 \frac{\beta^2 p_L (p_S + t_i) r_i}{\gamma_L^2 (p_S + t_i) + \gamma_S^2 p_L} di$$

When the stock supply $S$ is non-stochastic, and so its variance is zero, the model collapses to the Hellwig (1980) 'large market' case. The solution for this is given in the appendix (B2).

Since it is not clear how a solution can be isolated for this general case, we will therefore now look at special cases, starting with a situation in which there are two types of agent present.

## 3.3 Two agent types

Suppose that there are two groups of agents, present in the market in measure $\mu_f$ and $\mu_g$ and that within each group the agents are homogeneous in that they have the same degree of risk tolerance and receive information of the same precision. The price function can now be expressed as follows:

$$\begin{aligned}\tilde{P} = \frac{1}{rK}\Bigg\{ & rh_o\bar{u} + \beta\gamma_L p_L\left(\frac{\mu_f r_f(p_S+t_f)}{\gamma_L^2(p_S+t_f)+\gamma_S^2 p_L} + \frac{\mu_g r_g(p_S+t_g)}{\gamma_L^2(p_S+t_g)+\gamma_S^2 p_L}\right)\bar{L} \\ & + \left(rs + \beta^2 p_L\left\{\frac{\mu_f r_f(p_S+t_f)}{\gamma_L^2(p_S+t_f)+\gamma_S^2 p_L} + \frac{\mu_g r_g(p_S+t_g)}{\gamma_L^2(p_S+t_g)+\gamma_S^2 p_L}\right\}\right)\tilde{u} \\ & - \left(1 + \beta\gamma_S p_S p_L\left\{\frac{\mu_f r_f}{\gamma_L^2(p_S+t_f)+\gamma_S^2 p_L} + \frac{\mu_g r_g}{\gamma_L^2(p_S+t_g)+\gamma_S^2 p_L}\right\}\right)\tilde{S} \\ & - \left(1 + \beta\gamma_L p_L\left\{\frac{\mu_f r_f(p_S+t_f)}{\gamma_L^2(p_S+t_f)+\gamma_S^2 p_L} + \frac{\mu_g r_g(p_S+t_g)}{\gamma_L^2(p_S+t_g)+\gamma_S^2 p_L}\right\}\right)\tilde{L}\Bigg\}\end{aligned} \tag{3.8}$$

where:

$$\mu_f + \mu_g = 1$$

$$r \equiv \mu_f r_f + \mu_g r_g$$

$$s \equiv (1/r)\{\mu_f r_f s_f + \mu_g r_g s_g\}$$

$$K \equiv (1/r)\{\mu_f r_f K_f + \mu_g r_g K_g\}$$

$$= h_o + s + \frac{\beta^2 p_L}{r}\left\{\frac{(p_S+t_f)\mu_f r_f}{\gamma_L^2(p_S+t_f)+\gamma_S^2 p_L} + \frac{(p_S+t_g)\mu_g r_g}{\gamma_L^2(p_S+t_g)+\gamma_S^2 p_L}\right\}$$

$$K_f \equiv h_o + s_f + \frac{\beta^2 p_L(p_S+t_f)}{\gamma_L^2(p_S+t_f)+\gamma_S^2 p_L}$$

Comparing this to the price function originally hypothesised produces the following equations for the coefficients:

$$\alpha_u = \frac{h_o}{K}$$

$$\alpha_L = \frac{\beta\gamma_L p_L}{rK}\left(\frac{\mu_f r_f (p_S + t_f)}{\gamma_L^2 (p_S + t_f) + \gamma_S^2 p_L} + \frac{\mu_g r_g (p_S + t_g)}{\gamma_L^2 (p_S + t_g) + \gamma_S^2 p_L}\right)$$

$$\beta = \frac{1}{rK}\left\{ rs + \beta^2 p_L \left(\frac{\mu_f r_f (p_S + t_f)}{\gamma_L^2 (p_S + t_f) + \gamma_S^2 p_L} + \frac{\mu_g r_g (p_S + t_g)}{\gamma_L^2 (p_S + t_g) + \gamma_S^2 p_L}\right)\right\}$$

$$\gamma_S = \frac{1}{rK}\left\{ 1 + \beta\gamma_S p_S p_L \left(\frac{\mu_f r_f}{\gamma_L^2 (p_S + t_f) + \gamma_S^2 p_L} + \frac{\mu_g r_g}{\gamma_L^2 (p_S + t_g) + \gamma_S^2 p_L}\right)\right\}$$

$$\gamma_L = \frac{1}{rK}\left\{ 1 + \beta\gamma_L p_L \left(\frac{\mu_f r_f (p_S + t_f)}{\gamma_L^2 (p_S + t_f) + \gamma_S^2 p_L} + \frac{\mu_g r_g (p_S + t_g)}{\gamma_L^2 (p_S + t_g) + \gamma_S^2 p_L}\right)\right\}$$

(3.9)

The isolation of the solution to the above equation set can be facilitated by defining the following new variables:

$$Q_\beta \equiv \frac{\beta}{\gamma_L}, Q_S \equiv \frac{\gamma_S}{\gamma_L} \tag{3.10}$$

The above equations reveal that:

$$Q_\beta = rs \tag{3.11}$$

$Q_S$, however, is now a solution to the following fifth-order polynomial:

$$\begin{aligned} &\{Q_S - 1\}(Q_S^2 p_L + p_S + t_f)(Q_S^2 p_L + p_S + t_g) \\ &\quad + Q_S rsp_L \{(Q_S^2 p_L + p_S + t_g)\mu_f r_f t_f + (Q_S^2 p_L + p_S + t_f)\mu_g r_g t_g\} = 0 \end{aligned} \tag{3.12}$$

Since the coefficients can be expressed in terms of $Q_\beta$ and $Q_S$, they can be produced readily. After substitution for $Q_\beta$ they are as follows:

$$\alpha_u = \frac{h_o}{K}$$

$$\alpha_L = \frac{sp_L}{K}\left(\frac{\mu_f r_f(p_S+t_f)}{p_S+t_f+Q_S{}^2 p_L}+\frac{\mu_g r_g(p_S+t_g)}{p_S+t_g+Q_S{}^2 p_L}\right)$$

$$\beta = \frac{1}{K}\left\{s+rs^2 p_L\left(\frac{\mu_f r_f(p_S+t_f)}{p_S+t_f+Q_S{}^2 p_L}+\frac{\mu_g r_g(p_S+t_g)}{p_S+t_g+Q_S{}^2 p_L}\right)\right\}$$

$$\gamma_S = \frac{1}{K}\left\{\frac{1}{r}+Q_S s p_S p_L\left(\frac{\mu_f r_f}{p_S+t_f+Q_S{}^2 p_L}+\frac{\mu_g r_g}{p_S+t_g+Q_S{}^2 p_L}\right)\right\}$$

$$\gamma_L = \frac{1}{K}\left\{\frac{1}{r}+sp_L\left(\frac{\mu_f r_f(p_S+t_f)}{p_S+t_f+Q_S{}^2 p_L}+\frac{\mu_g r_g(p_S+t_g)}{p_S+t_g+Q_S{}^2 p_L}\right)\right\} \tag{3.13}$$

where:

$$K \equiv (1/r)\{\mu_f r_f K_f + \mu_g r_g K_g\}$$

$$= h_o + s + rs^2 p_L\left\{\frac{\mu_f r_f(p_S+t_f)}{p_S+t_f+Q_S{}^2 p_L}+\frac{\mu_g r_g(p_S+t_g)}{p_S+t_g+Q_S{}^2 p_L}\right\}$$

$$K_f \equiv h_o + s_f + \frac{r^2 s^2 p_L(p_S+t_f)}{p_S+t_f+Q_S{}^2 p_L}$$

From this we can write:

$$K = h_o + s + \frac{r^2 s^2 p_L(p_S+t)}{Q_S^2 p_L + p_S + t}$$

These results are still rather unwieldy: the nature of the solution can be analysed more easily if we assume that the agents are entirely homogeneous.

## 3.4 Homogeneous agents

Assume now that the agents are homogeneous, and so all have the same degree of risk-aversion, and receive information of the same precision. The signals they receive are therefore taken from the same distribution, although the signal of each agent represents a different drawing. In this case, the price equation collapses to the following:

$$\tilde{P} = \frac{1}{K}\left\{ h_o\bar{u} + \frac{\beta\gamma_L p_L(p_S + t)}{\gamma_L^2(p_S + t) + \gamma_S^2 p_L}\bar{L} + \left(s + \frac{\beta^2 p_L(p_S + t)}{\gamma_L^2(p_S + t) + \gamma_S^2 p_L}\right)\tilde{u} \right.$$
$$\left. - \left(\frac{1}{r} + \frac{\beta\gamma_S p_S p_L}{\gamma_L^2(p_S + t) + \gamma_S^2 p_L}\right)\tilde{S} - \left(\frac{1}{r} + \frac{\beta\gamma_L p_L(p_S + t)}{\gamma_L^2(p_S + t) + \gamma_S^2 p_L}\right)\tilde{L} \right\} \tag{3.14}$$

where:

$$K \equiv h_o + s + \frac{\beta^2 p_L(p_S + t)}{\gamma_L^2(p_S + t) + \gamma_S^2 p_L}$$

Comparing this to the price function originally hypothesised produces the following equations for the coefficients:

$$\alpha_u = \frac{h_o}{K}$$

$$\alpha_L = \frac{1}{K}\left\{\frac{\beta\gamma_L p_L(p_S + t)}{\gamma_L^2(p_S + t) + \gamma_S^2 p_L}\right\}$$

$$\beta = \frac{1}{K}\left\{s + \frac{\beta^2 p_L(p_S + t)}{\gamma_L^2(p_S + t) + \gamma_S^2 p_L}\right\} \tag{3.15}$$

$$\gamma_S = \frac{1}{K}\left\{\frac{1}{r} + \frac{\beta\gamma_S p_S p_L}{\gamma_L^2(p_S + t) + \gamma_S^2 p_L}\right\}$$

$$\gamma_L = \frac{1}{K}\left\{\frac{1}{r} + \frac{\beta\gamma_L p_L(p_S+t)}{\gamma_L^2(p_S+t)+\gamma_S^2 p_L}\right\}$$

Identifying the solution to the above equation set can once again be facilitated by using the variables $Q_\beta$ and $Q_S$, which we defined in equation 3.11 above. This reveals that:

$$Q_\beta = rs \tag{3.16}$$

and that $Q_S$ is now the solution to the following cubic:

$$(Q_S - 1)(Q_S^2 p_L + p_S + t) + Q_S p_L r^2 st = 0 \tag{3.17}$$

The following section looks at the properties of the solutions to this equation. The coefficients can be expressed as follows:

$$\begin{aligned}
\alpha_u &= \frac{h_o}{K} \\
\alpha_L &= \frac{1}{K}\left\{\frac{rsp_L(p_S+t)}{Q_S^2 p_L + p_S + t}\right\} \\
\beta &= \frac{1}{K}\left\{s + \frac{r^2 s^2 p_L(p_S+t)}{Q_S^2 p_L + p_S + t}\right\} \\
\gamma_S &= \frac{1}{K}\left\{\frac{1}{r} + \frac{Q_S rsp_S p_L}{Q_S^2 p_L + p_S + t}\right\} \\
\gamma_L &= \frac{1}{K}\left\{\frac{1}{r} + \frac{rsp_L(p_S+t)}{Q_S^2 p_L + p_S + t}\right\}
\end{aligned} \tag{3.18}$$

where:

$$K = h_o + s + \frac{r^2 s^2 p_L(p_S+t)}{Q_S^2 p_L + p_S + t}$$

As in the previous chapter, the expression for the mean level of the stock price is given by the following:

$$
\begin{aligned}
\overline{P} &= \alpha_u \overline{u} + \alpha_L \overline{L} + \beta \overline{u} - \gamma_S \overline{S} - \gamma_L \overline{L} \\
&= (\alpha_u + \beta)\overline{u} - (\gamma_L - \alpha_L)\overline{L} \\
&= \overline{u} - \frac{1}{rK}\overline{L}
\end{aligned} \tag{3.19}
$$

We use this result later in this chapter. A similar expression can be derived for the general case given in section 3.2.

## 3.5 Solutions for $Q_S$

We would expect the ratio of the coefficients of $S$ and $L$ ($Q_S$) to be positive - since higher supply shocks should lead to lower price and vice versa - and less than one when the unconditional precisions are the same - since the agents have some information about this for $S$, thus reducing its impact. The equation for $Q_S$ can be written as:

$$F \equiv Q_S^3 p_L - Q_S^2 p_L + Q_S\left(p_S + t + p_L r^2 st\right) - \left(p_S + t\right) = 0 \tag{3.20}$$

This can be re-written as:

$$F \equiv Q_S^3 a - Q_S^2 a + Q_S(b + c) - b = 0 \tag{3.21}$$

where:

$$
\begin{aligned}
a &\equiv p_L \\
b &\equiv p_S + t \\
c &\equiv p_L r^2 st
\end{aligned}
$$

These new variables - $a$, $b$ and $c$ - can vary independently of each other.

For non-positive values of $Q_S$ the value of this function is always negative. For values of $Q_S$ greater than one, the value of this function is always positive. When the precision of supply information ($t$) is zero, there is a solution where $Q_S$ takes the value of one. This ensures that all the (real) roots of the function must lie in the following range:

$$0 < Q_S \leq 1 \tag{3.22}$$

This result can be replicated intuitively. Given a set of estimates about the supply components held by the agents, unit changes in either of the components will affect the price to the same degree, regardless of the prior distributions, since they will both affect the price only through aggregate supply. $Q_S$ can therefore take the value of one. Whenever agents have some information about a supply component, however, a rise in the level of this component will be heralded by the information signals, and so its price effect will be partly offset. This shows that one is therefore the maximum value of $Q_S$. When the information is noiseless, the supply will affect the price in the same way that it is affected by a change in the mean level of the stock supply. The minimum value of $Q_S$ must therefore exceed zero.

We can examine the possibility of finding more than one solution by looking at the first derivative of this function with respect to $Q_S$:

$$\begin{aligned}\frac{dF}{dQ_S} &= 3Q_S{}^2 p_L - 2Q_S p_L + p_S + t + p_L r^2 st \\ &= 3Q_S{}^2 a - 2Q_S a + b + c\end{aligned} \tag{3.23}$$

At $Q_S = 0$, the value of this function is positive. At $Q_S = 1$, the value of this function is also positive. If the value of this function turns negative between zero and one, this implies that the original function, *F*, has two turning points in this range; and since the constant term in the original function can vary independently from the other coefficients, it also implies that for certain values of this constant term there will be three roots. The roots of the quadratic function in equation 3.23 are given by:

$$Q_S = \frac{1}{3} \pm \frac{\sqrt{4p_L^2 - 12p_L\left(p_S + t + p_L r^2 st\right)}}{6p_L} \tag{3.24}$$

This will have real roots whenever:

$$p_L > 3\left(p_S + t + p_L r^2 st\right) \tag{3.25}$$

or, equivalently:

$$a > 3(b + c) \tag{3.26}$$

This shows that there are values of the parameters for which the slope of the original function turns negative, and hence that there is a sub-set of these values for which there is more than one root.

Since all the parameters are positive, we can work out the range of values in which real roots must lie. This is as follows:

$$0 < Q_S < \frac{2}{3} \tag{3.27}$$

For each value of $Q_S$ the price function will be linear: however, the relationship the price bears to outcomes of the liquidation value and the stock supply will be different. Hence the effect of moving from one equilibrium position to another would be in terms of altering the volatility of prices, the degree to which prices anticipate the following-period liquidation value, and the degree to which noise shocks affect the price. Since the amount of information the price reveals about the liquidation value affects the riskiness of holding stock, it will also affect the general price level, as revealed by the mean of the price equation. Discontinuous shifts in the conditional variance will lead to discontinuous shifts in the mean of the price; and this is our definition of a crash.

## 3.6 A numerical example

### 3.6.1 Finding the potential equilibria

Figure 3.1 shows the $Q_S$ function $F$ for the parameter values shown in table 1. The three roots can easily be seen, and are also given in table 3.1.

*Fig. 3.1: The $Q_S$ function.*

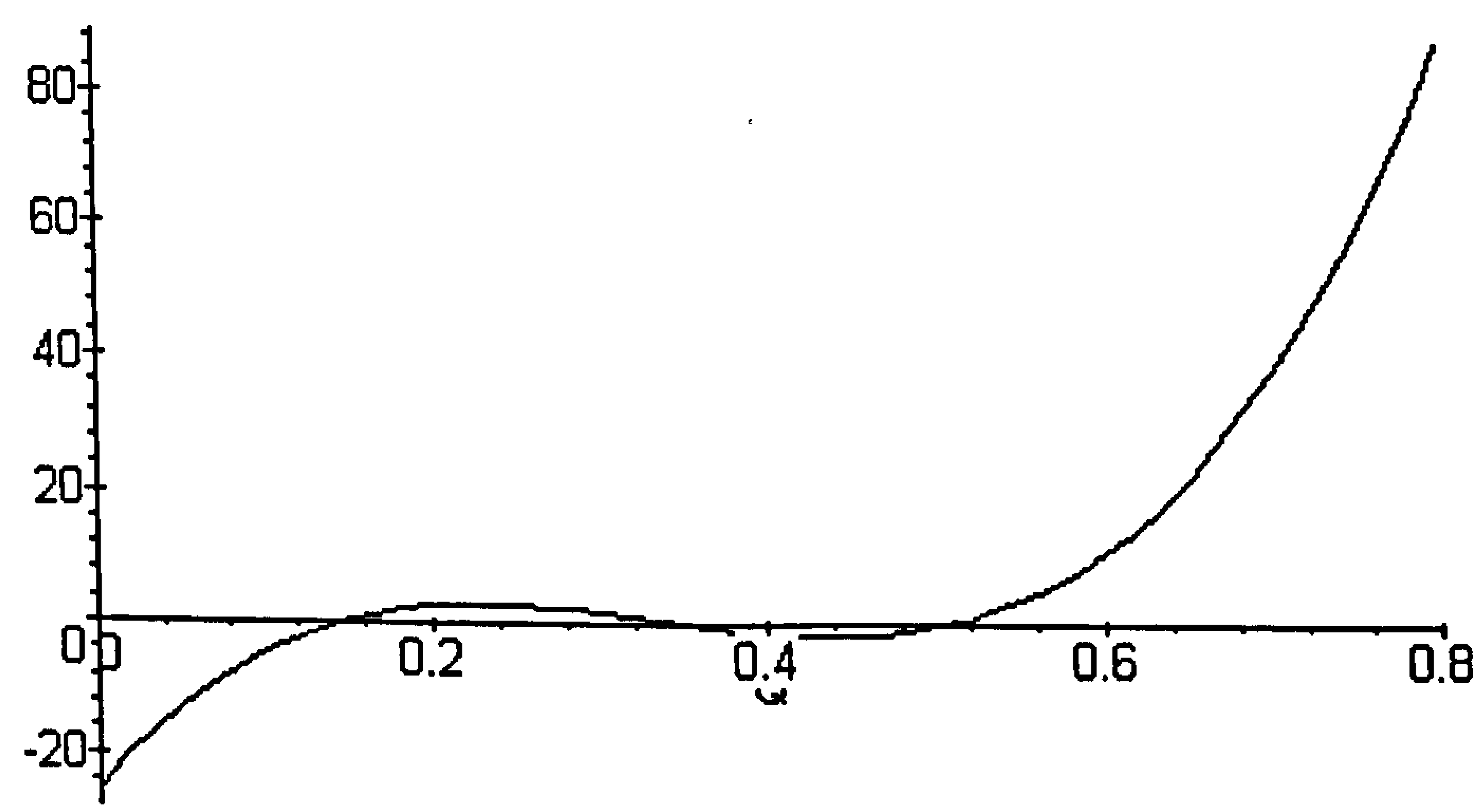


*Table 3.1: Parameter values and roots.*

| Parameter Values. | | Roots for $Q_S$. |
|---|---|---|
| | | |
| $p_L$ | 1000 | 0.14428 |
| $p_S$ | 25 | 0.35055 |
| $s$ | 0.5 | 0.50517 |
| $t$ | 0.55 | |
| $h_o$ | 12 | |
| $r$ | 1 | |

The price functions given by the three roots are shown below in table 3.2:

*Table 3.2: The nature of the equilibria.*

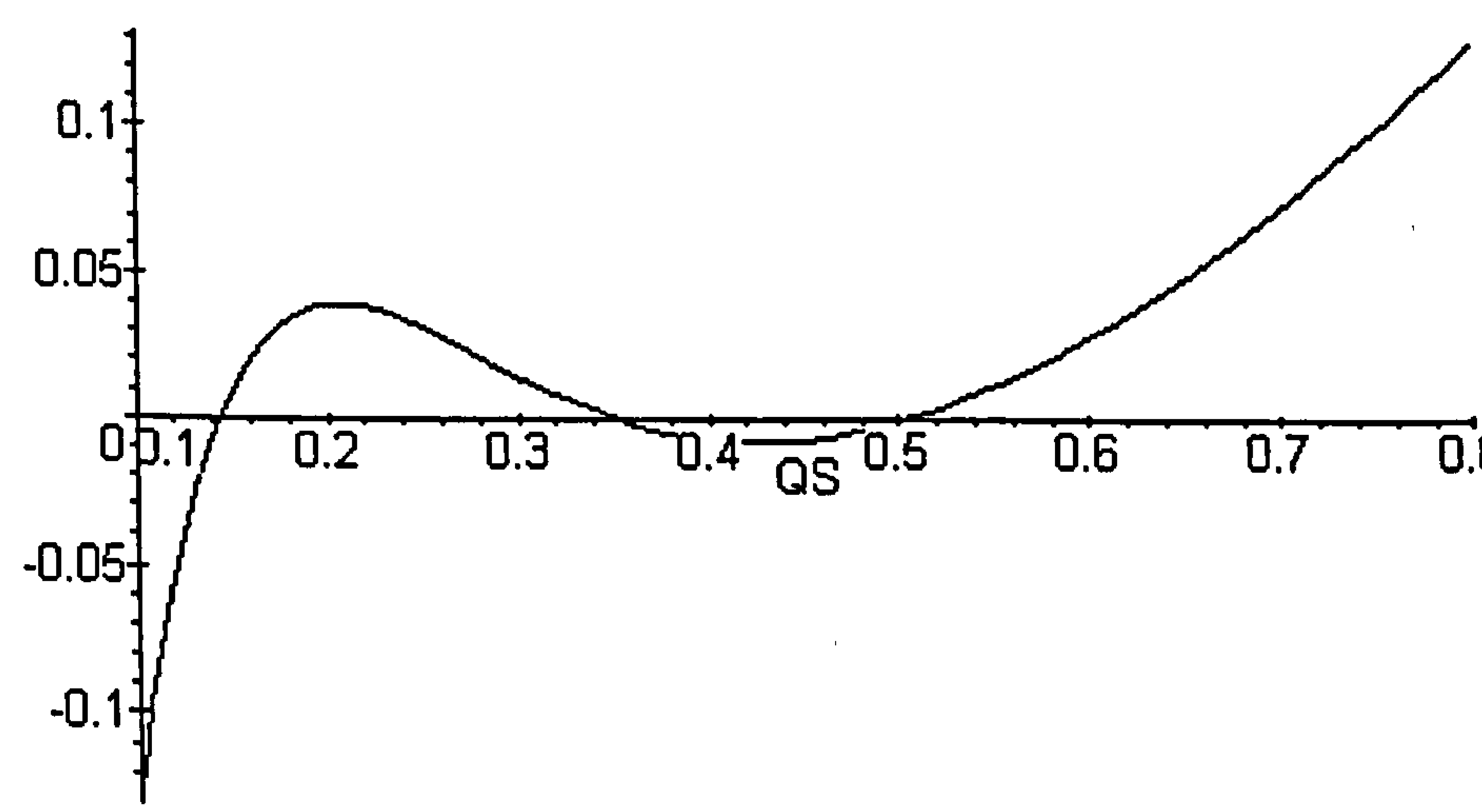
| $Q_S$ | 0.14428 | 0.35055 | 0.50517 |
|---|---|---|---|
| $\alpha_u$ | 0.07986 | 0.21609 | 0.34041 |
| $\alpha_L$ | 1.83362 | 1.54981 | 1.29081 |
| $\beta$ | 0.92014 | 0.78391 | 0.65959 |
| $\gamma_S$ | 0.26551 | 0.54960 | 0.66641 |
| $\gamma_L$ | 1.84028 | 1.56782 | 1.31918 |
| $\text{var}\left(\tilde{u}\middle\vert\tilde{P},\tilde{y},\tilde{w}\right)$ | 0.00666 | 0.01801 | 0.02837 |
| $\text{mean}\left(\tilde{P}\right)$ | $\bar{u}-0.00666\bar{L}$ | $\bar{u}-0.01801\bar{L}$ | $\bar{u}-0.02837\bar{L}$ |
| $\text{var}\left(\tilde{P}\right)$ | 0.07676 | 0.06575 | 0.05576 |

For each of the values of $Q_S$ an equilibrium is possible; and each of these equilibria is associated with a different price function, with the equilibrium price bearing a different relationship to the underlying variables. The above table shows that beta, the coefficient of the liquidation value in the price function, falls, and therefore reveals less about the price, as $Q_S$ rises, which causes the variance of the price of the risky asset to fall - since more weight is placed on the constants - and the conditional variance of the liquidation value to rise. The actual variance and the conditional variance therefore move in opposite directions. For a positive stock supply, the mean price level will move in the opposite direction to the conditional variance.

### 3.6.2 Assessing the stability of the roots

There are three values of $Q_S$ for which initial beliefs can be borne out in the equilibrium price. But what happens at other values of $Q_S$? For each possible value for $Q_S$, we can work out the amount of stock that would be demanded, and hence the excess demand, if this was the value hypothesised by the agents. Figure 3.2 shows the excess demand function, and reveals that it behaves similarly to the $Q_S$ function itself: the roots are the same as for that function; and for values of $Q_S$ below the first root, excess demand is negative; between the first two roots it is positive; between the second and third roots it is negative again; and above the third root it is positive.

*Fig. 3.2: Excess demand.*




The agents can work out the effect on excess demand of changing $Q_S$, so it is perhaps legitimate to assume that they revise their assumptions about $Q_S$ in the direction that would lead to a reduction in excess demand. This ensures that any of the three roots could sustain an equilibrium, with the starting position determining which one prevails. Thinking in terms of a price adjustment mechanism, however, we would expect the price to rise (fall) if the excess demand is positive (negative). By looking at the first derivative of the mean-price function with respect to $Q_S$, we can see in which direction $Q_S$ would have to move to bring about the appropriate price reaction (on average).

It can be shown that:

$$\frac{\partial \overline{P}}{\partial Q_S} = -\frac{2rs^2 p_L^2 (p_S + t) Q_S \overline{L}}{\left[(h_o + s)(p_S + t + p_L Q_S^{\ 2}) + r^2 s^2 p_L (p_S + t)\right]^2} \tag{3.28}$$

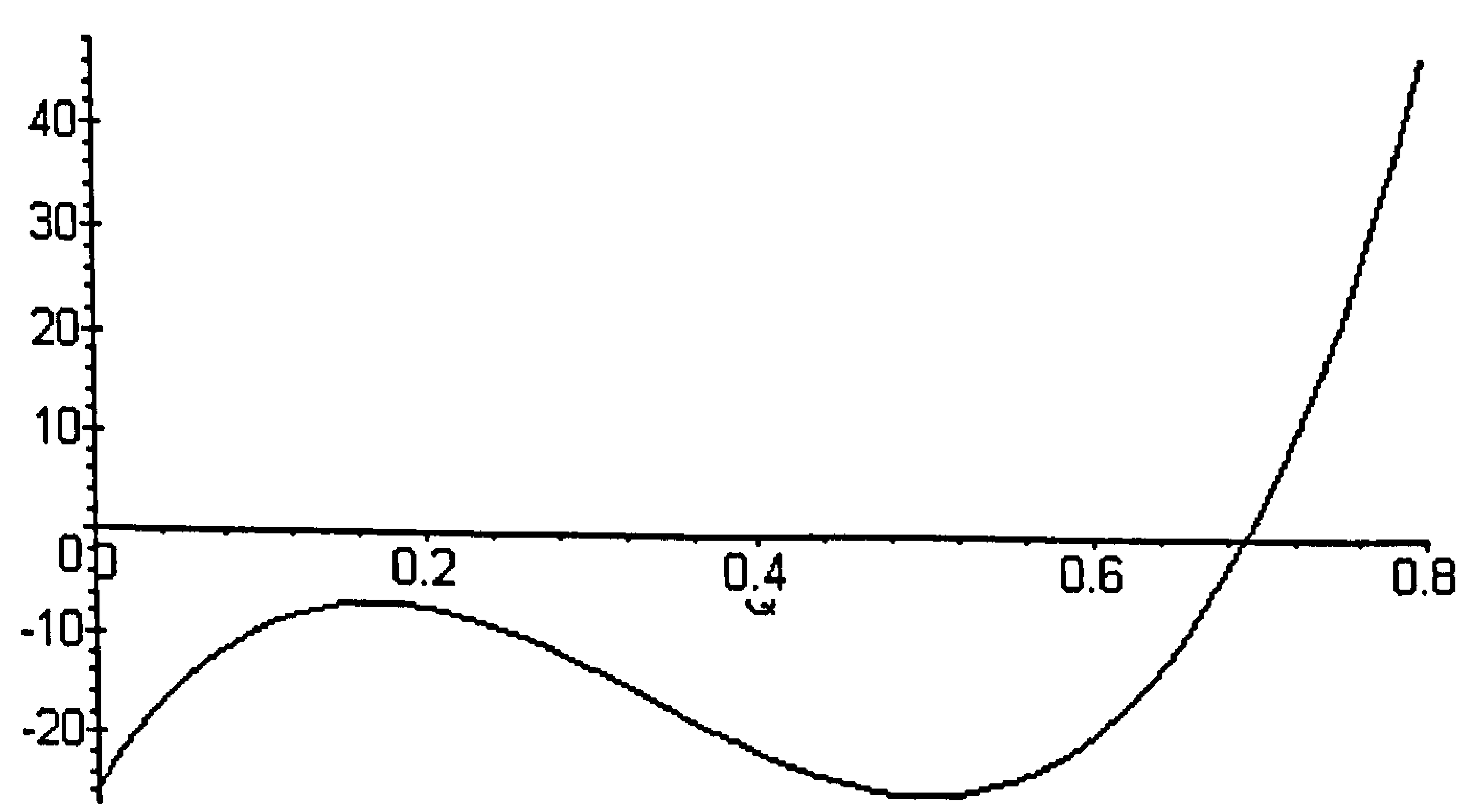
Assume the mean stock supply is positive. The above term is therefore negative, which implies that when excess demand is negative the required fall in price must come about by a rise in $Q_S$, and when it is positive the required rise in price must come about by a fall in $Q_S$. Returning to figure 3.2, we can see that if $Q_S$, is below the first root excess demand is negative, and so price would need to fall to induce additional demand, requiring a rise in $Q_S$; if $Q_S$ is between the first and second roots the positive excess demand can be remedied through the price by a fall in $Q_S$; between the second and third roots $Q_S$ would need to rise; and above the third root $Q_S$ must fall. Hence below the second root the system converges on the first root, and above the second root the system converges on the third root; the first and third roots therefore represent stable roots, while the second one is unstable, since slight deviations from this equilibrium will lead to a move towards one of the other two equilibria, from which a return would not occur.

### 3.6.3 Discontinuous changes in the price function

Imagine that the parameters change over time. We have shown above that the constant term in the $Q_S$ function can vary independently of the other parameters. Were this to occur, the function would shift vertically, either upwards or downwards. This would involve more than one variable changing, with the value of one variable in some way 'compensating' for the change in the other. It seems more sensible to look at movements in the curve brought about by changes in one variable alone. Below we will look at the effects of changes in the value of $t$ (keeping $p_S$ constant). In using a single period model to tell a dynamic story in this way we are following the example set by Gennotte and Leland (1990).

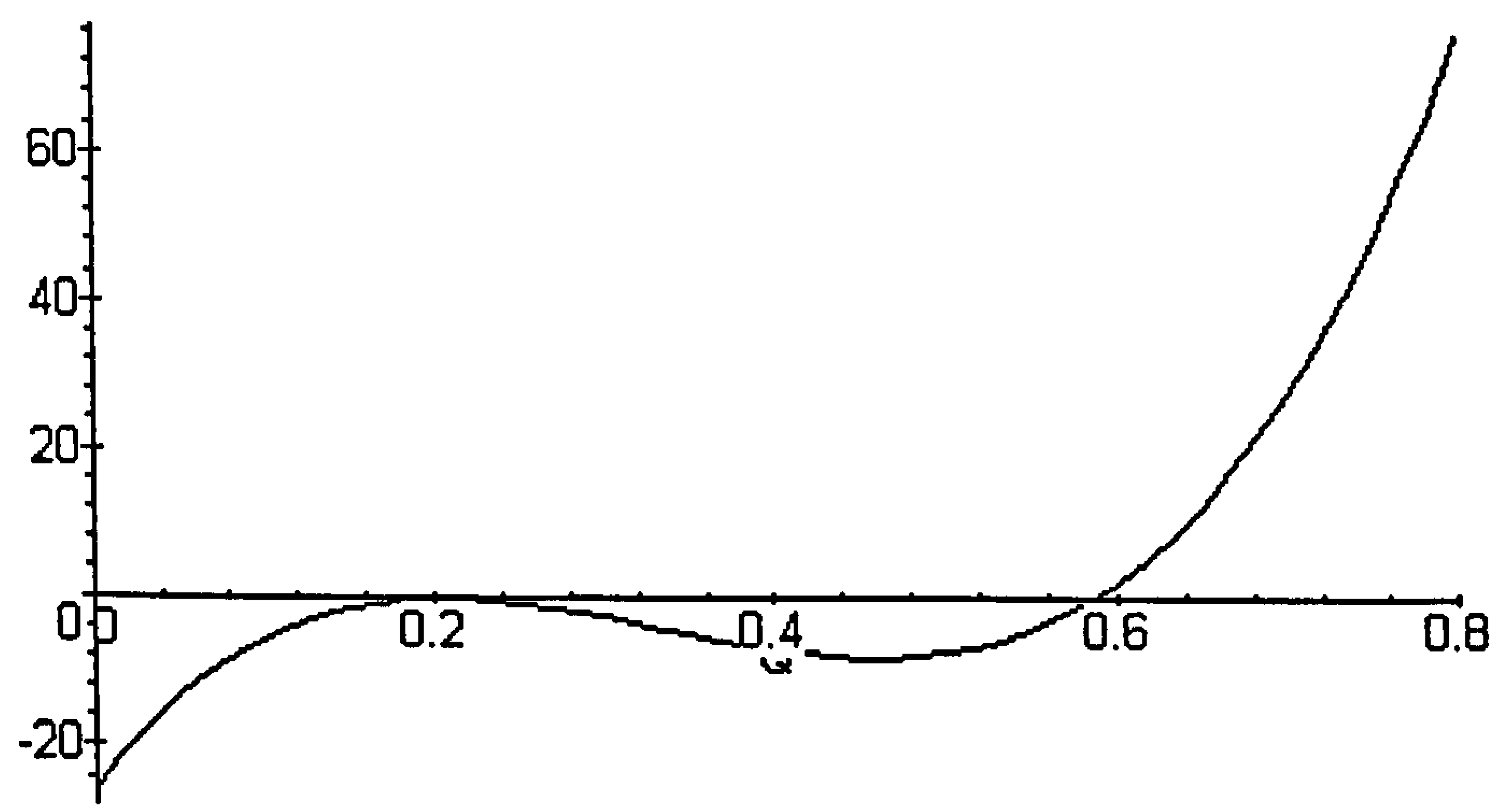
Assume that the value of $t$ - the precision of the information about the stock supply - increases. We may start in a situation such as in figure 3.3, with only one feasible equilibrium value for $Q_S$. This functional form is that given by the values used above, the one exception being that the value for $t$ is now 0.45.

*Fig. 3.3: One root (t=0.45).*



As this precision rises, the curve moves upwards, until we reach the point, as in figure 3.4, at which the curve representing the function is tangential to the x-axis, giving the system two potential equilibria; although we would expect the actual equilibrium position to remain at the higher of the two, with $Q_S$ decreasing smoothly as the curve rises.

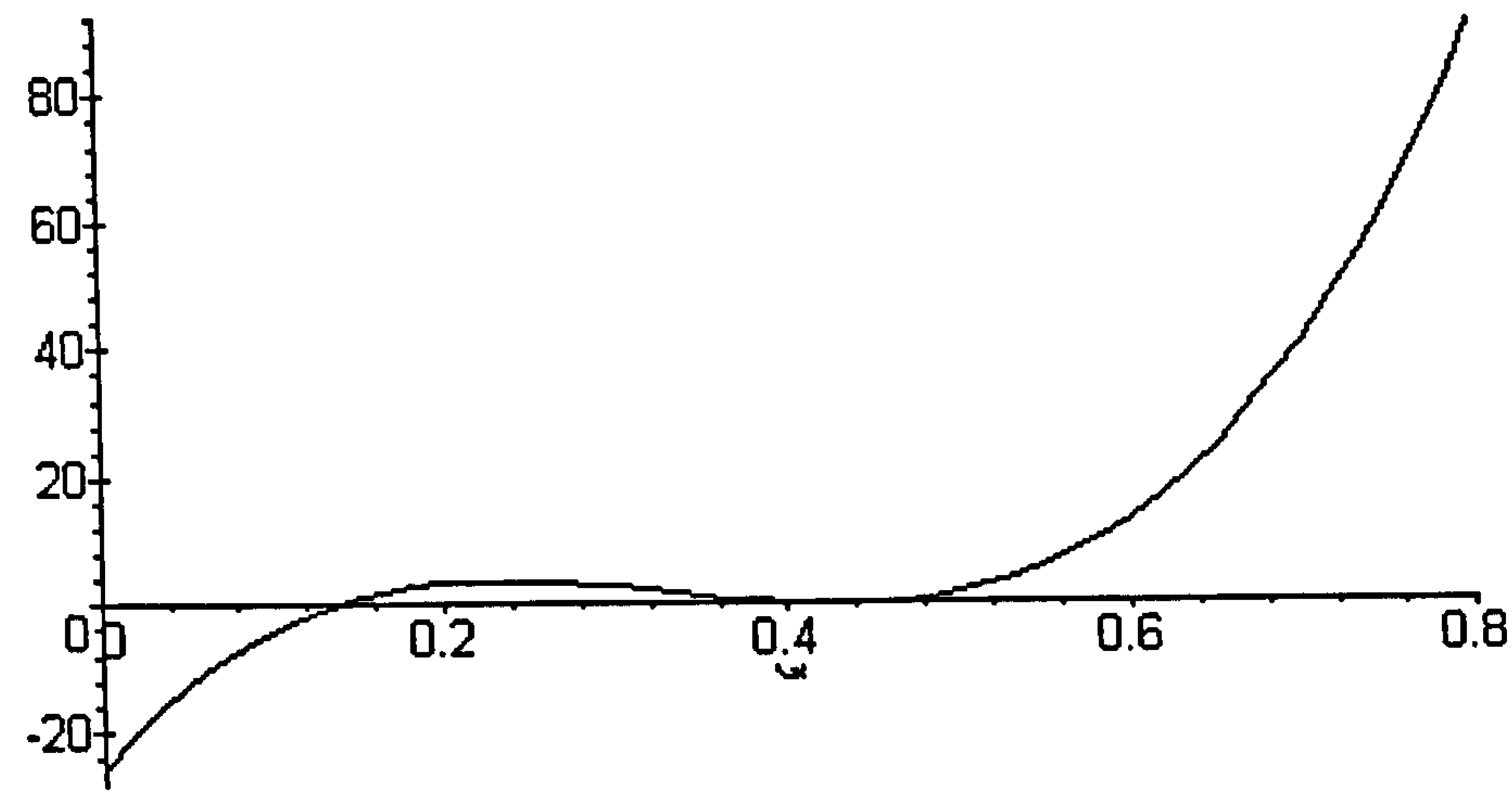
*Fig. 3.4: Initial tangency.*




Once the value of $t$ reaches 0.5 we arrive at the situation given by figure 3.1, with three potential equilibria, two of which could be described as stable. Eventually, with $t$ continuing to increase, another tangency position is reached, as shown in figure 3.5

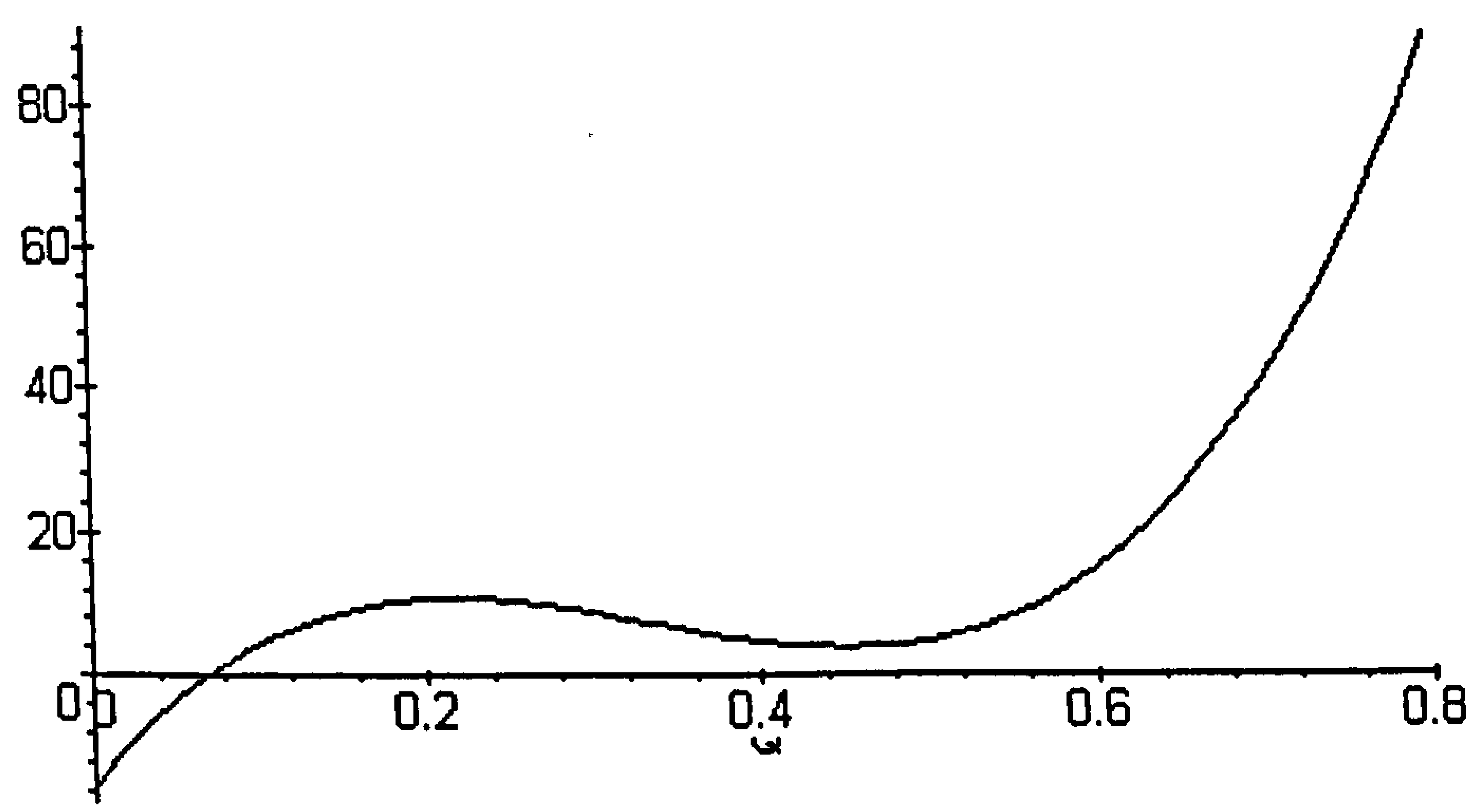
Once the curve moves beyond this point, the equilibrium value for $Q_S$ is forced to switch to what was previously the potential lower equilibrium, and is now the only equilibrium position. Of course, the equilibrium position may switch before the tangency position is reached, since the 'old' equilibrium becomes more and more unstable, in that the size of the disturbance required to send the system towards the other equilibrium becomes smaller and smaller as tangency is approached. For a value

*Fig. 3.5: Final tangency.*




of $t$ of 0.6 the situation is as figure 3.6.

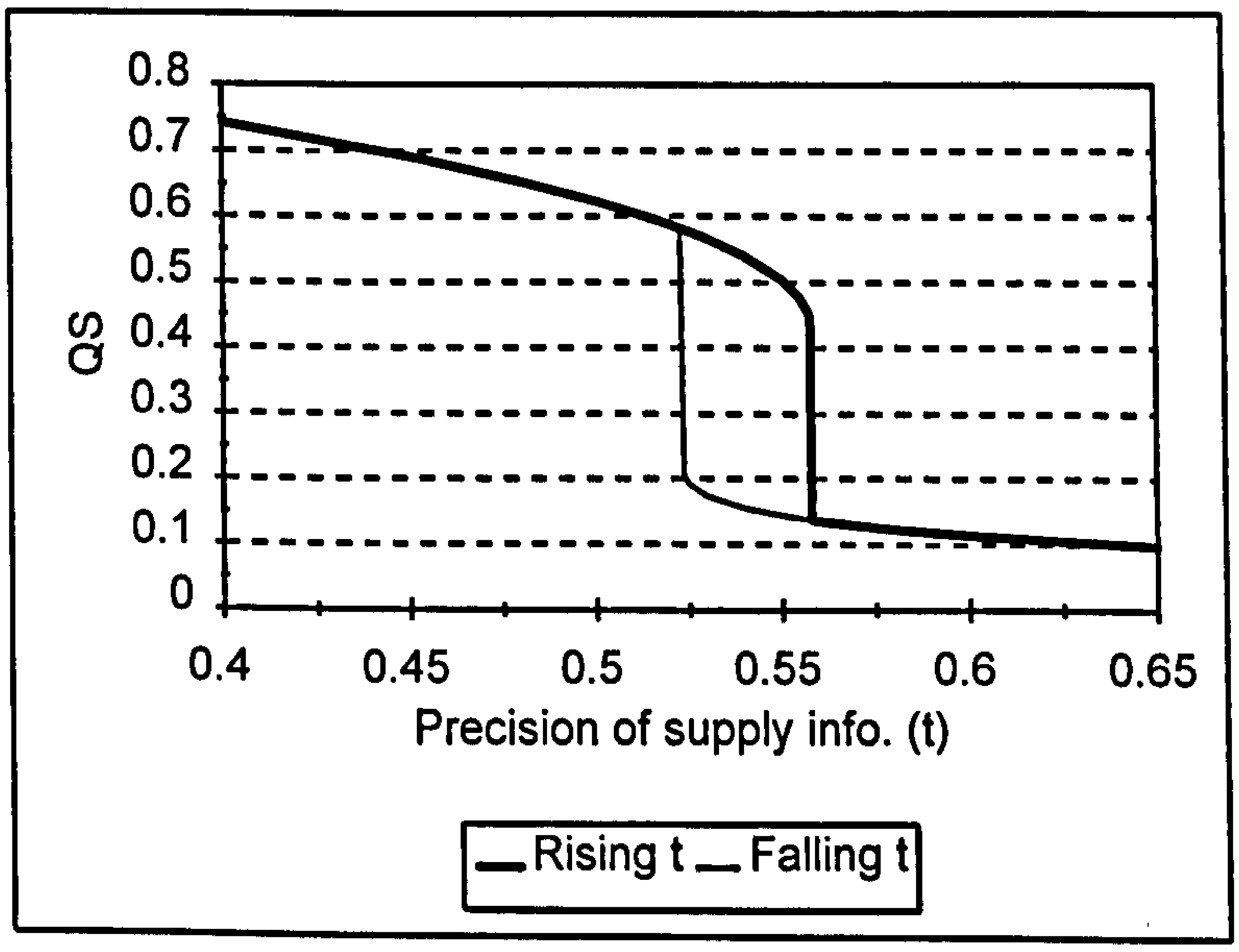
*Fig. 3.6: Back to one root (t=0.6).*




The same effect - the emergence and disappearance of three real roots - can also be shown with a falling value for $p_S$.

Figure 3.7 shows the path that would be followed by $Q_S$ given changes in the value of $t$. At low values of $t$ there is only one real root. As $t$ rises, the value of this root falls, with inertia keeping the equilibrium at the upper of the two potential equilibria when the other real roots emerge. When the point of the final tangency is reached, the equilibrium value of $Q_S$ is forced to switch to the (previously) lower equilibrium. The path of $Q_S$ is thus discontinuous. When $t$ is falling, the lower of the two lines is followed, with inertia keeping the value of $Q_S$ low until the upward switch is forced.

*Fig. 3.7: $Q_S$ equilibria with respect to changes in t.*




Figures 3.8 and 3.9 show the paths followed by $K$ - the precision for each agent of the liquidation value conditional on all the information available to them - and the variance of the price, respectively. As would be expected, these contains discontinuities. They are also inversely related: when the precision of the supply information increases, the conditional precision increases, as does the volatility of the price. Figure 3.10 shows the corresponding path for the mean price level, for a mean stock supply of 10, and a liquidation value equal to its unconditional expected value of 1. It confirms that the price function contains discontinuities, and that over a range there are two potential equilibria for the price. Following the story we have told in this

*Fig. 3.8: Precision of information with respect to changes in t.*

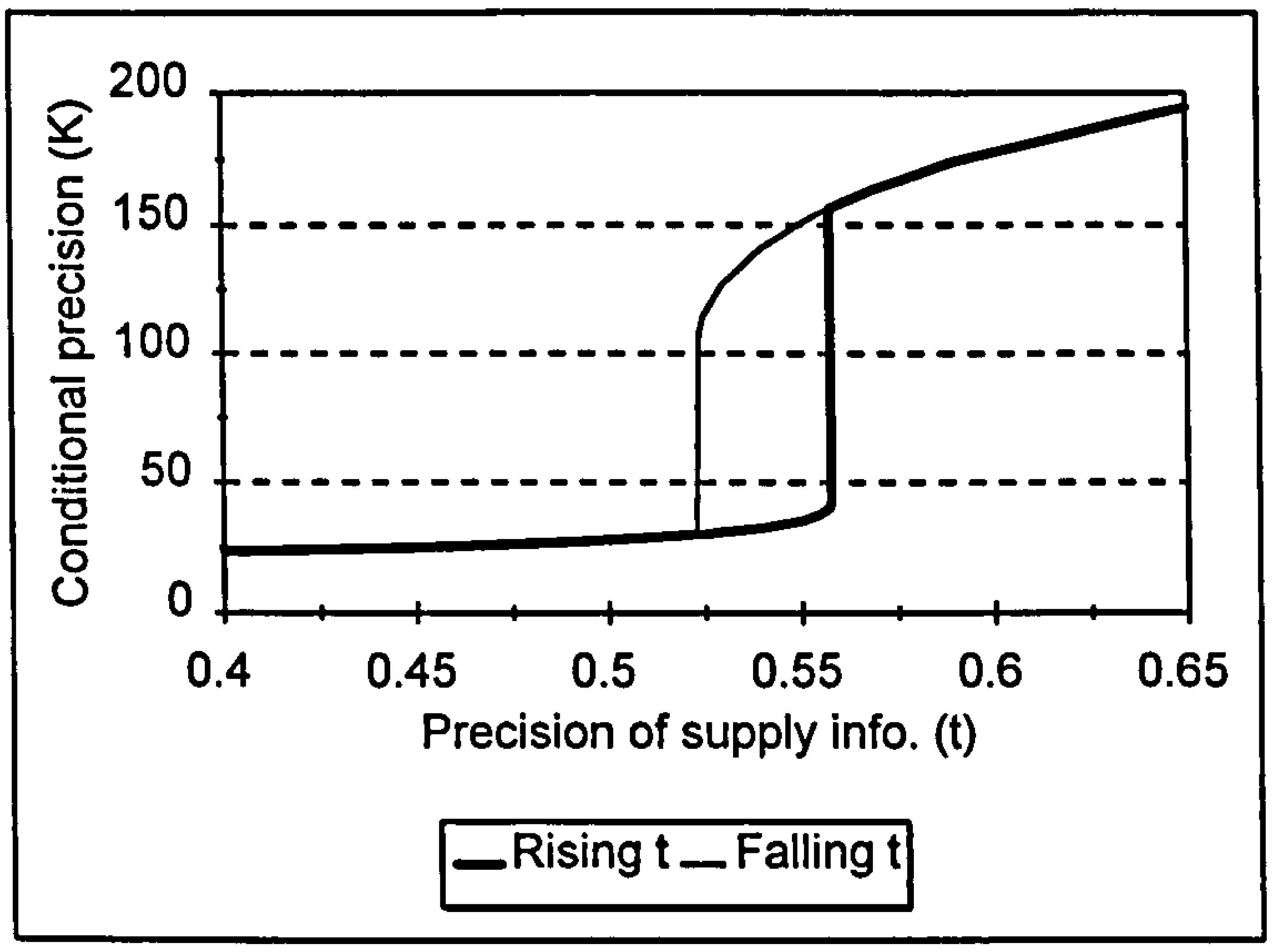



*Fig. 3.9: Volatility of price with respect to changes in t.*

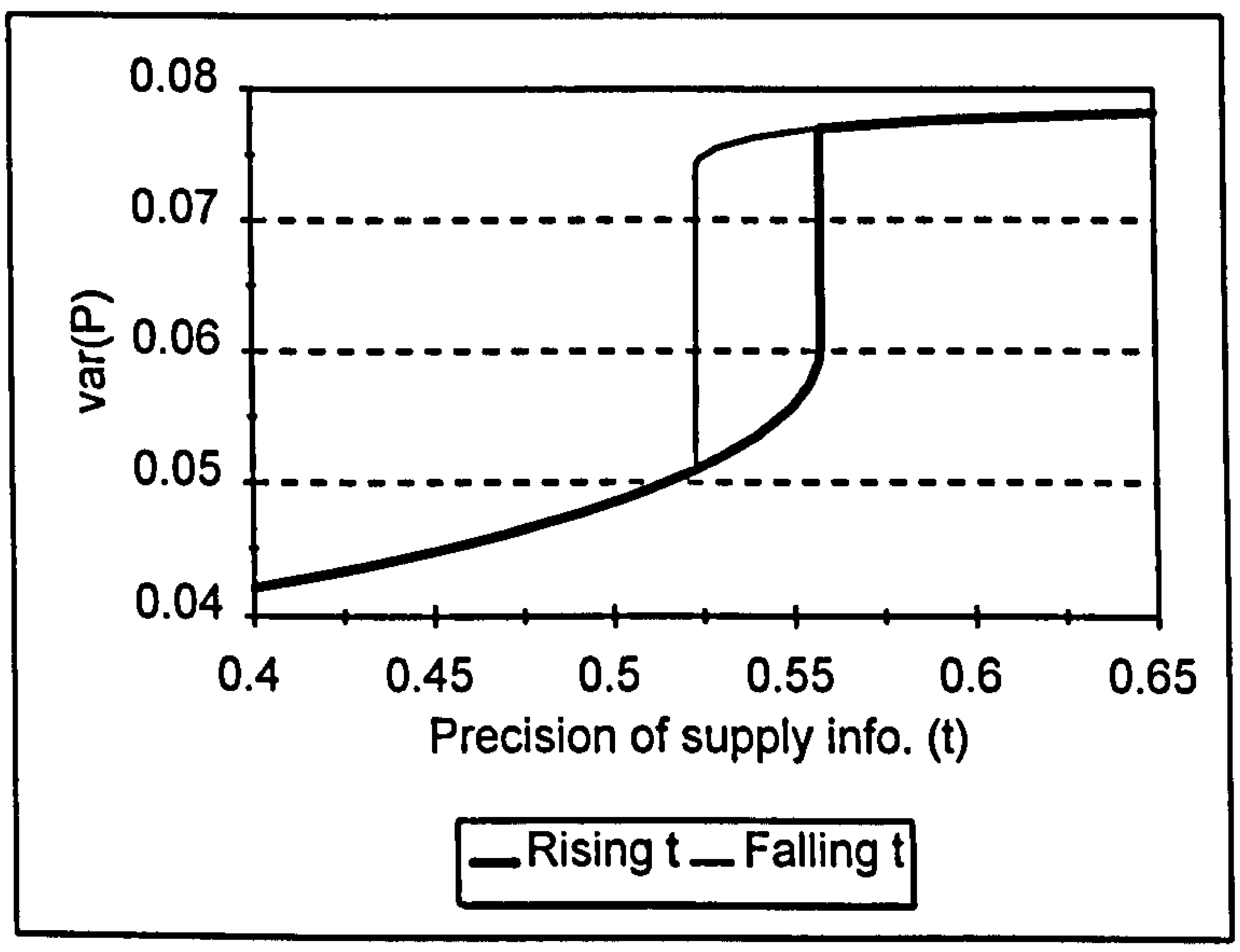
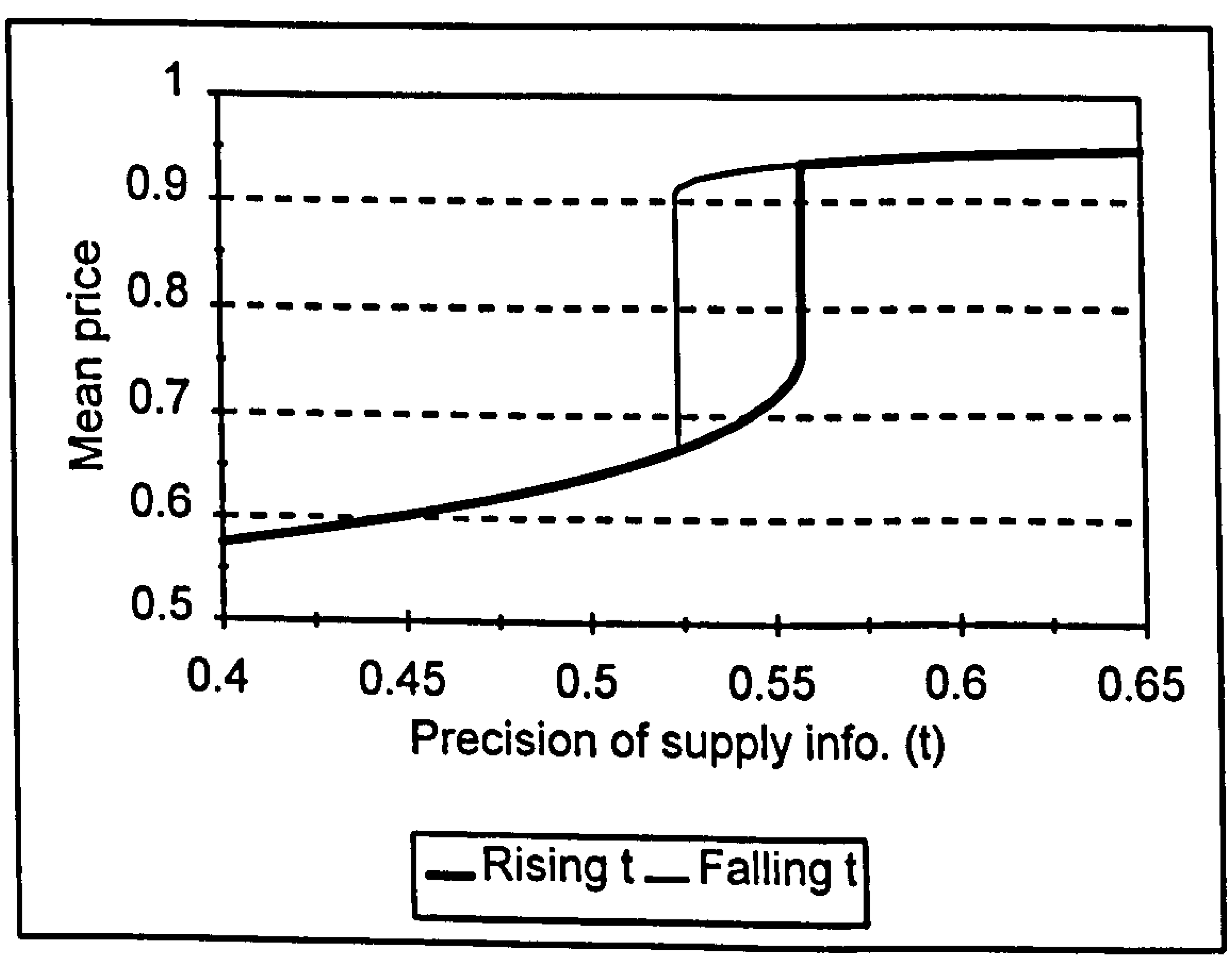



*Fig. 3.10: Price and precision.*



section, we can say that the particular equilibrium that will prevail is determined by a hysteresis effect.

# 3.7 Conclusion

The model in this chapter serves to demonstrate that since stock prices aggregate information of more than one type there may not be a unique equilibrium, as it may be possible for the price to reflect a given set of information signals in more than one way. This result was achieved in the strict world of the information aggregation literature, where the price function is defined to be linear, and all the variables are taken from normal distributions.

The possibility of multiple equilibria is in no way dependent on the assumption of agent homogeneity: it was shown (in section 3.3, equation 3.12) that for two agent types the equivalent value of $Q_S$ is the solution of a fifth-order polynomial, which gives even more scope for multiple equilibria; and it is possible to surmise that as the number of different agent types increases, the number of different potential equilibria will increase.

The existence of multiple equilibria in this model is associated with discontinuities in the price function. Following Gennotte & Leland (1990) a story was developed in which small changes in the precision of the information can induce price crashes or sharp upwards price revisions by causing the equilibrium to jump across the discontinuity. Unlike in Gennotte & Leland, however, where the cause of the crashes can be attributed to the portfolio insurers, the price revisions in our model are endemic to the system. This is an important result. In addition, our model is arguably more internally consistent than that of Gennotte & Leland, since the portfolio insurers in the latter act as though prices are distributed log-normally, although under the model structure prices are actually distributed normally. The use of a single period model is not the ideal way to obtain results about dynamic behaviour; but unfortunately the models that have extended the numbers of periods analysed - such as Kim and Verrecchia (1991a, 1991b), Brennan and Cao (1996) and Naik (1993) - are not capable of dealing with situations in which there are multiple potential equilibria, and so rule out by design the type of behaviour found here. The approach used here is therefore likely to be the most appropriate available.

This work provides two reasons why the volatility of market prices may exceed the volatility of the determinants of the underlying stock value. These are that: firstly, there may be discontinuous movements between equilibria independent of changes in underlying value; and secondly, there may be smooth movements between equilibria as parameter values, such as the quality of information, change. This provides a potential explanation for the empirical evidence of such 'excess' volatility discovered initially by Shiller (1981) and LeRoy and Porter (1981) and supported - albeit to a milder extent - by subsequent studies.[1]

[1] See Campbell, Lo and MacKinlay (1997).

# Appendix B: For Chapter 3

## B1 General case

The vector $Y_i = (\tilde{u}, \tilde{y}_i, \tilde{w}_i, \tilde{P})$ follows a normal distribution with mean $M = \{\bar{u}, \bar{u}, 0, (\alpha_u + \beta)\bar{u} + (\alpha_L - \gamma_L)\bar{L}\}$ and variance-covariance matrix $V_i$, where $V_i$ is:

$$V_i = \begin{pmatrix} 1/h_o & 1/h_o & 0 & \beta/h_o \\ 1/h_o & 1/h_o + 1/s_i & 0 & \beta/h_o \\ 0 & 0 & 1/p_S + 1/t & -\gamma_S/p_S \\ \beta/h_o & \beta/h_o & -\gamma_S/p_S & \beta^2/h_o + \gamma_S^2/p_S + \gamma_L^2/p_L \end{pmatrix} \quad \text{(B1)}$$

We will assume that all rational investors observe both price and supply signals. Following Appendix A (A1), for rational investor $i$ who receives the signal vector $(\tilde{y}_i, \tilde{w}_i, \tilde{P})$, the conditional mean ($\mu_i$) and precision ($K_i$) are given by:

$$\begin{aligned} K_i &\equiv \operatorname{var}^{-1}\left(\tilde{u} \middle| \tilde{y}_i, \tilde{w}_i, \tilde{P}\right) \\ &= h_o + s_i + \frac{\beta^2 p_L (p_S + t_i)}{\gamma_L^2 (p_S + t_i) + \gamma_S^2 p_L} \end{aligned} \quad \text{(B2)}$$

$$\begin{aligned} \mu_i &= E\left(\tilde{u} \middle| \tilde{y}_i, \tilde{w}_i, \tilde{P}\right) \\ &= \frac{1}{K_i}\left\{ h_o\bar{u} + \frac{\beta\gamma_L p_L (p_S + t_i)}{\gamma_L^2 (p_S + t_i) + \gamma_S^2 p_L}\bar{L} + s_i\tilde{y}_i \right. \\ &\quad \left. + \frac{\beta\gamma_S p_L t_i}{\gamma_L^2 (p_S + t_i) + \gamma_S^2 p_L}\tilde{w}_i + \frac{\beta^2 p_L (p_S + t_i)}{\gamma_L^2 (p_S + t_i) + \gamma_S^2 p_L}\tilde{q} \right\} \end{aligned} \quad \text{(B3)}$$

where:

$$\tilde{q} \equiv \frac{1}{\beta}\left(\tilde{P}-\alpha\right)=\tilde{u}-\frac{\gamma_S}{\beta}\tilde{S}-\frac{\gamma_L}{\beta}\tilde{L}$$

and:

$$\text{var}^{-1}(\tilde{q})=1\Big/\left\{\frac{\gamma_L^{\,2}}{\beta^2 p_L}+\frac{\gamma_S^{\,2}}{\beta^2(p_S+t_i)}\right\}$$

$$=\frac{\beta^2 p_L(p_S+t_i)}{\gamma_L^{\,2}(p_S+t_i)+\gamma_S^{\,2}p_L}$$

This signal $(\tilde{q})$ is a substitute for the price signal.

The relationship between the posterior distribution and the demand for stock of agent *i* ($D_i$) is as follows:

$$D_i=\frac{r_i\left\{E\left(\tilde{u}\middle|\tilde{y}_i,\tilde{w}_i\tilde{P}\right)-\tilde{P}\right\}}{Var\left(\tilde{u}\middle|\tilde{y}_i,\tilde{w}_i\tilde{P}\right)}=r_iK_i(\mu_i-P)$$

$$=r_i\left\{h_o\bar{u}+\frac{\beta\gamma_L p_L(p_S+t_i)}{\gamma_L^{\,2}(p_S+t_i)+\gamma_S^{\,2}p_L}\bar{L}+s_i\tilde{y}_i\right. \tag{B4}$$

$$\left.+\frac{\beta\gamma_S p_L t_i}{\gamma_L^{\,2}(p_S+t_i)+\gamma_S^{\,2}p_L}\tilde{w}_i+\frac{\beta^2 p_L(p_S+t_i)}{\gamma_L^{\,2}(p_S+t_i)+\gamma_S^{\,2}p_L}\tilde{q}-K_i\tilde{P}\right\}$$

In equilibrium the average demand per investor must equal the per capita supply. As the number of investors becomes large, the mean liquidation-value signal tends to the actual liquidation value; and the mean supply signal tends to the actual outcome for the part of supply to which the signal relates. The following expression must therefore hold:

$$\tilde{x} = \tilde{S} + \tilde{L} = \tilde{D}$$

$$\equiv \int_0^1 \tilde{D}_i \, di$$

$$= \int_0^1 r_i \left( h_o \tilde{u} + \frac{\beta \gamma_L p_L (p_S + t_i)}{\gamma_L^2 (p_S + t_i) + \gamma_S^2 p_L} \tilde{L} + s_i (\tilde{u} + \tilde{\varepsilon}_i) \right.$$

$$\left. + \frac{\beta \gamma_S p_L t_i}{\gamma_L^2 (p_S + t_i) + \gamma_S^2 p_L} (\tilde{S} + \tilde{\eta}_i) + \frac{\beta^2 p_L (p_S + t_i)}{\gamma_L^2 (p_S + t_i) + \gamma_S^2 p_L} \tilde{q} - K_i \tilde{P} \right) di \tag{B5}$$

$$= r h_o \tilde{u} + \tilde{L} \int_0^1 \frac{\beta \gamma_L p_L (p_S + t_i) r_i}{\gamma_L^2 (p_S + t_i) + \gamma_S^2 p_L} di + r s \tilde{u}$$

$$+ \tilde{S} \int_0^1 \frac{\beta \gamma_S p_L r_i t_i}{\gamma_L^2 (p_S + t_i) + \gamma_S^2 p_L} di + \tilde{q} \int_0^1 \frac{\beta^2 p_L (p_S + t_i) r_i}{\gamma_L^2 (p_S + t_i) + \gamma_S^2 p_L} di - r K \tilde{P}$$

where $r \equiv \int_0^1 r_i \, di$ , $s \equiv (1/r) \int_0^1 r_i s_i \, di$ , and:

$$K \equiv (1/r) \int_0^1 r_i K_i$$

$$= (1/r) \int_0^1 r_i \left( h_o + s_i + \frac{\beta^2 p_L (p_S + t_i)}{\gamma_L^2 (p_S + t_i) + \gamma_S^2 p_L} \right) di$$

$$= h_o + s + \frac{1}{r} \int_0^1 \frac{\beta^2 p_L (p_S + t_i) r_i}{\gamma_L^2 (p_S + t_i) + \gamma_S^2 p_L} di$$

Re-arrangement and substitution for $\tilde{q}$ reveals the expression for the price given in the main text.

# B2 Heterogeneous agents with non-stochastic stock supply *S*

With heterogeneous agents and a stock supply component *S* that is non-stochastic, the situation is as given by the large-market form of Hellwig (1980). The equilibrium price function takes the following form:

$$\tilde{P} = \alpha_u \bar{u} + \alpha_L \bar{L} + \beta \tilde{u} - \gamma_L \tilde{L} \tag{B6}$$

where:

$$\alpha_u = \frac{h_o}{h_o + s + r^2 s^2 p_L}$$

$$\alpha_L = \frac{rsp_L}{h_o + s + r^2 s^2 p_L}$$

$$\beta = \frac{s + r^2 s^2 p_L}{h_o + s + r^2 s^2 p_L} = 1 - \alpha_u$$

$$\gamma_L = \frac{1/r + rsp_L}{h_o + s + r^2 s^2 p_L} = \frac{1/r}{h_o + s + r^2 s^2 p_L} + \alpha_L$$

and:

$$r \equiv \int_0^1 r_i \, di \,, \quad s \equiv (1/r) \int r_i s_i \, di \,.$$

# B3 Two agent types

The demand of agent $i$ in group $f$ is:

$$D_{fi} = r_f\left( h_o\bar{u} + \frac{\beta\gamma_L p_L(p_S + t_f)}{\gamma_L^2(p_S + t_f) + \gamma_S^2 p_L}\bar{L} + s_f\tilde{y}_{fi} + \frac{\beta\gamma_S p_L t_f}{\gamma_L^2(p_S + t_f) + \gamma_S^2 p_L}\tilde{w}_{fi} + \frac{\beta^2 p_L(p_S + t_f)}{\gamma_L^2(p_S + t_f) + \gamma_S^2 p_L}\tilde{q} - K_f\tilde{P}\right) \tag{B7}$$

The demand per agent in group $f$ is therefore:

$$D_{fi} = r_f\left( h_o\bar{u} + \frac{\beta\gamma_L p_L(p_S + t_f)}{\gamma_L^2(p_S + t_f) + \gamma_S^2 p_L}\bar{L} + s_f\tilde{u} + \frac{\beta\gamma_S p_L t_f}{\gamma_L^2(p_S + t_f) + \gamma_S^2 p_L}\tilde{S} + \frac{\beta^2 p_L(p_S + t_f)}{\gamma_L^2(p_S + t_f) + \gamma_S^2 p_L}\tilde{q} - K_f\tilde{P}\right) \tag{B8}$$

The stock supply, as in the previous section, now represents that available to each investor. Taking $\mu_f$ as the measure of agent-group $f$ in the economy, the market-clearing condition is given by:

$$\begin{aligned}
\tilde{S} + \tilde{L} &= \tilde{D}_f + \tilde{D}_g \\
&= \mu_f r_f\left( h_o\bar{u} + \frac{\beta\gamma_L p_L(p_S + t_f)}{\gamma_L^2(p_S + t_f) + \gamma_S^2 p_L}\bar{L} + s_f\tilde{u} + \frac{\beta\gamma_S p_L t_f}{\gamma_L^2(p_S + t_f) + \gamma_S^2 p_L}\tilde{S} + \frac{\beta^2 p_L(p_S + t_f)}{\gamma_L^2(p_S + t_f) + \gamma_S^2 p_L}\tilde{q} - K_f\tilde{P}\right) \\
&\quad + \mu_g r_g\left( h_o\bar{u} + \frac{\beta\gamma_L p_L(p_S + t_g)}{\gamma_L^2(p_S + t_g) + \gamma_S^2 p_L}\bar{L} + s_g\tilde{u} + \frac{\beta\gamma_S p_L t_g}{\gamma_L^2(p_S + t_g) + \gamma_S^2 p_L}\tilde{S} + \frac{\beta^2 p_L(p_S + t_g)}{\gamma_L^2(p_S + t_g) + \gamma_S^2 p_L}\tilde{q} - K_g\tilde{P}\right)
\end{aligned} \tag{B9}$$

where:

$$K_f \equiv h_o + s_f + \frac{\beta^2 p_L (p_S + t_f)}{\gamma_L^2 (p_S + t_f) + \gamma_S^2 p_L}$$

Re-arrangement and substitution for $\tilde{q}$ reveals the expression given in the main text.

# Chapter 4

# Acquisition of information about stock value and liquidity trading

## 4.1 Introduction

The stock demand of investors other than those trading due to liquidity considerations reflects the information they have about the future stock value. Learning about the demand from other investors therefore provides an alternative way of learning about future value distinct from doing this directly. In this chapter we look at the relative incentives to acquire information of these different types in an information aggregation framework. More specifically we look at the acquisition of information when investors have information about the activities of liquidity traders, as well as the future stock value, as in the previous chapter. Since the concept of supply information, as developed in the previous chapter, is new to the literature, what follows also represents original work, although it draws heavily on Verrecchia (1982) - which endogenises the acquisition of information in the Hellwig (1980) framework - and uses similar cost functions for information that relate cost and the precision of the information acquired. Our treatment of the endogenous acquisition of the two information types is necessarily more mathematically complex than, but is also conceptually similar to, the treatment of the issue in Verrecchia. For the sake of tractability we will assume, unlike Verrecchia, that the agents are homogeneous, as we did previously when investigating the possibility of price discontinuities.

Once we have developed the model analytically, we go on to look at some numerical examples, to see how the amounts of information acquired change with the cost of information, and attempt to assess the validity of the example of dual equilibria and price 'crashes' given in the previous chapter.

## 4.2 Model structure

As in the previous chapter we will assume, following Hellwig (1980), that the investors are endowed with an initial wealth ($W_0$), and not a quantity of stock and bonds as in Verrecchia (1982). However, since in a 'large' market the agents will learn nothing from their endowments, the difference "turns out to be irrelevant" (Verrecchia, footnote 7, p. 1420), and so the (slightly) more tractable assumption is used.

Different agents receive different signals, although these are taken from the same distributions, and so have the same precisions. The random vector $\left(\tilde{u},\tilde{L},\tilde{S},\tilde{\theta}_i,\tilde{\theta}_j,\tilde{\varepsilon}_i,\tilde{\varepsilon}_j\right)$ is therefore distributed Normally with mean $\left(\bar{u},\bar{L},0,0,0,0,0\right)$ and variance $\left(\frac{1}{h_o},\frac{1}{p_L},\frac{1}{p_S},\frac{1}{t},\frac{1}{t},\frac{1}{s},\frac{1}{s}\right)$. We will continue to assume that these random variables are distributed independently, and so all covariances are zero.

There is an equilibrium price function of the following linear form:

$$\tilde{P}=\alpha_u\bar{u}+\alpha_L\bar{L}+\beta\tilde{u}-\gamma_S\tilde{S}-\gamma_L\tilde{L} \tag{4.1}$$

In the previous chapter we showed that the coefficients of this price function are given by the following:

$$\begin{aligned}
\alpha_u &= \frac{h_o}{K}\\
\alpha_L &= \frac{1}{K}\left\{\frac{rsp_L(p_S+t)}{Q_S{}^2p_L+p_S+t}\right\}\\
\beta &= \frac{1}{K}\left\{s+\frac{r^2s^2p_L(p_S+t)}{Q_S{}^2p_L+p_S+t}\right\}\\
\gamma_S &= \frac{1}{K}\left\{\frac{1}{r}+\frac{Q_Srsp_Sp_L}{Q_S{}^2p_L+p_S+t}\right\}\\
\gamma_L &= \frac{1}{K}\left\{\frac{1}{r}+\frac{rsp_L(p_S+t)}{Q_S{}^2p_L+p_S+t}\right\}
\end{aligned} \tag{4.2}$$

where:

$$K=h_o+s+\frac{r^2s^2p_L(p_S+t)}{Q_S{}^2p_L+p_S+t}$$

$$Q_\beta\equiv\frac{\beta}{\gamma_L}=rs$$

$$Q_S \equiv \frac{\gamma_S}{\gamma_L}$$

and $Q_S$ is the solution to the following cubic:

$$(Q_S - 1)(Q_S^2 p_L + p_S + t) + Q_S p_L r^2 st = 0 \tag{4.3}$$

## 4.3 Endogenous acquisition of information

Fortunately, the above equilibria still hold when information acquisition is made endogenous, since these are the relevant equilibria once the information has been acquired.

Assume that the traders are faced with convex cost functions ($c(s)$, $d(t)$) which give the relationship between the outlay and the precision of the information that will be acquired. Each trader wishes to maximise his utility at liquidation, which, as we demonstrate in Appendix C (sections C1-C3), is as follows:

$$-\left(\frac{h_o p_S p_L(\gamma^* + \gamma_L^2 t)}{(h_o\gamma^* + p_S p_L \alpha^2)\{(h_o + s)(\gamma^* + \gamma_L^2 t) + \beta^2 p_L(p_S + t)\}}\right)^{\frac{1}{2}} \times \exp\left(\frac{-W_0 + c(s) + d(t)}{r}\right) \tag{4.4}$$

where:

$$\gamma^* \equiv \gamma_L^2 p_S + \gamma_S^2 p_L$$

and

$$\alpha \equiv \alpha_u + (\alpha_L - \gamma_L)\frac{\bar{L}}{\bar{u}}$$

The optimal values of *s* and *t* satisfy the following Kuhn-Tucker conditions:

$$s \geq 0, t \geq 0, \frac{\partial()}{\partial s} \leq 0, \frac{\partial()}{\partial t} \leq 0, s\left(\frac{\partial()}{\partial s}\right) = 0, t\left(\frac{\partial()}{\partial t}\right) = 0. \tag{4.5}$$

The partial derivatives with respect to the information precisions are given in the appendix (C4), and reveal that the agents will wish to purchase information with the following precision:[1]

$$s = \max.\left\{\bar{s}, \hat{s}\left|\frac{2\left(c_s' + d_s'\right)}{r}\left(h_o + s + \frac{\beta^2 p_L (p_S + t)}{\gamma_L^2 (p_S + t) + \gamma_S^2 p_L}\right) = 1\right.\right\} \tag{4.6}$$

$$t = \max.\left\{\bar{t}, \hat{t}\left|\frac{2\left(c_t' + d_t'\right)}{r\beta^2 \gamma_S^2 p_L^2}\left\{ (h_o + s)\left(\gamma_L^2 (p_S + t) + \gamma_S^2 p_L\right)^2 + \beta^2 p_L (p_S + t)\left(\gamma_L^2 (p_S + t) + \gamma_S^2 p_L\right) = 1 \right\}\right.\right\} \tag{4.7}$$

where $\bar{s}$ and $\bar{t}$ represent minimum, or 'base-line', precisions, which the agents will automatically receive free of charge. Verrecchia implicitly assumed that these base-line levels were set at zero; but we allow the possibility that they can take positive values in order to make it easier to interpret the results of the previous chapter.

[1] The convexity of $c(s)$ and $d(t)$ ensures that a maximum has been found.

The decisions concerning the quality of information to acquire of the two types must be taken jointly with each other - since the precision of one information type enters the maximisation equation for the other - and also with the equations determining the equilibrium price function.

## 4.4 Linear cost functions

At this point we will make the simplifying assumptions that the cost functions *c(s,t)* and *d(s,t)* are linear and independent; and in particular that they take the following form:

$$c(s,t) = c(s) = \frac{s-\bar{s}}{2a_s} + b_s$$

$$d(s,t) = d(t) = \frac{t-\bar{t}}{2a_t} + b_t \tag{4.8}$$

The cost function for value information takes the same form as in Verrecchia, with the added possibility that the base-line precision level may exceed zero.

The first derivatives of the cost functions with respect to the precisions are:

$$\begin{aligned} c'(s) &= 1/2a_s \\ d'(t) &= 1/2a_t \end{aligned} \tag{4.9}$$

We can express the precision of the information the agents receive as the sum of the precision of their endowment and the precision of the information purchased:

$$s = \bar{s} + s^*$$

$$t = \bar{t} + t^{*} \tag{4.10}$$

Given the above cost functions, and after substituting in the expressions for the coefficients of the price function, the precisions of the information acquired can be expressed as:

$$s^{*} = \max.\left\{0, \hat{s}^{*} \left| \frac{1}{ra_s}\left( h_o + \bar{s} + \hat{s}^{*} + \frac{r^2\left(\bar{s} + \hat{s}^{*}\right)^2 p_L\left(p_S + \bar{t} + t^{*}\right)}{p_S + \bar{t} + t^{*} + Q_S{}^2 p_L}\right) = 1 \right.\right\} \tag{4.11}$$

$$\begin{aligned} t^{*} = \max.\Bigg\{0, \hat{t}^{*} \Bigg| & \frac{1}{r^3\left(\bar{s} + s^{*}\right)^2 Q_S{}^2 p_L{}^2 a_t}\Big[\left(h_o + \bar{s} + s^{*}\right)\left(p_S + \bar{t} + \hat{t}^{*} + Q_S{}^2 p_L\right)^2 \\ & + r^2\left(\bar{s} + s^{*}\right)^2 p_L\left(p_S + \bar{t} + \hat{t}^{*}\right)\left(p_S + \bar{t} + \hat{t}^{*} + Q_S{}^2 p_L\right)\Big] = 1 \Bigg\} \end{aligned} \tag{4.12}$$

The equations for $Q_S$, $s$ and $t$ can be solved simultaneously subject to the Kuhn-Tucker conditions given previously. This can be done once we have assigned numerical values to the parameters of the model.

One result we can immediately obtain from the above is that when the base-line precision of value information ($\bar{s}$) is zero, and no information is obtained about value ($s^{*} = 0$), there will also be no information obtained about supply ($t^{*} = 0$). This can be expressed as:

$$s^{*} = 0 \ \textit{implies} \ t^{*} = 0 \ \textit{when} \ \bar{s} = 0. \tag{4.13}$$

As the cost of value-information decreases, traders will acquire more of it. As the cost becomes 'small', traders will acquire a 'large' amount, and will therefore know the liquidation value with certainty. At this point supply-information becomes redundant, and so we have:

$$1/a_s = 0 \ \textit{implies} \ s^* = \infty \ \textit{and} \ t^* = 0. \quad (4.14)$$

An increase in the cost of value-information has three effects on the acquisition of supply-information: firstly, there is a 'substitution' effect, leading to an increased desire to purchase the now-cheaper supply-information; secondly, there is an 'income' effect, which tends to reduce the amount of supply-information purchased; and thirdly, there is a 'usefulness' effect, under which a cheaper cost of value-information - since it leads to a greater amount of value-information purchased, and consequently an improved level of knowledge about the liquidation value - reduces the need for supply-information. When the cost of value-information is 'small', the usefulness effect dominates, and no supply-information is acquired. As the cost rises, the substitution effect can lead to the purchase of some supply-information, provided its cost is below a threshold; but eventually the income effect will dominate and lead to supply-information being sacrificed for value-information.

# 4.5 Threshold cost levels

Assume that the base-line precisions ($\bar{s}$ , $\bar{t}$ ) are zero. In this situation, traders will purchase a positive amount of value-information when the following condition holds:

$$a_s > \frac{h_0}{r} \tag{4.15}$$

When a positive amount of value-information is purchased, supply-information will be purchased provided that the following condition holds:

$$a_t > \left(\frac{p_S + p_L}{r^3 s^2 p_L}\right)\left(h_0 + s + r^2 s^2 p_S p_L\right) \tag{4.16}$$

A necessary requirement for this to hold is that it holds for large amounts of value-information, and that therefore the following holds:

$$a_t > \left(p_S + p_L\right)\frac{p_S}{r} \tag{4.17}$$

# 4.6 Information cost, conditional precision and prices

Comparing the expression for the precision of value-information acquired with that for the precision of the liquidation value conditional on the price and the private information signals reveals that we can express the equilibrium level of value-information as:

$$\begin{aligned} s &= \max.\left\{\bar{s}, \bar{s} + \hat{s}^* \middle| \frac{K}{ra_s} = 1\right\} \\ &= \max.\left\{\bar{s}, \bar{s} + \hat{s}^* \middle| K = ra_s\right\} \end{aligned} \tag{4.18}$$

This shows that, whenever a positive amount of value-information is acquired over and above the base-line levels, the conditional variance of the liquidation value is determined solely by the unit cost of value-information and the risk-tolerance of the agents, in the following way:

$$K = ra_s \tag{4.19}$$

Changes in the cost of supply-information, which affects the amount of supply-information acquired, will therefore cause a compensating change in the amount of value-information acquired. Obtaining information about the stock supply therefore allows the agents to reach the desired level of informativeness of prices in a more desirable way, but does not alter the market equilibrium.

The above result also implies that the informativeness of price is a non-increasing function of the cost of value-information (and is a decreasing function when some (additional) value-information is being acquired), which confirms Corollary 4 of Verrecchia (1982, p. 1425-26). In addition, it shows that the informativeness of price is a non-decreasing function of the risk tolerance of the agents, confirming Corollary 5 (*ibid.* p. 1426-27).

When positive amounts of value-information are purchased, the mean price level can be expressed in the following way:

$$\begin{aligned}\overline{P} &= \overline{u} - \frac{1}{rK}\overline{L} \\ &= \overline{u} - \frac{1}{r^2 a_s}\overline{L}\end{aligned} \tag{4.20}$$

The mean price level is therefore not affected by the cost of supply-information, and hence the amount of supply-information acquired, when some (additional) value-information is also being acquired.

# 4.7 Numerical examples

By attaching numerical values to the underlying parameters we can observe the nature of the equilibria for various levels of the information costs. We will look at five examples. First, we look at an example with very simple parameters, including base-line precisions of zero, and observe the effect of varying the cost of acquiring each type of information on the amount of information acquired and the mean price level. We then repeat the procedure using the parameter values given in Gennotte & Leland (1990), in order to demonstrate that the nature of the results in the first example were not entirely dependent on the simplistic parameter values. In the third example, using the simple parameter values, we use a positive base-line level and a high marginal cost for value-information ($1/2a_s$) to effectively fix the precision of the value-information ($s$), which allows us to isolate the effect of cost on the acquisition of supply-information ($t$). This third example provides a comparative case for the fourth example, in which the parameter values used in the 'price-crash' example of the previous chapter are used to show how 'crashes' can occur under this scenario. The final example illustrates a case of multiple equilibria.

### 4.7.1 Example #1: Zero base-line, simple parameters

We will start by looking at the equilibria for different levels of the cost of supply-information. We assume that the base-line precision levels are zero. The parameter values used are as given in table 4.1.

*Table 4.1: Zero base-line, simple parameters, fixed cost of s-information.*

| $h_0$ | $p_S$ | $p_L$ | $r$ | $\bar{u}$ | $\bar{L}$ | $\bar{s}$ | $\bar{t}$ | $a_s$ |
|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 3 |

Figure 4.1 shows the levels of *s* and *t* that will be acquired in equilibrium for a given unit cost of information about supply (*t*-information). As would be expected, the amount of *t*-information acquired decreases with its cost, until a threshold is reached, above which none is acquired. The amount of *s*-information acquired rises to offset the fall in *t*-information, maintaining a constant overall precision in the way demonstrated above.

*Fig. 4.1: Varying the cost of t-info.*

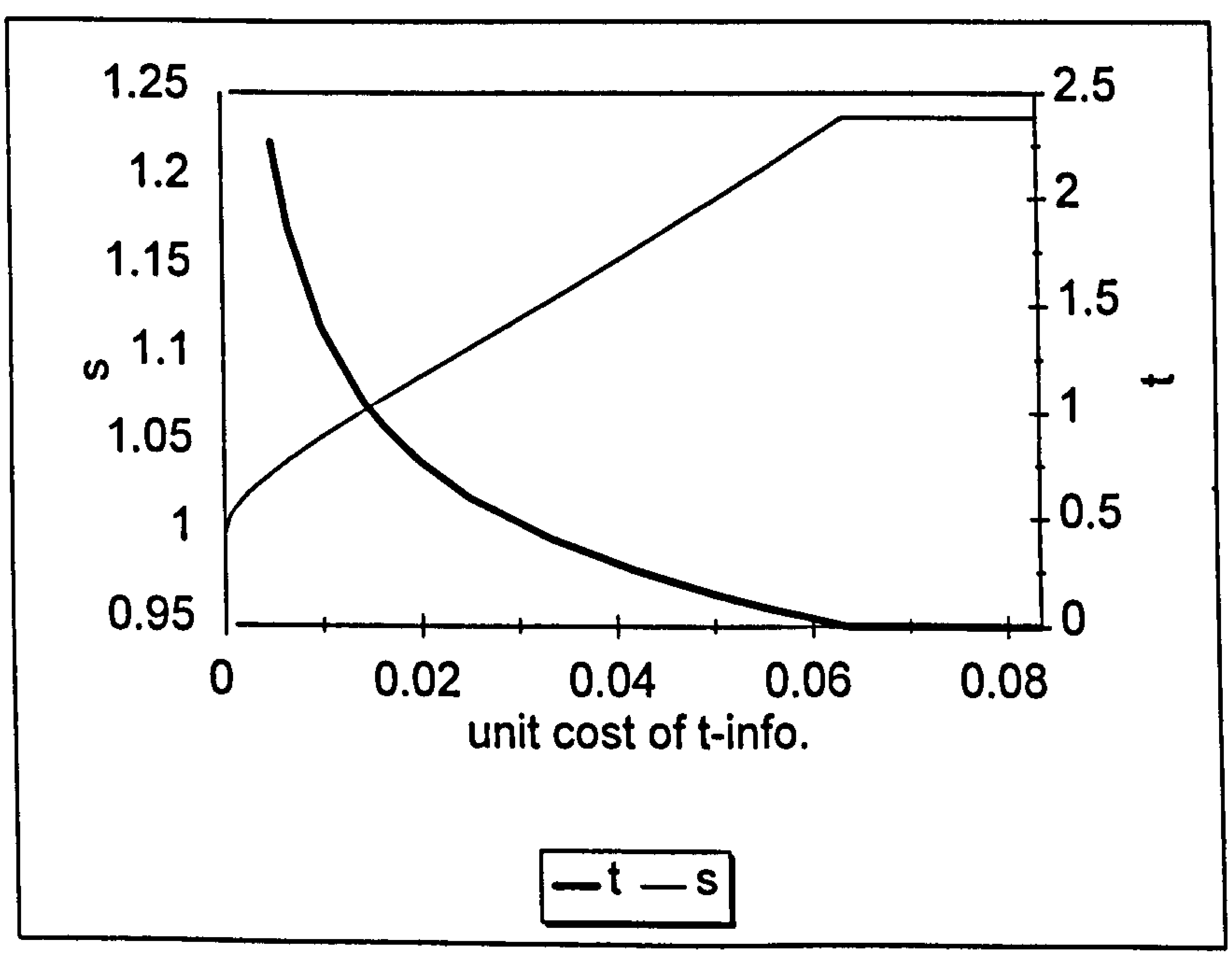
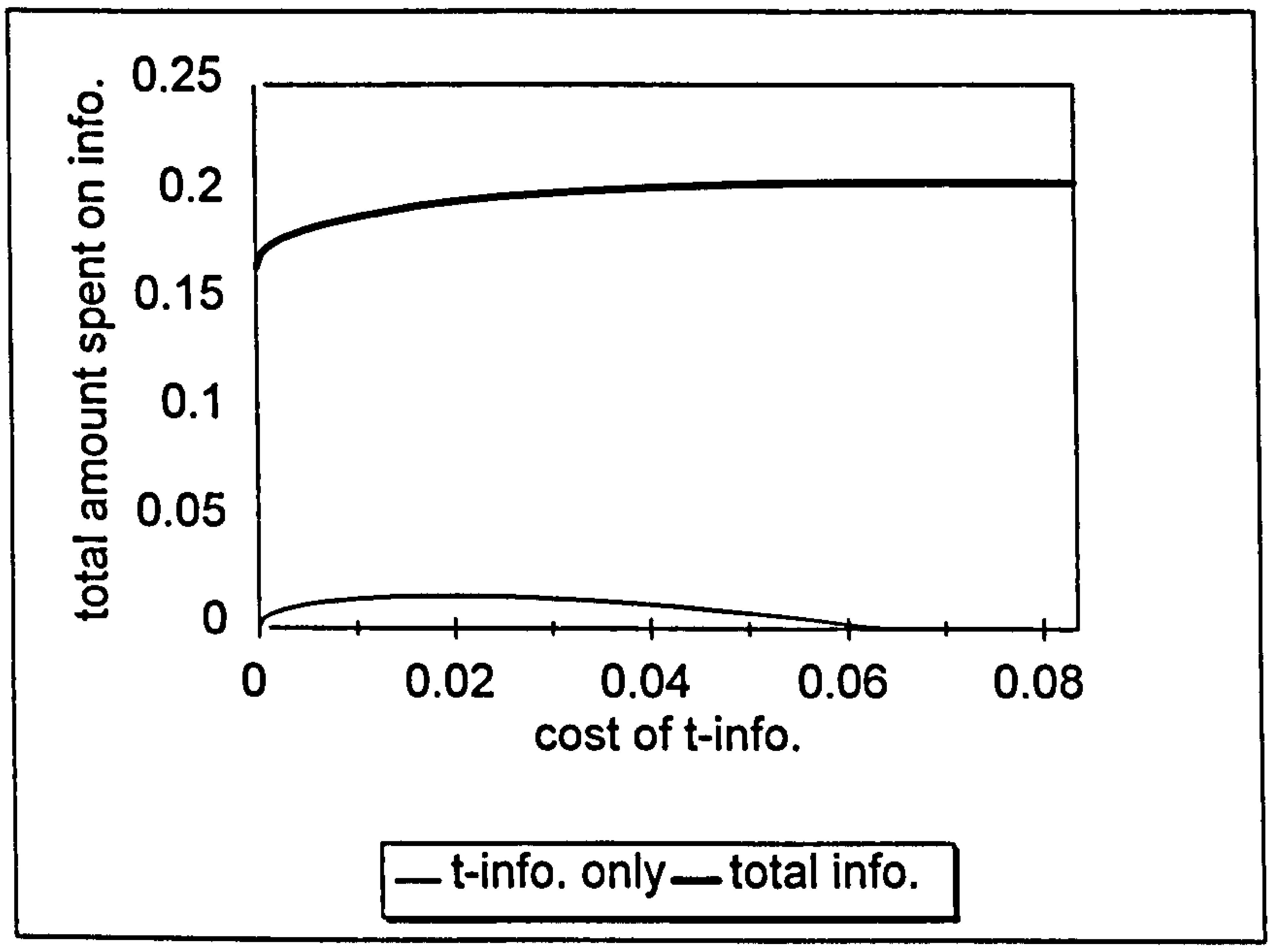



Figure 4.2 shows the amount spent on acquiring information, for both *t*-information alone and *s*- and *t*-information together.

*Fig. 4.2: Amount spent on information, with varying cost of t-information.*




As can be seen, the $t$-information spend follows a Laffer-curve, with both zero cost and zero purchase implying zero spend, with positive levels between these two extremes. The total information spend must rise with the cost of $t$-information, since the same overall precision must be maintained with a more expensive information mix. Since the same precision is maintained for all $t$-information cost levels, the mean price is also invariant.

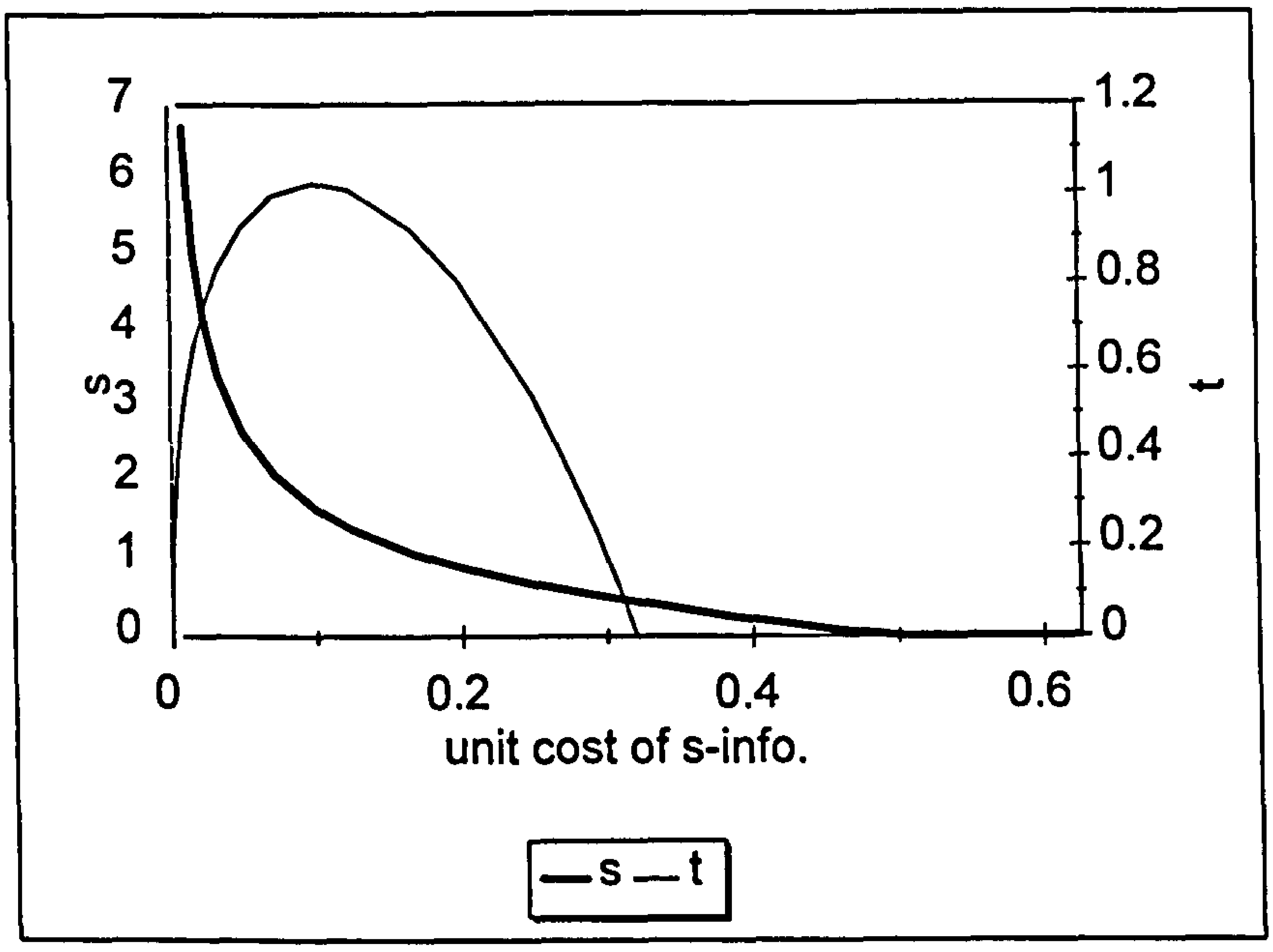
We will now look at the effects of varying the cost of information about value ($s$-information). Similar parameters to the above will be used, but with a fixed cost of t-information. The precise values are given in table 4.2.

*Table 4.2: Zero base-line, simple parameters, fixed cost of t-information.*

| $h_0$ | $p_S$ | $p_L$ | $r$ | $\bar{u}$ | $\bar{L}$ | $\bar{s}$ | $\bar{t}$ | $a_t$ |
|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 30 |

Figure 4.3 shows the equilibrium precision levels.

*Fig 4.3: Varying cost of s-information.*




These are as we would expect, with the level of *s*-information acquired falling with its cost until it reaches zero, and the level of *t*-information acquired again starting at zero, becoming positive - since the cost of *t*-information is below the threshold - and then falling back to zero while some *s*-information is still being acquired. Figure 4.4 shows the amounts spent on the information. With a varying cost of *s*-information, the amounts spent on both types of information (and the total amount spent on information) follow Laffer curves.

Since the precision of information as a whole is affected by the cost of *s*-information, so is the mean price level. Figure 4.5 shows the mean price levels corresponding to this example. As can be seen, the mean price level uniformly decreases with the cost of *s*-information up to the threshold above which no *s*-information is acquired.

*Fig. 4.4: Amount spent on information, with varying cost of s-information.*

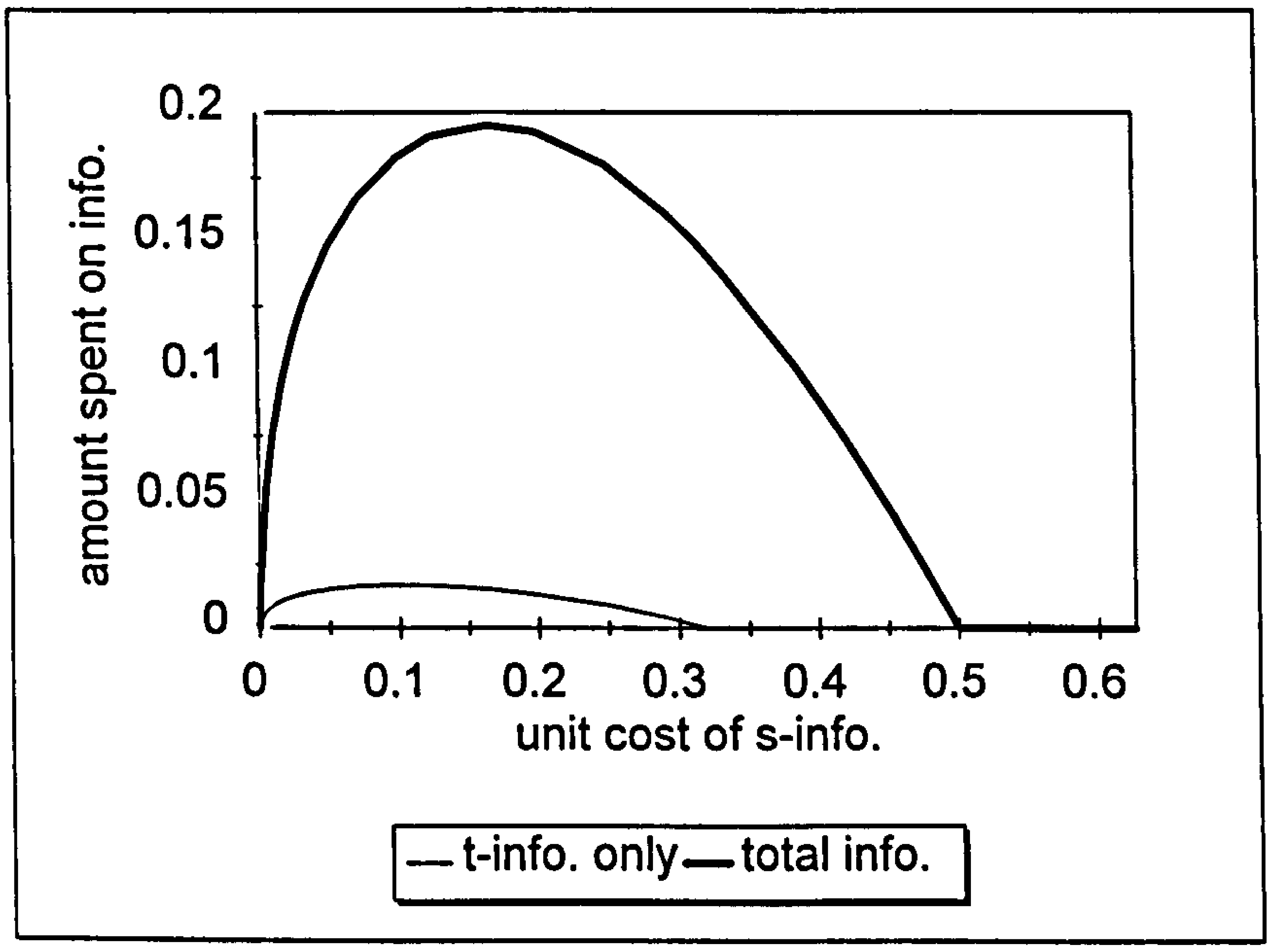
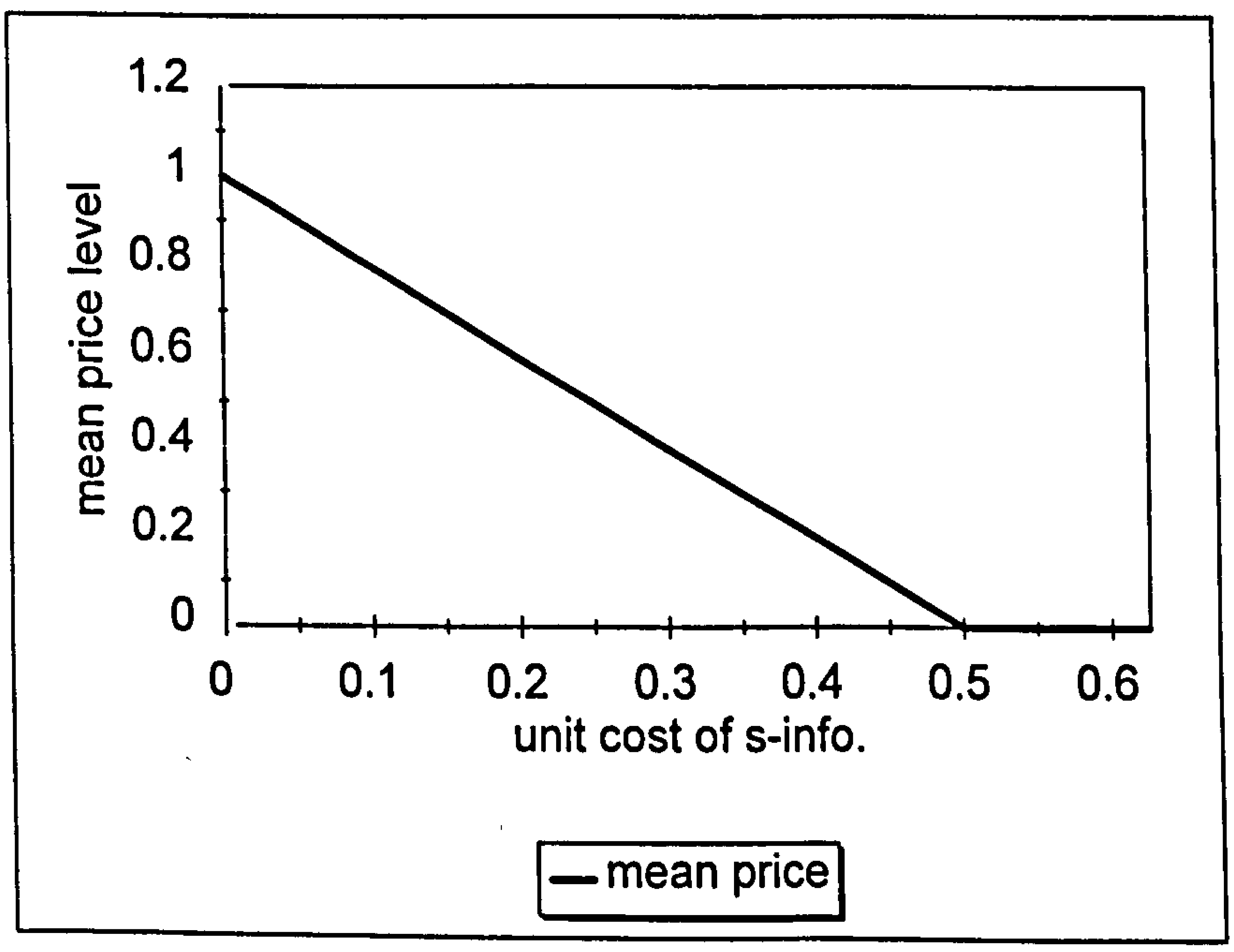



*Fig. 4.5: Mean price level with varying cost of s-information.*




## 4.7.2 Example #2: Gennotte & Leland inputs

In order to show that the nature of the above results is not heavily dependent on the (ad-hoc and simplistic) parameter values we used above, we can study the equilibria for an alternative set of parameters given in the literature by Gennotte & Leland

(1990). The cost of acquiring information about the liquidation value can be chosen so that the precision is as given in that paper: in particular, the target value of 2.5 is achieved, when $t$ is zero, at a value for $a_s$ of 18402. Our full set of parameter values is therefore as given in table 4.3.

*Table 4.3: Gennotte & Leland inputs.*

| $h_0$ | $p_S$ | $p_L$ | $r$ | $\bar{u}$ | $\bar{L}$ | $\bar{s}$ | $\bar{t}$ | $a_s$ |
|---|---|---|---|---|---|---|---|---|
| 12.5 | 5884 | 5884 | 1 | 1 | 0 | 0 | 0 | 18402 |

Figures 4.6 and 4.7 show the effect of varying the cost of acquiring supply information on both the amounts of value- and supply-information acquired, and the information spend.

*Fig. 4.6: Varying cost of t-information.*

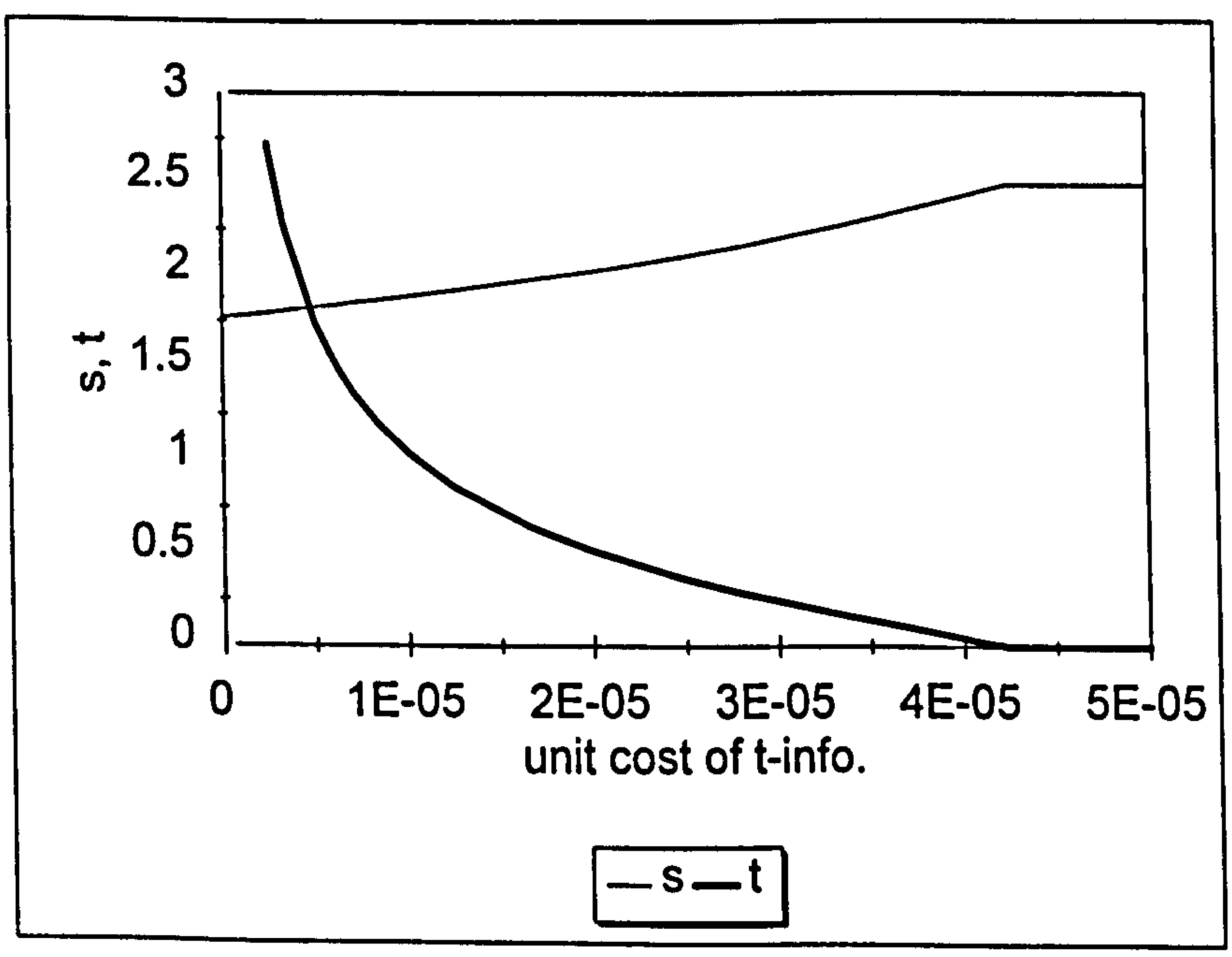
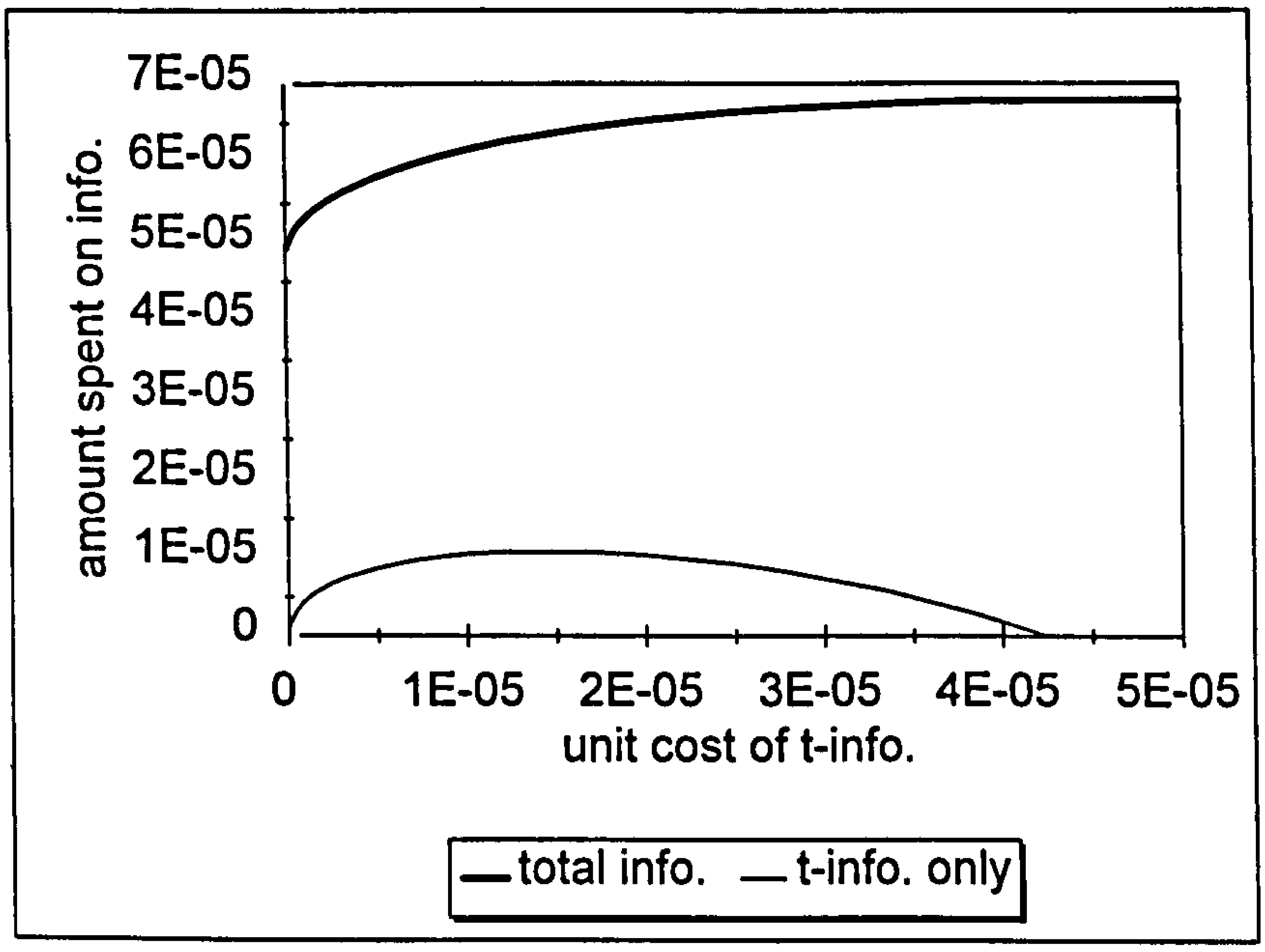


*Fig. 4.7: Amount spent on information, with varying cost of t-info. and G&L inputs.*




These closely resemble those for our previous example.

### 4.7.3 Example #3: Fixed *s*

The following two examples look at scenarios in which the precision of value-information is effectively fixed due to its high marginal cost. The parameters used are as shown in table 4.4.

*Table 4.4: Positive base-line for value information.*

| $h_0$ | $p_S$ | $p_L$ | $r$ | $\bar{u}$ | $\bar{L}$ | $\bar{s}$ | $\bar{t}$ | $a_s$ |
|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 'high' |

Figures 4.8 and 4.9 show the levels of supply-information acquired, and the resultant mean price level.

*Fig. 4.8: Equilibrium t with varying cost of t-information and fixed s.*

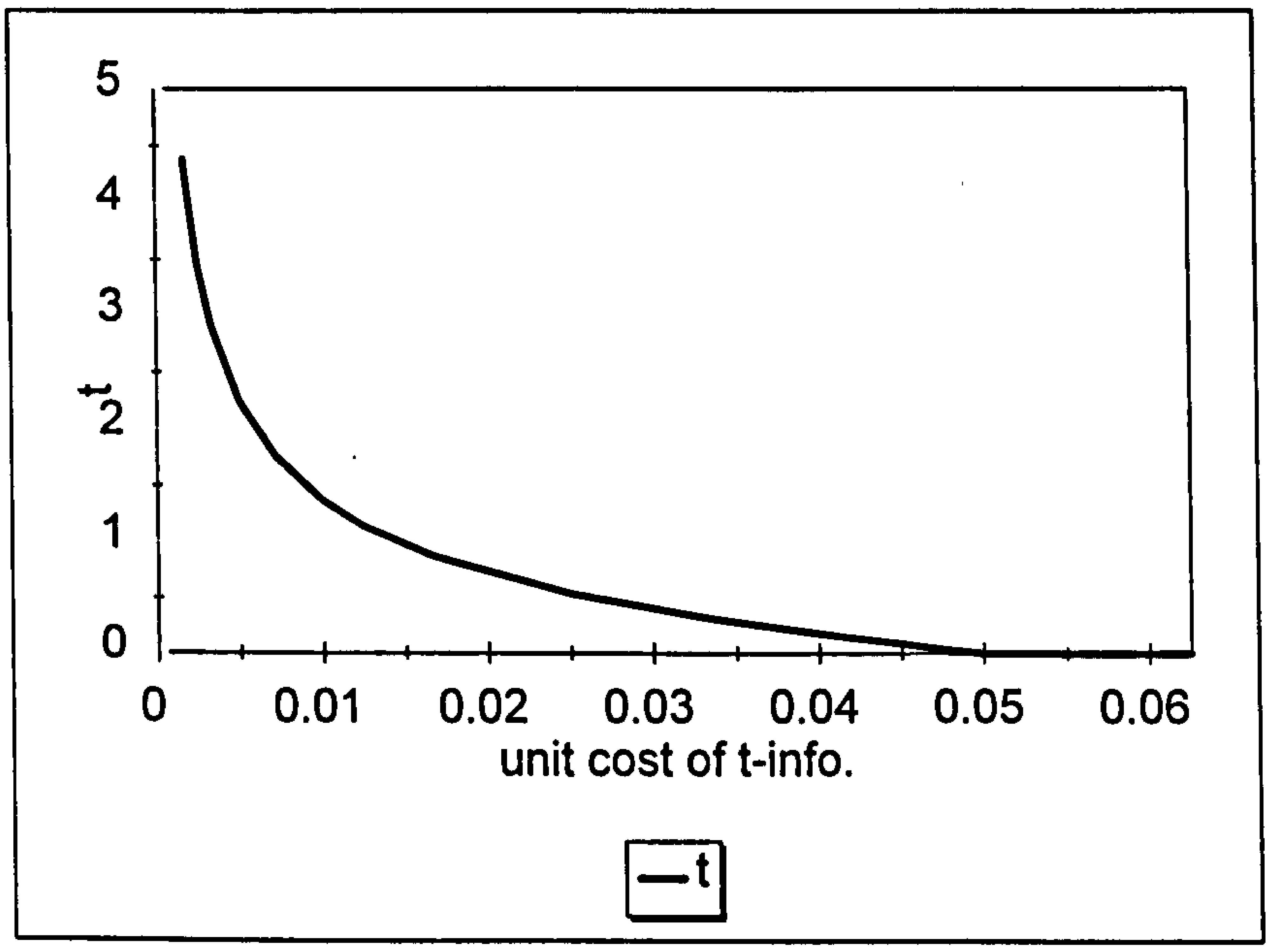
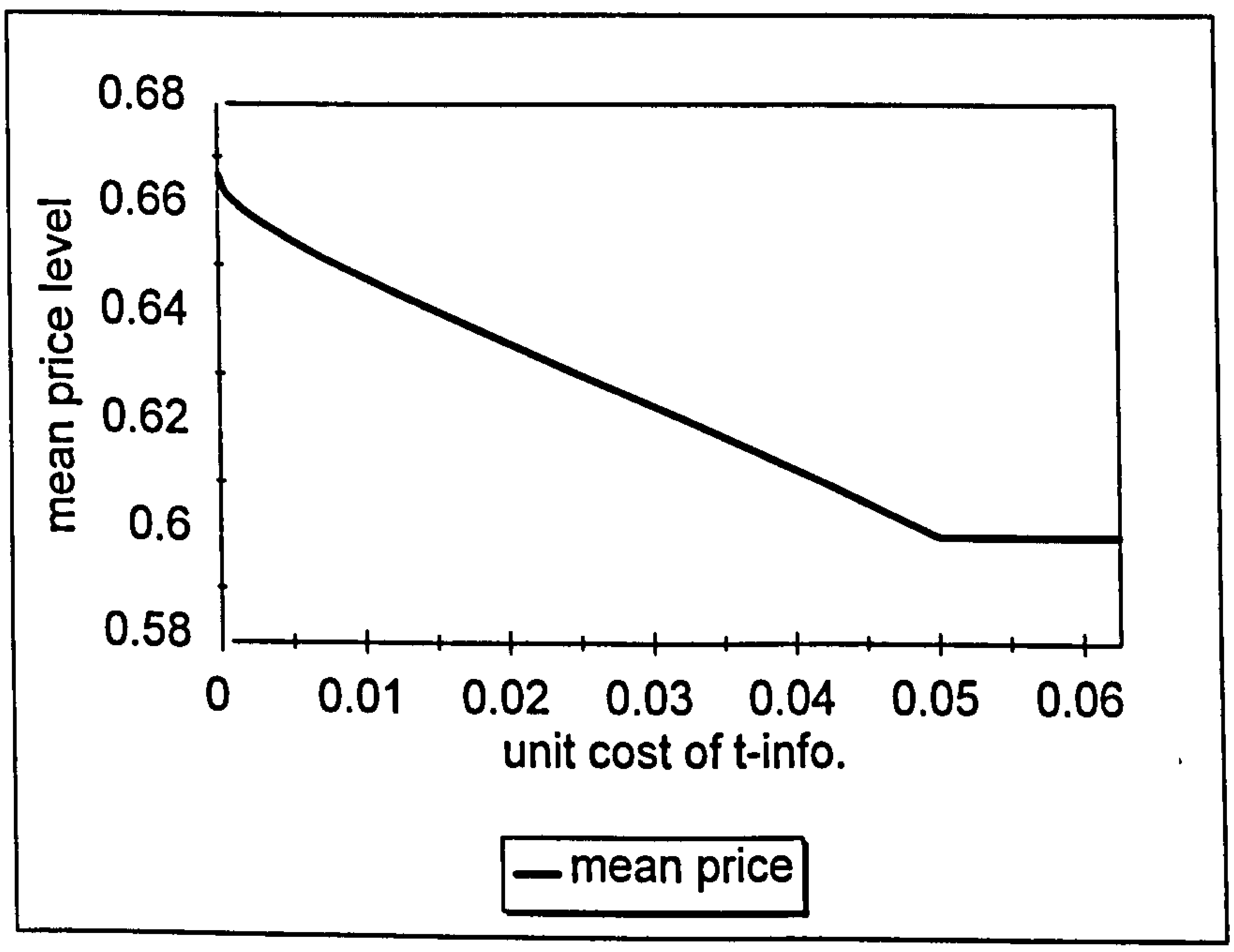



*Fig. 4.9: Mean price level with varying cost of t-information and constant s.*




Now that the reduction in the level of $t$-information acquired, brought about by its rising cost, can no longer be offset by additional investment in $s$-information, the overall precision of information is reduced, and hence so is the mean price level.

### 4.7.4 Example #4: Price crashes

In a previous paper, in which $s$ and $t$ were not determined endogenously, we showed that 'price crashes', in the sense of discontinuities in the price function,[2] could occur as a result of changes in the information precisions. In this example we show that they can also occur as a result of changes in the cost of information, provided that there is a positive base-line level of value-information, and the marginal cost of such information is high.

This example shows that by effectively fixing the precision of value-information ($s$), as in the previous example, changes in the cost of supply-information can induce price crashes for certain parameter values. We will use the same parameters as in the previous chapter, which are those given in table 4.5.

*Table 4.5: Parameter values from chapter three with zero supply info. base-line.*

| $h_0$ | $p_S$ | $p_L$ | $r$ | $\overline{u}$ | $\overline{L}$ | $\overline{s}$ | $\overline{t}$ | $a_s$ |
|---|---|---|---|---|---|---|---|---|
| 12 | 25 | 1000 | 1 | 1 | 10 | 0.5 | 0 | 'high' |

Figure 4.10 shows the precision of the supply-information acquired at each cost level. We now have a situation in which the precision of the supply-information acquired does not move smoothly to zero as its cost increases, but falls at an ever-diminishing rate, before suddenly slumping to zero. Figure 4.11 shows the associated mean price levels.

[2] Following Gennotte & Leland (1990).

*Fig. 4.10: Varying cost of t-information.*

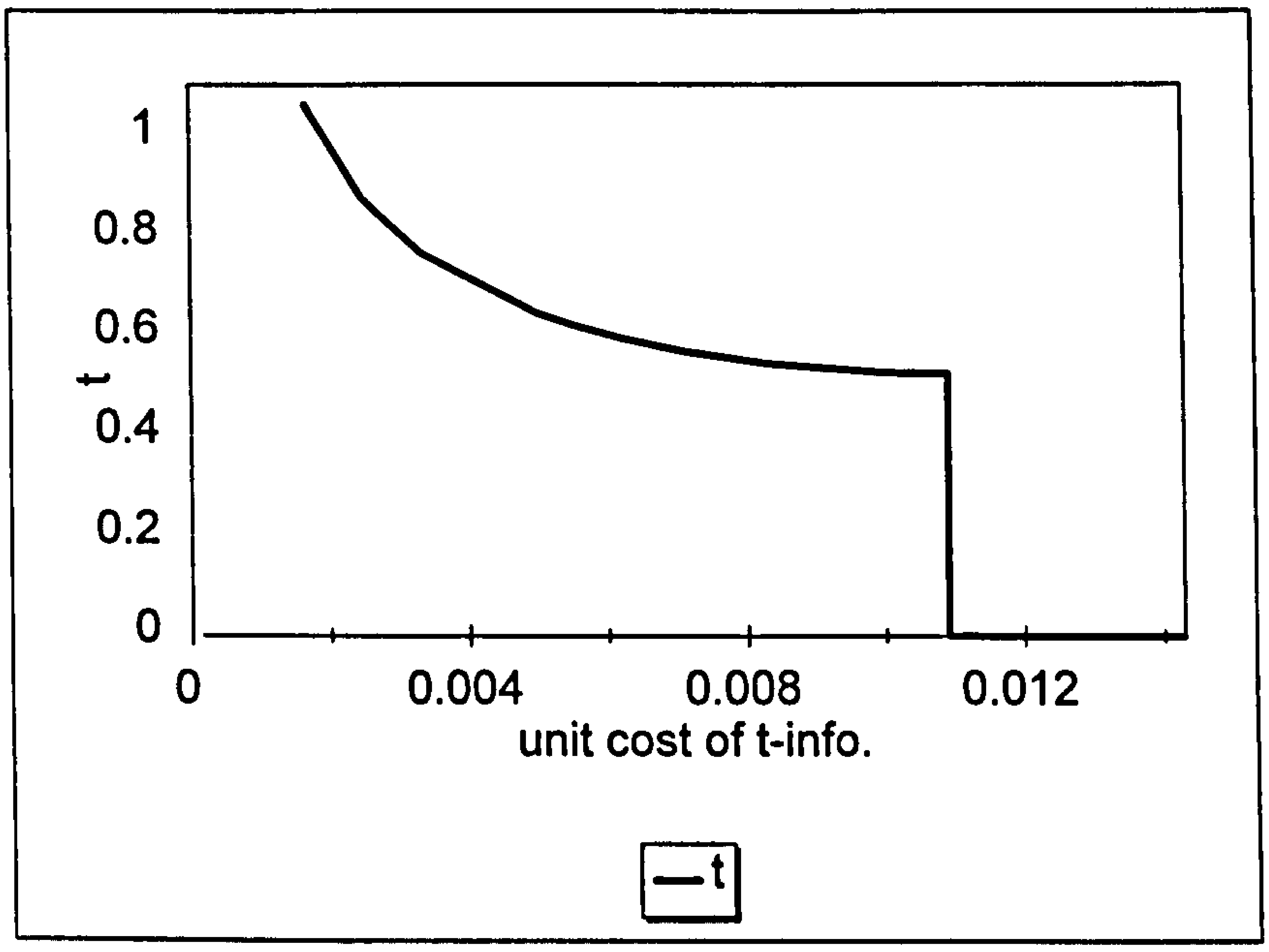
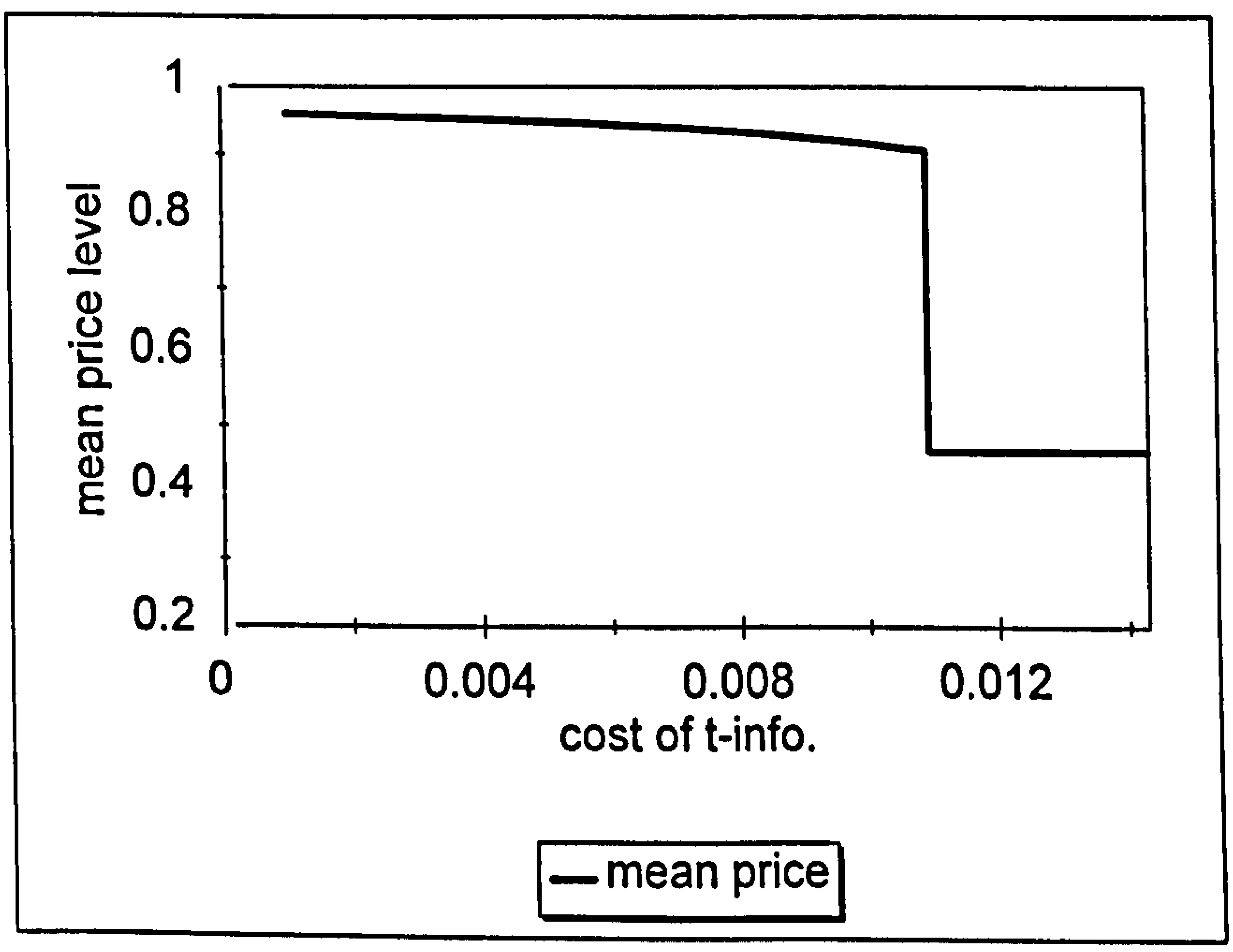



*Fig. 4.11: Mean price level, with varying cost of t-information and constant s.*




This reveals that the abrupt cessation of supply-information acquisition causes a corresponding discontinuity in the mean price level. In this example, it is therefore the crossing of a threshold for the cost of supply-information to either induce agents to purchase their first information units, or to cease purchasing any, that causes the equilibrium to move from one side of the discontinuity to the other.

### 4.7.5 Example #5: Dual equilibria

Although the previous example showed that price crashes can occur, it did not reveal a possibility for multiple equilibria. We know that multiple equilibria are possible, since we can set the base-line levels of precision equal to the values that induced multiple equilibria in the previous chapter, and impose high marginal costs of information. In the following example, by positing a positive base-line precision level for supply-information, as well as value-information, we demonstrate that multiple equilibria can also occur with agents purchasing positive extra amounts of supply-information. The parameters used are as given in table 4.6.

*Table 4.6: Parameter values from chapter three with supply info. base-line.*

| $h_0$ | $p_S$ | $p_L$ | $r$ | $\bar{u}$ | $\bar{L}$ | $\bar{s}$ | $\bar{t}$ | $a_s$ |
|---|---|---|---|---|---|---|---|---|
| 12 | 25 | 1000 | 1 | 1 | 10 | 0.5 | 0.54 | 'high' |

Depending on which of the two potential equilibria prevail, the agents will either acquire no additional supply-information, or will acquire an amount that is determined by the relationship given in figure 4.12. Figure 4.13 shows the two possible mean price levels.

*Fig. 4.12: Acquisition of t-information.*

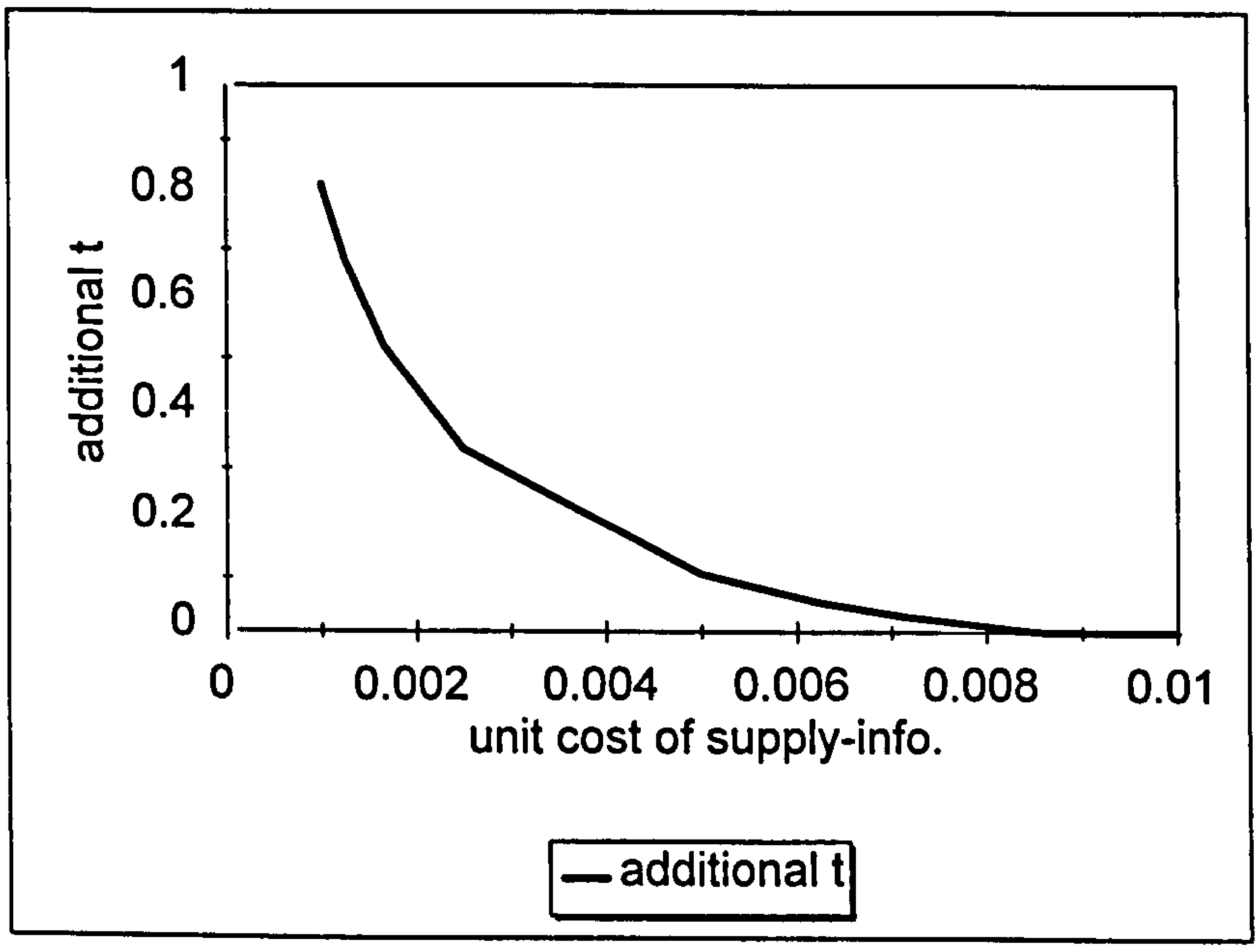
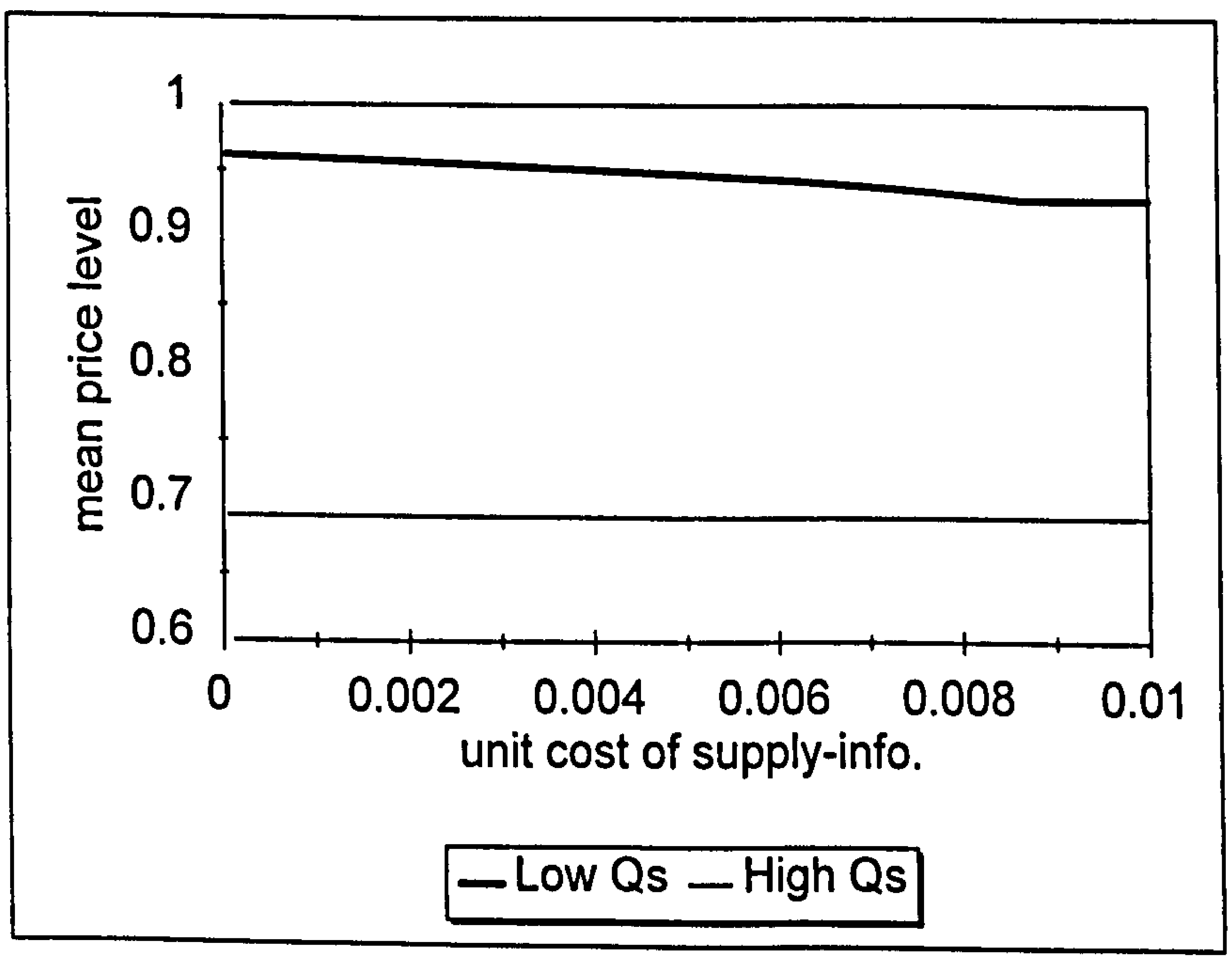



*Fig. 4.13: Dual equilibria.*



# 4.8 Conclusion

This chapter has investigated the incentives of traders to acquire information about the supply of stock emanating from liquidity traders as well as information directly relating to future stock value. We have found that with linear cost functions for information, and no free information endowment, the informativeness of the price is determined solely by the cost of value-information and the risk tolerance of the traders. In this situation, the acquisition of supply-information can sometimes provide a cheaper way of obtaining the desired level of informativeness, but will not alter this desired level.

We have also seen that, if the endowment of free value-information is assumed to be positive, price discontinuities can occur at the point at which supply-information ceases (or begins) to be acquired, following changes in the cost of this information; but there is only one potential equilibrium for each cost level, and so prices do not display hysteresis. In addition, there may also be dual equilibria, but we have found no evidence that changes in (supply-) information cost alone can precipitate a forced movement from one equilibrium to another: this remains the preserve of changes in the base-line precision levels, as illustrated in the previous chapter.

# Appendix C: For Chapter 4

## C1 Maximising traders' expected utility

The traders aim to maximise the expected utility of their consumption at liquidation, which is as follows:

$$\underset{\{t,s,D,B\}}{\text{maximise}}\, E_{\tilde{u},\tilde{w},\tilde{y},\tilde{P}}[U(\tilde{u}D+B)]$$

$$= \underset{\{t,s,D,B\}}{\max.}\, E_{\tilde{w},\tilde{y},\tilde{P}} \left\{ E_{\tilde{u}} \left( -\exp(-(\tilde{u}D+B)/r) \middle| \tilde{w}=w, \tilde{y}=y, \tilde{P}=P \right) \right\}$$

$$= \underset{\{t,s\}}{\max.}\, E_{\tilde{w},\tilde{y},\tilde{P}} \left\{ E_{\tilde{u}} \left( -\exp\left( -(\tilde{u}-P)\left(\frac{\mu-P}{\sigma^2}\right) - \frac{W_0 - c(s) - d(t)}{r} \right) \middle| \tilde{w}=w, \tilde{y}=y, \tilde{P}=P \right) \right\}$$

$$= \underset{\{t,s\}}{\max.}\, E_{\tilde{w},\tilde{y},\tilde{P}} \left\{ -\exp\left( -\frac{1}{2}\frac{\left(\mu\left(\tilde{w},\tilde{y},\tilde{P}\right)-\tilde{P}\right)^2}{\sigma^2} - \frac{W_0 - c(s) - d(t)}{r} \right) \right\} \quad \text{(C1)}$$

To analyse this further, we need to develop some background theory.

## C2 Multivariate normal distributions

The density function for the multivariate normal distribution can be expressed as:

$$\frac{1}{(2\pi)^{3/2} D^{1/2}} \exp\left( -\frac{1}{2} v M^{-1} v \right) \quad \text{(C2)}$$

where $M$ is the $n$-variable variance-covariance matrix, $D$ is the determinant of $M$, $v$ is the vector of variables or, as here, the vector of the differences between variables and their mean value.

The density function for the trivariate normal distribution is thus:

$$\frac{1}{(2\pi)^{3/2} D_3^{1/2}} \exp\left\{ -\frac{1}{2D_3} \Big[ \left(\sigma_y^2\sigma_z^2 - \sigma_{yz}^2\right)(x - x_o)^2 + \left(\sigma_x^2\sigma_z^2 - \sigma_{xz}^2\right)(y - y_o)^2 + \left(\sigma_x^2\sigma_y^2 - \sigma_{xy}^2\right)(z - z_o)^2 - 2\left(\sigma_z^2\sigma_{xy} - \sigma_{xz}\sigma_{yz}\right)(x - x_o)(y - y_o) - 2\left(\sigma_y^2\sigma_{xz} - \sigma_{xy}\sigma_{yz}\right)(x - x_o)(z - z_o) - 2\left(\sigma_x^2\sigma_{yz} - \sigma_{xy}\sigma_{xz}\right)(y - y_o)(z - z_o) \Big] \right\} \tag{C3}$$

where:

$$D_3 = \sigma_x^2\sigma_y^2\sigma_z^2 - \sigma_x^2\sigma_{yz}^2 - \sigma_y^2\sigma_{xz}^2 - \sigma_z^2\sigma_{xy}^2 + 2\sigma_{xy}\sigma_{xz}\sigma_{yz}$$

and $\sigma_i^2$ represents the variance of variable $i$, $\sigma_{ij}$ represents the covariance between variables $i$ and $j$, and $i_0$ represents the mean of variable $i$.

For the bivariate normal the density function is:

$$\frac{1}{2\pi D_2^{1/2}} \exp\left\{ -\frac{1}{2D_2} \left( \sigma_z^2 (y - y_o)^2 + \sigma_y^2 (z - z_o)^2 \right) - 2\sigma_{yz}(y - y_o)(z - z_o) \right\} \tag{C4}$$

where:

$$D_2 = \sigma_y^2\sigma_z^2 - \sigma_{yz}^2$$

And for the univariate normal the density function is:

$$\frac{1}{(2\pi)^{1/2}\sigma_z}\exp\left\{-\frac{(z-z_o)^2}{2\sigma_z^2}\right\} \qquad \text{(C5)}$$

The integral of the density function of the *n*-variate normal distribution over one of the variables is the marginal distribution, which is the density function for the *n*-minus-1-variate case. Hence:

$$\begin{aligned}
&\int_y\int_x \frac{1}{(2\pi)^{3/2}D_3^{1/2}}\exp\left\{-\frac{1}{2D_3}\Big[\left(\sigma_y^2\sigma_z^2-\sigma_{yz}^2\right)(x-x_o)^2\right.\\
&\quad+\left(\sigma_x^2\sigma_z^2-\sigma_{xz}^2\right)(y-y_o)^2+\left(\sigma_x^2\sigma_y^2-\sigma_{xy}^2\right)(z-z_o)^2\\
&\quad-2\left(\sigma_z^2\sigma_{xy}-\sigma_{xz}\sigma_{yz}\right)(x-x_o)(y-y_o)\\
&\quad-2\left(\sigma_y^2\sigma_{xz}-\sigma_{xy}\sigma_{yz}\right)(x-x_o)(z-z_o)\\
&\qquad\qquad\left.-2\left(\sigma_x^2\sigma_{yz}-\sigma_{xy}\sigma_{xz}\right)(y-y_o)(z-z_o)\Big]\right\}dx\,dy
\end{aligned}$$

$$\begin{aligned}
&=\int_y \frac{D_2^{-1/2}}{2\pi}\exp\left\{-\frac{1}{2D_2}\Big[\sigma_z^2(y-y_o)^2+\sigma_y^2(z-z_o)^2\right.\\
&\qquad\qquad\left.-2\sigma_{yz}(y-y_o)(z-z_o)\Big]\right\}dy
\end{aligned}$$

$$=\frac{1}{(2\pi)^{1/2}\sigma_z}\exp\left\{-\frac{(z-z_o)^2}{2\sigma_z^2}\right\} \qquad \text{(C6)}$$

where:

$$D_2=\sigma_y^2\sigma_z^2-\sigma_{yz}^2$$

Hence any expression of the form:

$$\frac{D_3^{-1/2}}{(2\pi)^{3/2}}\exp\left\{-\frac{1}{2}\left[b_1(x-x_o)^2+b_2(y-y_o)^2+b_3(z-z_o)^2\right.\right.$$
$$-2b_4(x-y_o)(y-y_o)-2b_5(x-x_o)(z-z_o)$$
$$\left.\left.-2b_6(y-y_o)(z-z_o)\right]\right\}$$

$$=\frac{D_3^{-1/2}}{(2\pi)^{3/2}}\exp\left\{-\frac{1}{2D_3}\left[b_1D_3(x-x_o)^2+b_2D_3(y-y_o)^2+b_3D_3(z-z_o)^2\right.\right.$$
$$-2b_4D_3(x-y_o)(y-y_o)-2b_5D_3(x-x_o)(z-z_o) \tag{C7}$$
$$\left.\left.-2b_6D_3(y-y_o)(z-z_o)\right]\right\}$$

with:

$$D_3=1/\left(b_1b_2b_3-b_3b_4^{\,2}-b_2b_5^{\,2}-b_1b_6^{\,2}+2b_4b_5b_6\right) \tag{C8}$$

is identical to the density of a trivariate normal distribution with:

$$\begin{aligned}
\sigma_x^{\,2}&=D_3\left(b_2b_3-b_6^{\,2}\right)\\
\sigma_y^{\,2}&=D_3\left(b_1b_3-b_5^{\,2}\right)\\
\sigma_z^{\,2}&=D_3\left(b_1b_2-b_4^{\,2}\right)\\
\sigma_{xy}&=D_3\left(b_3b_4+b_5b_6\right)\\
\sigma_{xz}&=D_3\left(b_2b_5+b_4b_6\right)\\
\sigma_{yz}&=D_3\left(b_1b_6+b_4b_5\right)
\end{aligned} \tag{C9}$$

and

$$\int_y\int_x\frac{D_3^{-1/2}}{(2\pi)^{3/2}}\exp\left\{-\frac{1}{2}\left[b_1(x-x_o)^2+b_2(y-y_o)^2+b_3(z-z_o)^2\right.\right.$$
$$-2b_4(x-y_o)(y-y_o)-2b_5(x-x_o)(z-z_o)$$
$$\left.\left.-2b_6(y-y_o)(z-z_o)\right]\right\}\,dx\,dy$$

$$= \int_y \int_x \frac{D_3^{-1/2}}{(2\pi)^{3/2}} \exp\left\{ -\frac{1}{2D_3} \Big[ b_1 D_3 (x-x_o)^2 + b_2 D_3 (y-y_o)^2 + b_3 D_3 (z-z_o)^2 \right.$$

$$- 2b_4 D_3 (x-y_o)(y-y_o) - 2b_5 D_3 (x-x_o)(z-z_o)$$

$$\left. -2b_6 D_3 (y-y_o)(z-z_o) \Big] \right\} dx\, dy$$

$$= \int_y \frac{D_2^{-1/2}}{2\pi} \exp\left\{ -\frac{D_3}{2D_2} \Big[ (b_1 b_2 - b_4{}^2)(y-y_o)^2 + (b_1 b_3 - b_5{}^2)(z-z_o)^2 \right.$$

$$\left. - 2(b_1 b_6 + b_4 b_5)(x-x_o)(y-y_o) \Big] \right\} dy$$

$$= \left( \frac{1}{2\pi D_3 (b_1 b_2 - b_4{}^2)} \right)^{\frac{1}{2}} \exp\left( - \frac{(z-z_o)^2}{2 D_3 (b_1 b_2 - b_4{}^2)} \right) \qquad \text{(C10)}$$

where:

$$D_2 = \frac{1}{D_3{}^2} \Big[ (b_1 b_3 - b_5{}^2)(b_1 b_2 - b_4{}^2) - (b_1 b_6 + b_4 b_5)^2 \Big] = \frac{b_1}{D_3}$$

# C3 Application to our model

Our model gives us the following:

$$\nu \equiv \begin{pmatrix} \tilde{w}_i \\ \tilde{y}_i - \bar{u} \\ \tilde{P} - (\alpha_u + \beta)\bar{u} - (\alpha_L - \gamma_L)\bar{L} \end{pmatrix} = \begin{pmatrix} \tilde{w}_i \\ \tilde{y}_i - \bar{u} \\ \tilde{P} - \bar{P} \end{pmatrix} \qquad \text{(C11)}$$

$$M = \begin{pmatrix} p_S^{-1} + t^{-1} & 0 & -\delta p_S^{-1} \\ 0 & h_o^{-1} + s^{-1} & \beta h_o^{-1} \\ -\gamma_S{}^2 p_S^{-1} & \beta h_o^{-1} & \beta^2 h_o^{-1} + \gamma_S{}^2 p_S^{-1} + \gamma_L{}^2 p_L^{-1} \end{pmatrix} \qquad \text{(C12)}$$

The associated density function can be expressed as:

$$\frac{A^{1/2}}{(2\pi)^{3/2}}\exp\left\{-\frac{1}{2}\left[a_1w^2+a_2(y-\bar{u})^2+a_3(P-\bar{P})^2 -2a_4w(y-\bar{u})-2a_5w(P-\bar{P})-2a_6(y-\bar{u})(P-\bar{P})\right]\right\} \tag{C13}$$

where:

$$A=\frac{1}{D_A}=\frac{h_o stp_S p_L}{(p_s+t)(\gamma_L{}^2(h_o+s)+\beta^2 p_L)+\gamma_S{}^2 p_L(h_o+s)}$$

The coefficients $a_1, \ldots, a_6$ are too cumbersome to be expressed here.

Using the technique given in Appendix A (section A1, expressions A3, A4) and rearranging, we can write the expression incorporating the conditional mean and variance as follows:

$$\frac{\{\mu(w,y,P)-P\}^2}{\sigma^2}=a_7w^2+a_8(y-\bar{u})^2+a_9(P-\bar{P})^2 -2a_{10}w(y-\bar{u})-2a_{11}w(P-\bar{P})-2a_{12}(y-\bar{u})(P-\bar{P}) \tag{C14}$$

where we once again do not give the coefficients ($a_7, \ldots, a_{12}$). Thus:

$$E_{\tilde{w},\tilde{y},\tilde{P}}\left[-\exp\left(-\frac{1}{2}\frac{\{\mu(\tilde{w},\tilde{y},\tilde{P})-P\}^2}{\sigma^2}-\frac{W_0}{r}\right)\right]$$

$$=\int_P\int_y\int_w -\frac{A^{1/2}}{(2\pi)^{3/2}}\exp\left\{-\frac{1}{2}\left[a_1w^2+a_2(y-\bar{u})^2+a_3(P-\bar{P})^2 -2a_4w(y-\bar{u})-2a_5w(P-\bar{P})-2a_6(y-\bar{u})(P-\bar{P})\right] -\frac{1}{2}\left[a_7w^2+a_8(y-\bar{u})^2+a_9(P-\bar{P})-2a_{10}w(y-\bar{u}) -2a_{11}w(P-\bar{P})-2a_{12}(y-\bar{u})(P-\bar{P})\right]-\frac{W_0}{r}\right\}dw\,dy\,dP$$

$$= \int_P\int_y\int_w -\frac{A^{1/2}}{(2\pi)^{3/2}} \exp\left\{-\frac{1}{2}(a_1+a_7)w^2 - \frac{1}{2}(a_2+a_8)(y-\bar{u})^2 \right.$$
$$-\frac{1}{2}(a_3+a_9)(P-\bar{P})^2 + (a_4+a_{10})w(y-\bar{u})$$
$$\left. +(a_5+a_{11})w(P-\bar{P}) + (a_6+a_{12})(y-\bar{u})(P-\bar{P}) - \frac{W_0}{r}\right\} dw\, dy\, dP$$

$$= -(AD_3)^{\frac{1}{2}} \exp\left(-\frac{W_0}{r}\right) \int_P\int_y \left\{ \int_w \frac{D_3^{-1/2}}{(2\pi)^{3/2}} \exp\left\{-\frac{1}{2}(a_1+a_7)w^2 \right.\right.$$
$$-\frac{1}{2}(a_2+a_8)(y-\bar{u})^2 - \frac{1}{2}(a_3+a_9)(P-\bar{P})^2 + (a_4+a_{10})w(y-\bar{u})$$
$$\left.\left. +(a_5+a_{11})w(P-\bar{P}) + (a_6+a_{12})(y-\bar{u})(P-\bar{P}) \right\} dw \right\} dy\, dP$$

$$= -(AD_3)^{\frac{1}{2}} \exp\left(-\frac{W_0}{r}\right) \int_P \left\{ \int_y \frac{D_2^{-1/2}}{2\pi} \exp\left\{ -\frac{D_3}{2D_2}\Big[c_1(y-\bar{u})^2 + c_2(P-\bar{P})^2 \right.\right.$$
$$\left.\left. - c_3(y-\bar{u})(P-\bar{P}) \Big]\right\} dy \right\} dP$$

$$= -(AD_3)^{\frac{1}{2}} \exp\left(-\frac{W_0}{r}\right) \int_P -\frac{1}{(2\pi D_1)^{1/2}} \exp\left(-\frac{(P-\bar{P})^2}{2D_1}\right) dP$$

$$= -(AD_3)^{\frac{1}{2}} \exp\left(-\frac{W_0}{r}\right) \tag{C15}$$

where:

$$D_3 = 1 \Bigg/ \left\{ \begin{array}{l} (a_1+a_7)(a_2+a_8)(a_3+a_9) - (a_3+a_9)(a_4+a_{10})^2 - (a_2+a_8)(a_5+a_{11})^2 \\ \qquad - (a_1+a_7)(a_6+a_{12})^2 + 2(a_4+a_{10})(a_5+a_{11})(a_6+a_{12}) \end{array} \right\}$$

$$\frac{1}{D_3} = \left[(a_1+a_7)(a_2+a_8) - (a_4+a_{10})^2\right] \times \left\{ (a_3+a_9) \right.$$
$$\left. - \frac{(a_2+a_8)(a_5+a_{11})^2 + (a_1+a_7)(a_6+a_{12})^2 - 2(a_4+a_{10})(a_5+a_{11})(a_6+a_{12})}{(a_1+a_7)(a_2+a_8) - (a_4+a_{10})^2} \right\}$$

$$\frac{1}{D_3 c_1} = a_3 + a_9$$

$$-\frac{(a_2 + a_8)(a_5 + a_{11})^2 + (a_1 + a_7)(a_6 + a_{12})^2 - 2(a_4 + a_{10})(a_5 + a_{11})(a_6 + a_{12})}{(a_1 + a_7)(a_2 + a_8) - (a_4 + a_{10})^2}$$

$$D_2 = \frac{(a_1 + a_7)}{D_3}$$

$$c_1 = (a_1 + a_7)(a_2 + a_8) - (a_4 + a_{10})^2$$

$$c_2 = (a_1 + a_7)(a_3 + a_9) - (a_5 + a_{11})^2$$

$$c_3 = (a_1 + a_7)(a_6 + a_{12}) - (a_4 + a_{10})(a_5 + a_{11})$$

$$D_1 = D_3 c_1$$

Substituting for $a_1$, $a_2$,...., $a_{12}$, equation C15 becomes:

$$-\left(\frac{h_o p_S p_L(\gamma^* + \gamma_L^2 t)}{(h_o\gamma^* + p_S p_L \alpha^2)\{(h_o + s)(\gamma^* + \gamma_L^2 t) + \beta^2 p_L(p_S + t)\}}\right)^{\frac{1}{2}} \times \exp\left(-\frac{W_0}{r}\right) \tag{C16}$$

where:

$$\gamma^* \equiv \gamma_L^2 p_S + \gamma_S^2 p_L$$

and

$$\alpha \equiv \alpha_u + (\alpha_L - \gamma_L)\frac{\bar{L}}{\bar{u}}$$

# C4 Partial derivatives

The partial derivatives of the expected utility function with respect to the precisions of the value- and supply-information are as follows:

$$
\begin{aligned}
\frac{\partial()}{\partial s} = -\left\{\frac{h_o p_S p_L\left(\gamma_L^2(p_S+t)+\gamma_S^2 p_L\right)}{\left(h_o\left(\gamma_L^2 p_S+\gamma_S^2 p_L\right)+p_S p_L \alpha^2\right)}\right\}^{\frac{1}{2}} \\
\times\left\{\left\{(p_S+t)\left\{\gamma_L^2(h_o+s)+\beta^2 p_L\right\}+\gamma_S^2 p_L(h_o+s)\right\}^{-\frac{1}{2}}\left(\frac{c_s'(s)}{r}\right)\right. \\
-\frac{1}{2}\left\{\gamma_L^2(p_S+t)+\gamma_S^2 p_L\right\} \\
\left.\times\left\{(p_S+t)\left\{\gamma_L^2(h_o+s)+\beta^2 p_L\right\}+\gamma_S^2 p_L(h_o+s)\right\}^{-\frac{3}{2}}\right\} \\
\times\exp\left(\frac{c(s)+d(t)-W_0}{r}\right)
\end{aligned}
\tag{C17}
$$

$$
\begin{aligned}
\frac{\partial()}{\partial t} = -\left\{\frac{h_o p_S p_L\left(\gamma_L^2\{p_S+t\}+\gamma_S^2 p_L\right)}{\left(h_o\left\{\gamma_L^2 p_S+\gamma_S^2 p_L\right\}+p_S p_L \alpha^2\right)}\right\}^{\frac{1}{2}} \\
\times\left\{\left\{(p_S+t)\left\{\gamma_L^2(h_o+s)+\beta^2 p_L\right\}+\gamma_S^2 p_L(h_o+s)\right\}^{-\frac{1}{2}}\right. \\
\times\left(\frac{\gamma_L^2}{2\left(\gamma_L^2\{p_S+t\}+\gamma_S^2 p_L\right)}+\frac{d_t'(t)}{r}\right) \\
-\frac{1}{2}\left\{\gamma_L^2(h_o+s)+\beta^2 p_L\right\} \\
\left.\times\left\{(p_S+t)\left\{\gamma_L^2(h_o+s)+\beta^2 p_L\right\}+\gamma_S^2 p_L(h_o+s)\right\}^{-\frac{3}{2}}\right\} \\
\times\exp\left(\frac{c(s)+d(t)-W_0}{r}\right)
\end{aligned}
\tag{C18}
$$

# Appendix D: Glossary of terms used in chapters 2-4.

**Terms with equivalents in the basic Hellwig-type model.**

| | |
|---|---|
| $W_i$ | Wealth of agent *i*. |
| $r_i$ | Risk tolerance of investor *i*. |
| $\tilde{P}$ | Market price. |
| $\tilde{u}$ | Liquidation value (L.V.). |
| $\tilde{L}$ | Unobserved stock supply (per investor). |
| $\tilde{y}_i$ | Investor *i*'s information signal concerning the liquidation value. |
| $h_0$ | (Unconditional) precision of the liquidation value. |
| $p_L$ | Precision of the unobserved stock supply (*L*). |
| $s_i$ | Precision of investor *i*'s L.V. signal. |
| $\bar{u}$ | (Unconditional) mean of the L.V. |
| $\bar{L}$ | Mean of the unobserved stock supply. |
| $K_i$ | Precision of L.V. conditional on price & investor *i*'s information. |

**Terms added in Chapter 2.**

| | |
|---|---|
| $N$ | Number of L.V. components. |
| $\mu_i$ | Measure in market of investors receiving info. of component *i*. |

**Terms added in Chapter 3.**

| | |
|---|---|
| $\tilde{S}$ | (Noisily-) observed stock supply (per investor). |
| $\tilde{w}_i$ | Investor *i*'s information signal concerning the stock supply. |
| $p_S$ | Precision of the observed stock supply (*S*). |
| $t_i$ | Precision of investor *i*'s stock supply signal. |
| $\bar{S}$ | Mean of the observed stock supply. |

**Terms added in Chapter 4.**

| | |
|---|---|
| $a_s$ | Cost co-efficient for L.V. information. |
| $a_t$ | Cost co-efficient for supply information. |
| $s_*$ | Additional purchase of L.V. information precision. |
| $t^*$ | Additional purchase of supply information precision. |
| $\bar{s}$ | Base-line endowment of L.V. information precision. |
| $\bar{t}$ | Base-line endowment of supply information precision. |

# Chapter 5

# Speculation and ill-informed investors

## 5.1 Introduction

In chapter two we saw how the presence of liquidity traders can allow a partially-revealing equilibrium to occur in an information aggregation framework. In chapters three and four we saw how the existence of private information about the level of liquidity trading can give rise to the potential for multiple equilibria and price 'crashes.' In the remainder of this thesis we investigate the potential behavioural attributes of liquidity - or ill-informed - trading, and investigate the implications of this for security prices.

In the models of the previous three chapters, prices in the absence of rational investors, although defined, are meaningless, and do not allow for a comparison with and without rational speculators. In what follows we begin to look on liquidity traders more as a group of ill-informed investors that are potentially capable of supporting prices on their own. In this context, there are three possibilities for the effect of informed investor behaviour on ill-informed investor activity: it can mitigate its effects; it can unwittingly exacerbate its effects; or it can deliberately induce destabilising behaviour. We will see below that the particular type of behaviour postulated for the ill-informed investors crucially influences the effect of informed trader behaviour. This issue is similar to that of the effect of speculators on prices in the presence of non-speculators. In some instances, the informed / ill-informed categorisation will correspond to the speculator / non-speculator categorisation; but this need not always be the case.

In the following section we look at the concept of speculation, and assess the effect speculators are likely to have on market prices. In section 5.3 we investigate the effect on prices of fads in ill-informed investor behaviour that not influenced by market prices, and in particular utilise a version of the model given in Shiller (1984). In section 5.4 we go on to look at the implications of ill-informed investor behaviour that is influenced by market prices, focusing particularly on Hart (1977).

# 5.2 Speculation

## 5.2.1 Defining speculation

Speculation can be thought of as trading activity motivated by expectations of abnormal returns resulting from subsequent price shifts, and will generally involve a reversal of the initial position taken. This is compatible with the definition given by Kaldor, who defined speculation as:

> "the purchase (or sale) of goods with a view to resale (repurchase) at a later date, where the motive behind such action is the expectation of a change in the relevant prices relatively to the ruling price and not a gain accruing through their use, or any kind of transformation effected in them, or their transfer between markets" (1939, p. 1).

Hirshleifer used a similar definition, noting that:

> "Speculation is ordinarily understood to mean the purchase of a good for later re-sale rather than for use, or the temporary sale of a good with the intention of later re-purchase - in the hope of profiting from an intervening price change" (1977, p. 975).

The expected price movement motivates speculators to pay more for the asset than they would if they were prevented from reselling, which provides the definition of speculative behaviour given by Harrison & Kreps (1978), as we saw in the first chapter of this thesis.

These definitions of speculation, and the associated implicit definitions of non-speculators, seems fairly suitable for commodities markets, since producers buy inputs with no intention of selling them back at a later date and sell outputs with no intention of buying them back, and so could be thought of as non-speculators. Even this is not totally clear-cut, however, since any attempt to 'time' purchases or sales would take such traders into the realm of speculation.

In asset markets such as the stock market, separating investors into speculators and non-speculators is more difficult. This is because the possibility of re-sale is generally an important consideration of an investment decision. Few investors are likely to be willing to pay as much for an asset they cannot resell as for one they can. Telser recognised the problem as it related to his definition:

> "the stock market is not one for which the theory described seems applicable, because no reasonable distinction between speculators and other traders can be made. Perhaps the only non-speculators in that market are those corporations engaged in a new stock issue" (p. 295).

Thus for the stock market the problem is no longer one of defining the behaviour of speculators and non-speculators, but rather one of defining the behaviour of the different types of speculators. As Baumol (1957) argued, in this situation:

> speculation "just amounts to some more skilful speculators profiting at the expense of others" (p. 264).

Baumol also argued, however, that in practice the problem of defining speculators and non-speculators may not pose a serious problem:

> "For the relevant dichotomy may not be between pure speculators and pure non-speculators, but rather it may involve conscious *vs.* unconscious speculators or professional *vs.* amateur speculators, or even pure speculators

> *vs.* those whose market behaviour is not primarily influenced by speculative considerations" (p. 264).

For the stock market, it may therefore be more relevant to focus on the informed / ill-informed distinction, as we do in this thesis. This provides a get-out clause for followers of the Friedman school, who believe that profitable speculation must stabilise market prices, since their argument is couched in terms of speculators and non-speculators. The most important implication of our alternative focus is that, while, as Friedman pointed out, non-speculators will only be influenced by current prices, and not by past prices or trends in prices,[1] ill-informed investors are not so constrained. We will see below that this distinction is crucial.

### 5.2.2 Categorising speculation

A useful starting point for the categorisation of speculation is Irwin (1937), who delineated three broad categories for the motivations for the trading activities of investors, which are 'speculation,' 'movement trading,' and 'manipulation,' where:

> "It is essential in the concept of speculation that profit to the speculator coincide with benefit to society" (Irwin, 1937, p. 268).

Under this categorisation, speculation therefore stabilises the price by definition. Movement trading involves traders chasing expected movements away from fundamentals, and manipulation, as had previously been recognised, involves:

> "the creation of an artificial price by planned action, whether by one man or a group of men.... Manipulation always implies the use of special power and ingenious methods in handling the market" (Dice, 1929, p. 414).

[1] In a letter to Baumol. As related in Baumol (1957).

Since manipulation involves traders influencing market conditions and prices in such a way as to induce a profit, it involves the use of market power, something that is not implied by Irwinian speculation or movement trading. Allen & Gale (1992) divide manipulation itself into three categories: action-based; information-based; and trade-based. Action-based manipulation involves taking actions that will change the value of the stock after first taking stock positions. Information-based manipulation involves the spreading of false rumours and information. Trade-based manipulation simply involves trading the stock: it thus relies on the characteristics of the market and its participants for its success.

### 5.2.3 The effect of speculators on the market

Friedman famously argued that the effect of ill-informed trader demand on prices will be mitigated by the speculative activity of informed traders, claiming (in the context of a foreign currency market under flexible exchange rates) that:

> "People who argue that speculation is generally destabilising seldom realize that this is largely equivalent to saying that speculators lose money, since speculation can be destabilising in general only if speculators on the average sell.... low.... and buy.... high." (Friedman, 1953, p 175).

It is fairly clear that the definition of speculation used by Friedman encompassed Irwin's categories of speculation and movement trading, and also trade-based manipulation. Friedman's argument echoes that of Mill a century before, that described the behaviour of speculators as:

> "naturally buying things when they are cheapest, and storing them up to be brought again into the market when the price has become unusually high; the tendency of their operations is to equalize price, or at least to moderate its

> inequalities. The prices of things are neither so much depressed at one time, nor so much raised at another, as they would be if speculative dealers did not exist" (1848, Book IV, chapter II, section 4, pp 67).

Against the view that speculators may destabilise the price, Mill argued that:

> "All that part of the rise of price by which it exceeds what there are independent grounds for, cannot give to the speculators as a body any benefit, since the price is as much depressed by their sales as it was raised by their purchases; and while they gain nothing by it, they lose, not only their trouble and expenses, but almost always much more, through the effects incident to the artificial rise of price, in checking consumption, and bringing forward supplies from unforeseen quarters. The operations, therefore, of speculative dealers, are useful to the public whenever profitable to themselves.... The interest, in short, of the speculators as a body, coincides with the interest of the public; and as they can only fail to serve the public interest in proportion as they miss their own, the best way to promote the one is to leave them to pursue the other in perfect freedom" (Book IV, chapter II, section 4, pp 69).

Mill was therefore of the opinion that only when the activity of informed traders is stabilising can it be profitable. Insofar as speculators possess information of a higher than average quality, and actively seek out even better information, they will tend to drive prices towards 'fundamental' levels; which ensures that other investors trade at 'fair' prices, and that the degree of price fluctuation in the market will be minimised. Speculators that trade on the basis of poor information will perform badly and their long-term losses will ensure their elimination from the market. Hence speculators reduce uncertainty; and although they may be able to make profits by utilising their informational advantage, this profit does not come at the expense of other market agents, but rather as a natural consequence of their price-stabilising activities. In addition, the greater the degree of competition between speculators, the lower will be their profits, and the closer the relationship between prices and fundamentals; so efforts should be made to encourage speculation rather than hinder it. If exploitation by speculators of the ignorance of other traders is theoretically possible, its potential threat can thus be minimised by minimising the market power of individual speculators, and by maximising the other traders' relevant knowledge.

This view of speculation contrasts with the widely held opinion that speculators can destabilise markets and so increase uncertainty, and also profit at the expense of other traders who are not primarily engaged in speculation.

### 5.2.4 Speculation and price stabilisation

Irwin argued that some of the activity possible under our definition of speculation could be destabilising due to:

> "the technical conditions of the market and especially.... the ways in which the public enters and leaves the market" (Irwin, 1937, pp 269).

Such 'technical conditions' could pave the way for profitable trade-based manipulation. Keynes and Kaldor have both given the argument that speculators may dominate and destabilise a market. As we have said elsewhere, Keynes argued that this was due primarily to the inherent uncertainty concerning the future, but he also assumed that many market participants behaved in a relatively unsophisticated way, and were ripe for exploitation. In a similar vein, Kaldor noted that:

> "If the proportion of speculative transactions in the total is large, it may become, in fact, more profitable for the individual speculator to concentrate on

> forecasting the psychology of other speculators, rather than the trend of the non-speculative elements" (Kaldor, 1939, p. 2).

This again assumed that more sophisticated speculators could concentrate on the exploitation of less sophisticated ones. Kaldor is thus arguing that the presence of unsophisticated speculators may prevent sophisticated speculation from being stabilising.

To the extent that speculators keep prices in line with the future underlying asset value, it may well have a stabilising effect. Once the behaviour of other investors is taken into account, however, as it must be, since speculators trade on the basis of expected future price not value, the situation changes. It is not that there are two different types of speculation, but rather that there are two different scenarios under which the behaviour of the other investors either causes speculators to act as though they are concerned solely with chasing value, or induces an alternative effect. The Friedman proposition can only retain its validity in a general setting if we classify the other investors as speculators.

Natural selection is not likely to reduce the effect of ill-informed investors to negligible proportions since there may be a constant influx of new funds, and the returns of these investors may not be much smaller than those of more informed investors. In fact, as De Long, Shleifer, Summers & Waldmann (1990b) demonstrate, ill-informed investors may actually earn greater profits than rational investors, as a result of taking more risky positions.

# 5.3 Ill-informed investor behaviour

In remainder of this chapter we will look at some of the effects of different forms of ill-informed investor behaviour. We begin in this section by utilising the framework of Shiller (1984) to look at the effect of fads, under which ill-informed investor behaviour is determined independently of the price. In section 5.4 we go on to look at the effect of ill-informed investor behaviour that is affected by current and some past prices, focusing particularly on Hart (1977).

## 5.3.1 Fads in ill-informed investor behaviour

When informed rational investors are risk averse, their required rate of return is determined by the aggregate amount of stock they hold, and so fluctuations in the stock demand of the ill-informed investors will affect prices, even if fully anticipated. Ill-informed investor demand can vary for a number of reasons, such as fluctuations in wealth. Shiller (1984, 1988) argues that it can also vary as a result of *fads* and *fashions*, as the amount of interest in the stock market in general, or specific sectors, or specific stocks, fluctuates. *Contagion* can occur as more and more investors become interested in investment due to contact with, or the observation of, others that are interested. The stock market differs from other examples of contagion in that the price level is positively related to the amount of interest in stocks, and the amount of interest in stocks can also be positively related to the price level, which means that the mechanism of the contagion can shift onto the price. This, according to Shiller (1988) is the point at which a fad becomes a bubble. In the following section we analyse the effect of changes in the level of ill-informed investor demand that is independent of the price, using the framework of Shiller (1984), and so under the above definition we look at the possibility of fads and not bubbles.

### 5.3.2 Modelling faddish behaviour

The model in Shiller (1984) is a simple adaptation of the dividend discount model, in which rational 'smart money' investors must estimate the future stream of demand from ordinary investors, which is assumed to be determined independently of the price, as well as future dividends. This polarisation of investors is clearly a gross oversimplification, but since the aim here is to illustrate the possible effect on market prices of ordinary investor demand that is known by informed investors in advance, it serves its purpose acceptably.

It is assumed that the supply of stock is fixed, beliefs about the future dividend stream are homogeneous within the group of informed investors, and the risk-aversion of the group of informed investors as a whole is expected to remain constant over time. The proportion of the total stock outstanding demanded by informed investors ($Q_t$) is assumed to take the following form:

$$Q_t = \frac{E_t R_t - \rho}{\varphi} \tag{5.1}$$

where $E_t$ is the expectations operator using all the information available in period $t$, and $R_t$ is the return expected for holding the stock from period $t$ to period $t+1$. From this we can see that when the expected return is $\rho$ the insiders hold no stock, and when the expected return is $\rho+\varphi$ they hold the entire market.

The ordinary investors are assumed to desire to have stock holdings worth $Y_t$ in period $t$. This amount could depend upon the price of stock and past stock price movements. The total of the two sources of demand must sum to the amount of stock available:

$$Q_t + \frac{Y_t}{P_t} = 1 \tag{5.2}$$

This gives a price in period $t$ of:

$$P_t = \sum_{k=0}^{\infty} \frac{E_t D_{t+k} + \varphi E_t Y_{t+k}}{(1+\rho+\varphi)^{k+1}} \tag{5.3}$$

This collapses to the dividend discount model when $\varphi$ becomes small.

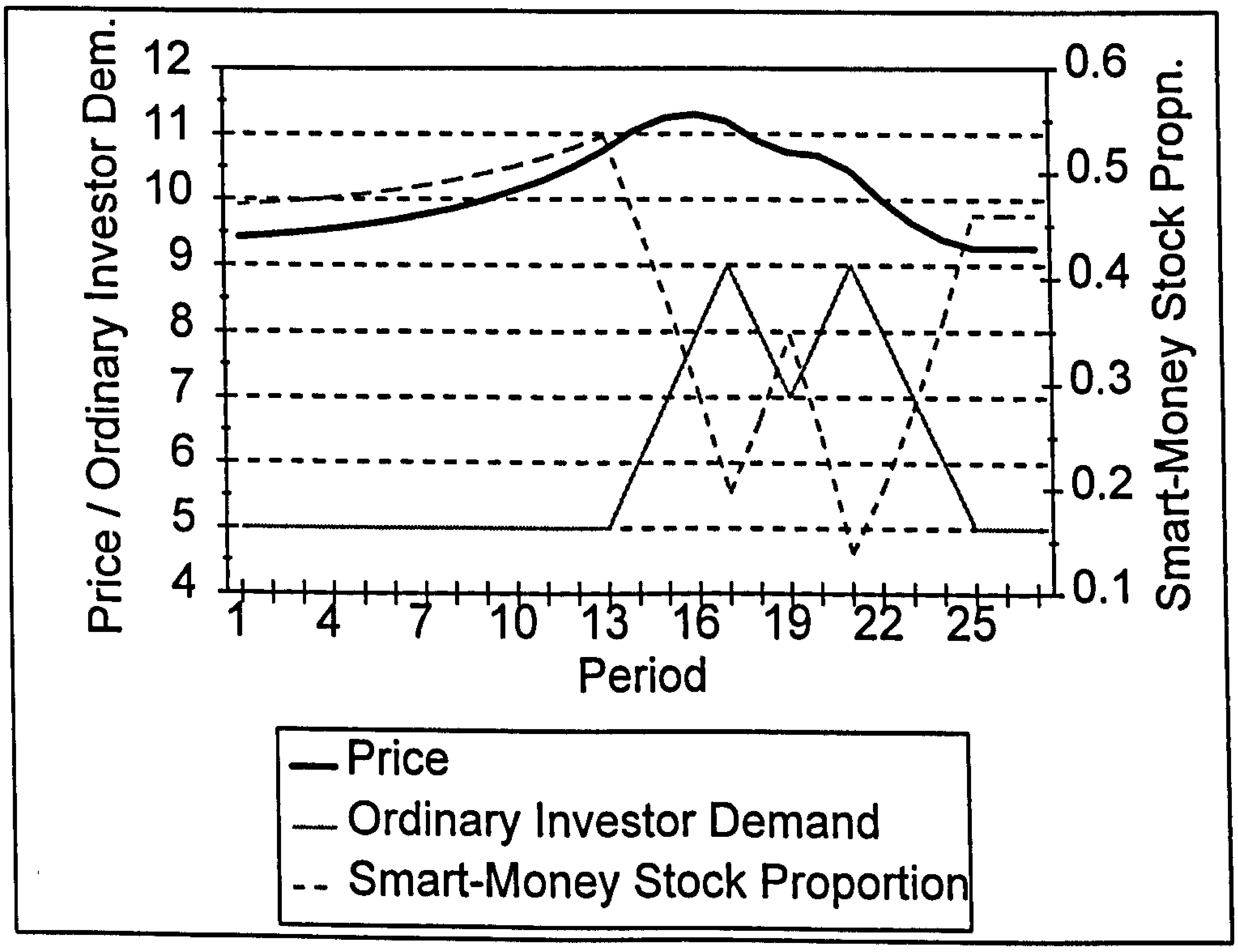
As the demand from ordinary investors rises temporarily, the insiders allow the price to rise, but at the same time sell some of their holdings in the anticipation of being able to buy them back at a cheaper price. The behaviour of the insiders serves to smooth the effect on the stock price of the changes in ordinary investor demand. Although the expected return can never be negative when insiders are still in the market, the stock price can fall since the expected return can be lower than the dividend payment. If ordinary investor demand is large enough, insiders may sell short; when this happens, the expected return can be negative and of unlimited size. If insiders were unable (or unwilling) to sell stocks short, the total stock value would equal the nominal ordinary investor demand.

Numerical examples can show the effect on price of various patterns for ordinary investor demand. This serves to illustrate the intuition, and give a feel for the model behaviour. It also allows us to assess the importance of the willingness of smart money investors to sell short. We will assume in the examples below that the actual series turns out to be as expected by the insiders, and that dividends are constant, thus ensuring that under the dividend discount model with a constant discount factor the stock price would be constant.

### 5.3.3 Example #1: The effect of ordinary investor demand

Figure 5.1 shows the price path for the given two-humped ordinary investor demand function, with $\rho=0.03$, $\varphi=0.17$ and a constant dividend of 1.

*Fig. 5.1: Ordinary investor demand and price.*




This reveals that ordinary investors can induce bubble-like behaviour in the price, although in this case the bubble does not burst, but gradually deflates. The smart-money investors will be short in the market when the expected return over the following period is less than 3 per cent. In this example, the expected return falls to a low of 6.33 per cent in period 17, at which level the smart-money investors still hold almost twenty per cent of the market, down from a maximum of 53.5 per cent in period 13 corresponding to a return of 12.1 per cent, just before the ordinary investor demand begins to increase. Smart money holds 45.9 per cent of the market when ordinary investor demand is expected to remain permanently at its 'normal' level of 5. In the absence of smart-money investors, the price would correspond to the nominal value of ordinary investor demand; in this case the price would be lower in all periods, as would be expected when an important source of demand for stock is eliminated.

The existence of smart-money investors clearly smoothes the price series relative to that which would prevail in their absence. However, if smart-money traders are risk-averse, as we have assumed them to be, they will not entirely eliminate the effects of changes in the nominal value of ordinary investor demand.

The extent to which the price is smoothed will depend on the willingness of the smart money investors to sell stock short. Up to now we have assumed that these investors are as willing to sell short as they are to take long positions. In reality, however, short-selling may not be pursued as vigorously, especially over long periods, and so it is worth investigating the differences in the price series that will result when short selling does not take place, and instead the smart money simply leaves the market entirely when expected return falls below the threshold for participation given by the value of $\rho$. A price series for the 'no short-selling' case of the above model can be compiled by starting at the point at which ordinary investor demand levels off, working recursively back to the start of the series, and for each period setting price as follows:

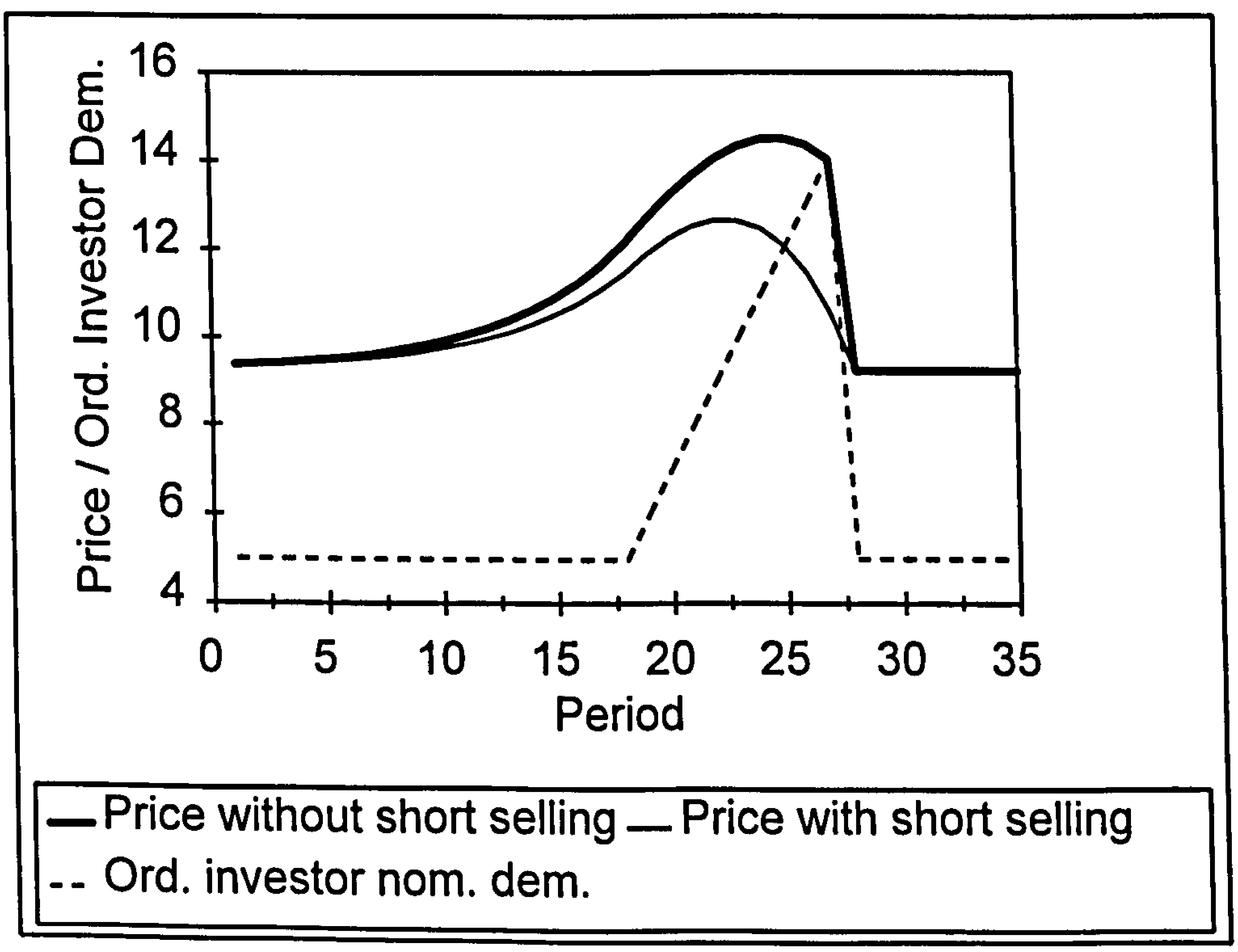
$$P_t = \begin{cases} \dfrac{D_t + \varphi Y_t + P_{t+1}}{1+\rho+\varphi} & \text{for} \quad Q_t > 0 \\[2ex] Y_t & \text{for} \quad Q_t \leq 0 \end{cases} \tag{5.4}$$

where $Q_t$ here represents the proportion of total stock holdings accounted for by smart-money investors as given by formula 5.2: the actual proportion will be zero for $Q_t \leq 0$.

### 5.3.4 Example #2: Price crashes induced by ordinary investors

Figure 5.2 shows the price paths that will result from nominal demand from ordinary investors that rises monotonically before suddenly falling back to its base-line level.

*Fig. 5.2: Price crashes.*



Although we have stated above that we are working under the premise that the ordinary investor demand is independent of the price level, this example is consistent with ordinary investors that are alienated by a price crash. The advantage of using this specification of ordinary investor demand is that it gives us a plausible example of how smart investors can ride a price 'bubble' before selling out to less smart investors at or near the peak.

This example reinforces the intuition that crashes are possible when short selling does not occur, but are not when it does. In fact, short selling leads to greater price smoothing in general, and prevents the price from being pushed as high as it would be otherwise. Even if short-selling does not take place, however, the presence of smart-money investors causes the ordinary investor demand to have a smaller effect on the deviation of prices from their long-term level, and so can be considered to be stabilising.

If ordinary-investor demand always remains a small proportion of the level required to displace smart-money investors from the market, smart-money will ensure that prices follow a smooth path, and do not fluctuate greatly as a result of changes in this demand. It is possible, however, that at certain times, for example in great bull markets, the size of the group of smart-money investors can become relatively small, in which circumstances the effect of ordinary-investor demand will be relatively large, and may induce price crashes, as in our example.

Although in the Shiller model ill-informed investors can affect market prices, the effect of their varying demand on prices is at least partially smoothed by the actions of the smart-money investors, and causes prices to remain closer to fundamentals when short-selling takes place. This model therefore lends some support to Friedman's proposition. We stated above that, under Shiller's definition, this model deals with the possibility of fads, and not bubbles, since the contagion mechanism for the ill-informed investors is independent of the market price (although our latter example muddied the waters a little). It is clear, however, that the smart money traders purchase ahead of increases in ill-informed investor demand in anticipation of a price rise, and sell in anticipation of demand-induced price falls: this demonstrates that stabilising speculation and movement trading are not incompatible.

It is also clear that sunspots in prices will occur in the above model when they affect the demand of ill-informed investors. Once again, however, the greater the market power of the informed investors, the smaller will be the effect on the market price; and when there are no ill-informed investors, sunspots in the price will not occur.

# 5.4 Ill-informed investor demand as a function of price

## 5.4.1 Models with mechanistic ill-informed investor demand

In Shiller's model, the stock demand of the ill-informed investors was assumed to be determined independently of the price. In reality, this demand is likely to be affected in some way by the price, with a number of different relationships being possible. In what remains of this chapter we will look at the implications of ill-informed investor behaviour that is a function of current and/or past prices, and the relevance of the particular form of the relationship. We begin by examining Hart (1977), which gave the conditions under which ill-informed manipulation that disturbs the steady state could be profitable in such a system, although did not isolate the best speculative strategies. Hart found that it can always be profitable to disturb the steady state when the system is asymptotically unstable, and can sometimes be profitable when past prices affect current prices.

An awareness of Hart's work can enhance the understanding of a number of models that appeared in the literature prior to this aiming to assess the validity of the Friedman proposition that profitable speculation must be stabilising. These began with Baumol (1957), and continued with Telser (1959), Kemp (1963), Farrell (1966), Schimmler (1973) and Williamson (1972). We take a look at these models following our analysis of Hart (1977), and analyse them in the context of this model. Hart

(1977) can also be used to better understand the De Long, Shleifer, Summers & Waldmann (1990a) framework, as we will also demonstrate.

Jarrow (1992) has developed Hart's idea into a stochastic framework, in which the equilibrium price process in the absence of speculators is exogenously specified. He attempts to find potentially-profitable arbitrage strategies for a manipulator, and derives results similar to those of Hart: in particular, it is shown that a sufficient condition for there to exist no arbitrage opportunities is that the stock price process depends only on the current stock holdings of the manipulator, and not on its history.

### 5.4.2 Hart (1977)

Hart (1977), starting with a system in an initial steady state, attempts to identify conditions for profitable speculation. In such a situation, any speculative activity must be destabilising, and hence the requirement here for a refutation of the Friedman position is to find any scope for profitable speculation. The profitability of a speculative strategy depends on the reaction of the non-speculators to the speculation; Hart points out that if non-speculative demand is positively related to price (as is the case in Williamson (1972), as we shall see below), and so embodies positive feedback trading in the aggregate, "it is not difficult to show that it is always possible for the speculator to make money," (footnote 5, p 583); so the more interesting question concerns the weaker conditions for which similar results can be obtained. Speculation is profitable in a system which is explosively unstable; but avoiding disaster in such a (highly implausible) system would be impossible even in its absence. For the case where non-speculator demand is a linear function of current and past prices, the requirements are completely characterised. Hart justifies the inclusion of lagged components in the non-speculative demand function as follows:

> "First, in the absence of forward markets, non-speculative demand will in general depend on expected future prices, and expected future prices may in turn depend on past prices. Second, since we are concerned with the demand for a stock rather than for a flow, demand at date *t* will be influenced by stock decisions made at previous dates, and these decisions will have been made on the basis of prices ruling before date *t*" (p. 582).

Non-speculative demand is independent of wealth, and the non-speculators themselves are assumed never to learn about speculative behaviour. Hart considers a non-speculator demand function, which for the linear case is of the following form:

$$F(P_t, P_{t-1}, \ldots, P_{t-n}) = \sum_{i=0}^{n} a_i P_{t-i} + b \tag{5.5}$$

where $a_0, \ldots, a_n$, $b$ are constants, and $a_0 < 0$. The stationary-state price is therefore given by:

$$P^* = -\frac{b}{\sum_{i=0}^{n} a_i} \tag{5.6}$$

Hart finds it useful to define a variable as the deviation of price from the stationary price level:

$$\pi_t = (P_t - P^*) \tag{5.7}$$

A speculator is assumed to enter the market in period one, following a period in which the market was in a stationary state:

$$\pi_0 = \pi_{-1} = \ldots = \pi_{1-n} = 0 \tag{5.8}$$

The speculator is assumed to formulate a strategy that involves leaving the market in or before period $T$, where $T \leq t$. The sales of the speculator must equal the non-speculative demand in each period:

$$\begin{aligned} S_t &= F(P_t, P_{t-1}, \ldots\ldots, P_{t-n}) = \sum_{i=0}^{n} a_i P_{t-i} + b \\ \Rightarrow S_t &= \sum_{i=0}^{n} a_i \pi_{t-i} \end{aligned} \qquad (5.9)$$

The profits of a speculator following strategy S are as follows:

$$\begin{aligned} M &= \sum_{t=1}^{T} P_t S_t = \sum_{t=1}^{T} \left(P^* + \pi_t\right) S_t = \sum_{t=1}^{T} \pi_t S_t \\ &= \sum_{t=1}^{T} \sum_{i=1}^{n} a_i \pi_{t-i} \pi_t \end{aligned} \qquad (5.10)$$

It is assumed that the speculator realises his profits, which means that the cumulative sum of his sales over the period must be zero:

$$\sum_{t=1}^{T} S_t = \sum_{t=1}^{T} \sum_{i=1}^{n} a_i \pi_{t-i} = 0 \qquad (5.11)$$

Hart shows that the condition for the speculator to make positive profits subsumes the latter condition, and so we arrive at Hart's Lemma 3.2:

> The speculator can make money if and only if there exist a positive integer $T$ and real numbers $\pi_1, \ldots\ldots, \pi_T$ such that
>
> $$M = \sum_{t=1}^{T} \sum_{i=1}^{n} a_i \pi_{t-i} \pi_t > 0 \qquad (5.12)$$

For the speculator to be unable to make money, this quadratic form must be negative semi-definite for all values of $T$. Hart shows that the quadratic form can be represented by a matrix in which each element is determined by the distance from the diagonal, and in particular takes the following form:

$$\begin{bmatrix} a_0 & a_1/2 & a_2/2 & \cdots & a_n/2 & 0 & \cdots & 0 \\ a_1/2 & a_0 & a_1/2 & a_2/2 & \cdots & a_n/2 & 0 & \cdots \\ a_2/2 & a_1/2 & a_0 & a_1/2 & a_2/2 & \cdots & a_n/2 & 0 \\ \cdots & a_2/2 & a_1/2 & a_0 & a_1/2 & a_2/2 & \cdots & a_n/2 \\ a_n/2 & \cdots & a_2/2 & a_1/2 & a_0 & a_1/2 & a_2/2 & \cdots \\ 0 & a_n/2 & \cdots & a_2/2 & a_1/2 & a_0 & a_1/2 & a_2/2 \\ \cdots & 0 & a_n/2 & \cdots & a_2/2 & a_1/2 & a_0 & a_1/2 \\ 0 & \cdots & 0 & a_n/2 & \cdots & a_2/2 & a_1/2 & a_0 \end{bmatrix} \tag{5.13}$$

Quadratic forms with this property are known as a Toeplitz forms. Making use of the mathematics literature on Toeplitz forms, Hart characterises the solutions. The main result is found in his Theorem 3.4, in which he states that (for $a_0 < 0$) the speculator can profit from disturbing the steady state if and only if the equation, $\operatorname{Re} f(z) = 0$, has a solution $z$ satisfying $|z| < 1$,[2] where:

$$f(z) = \sum_{t=0}^{n} a_t z^t . \tag{5.14}$$

Hart's Lemma 3.5 shows, in addition, (for $a_0 \neq 0$) that the system is explosive in the absence of speculators if and only if the equation $f(z)=0$ has a solution $z$ satisfying $|z| < 1$. As Hart points out (in Theorem 3.6), if the condition for explosiveness is satisfied, the condition for profitable speculation must also be satisfied, and so speculation will always be profitable when the system is explosive.

Explosiveness is both a necessary and sufficient condition for profitable speculation in the following situations: when the coefficients of all the lagged terms in the non-speculator demand function are non-negative; when the coefficients of the terms with even lags are non-negative and the coefficients of the terms with odd terms are non-positive; and when the maximum lag in the non-speculator demand function is one.

[2] Where Re $f(z)$ is used to denote the real part of $f(z)$.

For the case where the maximum lag is of two periods, Hart's Theorem 3.9 shows that one or both of the following conditions must hold:

1. The difference equation is explosive.
2. The coefficients $a_0$, $a_1$, $a_2$ satisfy: $a_2 < 0$, $|a_1| \le 4|a_2|$, *and* $a_1^2 - 8a_0a_2 + 8a_2^2 > 0$

Although not given by Hart, sufficient conditions for speculation to be unprofitable can be found relatively easily. For lags of up to $n$ periods, a sufficient condition for speculation to be unprofitable is that:

$$\begin{aligned} & a_0 < 0 \\ & \textit{and } |a_0| \ge |a_1| \\ & \textit{and } |a_0| \ge |a_1| + |a_2| \\ & \textit{and } |a_0| \ge |a_1| + |a_2| + |a_3| \\ & \textit{and....} \\ & \textit{and } |a_0| \ge |a_1| + |a_2| + |a_3| + \ldots + |a_n| \end{aligned} \tag{5.15}$$

A necessary condition for speculation to be unprofitable, again not given by Hart, is that the following conditions hold:

$$\begin{aligned} & |a_0| \ge \left|\frac{a_1}{2}\right| \\ & \textit{and } |a_0| \ge \left|\frac{a_2}{2}\right| \\ & \textit{and } |a_0| \ge \left|\frac{a_3}{2}\right| \end{aligned} \tag{5.16}$$

Since Hart looks at the profitability of (destabilising) 'strategies' followed by a body of speculators, it does not indicate the conditions required for competitive speculation to be stabilising. The presumption would be that competitive speculation would tend to stabilise prices. We once again reach a conclusion that monopolistic speculation can sometimes be destabilising, but that fully competitive speculation cannot be.

### 5.4.3 Baumol (1957)

Baumol (1957) represents an early attempt to produce a counter-argument to the Friedman position. Baumol was well aware, however, that its application to the stock market may have posed more problems than its application to other markets such as the foreign exchange market. The first set of key assumptions underpinning the work is that:-

> "there exists a group of non-speculators on some unspecified definition and that its activities somehow result in cyclical behaviour in the price of some commodity" (p. 264).

To a modern finance theoretician, the latter assumption may perhaps be enough to immediately discredit the results. This should not be the case, however, unless one believes whole-heartedly that all agents are fully rational and that the performance in this market is the only issue of concern to them. Baumol makes an additional assumption, however, that is more controversial: it is that the demand of the non-speculators is a function of past as well as current prices. It is this that drives prices in cyclical manner. It is therefore probably the first model to explicitly use the concept of feedback trading.

The particular non-speculative excess demand function used by Baumol is as follows:

$$E_t = K - UP_t + V(P_t - P_{t-1}) + W(P_{t-1} - P_{t-2}) \tag{5.17}$$

where W is a positive constant, and the constants K, U and V are given by the following:

$$\begin{aligned} V &= W(1-2a) \\ U &= 2W(1-a) \\ K &= Wk \end{aligned}$$

Setting the excess demand equal to zero produces a second-order difference equation for the price:

$$P_t = 2aP_{t-1} - P_{t-2} + k \tag{5.18}$$

Restricting the magnitude of $a$ to be less than unity and solving produces the following:

$$P_t = c\cos qt + s\sin qt + R = p\cos(qt + r) + R \tag{5.19}$$

In the absence of speculators, prices would therefore follow a cyclical path. It should be noted that the non-speculator demand was set up in such a way, indeed the only way, that would ensure that a cycle of constant amplitude was produced: if the coefficient on the twice-lagged price term differs from unity, the amplitude of the cycle will either grow or shrink over time. That prices would follow a cyclical path in the absence of speculators should not provoke too many objections by more modern market theorists, since they would implicitly consider the situation when speculators are present: we must therefore wait until the with-speculator behaviour is revealed before judgement is passed.

Baumol shows that speculators can make profits by trading in a manner that increases both the frequency and the amplitude of the price cycle, and is thus destabilising. In particular, Baumol assumes that the speculators attempt to concentrate their purchases immediately after an upturn in the price, and their sales immediately after a downturn. The specific speculator excess demand function chosen is as follows:

$$\begin{aligned} E_{s,t+1} &= C\left[(P_{t+1} - P_t) - (P_t - P_{t-1})\right] \\ &= C\left[(P_{t+1} - 2P_t + P_{t-1})\right] \end{aligned} \tag{5.20}$$

where $C$ is a positive constant. This gives the appropriate pattern of demand since, should $t$ represent a turning point of the cycle, the $(P_{t+1} - P_t)$ and $(P_t - P_{t-1})$ terms will be of the same sign, and thus reinforce each other; while at other points in the cycle they (at least partially) cancel each other out.

Combining the excess demand functions of the speculators and the non-speculators produces the following:

$$\begin{aligned} 0 &= K - WP_t + 2WaP_{t-1} - WP_{t-2} + C(P_t - 2P_{t-1} + P_{t-2}) \\ &= K - (W - C)P_t + (2Wa - 2C)P_{t-1} - (W - C)P_{t-2} \end{aligned} \tag{5.21}$$

The expression for the price process therefore becomes:

$$P_t = \frac{K}{W - C} + 2\frac{Wa - C}{W - C}P_{t-1} - P_{t-2} \tag{5.22}$$

This produces stable oscillations when $Wa > C$. This can only occur when $a > 0$. Baumol shows (as Property 2) that this type of speculative behaviour will be profitable provided that the cycle followed by prices lasts longer than four periods.

What this model shows is that, in the presence of feedback trading, speculative behaviour may be both destabilising and profitable. One weakness of the model is that the speculative demand is not derived from optimising behaviour. Another concerns the assumption that speculators concentrate their purchases and sales just after turning points have been reached: if speculators could predict these turning points, speculator behaviour is likely to be stabilising. In addition, the presence of large numbers of competitive 'rational' speculators would severely dampen, if not eliminate, the cyclical behaviour. The continued existence of cyclical price behaviour when speculators are present may be enough to discredit this approach, but not necessarily the inclusion of feedback trading.

Telser (1959) defended the Friedman position against Baumol, focusing his criticism on the feedback trading assumption, on the grounds that such behaviour, following Friedman's definition, would make all the agents speculators. As Baumol (1959) countered, this boils down to an uninteresting question of definitions; the real issue concerns the question of whether such feedback traders actually exist.

Baumol's specification of non-speculator behaviour can be analysed in Hart's framework.

*Application of Hart (1977) to Baumol (1957).*

Comparing the non-speculator excess demand used in Baumol to the general form given in Hart, tells us the following:

$$a_0 = -W$$

$$a_1 = 2Wa$$
$$a_2 = -W \tag{5.23}$$

Hart's Theorem 3.9 tells us that the speculator can make money from disturbing the steady state when the following conditions hold:

$$
\begin{aligned}
&-W < 0 \\
&|W| \leq 4|W| \\
&4W^2a^2 - 8W^2 + 8W^2 > 0 \\
&\Rightarrow 4a^2 > 0
\end{aligned}
\tag{5.24}
$$

These always hold (for positive *W*) and so speculation is always potentially profitable.

### 5.4.4 Telser (1959)

Telser (1959) looks at the effect on price variability of adding a monopolistic speculator to a system in which the non-speculator demand function contains only two terms: the current price, with a negative coefficient; and a component that is a pure function of time:[3]

$$E(t) = -aP + h(t) \qquad a > 0 \tag{5.25}$$

Given this non-speculator excess demand function, a monopolistic speculator would maximise profits by following an excess demand function of the form:

$$S = a\left(P - \overline{P}\right) \tag{5.26}$$

Telser showed that the addition of the speculator to the market would not alter the mean price, but would reduce the variance of prices to a quarter of its previous level.

[3] The notation used here differs from that in the original paper.

He then generalised the model slightly to a situation where the speculators were not perfectly informed about the mean price level and found that positive profits were a sufficient but not necessary condition for prices to have been stabilised.

As we saw above, Hart's analysis did not encompass situations where the non-speculative demand function contains a component that is a pure function of time. In order to make use of Hart's results in this context we must content ourselves to look at the special case where the time-dependent component reduces to a constant:

$$E(t) = -aP + K \tag{5.27}$$

When $a$ is positive, as Telser assumes, this system satisfies the sufficient condition for speculation to be unprofitable. When $a$ is negative, the system is explosive, and so speculation must be potentially profitable.

### 5.4.5 Kemp (1963)

Kemp (1963) begins by introducing a 'Giffenesque' (reversed-S-shaped) non-speculative excess demand curve, which has reappeared, in a more rigorous form, in both Gennotte & Leland (1990) and chapter three of this thesis. There are two stable equilibria, between which is a non-stable equilibrium, and Kemp shows how it would be profitable for a speculator to move the system from one stable equilibria to the other and back again. Williamson (1972) was unwilling to give this credibility, arguing that:

> "Since.... most economists would regard the possibility of multiple equilibria in the exchange market as a pathological case that is unlikely to be realised in practice, the force of this ingenious counter-example is limited" (p. 78).

In the light of the (information aggregation) model constructed in chapter three (as well as the work of Gennotte & Leland) which shows the possibility of multiple equilibria, and also our argument for the non-existence of 'fundamental' values, this opinion may need to be revised.

In a second example, Kemp specifies a non-speculator excess demand function of the form used by Telser, and attempts to find a relationship between speculator profits and price variability directly. The price in the absence of speculators would be as follows:

$$P(t) = \frac{h(t)}{a} \tag{5.28}$$

The only restriction placed on the speculative excess demand function is that their stock holding at the end of a given period ($t_1$) is the same as at the beginning ($t_0$). This ensures that all profits are realised. Therefore:

$$\int_{t_0}^{t_1} S(t)\,dt = 0 \tag{5.29}$$

The price path in the presence of speculators is:

$$\begin{aligned} P_S(t) &= \frac{h(t)+S(t)}{a} \\ \Rightarrow P_S(t) - \frac{S(t)}{a} &= P(t) \end{aligned} \tag{5.30}$$

The mean price over the period will be the same as when speculators are not present:

$$\overline{P} = \overline{P}_S \tag{5.31}$$

The profits of the speculators are:

$$\begin{aligned}\Pi &= -\int_{t_0}^{t_1} P_S(t)\, S(t)\, dt \\ &= -(t_1 - t_0)\operatorname{cov.}(P_S, S)\end{aligned} \tag{5.32}$$

Squaring equation 5.30 and substituting produces:

$$\frac{1}{a^2}\operatorname{var.}S + \operatorname{var.}P_S + \frac{2\Pi(t_1 - t_0)}{a} = \operatorname{var.}P \tag{5.33}$$

As Kemp shows, this indicates that unprofitable speculation may still be stabilising, and that profitable speculation must be stabilising, which indicates a result akin to that of Telser: that positive profits are a sufficient but not a necessary condition for the presence of speculators to have stabilised prices.

Kemp's analysis can be extended to derive some other results. Assume, for example, that the time-dependent variable in the non-speculator demand function is once again replaced by a constant:

$$E(t) = -aP + K \tag{5.34}$$

In the absence of speculators, the price is constant, and therefore has zero variance. The term relating the variances and profit is now as follows:

$$\frac{1}{a^2}\operatorname{var.}S + \operatorname{var.}P_S + \frac{2\Pi(t_1 - t_0)}{a} = 0 \tag{5.35}$$

This shows that, for a positive value of *a*, the profit must be negative, and vice versa. Hence when non-speculative demand is a negative linear function of price, any speculative activity must result in a loss and destabilise prices, which, as we have already seen, is implied by Hart (1977). When non-speculative demand is a positive function of price, any speculative activity must be profitable (and will, of course, still be destabilising): this is a stronger result than we could gain from Hart, since that work only indicates when speculation is potentially profitable.

### 5.4.6 Farrell (1966)

As we have seen, Telser and Kemp looked at the link between profitability of speculation and price stabilisation in a system in which the demand function of the non-speculators is a linear function of the current price. Farrell (1966) attempted to generalise the analysis to a situation in which the non-speculative demand function is not necessarily linear.

Farrell started by making the what he calls the 'independence assumption,' which is that the divergence of the price when speculation is present from the price when it is not is a function of the speculative demand in that period alone. As a consequence of this, non-speculative demand is not affected by prices in previous periods. The non-speculative demand function under this assumption can be written as follows:

$$E(t) = d(P_s - P_n) \tag{5.36}$$

where $P_s$ represents the actual market price (in the presence of speculators), and $P_n$ represents the price that would prevail in the absence of speculator activity.

Farrell also invokes the *Law of Demand* to impose the condition than this function is a monotonically decreasing function. To see that the Telser and Kemp models represent a special case of this for a linear function, we can re-arrange the functional forms used in those papers:

$$
\begin{aligned}
E(t) &= -aP_s + K \\
&= -a \times \left(P_s - \frac{K}{a}\right) \\
&= -a \times (P_s - P_n)
\end{aligned}
\tag{5.37}
$$

Farrell shows that a necessary and sufficient condition for positive speculative profits to guarantee price stabilisation is that the non-speculative excess demand function is a (negative) linear function of price, such as we have given above. The results given by Telser and Kemp therefore do not apply to any other forms of the non-speculative excess demand function.

Farrell does show, however, that with transactions costs profitable speculation can still imply stabilisation for non-speculative excess demand functions that are not perfectly linear. Farrell also shows, however, that temporal interdependence, which in this framework involves a non-speculative demand function that depends on past as well as current prices, prevents a clear link between the profitability of speculation and stabilisation from being established, even with linear specifications. This should come as no surprise following Hart's results.

Farrell concludes that his analysis using the independence assumption:

> "does not take us very far, for we have found reasons for expecting that many real-world markets will display some measure of temporal interdependence, so invalidating our sufficient conditions. Thus our search for reasonably simple and plausible sufficient conditions for the validity of our basic proposition seems to have been in vain. But the analysis of this paper will not have been wasted if it has persuaded economists that our basic proposition is too strong to hold with any great generality and that they should therefore seek to

establish weaker propositions concerning the properties of speculative markets" (p. 192).

Schimmler (1973) demonstrated that the models of Telser, Kemp & Farrell can be unified and compared in an appropriate Hilbert space, and generalised Farrell's result by finding that Friedman's theory is not valid for any system that exhibits temporal interdependence in the nonspeculative excess demand function.

### 5.4.7 Williamson (1972)

Williamson (1972) showed that the lag structure of foreign exchange markets gives scope for profitable destabilising speculation. Following Williamson's argument, but expressing the model in terms compatible with Hart (1977), we can represent the non-speculator excess demand function as an expression for the balance of payments, and postulate that the Marshall-Lerner condition holds, which produces the following:

$$E(t) = a_0 P_t - a_1 P_{t-1} \tag{5.38}$$

where $a_0$, $a_1 > 0$, the Marshall-Lerner condition implies $a_1 > a_0$, and $P$ represents the exchange-rate expressed as the number of units of foreign currency per unit of domestic currency.

Since this system is explosive in the absence of speculators, Hart's results immediately tell us that speculation that disturbs an initial steady state can be profitable; but since the system is naturally unstable, the destabilising effect of speculation is difficult to assess. One thing that once again seems clear is that "a marginal addition to speculation would tend to be stabilising" (p. 82), since competition between speculators is likely to push the price down to its steady-state level. Other less stabilising speculative strategies can be profitable, however: for example, cyclical behaviour can be induced. The overall effect of speculators on stability is determined by the strategy followed by the speculators:

> "If they know the equilibrium rate and buy (sell) when the price is below (above) equilibrium, then clearly their activity will be stabilising. It has often been asserted, however, that speculators are more likely to jump on a bandwagon than to lean into the wind. At some point, to be sure, they will jump off the bandwagon; but if this point is delayed, so that speculative activity produces large price gyrations, it does not follow that the speculators will lose money in the presence of a lag structure of the type postulated in this paper" (p. 83).

The profits of the speculators in the Williamson model can be thought of as being made at the expense of other traders whose commitments in other markets induce a lagged reaction in this one. The specific form of the lagged response is perhaps only really applicable to the foreign-exchange market.

Price & Wood (1974) note that in the Williamson model with cyclical exchange-rate movements the most profitable strategy for speculators would be to jump off the bandwagon before others; which would have a stabilising effect on prices. They also point out that a central bank could profit from stabilising the exchange-rate, if it was aware of the long-run equilibrium level. Competitive speculation, by well-informed investors, would therefore seem to be extremely desirable.

### 5.4.8 Competitive speculation

One model that claims to illustrate the potential for destabilising competitive speculation is that of De Long, Shleifer, Summers & Waldmann (1989, 1990a). The crucial assumption made in this model is that of the existence of positive feedback trading: in other words, it is assumed that there exists a group of investors whose demand bears a positive relationship to the market price. The demand of these agents is counter-balanced by other (more powerful) traders, whose demand is related to the distance of the price from their estimate of the stock's value, which ensures that in the absence of rational investors the market price is dynamically stable. In this framework competitive speculation may not stabilise prices at times of new information arrival (De Long et. al. 1990a) or an exogenous demand shock (De Long et. al. 1989), since the anticipation by rational investors of the effect of this on the price can cause prices to fluctuate more than they would have otherwise.

The De Long et. al. model is important since it represents perhaps the only example of competitive speculation in the stock market having a destabilising effect. In a storage model applicable to commodities markets Hart & Kreps (1986) showed that rational competitive speculators can destabilise prices, but the argument requires the presence of other agents who purchase for immediate consumption, and so is inapplicable to the stock market. It is also set in an overlapping generations framework, which is arguably not as suitable for analysing the stock market. Its uniqueness makes the De Long et. al. model worth investigating in depth, which we do in chapter eight.

# 5.5 Conclusions

In section 5.3 we analysed a model in which faddish behaviour by ordinary investors affects the price when smart-money investors are risk-averse, although the presence of smart-money investors tends to smooth the price series and reduce the extent of the price fluctuations. This model also highlighted the importance of the assumption regarding short-selling, with an unwillingness of informed investors to sell short leading to greater price fluctuations and the possibility of price crashes following price bubbles.

For the case where non-speculator demand is not influenced by past prices Farrell (1966) showed that the Friedman proposition that speculation is stabilising is only sure to hold when the relationship between this demand and the current price is linear; and if non-speculator demand is influenced by past prices the Friedman proposition does not necessarily hold. Competitive speculation, however, would tend to stabilise prices. This ties in with the results we found using the Shiller (1984) model, and is also consistent with other models that illustrate that in the presence of noise traders speculative activity may be less than fully effective in stabilising prices, but it will make prices more stable than they would have been otherwise.[4] The only evidence in this type of framework that indicates that speculation may not tend to be stabilising is given by De Long et. al., which will be evaluated in chapter nine.

[4] See Figlewski (1979), Kyle (1985), De Long, Shleifer, Summers & Waldmann (1990b) and Campbell & Kyle (1988).

It should be noted that we have been assessing the effect of informed speculation. If we assume that speculators are actually ill-informed and boundedly rational, then it should be clear that their activities are likely to be destabilising in any framework. Stein (1987) demonstrated that even an influx of ill-informed speculators who are rational in the terms of the R.E. literature can destabilise a (commodities) market.

We have seen, following Hart (1977), that speculation that disturbs the steady state can be profitable when non-speculator demand is influenced by past prices. Baumol's model shows that manipulation can be both profitable and destabilising in the presence of positive feedback traders, although the particular model formulation used was not that general. An alternative formulation for positive feedback trading is that given by De Long et. al. (1989, 1990a) and Cutler et. al. (1990), and it is in this framework, utilising Hart (1977), that we analyse, in the following chapter, the possibilities for manipulation.

# Chapter 6

# Positive feedback trading

## 6.1 Introduction

Positive feedback trading can result from investment strategies, techniques of estimating value, information paucity and psychological factors. In this chapter we investigate some of the theoretical, anecdotal and empirical evidence of its presence. We begin by looking at the consequences of some investors following trading rules that are dependent on the price, and in particular dynamic portfolio insurance strategies. We then look at the effect of information shortages and psychological factors, before examining anecdotal evidence of bubbles and incidences of manipulation. We finish by looking at empirical evidence that may indicate the presence of positive feedback trading.

# 6.2 Price-influenced trading rules

## 6.2.1 Portfolio insurance

We have seen that positive feedback trading involves selling after price falls, and buying after price rises. Such behaviour would also characterise a dynamic portfolio insurance strategy. Leland (1980) argued that a strategy of portfolio insurance is rational for investors whose level of (absolute) risk tolerance increases with wealth more rapidly than it does for the average investor,[1] as well as for optimists. This implies that positive feedback trading is not necessarily the result of irrationality or ill-informedness. However, this is not the same as arguing that simple mechanistic feedback strategies can legitimately be used in models to reflect this since, clearly, rational investors will still take into account expected returns, and will not allow themselves to be consistently exploited by other investors if the costs of this exceed the benefits from the insurance strategy itself.

There are several situations in which simple mechanistic rules may be relevant, at least on the down-side. It is sometimes important for investors that the value of their portfolio exceeds some minimum value. This reduces the discretion surrounding investment decisions as the portfolio value approaches this level as prices fall, and may force sales to be made 'at any price.' Institutions, for example, may have commitments of a fixed size to meet. Leveraged positions are especially vulnerable, since lenders are likely to demand payment if the value of the collateral falls close to

[1] Black (1988) argued that rising levels of risk tolerance were a contributory factor in the market break of 1987.

the loan value. This was perhaps best illustrated in 1929, when the extent of margin trading - which involved purchasing stock with payment of a deposit for less than the full amount - increased enormously.[2] Once prices began to fall, and margins became exhausted, the positions of investors that could not afford to provide more capital were liquidated: these investors were forced to accept whatever price could be got in the market, regardless of their own opinions as to the stock's true worth. Leveraged positions also tend to follow positive feedback behaviour in bull-markets, since the collateral provided by the stock value, and hence the scope for borrowing, increases in line with it. This (partly) explains the rise in margin trading as the market rose in the late 1920s.

The modelling of portfolio insurance in the market has tended to assume mechanistic rather than explicitly modelled rational behaviour. This includes the information aggregation model of Gennotte & Leland (1990) - which we came across in chapter one, and will look at in greater depth in the concluding chapter - and Brennan & Schwartz (1989).

### 6.2.2 Other trading rules

Past price movements are used by some investors to assess the likely future direction of prices, and therefore to determine the appropriate investment strategy. Such *chartism* can lead to price rises inducing further buying, and hence positive feedback: this can happen when, for example, psychological barriers are broken. As we stated in

[2] See Galbraith (1975).

the introductory chapter, the root cause of such behaviour is the uncertainty surrounding the future stock value and price, which also has a number of other effects.

## 6.3 Consequences of uncertainty, ill-informedness and psychological factors

### 6.3.1 Uncertainty and the effectiveness of speculation

As we argued in chapter one, the greater the degree of uncertainty about the future, the more influence the trading activity of ill-informed investors will have, since the relatively well-informed investors will be less inclined to trade strongly against them. The effectiveness of speculation in neutralising the effects of ill-informed trading is therefore naturally restricted.

### 6.3.2 Trading on noise

Information is incorporated into prices as investors act on it. For this to work effectively, however, the investors must be aware of how much of the information has already been incorporated into the price when they trade; and so they must either know the appropriate price which the new information implies, or, if the information is noisy, the trading activity and information of the other investors. If investors act on information that is new to them but that has already been incorporated into the price, they are trading on noise in the sense used by Black (1986), and will exhibit positive feedback trading, since their demand will be in the same direction as the recent price changes induced by the new information.

### 6.3.3 Price-influenced fads

We have argued previously that investor interest in the stock market, and as a consequence stock demand, can be affected by social factors inducing contagion among investors. In the Shiller (1984) model of the previous chapter the contagion and the resultant fads were assumed to be independent of market prices, both past and present. It is more likely, however, that the contagion is reinforced by price movements. Even if the existing investors do not increase the level of their holdings as prices rise, the stock demand from newly-interested investors provides a source of positive feedback trading.

The behaviour of individual ill-informed investors may be subject to fads that are influenced by the path of market prices, especially if these investors are sufficiently ill-informed as to be unable themselves to produce meaningful estimates of the value of stock. Such investors must rely on Keynes' 'animal spirits' to make investment decisions, with trading activity the result of a preference for action over inaction. The decisions taken as to whether or not to invest in this scenario must be based on factors such as their judgement of how good an investment the stock market has been in the past, or simply the degree of 'goodwill' they feel towards the market.[3]

[3] This echoes an argument found in Black (1988).

Investors' goodwill is likely to be far less volatile in the short-term than prices, and changes in it are likely to lag (ex post) price trends. The length of the lag is not likely to be constant; it is certainly feasible to assume, for example, that a price crash, such as occurred in 1929, and to a lesser extent in 1987, can have a dramatic impact on the goodwill towards the stock market felt by society as a whole, depending on the general perception of the permanence of the price changes.[4] The idea that a prolonged bull period can, perhaps via the media, lead to greater participation, and thus positive feedback trading by this group of investors as a whole, bears comparison to the 'visibility' idea of Miller (1977), whereby the demand for individual stocks is positively related to the degree to which the companies are known to the public at large. There must surely be some psychological effect of constant exposure to stories of high prices, perhaps 'record highs'.

Even if these investors are aware of their lack of informedness, their behaviour is not necessarily irrational, since over long periods stock market investment has always proved profitable in markets that are currently well-established, and so avoiding the stock market entirely would not be rational. Ill-informed trading activity may actually be relatively profitable, as demonstrated by De Long et. al. (1990b). The current broad acceptance of the efficient markets hypothesis serves further to justify their behaviour, since it implies that the market prices are always 'fair,' and so investors should be happy to accept whatever the price happens to be. Of course, if the efficient markets hypothesis is true, and the market prices do actually reflect fundamentals, then this

[4] See Allen (1931) and Galbraith (1975) for a discussion of the breadth of stock-market participation in 1929.

class of investor is not exploited. The danger is that it is not true, but that some investors believe that it is, since this may lead to investors ignoring their instincts, and blindly accepting any price. So once (potential) investors have built up their level of goodwill from previous price rises, the belief that prices are always 'fair' will lead them to be fairly insensitive to current prices. Thus if other agents can manage to induce goodwill, there will be scope for strategic behaviour to exploit the poorly-informed feedback traders; and the greater the belief that prices are 'fair,' the more scope there is for price manipulation. Even without the presence of manipulators, there is an in-built tendency for the demand of poorly-informed investors, and hence perhaps prices, to follow trends. Perhaps we should label this effect - that the greater the belief that prices reflect fundamentals, the less likely this is to be true - as the 'Paradox of Efficient Markets.'

Of course, if belief in the efficient markets hypothesis prevents ill-informed investors from investing on the basis of personal assessments as to whether the market is over- or under-valued, and believing that high returns in the recent past are likely to continue, it may have a stabilising influence, and so the net effect of such a belief is ambiguous, and will depend on its relative effectiveness in eliminating the fear of losses and the hope of large gains.

### 6.3.4 Psychological evidence

Our 'goodwill' theory does not necessarily imply that ill-informed investors' beliefs about future returns are influenced by recent returns. This type of belief formation may, however, be important. For example, Tversky & Kahneman (1982) have found that excessive emphasis is placed on the recent past when expectations about the future are formed. Evidence that past returns influence expectations of the future is given by the survey conducted by Case & Shiller (1988),[5] which found that expectations of future house price appreciation in different U.S. cities was positively related to past price changes. Andreassen & Kraus (1990) conducted experiments using authentic stock-price data, and found that the subjects tended to chase trends once they had identified them. The demand behaviour that this results in, with rising prices leading to an increase in demand, and *vice versa,* is known as positive feedback trading. Evidence concerning gambling behaviour may also be relevant, since investment activity can similarly be considered as a game involving risk-taking from which the investors derive enjoyment.[6] Established attributes of the psychology of gambling can help to explain speculative bubbles:

> "If the price of an asset has gone up and made some of one's friends considerably richer, one's attention is drawn to that asset. The gamble posed then by investing in the asset will certainly seem interesting. On reflection, one may well realize that one has no way of knowing whether the price of the asset will continue to go up or even reverse itself and drop. The 'chain letter' nature of the speculative bubble may even be readily apparent to market participants. But by the time one has realized this, the game may have so captured one's ego that one is sorely tempted to play" (Shiller, 1988, p. 63).

Thus there are many reasons to believe that ill-informed investors may be subject to contagion generated by price changes. The following section looks at evidence that such contagion has existed.

[5] As reported in De Long et. al. (1990).
[6] See Lease, Lewellen & Schlarbaum (1974).

## 6.4 Evidence of price-based contagion amongst ill-informed investors

### 6.4.1 Anecdotal evidence

There have been many occasions in the past when prices in asset markets have risen rapidly before falling even more rapidly, and for which there is anecdotal evidence indicating that many investors lost sight of reality, and were prepared to buy stock at levels that informed assessment could not justify. Indeed:

> "The pages of history are strewn with language, admittedly imprecise and possibly hyperbolic, that allows no other interpretation than occasional irrational markets and destabilising speculation" (Kindleberger, 1978, p. 27).

A commonly-used example is that of the 'tulipomania' that broke out in Holland in the 1630s. Apparently, during this:

> "People of all grades converted their property into cash, and invested it in flowers.... Everyone imagined that the passion for tulips would last for ever, and that the wealthy from every part of the world would send to Holland and pay whatever prices were asked for them. The riches of Europe would be concentrated on the shores of the Zuyder Zee, and poverty banished from the favoured clime of Holland" (Mackay, 1841).

The Mississippi and South Sea bubbles occurred in France and England respectively in 1719/20. During the South Sea bubble incident:

> "Favorable rumours... intoxicated the country with the thought of instant wealth. Visions of glory danced in the investors' heads when they heard England might be granted the right of free trade with all of Spain's colonies. Mexicans supposedly were waiting for the opportunity to empty their gold

> mines in return for England's abundant supply of cotton and woollen goods" (Malkiel, 1973, ch. 2, p. 34).

Stock markets have exhibited bubble-like behaviour at intervals ever since. For example, the U.S. experienced bull markets associated with the development of canals and then railways, in the first half of the nineteenth century, and the Californian gold strikes of 1849. The 'hot' issues then became those associated with the telegraph (late 1850s), steel, chemicals and munitions (1915), clothing, machinery and food processors (1919), aviation, radio, vehicles, mass production (1920s).[7] Post-war there have been stock booms associated with uranium, electronics, information technology, conglomerisation, and more recently bio-technology. The two best-remembered boom-and-crash scenarios of recent times terminated in 1929 and 1987.

For each of these periods it is possible to find anecdotal evidence of speculative manias, fuelled by the rising prices.[8] The stocks of those companies engaged in these new areas have understandably been affected most, and have been endowed with a glamour that have made investment in them seem more attractive. This glamour has been combined with an increase in uncertainty concerning the future, since the new developments are always unprecedented, and so the degree of subjectivity in estimating 'fundamental' values is greater than usual. It is easy to argue that this is likely to lead to a greater level of volatility in beliefs and prices, since:

> "In abnormal times in particular, when the hypothesis of an indefinite continuance of the existing state of affairs is less plausible than usual even though there are no express grounds to anticipate a definite change, the market

[7] See, for example, Sobel (1965).
[8] See, for example, Galbraith (1975) on 1929, and Soros (1987) on more recent bubbles.

> will be subject to waves of optimistic and pessimistic sentiment, which are unreasoning and yet in a sense legitimate where no solid basis exists for a reasonable calculation" (Keynes, 1964, V, p. 154).

To use the terminology of Malkiel (1973), the uncertainty and glamour may lead investors to 'build castles in the air.' In addition, informed investors will be less willing to trade against ill-informed investors at these times of greater uncertainty, due to the increased riskiness of such actions. The combination will lead to more influence for ill-informed behaviour in bull markets, and occurrences of bubbles.

It is tempting to think that, with the steady shift of direct share-holding from individuals to institutions, the problems associated with ill-informedness will have diminished, and that stock-buying will have become more 'rational'. There is, however, little evidence that this is the case. Referring to the 1960s:

> "Bradley K. Thurlow, a well-known professional, summed it up by noting, 'The funds behaved like the worst of small investors, showing speculative exuberance at the top, dire forebodings at the bottom, and steadfast timidity during the recovery'" (Malkiel, 1973, ch. 7, p. 154).

An important feature of institutional fund management that makes contagion possible is the way in which participants form a fairly tightly-knit group. According to the protagonists themselves, exploitation of the fads of other fund managers is the key to the success of Warren Buffett and George Soros.[9]

[9] See Soros (1987).

### 6.4.2 Disputing irrationality

The claim of irrationality during bull markets has, however, been disputed by Garber (1990), who argued, with specific reference to the cases of the tulipomania and the Mississippi and South Sea bubble incidents, that such events can be explained with reference to beliefs about fundamentals by rational investors. In particular, Garber claimed that the tulip market may simply have displayed the standard behavioural patterns of bulb markets; and the South Sea stock price may have risen due to a rational expectation of a rise in economic activity.

A crash is not itself evidence that prices have moved away from fundamental values. At times when it seems possible that a shift in the growth rate of economic activity may be taking place, stock prices should move to reflect this. The two main possibilities affecting the price of the South Sea Company were that rapid commercial expansion would result, or would not. The possibility of rapid commercial expansion led to the initial rise in the price of stock, and would inevitably have been followed by either another rise or a crash as the outcome became clearer. A crash is therefore possible when the likely outcome is revealed rapidly; quite possible for such a polar situation. This could happen when the price affects the likely outcome, and so price movements become self-sustaining.

### 6.4.3 Self-sustaining stock prices

The stock market can influence the wider economy, since it determines one set of terms by which capital can be raised. In addition, stock prices, since they aggregate information about future dividends, can provide information about the beliefs of others concerning the future levels of economic activity, and so provide useful information for investors in the real economy. High stock prices, associated with a relative ease of capital-raising and optimism concerning the future, can stimulate the economy; while low prices can have the opposite effect. If the information about the effect of current prices on the future economic situation is reflected in the price, however, prices will not follow trends.

This interaction between the stock market and the economy may lead to a possibility of multiple equilibria, especially since the potential for this is known to exist on an economic level.[10] It could be possible that high levels of stock prices are justifiable given expectations about the future state of the economy; but that once prices begin to fall, the expected effect of these falls on value may cause prices to fall discontinuously until an equilibrium is reached at a lower level. The economic stagnation following the crash of 1929 could perhaps be used as evidence of this, with the sharpness of the price falls arguably both causing and reflecting this. Theoretically, the effect could work in the opposite direction as well, with rising prices raising expectations of future levels of investment and output.

Even if the direct 'economic' link between stock prices and economic activity were slight, in that the role of the stock market as a capital provider is relatively insignificant, and in addition market prices provide little useful additional information concerning the real economy, the stock market may still exert an influence. This can

[10] For example, see Shiller (1978).

be for two reasons: firstly, some agents may believe that the influence is greater than it actually is; and secondly, some agents may believe that other agents, though not themselves, believe the influence to be greater than it actually is. This perhaps better illustrates the crash of 1987, when much talk of recession followed the crash, but prices subsequently recovered ground when this did not materialise. This latter argument is akin to that concerning sunspots, and illustrates that, in the presence of multiple potential equilibria resulting from the interaction - or the perceived interaction - of the market with the real economy, sunspot activity can lead to movements between equilibria, and can therefore affect both current and future prices.

## 6.5 Lessons from market manipulation

In this section we look at examples of manipulations that have taken place, taking care to draw out the possible implications for ill-informed investor behaviour in the light of explanatory models in a rational expectations framework. Incidences of past stock-market manipulations, like incidences of market bubbles, can help to reveal properties of investor behaviour; and even if the scope for such manipulation is removed by legislation, the insights it can give into non-speculator behaviour may still remain valid.

### 6.5.1 Action-based manipulation

As we have seen previously in chapter five, Allen & Gale (1992) divide manipulation into three categories: action-based; information-based; and trade-based. To illustrate action-based manipulation, Allen & Gale give the 1901 example of the American Steel and Wire Company, the managers of which took short positions in the company, closed its steel mills, watched as the stock price plunged, then covered their positions before reopening the mills. A more recent example concerns Ramiro Helmeyer, a Venezuelan investor; who in July 1993 sold stock short on the Venezuelan exchange, then reputedly set off explosions around Caracas to increase the perceived level of political instability, and to profit from the resulting price falls.[11]

Vila (1989) provides a simple model of this type of behaviour in a game-theoretic framework. Here the market participants consist of two agents that participate in an auction plus a manipulator and a potential noise trader. The manipulator formulates a strategy as to how often to enter the market, and how often to take 'action' - which could be the initiation of a takeover bid - that affects the value of the assets. The existence of noise trading is crucial for the ability of the manipulator to operate in this context. Unfortunately, interesting as cases of action-based manipulation may be, they tell us very little about the nature of other investors.

### 6.5.2 Information-based manipulation

Information-based manipulation goes back at least as far as the South Sea bubble incident, in which favourable rumours were said to have been "purposely and widely

[11] Wall Street Journal, 2nd Dec. 1993.

spread by the directors" (Malkiel, 1973, p. 34). This incident also provides an example of trade-based manipulation combining with information-based manipulation, since after the bubble appeared to burst in June 1720, when the stock price fell from 890 to 640 in a single day, the directors "gave their agents orders to buy, thus stabilising the price, and then - through a combination of manipulation and rumor - drove the price all the way up to 1,000" (Malkiel, 1973, p. 34). This type of activity came into its own with the growth in the importance of the media, and perhaps reached its peak in the 1920's with the partnerships between traders and stock tipsters:

> Such a partnership was formed between "John J. Levinson, a free-lance trader who... made profits of over one million dollars a year... [and] Raleigh T. Curtis, who wrote a column entitled 'The Trader' for the *New York Daily News*. Levinson would buy a stock, Curtis would speak highly of it in his column, the stock would rise, Levinson would sell, and the two friends would share the profit. Each time the procedure was repeated, it was easier, for as Curtis' readers bought on his recommendations, they made them come true, and the next time they trusted him all the more. David Lion, a market manipulator, and William J. McMahon, president of the McMahon Institute of Economic Research and for a while, a widely-followed radio commentator on stock market affairs, worked a similar dodge" (Sobel, 1965, ch. 12, p. 248-9).

The scope for information-based manipulation has received a boost with the development of the Internet: already there have been allegations that an on-line stock-tipping service, SGA Goldstar Research, was used to release false information about a company called Systems of Excellence, and to recommend its shares.[12] Since the scope for the dissemination of information is so much greater on the Internet than in

[12] Financial Times, 9th Nov. 1996.

other media, it is possible that information-based manipulation may receive a new lease of life.

Vila (1989) and Benabou & Laroque (1992) provide models illustrating information-based manipulation. Benabou & Laroque (1992) used a model of strategic communication to look at the potential for agents with privileged but imperfect information to earn trading profits by manipulating prices over the long term through releasing strategically-distorted announcements.[13] Benabou & Laroque found that such manipulation is a real possibility, since the 'dishonesty' cannot immediately be distinguished from genuine error, and so the credibility of those revealing false information may diminish only slowly. This shows that the presence of information-based speculation does not necessarily imply the existence of irrational investors. The ease with which it can occur does, however, indicate the extent of the ill-informedness of many investors, the lack of common knowledge of beliefs, or the disparity in prior beliefs: such manipulation can only have a large effect if others do not realise the motivation behind the announcement and have little confidence in their initial estimate of the fair stock price.

### 6.5.3 Trade-based manipulation exploiting external links

All three types of manipulation can involve external links, with the market price having some significance outside of the market itself; but for trade-based manipulation this possibility arguably becomes more important, and so we will look

[13] As we saw above (in chapter four), this issue was not addressed by Admati & Pfleiderer (1988), since the information seller in that model was assumed to be honest in his dealings.

at the possibilities with and without external links. In this section we look at the former.

In the foreign exchange markets, central banks trade in currency in order to influence the exchange rate for economic (or political) reasons. Stein (1961) gave an example prophesying the collapse of the European Exchange-Rate Mechanism, that showed how such central bank behaviour can allow destabilising speculation to be profitable: when it attempts to retain the exchange rate within certain bounds through market transactions, there will always be a threshold of market pressure which, if exceeded, will force the bank to devalue the currency in order to prevent further outflow from its foreign exchange reserves. Although of a very specific nature, this example highlights the fact that potential opportunities for profitable speculation are likely to exist whenever an agent without unlimited resources attempts to maintain market prices through trading activity.

In the stock market, there is scope for manipulation around rights issues, seasoned equity issues and takeovers. In a seasoned equity offering it may be possible to profit by driving down the market price, thereby inducing a reduction in the offer price. Gerard & Nanda (1992) illustrates how the losses the traders make by selling stock in the initial action can be more than made up for by purchasing stock at a reduced price in the offering.

The situation is less clear-cut for rights issues, since the price at which the new shares are issued does not affect the value of the holdings of shareholders who take up their allocation. Underwriters that wish to minimise the risk of having to take up any shares, however, can reduce their risk by reducing market prices prior to the setting of the rights price; and institutions that wish to purchase shares not taken up in the rights issue can increase both the number of these shares and the degree of under-pricing of each one by depressing the market price prior to the expiry of the rights offer.

There has been a long-running investigation into activity surrounding the Eurotunnel rights issue of 1994. There is evidence of heavy short-selling both prior to the setting of the rights price,[14] and the closing of the offer.[15] Institutions have been accused of forcing the offer price down to reduce their underwriting risk,[16] but proving this has been difficult. Institutions have also been accused of using insider information to profit from selling short before the plan to make the rights issue was announced;[17] but again, this has not been proven. Suspicions about price manipulation through short-selling prior to stock issues have also been raised in relation to the EuroDisney rights issue of 1994, and the sale of the second and third tranches of B.T. stock.[18] Banks in India have also been accused in the past of keeping stock prices down prior to privatisations, to enable them to pick up shares in them at 'favourable' rates. All such activity requires collusion, or at least sufficient uncertainty that allows a powerful institution to act in this way undetected and without engendering a feeling of undervaluation.

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[14] Financial Times, 13th June 1994.
[15] Financial Times, 21st June 1994.
[16] Financial Times, 17th June 1994.
[17] Financial Times, 15th Aug. 1995.
[18] Financial Times, 3rd Oct. 1994.

An example of manipulation in the opposite direction to the above is that alleged to have been perpetrated by a trader at Drexel Burnham Lambert in 1986: Pamela Monzert was accused by the Securities and Exchange Commission of supporting the share price of Stone Containers so that a public offering underwritten by DBL could go ahead.[19]

It may also be possible for managers involved in real take-over bids to profitably manipulate the stock price. As with seasoned equity issues, bidders can 'lock in' profits by strengthening the share price of their company during a (paper) bid, as can targets by strengthening theirs; although in the latter case too much success may raise the price to a level that frightens off the bidder to the detriment of the target firm's shareholders. The court case following the Guinness bid for Distillers was based on the assumption that such behaviour could take place; although in the end it was not proven that the shares were supported,[20] although compensation to Argyll, the rival bidder, was paid.[21] More recently the shares of Unichem were suspected of being supported to enhance the value of its bid for Lloyds Chemists.[22] Robert Maxwell was alleged to have illegally used money from pension funds to support the share prices of his companies,[23] but this was partly due to his use of stock as collateral.

Bagnoli & Lipman (1996) show how the possibility of takeover bids can allow manipulation to take place. They showed how a manipulator can profit by purchasing

[19] Wall Street Journal, 5th April 1993.
[20] Economist, 15th Feb. 1992.
[21] Economist, 30th Nov. 1991.
[22] Financial Times, 23rd Feb. 1996.
[23] Economist, 13th June 1992.

stock, announcing a takeover bid, and then selling stock before abandoning the bid. Chatterjea, Cherian & Jarrow (1993) have shown how the corporations can be viewed as manipulators of their own stock for the benefit of their shareholders.

There is an incentive to manipulate prices that determine payoffs to other contracts. The most obvious example relates to derivatives, where manipulating the underlying asset price at the time when the payoffs on the derivatives are determined may be profitable. This type of activity appears to be fairly common, arguably taking place in stock markets such as Tokyo[24] and Hong Kong.[25] Nick Leeson appears to have attempted a version of this, by buying Nikkei futures in the hope of supporting the falling market and hence protecting the value of the large number of straddles he had sold: but in the end this just compounded the problem.[26] In Belgium, index-linked funds, that pay out when the Belgian index surpasses a reference level, have been accused of manipulating closing prices, which determine the reference prices, in order to minimise the payout.[27] Index manipulation can also be used by dealers to generate interest among general investors in the hope of securing more business.[28]

A prime example of the potential for speculation in a market where market prices determine contract prices elsewhere concerns recent events at the National Cheese Exchange in Green Bay, Wisconsin. The prices set on this exchange for cheddar cheese determine national U.S. prices for cheese, and also, indirectly, milk, although

[24] Wall Street Journal, 12th March 1993.
[25] Financial Times, 29th Nov. 1996.
[26] Economist, 4th March 1995.
[27] Financial Times, 30th March 1996.
[28] Wall Street Journal, 6th Oct. 1993.

little actual trading takes place there. Kraft Foods, despite being a large net purchaser of these products outside the exchange, accounted for 74 percent of sales on the exchange between 1988 and 1993, and has been accused of a price manipulation that has cost U.S. dairy producers about $1 bn.[29]

Kumar & Seppi (1993) used a model based on Kyle (1985) to show how it can be profitable for a manipulator, after taking a position in a futures market, to profit from manipulating the spot price used to determine the cash settlement price in this futures market. It is clear that manipulation that exploits agents external to the market itself does not necessarily imply any particular behavioural traits of these investors. A more relevant consideration is the potential for some investors to manipulate prices at the expense of other market participants.

### 6.5.4 Pure trade-based manipulation

It is possible that there is scope for manipulation even when there are no incentives for such behaviour resulting from external links to the market. Pure trade-based manipulation has been around from the early days of organised markets. Dutch traders at the beginning of the seventeenth century had already discovered the benefits of the 'bear raid':

> "some brokers began to realize that concentrated selling over a relatively short period of time might cause the less wary to offer their shares at low prices, in this way spreading and intensifying the decline. If prices could be beaten down artificially at first, frightened investors could be counted upon to join in and push prices still lower. Those who realized what was happening could then

[29] Financial Times, 6th June 1996.

> step in and buy shares at bargain prices. 'Bear raids' of this nature were soon common in Amsterdam" (Sobel, 1965, ch. 1, p. 6).

Trade-based manipulation can also take the form of 'wash sales', which involves "the sale of a security by one broker to another who [acts] for him or his client, in order to give the impression of a transaction when one [has] not taken place... This device [has] been used to simulate a bull or bear market, when one actually [does] not exist" (Sobel, 1965, ch. 3, p. 30-1). This practice became such a feature of early stock markets that by 1817 the New York Stock and Exchange Board had already legislated against it. This legislation, however, proved to be ineffective in preventing the spirit of wash sales to live on, in the activities of 'stock pools' which were common during the 1920s.

> "The point of a pool manipulation was simplicity itself: it was a way of inducing the Stock Exchange ticker tape to tell a story that was essentially false, and thus to deceive the public" (Brooks, 1970, ch. 4, p. 69).

> "Generally such operations began when a number of traders banded together to manipulate a particular stock. They appointed a pool manager.... and promised not to doublecross each other through private operations.
>
> "The pool manager accumulated a large block of stock through inconspicuous buying over a period of weeks. If possible, he obtained an option to buy a substantial block of stock at the current market price within a stated period of, say, three or six months. Next he tried to enlist the stock's specialist on the exchange floor as an ally....
>
> "Generally, at this point the pool manager had members of the pool trade between themselves.... These sales were recorded on ticker tapes across the country and the illusion of activity was conveyed to the thousands of tape watchers who crowded into the brokerage offices of the country. Such

> activity... created the impression that something big was afoot" (Malkiel, 1973, ch. 2, p. 39-40).

Pools would often enlist the help of stock tipsters to spread favourable rumours, and sometimes even enlisted the help of the managers of the firm involved, to prevent the revelation of bad news while the operation was being conducted.

> "in a skilfully conducted manipulation, the thing would become self-sustaining; the public would in effect take the operation over, and in a frenzy of buying at higher and higher prices would push the stock on up and up with no help from the pool manager at all. That was the moment for the final phase of the maneuver, the pool's liquidation of its own stock, often spoken of as 'pulling the plug'" (Brooks, 1970, ch. 4, p. 70).

Wash sales and pool trading continue to be a problem in developing stock markets, for example the Stock Exchange of Thailand where in 1993 a group of investors was alleged to have moved shares between different accounts to simulate buying interest.[30] It also continues to occur in fully-developed markets such as Japan where, also in 1993, a speculator, Makoto Araya "allegedly placed large buy and sell orders in Nihon Unisys simultaneously to create the false impression among average investors that the stock was popular" (Wall Street Journal, 28th July 1993).

The above examples show that trade-based manipulation can be split into two components: manipulation based on net trades; and manipulation based on gross trades. Bear raids are a pure example of the former, since it is the net sales by the speculators that depress the share price and induce the panic in the ranks of the other

[30] Wall Street Journal, 30th April, 1993.

investors. Pool trading involves both types, but the defining characteristic of this form of manipulation, that of trading between pool members to arouse the interest of other investors, falls into the latter category.

That trade-based manipulation can be profitable in a market exhibiting asymmetric information should come as no surprise, and fits with the models we have already seen of Vila (1989), Allen & Gorton (1992) and Allen & Gale (1992), in which the inability of ill-informed investors to distinguish the trades of uninformed manipulators from those of informed investors creates profitable openings. Another example is provided by Fishman & Hagerty (1995), who used a specialist-based model with noise traders, in which the specialist agrees to trade any quantity at the bid and ask prices, to show how a trader who it is thought might be informed can benefit from the mandatory disclosure of her trades. This occurs since the announcement of the trades affects the market perception of the stock value, and hence the price, allowing a strategy of buy-disclose-sell or sell-disclose-buy to be profitable. As with information-based manipulation, however, the scale of the price movements and profits that have been possible in the past illustrates the extremely low level of informedness of many investors about both the underlying stock values and the structure and operation of the market.

### 6.5.5 Corners and squeezes

Corners, and the subsequent squeezes, can be thought of as special cases of (net) trade-based manipulation. In the stock market these involve buying up all or most of a stock issue, in order to force those holding short positions to cover them at an artificially inflated price. Theoretically, a successful cornerer can charge any price to the short-sellers, and so guarantee a profit. Corners were extremely common in the nineteenth century; for example Jacob Little cornered the market for stock in the Morris Canal and Banking Company in the bull market of 1834:

> "By gaining control of almost the entire floating supply of stock, Little was able to force short sellers to pay his price for shares they had contracted to deliver and could not buy in the open market. Little sold Morris Canal stock that he had bought in December at $10 a share for an average price of $185 a month later" (Sobel, 1965, ch. 3, p. 42-3).

The following account, concerning Allan A. Ryan and the Stutz Motor Car Company of America illustrates in more detail how a market can be cornered. It is taken from John Brooks, "Once in Golconda", pages 26 to 35.

The price of stock in the Stutz Motor Car Company of America rose rapidly from around $100 per share at the start of 1920 to $134 on February 2. At this point, Allan A. Ryan, who had a controlling interest in the company - and was the son of the more famous Wall Streeter Thomas Fortune Ryan - heard that organised short selling had appeared in the market, and decided to fight it. It is possible that the short-sellers believed the stock to be over-valued at the new levels, but it may have been a 'bear raid' relying on driving the price below its true value in order to buy the stock back at lower prices. In any case, Ryan borrowed millions of dollars, and attempted to squeeze the short-sellers by buying all the stock appearing in the market and, in so doing, forced the price ever upwards.

> "At first, Ryan lost ground. So great was the short-selling pressure that, despite his efforts, by early March the price of Stutz had dropped back to near

> 100. But then the tide turned decisively. By the morning of March 24 Stutz was up to 245; that day it shot up to 282, and a week later had skyrocketed to 391. In the course of the startling rise, practically all Stutz stockholders except Ryan, his firm, and members of his family decided to take their profits, and sold their stock - which was snapped up in every case by Ryan; meanwhile, the opportunity to get an inflated price for Stutz appealed more and more to the short sellers, whose number and activity increased, and Ryan bought their offerings, too. Toward the end of the month, the stock that they were selling *to* Ryan had first to be borrowed *from* him, since there was no longer anyone else who had any. Confident that he was winning, he gladly went on lending and then buying it, and the wild, uncontrolled rise to 391 on March 31 sealed his victory. The short sellers, it was clear, had disastrously underestimated his strength; they were overpowered, and their remaining choices were to buy back the stock they owed him, at his price, thereby incurring huge losses, or, alternatively, to face professional ruin and perhaps a prison term for breach of contract. Ryan.... had engineered in Stutz what Wall Street calls a corner" (Brooks, 1970, ch. 2, p. 26-7).

The short-sellers eventually bought back their borrowed stock for $550 per share, ensuring that they suffered an average loss of several hundred per cent, much more than is theoretically possible for long positions of an equivalent size. The only gainers from this episode, however, were the original stock holders, who were able to sell to Ryan at the hugely inflated prices. Ryan himself became bankrupt, since the value of the loans he had taken out to purchase the Stutz stock far exceeded the payments from the short-sellers and the value of his Stutz stock. Had he not grossly overestimated the value of Stutz, he might have held out for a higher price from the short-sellers, and secured his actual fortune, rather than a paper one based on the unrealistic share price.

Corners were possibly the most risky form of speculation prevalent in the market, although they were potentially the most profitable. The above example shows that even a successful operation does not guarantee profits for the cornerer, although, barring bankruptcy of all the shorts, this need not be the case. Since to successfully corner the market almost all the outstanding stock must be purchased, the price paid per stock to complete the operation is likely to be extremely high, and failure to secure victory ensures financial ruin: bankruptcy following unsuccessful cornering attempts was not at all uncommon.

These days, corners are much more likely to be attempted in a commodities market than in a stock market. A good example is the activity in the silver market in 1979-80.[31] It appears that Yasuo Hamanaka, the Sumitomo copper trader, attempted over a decade to manipulate the copper market, and established corners in 1993[32] and 1996, on the latter occasion wrestling with the might of George Soros's Quantum fund.[33] By exerting some degree of control over the price he should also have been able to profit from derivatives dealings.[34] The U.S. bond market has also been cornered in recent years: Salomon admitted trying (and to some extent managing) to corner the May Treasuries auction in 1991;[35] and Steinhardt Management and Caxton were accused of cornering the market for two-year Treasury notes a month prior to this.[36] An example

[31] See, for example, Gastineau & Jarrow (1991).
[32] Financial Times, 15th June 1996.
[33] Financial Times, 28th May 1996.
[34] Wall Street Journal, 15th July 1996.
[35] Economist, 17th Aug. 1991.
[36] Financial Times, 17th Dec. 1994.

of a recent corner on a stock market is that in 1993 of Union Paper on the Kuala Lumpur Stock Exchange.[37]

Since corners do not succeed at the expense of ordinary investors, but rather short-sellers who are likely to be speculators themselves following a different agenda, this type of speculation does not directly require ordinary investors to have any specific behavioural characteristics. Indirectly, however, this may not be the case, since the appropriate conditions for other types of speculation may be required, in order to induce the short-selling.

### 6.5.6 Lessons from manipulation

Legislation in the U.S. in the aftermath of the crash of 1929, for example the Securities Act 1933 and the Securities Exchange Act 1934, was designed to eliminate the worst of the speculative excesses from the market, and as a consequence "Pool operations, wash sales, the dissemination of tips or patently false information and other devices for rigging or manipulating the market were prohibited" (Galbraith, 1975, ch. IX, p. 184). In contrast to the legislation of 1817, this seems to have been fairly successful in meeting its objectives. This does not mean, however, that the nature of the non-speculators has changed, and so the lessons we have learned about investor behaviour in the past may still be relevant.

Manipulation seems to have been more prevalent during bull markets. This indicates that either speculation is solely responsible for such markets; or that the conditions

[37] Wall Street Journal, 30th Aug. 1993.

creating them, or induced by them, are conducive to speculation. The former is unlikely to be true, although there may be some causality in this direction. As we have already mentioned, the increased uncertainty surrounding new economic or technological developments may provide the link.

The potential for much speculation, notably trade- and information-based speculation, seems to be positively related to the extent of the participation in the market of relatively ill-informed investors. In the 1920s, for example:

> "people entered the stock market in greater numbers and with larger amounts of money than ever before. Their purchases and sales made the gigantic [Arthur W.] Cutten and [Jesse] Livermore pools possible." (Sobel, 1965, ch. 12, p. 252).

It seems as though the presence in the market of ill-informed investors is an important feature of markets, and that this provides the key to understanding the process of manipulation.

## 6.6 Further evidence relating to investor behaviour

### 6.6.1 Direct evidence

Direct evidence of supposedly informed investors participating in a bubble has been unearthed by a survey conducted by Shiller (1988), which found that many investors believed the stock-market to be over-valued before the crash of October 1987, but stayed in the market in the belief that it would become even more over-valued in the future.[38] In a similar vein, Frankel & Froot (1990) give survey results revealing that traders in the foreign exchange markets can believe that short-term and long-term movements can be in the opposite direction. These results lend support to the claim that bubbles can exist; although, as we have seen, this would not necessarily imply the presence of irrationality on the part of the agents, and could instead simply indicate a lack of informedness. The existence of bubbles is, however, compatible with contagion amongst ill-informed investors, which could lead informed investors to ride the bubbles in the hope of being able to sell out before they burst.

### 6.6.2 Evidence from stock prices of the influence of positive feedback trading

The presence of positive feedback trading that is not fully countered by informed trading will lead to: stock prices that vary more than is warranted by changes in fundamentals; overreaction to changes in fundamentals, perhaps causing deviations from fundamentals that are persistent; and, linked to these, price trends that are subsequently reversed.

LeRoy & Porter (1981) and Shiller (1981) were the first to find that the variance of stock prices has exceeded the variance of dividends, thus violating an implication of the standard model of efficient markets with a constant expected return. It has since been shown that the assumption of stationarity in these studies subjects the results to a significant degree of small-sample bias; however, it is possible to conclude, as in the survey by West (1988), that although:

[38] A good description of some potential bubble forms can be found in Blanchard & Watson (1982).

> "Subsequent studies[39] that explicitly allow for unit roots find excess volatility that is typically an order of magnitude smaller than for studies that assume stationarity.... they do still tend to find substantial excess volatility" (p. 639).

A consequence of excess volatility is that prices will move away from their fundamental levels; and the return to these levels will cause stock returns to exhibit negative correlations over longer horizons. Several studies have claimed to have found evidence of serial autocorrelation in stock prices. Lo & MacKinlay (1988) found evidence in short-horizon returns. Fama & French (1988), Poterba & Summers (1988) and Cutler, Poterba & Summers (1990) found evidence of positive autocorrelation at short horizons (up to a year) and negative autocorrelations beyond this.

The estimation of long-term autocorrelation is subject to the difficulties associated with small samples. One consequence of this is that the overall results will be sensitive to the inclusion of particular sub-sections of the sample. Kim, Nelson & Startz (1988) show that the findings of long-term negative autocorrelation in Fama & French (1988) and Poterba & Summers (1988) depends heavily of the first ten years of the 1926-1985 sample. Richardson & Stock (1989) and Richardson (1993) show that when the appropriate adjustments are made for small sample sizes and long-horizon returns, much of the power of these studies are lost. As Cutler, Poterba & Summers (1990) show, weak evidence of negative autocorrelation may still remain, although the statistical tests do not have the power to confirm this conclusively (Campbell, Lo & MacKinlay, 1997).

[39] Such as Mankiw, Romer & Shapiro (1985), French & Roll (1986) and Roll (1988).

Although the studies of mean reversion are subject to such econometric difficulties, the conclusions from the volatility tests are more established, and indicate that the standard constant return model is not appropriate. This, however, merely indicates that the returns are not constant, and does not provide an explanation:

> "What is left - which is non-trivial - is the source of the non-constant expected returns" (Kleidon, 1988, p. 659).

One area of the literature which may shed light on the source of the non-constant returns is that which looks for evidence of overreaction to new information. Evidence of overreaction to adverse world events and news of presidential illnesses was found by Niederhoffer (1971), and subsequent evidence indicating (longer-lasting) overreaction has been provided by DeBondt & Thaler (1985, 1987), Howe (1984), Renshaw (1984, 1990), Good (1991), Lehmann (1990) and Campbell & Kyle (1993). This type of price behaviour is consistent with excess volatility and mean reversion.

The empirical evidence of excess volatility and mean reversion has been interpreted by Shiller (1984) and Poterba & Summers (1988) as indicating the presence of fads. Such a conclusion, although consistent with the evidence, lacks strong specific endorsement, since:

> "There is little direct evidence that trading by naive investors plays a substantial role in stock determination. Such evidence as there is in favor of fads is largely indirect, and consists of negative verdicts on traditional present-value models" (West, 1988, p. 640).

The weakness of statistical tests makes it impossible to distinguish between a wide variety of competing models of stock price behaviour, and to identify instances of deviations of prices from fundamentals if and when these occur (Summers, 1986). It is, however, possible to interpret the empirical evidence, as well as other evidence, such as the high volume of trade that takes place, as indicating a failure of the efficient markets model on a fundamental level, and that:

> "Regrettably, it appears as if it is the assumptions of rationality and rational expectations that require reformation" (LeRoy, 1989, p. 1616).

Cutler, Poterba & Summers (1990) showed that the mean reversion they seem to have discovered is consistent with long-term deviations of prices from their mean path caused by the dynamics of speculation in a market containing naive (or non-rational) investors. They postulated a model of investor behaviour that contains three types of investors that are similar to those that reappear in De Long, Shleifer, Summers & Waldmann (1989, 1990a), which we examine in detail below; and demonstrated that this framework can produce positive serial correlation in stock price series in any of three ways: firstly, when there is a lag before fundamental traders learn about value; secondly, when there is negative feedback trading; and thirdly, when there is positive feedback trading influenced by returns in several previous periods. The first two alternatives simply delay the incorporation of new information into prices, and so lead to positive serial correlation until the information is fully incorporated. The third alternative, however, that of positive feedback trading, as well as initially leading to positive serial correlation, can also lead to negative serial correlation at longer horizons, and so is arguably more consistent with the empirical evidence, although, as we have seen, the empirical evidence is as yet inconclusive.

## 6.7 Conclusion

In this chapter we have outlined some of the theoretical, anecdotal and empirical evidence that points towards the existence of positive feedback trading. Theoretical causes ranged from trading strategies such as portfolio insurance and chartism, to uncertainty, bounded rationality and investor psychology. Anecdotal evidence related to past instances of bubbles and market manipulation. The empirical evidence, although not providing strong support for the existence of positive feedback trading, is at least consistent with it, and indicates a failing in the standard efficient markets model. Overall, the evidence allows us to follow Shiller in concluding that:

> "Despite the weaknesses of the anecdotal evidence, it does *suggest* that there is an important influence of faddish behaviour in financial markets" (1988, p. 58).

Since the possibility of positive feedback trading exists, its effects should be studied. The following chapter looks at the implications for prices of the form of positive feedback trading and passive investor behaviour described in De Long et. al. (1989, 1990a) and Cutler et. al. (1990).

# Chapter 7

# Manipulation with positive feedback traders

## 7.1 Introduction

In this chapter we develop and analyse a simple model featuring positive feedback trading. There are a number of problems associated with incorporating positive feedback trading into models, since it is likely to vary in importance through time and with phases in market behaviour. It is also likely to follow a relatively complicated form that would be difficult to model. Since, however, there is evidence that it forms an important feature of investor behaviour, it should not be ignored, and we should attempt to learn as much about its implications as we can. This simple model should therefore be seen as a preliminary attempt to shed light onto relatively uncharted territory. The use of such a simple model ensures that the results obtained must be interpreted with care, and the more extreme implications discounted: but some benefits from the exercise will remain.

In this chapter we look at the scope for trade-based manipulation in the presence of positive feedback traders. In particular, we make the assumption that the feedback traders exist in a market along with value-investors, whose demand is determined by the discrepancy between the market price and the estimated stock value. As the modelling of the behaviour of these agents is done in a similar way as in De Long, Shleifer, Summers & Waldmann (1989, 1990a), the analysis of this chapter helps to facilitate the assessment of that model which is carried out in chapter eight. The specification of the value- and demand-shocks introduced in section 7.4 also have their counterparts in De Long et. al.; but here these are placed in a much more general setting, with no assumption of stock liquidation which, as we shall see later, is an integral part of that work. In section 7.5 results from Hart (1977), which we have highlighted in the previous chapter, are applied to our specification in order to determine the conditions necessary for manipulation in such a situation; and we finish, in section 7.6, by using a numerical example of manipulators following the strategy proposed by Baumol (1957) to show how manipulation could occur in an appropriate setting.

## 7.2 Modelling feedback trading

As in De Long et. al. (1989), we assume that the positive feedback trader demand $\left(D^f\right)$ is positively affected by the price changes in the preceding two periods. It can also provide the market with a demand shock. In particular, we assume that it takes the following form: [1]

$$\begin{aligned} D_t^f &= \beta\left(p_{t-1} - p_{t-2}\right) + \delta\left(p_t - p_{t-1}\right) + f_t(t) \\ &= \delta p_t + \left(\beta - \delta\right)p_{t-1} - \beta p_{t-2} + f_t(t) \end{aligned} \tag{7.1}$$

Beta and delta are assumed to be non-negative. The demand shock is a function of time, and is unrelated to market prices. We could think of this as deriving from a separate group of noise traders, but is attributed to the positive feedback traders for convenience. De Long et. al. (1990a) used only 'delayed' feedback, corresponding to a delta of zero, and did not incorporate a demand shock.

It is not difficult to see that if this defined the behaviour of the only group of agents present in the market, the market price could be extremely unstable. Setting this demand equal to a fixed stock supply, and assuming a strictly positive value for delta, reveals the following second-order difference equation for the price series:

$$p_t = -\frac{(\beta - \delta)}{\delta} p_{t-1} + \frac{\beta}{\delta} p_{t-2} + \frac{S - f_t(t)}{\delta} \tag{7.2}$$

If we assume that the stock supply is zero and that there is no time-varying demand component this can be simplified to the following:

[1] Note that the use of beta and delta here differs from that in De Long et. al. (1989). Our beta is equivalent to beta plus delta in that model.

$$p_t = -\frac{(\beta - \delta)}{\delta} p_{t-1} + \frac{\beta}{\delta} p_{t-2} \tag{7.3}$$

We show in Appendix E (sections E1 and E2) that this system does not oscillate, but is dynamically unstable.

# 7.3 Feedback traders and passive investors

## 7.3.1 Introducing passive investors

Analysing this situation is unsatisfactory in a number of ways: speculative activity would in a sense be 'too easy'; and it is difficult to justify the assumption that ALL 'non-speculators', rather than just some, are positive feedback traders in the sense that their demand is a positive function of the current price; it also makes an assessment of the destabilisation issue problematic, since we have not mentioned fundamentals. De Long et. al. therefore assume that there exists another type of agent, denoted 'passive' investors. By assumption the passive investors are not capable of formulating strategies based on the behaviour of the other agents, but simply act on their estimates of underlying stock value, which is unaffected by the market price. In particular, their stock demand in period $T$ $\left(D_T^p\right)$ is given by the following:

$$D_T^p = \alpha\left(V_T - p_T\right) \tag{7.4}$$

where $V$ represents the passive investors' estimate of the fundamental stock value.

The passive investor demand therefore increases at a constant rate as the price moves away from the value estimate.

### 7.3.2 Equilibrium with passive investors and feedback traders

Combining the demand functions of the feedback traders and passive investors gives us a non-speculative excess demand function which is as follows:

$$\begin{aligned} E(t) &= D_t^f + D_t^p \\ &= \alpha(V_t - p_t) + \beta(p_{t-1} - p_{t-2}) + \delta(p_t - p_{t-1}) + f_t(t) \\ &= -(\alpha - \delta)p_t + (\beta - \delta)p_{t-1} - \beta p_{t-2} + \alpha V_t + f_t(t) \end{aligned} \tag{7.5}$$

Setting this equal to zero and rearranging:

$$p_t = \frac{(\beta - \delta)}{(\alpha - \delta)} p_{t-1} - \frac{\beta}{(\alpha - \delta)} p_{t-2} + \frac{\alpha V_t + f_t(t)}{(\alpha - \delta)} \tag{7.6}$$

If we assume that the value estimate of the passive investors is constant, and that there are no demand shocks, this can be simplified to:

$$p_t = \frac{(\beta - \delta)}{(\alpha - \delta)} p_{t-1} - \frac{\beta}{(\alpha - \delta)} p_{t-2} + \frac{\alpha V}{(\alpha - \delta)} \tag{7.7}$$

This bears more than a passing resemblance to the model used in Baumol (1957). The time path represented by this second-order difference equation will have different properties depending on the strengths of the two coefficients of feedback trading.

### 7.3.3 Oscillations

We show in the appendix (E1 and E3.1) that the price path will oscillate when the following condition holds:[2]

$$4\alpha\beta > (\beta + \delta)^2 \tag{7.8}$$

and that this condition is equivalent to:

$$\delta < -\beta + 2\sqrt{\alpha\beta} \tag{7.9}$$

[2] See, for example, Levy (1992).

A sufficient condition for this to hold is:

$$\delta < \beta < \alpha. \tag{7.10}$$

This clearly cannot hold for a beta of zero, and so the price will not oscillate when there is no delayed feedback.

### 7.3.4 Stability in an oscillatory system

When the above condition holds, and thus the path oscillates (as it does in Baumol, 1957), the oscillations will either be damped, regular, or explosive, depending on whether the magnitude of the coefficient of the twice-lagged term is less than, equal to, or greater than one. The system will therefore be asymptotically stable when the following condition holds:

$$\begin{aligned} &\left|\frac{\beta}{\alpha-\delta}\right| \leq 1 \\ &\Rightarrow \alpha \geq \beta + \delta \qquad \textit{for} \;\; \alpha > \delta \end{aligned} \tag{7.11}$$

The system will exhibit regular cyclical behaviour as in Baumol when this is an equality. This condition is derived in an alternative way in the appendix (E3.2).

### 7.3.5 Stability in a non-oscillatory system

We show in the appendix (E3.3) that a non-oscillatory system will be stable when the following conditions hold:

$$\begin{aligned} &\textit{EITHER} \quad \beta \geq \delta \\ &\textit{OR} \qquad\;\; \beta < \delta \\ &\qquad\qquad\;\; \textit{AND} \;\; 2\alpha + \beta - 3\delta \geq 0 \\ &\qquad\qquad\;\; \textit{AND} \;\; \beta \geq \delta - \frac{\alpha}{2} \end{aligned} \tag{7.12}$$

### 7.3.6 Regions of oscillations and stability

Figure 7.1 shows the combinations of the coefficients that produce systems that oscillate; and also those that are asymptotically stable, oscillatory or otherwise, as derived from the above conditions. The equations for the boundary lines can be found by normalising the relevant conditions (by dividing through by alpha) and expressing them as equalities. Stability occurs when one of the following conditions hold:

$$1) \quad \rho_\delta < -\rho_\beta + 2\sqrt{\rho_\beta} \quad AND \quad \rho_\delta \le 1 - \rho_\beta$$

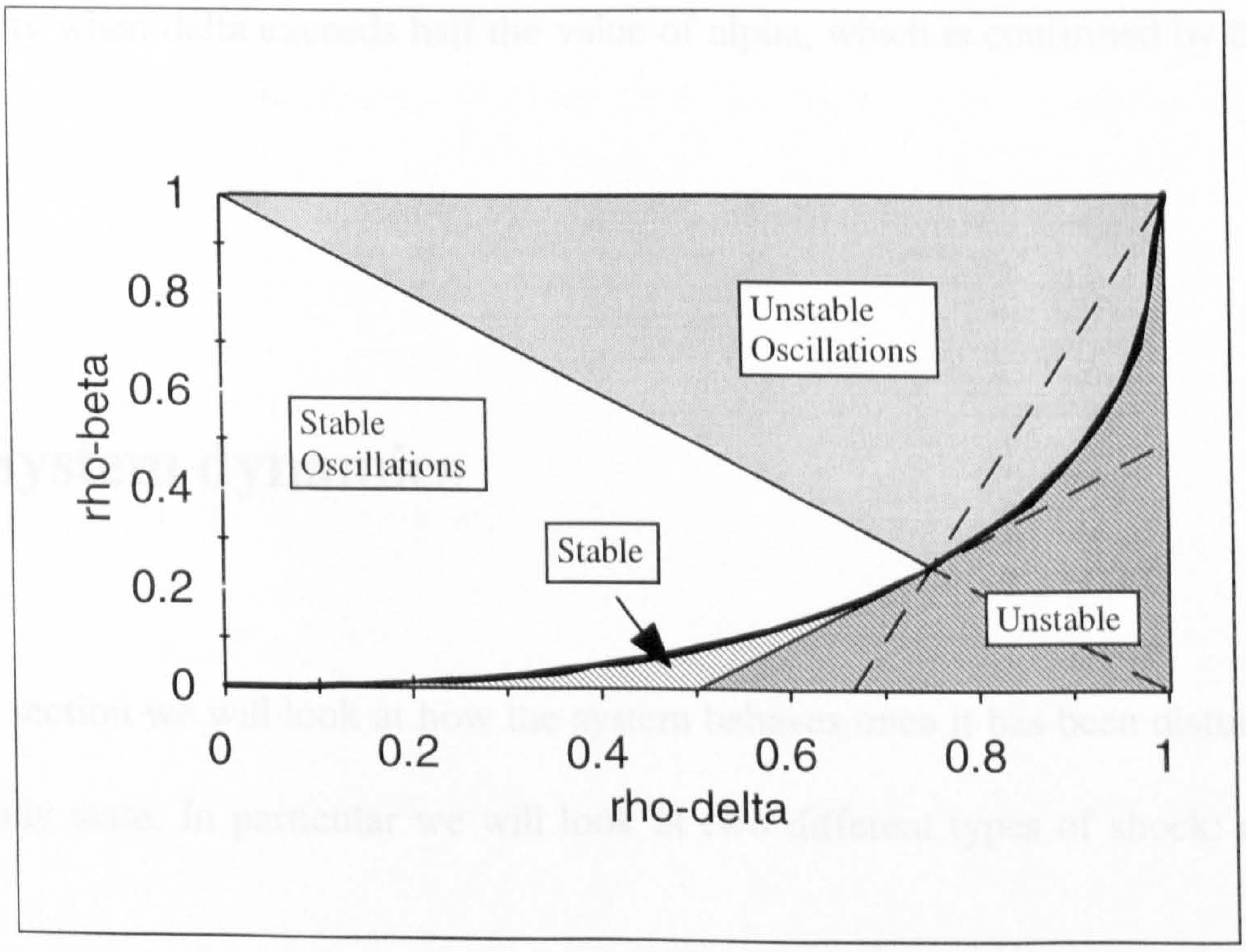
$$OR \tag{7.13}$$

$$2) \quad \rho_\delta \ge -\rho_\beta + 2\sqrt{\rho_\beta} \quad AND \quad \rho_\delta \le \rho_\beta + \frac{1}{2} \quad AND \quad \rho_\delta \le \frac{\rho_\beta}{3} + \frac{2}{3}$$

where: $\rho_\beta = \beta/\alpha$ *and* $\rho_\delta = \delta/\alpha$.

The first condition corresponds to the plain white region in figure 7.1, while the second corresponds to the white hatched region.

*Fig. 7.1: Regions of stability.*



Cases where the system in the absence of speculation would be asymptotically unstable are not particularly interesting, since the destabilising effect of speculation would be difficult to assess in such a situation. For the case of delayed feedback only ($\delta = 0$) as in De Long et. al. (1990), figure 7.1 shows that the system will exhibit asymptotically stable oscillations.

When there is no delayed feedback trader response to price changes ($\beta = 0$) the situation collapses to the following first-order difference equation:

$$p_t = -\frac{\delta}{(\alpha - \delta)} p_{t-1} + \frac{\alpha V}{(\alpha - \delta)} \tag{7.14}$$

This exhibits asymptotic stability when the magnitude of the coefficient is less than (or equal to) one:

$$\left| \frac{\delta}{(\alpha - \delta)} \right| \leq 1 \tag{7.15}$$

This shows that when there is no delayed feedback the system exhibits asymptotic instability when delta exceeds half the value of alpha, which is confirmed by the figure 7.1.

# 7.4 System dynamics

In this section we will look at how the system behaves once it has been disturbed from its steady state. In particular we will look at two different types of shock: a demand shock; and a shock to the underlying stock value. These mirror those found in De Long et. al.

### 7.4.1 Demand shock

A demand shock occurs as the result of liquidity trades. This is the type of shock present in De Long et. al. (1989). Assume that the system is in its steady state, with past prices equal to $V_o$, when a one-off demand shock N hits the market in period $T$. The price in this period is determined as follows:

$$\begin{aligned} -N &= -(\alpha-\delta)p_T + (\beta-\delta)p_{T-1} - \beta p_{T-2} + \alpha V_o \\ &= -(\alpha-\delta)p_T + (\beta-\delta)V_o - \beta V_o + \alpha V_o \\ \Rightarrow p_T &= V_o + \frac{N}{(\alpha-\delta)} \end{aligned} \tag{7.16}$$

For simplicity, we will now assume that delta is zero. The system reduces to the following:

$$p_t = \frac{\beta}{\alpha} p_{t-1} - \frac{\beta}{\alpha} p_{t-2} + V_o \tag{7.17}$$

The path of prices in this system will take the following form, for discrete values of $t$:[3]

$$p_t = Ar^t \cos(t\theta + B) + V_o \tag{7.18}$$

where: $r = \sqrt{\frac{\beta}{\alpha}}$

and $\theta$ is such that the following holds:

$$\cos\theta = \frac{1}{2}\sqrt{\frac{\beta}{\alpha}}$$

This shows that the periodicity of the cycle is proportional to the square root of beta.

[3] See, for example, Goldberg (1958).

### 7.4.2 Value shock

When a value shock occurs, the system must move to a new steady-state equilibrium, motivated by the realisation by the passive investors that, with a new fundamental value of $V_n$, the stocks are now mispriced. The general form of the price process, again for discrete values of $t$, is similar to that for the case with the demand shock, but centred around the new fundamental value:

$$p_t = Cr^t \cos(t\theta + D) + V_n \tag{7.19}$$

where $r$ and $\theta$ is the same as in the previous example.

### 7.4.3 Numerical example of a demand shock

Assume now that beta is a three-quarters, alpha is one, and the value estimate is initially ten. This gives the following:

$$\begin{aligned} r &= \frac{\sqrt{3}}{2} \\ \theta &= \cos^{-1}\left(\frac{\sqrt{3}}{4}\right) \end{aligned} \tag{7.20}$$

Assume also that a demand shock of one hits the market in period one. The subsequent path of prices is given by the following:

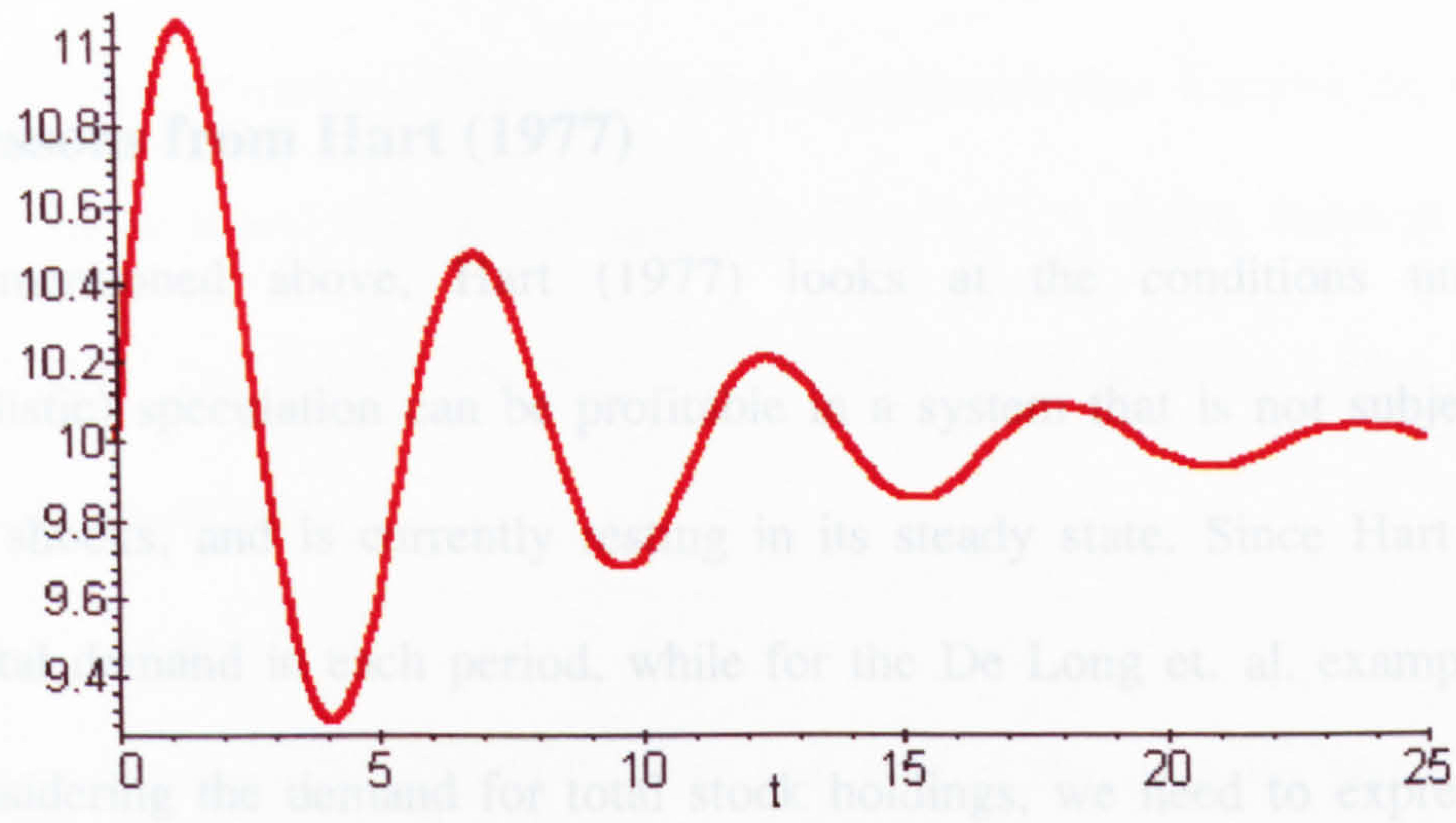
$$p_t = -\frac{8}{\sqrt{39}} r^t \cos\left(t\theta + \frac{\pi}{2}\right) + 10 \tag{7.21}$$

This generates the price path shown in figure 7.2.

### 7.4.4 Numerical example of a value shock

Given the same system as in the previous example resting in a steady-state position, assume that the value estimate of the passive investors changes from ten to eleven. If

*Fig. 7.2: Price with demand shock.*

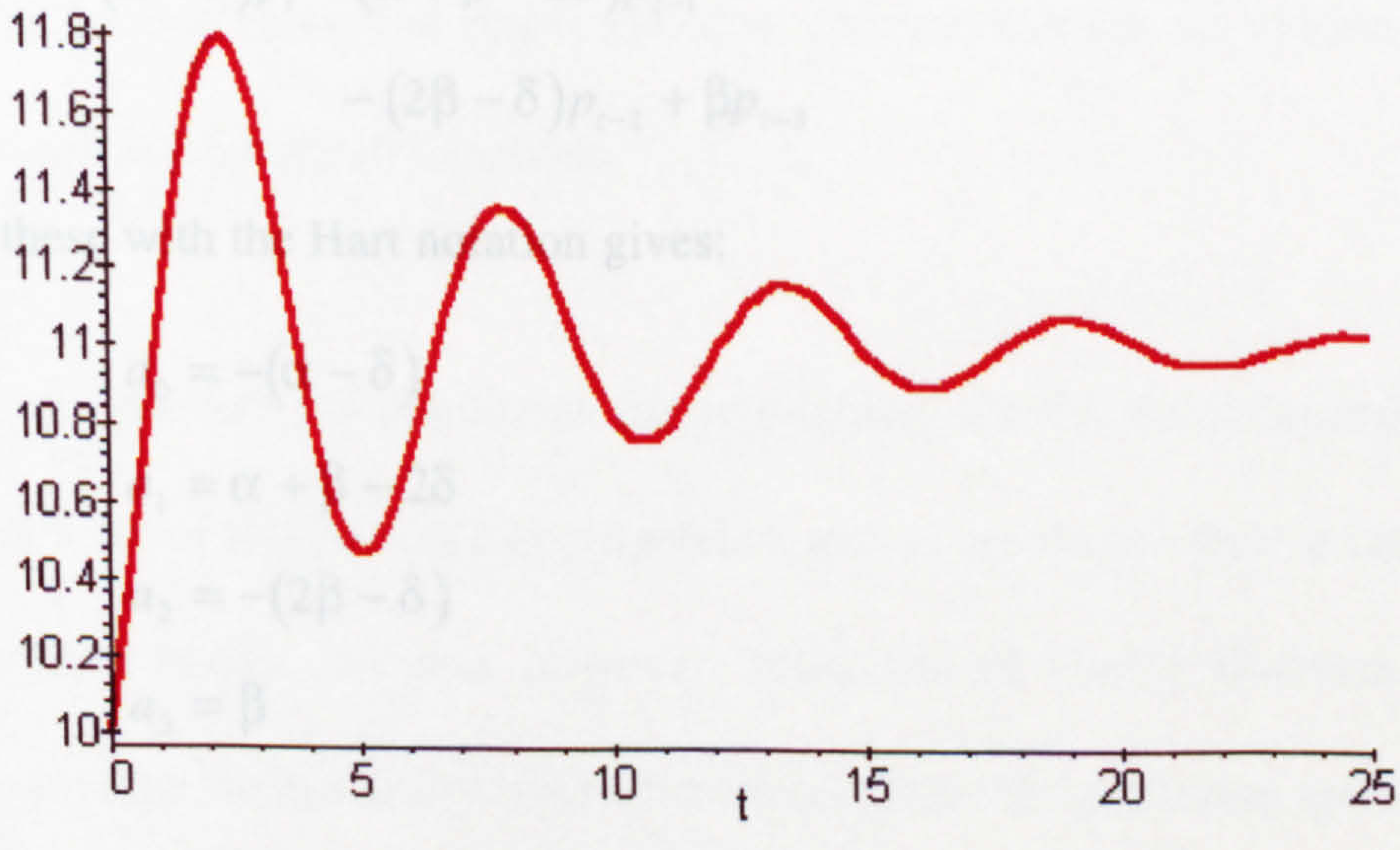



we assume that this occurs in period one, the price will now begin a cycle that can be characterised by equation 7.22.

$$p_t = -\frac{4}{\sqrt{13}} r^t \cos(t\theta + 0.4478) + 11 \tag{7.22}$$

The associated price path is given in figure 7.3.

*Fig. 7.3: Price with change in value.*



# 7.5 Profitability of disturbing the steady state

## 7.5.1 Lessons from Hart (1977)

As we mentioned above, Hart (1977) looks at the conditions under which (monopolistic) speculation can be profitable in a system that is not subject to other types of shocks, and is currently resting in its steady state. Since Hart deals with incremental demand in each period, while for the De Long et. al. example we have been considering the demand for total stock holdings, we need to express the non-speculator excess demand function in a new form. The incremental demand of the non-speculators is as follows:

$$\begin{aligned} F_t &= E_t - E_{t-1} \\ &= -(\alpha-\delta)(p_t - p_{t-1}) + (\beta-\delta)(p_{t-1} - p_{t-2}) \\ &\qquad\qquad - \beta(p_{t-2} - p_{t-3}) + \alpha(V_t - V_{t-1}) \\ &= -(\alpha-\delta)p_t + (\alpha+\beta-2\delta)p_{t-1} - (2\beta-\delta)p_{t-2} \\ &\qquad\qquad + \beta p_{t-3} + \alpha(V_t - V_{t-1}) \end{aligned} \tag{7.23}$$

When $V$, the value estimate of the passive investors, is constant, this reduces to:

$$\begin{aligned} F_t = -(\alpha-\delta)p_t + (\alpha+\beta-2\delta)p_{t-1} \\ - (2\beta-\delta)p_{t-2} + \beta p_{t-3} \end{aligned} \tag{7.24}$$

Equating these with the Hart notation gives:

$$\begin{aligned} a_0 &= -(\alpha-\delta) \\ a_1 &= \alpha+\beta-2\delta \\ a_2 &= -(2\beta-\delta) \\ a_3 &= \beta \end{aligned} \tag{7.25}$$

We can define a new function, as in Hart:

$$f(z) = -(\alpha - \delta) + (\alpha + \beta - 2\delta)z - (2\beta - \delta)z^2 + \beta z^3 \tag{7.26}$$

As we have seen previously, Hart shows that speculation that disturbs the steady state can be profitable if and only if the equation, $\operatorname{Re} f(z) = 0$, has a solution $z$ satisfying $|z| < 1$. The real part of the above can be found by first splitting $z$ into its real and imaginary components:

$$z \equiv b + ci \tag{7.27}$$

We show in the appendix (E3.4) that the real part of the function can be written as follows:

$$\begin{aligned} \operatorname{Re} f(z) \; &= (2\beta - \delta)x - (\alpha - \delta) + (\alpha + \beta - 3\beta x - 2\delta)b \\ &\qquad - 2(2\beta - \delta)b^2 + 4\beta b^3 \end{aligned} \tag{7.28}$$

where $0 \leq x < 1$ *and* $b^2 < x$.

For speculation that disturbs the steady state to be profitable, there must exist a root $b$ that lies within the unit circle for some feasible value of $x$. A sufficient condition for this is that the original function $f(z)$ has a real root that satisfies the condition. In the appendix (E3.5) we show that this is the same condition as for the system to be non-oscillatory and asymptotically unstable.

Hart does not provide the conditions for profitability for the thrice-lagged case, since the general form of this proves too intractable, and so we do not have a ready solution to the problem handy. We can, however, make use of Hart's *Theorem 3.6*, which states that asymptotic instability is a sufficient condition for profitable speculation.

### 7.5.2 Sufficient conditions for non-profitability

Expression 5.15 of chapter five reveals that, for our example, a sufficient condition for manipulation that disturbs the steady state to be unprofitable is as follows:

$$\begin{aligned} &\alpha - \delta \geq |\alpha + \beta - 2\delta| \\ &\textit{and}\ \ \alpha - \delta \geq |\alpha + \beta - 2\delta| + |2\beta - \delta| \\ &\textit{and}\ \ \alpha - \delta \geq |\alpha + \beta - 2\delta| + |2\beta - \delta| + \beta \end{aligned} \tag{7.29}$$

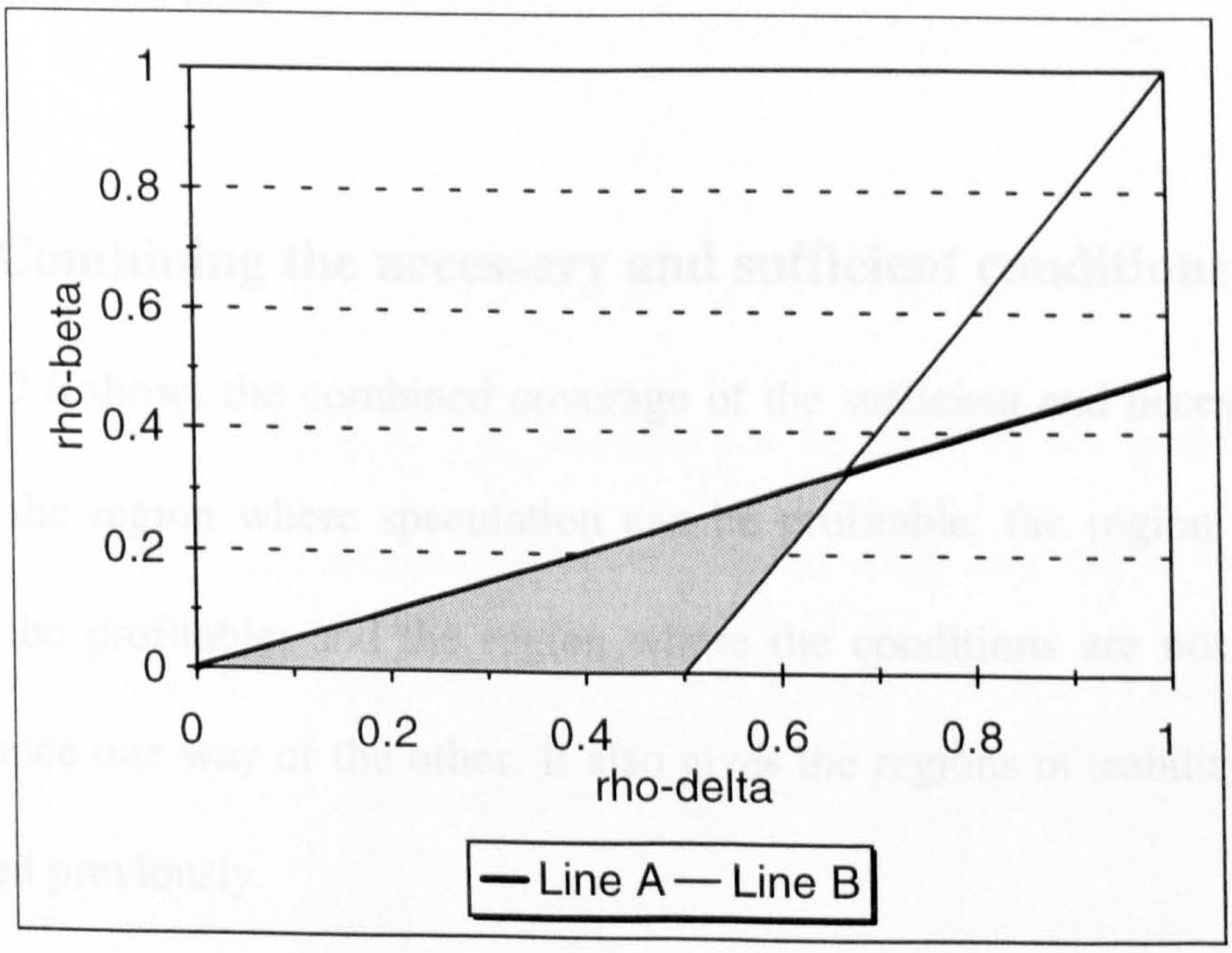
These are clearly increasing in severity, implying that we need only consider the final condition, which can be expressed as:

$$\rho_\delta \geq 2\rho_\beta \qquad AND \qquad \rho_\delta \leq \frac{1}{2} + \frac{\rho_\beta}{2} \tag{7.30}$$

where $\rho_\beta = \beta/\alpha$ *and* $\rho_\delta = \delta/\alpha$.

This condition is illustrated in figure 7.4. The intersection of the two lines, which marks the apex of the shaded triangle, occurs at the point (2/3, 1/3).

*Fig. 7.4: Sufficient condition.*



### 7.5.3 Necessary conditions for non-profitability

Expression 5.16 of chapter five reveals that a necessary condition for manipulation to be unprofitable is that the following hold:

$$\begin{aligned}&\alpha-\delta \geq \left|\frac{\alpha+\beta-2\delta}{2}\right| \\ &\textit{and}\ \ \alpha-\delta \geq \left|\frac{2\beta-\delta}{2}\right| \\ &\textit{and}\ \ \alpha-\delta \geq \frac{\beta}{2}\end{aligned} \tag{7.31}$$

These can be expressed alternatively as:

$$\begin{aligned}&4\rho_\delta \leq \rho_\beta + 3 \\ &\textit{AND}\ \ \begin{cases} \textit{EITHER}\ \ \rho_\beta \geq \dfrac{\rho_\delta}{2} \ \ \textit{and}\ \ \rho_\beta + \dfrac{\rho_\delta}{2} \leq 1 \\ \textit{OR}\ \ \rho_\beta \leq \dfrac{\rho_\delta}{2} \ \ \textit{and}\ \ \dfrac{3}{2}\rho_\delta - \rho_\beta \leq 1 \end{cases} \\ &\textit{AND}\ \ \rho_\delta + \frac{\rho_\beta}{2} \leq 1\end{aligned} \tag{7.32}$$

The combined condition is shown in figure 7.5. Speculation can be profitable for values in the unshaded area.

### 7.5.4 Combining the necessary and sufficient conditions

Figure 7.6 shows the combined coverage of the sufficient and necessary conditions. It shows the region where speculation can be profitable; the region where speculation cannot be profitable; and the region where the conditions are not strong enough to pronounce one way or the other. It also gives the regions of stability and instability, as revealed previously.

*Fig. 7.5: Necessary condition.*

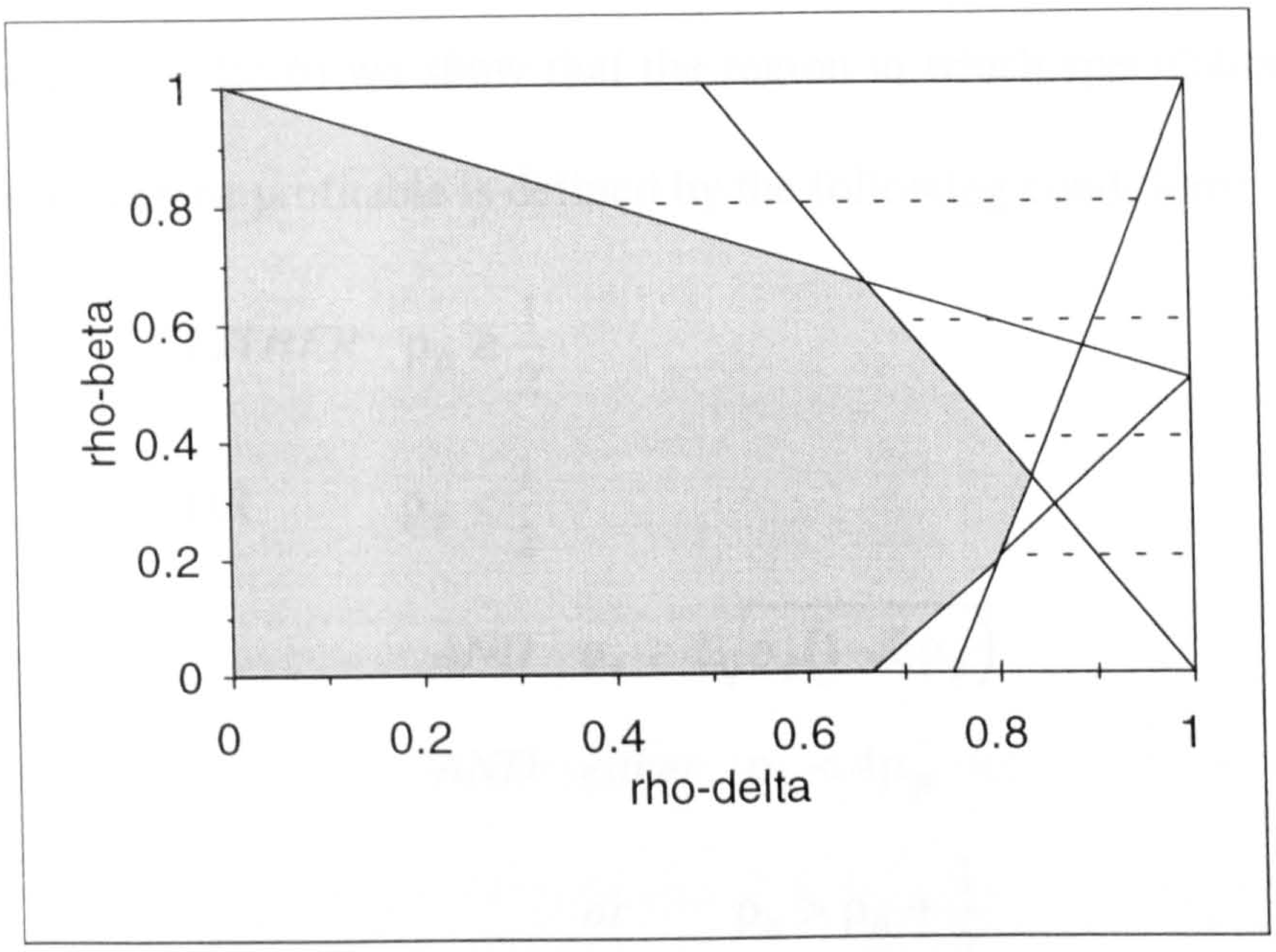
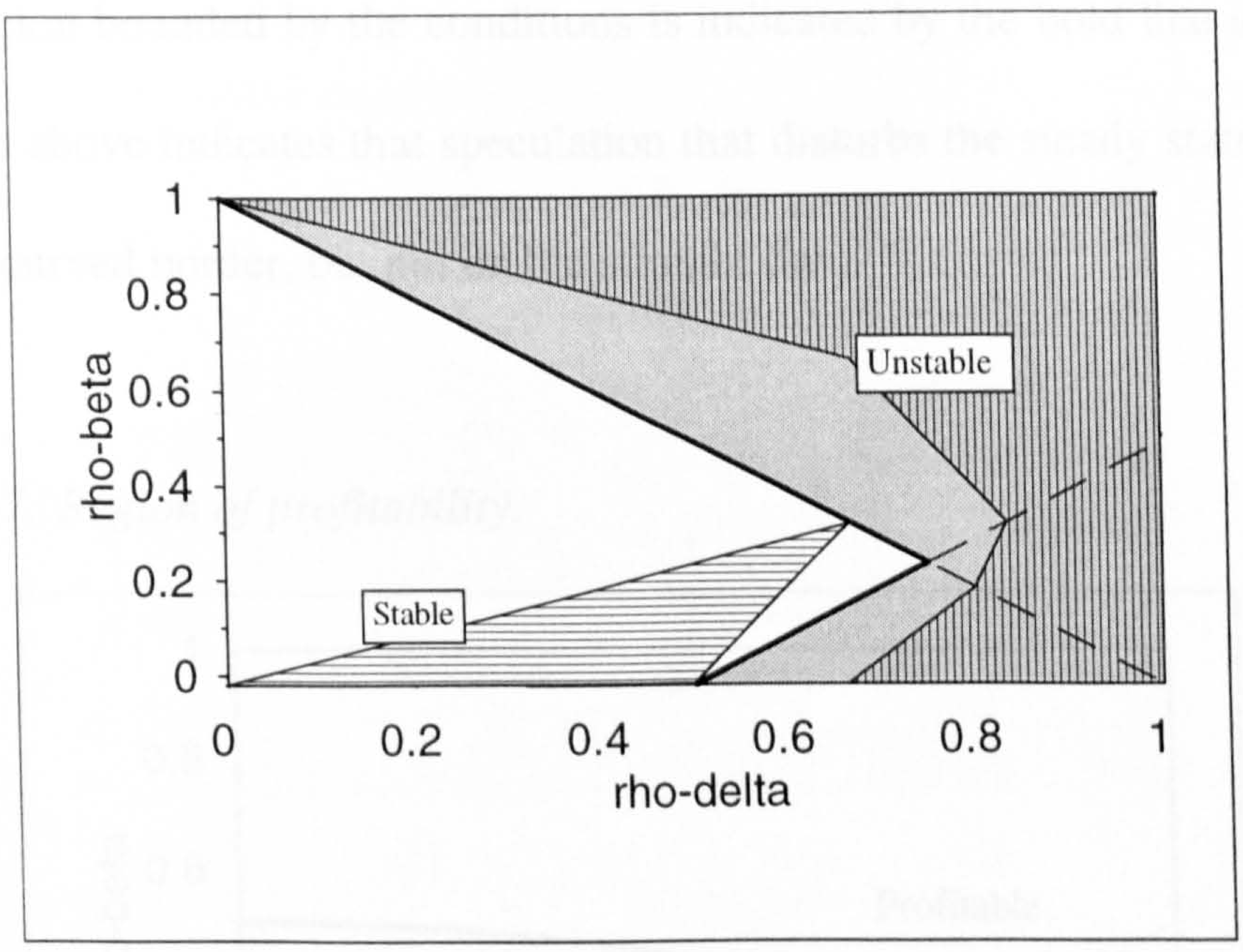



*Fig. 7.6: Combining the conditions.*




As can be seen from figure 7.6, the area not covered by the necessary and sufficient conditions represents approximately half of the total area. In addition, the necessary condition is entirely superseded by the combination of the conditions for asymptotic instability and Hart's *Theorem 3.6*, which, as we have seen, states that asymptotic instability implies profitability.

## 7.5.5 Isolating the boundary of profitability

In the appendix (E3.6) we show that the region in which speculation that disturbs the steady state can be profitable is defined by the following conditions:

$$\textit{EITHER} \quad \rho_\beta \geq \frac{1}{2}$$

$$\textit{OR} \quad \rho_\beta < \frac{1}{2}$$

$$\textit{AND} \quad \rho_\delta \geq 2\sqrt{\rho_\beta(1-2\rho_\beta)}$$

$$\textit{AND} \quad \textit{either} \quad \rho_\delta < 4\rho_\beta$$

$$\textit{or} \quad \rho_\delta > \rho_\beta + \frac{1}{2}$$

(7.33)

The region bounded by the conditions is indicated by the bold line in figure 7.7. Note that the above indicates that speculation that disturbs the steady state can be profitable on the curved border, but not on the straight line.

*Fig. 7.7: Region of profitability.*

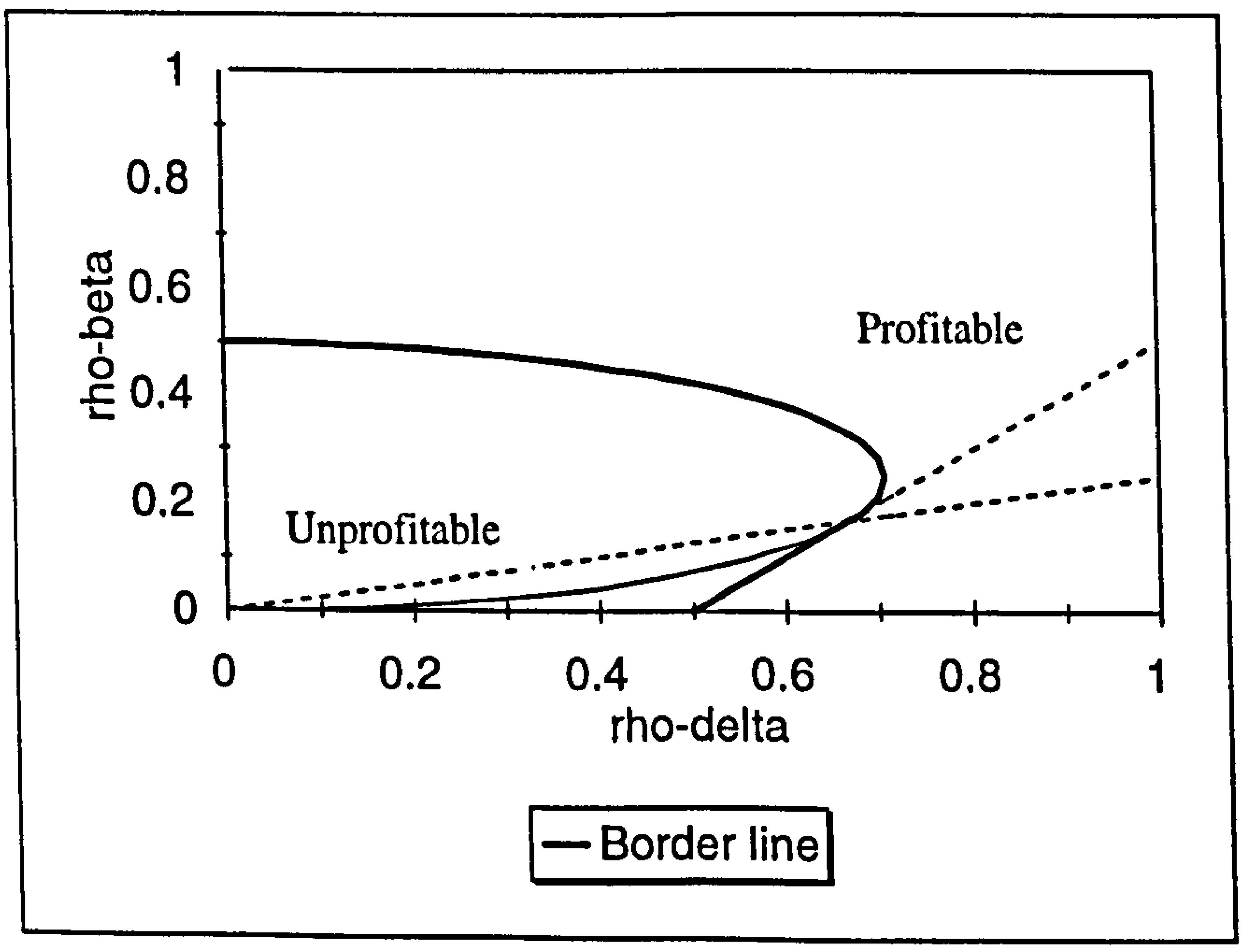
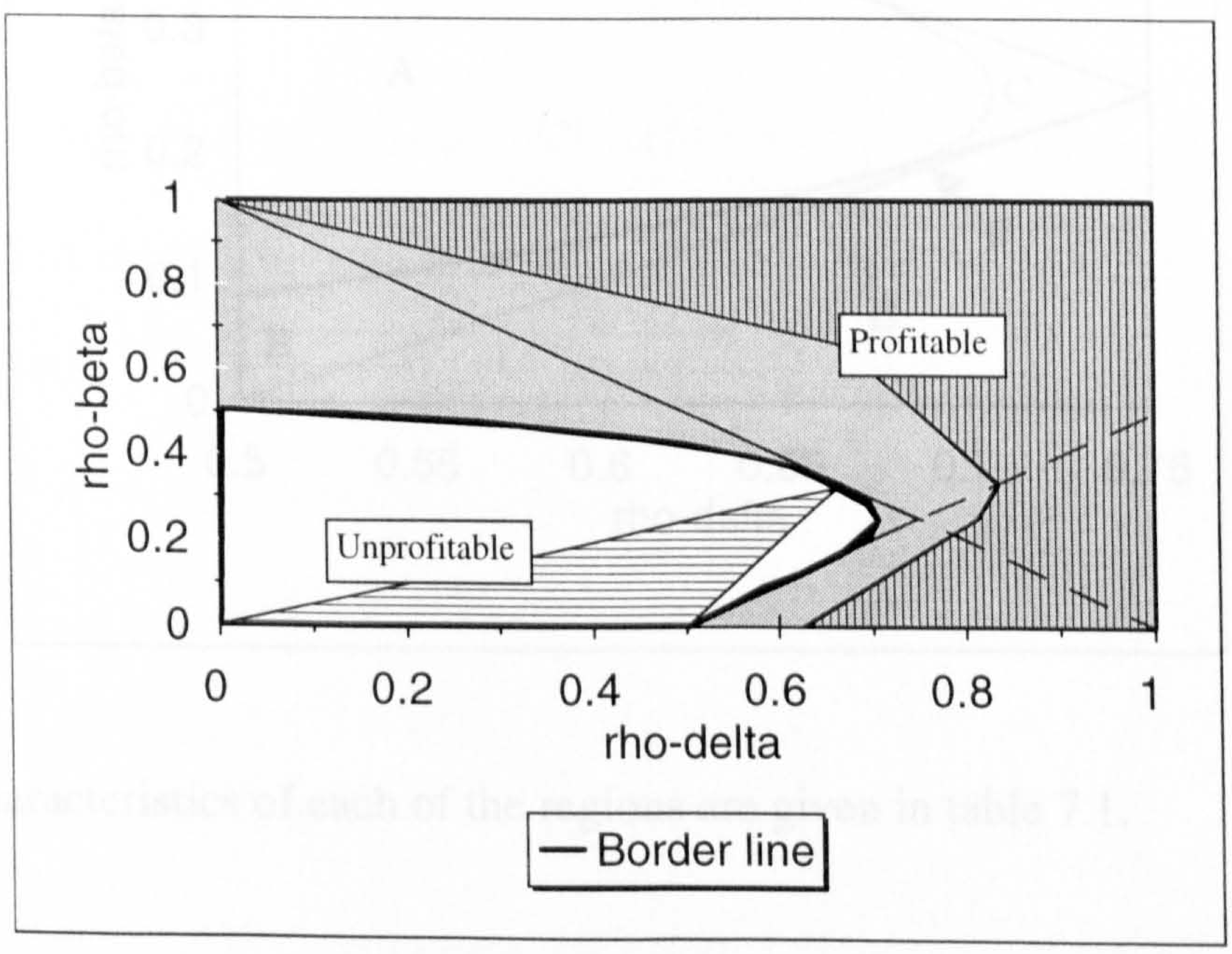


Figure 7.7 shows that an increase in feedback trading can, in some circumstances, move the system from a situation in which speculation is profitable to one in which it is not. For example, with an alpha of one, and a delta of 0.6, speculation can be profitable when beta is zero; but if beta rises to, say, 0.2, speculation can no longer be profitable.

Combining figure 7.7 with the necessary and sufficient conditions allows us to assess more clearly the performance of these. This is done in figure 7.8, and reveals that: the sufficient condition identifies only about a third of the unprofitable region, but gives two of the boundary points, (1/2, 0) and (2/3, 1/3); and the boundary of stability gives a reasonable approximation of the boundary of profitability for values of $\rho_\beta < 1/2$.

*Fig. 7.8: Power of the conditions.*




The boundaries of the regions displaying unique combinations of oscillatoriness, stability and profitability are shown in figure 7.9. The situation can be seen more clearly if we focus on the central section, which is shown in figure 7.10.

*Fig. 7.9: Combinations of oscillatoriness, stability and profitability.*

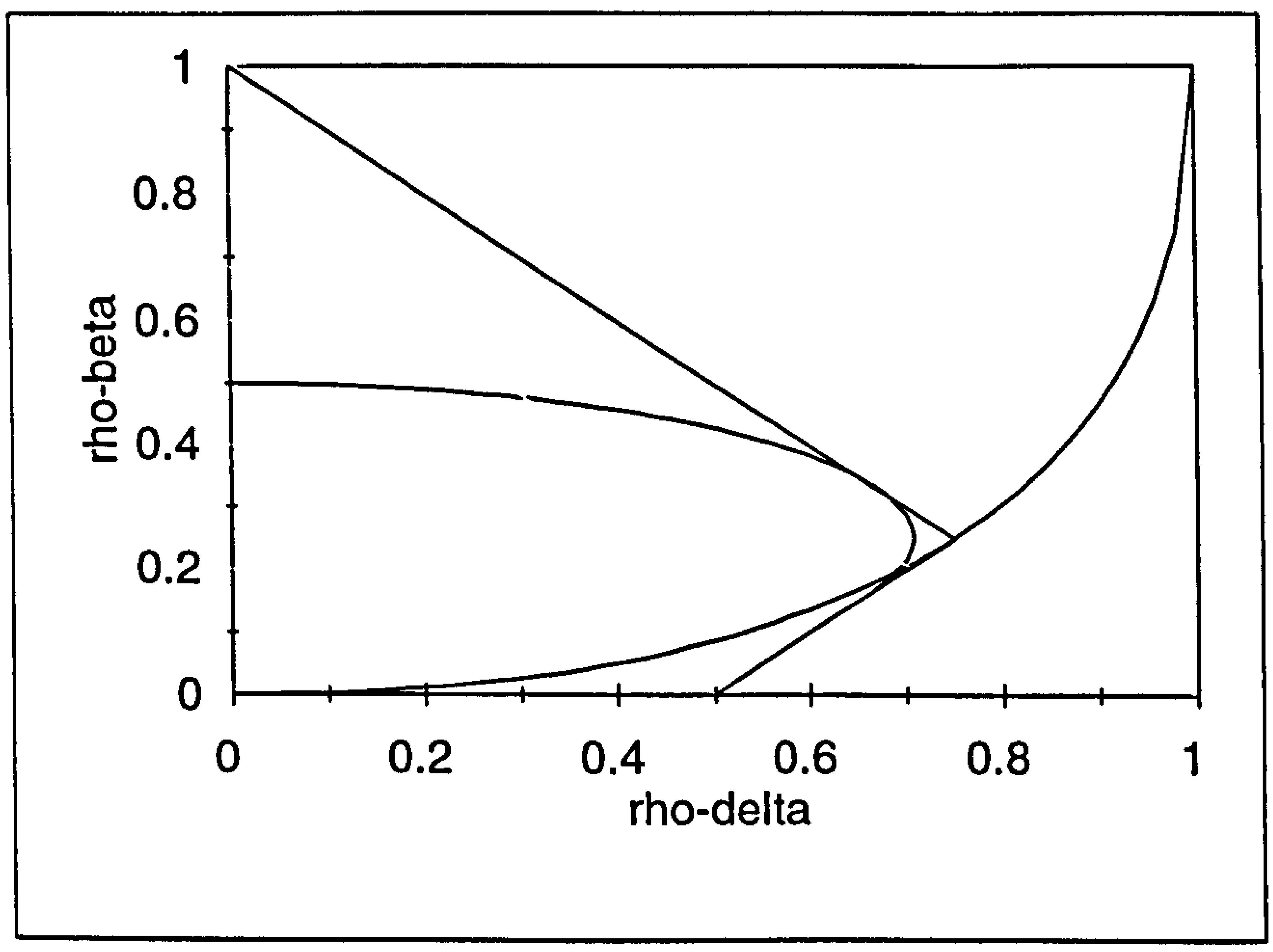
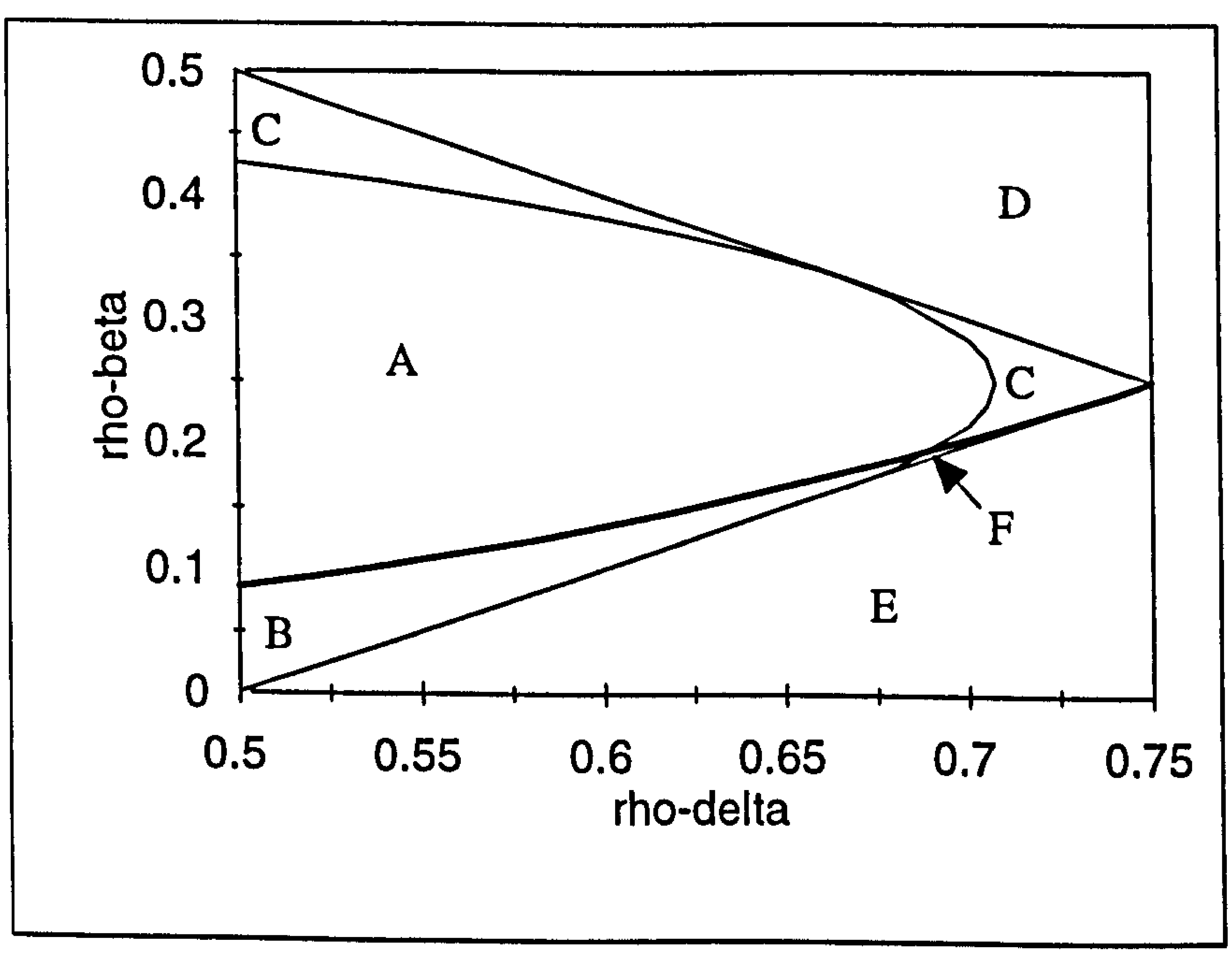



*Fig. 7.10: Close-up of combinations of oscillatoriness, stability and profitability.*




The characteristics of each of the regions are given in table 7.1.

*Table 7.1: Characteristics of the regions.*

| | Oscillations. | Stability. | Profitability. |
|---|---|---|---|
| A | √ | √ | X |
| B | X | √ | X |
| C | √ | √ | √ |
| D | √ | X | √ |
| E | X | X | √ |
| F | X | √ | √ |

### 7.5.6 Comparison with Baumol (1957)

As we have seen in the previous chapter, Baumol (1957) contains a model that encompasses positive feedback. That model can be compared with the one we have analysed above. In Baumol the coefficients of the non-speculator excess demand function are as follows:

$$\begin{aligned} a_0 &= -W \\ a_1 &= 2Wa \\ a_2 &= -W \end{aligned} \tag{7.34}$$

where $W$ is a positive constant. All further coefficients ($a_3$, $a_4$, etc.) are zero. To re-cap: in the model we have been using, the coefficients are as given in equation 7.25, which are:

$$\begin{aligned} a_0 &= -(\alpha - \delta) \\ a_1 &= \alpha + \beta - 2\delta \\ a_2 &= -(2\beta - \delta) \\ a_3 &= \beta \end{aligned}$$

A comparison of the coefficients reveals that these models are only compatible for the following parameter values:

$$\begin{aligned} \alpha &= \beta = 0 \\ \delta &= -W \\ a &= 1 \end{aligned} \tag{7.35}$$

This indicates that the De Long et. al. formulation is not as general as might be supposed.

# 7.6 Profitable speculator strategies

Previously we found the values for beta and delta under which speculators can make money from disturbing the steady state. In this section we look at an example of a profitable speculative strategy that can be employed in such a situation. The particular strategy we use is taken from Baumol (1957), and represents a rule-of-thumb approach rather than one based on optimisation. Since in De Long et. al. the optimal behaviour of the speculators is determined by working back from a fixed end-point, the approach in that paper is not transferable to this situation.

## 7.6.1 Speculator strategy from Baumol (1957)

We saw in the previous chapter that in the Baumol (1957) model speculators can make money by utilising the following excess demand function, which, although not explicitly derived from optimising behaviour, is designed to mimic speculator behaviour that concentrates purchases just after a price trough, and sales just after a peak:

$$\begin{aligned} E_{s,t+1} &= C[(P_{t+1} - P_t) - (P_t - P_{t-1})] \\ &= C[(P_{t+1} - 2P_t + P_{t-1})] \end{aligned} \tag{7.36}$$

The speculator demand will therefore follow a cyclical pattern of the same frequency as the price. Although Baumol assumes that the prices before any speculator involvement are fluctuating cyclically, we know from the previous chapter that speculators can profit from disturbing the steady state to induce this cyclical behaviour.

Suppose that the speculators in our system behave in the same way that they do in Baumol, with the exception that they have to initially disturb the steady state. We can therefore express the speculators' excess demand function in the following way:

$$\begin{aligned} E_{s,t} &= C[(p_t - p_{t-1}) - (p_{t-1} - p_{t-2})] + X_t - X_{t-1} \\ &= C[(p_t - 2p_{t-1} + p_{t-2})] + X_t - X_{t-1} \end{aligned} \tag{7.37}$$

where $X$ is the demand shock required to jerk the system out of its steady state, and is assumed to be non-zero only in period one, the first period of speculative activity. The total demand from the speculator in period $t$ is therefore given by the following:

$$F_{s,t} = C(p_t - p_{t-1}) + X_t \tag{7.38}$$

Adding this to the non-speculator total demand function and setting it equal to the stock supply, which we set equal to zero for convenience, allows us to determine the new price process:

$$\begin{aligned} &-(\alpha - \delta - C)p_t + (\beta - \delta - C)p_{t-1} - \beta p_{t-2} + \alpha V_t + X_t = 0 \\ &\Rightarrow p_t = \frac{(\beta - \delta - C)}{(\alpha - \delta - C)} p_{t-1} - \frac{\beta}{(\alpha - \delta - C)} p_{t-2} + \frac{\alpha V_t + X_t}{(\alpha - \delta - C)} \end{aligned} \tag{7.39}$$

The price will oscillate when the following condition holds:

$$(\beta - \delta - C)^2 < 4\beta(\alpha - \delta - C) \tag{7.40}$$

A sufficient for this to hold is that:

$$\beta > \delta + C \tag{7.41}$$

When the price oscillates, the cycles are of the following form:

$$p_t = A\left(\frac{\beta}{\alpha-\delta-C}\right)^{\frac{t}{2}}\cos(t\theta+B) \tag{7.42}$$

where θ is given by $\cos\theta = \frac{\beta-\delta-C}{2\sqrt{\beta(\alpha-\delta-C)}}$ and $A$ and $B$ are given by the initial conditions.

### 7.6.2 Setting up the numerical examples

Let us now look at the effect of speculative involvement of this sort, within the system used to provide the previous examples. For simplicity we will set the fundamental value to zero, which does not alter the nature of the cycles illustrated. The parameters are therefore as follows: $\beta = 3/4$, $\delta = 0$, $\alpha = 1$, $V = 0$. Assume that the speculator purchases one unit of the stock, in period one, to disturb the system from its steady state. The speculator demand in subsequent periods is determined, via prices, by the value of $C$. The lines in the subsequent plots follow the colour-scheme given in table 7.2.

*Table 7.2: Colour-code for plots.*

| | |
|---|---|
| red | deviation of price from mean |
| green | contribution of current period to total profit |
| black | incremental demand of speculators |

The deviation of the price (from its mean of the fundamental value $V$) is plotted, rather that the price itself, in order to aid comparison of this cycle with the others given. The contribution of each period to the total profit is calculated in the following way:

$$\textit{Single period profit} = -E_{s,t}p_t \tag{7.43}$$

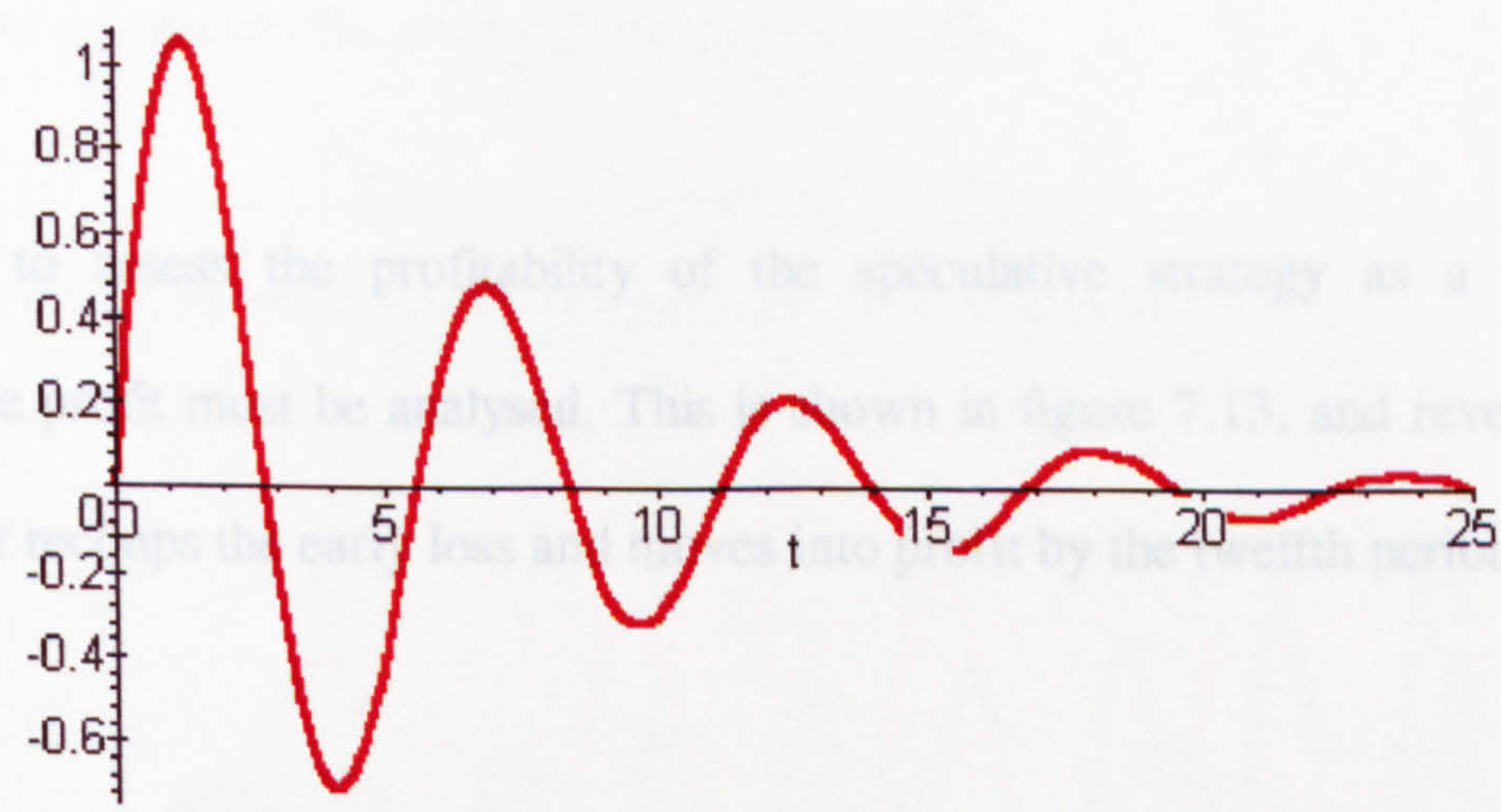
The cumulative profit of the speculators is therefore given by:

$$\textit{Cumulative profit to period } T = \sum_{t=1}^{T} -E_{s,t}p_t \tag{7.44}$$

### 7.6.3 Numerical example #1, with C of zero

When *C* is zero, the price will follow a path totally determined, beyond the first period, by the non-speculator demand function; and so will resemble that given previously for the demand shock. This is shown in figure 7.11. Such a strategy would clearly not be followed by a speculator, since he would have no way of recouping the initial loss from disturbing the steady state.

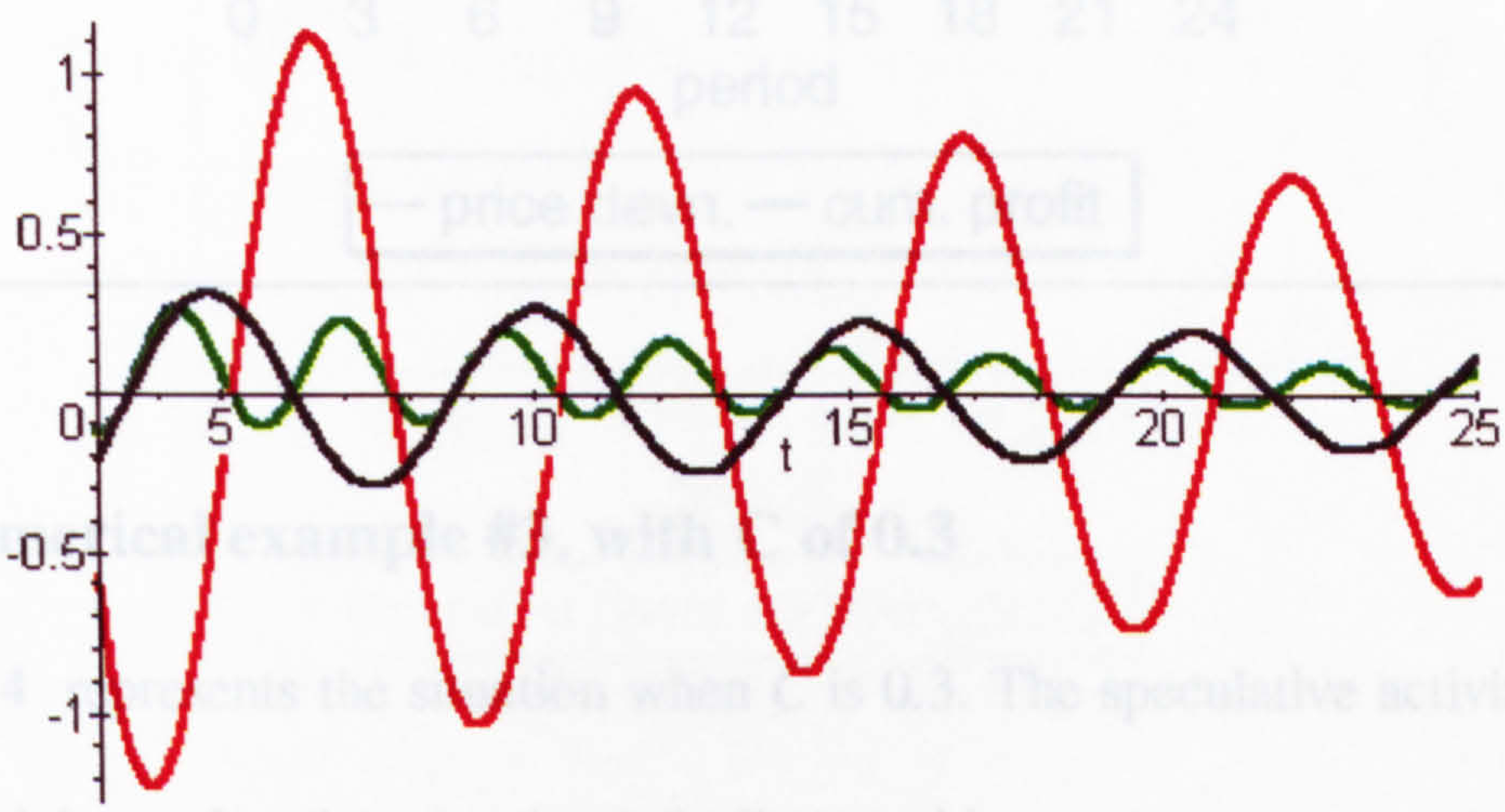
*Fig. 7.11: Price path with C of zero.*



### 7.6.4 Numerical example #2, with C of 0.2

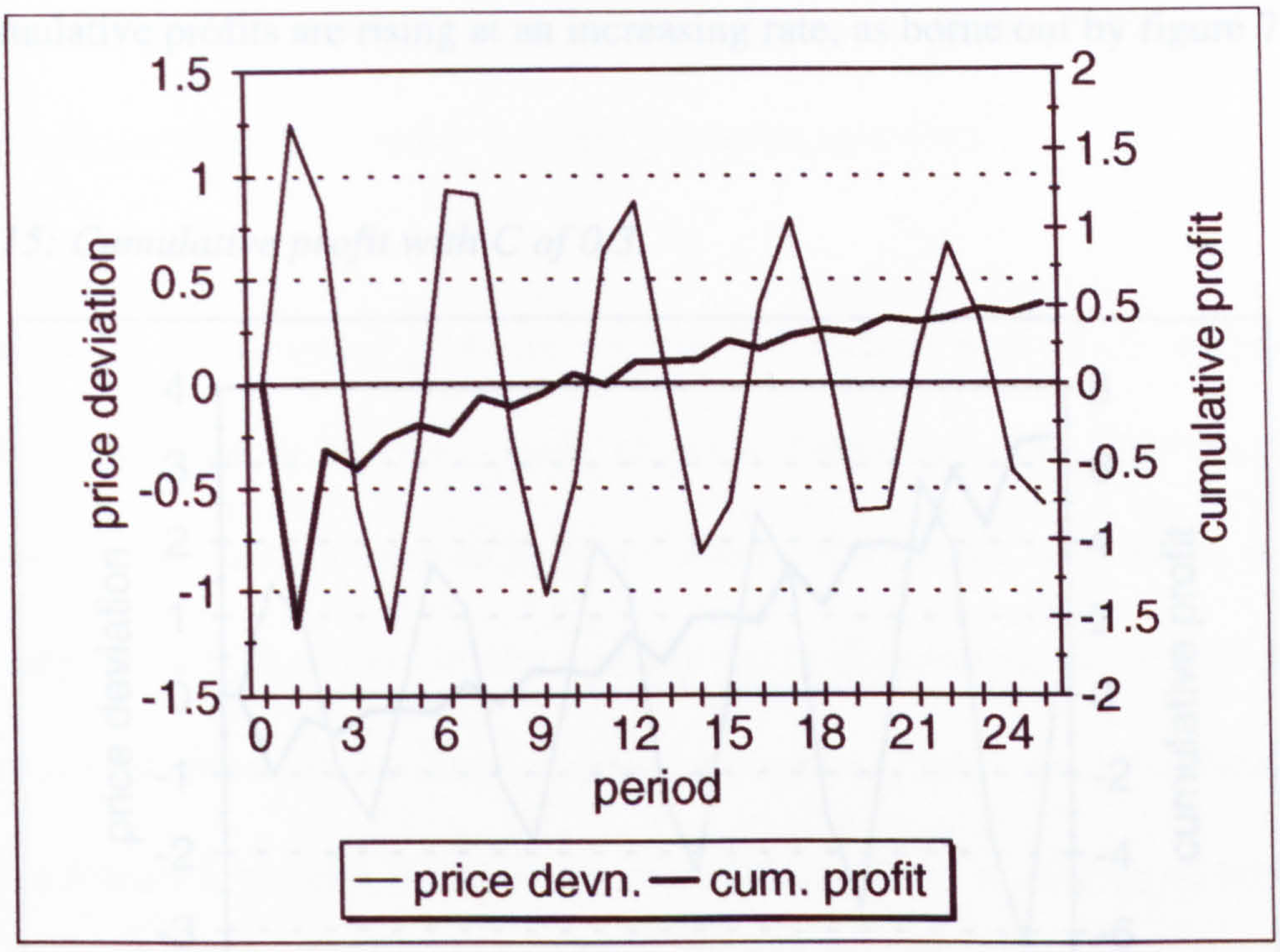
Figure 7.12 shows the situation with *C* set at 0.2, and shows that the speculative activity has reduced the rate at which the price cycle is damped. Since the green line - representing the incremental change in profits - has a mean greater than zero, the speculator earns positive profits once the system has been destabilised.

*Fig. 7.12: Price, demand and incremental profit with C of 0.2.*




In order to assess the profitability of the speculative strategy as a whole, the cumulative profit must be analysed. This is shown in figure 7.13, and reveals that the speculator recoups the early loss and moves into profit by the twelfth period.

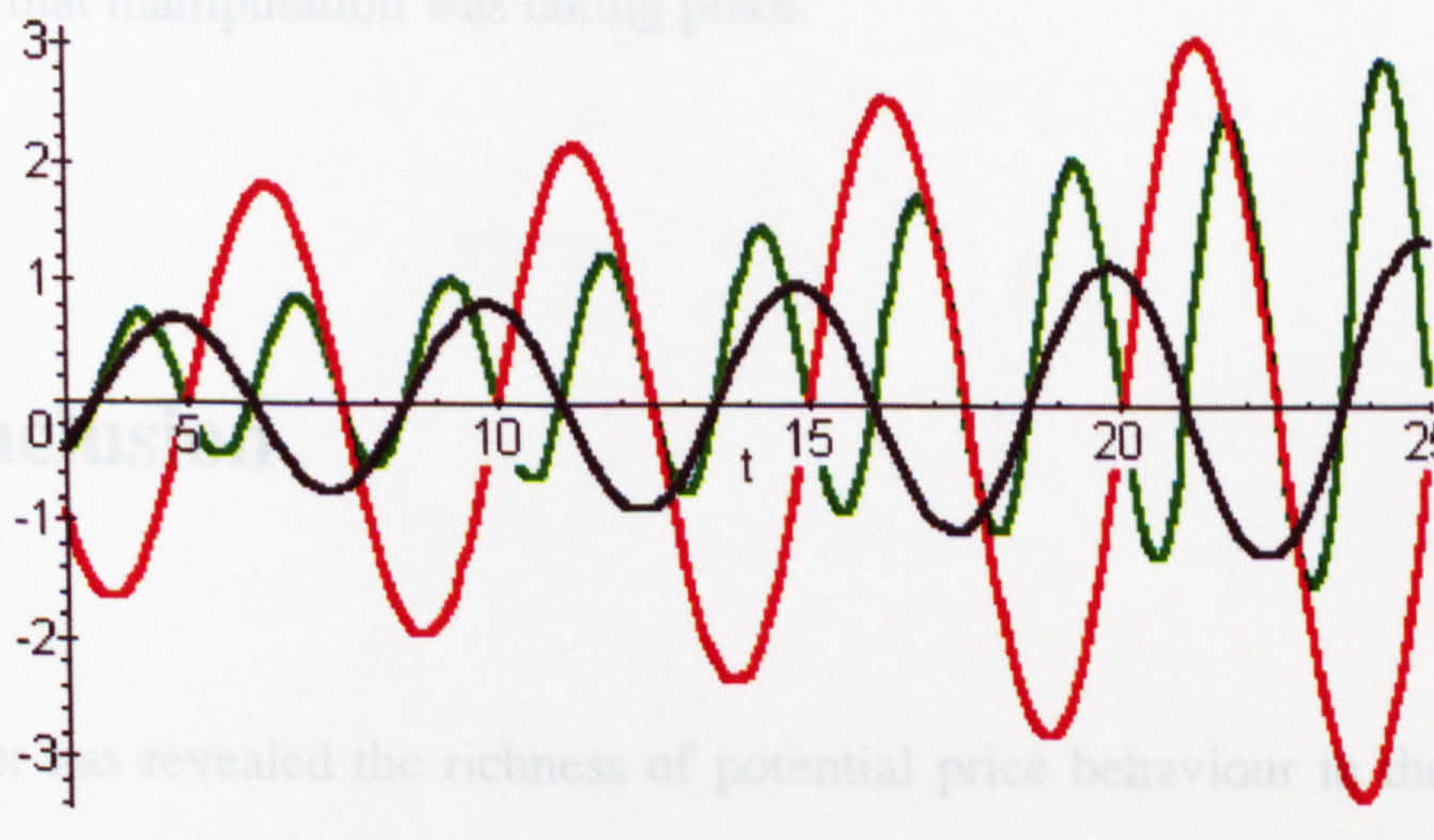
*Fig. 7.13: Cumulative profit with C of 0.2.*




### 7.6.5 Numerical example #3, with C of 0.3

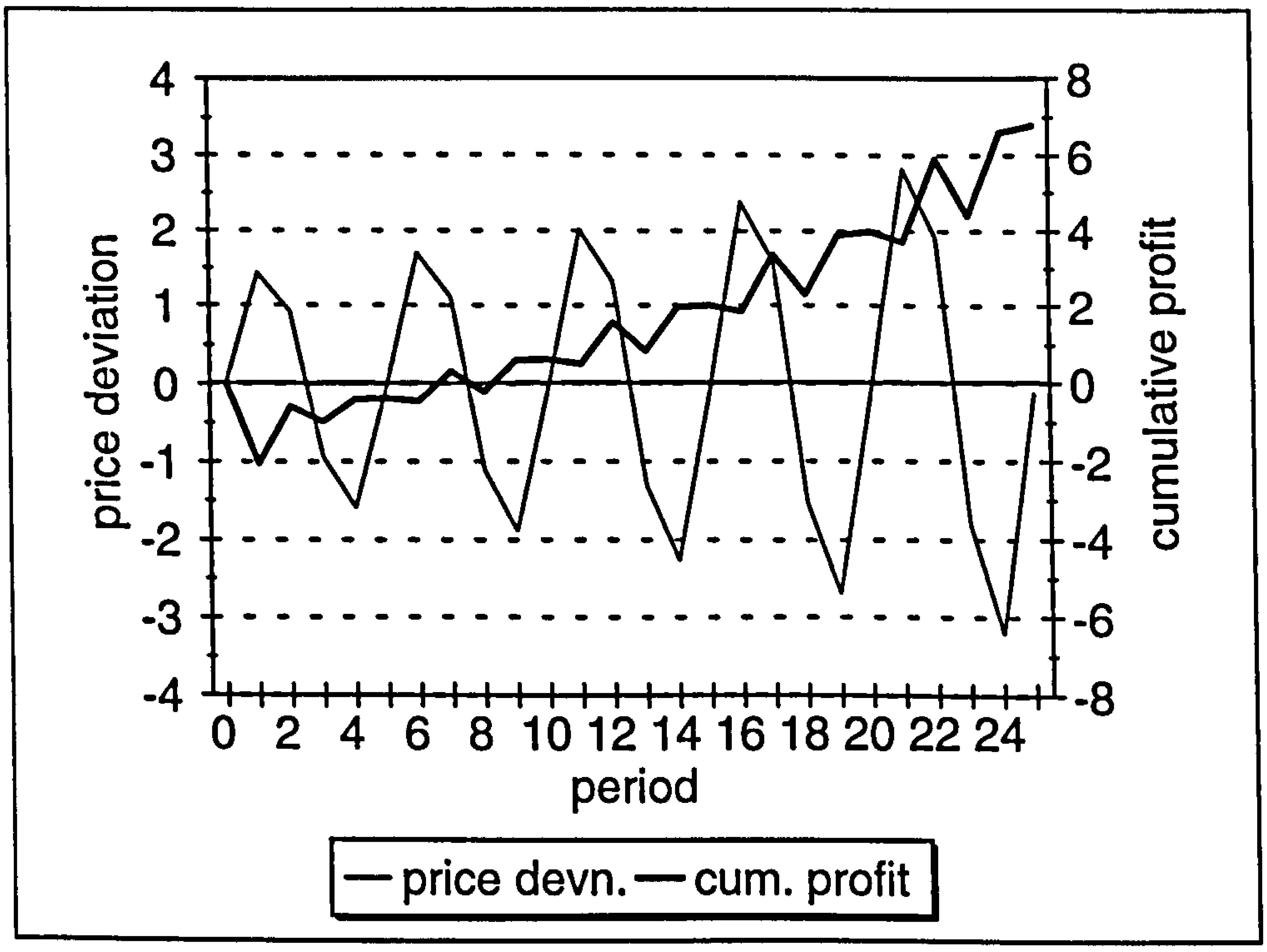
Figure 7.14 represents the situation when $C$ is 0.3. The speculative activity is now at such a level that makes the price dynamically unstable.

*Fig. 7.14: Price, demand and incremental profit with C of 0.3.*



The mean profits over each cycle are still positive, but are now increasing, ensuring that cumulative profits are rising at an increasing rate, as borne out by figure 7.15.

*Fig. 7.15: Cumulative profit with C of 0.3.*




This example highlights a limitation of the model, since it is clear that the most profitable form of speculative behaviour is dynamically unstable. We would not expect to see this type of behaviour in reality, since it would become clear to the market participants that manipulation was taking place.

## 7.7 Conclusion

This chapter has revealed the richness of potential price behaviour in the presence of positive feedback trading. It has also shown that the presence of positive feedback trading does not necessarily provide an opening for manipulators, since the potential for this to be profitable will depend on the delay with which prices feed back into demand and the strength of value-trading by passive investors.

Unfortunately, the example given in section 7.6 reveals that when manipulation is possible its most profitable form leads to price fluctuations that increase over time. However, if other factors were to be taken into consideration, such as the riskiness of the strategy or the likelihood of the strategy being discovered, this conclusion might change. A non-linear passive investor demand function might even be sufficient. In any case, this feature highlights the need for more sophisticated modelling of the effects of positive feedback trading.

Now that we understand better how a market with passive investors and positive feedback traders behaves, it is time to examine the De Long et. al. claim that competitive speculation can be destabilising.

# Appendix E: For Chapter 7

## E1 Second-order difference equations

Take a general second-order difference equation such as the following:

$$p_t = a_1 p_{t-1} + a_2 p_{t-2} \tag{E1}$$

The characteristic roots are the roots of the following expression, which is derived from the above:

$$m^2 - a_1 m - a_2 = 0 \tag{E2}$$

The characteristic roots are therefore:[1]

$$\mu_{1,2} = \frac{1}{2}\left(a_1 \pm \sqrt{a_1^2 + 4a_2}\right) \tag{E3}$$

The system will oscillate around the stationary level of zero when the characteristic roots are complex, which occurs when the discriminant is negative, which is when the following condition holds:

$$a_1^2 + 4a_2 < 0 \tag{E4}$$

When the system does not oscillate, it is asymptotically stable whenever both the characteristic roots lie within the unit circle.

[1] See, for example, Levy (1992).

# E2 Feedback traders only

Substituting in the values from our example with feedback traders only, reveals that the characteristic roots are given by:

$$\begin{aligned} \mu_{1,2} &= \frac{1}{2}\left(-\frac{(\beta-\delta)}{\delta} \pm \left(\frac{\beta+\delta}{\delta}\right)\right) \\ \Rightarrow \mu_1 &= 1, \mu_2 = -\frac{\beta}{\delta} \end{aligned} \tag{E5}$$

Since the roots are always real, we know that the price path does not oscillate. For asymptotic stability we require that both of these roots lie within the unit circle. This is not the case here, and so a perturbation from the steady state will lead to the magnitude of the price increasing indefinitely.

# E3 Feedback traders and passive investors

## E3.1 Generalities

When both feedback traders and passive investors are present, the characteristic roots are given by:

$$\begin{aligned} \mu_{1,2} &= \frac{1}{2(\alpha-\delta)}\left(\beta-\delta \pm \sqrt{(\beta-\delta)^2 - 4\beta(\alpha-\delta)}\right) \\ &= \frac{1}{2(\alpha-\delta)}\left(\beta-\delta \pm \sqrt{(\beta+\delta)^2 - 4\alpha\beta}\right) \end{aligned} \tag{E6}$$

This tells us that the price path will oscillate when the following condition is met:

$$4\alpha\beta > (\beta+\delta)^2 \tag{E7}$$

The boundary, beyond which the system oscillates can be found by solving the above as an equality. This gives:

$$\delta = -\beta \pm 2\sqrt{\alpha\beta} \tag{E8}$$

Re-inserting the inequality, and noting that we are only interested in positive values of delta, allows the condition for oscillations to be written as:

$$\delta < -\beta + 2\sqrt{\alpha\beta} \tag{E9}$$

## E3.2 Condition for stability in an oscillatory system

When the characteristic roots have an imaginary component, the condition for stability becomes:

$$\begin{aligned}
&\frac{(\beta-\delta)^2}{4(\alpha-\delta)^2}+\frac{4\beta(\alpha-\delta)-(\beta+\delta)^2}{4(\alpha-\delta)^2}\leq 1\\
&\Rightarrow \frac{4\beta(\alpha-\delta)}{4(\alpha-\delta)^2}\leq 1\\
&\Rightarrow \alpha \geq \beta+\delta
\end{aligned} \tag{E10}$$

## E3.3 Condition for stability in a non-oscillatory system

For stability we require the largest root to lie within the unit circle. When $\beta \geq \delta$, the condition becomes:

$$\begin{aligned}
&\frac{1}{2(\alpha-\delta)}\left(\beta-\delta+\sqrt{(\beta+\delta)^2-4\alpha\beta}\right)\leq 1\\
&\Rightarrow \sqrt{(\beta+\delta)^2-4\alpha\beta}\leq 2\alpha-\beta-\delta\\
&\Rightarrow (\beta+\delta)^2-4\alpha\beta \leq 4\alpha^2+(\beta+\delta)^2-4\alpha(\beta+\delta)\\
&\Rightarrow \alpha-\delta \geq 0
\end{aligned} \tag{E11}$$

This always holds. When $\beta < \delta$, the condition becomes:

$$\frac{1}{2(\alpha - \delta)}\left(\beta - \delta - \sqrt{(\beta + \delta)^2 - 4\alpha\beta}\right) \geq -1$$
$$\Rightarrow \sqrt{(\beta + \delta)^2 - 4\alpha\beta} \leq 2\alpha + \beta - 3\delta \qquad \text{(E12)}$$

When $2\alpha + \beta - 3\delta < 0$, this never holds. When $2\alpha + \beta - 3\delta \geq 0$, the condition becomes:

$$(\beta + \delta)^2 - 4\alpha\beta \leq (2\alpha + \beta - 3\delta)^2$$
$$\Rightarrow \beta \geq \delta - \frac{\alpha}{2} \qquad \text{(E13)}$$

Combining these conditions gives the result found in the main text.

### E3.4 Profitability

As we noted in the main text, following Hart (1977), we can define a new term $f(z)$:

$$f(z) = -(\alpha - \delta) + (\alpha + \beta - 2\delta)z - (2\beta - \delta)z^2 + \beta z^3 \qquad \text{(E14)}$$

For speculation that disturbs the steady state to be profitable, we require the existence of a solution to:

$$\operatorname{Re} f(z) = \operatorname{Re}\left[-(\alpha - \delta) + (\alpha + \beta - 2\delta)z - (2\beta - \delta)z^2 + \beta z^3\right] = 0 \qquad \text{(E15)}$$

that satisfies $|z| < 1$.

We can split $z$ into its real and imaginary components as follows:

$$z \equiv b + ci$$

Substituting this into the function gives:

$$
\begin{aligned}
\operatorname{Re} f(z) &= \operatorname{Re}\Big[-(\alpha-\delta)+(\alpha+\beta-2\delta)(b+ic) \\
&\qquad -(2\beta-\delta)(b+ic)^2+\beta(b+ic)^3\Big] \\
&= \operatorname{Re}\Big\{-(\alpha-\delta)+(\alpha+\beta-2\delta)b \\
&\qquad -(2\beta-\delta)\left(b^2-c^2\right)+\beta\left(b^3-3bc^2\right) \\
&\qquad +i\Big[(\alpha+\beta-2\delta)c-2(2\beta-\delta)bc+\beta\left(3b^2c-c^3\right)\Big]\Big\} \\
&= -(\alpha-\delta)+(\alpha+\beta-2\delta)b-(2\beta-\delta)\left(b^2-c^2\right) \\
&\qquad +\beta\left(b^3-3bc^2\right)
\end{aligned}
\tag{E16}
$$

The condition for the magnitude of the root to lie within the unit circle can be re-written as:

$$
\begin{aligned}
&|z|<1 \\
&\Rightarrow |b+ic|<1 \\
&\Rightarrow b^2+c^2<1 \\
&\Rightarrow c^2<1-b^2 \\
&\Rightarrow c^2=1-b^2-y \\
&\Rightarrow c^2=x-b^2
\end{aligned}
\tag{E17}
$$

where $x \equiv 1-y, \quad 0 \le x < 1 \quad \textit{and} \quad b^2 < x.$

Substituting this into the above gives the expression found in the main text (7.28).

## E3.5 Sufficient condition for profitability

A sufficient condition for profitability is that one of the roots of the full function (E14) is real and lies within the unit circle. The roots are:

$$
1, \textit{and}\ \frac{\beta-\delta \pm \sqrt{(\beta+\delta)^2-4\alpha\beta}}{2\beta}
\tag{E18}
$$

The non-unitary roots are therefore real when:

$$4\alpha\beta \le (\beta+\delta)^2 \tag{E19}$$

This is the same condition as for the system not to exhibit oscillations.

Remember that speculation will be profitable when the smallest root (in magnitude) lies within the unit circle. We can consider two cases: $\beta \ge \delta$ and $\beta < \delta$. First, it is worth noting that the magnitude of the term inside the square root is less that the magnitude of the square of the term outside. This can be shown as follows:

$$\begin{aligned} 0 &\le (\beta+\delta)^2 - 4\alpha\beta \\ &= (\beta-\delta)^2 - 4\beta(\alpha-\delta) < (\beta-\delta)^2 \end{aligned} \tag{E20}$$

When $\beta \ge \delta$, the condition for the smallest root to be within the unit circle is:

$$\begin{aligned} &\frac{\beta-\delta-\sqrt{(\beta+\delta)^2-4\alpha\beta}}{2\beta} < 1 \\ &\Rightarrow \beta+\delta+\sqrt{(\beta+\delta)^2-4\alpha\beta} > 0 \end{aligned} \tag{E21}$$

This always holds. When $\beta < \delta$, the condition for the smallest root to be within the unit circle is:

$$\begin{aligned} &\frac{\beta-\delta+\sqrt{(\beta+\delta)^2-4\alpha\beta}}{2\beta} > -1 \\ &\Rightarrow \sqrt{(\beta+\delta)^2-4\alpha\beta} > \delta-3\beta \end{aligned} \tag{E22}$$

This always holds for $\delta < 3\beta$. When $\delta \ge 3\beta$, the condition becomes:

$$\begin{aligned} &(\beta+\delta)^2 - 4\alpha\beta > (\delta-3\beta)^2 \\ &\Rightarrow 2\beta - 2\delta + \alpha < 0 \\ &\Rightarrow \delta > \beta + \frac{\alpha}{2} \end{aligned} \tag{E23}$$

Combining the above gives sufficient conditions for speculation that disturbs the steady state to be profitable:

$$\begin{aligned}
&4\alpha\beta \le (\beta+\delta)^2 \\
&\textit{AND EITHER} \quad \beta \ge \delta \\
&\qquad \textit{OR} \quad \beta < \delta \\
&\qquad\qquad \textit{AND either} \quad \delta < 3\beta \\
&\qquad\qquad\qquad \textit{or} \quad \delta \ge 3\beta \\
&\qquad\qquad\qquad\qquad \textit{and} \quad \delta > \beta + \frac{\alpha}{2}
\end{aligned} \tag{E24}$$

Plotting these conditions reveals that they are equivalent to the conditions for the system to be non-oscillatory and asymptotically unstable.

## E3.6 General conditions for profitability

We showed in the main text that the real part of the function can be written as:

$$\begin{aligned}
&\operatorname{Re} f(z) \\
&= (2\beta-\delta)x - (\alpha-\delta) + (\alpha+\beta-3\beta x-2\delta)b - 2(2\beta-\delta)b^2 + 4\beta b^3
\end{aligned} \tag{E25}$$

where $z \equiv b + ci, \quad 0 \le x < 1 \quad and \quad b^2 < x.$

This will have a root (for $b$) that lies within the unit circle when the following expression has a real root that lies within the unit circle:

$$2\beta - \alpha + (\alpha - 2\beta - 2\delta)b - 2(2\beta-\delta)b^2 + 4\beta b^3 \tag{E26}$$

This expression is the above expression under a value for $x$ of one. The roots of this are as follows:

$$1, \textit{and } \frac{-\delta \pm \sqrt{\delta^2 + 8\beta^2 - 4\alpha\beta}}{4\beta} \tag{E27}$$

A requirement is that the term inside the square-root is non-negative, since $b$ is real:

$$\delta^2 + 8\beta^2 - 4\alpha\beta \geq 0 \tag{E28}$$

This always holds when $\beta \geq \alpha/2$. When $\beta < \alpha/2$, this will hold when:

$$\begin{aligned} &\delta^2 \geq 4\alpha\beta - 8\beta^2 \\ &\Rightarrow \delta \geq 2\sqrt{\beta(\alpha - 2\beta)} \end{aligned} \tag{E29}$$

Assume that the roots are real. When $\beta \geq \alpha/2$, the condition for the smallest root to lie within the unit circle becomes:

$$\begin{aligned} &\frac{-\delta + \sqrt{\delta^2 + 8\beta^2 - 4\alpha\beta}}{4\beta} < 1 \\ &\Rightarrow \sqrt{\delta^2 + 8\beta^2 - 4\alpha\beta} < 4\beta + \delta \\ &\Rightarrow \delta^2 + 8\beta^2 - 4\alpha\beta < (4\beta + \delta)^2 \\ &\Rightarrow 2\beta + 2\delta + \alpha > 0 \end{aligned} \tag{E30}$$

This always holds. When $\beta < \alpha/2$, the condition for the smallest root to lie within the unit circle becomes:

$$\begin{aligned} &\frac{-\delta + \sqrt{\delta^2 + 8\beta^2 - 4\alpha\beta}}{4\beta} > -1 \\ &\Rightarrow \sqrt{\delta^2 + 8\beta^2 - 4\alpha\beta} > \delta - 4\beta \end{aligned} \tag{E31}$$

This always holds when $4\beta > \delta$. When $4\beta \leq \delta$, the condition becomes:

$$\begin{aligned} &\delta^2 + 8\beta^2 - 4\alpha\beta > (\delta - 4\beta)^2 \\ &\Rightarrow 2\beta - 2\delta + \alpha < 0 \\ &\Rightarrow \delta > \beta + \frac{\alpha}{2} \end{aligned} \tag{E32}$$

The over-all conditions for speculation that disturbs the steady state to be profitable are therefore:

$$
\begin{aligned}
&\textit{EITHER} \quad \beta \geq \frac{\alpha}{2} \\
&\textit{OR} \qquad \beta < \frac{\alpha}{2} \\
&\qquad\qquad \textit{AND} \quad \delta \geq 2\sqrt{\beta(\alpha - 2\beta)} \\
&\qquad\qquad \textit{AND} \quad \textit{either} \quad \delta < 4\beta \\
&\qquad\qquad\qquad\quad \textit{or} \qquad \delta > \beta + \frac{\alpha}{2}
\end{aligned}
\tag{E33}
$$

Dividing through by alpha produces the conditions given in the main text.

# Chapter 8

# Speculation surrounding exogenous shocks

## 8.1 Introduction

In the previous chapter we saw, with the help of Hart (1977), the conditions under which speculation that disturbs the steady state can be profitable in an economy with feedback traders and passive investors. We also found a speculative strategy that could be profitable, although we did not isolate the optimal speculator strategy. This previous work does not, however, tell us anything about the speculative possibilities associated with the incidence of shocks to the system. If a system suffers a change in the fundamental stock value, or receives a demand shock, it may be vulnerable to speculative activity when such activity could not profitably disturb the steady state:

and if this is the case, it would be important to see what form such speculation would take. It is the broad aim of this chapter to investigate these issues.

The analysis of the previous chapter dealt with the possibilities for monopolistic speculation. It is apparent that in such a framework competitive speculation would lead to prices remaining firmly at fundamental levels. We shall see below that the incidence of shocks may prevent competition from having such a stabilising role. In fact, competitive speculation may be more destabilising than collusive speculation. The structure we use shares many characteristics with the models found in De Long et. al. (1989, 1990a), which deal exclusively with competitive speculators. Throughout we will compare our model with these, and produce new results. In the main body of the text we will assume that the shock concerns the fundamental stock value, as in De Long et. al. (1990a), rather than a demand shock as in De Long et. al. (1989). In the appendix, however, we produce results for the more general case with both types of shock.

We begin by detailing the basic model framework, and examine the implications of this in the context of the previous chapter; in particular, we try to assess the importance of the assumption, explicitly made in De Long et. al., that the stock is liquidated. We then introduce a utility-maximising speculator to the system, derive the market prices, and look at some special cases. Competitive speculation is analysed next, with a slightly richer information structure. This then allows us to compare prices under the alternative assumptions that speculators act competitively or collude (and hence act jointly in the same way as would a monopolist). We finish by attempting to draw some conclusions from the analysis.

## 8.2 Model structure

### 8.2.1 Basic framework

As in De Long et. al., we will assume that the stock is liquidated in period three, and that a steady state has existed prior to period one, with prices and value equal to zero. When liquidated the stock pays out an amount equal to a pre-determined 'mean' value disturbed by noise ($\Phi + \theta$). The mean of the liquidation value ($\Phi$) can take three possible values ($\Phi = -\phi$, 0 or $\phi$). The noise term ($\theta$) has a mean of zero and a variance of $\sigma^2$. We will initially assume that the speculator receives information in period one that reveals the mean of the liquidation value: this corresponds to the De Long et. al. case of 'noiseless' information. The speculator may receive this information in either period one or period two. To re-cap, the assumptions made are as follows:

*Assumption 1: A steady state, with prices equal to zero, exists prior to period 1.*

*Assumption 2: The stock will be liquidated in period 3, paying an amount* $\Phi + \theta$.

*Assumption 3: The mean of the liquidation value* ($\Phi$) *is revealed to the speculators in period 1, and can take one of three values* ($\Phi = -\phi$, *0 or* $\phi$).

Before analysing the model, we will first assess the implications of adding the assumption of liquidation.

### 8.2.2 The implications of the liquidation assumption

The assumption of third-period stock liquidation is clearly an unrealistic one. It is, however, necessary for the isolation of the equilibria once rational speculators are added to the system. It is important to assess how much the results are compromised by such an assumption; either by the restrictions it effectively imposes, or by the freedom given to speculators not to have to liquidate their holdings at market-determined prices.

We can replace the assumption of liquidation with the assumption that competitive speculators peg the price at the revealed 'fundamental value' in period three and beyond. This means that the speculators must offset the trades of the feedback traders, since the passive investors will purchase nothing, and so the speculator demand follow the following sequence:

$$\begin{aligned} D_3^r &= \beta(p_1 - p_2) + \delta(p_2 - \Phi - \theta) \\ D_4^r &= \beta(p_2 - \Phi - \theta) \\ D_5^r &= 0 \end{aligned} \tag{8.1}$$

From period five onwards the holdings of each agent type will be zero, and so the speculators will have reversed all their speculative trades. Since the price has been kept constant at the fundamental value, this has been achieved costlessly.

The liquidation assumption is therefore equivalent to an assumption that the stock is not liquidated, but that the market price equals the new stock value once this has been revealed. For simplicity, it would also be necessary to assume that it is known (or at least believed) that the new stock value will prevail indefinitely. It seems reasonable to assume that arbitrage, possibly from other speculators or the passive investors, ensures that this occurs: indeed, this is a necessary assumption when speculators are competitive. This assumption therefore marks a clear divergence with the previous chapter, since there we were working under the assumption that, although the fundamental value was unchanging, the price was determined by the monopolistic speculator, and need not necessarily reflect it.

Stock liquidation can therefore be thought of as a simplifying assumption that enhances model transparency, and that is relatively benign in a world of competitive speculators, since in this environment prices at all times other than when a change in this fundamental value is imminent would be kept at fundamental values by arbitrage even in the absence of liquidation. When there is a monopolistic speculator the assumption of stock liquidation is justifiable, due to improved modelling tractability, but also less benign, since an equivalent alternative assumption is more difficult to construct than for the competitive case.

The immediate consequence of the liquidation assumption is that the scope for speculation is limited to the periods immediately preceding a change in the fundamental value. For non-competitive speculators, this limits the potential length of speculative strategies. Another way of looking at the liquidation value assumption with a monopolistic speculator is therefore as a restriction on the range of speculative strategies that can be followed. It must therefore reduce the scope for profitable speculation relative to a situation where such a liquidation assumption is not made.

### 8.2.3 Feedback traders and passive investors

The stock demand of the feedback traders and passive investors at dates one and two are determined in the same way as in the previous chapter, and are therefore as follows:

$$\begin{aligned} D_1^f &= \delta(p_1 - p_0) = \delta p_1 \\ D_2^f &= \beta(p_1 - p_0) + \delta(p_2 - p_1) \\ &= (\beta - \delta)(p_1 - p_0) + \delta(p_2 - p_0) \\ &= (\beta - \delta)p_1 + \delta p_2 \end{aligned} \tag{8.2}$$

$$\begin{aligned} D_1^p &= \alpha(V_1 - p_1) \\ D_2^p &= \alpha(V_2 - p_2) = \alpha(\Phi - p_2) \end{aligned} \tag{8.3}$$

where $V_1$ represents the value estimate of the passive investors in period one, and $\Phi$ represents the liquidation value, or alternatively the new 'fundamental' stock value. Note that the passive investors are not necessarily 'rational' in the sense used in the rational expectations literature - although their pattern of behaviour may turn out to be rational in certain circumstances if modelled more explicitly - and so they are not insulated from 'exploitation' by the speculators.

De Long et. al. assume that information about the liquidation value becomes available in period one, but that the passive investors discover this only in period two - hence $V_1$ is zero - and so they are 'uninformed' in period one. We will leave open the possibility that $V_1$ may take other values: in particular, that the passive investors may be 'informed' in this period, in which case they will not suffer at the hands of the speculators, since they will always purchase stock when the price is below its true value, and sell when it is above. Making the assumption that the passive investors do not receive an information signal in period one enshrines in this model an informational advantage for the rational speculators. De Long et. al. do not see this as a problem, since their paper was motivated by the question of whether the addition of better-informed and rational agents could make market prices more unstable. But this does not mean that studying a situation in which the passive investors are not informationally disadvantaged in this way would not be of interest, since this would allow us to see which aspects of the price behaviour are due to the information advantage, and which are due to the lack of rationality on the part of the passive investors.

### 8.2.4 Introducing speculators

Since it is known that an increase in the risk-bearing capacity will exert a stabilising influence on price, we follow De Long et. al. in introducing speculators into the market in a way that keeps this constant. Hence the effects of an increase in the measure of informed rational speculation are only analysed for cases where the measure of passive investors is correspondingly reduced: introducing speculators of measure $\mu$ into the market entails a reduction of the measure of passive investors to $(1 - \mu)$.

The speculators are assumed to have a mean-variance utility function, the sole argument being final-period wealth. In period two the variance of the distribution of the liquidation value determines the aggressiveness with which they push the price towards fundamentals. In period one the quality of the information signal is also important. We will deal in turn with the alternative assumptions that the information signal in period one is 'noiseless' or 'noisy.'

### 8.2.5 Informational assumptions

De Long et. al. make two alternative assumptions concerning information. The first of these, whereby in period one the speculators receive information revealing the mean of the liquidation value, is termed 'noiseless' information. The second assumption is that of 'noisy' information.

For the case of noisy information, De Long et. al. (1990) assume that in period one the speculators receive a signal concerning the mean liquidation value ($\varepsilon_\phi$) which can take two possible values, symmetric about zero and both equally likely ($\phi$ or $-\phi$); and that the outcome will be either the value of the signal or zero, with equal probability. The informational set-up, for both informational assumptions, is represented in the following table:

*Table 8.1: Information assumptions.*

| Value signal ($\varepsilon_\phi$). | Liquidation value mean ($\Phi$). Noiseless info. | Noisy info. |
|---|---|---|
| $\phi$ | $\phi$ | 0 or $\phi$; 1/2 |
| 0 | 0 | n/a |
| $-\phi$ | $-\phi$ | 0 or $-\phi$; 1/2 |

## 8.2.6 Prices in a world without speculators

The prices in the absence of speculators, following receipt by the passive investors of information relating to the new fundamental stock value, are the same as those given in the previous chapter. The difference now is that in period three the price becomes equal to the new fundamental value, rather than continuing its cyclical behaviour. The prices at dates one and two are therefore given by the following:

$$\begin{aligned} p_1 &= \frac{\alpha V_1}{\alpha - \delta} \\ p_2 &= \frac{\alpha(\beta - \delta)V_1}{(\alpha - \delta)^2} + \frac{\alpha \Phi}{\alpha - \delta} \end{aligned} \tag{8.4}$$

In Appendix F (section F1) we give the prices for the two cases corresponding to when the passive investors are uninformed ($V_1 = 0$) or informed ($V_1 = \Phi$) in period one.

The impact of speculators on market prices can be assessed by comparing the prices when they are present to those given above.

## 8.2.7 Defining what we mean by a 'destabilising' effect on prices

Since we are interested in looking at the destabilising effect of competitive speculation, we must make clear what we mean by this. We follow De Long et. al. in using the following definition:

> *Definition: the presence of speculators is 'strongly destabilising' when it leads to the price in both periods one and two being further from fundamentals than it would have been in the absence of speculators.*

An alternative to this would be to look at the total fluctuation in price: this approach may produce results that are more reliant on the outcome of the liquidation value; but this could be overcome by taking the expected values. The total price fluctuation could be defined as the sum of the magnitudes of the price changes for the three periods. We do not consider these alternatives.

### 8.2.8 Fundamental stock values

We are now in a position to be able to determine the fundamental stock values. The following table shows the fundamental stock value in periods one and two (1,2) for the alternative assumptions of information signals that are noiseless or noisy. It also gives the estimates of this fundamental value made by the passive investors ($V_1$, $V_2$) under the alternative assumptions that in period one they are informed (I) or uninformed (U) in period one.

*Table 8.2: Fundamental stock values.*

| | Noiseless info. | Noisy info. |
|---|---|---|
| Fund. Val. | $\Phi$, $\Phi$ | $\varepsilon/2$, $\Phi$ |
| *V(U)* | 0, $\Phi$ | 0, $\Phi$ |
| *V(I)* | $\Phi$, $\Phi$ | $\varepsilon/2$, $\Phi$ |

We have defined the 'fundamental value' to be the value that would reflect all the information that could theoretically be known about the liquidation value in that period. Until liquidation the size of the noise term ($\theta$) is undetermined.

# 8.3 Monopolistic speculation

## 8.3.1 Introducing monopolistic speculators

We will now introduce a (monopolistic) speculator into the system. Such a situation was not covered by De Long et. al. (1989, 1990). For the purposes of analytical tractability, the model structure, barring the monopolistic speculator assumption, will broadly follow that of De Long et. al. (1990), with information signals being received in period one, no supply shock in period two, and no fast-acting feedback trading (i.e. delta is zero); although we allow for the possibility that the passive investors may be as informed as the speculator in period one, unlike De Long et. al., who assume that the speculators are more informed than the passive investors in this period.

## 8.3.2 Equilibrium prices

We show in the appendix (F2.1) that the price in period two for this formulation will equal the following:

$$p_2 = \frac{1}{\alpha_m(2+\lambda)}\left\{p_1\left(\alpha_m + \beta(1+\lambda)\right) - V_1\alpha_m\right\} + \Phi \tag{8.5}$$

where: $\alpha_m \equiv (1-\mu)\alpha$ and $\lambda \equiv 2\alpha_m \gamma_m \sigma^2$ .

At this point we will assume that the monopolistic speculator receives a *noiseless information signal* in period one, revealing the mean of the liquidation value ($\Phi$). The equilibrium prices for a system with a monopolistic speculator and noisy information is given in the appendix (F2.1, F2.3), but they are sufficiently complex to make a close study of their properties unrewarding.

Under noiseless information, since the monopolist knows in period one the information he will have in period two, he knows the price that will prevail in period two as a function of his own period one demand. This makes his task of choosing this period one demand relatively straightforward. We show in the appendix (F2.2) that the equilibrium price in period one is:

$$p_1 = \frac{\Phi\alpha_m^{\,2}(2+\lambda)+V_1(1+\lambda)\alpha_m(\alpha_m-\beta)}{(\alpha_m-\beta)\{\alpha_m(3+2\lambda)+\beta\}} \qquad \textit{for } \alpha_m > \beta \qquad (8.6)$$

This shows that the price bears a positive relationship to both the liquidation value signal and the value estimate of the passive investors.

When the liquidation value (and consequently the period one information signal of the passive investors) is zero, the price in each period will be zero. This gives the following result:

*Result 8.1: Monopolistic speculation is not profitable in the absence of a shock.*

This lack of speculative potential must be due to the assumption of stock liquidation, with its associated truncation of the time-horizon.

Since speculators always enter the market when a shock occurs, we have another result:

*Result 8.2: Unlike speculation that disturbs the steady state, speculation surrounding a shock is always profitable.*

As we mentioned above, we know that the unfailing profitability of speculation in the presence of shocks is not due to the assumption of liquidation, because the liquidation assumption is equivalent to a five-period speculative strategy that involves choosing demand to maximise profits in periods one and two, given that prices will be set equal to the fundamental value from the third period onwards, and so can be thought of as a restriction on the range of strategies available to speculators. This indicates that the driving force behind the profitability is the existence of the value shock itself: the presence of shocks that disturb the steady state is sufficient for speculation to be profitable.

### 8.3.3 No feedback trading

When there is no positive feedback trading, the equilibrium prices are as follows:

$$
\begin{aligned}
p_2 &= \frac{p_1 - V_1}{2+\lambda} + \Phi \\
p_1 &= \frac{\Phi(2+\lambda) + V_1(1+\lambda)}{3+2\lambda}
\end{aligned}
\tag{8.7}
$$

These show that, as we would expect, in the absence of feedback traders it will be profitable for the speculator to move prices away from fundamental values only when he has an information advantage over the passive investors. The expected profit of the speculator (and the corresponding loss to the passive investors as a group) when the passive investors are uninformed in period one is as follows:

$$E\left(R^r\right) = -E\left(R^p\right) = \frac{\Phi^2 \alpha_m \left[(2+\lambda)^2 - 1\right]}{(3+2\lambda)^2} \tag{8.8}$$

This loss is not foreseen by the passive investors, and is proportional to the measure in the market of passive investors and the square of the liquidation value signal, which represents the degree of uninformedness of the passive investors.

# 8.4 Competitive speculation

## 8.4.1 Introducing competitive speculators

The previous section has shown that a shock to the system will allow monopolistic speculation to become profitable, when it is not necessarily profitable for speculators to disturb the steady state themselves. So far, however, we have said nothing about equilibrium in the presence of competitive speculators. As we have seen previously, the addition of competitive speculators to a system usually tends to push prices towards fundamental levels, as would be the case in the system without shocks that we looked at previously. In a system with shocks, however, things may be different, as revealed by the De Long et. al. (1989, 1990a) models. The De Long et. al. models are designed to show that, *even when speculators behave competitively* they can destabilise prices.

In this section we will develop the model with competitive speculators, following De Long et. al. (1989, 1990a). This involves little more than adding the assumption of competitive speculation to the above model. In the subsequent section we compare these competitive results with those we obtained under monopolistic speculation. We show that increasing the degree of competition between speculators may actually destabilise prices. This is a new result.

The De Long et. al. 1989 and 1990a models are extremely similar to each other in form, but differ in their focus: in the initial working paper, the speculators are given a signal about a future demand shock emanating from the feedback traders (or an additional group of noise traders), but have no special information concerning the liquidation value; while in the later Journal of Finance paper, the signal relates to the liquidation value, and there is no demand shock. So if we are interested in studying the effects of a group of investors anticipating the future demand from other agents, the appropriate model to study would be the first one; while if we are interested in studying the effects of the appearance in the market of new information, we should study the second one. The 1990a model also makes the simplifying assumption that the feedback trading depends solely on the price change in the previous period; or in other words that delta is zero.

In order to simplify the exposition, the main body of the text will analyse the effect of a shift (or a potential shift) in the underlying stock value, as in De Long et. al. (1990a), rather than a demand shock, as in De Long et. al. (1989). For completeness, the results for the general model form, in which there is a change in value and a demand shock, along with the special case of a demand shock alone, are given in Appendix G. The feedback trading will be of the form used in the previous chapter, determined by both current and lagged price changes, as in De Long et. al. (1989), rather than just the lagged price change, as in De Long et. al. (1990a) and the previous section of this thesis (8.3).

### 8.4.2 Noiseless information

When the information signal is noiseless, the rational speculators are informed, in period one, of the expected value of the liquidation value ($\Phi$), and so the information available to them is the same in both periods one and two. These investors therefore know in period one the price that will prevail in period two conditional on the price in period one; and since they know the period one price when making their decisions in period one, they therefore know for certain the price that will prevail in period two. From the point of view of the rational speculators, then, holding stock between periods one and two is riskless; and so, when they are present in the market, arbitrage will ensure that the following result holds:

*Result 8.3: As recognised by De Long et. al., competitive speculators under noiseless information ensure that the prices in periods one and two will be identical.*

We show in the appendix (F3.1.1) that the stock demand of the rational investors in period two is given by the following:

$$D_2^{rc} = \frac{\Phi - p_2}{2\gamma\sigma^2} \tag{8.9}$$

The rational agent demand (per agent) in period two is therefore simply a constant multiplied by the excess of the expectation of the liquidation value over the period two price. We will follow the De Long et. al. convention of setting this constant - $(1/(2\gamma\sigma^2))$ - equal to the demand coefficient for the passive investors ($\alpha$), which ensures that the period two demand for agents from these two groups will always be the same:

$$D_2^{rc} = D_2^{p} = \alpha(\Phi - p_2) \tag{8.10}$$

Hence the period two behaviour of the passive investors is (assumed to be) the same as that of the rational investors, so it is only the period one behaviour that sets them apart. In the appendix (F3.1.1) we show that the market prices are given by the following:

$$p_1 = p_2 = \frac{\alpha\Phi}{\alpha - \beta} \qquad \textit{for } \mu > 0 \tag{8.11}$$

Since these prices will be zero when the liquidation value remains at zero, we have the following result:

*Result 8.4: Competitive speculation under noiseless information is not profitable in the absence of a shock.*

Other results are also apparent:

*Result 8.5: As recognised by De Long et. al., under noiseless information the market prices are not affected by the measure of competitive speculators in the market ($\mu$), once this rises above zero.*

*Result 8.6: As delayed feedback develops under noiseless information, the price rises from its fundamental level, becoming unbounded as beta approaches alpha.*

We show in the appendix (F3.1.2) that when the passive investors are *informed* in period one, a sufficient condition for the presence of speculators to be strongly destabilising under our definition is that beta exceeds delta. We also show in the appendix (F3.1.3) that when the passive investors are *uninformed* in period one, the presence of speculators is strongly destabilising when beta exceeds both delta and half the value of alpha. Combining these gives the following result:

*Result 8.7: Under noiseless information, competitive speculation will be strongly destabilising when the following conditions hold:*

$$\begin{aligned} &\beta > \delta && \textit{for} \;\; V_1 = \Phi \\ &\beta > \delta \quad \& \quad \beta > \frac{\alpha}{2} && \textit{for} \;\; V_1 = 0 \end{aligned} \tag{8.12}$$

Since when the passive investors are uninformed the magnitude of the price in period two will be at least as great as in period one, and will exceed the magnitude of the liquidation value, we can legitimately say for this case that if beta exceeds delta the price fluctuates more when rational speculators are present. Hence we have another result:

*Result 8.8: Under noiseless information, the presence of competitive speculators will cause prices to fluctuate more when the amount of delayed feedback (beta) exceeds* *the amount of immediate feedback (delta), regardless of the level of informedness of the passive investors in period one.*

When the feedback trading is all immediate ($\beta = 0$) the presence of speculators is stabilising; and, in fact, moves prices to their fundamental levels. When the feedback trading is all delayed ($\delta = 0$) the presence of speculators is always strongly destabilising when the passive investors are informed; and is strongly destabilising under uninformed passive investors when the value of beta exceeds half the value of alpha. This shows that the form of feedback trading is crucially important when considering questions of destabilisation: a higher degree of immediate feedback trading ($\delta$) makes speculation more stabilising; while a higher degree of delayed feedback trading ($\beta$) makes speculation more destabilising.

The main drawback with this assumption of noiseless information is that, although it provides an extremely tractable model, the arbitrage it induces may mask interesting results we may be able to glean from a weaker assumption. To this effect we will now look at the situation in which the information in period one is less complete than in period two.

### 8.4.3 Noisy information

Assume that the signal received by the rational speculators in period one contains some noise. In period one the price that will prevail in period two is not known, since the rational speculators have less information than they will use to choose their demand in period two. This ensures that there is risk associated with holding stock between periods one and two, which is not the case when the information is noiseless. In period one the rational speculators, given their information signal ($\varepsilon_\phi$), know that the expectation of the liquidation value in period two will be one of two values ($\varepsilon_\phi$ or 0), with each outcome equally likely, although they will not know which of these outcomes will occur until period two; and given their optimal investment rule, they thus know in period one the two possible prices that will prevail in period two, conditional on the period one price. For each of these prices, the certain-equivalent value of their holdings conditional on the price in period one and their demand in the same period can be calculated by the rational speculators; their period one problem becomes one of choosing their demand to maximise the certainty-equivalent of their period two wealth given these two possible levels.

As we show in the appendix (F3.2), the equilibrium market prices are as follows:

$$p_{2a} = \frac{(\beta-\delta)p_1 + \alpha\varepsilon_\phi}{\alpha-\delta}$$
$$p_{2b} = \frac{(\beta-\delta)p_1}{\alpha-\delta} \tag{8.13}$$

where:

$$p_1 = \frac{\alpha\varepsilon_\phi}{2}\frac{2V_1\varepsilon_\phi\alpha(\alpha-\delta)(1-\mu)+\mu\left(4\sigma^2(\alpha-\delta)^2-\delta^2\varepsilon_\phi^{\ 2}\right)}{\alpha\varepsilon_\phi^{\ 2}(\alpha-\delta)^2(1-\mu)+\mu(\alpha-(\beta-\delta))\left(4\sigma^2(\alpha-\delta)^2-\alpha\delta\varepsilon_\phi^{\ 2}\right)}$$

From this it is straightforward to derive the period one price for the cases where the fundamental value estimate of the passive investors in period one ($V_1$) is either zero or the expected liquidation value given the information signal ($\varepsilon_\phi/2$).

The price equations clearly show that when the information received by the informed speculators in period one is not noiseless, the prices in periods one and two are influenced by the measure of these investors in the market ($\mu$). This two-feedback-type case does not readily provide clear-cut results. We can, however, say the following:

*Result 8.9: Under noisy information, and with competitive speculators and informed passive investors, prices will be at their fundamental levels in the absence of feedback trading.*

*Delayed feedback.*

When there is a lag between price changes and feedback trading ($\delta = 0$) the market prices are as follows:

$$p_1 = \frac{\alpha\varepsilon_\phi\left(V_1\varepsilon_\phi(1-\mu)+2\sigma^2\mu\right)}{\alpha\varepsilon_\phi^{\,2}(1-\mu)+4\sigma^2\mu(\alpha-\beta)}$$

$$p_{2a} = \frac{\beta}{\alpha}p_1 + \varepsilon_\phi \qquad (8.14)$$

$$p_{2b} = \frac{\beta}{\alpha}p_1$$

The following result is immediately apparent:

*Result 8.10: Under noisy information, and with competitive speculators and delayed feedback, the magnitudes of the prices in both periods are positively related to the value estimate of the passive investors in period one.*

This gives another result:

*Result 8.11: Under noisy information, and with competitive speculators and delayed feedback, increases in the informedness of the passive investors are strongly destabilising.*

In the appendix (F3.3.3) we prove the following:

*Result 8.12: Under noisy information, and with delayed feedback and informed passive investors, the presence of competitive speculators is always strongly destabilising.*

In the appendix (F3.3.2) we also show that when the passive investors are not informed, the condition for strong destabilisation is as given in De Long et. al. (1990a, expression 21). In this scenario, increasing the measure of speculators in the market ($\mu$) initially moves the period one price towards the fundamental value from below; and always moves the period two price away from fundamentals. Increasing the measure of speculators in the market is therefore not always strongly destabilising under our definition of the term when the passive investors are uninformed, although we can say the following:

*Result 8.13: Under noisy information, and with delayed feedback and uninformed passive investors, the presence of competitive speculators leads to a greater degree of price fluctuation over the period as a whole, causing prices to fluctuate more.*

As De Long et. al. acknowledge, for the case of uninformed passive investors, provided that beta exceeds half the value of alpha there exists a value for the measure of speculators in the market above which additional speculators will move the price further away from fundamentals than it would have been in the absence of speculators.

Combining results 8.12 and 8.13 gives a further result:

*Result 8.14: Under noisy information, and with competitive speculators and delayed feedback, the presence of competitive speculators leads to a greater degree of price fluctuation over the period as a whole (regardless of the level of informedness of the passive investors).*

The price in each period will have the same sign as the liquidation-value signal. We show in the appendix (F3.3.5) that this implies the following result:

*Result 8.15: Under noisy information, and with delayed feedback, an increase in the measure of competitive speculators in the market ($\mu$) increases the magnitudes of the prices in both periods one and two.*

In the appendix (F3.3.6) we also prove the following:

*Result 8.16: Under noisy information, and with competitive speculators and delayed feedback, the prices in both periods one and two are positively related to the degree* *of feedback trading (β). Prices will become unbounded at a finite level of positive feedback activity.*

*Immediate feedback.*

When the feedback trading reacts immediately to price changes ($\beta = 0$) there is an inverse relationship between the prices in periods one and two, the size of which depends on the strength of the positive feedback trading. Further analysis of this case is rather less straightforward than for the case of delayed feedback trading, and does not yield results sufficiently tractable for our purposes. We will therefore resort to numerical examples. The examples below are sufficient to validate the following proposition:

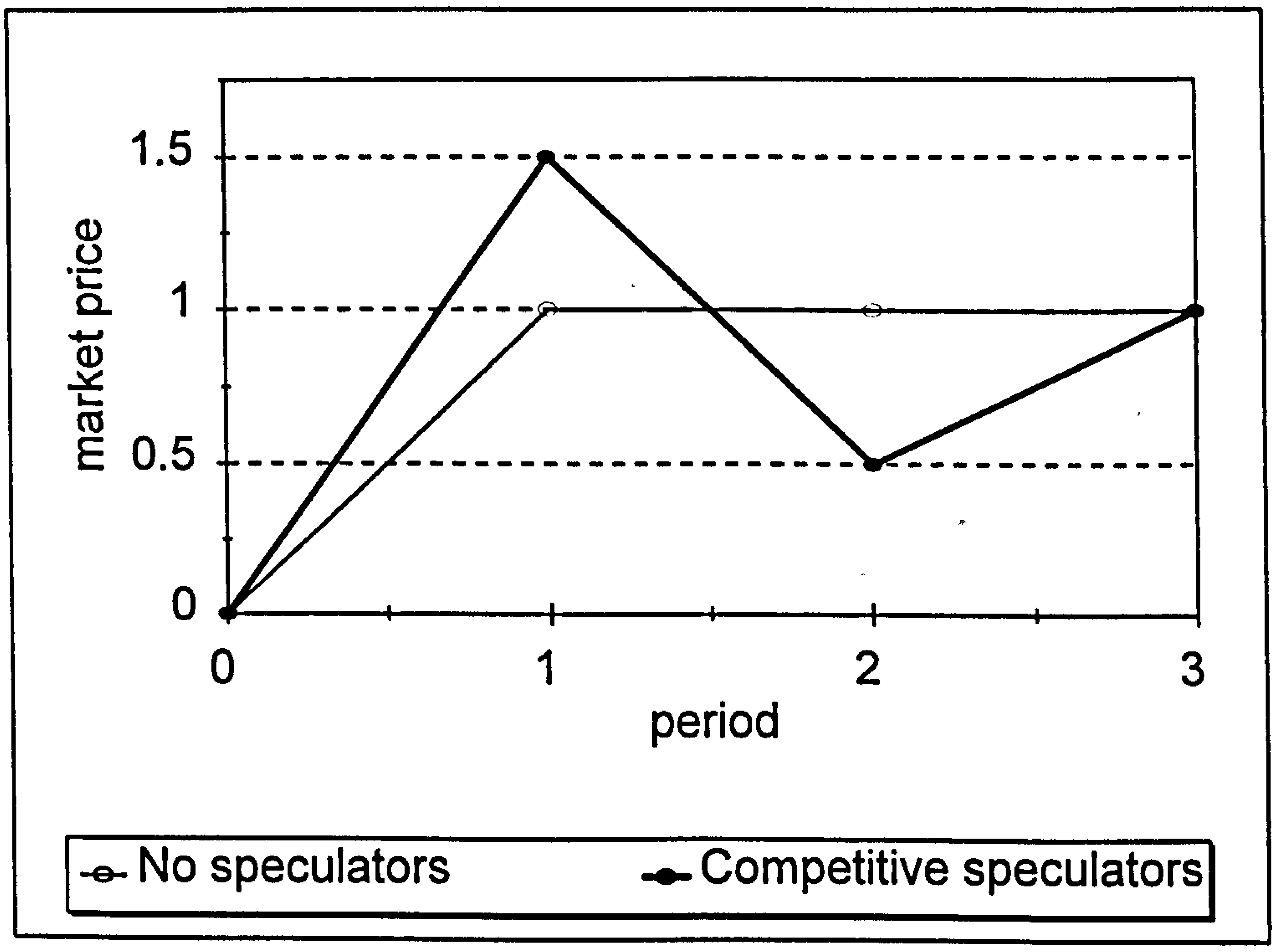
*Proposition 8.1: Under noisy information, and with immediate feedback trading, the introduction of competitive speculators can destabilise prices whether or not the passive investors are informed in period one.*

In the first example the passive investors are assumed to be informed in period one, and the model parameters used are as given in table 8.3. The market prices with and without competitive speculators are shown in figure 8.1.

*Table 8.3: Parameter values for example #1.*

| $\alpha$ | $\delta$ | $\mu$ | $\varepsilon$ | $\Phi$ | $\sigma^2$ |
|---|---|---|---|---|---|
| 1 | 0.5 | 0.5 | 1 | 1 | 0.5 |

*Fig. 8.1: Price with informed P.I.s, noisy info. & no delayed feedback.*

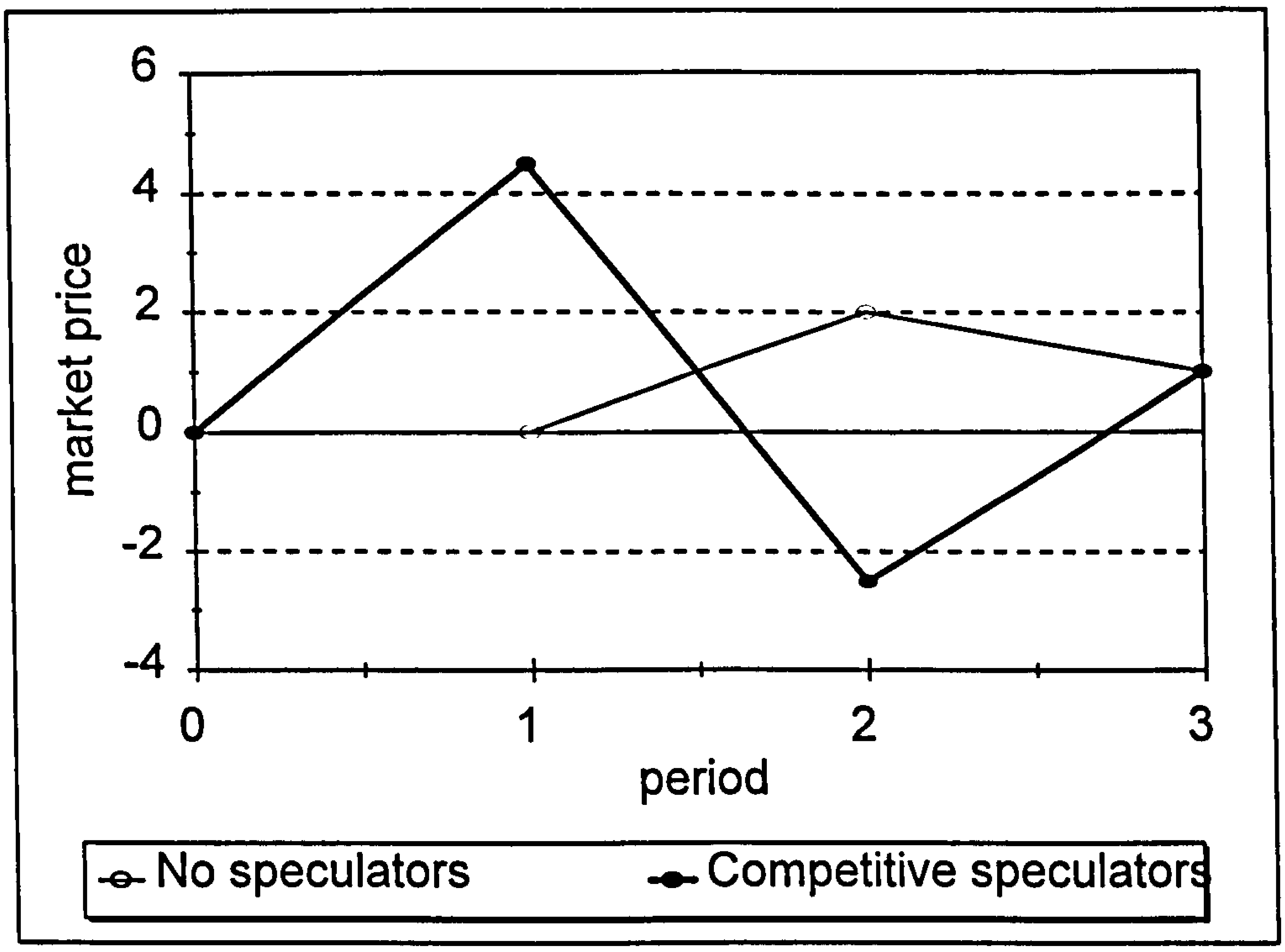



The second example demonstrates that the impact of competitive speculators when the passive investors are uninformed in period one - as is assumed in De Long et. al. (1990) - can be destabilising. The parameters used are given in table 8.4, and the resulting prices are shown in figure 8.2.

*Table 8.4: Parameter values for example #2.*

| $\alpha$ | $\delta$ | $\mu$ | $\varepsilon$ | $\Phi$ | $\sigma^2$ |
|---|---|---|---|---|---|
| 1 | 0.5 | 0.9 | 1 | 1 | 0.5 |

*Fig. 8.2: Price with uninformed P.I.s, noisy info. & no delayed feedback.*




These show that even if positive feedback traders respond only to current price changes, rational informed speculation can destabilise prices. This result contrasts with the conclusion drawn in footnote six of De Long et. al. (1990a) which reads:

> "Our working paper version.... shows that it is the responsiveness of positive feedback traders to past price changes - the coefficient β - and not the responsiveness of demand to current price changes that leads to the possibility of destabilizing rational speculation." (p. 385)

We have shown that, although the above conclusion is valid for the model contained in De Long et. al. (1989), it is not transferable to the later work, and so the simplification of allowing delta to be zero is not as benign as it is portrayed.

# 8.5 Price comparison: competition versus monopoly

We have already seen that in this model the introduction of competitive speculators may move prices away from fundamentals; but this does not tell us whether or not prices will be closer to fundamentals when the speculators are competitive than they would be if they had market power. In this section we will compare the situations with monopolistic and competitive speculators.

In order to be able to compare the results, we must make the effective 'size' in the market of the three groups the same for each case. This only poses a problem for the rational speculators. We can think of the monopolist as the figurehead of a cartel consisting of colluding speculators. If the cartel was unable to affect market prices, the individual speculators would wish to demand the same amounts each as they would acting individually. To achieve the desired specification, we need only to set the period two demand of a monopolistic speculator that ignores the effect his demand has on prices equal to the aggregate demand from speculators acting competitively, and solve for the coefficient of risk-aversion for the monopolist. This produces the following:

$$\frac{\Phi - p_2}{2\gamma_m \sigma^2} = \mu \frac{\Phi - p_2}{2\gamma_c \sigma^2}$$
$$\Rightarrow \gamma_m = \frac{\gamma_c}{\mu} \tag{8.15}$$

The remainder of this section consists of a numerical example that is sufficient to validate the following proposition:

*Proposition 8.2: Competitive speculation can be more destabilising than monopolistic speculation, for the same degree of market power, regardless of whether or not the passive investors are informed in period one.*

The parameter values used in the example are given in table 8.5.

*Table 8.5: Parameter values for example 3.*

| $\alpha$ | $\beta$ | $\mu$ | $\gamma_c$ | $\sigma^2$ | $\Phi$ |
|---|---|---|---|---|---|
| 1 | 0.5 | 0.1 | 1 | 0.5 | 1 |

Figure 8.3 shows the resulting market prices under the different assumptions of speculative behaviour when information is noiseless and the passive investors are uninformed in period one. The relative instability of prices in the presence of competitive speculators is clearly illustrated.

*Fig. 8.3: Price comparison, competition vs. monopoly.*

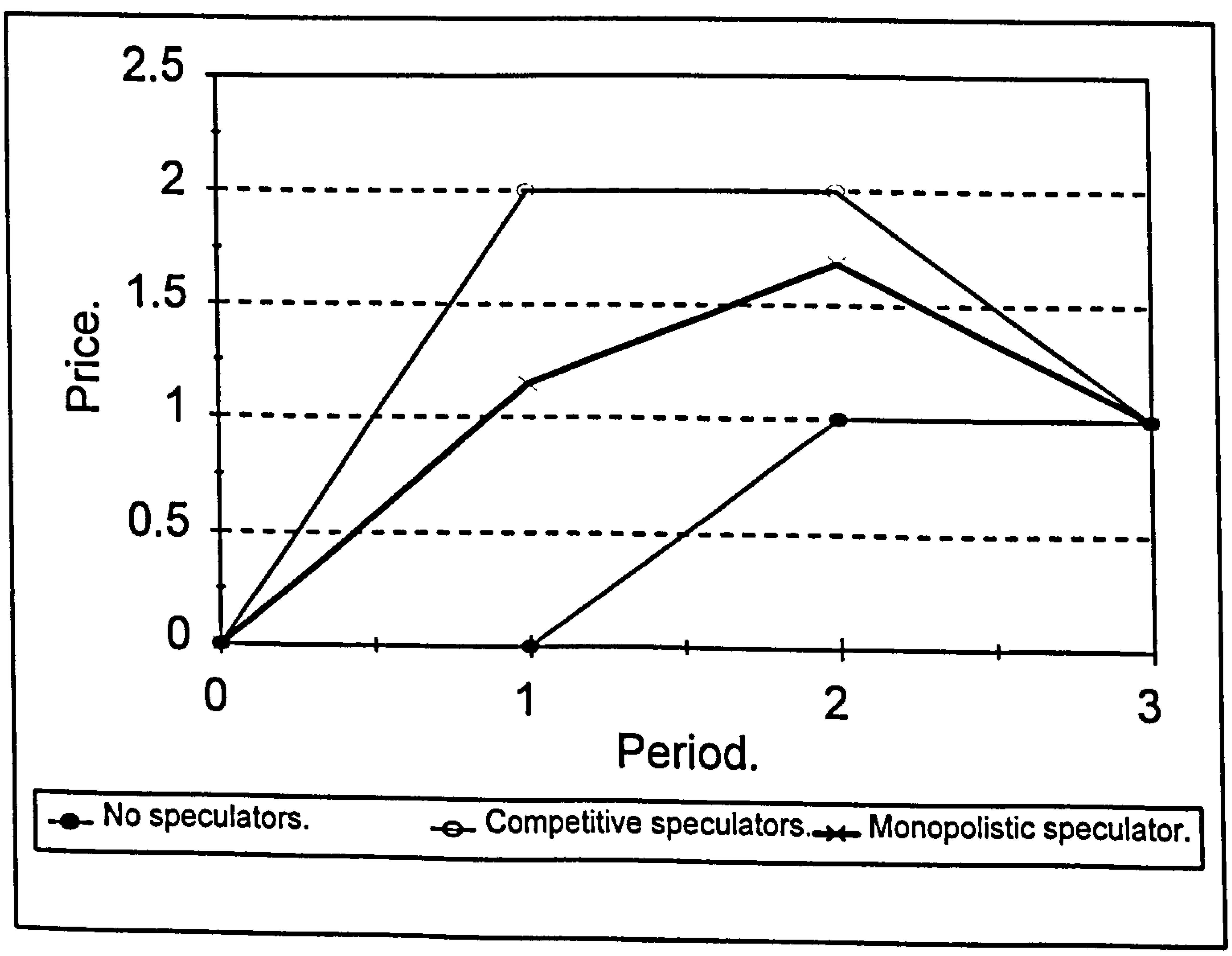


Table 8.6 gives the profits for each of the agent groups as a whole, for the above example. It also gives the 'standardised' profit, which is simply the group profit divided by the measure of that group in the market, and could be thought of as indicating the relative profit per investor.

*Table 8.6: Profits under example #3.*

| | Competitive | | Monopolistic / collusive | |
|---|---|---|---|---|
| | Group | Standardised | Group | Standardised |
| Feedback traders. | -1 | -1 | -0.39 | -0.39 |
| Passive investors. | 0.9 | 1 | -0.13 | -0.15 |
| Speculators. | 0.1 | 1 | 0.53 | 5.25 |

This clearly shows that, although the monopolistic behaviour causes prices to fluctuate less, it dramatically increases the profits of the speculators: in this example, collusion allows the speculators to increase their profits five-fold. This increase in profits comes solely at the expense of the passive investors, with the feedback traders actually benefiting significantly. The losses of the passive investors highlight the failing of their rationality.

# 8.6 Conclusions

## 8.6.1 General conclusions

We have seen (results 8.1 & 8.2) that in a system in which shocks hit the system, speculation can be profitable when speculation that disturbs the steady state of an otherwise identical system would not be. This shows that, with new information and demand shocks impacting the market all the time, the scope for speculation may actually be greater than that implied by the Hart (1977) analysis. We have seen (proposition 8.1 and example) that the addition of competitive speculators to a system in which positive feedback trader demand reacts immediately to price changes can be destabilising. This contradicts an assertion made in De Long et. al. (1990).

Perhaps the most interesting conclusion (proposition 8.2 and example) is that competitive speculation may be more destabilising than monopolistic speculation, keeping the market power of the speculators constant: this tells us that collusion between speculators may actually exert a stabilising influence on market prices.

### 8.6.2 Conclusions from the competitive equilibria

There are a number of conclusions we can draw about the market behaviour in the presence of competitive speculators.

*Increasing the measure of speculators in the market.*

Under the assumption of noiseless information, result 8.5 revealed that the prices are not affected by changes in the measure of the rational speculators in the market relative to passive investors ($\mu$), providing this measure remains positive: the price function jumps discontinuously as the measure of rational speculators becomes positive, then remains unchanged as the measure increases further.

Under the assumption of noisy information we saw (result 8.15) that when there is no fast-acting feedback ($\delta = 0$) an increase in the measure of speculators in the market increases the magnitude of the prices in periods one and two, and therefore causes prices to fluctuate more. The situation is less clear-cut when the feedback trading kicks in immediately, and the effect can work in both directions.

It must be remembered that these results were obtained by keeping the measure of positive feedback traders in the market constant: we were in effect assuming that more and more passive investors begin to behave as speculators. The effects of increasing the joint measure of passive investors and competitive speculators can be found by reducing the size of the coefficients of positive feedback trading. Increasing the coefficient of feedback trading is equivalent to reducing the extent of speculation.

*Increasing the extent of positive feedback trading.*

Under the assumption of noiseless information, the price in each period equals the 'fundamental' value ($\Phi$) when there is no delayed positive feedback trading (beta is zero), regardless of the degree of immediate feedback trading. As result 8.6 showed, once feedback trading develops, and the value of beta becomes positive, the period one and two prices rise, until, as beta approaches alpha, the magnitude of the equilibrium price becomes unbounded.

Result 8.16 showed that under the assumption of noisy information, when the feedback trading is delayed ($\delta = 0$), the prices in both periods are positively related to the degree of feedback trading (for a positive information signal), which means that a marginal decrease in the feedback coefficient ($\beta$) leads to prices becoming more stable. Once again the situation when some feedback trading responds immediately to price changes ($\beta = 0$) is not straightforward to analyse. One thing we can say, following result 8.9, is that as the amount of positive feedback activity becomes 'small' in the context of the market as a whole, prices approach fundamental values when the passive investors are informed.

*The relative effects of the different forms of feedback.*

Under noiseless information, the prices are not affected by the degree of immediate positive feedback trading. This is because the equilibrium price is determined by the situation in period two, and, since the price change from period one to period two is zero, there is no channel through which immediate feedback can exert an influence.

In the noiseless information case we could also see that, since the price fluctuation in the absence of speculators is positively related to the degree of immediate feedback trading, the destabilising effect of speculators is negatively related to immediate feedback, but positively related to delayed feedback: in fact, we can see from result 8.7 that when more feedback is immediate than delayed ($\delta > \beta$), the presence of competitive speculators is actually stabilising.

*Increasing the degree of informedness of the passive investors.*

Under noiseless information, the prices are not affected by the fundamental value estimate of the passive investors in period one. Result 8.10 showed that under noisy information with delayed feedback, the price in both periods is positively related to the period one value estimate of the passive investors.

Speculation in the absence of feedback traders is not profitable under noiseless information, since arbitrage keeps prices in line with fundamentals. Speculation in the absence of feedback traders is only profitable under noisy information when the passive investors are informationally disadvantaged in period one. When feedback traders are present, speculation is always profitable. This shows that the presence of uninformed passive investors is not a necessary requirement for profitable speculation, but is sufficient when information is noisy. The presence of feedback traders is a necessary and sufficient condition for profitable speculation when information is noiseless; and is a sufficient, but not necessary, condition for profitable speculation when information is noisy.

### 8.6.3 Summary of the conclusions

The main conclusions can be stated briefly as follows:

- Demand and value shocks give additional scope for profitable speculation.
- Competitive speculation may be more destabilising than monopolistic speculation.
- Competitive speculation can be destabilising without delayed feedback.
- Reducing delayed feedback increases price stability.
- Delayed feedback is more destabilising than immediate feedback.
- As feedback trading becomes small, prices become more stable.

- As the market power of speculators becomes large, prices become more stable.

### 8.6.4 The destabilising effect of competitive speculation

It is now time to see how the De Long et. al. claims of a destabilising effect of competitive speculation have stood up to our examination.

De Long et. al. look at a world in which shocks hit the system. This makes it distinct from the shock-free situation in Hart (1977). In a world with shocks, the assumption of liquidation makes sense. We have seen that the informedness of the passive investors does not affect the conclusions concerning the destabilising effect of speculation: indeed, with informed passive investors and delayed feedback, competitive speculation is guaranteed to be destabilising.

De Long et. al. seems to provide a strong argument that competitive speculation in the lead-up to a shock can be destabilising, but this is with one proviso: which is that the competitive speculators are constrained to be present in the market in a fixed measure; or in other words, that they have a limited market power. Increases in the market power of competitive speculators will eventually lead to prices becoming more stable.

# Appendix F: For Chapter 8

## F1 No speculators

When the passive investors are uninformed in period one ($V_1 = 0$), the prices will be as follows:

$$p_1 = 0$$
$$p_2 = \frac{\alpha\Phi}{\alpha - \delta} \tag{F1}$$

When the passive investors are noiselessly informed in period one ($V_1 = \Phi$), the prices are:

$$p_1 = \frac{\alpha\Phi}{\alpha - \delta}$$
$$p_2 = \frac{\alpha(\alpha + \beta - 2\delta)}{(\alpha - \delta)^2}\Phi = \frac{\alpha^2 + \alpha\beta - 2\alpha\delta}{\alpha^2 + \delta^2 - 2\alpha\delta}\Phi \tag{F2}$$

And when the passive investors are noisily informed in period one ($V_1 = \varepsilon_\phi/2$), the prices are:

$$p_1 = \frac{\varepsilon_\phi}{2}\frac{\alpha}{\alpha - \delta}$$
$$p_2 = \frac{\varepsilon_\phi}{2}\frac{\alpha(\beta - \delta)}{(\alpha - \delta)^2} + \frac{\alpha\Phi}{\alpha - \delta} \tag{F3}$$

# F2 Monopolistic speculator

## F2.1 Derivation of the period two price

In period two, the monopolist maximises utility, given his demand in period one and his information set in period two. We have kept the De Long et. al. assumption of a mean-variance utility function, which in this case gives an expected utility in period two of the following:

$$E(U|I_2) = E\left\{D_1^{m}(p_2 - p_1) + D_2^{m}(p_3 - p_2) - \gamma_m (D_2^{m})^2 \sigma^2 \right\} \tag{F4}$$

The monopolist knows the level of demand from the other investor types. As before, the demand from the group of passive investors is given by a constant ($\alpha$) multiplied by the difference between the fundamental value estimate of this group and the price in this period ($p_2$), and weighted by the measure of these agents in the market.

$$D_2^p = (1-\mu)\alpha(\Phi - p_2) \tag{F5}$$

At this point it would be convenient to define a new term for the sake of expositional simplicity:

$$\alpha_m \equiv (1-\mu)\alpha \tag{F6}$$

The demand of the positive feedback traders under our assumption of zero delta is:

$$D_2^f = \beta p_1 \tag{F7}$$

Since in equilibrium the demand from the monopolist must be exactly counterbalanced by the demand from the other two agent types, the above demand equations give us the relationship between the monopolist demand and the period two price.

$$\begin{aligned} D_2^{rm} &= -\left\{D_2^p + D_2^f\right\} \\ &= -\alpha_m(\Phi - p_2) - \beta p_1 \end{aligned} \tag{F8}$$

Therefore:

$$p_2 = \frac{D_2^{rm} + \beta p_1}{\alpha_m} + \Phi \tag{F9}$$

Substituting the expression for the period two price into the monopolist's utility function gives:

$$E(U|I_2) = D_1^{rm}\left(\frac{D_2^{rm} + \beta p_1}{\alpha_m} + \Phi - p_1\right) - D_2^{rm}\left(\frac{D_2^{rm} + \beta p_1}{\alpha_m}\right) - \gamma_m \sigma^2 \left(D_2^{rm}\right)^2 \tag{F10}$$

The monopolist maximises this with respect to his period two demand to discover the utility-maximising level.

$$\frac{dE(U)}{dD_2^{rm}} = \frac{D_1^{rm} - 2D_2^{rm} - \beta p_1}{\alpha_m} - 2\gamma_m \sigma^2 D_2^{rm} \tag{F11}$$

Setting this equal to zero gives the following:

$$D_2^{rm} = \frac{D_1^{rm} - \beta p_1}{2\left(1 + \alpha_m \gamma_m \sigma^2\right)} \tag{F12}$$

This expression can be simplified to:

$$D_2^{rm} = \frac{D_1^{rm} - \beta p_1}{2 + \lambda} \tag{F13}$$

where:

$$\lambda \equiv 2\alpha_m \gamma_m \sigma^2$$

The second derivative of the utility function with respect to the period two demand is:

$$\frac{d^2E(U)}{\left(dD_2^{rm}\right)^2} = -\frac{2}{\alpha_m} - 2\gamma_m\sigma^2 < 0 \tag{F14}$$

Since this is always negative, we have found the level of demand that always maximises the expected utility of the monopolist.

In period one there will be no demand from the positive feedback traders, so the market clearing condition is:

$$D_1^{rm} = -D_1^p \tag{F15}$$

We will again denote the fundamental value estimate of the passive investors in period $i$ as $V_i$. The period one market clearing condition can now be written as:

$$D_1^{rm} + \alpha_m\left(V_1 - p_1\right) = 0 \tag{F16}$$

or equivalently:

$$p_1 = \frac{D_1^{rm}}{\alpha_m} + V_1 \tag{F17}$$

By combining equations F8, F12 and F15 the price in period two can now be expressed as a function of the price in period one, as in the main text.

## F2.2 Derivation of the period one price under noiseless information

Under noiseless information, the rational speculators' certainty-equivalent wealth in period two ($W_2$) is:

$$W_2 = D_1^{rm}\left(p_2 - p_1\right) + D_2^{rm}\left(\Phi - p_2\right) - \gamma_m\left(D_2^{rm}\right)^2\sigma^2 \tag{F18}$$

Since this certainty-equivalent wealth is known for certain in period one, the expectation of this in period one represents expected utility for the monopolist:

$$E(U|I_1) = E_1\left\{D_1^{rm}(p_2 - p_1) + D_2^{rm}(\Phi - p_2) - \gamma_m (D_2^{rm})^2 \sigma^2\right\} \tag{F19}$$

The price and monopolist demand in period two can be expressed in terms of the monopolist demand in period one:

$$\begin{aligned} D_2^{rm} &= \frac{1}{2+\lambda}\left\{D_1^{rm}\left(\frac{\alpha_m - \beta}{\alpha_m}\right) - \beta V_1\right\} \\ p_2 &= \frac{1}{\alpha_m(2+\lambda)}\left\{D_1^{rm}\left(1 + \frac{\beta(1+\delta)}{\alpha_m}\right) + V_1\beta(1+\lambda)\right\} + \Phi \end{aligned} \tag{F20}$$

Substituting these into the expression for the expected utility gives the following:

$$\begin{aligned} E(U|I_1) = {} & \frac{(D_1^{rm})^2}{2\alpha_m^{\,3}(2+\lambda)}\left\{2\alpha_m\beta(1+\lambda) - \alpha_m^{\,2}(3+2\lambda) + \beta^2\right\} \\ & + \frac{D_1^{rm}V_1}{\alpha_m^{\,2}(2+\lambda)}\beta\left\{\alpha_m(1+\lambda) + \beta\right\} + D_1^{rm}(\Phi - V_1) \\ & + \frac{\beta^2 V_1^2}{2\alpha_m(2+\lambda)} \end{aligned} \tag{F21}$$

The first derivative of this is:

$$\begin{aligned} \frac{dE(U|I_1)}{dD_1^{rm}} = {} & \frac{D_1^{rm}}{\alpha_m^{\,3}(2+\lambda)}\left\{2\alpha_m\beta(1+\lambda) - \alpha_m^{\,2}(3+2\lambda) + \beta^2\right\} \\ & + \frac{V_1}{\alpha_m^{\,2}(2+\lambda)}\beta\left\{\alpha_m(1+\lambda) + \beta\right\} + \Phi - V_1 \end{aligned} \tag{F22}$$

Setting this equal to zero gives us the following expression for the period one demand:

$$D_1^{rm} = \frac{V_1\alpha_m\beta\left\{\alpha_m(1+\lambda) + \beta\right\} + \alpha_m^{\,3}(2+\lambda)(\Phi - V_1)}{\alpha_m^{\,2}(3+2\lambda) - 2\alpha_m\beta(1+\lambda) - \beta^2} \tag{F23}$$

This gives an expression for the price in period one of the following:

$$p_1 = \frac{V_1\beta\left\{\alpha_m(1+\lambda)+\beta\right\}+\alpha_m^{\,2}(2+\lambda)(\Phi - V_1)}{\alpha^2(3+2\lambda)-2\alpha_m\beta(1+\lambda)-\beta^2}+V_1 \tag{F24}$$

This can be re-arranged to give the expression found in the main text.

The second derivative of the utility function is:

$$\frac{d^2E(U|I_1)}{\left(dD_1^{rm}\right)^2} = \frac{2\alpha_m\beta(1+\lambda)-\alpha_m^{\,2}(3+2\lambda)+\beta^2}{\alpha_m^{\,3}(2+\lambda)} \tag{F25}$$

The sign of this follows the sign of the numerator, which can be re-written as:

$$\left(\beta^2+2\alpha_m\beta-3\alpha_m^{\,2}\right)+2\delta\alpha_m(\beta-\alpha_m) \tag{F26}$$

It is clear that both of the terms will be negative when the weighted alpha term ($\alpha_m$) exceeds beta, and positive when the opposite is true. Hence a necessary and sufficient condition for the expression for the period one demand given in the text to maximise expected utility is that weighted alpha exceeds beta.

## F2.3 Derivation of the period one price under noisy information

Assume that the information signal is noisy in period one. The certainty-equivalent wealths for the two possible outcomes in period two of the mean of the liquidation value are as follows:

$$\begin{aligned} W_{2a} &= \frac{\left(D_1^{rm}\right)^2}{2\alpha_m^{\,3}(2+\lambda)}\left\{2\alpha_m\beta(1+\lambda)-\alpha_m^{\,2}(3+2\lambda)+\beta^2\right\} \\ &\quad + \frac{D_1^{rm}V_1}{\alpha_m^{\,2}(2+\lambda)}\left\{\alpha_m\beta(1+\lambda)+\beta^2\right\}+D_1^{rm}(\varepsilon - V_1) \\ &\quad + \frac{\beta^2V_1^2}{2\alpha_m(2+\lambda)} \end{aligned} \tag{F27}$$

$$W_{2b} = \frac{\left(D_1^{rm}\right)^2}{2\alpha_m^{\ 3}(2+\lambda)}\left\{2\alpha_m\beta(1+\lambda)-\alpha_m^{\ 2}(3+2\lambda)+\beta^2\right\}$$
$$+\frac{D_1^{rm}V_1}{\alpha_m^{\ 2}(2+\lambda)}\left\{\alpha_m\beta(1+\lambda)+\beta^2\right\}-D_1^{rm}V_1 \tag{F28}$$
$$+\frac{\beta^2V_1^2}{2\alpha_m(2+\lambda)}$$

$$E(W_2|I_1)=\frac{W_{2a}+W_{2b}}{2}$$
$$=\frac{\left(D_1^{rm}\right)^2}{2\alpha_m^{\ 3}(2+\lambda)}\left\{2\alpha_m\beta(1+\lambda)-\alpha_m^{\ 2}(3+2\lambda)+\beta^2\right\}$$
$$+\frac{D_1^{rm}V_1}{\alpha_m^{\ 2}(2+\lambda)}\left\{\alpha_m\beta(1+\lambda)+\beta^2\right\}+D_1^{rm}\left(\frac{\varepsilon}{2}-V_1\right) \tag{F29}$$
$$+\frac{\beta^2V_1^2}{2\alpha_m(2+\lambda)}$$

$$Var(W_2|I_1)=\left(D_1^{rm}\right)^2\left\{\frac{1}{2}\left(\varepsilon-\frac{\varepsilon}{2}\right)^2+\frac{1}{2}\left(-\frac{\varepsilon}{2}\right)^2\right\}$$
$$=\left(D_1^{rm}\right)^2\frac{\varepsilon^2}{4} \tag{F30}$$

The monopolist maximises as follows:

$$\max_{D_1^{rm}}\left\{E(W_2|I_1)-\gamma_m\,Var(W_2|I_1)\right\} \tag{F31}$$

The first derivative is:

$$\frac{d\{.\}}{dD_1^{rm}}=\frac{D_1^{rm}}{\alpha_m^{\ 3}(2+\lambda)}\left\{2\alpha_m\beta(1+\lambda)-\alpha_m^{\ 2}(3+2\lambda)+\beta^2\right\}$$
$$+\frac{V_1}{\alpha_m^{\ 2}(2+\lambda)}\left\{\alpha_m\beta(1+\lambda)+\beta^2\right\}+\frac{\varepsilon}{2}-V_1-\frac{\gamma_m\varepsilon^2D_1^{rm}}{2} \tag{F32}$$

Setting this equal to zero and re-arranging gives the following expression for demand:

$$D_1^{rm} = \frac{2\alpha_m\left\{V_1\left\{\alpha_m\beta(1+\lambda)+\beta^2\right\}+\alpha_m^2(2+\lambda)\left(\frac{\varepsilon}{2}-V_1\right)\right\}}{\gamma\varepsilon^2\alpha_m^3(2+\lambda)-2\left\{2\alpha_m\beta(1+\lambda)-\alpha_m^2(3+2\lambda)+\beta^2\right\}} \tag{F33}$$

The equilibrium price is therefore:

$$p_1 = \frac{2\left\{V_1\beta\left\{\alpha_m(1+\lambda)+\beta\right\}+\alpha_m^2(2+\lambda)\left(\frac{\varepsilon}{2}-V_1\right)\right\}}{\gamma\varepsilon^2\alpha_m^3(2+\lambda)-2\left\{2\alpha_m\beta(1+\lambda)-\alpha_m^2(3+2\lambda)+\beta^2\right\}}+V_1 \tag{F34}$$

This can be re-arranged to give the following:

$$p_1 = \frac{\alpha_m\left\{V_1\varepsilon^2\gamma_m\alpha^2(2+\lambda)+\varepsilon\alpha_m(2+\lambda)-2V_1\left(\alpha_m+\beta(1+\lambda)\right)\right\}}{\gamma_m\varepsilon^2\alpha_m^3(2+\lambda)-2\left\{2\alpha_m\beta(1+\lambda)-\alpha_m^2(3+2\lambda)+\beta^2\right\}} \tag{F35}$$

The second derivative of expected utility with respect to the period one demand is as follows:

$$\frac{d^2\{\}}{d\left(D_1^{rm}\right)^2} = \frac{2\alpha_m\beta(1+\lambda)-\alpha_m^2(3+2\lambda)+\beta^2}{\alpha_m^3(2+\lambda)}-\frac{\gamma_m\varepsilon^2}{2} \tag{F36}$$

A comparison of this with the corresponding expression for the case of a noiseless information signal shows that a sufficient condition for it to be negative is that weighted alpha exceeds beta. The condition for a maximum can be written as:

$$2\alpha_m\beta(1+\lambda)-\alpha_m^2(3+2\lambda)+\beta^2 < \frac{\alpha_m^3(2+\lambda)\gamma_m\varepsilon^2}{2} \tag{F37}$$

# F3 Competitive speculators

## F3.1 Noiseless information

*F3.1.1 Derivation of equilibrium market prices.*

Since the rational investors are atomistic, they are correct in thinking that they have no power to influence the equilibrium market prices, and so take them as given. They maximise their expected utility, which is as follows:

$$\begin{aligned} E(U|I_2) &= E_2\left\{D_1^{rc}(p_2 - p_1) + D_2^{rc}(\Phi + \theta - p_2) - \gamma\left(D_2^{rc}\right)^2\sigma^2\right\} \\ &= D_1^{rc}(p_2 - p_1) + D_2^{rc}(\Phi - p_2) - \gamma\left(D_2^{rc}\right)^2\sigma^2 \end{aligned} \tag{F38}$$

Hence the derivative with respect to the period two rational agent demand is:

$$\frac{\partial\{.\}}{\partial D_2^{rc}} = \Phi - p_2 - 2\gamma D_2^{rc}\sigma^2 \tag{F39}$$

Setting this equal to zero gives the expression for demand found in the main text. The second derivative is as follows:

$$\frac{\partial^2\{.\}}{\partial\left(D_2^{rc}\right)^2} = -2\gamma\sigma^2 \tag{F40}$$

Since this is always negative, we know that a maximum has been found.

In equilibrium, the stock demand must equal the stock supply - which is assumed to be net zero for convenience - so the sum of the demand from the three investor types must be zero.

$$\begin{aligned} &\mu D_2^{rc} + (1-\mu)D_2^{p} = -D_2^{f} \\ &\alpha(\Phi - p_2) = -((\beta - \delta)p_1 + \delta p_2) = -\beta p_2 \end{aligned} \tag{F41}$$

From this we can derive the price expressions given in the main text.

*F3.1.2 Informed passive investors.*

Comparison with equation 8.4 reveals that when passive investors are informed in period one, the price in period one when speculators are present will be further from the fundamental value when beta exceeds delta.

In the absence of rational speculators, the price in period two will exceed the liquidation value when the following condition holds:

$$
\begin{aligned}
&\frac{\alpha\Phi}{(\alpha-\delta)^2}(\alpha+\beta-2\delta)>\Phi \\
&\Rightarrow(\alpha-\delta)^2<\alpha\{(\alpha-\delta)+(\beta-\delta)\} \\
&\Rightarrow\alpha^2-2\alpha\delta+\delta^2<\alpha^2-2\alpha\delta+\alpha\beta \\
&\Rightarrow\delta^2<\alpha\beta
\end{aligned}
\tag{F42}
$$

A sufficient condition for this to hold is that beta exceeds delta. When the price in period two in the absence of rational speculators exceeds the liquidation value, the condition for the presence of rational speculators to move prices further away from fundamentals is the following:

$$
\begin{aligned}
&\frac{\alpha\Phi}{\alpha-\beta}>\frac{\alpha\Phi}{(\alpha-\delta)^2}(\alpha+\beta-2\delta) \\
&\Rightarrow(\alpha-\delta)^2>(\alpha-\beta)\{(\alpha-\delta)+(\beta-\delta)\} \\
&\Rightarrow\alpha^2-2\alpha\delta+\delta^2>\left(\alpha^2-\alpha\delta-\alpha\beta+\beta\delta\right) \\
&\qquad\qquad\qquad\qquad+\left(\alpha\beta+\beta\delta-\alpha\delta-\beta^2\right) \\
&\Rightarrow\beta^2-2\beta\delta+\delta^2>0 \\
&\Rightarrow(\beta-\delta)^2>0
\end{aligned}
\tag{F43}
$$

This shows that a sufficient condition for the price in period two to be further away from fundamentals in period two when rational speculators are present is that beta exceeds delta. We can therefore say that a sufficient condition for the presence of speculators to be strongly destabilising when passive investors are informed in period one is that beta exceeds delta, or in other words that the strength of the delayed feedback exceeds that of the immediate feedback.

*F3.1.3 Uninformed passive investors.*

Comparison of the prices given above with equation 8.4 reveals that when passive investors are uninformed in period one, the price in period two when speculators are present will be further from the fundamental value when beta exceeds delta.

The price in period one would be zero in the absence of rational speculators. The condition for the price to be further from fundamentals when rational speculators are present is therefore that it exceeds twice the fundamental value:

$$\begin{aligned} &\frac{\alpha\Phi}{\alpha-\beta} > 2\Phi \\ &\Rightarrow \beta > \frac{\alpha}{2} \end{aligned} \tag{F44}$$

Combining the conditions for the two periods reveals that when passive investors are informed in period one the presence of speculators will be strongly destabilising when beta exceeds both delta and half the value of alpha.

## F3.2 Noisy information, general case

The alternative market-clearing conditions for period two are:

$$\begin{aligned}\alpha\left(\varepsilon_{\phi}-p_{2a}\right)&=-(\beta-\delta)p_1+\delta p_{2a}\\ -\alpha p_{2b}&=-(\beta-\delta)p_1+\delta p_{2b}\end{aligned} \tag{F45}$$

These give the relationship between the price in period two and that in period one for each of the possible outcomes, which can be expressed more clearly as:

$$\begin{aligned}p_{2a}&=\frac{(\beta-\delta)p_1+\alpha\varepsilon_{\phi}}{\alpha-\delta}\\ p_{2b}&=\frac{(\beta-\delta)p_1}{\alpha-\delta}\end{aligned} \tag{F46}$$

The period one market-clearing condition becomes:

$$\begin{aligned}&\mu D_1^{rc}+(1-\mu)\alpha D_1^{p}+D_1^{f}=0\\ &\Rightarrow \mu D_1^{rc}+(1-\mu)\alpha\left(V_1-\frac{\varepsilon_{\phi}}{2}\right)+\delta p_1=0\end{aligned} \tag{F47}$$

As for the case of noiseless information, the rational investor demand in period two will be as follows:

$$D_2^{rc}=\frac{\Phi-p_2}{2\gamma\sigma^2}=\alpha\left(\Phi-p_2\right) \tag{F48}$$

The certainty-equivalent wealths for the two possible states are:

$$\begin{aligned}W_{2a}&=D_1^{rc}\left(p_{2a}-p_1\right)+D_{2a}^{rc}\left(\varepsilon_{\phi}-p_{2a}\right)-\gamma D_{2a}^{2}\sigma_{\theta}^{2}\\ &=D_1^{rc}\left(p_{2a}-p_1\right)+\alpha\left(\varepsilon_{\phi}-p_{2a}\right)^2-\frac{\alpha^2\left(\varepsilon_{\phi}-p_{2a}\right)^2}{2\alpha}\\ &=D_1^{rc}\left(p_{2a}-p_1\right)+\frac{\alpha\left(\varepsilon_{\phi}-p_{2a}\right)^2}{2}\\ W_{2b}&=D_1^{rc}\left(p_{2b}-p_1\right)+\frac{\alpha p_{2b}^2}{2}\end{aligned} \tag{F49}$$

The expected value of the period two certain equivalent wealth and its variance in period one are given by the following relationships:

$$E\left(W_2 \middle| \varepsilon_\phi\right) = \frac{W_{2a} + W_{2b}}{2}$$

$$Var\left(W_2 \middle| \varepsilon_\phi\right) = \frac{\left(W_{2a} - E\left(W_2 \middle| \varepsilon_\phi\right)\right)^2 + \left(W_{2b} - E\left(W_2 \middle| \varepsilon_\phi\right)\right)^2}{2} \tag{F50}$$

The expected utility of the rational speculators is given by:

$$E\left(U \middle| \varepsilon_\phi\right) = E\left(W_2 \middle| \varepsilon_\phi\right) - \gamma Var\left(W_2 \middle| \varepsilon_\phi\right) \tag{F51}$$

Differentiating this with respect to the period one demand gives:

$$\frac{\partial E\left(U \middle| \varepsilon_\phi\right)}{\partial D_1^{rc}} = -\frac{\alpha\gamma}{4}\left(p_{2a} - p_{2b}\right)\left(\left(\varepsilon_\phi - p_{2a}\right)^2 - \left(p_{2b}\right)^2\right)$$
$$-\frac{\gamma}{2} D_1^{rc}\left(p_{2a} - p_{2b}\right)^2 + \frac{p_{2a} + p_{2b}}{2} - p_1 \tag{F52}$$

Setting this equal to zero gives us the following expression for the rational speculator demand:

$$D_1^{rc} = \frac{\frac{p_{2a} + p_{2b}}{2} - p_1}{\frac{\gamma}{2}\left(p_{2a} - p_{2b}\right)^2} - \frac{\alpha}{2}\frac{\left(\left(\varepsilon_\phi - p_{2a}\right)^2 - \left(p_{2b}\right)^2\right)}{p_{2a} - p_{2b}}$$
$$= \frac{\frac{p_{2a} + p_{2b}}{2} - p_1}{\frac{\gamma}{2}\left(p_{2a} - p_{2b}\right)^2} - \frac{\alpha}{2}\frac{\left(\varepsilon_\phi^{\ 2} - 2\varepsilon_\phi p_{2a}\right)}{\left(p_{2a} - p_{2b}\right)} - \frac{\alpha\left(p_{2a} + p_{2b}\right)}{2} \tag{F53}$$

The period one market clearing condition is:

$$\mu D_1^{rc} + (1 - \mu)\alpha\left(V_1 - p_1\right) + \delta p_1 = 0 \tag{F54}$$

Solving this simultaneously with the expression for demand, and substituting in for the period two demands gives an expression for the period one price of:

$$p_1 = \frac{\alpha\varepsilon_\phi}{2}\frac{2V_1\varepsilon_\phi\alpha(\alpha - \delta)(1 - \mu) + \mu\left(4\sigma^2(\alpha - \delta)^2 - \delta^2\varepsilon_\phi^{\ 2}\right)}{\alpha\varepsilon_\phi^{\ 2}(\alpha - \delta)^2(1 - \mu) + \mu(\alpha - (\beta + \delta))\left(4\sigma^2(\alpha - \delta)^2 - \alpha\delta\varepsilon_\phi^{\ 2}\right)} \tag{F55}$$

The second derivative of the expected utility function with respect to the rational speculator period one demand is:

$$\frac{\partial^2 E\left(U \middle| \varepsilon_\phi\right)}{\partial\left(D_1^{rc}\right)^2} = -\frac{\gamma}{2}\left(p_{2a} - p_{2b}\right)^2 < 0 \tag{F56}$$

Since this is always negative, the above price always corresponds to rational speculator maximising behaviour.

## F3.3 Special case: delayed positive feedback

*F3.3.1 The condition for the period one price to exceed the fundamental value estimate of the passive investors.*

This condition is one we have seen before:

$$V_1 < \frac{\varepsilon_\phi}{2}\frac{\alpha}{(\alpha - \beta)} \tag{F57}$$

and is satisfied for our both our 'informed' and 'uninformed' assumptions, hence the result given in the main text.

*F3.3.2 Uninformed passive investors.*

In the absence of speculators, and with no immediate feedback, the price in period two will equal the liquidation value when passive investors are uninformed in period one. Since the price in period one in the absence of speculators will be zero, and the price in period one in their presence will be greater than the fundamental value, the issue of whether or not speculation is destabilising therefore depends on whether or not the price in the presence of speculators is greater than twice the fundamental value:

$$\frac{\alpha\phi\left(V_1\phi(1-\mu)+2\sigma^2\mu\right)}{\alpha\phi^2(1-\mu)+4\sigma^2\mu(\alpha-\beta)}>\phi \qquad \text{(F58)}$$

This can be re-arranged to form the condition, given in De Long et. al. (1990a, expression 21):[1]

$$\frac{1-\mu}{\mu}<\frac{2\sigma_\theta^2}{\phi^2}\left\{1-2\left(\frac{\alpha-\beta}{\alpha}\right)\right\} \qquad \text{(F59)}$$

*F3.3.3 Informed passive investors.*

In the absence of speculators, and with no immediate feedback, the price in period one will equal the fundamental value when passive investors are informed in period one, and will exceed the liquidation value in period two, and so the presence of speculators will be strongly destabilising when the price in period two in the presence exceeds the price in their absence, and therefore that the following condition holds:

$$\begin{aligned}&\frac{\beta}{\alpha}p_1+\varepsilon_\phi>\frac{\beta}{\alpha}\frac{\varepsilon_\phi}{2}+\varepsilon_\phi\\&\Rightarrow p_1>\frac{\varepsilon_\phi}{2}\end{aligned} \qquad \text{(F60)}$$

We have already seen that this always holds (F3.3.1), and so we can say that when the passive investors are informed in period one the presence of speculators is strongly destabilising.

*F3.3.4 Market prices in the absence of feedback traders.*

The price in period one will equal the following:

[1] Since $\varepsilon^2 = \phi^2$.

$$p_1 = \frac{\varepsilon_\phi}{2}\frac{2V_1\varepsilon_\phi(1-\mu)+4\sigma^2\mu}{\varepsilon_\phi^2(1-\mu)+4\sigma^2\mu} \leq \frac{\varepsilon_\phi}{2} \qquad for\ V_1 \leq \frac{\varepsilon_\phi}{2}$$

$$V_1 = \frac{\varepsilon_\phi}{2} \ \ implies \ \ p_1 = \frac{\varepsilon_\phi}{2} \tag{F61}$$

The price in period two will be equal the liquidation value, and so when there is no feedback trading the price will equal its fundamental value at all times provided that the passive investors are not informationally disadvantaged.

*F3.3.5 Varying the measure of informed speculators.*

The derivative of the period one price with respect to the measure of informed speculators in the market is given by:

$$\frac{\partial p_1}{\partial \mu} = \frac{2\varepsilon_\phi^2\sigma^2\alpha\left(\varepsilon_\phi\alpha - 2V_1(\alpha-\beta)\right)}{\left\{4\mu\sigma^2(\alpha-\beta)+\alpha\varepsilon^2(1-\mu)\right\}^2} \tag{F62}$$

This is positive for $0 \leq V_1 \leq \frac{\varepsilon_\phi}{2}$, which encompasses our cases of informed and uninformed passive investors. It is negative for $\frac{\varepsilon_\phi}{2} \leq V_1 \leq 0$. These results, combined with the fact that the price in each period has the same sign as the liquidation value signal, indicate that an increase in the measure of speculators will increase the magnitude of price in period one, and consequently the price in period two as well.

*F3.3.6 Varying the degree of feedback trading.*

The rate of change of the price in period one with respect to an increase in the degree of feedback trading is as follows:

$$\frac{\partial p_1}{\partial \beta} = 4\frac{\varepsilon_\phi \sigma^2 \alpha \mu \left( V_1 \varepsilon_\phi (1-\mu) + 2\sigma^2 \mu \right)}{\left\{ \mu \varepsilon_\phi^{\,2} (1-\mu) + 4\sigma^2 \mu (\alpha - \beta) \right\}^2} \tag{F63}$$

This is always positive (for a non-negative passive investor value estimate), and so proves the result given in the main text. The price in period one will equal the true fundamental value of the stock when beta is of the following magnitude:

$$\beta = \varepsilon_\phi (\varepsilon_\phi - 2V_1) \frac{\alpha}{4\sigma^2} \frac{(1-\mu)}{\mu} \qquad \textit{gives } p_1 = \frac{\varepsilon_\phi}{2} \tag{F64}$$

Further increases in beta continue to increase the price in period one, until this price becomes unbounded at the following level of beta:

$$\beta = \alpha \left\{ 1 + \frac{\varepsilon^2}{4\sigma^2} \frac{(1-\mu)}{\mu} \right\} \qquad \textit{gives } p_1 = \infty \tag{F65}$$

This clearly shows that, unlike the case with noiseless information, the price does not become unbounded when beta equals alpha, but at a value of beta greater than this.

# Appendix G: For Chapter 8

## G1 Monopolistic speculator and noisy information

When the information concerning the liquidation value is noisy, the certainty-equivalent wealths for the two possible outcomes in period two of the mean of the liquidation value are as follows:

$$
\begin{aligned}
W_{2a} = & \frac{\left(D_1^{rm}\right)^2}{2\alpha_m^{\ 3}(2+\lambda)}\left\{2\alpha_m\beta(1+\lambda)-\alpha_m^{\ 2}(3+2\lambda)+\beta^2\right\} \\
& +\frac{D_1^{rm}V_1}{\alpha_m^{\ 2}(2+\lambda)}\left\{\alpha_m\beta(1+\lambda)+\beta^2\right\}+D_1^{rm}(\varepsilon-V_1) \\
& +\frac{\beta^2V_1^2}{2\alpha_m(2+\lambda)}
\end{aligned}
\qquad (G1)
$$

$$
\begin{aligned}
W_{2b} = & \frac{\left(D_1^{rm}\right)^2}{2\alpha_m^{\ 3}(2+\lambda)}\left\{2\alpha_m\beta(1+\lambda)-\alpha_m^{\ 2}(3+2\lambda)+\beta^2\right\} \\
& +\frac{D_1^{rm}V_1}{\alpha_m^{\ 2}(2+\lambda)}\left\{\alpha_m\beta(1+\lambda)+\beta^2\right\}-D_1^{rm}V_1 \\
& +\frac{\beta^2V_1^2}{2\alpha_m(2+\lambda)}
\end{aligned}
\qquad (G2)
$$

$$
\begin{aligned}
E(W_2|I_1) = & \frac{W_{2a}+W_{2b}}{2} \\
= & \frac{\left(D_1^{rm}\right)^2}{2\alpha_m^{\ 3}(2+\lambda)}\left\{2\alpha_m\beta(1+\lambda)-\alpha_m^{\ 2}(3+2\lambda)+\beta^2\right\} \\
& +\frac{D_1^{rm}V_1}{\alpha_m^{\ 2}(2+\lambda)}\left\{\alpha_m\beta(1+\lambda)+\beta^2\right\}+D_1^{r}\left(\frac{\varepsilon}{2}-V_1\right) \\
& +\frac{\beta^2V_1^2}{2\alpha_m(2+\lambda)}
\end{aligned}
\qquad (G3)
$$

$$Var(W_2|I_1) = \left(D_1^{rm}\right)^2 \left\{ \frac{1}{2}\left(\varepsilon - \frac{\varepsilon}{2}\right)^2 + \frac{1}{2}\left(-\frac{\varepsilon}{2}\right)^2 \right\} = \left(D_1^{rm}\right)^2 \frac{\varepsilon^2}{4} \tag{G4}$$

The monopolist maximises as follows:

$$\max_{D_1^{rm}} \left\{ E(W_2|I_1) - \gamma_m \, Var(W_2|I_1) \right\} \tag{G5}$$

The first derivative is:

$$\begin{aligned} \frac{d\{\}}{dD_1^{rm}} &= \frac{D_1^{rm}}{\alpha_m^{\ 3}(2+\lambda)} \left\{ 2\alpha_m \beta(1+\lambda) - \alpha_m^{\ 2}(3+2\lambda) + \beta^2 \right\} \\ &+ \frac{V_1}{\alpha_m^{\ 2}(2+\lambda)} \left\{ \alpha_m \beta(1+\lambda) + \beta^2 \right\} + \frac{\varepsilon}{2} - V_1 - \frac{\gamma_m \varepsilon^2 D_1^{rm}}{2} \end{aligned} \tag{G6}$$

Setting this equal to zero gives the following expression for demand:

$$D_1^{rm} = \frac{2\alpha_m \left\{ V_1 \left\{ \alpha_m \beta(1+\lambda) + \beta^2 \right\} + \alpha_m^{\ 2}(2+\lambda)\left(\frac{\varepsilon}{2} - V_1\right) \right\}}{\gamma \varepsilon^2 \alpha_m^{\ 3}(2+\lambda) - 2\left\{ 2\alpha_m \beta(1+\lambda) - \alpha_m^{\ 2}(3+2\lambda) + \beta^2 \right\}} \tag{G7}$$

The equilibrium price is therefore:

$$p_1 = \frac{2\left\{ V_1 \left\{ \alpha_m \beta(1+\lambda) + \beta^2 \right\} + \alpha_m^{\ 2}(2+\lambda)\left(\frac{\varepsilon}{2} - V_1\right) \right\}}{\gamma \varepsilon^2 \alpha_m^{\ 3}(2+\lambda) - 2\left\{ 2\alpha_m \beta(1+\lambda) - \alpha_m^{\ 2}(3+2\lambda) + \beta^2 \right\}} + V_1 \tag{G8}$$

This can be re-arranged to give the following:

$$p_1 = \frac{\alpha_m \left\{ V_1 \varepsilon^2 \gamma_m \alpha^2 (2+\lambda) + \varepsilon \alpha_m (2+\lambda) - 2V_1\left(\alpha_m + \beta(1+\lambda)\right) \right\}}{\gamma_m \varepsilon^2 \alpha_m^{\ 3}(2+\lambda) - 2\left\{ 2\alpha_m \beta(1+\lambda) - \alpha_m^{\ 2}(3+2\lambda) + \beta^2 \right\}} \tag{G9}$$

The second derivative of expected utility with respect to period one demand is:

$$\frac{d^2\{\}}{d\left(D_1^{rm}\right)^2} = \frac{2\alpha_m \beta(1+\lambda) - \alpha_m^{\ 2}(3+2\lambda) + \beta^2}{\alpha_m^{\ 3}(2+\lambda)} - \frac{\gamma_m \varepsilon^2}{2} \tag{G10}$$

Comparing with the corresponding expression for the case of a noiseless information signal shows that a sufficient condition for this to be negative is that alpha exceeds beta. The condition for a maximum can be written as:

$$2\alpha_m\beta(1+\lambda)-\alpha_m^{\,2}(3+2\lambda)+\beta^2<\frac{\alpha_m^{\,3}(2+\lambda)\gamma_m\varepsilon^2}{2} \tag{G11}$$

# G2 Adding a demand shock to a model with competitive speculators

## G2.1 Model structure

Assume that in period two the feedback traders provide a demand shock of N.[1] The feedback demand therefore takes the following form:

$$\begin{aligned} D_1^f &= \delta(p_1-p_0)=\delta p_1 \\ D_2^f &= \beta(p_1-p_0)+\delta(p_2-p_1)+\mathrm{N} \\ &= (\beta-\delta)(p_1-p_0)+\delta(p_2-p_0)+\mathrm{N} \\ &= (\beta-\delta)p_1+\delta p_2+\mathrm{N} \end{aligned} \tag{G12}$$

The prices in the absence of speculators will therefore be as follows:

$$\begin{aligned} p_1 &= \frac{\alpha V_1}{\alpha-\delta} \\ p_2 &= \frac{\alpha(\beta-\delta)V_1}{(\alpha-\delta)^2}+\frac{\alpha\Phi+\mathrm{N}}{\alpha-\delta} \end{aligned} \tag{G13}$$

We again assume that information concerning the demand shock can be either noiseless or noisy, in the way given in the table below. For the case of noisy

[1] In De Long et. al. (1989) this is represented as $\overline{\mathrm{v}}$.

information, we assume that the structure resembles that for the mean of the liquidation value, with the outcome taking one of three *possible values (-ν, 0, ν), and* the signal one of two (-ν or ν). This is illustrated in table G1.

*Table G1: Demand shock.*

| Demand shock signal ($\varepsilon_\nu$). | Demand shock (N). Noiseless info. | Noisy info. |
|---|---|---|
| ν | ν | 0 or ν; 1/2 |
| 0 | 0 | n.a. |
| -ν | -ν | 0 or -ν; 1/2 |

## G2.2 Noiseless information

The price in periods one and two when information is noiseless is as follows:

$$p_1 = p_2 = \frac{\alpha\Phi + N}{\alpha - \beta} \qquad \textit{for } \mu > 0 \qquad \text{(G14)}$$

## G2.3 Noisy information

We will assume that the information about fundamentals is also noisy. The alternative market-clearing conditions for period two are:

$$\begin{aligned} \alpha\left(\varepsilon_\phi - p_{2a}\right) &= -\left((\beta - \delta)p_1 + \delta p_{2a} + \varepsilon_\nu\right) \\ \alpha\left(\varepsilon_\phi - p_{2b}\right) &= -\left((\beta - \delta)p_1 + \delta p_{2b}\right) \\ -\alpha p_{2c} &= -\left((\beta - \delta)p_1 + \delta p_{2c} + \varepsilon_\nu\right) \\ -\alpha p_{2d} &= -\left((\beta - \delta)p_1 + \delta p_{2d}\right) \end{aligned} \qquad \text{(G15)}$$

These give the relationship between the price in period two and that in period one for each of the possible outcomes, which can be expressed more clearly as:

$$p_{2a} = \frac{(\beta - \delta)p_1 + \alpha\varepsilon_\phi + \varepsilon_\nu}{\alpha - \delta}$$

$$p_{2b} = \frac{(\beta - \delta)p_1 + \alpha\varepsilon_\phi}{\alpha - \delta}$$

$$p_{2c} = \frac{(\beta - \delta)p_1 + \varepsilon_v}{\alpha - \delta} \tag{G16}$$

$$p_{2d} = \frac{(\beta - \delta)p_1}{\alpha - \delta}$$

The period one market-clearing condition becomes:

$$\mu D_1^{rc} + (1 - \mu)\alpha D_1^p + D_1^f = 0$$
$$\Rightarrow \mu D_1^{rc} + (1 - \mu)\alpha\left(V_1 - \frac{\varepsilon_\phi}{2}\right) + \delta p_1 = 0 \tag{G17}$$

As for the case of noiseless information, the rational investor demand in period two will be as follows:

$$D_2^{rm} = \frac{\Phi - p_2}{2\gamma\sigma^2} = \alpha(\Phi - p_2) \tag{G18}$$

The certainty-equivalent wealths for the four possible states are:

$$\begin{aligned}
W_{2a} &= D_1^{rc}(p_{2a} - p_1) + D_{2a}^{rc}(\varepsilon_\phi - p_{2a}) - \gamma\left(D_{2a}^{rc}\right)^2\sigma_\theta^2 \\
&= D_1^{rc}(p_{2a} - p_1) + \alpha(\varepsilon_\phi - p_{2a})^2 - \frac{\alpha^2(\varepsilon_\phi - p_{2a})^2}{2\alpha} \\
&= D_1^{rc}(p_{2a} - p_1) + \frac{\alpha(\varepsilon_\phi - p_{2a})^2}{2} \\
W_{2b} &= D_1^{rc}(p_{2b} - p_1) + \frac{\alpha(\varepsilon_\phi - p_{2b})^2}{2} \\
W_{2c} &= D_1^{rc}(p_{2c} - p_1) + \frac{\alpha p_{2c}^2}{2} \\
W_{2d} &= D_1^{rc}(p_{2d} - p_1) + \frac{\alpha p_{2d}^2}{2}
\end{aligned} \tag{G19}$$

The expected value of the period two certain equivalent wealth and its variance in period one are given by the following relationships:

$$E\left(W_2 \middle| \varepsilon_\phi, \varepsilon_v\right) = \frac{W_{2a} + W_{2b} + W_{2c} + W_{2d}}{4}$$
$$Var\left(W_2 \middle| \varepsilon_\phi, \varepsilon_v\right) = \frac{1}{4}\sum_i \left(W_{2i} - E\left(W_2 \middle| \varepsilon_\phi, \varepsilon_v\right)\right)^2 \qquad i = a \ldots d \tag{G20}$$

The expected utility of the rational speculators is given by:

$$E\left(U \middle| \varepsilon_\phi, \varepsilon_v\right) = E\left(W_2 \middle| \varepsilon_\phi, \varepsilon_v\right) - \gamma Var\left(W_2 \middle| \varepsilon_\phi, \varepsilon_v\right) \tag{G21}$$

Taking the derivative of this with respect to the period one demand and setting this equal to zero provides one equation in terms of period one price and rational speculator demand. The period one market clearing condition provides a second equation:

$$\mu D_1^{rc} + (1-\mu)\alpha(V_1 - p_1) + \delta p_1 = 0 \tag{G22}$$

Substituting in the relationships between the period two and period one prices and solving gives a value for the period one price of:

$$p_1 = \frac{\alpha}{2} \frac{2V_1(\alpha-\delta)(1-\mu)\left(\alpha^2\varepsilon_\phi^{\ 2} + \varepsilon_v^{\ 2}\right) + \mu\left(4\sigma^2(\alpha\varepsilon_\phi + \varepsilon_v)(\alpha-\delta)^2 - \left(\varepsilon_v^{\ 3} + \varepsilon_v^{\ 2}\varepsilon_\phi\delta + \varepsilon_v\varepsilon_\phi^{\ 2}\alpha\delta + \varepsilon_\phi^{\ 3}\alpha\delta^2\right)\right)}{\alpha\mu\{\alpha - (\beta+\delta)\}\left\{4\sigma^2(\alpha-\delta)^2 - \left(\alpha\delta\varepsilon_\phi^{\ 2} + \varepsilon_v^{\ 2}\right)\right\} + (\alpha-\delta)^2\left\{\alpha^2\varepsilon_\phi^{\ 2}(1-\mu) + \varepsilon_v^{\ 2}\right\}} \tag{G23}$$

The second derivative of the expected utility function is:

$$\frac{\partial^2 \left\{E\left(U \middle| \varepsilon_\phi, \varepsilon_v\right)\right\}}{\partial\left(D_1^{rc}\right)^2} = -\frac{\gamma}{8}\left\{(p_{2a} - p_{2b})^2 + (p_{2a} - p_{2c})^2 + (p_{2a} - p_{2d})^2 + (p_{2b} - p_{2c})^2 + (p_{2b} - p_{2d})^2 + (p_{2c} - p_{2d})^2\right\} \tag{G24}$$

This is always negative.

## G2.4 Special case: supply shock only

The fundamental value of the stock is zero at all times, and so the deviation of the price from fundamentals is simply given by the price itself. The period two prices are given by:

$$p_{2a} = \frac{(\beta - \delta)p_1 + \varepsilon_v}{\alpha - \delta} \qquad p_{2b} = \frac{(\beta - \delta)p_1}{\alpha - \delta} \tag{G25}$$

where:

$$p_1 = \begin{cases} \dfrac{\dfrac{\alpha - \delta}{\gamma \varepsilon_v} - \dfrac{\alpha \varepsilon_v}{2(\alpha - \delta)}}{\dfrac{\alpha - \delta}{\mu} + (\alpha - \beta)\left\{\dfrac{2(\alpha - \delta)}{\gamma \varepsilon_v^{\,2}} - \dfrac{\alpha}{\alpha - \delta}\right\}} & \text{for } \mu > 0 \\[2ex] 0 & \text{for } \mu = 0 \end{cases}$$

The price in period one will always be zero when rational speculators are absent, so the price will be further from fundamentals when they are present. However, the total price fluctuation when rational speculators are present will be the same as when they are absent in the event that beta equals delta, and will only be greater when beta exceeds delta.

# Chapter 9

# Conclusion

## 9.1 General remarks

The failure of existing models in the finance literature to take account of the high degree of uncertainty surrounding the determinants of underlying value and the lack of information that prevents many investors from fulfilling the requirements of the rational expectations literature, has led to an inability to adequately explain certain aspects of market behaviour, such as crashes. In this thesis we have focused on investigating the consequences of relaxing some of the relatively strict informational assumptions made in the literature.

We began with a relatively broad investigation of the art of model-building, focusing particularly on the importance of the assumptions regarding liquidity- and noise-traders. In chapter two we constructed, from the framework of Hellwig (1980), an information aggregation model in which each investor receives information about only one of the factors that determine the payoff, and showed that with no improvement in information quality the informativeness of the price is necessarily reduced as the number of factors is increased, and that there is a finite limit to the number of factors beyond which improvements in information quality generated by the specialisation cannot offset this effect. This shows that the way in which information is distributed in an economy is important in determining the ability of prices to aggregate information, and that this is therefore not determined solely by the quality of the individual pieces of information available and investor characteristics. We found that the model structure has similarities with the A.P.T.-based model of Handa & Linn (1991); but its motivation, and the use to which it has been put, differs significantly.

In chapter three, we constructed a model, again based on Hellwig (1980), in which the investors have private information about the extent of liquidity trading. This is the first model in the information aggregation literature that has investigated the consequences of the price aggregating more than one distinct type of information. We showed that such a structure can lead to multiple equilibria and price crashes. These results did not rely on adverse selection, as in Wilson (1980), or the presence of portfolio insurers, as in Gennotte & Leland (1990), and so provide the first illustration of such a possibility for seasoned-equity markets.

In chapter four we developed the model presented in chapter three, in a similar manner to the way Verrecchia (1982) developed the model of Hellwig (1980), and showed that multiple equilibria can still exist, and crashes still occur, when information acquisition is made endogenous. We also found that for certain specifications of the cost function the overall quality of the information acquired is independent of the cost of information about liquidity trading. The former result supports the results of chapter three by showing that the key results remain when the model is made more realistic. The latter result, although dependent on the particular specification of the cost function, indicates that the availability of information about underlying value plays the dominant role in determining the character of prices.

In chapter five we began to look at the implications of the presence in the market of what could be classed as 'naive' investors, who perhaps do not fulfil the requirements for rationality of the rational expectations literature. We started by looking at (and developing) the simple 'smart-money' model of Shiller (1984) in which the stock demand from naive investors affects prices via the required return of the smart investors. It was shown that when short-selling is restricted, bubbles and subsequent crashes can occur as a result of smart money taking advantage of temporary increases in demand from naive investors to buy on an upswing and sell out at or near the top. We then went on to look at models from the literature that incorporate naive investors whose demand is a function of present and/or past prices, focusing particularly on Hart (1977), and found that the scope for profitable destabilising speculation is heavily dependent on the nature of the naive investor behaviour.

In chapter six we brought together various pieces of evidence - anecdotal, theoretical and empirical - that suggest that positive feedback trading may be a feature of naive investor behaviour. The evidence ranged from the existence of trading rules resulting from portfolio insurance and chartism, through trading on noise and psychological factors, to historical episodes of bubbles and market manipulations. This evidence was extremely important in determining the credibility of the work in chapter seven.

In chapter seven we looked at the link between the extent of positive feedback trading and destabilising manipulation in the presence of unsophisticated value-seeking investors. Using the results of Hart (1977) and the specification of naive investors given by De Long et. al. (1990a), we found a rich variation in the types of market behaviour and the potential for manipulation that can occur with different levels and types of positive feedback trading. It thus appears that the effect of positive feedback trading is heavily dependent on the precise nature of the market structure, again indicating that such behaviour will be difficult to discern empirically.

Chapter seven concludes with an example, based on the speculator behaviour in the counter-example to Friedman (1953) given by Baumol (1957), that shows how manipulation can be both destabilising and profitable under the appropriate conditions. This model was not meant to describe accurately the general nature of stock price movements, but it does indicate a type of behaviour that may become profitable as conditions in the market change, and so may prevail for a short time before observed by others. It may be more generally relevant for individual stocks at times of high levels of positive feedback activity, such as in strong bull markets.

Provided that their market power is not unlimited, competition between manipulators may not completely eliminate this type of behaviour, although there will be a strong disincentive to create the initial price disturbance, and so there may be more reliance on reacting to exogenous shocks.

In chapter eight we used the De Long, Shleifer, Summers & Waldmann (1989, 1990a) framework to investigate further the implications of positive feedback trading, this time in the presence of exogenous shocks. We also attempted to assess, in the light of previous chapters, the De Long et. al. conclusion that competitive speculation can be destabilising, and found that in the presence of shocks there is greater scope for manipulation than there would be otherwise. We also found that the presence of a positive feedback from prices to demand that occurs after a delay is more destabilising than one that kicks in immediately. Increases in the extent of speculation were shown to be stabilising as the market power of speculators increases beyond a certain point, and so there is support for both opponents and proponents of the Friedman position. Perhaps the most surprising result is that competitive speculation can be more destabilising than monopolistic speculation.

## 9.2 Suggestions for future research

### 9.2.1 General comments

This thesis demonstrates that there is much work still to be done in assessing the impact of the nature of information on stock prices, and in modelling the behaviour of uninformed investors. The modelling of positive feedback trading in the latter part of the thesis is necessarily crude, given the state of our knowledge and the available modelling technology, and this illustrates that there is ample scope for future research in these areas. More work needs to be done in dealing appropriately with the uncertainty present in the real world, and with the bounded rationality of investors who do not have the level of information assumed in rational expectations frameworks. There must be scope for integrating psychological factors, including contagion, into more rigorous frameworks. Until these factors are addressed, models will continue only to partially reflect reality.

On a more specific level, we have discovered the need for a multi-period information aggregation framework that does not rule out multiple equilibria in its construction, and more sophisticated ways of modelling the presence of positive feedback trading.

### 9.2.2 Specific models

During the course of this research a number of modelling ideas came to light that were not pursued. We outline some of these below.

*Restricting common knowledge.*

One way in which a lack of common knowledge can be important is through uncertainty about the information that has already been incorporated into the price. Indeed, investors acting on information that has already been incorporated into the price are the 'noise traders' of Black (1986). It may be possible in an information aggregation framework to introduce uncertainty about the proportion of investors who have received an information signal; or perhaps even uncertainty about the timeliness of information individual investors have received.

*Introducing differences in priors.*

Differences in priors could be introduced in an information aggregation framework, and could provide results to compare with those of Pfleiderer (1984) who looks at changes in the quality of information, and Holthausen & Verrecchia (1990) who look at the effects of changes in *informedness* and *consensus*, both under the common prior assumption. As with Harrison & Kreps, it must be clear that the differences in prior beliefs are robust to the sharing of information in such a sophisticated model of learning from prices. It is anticipated that the information aggregation framework would be robust to such a generalisation: a simple possibility could be that there are two groups of agents each with their own estimate of the mean value of the liquidating payoff.

*Endogenous stock supply.*

Following the spirit of Chatterjea, Cherian & Jarrow (1993), we could allow the corporations to which the stock relates to time their decisions concerning the issuing of new stock and repurchasing of stock already outstanding in a framework similar to Shiller (1984). The variations in the return resulting from fluctuations in the holdings of uninformed investors ensure that such timing will enhance corporate value.

## 9.3 Final comments

In extending the information aggregation literature by adding information about the trading of noise traders, we have revealed some of the problems stock market prices will have in aggregating more than one type of information, and provided results that are consistent with empirical evidence such as 'excess' volatility and the occurrence of crashes. The evidence we found in chapter two, of the difficulty prices will have in aggregating information when this is dispersed more widely amongst agents, is consistent with the observation that prices do not accurately reflect the aggregate stock of information present in the economy. These results indicate the benefits that can be gained from relaxing assumptions in realistic ways.

The modelling of positive feedback trading we carried out in the latter half of the thesis is perhaps a first step in assessing the susceptibility of markets to manipulation, and the underlying structure that allows this to be profitable. The result that monopolistic manipulation may be less destabilising that competitive speculation is an interesting one, and deserves further investigation.

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