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Article Title: Scaling in long term data sets of geomagnetic indices and solar wind ϵ as seen by WIND spacecraft
Year of publication: 2003
Link to published article:
http://dx.doi.org/10.1029/2003GL018209

Scaling in long term data sets of geomagnetic indices and solar wind ϵ as seen by WIND spacecraft

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Received 20 July 2003; revised 7 October 2003; accepted 14 October 2003; published 28 November 2003.

[1] We study scaling in fluctuations of the geomagnetic indices (*AE*, *AU*, and *AL*) that provide a measure of magnetospheric activity and of the ϵ parameter which is a measure of the solar wind driver. Generalized structure function (GSF) analysis shows that fluctuations exhibit self-similar scaling up to about 1 hour for the *AU* index and about 2 hours for *AL*, *AE* and ϵ when the most extreme fluctuations over 10 standard deviations are excluded. The scaling exponents of the GSF are found to be similar for the three *AE* indices, and to differ significantly from that of ϵ. This is corroborated by direct comparison of their rescaled probability density functions. *INDEX TERMS:* 2159 Interplanetary Physics: Plasma waves and turbulence; 2704 Magnetospheric Physics: Auroral phenomena (2407); 2784 Magnetospheric Physics: Solar wind/magnetosphere interactions; 2788 Magnetospheric Physics: Storms and substorms; 3250 Mathematical Geophysics: Fractals and multifractals. **Citation:** Hnat, B., S. C. Chapman, G. Rowlands, N. W. Watkins, and M. P. Freeman, Scaling in long term data sets of geomagnetic indices and solar wind ϵ as seen by WIND spacecraft, *Geophys. Res. Lett.*, *30*(22), 2174, doi:10.1029/2003GL018209, 2003.

1. Introduction

[2] The statistical properties of fluctuations in geomagnetic indices and their relation to those in the solar wind, is a topic of considerable interest (see, e.g., [*Sitnov et al.*, 2000; *Tsurutani et al.*, 1990; *Ukhorskiy et al.*, 2002; *Vörös et al.*, 1998]). Scaling has been identified as a key property of magnetospheric energy release in the form of bursty bulk flows in the magnetotail [*Angelopoulos et al.*, 1992], "blobs" in the aurora [*Lui et al.*, 2000], non-Gaussian fluctuations in geomagnetic indices [*Hnat et al.*, 2002, 2003a; *Consolini et al.*, 1996] and in single station magnetometer data [*Kovács et al.*, 2001; *Vörös et al.*, 1998]. Models include Self-Organized Criticality (SOC) [*Chang et al.*, 2003] (see also the review [*Chapman and Watkins*, 2001]) and multi-fractal models [*Kovács et al.*, 2001] related to those of turbulence [*Consolini et al.*, 1996; *Vörös et al.*, 1998].

[3] These measures of scaling and non-Gaussian fluctuations in magnetospheric output need to be understood in the context of the system's driver, the solar wind, which is turbulent and thus also scaling. Other work has focussed on comparing properties of input parameters such as ϵ and the indices (*AE*, *AU*, and *AL*) to establish whether they are directly related. However, these studies have not provided a consistent answer. While *Freeman et al.* [2000] found that both the ϵ and the *AU* and *AL* indices exhibited nearly identical scaling of burst lifetime probability density functions (PDFs), *Uritsky et al.* [2001] obtained quite different scalings for *AE* and the solar wind quantity $v_x B_{yz}$ using spreading exponent methods motivated by SOC. *Hnat et al.* [2002, 2003a] used a PDF rescaling technique to characterize the fluctuation PDF of 4 years ϵ data from WIND and a 1 year data set of *AE* indices with fluctuations over a few standard deviations. Direct comparison of the PDF's functional form suggested close similarity to within statistical error.

[4] In this paper we use a larger 10-year data set for the *AE* indices to obtain a more accurate statistical determination of the functional form of the PDF of fluctuations over a more extensive dynamic range, including characterization of extremal events up to 10 standard deviations for the first time. We apply structure functions to characterize and compare both the low and higher order moments for all quantities. A 4-year subset of the index data, corresponding to the same period in the solar cycle as that used to produce ϵ, is used to facilitate this comparison. We then verify these results by direct examination of the fluctuation PDF using the full 10-year *AE* indices dataset.

2. Data Sets

[5] The *AL*, *AU*, and *AE* index data sets investigated here comprise over 5.5 million, 1 minute averaged samples from January 1978 to December 1988 inclusive. The ϵ data set is identical to that used in *Hnat et al.* [2002, 2003a] and extends from January 1995 to December 1998 inclusive. It includes intervals of slow and fast speed streams. ϵ is defined (see [*Hnat et al.*, 2002]) in SI units as $\epsilon = v(B^2/\mu_0)l_0^2 \sin^4(\Theta/2)$, where $l_0 \approx 7R_E$ and $\Theta = \arctan(|B_y|/B_z)$, and was calculated from the WIND spacecraft key parameter database [*Lepping et al.*, 1995; *Ogilvie et al.*, 1995]. The indices and ϵ are from different time intervals and here we assume statistical stability over these long time intervals.

3. Generalized Structure Functions

[6] Generalized structure functions (GSF), or generalized variograms, can be defined in terms of an average over time of a differenced variable $\delta x(t, \tau) = x(t + \tau) - x(t)$ as $S_m(\tau) = \langle |\delta x(t, \tau)|^m \rangle$ [*Rodríguez-Iturbe and Rinaldo*, 1997]. If δx exhibits scaling with respect to τ, then $S_m \propto \tau^{\zeta(m)}$. A log-log plot of S_m versus τ should then reveal a straight line for each m with gradients $\zeta(m)$. If $\zeta(m) = \alpha m$ (α constant) then the time series is self-similar with single scaling exponent α.

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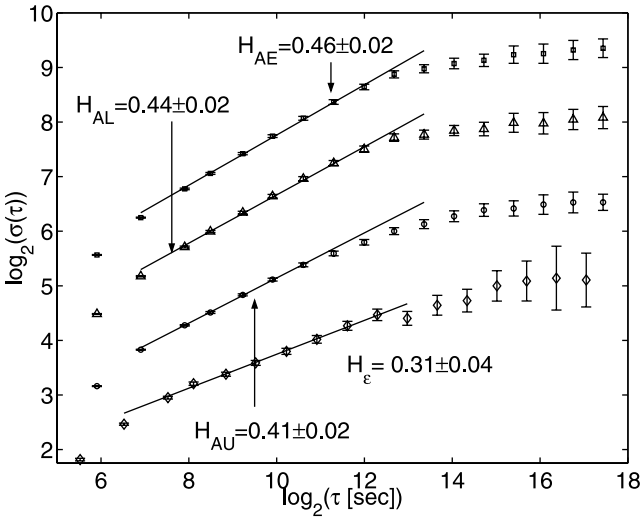

Figure 1. Scaling of the standard deviation of the PDFs of: ◇-ϵ, ○-*AU* index, △-*AL* index and □-the *AE* index. The plots have been offset vertically for clarity. Error bars are estimated assuming Gaussian statistics for the binned data.

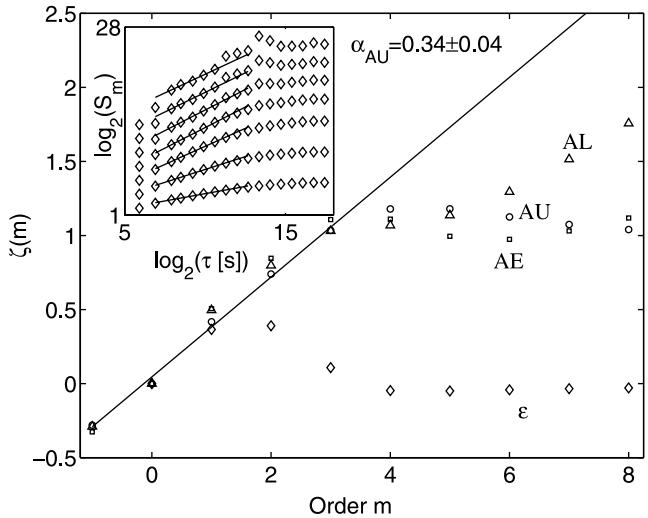

Figure 2. Dependence of the scaling exponent $\zeta(m)$ of the raw GSF on moment order m. Inset shows the GSF S_m versus time lag τ for *AU*.

[7] In order to compare the scaling properties of the non-contemporaneous ϵ and *AE* indices time series, we select a 4-year subinterval 1984–1987 from the *AE* indices at the same phase in the solar cycle as the ϵ data. Figure 1 shows the second order GSFs as measured by the standard deviations $\sigma(\tau) = [S_2(\tau)]^{1/2}$ of the fluctuation $\delta x(t, \tau)$. A scaling region is apparent between 2^7 and 2^{12} s where $\sigma(\tau) \propto \tau^H$, where H is the Hurst exponent [$\zeta(2)/2$]. The R^2 goodness of fit analysis was performed to select the optimal power law region and gradient and results are summarized in Table 1. The upper limits of the scale regions τ_{max} are in good agreement with values reported previously [*Consolini and De Michelis*, 1998; *Takalo et al.*, 1993; *Takalo and Timonen*, 1998].

[8] Any such single estimate of the H, whilst establishing the region of τ over which there is scaling, does not fully characterize the properties of the time series. For example, a fractional Brownian motion (fBm) can be constructed to share the same H value as *AE*, but the fBm series has Gaussian distributed increments δx by definition [*Mandelbrot*, 2002] whereas those of *AE* are non-Gaussian [*Consolini and De Michelis*, 1998; *Hnat et al.*, 2002]. As discussed by *Mandelbrot* [2002] the similar values arise because H aggregates two sources of scaling in monofractal random walks: persistence (the "Joseph" effect) and heavy tails in the increments (the "Noah" effect). In the above example the anomalous value of H for fBm comes just from the Joseph effect, whilst for *AE* the Noah effect must be at work. Furthermore, estimating H by only one method may not distinguish a fractal time series from a discontinuous one [*Watkins et al.*, 2001; *Katsev and L'Heureux*, 2003]. We thus turn next to the higher order m values of $\zeta(m)$.

[9] Figure 2 shows scaling exponents $\zeta(m)$ derived from raw GSFs with m varying between -1 and 8 for the $\delta\epsilon$ and *AE* indices fluctuations. These suggest the departure of higher orders from self-similarity, i.e., $\zeta(m)$ departs from a straight line. The inset of this figure shows the origin of these $\zeta(m)$ values for δ *AU* and $m = 1, \ldots, 7$. Only the first four orders exhibit clear linear behavior expected in the scaling region. For higher orders, the value of ζ very strongly depends on the assumed extent of the scaling region to which one fits a straight line. In principle, $\zeta(m)$ can be obtained for any m. However, errors do not contribute uniformly over m, for example, the largest fluctuations that affect large m, are statistically poorly resolved, whereas the smallest fluctuations ($\delta x \to 0$) are dominated by instrument thresholds. For the latter reason we will exclude $m = -1$ for $\delta\epsilon$ as $\delta\epsilon \to 0$ is not well determined through its definition.

[10] Conditioned GSFs quantify the impact of intermittency on fluctuations of different sizes by imposing a threshold A on the event size [*Kovács et al.*, 2001]. Here, this threshold will be based on the standard deviation of the differenced time series for a given τ, $A(\tau) = 10\sigma(\tau)$. This procedure allows us to exclude rare extreme fluctuations with large statistical errors which, for large m, could lead to a spurious departure from self-similar behavior. Alternatively, conditioning with different thresholds estimates a maximum size for the fluctuations for which self-similarity is still valid.

[11] Following conditioning, log-log plots of $S_m^c(\tau)$ show good correspondence with straight line fits, shown for δAU in the inset of Figure 3. This power law dependence holds between times already obtained from the R^2 analysis performed for $\sigma(\tau)$. The main plot then shows $\zeta(m)$ obtained from the conditioned $S_m^c(\tau)$. All lines in the figure were fitted for moments between -1 (0 for ϵ) and 6 and then extended to the entire range of data. Scaling exponents

Table 1. Scaling Indices Derived from $P(0, \tau)$, $\sigma(\tau)$ and GSF Power Laws

Quantity	α from $P(0, \tau)$	α from $\sigma(\tau)$	α from GSF	τ_{max} [*min*]
ϵ	–	0.31 ± 0.04	0.25 ± 0.04	~100
AE	−0.47 ± 0.03	0.46 ± 0.02	0.40 ± 0.02	~100
AU	−0.46 ± 0.03	0.41 ± 0.02	0.37 ± 0.02	~60
AL	−0.45 ± 0.03	0.44 ± 0.02	0.36 ± 0.03	~100

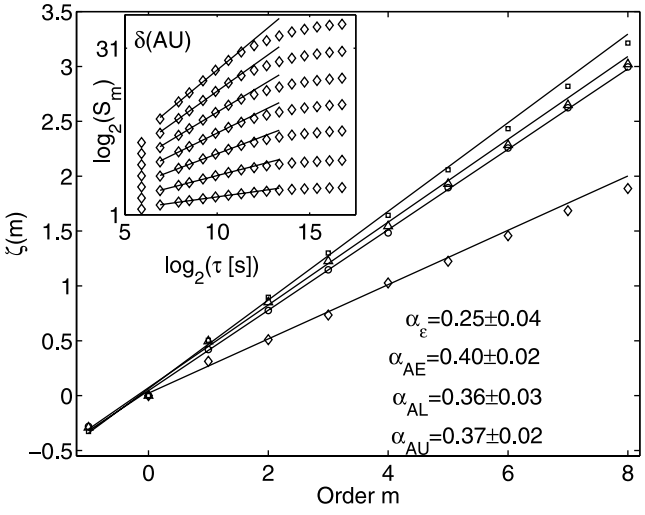

Figure 3. Dependence of the scaling exponent of the conditioned GSF on moment order. Inset shows the conditioned GSF S_m^c versus time lag τ for AU.

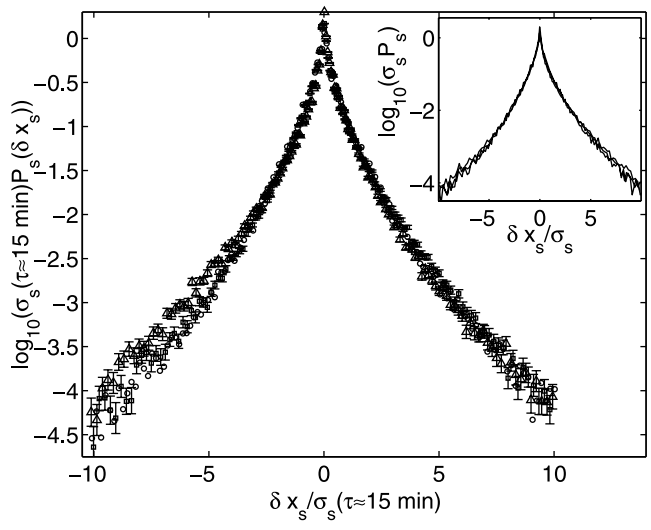

Figure 4. Direct comparison between the fluctuation PDFs for AE(□), AU(○) and AL(△), again at τ = 15 minutes. Inset shows overlaid PDFs of AU, AE and $-AL$ fluctuations. Error bars as in Figure 1.

obtained from this technique were unchanged for thresholds $A(\tau)$ between 6σ and 12σ.

[12] Firstly, our analysis suggests that the statistics of the fluctuations for all four quantities are self-similar for times between 2 and ~100 minutes and fluctuations of size $\delta x \leq 10\sigma(\tau)$. Secondly, the scaling exponent α in $\zeta(m) = \alpha m$ that characterize this self-similar behavior, are identical within errors for fluctuations in the AE indices but different to that in ϵ at the 1σ level.

4. Rescaling of Fluctuation PDFs

[13] Scaling of the GSFs can be related to scaling properties of the fluctuation PDFs [*Hnat et al.*, 2002, 2003a] using the generic, model-independent rescaling method (e.g., [*Mantegna and Stanley*, 1995; *Hnat et al.*, 2003b]) based on the rescaling of the PDFs $P(\delta x, \tau)$ of $\delta x(t, \tau)$ on different time scales τ. If a time series exhibits statistical self-similarity, a single argument representation of the PDF can be found that is given by $P(\delta x, \tau) = \tau^{-\alpha} P_s(\delta x \tau^{-\alpha})$, where α is the rescaling exponent. We now express S_m using the fluctuations' PDF, $P(\delta x, \tau)$ as follows:

$$S_m(\tau) = \int_{-\infty}^{\infty} |\delta x|^m P(\delta x, \tau) d(\delta x). \quad (1)$$

Expressing the integral in (1) in terms of rescaled variables P_s and $\delta x_s = \delta x \tau^{-\alpha}$ shows that the scaling exponent $\zeta(m)$ is a linear function of m, $\zeta(m) = m\alpha$, for a statistically self-similar process, as suggested here by Figure 3.

[14] The exponent α is ideally obtained from the scaling of the peaks of the PDF $P(0, \tau)$. However, the finite accuracy of the measurement may discretize the amplitude leading to errors in the peak values. Table 1 gives all scaling exponents, obtained by different methods. These yield consistent values of α, to within the errors. We will use α from the scaling of $\sigma(\tau)$ versus τ. If the fluctuations are statistically self-similar, as suggested by our GSF analysis, then the unscaled PDFs $P(\delta x, \tau)$ should collapse onto a single curve $P_s(\delta x_s)$. We applied PDF rescaling to the fluctuation PDFs of all quantities and obtained satisfactory collapse of the curves within the scaling regions. The χ^2 test applied to all quantities revealed that, for the scaling regions given above, the collapsed curves lie within 5–7% error band.

[15] Figure 4 shows the re-scaled fluctuation PDFs for the indices alone for $\tau \approx 15$ min. The δx variable has been normalized to the rescaled standard deviation $\sigma_s(\tau \approx 15$ min.) of P_s in each case to facilitate this comparison. The inset of this figure shows the comparison for AU, AE and $-AL$ fluctuations and these PDFs are nearly identical. These results are consistent with conclusions of the GSF analysis at the 1σ level.

[16] Figure 5 shows the normalized PDFs $P_s(\delta x_s)$ for $\delta x = \delta\epsilon$, δAE and $\tau \approx 15$ min overlaid on a single plot. We can clearly distinguish between the PDFs of the $\delta\epsilon$ and AE

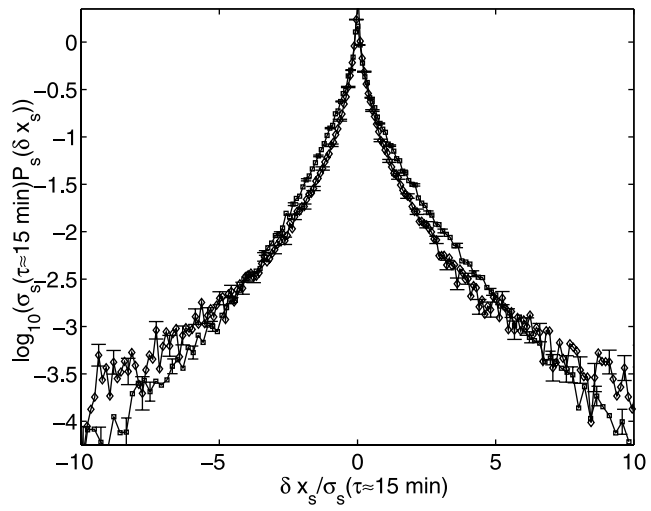

Figure 5. Direct comparison, for the particular choice τ = 15 minutes, of the fluctuation PDFs for ϵ (◇) and AE index (□). Error bars as in Figure 1.

indices' fluctuations. We obtain the same result repeating this comparison for several values of τ, within the scaling range τ_{max}. We have also verified that the functional form of the PDF are insensitive to the solar cycle within errors. The use of a larger, 10 year data set for the indices has reduced statistical scatter and expanded the dynamic range of the considered fluctuations as compared to the analysis given in [*Hnat et al.*, 2002, 2003a], and would lead us to draw the opposite conclusion, that on time scales less than $\approx$1 hour the *AE* index amplitude fluctuations are not driven linearly by those of the solar wind. We would also conclude that the difference seen at the 1σ level in the scaling of the ϵ and the indices is significant, even though they agree at the 2σ level [*Freeman et al.*, 2000].

5. Summary

[17] In this paper we have addressed an open question of the possible connection between the scaling properties of fluctuations in the solar wind driver and those observed in global measures of magnetospheric dynamics. We applied two statistical methods, generalized structure functions and PDF rescaling, to study the scaling of fluctuations in the ϵ parameter and the magnetospheric indices *AU*, *AL* and *AE*. We find that, statistically, fluctuations in all four quantities are approximately self-similar when their size is limited to $\sim 10\sigma$. This self-similarity extends to $\sim$1–1.5 hours. The scaling exponents of the *AE* indices are close to each other and are appreciably different to that of the ϵ parameter.

[18] The fluctuation PDFs of the *AE* indices, unlike that of $\delta\epsilon$, are asymmetric. Direct comparison of the PDFs for the fluctuations in the *AU*, *AE* and $-AL$ index indicates that they are nearly identical. Whilst the low frequency behavior of the solar wind and the indices may be well correlated [*Tsurutani et al.*, 1990], here we have concluded that, on time scales smaller than 1 hour the properties of the fluctuations in the solar wind and the indices differ in *both* amplitude and persistence. If the underlying physical origin of the auroral scaling is turbulence, then different scaling behavior implies a different type of turbulence, i.e., different dimensionality/topology or different relevant physics [*Frisch*, 1995]. If the underlying physics is SOC or similar [*Chang et al.*, 2003] then similar conclusions would still be drawn (c.f. [*Uritsky et al.*, 2001]). However, at this point we also can not rule out the possibility that the way in which the indices are constructed "burns" information still present in the magnetometer data about the solar wind scaling, here possibly by changing either or both of the degree of persistence (power spectral slope) and the heavy-tailed property (see [*Edwards et al.*, 2001] for a related preliminary investigation).

[19] **Acknowledgments.** SCC and BH acknowledge the PPARC and GR the Leverhulme Trust. We thank R. P. Lepping and K. Ogilvie for provision of data from the NASA WIND spacecraft and the World Data Center C2, Kyoto for geomagnetic indices.

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