COGNITIVE - AFFECTIVE INTERACTION IN THE TEACHING AND LEARNING OF MATHEMATICS.

(One volume)

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ABSTRACT: "COGNITIVE-AFFECTIVE INTERACTION IN THE TEACHING AND LEARNING OF MATHEMATICS."

This is a study of extreme emotional reactions to the learning of mathematics. It is conducted among mature adults, mostly well-qualified in some academic area, but with relative failure in mathematics, and showing a marked distaste for the subject.

It first explores, through interviews with twenty-four people, their memories and feelings about their experiences in the mathematics classroom.

Two modes of investigation follow. One is with a group of seven who, with the experimenter, seek both to learn some mathematics and examine their negative reactions to it. The group met for two-hour sessions through thirty-six weeks of an academic year. Side by side with this ran individual studies of three people, who also worked on mathematics, re-experiencing and discussing their fears. These studies lasted respectively through twelve, twenty-four, and twenty-six hourly sessions.

Starting with Skemp's model of intelligence, and in particular its view of the emotions, six situations are defined which inhibit learning. A model of the most extreme reaction, panic, leads to the speculation that it is caused by authority and time pressures.

This is tested, both in large groups and in individual cases. The belief that the explicit avoidance of them can lead to rewarding mathematical experiences is also examined.

CONTENTS

CHAPTER ONE

INTRODUCTION AND OVERVIEW

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The starting point of this study is the recognition of the very great difficulty that many people experience in learning mathematics. In particular it is evident that many who have demonstrated great ability in other academic studies have a 'blind spot' in this area. What is less evident is the strength and depth of emotional reaction that may occur.

The general concern about failure in mathematics over the population at large is of long standing; reports from various bodies, particularly those in industry and commerce, regularly speak of the inability of their recruits to do what they refer to as the most simple of numerical calculations.

The response of those involved in mathematics education initially focussed on a better understanding of the structures of mathematics, and attempts to explain them more clearly. New ways of presenting mathematical ideas were and are being developed. The professional associations such as the Mathematical Association and the Association of Teachers of Mathematics have, through their publications, encouraged good practice, and no teacher need nowadays lack for a variety of approaches to the exposition of any mathematical idea.

Piaget's work points in a different direction and led gradually to a deeper consideration of the way children think. This concern with the cognitive process rather than with the structures of mathematics is developed by Skemp (1971) and in recent work on relational and instrumental understanding.

We thus have two approaches, one starting with mathematics and the other with the way we think. That is not to suggest they are completely independent. It is the standpoint of this thesis that another, relatively unexplored area, plays a large role in learning failures in mathematics. This is the effect of emotional responses, and the influence these may have upon the purely cognitive processes.

In the last few years this has become increasingly recognised, and the review of the literature in chapter 2 details this.

"Intelligence, Learning, and Action." (Skemp, 1980), provides a general theoretical background and a starting point for this present study of the effect of the emotions on learning. A resume of this theory is given in chapter 3, and it is then developed in that chapter to offer a model for the complete breakdown of the cognitive process that those who experience it call panic.

In chapter 4 a rationale is provided for the method of approach, which is compared with that in other research.

The field work is described in chapter 5, and is conducted in an exploratory and investigatory style, without presumptions as to what we may find. The individuals chosen do not represent the population at large, and the limitations of this approach are discussed both here and in our conclusions.

From this field work we adduce a variety of situations (in chapter 6) in which emotions may interfere with the cognitive process.

In chapter 7 we initiate a discussion of the extreme case of 'panic' and suggest ways in which it is brought about. These external causes will be shown to be at least compatible with the extension of Skemp's model given in chapter 3, and to offer reasons for the triggering of the process indicated there.

The external pressures (of authority and time) are then used, both on groups of people, and on individuals, to see whether 'panic' can be induced. This will support the assertion of the ease with which strong negative reactions can be produced (and are often produced unwittingly), and the strength of these responses.

In the final chapter the conclusions are assessed in terms of the various limitations of the methodology. Implications for teaching are given, together with a brief description of some teaching conducted with the implications of this work in mind.

CHAPTER TWO

REVIEW OF THE LITERATURE

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This review falls into seven categories, starting with those that influence the more general positions, and moving towards those that examine phenomena similar to those explored in this thesis.

These categories are:-

1) The relationship between mind and mathematics.
2) Positions in cognitive psychology.
3) Issues in methodology.
4) The nature of understanding.
5) General issues in anxiety.
6) Emotion.
7) 'Math. Anxiety'.

i) The Relationship Between Mind and Mathematics

The most general statement to influence my thinking has been Popper's concept of Three Worlds (1973). This has proved valuable both as a background framework and in the more detailed analysis of chapter 6. The idea of mathematics as an important part of World 3 has been most helpful. It is , however, more appropriate to elaborate Popper's ideas in chapter 4, where its influence on my methodology is explained.

The subjects examined in this thesis had their general cognitive powers seriously interfered with by negative emotional reactions developed long ago, and sometimes in early childhood. Some of these emotional problems derived from an early inability to match the way mathematical issues were presented with their own internal logical processes. It is for this reason that the work of Piaget, Chomsky and Donaldson is relevant.

As Piaget (1952) first showed, in working on conservation of number, a most striking feature of children's behaviour before they conserve is their willingness to count again and again the same objects. Once they do conserve, their evident surprise that an adult should ask, on rearranging the objects, how many are present, is as marked as their previous inability to observe that fact.

It is characteristic of many advances in conceptual understanding that they seem insuperable before they are achieved, and trivial afterwards. The subjects of this study, when they had grasped a new idea, were at first elated but later re-established their low self-esteem by reflecting how simple the matter was, and how difficult they had found it.

In their mathematical education, it is clear that they had been forced into work only appropriate for those at the formal-operational stage when they were not ready. As adults they remained below this stage in mathematics, while showing skill and enjoyment in activities such as timetabling, provided it was not seen as mathematical.

Chomsky's study of languages led him to see an underlying similarity in their structures. He proceeded to the speculation that these structures reflect certain common structures in our brains, and that there is an innate pattern that facilitates the acquisition of language. In "Language and Mind" (pp 74-75) he meets the counter suggestion that all languages may have developed from one original spoken tongue. He argues that this "contributes nothing to explaining how the grammar of a language is discovered by the child from the data presented to him." He goes on to say "The language is 're-invented' each time it is learned, and the empirical problem to be faced by the theory of learning is how this invention of grammar can take place."

Arising from Chomsky's position is the fundamental question of the relationship between mind, language and mathematics. It may be that the mind is not only adapted to the acquisition of language, but to the development of mathematics, in that mathematics, like language, is commonly developed throughout the world. Indeed, it is much nearer uniform than is language. Yet inhibitions in learning mathematics seem much stronger than in language. Certainly early experiences of Elaine, one of my subjects, indicated very rapid language development, accompanied by a very early hampering of mathematical understanding.

In "Children's Minds"(1977) Margaret Donaldson challenges some of the claims of Piaget and Chomsky. She claims (p.36), quoting Macnamara (1972) that the undeniable fact that children learn complicated grammatical forms does not involve a 'language acquisition device', but is based on their skill in understanding situations, to which adults later supply descriptive language. She further argues that children's reasoning is often more advanced than Piaget's experiments with subsets suggests, and that their ability to decentre is not as limited as he supposes. Experiments of my own concerning children's perceptions of 'general' and 'specific' support her view. The same experiments show that some adults have a more limited grasp of these logical connections than some quite young children. In this present study, however, the only examination of this sort of reasoning used the rather sophisticated experiment devised by Wason and Johnson-Laird (1972), in which most adults fail (quoted by Donaldson (p. 80)).

Perhaps most interesting is that my subjects generally exhibited powerful cognitive skills when using language, and even other symbols when it was not perceived to be mathematics. The transfer of these skills to mathematics appeared to be blocked by emotion.

Krutetskii (1976) approaches the issue of mathematics and the mind in a different way. In a closely argued first chapter he asserts that innate abilities cannot be accepted but that innate inclinations can be. In line with most, if not all, workers in mathematical education he agrees that all pupils could learn much more than they do, and that limits cannot easily be drawn. He has to accept that some children learn more quickly in the same environment than others. In chapter 11 he investigates mathematically gifted children and later (p350) analyses the characteristics of a mathematical cast of mind.

2) Positions in Cognitive Psychology

Ryle, in "The Concept of Mind" (1949) set out a position welcomed by materialists as expounding their creed. He writes of the book that its "general trend....will undoubtedly, and harmlessly, be stigmatized as behaviourist" (p327). He also explicitly declares (p328) "that the two-worlds story is a myth." and creates a memorable phrase by denying "the ghost in the machine".

Popper (1977) in "The Self and its Brain." remarks of Ryle (p105)
"However, he also wishes no doubt to deny the (Socratic and Platonic) idea of the mind as the pilot of the ship - the body; a simile which I regard as in many ways excellent and adequate; so much that I could say of myself 'I believe in the ghost in the machine.'."

At a more popular level Koestler in "The Ghost in the Machine." (1976) presents in the first chapter a strong attack on the behaviourist position.

My own position here follows Popper's "interactionism", with the three worlds open one to another, interactive, but with no suggestion that the mind can continue without the body.

In any testing of psychological theory it is important to state whether or not a stimulus-response model is accepted. Clearly some low-level responses might be modelled in this way, but we cannot here accept the general position.

The first substantial criticisms of the approach, which had gained a wide following, appeared in Chomsky's review (1959) of Skinner's "Verbal Behaviour" (1957). In it he attacks the language of behaviourism as being so loose as to cover almost anything, and not therefore lending itself to testing. He further claims that the creativity in language exhibited by young children, in the way they form an indefinite number of new utterances, refutes the stimulus-response model. Later, in "Language and Mind" (1968, p22) he challenges

".......the belief that the mind must be simpler in its structure than any known physical organ, and that the most primitive of assumptions must be adequate to explain whatever phenomena can be observed."

My own standpoint in general learning theory is best expressed in the following description of cognitive psychology versus traditional learning theory, (Farnham-Diggory 1977).

"By traditional learning theory I mean the theories of Hull, Spence, Guthrie, Pavlov, Skinner, and their descendants. These theories were based on the general assumption that a stimulus goes in, a response comes out and what happens in between is summarised by a hyphen. For several decades, American psychological

research reflected this theoretical philosophy. The result, as might have been predicted, was a proliferation of methodological techniques for controlling stimuli and for measuring responses, but no new information about the hyphen. It is important to understand that all this resulted in little new theoretical information about human cognition.
The whole point about the nature of a response and the precision of its measurement is what it tells us - not about the nature of the stimulus and the precision of its measurement - but about the processes in between; the activities in the head of the behaving organism. With the emergence of cognitive psychology in the 1960's S-R technology was finally applied to the understanding of the hyphen. Now, instead of a hyphen, we have mental structures and processes that are gradually being integrated theoretically into a general picture of mind. Traditional learning technology depends upon the systematic impoverishment of natural learning environments. The human organism has been evolved to cope with a rich diversity of stimuli. We have the capacity to sample from a stimulus flux, and to put those bits of information purposefully together. We make something of the welter of information that surrounds us. It is our ability to organise diversity that needs to be educated. Mental structuring is what educators need to know about."

3) Issues in Methodology

A full rationale for the methodology is presented in chapter 4, but certain brief references should be included here.

The concentration on detailed examination of individuals follows the pattern used by Piaget and, more recently by Krutetskii (1976). Many of Krutetskii's conclusions about the nature of mathematical thinking rest on the problem solving activities of individual children whom he has studied. Unlike this present work, he is concerned almost entirely with the cognitive area, particularly in that (Ch.11) conducted with the mathematically gifted.

Most closely allied to my own approach is that of the constructivists, among whom Steffe is prominent; this connection is pursued in chapter 4.

Another influence, which led to the "group" as an important vehicle for investigation, came from earlier work on school management which I had initiated in the mid 60's. I worked with the Grubb Institute of Behavioural Studies on human behaviour in authority structures, and based some elements of the training for headship on issues in group dynamics. A feature of this work was the examination of interpersonal relationships in leaderless groups. After a while I became concerned about the stresses present when there was no overt task, and the design of the group in my study sought to allow emotional responses (which were of course being studied), without the heightened and artificial responses that can be generated by these techniques.

A work of major relevance in this area is that of Bion (1961) on "Experiences in Groups". In the "Preview" (p7ff) he describes working in the psychiatric wing of a military hospital, where by devising groups with an aim and task he was able to establish coherence and purpose. In my group there was a common purpose: to come to an understanding of the origins of their mathematical difficulties and to overcome them. As with his groups, I established clear boundaries as to where and when they met, and the range of matters available for discussion. Appendix B analyses the nature of ~~issues~~ the topics pursued.

In the main section of his book (p29ff) Bion describes his role in therapy groups, where his interventions are mainly observations on the way the group is behaving. It was in using this technique in a course for secondary heads that I first became aware of how powerful feelings about authority could be. With the 'therapy' that my subjects needed, it was unwise to allow stresses to develope that might result in someone leaving the group. In fact, L , one of the individual subjects, did withdraw for a while from her weekly attendances. The exploration of feelings about authority proved eventually very important.

Some of Bion's comments were mirrored in my work in the group. He says (p78)

"It is common at this point to see a group insisting that the doctor is the only person to be regarded, and at the same time showing by its behaviour that it does not believe that he, as a doctor, knows his job."

In both the group described here and in later groups the members constantly appealed to me to confirm mathematical points, and refused to accept my idea that "the authority lies within the subject (mathematics)" When I refused to answer questions, which they felt I should in my role as teacher, they responded either by ignoring me, or by overt complaints at the way I worked. The distinction between Bion's position and mine was that, while I knew more mathematics than the group members, they at least shared my professional competence as a teacher.

4) The Nature of Understanding

Recent work on the nature of understanding has been of great importance. We shall later arrive at a position which sees understanding as being finally secured and established in a manner which has an affective dimension. We shall examine the notion of 'emotional acceptance' of a piece of learning. Prior to that we need to clarify the meaning of understanding as far as we may.

One particular strand, which is of very direct relevance to classroom practice, is the distinction between relational and instrumental understanding initiated by Mellin-Olsen and developed by Skemp (1976) in an article in 'Mathematics Teaching'. The importance of the distinction lies in the greater ability of those who have relational understanding to adapt and make new plans for related problems, while those whose understanding is instrumental have only the ability to obtain routine answers in a specified area.

Byers and Herscovics (1977) took this further in the same publication in an article entitled 'Understanding School Mathematics'. This was followed by my own article 'Four Levels of Understanding' (1978a) in 'Mathematics in Schools.'

Ausubel (1963) sees reception learning in a very different light from the passive posture that is encouraged by those teachers who are insistent on their own authority, and who strongly espouse rote learning. He sees the activity of building new ideas and concepts as demanding, and far from passive. In building schemas an important criterion is their adaptability; new ideas must be worked on before they can be flexibly integrated with existing structures.

Though this activity is not one where the memory is consciously called into operation, it is, paradoxically, the best way to remember. The distinction in the 'mind postures' involved is pursued in "Remembering and Understanding" (Buxton,1979).

There are clear affective responses to reception learning. Often there is initial inertia to trying to grasp something new, but once firmly tackled, the feeling 'Why should I know?' gives way to the response 'It is good to know.' Recently I heard a colleague speak most feelingly to a group of people of the deep satisfaction she got when she 'sorted out' something in her mind.

This is well matched in Ausubel's comment (p.20-21):
"The extent to which meaningful reception learning is active is also a function of the learner's drive for integrative meaning and of his self-critical faculty."

This latter point is developed by Eagle (1978) in an article called "Self-Appraisal in the Learning of Mathematics" in which she argues for the assessment of pieces of work to be done by the student rather than the teacher.

This present thesis will show how people have suffered from the judgements of some teachers - yet one's work must surely be assessed. The points she makes are valuable, and in line with work I have developed in the teaching of adults. Three more quotations from her article will illuminate her approach:-

"The situation in which the answer book and the teacher are ever-present authorities is not a very good preparation for the tackling of real problems."

"Cultivating his own sense of judgement can be for the child both a symptom and a cause of a more balanced view of the task."

"To see a teacher make errors and then cope with them can be reassuring. It also emphasises the importance of monitoring your own work."

This area has also been explored in "What Goes On In The Mind?" (Buxton, 1978b).

The bulk of studies emanating from workers within mathematics education is designed to explain mathematical ideas more clearly, on the assumption that it is only on the cognitive level that difficulties need to be approached. Only more recently has 'maths. anxiety' been directly studied, and we shall shortly look at the literature there. However, before we do, there is some more general work on anxiety, which in passing notes the problem in mathematics.

5) General Issues in Anxiety

Naturally, there is a wealth of material in Freud on anxiety, and its possible springs in early childhood that he demonstrates must form a background to any work on anxiety. However, he does not specify the feeling of 'panic' on which we hope eventually to fix, nor does he refer to mathematics.

Bruner does have some very relevant things to say in a chapter on "Coping and Defending" in "Towards a Theory of Instruction" (Bruner, 1975). He defines the positions thus (p.129):
"Coping respects the requirements of problems we encounter while respecting our integrity.
Defending is a strategy whose objective is avoiding or escaping from problems for which we believe there is no solution that does not violate our integrity of functioning."

It was evident from the subjects of the present study that they regarded mathematics as a minefield of problems -- and that it was in fact, the only subject that overtly claimed to be about 'problems'. They spoke of being unable to cope and were often in full flight from mathematics.

Bruner arrives at the conclusion that (p.148):
"in a word, then, coping and defending are not, in my opinion, processes of the same kind that merely differ in degree. They differ in kind."

This is supported by the polarisation of attitudes about mathematics, with relatively few at the positive pole.

He describes work with children disturbed for reasons other than the learning of mathematics, and says of them (p131):

"They could not, in short, cope with the demands of school-work unless and until they were able to defend themselves against the panic (my underlining) of impulse and anxiety that the demands of schoolwork set off in them."

These children were not cognitively weak. Judged on any criterion of cognitive ability it seems certain that the tasks were well within their range. He describes (p.137) a child (IQ 125) with clearly defined emotional attitudes to certain mathematical topics.

"He dislikes fractions, for example, and cannot work readily with them, for he sees them as 'cut-up' numbers. The elementary operation of cancelling in algebra symbolises for him the act of 'killing off numbers and letters on each side of the equals sign'."

Bruner describes a therapeutic approach to a child who had been yelled at by a teacher (in reading, not mathematics). His tutor made a joke of the whole thing by trying to yell louder at him than had the teacher, and then reversing roles as to who yelled at whom. This led to greatly improved learning by the child while with the tutor. Whether he could have then been able to face the rigours of a classroom similar to that which caused the problem is not clear.

In some very direct work on learning difficulties Hart (1980) provides a wealth of information concerning the performance of children on specific mathematical questions. Of great interest is the demonstration that in some areas, such as ratio and proportion, there is very little advance in understanding over the five years of secondary school, while in computation it does not advance at all. The extremely low facility shown by pupils on a range of topics regularly taught in all our schools raises serious questions. It may well be that some should not be taught at all.

These results lead us to wonder about the cognitive and affective difficulties these children experienced. There is much valuable material in the individual responses she gained from children in the interviews, and the data lying here can tell us much of the reasons why they performed so badly.

6) Emotion

The issue eventually discussed in this thesis is one specific reaction, that of panic. Naturally, many different reactions were offered by subjects, and it would be an interesting, but very extensive investigation to try to assign certain emotions to certain classroom situations. I have done some work in this field, branching off from this particular study. In such a pursuit, and to some degree in this present work, it would have been helpful to have some taxonomy of emotion, but little seems available.

Nicholson (1977) discusses "worrying" (Ch. 17) and equates worry with anxiety. A taxonomy would help in deciding whether this is an appropriate identification. He makes the unsurprising statement that some worry is necessary to get anything done, but that carried beyond a certain point it is unproductive.

In fact this last comment is a specific example of the well-established Yerkes-Dodson law, which states that the optimal degree of motivation for a given task decreases with the complexity of the task.

Being chased by a bull in a field is strongly motivating and leads to good performance in running away! Opening the gate to get out may be done less well with the bull approaching than without. Running is a task of low complexity, even if we do not do it well, but finding out how a lock works is certainly a level or two higher in difficulty.

Doing mathematics is a high level task, and strong external pressures are not conducive to good performance. Zeeman (1977, p.613) sees learning as an example of catastrophe theory, where an increase of pressure beyond a certain level results in a sudden and dramatic drop in performance.

Nicholson (1977, p.185) has some comments, however, which do bear upon the present work:-

"In its extreme form, anxiety may be experienced either as the generalised, free-floating state mentioned earlier, or it may be focussed specifically on certain objects or situations. Most people have experienced the former - the taut muscles and dry mouth accompanied by a feeling of agitation, dread, or even panic."

The subjects studied here did react to specific situatons, and the reaction was one of panic.

We shall later hear from subjects their views as to the strengths of various emotions, and to the connections between them, as suggested in this last reference. They do see the movement to panic as sometimes (but by no means always) involving a progress through anxiety to fear and then to panic.

The nearest to a taxonomy seemed to be "The Language of Emotion." (Davitz, 1969). In this work he describes the clusters of words used by his subjects to describe their emotional reactions to various situations. I have done this with groups myself, and like Davitz, obtained clusters of words describing experiences they were asked to recall. The result is more like a thesaurus than a taxonomy, but it is an area worth pursuing.

7) '<u>Math. Anxiety</u>'

At this stage the known references were supplemented by a computer search (ERIC). One such search sought the intersection of "Learning Disabilities" and "Emotional Problems". Though it provided 128 references, none proved suitable. They tended to be too heavily psychotherapeutical, and to discuss learning problems proceeding from maladjustment.

As expected, a search for "Math. Anxiety" produced many appropriate references. The 187 references were reduced considerably using various criteria:-

1) Preference was given to those in research level journals
2) Rating scales were not generally included.
3) Those whose emphasis was remediation without a clear theoretical background were omitted.
4) Repeated references to a particular author (such as Tobias) were condensed.
5) Though some on 'attitude' are included, this was sometimes too general.
6) Gender in mathematical achievement is of great importance, and some references pursue this line, but it is not seen as central to this present study.

The Cockcroft Report (1982) includes a study of the mathematical needs of everyday life (Sewell). Carrying out the study did not prove easy. In her words (sect 16)

"Both direct and indirect approaches were tried, the word 'mathematics' was replaced by 'arithmetic' or 'everyday use of numbers', but it was clear that the reason for people's refusal to be interviewed was simply that the subject was mathematics....... Several personal contacts pursued by the enquiry officer were also adamant in their refusals. Evidently there were some painful associations which they feared might be uncovered. This apparent widespread perception among many adults of mathematics as a daunting subject pervaded a great deal of the sample selection; half the people approached as being suitable for inclusion in the sample refused to take part."

The main report then adds:-
"The extent to which the need to undertake even an apparently simple and straightforward piece of mathematics could induce feelings of anxiety, helplessness, fear and even guilt in some of those interviewed was, perhaps, the most striking feature of the study."

Most germane to the research undertaken in this thesis is the following (sect.21)
"The feelings of guilt to which we referred earlier appeared to be especially marked among those whose academic qualifications were high and who, in consequence of this, felt they 'ought' to have a confident understanding of mathematics, even though this was not the case. Furthermore, they were aware that others, to

to whom it was evident that they were well-qualified in general terms, took it for granted they would be mathematically competent. 'People assume you are good at maths. if you are good at other things.' Those who were not academically well-qualified did not feel guilty in the same way. Some arts graduates who had gained O-level passes in mathematics were nevertheless so aware of a lack of confident understanding of the subject that their career choices were seriously reduced as a result of their determination to avoid mathematics."

The term 'Math. anxiety' was coined by Tobias (1978) in her book "Overcoming Math. Anxiety". The evidence of her subjects accords very directly with that offered by the subjects in this thesis, and many of her quotations have a familiar ring. It is interesting to note that her work was probably done at the same time as the investigations described here, which were pursued through the academic year 1977-8. The publication of her work and of my own popular book (Buxton, 1981) have both stimulated much response from readers who confirm the nature, depth, and extent of the emotional reaction that people may have to the study of mathematics.

Tobias' work makes a strong point about the way girls and women are deflected from learning mathematics. It is clear this happens, yet certain arguments she puts forward are stronger in the U.S. context than here. Through the elective system in the U.S. girls can give up mathematics much earlier than here, where they nearly all pursue it to the school leaving age. Yet girls still perform worse at public examinations. The gender issue dominates her writing. I accept it here as an important issue, but it is not the issue of the thesis. I believe that serious anxiety about mathematics is present in many men, even if they tend to conceal it more.

Tobias and Weissbrod (1980) produced "An Update" on maths. anxiety where they argue for new techniques more closely linked to theories of learning. In this, as in the earlier work, theory is applied to remediation rather than to root causes, but some important points are made. She quotes (p66) various researchers who are not sure that maths. anxiety is more common among females than among males; this certainly seems an open question.

In connection with the 'workshops' (which seem to offer less structure than the group in this work) she comments (p.67):

".....there is no typical technique, but there is a common goal, namely, to change the classroom atmosphere from one of tension and competition, and a resulting unwillingness to ask 'dumb' questions, to one of trust."

This aspect of trust, and the assurance that all statements made by subjects will be respected has been an important issue in my work and is an essential feature of the 'textured learning' method I have developed from the work of the group.

Another point made in this article (p67) is supported by comments in this present study:-

"Even after getting the right answer, a learner might dismiss her success by saying 'It was easy' or 'It wasn't really math.' or 'I could never do it again.'

Perhaps the most significant comment in this article however, relates to the need to recall the past (p68)

"Throughout these interventions, the subject is invited to re-experience past failures and humiliations before the group or while writing the math. diary. The purpose of this is to eliminate the continuing and disabling effects of earlier experiences. The strategy assumes that it is the inter-personal dimension of mathematics that is at issue in math. anxiety, and not mathematics itself."

Michaels and Forsyth (1978) distinguish, very properly, (as does the last quotation) between attitudes to mathematical topics and to methods of teaching maths. The eventual conclusions of this thesis emphasise the effect of authority pressures, which are clearly inter-personal, and do not lie within mathematics. In assessing attitude, Michaels and Forsyth advocate simple wording of questions; this may well be necessary in broad surveys, but the danger of simple questions is that they may elicit only superficial answers.

Sandeman (1979) used his own attitude inventory in testing anxiety over some 413 classes. He came to the conclusion that boys have higher maths. anxiety than girls. At first this may seem surprising, but it may well be that the girls are more able through 'math. avoidance' to escape from the anxiety, while boy-girl stereotypes in career choice lead the boys to believe that they cannot escape in the same way.

Hilton (1980a) makes some important distinctions, and recognises how severe some of the reactions can be. He comments (p175):-

"In some cases the avoidance is the deliberate, conscious, and well-worked-out procedure of a mature individual who has coolly assessed his or her capacities; sometimes, at the other end of the spectrum, mathophobia is a panic reaction." (my underlining)

He goes on to point out that math. avoidance, math. anxiety, and math. incompetence are all different. Later in the article, he categorises in the following manner (p177):-

"I distinguish three primary classes: Class A consists of those who, as a result of their mathematics education know some mathematics; Class B consists of those who know they don't know any mathematics; Class C consists of those who don't know they don't know any mathematics..."

Hilton continues:-

"I further divide Class B into three subclasses. Class B1 consists of those who react to their ignorance of mathematics with defiance......Class B2 consist of those rendered timid by their awareness of their mathematical ignorance. It is in Class B2, then, that we typically find those who are both anxious about mathematics and seek to avoid it. These people, like all those in Class B, are unable to function in any situation in which some understanding of mathematics and ability to use it are required, but they are distinguished by feeling disquiet as a consequence.... Class B3 consists of those who are indifferent to the fact that they know no mathematics. They are aware, but they do not care."

Our main concern here is with Class B2. It is an interesting classification; I would accept the existence of B3, but feel that it is seldom or never that the defiance exhibited by those in B1 is a genuine denial of the importance of mathematics.

In Class C he sees some who may become aware of their deficiency and then enter Class B2. The remainder of them, Class C2, he characterises thus:-

"The members of this class tend to be entirely insulated from awareness of their ignorance. They are likely to be confident people and, in adult life, they are apt to be found in many positions of authority. Protected from an awareness of the abyss of ignorance in which they pass their lives, they will not hesitate to take decisions affecting the lives of other people as well as themselves."

We must hope that not too many mathematics teachers fall into this last category.

He next touches (p182) on the ill effects of an education based on memory rather than on reasoning.

".....There thus begins an inexorable process in which the reasoning faculty lapses into disuse and the memory becomes overloaded..... It is a supreme irony that the measure of progress in any science is its independence of the memorisation of brute fact, yet such memorisation is endemic in current education."

Certainly many of my subjects complained of overload in their own education. It is interesting that this was often particularly attached to the learning of theorems in geometry, where it is possible to limit memory greatly by an emphasis on process.

Bearing very directly on this thesis is his observation (p185)

"(e) Authoritarianism. Typically in a traditional mathematics education, students are told what to do, told that it will turn out to be useful, told that it works, told that they will understand it later. They learn early on the advantage of accepting argument by authority...... It is a superb irony that it is in mathematics education, so called, that students learn to discount the value of the reasoning process. It is natural that this authoritarianism breeds anxiety and distaste."

All of this is strongly supported throughout this present study by the reports from my subjects.

A study by Betz (1977) examined the prevalence and intensity of math. anxiety in college students. The students were selected from mathematics and psychology courses at the Ohio State University, and even though they had elected to take these courses, math. anxiety was frequent, and more likely to occur in women than in men.

Contrasting with this study is one by Chavez and others (1982) which showed that most elementary teachers do not dislike mathematics and that there is no significant difference between male and female attitudes to mathematics.

While the two sample populations are not the same, it is surprising to see this difference. As with much testing, it may be the nature of the questions asked that is important.

Sovchik (1981) accepts the existence of anxiety in those recruited to courses for elementary teachers, and describes pre-service courses designed to reduce anxiety. Kogelman (1981) describes a remedial program at the Bronx Community College, serving a black and Hispanic population. The population is very different from that used in this thesis, though some of the techniques, aimed at self-esteem as well as new cognitive approaches are similar. Chapline's attempts to reduce mathematics anxiety, described in a paper (1981) by her, shows a highly structured program with frequent requests to students for feedback and evaluative reactions. It is this aspect that is significant in overcoming the authoritarianism described by Hilton.

All these programs tackle the need for a trusting, non-authoritarian climate, with problems discussed at length, and an emphasis on understanding rather than on memory. One of the best descriptions of the situation students find themselves in is given by Morris (1981). Her descriptions of some of the more severe states is exactly in line with some of my subjects. She writes (p413ff)

"Physically you might have experienced sweating palms, clenched fists, queasy stomach, dry mouth, and cold sweat. Psychologically you might have felt <u>panic</u>, tension, helplessness, fear, distress, shame, and an inability to cope." (my underlining)

"The math. anxious are especially sensitive to criticism and would rather sit through a whole class without understanding than to risk ridicule by asking a 'dumb' question."

"For the math. anxious, then, mathematics becomes a rigid, authoritarian subject consisting of rules to be memorised and obeyed, and of formulas to be applied blindly. Memorisation replaces understanding."

"For example, consider timed tests. Many, especially the anxious, tend to freeze up under pressure."

A most unusual approach is developed by Elliott(1982) who examines a neurological approach to the issue. She first postulates that anxiety and the necessary creativity in working at mathematics cannot exist side by side. Taking creativity as being a function of the right hemisphere of the brain, she writes that it can only function when there is harmony between the two hemi-spheres; anxiety is a function of disharmony.

In the same terminology used by Bion she writes(p779) "So many of them set limits on themselves and their mathematical abilities by trying to <u>fight</u> learning mathematics or by trying to <u>flee</u> from it entirely." (her underlining)

In discussing psychological profiles of the math-anxious she writes (p779)
"otherwise successful and intelligent people 'go blank'

when they see problems involving decimals, or 'freeze' when asked to figure a 15% tip or 'panic' at the sight of a story problem."

All three of these statements - 'go blank', 'freeze' and 'panic' are seen in this thesis as a mechanism explainable in terms of Skemp's model of intelligence.

Elliott picks up Kogelman's suggestion that convergent and divergent thinkers both develope anxiety when confronted with the need for the other type of thought. Since mathematics requires both forms of thinking, and they may be associated with the two hemispheres of the brain, anxiety occurs in both types of thinkers. This is certainly a plausible suggestion, and would be linked with convergent thinking representing an acceptance of authority and divergent a rebelling against it. Since girls are encouraged to be conformist, they perform well when the tasks are convergent and routine, but fail when asked to break rules. Boys, although punished for breaking rules, are nevertheless expected to do so; this conditioning may lead to their greater creativity and better performance at problem-solving in mathematics.

Bearing directly on 'Situation 2' of chapter 6, where we have a pathological response to mathematics is the following passage (pp780-781):-

"Self-professed math-anxious students reveal that one unsuccessful experience after another form the foundation on which they have attempted to build their mathematical knowledge. With rote-memorised algorithms serving as the mortar for thes constructions, it is little wonder that, during the first problem-solving storm, their mathematical knowledge topples. Successive failures produce negative physiological arousal patterns which give rise to negative psychological behaviours; these include defensive attitudes, fearful feelings and, finally, total avoidance. The psychophysiological downward spiral is set in motion. Physiologically speaking, the math-anxious person reverts to the primitive survival instincts which have their origin in the mammalian and reptilian portions of our evolved human brain. The mammalian centre, called the limbic system, has two primary functions which affect math. anxiety: (1) it stores long-term memory, and (2) it prepares the body for 'flight or fight'.

Memories of a string of unsuccessful experiences are stored in the hippocampus of the limbic system. The amygdala (another limbic system) is triggered and the math-anxious person goes into a 'rage' when asked, 'How much time will it take two men to dig a ditch when.......?'. Finally, the central structure in the limbic system, the hypothalmus, is activated and the fight or flight mechanism is set in motion. Generally, the math-anxious person may have sweaty brows and palms, increased heartbeat, increased respiration, and desynchronisation in cortical and subcortical brain waves."

This is a most interesting extension into physiological terms of the psychological interpretations and models we have used. While expressed differently, the terms are compatible with, and characterise the same experiences that we have described in other ways.

One of the most important elements in alleviating acute symptoms is the removal of guilt. Elliott says of mathematical incompetents (p783):-

"They feel the onus of this inability to do mathematics lies with them. They never suspect that teachers or teaching strategies could be at fault. They believe there is something 'wrong' with them."

A very interesting, and directly relevant piece of work is by Hoyles (1982). In it she elicits from 14 year old pupils their perception of good and bad learning experiences in school. Although they were free to comment on any part of the curriculum, about 40% were concerned with mathematics, many more being bad than good. The pupils were asked to describe the situation, the feelings, and the reasons for these feelings. The individual interviews are very revealing. Hoyles analyses feelings within self, feelings towards the outside world, and feelings about self. In these the pupils reveal the same attitudes that the adults of this study showed.

Certain results point up the effect of time and authority pressures. She writes (p364-5):-

"One further finding of interest was that nearly 22% of all bad stories contained statements categorised in the sub-category called Teacher Pace, Pressure."

"The pupil might also have described how the teacher put him under pressure by, for example, demanding instant answers or telling him to work something out in front of the class. It should be noted that the statements in this category were about pressure thought to be imposed by the teacher. They were distinguished in the categorial scheme from statements describing either pressure thought to emanate from the work itself and its level of difficulty or pressure felt because of lack of confidence on the part of the pupil himself. The statements were also concerned with teacher-imposed pressure which was seen to be associated with learning and were distinguished from statements concerned with teacher-imposed pressure, seen to be associated with discipline or control."

This last distinction, between the authority of control, and authority in the teaching, is an important one.

Conclusion

There is much literature in the general area, but little that touches upon the most specific issues discussed here. The general reports from subjects in all the research projects are very similar, and support the evidence given here. Theory, however, seems to enter only in the remedial teaching, ahd not in discussions of root causes, save in Elliott and Hoyle.

However, many references specify panic, without analysing it. Such references appear in Bruner, Nicholson, Hilton, Morris, and Elliott. One of the subjects in Skemp's doctoral thesis (1958) reported 'waves of panic' when given a timed test.

In chapter 6 a general analysis is given of situations causing various degrees of anxiety. In particular, the second such situation can be read in conjunction with the quotations from Elliott.

From there an analysis of panic is given based upon the Skemp model. Despite the relevance of some of the authorities quoted here, it is upon this model that the theoretical basis of this work is founded. We there now proceed to examine this, devoting the next chapter to it.

CHAPTER THREE

SKEMP'S MODEL OF INTELLIGENCE

SKEMP'S MODEL OF INTELLIGENCE

This work is based on the model put forward in "Intelligence, Learning and Action." (Skemp, 1979) There follows a summary of those elements in his theorising relevant to this thesis. We start with his general view of intelligence, note the original way in which he links the cognitive and the affective, and then interpret, in terms of his model, the particular dysfunction known as 'panic'.

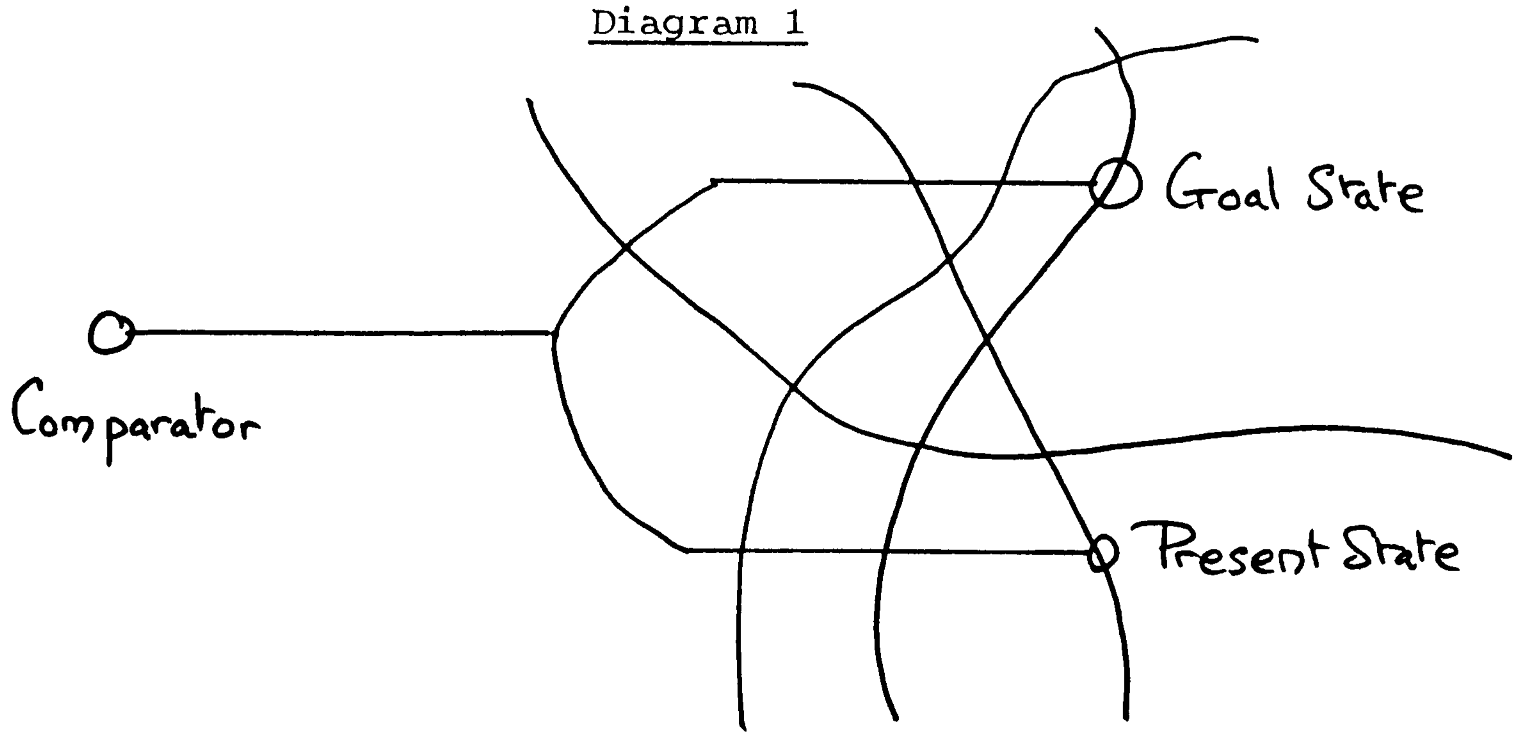
Skemp sees many of our activities as goal-directed (though others are simply expressive). In seeing these as to some marked degree under our own control, he adopts a very different stance from the behaviourists. The extent to which we are goaldirected depends on our definition of a goal, but certainly in the pursuit of mathematics goals (all too often set by others) are constantly present.

The central activity of intelligence is seen as the building of mental models. It is the quality of these models and of the plans we devise using them that distinguishes us from other living species. Skemp sees these activities as strongly linked with survival. Certainly these abilities have led not just to survival but to our present dominance over other species.

The next notion we need is that of a director system. This is an idea which owes much to cybernetics and has analogues in thermostats and stabilising devices. The director system (delta) contacts the environment through sensors, and has a comparator which records the relationship of our present state to our goal state (where we want to get). A thermostat must record both present temperature and desired temperature. That does not mean than the human model is mechanistic, for it is the way we choose and reach our goals in which we differ from these devices.

In order to determine where we are and where we should be, we make mental models (schemas) on which we plot these positions.

Diagram 1



The comparator records the present state and the desired state on the schema in our mind. It is convenient to think of schemas as cognitive maps; such geographical analogies are deeply woven into our language.

Our ability to achieve goals depends on two features

(a) the quality of the schemas we have

(b) our skill in making plans to get between points in them.

We can think of this as defining our 'intelligence', but it is important to recognise that it is not measurable, and that in any individual the ability to achieve goals depends very heavily on the area of activity.

We now distinguish between the exercise of our intelligence in dealing with the outside world and its function in building schemas and making plans associated with them. We shall refer to delta-one when our reason is tackling the outside world and to delta-two when it is modelling and planning internally.

Delta-one exhibits three levels of function

(1) Instinctual, or genetically determined responses

(2) Automatic responses, once learned but not now thought about.

(3) Responses to new situations.

The first category includes the more basic response to hunger and sex drives, though we have developed more complex learnt ones in these areas. This category we share with all other animals.

The second category is learned, but not necessarily thought about any longer. When a telephone rings, my hand stretches out to answer it. When a child runs in front of my car I brake sharply. In neither case do I 'think' about what I am doing, yet the reactions had to be learned at one time. The plans for these actions were once devised by internal cerebration (delta-two) but are now immediately available for delta-one in its dealings with the outside world.

Some actions that we wish to improve, such as our golf swing or our skill in cornering in a car need regular thought and practice, with delta-two fully engaged. Yet to 'think' too precisely on a dangerous corner, or to seek to improve a swing when we are in a match is a recipe for disaster. The aim of delta-two is to make delta-one automatic.

We shall see that many of my subjects had been taught on the assumption that mathematics was made up largely of such skills and that the aim was that they should respond quickly and accurately when called upon to do so.

In a completely new situation, delta-two might need to create a schema and then devise a plan. More commonly, a schema of some sort exists, but a new plan is needed urgently.

As an everyday example, when we make a regular car journey we do not 'think' where we are going. We are able to leave delta-one to conduct the intricate business of controlling the car, avoiding standard hazards, and getting where we want to go, while we may be 'thinking' about something quite else. This latter activity is by delta-two. However, if a quite unusual danger presents itself, our consciousness switches to this danger, and delta-two devises plans for its avoidance, which delta-one then operates. It may be as simple a matter as a new 'no right turn' sign. We immediately move delta-two from its unrelated activity to drawing up a new plan as to where we go.

The student in the classroom is in a very demanding environment,
one where new schemas and plans are in constant demand. The aim of the teacher must be to aid the formation of schemas and to offer opportunities for the student to plan routes. In the first of these, particularly if there is a strong didactic element, the student is engaged in reception learning, and in the second in problem solving. We shall see in our investigation the crucial importance of allowing delta-two to do its work without constant demands being made from the external world through delta-one. Many of my subjects complain of 'demands' being made.

To sum up then as far as the deltas are concerned; they are distinguished by their 'operands' (that on which they operate) - with delta-one dealing with external matters and delta- two with internal formulations. Delta-one ultimately is highly dependent in its performance on the quality of the work done by delta-two, particularly in 'learning', with which we are here concerned.

Of central signifigance for this thesis is the atarting point Skemp offers for the role of the emotions. Freud used the phrase 'hedonic tone' to describe our position on a spectrum pleasure - unpleasure,determined by our satisfaction or otherwise of basic drives such as hunger or sex. The following comment (Koestler, Arthur. (1976) "The Ghost in the Machine.". Hutchinson. London.) extends the definition and matches the position in Skemp very closely (p.261):

"The hedonic tone depends on several factors and could be described as the feedback report on the progress or otherwise of the drive towards its real, anticipated or imaginary target."

Skemp, as we have said, sees us as goal-seeking organisms, and supplements this idea with the extra concept of an 'anti-goal' or threat. We pursue goals and try to avoid anti-goals.

The 'comparator', which assesses our approach or recession from a goal has outputs of pleasure or unpleasure accordingly. It is possible, as my subjects showed, to have within these broad terms, more specific feelings related to other elements in their situation. For example, the knowledge that they were on the verge of a solution of a problem that had caused them some considerable difficulty gave rise to 'elation'; failure, to a variety of responses such as frustration, anger or a very deep disappointment. The analysis of what emotions are experienced in what situations will prove a large field for the future.

An anti-goal is not simply the failure to meet a goal. A mathematics problem in the classroom offers a goal (though if set by the teacher as such, but not so accepted by the pupil, we may question whether that is indeed what it is), but failure to solve it may not be in any sense an anti-goal for the pupil. His or her anti-goal is the disapproval shown by the teacher and the possible punishment ensuing. Skemp sees the output from the comparator at the approach of an anti-goal as being fear, and the avoidance of it as being relief.

It is worth commenting here that if goals are presented, and are achieved at least some of the time by the student, the very pleasant experiences such as elation will be valued and more learning attempted in the hope of repeating them. If however the mathematics classroom is seen (as it was by my subjects) as being a minefield of threats, the only pleasurable experience is one of relief at their avoidance. Relief, unlike elation, is not something most of us set out to experience.

We all of us have certain areas where we know ourselves to be competent, whether it be in gardening, driving a car, or in swimming. Skemp refers to such a part of our environment as a 'prohabitat', and goes on to characterise the feelings we have here as 'confidence' (arising from our belief that we can achieve goals) and 'security' (in that we think we can avoid anti-goals). Outside a prohabitat a person may feel 'frustration' in the expectation that goals will not be reached and 'anxiety' in that threats may not be avoided.

Some of my subjects, as we shall see, experienced very strong and unpleasant feelings once the area was characterised as mathematical, even before a task was proposed.

We now present this in table form (p81. "Intelligence Learning and Action")

	State perceived as changing	Signals from comparator	Knowledge of ability to change state	Knowledge of inability to change state
Goal state	Towards	*Pleasure*	*Confidence*	*Frustration*
	Away from	*Unpleasure*	-	-
Anti-goal state	Towards	*Fear*	-	-
	Away from	*Relief*	*Security*	*Anxiety*
		Signals that organism is	within prohabitat	outside prohabitat

Table 1

Skemp sees the emotions emanating from the comparator as signals to us which attract attention to a particular area where action may be needed.

This is clearly a function, though I do not share his view that it is the only or even main one. However, once an alarm is raised about an approaching anti-goal, the fear induced will cause us to try to solve the problem of avoiding it, and in this both delta-one and delta-two may be involved.

This direction of the consciousness by the emotions may be counterproductive to learning. In chapter 6 we analyse six different ways in which the signals from the emotions inhibit learning.

Let us now interpret in terms of the model how we cope (or fail to cope) with threats from the environment. The appearance of an anti-goal leads the comparator to emit signals (emotions). This attracts consciousness to delta-one, which then seeks to take avoiding action. This may be as mundane as avoiding collision with another pedestrian. This is a routine task accomplished (with a background of long experience) by delta-one without the formulation of new plans. Here the threat is minor and its evasion simple. Even a very severe threat (say a wild animal in the room) may be avoided by an obvious action (jumping out of the window) in which no new sort of plan is needed. In a number of cases, therefore, delta-one copes without referring to delta-two, and it is not the level of threat that determines this.

The appearance of a 'No right turn' sign preventing us from taking a usual route is a minor worry, yet we may have to think quite hard as to what we should now do, and delta-two may be fully engaged in the preparation of a new plan. This is a normal functioning of the two deltas; when the first has no ready-made plan, it is the job of the second to prepare one.

The major problem arises when the threat is seen as serious enough not to want to suffer the consequences of ignoring it, when delta-one does not have a routine, and when delta-two fails. It may fail:-

(a) because it cannot make a plan.

(b) because it cannot make a plan <u>in time</u>.

(c) because it has <u>no expectation</u> of being able to make a plan.

The threat directs the consciousness into delta-one, which then refers it to delta-two. When a plan does not result, the emotions generated through the comparator again bring the consciousness to delta-one, asking that it deal with the threat, but again it is referred to delta-two. The process then becomes a flicker of the consciousness between the two deltas; the result is paralysis and a state of complete dysfunction.

This state - referred to by one subject as 'mind in chaos' - is what we shall call in terms of the model 'Panic'. It is of course what many of my subjects have experienced, and the sensations attached to which are discussed in the second session of the 'group' in the chapter on the field work.

Let us illustrate this from the classroom. A teacher poses a question in mathematics. He or she may see this as the calm setting of a goal, and this view will be shared by some of the pupils. But some will most certainly see it as a threat - not the question, but their expectation of what the teacher will do when they fail.

If the question requires a routine memory or rote-learnt answer, then delta-one should have that to hand, and the threat is avoided. It may be that the answer is not known, but that delta-two manages to work it out, and again the threat is avoided.

When, however, delta-two fails, either through lack of cognitive power, or through the pressure of the teacher standing waiting, or, as with many of my subjects, a feeling of complete hopelessness at tackling any such problem (being out of their prohabitat), then many have described the state of mental paralysis, (that they name 'panic') which ensues.

Though, en route, we shall discuss various forms of malfunction in the reason-emotion complex, it is the creation of this state that we shall finally examine. We shall use the material from the field work to speculate on the conditions that induce it, and demonstrate that we can artificially bring it about with the use of certain pressures.

CHAPTER FOUR

A RATIONALE OF THE METHODOLOGY

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In this chapter we first discuss distinctions between theories in the natural sciences and in psychology. We then observe upon the nature of models, and in particular that used (and developed) as the theoretical basis to this work. We then look at the different methodologies used in psychological studies and their relationship to the theories and models which they explore. Against this background we develope the rationale for this particular study.

The evident power and success of the approach used in the natural sciences has led to the assumption by some that it is appropriate for all investigatory work. This assumption needs to be questioned.

We use a helpful classification by Sir Karl Popper (Popper,1973 p20ff). He postulates three "worlds". World 1 is the physical world of concrete objects, including our own physical bodies (and their brains). It also contains less tangible objects such as waves and forces. World 2 is the psychological world, of thoughts and feelings, of dispositions to act, and all kinds of subjective experiences. The position is dualist, but Popper asserts the importance of the fact that they are interactive (Popper, 1973 p25.).

His innovation is World 3. This is the world of human utterances, our words (written or spoken), our art and our theories of all sorts. There is no criterion for admission to World 3 other than utterance. Truth or validity of a statement is not relevant. Following his definitions, Popper points some distinctions by considering two copies of the same book. In World 1 they are distinct objects; in World 3 they are a single utterance. It may well be that two different people on reading the book may arrive at different conclusions as to its meaning, but it remains a single utterance, though replicated in World 1.

Theories about the physical world and about psychology both lie in World 3. Yet there is a crucial question to be asked - what are they about? The natural sciences are about World 1; psychology is about World 2. The fact that they are about essentially different things is important. It may well mean that we have to approach them differenly.

The next question is how theories in these areas are to be tested. Scientific theories are eventually tested in World 1; those in psychology are not. How then must we test them?

Popper clearly defines the characteristics of a scientific theory, the main criterion being succinctly put by Brian Magee thus:-

"Falsifiability is the criterion of demarcation between science and non-science." (Magee 1973 p43)

This means that to be scientific a theory must make predictions which subsequent tests could show to be false. The strength of Einstein's General Theory of Relativity lay in the fact that it made three clearly defined predictions of astronomical occurences which were different from those made by Newton's gravitational theory. Though the differences predicted were in fact quite small, when the opportunities arose to make suitable tests, Einstein was found to be more accurate in the predictions made than Newton. This led to the abandonment of Newton's underying theory (but not of its use in many calculations). The results supported Einstein's theory, though it is important to realise that no observations prove a theory. It is the nature of science that its theories cannot be proven.

Only an extreme behaviourist would believe that the actions of human beings can be (or are ever likely to be) predictable in the way that we predict the positions of the planets or the balls on a snooker table. It is not merely that we are more complicated, but if we assume some degree of freewill we may well exhibit different responses at different times to the same stimuli. Essentially we are not predictable in the way that scientific theories (despite the Uncertainty Principle) demand of snooker balls.

In psychological theories we are not likely to be able to predict events with any certainty. We hope at the end of this thesis to demonstrate that certain pressures

are likely to evoke a specific response in some people. Such theories are not 'scientific' in Popper's sense. In fact he would not regard Freudian theory as 'scientific', though that is not to imply he would think it unimportant. As he comments (Popper 1963 p37)
"This does not mean that Freud and Adler were not seeing certain things correctly:I personally do not doubt that much of what they say is of considerable importance,.."

Skemp's views, while expressed in a rather different form, make many of the same distinctions. For him there are Type 1 and Type 2 theories. (Skemp 1981 p28ff). Type 1 theories are general mental models of regularities in the physical world. He sees them as important in providing delta-one with bases for achieving goals in the physical environment.

A Type 2 theory is a model of the ways in which Type 1 theories are constructed. If we study how someone learns (or in this thesis, fails to learn), and make a mental model of that process, we are making a Type 2 theory. An important aspect of Type 2 theories is that they must allow for a considerable degree of autonomy in the subjects studied. The position adopted is similar to that arising from Popper's classification.

Skemp sees theories as very general models. My position differs slightly. I regard the statement of Newton's Law of Universal Gravitaion as a theory. He postulates that the attraction between two bodies is proportional to the product of their masses, and inversely to the square of their distance apart. That is a theory. From it we make models of how the planets, say, move. The models have had quite remakable success - such as the prediction of the existence of hitherto unknown planets. The theory has now been replaced by Einstein's, and is now regarded as false; yet it still predicts as well as it did. It is interesting to observe that a Ptolemaic view of the universe might well be very practical in programming a telescope to follow a particular object in the sky. Yet we do not believe in the theory that the Earth is at the centre of the universe.

It may well be that distinctions cannot always be made between theories and models, yet in the cases mentioned it seems useful to do so.

Theories in psychology are not likely to show the same sharpness in conception as the inverse square law. In part this may reflect the relative newness of the study, but the main reason is that people are less tractable than planets.

Nevertheless, from our theories we may construct specific models. Skemp theorises that intelligence is the power to make models of the world, and the ability to reach goals using them. One model arising from this is the director system explained in the last chapter. We must not expect the diagrams of director systems, with their schemas and comparators, to resemble our inner psychological processes in the way that a system of ellipses does reasonably represent the solar system. What the model does offer is a picture we can work with, communicate to others about, and perhaps predict for someone the consequences of certain actions he may contemplate. The diagrams do look, of course, almost as clearcut as those of the solar system. This may well be misleading, and when we look, for instance, at the diagrams of the 'six situations' of chapter.6, we must accept that they are far more stylised representations than many of those in the natural sciences.

Models lead us to speculate on further models. In the last chapter we developed a model of panic, as a flicker of consciousness between delta-one and delta-two. This arose from the original theory and in particular from the characteristic of the emotions in attracting attention. This new model both suggests a mechanism by which the reason can be severely inhibited, and provides an acceptable analogue for those descriptions of the state of panic offered by those who had experienced it. Our final testing demonstrates that certain threats can trigger the 'flicker' suggested by the model.

The value of our models lies in their giving us mental pictures of the mind at work, to which we can attach items of observed behaviour. We then hope to establish regularities in what we do observe.

Psychology is the study of Popper's World 2. Skemp uses the term 'reality', or 'inner reality' to contrast with the physical world, which he calls 'actuality'. The only part of World 2 to which we have direct access is our own mind-emotion complex. Despite the difficulties of studying oneself, this is a most valuable resource. It is unusual to learn of a response in another person which we cannot mirror to some degree within ourself. It seems that the balance of the elements of others may differ from our own, yet we can empathise to some extent with their thoughts and feelings. Some checking of psychological theories can therefore be done internally. A flat contradiction between a theory and our own internal perception of the way we work should encourage us to look closely at the theory - and then perhaps at ourself.

When we look at others we have no direct access to their inner world. We must rely on their utterances. We are studying World 2 through World 3. There are important distinctions, however, in the utterances we seek to study. One obvious and sensible thing to do is to observe how subjects tackle pieces of mathematics. A second way is to ask them what is going on inside while they are doing it. The methods are distinct, but in no way incompatible; in fact they should be used in conjunction.

It is only recently through the work of the Concepts in Science and Mathematics Project and the reports of the Assessment of Performance Unit that we have substantial information about children's performance in mathematical tasks. Consider the following question and its response. (Hart, 1981 p55 12e)

"How many different numbers could you write down which lie between 0.41 and 0.42?"

Percentage Responses

	12yrs.	13yrs.	14yrs.	15yrs.
Infinitely many, more than you can count.	7	7	16	16
Lots, hundreds.	5	3	5	4
8, 9, or 10	22	39	36	38
1	17	8	8	9
0	9	5	4	2

Here we have a question aimed at an important concept, used with a large number of children, and clearly replicable with other groups. Not only is this hard information, but in the interviews with individual children, they often give reasons for their beliefs, or tell of the way they came by the knowledge. One child is described here whose grasp of the idea of infinity came through his father's interest in astronomy.

The intent of the CSMS work is to form a hierarchy or 'concept tree', from which authors or teachers could develope better sequencing of topics. The conclusions reached show us, among other things, that many mathematical ideas now taught early on are in fact too hard for that age-group.

In this style of research the central feature is the children's performance in significant mathematical tasks. The descriptions by the children of how their minds work lie in the wealth of information in the reports of individual interviews. This material could provide an equally interesting view to that already published.

In looking at the affective, it is to that sort of material we must turn. We are heavily dependent upon what the subjects tell us of their feelings. Occasionally we have external evidence. The tapes of the individual interviews described in chapter 8 reveal stress in the subjects' voices, for instance. Nonetheless, in general it has been necessary to use what the subjects say of themselves, with all the reservations about possible concealments, or even lies, that people under stress may use.

Results such as those given in 12(e) above can, of course, be replicated. The same formulation of the question presented to any balanced sample of a particular age-group will doubtless produce very similar percentage results. It is not certain that a variation of the formulation, aimed at the same concept, would give the same results. However, the methodology permits of replicable experiments.

The experiments we describe to test affective responses (ch.8) seek to show certain likelihoods, but we must not expect the statistical precision offered by tests of particular tasks. We shall show that a threat of a mathematics test produces many negative reactions, some in the extreme range, such as 'panic'. We cannot predict that with, say, 30 people, we can expect 5 to record 'panic' as their reaction. But we do predict that in most groups of this side, there will be some who recrd feelings at that end of the spectrum.

We shall also show,with the explicit removal of the pressures we used, that people will record very positive emotional experiences. We cannot, again, predict how many.

In the individual testing in that chapter, we try to create situations where the subjects panic, and then try to retrieve them, so that they enjoy a piece of work. Not surprisingly, the subjects prove to be highly individual, and they do not always react as the experimenter predicts. That is the essential difference in working with people rather than things.

Having said all that, we do hold that the results can be replicated by others using the same pressures.

This thesis builds models of the way emotion may inhibit reason (ch.6) and in particular a model of a state of panic (ch.3). It asserts that two particular pressures, those of time and authority, trigger this state of panic. We have described how we test this. We now turn to the investigatory work (ch 5) that led to those models, and the isolation of the particular inimical pressures.

The three modes of field work, the 'one-off' interviews, the 'in-depth' studies and the 'group' all proved very interesting to the experimenter. From the way they are named, it should not be assumed that the 'one-off' interviews lacked depth. In each of them, the subject was closely questioned for upwards of an hour; it is simply that the 'in-depth' sessions went on for the whole year.

Skemp's general theory was used from the start, but particular models for panic and other inhibitors grew as the work progressed, and their validity was constantly discussed with all the subjects. All the work at this stage was investigatory, and it was therefore not sensible to work with a protocol, such as was used in the testing. As concerns emerged, reasons were adduced for their origins, and then discussed with all my helpers. Methods for correcting these concerns were also considered. Essentially, investigation and teaching went hand in hand.

This method is very much in line with Steffe's constructivist approach. He summarises the approach as (Steffe 1977)*

1) daily teaching of small groups of children by the experimenters
2) intensive study of individual children as they engage in mathematical behaviour
3) prolonged involvement with the same children over periods ranging from about six weeks to the academic year
4) clinical interviews with children, and
5) detailed records of observations through video-taping and the written work of children.

The similarities of this style to that adopted in this thesis is evident.

* see also Cobb & Steffe (1983)

An important Soviet study (Krutetskii 1976) also bases some conclusions on the study of individual subjects. This is particularly true of ch.11 - "An Analysis of Individual Cases of Giftedness in Children".

Contrast the style of Steffe and of this thesis with Opper's advice (Opper 1977 p90-107) on interviews.
"A particularly delicate aspect of the method, and one against which every interviewer must be on the alert, is the tendency to suggest answers to the child. Inexperienced interviewers, and sometimes even experienced ones, often forget how easy it is to convey to the child cues as to how they expect hiw to react...."
It was diagnostic interviews to which Opper referred, and those of this work are not purely diagnostic. However, the dangers commented upon are important. It was hoped to avoid them by two features of this study. The first was that adults were used, and they proved openly resistant on many occasions when suggestions were made. Perhaps more importantly, such possibilities in the method were regularly discussed; it was explicit in the approach that the subjects commented on

1) maths
2) the way they thought about it
3) the way they felt about it
4) the way the experimenter was working.

The discussion, the analysis of difficulties, the teaching and model-building all went on together, and they did not do so according to strict protocols, which at this stage would have been inappropriate.

There are other issues in the methodology, some of which may impose limitations on the conclusions reached.

The subjects chosen were adults of high general intelligence (insofar as we can define that). They were mainly in the educational world, and claimed specific 'failure' in mathematics.

The choice of adults rather than children was based on several reasons. It takes a certain degree of maturity to discuss emotional responses. It is doubtful whether many children have come to sufficient understanding of the affective dimension of their personalities to be able to discuss it in sufficient depth. Children may, of course, be more open than adults. However, the length of the experiment and the emphasis on openness and trust in all the work will have overcome some of the defences present in all adults. The final point in choosing adults was an ethical one. Some of the situations created were stressful, and we could not justify exposing children to them (though it must be said that many regularly face worse pressures in the classroom)

The high level of intelligence, perhaps better characterised as articulateness, proved very useful. It was most helpful not to have to help formulate (and in doing so perhaps modify) what the subjects wished to say. Some statements, particularly by Elaine, clarified ideas remarkably through the sheer precision of language.

'Failure' in mathematics was their own perception of their background. In any objective sense the fact that most took (even if they failed) 'O' level implies that all were probably well above average in mathematical performance, a fact they certainly did not credit. All were more competent in other academic work, and saw mathematics as their great weakness.

They came, largely, from the educational world. This helped communication, for we all had much in common. The reason were pragmatic; they could commit the time; they felt it relevant to their work& they knew the experimenter. It is not claimed that they were a balanced sample: What they said might only be relevant to that type of person. If so, it remains important, but there is evidence that what they say has certainly wider relevance.

In fact, depth investigations, even of so relatively few people, may well have wide applications. The number of patients seen by Freud in his lifetime might well not prove a good statistical justification for his theories.

Subsequent reactions to the popular version of this work (Buxton 1981), from various parts of the world, show that the responses elicited from those investigated are replicated in many others. It is interesting to note that the Cockcroft Report suggests (sect 21)
"The feelings of guilt to which we referred earlier appeared to be especially marked among those whose academic qualifications were high..."

It may well be that the responses described in this work are stronger in those studied than in the population at large. It is enough that they are present in an important minority.

As to specificity in mathematics, this is not claimed. Indeed it is to be hoped that the recognition of the damaging effects of time and authority pressures might be of help across the curriculum. There were sufficient comments specifically related to mathematics, however, to suggest that, while extremely negative reactions can develope in many teaching situations, there are certain features of mathematics that increase the likelihood of such feelings being generated.

We shall return to some of these issues at the end of this thesis.

CHAPTER FIVE

THE FIELD WORK

THE FIELD WORK

In all there were three modes of enquiry. In the first, which was mainly preparatory, I talked with twenty-four people who claimed to have difficulty with mathematics. The discussions were not heavily structured, and, indeed, were at times rather discursive. In the discussions a number of hints and suggestions of matters that I might pursue emerged.

The second mode was the group work, which was central to the whole investigation, and involved two hours a week with the group over the whole year. The style had structure, as we shall shortly describe, but the discussions were still intended to be investigatory and were in no sense according to any strict protocol.

The third mode was the individual 'in-depth' studies. These had not been part of the original plan, but so valuable were the initial 'one-off' sessions, that this was a natural development.

As ideas and speculations arose, I wrote a series of papers, which were then fed to all the subjects for their comments. This process unified the work, and meant, for instance, that as the situations of chapter 6 emerged, they were considered and amended through regular discussion.

The 'One-off' Interviews

Those interviewed were effectively self-selected, in that when it was known that I proposed to conduct this enquiry, many people spoke to me of their interest in it, and offered to be interviewed. They were in no way a balanced sample of the population, mostly coming from the group described in Cockroft as being well-qualified in general terms, but with deep-seated fears about mathematics. Had it not been that it was the fears I wished to pursue, rather than their knowledge of mathematics, many of them might well have reacted as did those in the report, and refused to discuss the matter.

The number included the heads of three major London secondary schools, several other heads of primary schools, a range of people in advisory work (including two of my colleagues in the ILEA Inspectorate), two well-known educational journalists, a number of other teachers, and some unconnected with the educational world.

All of these people, as far as I could judge, were successful in their work; their fears of mathematics did not appear to have hampered their careers, though in some cases it had had a marked effect on their personal self-esteem.

The interviews were mainly conducted in some privacy, in my office or in theirs, though one was pursued over lunch in a restaurant. The subjects were encouraged simply to talk freely of their experiences in mathematics classes. Despite the fact that for most of them these were many years in the past, their memories, particularly of unpleasant events, were vivid. As points of interest occurred, I would question them more closely on those matters, particularly, but not solely, those with a strong affective element. As I gained more experience in these interviews, my questions centred around the issues I saw emerging, but at no time did I ask about such issues unless it had arisen in what they said. I was anxious not to feed in ideas which might be too readily agreed with.

Some of the memories were clearly stressful. One head of a large school confessed that my coming to see him about mathematics had created great tension for him leading up to my arrival.

The issues arose, as I have indicated, in an unstructured way. Detail of some of them now follows under general headings that seemed relevant at this stage.

The first category is their general view of school life as they experienced it. Their attitudes varied from affection to horror. For some, their (relative) failure in mathematics was a matter of puzzlement in an otherwise enjoyable school life; for others it was the lowest point of a generally disastrous school life. Some of them liked their teachers, others did not. The reflections of matire adult minds on these teachers are telling:-

"All assumed that what they were saying was comprehensible. The maths. teacher only explained anything once and then it was very difficult for anyone to ask questions because she always gave the impression that anyone who didn't understand the thing the first time was quite stupid."

"If you responded correctly to the marks he made on the blackboard you evidenced understanding; if you responded incorrectly you were somehow breaking a code that he had established that you were supposed to conform to."

Thes are sophisticated observations, made in retrospect, but the attitudes about authority held by those teachers is evident. Sanctions were directed at a lack of understanding, rather than at laziness or disobedience, where they might have been more appropriate. There were powerful memories of parental reinforcement of these attitudes.

Punishment varied from mild and oblique expressions of dismay or disappointment (sometimes carrying great weight in a child's mind), through blatant derision ('I was being singled out as a ludicrous object') to savage beatings.

One said of his father:-

"when I was between seven and ten he must have realised my arithmetic was very bad and he set me long division while my friends were playing outside. I was crying my eyes out. He locked me up for two or three hours, but I just couldn't do them."

Another, of school:-

"I've always associated maths. with fear and trembling. I managed to escape with a beating once a term."

The authority stretched to matters of presentation:-

"It was the whole school ethos. You got your figures in nice lines and you had vertical lines in maths. books (which my writing didn't fit anyway) and if you didn't do it, it was a mark of insolence and deviation and all sorts of things."

Many people had very clear memories of their performance at school: some could remember marks in particular examinations, others exactly when they gave up in the subject. One person claimed he gave up when asked to learn the two times table; others could also define exactly when:-

"Well, I know the point where understanding switched off. It was somewhere just beyond quadratic equations, and I learnt the formula."

This is a nice comment on remembering and understanding. For another the appearance of the letter θ was critical. He asked the teacher what it was and was told it was a circle with a line across. He felt it must be 'something rather grand', but it was the final straw for him.

One story of long frustration is worth telling at some length. The person involved has a good degree and a senior post in the educational world; he had a very detailed recall.

His father was a carpenter, who believed that mathematics was the only school subject that mattered. He himself accepted this view and worked very hard at mathematics, with, he says, no distaste . The experience was frustrating, and yet not upsetting. He wanted to do mathematics, he still wants to do it and he does not know why he cannot do it.

At primary school he had no difficulty with the 'four rules' but 'problems' were another matter. In the 11+ he got three right and he remembers to this day the mistake he made in another. His father was very cross and he was very upset.

At his grammar school there were four sets in maths. and he was shocked to find himself in the fourth. He had to conceal it from his father, regarding it as shameful. He was determined to work hard to get out of it. The following year there were five maths. sets and he was in the fifth.

At General Schools he could not get a pass, let alone the credit he needed for matriculation. In the sixth form he took mathematics as a subsidiary, slaved at it, spending more time on it than on his main studies, but again with no success.

The second category we now look at contains more personal observations, in that they look inside themselves, or reflect upon sensitive areas with parents or teachers. It helps in discussing emotion that people can reflect upon it and express in an articulate and cognitive way what their feelings are. A typical panic reaction was:-

"A string of figures and my brain seizes up."

A less strong response, but one in the long-term that it very damaging is that of frustration:-

"My frustration at not being able to do maths. then led to a frustration at being obliged to spend time on it which might be better used elsewhere."

A sense of bewilderment was often expressed:-

"....and the thing I remember about maths. is of course a fantastic lack of comprehension."

When pressed for his reaction to a particular problem this subject said:-

"Well, it's a bugger's muddle is what goes on in my head when I'm faced with that."

It was clear that their was often disquiet about early authority relationships, with teachers and parents, but some was not possible to follow up because of its very personal nature.

Essentially authority must relate to inter-personal issues, but by people attaching great importance to mathematics, it attracted to itself as a study the standard reactions. Some were submissive, and tried to conceal failure, others rebelled, and claimed that mathematics was not important (note Hilton's classifiacation in chapter 2). One interesting comment showed a detachment that was probably not achieved at the time:-

"Authority figures often ask about unimportant matters and thereby give them a cachet they do not deserve."

The concern with time pressure that later I came to believe to be significant did not emerge strongly at this stage, but it was clear that people believed that maths. had to be done quickly, and did not like that aspect. One person did say that when she was rushed

"I just blank off."

One interesting feature of a string of interviews like this is the number of times the same phrases are used, mirroring an internal event which we finally hope to examine.

A third category of response concerned the view people took about the nature of mathematics. It was spoken of as cold, precise, formal and abstract:-

"I can't think abstractly enough for this sort of thing."

There was a general belief in the firmness, fixity and precision of mathematics. One subject thought the last two notions the same, and this may have been at the root of his inability to come to terms with the idea of variation. These feelings were linked with the belief that mathematics is not man-made but is external and intractable. A curious observation may represent a concern that many have:-

"I felt I could not actively impose my personality on the material which was being presented."

There was an acceptance that there were 'rules of the game' in mathematics, but it was not understood how many were at our own choice.

"Mathematics has been presented to me as a fixed body of knowledge, which has been handed down, which embodies certain abstract truths and which one has been expected to imbibe of and reproduce."

An understanding of certain features was revealed in this comment:-

"It seems to me a very private occupation, whereas writing, speaking, drawing are partly communicative."

The notion of a mystique was widespread:-

"I dislike doctors because they seem to have wrapped themselves round in magic - mystique - and I feel mathematicians have the same quality."

This remark I followed by asking if he saw mathematicians as priests. He replied very firmly "Yes".

".....and do they have power?"
"Priests always have power, don't they?"

Others had images of gurus (who knew maths..) and were surrounded by disciples. This touches upon the issue of authority, but would have been interesting to follow up in more depth.

The question of relevance to the real world was seen in various ways. It seemed possible for people to hold the views already expressed, and still recognise how important it might be in practical terms.

"The important distinction here is that in the classroom maths. was a matter of juggling around with symbols and numbers, where it had no meaning at all in my mind. In physics your u's and v's represented certain quantities, and you had a purpose in working out a formula to find a focal length."

A geographer took a similar view. He had been involved in a study of a projected line on London Transport, and its likely effects. Despite his general distaste for mathematics, he even enjoyed it in this context.

A number of similar views at the more mundane level of household accounts suggested that mathematics might be liked if seen as 'useful' or 'relevant'.

There were some counter-arguments to this being the main criterion. I asked a number of the subjects if the thought the question as to whether the prime numbers went on for ever was intrinsically interesting.

Though it is difficult to see how this could ever be useful, most regarded it as definitely interesting.

Mathematics was seen by nearly all of my subjects as a movntain of material, formulae, routines and facts. This view resulted from them all having moved from trying to understand to trying to remember at some atage in their learning.

"We had to learn the theorems by heart (fifty or so) and then regurgitate them. In no way was I going to pass that geometry exam."

"You just had to remember them.... lots of them."

"I think of maths. as an attempt at remembering routines, and I gained no comprehensive view of the subject. "

The nature of mathematics left people with concerns about whether they could 'control' it. This was clearly related to psychological concerns about inner stability.

"That is why I have an allotment. It is the one place where I am absolute boss and absolutely independent, and I can order things exactly as I want. In a tiny way I get that pleasure in adding up a column of figures or in working out exactly how much income-tax I should pay in a year. I get real pleasure out of doing simple calculations."

The last category we shall deal with here is that of particular issues in the mathematics, but it was the reactions to them that are most significant.

The first big hurdle for many had been long division, and some remarkably strong feelings were attached here. In the case of Elaine, which we shall report shortly, the purely cognitive explanation that it was simply a method of taking away one number from another as often as one could resolved a long-standing emotional problem.

Place value had caused many intellectual difficulties, which then developed to emotional blocks.

"I missed out on hundreds, tens and units and never understood it, and it had to be done in vertical columns, which I couldn't do anyway."

The most serious stumbling block for many came with the introduction of x, which they knew as the 'unknown' and was in consequence invested with mystery. There were, of course, genuine cognitive problems in the shift from x as something unknown but findable, to a multivalued x, to the notion of a variable.

I also found other examples where the notation inhibited learning; there can be strong emotional reactions to certain symbols (Buxton, 1982). To one subject I offered the phrase 'Let n be the number such that...' and he interrupted with:-

"The shutters come down."

Here the phraseology may be as important as the use of a letter for a number.

As expected from other studies, people varied greatly in their spacial perceptions. When asked for the number of small cubes in a larger one made up of three small ones each way, some answered immediately, some took a fair time, but were usually then correct, while one person had no idea at all. When then asked how many were on the 'ground floor' of the large cube she replied '36'. Then asked to reconsider she said:-

"There are four on each side - that's twelve round the perimeter - and them some in the middle."

Related to this issue was the reaction to diagrams of an explanatory type. Some people obviously find them helpful; for others, they positively inhibit learning.

Krutetskii has pointed out how marked can be the imbalance in a person between spacial and analytic ability. Since we do not expressly seek to develope it, we may wonder how much is innate.

These preliminary interviews therefore offered many leads, only some of which bear upon the eventual task here - to examine panic. Others were the starting point for the analysis of chapter 6, and all gave me material worth using in the lengthier work of the other two modes.

There is perhaps one last quotation, however, and this the only one from a child:-

"I think in the playground, when I go out to play."

The Group Investigation

Because of my experience in group dynamics, and its particular value in exploring emotional issues, I had determined to use a group teaching base for the main investigation. The structure needed for this work was fairly clear when I started, but there were changes in emphasis as we proceeded. The techniques since developed from here show even more structure.

The first important matter was the size of the group. There are quite close limits here as to what is suitable. A group of six or less does not seem to work as a group, and is more likely to split. In a group of ten or more there will be some people who find it extremely hard to participate, particularly if they are describing their feelings.

In the event I recruited seven people, which with myself gave a group of optimum size. It was also balanced in terms of sex, with four men and four women. I had looked for people who were on the whole well-qualified, but with relatively poor performance at mathematics represented by a failure at 'O' level. In fact, two had managed to get 'O' level, but in both cases by scraping past at the age of eighteen, having scored five failures between them before succeeding. More important than the failures were the feelings they expressed about maths. All were fearful of it, but curious enough to explore this fear with me.

Before the sessions, I knew three of the people well and two slightly. In some ways it would have been easier to deal with a group none of whom knew me and none of whom had relationships with me in our daily work. This would have meant that for all of us the group was a weekly experience unrelated to any other activity. In this way no preconceived authority relationships would have existed.

All of the people had heavy work commitments, some of a very responsible nature. It was therefore important to me that they were prepared to give a very high priority to attendance at the weekly sessions. In the event, the attendance rate was extremely good. (for detail, see Appendix A) During the first two terms, out of a possible 184 attendances there were 176. In the summer term, one member had to leave altogether, owing to the demands of a new post, but despite the fact that two others had to take school journeys, there were still 81 attendances out of a possible 91.

There are a number of important principles in running groups of this sort. Some derive directly from the absolute need to trust one another. Without this trust, very little that is reliable will be said. One is that what goes on in the group is private to the group. This does not apply to mathematical content, but to effectively everything else. Naturally clearance was sought after the year from all the members before such material as has been published was released.

It is also important that no-one enters the room when the group is in session, or the feeling of being a group can be lost. At a very practical level this would make sense in all classrooms. The assumption made in most schools that anyone can come in to get a pencil or deliver a trivial message is quite counter to good learning practice. In work with a group where the dynamic is considered, the need for it to be well-defined is represented by its closure to others.

Another defining feature, not possible to ensure during this study, is a very prompt beginning and end. The fact that it exists for that time, and only then, is significant.

The sessions were of two hours, with a break. This is not ideal, and an hour and a quarter might have been better, with no break.

The work proceeded mostly by discussion; we sat in a circle. We were not precluded from using materials. The venue was a mathematics centre, and a wide range of aids was available, but they were not much used.

The most important issue in such a group is the definition of what may be talked about. Initially I was not as clear about this as I later became, but in essence the permitted range was:-

1) Maths. content

2) How we thought about it

3) How we felt about it

4) Teaching issues arising from the discussions.

There was no set pattern as to who spoke when, and I tried not to exercise a chairman's role. On the whole this worked; certainly the men, as is usual, tended to talk more, but all contributed regularly. It was accepted that that there was freedom to move from one of the defined areas to another at will.

An example may help understand the sort of thing that happens, though people who have worked in these groups find that they are not really in tune with the style of working until they have been in it for some months.

If we considered, for example, operations with directed numbers, they might first struggle with finding an answer to something specific, such as (-3)-(-5). All would attempt the problem, and arrive at various answers They would then argue about them. Whether or not they then reached agreement, each would say how they had thought about it, trying, not to get the answer right, but to get the thinking right. There might then be a statement from someone about feelings they had always had about negative numbers, or about being made to feel stupid, perhaps in the past, but sometimes in the present situation. This would lead to a discussion af what had been said to make the person feel uncomfortable, and this would then be interpreted as a pedagogical point. The flavour of such discussions, particularly when it is emotional responses that are being talked about, is varied and can be stressful. It is the development of trust that leads to the proper exposure of what is really going on in a learning situation.

My role was to indicate areas we might work on, and sometimes to interpret what was happening in the group dynamics. Generally, the less I intervened, the better the session.

The actual topics discussed in each session are listed in the Appendix B, together with those stages where we developed ideas relating to the situations of the next chapter.

One further matter for discussion in the group was the developing theory, presented by me in a series of papers which, as they were prepared, were circulated and considered (mostly between sessions) and then reviewed in the session.

As we worked, we became more and more aware of how much time is needed before an idea has really been internalised by all those present, and at times we worked, it seemed, very slowly. Yet the work was continuous, and that must be the real criterion. We found that the social pressure on a person not to hold up progress was very strong, until we could accept that we were not heading to a particular goal, but that learning and understanding was the goal. If one person lagged behind in grasping something, it was the job of the others to help them through the difficulty.

A feature of the slowness was that it seemed even more essential in shifting attitudes, than in gaining understanding.

This then, describes the style of the group: the list of matters discussed appears, as we said, in the appendix. However, it is necessary to illuminate the proceedings with some quotations, and we shall group these under categories, as before.

In the first, we detail some of the mathematics which was done, and their reactions to it.

We made a very curious start by plunging into conic sections. The only reason for this was that one of the group was designing helmets for a film production and the designer wanted them to be elliptical. We used it to explore the notion of a point moving under constraint, and when I told them of the focus-directrix property, they drew the curve. This involved more direction on my part than I wished, but when we looked at points on moving wheels, they did much more by themselves. One of them was a railway enthusiast, and when they looked at the locus of a point on the rim when the wheel was s slipping, he commented:-

"If it came in at more than a right angle it would have to have a loop to get on to the next revolution."

This is the sort of conceptual understanding often impeded by symbolism.

The multiplication of directed numbers was grasped, paradoxically, because a separate issue had distracted one member from the maths. (situation 3 of chapter 6) She had just bought a house, and this totally absorbed her thinking. We then made a breakthrough; using the appreciation and depreciation of cars and houses we modelled negative numbers in a way that they found acceptable.

At first this seemed a strong argument for 'relevance', yet before long we were discussing the fundamental theorem of arithmetic, and then doing (because they asked to see) the square root by the long method. This seemed to afford equal interest.

On the whole, geometry intrigued them. We did the angle sum of a triangle

"That's the nicest thing we've done the whole year - I love that."

I showed them the theorem as stated in an old textbook

"In one sense it has a lilt - on the other hand it is gobbledegook."

It was not only that understanding was coming, but we were reassessing the past that had led to most of them being in situation 2 of the next chapter.

We managed to establish that some of the harder questions at 'O' level were within their grasp, but more importantly we altered views as to what mathematics was about.

The second category contains teaching points. With most of us teachers, we often dwelt on these.

We returned again and again to the question of relevance as a motivator. In terms of our model we are seeking to have the emotions lending weight to the reason, rather than impeding it. Often a external desire or need gave great impetus. This was the motivator for the conics, it led to the breakthrough on directed numbers, and graphs generally were seen as media communicators, and with socio-political applications. There was a second-hand relevance in the theory of indices and the fact that they led to logarithms. Though these are now obsolete as a calculating tool, the relevance in the period when science was becoming overwhelmed with calculation was fully accepted.

Another sort of relevance lay in the psychological area. It was clear that some of the backlog of failure that led to their present state lay in the fact that they could not match what they were told with their own intuitions. The resolution of some old difficulties of this sort was important. One member exclaimed about logs.

"It was never taught in that way. That actually makes it simple. Ludicrous! "

It was not just logs. He now drew the conclusion that there might be much else that he could now tie to his own thinking.

Two more quotes show the need to make the information fit inside, and the satisfaction when it does.

"Why do you double the last figure?" (of square rooting)

"I don't like applying something when I don't know why it does what it does."

A further example allied to this came when two of the group reached different results. One then convinced the other that his method was correct. The other then complained that while he saw the correctness of the method, he still needed to be shown why his was wrong.

So relevance can be extended to sorting out one's mind. There still remains the sheer joy of knowing something. One member, when she had grasped the focus-directrix property revelled in describing it to a mathematician friend.

We worked on understanding a good deal, and sought a measure of when we could say that something was understood. At one stage we wondered if the ability to defend what one knew against questioning by someone clearly more able at mathematics than oneself was a measure. It turned out that it might depend more on the confidence of the person than his grasp of the topic.

The methods adopted were designed to give a deeper grasp than was usual, and I had hoped that retention would be much better. We had no testing of this, only subjective views. Ideas were still forgotten, but it did seem that retrieval was easier, and a very valuable memory of 'mastery' of the sort described by Eagle (1978).

The central issue for this thesis is the way the emotions affect learning. We spent the whole of an early session on the meaning of panic. During this session we looked at the physiological manifestations, the distinction between the reponse of frenzied action and paralysis when in panic. We looked at critical survival crises (such as racing downhill without brakes) where the very reverse of panic was induced, with an unnatural calm enabling effective action. After a long while, everyone came to agree that, though the issues in the maths. classroom are objectively much less serious than others discussed, they do on occasion induce genuine panic. Some of the group were so far advanced in their fear of the subject that the mere threat of having to do mathematics induced immediate panic

"Panic is when you dry up before you start. Finished - done! I've never really tackled anything in maths."

With every point in maths. we checked how we felt about it. Tensions were mostly remebered from school rather than experienced in the group, but there were occasions when people became very upset. On two occasions members experienced deep disappointment at having thought they had been successful and then finding they were not. It is not the fact that they were experienced that is surprising, but the depth of the experience is. One person spent a whole evening in deep despair, at this objectively small issue.

These feelings were sometimes too strong to be openly discussed, and I had twice to see people outside the sessions. On another occasion a member felt that a very sore point was emerging from har teens, and had to avoid the issue. This was a relevant matter for situation 6 of the next chapter, and the notion of a 'ladder to the past'.

Strong feelings of inertia and unwillingness to work manifested themselves once or twice. We attempted to discuss and analyse them, for they were of great importance for the classroom, but we made little headway.

It remained fairly easy to create anxiety even in so generally comfortable environment. I did not set out to do this, but I occasioned it at times. Once I thought that it would be a good idea to demonstrate that someone understood something by getting them to explain it to everyone else. As I looked for someone to do this I realised that all were in panic.

The existence of these strong emotions, and the fact that we were expected to discuss them, did eventually lead them to talk of their feelings towards the teacher, myself. These revealed that there were still very negative reactions, not at a conscious level. In particular, the presentation of problems was still seen as an attempt to trick or embarrass, which prevented them being worked at properly. Even whensomething was explained by me on one occasion, there was a comment:-

"It's clever.......but it's still a 'con'"

A great number of issues arose therefore, some well worth following, but the work here and in the three individual studies formed the basis for the rest of this work.

The Individual Studies

Three people, L, S, and E, worked with me on an individual basis. L worked as an advisory teacher in the same building as myself, S as a primary teacher at a local school, and E was a secondary head. The first two came to my office, roughly once a week. I visited E at her school and we used her office.

In most weeks there were three sessions, one each with L and S, and one with the group, during which the same ideas and mathematical material might be used. This focussed thinking and allowed each to serve as sounding boards for the others.

The 'in-depth' sessions, as we began to call them, yielded rather deeper personal insights into these three people than were gained in the group. There may have been idiosyncratic reactions at times, but any speculations arising from such responses were always checked with everyone.

With all three subjects it became apparent that there were issues in early chidhood, connected with authority relations with their parents, which were liable to surface. At first this seemed surprising in a study of maths. anxiety; it led to the formulation of situation 6 of the next chapter. I had not been prepared for psychotherapy, which some of the sessions verged upon. However, as mature adults they were able to control what they would discuss and what not.

In describing now some of the sessions with L and S we shall see those issues that came up in the group repeated with different emphases. The study of E(laine) is given in Appendix C. The same issues arise, often with greater sharpness, but it is placed in the appendix to stand on its own as an approach to dealing with a very deep-seated fear of mathematics.

"L"

At 'O' level L gained seven passes and failed only in mathemtics. She remembers this 'failure' beginning in the final year of junior school (though there was no suggestion she could fail the 11+). Her junior school was a large one, streamed, with three classes in each age group. Even within a class they were streamed by ability, again into three groups. Effectively, then, there was tight streaming into nine sets. L was in the top class, and within that class in the top group for everything until she was 'put down' into the second one for mathematics. This separated her from some friends and she regarded it as failure and felt diminished. When I pointed out that she had been in the second set of nine, it seemed a new idea, despite her clear knowledge now that this was so.

She passed into a high-powered grammar school, entered the lower sets in mathematics, and began to fail. She said that if she did not understand, she did not bother to ask

"Well, not bother, a certain embarrassment."

Corrections of this sort may have a special significance, and an analysis of the reason for the embarrassment should perhaps have been sought.

L was very inclined to use 'can't remeber' as a defence. We both acknowledged this, but accepted that the memories that were surfacing were not ones she wanted to discuss. Her tentativeness at times (though she was very frank at others) led me to the interpretation that the encounter was seen as mirroring earlier one-to-one meetings which she had not enjoyed. (Situation 5)

We discussed what interested her in mathematics, and she probably had more mathematical intuitions than the other subjects, particularly in geometry. When I asked if she though it interesting whether the angles in the same segment of a circle were equal (in a diagram, not in words) she thought it 'obvious'. When similar angles were drawn in an ellipse, and she was asked if they were equal, she said:-

"No, because it narrows down here."

On the other hand, asked if it were interesting whether the primes went on for ever,

"No, I don't care - it doesn't matter to me or my life."

L's replies sometimes reflected her state of mind; the last answer was given after she had just failed to do an earlier problrm on primes.

L's responses were fluid, and showed real interest at times. She thought the idea of a mathematical principle in Greek statues was 'magical'.

Despite her spacial ability, L did not even try to see how many small cubes there were in a 3x3x3 large one. She responded very rapidly '54'. There were nine cubes visible on each of six faces. She believed there was a formula for everything, and in connection with this:-

"But I see the whole of maths. as remembering."

An important aim of this work was to interest people (at least occasionally), and then to reflect upon this enjoyment. I got her to look for the first perfect number after 6 ... she worked at it 'began to get curious' and then finished it as a chore and did not believe that the answer she got was right.

During the year we worked on the issue of enjoyment, eventually with marked success, when she explained to a friend that she was doing fractional indices, and 'quite enjoying them'. Her friend remarked that 'even: looking at those fractions, my stomach turns.'

L was interested in philosophical points, and was interested whether mathematics was invented or created.

"We recognised 'five' but it was always there."

Talking to L led to one important distinction. If we have had a pleasurable emotion, we seek to have it again. Inprinciple, therefore, if success brings pleasure, we should want to do more to get this pleasure. In fact, for those with repeated previous failure, there is a different reaction possible:-

"Yes, but that is just a relief."

Relief is the pleasurable emotion we may not seek to re-experience - in Skemp terms, we are not reaching goals, but avoiding anti-goals.

L had some clear points on judgement and authority, which we shall see finally as some of the most inhibiting factors in learning. One also touched an her view of mathematics, as opposed to writing an essay:-

"You have a certain freedom in that. If you get a low mark you can think they just don't understand. You can question their judgement because it is so uniquely their judgement. "

But with a maths. teacher...

"No, he is totally right and you are totally wrong."

The authority was clearly oppressive to her.

This distrust of the teacher-pupil relationship was sometimes directed at me. In giving her some problems she felt I had been dishonest and manipulative. I had said she might not be able to do them (as a guard against failure, as I thought), and she had found them easy and was disappointed.

L always felt that questions had to be done quickly. There was a self-imposed time pressure, even when I insisted there was no hurry. Once recognised, its importance became more and more evident, and a point with E brought it home very strongly.

An issue perhaps at the other end of a spectrum from the chaos of panic is that of control - or perhaps one aspect of panic is being out of control. The word 'control'kept coming up (as with the 'allotment' man). L is a very organised and controlled person.

"Everything has its place, and it's put back in it, and maths. is the one area where things don't have their place. That's obviously why I blocked it out for so long."

There was an interesting happening with L which did not occur with the others, and relates to situation 6. We were obviously touching on early relationships with her mother at one stage, and she broke off the sessions for several weeks. She said that I was probing where she did not wish to be probed.

'S'

Originally S had been very keen to join the group, but fixing a day had proved difficult. S was thirty-two and at the beginning of her third year of teaching. She had taken three 'O' levels at school, spent two years in France becoming fluent in the language, worked for a photographer, and then decided to train for teaching. She had taken two more 'O' levels (not maths.).

S had come to see learning as competitive, and did not want to compete. Throughout the interviews, she often saw the encounter as a competition, which she was going to lose. She also wished not to promote expectations inothers which she could not then meet. We did not get at the underlying causes, which almost certainly lay outside mathematics, but as far as maths. was concerned,'it was almost as if I can't bear to be right' This was most noticeable when demands for quick answers were made (she broke up rapidly under time pressure), was always present when I was explaining something, but was much lessened when she was free to explore.

She enjoyed wood-block puzzles, and liked fitting the pieces together, she herself initiated an exploration with cubes getting larger and larger, with the outside painted black,to find when there were more unpainted small cubes than painted. It was perhaps the fact that there was no person there that helped. They provided authority pressures, and reminded her of unhappy one-to-one encounters of all sorts.

It was a mathematical point that few would regard as 'relevant' that had prompted S to get involved with mathematics again She had read the well-known story of Hardy and Ramanujan; Hardy remarked on visiting Ramanujan that the taxi that brought him had the number 1729, which he could see no interest in at all. 'Not at all' said Ramanujan 'it is the smallest number that can be written as the sum of two cubes in more than one way.' This story had caught her imagination. It was not so much the fact, as that someone could have a mind that thought in that way. She made the important statement on relevance:-

"It's what has connections for you."

It is the satisfaction of adding to existing schemas, rather than external relevance that is here at issue.

The issue of control, mentioned by others, was important for S:-

"I'm a great one for control."

She added that a sudden demand can break this control. She agreed that a displaced anxiety (situation 6) could be 'pushed into that area because it is such a demanding area', and that the frenzied activity she sometimes showed was to avoid experiencing such demands.

S expressed very strong feelings at times, and interpreting what is behind her statements is not easy. The fact that 'minus times a minus gives a plus' was found unacceptable by most of the subjects - but S says:-

"Forced to accept something when you don't want to, the whole of you revolts against it, emotional, authority things, you've never really assimilated it."

At times I deliberately applied pressures. On one occasion I asked S to factorise 12, and then add up the factors. I then indicated urgency with a slightly demanding gesture of the hand.

"S. Who? Me? Yes? At the sudden demand. You see, of course I can add that up. Of course I knew what the factors were, but as I mentioned before the sudden pressure of the expectation to do it quickly - immediately I can't.

Self. How would you describe this feeling?

S. Panic. Having panic at this level is stupid really. Yes, panic in the mind, it was almost a panic in the mind with those figures dropping away. You know I had to keep recording them to be able to add them up."

At this time I had not introduced the word panic to S. Later I wrote a discussion paper on it for the group and the individuals. S had remarks worth quoting at length.

"In your paper on panic there seemed no very precise differe difference between PANIC in the MIND and an emotional feeling in the stomach, sweating, numbing, etc. I feel these are very distinct things. With me, the mind itself is thrown into a confusion and cannot make the necessary connections. I am not necessarily sweating or stomach-churning but am trying desparately to calm the mind so it will perform the necessary switchboard connections to get the right message across to me (the one I have asked for). Stop! Perhaps this is the crux of the panic. I haven't asked for the right one, I haven't made the right demand of my brain and the brain knows it and consequently, because of all previous input and store of information, it rebels. Is this an INPUT-OUTPUT dilemma? At the time of making the demand I could not consciously (or anyway immediately) recall all the information stored over the years. I might make a demand which is not compatible with the information stored. This in turn might be caused by my misinterpreting the demand made on me which I then pass on to my brain in a confused state. It may be the phrasing of the demand made on me that throws the switchboard in confusion, trying to plug a non-existent hole! Why am I making this distinction between myself and my brain? Am I really talking about my emotions and my brain? The one paralysing the other?"

This extended statement has many links with the model of panic, with the two deltas unable to function together.

CHAPTER SIX

PATTERNS OF INHIBITION

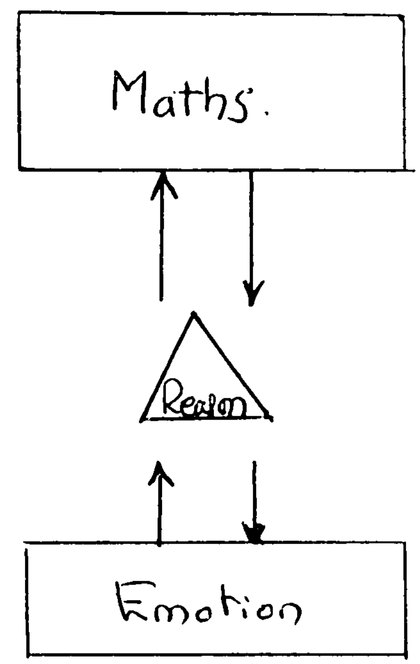
PATTERNS OF INHIBITION

In this chapter begins an analysis of the material gathered in the various activities. It does not postulate what goes on in the reason-emotion complex inside ourselves; that has been discussed in chapter three. Instead it separates certain types of external conditions that may result in emotional responses, generally of a negative sort. If we refer back to the comments of Farnham-Diggory (chapter 2) on the S-R model, we are not yet looking at the important issue, the hyphen, but at several S's and the accompanying R's.

As the year proceeded it became clearer when my various subjects were advancing in their mathematical understanding, and when emotions were induced that stopped further progress dead in its tracks. My position was the opposite of most teachers, for it was the latter that interested me, and I was happy enough when it occurred. A pattern of situations began to form, each category backed by remembered statements and reactions. While this categorisation did not advance my knowledge of the eventual target, the internal mental structuring, it was of value in listing situations that the teacher should seek to avoid.

Situation 1

Diagram 2



We start with a basic picture. In the diagram, the reason, denoted by a triangle which we shall call "delta" is applying itself to a piece of mathematics, in an attempt either to understand it, or perhaps to solve a problem. We have seen that the feedback, whose mechanism is explained in chapter 3, induces emotions. This is the point in which Skemp's theory leads into the present study. In general, a feedback that success is likely is pleasureable; this increases motivation and the thrust of reason is enhanced. A feedback indicating likely failure produces feelings of frustration and possibly, anger. Reaction to these may be highly individualistic. Motivation may be decreased, leading to withdrawal; for some, anger may increase motivation and the chance of eventual success. The internal mechanisms that determine what exactly happens must be very complex, and while we have given a theoretical model of one specific negative reaction (panic), most of the observations made here are simply that - observations that will not be fully explained by our model. It is an area in which much remains to be done.

Most classroom mathematics sets tasks with very well-defined goals; whether or not they have been reached is seldom in doubt. Some more modern developments in mathematics teaching present it, very properly, as investigatory. In the past, the emphasis has been on questions with single answers, right or wrong. This clarity tends to enhance the sharpness of emotional response. There is a nakedness about the success or failure in reaching a goal that evokes clearly defined emotions whose nature one cannot disguise to oneself. There may be no immediately obvious reason why pleasure from success has been less apparent than frustration and disappointment at failure in our classrooms, but the evidence is that more people end up anxious about mathematics than enthusiastic.

This may lead us to the widely accepted view that in teaching we should try to ensure a high success rate, and so arrange problems that students seldom fail. There are some reservations to be made about this approach, although in general it is a sound principle. Many teachers claim, perfectly correctly, that their classes enjoy doing repetitive work of a sort at which they are competent, such as columns of addition. The class is, however, engaged in occupational therapy, not learning, which we might have supposed to be the primary task of an educational institution. The circuit in the diagram is being pursued painlessly, or even with mild enjoyment at continued success, but without any development of mathematical knowledge.

On a number of occasions my subjects denied themselves pleasure on the grounds that a task was too simple. Goals must not be too simply achieved. The pleasure arising from the process does depend on the difficulties overcome. It is even possible to gain considerable pleasure from a lengthy attack on a problem, with many avenues explored but the goal of a solution not achieved.

"I sat down and spent about twenty minutes on it and then I thought quite calmly, with no sense of humiliation and certainly not sadness or despair "I'm blocked. I've gone down so many blind alleys and I'm not going to be able to do this. I must ask Laurie what the trick is. Once he tells me the trick I'll have got it. Just like 7 x 8. I'll never forget it because I've explored so many blind alleys."

Failure must be handled sensitively, however. The early realisation of this is the essence of good teaching. The central problem becomes the adjustment of the level of difficulty for a particular individual at a particular time.

Situation 2

The first situation assumed we were approaching the task for the first time, or at least with limited previous experience. The emotional reactions are then assumed to be largely related to the task in hand at the moment. We are, of course, seldom faced with work that is unconnected with previous experience, though sometimes it is possible to create learning environments where a 'fresh start' is made. Ideally reactions to the task are then simply related to the difficulties experienced.

We may, and many people do, arrive at a state where this is not so. If we have had a considerable amount of previous experience, strongly charged with emotion, then it may become this previous experience, and not the present task, that is the main determinant of how we feel. It might have been positive enough to produce great drive and determination in both learning and problem-solving. Unfortunately, the more common experience is reiterated failure, and people for whom this is true find there is a great inertia in even considering a problem.

It is worth analysing three stages in approaching a problem. The first is well described in Krutetskii (1976). It is the receiving, sorting and imprinting of the information. This is still occupation of the mind, which does not have to go anywhere or conduct a search

The second stage involves a search in the memory for relevant facts and tools for attacking the problem. Generally the tools or methods of approach are more significant than the facts, though some of the latter need to be known. The process requires judgment as to what might be relevant or not. It can still be seen as preparatory and not as the actual attack, though both it and the first stage are essential. The third stage may go hand in hand with the second but appears to be a distinct activity. It is what most people refer to as 'thinking', and involves the movement from a present state to some new goal. This may be reached by careful construction of new conclusions from our initial information using standard tools and/or by insightful and intuitive leaps. It is more active and demanding than the first two stages.

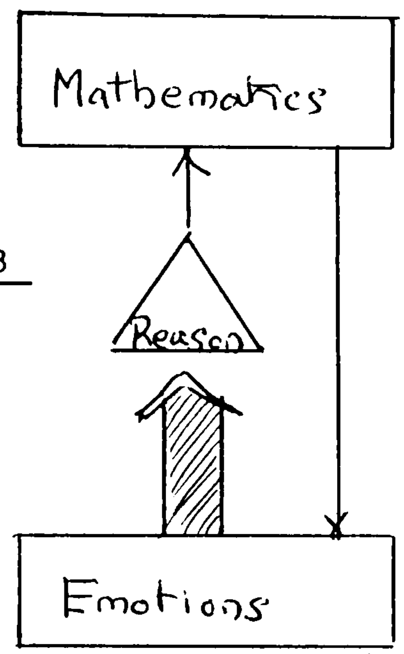
To people who are in the situation I describe, the mere suggestion of a maths. problem produces feelings of anxiety and an unwillingness even to enter the area. This was demonstrated by an inability to receive and stabilise information about a problem. I had asked Sarah to find any whole numbers that went exactly into 12. She started them, cautiously, but at a demand from me (introducing other elements into the situation), 'the numbers toppled out of my mind'. Within the group, too, there were often demands made:-

"What was it you said, Laurie? I've forgotten the problem."

This is apparent with people who are trying to be cooperative. With those who are not even willing, the reaction is to claim that the facts, however few, cannot be remembered, or even that the question has been badly explained. If in a position to do so, the person may say that the whole thing is too boring, and that they have something better to do. Some children nowadays may say this - many more may think it.

It seems that a violent reaction from the emotions completely ruins the reasoning process. What exactly the internal mechanism is, we cannot say, yet we certainly observe the effect. The feedback is not connected with possible difficulties in reaching goals. The reason simply does not get to work at all. Once the message is received that it is maths. that confronts us, no start is made.

Diagram 3



Taking terminology from the medical field, situation 1 is 'normal' in that everyone should experience emotion when tackling a problem. Situation 2 is completely dysfunctional, and we may think of it as genuinely 'pathological'. Repeated attempts to encourage further efforts may well result in further failure and a reinforcement of the emotional inability to tackle mathematics. The process simply becomes counter-productive. Teachers will evidence pupils whose ability at maths. seems less when they leave secondary school than when they entered it. This raises serious questions.

An adult who has developed this distaste for mathematics and who feels that knowledge of it is not relevant to the conduct of his or her life has a perfect right to refuse to think about it. It may be a wrong view in the event, and it may shut off areas that could be enlightening, but it is a decision a responsible adult can properly make. Some would extend this to children of school age. Here I sharply disagree with views expressed by Skemp. I fully accept that the best learning situation is one where the learner freely enters into it and engages in genuinely cooperative activity with the teacher, and I have been fortunate in experiencing it with many individuals. I regard it as naive and idealistic, however, to believe it can be in any way the basis for the mass education of the whole population. That is not to say that every child must pursue compulsory mathematics until the age of sixteen - but total freedom of choice cannot be theirs, and we should continue to expect the vast majority to be taught mathematics to the age of sixteen.

The evidence from my subjects is that they spent much of their secondary school lives learning no mathematics and becoming increasingly more bewildered, frustrated and anxious about it. The answer is not to opt out, but to seek remedies, conscious that they will not be universally successful. Some of these ideas will be expanded later, but we can indicate some guidelines.

The first and most obvious suggestion is not to allow people to reach situation 2, and this is explained in situation 1. We must provide people with a high level of success and a level of failure which they can tolerate. On the whole, our primary schools are very effective in this, and the most likely stage of breakdown is at the beginning of secondary school. In part this derives from an attitude sometimes found in secondary, that the pupils' previous experience is of little consequence, and only now are they getting down to 'real' work. Once this real work has afforded a certain amount of failure and the previous success is ignored, we are well on the downward path. Clearly past success should be treasured and past failure regarded as of little importance. This is a sensible maxim for all stages where a fresh start is being made - be it a new school, a new teacher or the beginning of a new session,

How then to escape from the situation, once in it? One of the most useful starts came from the idea of stages in solving as problem, and was found very useful in the group sessions. It was suggested that everyone simply tackle the first stage - no more. The information should be received and stabilised. When this is seen as the goal, emotional disturbance is minimised and the task does not appear as demanding. There is a certain sleight-of-hand involved in that the students believe that they are not being asked to do the problem (they are not being asked to complete it) while the teacher has overcome the objection to even getting involved, while recognising that the stage is part of problem-solving. Once the students know what the problem is they are allowed to rest. With certain well-chosen problems the statement together with the second stage. "What might be useful here?", may suddenly produce a position where the answer is obvious. The feeling of success must then be relished, to remove the taste of unrelieved failure. There was a very favourable response from most of my subjects to the 'staging' approach. One group member commented that when she was attempting some homework she would feel the onset of the paralysis to which she was accustomed, and would way to herself:-
"Now just receive the information and stabilise it."
We shall shortly see panic as an essentially destabilised state of mind, and in our eventual testing again use the word stabilise in offering positive experiences.

There is, I believe, something in the very use of the word stabilise, for in stabilising the information, one seems to stabilise the mind.

Two other guidelines, more relevant in later situations, should be mentioned. One is the request that the process be started with full attention but slowly. The other is to provide material where the answer can be checked by internal means or by an external method not involving the teacher. The rationale of this is that it is not failure, but its exposure to judgement that matters.

Another route that may help people to return to mathematical activity is a little devious. Some activities that mathematicians might consider to have a mathematical element are not so labelled. Elaine happened to get intense pleasure from timetabling for a large school, and was most surprised to find that I felt it had a large mathematical element - not in any calculations involved, but in the ordering, arranging and assertions of priority. She was fairly resistant to any classification that put timetabling (which she could do) with mathematics (which she could not do). If, however, it could be established and believed by her that she had had success in a mathematical-type operation, her certainty of failure in such things might have been dented a little. "S" enjoyed fitting together wood-block puzzles, and because she enjoyed them and was successful with them, refused to acknowledge that this perception and matbhing of shape had any mathematical connection.

She did not accept that that was one of the things geometry was about. If she had done, it might have been the beginning of a way through.

In some senses the recognition by the teacher of the real nature of the difficulty is the most important step. It is all too easy to believe that if one explains once more, and gets the pupil to make yet another assault on the problem, the breakthrough will be achieved. The pressure to achieve this, and the anxiety of the teacher, convey themselves all too readily, and frustrate the desired end. Stop, recognise what is happening, and respond through one of the suggested modes, and attitudes may change.

Is there a chance that one can shift one's own attitude without the help of a sensitive teacher? Certainly there is. We all have the capacity to reflect upon our past experiences and actions. This reflective intelligence is one of the most powerful weapons we have. It is the teacher in the mind. Reading, understanding and recognising emotional blockages in ourselves can lead us far on the way to their removal.

To recapitulate the first two situations. Success or failure in cognitive tasks brings about emotional response which then affects the performance of the reason. Success that comes too easily does not have long-term payoff, but the level of failure must be carefully monitored so that it can be tolerated. Repeated failure can lead from the normal situation desctibed into a pathological one where problems cannot be tackled, irrespective of their difficulty. Remediation is then more difficult, but should be attempted, certainly while the student is in full-time eductaion. The attempted remedy may come from the teacher or from the student's own reflective intelligence.

The assumption of the analysis so far is that it is the cognitive difficulty of the mathematics that starts the process and that we are throughout concerned purely with mathematics and with success or failure in it. Our further situations will indicate that other matters may intrude themselves.

Situation 3

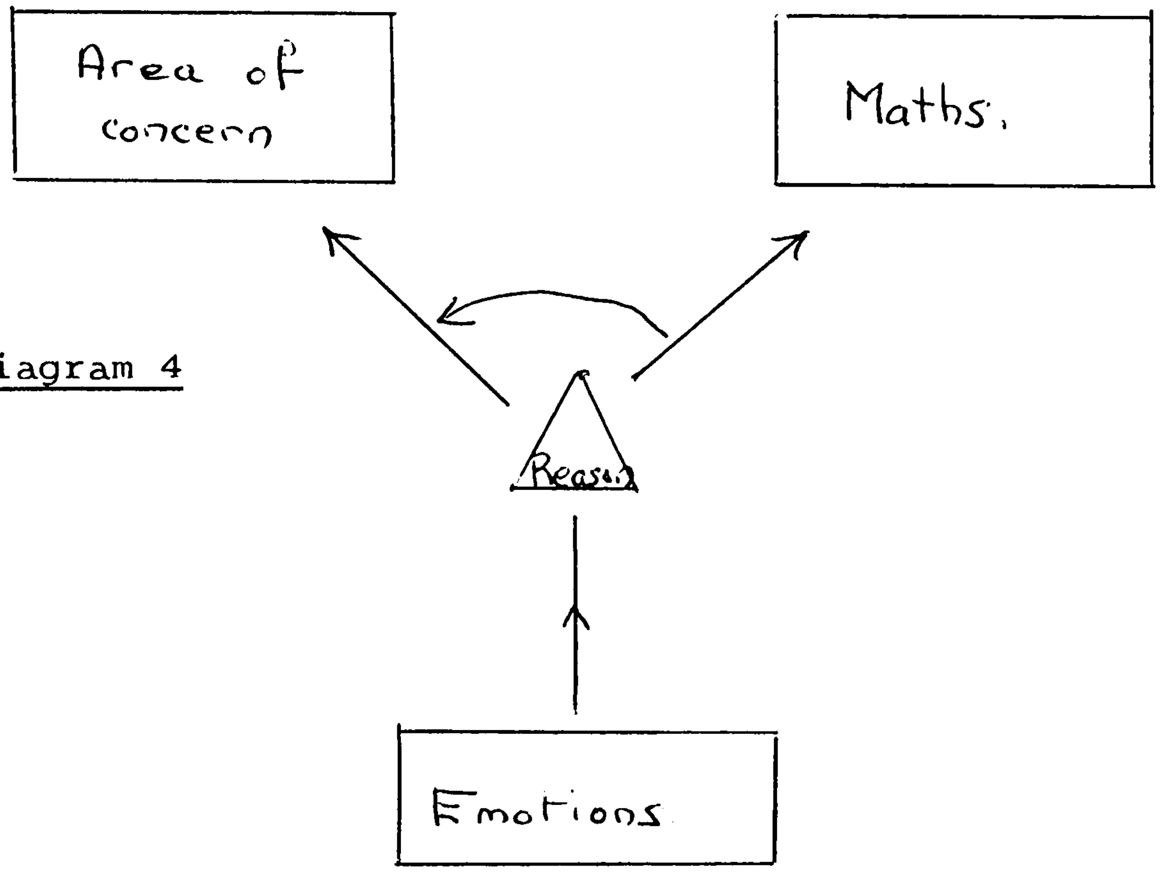
The emotions may directly help or hinder our attack on a problem through their role in altering motivation. They may even deflect us from a particular task. They have an extremely important role in ensuring survival, and the alarm bells sounded at the approach of danger allow our reason to plan for its avoidance. Skemp sees goals in different universes of discourse, with some being of more importance for survival than others. He sees emotions as calling attention to those of greater priority. There is evidence, however, from various of my subjects, that the priorities they assert should not always be regarded.

One evening "S" arrived for her weekly interview obviously disturbed about an incident at school. One of her class had got into trouble of some sort at school and had lied to his mother about it. S had been involved and had told the child to tell his mother. The child had done so at the end of school, but S was not sure what the mother's reaction had been. She felt a sense of responsibility, even guilt, and could not settle to the work I had intended that we do. The was nothing S could do about the matter; there was no preparation or action that would affect the situation the next morning, when she would learn how things had worked out. She understood this rationally, was concerned to be helpful and not to waste my time, but could not settle to doing any mathematics. We talked about her problem.

One evening at the group H arrived late, having been buying a house. She was clearly unable to think or talk of anything else. Here the emotions were entirely pleasurable but equally demanding. In fact, starting from her house, and considering appreciation and depreciation, forwards and backwards in time, we reached a good model for directed numbers.

There is no need to labour the point. It is commonplace that other concerns may deflect us from what we should be doing. Diagrammatically we may show it thus:-

Diagram 4



Emotions direct attention, and here we see them directing delta away from the mathematics to another area of concern. Naturally this deflection does not only occur in mathematics lessons, but shortly we shall claim that certain attitudes are specially important in mathematics.

As we have said, despite the survival quality of emotions asserting priorities in what we attend to, the importance of this other area of concern does not always indicate the degree of distraction it may cause. A very weighty concern, such as the loss of someone close, a deep emotional involvement, or major issues in one's career, may demand to be thought about to the exclusion of routine cognitive tasks. This may be entirely sensible. However, if the thinking and plan-making we feel obliged to do become evidently fruitless, a switch to unrelated tasks, such as mathematics, may prove a therapeutic diversion.

These major concerns cannot be taken account of in our brief. We now need to say something of how mathematics is best learnt, and how we may cope with lower level distractors.

Mathematics is a deeply contemplative activity, and is best tackled with deep and undivided attention, even if that can be given for only relatively short periods. The benefit of this in comparison with more half-hearted attempts over a much longer time is considerable. That does not deny the importance of the periods of gestation in between bouts of thought, when the subconscious seems to work on the material.

Most of us face a number of what we might call middle-range concerns, such as S.[illegible] These can prove very difficult. Either the teacher or the student's own reflective intelligence must allow the problem to be worked through to a stage where it can be left for the time, and then attention can be re-directed to the learning task. The ability to compartmentalise and thrust aside the distraction once it has been thought through to a certain stage, is one that needs consciously to be developed. That does not imply that the moment a new task presents itself we can put aside other matters, but we do need to recognise when something is tidied to an extent where we can leave it and engage fully on the new task.

People whose life style or job yields a variety of these middle-range concerns become accustomed to them overlapping and rarely give attention to one for long. It may be a perfectly satisfactory way of operating in many areas, but is inappropriate for mathematics, or, indeed other matters which need a high level of concentration. Most of my subjects were very busy people, yet it is significant that some claim to be and others do not. The most effective workers are able to compartmentalise, and think hard about one thing at a time. They seem to monitor their own performance all the while. In our later chapter we see this as a delta-three function.

There is even a danger, with many things to consider, that we spend our time keeping track of what has to be done, and not doing it! A useful technigue is to write or record lists of things to be done. This at least moves them from W2 to W3, leaving space in W2. It also tempts one to think them already done!

Even essentially trivial matters may produce emotional interference with our reasoning processes. During any day such issues regularly assail us. We leave our cheque-book at home, wonder if we left a window open, fuss about whether we shall be able to park when we reach our destination. Some of these we submerge but they still irritate at a subliminal level. Again it can be worth clearing the mind by spending a minute or two determining and then externalising them.

A last brief example of deflection. The expectation of some event, pleasurable or otherwise, will readily interfere with our concentration on the matter in hand. Sitting in one's office, happily working, but with the expectation of someone arriving at eleven o'clock, there are minor impediments in the smooth flow of thought as the hour approaches, and more serious ones if the caller is decidedly late. Perhaps it is the obsession with time our civilisation has.

Situation 4

The first two situations concentrated on reactions to mathematics. The next looked at external and unrelated matters, and their influence. We now look at the interpersonal issues between teacher and taught. That is not to say they were not at the root of, say, situation 2, but the separate statement is needed, and is the central one.
It is well-known that when pupils have a choice of subjects, it is as much dictated by who

is teaching what as by any genuinely curriculum issue. While many teachers would agree that this is wrong, they still assert the primacy of the relation of the teacher to the taught in the classroom. Accepting that there is truth in this, there are some dangers to be avoided. The central one is that teacher-approval rather than learning satisfaction is sought, and this is a serious deviation from what education should be about. It may result in the subject (mathematics) becoming almost an irrelevance.

M reflected on her state of mind when posed a mathematical problem by me:-
"You see, I've still got this, if I use the word 'eagerness' again, still trying to show that I can do it. It's partly wanting to be right, it's partly wanting to be the bright girl, it's partly wanting to show off, it's .... and I think at the same time as the show-off part, there is also the fear part, that if you do it, and do it in a hurry it will be over and done with."

Many of the problems of satisfaction and dissatisfaction are expressed in this statement: the teacher and the other pupils are relevant but not what is supposed to be studied. Both L and S in early sessions with me exhibited the anxious side of this polarisation of feeling, and often offered answers at random in the hope that the question (and questionner)

could be disposed of.
The balance between eagerness to respond, usually seen by teachers as a positive feature, and the fearful anxiety that can be produced by an expectation of quick response, is an important issue. Evidence from the subjects of this present study suggests that there is a much higher level of anxiety among students than most teachers would credit, and that is true even when the relationships are notably good.

Time pressure and constant questioning will be examined at length later, but let us look at the role played by questioning in the emotional life of the classroom. The sight of another human being trying to attract your attention has obvious satisfaction, and we need not be surprised that it is sought and the structure of the lesson designed to give it. Though it is not necessarily bad, there are obvious reservations when the teacher appears to be satisfying strong emotional needs in this way. We are getting pupil-response giving teacher-satisfaction and teacher-approval giving pupil-satisfaction. Where is the satisfaction inherent in learning mathematics? It may in fact be that a powerfully affection-seeking personality achieves good results in mathematics with a class; these may often not be sustained by a teacher of different style.

Mathematics is a largely contemplative and exploratory study, and the extent to which it can be an inner pursuit is not always understood. It cannot be pursued without considerable input, but the satisfactions can lie in success in pursuing it, not in the teacher telling you you are right. The "authority" lies within the mathematics, and the aim of the operation is that the student should learn, understand and actually do mathematics. Satisfaction should be strongly related to the primary task, and not hinge on personal factors largely independent of the material. Diagrammatically the danger is that we pursue the right-hand arrowed circuit rather than the left.

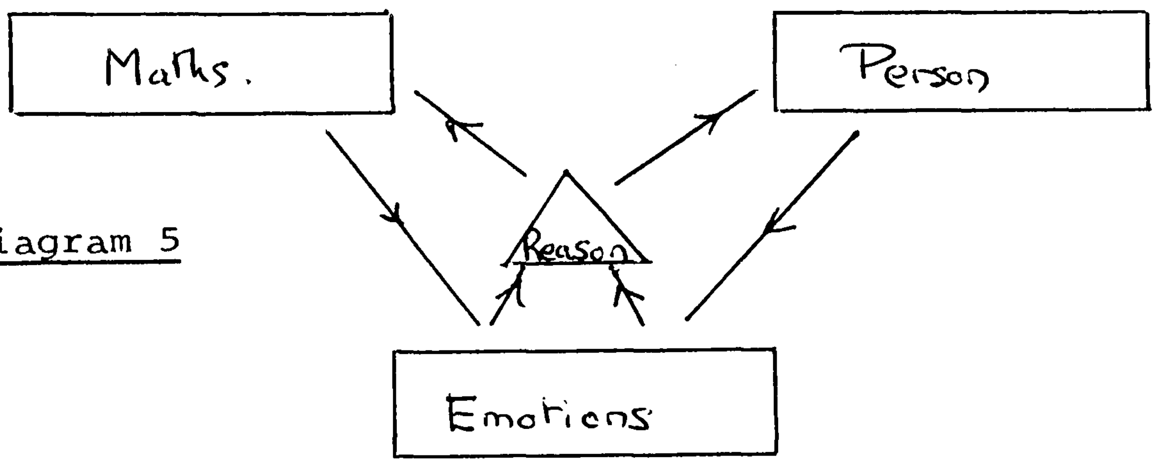


Diagram 5

A number of my subjects so lacked confidence that they remained dependent on my agreement of their correctness. It seemed (my subjective view) that they became less dependent on my approval as we progressed, but the need for agreement remained. My insistence that a problem could not only have satisfaction within it, but also the certainty of correctness, was simply not accepted. L was given a series of problems on a cube. They involved visualising a 3 x 3 x 3 cube, like this:-

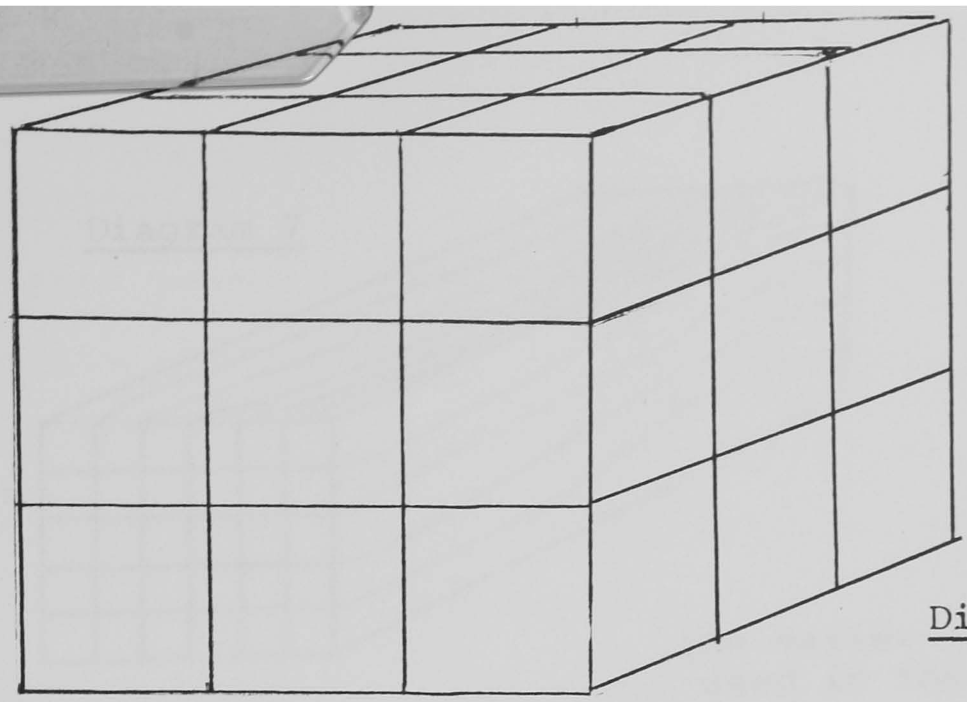


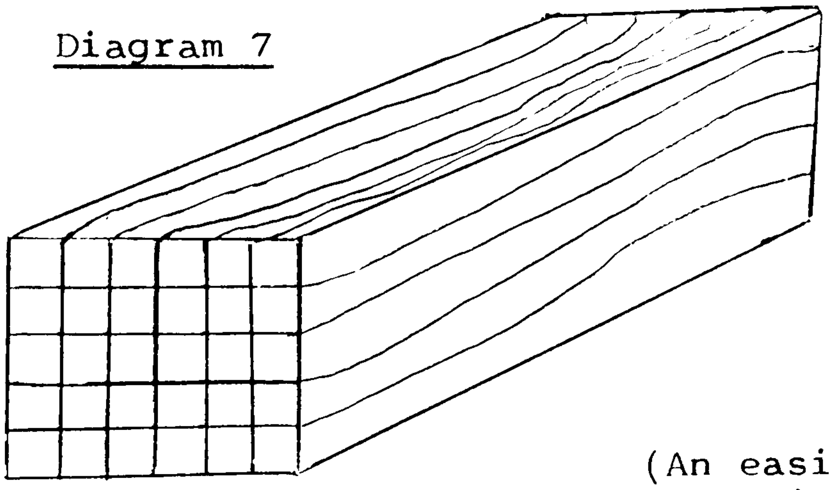
Diagram 6

The outside is painted black, and the question is "How many small cubes have no paint on them?" Once the answer is seen, it should be evident that it is correct. L did reach it: she found it interesting and discussed it with a friend who is a mathematician. She claimed to be confident of the answer. The critical test for this confidence seems to be the willingness to defend it against attack. I asked how she would respond to someone knowledgeable who said she was wrong. Her reply, she said, would be:-

"Two people I know, who are better at maths. than you, agree that I am right."

A more successful attempt, with S involved a wood-block puzzle. A cuboid of wood is cut two ways with a band-saw, using wavy lines. It is like a three-dimensional jigsaw.

Diagram 7



(An easier version was used in the testing later)

The block is dismantled and the problem is to reassemble it. The matching of shapes that is involved is a worth-while geometrical exercise, and simple curves need to be appreciated. An important feature of such a puzzle (indeed of many puzzles) is that when they are done, there is certainty that the solutions are right. S gained satisfaction from doing it, and had no need to refer to me to confirm that her solution was correct (though she was pleased to tell me she had done it), but decided that as she could do it, it must be (a) very easy, and (b) not connected with maths.

If you are sure you cannot do mathematics, there are many ways of refusing to admit that you might. Difficult though it may be to achieve, it is necessary that people are given the opportunity of experiencing pleasure in doing maths. for itself, and to get the satisfaction of knowing they are right, without referring to anyone. Only with that can we carry through the periods of bad teaching we are almost certain to get.

Situation 5

In some senses the situation we now describe could be subsumed under the last heading. Again we have emotional responses inhibiting the reason, where these responses relate not to the task of doing mathematics, but to a person engaged in the teaching process. The distinction is that the student does not see the relationship simply as a teacher-pupil one, but is reminded of other encounters of an unhappy kind. One person I interviewed is the head of a large comprehensive school. He is a man of powerful intellect, sensitive and highly perceptive. I arranged an interview at his school and expected to learn much from him; and so it proved. An interesting idea, described in the next section, started with him, but so also did an understanding of how a discussion about mathematics, one-to-one, might be perceived by people with unfortunate early experiences. His own early schooldays were dominated by the arbitrary distribution of punishment. Though he was very able at English, and by diligence and punctuality managed to avoid many pitfalls, his weakness at mathematics regularly put him in danger of a beating. Thus our meeting brought back to him the deep-seated fears of childhood, and he acknowledged to me afterwards that he had felt most anxious at the prospect of our talk, though he knew me quite well, felt friendly, and in no rational sense should have found it difficult.

Talking about mathematics seems to be found demanding , a term often used by my subjects. Perhaps more questions and challenges are put in mathematics, but it was surprising how often other matters surfaced. In several cases the interviews clearly brought back memories of early parental relationships. At times the parents had been insistent about mathematics, but the interviews triggered off more general concerns in this area, with the result that they could not tackle any mathematics till some of these problems were shifted a little. One particular interview led to a subject experiencing several days' disturbance which she was in fact well able to handle, and which it was appropriate to handle at that stage. With someone else we uncovered a serious and lengthy struggle with her mother, reinforced by experience with a teacher at the top junior stage. Working through these led to marked improvement in her attitude.

Generally, I felt that interviews in a one-to-one context about mathematics, were often coloured by memories of other encounters, not necessarily about mathematics. Skemp uses the term "resonance" to describe this type of displacement. The term is that used in the physical sciences, where a particular note can set a piano string vibrating, though at some distance away. Here too, touching some part of our inner world evokes resonances elsewhere.

We shall later see the importance of these relationships being of an authority type. Certainly in the cases just quoted there were clear authority problems. With one other subject, the mere assumption of a teaching role on my part would sometimes make her bristle.

The interpersonal situation here, then, unlike that in situation 4, is seen as resembling some other interpersonal relationship of an unsatisfactory kind - and again the mathematics does not get done.

Situation 6

In this situation anxiety, in fact unrelated to mathematics, has become arbitrarily associated with that area. It is now not the immediate interpersonal relationship which is wrongly seen, but the whole of mathematics.

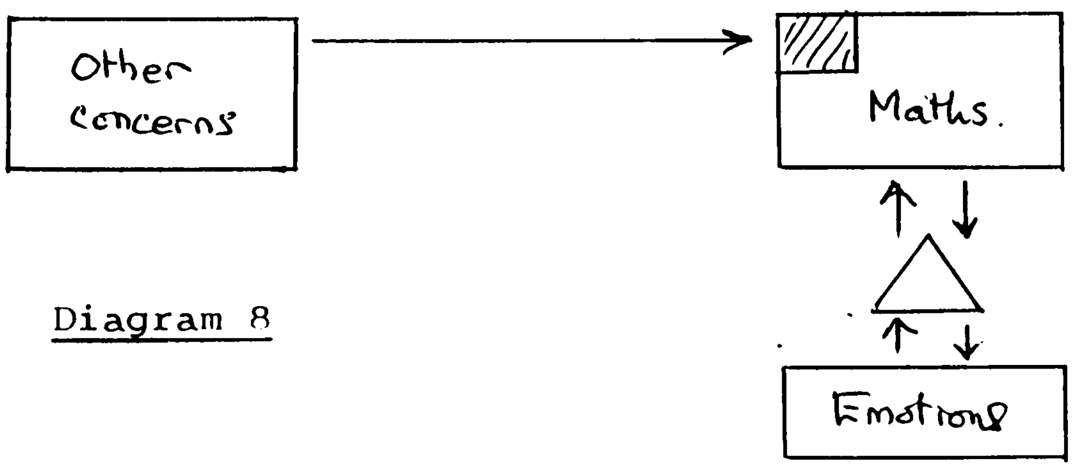


Diagram 8

Our diagram shows a concern well away from mathematics, which has been wrongly stored in that boundary. Now when mathematics is approached, the negative emotional response is due to this concern, in the mathematics box, where it should not be.

We have already seen the effect of some deep-seated early relationships becoming attached to mathematics. Once that has been done, the section labelled 'maths.' in one's mind may become a dumping ground for difficult and unresolved problems of all sorts. It may simply be the way the mind chooses to pigeon-hole items.
A number of people have, curiously enough, expressed the view that mathematics was all right until they had to do 'problems'. These loomed large in the 11+ examination, and it may well be that mathematics then became identified with problems in general. As Sybil put it very succinctly:-
"Category - problems".
A particular stumbling block for many of my subjects was their first encounter with x, the unknown. It is unfortunate that a word like 'unknown' , with its built-in anxiety, should be so firmly attached to x. It may also open the door to feelings about sex - regarded at that stage as mysterious - also being classified in that area.

This may seem a little far-fetched, but some signposts are there, and for those whose reactions to mathematics are extreme, it might signify that a more sensitive area was somehow involved. For some people, any approach to this 'no go' area is strongly negated by the emotions, and their mathematical understanding cannot grow. Such a situation may lie behind the many statements that 'I can't do maths.' or that it is 'boring'. This latter statement may be all too true of some people's experience, but it is also used as a defence mechanism.

There is an interesting feature about mathematics and our early experience. It is the main substantial piece of explicit learning that stretches back into early childhood. It is true that language also does, but it is doubtful if it is seen by the individual as either so contained or so obviously developmental as is mathematics. In mathematics we have a structured subject, seen as having considerable importance, stretching back as a ladder to the past within each human personality. If I speak to someone about learning to count, I expect this to have happened at say four or five years old; if I ask about quadratic equations, at about fourteen. Experiences contemporaneous with this, and particularly those with heavy emotional charge, may then surface into the consciousness. Touching on mathematical experience does therefore release other memories, as I have found with a number of people. Memories which people may wish to suppress because of the unpleasant

PAGE

NUMBERING

AS ORIGINAL

emotions associated with them may be associated also, purely in time and in no other way, with a piece of mathematics. If that, too, gets blanked out, it may affect the development of mathematical understanding, because of the structured nature of mathematics, but we may also find that later attempts to reme this are seen as threatening to the matter which is not to be thought about.

So there are some special reasons why unrelated anxieties become attached to mathematics. It is not easy to suggest a cure, which seems to lie within psychotherapy rather than the teacher's normal skills.

Finally for this chapter, let us review the situations:-

1). The standard situation arising from the Skemp model, with emotional response to success or failure in the task at the time..

2). A pathological reaction to previous repeated failures that stops the student before he or she starts.

3). Emotional distractions of various levels which prevent the reason from applying itslef to the problem in hand.

4). An over-emphasis on teacher approval rather than intrinsic interest.

5). False interpretations of the interpersonal relationship in a teacher-pupil contact.

6). Transferred anxieties from areas unrelated to mathematics.

CHAPTER SEVEN

THE SPECIFIC ISSUE OF PANIC

THE SPECIFIC ISSUE OF PANIC

The various distractors which inhibit learning, listed in the last chapter, involve unpleasurable feelings of different degrees. Our intention now is to narrow the field of discussion to a particular reaction, that of panic.In everyday terms this may manifest itself either in flurried and inappropriate action or in a form of paralysis that prevents all action. It is the latter we shall observe upon. An interpretation of the mechanism of panic, provided by Skemp's theory, is discussed in chapter 3.

The study of mathematics, seen simply as a relationship between the subject matter and the student (situation 1 of the last chapter) certainly generates feelings according as goals are reached or not. At the positive end there may be great elation at the successful solution of a problem, and at the other end of the scale feelings of frustration and annoyance at continued failure. Disappointment may be strongly felt if an apparent solution is reached, and then found to be false. But in normal emotional responses of this sort we shall not meet panic.

It is in situation 2 that we meet what we describe as a pathological reaction to mathematics. Here certainly we may meet panic reactions, not just when engaged in mathematics but sometimes at the mere threat of having to do some. From the people who do behave so we must find the pressures which trigger in them the 'flicker of consciousness' that the Skemp model offers.

We shall shortly suggest what these pressures are, and then, in the next chapter, demonstrate that, by applying them even in an artificial situation, we can induce panic in some people.

In the third situation of our last chapter, the distractions were outside the study area and were known to be so by the subject. Certainly the study was impeded, and the emotions could well be strong, but the issues are not relevant to our present concern.

Situation 4 is very relevant. Here the student sees the relationship with the teacher as more important than that with mathematics, and there is much evidence from my subjects of the strength of the emotions that can be generated. It is only to be expected that interpersonal relationships, rather than interactions between a student and what he studies, should evoke the strongest emotions.

Many observations by my subjects speak of the relations with teachers (and sometimes parents), and the fear of authority is manifest in these statements. Authority is seen as 'cross', 'demanding', and has in their experience often been used to ridicule them. Even when they can recall good experiences with authority figures, it is the unpleasant episodes that appear to be most deeply etched. Layers of such experiences lead to the condition of situation 2.

This fear of authority was enhanced greatly by time pressures. We indicate in chapter 3 how it is not merely the threat posed by an outside agency, but the urgency of the threat that can lead to the dysfunction between Δ_1 and Δ_2 that we call panic. It is not only if Δ_2 cannot provide a plan that anxiety grows, but if it cannot provide a plan in time.

It is therefore elements present in situation 4 that are most relevant to our final testing, where we shall show that the imposition of authority and time pressures do create panic. We shall also show that their explicit removal can lead to rewarding mathematical experiences.

Situation 5 can lead to very strong reactions and to hasty attempts at withdrawal by the subject. It can also lead to panic, again related to the interpersonal relationship. Here, however, it was general unpleasant experiences in relationships that were involved, not specific either to learning or to mathematics. It is clear that in the past some of my subjects had felt their territory invaded and unwanted demands made upon them; the teaching situation they were in with me somehow mirrored this. The demands seemed to be of submission to authority, of giving up something of themselves, of being exposed to ridicule, or even of demands of a sexual nature. The one-to-one encounter had evoked memories of such earlier encounters. This situation, then, might be panic-inducing, but is not relevant to the learning of mathematics.

In situation 6 it seems that mathematics has become arbitrarily associated with some other very sensitive area, perhaps owing to some broad classification in the subject's mind such as 'problems'. Reactions here may be strong, but again are not the substance of our study.

It is of course important to recognise that people may panic for reasons other than those pressures of authority and time that we posit. In the testing that follows we use those pressures. In showing that panic results we do not establish that in all cases it is those threats that are responsible. It may well be that some whose sensitivities are in the categories just lasted will react in this way in any case. However, the claim is only that some respond to these threats. In fact, the good experiences that the same people have when these pressures are removed might be better evidence of the claim.

We have therefore narrowed down the investigation to one aspect of situation 4, which we say leads to the state characterised in situation 2. We postulate that the specific response of panic is often caused by the fear of authority coupled with the pressure of time.

It is important to observe that it is not only in the teaching of mathematics that we shall see this phenomenon. Certainly there is evidence from music and particularly from physical education. If conclusions about authority are relevant in other areas, that is to be welcomed, yet there are issues peculiar to mathematics that suggest it more often creates this extreme sensitivity. To enumerate some:-

1) Mathematics is seen as 'important' and bestowing more authority on those teaching it.
2) Mathematics is seen as the main measure of cognitive power, and that power is seen as the quality most valued by the system.
3) Answers in mathematics are seen as being firmly right or wrong. As a result clearer and often harsher judgements are made upon one.
4) The lack of discussion of emotional elements in the the teaching of mathematics leads to feelings being repressed and becoming eventually more intense.

So in asserting as we shall that particular threats, often present in mathematics lessons, create panic, and inducing this in a slightly artificial way in our testing, we are not limiting this to the study of mathematics, but saying that most of our evidence comes from there and that there are good reasons why such tensions are more prevalent there.

CHAPTER EIGHT

TESTING

TESTING

Our theory is that many of the most negative reactions to mathematics teaching arise from feelings about authority and the fear of being judged. When the authority figure also imposes time pressures, then a state of panic may result. The theoretical model of panic was offered in chapter 3: we now propose to show that it can be rapidly induced by the stated pressures.

In the first series of experiments, audiences of up to forty people were offered two experiences. The first involved a threat that they were to do some mathematics under time pressure. The prediction was that even under a threat, a proportion of people would report emotional responses in the area of panic, with some using that actual word.

It was further predicted that these same people could be given an enjoyable and rewarding mathematical experience, in part by the explicit removal of those particular pressures.

In the second series, I worked with individuals, where I sought to move them to and from a state of panic over a continuous period of time. This was achieved, as before, by the application or removal of the stated pressures.

Group Testing

These tests were conducted on public occasions when I had been invited to address a gathering of people. The title of the talk would indicate that some psychological aspects of the teaching of mathematics would be discussed, but nothing beyond that. The settings were mostly those of a formal lecture, with the speaker at the front. The audiences were all adult, most often teachers, though on occasion the parents of the children at a particular school. They were sitting in rows, facing the front.

I now describe the general way in which these experiments proceeded, and report on the broad reactions. This is followed by a detailed account of one such session. Further support is then to be found in Appendix D, giving the reactions on other occasions, though by no means all the work is recorded.

I opened the proceedings with a statement of this sort:-

"I am Staff Inspector for Mathematics and I am going to give you a test. I shall want to see your answers so that I can judge them. There is a strict time limit." (Takes off watch and holds it.)

After a slight pause, I would then say:-

"Now I want you to close your eyes, find the single emotional word that best describes your feelings and write it down. Do not put your name on the paper."

The release of tension was usually very marked, and accompanied by gusts of nervous laughter. When the results had been collected, I read them back to the audience. There were nearly always a proportion in the panic region. One such list of responses is given after the account of a single occasion later in this chapter, others appear in Appendix D.

The prepared statement is designed to point the pressures we have indicated. My structural authority is emphasised by giving my position. Judgement is explicitly promised, and the time restriction stated and reinforced by an action.

The induction of panic is therefore a very short process. That to engage those same adults who reported panic, or something allied to it, in an enjoyable mathematical experience is much longer. We now state some principles in doing so.

The first stance is towards authority. In the second part of the experiment, I state that I shall not want to see their answers: this removes the fear of judgement by a structural authority. The reason I give is that the answer lies within the material - "The authority lies within the subject (maths.)" - and when they have found it there will be no need to check with my (sapiential) authority.

A relaxed atmosphere must be achieved, and the fact that there is no time limit must be stated, often several times.

This is helped by avoiding them knowing that others had finished. This was achieved by getting them to close their eyes while working. (A few people do not like shutting their eyes)

The material is important, but the criterion not particularly restrictive. It is that the answer should be one that can be 'seen' and that one can be sure of. In the experiment we shall now describe, I used a series of exercises on the cube.

The audience here were twenty primary teachers on a six week mathematics course at a teachers' centre. They were all supplied with a sheet of paper, and after the first part of the experiment handed in a torn off part with their single word emotional response on it. The second part is now described almost verbatim. Though at times it seems rather discursive, most of the statements were calculated, and notes are supplied at the end as to the use of certain words. I said that I wished them to take a cube in their minds:-

"I don't mind what sort of cube it is. It can be a small one you can pick up and handle and turn over in your mind. Or it can be a big one if you like (1) made of constructa-straws, that you can walk through(2)

I just want you to........I think the word I use (and the word I want to talk about later), the word I want to use, I want you to stabilise (3) it in your mind. I want you to sort of walk round it and touch it and feel its corners and I want you to get it there, so that whatever size it is you've got it firmly in your mind and you're just settled (4) with it."

"Now we can do some work on it, but at no stage, at no stage does anyone give me an answer (5). The answers are for you to find out and when you have an answer it is for you and you alone (6). And take as long as you like (7).

Now I want you first of all to put a hand on a face. The face is one of the flat bits. (Laughter). Well, I don't mind - you don't have to know about a cube, do you? I want you to count how many there are. You may already know, I don't mind. But I don't want you just to know. I want you to go round and actually count them. Even though you know. Have it in your mind. It will help stabilise it, go round and count them and see them all. (9)

Now you've got your eyes shut. I want you to flap your hand at me if you haven't done it, because this means that no-one else will know you haven't done it (10), and you can quite easily flap at me and respond - there may be several of you (11).

Nobody's flapped on that one. We'll move on. Run your finger along an edge (12) until you come to a point at the corners. Feel that point. It's sharp - press your palm on it. About those points at the corners - again I know you may know, but I want you to go round and count them. I want you to establish (13) to yourself how many there are there. I want you to be totally certain of it...and if you are totally certain of it, what would be the point of giving anyone an answer? (14)

Now, take your time, go round and count them. Count them again. If you are finished and getting a bit im impatient (15) about waiting for it, just browse round it, the cube, because lots more is going to be asked about it soon, so just walk in and out of it. (16)

It's a relaxing occupation. (17)

Anyone not seen how many vertices there are? Flap your hand if you haven't. Now I can move on. Run your hand dowm the edges. Slightly more complicated, the edges. And count them, taking each end as you go, running your hand along it, or a finger along it. Make absolutely sure you've got it, and that usually takes a bit longer.

Right. Flap your hand if you haven't done that. Must be totally honest about it (18) - it's only me that knows, and so it doesn't matter if you flap your hand -noone else will know - no peer group pressure. (19)

Now we're taking that cube. And that cube (you've got it firmly established in your mind) is three units each way, cm., m., whatever you like. I don't mind if it's a small one or a big one, but it's three units each way. I want you to do one of two things. I either want you, if its solid, to cut it up...or if you want to you can take those little cubes you get in the classroom and build it up. It's three units each way and I want to build it up with little cubes, or if you like chop it down into little cubes.

Now some of you may know how many there are in the cube; probably quite a lot of you know. I want you to see how many there are. Although you may have known it, I want you to see it. It's quite common for people to get stuck at this stage because visualisation is not something everyone finds easy.

The cube is three inches each way. We want it made up of small cubes which are only one inch each way. Three inches each way built up of smaller cubes which are in fact only one inch each way (20)

If you are still a bit stuck on it (and I think one or two of you are) take the bottom layer and see how many there are there, and then add in the other layers. Or alternatively, chop it into walls. How many in a wall? Chopping it down vertically. All right. Who has not got the number yet? Don't tell me because I know it already.

Now hold that cube in your minds, made up of all the little cubes. Paint the outside black (21). Pick it up carefully or it will all fall apart (22), and paint the underneath bit as well. Get your fingers dirty, but there, that's life! Paint all the outside black. Now, of those little cubes, can you find how many of those little cubes have got no black paint on them at all?

Gradually increasing difficulty, but you've got endless time. Take just as long as you like about it. (Long pause). This time, would you flap your hand if you've got the answer.....few of you have, number more to go.

If you've got it and want to do something while the others are sorting it out, you can wander off and do something I'm not doing with the others, and see how many have **one** face, two faces etc., painted black. Those who are still on it concentrate on how many are unpainted."

In fact, we continued on to problems about cutting up the cube. It was necessary that it was a fairly long piece of work, that in itself emphasising that there was no hurry. We now indicate the intentions of the phrasing at the marked places.

1) Phrases such as 'if you like' are reiterated to diminish the sense of compulsion.
2) Throughout there is an emphasis on the concreteness of the cube, although it is all done in the head.
3) "Stabilise" is an important word - almost an antidote to "panic". It is also a rest before

5) Explicitly removing the element of judgement.

6) Returning to the theme of 'the authority of the subject'.

7) Explicitly denying time pressure.

8) The appeals to sensory experience seem to be effective in aiding visualisation.

9) This sought to make things 'visually apparent' - a notion explored in geomtrical work with the subjects in the field work.

10) Exposure to the group forces people to move on when not ready.

11) Removes a sense of isolation. People often believe, falsely, that they are the only one in the group who does not understand.

12) The tactile experience again.

13) 'Establish' is a word like 'stabilise' and has a similar function.

14) Another reference to 'the authority of th subject'.

15) One way of coping with the quicker ones.

16) Another appeal to 'pseudo-concrete' experience.

17) The need is to achieved relaxed attentiveness.

18) There is no guarantee that people will be honest, even in this setup.

19) I believe explicit statements of what one is trying to do to be helpful generally in teaching.

20) Repetition is used deliberately to get the problem stabilised, and to offer security. It also has a slightly hypnotic effect, which may reduce anxiety.

21) Repetition - said soothingly.

22) More tactile recall.

The group were now asked to record, briefly, but not this time in a single word, their emotional response to this second experience.

The table that now follows records their reactions. They are listed in the order of severity of their first rsponse (my perception). Of particular interest are the replies of the first five people to the second exercise. We shall discuss these and other such replies to the group experiment later.

TABLE 2

Comparative results of subjects in two tests

Sex	Test 1	Test 2
M	Terror	Much more relaxed. Interested.
F	Panic	Quiet fun, which I really enjoyed.
F	Panic	Happy, calm and relaxed.
F	Panic	Calm, more secure but still had moments of feeling not confident.
F	Sweaty/Palpitating	Pleasurably interested, wanted to go on.

M	Fear	Inquisitive - curious.
F	Fear	Satisfaction as each stage was completed because I was fairly confident that I was correct. Happy there was no time pressure.
F	Fear	Sensual.
M	Ugh!	Soothing and stimulating.
F	Apprehension	Interesting - nice it could be private. Could scrap 'muddle' and start again without shame.
M	Apprehension	Cosy - Womblike.

Sex	Test 1	Test 2
	Tense	Relaxed. Confused in the half-way but did not lose heart. Got back again to struggle to find answer.
M	Tension	Suspicion of potential trickery.
F	Tension	Relaxing
M	Nervous	Involved - wanted to do it all enjoyment.

Sex	Test 1	Test 2
F	Hilarity	Surprised at how clearly I could visualise each stage.
	Ridiculous	Reasonable purposeful activity.
F	Stupid	I don't like shutting my eyes but having got over that hurdle I liked the exercise and thought it was sensible if a bit slow.
M	Joy	Dark

Individual Testing

As we have seen, emerging most forcibly from the study was the fact that panic was created by authority and time pressures. In the five studies in this chapter we see how little difficulty there was in inducing panic, though there were also some interesting ways in which they managed to develope resistance at some points. It is interesting to speculate whether even people who are able at mathematics, and with positive attitudes, might be reduced to a state of anxiety by suitably devised experiments.

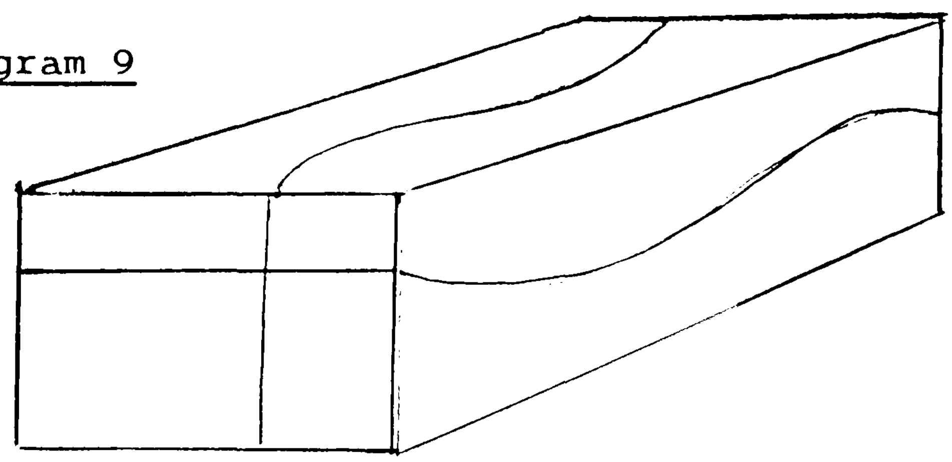
Of the five subjects, two were junior to me, and directly responsible to me for their professional work. This is important to state, since the weight of authority is so significant. The third worked in the same general area as myself, regarded me as a senior person (and something of an authority), but was not responsible to me. The final two were people who had never met me before. The first three subjects were seen in private, in surroundings familiar to them. The last two were in very public situations at conferences. We might therefore say that the first two might feel more personal pressure from me, and the others more peer-group pressure. An added factor with the first two was that they already knew a good deal of my theories.

All five were women. This is a disadvantage, in some senses, but it has emerged that women seem better able, and perhaps more willing, to discuss emotional response. However, the thesis seeks to show what causes panic: it does not seek to discuss sex-differences in attitudes to mathematics. It may provide some bases for such work, but that is another very extended enquiry.

Firstly, then, let us describe the experiment. It consisted of one-to-one interviews conducted, as we have said, some in private and others very publicly with an audience who then commented on what had happened. I sat on one side of a table, with the subject on the other, and I asked questions for about ten minutes or a quarter of an hour.

The first questions were numerical calculations, carried out mentally, and of increasing difficulty. The nature of these will become clear as we describe the separate interviews. These were followed by a spacial problem, which must be described. The wood sculptor, Brian Willsher, makes puzzles rather like three-dimensional jigsaws:-

Diagram 9



A cuboidal block of wood is cut length-wise in curved lines using a band-saw. The one used in the experiment had only four pieces, as shown in the diagram. It is simple, and chosen for that reason. Many people find this type of puzzle intriguing, and if left alone, with no pressure, would enjoy attempting it, and would probably be successful quite quickly. It did not always prove easy under the conditions of the experiment.

The interviews were audio-taped, and as much can be deduced from the subjects' voices and manner about their emotional state, as from what they actually say. An ingenious attempt (devised by Skemp) was made to introduce more precision into the experiment. The subjects had two buttons to press, one indicating pleasure and the other anxiety. The buttons gave high or low-pitched buzzes, not heard by by myself, but recorded on the tape. Their noise level could be controlled separately from the conversation, so that on playback they could be brought to a suitable level to punctuate the recording. My questioning was devised with the expectation of certain responses at certain stages, and the buzzes showed whether these expectations were met.

This method was effective and revealing but was not without difficulty. Practice was necessary for the subject, and the presence of electronic equipment was itself a factor in creating emotional tension. At times comparisons between what was said and the button pressed made it evident that the wrong button had been pressed. There was a tendency to press the 'pleasure' button when an answer had been reached, as a measure of relief rather than pleasure. Furthermore, the need to use the hands on the spacial problem made it difficult to work the buttons as well. Efforts to replace the hand buttons by foot pedals ran into technical difficulties.

We now give the detail of the separate experiments:-

Subject No. 1

The first subject was an advisory teacher, since promoted to headship. She had taken 'A' level mathematics at school (not successfully, I think) and main mathematics at college in teacher training. She could therefore be thought of as someone 'good' at mathematics. She was very conscious that I knew more mathematics than she. She also worked directly to me, so I was very much her 'boss'.

Despite what I hoped was an easy and relaxed manner at the start, she immediately pressed the 'panic' button when told we were about to begin. This was repeated when I said that it was to be calculation. When I said that I did not need to know the answer, she hastened to have this confirmed.

The first question was '17+8'. She asked for it to be repeated. She then settled a bit and did a series of questions, not giving the answers, but pressing the pleasure button with every answer. I then sought to impose pressure. I asked for 117 + 84, wanted the answer, and said 'move along'. She replied 'Oh God!', presssed the pleasure button (clearly wrong) and said:-

"You see that's panic - I'd have to go back."

I consoled her, slowed down and did some multiplication 3 x 8.....5 x 7... and she settled again. I tried to keep things stable by saying:-

"Take your time over this - it's not in the tables - do it in your head...7 x 13."

she says 'Uh-mm' rather quietly, and apparently happily, but without pressing a button. She then got 8 x 13. Then:-

"Speed it up again 7 x 14, and I want it as quickly as you can."

She again sought to save time by repeating the question. This is characteristic of the behaviour of various subjects throughout this study, when they are fussed. They either repeat the question themselves, in a questioning tone, or ask the experimenter to do so. In this case she gave 117 in a rush, then managed to remain calm and corrected to 98. It was clear from the tone of voice that she was holding herself firmly in check.

Again I hustled her, asking for 11 x 18 'As fast as you can'......then.....'come on, come on'. She pressed the panic button and dissolved into nervous laughter.

In the second half I used not only a four-piece puzzle but one of a similar type, but with nine pieces, which was much harder.

We started with the four pieces, and she was asked to work with her right hand only, so that she could work the buttons with the other hand. The recording showed that she had pressed the pleasure button early on, laughing (it seemed nervously) at the same time. She took some time, but got the problem out. I applied no pressure.

She explained to me that she did not like the fact that it was not a cube (which had been her expectation).

"I didn't like it when they didn't fit like I thought they were going to.... Yet my first reaction when I saw it was to press the pleasure button when I saw the shape. I like shapes. It doesn't mean I was going to be able to do it, but I felt differently from the numbers."

I then asked her to do it again, saying:-

"You ought to be able to do it faster the second time, so do it now, as quickly as you can."

She pressed the panic button, but got through quite quickly.

Next, I used the nine-piece puzzle, and assured her that it was bound to take much longer, and was generally soothing. As she progressed, however, I gave more emphasis to speed, and was somewhat derisive about her efforts:-

"That's all wrong. Only three to do and you can't."

She finished in good time and seemed to have cut out the interventions. She then commented:-

"I didn't press any buttons then. I wanted to do it. I got involved in doing it. Because I liked the shapes and because I felt more confident in dealing with it, I was prepared not to listen to what you were saying. It isn't that I can't do mental arithmetic - I was good at it at school - but I'm out of practice and it's that feeling of being tested. With the puzzle, I could cut you out. I like it, and was confident of doing it in my own way. And there was a right answer in the sense that it had to be finished but if I made a mistake and put the wrong thing down, I was learning by that because I was eliminating things. When you were asking the answer to a sum and you wanted me to be quick, I did not find it easy if I said the wrong thing."

Subject No. 2

This subject also worked to me, and has since been promoted to a deputy headship. At school, she had been successful and confident in mathematics until she ran into difficulty, particularly with calculus, in the 'A' level course. Having entered teaching, she regained interest, and held a responsibility post for mathematics in a primary school.

The opening remark, that I was going to ask her to do simple problems, in her head, but that I did not want answers, merely an indication when she had finished each sum, was accompanied by the 'pleasure buzz' on the recording. First question was 8 + 3. Then 17 + 35 , then 83 + 41 , and the second two drew the 'pleasure buzz' when she had reached the answer. The next question was 'seven eights' and this was misheard (genuinely) as 'seven-eighths' and she asked 'Of what?'

"Now, specifically work faster, and tell me the answer..."

This immediately produced the panic button. I then asked 23 + 18 , which she repeated, pressed the panic button, laughed nervously and then got it right. Next I asked 29 + 36 and she said 75. Rather aggresively I said 'Are you sure?' and she showed confusion, repeated the question, and then got 65. The repetition was said firmly and slowly, seeming to show that she was seeking to control her reactions.

"Got that wrong, didn't you?"

This was more hostile, sensing that she was consciously resisting.

"7 x 13 in your head, as fast as you can do it." ..."91"
"Are you sure about that?"........"Yes"
"Checked it?" ....."Yes"

The answers came slowly and with control.

When the blocks were produced, the panic button was pressed. When she was told she could take as long as she liked she took the blocks and seemed happy. As she worked she pressed the pleasure button. She succeded in doing the puzzle, and I then asked her to do it again, under some pressure.

She complained at the use of the button:-

"I can't think straight"
"What's wrong with your thinking? (Panic button)
.....bet you'll get the rest wrong."
"I don't think of these as being serious; it doesn't bother me now."

With this claim to be cool, she was not put out by further attempts to affect her, and completed the task.

Subject No. 3

This, the third of the private sessions, was with another advisory teacher, not in the mathematics team. A well-qualified person, with a higher degree, but no special interest in mathematics.

We started as before, slowly and not needing to know the answers:-

"17 + 8 ?"....and a very quick response "Done that"
"25 + 29 ?", and again a quick answer was indicated.
"Feel happy?"........"Yes"
"I'd like to make it more difficult... 87 + 49.... quickly now!"........"Yes" (quite quickly)
"Like to tell me the answer?".......".138"
"Are you quite sure of that?"...."Can you repeat the sum?"

At this stage I become more aggressive....

"Why has it gone out of your head?"....."Because I thought I'd got the answer."

Here I appear cross and hustling...

"It's only two two-figure numbers."

I then gave the question again, and she first gave 139, then questioned the 87. There was evidence of confusion, but she then got 136, and was confident.

Next 115 + 95....

"Should be able to do that quickly....have you got it yet?.....Can you do it?"........"200" (doubtfully)

"200 - Really....exactly?" "210" (Pleasure)

As with the others, we now moved to multiplying, and her confused appearance led me to try to re-establish calm.

"Let your mind go flat. Take an image of a calm lake inside the mind. Just remember a few facts, like five fives. 7 x 9, that's supposed to be a hard one in the tables." ...... "63"

"Yes, no problem about that. Another one, relaxed and easy, take as long as you like. Don't offer me an answer till you are totally certain you are right.... 7 x 14."

She replied, quite fast, 98"

"Faster 15 x 12 . Any way, but quick. "

She repeated fifteen twelves. Then hesitated.

"144 and 36....... disturbed by the noise next door.... ..ummmm..."

"15 x 12 !" (sharply) (Panic button)

There was now a long delay and then she said:-

"Gone completely blank."

"Right.... 7 x 13 . Quick." (tapped on desk) (Panic)
"103...no, that isn't right."
There were distinct signs of fluster, and we left the numerical work.

We moved to the four-piece puzzle, and I sought again to produce calm, by asking her to handle the pieces, experience them, recognise that they will form a brick shape.

"Take your time and do it. The shapes are pleasant and attractive, aren't they? It usually takes quite a time, that one. Not to be expected that you get it very quickly."

She worked at it,asked when she has two pairs fitting if they all go together, and then did it. (Pleasure)

I then broke up the puzzle, and said:-

"Now you've had practise with those, you should be able to do it decidedly quicker. Now move on and get it done. ....That's crazy, that doesn't fit like that."

Despite my harassment, and with a nervous laugh, she did the puzzle quickly.

This subject, too, is highly articulate, and able to express her feelings with some precision:-

"I've had very little experience of mental arithmetic. It made it more difficult to hold onto the numbers with the pressure. I think the mistake was to set it out in my head as though it were a paper calculation. I couldn't keep it when I was carrying."

"When you did the spacial problem you did it much more quickly the second time despite the pressures I was putting on you. What was your reaction to them?"

"Far less, because I knew what the goal was and I knew I'd recognise it when I'd got it. I gave you one calculation that was way out because it ended with a 1 and it shouldn't have done. And so I knew I had to go right back to the beginning. With this (the puzzle) it was simply a case of manipulating."

"Do you like spacial problems?"

"Yes, much more. Unless I have paper."

We now look at two rather different situations. All these three subjects were seen privately, in the same room, well-known to them. The next two were public demonstrations of the experiment.

Subject No. 4

This interview and the next were conducted publicly at meetings of the British Society for the Psychology of Learning Mathematics. At the opening session of each of these conferences descriptions of 'research reports' were given , very briefly, by those offering them. The purpose was to allow members to decide which of several sessions going on at the same time they would attend. I therefore said that I wanted to apply pressure to a volunteer to produce panic. This proved an attractive idea, and there were good audiences. More fortunate was the willingness of an individual volunteering in each case. A secretary in the local college 'offered'.

With this subject, a woman I had never met, I started as usual, with time no object and the answer not demanded. The first two questions were dealt with confidently, and with the pleasure button pressed when the answer was reached. Then:-

" 53 + 84 " ...." Bit quicker, get it now."

Again, the tape indicated pleasure when she got the answer, which did not have to be given. I applied more pressure:-

"Sure of it?"........."I think so." (nervously)
"Sure?" .... "Yes" (with a giggle)

I now increased the pressure:-

"Can you get really rapid responses. Let me know the answer and I'll be able to see if it is right or not"

"31 + 59" .... "70 .... Oh! (nervous exclamation) No, no .... That's wrong...." (The voice weak and hesitant)

"Come on, what was it I told you?" ....."31 + 59 ...90..90"

"Sure of that?"........"Yes"....... (Panic)

"83 + 121" (sharply) ..... "204" (firmly)

We moved on to multiplication, but I remained hostile:-

"13 x 17 , quickly as you can, by any manner you like, but quick!" ...... "181"

"Really? .. Really?? ... You sure? "

At this stage she began to mumble in a low strained voice.....'not sure...... 170, 230, 221" (correct)

We broke at this stage and discussed what had happened with both the subject and the audience . There was a great change in her voice, which became firm and clear, in marked contrast to the tone when she was most anxious.

She said that she would have done much better with pencil and paper for she was 'absolutely dependent' on them for doing mathematics.

"What was your reaction to my manner?"
"Slightly threatening, if that's the right word. I don't want to be nasty, but slightly threatening."

A member of the audience commented that it was more than 'slightly' threatening!

She then recalled her emotional state through the experiment so far and said, freely and without prompting:-

"The emotions came over me. Firstly with the speed with which you asked me to do it, and then the manner you attached to it."

She was now invited to handle the separate pieces of the puzzle, to take her time, and generally relax. She almost solved the puzzle, veered off, and tried all sorts of impossible arrangements. The effect on those watching was to create intense frustration and an urgent desire to help. The neutrality of the audience was completely lost. I asked her to monitor her emotional state.

"I can see that they fit together in pairs, but that's not the way they are going to fit together as a four. At the moment I'm looking for an alternative."

" Do you feel content with the process?"

"No, far from it. I'm not making any progress and I haven't got anything visualised in my mind as to where I am going. I've a feeling that somehow the outside should be straight edges."

She completed the task, with satisfaction, but feeling she had taken too long. I then gave her the puzzle, and tried to put the pressure on. This was not easy, and there was some levity from the audience. She finished, and said:-

"I am getting used to you, and you are not such a threat to me. I can feel myself saying 'Never mind what he says in the background; try to ignore him'"

Then, observing on the whole experiment:-

"I didn't feel the pressure I did on the number work because I had a thing there to get a hold of. Something to hold on to, to move about with, to keep my mind on. There is a tangible object to work with - not listening to you on multiplication when you broke in and I had to start again."

Subject No. 5

This was at another BSPLM conference, and the volunteer was a woman remedial teacher. The same process was used.

With the initial 'comfortable' numerical work there were firm clear responses, with the pleasure button used as each answer was reached. I increased the pressure and reached a higher level of urgency, and greater hostilty in demands than with any other subject. All the pressure was successfully resisted, wothout any overt or covert (through the button) show of anxiety.

She explained that her experience in teaching was in numerical work, at which she was confident and assured. I then moved to the puzzle, and produced the four pieces, intending to be helpful and soothing. At the mere sight of them, the subject immediately showed symptoms of acute anxiety, both in manner and in the use of the panic button. Despite efforts to encourage her, she took a very long time eventually to fit the pieces together.

Later in the day, well after the session, the subject approached me and asked to borrow the puzzle. She then took it off, worked at it by herself until she had mastered it and reported this back to me later still in the day. She was delighted when I gave her the puzzle as a present.

CHAPTER NINE

CONCLUSIONS, LIMITATIONS AND FURTHER WORK

CONCLUSIONS, LIMITATIONS AND FURTHER WORK

The original intent was to look at the more extreme manifestations of 'maths. anxiety' (though this name was not current when the work was started). With this in view, I simply let it be known that that was what I proposed. In contrast to the experience of Sewell (1982, Cockcroft Report), who found that people were unwilling to discuss mathematics, I found many willing to discuss their distaste for it. The willingness was the main criterion, so no claims for a representative sample of the population are made.

The initial investigation, conducted with twenty-four people in interviews of an hour each, gave many leads, and would provide a base for other studies, with a different direction. A wide range of affective responses were offered, the most surprising feature being the great strength of feeling, at levels which both the subjects and I felt was out of proportion to the issue. However, as one of them commented:-

"There's no proportion in it."

and I was driven to accept that the extremely high levels of anxiety were genuinely felt.

Naturally there were cognitive issues raised, but it became clear that one source of later emotional rejection of mathematics had its origin in the things that the subjects had been told in mathematics did not accord with their own intuitions. This leads to the important matter of the need for 'emotional acceptance' as well as cognitive understanding of a piece of work. There is scope for more investigation of this idea.

Certain words seemed to crop up often in these preliminary investigations, and to be invested with particular significance by those pffering them. Some, like 'control' and 'demand' need further study, but the specific reaction on which I focussed was that of 'panic'. This was an odd word for people to use; it seems to have a specific meaning and not be well related to other emotional words, and it is clearly a very powerful reaction.

In conducting what is now referred to as a 'teaching experiment' I had this specific rection in mind from the beginning, bot needed to place it in the context of other affective responses, and the typical situations that occasion them.

The two modes of this part of the work have been fully described. They were still investigatory, and were not testing a theory, though I had Skemp's model of intelligence in my own thinking as I worked. Part of my intent was to develope a style of teaching that constantly recognised the affective dimension of what was happening. It was not designed to make things easy, either in understanding mathematics or in avoiding emotional problems. The intent was to examine how we think (which does not always make the mathematics easier) and how we feel (which sometimes meant that stresses were allowed to develope in order to study them).

The work both in the group and with individuals led to the isolation of a number of ways in which the action of the reason might be inhibited by the emotions. As this analysis (detailed in chapter 6) developed it was checked with the s subjects for their observations on it. Appedix B indicates where and when in these sessions such issues arose.

These situations form a broad base for a study of emotional interference, some of which may be at a relatively low level. However, both in the group and with individuals there had been a great deal of discussion of the particlar reaction of panic, and from there it seemed sensible to pursue the more limited aim of looking at that response. It was not only that throughout, the word had often been used, but it also regularly occurs, as we have detailed, in the literature, but without an analysis of what it might mean.

An interpretation of panic on Skemp's model proved possible and was outlined in chapter 3 . Its virtue lay, not only in proceeding from his theory, but in matching in many ways the things the subjects said of their own experience of panic. (In particular, note the description given by S on p 111)

In his model, Δ_1 is under what it perceives as threats from outside, which will materialise if an answer is not produced, or not produced quickly enough. Should Δ_2 not produce a plan, or not produce it quickly enough, the consciousness, directed by the emotions, moves back and forth between the two deltas, resulting in the paralysis we know as panic.

The field work showed us that it was authority pressures, particularly when they demanded that a time limit be met, that were the most potent threats that people described.

The testing, therefore, of chapter 8 sought to show how easily, by the use of these pressures a state of panic could arise, and that this particular word would be used. It was then contrasted with the very agreeable experience that these same people could enjoy, largely through the explicit removal of these pressures.

In the tests with large groups, the number of people offering the specific word 'panic' was as follows

Table 2	3/20
Table A	3/27
Table B	5/18
Table C	0/14
Table D	6/39

Overall ,therefore,some 14% of those subjected to this very short, and artificial, pressure, reported that they experienced panic. One of the groups listed was a 'failure' for the experimenter, and there may have been special reasons obtaining there. This is strong evidence of the proportion of people whose mathematical education has left them very vulnerable to the very pressures they suffered at school.

There were other very strong negative reactions, such as 'horror' ar 'terror'. It would need a close examination of what the subjects meant by these words to discover whether they are in the same emotional region as panic.

In the second part of the experiment, it is particularly significant that some very positive responses were offered by those who had had the strongest negative reactions to the first part.

They speak of 'Quiet fun which I really enjoyed.' or 'Happy, calm and relaxed.', and comment on feeling much more confident. Since the remarks made while giving this problem were aimed at the removal of those negative pressures, this provides extra evidence of their importance.

The intent in the individual testing was to create in succession states of calm, relaxed enjoyment and of panic by alterations in manner (authority pressure) and in attitude to time. Each individual was taken through three tasks, and an attempt was made to produce six states, alternating calm with panic. It would be a sad commentary on human beings if this form of control by brief mechanisms (however carefully devised) could switch them back and forth in this way. We would appear more like billiard balls, under a stimulus-response model. I consider it enough, therefore, that marked changes are produced, generally in line with the predictions.

In the following table, success or failure in achieving the desired change are marked by ticks and crosses.

Table 3	Adding		Multiplying		Puzzle	
Pressure	Off	On	Off	On	Off	On
1	✗	✓	✓	✓	✓	✓
2	✓	✓	—	✓✗	✓	✓
3	✓	✓	✓	✓	✓	✗
4	✓	✓	—	✓	—	✓✗
5	✓	✗	✓	✗	✗	✓

Where the panic rose, but was then controlled, both a tick and a cross appear. The results show that in general panic could be caused. There was a total dysfunction in the task performance, and the tapes of the interview reveal very marked changes in the voices as the sujects moved through the tasks and the pressures.

The experiments were not conducted in standard situations. The important differences were that some occasions were very public, and others completely private; that some of the people were my juniors in the normal work situation and others had never met me; some may have known of my thinking and others clearly did not. Whatever the situation, each was moved back and forth, some with marked changes in performance.

One interesting quotation recognised the pressures I was imposing:-

"The emotions came over me. Firstly with the speed with which you asker me to do it, and then with the manner you attached to it."

From these experiments I conclude that the presence of an authority figure imposing time pressures and overtly sitting in judgement can rapidly induce panic. Furthermore, I conclude from the many observations made to me that this was the root cause of the disablities many of my subjects suffered in mathematics. I further conclude that the explicit removal of these pressures can rapidly lead to the same people having rewarding mathematical experiences.

The panic reaction is not peculiar to the study of mathematics, but there are features of its study that can make it singularly productive of such negative experiences.

Certain limitations in this study have been indicated. In no sense can it be claimed that a representative sample of the population was used. The subjects were nearly all well-qualified and articulate. They form part of the community mentioned in the Cockcroft Report (sect. 21) as being particularly susceptible to guilt about their mathematical incompetence. Hilton (1980a) lists them as his class B2.

In looking at people of this type, this study shows that certain particular pressures are responsible for this condition, and indicates means of avoiding or even recovering from this state. Were it only to apply to such people, its importance would be considerable, for there are many such. This has become much more widely recognised since this work was begun.

The extent to which the self-esteem of these otherwise competent people had been attacked was quite remarkable (the classic case was Elaine). An essential element of all remediation is the re-establishment of this esteem. Kogelman, in his program at the Bronx Community College, found this with a very different population.

There is an issue as to whether it is the mathematics that is central to the problem. Certainly the worst pressures are inter-personal, but that does not mean they are independent of the subject.

There are features of the mathematics classroom that may make the pressures more potent than elsewhere. Schools generally emphasise the importance of two subjects, mathematics and English, and this means that things said by teachers there may carry more weight. Specific to mathematics, however, is the equating of ability at it to general intelligence. Since schools have as their main function learning, they tend to regard more highly those pupils who learn well, even when they claim this is not so. The attachment of moral worth to intellectual ability, and the equating of this to mathematical ability produces a situation (however false) where failure in mathematics is seen as striking much more at self-esteem even than ability in English. So people are more at risk in a mathematics lesson.

The treatment of mathematics in many schools as a subject where you are either right or wrong produces an even greater imbalnce between pupil and teacher than elsewhere. As several of my subjects saw it, in the mathematics lesson, the teacher was always right and they were always wrong.

It may also be, though this is purely subjective, that there is a higher incidence of questioning in the mathematics classroom than elsewhere. This is a feature that many find too 'demanding'.

These various features enhance the dangers we have specified; they do not mean that they are absent in other areas. A number of people saw games and P.E. as subjects where they might suffer similar traumas - though once they had escaped from school the failure seemed to matter less.

It would, of course, be pleasing rather than otherwise if the results were more widely applicable, though the study only looked at mathematics. As we noted in chapter 2 Hoyles (1982) found that children reported disproportionately many 'bad' experiences in mathematics.

Although I was concerned to work with both men and women, and saw a fair balance in the initial interviews, had a balanced group, and only failed to get balance in the individuals, the results do not differentiate between men and women. My subjective impression was that both were subject to very similar negative feelings, and at the same level, but that it took longer for the men to talk about it. A speculation did arise however, which we shall now discuss in the suggestions for further work.

There are many and varied reasons given for the difference in performance at school leaving age and in higher level examinations in mathematics between boys and girls. It would not be appropriate to survey them here. However, this study indicates the importance of feelings about authority, and our society certainly encourages quite different responses to authority by girls and boys. Girls are encouraged to conform; if they do not, they are usually 'tutted' at, and shown by small gestures that their behaviour is inappropriate for their sex. Boys, even when punished much more severely, are given to understand that this behaviour is expected of boys. The speculation is that as one advances in mathematics, the routines are better performed by girls, and as more venturesome material in the form of problems arises, the boys perform better. This is a speculation which clearly would need much work to support or refute it.

It became apparent from stories about their school lives that certain regularly occurring incidents resulted in particular emotional responses. I have startes to detail such common situations and test with groups what their reactions are. Any such investigation might depend on a better analysis of emotion than seems to be available.

A most important area is how we may best introduce an affective dimension into the mathematics classroom. I have over the last few years developed a method, called 'textured learning' which had its origins in the group of this study. This is an area where there needs to be a wide range of experiments.

Certain of the subjects, particularly those in the individual testing managed to control the rise of panic and to work effectively. In Skemp's theory we may postulate the action of a delta-three, but evidence from people would be needed. Were we to analyse how it is done, it would be an important step.

Finally, while we may have done something to demonstrate the adverse effects that emotions may have upon the reason, it might be more important to find out how the emotions can aid one's reason. We are all aware of the added drive we have in tackling certain pieces of mathematics that appeal to us.

If we could just occasionally allow our pupils to experience that drive, we would have made a great step forward.

APPENDIX A

ANALYSIS OF ATTENDANCE AT THE GROUP

ANALYSIS OF ATTENDANCE AT GROUP

Week	A	B	C	D	E	F	G	H
1								O
2								
3								
4				O				
5								
6								
7	O							
8								
9								
10								
11								
12								
13								
14								
15								
16								
17								O
18								
19								
20		O						
21								
22				O			O	
23				O				
24								
25								
26								
27								
28								
29								
30				O				O
31					O			O
32		O						
33				O				
34	O						O	O
35				O				
36	O							

O signifies absence.

APPENDIX B

~~ISSUES~~ TOPICS DISCUSSED IN THE GROUP

Mathematical ~~Issues~~ Topics

SESSIONS 2-4

Conic sections, and the notion of movement under constraint.

SESSION 5

The cycloid. (An extension of movement under constraint)

SESSIONS 6-7

Number bases.

SESSIONS 8-9

Straight line equations; solution of simultaneaous equations.

SESSIONS 10-11

Logical problems.

SESSIONS 12-15

Various graphs, leading to solution of harder problems on 'O' level papers.

SESSION 16

Interpreting numerical facts.

SESSIONS 18-22

Extending the number system, with special reference to directed numbers.

SESSION 23

Symbols.

SESSION 24

Square roots.

SESSION 25

Factorising, and the fundamental theorem of arithmetic.

SESSION 26

Indices

SESSIONS 27-28

Papallel lines and the angle sum of a triangle.

SESSION 29-30

Pythagoras' theorem.

SESSIONS 31-33

Angle properties of a circle.

SESSION 34

Radian measure: trig ratios.

SESSION 35

Problems from papers.

SESSION 36

Series.

Psychological and Pedagogical ~~Issues~~ Topics

SESSION 1

Reasons for interest on noninterest in learning.

Effect of physical environment.

Adoption of clown's role to escape classroom threats. (2)

SESSION 2

The meanings, mentally and physically, of 'panic', in a range of situations, not all learning ones.

The need to know why something works, as well as how.

SESSION 3

The difficulty of receiving information about a problem. (2)

The issue of 'relevance' in creating interest.

SESSION 4

The pleasure in having understood something, as demonstrated by being able to explain it to someone else.

Previous failures in seeing structure in maths. - all detail. (No satisfactory schemas)

Issues in geography where mathematical structuring made remembering easier.

SESSION 5

The need to discuss qualitatively as well as quantitatively.

A short excursion into 'visual logic', later discussed in geometry in terms of whether something is 'visually apparent'.

SESSION 6

Extent to which members are now talking about maths. with friends on social occasions.

Distress caused to Rosita when she did not understand something. (2)

SESSION 7

Relation between concept and symbol.

Relative and objective knowledge.

SESSION 8

'Demanding' situations (the issue of being judged) (5)

- my bringing in paper and pencil as if for a test, or asking someone to explain to another.

Test of understanding in defending an issue against someone known to be more expert in the field than oneself.

Unwillingness to accept proof based on internal consistency.

Retention of earlier work.

Time pressures experienced by those who see these sessions as a last chance.

Reinforcement by thinking about issues between sessions.

SESSION 9

More concern with demanding situations. (5)

An exploration of how many were still refusing to admit when they had not understood.

Explanations of correct method not helpful in showing why one's incorrect thinking was wrong.

View of teacher (or authority figure in general) as aiming to trick or embarrass one. (4)

SESSION 10

Conflict in mind as to whether numbers are fixed or variable.

Tension at introduction of problem (Wason) characterised as 'logical'.

SESSION 11

Intensity of disappointment when confident of solution, which then proves wrong. (4)

SESSION 12

Reaction to test paper.

Controlled calm and relaxed calm.

SESSION 13

The security of holding on to one point.

SESSION 14

Pressing on an earlier concern, which still does not surface. (6)

Pleasure from slow pace which yet advances. Progress, not speed creates the pleasure.

SESSION 15

Pleasure at thinking way through then destroyed for oneself by belief that it was very simple anyway.

SESSION 16

Statement of positive enjoyment at working on problem.

Level of absorption in 'relevant' problem and pure number one no different.

SESSION 17

Degree of upset over mistake. (4)

Feeling of inertia and boredom expressed by Barbara; we attempt to analyse the feeling, but boredom difficult to characterise.

SESSION 18

Furhter talk on inertia.

Example of reducing child to tears by timed reading test.

SESSION 21

Results clever, but a 'con'. Distrust again. (5)

Do variety of methods help or confuse?

SESSION 22

Inability to distinguish levels of understanding.

Growing willigness to slow pace if necessary.

SESSION 24

Pleasure at getting quick, even if instrumental understanding.

Belief that relationally understood is still forgotten, but is perhaps easier to recall.

SESSION 26

Astonishment at clarification of mystery of logs.

Symblos - "So much is going on there" - interiority.

Statement of cutting out at one stage.

SESSION 27

Reactions when they are supposed to have read some papers - frightened if not read, wanting to be asked if they had.

More on what is visually apparent in a diagram.

SESSION 28

Negative feelings generated by formal language used in maths. (2)

Undirected discussion on emotion - is it interpersonal always? (4&5)

SESSION 29

Pleasure at grasping something, and request to go through once more.

SESSION 30

What proportion of teacher out-put is received?

Constant losing of aim in a problem.

Difficulty of getting whole line of proof in ones mind at once.

SESSION 31

Schools knocking out 'commonsense' way of seeing things.

Insistence of single method as well as single answer.

SESSION 33

Understanding principle does not mean you can do problem.

Attachment of other fear to maths. at some stage. (6)

SESSIONS 34-36

Analsing shifts; some more confident, all have experienced some pleasure at maths. Still doubt about distinction between ability at reception learning and problem solving.

Note

In chapter 6 we analysed a series of situations in which the reason may be impeded by emotion. The numbers in brackets against some of the discussions just listed indicate that there was evidence in that session for one of these six cases.

To recapitulate, the cases are:-

1) The normal feedback from emotion in a standard learning situation.
2) A 'pathological' response which does not allow thinking once the area is designated as mathematics.
3) Outside matters causing the emotions to send signals which distract the reason from its task.
4) Over emphasis on teacher approval.
5) Misinterpretation of the teacher-pupil relationship.
6) Transferred anxieties from areas other than maths.

APPENDIX C

ELAINE

ELAINE

Mrs. Elaine Dunford is seen as a highly successful person. At twenty she took a first in English, in her thirties became headmistress of Central Foundation Girls' Grammar School, which she later led into an amalgamation with Bowbrook School to form a comprehensive. She was then appointed as a divisional inspector for the I.L.E.A. All her career and personal presentation betoken confidence in herself. Yet throughout she suffered from one area of deep insecurity that greatly coloured her view of herself. She could not do long division. It seems a joke, but the issue is that it was not a joke to her. The inability to deal with simple arithmetic calculations was an intensely serious matter. It was not helped by the fact that no-one else took it at all seriously. When we started these interviews I had known Elaine for ten or twelve years. Initially I had wondered if her fear of numbers was slightly affected, but I had soon rejected the notion.

First Interview

We talked at Elaine's home, setting aside an hour, with a tape running.

Laurie: What I want to get at particularly is this terrific anxiety.

Elaine: Yes.

Laurie: Do you feel that at this moment?

Elaine: No, because I've made a conscious effort not to, because I was preparing for you coming. I thought 'Now I'm not going to get nervous about something as irrational as this.' But I was also thinking how people boast about not being any good at maths. Now I've never felt like that Laurie. I've always felt ashamed, it's always worried me. I've always felt it was a worse weakness than in fact it is. I have honestly thought 'How can I lay claim to intelligence when I can't add up a row of figures?"

At that time a pass in mathematics was almost essential in School Certificate if University entrance was in view. Elaine did no mathematics after the fourth year, but with a certain combination of passes at Higher Schools was able to avoid the mathematics. She had not liked her mathematics teacher, but it was not a very big issue. In fact the teacher had been very sensible in telling Elaine that the arithmetic did not matter much if the method was right.

Many people recalled particularly painful episodes. Elaine remembered only one, where because her term and exam. marks were so much at variance, that the teacher had publicly accused her of getting her father to do the homework.

I asked Elaine about time pressures, when it might have been better to let it arise.

"Yes, I think the worst moments of my school career were mental arithmetic tests, which had to be done to time, and my father , to try to teach me my tables, quite often he would, before I went to bed, jump through tables...quick,quick..7x7."

It was apparently the mere existence of a time limit that caused problems, even when it was a very generous one.

Laurie: Supposing I were to give you a maths. problem within your competence, and I said to you, 'Do those whenever you choose. and perhaps I'll see you in three weeks time', where there is obviously no time thing, you would still feel pressured by time?

Elaine: Yes, slightly.

Laurie: Even if they are the sort of thing that in fact would take you say twenty minutes and you had three weeks to do them in ......

Elaine: Because I would begin to build up such a dread about them, Laurie, that I would do them as soon as you had gone because I would rather do that than get worried about them. But it wouldn't be as bad a pressure as if you said 'Now I'm just going to read the paper, and do them while I'm doing it, that would make me - really bring me near to tears and I don't cry easily - the feeling that I'd got to do it within even twenty minutes even if it was something really you knew I could do in five.

Laurie: And my presence would make a good deal of difference one way or the other?

Elaine: Yes, it would; though I'm fond of you, it's the presence of the authority figure in the maths. subject.

Part of the rationale for talking to adults rather than children was that there was not too great an ethical problem in inducing unpleasant feelings.

Laurie: If somebody snaps things at you what happens, do you go into a fog? Freeze?

Elaine: I feel my throat tighten.

Laurie: (sharply) 7x8!

Elaine: Yes, that's right, ny throat tightens and then I calm myself and I think, 'I know that 7x7 is 49; all I have to do is add 7 - it's 56.' Oh, I suppose I would say 56 in the end without working it forwards or backwards but that's how my father used to try to teach me....

But the aim throughout her schooling was not the p pleasure of the study, or any value or purpose in it. Elaine represents an extreme case of 'situation 4' in ch. 6., where the interpersonal and not the study is completely dominant. The next comment is one of the saddest that can be imagined.

Elaine: Oh, no, it was simply to please my father or my teacher. But that was, as far as I was concerned what school was about. You went through meaningless hoops and they had meaning for me because I pleased someone, my mother stopped crying and my father was pleased.

Laurie: And that was the total purpose of the operation?

Elaine: Yes.

We looked for things at all allied with mathematics that might interest Elaine, and a link with S. arose. Secondary school timetabling is a complex problem, often delegated by heads to their head of mathematics. Elaine did it herself.

Laurie: You like time-tabling?

Elaine: Oh, yes, yes, that's a very positive pleasure to me.

Laurie: The actual arranging of the small squares, not just devising the curriculum?

Elaine: Both. I find both intensely satisfying, but the actual labelling of the small squares I find.... I was going to say almost, but it is, a physical satisfaction. I get a physical glow as I do it. And occasionally I'll stay home for one day to do it because I get so many interruptions at school. And I wake in the morning and I think, 'I can do the time-table <u>all</u> day'

I had always, partly because of an interest in operational research, thought of time-tabling as rather mathematical, (and this is a common perception). In exploring areas which might be thought mathematical....

Elaine: Would you say that timetabling was?

Laurie: Oh, yes.

Elaine: I always think it can't possibly have any relationship with maths. because I enjoy doing it and do it well.

An interesting response, showing great confidence in her own skills, when it was not something she considered mathematical. Her attitudes in and outside mathematics are so markedly different.

After a discursive chat about Greek civilisation, I returned to some mathematics.

Laurie: If you take the first few numbers there 1, 4, 9, 16.... How big are the gaps?

Elaine: 3

Laurie: 4 and 9?

Elaine: 5

Laurie: 9 and 16 ?

Elaine: Oh, 7 - now you see I've started worrying.

Laurie: What I want to do with anxiety is edge in on it. If you go into it, then I will detach you from it if possible. Because, you can't learn about what it is until you actually get at it. Well, on the other hand if you get it too strongly it finishes you, you get screwed up. The first gap was 3 and the second gap was 5 and the next one was 7 and the gap between 16 and 25 is 9.

Elaine: Oh! Yes, I don't think that has ever been pointed out to me. It may have been...

She was interested. The fact has intrinsic interest for many people, however hackneyed it may seem to the knowledgable.

By the second interview I had discussed panic with the group. I had returned with Elaine to the issue of time, a and another feeling expressed by some that there was a mountain of material which could never be dealt with. I asked if that rang a bell:-

"No, it doesn't really, Laurie - when I try to think back to it, it was always that dreadful numbing panic. I remember distinctly in one exam. knowing that I knew how to do a problem or whatever it was and being quite paralysed so that I could not remember what - I think it was seven sevens, so I remember writing down seven sevens and adding them up and getting stuck two thirds of the way up. You know, thinking. 'I'm not going to get beaten by this, I can work out what seven sevens are,' and yet I couldn't because of this awful panic. When I came out of the exam.

I don't think I would have been able to say 7x7 = 49 but I would have been able to add up seven sevens. It's the hysteria that I think about, because the homework was often easy - it's linked with exams. certainly."

I was encouraged that Elaine had swept aside a suggestion of mine. The danger of getting back what has been fed in is evident. It is lessened by dealing with strong adults, and there is much evidence that ny views were not always accepted readily. In the reply just quoted she was clearly remembering vividly the emotions of that time, and the fact that they are remembered so is an indication of their intensity. We went on to discuss the physical symptoms and arrived at the surprising conclusion, mentioned in the third chapter, that they were very similar to those she felt concerning a much more serious matter - the much delayed return of her husband Stephen from abroad and her intense fears for his safety. .

It is strange that anyone should see ability at mathematics as related to personal worth: the following passage explores this.

Laurie: Does the maths. thing threaten part of you in any way ?

Elaine. (long pause).....self-respect ?

Laurie: Yes. You mentioned in the first interview you felt shamed.

Elaine. Oh yes.

Laurie: Now, what does shame mean ? In relation to self ?

Elaine: Inferiority. Disgrace. A low opinion of you in other people's eyes.

Laurie: So perhaps the threat was of diminishing you in others' eyes. Am I... I'm leading you again aren't I ?

Elaine: I don't think it was in others'...it's somehow in my own.

Yet it needed to be made clear to others that she had this weakness.

Laurie: You'll feel impelled to tell them.

Elaine: Oh, yes,oh yes, puritan.

Laurie: 'I must be seen how bad I am.' ?

Elaine: Yes, certainly. You see I always think people think too well of me and it is important to me that they shouldn't like me for things that I believe I do not possess and they therefore must not think I am intelligent or nice or the things that people tend to think I am. I have to go out of my way to prove that while I'm quite intelligent I'm not <u>very</u> intelligent; whereas, yes, I've got a kind nature, I'm capable of very unpleasant things and therefore if they think I'm a good head it is important that they know that I can't add up. That's putting it rather crudely, that is exactly what I feel, as far as language can convey it that's what I feel.

Laurie: Don't you see that sometimes other people understand all that, don't necessarily agree with your assessment of yourself and like you for that particular....

Elaine: Yes, that makes me all the angrier that I can't get it into their heads.....

Laurie: That you are not as you seem...

Elaine: That's right.

Mathematics seemed to play an important part in Elaine's view of herself.

Laurie: ....if maths. demonstrates to you - to your satisfaction - that you're not intelligent

Elaine: Yes, yes.

Laurie: It's the thing that really confirms..... that you are not intelligent.

Elaine: Yes, yes.

Laurie: It's not acceptable that I think you're intelligent ?

Elaine: No, I'd feel I'd fooled you.

laurie: How could you if I'm that much brighter than you!

This obtruding of her inability to do mathematics had been characteristic of Elaine since I had known her. Such statements as the underlined one about being a good head were treated by most people with casual amusement, because it showed such imbalance. Yet it was not out of balance to her. Her failures in mathematics were an essential part of her character.

The next passage is a very significant one. At a crucial stage, by a lucky chance, I dispel the panic as it is rising. The issues of time and authority, which we shall later see as central, are both present. The release of time pressure and the lack of crossness of an authority figure trigger an important change.

Laurie: I'd like to see if you could do mental arithmetic.... in the sense of doing something in your head, but something that does not just involve recall. So supposing I asked you for instance, what's 15 x 12 ?

Elaine: Oh, I couldn't possibly do it.

Laurie: Not in your head.

Elaine: No. I would have to say 12 x 12 = 144 and write it down and add on the others, oh I suppose the next step would be.... you don't mind how slow I am ?.... How many 12's did you say ?

Laurie: 15.

Elaine: 15. (mumbles).... 144. Oh, I can't do it, I can't carry the, you know, the one on.

Laurie: What you are doing is visualising the pencil and paper method.

Elaine: Yes, yes, I see exactly what I am doing.

Laurie: And that's what you believe mental arithmetic to be about ?

Elaine: Well, I remember when I did mental arithmetic I knew it shouldn't be. I knew it should be instant recall.

Laurie: No, no, instant recall is for facts, not for calculations. Supposing I said 87 and 15, try it.

Elaine: 102

Laurie: How did you do it ?

Elaine: Added 10 and then added 5.

Laurie: Now that isn't the way you were taught at school ?

Elaine: No, no that's the way I protected myself inadequately but that's the way I taught myself to do it.

Laurie: It's the way everybody does it.

Elaine: Is it ? Oh....

Laurie: Now return to the other one, and you had... you'd got 144 and you'd got 36.

Elaine: Yes.

Laurie: And you proceeded to set it out in your mind, you tried to picture it...

Elaine: Instead of adding 44 and 36.

Laurie: Can you do that ?

Elaine: 70 ? No that's....

Laurie: No hurry....

Elaine: 80

Laurie: So what are 15 twelves ?

Elaine: 180. Now I did that quite calmly. Now you know why I did it calmly, it's because you said in that nice voice, 'No hurry'. That was the key. I was beginning to get panicky now if you had said 'Don't worry', that wouldn't have mattered, but you said 'No hurry'. You said it in a kind way.

In fact, a great deal is said by Elaine in those last two lines about the mechanism of panic. We shall shortly pursue the 'crossness' in greater depth. L: had been very tentative with me for a long time, because she expected me to show anger, and in fact had a dream in which I did so, which enabled her to think 'So that's what he is really like.'

At the end of this particular interview with Elaine we returned to talking about the mental arithmetic.

Elaine: ....because I felt just the very beginnings only, a ghost of panic - and it went, linked with a confidence that you weren't going to think badly of me for being slow.

Laurie: No.... Well, that's very ambivalent because you don't want me to think badly of you, but

but you want me to know how bad you are.
And then accept you ?

Elaine: Yes, yes, that's it exactly. That's what I want people to do.

I gave her two wood-block puzzles (described on p. 134/5) She seemed to welcome then, and admire their shape. We now established a new element in the methodology, and between interviews wrote independent comments on the last session. Elaine described how she had added up the prices of a few items in a supermarket, and then did a small sum every day. Success left her positively euphoric.

We moved further in calculation in the next session and Elaine , in her words 'reacted with far more strength'. Yet something happened in this interview which we could not later track down, concerning her relations with her parents. The mathematical work was on visualising a cube, in which she was successful. At the end of the interview she reinforced the earlier concern (perhaps related to early parental attitudes)

Elaine: Do you know, as you're talking I'm finding it remarkable that you're not cross with me."

My notes after this interview indicated great pleasure at the advances made. Elaine's showed great turbulence about her parents, pleasure at a wooden puzzle, and

"Tables are soon not going to frighten me any more and they are not an important part of maths. anyway"

"If I overcome my feelings of inferiority over maths. how am I going to 'stop feeling superior'?"

"I've wasted years of my life over a chimera! It doesn't matter whether I can add up or not."

"Towards the end of the session I found myself wanting to grasp Laurie's arm and say in an almost grovelling burst of emotion, 'Thank you, thank you, for not being cross!'. Yet I don't remember anyone actually ever being so - more puzzled than cross.

"Difference in my vocabulary between fear and panic. Fear is controllable. It can't be obliterated, but it prompts action and can sometimes sharpen intellect. Panic inhibits rationality completely. It is entirely a gut feeling and shatters both intellect and personality."

Most of what is important to this thesis has now been brought out, although there is a wealth of other issues that could be pusued, as Elaine became stronger, and overcame many of the hurdles she had baulked at before. There was a sense of waiting in the two interviews before we tackled the dragon we knew we must face - long division.

I rather muddled doing long division, which Elaine found reassuring, but commented that if I wanted to divide 345 by 10 I could take off one 10 at a time, keeping track of how many I had taken off.

Elaine: Do you know, I've never thought that that's what division is.

Laurie: You'd never thought that division was repeated subtraction?

Elaine: Never. I thought that multiplication was adding but I've never thought of division. That's why long division has always defeated me because I hadn't known what I was doing. Do you follow ?

I then tackled 345 divided by 29, and said again that we could take away one 29 at a time.

Elaine: You've no idea how reassuring that is to me, that there is a way to do it, however long ..... well it's what you said, it's the first stake I put in, if all else fails I can do it that way.

I went on to say that it made life simpler to take off larger chunks, rather than one at a time.

Elaine: Why didn't I see that, I wasn't a fool, I suppose again it's just emotional blockage I was too frightened to think about it.

We moved on to a more difficult one, and suddenly she said 'I did that in my mind before you did it, I understand!!! '

There is much else, but this is a good place to stop. The change in attitude was what was important. Not much mathematics may have been learnt, but two great horrors, the tables and long division were seen for what they were.

Later, in the summer term, after several months in her new job, Elaine faced a situation, totally unconnected with mathematics, in which she felt some uncertainty. She suddenly thought, '...and I expect I can't do long division any more. '

So she went home, found she could, and was settled again.

APPENDIX D

ADDITIONAL MATERIAL ON EMOTIONAL RESPONSES

WORKING WITH LARGE GROUPS

Additional material on emotional responses, working with large groups.

Four separate occasions are recorded here. They are in slightly different contexts, and there is some variation from the experiment discussed in chapter 8. One theme, that of an authority figure threatening a test, with a time limit, ran through them all. They confirm the result of the experiment already reported, and are included in the general assessment of the results which appears in chapter 9.

The first one of this four (Table A) shows the responses of a group of teachers on another six-week course at a mathematics centre. The audience was therefore similar to that of the one described in chapter 8. They would be interested but not necessarily confident in mathematics, perhaps with no formal qualification, but with experience of teaching it at primary school level. They would have been conscious of my position as Staff Inspector.

In this table the last two columns are those corresponding to the earlier experiment. The middle one, therefore, is the reaction to the threat, and the last column that to the leisurely work on the cube. The first column has been retained: it was part of my experiment on that occasion. It is therefore necessary that I explain it. I presented the rules governing fractions (treated as number couples) in a very formal way, not even explaining that they were fractions, and refusing to answer questions, merely telling them how to get the answers. The point was a discussion of instrumental and relational understanding and the reactions those to good-humoured but totally instrumental teaching.

Table B records the reactions of an exactly similar group of teachers to an experiment exactly in line with that in chapter 8 .

Table C shows reactions to just the first part. The teachers were also on a six-week course, concerning slow learning and difficult children, had not necessarily any interest in mathematics, and may have been none-too pleased at having to discuss it.

Table D shows the relevant experiment (the one with a threat) in a series concerning emotional reponses, of which the others are outside the scope of this thesis. This took place on a three-day residential course for teachers holding posts of special responsibility for mathematics in primary schools. They would therefore have a keen interest, but not necessarily much in the way of formal qualification in mathematics.

Table A

Uneasy	Blind panic	A dream - blissful. A quiet unhurried walk in my head. I must go there again sometime.
Confused	Fear and panic	Restful
Non-comprehension	Panic	Enjoyable
Irritating	Horrified	Challenging, satisfying
Why?	Horror	Irritation
Disorientated	Despair	Relaxed interest
Rubbish	Please sir, I feel sick.	Too much time allowed Would have preferred to check ideas with real things.
Initial satisfaction, like doing an easy crossword, but sterile.	Resign, or Oh hell!	Quiet.
Nuisance	Hell!	Vulnerable.
Apprehension	Dismay	Too slow.
Suspicious	Dismay	A headache.
Golly	Wish I didn't come.	Sorry, can't do it.
Help!	My God	Easier except for the last bit.
Pointless	Some anxiety!	Stimulating
Interesting	Very scaring	Hard thinking

Table A (continued)

Interest (why?)	aagh!!	A.O.K.
Meaningless	Fear	Bored - too slow.
Nervous	Fear	Better, though I was still worried about being right.
Pointless	Fear	Relaxing
Confusion, frustration	Fear	Interesting in perception.
Confusing	Fear	Peaceful
Boring	Threatened	Thoughtful
I do not understand what it is all about.	Worry	A tax on the memory.
Happy	Irritation	Interested
Interested	Annoyed	Peaceful
So?	Total non-cooperation	Interested
Insufficient reasoning. Pointless.	Arrogant sod.	Peaceful, tranquil, relaxed.

Table B

Panic	Relaxed, able to concentrate more, and 'see' problem.
Panic/confusing	Relaxed
Panic!!!	Confident
Panic	Difficulty in concentrating on the cube.
Was not here, but would have been panic.	(a) Initially v. worried at idea of any mathematical problem. (b) Slightly unsteady at beginning of visual working out. (c) Eventually quite enjoyed.
Anxiety	A feeling of really looking at a cube for the first time. I could 'see' my hands performing the various tasks and satisfaction with my answers.
Threatened	Would have found it helpful to be able to draw diagram of last problem after attempting to visualise it (unsuccessfully)
Apprehension	I was able to see (almost feel) the cube immediately, turn it in my mind and 'see' faces, corners and edges easily, and then split it into 27 smaller with one in the middle unpainted. I enjoyed it.
Worried	A very comfortable feeling of knowing the cube and a gradual growth of confidence in completing the tasks, also pleasure at not having to

Table B (continued)

	produce the answers, although I was memorising the numbers.
Nervous	I don't like having my eyes closed - embarrassed.
Collywobbles	Restful
Fear	I found the length of time given was more than enough and this gave me confidence.
Apprehension	Quite relaxed at first. Found beginning easy. Began to be too relaxed and so found it annoying to have to work out the more complicated sections at the end.
Guile	This imagining reproduced the way in which I rely on doing my own maths. problems in real life. I like to shut my eyes, turn the idea round and up and down, and then feeling certain of what my opinion of the answer is based on.
Amusement	An enjoyable experience! I now really understand all the features of a cube. The beginning was easily understood by me, the middle tasks more difficult, but the last task I understood and enjoyed my immediate answer.

Table B (continued)

Equanimity	Too protracted, frustrating - not enough activity. Attention wandered to driving a fast car.
Resignation	Interest, eagerness to do problems, admiration of the new teaching ideas, impatience wuth the slowness - and disappointment of no feed-back of praise.
Disbelief	Impatient to get on when I had reached an answer and next part had not been given. Could have drifted off to sleep in the waiting periods.
Absent	Slightly apprehensive towards the end, but excited that I might be able to do it.
Absent (hated maths.)	Comfortable process of imagining the cube. I found it easy to do each action, and if anything, was slightly impatient to get on.
Absent	Bored - gaps between questions too long.
Absent	Puzzlement - in construction Satisfaction - after construction Realisation - when it began to perform.

Table C

Anxiety

Pressure

Apprehension

Apprehension

Apprehension

Uncomfortable

Uneasy

Blank

Relaxed

Interest

Great

Good/Elation

Excitement

Pleasure

Table D

M	Panic. Doom. Help.
F	Panic
F	Panic
M	Panic
F	Panic
F	Panic
F	Horror
F	Horror
F	Help!
M	Help
M	Ugh
F	Sick
F	Good God!
F	Fear and trepidation
M	Fear
M	Fear
F	Fear
F	Fear
M	Fear
F	Apprehension
M	Apprehension
F	Anxious
M	Worried
F	Worried
F	Uncomfortable
F	Hotness
M	Why?
M	Uncertainty
F	Doubt
M	Not again
F	Antagonism
F	Defensive
F	Interesting
F	Surprise
M	Surprise
F	Anticipation
F	Excitement
M	Excitement
F	Excitement
M	Enjoyment

BIBLIOGRAPHY

Ausubel, D.P. (1963) "The Psychology of Meaningful Verbal Behaviour." Grune and Stratton. New York.

Betz, Nancy E. (1977) "Math. Anxiety: What Is It?" (Paper presented at the Annual Convention of the American Psychological Association.)

Bion, W.R. (1961) "Experiences In Groups." Tavistock Publications. London.

Bruner, J. (1975) "Towards a Theory of Instruction." Harvard U.P. Cambridge, Mass.

Buxton, Laurie (1978a) "Four Levels of Understanding." Mathematics in Schools. V7 4 Sept.1978 p36.

Buxton, Laurie (1978b) "What goes on in the Mind?" Times Educational Supplement (Extra). July 1978.

Buxton, Laurie (1981a) "Remembering and Understanding." Mathematics in Schools. V10 1 Jan. 1981 pp32-33

Buxton, Laurie (1981b) "Do You Panic About Maths?" Heinemann Educational Books. London.

Buxton, Laurie (1982) "Emotional Responses to Symbolism." Visible Language XVI 3 pp 215-220 Summer 1982

Byers, V. and Herscovics, N. (1977) "Understanding School Mathematics." Mathematics Teaching. 81 pp24-27

Chapline, Elaine Burns. (1981) "Formative Evaluation in the Development of a Math. Anxiety Reduction Program." (Paper presented at the Annual Meeting of the American Education Research Association)

Chavez, Annette; Widmer, Connie Carroll. "Math. Anxiety: Elementary Teachers Speak for Themselves." Educational Leadership. V39 5 pp387-88 Feb. 1982

Chomsky, N. (1959) Review of B.F.Skinner "Verbal Behaviour" in "Language 35" pp26-58

Chomsky, N. (1968) "Language and Mind." Harcourt Brace. New York and London,

Cockcroft Report (1982) "Mathematics Counts." Her Majesty's Stationery Office. London.

Cobb, Paul; Steffe, L.P. "The Constructivist Researcher as Teacher and Model Builder." (1983) Journal for Research in Mathematics Education. V14 2 pp81-94.

Davitz, J.R. (1969) "The Language of Emotion." Academic Press. New York.

Donaldson, Margaret (1977) "Children's Minds." Penguin. London.

Eagle, Ruth (1978) "Self-Appraisal in the Learning of Mathematics." (Personal communication)

Elliott, Portia C. (1982) "QUESTION: Is math. anxiety a figment of the imagination? ANSWER: Never!(A neurological glimpse at mathematics anxiety)" Int. J. Math. Educ. Sci. Technol., 1983, V14 6 pp777-785

Farnham-Diggory, S. (1977) "The Cognitive Point of View." in Handbook of Teaching Educational Psychology, eds. Treffinger, Davis, and Ripple. Academic Press. New York.

Hart, K. (1980) "Children's Understanding of Mathematics." ~~National Foundation for Educational Research. Slough~~. J Murray, London

Hilton, Peter. (1980' "Math. Anxiety: Some Suggested Causes and Cures: Part 1." Two-Year College Mathematics Journal. V11 3 pp174-88 Jun. 1980

Hilton, Peter. (1980 "Math. Anxiety: Some Suggested Causes and Cures: Part 2. 3. The Constituents of a Sound Mathematics Education." Two-Year College Mathematics Journal. V11 4 pp246-51 Sept. 1980.

Hoyles. C. (1982) "The Pupil's View of Mathematics Learning." Educational Studies in Mathematics. 13 (1982) pp 349-372.

Koestler, Arthur. (1976) "The Ghost in the Machine." Hutchinson. London.

Kogelman, Stanley; and others. (1981) "Math. Anxiety." American Educator. V5 3 pp30-32 Fall 1981

Krutetskii, V.A. (1976) "The Psychology of Mathematical Abilities in Schoolchildren." Chicago Press. Chicago.

Macnamara, John. (1972) Quoted in "Children's Minds." (Donaldson 1978) p36ff re Chomsky's language acquisitonn device

Magee, Brian. (1973) "Popper." Fontana. London.

Michaels, Linda and Forsyth, Robert. (1978) "Measuring Attitudes to Mathematics. Some Questions to Consider." Arithmetic Teacher. Dec. 1978 pp22-25

Morris, Janet. (1981) "Math. Anxiety: Teaching to Avoid it" Mathematics Teacher. Sept. 1981. pp413-417

Nicholson, John. (1977) "Habits." Macmillan. London.

Opper, S. "Piaget's Clinical Method."Journal of Children's mathematical Behaviour. 1977. V1 4, pp97-98.

Piaget, J. (1952) "Child's Conception of Number." Routledge and Kegan Paul. London.

Popper, Sir Karl. (1963) "Conjectures and Refutations: the Growth of Scientific Knowledge." Routledge and Kegan Paul. London.

Popper, Sir Karl. (1973) "Indeterminism is not Enough." Encounter. XL 6 pp22-25

Popper, Sir Karl; Eccles, Sir John. (1977) "The Self and its Brain." Springer International. Berlin. New York. London.

Ryle, Gilbert. (1949) "The Concept of Mind." Peregrine. London.

Sandman, Richard S. (1979) "Factors Related to Mathematics Anxiety in the Secondary School." (Paper presented to the Annual Meeting of the American Educational Research Association.)

Sewell, Brigid. (1982) As quoted in the Cockcroft Report "Mathematics Counts." Sections 20-21.

Skemp, R.R. (1958) "Difficulties of Learning Mathematics by Children of Good General Intelligence." Unpublished Ph.D. Thesis of the University of Manchester.

Skemp, R.R. (1971) "The Psychology of Learning Mathematics." Penguin. London.

Skemp, R.R. (1976) "Relational and Instrumental Understanding." Mathematics Teaching. 77 pp20-26

Skemp, R.R. (1980) "Intelligence, Learning and Action." John Wiley. Chichester.

Skemp, R.R. (1982) "Type 1 and Type 2 Theories in Relationship to Mathematical Learning." in Addition and Subtraction: A Cognitive Perspective. Eds. T.Carpenter. E.Moser. T.Romberg. Lawrence Erlbaum Associates. New Jersey.

Skinner, B.F. (1957) "Verbal Behaviour." Appleton-Century-Crofts.

Sovchik, Robert; and Others. "Mathematics Anxiety of Preservice Elementary Mathematics Methods Students." School Science and Mathematics. V81 8 pp643-48 Dec. 1981

Steffe, L.P. "The Teaching Experiment." Unpublished manuscript prepared for a meeting of the models working group of the Georgia Center for the Study of Learning and Teaching Mathematics, presented at the University of New Hampshire, Fall 1977

Tobias, Sheila. (1978) "Overcomong Math. Anxiety." W.W.Norton and Co. New York.

Tobias, Sheila; Weissbrod, Carol. (1980) "Anxiety and Mathematics: An Update." Harvard Educational Review. V50 1 pp63-70 Feb. 1980

Wason, P.C., and Johnson-Laird, P.N., (1972) "Psychology of Reasoning: Structure and Content." Batsford. London.

Zeeman, E.C. (1977) Catastrophe Theory: Selected Papers 1972-77 Addison Wesley, New York.