Analysing the Familiar:

Reasoning about Space and Time in the Everyday World.

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Thesis submitted in partial fulfilment for the degree of Doctor of Philosophy.

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June 1991

Abstract

The development of suitable explicit representations of knowledge that can be manipulated by general purpose inference mechanisms has always been central to Artificial Intelligence (AI). However, there has been a distinct lack of rigorous formalisms in the literature that can be used to model domain knowledge associated with the everyday physical world. If AI is to succeed in building automata that can function reasonably well in unstructured physical domains, the development and utility of such formalisms must be secured.

This thesis describes a first order axiomatic theory that can be used to encode much topological and metrical information that arises in our everyday dealings with the physical world. The formalism is notable for the minimal assumptions required in order to lift up a very general framework that can cover the representation of much intuitive spatial and temporal knowledge. The basic ontology assumes regions that can be either spatial or temporal and over which a set of relations and functions are defined. The resulting partitioning of these abstract spaces, allow complex relationships between objects and the description of processes to be formally represented. This also provides a useful foundation to control the proliferation of inference commonly associated with mechanised logics. Empirical information extracted from the domain is added and mapped to these basic structures showing how further control of inference can be secured.

The representational power of the formalism and computational tractability of the general methodology proposed is substantiated using two non-trivial domain problems - modelling phagocytosis and exocytosis of uni-cellular organisms, and modelling processes arising during the cycle of operations of a force pump.

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Acknowledgements

There are many people who helped me during my period of research and during the writing of this thesis. Some of the help was academic, some practical, and some both. I only hope I can clearly recall all who have contributed, and apologise in advance if I miss any who justifiably should have been acknowledged.

First I would like to thank the Department of Computer Science (Warwick University) for given me the opportunity to do PhD research, and to the School of Computer Studies (Leeds University) which allowed me generous time to finally complete this thesis. In this respect I am particularly grateful to Tony Cohn (my thesis supervisor) who directed the research. When times got tough, he provided much needed practical support and encouragement when it really mattered.

Next, I would like to give especial thanks to my postgraduate colleages at Warwick. In particular I would like to thank Felix Hovsepian whose intuitive insights and broad knowledge of mathematics and physics, helped me recognise and realise the scope and complexity of the subject I had broached. Thanks also to Ian Gent for his formal and critical insights - of pointing out more than once that an alleged theorem on my part was unlikely to be so - and which turned out to be the case! Thanks also to Guy Saward for many fruitful discussions, and to Kathy Courtney who did not buy my logicist approach to AI. I must also mention David Mond who helped in my understanding of general topology, and to Tony Richards who always took an interest in what I was currently up to. All these people helped shape my ideas, but above all proved to be very good friends.

I would like to thank Roger Teichmann (Oxford University), Tony Halbert and Nick Denyer (Cambridge University) for the many valuable philosophical discussions I have had with them over the years. Thanks also to Adrian Larner (IBM Warwick) who shared my interest in calculi of individuals, and to my colleague Zhan Cui (Leeds University) for his astute comments and friendship.

Especial thanks are extended to the neurosurgical team at the Walsgrave Hospital (Coventry) who cured me of a protracted period of acute sciatica that arose during my period of research, to my current General Practioner Dr Campbell and to Graham Martin who gave much appreciated practical help during this very difficult period.

Finally, I would like to thank my wife Sarah for her love and support during the time this thesis was researched and written. I only hope I can now make the countless sacrifices made, worthwhile. Research theses frequently acknowledge spouses or partners who supported the author during their research period. I can only say from my own experience that such acknowledgements cannot be emphasised strongly enough.

This research was made possible by funding from the SERC for which I am particularly grateful, first from a PhD research student award, and secondly from the research grant award GR/F/64380 and GR/G/36852.

Declaration

The material presented for this thesis is (unless explicitly stated otherwise) my own work.

"...There are an indefinite number of purely abstract sciences, with their laws, their regularities, and their complexities of theorems - all as yet undeveloped. We can hardly avoid the conclusion that Nature in her procedures illustrates many such sciences. We are blind to such illustrations because we are ignorant of the type of regularities to look for. In such cases, we may dimly sense a sort of familiarity attached to novel circumstances, without any notion of how to proceed in the analysis of the vague feeling."

A.N. Whitehead

Chapter 1: Analysing the Familiar

1.1 General introduction to and outline of the thesis

Artificial Intelligence (AI) has long sought computationally efficient and expressive ways to represent non-trivial domain knowledge in a formal framework. However, despite the importance given to the development of formal theories that can be used to encode rich domain knowledge associated with the everyday (commonsense) world, few worked examples have appeared in the literature.

This thesis motivates and describes a sorted first-order axiomatic theory that can be used to model intuitive spatial and temporal knowledge associated with the everyday world. The theory concentrates upon the explicit representation of topological information, although geometrical and metrical information is also used. The theory is notable for the minimal set of assumptions required in order to lift up a comprehensive theory that can be used to describe non-trivial modelling problems.

The ontological primitives of the theory include a set of regions which are interpreted so that they support either a spatial or temporal reading. A set of functions and relations are then defined on these regions. This enables complex spatial relationships between physical objects to be formally described, as well as providing the basis for describing physical processes by specifying particular sequences of state descriptions that change over time.

The theory gives rise to various abstract structures: in particular, a set of monadic predicates encoding taxonomic information is factored out and embedded in a special sort lattice, as are sets of higher-arity predicates which are embedded in a relational lattice. These and other structures are highlighted and factored out to show how the proliferation of inference commonly associated with automated reasoning programs can be more effectively controlled in an automated reasoning setting. Means to secure further control of inference is also secured by abstracting out empirical information from the modelled domain and using this to put contraints on these basic structures.

Although a direct implementation of the theory using a mechanised sorted logic is envisaged, special attention is given to the theoretical separation made between the formal theory and different ways the theory might be used or implemented.

The thesis falls into three main parts. The first part analyses the place of commonsense knowledge in AI research and concludes with a working methodology. The second part describes the formal theory, while the final part concentrates upon implementational questions - suggesting efficient ways of controlling inference using a resolution-based implementation of the theory.

The chapter outline is as follows. Chapter 1 (being the rest of this chapter) introduces and motivates the subject of common sense knowledge within AI research. A separate chapter (Chapter 7) compares and contrasts related work. Chapter 7 assumes some familiarity of the formal contribution of this thesis and should be read with this in mind. In Chapter 2 the bulk of the formal theory is covered in detail. Each relation, function and property is formally defined, and where appropriate discussed and illustrated with intuitive examples. This is extended in Chapter 3 where physical objects, states and events and the description of processes are introduced. The representational power of the theory is illustrated in Chapter 4 where two non-trivial modelling problems are tackled. Chapter 5 discusses implementational matters, while Chapter 6 considers ontological and epistemological questions raised by the theory and working methodology. A critical survey of related work appears in Chapter 7, while Chapter 8 discusses future work and summarises the main contributions of the thesis. Notes on the text, a bibliography and appendices are included. Appendix A is a glossary of specially defined symbols used in the thesis, while appendices B and C cover all the listings of proofs cited in the text. In the interest of space, full listings of the inference steps used in each proof is not given. This is reserved for the most interesting theorems only. In this case, a resolution-based proof strategy is used. The rest of the theorems are simply relegated to a list of axioms and definitions which together with the negation of the theorem to be proved, are sufficient to secure the stated theorem (again using a refutation-based proof strategy).

1.2 The need to encode commonsense knowledge in programs

It has long been maintained that if AI is to succeed in building machines that exhibit intelligent behaviour, their programs must encode large amounts of commonsense knowledge of the world [Hobbs et al 1985 p1-1], or at the very least must be seen to share our common knowledge and assumptions [Forbus 1988a p197]. Exactly what this commonsense knowledge consists of remains to be examined, as does the justification behind this assumption. However, the general view is that without some means of encoding general knowledge of the everyday world in computer programs, we will have to be content with the limited use of these in specialist applications only [Forbus 1988a p197].

The motivation for imparting very general knowledge of the world to programs can be identified with limitations encountered in the development of Expert Systems, and in the parallel development of Qualitative Physics. Cohn [Cohn 1989 pp180-82] discusses some of the common problems encountered. For example, expert system programs designed to do diagnosis seem inadequate when used to make predictions or tutor. Moreover, they seem unable to solve (what we identify as) simpler versions of the type of problem the program was originally designed to solve. Typically these programs have their knowledge implicitly represented as a set of shallow (or "compiled") rules, with no interactive means to demonstrate or justify why a conclusion was reached.

Given comparable tasks, human beings can give reasonable answers to general problems, whether or not they have specialist knowledge, whereas programs unable to reason from first principles, cannot.

In Qualitative Physics a similar trend can be seen. Traditional methods of problem solving using numerical modelling are claimed to be inadequate when precise numerical information is missing, a model cannot be provided, or where it is impossible to specify initial conditions. In many realistic settings some or all of these factors simply cannot be given or derived, yet in the face of such limitations human beings still seem able to make useful inferences about the modelled domain. In general, in the literature, such observations are used to argue that we are better advised to consider qualitative or symbolic representations in models rather than using standard quantitative modelling techniques. Two main thrusts are evident here. The first is grounded in the notion of cognitive validity, since it is argued that a qualitative representation better approximates the way we habitually describe and reason about the world, and the second appeals to more effective ways to encode and process information in a computational setting than that associated with numerical modelling methods.

1.3 Some history

The idea that AI should consider the need to impart common sense to programs is not new. In fact it can be traced to the very beginnings of AI with John McCarthy's classic paper "Programs with Common Sense" [McCarthy 1959]. Renewed interest appeared in the form of the Commonsense Summer workshop [Hobbs et al 1985], and in a collection of articles devoted to the formal treatment of commonsense theories of the world [Hobbs and Moore eds. 1985]. More recently, continuing interest in the subject has given rise to Davis's [1990] book on the representation of commonsense knowledge, and the initiation and development of the ambitious *Cyc* project [Guha and Lenat 1990, Lenat et al 1986 and Lenat et al 1990] with its central aim of codifying and using efficient means to reason with large amounts of general knowledge of the world. Hobbs [Hobbs et al 1985 p1-15] (concluding on the results of the Commonsense Summer workshop) remained optimistic about the general enterprise of encoding commonsense knowledge, but more recently, and particularly following the controversy centred on Patrick Hayes' Naive Physics programme [Levesque 1987], it must be said that the general view emerging is that the task of encoding commonsense knowledge is proving far more difficult than at first conceived. The difficulties encountered in the Cyc project [Guha and Lenat, 1990] provide further justification for this point.

1.3.1 Naive Physics

An influential attempt to stimulate research workers into building large scale formal theories that encoded commonsense knowledge appeared in Hayes' [Hayes 1979,1985a,1985b] Naive Physics programme. Hayes argued that one should concentrate upon the task of building large scale formal theories *before* considering how such a theory might be implemented. He envisaged this programme would proceed by first of all identifying and then linking together various sub-theories. This would include detailed knowledge about the nature of e.g. objects, substances, shape, space, movement and time.

The Naive Physics programme embodied the assumption that human beings rely upon a more general (hence "naive") view of the world than that found in current developed bodies of science. For example, we do not require knowledge of fluid dynamics in order to handle or reason about liquids in most everyday situations. Although in practice the modelling might well encode some non-naive concepts at its theoretical core, it was important that the theory reflected this assumed body of knowledge garnered from our everyday experiences. The same assumption applied to reasoning - "obvious" deductions were to coincide with "short proofs".

Hayes argued that a move had to be made away from the simple domains and sparse axiomatic theories which had dominated earlier AI research. In part this had been identified with a premature pressure to demonstrate the worth of some approach by holding aloft a working program. Simple domains had the virtue of helping to avoid the serious problem of uncontrolled inference using standard interpreters and extant automated theorem provers, but equally the modelling suffered. Hayes argued that implementational pressures should not detract from a good working methodology. In its place a call was made to first of all concentrate upon the task of building large scale formal theories, and then having done this, devise algorithms or heuristics to exploit anticipated structures that would be naturally embedded in any rich formal theory of the world.

Hayes argued that a formal theory should support a clean semantics, and in this respect advocated the use of first order logic (FOL) as a representational language. FOL also had the virtue of supporting a well understood proof and model theory. The model theory worked well by helping to shape the theory. Hayes argued that it was all too easy to develop a sparse axiomatisation that supported too many unintended models. To overcome this problem, alternative models were constantly considered, and ways suggested themselves to constrain the intended model by suggesting additional axioms which when added to the theory, would eliminate contenders. Hayes argued that a rich theory had to be both broad (i.e. have enough concept tokens to cover what one needed to say) and dense (i.e. support enough inferential links between the formal expressions supported by the theory). In practise the theory builder would eventually find that he or she would have enough formalised concepts to describe the chosen domain - what Hayes referred to as conceptual closure. Identifying this was taken as a measure of success, though it was deemed unlikely that complete closure could be actually achieved.

Despite the fact that Hayes' Naive Physics programme originally received much interest among AI researchers, few papers appeared in the literature based on Hayes' original contributions. This trend probably led McDermott to the conclusion that the whole programme (being characteristic of what he identified as "logicism") was unlikely to succeed. The main problem according to McDermott was the central assumption that deductive reasoning was deemed sufficient to model and reason about the domain. McDermott's paper appeared at the centre of a lively forum, but the respondents seemed divided on many points [Levesque 1987].

1.3.2 Qualitative Physics

In contrast to Naive Physics, Qualitative Physics (QP) seems to have generated much published material, and at the time of writing the subject is still burgeoning (see e.g. Weld and De Kleer 1990, and Struss and Faltings 1991). At best I can only outline its central characteristics here. Later in Chapter 8 I will discuss the different approaches in more detail.

Like Naive Physics, Qualitative Physics takes the physical world as its domain. Its adherents aim to provide the means to effectively represent and reason about the world that captures both the commonsense knowledge of the person on the street and the tacit knowledge used by both engineers and scientists [Forbus 1988b p239]. In contrast to the Naive Physics programme, QP places emphasis on how inferences are drawn and thus more attention is given to program development. The motivation for developing Qualitative Physics has already been sketched out above.

Forbus [1988a p198] characterises Qualitative Physics as "having to do with reasoning about continuous properties via discrete abstractions". This is preferred to the general comparison made where the term "qualitative" is associated with a non-numerical approach to modelling, since according to Forbus the term "symbolic" serves the same purpose

[Forbus 1988a p198]. QP uses finite sets of discrete symbols for modelling dynamical systems: for example, the signs "+", "0" and "-" are frequently used which reflect the important observation that important changes of state arise when certain signs of physical magnitude change. The path of a projectile travelling up, then down is a case in point, as is the prediction of what will happen if the temperature of a liquid continues to rise [Forbus 1988a p198]. Qualitative Physics does not necessarily seek to supplant traditional methods of numerical modelling but recognises the value for combining the two in a complementary role.

1.4 Knowledge of the commonsense world

Despite the fact that the addition of commonsense knowledge to programs is generally regarded as an important problem that needs to be solved, it is notable that little work in the literature seems to be done establishing exactly what commonsense consists of. Indeed given the points discussed below, it would seem that the paucity of work is a direct consequence of this, since if anything it is difficult to establish exactly what can or should be *excluded* from a program. In other words the theoretical underpinning remains weak from the fact that commonsense knowledge is taken to be too inclusive.

1.4.1 Commonsense knowledge and reasoning

Research with commonsense as the central subject matter can be split into two distinct but complementary strands: modelling commonsense knowledge and modelling commonsense reasoning, although the distinction is easily conflated. For example, Forbus [Forbus 1988a p197] correctly points out the somewhat loose contrasts made between commonsense reasoning and "expert reasoning". He also criticizes the inadequate characterisation of commonsense reasoning with default or nonmonotonic reasoning, since such modes of reasoning also appear in many areas of expertise. However, he himself merges the two by characterising commonsense reasoning by the domain it is applied to. In this case this is said to cover the physical, social and mental world [Forbus 1988a p197-198]. But here it is difficult to see exactly what knowledge Forbus intends to *exclude*.

While one can agree with Forbus that default and nonmonotonic reasoning is not a defining property of commonsense reasoning, one can still separate out commonsense knowledge from the reasoning component. Modelling commonsense knowledge brings *ontological* questions to the fore. That is to say, it draws attention to the set of entities assumed by a theory which cannot be eliminated or analysed out. And, moreover, by expressing commonsense knowledge in the form of a theory (where by "theory", here I mean nothing more than a set of declarative sentences closed under implication), we also have the means to check the sufficiency of the ontology and the conditions built into the theory by examining the theories' formal consequences. That everyday reasoning involves all manner of inference, e.g. deduction, abduction (i.e. reasoning to the best explanation) and induction is beside the point if one is interested in *codifying knowledge*. Whether that knowledge takes the form of simple know how or knowing that something is the case, in either case the ontology must be first made clear.

1.4.2 Characterising commonsense knowledge

If commonsense knowledge can be clearly isolated from other bodies of knowledge we have of the everyday world, it must be very general in nature. But exactly what that knowledge consists of seems difficult to state, despite its seemingly obvious nature once articulated.

For Hobbs [Hobbs and Moore 1985 pxi-xiii], and Forbus [Forbus 1988a p198] commonsense knowledge covers a large body of material drawn from the physical, psychological and social world. To illustrate the scope of this knowledge Hobbs uses the example of a robot journeying between buildings to get salad and a sandwich from a refectory, and asks himself what that robot would have to know in order to carry out the task. It would require knowledge about location, shape, motion and causality in the recognition of buildings, offices, elevators and elevator buttons. The robot would need some concept of itself (e.g. comparative notions of size) in order to negotiate doorways or staircases. Outside the building it would encounter paths and lawns, and would need to correctly identify the former for ease of travel and avoid difficulties arising from prescriptions made by humans about not travelling across the grass. Inside the refectory, it would need to know how to deal with flexible material (lettuce), and certain tools (salad tongs), about the handling of liquids and viscosity (salad dressings) and the importance of monetary transactions.

Given a central aim of AI is to eventually produce programs capable of giving rise to flexible intelligent behaviour, the enormous scope and importance of effectively encoding such knowledge becomes quickly apparent. But equally this could be taken to indicate the sheer difficulty imposed by and the practical impossibility of succeeding in such a project [Hobbs in Hobbs and Moore 1985 pxii]. Unfortunately, the explicit identification of commonsense knowledge with such large body body of knowledge does little when it comes down to the actual process of knowing *what* to encode in a machine's program. In this respect it is useful examining some common assumptions and misconceptions to be found in the literature; this at least suggests a way forward by constraining the subject matter.

1.4.2.1 The problem posed by familiarity.

Perhaps the most difficult problem met when trying to understand exactly what commonsense consists of, is simply breaking through the element of familiarity we habitually associate with commonsense knowledge. Paradoxically, it is the element of familiarity itself that is the problem.

That much commonsense knowledge is familiar, does not make it any the easier to develop a theory that yields a set of plausible consequences. As Whitehead noticed in the quote with which this thesis begins, the very air of familiarity about a subject frequently makes it very difficult to know how to proceed in the analysis. It is all too easy to assume that if something is familiar and not requiring much deliberation or sustained thought (typically said to be "intuitive"), that the subject matter or process involved is simple in nature, or can be adequately characterised using everyday concepts. Marr [Marr 1982 p30] gives a good example of this; pointing out how the simplicity of the act of seeing had misled Gibson to vastly underate the complexity of visual information processing simply required in order to detect physical invariants. The very same difficulty arises when building a theory said to encode commonsense knowledge, since in spite of the familiarity of the subject matter, it is not at all obvious what invariants extracted from the environment are most likely to be exploited, what processes act on them, and how these link in with our articulated responses to the world.

Although it is important to choose a good working ontology, care is needed not to assume that the ontology of some proposed theory actually uncovers the same set of entities 'posited by the brain' which accounts for us having a particular body of knowledge of the world. In this respect Hayes [Hayes 1985b p21 footnote] was correct to emphasize that while he used non-intuitive mathematical concepts in his theory of space, it was the match between the formal theory and the world that mattered. One should not reject a theory as inappropriate simply on the grounds that one has difficulty has in understanding its central concepts. To do so is to already assume that whatever underlies commonsense is simple in nature, but if anything the opposite is more likely to be true.

1.4.2.2 Commonsense knowledge as "core knowledge"?

A second problem characterising common sense stems from the simple conviction that commonsense knowledge can be readily identified as a coherent body of knowledge shared among large groups of people. Although one can agree with Hobbs [Hobbs and Moore 1985 pxi] that any "reasonably sophisticated intelligent agent" must have a certain minimum of "core knowledge" to make its way around the world, it is not at all clear what this core actually consists of, nor is it as ubiquitous as Hobbs seems to suggest. Take for instance the not unreasonable assumption that commonsense knowledge is intuitive, and that anything that is considered an affront to intuition is thereby excluded. Han [Newman 1956 p1976] points out that not only does intuitive knowledge change with time, at any one time it differs across different groups of people. For example, the hypothesis that the Earth was spherical was once considered unintuitive, but is taken as a given now. While the notion of bodies having weight is commonly held, and the notion of intertia less so, this changes if one is an engineer or a physicist where regular use of such concepts make them equally familiar hence intuitive. Indeed it is difficult to see how one can clearly maintain the purported distinction between commonsense knowledge, and that culled from the sciences despite the fact that this seems commonly believed.

1.4.2.3 Letting the term "naive" do too much work

There is no clear reason why we should assume that a theory of commonsense will be any the simpler in structure, or will require less work to refine than those currently used in science and philosophy. In this respect, my colleague Ian Gent once remarked that no scientist ever intentionally starts out to build a complicated theory to account for some state of affairs, where a simple one would do. So why should we expect a naive physical theory will turn out any the simpler and more tractable in practice. Despite the underlying attraction naive theories might hold for some, it would be unreasonable to expect a noticable difference in complexity between a comprehensive commonsense theory and any other scientific theory. This being so it would be difficult simply justifying on these grounds why a commonsense theory will more naturally find itself at the core of a program instead of the latter.

Unfortunately, the ubiquity of the term "naive" in AI literature, e.g. "naive physics", "naive botany" and naive meteorology" [see e.g. Hobbs in Hobbs and Moore 1985 pxiv, and Legrenzi and Sonino 1991] does little to clarify exactly what "naive" covers. Some seem simply content to characterise "naive" in terms of "what ordinary people know", prefixing the term to the name of any scientific discipline as though this demarcates a viable research area [Legrenzi and Sonino 1991]. If anything, we would be better advised to recognise the danger of muddying the research topic by overuse of a term. How then should "naive" be characterised? Halmos [Halmos 1960] in his book 'Naive Set Theory', provides a simple but sufficient characterisation of the term. For Halmos, the term "naive" is justified by using an informal language and notation, but on the condition that the subject matter is formalizable. For Halmos geometry is naive if it proceeds on the paper-folding kind of intuition alone. This use of the term "naive" agrees with Hayes' implicit use of the same term and this seems perfectly adequate. Thus a naive theory of common sense knowledge should be formalizable, but should aim to proceed from some simple intuitions, for example, that bodies occupy space, and that no two distinct bodies can occupy the same place at the same time.

1.4.2.4 To what extent should a commmonsense theory reflect current bodies of scientific knowledge?

There is some evidence (see e.g. Gentner and Stevens 1983) that a significant number of people tend to give Aristotelian or at least pre-Newtonian explanations to account for physical events. Such observations have been used in AI and Cognitive Science to motivate research to uncover and codify this class of pre-scientific beliefs, with the view of incorporating this knowledge into programs. However, even assuming a significant number of peoples' beliefs do indeed cohere more with a pre-Newtonian world view, there is no reason why the commonality of such beliefs should be used as the basis of some knowledge base in a program. Belief certainly is a necessary condition for having knowledge, but not sufficient. We would do well to make sure that our programs actually embody a model that give rise to sound predictions grounded in the world. A program driven machine that embodied an archaic physics might be viewed as sharing a form of life with us, but would require extensive defaults to be of general use and considered "safe". Moreover, given the body of scientific knowledge that has already been developed and used, it would be advisable to seek ways to incorporate this into our programs than seek an alternative physics, and encode that.

A common distinction drawn in the literature between an "engineering" and a "psychological" approach to knowledge acquisition may well be thought appropriate here, i.e. whether we are aiming to model the world as described by science, or non scientists' beliefs about the world. The idea that this distinction can be effectively maintained, and that psychological validity can be simply put aside for an engineering solution when developing a theory of commonsense knowledge is a mistaken position. Firstly, the motivation to develop e.g. Naive Physics, made an implicit appeal to the psychology of the human being, i.e. identifying the lacunna with the lack of commonsense knowledge. Secondly, it is all too easy to adopt an engineering solution when facing problems associated with programs using large knowledge bases e.g. efficient retrieval of information, or uncontrolled inference in automated theorem proving. Psychological validity of a theory may well indicate that the assumed model is unwieldly, and mask the distinct possiblility that the human beings may well use sparse (as opposed to rich) mental models and exploit fast and shallow chains of inference when solving problems.

1.4.2.5 Do we include the paradoxical into a commonsense theory of the world?

Another difficulty characterising commonsense knowledge is that the paradoxical will most certainly be associated with falsehoods, and not readily incorporated into some proposed theory. But very often reflection on some original statement will reveal that we were mistaken. Take for instance a naive theory of motion for rolling wheels, which might say that whenever a wheel rolls forward every part of that wheel will do so too. This is intuitive, but in fact it is not true of all rolling wheels. Kasner and Newman [Newman 1956 p1941] give an example where at any instant of time, a railway engine never moves entirely in the direction in which the train pulls. The paradox arises from the simple fact that a point on the flange of a moving railway engine wheel traces out a curtate cycloid curve which moves back on itself, rather like the greek letter "γ". In other words, a part of the wheel flange which lies below the top of the rail, will move in the opposite direction to the general direction assumed by the moving wheel. Although this fact is clearly unintuitive, it is difficult to see how a useful theory of motion for rolling wheels could be stated *without* incorporating the paradoxical. Thus, once again the "naive" element reveals complexity at its core. Given a desiderata where formal naive theories should be both broad and dense, it becomes difficult to see how unintuitive concepts and the paradoxical can, or indeed should be avoided.

1.4.2.6 Commonsense knowledge as "deep knowledge" and solipsism.

A common assumption underpinning much research work in AI, is to endorse a position known in philosophy as solipsism. To endorse solipsism (literally 'only-oneself-ism') is to hold the view that nothing exists outside one's mind, or that nothing such can be known [Lacey 1976].

In AI, solipsism appears in the tendancy to incorporate not only the means to reason about the world in a program, but also to want to encode a very *rich symbolic model* or theory of the world in that program too. In other words the program and the machine running it is taken to be a world unto itself. Solipsism not only appears in the general view that intelligent machines can be effectively driven by programs that have little or no recourse to either artifical sensory or perceptual mechanisms, its influence can be seen in the motivation behind, and the the common distinction drawn in expert system literature between "deep" and "shallow" knowledge mentioned earlier.

The motivation for the distinction made between "shallow" and "deep" knowledge [see e.g. Bonissone and Valavanis 1985] draws off the same set of difficulties found in expert system development discussed earlier. In this case compiled knowledge is identified as "shallow" knowledge and "deep" knowledge as a complementary body of very general knowledge associated with the problem domain. The term "deep" refers to the fact, that for us this knowledge is rarely made explicit in our dealings with the world. According to Hobbs [Hobbs and Moore 1985], the provision of deep knowledge in a program allows machines to function effectively in an unstructured environment; and that this knowledge is clearly "deep", is supported by the use of protocol-based questionnaires and the general difficulty people have in eliciting such fundamental material. But such findings support a simpler explanation.

In the first case, I would argue that it is neither necessary nor always desirable to posit a complex model or theory to account for complex behaviour. That I can consistently catch a ball does not require me to have Newton's laws of motion encoded in my physiology, and similarly for the program of a machine. For example, pattern recognition, the detection of motion and 'textural explosion' may well be sufficient properties extracted from the external world to track a ball and initiate a successful catch. Secondly, much stated commonsense knowledge takes the form of rule-of-the-thumb know how, of what happens when something else happens, and that is all. The idea that human beings must have complex mental models or large bodies of "deep knowledge" to account for flexible intelligent behaviour can be identified with a failure to recognise problems stemming from an uncritical acceptance of representational theories of mind which can be recognised underpinning much research work in AI.

1.5. Standing back: establishing a working methodology

Having established directions in which we do not want to go, how then should one proceed? Below I outline a working methodology.

1.5.1 The use of first order logic.

First order logic (FOL) is chosen as the representation language for the following reasons. In the first instance logic can be effectively used to *model* a domain. It is important that any proposed theory be capable of being expressed in a formal framework, since without this foundation we have no reliable method to establish either the appropriateness of a given ontology or the content of the theory in terms of its consequence class of deductions. Admittedly, certain kinds of inference associated with commonsense reasoning do not fall neatly into the deductive mould, but it is important to stress that this is a point about modelling modes of reasoning rather than modelling domain knowledge associated with the everyday world. These different modes of reasoning actually presuppose a way of describing and ordering such domain knowledge, and for this deduction seems perfectly adequate. Despite the criticism FOL receives from the standpoint of capturing the various modes of reasoning human beings use, its central role in modelling domain knowledge still remains very much in evidence.

Following Hayes [1979,1985a] FOL is chosen for its well understood proof and model theory. FOL supports a clean semantics, a condition deemed essential if the formal theory is taken to describe a *theory of* the world. As Hayes [1985a] correctly points out, without a clean semantics we have no way to say what the formal inscriptions of a theory actually denote or what extension a predicate has, hence no way to say that the formalism is a formalism of anything. Non standard syntactical formal expressions, e.g. "cousins(x) = children(siblings(parents(x)))" [Guha and Lenat 1990], and (paradoxically) "roughly(height(Bill)) = tallish" [Hayes 1985b], require *explicit* readings to be first given in the metalanguage that interpret the set of object level expressions used. Failure to recognise the importance of this point can easily result in a muddied analysis. An example of this can be seen in Hobbs et al [1985 1-9] where we find the assertion: "When we *write* an axiom of the form $(\forall x)\ p(x) \supset q(x)$, we really *mean* an axiom of the form $(\forall x)\ p(x) \wedge \neg ab(x) \supset q(x)$"! - my italics.

An important point raised by Hayes (and judging by the repetition one not fully appreciated, e.g. Hayes 1977), is that representational languages can be implemented in a variety of ways. For example a frame representation language (see Minsky 1975), may well have desireable retrieval operations, but this is a point about implementation and does not touch the question of representation. Minsky developed Frames as an alternative representational language to FOL, however depite the popularity Frames has enjoyed, Hayes [1985c] has argued quite forcibly that Frames offers no real increase of expressive power nor modes of reasoning over that which it was assumed to replace. To criticise FOL as a representational language because e.g. current implementations of axiomatic theories incur problems of computational cost with the common problem of generating large search spaces, simply misses the point.

A third reason for choosing FOL is that the formal theory can be better compared with other theories. Again this is to adopt another recommendation by Hayes [1985a], using FOL as a reference language into which other representational formalisms should be capable of being translated.

FOL is also chosen from a computational standpoint. There is a well researched body of literature devoted to automated reasoning using FOL as the representational language. Thus implementing a first order theory is a relatively straightforward matter, even though (as argued above) the implementation of a theory need not be restricted to a resolution based automated reasoning setting, for example. Having the theory expressed in FOL allows for machine assisted development and testing of the theory, despite the fact one may well see how to factor out information, so that computationally expensive procedures in a simple resolution based implementation of the theory might well benefit using hybrid reasoning or other less expensive techniques. (This is covered in more detail in Chapter 5.)

Although FOL is semi-decidable, it is preferred to higher-order (and nonmonotonic) logics currently used in AI which are characteristically undecidable. Higher order logics gain in expressiveness, but when automated suffer from incomplete inference strategies. This immediately reduces the attractiveness of using an automated logic for theory development.

A mechanised sorted logic is actually used to describe and implement the general theory. The advantages of using sorted as opposed to unsorted computational logics are well known in automated reasoning literature. These are only briefly mentioned here. Firstly, sorted logics yield a more compact notation that their unsorted counterparts making the formal theory generally easier to read [Cohn 1989a]. Secondly, given a theory rich in taxonomic information, mechanised sorted logics used to implement the theory tend to score in terms of efficiency over their mechanised unsorted counterparts - see Cohn [1989a] for a review of relevant work. From the standpoint of developing the conceptual apparatus of the theory, the use of a sorted logic is not essential, but the added requirement of declaring what sorts constants, functions and predicates are defined on, suggests a third advantage. Using a sorted logic helps to constrain one's thinking and thereby reduce the risk of introducing spurious information into the developing theory [Cohn 1989a].

Finally, by expressing knowledge in an axiomatic framework, the primitivity of certain concepts and a minimal set of axioms is made explicit. A formal theory sporting few primitives and axioms frequently coincides with the need for long chains of deductive inference in order to secure a chosen theorem. However, such austerity can extend beyond aesthetic satisfaction in having reduced the theory to a minimum set of conditions. For example the theory might be of use to a Cognitive

Scientist looking for a minimal set of entities or conditions required in a particular theoretical construction of the world. An austere formal theory describing, for example, the ways in which objects tend to be related in space, can be used to constrain and direct research for physical correlates of the theory in terms of brain functioning. While the set of primitive concepts supported by a theory may not be necessarily encoded in perception, the sufficiency of the theory to generate a plausible set of consequences at least suggests a fruitful line of enquiry. Indeed without some theory to direct the research no method to interpret any set of data will be forthcoming.

This emphasis on ontological reduction within a theory is in direct contrast to that suggested by Hayes in the Naive Physics programme who argued for the use of a prolix ontology [Hayes 1985a]. However, it must be said that Hayes' recommendation that a rich theory should be both broad and dense, makes it very unlikely that a consistent formal theory using a prolix naive ontology will be forthcoming. While Hayes is correct to emphasise the theoretical importance of breadth, density and conceptual closure in a theory [Hayes 1985a p15], the dense web of inferential connections within any formal theory puts severe demands on the theory (and theory builder!), particularly if that theory is to have the scope which Naive Physics demands.

1.5.2 The need to represent and exploit topological information.

There is good reason to encode topological information into any theory used to describe the relationships between objects in space, and descriptions of states and events in time. In fact, much information used in our everyday dealings with the world appears to exploit topological rather than metrical or geometrical information [Barr 1964]. Saying whether something is inside or outside another thing, or whether some moment is before, after, or during another period of time uses topological information, certainly no (stated) metrical or geometrical information. Geometrical and metrical constraints imposed by the size and shape of objects, and physical contraints, for example rigidity and the degree of deformability, may well be expected to appear in any rich theory of the world, but given the importance of selecting out useful invariants in a changing world, there is good reason to concentrate upon a theory that captures topological information, since such properties remain relatively stable over sufficiently long periods of time. As with QP where the interpreted signs "+", "0" and "-" have proved particularly useful in modelling physical systems, and where changes in signs locate points where interesting things happen, so to with certain topological relationships holding between objects, as when one object is outside another object and then later inside that object as part of the process of ingestion.

5.3 Concentrating upon perceptual information

Given the broad spectrum of knowledge frequently associated with common sense and the difficulty identifying exactly what commonsense covers or at the very least, what it should cover, it is useful concentrating upon descriptions of the world that are grounded in perception. If common sense knowledge is to be sufficiently robust over time and at the foundation of many of our beliefs about the world, it is well to first consider the primitive basis for such beliefs and use that in a formal theory. The relationship between topological and perceptual information can be easily wedded together, since we have direct experience of, for example spatial relationships between bodies embedded in space that exhibit varying degrees of connectivity. Moreover, by concentrating upon perceptual knowledge (in this case describing the arrangement of bodies in space), it is easier to see how one can begin to build a rigorous theory using few primitive notions, rather than seeking to build a theory of the same rigour, incorporating many high level descriptions thought to embody commonsense notions.

It is to be expected that such a working methodology will naturally find an overlap with extant mathematical concepts and theories. Rather than avoiding such *foundations* (because the underlying concepts are in many cases non-naive), the overlap should be championed on at least two accounts. Firstly, that mathematics provides a rich source of well understood abstract models and theories that have been, and are still used with great success in describing and explaining aspects of the physical world. And secondly, that by working with sufficiently abstract, and non-naive concepts, the ontology of the theory will be sufficiently 'removed' from its interpreted correlates in the everday world, to allow the familar to be broached and analysed out.

6 Summary

In summary then, I argue that the better understanding and codification of commonsense and commonsense knowledge must be secured if machines are to be able to share a *form of life* with us. Earlier attempts to derive useful theories of commonsense knowledge have suffered from a general lack of analysis of exactly what commonsense consists of. This has led either to a vague characterisation of commonsense with the result that commonsense knowledge is readily associated with a too large body of knowledge, or a too rigid adherence to representational theories of the mind and solipsism. Taken together both have led to a common belief that a program must contain an extensive amount of this very general knowledge, together with the means to reason about it.

I argue that a formal theory is useful since it highlights a particular ontology, and that by using a reductionistic approach, that the end result can be better tested in terms of cognitive validity. The concentration upon perceptual information and topological concepts is argued to be a fruitful approach. Given the emphasis of FOL for modelling and theory refinement, this does not mean that a direct implementation of the theory in an automated reasoning setting follows. Various ways to implement a theory may be suggested, e.g. in the use of hybrid reasoning techniques where various parts of a theory are factored out and assigned to specialist procedures, or using other structures that have useful computational properties, e.g. planar graphs. Indeed the worth of a theory may be simply in its demonstration of the adequacy of its ontology and conditions to derive a set of plausible consequences.

In the following chapter I describe the formal theory that lies at the centre of this thesis. The theory is expressed in FOL, and concentrates upon the explicit representation of topological information. The correspondence between this information and that given immediately in perception is developed throughout the thesis and drawn together in chapter 6 where ontological and epistemological questions raised by the theory are discussed.

Chapter 2: The Basic Formalism

2.1: Introduction

The general theory outlined below is expressed in full first order predicate logic. It extends the conceptual apparatus outlined in Clarke's [1981,1985] calculus of individuals and uses Cohn's [1983,1987] many sorted logic LLAMA. The syntax of the general formal language is given in section 2.3 and the sort/type notation used in LLAMA in section 2.4. For readers unfamiliar with sorted logics, and in particular with the logic LLAMA, introductory material is also given in section 2.4. This is covered in more detail in Chapter 5. A complete description of LLAMA is given in Cohn [1983,1987].

2.2: Preliminaries (for Chapter 2)

The reader is assumed to be familiar with general first order predicate logic. Some familiarity with concepts drawn from general topology, and with partial orders and lattices is also assumed, but this is fairly elementary. A good introductory text to lattice theory is Rutherford [1965]. Further introductory material relating lattices to sorted logics, and their general application in automated reasoning can be found in Cohn [1987,1989].

2.3: The alphabet and syntax of the general formal language.

The *expressions* of the general formal language are strings (of finite length) of symbols which are classified as follows:

i) a set of individual variable symbols typically denoted by the lower case letters from 'u' through to 'z' with or without numerical suffixes.

ii) a set of individual constant symbols [1] typically denoted by lower case letters from 'a' through to 't' with or without numerical suffixes.

iii) a set of n-place function symbols typically denoted by strings of lower case letters, e.g. 'sum', 'compl'. These include a set of n-place skolem function symbols with numerical suffixes, for which the letter 'f' is reserved: i.e. 'f1, f2, f3, ..., fn'.

iv) a set of n-place predicate symbols either denoted by strings of upper case letters, e.g. 'C', 'DC', 'POINT', W_INSIDE, or by strings of lower case letters prefixed by an upper case letter, e.g. 'Open', 'Atom'. In both cases, the strings may include an underscore symbol, e.g. W_INSIDE.

v) a set of Boolean connective symbols: '¬' (not), '∧' (and), '∨' (or), '→' (materially implies), '↔' (if and only if).

vi) the two quantifier symbols: '∀' (for all) and '∃' (for some). In addition two other related symbols are used: a metalinguistic descriptive operator '℩' (the unique) [2] and the E-shriek operator '∃!' (there is exactly one) [3].

vii) a set of punctuation markers: paired square brackets '[' and ']', open brackets '(' and ')' , and the comma ',' as a term separator.

viii) a set of additional metalinguistic symbols: '≡def.' (is defined to be equivalent to) and '=def.' (is defined to be identical to). The former symbol denotes a defined equivalence between well formed formulae, the latter between terms. In general, Greek letters are reserved for metavariables.

DEFINITION: *Terms* are defined recursively as follows:

i) an individual constant is a term.

ii) an individual variable is a term.

iii) if α is an n-place function symbol, and x1,...xn terms, then α(x1,...xn) is a term.

iv) no other expression is a term.

DEFINITION: *Atoms* (i.e. atomic formulae) are defined as follows: if Φ is an n-place predicate symbol, and x1,...,xn terms, then Φ(x1,...,xn) is an atom. No other expression is an atom.

DEFINITION: a variable α occuring in a formula φ is *bound* if it lies within the scope of a quantifier using that variable, i.e. if either $\forall\alpha[\varphi]$ or $\exists\alpha[\varphi]$, or it lies within the scope of the $\exists!$ symbol, i.e. $\exists!\alpha[\varphi]$; otherwise it is *free*.

DEFINITION: *well-formed formulae* (wffs) are defined recursively as follows:

i) an atom is a wff.

ii) If φ is a wff then $\neg\varphi$ is a wff.

ii) If φ and ψ are wff's, then $[\varphi \vee \psi]$, $[\varphi \mathbin{\&} \psi]$, $[\varphi \rightarrow \psi]$ and $[\varphi \leftrightarrow \psi]$ are wff's. As is standard practice, where no danger of ambiguity arises, the outermost pair of brackets of a wff may be dropped.

iii) If φ is a wff and α a free variable in φ, then $\forall\alpha[\varphi]$, $\exists\alpha[\varphi]$ and $\exists!\alpha[\varphi]$ are wff's.

DEFINITION: If φ is an atom then both φ and $\neg\varphi$ are *literals*. φ is a *positive literal* and $\neg\varphi$ a *negative literal*.

DEFINITION: a *clause* is a finite disjunction of literals: the *null clause* is a disjunction of zero literals, a *unit clause* a disjunction of one literal.

2.4: A brief introduction to sorted logics and the logic LLAMA

In an unsorted or one-sorted logic, the universe or domain of discourse ranges over a single set of homogeneous entities. Further partitioning of this set is done by introducing a set of monadic predicate symbols into the formal language that are used to denote specific homogeneous subsets of domain. Further information about the relationship between these subsets, e.g. whether they are disjoint, or overlap, or whether one is a subset of the other, is then expressed in the logic by incorporating the predicate symbols into a set of axioms which define the theory.

Unlike an unsorted logic, a sorted or many-sorted logic takes as its starting point a universe of discourse that ranges over a heterogeneous rather than a homogeneous set of entities. The homogeneous subsets of this set are called sorts. Sorted logics differ from their unsorted counterparts by explicitly representing this and other sortal information embedded in the formalised theory. In terms of the sorts, a set S of sort symbols are first of all specified. Each sort in the theory is then denoted by a unique sort symbol. In a simple sorted logic such as that used by Enderton [Enderton 1972] the set of sort symbols simply denote a set of pairwise disjoint sorts, but in other sorted logics commonly used in AI, additional structure embedded in S also allows sorts to overlap or include one another. The usual technique is to add a binary subsort symbol '$\sqsubseteq$' to the formal language that imposes a partial ordering on pairs of sort symbols. The interpretation of $\sqsubseteq$ is taken to be set inclusion. Given the subsort relation is defined so that one sort can be a subsort of itself, an additional binary symbol '$\sqsubset$' is added, where $\tau1 \sqsubseteq \tau2$ and $\tau1 \neq \tau2$ and where $\sqsubset$ is interpreted as proper inclusion.

In the sorted logic LLAMA, the sort structure takes the form of a complete Boolean lattice $L_{\bullet}$. In addition to the binary subsort symbol, a further set of binary lattice theoretic operators are explicitly represented in the formal language; these are the least upper bound (lub), greatest lower bound (glb) and complementation operators which are denoted by the symbols '$\sqcup$','$\sqcap$' and '\' respectively. Two other symbols: 'T' (top) and '$\perp$' (bottom) complete the set of lattice theoretic operators; the sort T is that sort of which every sort is a subsort and the sort $\perp$ that sort which is a subsort of of every sort. The set theoretic interpretation of S identifies T with the universe of discourse, $\perp$ with the empty set, $\sqsubseteq$ with set inclusion, $\sqcup$ as set union, $\sqcap$ as set intersection, and \ as set negation (or relative complement). Any expression that is of sort $\perp$ is interpreted as "nonsense" and is classified as ill-sorted in the logic.

Although LLAMA's sort structure is a complete Boolean lattice, typically only a few of the nodes will be occupied by explicitly named sort symbols declared by the user. The remaining nodes of the lattice are named implicitly, and are constructed and maintained internally by LLAMA's sort algorithm using the lattice theoretic operators on combinations of the named nodes. In practise, all the user of the logic needs to do, is to identify the set of pairwise mutually exclusive set of base sort nodes that provide a cover for $\perp$, and indicate where in the sort hierarchy the other named sort symbols are to be found.

In addition to LLAMA's sort lattice $L_\bullet$ which encodes the subsort relationships between the non-logical sort symbols of the theory, another special Boolean sort lattice $L_\bullet$ is also used. The $L_\bullet$ lattice has as its elements the four sort symbols UU (top), TT, FF and EE (bottom). The interpretations of these sort symbols are fixed as "either true or false", "true", "false" and "nonsensical" respectively.

LLAMA uses a set of sorting functions which are defined on the set of constants, functions and predicates supported by the theory to separate out well-sorted and ill-sorted terms and formulae. The well-sorted expressions of the logic are interpreted as meaningful and the ill-sorted expressions as supporting no sense, or meaningless. The declarations are set up as follows:

The sort/type notation of LLAMA

Metavariables for sort symbols are denoted here and throughout this thesis by the set of symbols $\{\tau 1, \ldots, \tau n, \tau n{+}1\}$.

sort $\tau 1 \sqsubset \tau 2$ means sort $\tau 1$ is a (strict) subsort of $\tau 2$. The sorting functions of LLAMA already referred to are declared by means of *type* declarations. Thus, **type** $\alpha{:}\tau$ means constant symbol α is well-sorted and of sort τ, **type** $\alpha(\tau 1, \ldots, \tau n){:}\tau n{+}1$ means function symbol α is well-sorted when its argument sorts are $\tau 1, \ldots, \tau n$ with $\tau n{+}1$ as the result sort, and **type** $\alpha(\tau 1, \ldots, \tau n){:}\tau n{+}1$ means predicate symbol α is well-sorted when defined on argument sorts $\tau 1, \ldots, \tau n$, and where $\tau n{+}1$ is any element (except EE) of the special sort lattice $L_\bullet$, i.e. UU, TT or FF.

LLAMA differs from most sorted logics by having the quantifiers unsorted. The main reason for this is an increase in the expressiveness of the logic by allowing functions and predicates to range over different combinations of argument sorts. A simple sorted logic supporting restricted quantification (e.g. Enderton, 1972) requires each variable to be associated with a unique sort, and consequently disallows any function or predicate to range over several distinct argument sorts. With LLAMA sortal restrictions on variables are derived implictly from the argument positions of functions and predicates they occur in. Each non-logical symbol is accompanied with a sorting function which describes how the result sort varies with the given argument sorts. This facility allows ad hoc *polymorphic* functions and predicates to be handled by the logic, i.e. allowing more than one argument sort declaration to be made for function and predicate symbols arising within a given formalism. This formal feature is in keeping with the manner in which nouns and verbs acquire different meanings in natural languages (usually separated out by context) and enables compact expressions to be made in the formal language.

A set of *sort environments* is associated with every wff which specifies the combinations of sorts on variables for which the wff is well sorted. These are calculated using the sorting functions of the consituent non logical symbols.

LLAMA also allows the sort of a given term to be more general than the sort of the argument position where it occurs. This means wffs such as O(prod(a,a),a) (in the theory to be described) remain well sorted even though the sort of the term prod(a,a) is more general (or higher in the sort lattice) than that declared by the sorting function for the predicate O. This is called *overlapping* [Cohn, 1983,1987].

Each named sort symbol has a unary predicate assigned to it called a sort predicate (and by Cohn, a "characteristic predicate"); literals formed from these predicates are correspondingly called sort literals (Cohn's "characteristic literals"). The name of each sort symbol is used in the corresponding sort literal: e.g. the sort symbol 'REGION' appears in the sort literal REGION(x).

With respect to theorem proving in LLAMA, the explicit use of the Boolean sort lattice allows the detection and deletion of some sets of formulae without invoking general inference rules. Clauses with a sort environment evaluated as EE (illsorted) are ignored by the deductive machinery since they cannot support any interpretation in the domain, and are subsequently deleted in the proof run. Similarly, clauses with an environment evaluated as TT ("true") forces the whole clause to be tautologous: the whole clause can be deleted in the proof run since it cannot lead to the desired refutation. Clauses supporting a sort environment evaluated as FF ("false") in the proof run indicates a desired contradiction. This follows because the variables in the logic are universally quantified. Thus the desired refutation can sometimes be found by virtue of the sortal information only. In general however this will not be found for any interesting theorem where the sort environment for most of the clauses in the refutation set is evaluated as UU ("either true or false"), and requires normal inference on clauses to detect the contradiction.

In general sorted logics derive their computational power over unsorted logics by reducing the search space generated using general purpose inference for an unsorted logic in several ways. The notion of well-sortedness partitions wff's of an unsorted logic into those which are ill-sorted (and hence eliminable) and those which are well-sorted to which general purpose inference can be applied. Secondly, sortal information is separated out and is assigned to be used by special purpose inference machinery which does not get processed by the more general purpose first order rules of inference. In practice, the simple expedient of partitioning a theory into the sort theory and that which contains more general formule is more likely to make inference in a theory more efficient (see e.g. Abrams and Frisch 1991). Finally, sorted logics gain in terms of efficiency over unsorted logics, by exploiting sort information to allow partial functions to appear in the representational language used.

2.5: Pedagogic conventions

The following conventions have been adopted to assist the reader when reading the formalism. Sortal declarations for new constants, functions and predicates in a definition or axiom, will immediately follow the definition. Since the formalism includes a large number of potential sorts, these will be gradually introduced as the formalism is developed.

In general, an indication of the range of sorts associated with terms embedded in axioms, theorems or lemmas cited in the text are made explicit. For this, the notation $x1,\ldots xn{:}\tau$ is used, meaning terms $x1, \ldots xn$ are of sort τ. For example, in the following theorem:

(T5) $\forall xyz[[P(x,y) \wedge P(y,z)] \rightarrow P(x,z)]$
$x,y,z{:}\tau$, where $\tau \in \{SPATIAL, PERIOD\}$

the variables x,y and z can be either of sort SPATIAL or PERIOD. (Strictly speaking, of course it is generally incorrect to talk of the sorts of variables, since LLAMA does not sort variables explicitly. A literal may be well-sorted for different combinations of sorts mapped to the literals' individual component terms.) In the case of syntactically complex theorems, or where paraphrasing better captures the intended meaning, an informal reading is also given, e.g.

$\forall xyz[C(x,y,z) \leftrightarrow \forall u[P(u,z) \rightarrow C(x|u,y|u)]]$

x,y:PHYSOB, z:PERIOD, u:MOMENT, x|u,y|u:SPATIAL

(in words: Physical objects x and y connect at or throughout period z, iff the spaces they occupy connect for every moment of z.)

In the interest of brevity, where axioms and theorems are naturally grouped together in the text, and where the range of embedded sorts associated with terms are invariant, these are declared globally and immediately follow the list given, e.g.

(T29) $\forall xy[\neg EC(x,y) \leftrightarrow [C(x,y) \leftrightarrow O(x,y)]]$
(T30) $\forall xy[\neg\exists z[EC(z,x) \rightarrow [P(x,y) \leftrightarrow \forall u[O(u,x) \rightarrow O(u,y)]]]]$
(T31) $\forall x[NTP(x,x) \leftrightarrow \neg\exists y[EC(y,x)]]$
(T32) $\forall xyz[[NTP(x,y) \wedge C(z,x)] \rightarrow O(z,y)]$

$x,y,z,u{:}\tau,\ \tau \in \{SPATIAL, PERIOD\}$

The formalism provides a formal distinction between monadic predicate symbols that are sort symbols, and those monadic predicate symbols that are not. A subset of the sort symbols embedded in $L_{\bullet}$ which lie immediately above $\perp$, are the *base sorts* of the formalised theory. These base sorts correspond to a set of monadic predicates in the theory whose extensions are treated as pairwise disjoint sets. In contrast the *primitive sorts* of the theory correspond to the set of sort symbols which lie immediately below T. Sort predicate symbols are distinguished by the use of strings composed (with the possible addition of underscore symbols) entirely of upper case letters, e.g. 'REGION', and 'POINT' used in the sort literals REGION(x) and POINT(x). In contrast, monadic predicate symbols which are not sort symbols are strings composed of lower case letters prefixed by a single upper case letter (again with the possible addition of underscore symbols), e.g. 'Open' and 'Atom' in the literals Open(x) and

Atom(x). Normally only the non-sort monadic predicate symbols are explicitly represented in wff's, but sort literals also appear in defining axioms, e.g.

(A7) $\forall xy[\mathrm{NULL}(\mathrm{prod}(x,y)) \leftrightarrow \mathrm{DR}(x,y)]$
$x,y{:}\tau,\ \tau \in \{\mathrm{SPATIAL},\mathrm{PERIOD}\}$,
$\mathrm{prod}(x,y){:}\tau \sqcup \mathrm{NULL},\ \tau \in \{\mathrm{SPATIAL},\mathrm{PERIOD}\}$

Normally, monadic predicates arising in an unsorted axiomatisation are treated as sort predicates in LLAMA, and are thereby 'absorbed' into the sortal machinery. However, there are several reasons why only some monadic predicates are treated as sort predicates in this formalism. Presently formulated LLAMA requires a complete Boolean lattice encoding the sortal relationships between the sorts embedded in a formal theory. This means any translation of an unsorted theory to its sorted counterpart can only be done when the sort relationships have been hitherto established. The formalism supports many monadic predicates, over 20 of which are specialisations of the sort REGION alone. However, the task of extracting all the potential sorts and establishing their mutual relationships in a complete sort lattice that could be supported by the formalism is a non-trivial task, and has not been done. This is discussed further in Chapter 5. However, it should be pointed out that even given complete knowledge of the sort lattice, a large number of potential sorts supported by the formalism would require long listings of the sort declarations for the functions and predicates used. This alone would detract from the general readability of the formalism. In view of these main points, a minimal sort lattice using 13 named sorts (of which 9 are base sorts) is actually used to describe the general theory. As a final comment, the reader may be wondering why, given these difficulties, the formal theory has not been expressed as an unsorted theory. There are three main reasons. The first is that the set of defining axioms become significantly longer, and in many cases, simply prove very difficult to scan and read. The second is that listed proofs become significantly longer with much of the proof simply serving to restrict the sorts before the interesting part of the theorem is addressed. Again, this is discussed further in section 5. Finally, by relaxing the logic to an unsorted one, the standard difficulty met by incorporating improper or partial functions into the theory reappears.

Numbered definitions, axioms, theorems/lemmas and conjectured theorems are respectively indicated with the prefixes '(D...)', '(A...)', '(T...)' and '(C...)'. Proofs of all the theorems are assembled in appendices B and C.

2.6: The (minimal) sort lattice L_s

The *primitive* sorts of the theory cover sets of null objects, regions, points, physical objects (or bodies) and numbers; these are denoted by the sort symbols '*NULL*', '*REGION*', '*POINT*', '*PHYSOB*' and '*NUMBER*' respectively. As ontological primitives of the theory, no sort is taken to be reducible to another. This is reflected in the relative position of the primitive sort symbols in the sort lattice where the corresponding sort symbols are pairwise disjoint and immediately below T. Apart from regions (perhaps) and null objects, no explanation of the intended meaning of these named sorts need be given. Regions are simply viewed as either the spaces that could be conceivably be occupied by a physical body (being a region of space), or durations of time over which some conceivable state of affairs or an event could obtain or occur. The sort NULL is added for convenience and simply appears either to allow arbitrary Boolean combinations of regions to be expressed as functions in the formalism, in particular where two regions do not overlap and have no region as their intersection, or where physical objects pass out of existence.

I shall start the analysis by concentrating upon the sort REGION. These regions may be thought to be potentially infinite in number and capable of any degree of overlap (or mutual penetration) with other regions. Depending upon the general ontology selected, regions can either be spatial: denoted by the sort symbol 'SPATIAL' or temporal, denoted by the sort symbol 'PERIOD'. The sorts SPATIAL and PERIOD are disjoint. Informally, each spatial and temporal region coincides with a set of points and is contained in one of two special regions called the spatial universe (denoted by the constant u_s, of sort SPATIAL_UNIVERSE) and period universe (denoted by the constant u_T, of sort PERIOD_UNIVERSE). Spatial regions which are not identical with u_s are assigned to the sort SPATIAL\SPATIAL_UNIVERSE. Periods are split into moments (the sort MOMENT) and intervals (the sort INTERVAL). Intervals are further divided into the period universe (described above) and those intervals that are not the period universe: the sort INTERVAL\PERIOD_UNIVERSE. The sort hierarchy described here (and declared immediately below) is illustrated in Figure 1.

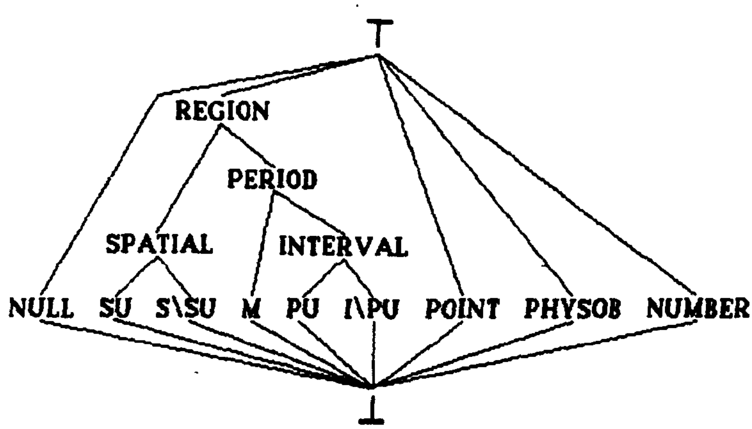
sort NULL $\sqsubset$ T
sort REGION $\sqsubset$ T
sort POINT $\sqsubset$ T
sort PHYSOB $\sqsubset$ T
sort NUMBER $\sqsubset$ T
sort SPATIAL $\sqsubset$ REGION
sort PERIOD $\sqsubset$ REGION
sort SPATIAL_UNIVERSE $\sqsubset$ SPATIAL
sort [SPATIAL\SPATIAL_UNIVERSE] $\sqsubset$ SPATIAL
sort MOMENT $\sqsubset$ PERIOD
sort INTERVAL $\sqsubset$ PERIOD

sort PERIOD_UNIVERSE $\sqsubset$ INTERVAL
sort [INTERVAL\PERIOD_UNIVERSE] $\sqsubset$ INTERVAL

For reasons of brevity, disjointness between sorts, e.g. **sort** [REGION $\sqcap$ POINT] = $\perp$, is not declared but is implicitly assumed by default unless otherwise inferrable. The same principle applies to functions and predicates e.g. **type** sum(POINT,POINT):$\perp$, and **type** C(POINT,REGION):EE, where ill-sortedness is not explicitly stated.

The sorting functions for the sort predicates are globally defined as follows:

type $\tau(\tau)$:TT
type $\tau(T\backslash\tau)$:FF
e.g. **type** REGION(REGION):TT
type REGION(T\REGION):FF



Key: SU	abbreviates	SPATIAL_UNIVERSE
S\SU	"	SPATIAL\SPATIAL_UNIVERSE
M	"	MOMENT
PU	"	PERIOD_UNIVERSE
I\PU	"	INTERVAL\PERIOD_UNIVERSE

Figure 1: The sort lattice L_s defining the positions of the sort symbols in the sort hierarchy used in the text.

2.7: The mereological relations

The word "mereological" used above, comes from a Greek root meaning *part*. The theory known as mereology [Lesniewski 1927-1931] reformulated as The Calculus of Individuals [Leonard and Goodman 1940] makes explicit use of the part whole relation. In Clarke's theory, a much weaker relation of *being connected with* is used from which the relation of part to whole is defined. However, Clarke still uses the term mereological when discussing these relations. I also follow this convention.

In order to help guide the desired intuition needed to understand this formalism, I follow Clarke's example [Clarke 1981, p.205] by providing intuitive interpretations for a sufficient number of relations given below. Clarke [1981] suggests that his basic variables be construed as ranging over spatio-temporal regions and any points deemed to coincide with a region to be spatio-temporal points. However I depart slightly from this by separating out that part of the formalism that can be given either a spatial or temporal interpretation. In most cases context will indicate the nature of the ontology being assumed, but unless indicated otherwise the reader is advised to read these relations in the light of a purely spatial reading.

Two primitive relations are introduced: 'C(x,y)' read as 'x connects with y' and 'B(x,y)' read as 'x is (temporally) before y'. In terms of points incident in regions, C(x,y) holds when two regions connect; of the incident points contained in both regions, at least one point is shared. Similarly B(x,y) holds between two regions when one region is (temporally) before the other and no incident point is shared [*].

A set of axioms governing the meaning of these relations is given below:

(A1) $\forall x\ C(x,x)$
(A2) $\forall xy[C(x,y) \rightarrow C(y,x)]$
(A3) $\forall xy[\forall z[C(z,x) \leftrightarrow C(z,y)] \rightarrow EQUAL(x,y)]$
(A4) $\forall x\ \neg B(x,x)$
(A5) $\forall xyz[[B(x,y) \wedge B(y,z)] \rightarrow B(x,z)]$
(A6) $\forall xy[B(x,y) \rightarrow [\forall zu[P(z,x) \wedge P(u,y)] \rightarrow B(z,u)]]$

type C(SPATIAL,SPATIAL):UU [*]
type C(PERIOD,PERIOD):UU
type B(PERIOD,PERIOD):UU

C(x,y) is totally reflexive and symmetrical, B(x,y) irreflexive, transitive (and by implication asymmetrical). The relations P(x,y) and

EQUAL(x,y) which are defined in terms of the primitive relation C are dealt with below.

C(x,y) covers all cases of connection between regions from external contact ('touching') to all instances of mutual penetration including mutual total overlap or identity. Figure 2 illustrates the intended meaning of C(x,y) with pairs of (topologically) closed regions that satisfy the relation.

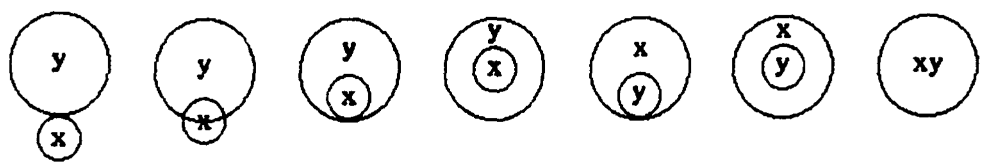


Figure 2: Pairs of connected spatial regions.

A basic set of mereological relations are defined and interpreted as follows: 'DC(x,y)' is read as 'x is disconnected from y','P(x,y)' as 'x is a part of y', 'EQUAL(x,y)' as 'x is identical with y', 'PP(x,y)' as 'x is a proper part of y', 'O(x,y)' as 'x overlaps y' and 'DR(x,y)' as 'x is discrete from y':

(D1) $DC(x,y) \equiv_{def.} \neg C(x,y)$

(D2) $P(x,y) \equiv_{def.} \forall z[C(z,x) \rightarrow C(z,y)]$

(D3) $EQUAL(x,y) \equiv_{def.} P(x,y) \wedge P(y,x)$

(D4) $PP(x,y) \equiv_{def.} P(x,y) \wedge \neg P(y,x)$

(D5) $O(x,y) \equiv_{def.} \exists z[P(z,x) \wedge P(z,y)]$

(D6) $DR(x,y) \equiv_{def.} \neg O(x,y)$

(D7) $PO(x,y) \equiv_{def.} O(x,y) \wedge \neg P(x,y) \wedge \neg P(y,x)$

type $\Phi(\tau,\tau)$:UU, where $\tau \in$ {SPATIAL,PERIOD} and $\Phi \in$ {DC,P,EQUAL,PP,O,DR,PO}.

Excepting the equality relation: EQUAL(x,y), the sortal declarations for the relations DC, P, PP, O, DR and PO defined above are identical to those declared for C. It should be pointed out that although the equality relation is defined on regions here, EQUAL is a polymorphic predicate: the sortal declarations being as follows:

type EQUAL(τ,τ):UU

type EQUAL$(\tau1,\tau2)$:FF, where $\tau1,\tau2 \in S$ and $\tau1 \sqcap \tau2 \neq \perp$

DC(x,y) is understood to mean that x and y share no incident point in common, P(x,y) when all the points incident in x are incident in y, EQUAL(x,y) when x and y share identical points, and PP(x,y) when all the points incident in x are incident in y, but not vice versa, O(x,y) when x and y share a common interior point, DR(x,y) when either x and y share no incident point in common or share a point in common but share no interior points (i.e. when x and y share only boundary points in common), and PO(x,y) when x and y share a common interior point, but not that every point incident in x is incident in y (and vice versa).

The axioms imply that DC(x,y) is irreflexive (T1) and symmetric (T2):

(T1) $\forall x\ \neg DC(x,x)$

(T2) $\forall xy[DC(x,y) \rightarrow DC(y,x)]$

$x,y{:}\tau,\ \tau \in \{SPATIAL, PERIOD\}$

P(x,y) is totally reflexive (T3), antisymmetric (T4) and transitive (T5):

(T3) $\forall x\ P(x,x)$

(T4) $\forall x[[P(x,y) \wedge P(y,x)] \rightarrow EQUAL(x,y)]$

(T5) $\forall xyz[[P(x,y) \wedge P(y,z)] \rightarrow P(x,z)]$,

$x,y{:}\tau,\ \tau \in \{SPATIAL, PERIOD\}$

EQUAL(x,y) totally reflexive (T6), symmetrical (T7) and transitive (T8):

(T6) $\forall x\ EQUAL(x,x)$

(T7) $\forall xy[EQUAL(x,y) \rightarrow EQUAL(y,x)]$

$$(T8)\ \forall xyz[\mathrm{EQUAL}(x,y) \wedge \mathrm{EQUAL}(y,z)] \rightarrow \mathrm{EQUAL}(x,z)],$$
$$x,y{:}\tau,\ \tau \in S$$

PP(x,y) irreflexive (T9), asymmetrical (T10) and transitive (T11):

$$(T9)\ \forall x\ \neg PP(x,x)$$
$$(T10)\ \forall xy[PP(x,y) \rightarrow \neg PP(y,x)]$$
$$(T11)\ \forall xyz[[PP(x,y) \wedge PP(y,z)] \rightarrow PP(x,z)],$$
$$x,y{:}\tau,\ \tau \in \{\mathrm{SPATIAL,PERIOD}\}$$

O(x,y) totally reflexive (T12) and symmetrical (T13):

$$(T12)\ \forall x\ O(x,x)$$
$$(T13)\ \forall xy[O(x,y) \rightarrow O(y,x)],$$
$$x,y{:}\tau,\ \tau \in \{\mathrm{SPATIAL,PERIOD}\}$$

DR(x,y) irreflexive (T14) and symmetrical (T15):

$$(T14)\ \forall x\ \neg DR(x,x)$$
$$(T15)\ \forall xy[DR(x,y) \rightarrow DR(y,x)],$$
$$x,y{:}\tau,\ \tau \in \{\mathrm{SPATIAL,PERIOD}\}$$

and PO(x,y) irreflexive (T16) and symmetrical (T17):

$$(T16)\ \forall x\ \neg PO(x,x)$$
$$(T17)\ \forall xy[PO(x,y) \rightarrow PO(y,x)].$$
$$x,y{:}\tau,\ \tau \in \{\mathrm{SPATIAL,PERIOD}\}$$

Note is drawn to the fact that DC(x,y) implies DR(x,y) but not vice-versa: two regions may be discrete yet can be disconnected or connected at their boundaries only. It is also worth emphasizing here that by overlap (and by implication, connection) we are not capturing the physical relation of covering, neither in the case of surface contact between objects, or optically as in the case when one object occludes another. The intended meaning of overlap is one of varying degrees of mutual penetration between regions [*]. Similarly, care is needed with the intended meaning given to the part/whole relation for some nuances of 'part' do not coincide with the meaning of part as captured in the formalism. In the case of an amoeba that engulfs a food particle, for that food to be part of the amoeba as dictated by the formalism, that food must assume the same relationship to the amoeba as the amoebal nucleus does to the whole [7]. If on the other hand by 'part' one construes this to mean containment, then additional formal machinery is required to capture this relation (covered in section 2.14). The important point being made here is that by 'part' I do not mean the latter notion.

The distinction Clarke draws between connecting and overlapping regions enables a set of relations to be defined that are not commonly associated with calculi of individuals, e.g. Eberle (1970). 'EC(x,y)' is read as 'x is externally connected with y', 'TP(x,y)' as 'x is a tangential part of y', NTP(x,y)' as 'x is a nontangential part of y', TPP(x,y) read as 'x is a tangential proper part of y' and 'NTPP(x,y)' read as 'x is a nontangential proper part of y':

(D8) $EC(x,y) \equiv_{def.} C(x,y) \wedge \neg O(x,y)$

(D9) $TP(x,y) \equiv_{def.} P(x,y) \wedge \exists z[EC(z,x) \wedge EC(z,y)]$

(D10) $NTP(x,y) \equiv_{def.} P(x,y) \wedge \neg\exists z[EC(z,x) \wedge EC(z,y)]$

(D11) $TPP(x,y) \equiv_{def.} TP(x,y) \wedge \neg P(y,x)$

(D12) $NTPP(x,y) \equiv_{def.} NTP(x,y) \wedge \neg P(y,x)$

type $\Phi(\tau,\tau)$:UU, where $\tau \in \{SPATIAL, PERIOD\}$ and $\Phi \in \{EC,TP,NTP,TPP,NTPP\}$

(Again, the sortal declarations for EC, TP, NTP, TPP and NTPP are identical to that declared for C.) EC(x,y) is understood to mean that when x and y share a point in common, they do not share any interior points, TP(x,y) when all the points incident in x are incident in y and some other region z exists such that x, y and z share a point in common but share no interior points in common, and NTP(x,y) when all the points incident in x are incident in y and no region z exists sharing a common boundary point with both x and y. The intuitive semantics for the rest of the mereological relations is dispensed with at this point owing to the linguistic demand made on the reader (and author!).

The following theorems arise: EC(x,y) is irreflexive (T18) and symmetrical (T19):

(T18) $\forall x\ \neg EC(x,x)$

(T19) $\forall xy[EC(x,y) \rightarrow EC(y,x)]$,

$x,y{:}\tau,\ \tau \in \{SPATIAL,PERIOD\}$

TP(x,y) weakly reflexive [*] (T20) and antisymmetric (T21):

(T20) $\forall xy[TP(x,y) \rightarrow TP(x,x)]$

(T21) $\forall xy[[TP(x,y) \wedge TP(y,x)] \rightarrow EQUAL(x,y)]$,

$x,y{:}\tau,\ \tau \in \{SPATIAL,PERIOD\}$

NTP(x,y) is antisymmetric (T22) and transitive (T23):

(T22) $\forall xy[NTP(x,y) \wedge NTP(y,x)] \rightarrow EQUAL(x,y)]$

(T23) $\forall xyz[[NTP(x,y) \wedge NTP(y,z)] \rightarrow NTP(x,z)]$,

$x,y,z,{:}\tau,\ \tau \in \{SPATIAL,PERIOD\}$

TPP(x,y) irreflexive (T24) and asymmetrical (T25):

(T24) $\forall x\ \neg TPP(x,x)$

(T25) $\forall xy[TPP(x,y) \rightarrow \neg TPP(y,x)]$,

$x,y{:}\tau,\ \tau \in \{SPATIAL,PERIOD\}$

and NTPP(x,y) irreflexive (T26), asymmetrical (T27) and transitive (T28):

(T26) $\forall x\ \neg NTPP(x,x)$

(T27) $\forall xy[NTPP(x,y) \rightarrow \neg NTPP(y,x)]$

(T28) $\forall xyz[[NTPP(x,y) \wedge NTPP(y,z)] \rightarrow NTPP(x,z)]$

$x,y,z{:}\tau,\ \tau \in \{SPATIAL,PERIOD\}$.

A substantial list of *stipulated* theorems involving most of the defined relations defined above can be found in Clarke [1981], although Clarke does not concentrate upon the formal properties of his defined relations as is done here. The exceptions in the set defined so far are the relations PO, TPP, and NTPP which are new. Of the theorems given by Clarke, a few important ones are given immediately below and are briefly discussed immediately following:

(T29) $\forall xy[\neg EC(x,y) \leftrightarrow [C(x,y) \leftrightarrow O(x,y)]]$
(T30) $\forall xy[\neg\exists z[EC(z,x) \rightarrow [P(x,y) \leftrightarrow \forall u[O(u,x) \rightarrow O(u,y)]]]]$
(T31) $\forall x[NTP(x,x) \leftrightarrow \neg\exists y[EC(y,x)]]$
(T32) $\forall xyz[[NTP(x,y) \wedge C(z,x)] \rightarrow O(z,y)]$
$x,y,z,u{:}\tau,\ \tau \in \{SPATIAL,PERIOD\}$

Theorems (T29) and (T30) are singled out by Clarke since they show the relationship between his and the classical calculus of individuals of Leonard and Goodman [1940]. With the absence of external connectedness in the domain Clarke's calculus collapses to the classical one; the relations C and O become indistinguishable as do DC and DR, P and NTP, PP and NTPP, and EQUAL and NTPI (defined below) [*]. Given the topological interpretation, the regions become open, which means that connection between regions implies the regions overlap - i.e. if a point is shared in common, a region is also shared in common. Theorem (T31) often proves puzzling at the first reading, but once it is understood that any region that is a nontangential part of itself is an open region, it becomes apparent that boundary connection with that region cannot be made. Finally with theorem (T32), once we recognise that a nontangential part of a region is part of the interior of that region, again connection of a region with part of the interior of a region implies regional overlapping.

A set of configurations satisfying a subset of the defined relations (together with the relation TPI(x,y) defined below) is given in Figure 3. For reasons of clarity the regions depicted include their boundaries, although the formalism supports both open and closed regions. An additional assumption adopted here is that each paired set of regions are deemed to be embedded in another region that acts as the externally connecting region z in order that the tangential relations be satisfied. The existence of this region is ensured by a closure operator (defined in section 2.9) and axiom (A8) $\forall x\ EC(cl(x),cl(compl(x)))$ - described in section 2.8 and 2.9.

Eight additional relations are added: 'TPI(x,y)' read as 'x is the identity tangential part of y', 'NTPI(x,y)' as 'x is the identity nontangential part of y'. Every nonsymmetrical mereological relation has an inverse: $P^{-1}(x,y)$, $PP^{-1}(x,y)$, $TP^{-1}(x,y)$, $NTP^{-1}(x,y)$, $TPP^{-1}(x,y)$ and $NTPP^{-1}(x,y)$. The inverse relations are named using a standard notation, but it is worthwhile pointing out here that they also admit intuitive names e.g. $P^{-1}(x,y)$ could equally be characterised as 'E(y,x)' read as 'y extends over x' [10].

(D13) $TPI(x,y) \equiv def.\ TP(x,y) \wedge P(y,x)$
(D14) $NTPI(x,y) \equiv def.\ NTP(x,y) \wedge P(y,x)$
(D15) $P^{-1}(x,y) \equiv def.\ P(y,x)$
(D16) $PP^{-1}(x,y) \equiv def.\ PP(y,x)$
(D17) $TP^{-1}(x,y) \equiv def.\ TP(y,x)$
(D18) $NTP^{-1}(x,y) \equiv def.\ NTP(y,x)$
(D19) $TPP^{-1}(x,y) \equiv def.\ TPP(y,x)$
(D20) $NTPP^{-1}(x,y) \equiv def.\ NTPP(y,x)$

type $\Phi(\tau,\tau)$:UU, where $\tau \in$ (SPATIAL,PERIOD) and $\Phi \in \{TPI, NTPI, P^{-1}, PP^{-1}, TP^{-1}, NTP^{-1}, TPP^{-1}, NTPP^{-1}\}$

Relations TPI(x,y) and NTPI(x,y) are weakly reflexive (T55) (T56), symmetrical (T57) (T58) and transitive (T59) (T60):

(T33) $\forall xy[TPI(x,y) \rightarrow TPI(x,x)]$
(T34) $\forall xy[NTPI(x,y) \rightarrow NTPI(x,x)]$
(T35) $\forall xy[TPI(x,y) \rightarrow TPI(y,x)]$
(T36) $\forall xy[NTPI(x,y) \rightarrow NTPI(y,x)]$

(T37) $\forall xyz[[TPI(x,y) \wedge TPI(y,z)] \rightarrow TPI(x,z)]$

(T38) $\forall xyz[[NTPI(x,y) \wedge NTPI(y,z)] \rightarrow NTPI(x,z)]$.

$x,y,z{:}\tau,\ \tau \in \{SPATIAL, PERIOD\}$

Figure 3 illustrates how the above set of relations can be embedded into a lattice, which is named L_C. The weakest and most general relations are directly linked to T and the strongest to $\perp$ which are interpreted as tautology and contradiction respectively. Theorems that define the structure in lattice L_C are given in appendix C. A virtue of this calculus is that intuitive names for many relations are relatively easy to find. The underlying significance of this point in relation to questions of cognitive validity of this approach to Naive Physics is dealt with in Chapter 6.

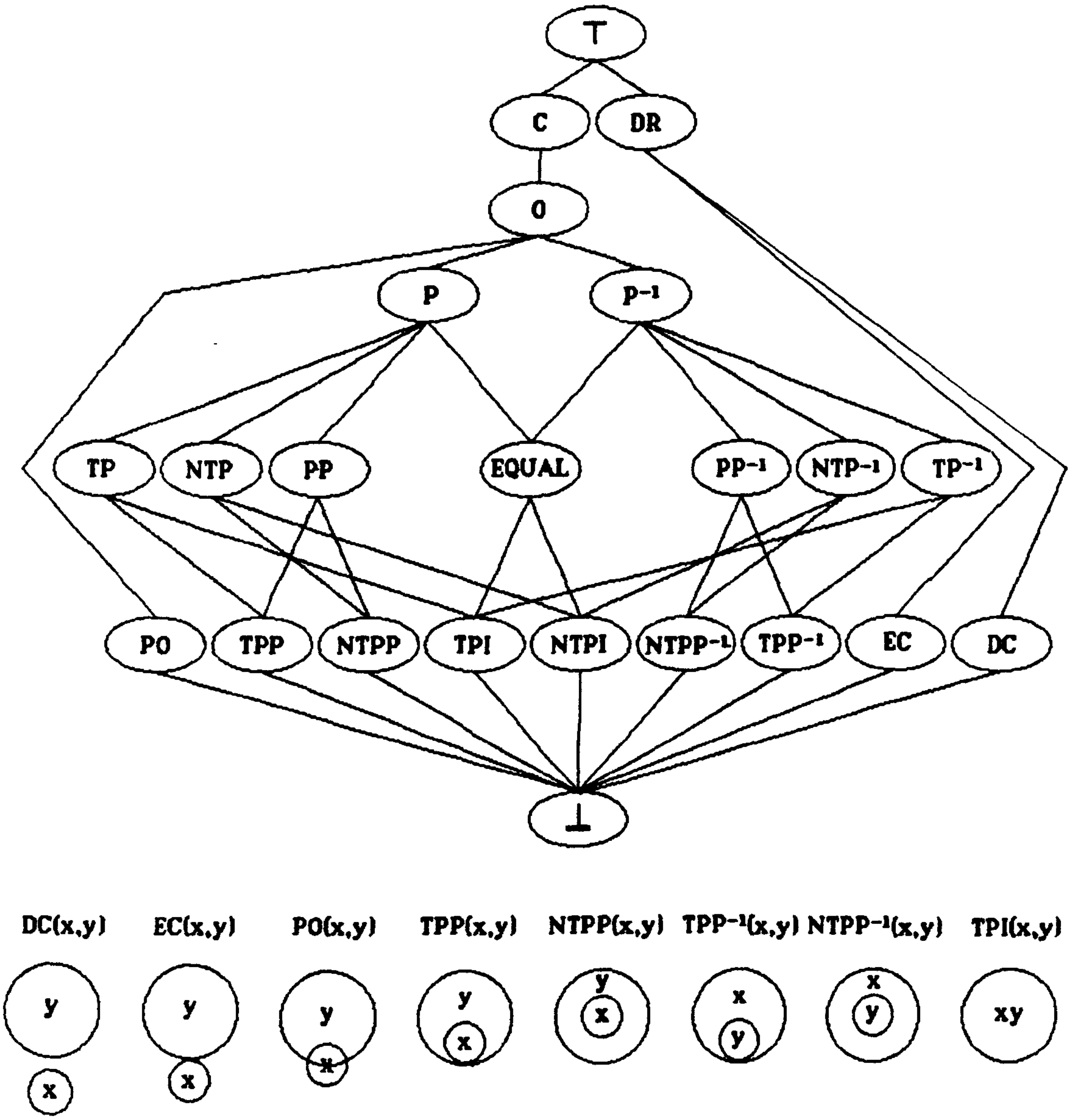


Figure 3: The relational lattice L_C defining the relative positions of the set of dyadic relations defined solely in terms of the primitive relation C(x,y). The set of configurations show pairs of closed regions that (excepting the relation NTPI(x,,y)) satisfy the set of base relations that lie immediately above ⊥. The relation NTPI(x,y) is satisfied when x and y are open regions.

2.8 The Boolean part

It has been pointed out by Leonard and Goodman [1940], Tarski [1956] and others that the linguistic domain of a classical calculus of individuals can be characterised as a Boolean algebra with the null element removed [11]. Clarke's calculus follows this pattern excepting that the distinction made between the relations C and O (and subsequent introduction of the relations EC, *TP and NTP missing in the classical calculus*) suggests that the linguistic domain of his calculus is a closure algebra with the null element and boundary elements removed [Clarke 1981 p.216]. For this reason, Clarke refers to the set of Boolean and topological operators outlined in his calculus as "quasi-Boolean" and "quasi-topological" respectively.

Unlike Clarke, the sort NULL is introduced so that all the Boolean functions in the sorted logic can be made total on regions. The decision to depart from the traditional position [12] and actually import a new sort into the domain functioning not unlike the null individual is justified as follows. In the first place although the explicit use of a sorted logic allows one to 'remove' some of the existential preconditions that arise in many of Clarke's theorems *without* introducing the sort NULL, this cannot be maintained for all the Boolean functions defined on regions. For example in Clarke's calculus [Clarke 1981 p210] (allowing for notational changes) we find the theorem:

$$\forall x[\exists y[\mathrm{EQUAL}(y,\mathrm{compl}(x))] \leftrightarrow \neg\mathrm{EQUAL}(y,a^*)]]$$

which states that the complement y of region x exists if and only if that region is not the universal region a*. By simply restricting the complement function so that it is well-sorted only when defined on regions that are not the universal region, the existential condition is not required. However, sortal restrictions alone cannot deal with the fact that, for example, disconnected regions have a null intersection. In Clarke's axiomatisation this restriction is met by treating intersection like complementation which requires an existential precondition to hold - in this case that the regions overlap. But this move complicates his proofs.

While it is possible to use a Russellian theory of descriptions [Russell 1905], to eliminate descriptive functions contextually in terms of relations, identity and quantifiers (thereby resolving the problem of non-existence for certain values of functions), this solution is not adopted. Any ontological gain using the theory of descriptions, must be offset against the fact that one's notation becomes correspondingly complex, and with it the related question of the computational cost incurred [13]. Instead, pure functor notation is chosen to represent descriptive functions. This is more compact and perspicuous than relational notation and is in keeping with the motivation to use a sorted logic and reduce the search space during mechanised inference. But the use of functor notation in classical treatments of first order logic requires the introduction of an object into the domain that acts as the null object to cope with improper functions [14]. I meet this requirement by the following strategy. Three sorts REGION, NULL and NULL $\sqcup$ REGION are first of all used as result sorts for the sorting functions declared for the set of improper Boolean functions; these depend upon whether the Boolean composition yields a region, no region, or possibly either respectively. (However, note that no function or predicate allows NULL as an argument sort, so that the sort NULL has a secondary role in relation to the sort REGION.) Although these sort symbols are disjoint, *overlapping* is used so that wffs with related improper functions as arguments become well sorted, e.g. the wff O(prod(a,a),a), as do the set of defining axioms that link the two sorts, in this case the axiom: (A7) $\forall xy[\mathrm{NULL}(\mathrm{prod}(x,y)) \leftrightarrow \mathrm{DR}(x,y)]$. This decision allows functor notation to be used, the ontological distinction between the sorts REGION and NULL to be preserved, yet allows a defining axiom for NULL in terms of regions. A similarity of this solution with Scott's [1967] analysis is worth noting; in Scott's case improper descriptions are given a value outside the domain; in this theory improper descriptions relating to regions are given a value outside the given sort domain. Ontological objections to a null object (or the sort null) still stand [15] (albeit to a lesser extent), but its use is motivated by pragmatic convenience.

Clarke introduces analogues of most of the standard operators characterised in a Boolean algebra: the universal region and the sum, complementation and intersection of regions. The universal region added as a single defined constant in Clarke's calculus (remembering that in Clarke's calculus the domain is over spatio-temporal regions) splits into two constants in this formalism: corresponding to the spatial and temporal universe respectively. A further difference arises with with the addition of the difference operator and the sort NULL. The function 'sum(x,y)' is read as 'the sum of x and y', 'compl(x)' as 'the complement of x', 'u_s' as 'the spatial universe', 'u_T' as 'the period universe', 'prod(x,y)' as 'the product (i.e. the intersection) of x and y', 'diff(x,y)' as 'the difference (or relative complement) between x and y'. The sort predicate 'NULL(x)' is read as 'x is null'. The Booleans are defined immediately following and then discussed.

(D21) $\mathrm{sum}(x,y) =_{\mathrm{def.}} \iota z[\forall u[C(u,z) \leftrightarrow [C(z,x) \vee C(z,y)]]]$
(D22) $\mathrm{compl}(x) =_{\mathrm{def.}} \iota y[\forall z[C(z,y) \leftrightarrow \neg P(z,x)]]$
(D23a) $u_s =_{\mathrm{def.}} \iota x[\forall y[C(y,x)]]$
(D23b) $u_T =_{\mathrm{def.}} \iota x[\forall y[C(y,x)]]$

(D24) $prod(x,y) =def. \iota z[\forall u[C(u,z) \leftrightarrow \exists v[P(v,x) \wedge P(v,y) \wedge C(u,v)]]]$ [16]

(D25) $diff(x,y) =def. \iota z[\forall u[C(u,z) \leftrightarrow C(u,prod(x,compl(y)))]]$ [16]

(D26) $SPATIAL\_UNIVERSE(x) \equiv def.\ EQUAL(x,u_s)$

(D27) $PERIOD\_UNIVERSE(x) \equiv def.\ EQUAL(x,u_T)$

type $sum(\tau,\tau):\tau,\ \tau \in \{SPATIAL,PERIOD\}$

type $compl(SPATIAL \backslash SPATIAL\_UNIVERSE):SPATIAL \backslash SPATIAL\_UNIVERSE$

type $compl(SPATIAL\_UNIVERSE):NULL$ [17]

type $u_s:SPATIAL\_UNIVERSE$

type $u_T:PERIOD\_UNIVERSE$

type $prod(\tau,\tau):\tau \sqcup NULL,\ \tau \in \{SPATIAL,PERIOD\}$

type $diff(\tau 1,\tau 2):\tau 2 \sqcup NULL,\ \tau 1 = SPATIAL,\ \tau 2 = SPATIAL \backslash SPATIAL\_UNIVERSE$

(A7) $\forall xy[NULL(prod(x,y)) \leftrightarrow DR(x,y)]$
$x,y:\tau,\ \tau \in \{SPATIAL,PERIOD\},$
$prod(x,y):\tau \sqcup NULL,\ \tau \in \{SPATIAL,PERIOD\}$

Any *region* returned by the complement operator compl(x) contains all the points incident in the universal region u_s not incident in x [18]. This informal characterisation is justified by the following theorem:

(T39) $\forall x[\neg C(compl(x),x)]$
$x,compl(x):SPATIAL \backslash SPATIAL\_UNIVERSE$

i.e. spatial region x and its complement have no incident points in common.

It may be thought (pace the discussion above) that a null region can equated with the null element in a Boolean algebra and that e.g. compl(x) be defined on both u_s and a new constant n_s (acting as the null spatial region), so that $EQUAL(compl(u_s),n_s)$. But given the definitions for the Boolean part of the formalism, this cannot be done without falling into an immediate contradiction: e.g. given u_s defined to be that spatial region that connects with every spatial region in the domain, and $compl(u_s)$ now returning the spatial region n_s, then $C(compl(u_s),u_s)$ follows from the definition of u_s. But by theorem (T39) (with the sortal restrictions suitably weakened) $\neg C(compl(u_s),u_s)$ equally follows - contradiction. This result not only provides a syntactic justification for declaring the sortal restrictions for complementation as given above and making the sorts REGION and NULL disjoint, it also justifies the intuitive semantics for the null region (were it to exist) having no incident point and hence *cannot connect* with any region.

It should be pointed out that the above problem cannot be eliminated with the removal of either u_s or u_T as defined constants. Within the formalism arbitrary names can be used to generate the contradition, e.g. the term sum(a,compl(a)) once admitted in the formalism (being well-sorted) leads to the contradiction. The term sum(a,compl(a)) is of course identical to u_s, hence the derived contradiction - thus compl(x) requires the restriction.

Definition (D24) given for prod(x,y) corrects that which appears in Randell and Cohn [1989b] and Clarke [1981] for which counterexamples have been found. Axiom (A7) now replaces the definition for Null(x) used in Randell and Cohn [1989a,b,c]. The definition for prod(x,y) (D24) and axiom (A7) linking the sort literal NULL(x) implies that intersecting regions must overlap, and that regions that do not overlap have a null product.

The characterisation of NULL(x) as a monadic predicate designating a class of objects rather than a unit class, is intentional. In the early development of this theory null was conceived as a constant of the domain which denoted a singular object that contained no incident points. An early definition of null was formulated as follows:

$\mathrm{NULL}(x) \equiv_{\mathrm{def.}} \exists!x\ \mathrm{EQUAL}(x,n)$
$\forall xy\left[\mathrm{EQUAL}(\mathrm{prod}(x,y),n) \leftrightarrow \mathrm{DC}(x,y)\right]$

But problems arose when considering the product of regions satisfying the EC relation. Forcing prod(x,y) to have the sortal declarations:

type prod(τ,τ):$\tau \sqcup$ NULL, $\tau \in$ {SPATIAL,PERIOD}

meant that the result sort for the product of any pair of regions satisfying the EC relation would be REGION. But given the intuitive semantics of product in terms of sets of points, this is turn meant that a single point, a set of unrelated points, partial boundaries, boundaries, part surfaces, and surfaces could all classify as members of the sort REGION. This in turn opened up a set of complications. With points construed as regions, and regions having parts, then if two objects EC, e.g. EC(a,b) (sharing a point in common) they have a part in common. But EC(a,b) implies $\neg$O(a,b) (by the definition of EC) and $\neg$O(a,b) implies $\neg\exists z[P(z,a) \wedge P(z,b)]$ (by the definition of O) - contradiction.

Originally points were explicitly introduced into the ontology to meet the problem of inversion discussed in section 2.14, and in terms of prod(x,y) the result sort was consequently expanded as follows. In this case a new sort BOUNDARY (conceived to be pairwise disjoint with the sorts REGION, POINT and NULL) was added:

type prod(τ,τ):$\tau \sqcup$ POINT $\sqcup$ BOUNDARY $\sqcup$ NULL, $\tau \in$ {SPATIAL,PERIOD}

This was eventually replaced with the declaration:

type prod(τ,τ):$\tau \sqcup$ POINT $\sqcup$ POINT* $\sqcup$ NULL

POINT* covered those cases of regions whose product was simply a set of points which did not constitute a region proper, e.g a boundary or a face of a geometrical figure.

The definition for prod(x,y) now assumed this form:

$$
\begin{aligned}
\mathrm{prod}(x,y) =_{\mathrm{def.}}\ \iota z[&DC(x,y) \rightarrow EQUAL(z,n) \wedge \\
&EC(x,y) \rightarrow [\exists! u[POINT(u) \wedge IN(u,x) \wedge IN(u,y)] \vee \\
&\quad \exists vw[POINT(v) \wedge POINT(w) \wedge \neg EQUAL(v,w) \wedge \\
&\qquad IN(v,x) \wedge IN(v,y) \wedge IN(w,x) \wedge \\
&\qquad IN(,y)] \leftrightarrow Point^{*}(z)] \wedge \\
&\forall x'[O(x,y) \leftrightarrow \exists y'[P(y',x) \wedge P(y',y) \wedge C(x',y')]
\end{aligned}
$$

Thus the product of two disconnected regions would be n (the null object), for two externally connected regions either a single point, a boundary or several 'unrelated' points, or a region for the overlapping case. The increase in the complexity of the ontology also required the introduction of a new relation IN(x,y) linking points to the formalism, and the addition of a new sort POINT declared to be pairwise disjoint with REGION. The latter decision to make points a distinct sort from region countered the problem mentioned above raised if points are equated with regions. But the increase in the complexity of the ontology resulted in a proliferation of sorts to cover the intermediate set of entities mentioned. This cast doubts on the gains to be had in terms of the practical expressiveness of the theory and the computational overheads anticipated. Given these considerations, null was relaxed from its status as an individual constant and the ontology simplified. The product of two regions was consequently defined to be null iff those regions were discrete. This means two regions that EC have a null product, even though on the point interpretation, points are shared. Discrete regions can share points in common, what they do not share is a region.

The standard definition for the boundary of a region (as a region) cannot be set up in this theory. The usual definition found in general topology, i.e.

bound(x) =def. prod(cl(x),cl(compl(x))

requires cl(x) (read as 'the (topological) closure of x') and cl(compl(x)) (read as 'the closure of the complement of x') to overlap (hence share a part in common) - see section 2.9 for clarification of these topological concepts. If this definition were sanctioned i.e. allowing for the overlap between cl(x) and cl(compl(x)), the formalism including the axiom:

(A8) $\forall x[EC(cl(x),cl(compl(x)))]$ (described below) would become inconsistent. The proof is trivial. Taking boundaries as regions, then bound(x) would be part of cl(x), and bound(x) would also be part of cl(compl(x)). But this implies O(cl(x),cl(compl(x))), which in turn implies $\neg$EC(cl(x),cl(compl(x))) - contradiction. This leaves axiom (A8) in question. But given that this axiom guarantees the existence of an externally connected region for any non-open region; and ensures that the tangential part relations are satisifled in the intended model, its excision cannot be made. The addition of the axiom: (A8) $\forall x$ EC(cl(x),cl(compl(x))), forces the following important theorems:

(T40) $\forall x\ PP(int(x),cl(x))$
(T41) $\forall x \neg \exists y[P(y,cl(x)) \wedge \neg O(y,int(x))]$
(T42) $\forall xy[\neg O(y,int(x) \rightarrow P(y,cl(compl(x)))]$
x,y,int(x),cl(x),compl(x),cl(compl(x)): SPATIAL

from which one can see (by T42) that any region discrete from the interior of a region is pushed out into the closure of the complement of that region. Thus although the interior of a region is a proper part of the closure of that region (T40), there is no other proper part remaining as part of the closure (T41). Thus boundaries cannot be regions. QED .

2.9 The topological part

The distinction Clarke draws between the relation C and O and subsequent introduction of the defined relations EC, TP and NTP enables a set of standard topological operators to be defined. This feature is missing in the classical calculus of individuals [Leonard and Goodman 1940]. A topological interpretation can be given for the classical calculus of indivduals, but it turns out that all the regions would be open [**] and with it the loss of many useful relations derived from the relation EC.

In general topology, an open region is classified as any region that does not contain any of its boundary points, and a closed region, one that does. Some regions are constructable that are neither open nor closed, which I name clopen regions. The interior of a region x is the maximal open region y that is included in x. If region x and its interior y are identical, x is open. The closure of a region x takes the interior of x and includes its boundary too. If then, region x and its closure are identical, then x is closed.

The functions:'int(x)' read as 'the interior of x', 'cl(x)' read as 'the closure of x', 'ext(x)' read as 'the exterior of x' and the predicates 'Open(x)' read as 'x is open', 'Closed(x)' as 'x is closed' are defined by Clarke; the predicate 'Clopen(x)' read as 'x is neither open nor closed' is added:

(D28) $\mathrm{int}(x) =_{\mathrm{def.}} \iota y[\forall z[C(z,y) \leftrightarrow \exists u[NTP(u,x) \wedge C(z,u)]]]$
(D29) $\mathrm{cl}(x) =_{\mathrm{def.}} \iota y[\forall z[C(z,y) \leftrightarrow \exists u[\neg C(u,\mathrm{int}(\mathrm{compl}(x))) \wedge C(z,u)]]]$
(D30) $\mathrm{ext}(x) =_{\mathrm{def.}} \iota y[\forall z[C(z,y) \leftrightarrow \exists u[NTP(u,\mathrm{compl}(x)) \wedge C(z,u)]]]$
(D31) $\mathrm{Open}(x) \equiv_{\mathrm{def.}} \mathrm{EQUAL}(\mathrm{int}(x),x)$
(D32) $\mathrm{Closed}(x) \equiv_{\mathrm{def.}} \mathrm{EQUAL}(\mathrm{cl}(x),x)$
(D33) $\mathrm{Clopen}(x) \equiv_{\mathrm{def.}} \neg\mathrm{Open}(x) \wedge \neg\mathrm{Closed}(x)$

type $\mathrm{int}(\tau){:}\tau,\ \tau \in \{\mathrm{SPATIAL},\mathrm{PERIOD}\}$
type $\alpha(\tau){:}\tau,\ \alpha \in \{\mathrm{cl},\mathrm{ext}\},\ \tau \in \{\mathrm{SPATIAL}\backslash\mathrm{SPATIAL\_UNIVERSE}\}$

type $\alpha(\tau)$:NULL, $\alpha \in \{cl, ext\}$, τ = SPATIAL_UNIVERSE
type Open(τ):UU, $\tau \in$ {SPATIAL,PERIOD}
type $\Phi(\tau)$:UU, $\Phi \in$ {Closed,Clopen}, $\tau \in$ {SPATIAL\SPATIAL_UNIVERSE}

It is useful to bear in mind that the definitional schema:

$$\phi(x) =\text{def. } \imath y[\forall z[C(z,y) \leftrightarrow \exists w[\Phi(x) \wedge C(z,w)]]]$$

used in these definitions (and for prod(x,y)) can be informally thought of as taking the sum fusion of all the regions that satisfy the metalogical predicate Φ, and mapping the region so formed to region $\phi(x)$. It is also useful to realise that the following wff's:

$$\forall xy[C(y,\text{int}(x) \leftrightarrow \exists z[NTP(z,x) \wedge C(y,z)]]$$
$$\forall x[P(\text{int}(x),x) \wedge \forall y[NTP(y,x) \rightarrow P(y,\text{int}(x)]]$$

are in actual fact formally equivalent. A little reflection on this can help in the understanding of these particular definitions.

Informally, the (topological) interior of a region x coincides with the set of points incident in x which are not incident in the boundary of x, while the closure of region x includes that set of boundary points. The exterior of a region x comprises the set of points that coincide with the complement of the closure of x. Note that the exterior of a region is not necessarily identical to the complement of a region: the exterior of a region is always open, while the complement of a region x can be either open or closed depending on whether x itself is respectively closed or open.

As discussed in the last section, this formalism does not support a boundary region. However it must be remembered that regions still have boundaries; what is denied is an explicit characterisation of them within the formalism, hence their existence can only be inferred implicity from the role they play in the intuitive semantics used to interpret the formal theory.

Within this formalism the predicates Open, Closed and Clopen are exhaustive and pairwise disjoint. This is forced by the addition of the following axiom:

(A8) $\forall x[EC(cl(x), cl(compl(x))]$.
$x, cl(x), compl(x), cl(compl(x))$:SPATIAL\SPATIAL_UNIVERSE

(T43) $\forall x[Open(x) \vee Closed(x) \vee Clopen(x)]$

(T44) $\forall x[Open(x) \rightarrow \neg Closed(x)]$

(T45) $\forall x[Closed(x) \rightarrow \neg Clopen(x)]$

(T46) $\forall x[Open(x) \rightarrow \neg Clopen(x)]$

(T47) $\forall x \exists y[EC(cl(x), y)]$
$x, y, cl(x), compl(x), cl(compl(x))$:SPATIAL\SPATIAL_UNIVERSE

Intuitively one can think of axiom (A8) expressing the fact that every region is embedded and completely surrounded by another region, both of which make up the whole of space, rather like a fish in an aquarium surrounded by water.

In general topology both the topological space X and the null set $\emptyset$ are defined to be both open and closed. However in this formalism the universal regions u_s and u_T are open only, while the sort NULL is false defined on the sort REGION, e.g.:

(T48) $Open(u_s)$
u_s:SPATIAL_UNIVERSE.

While open regions have no regions that are in external contact with them; in contrast, closed or clopen regions do:

(T49) $\forall x[Open(x) \leftrightarrow \neg\exists y[EC(y,x)]]$
$x, y : \tau,\ \tau \in$ {SPATIAL,PERIOD}

(T50) $\forall x[Closed(x) \rightarrow \exists y[EC(y,x)]]$
x:SPATIAL\SPATIAL_UNIVERSE, y:SPATIAL

(T51) $\forall x[\mathrm{Clopen}(x) \rightarrow \exists y[\mathrm{EC}(y,x)]]$

x:SPATIAL\SPATIAL_UNIVERSE, y:SPATIAL

In Clarke [1981 p.213, and 1985 note 4 p74] we find the following axiom:

$$\forall x[\exists y[\mathrm{NTP}(y,x) \wedge \forall zu[[[C(z,x) \rightarrow O(z,x)] \wedge [C(u,x) \rightarrow O(u,x)]] \rightarrow \forall v[C(v,\mathrm{prod}(x,y)) \rightarrow O(z,\mathrm{prod}(x,y))]]]$$

The first conjunct ensures every region has a nontangential part (and hence has an interior), the second that the product of two open regions is open [**]. I add this axiom:

(A9) $\forall x[\exists y[\mathrm{NTP}(y,x) \wedge \forall zu[[[C(z,x) \rightarrow O(z,x)] \wedge [C(u,x) \rightarrow O(u,y)]] \rightarrow \forall v[C(v,\mathrm{prod}(x,y)) \rightarrow O(z,\mathrm{prod}(x,y))]$

either x,y,z,u,v:SPATIAL, prod(x,y): SPATIAL ⊔ NULL, or

x,y,z,u,v:PERIOD, prod(x,y):PERIOD ⊔ NULL

Separated and Connected regions are definable in the formalism. Clarke introduces the relation 'SEPARATED(x,y)' read as 'x is separated from y' and the predicate 'Connected(x)' read as 'x is connected'. The predicate 'Disconnected(x)' as 'x is disconnected' is new:

(D34) $\mathrm{SEPARATED}(x,y) \equiv$def. $\neg C(\mathrm{cl}(x),y) \wedge \neg C(x,\mathrm{cl}(y))$

(D35) $\mathrm{Connected}(x) \equiv$def. $\neg\exists yz[\mathrm{EQUAL}(\mathrm{sum}(y,z),x) \wedge \mathrm{Separated}(y,z)]$

(D36) $\mathrm{Disconnected}(x) \equiv$ def. $\neg\mathrm{Connected}(x)$

type SEPARATED(τ,τ):UU, τ = SPATIAL\SPATIAL_UNIVERSE

type Connected(SPATIAL):UU

type Disconnected(τ):UU, τ = SPATIAL\SPATIAL_UNIVERSE

A region is connected if it cannot be divided into two exhaustive separated parts. This feature of the formalism illustrates that regions do not have to be construed as continuous or connected in the topological sense - although this has been implicitly assumed in the examples used to illustrate the meaning of the relations and functions. While the classical calculus of individuals equally allows for scattered and continuous individuals, Clarke's calculus defines these properties in relation to a topology.

The classical calculus of individuals has been used in the formal treatment of mass term extensions, e.g. by Moravcsik (discussed in Pelletier 1974). Mass terms e.g. 'water' and 'flour' (relating to stuffs) unlike count terms e.g. 'cup' and 'container' (relating to things) are often said to divide their reference. Mereology comes in by sanctioning individuals that may be discontinuous, e.g. all bodies of water e.g. drops, puddles, pools, lakes and so on are regarded as part of one watery individual. The possibility of using both continuous and discontinuous regions as the basis for modelling stuffs as well as things, raises several questions concerning the individuation of such objects and the adequacy of a formal semantics describing them. The issues involved are complex and are subsequently relegated to Chapter 6. However it is worth pointing out that there is no apriori reason why individuals must be continuous.

One other topological concept is introduced and then defined - that of a quasi-manifold. A manifold proper (in 3-space) is a connected surface such that for each point incident in the surface, all the points sufficiently near to the indexed point form a set topologically equivalent to an open disk. The definition ensures that any region that has point connected parts is not a manifold, e.g. in the case where two cones share a common vertex point, and where the two cones are regarded as a single object. A quasi-manifold is defined as a region that has a connected interior, remembering that a quasi-manifold need not be a manifold. 'Manifold(x)' read as 'x is a (quasi-) manifold' is defined as follows:

(D37) Manifold(x) ≡def. Connected(int(x))

type Manifold(SPATIAL):UU

Suppose then, that two regions externally connect. The definition of a quasi-manifold if true for that configuration, ensures that the composite region must be at least edge connected (in 2-space) or share a 'fused' surface (in 3-space).

In Randell and Cohn [1989b,c] a set of functions are defined that take the Boolean complementation and difference operators and map these to their respective closures. This is not reproduced here; instead composition of functions is used, e.g. cl(compl(x)) as the closure of the complement of x.

2.10 Atoms

An atom is a region that has no proper parts: the only part an atom has is itself. Every region contains an atom:

(D38) Atom(x) $\equiv$def. $\forall y[P(y,x) \rightarrow EQUAL(y,x)]$

type Atom(τ):UU, $\tau \in$ {SPATIAL,PERIOD}

(A10) $\forall x \exists y[Atom(y) \wedge P(y,x)]$
x,y:τ, $\tau \in$ {SPATIAL,PERIOD}

Neither the classical calculus of individuals nor Clarke's calculus contain atomic individuals. But atomic calculi of individuals do exist, e.g. Eberle [1970].

If atoms are denied an interior they can either be disconnected, externally connect or be identical. If as has been done, int(x) is defined on atoms, and atoms thereby allowed to have an interior, they become open regions. Pairs of atoms are either disconnected, or connected and identical:

(T52) $\forall x[Atom(x) \rightarrow Open(x)]$
x:τ, $\tau \in$ {SPATIAL,PERIOD}

(T53) $\forall xy[[\mathrm{Atom}(x) \wedge \mathrm{Atom}(y) \wedge C(x,y)] \rightarrow \mathrm{EQUAL}(x,y)]$
$x{:}\tau,\ \tau \in \{\mathrm{SPATIAL}, \mathrm{PERIOD}\}$

The following additional theorems concerning atoms are provable:

(T54) $\forall xy[[\mathrm{Atom}(x) \wedge P(x,y)] \rightarrow \mathrm{NTP}(x,y)]$
$x,y{:}\tau,\ \tau \in \{\mathrm{SPATIAL}, \mathrm{PERIOD}\}$

(in words: every atomic part of a region is a nontangential part)

(T55) $\forall xy[O(x,y) \leftrightarrow \exists z[\mathrm{Atom}(z) \wedge P(z,x) \wedge P(z,y)]]$
$x,y,z{:}\tau,\ \tau \in \{\mathrm{SPATIAL}, \mathrm{PERIOD}\}$

(in words: regions overlap if and only if an atom is shared in common)

(T56) $\forall xyz[C(z,\mathrm{int}(x) \leftrightarrow \exists u[\mathrm{Atom}(u) \wedge P(u,x) \wedge C(u,z)]]$
$x,y,z,u,\mathrm{int}(x){:}\tau,\ \tau \in \{\mathrm{SPATIAL}, \mathrm{PERIOD}\}$

(in words: a region z is connected with the interior of a region x if and only if z connects with an atom of x)

(T57) $\forall xy[\mathrm{EQUAL}(x,y) \rightarrow \forall z[\mathrm{Atom}(z) \rightarrow [P(z,x) \leftrightarrow P(z,y)]]]$

$x,y,z{:}\tau,\ \tau \in \{\mathrm{SPATIAL}, \mathrm{PERIOD})$

(in words: regions are identical *only if* they have the same atoms as parts.) [21]

Defining interiors over atoms produces an interesting deductive result, for although two regions may externally connect, none of their constituent atoms externally connect:

(C58) $\forall xy[\mathrm{EC}(x,y) \rightarrow \forall zu[[\mathrm{Atom}(z) \wedge \mathrm{Atom}(u) \wedge P(z,x) \wedge P(u,y)] \rightarrow \neg\mathrm{EC}(z,u)]]$
$x,y,z,u{:}\tau,\ \tau \in \{\mathrm{SPATIAL}, \mathrm{PERIOD}\}$

This formal result casts some light on the naive conundrum of how if (physical) atoms are construed as points with fields (topologically open?), and atoms make up objects, how is that objects comprised of these atoms touch? A similar conundrum arises when physical objects are simply construed as sets of points without a topological structure defined on them. The formalism supporting open atoms illustrates what may be seen as an informal fallacy at work, namely the fallacy of composition. This is the mistake to assume that all the properties of parts of a whole must belong to that whole.

2.11 Closures of atoms.

The closure of an atom is a closed atom. 'C_Atom(x)' is read as 'x is a closed atom':

(D39) $C\_Atom(x) \equiv def. \exists y[Atom(y) \wedge EQUAL(cl(y),x)]$

type C_Atom(SPATIAL\SPATIAL_UNIVERSE):UU

Closed atoms are not atoms in the way closed regions are regions. Atoms are open regions but their closures are not. Care is needed that the linguistic reading assigned to 'C_Atom(x)' does not mislead one into thinking otherwise.

Closed atoms and atoms cannot partially overlap, although closed atoms unlike atoms can externally connect. If two closed atoms overlap they become identical:

(C59) $\forall xy[[C\_Atom(x) \wedge C\_Atom(y)] \rightarrow [DC(x,y) \vee EC(x,y) \vee EQUAL(x,y)]]$

(T60) $\forall xy[[C\_Atom(x) \wedge C\_Atom(y) \wedge O(x,y)] \rightarrow EQUAL(x,y)]$

$x,y{:}\tau,\ \tau = SPATIAL \backslash SPATIAL\_UNIVERSE$

Atoms and their closures are employed in definitions that pick out 'surfaces' of non-atomic regions. This is covered in section 2.18.

2.12 Proper regions and atom strings

Intuitively, a proper region is any region x that consists of a cluster of atoms that completely pack around a nuclear one, all of which are part of x. This is defined formally as a region x that has some atom y as part such that the closure of y is not connected to the closure of the complement of x. 'Proper_Region(x)' is read as 'x is a proper region':

(D40) $\text{Proper\_Region}(x) \equiv_{\text{def.}} \exists y[\text{Atom}(y) \land P(y,x) \land \neg C(\text{cl}(y),\text{cl}(\text{compl}(x)))]$

type Proper_Region(SPATIAL):UU

Proper regions exclude regions that are atomic, or are composed of strings of atoms, although a proper region can have strings of atoms that extend out from the main body of the region. Strings of atoms (or atom strings) are defined as follows, 'String(x)' is read as 'x is a string of atoms':

$$
\begin{aligned}
(\text{D41})\ \text{String}(x) \equiv_{\text{def.}} \exists yz[&\text{Atom}(y) \land \text{Atom}(z) \land \\
&P(y,x) \land P(z,x) \land \neg\text{Equal}(y,z) \land \\
&\forall u[PP(u,x) \rightarrow C(\text{cl}(u),\text{cl}(\text{compl}(x)))]]
\end{aligned}
$$

type String(SPATIAL\SPATIAL_UNIVERSE):UU

Atom strings are composed of at least two atoms whose closures are connected. Isolated atoms are not the limiting case of an atom string.

2.13 Points

Some modelled domains do not require points to be explicitly represented. However it is instructive to provisionally include points in the general ontology in order to see what advantages and disadvantages arise with their introduction.

Clarke [1985] identifies three common methods by which points are defined: nesting definitions (identifying points with limiting cases of sets of nested individuals), algebraic definitions (e.g. the use of Boolean rings or distributive lattices) and atomic definitions which take basic individuals and defines points as atomic parts, i.e. individuals having only themselves as parts. Clarke actually adopts the second option but he expands his ontology to include the explicit representation of sets of regions as well as individual regions to construct his definition.

Of the three common methods cited above, we can immediately exclude the notion of identifying points with atomic regions in this theory. The proof is straightforward. Atoms are defined as regions that have no parts other than themselves, i.e. they have no proper parts. But given the role of points in providing the intuitive semantics for the defined relations, if atoms are identified with points, two externally connecting regions x and y sharing a boundary point in common must share an atom in common. But since atoms are regions, a region is shared between x and y; which given the definition for part entails the regions overlap. But this immediately introduces a contradiction, for externally connecting regions do not overlap (by definition). Were the quantifiers to range over open regions only, the difficulty cited would dissolve in part. Open regions that connect, overlap and overlapping regions share an atom in common. Thus far so good. The formal result meshes with the intuitive semantics provided by the model. But now points become open regions and as such have interiors. This is a less agreeable result given that the primitivity typically associated with the concept of a point requires it to have positional qualities only. That aside this move cannot be sanctioned. Restricting the domain to open regions only only serves to collapse Clarke's calculus to the traditional calculus of individuals, and in so doing one immediately looses the advantage gained by using the weaker relation C.

An alternative method of introducing points can be done by making points a primitive ontological category in the theory - which is in fact done with the introduction of the sort POINT. Following Clarke, points are linked to regions by introducing a new relation of incidence: '$IN(x,y)$' read as 'x is incident in y' as follows:

type $IN(POINT,\tau):UU$, $\tau \in \{SPATIAL, PERIOD\}$

The following axioms are needed:

(A11) $\forall xy[C(x,y) \leftrightarrow \exists z[IN(z,x) \wedge IN(z,y)]]$

$x,y:\tau$, $\tau \in \{SPATIAL, PERIOD\}$, $z:POINT$

(in words: two regions connect if and only if they share an incident point.)

and,

(A12) $\forall xy[P(x,y) \rightarrow \forall z[IN(z,x) \rightarrow IN(z,y)]]$

$x,y:\tau$, $\tau \in \{SPATIAL, PERIOD\}$, $z:POINT$

(in words: region x is part of region y *only if* every point incident in x is incident in y)

Added to the extant formalism, the following theorems arise:

(T61) $\forall x \exists y[IN(y,x)]$

$x:\tau$, $\tau \in \{SPATIAL, PERIOD\}$, $y:POINT$

(in words: every region has an incident point)

(T62) $\forall xy[DC(x,y) \leftrightarrow \neg\exists z[IN(z,x) \wedge IN(z,y)]]$

$x,y:\tau$, $\tau \in \{SPATIAL, PERIOD\}$, $z:POINT$

(in words: regions x and y are disconnected if and only if they share no incident point in common)

(T63) $\forall xy[O(x,y) \leftrightarrow \exists z[IN(z,int(x)) \wedge IN(z,int(y))]]$

$x,y:\tau$, $\tau \in \{SPATIAL, PERIOD\}$, $z:POINT$

(in words: regions x and y overlap iff their exists an interior point shared in common.)

(T64) $\forall xy[P(x,y) \leftrightarrow \forall z[IN(z,x) \rightarrow IN(z,y)]]$

$x,y:\tau$, $\tau \in \{SPATIAL, PERIOD\}$, $z:POINT$

(in words: region x is a part of region y if and only if every point in x is a point of y)

(T65) $\forall xy[\forall z[IN(z,x) \leftrightarrow IN(z,y)] \leftrightarrow EQUAL(x,y)]$

$x,y:\tau$, $\tau \in \{SPATIAL, PERIOD\}$, $z:POINT$

(in words: regions x and y are identical if and only if every point incident in x is incident in y and vice versa.)

$$\text{(C66)}\ \forall xy\left[EC(x,y) \leftrightarrow \exists z\left[IN(z,x) \wedge IN(z,y)\right] \wedge \neg\exists u\left[IN(u,int(x)) \wedge IN(u,int(y))\right]\right]$$

$$x,y{:}\tau,\ \tau \in \{SPATIAL, PERIOD\},\ z,u{:}POINT$$

(in words: regions x and y externally connect if and only if they share an incident point in common but share no interior point in common.)

The reader is invited to confirm that the above theorems mirror the intuitive semantics used to interpret the mereological relations.

2.14 The surround relations

The surround relations are motivated as follows. If one considers the set of configurations depicted in Figure 3 for the proper part relations as nested circles and not discs, they could be filled and then described in at least two ways. In the first case the inner circle could be filled to make a region, and then the other circle filled so as to make the inner a part of the outer. But equally the outer annulus or 'crescent' could be filled so that the inner is surrounded by the other. The latter case depicts the surround relation where neither is a part of the other - see Figure 4. This distinction is characterised between the relation of the nucleus of an amoeba to the whole organism, and the relation between the amoeba and some particle of food it has just enveloped.

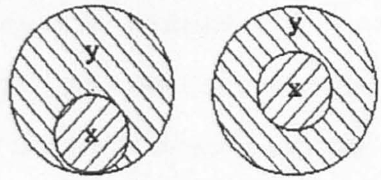


Figure 4: Surround analogues of the proper part relations

Because some notion of containment is being considered here, it seems possible to define a surround analogue of the relation NTPP(x,y), since clearly the relation between x and y is asymmetrical in the intended model. However the same cannot be said for the surround analogue of the relation TPP(x,y), which using the mereological relations only, is impossible to define so that only intended models are allowed. Given no metric or 'size' is being assumed here, the relation TPP(x,y) is satisfied by all the configurations depicted in Figure 5; from which it should be apparent that either region can be the surround of the other.

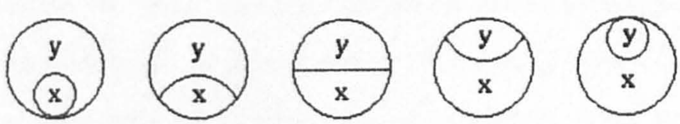


Figure 5: Configurations satisfying the relation TPP(x,y). In each case region y represents the whole figure and x the proper part.

There are several strategies that can be used to curtail the problem posed by inversion. The first makes use of points. In this case the degree of boundary connection between the surrounded and surrounding region is restricted to a single point. Then, having defined this relation a stronger relation for the TPP(x,y) relation is defined - again restricting the tangential connection to a single point. Thus we get two relations such that one is the clear surround analogue of the other. This strategy was made use of in Randell and Cohn [1989b] and is repeated below: 'NTS(x,y)' is read as 'x is the nontangential surround of y', 'TPPp(x,y)' read as 'x is a boundary point connected tangential proper part of y' and 'TSp(x,y)' read as 'x is tangentially surrounded by y (at a point)':

(D42) $\mathrm{NTS}(x,y) \equiv\mathrm{def.}\ \exists z[\mathrm{NTPP}(x,z) \wedge \mathrm{EQUAL}(y,\mathrm{prod}(\mathrm{cl}(\mathrm{compl}(x)),z))]$

(D43) $\mathrm{TS}(x,y) \equiv\mathrm{def.}\ \exists z[\mathrm{TPP}(x,z) \wedge \mathrm{EQUAL}(y,\mathrm{prod}(\mathrm{cl}(\mathrm{compl}(x)),z))]$

(D44) $\mathrm{TPPp}(x,y) \equiv\mathrm{def.}\ \mathrm{PP}(x,y) \wedge \exists z[\mathrm{EC}(z,x) \wedge \mathrm{EC}(z,y)] \wedge \exists!u[\mathrm{IN}(u,x) \wedge \mathrm{IN}(u,y) \wedge \mathrm{IN}(u,z)]]$

(D45) $\mathrm{TSp}(x,y) \equiv\mathrm{def.}\ \exists z[\mathrm{TPP}(x,z) \wedge \mathrm{EQUAL}(y,\mathrm{prod}(\mathrm{cl}(\mathrm{compl}(x)),z)) \wedge \exists!u[\mathrm{IN}(u,x) \wedge \mathrm{IN}(u,y) \wedge \mathrm{IN}(u,z)]]$

type $\Phi(\tau,\tau)$:UU, τ = SPATIAL\SPATIAL_UNIVERSE, $\Phi \in \{\mathrm{NTS},\mathrm{TS},\mathrm{TPPp},\mathrm{TSp}\}$

In Randell and Cohn [1989c] points ceased to be explicitly represented in the formalism; this resulted in a corresponding change in the above definitions. The role of points were replaced with atoms. Here an implicit notion of 'size' appeared in the intended model, i.e. equating spatial atoms with the spaces assumed by physical atoms. Without this restriction (and with no explicit use of a metric) atoms can of course assume any size, and the problem of inversion simply reappears. The readings of the set of relations given below are not given but should be readily understood:

(D46) $\mathrm{TPPa}(x,y) \equiv\mathrm{def.}\ \mathrm{PP}(x,y) \wedge \exists z[[\mathrm{EC}(z,x) \wedge \mathrm{EC}(z,y)] \wedge \exists!u[\mathrm{C\_Atom}(u) \wedge \mathrm{P}(u,x) \wedge \mathrm{P}(u,y) \wedge \mathrm{EC}(u,z)]]$

(D47) $TSa(x,y) \equiv def. \; \exists z[TPP(x,z) \wedge EQUAL(y,prod(cl(compl(x)),z) \wedge$
$$\exists !u[C\_Atom(u) \wedge P(u,x) \wedge P(u,y) \wedge EC(u,z)]]$$

type $\Phi(\tau,\tau)$:UU, τ = SPATIAL\SPATIAL_UNIVERSE, $\Phi \in \{TPPa,TSa\}$

Whatever way one may wish to defined these relations, a correspondence is set up between the proper part relations and their surround duals. This enables a rewrite rule to be used so that dual descriptions can be given for a given model - either in terms of proper part to whole, or one region being surrounded by another. The use of this feature is discussed in section 5.2 where two alternative descriptions of a model are given.

2.15: Inside and outside

We often talk about objects being inside or outside other objects, e.g. water might be said to be inside a cup or a dangerous animal put inside a cage with us outside it! These relations occur so frequently in everyday discourse, that it would seem very desirable to include them in any theory that aims to capture fundamental properties of space.

Despite their intuitive meanings, the relations of being inside and outside are difficult to define. One difficulty is that the function of certain objects have a clear bearing on what is then characterised as an objects inside or outside - see Figure 6.

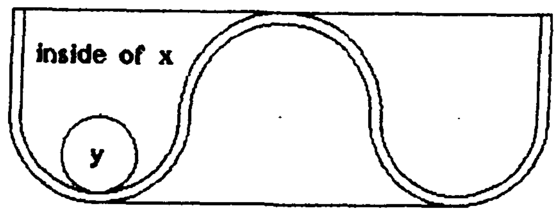


Figure 6: Problems characterising the inside of an object. Here for example, the inside of container x with respect to ball y can be seen to vary according to its orientation in space.

Despite such difficulties, we can begin to characterise the inside or outside of an object, or one object being inside or outside another by introducing and using the concept of a convex hull (or convex cover).

Intuitively, the convex hull of a body describes the region of space that is generated by completely enclosing that body in a taught 'cling film' membrane. In 2-space this would be akin to isolating that region of space described by a rubber band stretched to fit around some given figure. Formally, the convex hull is usually defined to be the smallest convex set of points that encloses a given set of points [22].

Although the convex hull can be applied to a heterogeneous set of points, or (using the ontology of regions) a set of regions, the function is actually restricted in this theory to individual connected ("one piece") regions. An object x is then said to be inside object y iff x and y are discrete and x is part of the convex hull of y. Conversely, object x is outside object y iff x and the convex hull of y are discrete. The function 'conv(x)' is read as 'the convex hull of x', 'INSIDE(x,y)' as 'x is inside y', and 'OUTSIDE(x,y)' as 'x is outside y':

(D48) INSIDE(x,y) ≡def. P(x,conv(y)) ∧ DR(x,y) [23]

(D49) OUTSIDE(x,y) ≡def. DR(x,conv(y)) [23]

type INSIDE(τ1,τ1):UU

type OUTSIDE(τ1,τ1):UU

type conv(τ1):τ1

type conv(τ2):NULL

where: τ1 = SPATIAL\SPATIAL_UNIVERSE, τ2 = SPATIAL_UNIVERSE

Using these definitions, other specialisations can be defined which capture the notion of one region either being wholly outside another, or partly inside, or being just inside or wholly inside. 'W_OUTSIDE(x,y)' is read as 'x is wholly outside y', 'J_OUTSIDE(x,y)' read as 'x is just outside y', 'P_INSIDE(x,y)' as 'x is partially inside y', 'J_INSIDE(x,y)' as 'x is just inside y' and 'W_INSIDE(x,y)' as 'x is wholly inside y':

(D50) W_OUTSIDE(x,y) ≡def. DC(x,con(y))

(D51) J_OUTSIDE(x,y) ≡def. EC(x,conv(y))

(D52) P_INSIDE(x,y) ≡def. PO(x,con(y)) ∧ DR(x,y) [23]

(D53) J_INSIDE(x,y) ≡def. INSIDE(x,y) ∧ TP(x,conv(y))

(D54) W_INSIDE(x,y) ≡def. INSIDE(x,y) ∧ NTP(x,conv(y))

type $\Phi(\tau,\tau)$:UU, $\tau \in$ {SPATIAL\SPATIAL_UNIVERSE},
$\Phi \in$ {W_OUTSIDE,J_OUTSIDE,P_INSIDE,J_INSIDE,W_INSIDE}

Figure 7 below depicts pairs of spatial regions that satisfy this set of defined relations together with a partial lattice that indicates how the defined relations would be embedded in a larger relational lattice that would also include the set of relations embedded in lattice L_C (compare with Figure 3).

Although omitted here, further specialisations of all these defined relations (with the exception of W_OUTSIDE) can be constructed. For example, given that regions x and y are discrete, x and y can either externally connect or be disconnected. Also omitted are the set of inverse relations, and the additional relations that are generated when the set is embedded in a relational lattice as was done for the set of relations defined solely in terms of the primitive relation C.

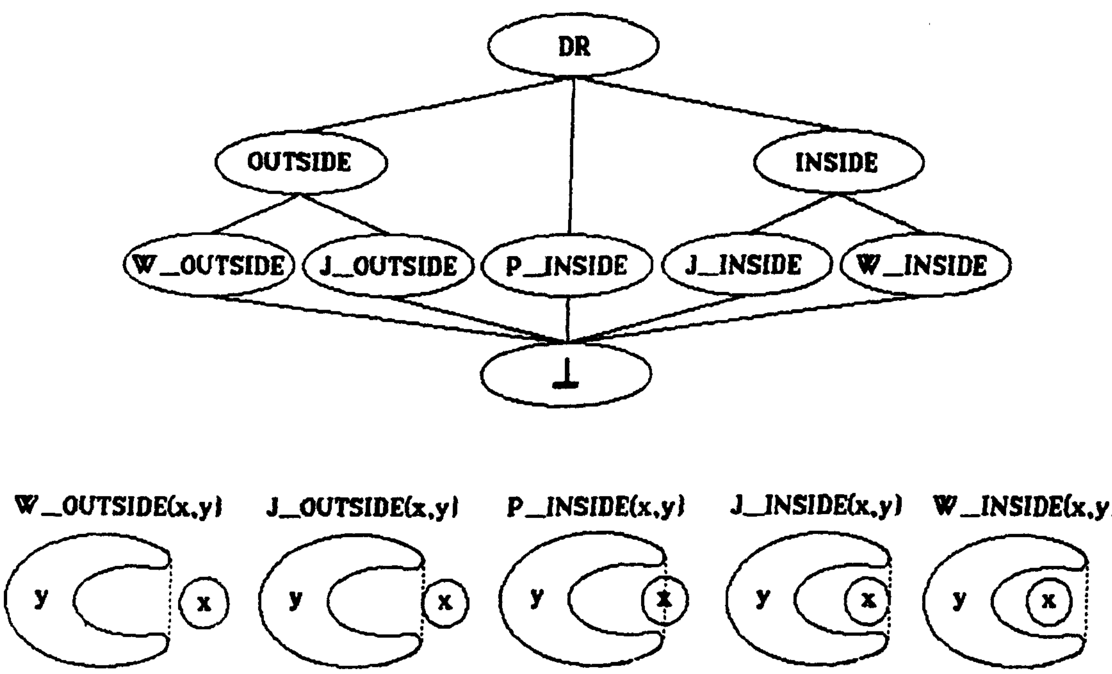


Figure 7: Partial lattice for the inside and outside relations. The set of configurations satisfy the set of base relations that lie immediately above ⊥. The dotted lines indicate the extent of the convex hull in each case.

As suggested above, in addition to relations, the concept of being inside and outside also appear as descriptions, i.e. when one talks about the inside or outside of a particular object. These appear as functions in this theory: 'inside(x)' and outside(x)' are read as 'the inside of x' and 'the outside of x' respectively. The definitions are as follows:

(D55) $inside(x)$ =def. $\imath y[\forall z[C(z,y) \leftrightarrow \exists w[INSIDE(w,x) \wedge C(z,w)]]]$

(D56) $outside(x)$ =def. $\imath y[\forall z[C(z,y) \leftrightarrow \exists w[OUTSIDE(w,x) \wedge C(z,w)]]]$

type inside(τ1):τ1 $\sqcup$ NULL,

type inside(τ2):NULL

type outside(τ1):τ1

type outside(τ2):NULL

where: τ1 = SPATIAL\SPATIAL_UNIVERSE, τ2 = SPATIAL_UNIVERSE

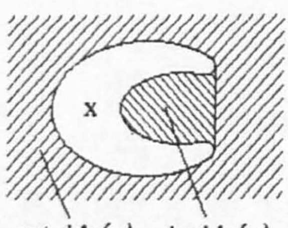


Figure 8: The inside and outside of a region.

As the convex hull function is primitive, it needs to be axiomatised. However, before this is done, some other axioms and definitions are given - 'Convex(x)' is read as 'x is convex':

(D57) $Convex(x) \equiv$def. $EQUAL(conv(x),x)$

type Convex(SPATIAL\SPATIAL_UNIVERSE):UU

(A13) $\forall x[Convex(x) \rightarrow Connected(x)]$

(A14) $\forall xy[C(x,y) \rightarrow C(x,conv(y))]$

(A15) $\forall xyz[[P(x,conv(y)) \wedge P(y,conv(z))] \rightarrow P(x,conv(z))]$

(A16) $\forall xy[P(x,conv(y)) \wedge P(y,conv(x))] \rightarrow O(x,y)]$

(A17) $\forall x$ EQUAL(conv(x),conv(conv(x)))

x,y,z:SPATIAL

The axioms imply that INSIDE(x,y) is irreflexive (T67), asymmetric (T68) and (with the condition that all the objects are pairwise discrete) transitive

(T67) $\forall x$ $\neg$INSIDE(x,x)

(T68) $\forall xy$[INSIDE(x,y) $\rightarrow$ $\neg$INSIDE(y,x)]

(T69) $\forall xyz$[[DR(x,y) $\wedge$ DR(y,z) $\wedge$ DR(x,z) $\wedge$ INSIDE(x,y) $\wedge$ INSIDE(y,z)] $\rightarrow$
INSIDE(x,z)]

x,y,z:SPATIAL\SPATIAL_UNIVERSE

while OUTSIDE(x,y) is irreflexive (T70):

(T70) $\forall x$ $\neg$OUTSIDE(x,x)

x:SPATIAL\SPATIAL_UNIVERSE

The following sample set of theorems are provable:

(T71) $\forall xy$[INSIDE(x,y) $\leftrightarrow$ [J_INSIDE(x,y) $\vee$ W_INSIDE(x,y)]

(T72) $\forall xy$[OUTSIDE(x,y) $\leftrightarrow$ [J_OUTSIDE(x,y) $\vee$ W_OUTSIDE(x,y)]

(T73) $\forall xy$[INSIDE(x,y) $\rightarrow$ $\neg$OUTSIDE(x,y)]

(T74) $\forall xy$[INSIDE(x,y) $\rightarrow$ $\neg$P_INSIDE(x,y]

(T75) $\forall xy$[INSIDE(x,y) $\rightarrow$ P(x,inside(y))]

(T76) $\forall xy$[OUTSIDE(x,y) $\rightarrow$ P(x,outside(y))]

(T77) $\forall x$ P(x,conv(x))

x,y,inside(y), outside(y),conv(x):SPATIAL\SPATIAL_UNIVERSE

2.16: Convexity and concavity

A figure in 2-space is called convex if it wholly contains the line segment that joins any two points incident in that figure. Similarly a body in 3-space is called convex if it wholly contains the line segment that joins any two points incident in that body. Convexity arose in the previous section - section 2.15 - where the convex hull function was introduced.

In order to capture the dual property of concavity, it is not sufficient to simply define this property as the negation of being convex. If a body is convex, that body has in addition to a surface of positive curvature the additional property of being simply connected, i.e. having no holes. This property is a primitive property in this formalism, and is covered in the next section. Suffice to say, the definition for a concave body, in the sense of a body having an outer surface of part negative curvature must incorporate the condition that the body is simply connected. 'Concave(x)' is read as 'x is concave':

(D58) Concave(x) ≡def. Simply_Connected(x) $\wedge$ ¬Convex(x)

type Concave(SPATIAL/SPATIAL_UNIVERSE):UU

Additional axioms are added:

(A18) Convex(x) → Simply_Connected(x)
(A19) ∀x[Simply_Connected(x) → Connected(x)]
x:SPATIAL\SPATIAL_UNIVERSE

The last axiom simplifies the theory by restricting convex bodies to "one piece" regions, and similarly by implication the same holds for concave regions.

2.17: Hollow, simply and multiply connected regions

Hollow regions are easily defined within the formalism given disconnected regions. 'Hollow(x)' read as 'x is hollow' is defined as:

(D59) Hollow(x) ≡def. Disconnected(compl(x))

type Hollow(SPATIAL\SPATIAL_UNIVERSE):UU

By way of examples, in 2-space an annulus is hollow, and in 3-space a soap bubble. In order to distinguish between the case where the body has some region of space completely surrounded by material (as in the case of the soap bubble), and the case where the body is said to have a hole, but where that hole is not completely surrounded by material (e.g. a torus) we introduce the property of simple connectedness.

A region x is simply connected iff every closed loop incident in x can be shrunk to a point also incident in x. If the region has a hole (although strictly speaking the hole is a property of the surrounding space) this operation cannot succeed - the act of shrinking a class of closed loops incident in that region would require them to pass through the boundary of the region. Regions satisfying the latter condition are said to be multiply connected. As an everyday example of simple connectedness, a potter initially aims to produce a well worked lump of clay with no air pockets - such an object is is simply connected. Subsequent pulling or compacting the clay will not alter this property, providing the potter by working the clay does not join any parts of the surface. Simple connectedness is assumed as a primitive property: 'Simply_Connected(x)' is read as 'x is simply connected'. The dual property of being multiply connected (i.e. having at least one hole) is defined immediately below. 'Multiply_Connected(x)' is read as 'x is multiply connected':

(D60) Multiply_Connected(x) ≡def. Connected(x) ∧ ¬Simply_Connected(x)

type Multiply_Connected(SPATIAL\SPATIAL_UNIVERSE):UU

It is common to distinguish between multiply connected objects in terms of the minimum number of cuts that are required to convert them into simply connected objects. For example an object with one 'hole' requires one cut to make it simply connected, and an object with two holes, two cuts. In general, if n-1 non-intersecting cuts from boundary to boundary are needed to convert a given multiply connected object into a simply connected object, the object is said to be n-tuply connected [Cournat and Robbins, in Newman ed 1956 p587-588]. By regarding the 'cut' as a region, n-tuply connected objects are readily defined - only one is given here: 'Doubly_Connected(x)' read as 'x is doubly connected':

(D61) Doubly_Connected(x) $\equiv$def. Multiply_Connected(x) $\wedge$
$\exists$y[Simply_Connected(y) $\wedge$
PP(y,x) $\wedge$ Simply_Connected(diff(x,y))]

type Doubly_Connected(SPATIAL\SPATIAL_UNIVERSE):UU

Being hollow is a sufficient condition for being multiply connected, but not necessary, e.g. the prototypical solid torus is multiply connected but not hollow. Filters or chambered vessels can be construed as n-tuply connected objects.

2.18: Modelling surfaces

Outside geometry proper, surfaces of everyday objects are often talked of as part of the outside aspect of a body as though the surface has materiality in the way the bodies they are surfaces of obviously do. We find it perfectly sensible to talk about touching such bodies, and in order to bring attention to the outward aspect we find it expedient to talk about surfaces of such objects which can also be touched. Intuitively this characterisation of a surface is quite unlike that ascribed to geometrical bodies embedded in 3-space. For one thing the relation of touching is clearly a physical concept which has no proper use in geometry, for another the notion of materiality associated with the surface of a physical object (e.g. we find it perfectly sensible to talk about staining the surface of a piece of wood) has no correlate with the geometrical concept of a surface having volumetric extension in 3-space.

While mathematics provides many useful structures and models by which aspects of the everyday world can be modelled, it is all too easy to forget the abstraction made. An example of the difficulty that can arise when a tension in the ontology of a theory is set up, can be seen in Hayes's [1985b] complex ontology of directed surfaces, which are introduced in order to make sense of wetted surfaces.

The outside (in the sense of the outside aspect) of a physical body is characterised by a function in this theory that picks out the outermost layer of atoms or 'skin' of that region of space the body occupies, and is so named: 'skin(x)' is read as 'the skin of x':

(D62) $\mathrm{skin}(x) =_{\mathrm{def.}} \iota y[\forall z[C(z,y) \leftrightarrow \exists u[C\_Atom(u) \wedge P(u,x) \wedge C(cl(u),cl(compl(x))) \wedge C(z,u)]]]$

type $\mathrm{skin}(\tau){:}\tau \sqcup \mathrm{NULL}$, τ = SPATIAL\SPATIAL_UNIVERSE

The function skin(x) is not defined directly on physical objects, but indirectly by mapping bodies to the regions of space they occupy. This is discussed in more detail in section 3.4.

2.19: Defining a nest of spheres

Many physical phenomena obey the inverse square law, e.g. the variation in amplitude of a radial wave propogating across the surface of a pond, or the drop in the level of illumination of a surface as the distance between a constant light source and that surface varies. The geometrical basis for describing such phenomena is rooted in the construction of a nest of solid spheres (or balls) sharing a common centre. The relative distance between 'shells' of the set can then be exploited to provide a basis from which estimates of the intensity of some energy source radiating through the nest can be made.

A primitive monadic predicate symbol 'Ball' to the formal language; the denotation of which is a set of spherical solids (in the geometrical sense of the term "solid"). Using this primitive in the theory, a sphere and a nest is constructed.

Apart from notational differences (and the explicit introduction of the monadic predicate 'Ball(x)'), the following set of definitions are identical to those that appear in Tarski's [Tarski 1956] axiomatisation of solid body geometry [24]. The distinction between the terms "ball" and "sphere" used below mirror the common useage in mathematics: by a ball I mean a spherical *solid*, while a sphere is surface only in the manner of a shell.

'EXT_TANGENT(x,y)' is read as (ball) x is externally tangential to (ball) y', 'INT_TANGENT(x,y)' as '(ball) x is internally tangent to (ball) y', 'EXT_DIAMETR(x,y,z)' as '(ball) x and (ball) y are externally diametrical to (ball) z', 'INT_DIAMETR(x,y,z)' as ' (ball) x and (ball) y are internally diametrical to (ball) z', and 'CONCENT_PART(x,y)' as '(ball) x is a concentric part of (ball) y':

$$\text{(D63) EXT\_TANGENT}(x,y) \equiv_{def.} \text{Ball}(x) \wedge \text{Ball}(y) \wedge \forall zu\big[[\text{Ball}(z) \wedge \text{Ball}(u) \wedge P(x,z) \wedge P(x,u) \wedge \neg O(z,y) \wedge \neg O(u,y)] \rightarrow [P(z,u) \vee P(u,z)]\big]$$

$$\text{(D64) INT\_TANGENT}(x,y) \equiv_{def.} \text{Ball}(x) \wedge \text{Ball}(y) \wedge PP(x,y) \wedge \forall zu\big[[P(x,z) \wedge P(u,x) \wedge P(z,y) \wedge P(u,y)] \rightarrow [P(z,u) \vee P(u,z)]\big]$$

$$\text{(D65) EXT\_DIAMETR}(x,y,z) \equiv_{def.} \text{Ball}(x) \wedge \text{Ball}(y) \wedge \text{Ball}(z) \wedge \text{EXTERNALLY\_TANGENT}(x,z) \wedge \text{EXT\_TANGENT}(y,z) \wedge \forall uv\big[[\text{Ball}(u) \wedge \text{Ball}(v) \wedge \neg O(u,z) \wedge \neg O(v,z) \wedge P(x,u) \wedge P(y,v)] \rightarrow \neg O(u,v)\big]$$

```
(D66) INT_DIAMETR(x,y,z) ≡def. Ball(x) ∧ Ball(y) ∧ Ball(z) ∧
                               INT_TANGENT(x,z) ∧ INT_TANGENT(y,z) ∧
                               ∀uv[[Ball(u) ∧ Ball(v) ∧ ¬O(u,z) ∧ ¬O(v,z) ∧
                                    EXT_TANGENT(x,u) ∧ EXT_TANGENT(y,v)] →
                                                                  ¬O(x,y)]
(D67) CONCENT_PART(x,y) ≡def. Ball(x) ∧ Ball(y) ∧
                              [EQUAL(x,y) ∨
                              [PP(x,y) ∧ ∀zu[[Ball(z) ∧ Ball(u) ∧
                                              EXT_DIAMETR(z,u,x) ∧
                                              INT_TANGENT(z,y) ∧
                                              INT_TANGENT(u,y)] →
                                                 INT_DIAMETR(z,u,y)]] ∨
                              [PP(y,x) ∧ ∀zu[[Ball(z) ∧ Ball(u) ∧
                                              EXT_DIAMETR(z,u,y) ∧
                                              INT_TANGENT(z,x) ∧
                                              INT_TANGENT(u,x)] →
                                                 INT_DIAMETR(z,u,x)]]]
(D68) SPHERE(x) ≡def. ∃y[Ball(y) ∧ EQUAL(x,skin(y))]

type Ball(SPATIAL):UU
type Φ(τ,τ):UU, τ = SPATIAL\SPATIAL_UNIVERSE,
                Φ ∈  {EXT_TANGENT,INT_TANGENT}
type Φ(τ,τ,τ):UU, τ = SPATIAL\SPATIAL_UNIVERSE,
                Φ ∈ {EXT_DIAMETR,INT_DIAMETR}
type CONCENT_PART(τ1,τ2):UU, τ1 = SPATIAL\SPATIAL_UNIVERSE, τ2 = SPATIAL
type SPHERE(SPATIAL\SPATAIL_UNIVERSE):UU

(A20) ∀x[Ball(x) → Convex(x)]
(A21) ∀x[Ball(x) → ∃y[CONCENT_PART(y,x)]]
(A22) ∀x[Ball(x) → ∃y[CONCENT_PART(x,y)]
      x,y:SPATIAL
```

2.20: The metric part and the relation of relative distance.

While estimates of distance in everyday reasoning do involve units of measure, and in many cases take the form of tentative estimates, there is good reason to introduce a relation of relative distance into this theory.

As van Benthem points out [van Benthem 1982 Appendix A], contextual comparative relations - of one thing being nearer to another thing than something else - not only has a certain naturalness about it, it turns out to be a powerful primitive relation, to use in a partial axiomatisation of Euclidean space.

Following van Benthem, I add his ternary relation 'N(x,y,z)' read as 'y is nearer to x than z' and give a set of defining axioms. First the sorting function for N(x,y,z):

type $N(\tau 1,\tau 2,\tau 3){:}UU,\ \tau 1,\tau 2,\tau 3 \in \{SPATIAL, POINT\}$,

i.e. I allow measures between points, points and regions or between regions.

Next the set of defining axioms:

(A23) $\forall xyzu[[N(x,y,z) \wedge N(x,z,u] \rightarrow N(x,y,u)]$
(A24) $\forall xy\ \neg N(x,y,y)$
(A25) $\forall xyzu[N(x,y,z) \rightarrow [N(x,y,u \vee N(x,u,z)]]$
$\quad x,y,z,u{:}\tau,\ \tau \in \{SPATIAL, POINT\}$

Using the relation N, equidistance is immediately definable, 'E(x,y,z) is read as 'y is as near to x as z':

(D69) $E(x,y,z) \equiv def.\ \neg N(x,y,z) \wedge \neg N(x,z,y)$
type $E(\tau 1,\tau 2,\tau 3){:}UU,\ \tau 1,\tau 2,\tau 3 \in \{SPATIAL, POINT\}$

van Benthem also adds the following set of axioms: the first simply states that x is closer to itself than any other y, while the last two axioms express triangle inequalites:

(A26) $\forall xy[EQUAL(x,y) \vee C(cl(x),cl(y)) \vee N(x,x,y)]]$ [25]
(A27) $\forall xyzu[[N(x,y,z) \wedge N(z,x,y)] \rightarrow N(y,x,z)]$
(A28) $\forall xyzu[[E(x,y,z) \wedge E(z,x,y)] \rightarrow E(y,x,z)]$
$\quad x,y,z,u{:}\tau,\ \tau \in \{SPATIAL, POINT\}$

With the introduction of the distance function 'd(x,y)', the constant '0' (zero) and the standard set of ordering relations defined on numbers:'$\leq$','$<$','$\geq$' and '$>$':

$$\textbf{type}\ d(\tau 1,\tau 2):\mathrm{NUMBER},\ \tau 1,\tau 2 \in \{\mathrm{SPATIAL},\mathrm{POINT}\}$$
$$\textbf{type}\ 0:\mathrm{NUMBER}$$
$$\textbf{type}\ \leq(\mathrm{NUMBER},\mathrm{NUMBER}):\mathrm{UU}$$
$$\textbf{type}\ <(\mathrm{NUMBER},\mathrm{NUMBER}):\mathrm{UU}$$
$$\textbf{type}\ \geq(\mathrm{NUMBER},\mathrm{NUMBER}):\mathrm{UU}$$
$$\textbf{type}\ >(\mathrm{NUMBER},\mathrm{NUMBER}):\mathrm{UU}$$

and the axiom:

(A29) $\forall xy\ d(x,y)\geq 0$
$x,y:\tau,\ \tau \in \{\mathrm{SPATIAL},\mathrm{POINT}\}$

one can immediately define the following equivalences:

(A30) $\forall xyz[N(x,y,z) \leftrightarrow [d(x,y) < d(x,z)]]$
(A31) $\forall xy[\mathrm{EQUAL}(d(x,y),0) \leftrightarrow [\mathrm{EQUAL}(x,y) \vee C(cl(x),cl(y)) \vee \mathrm{IN}(x,cl(y)) \vee \mathrm{IN}(y,cl(x))]]$
$x,y,z:\tau,\ \tau \in \{\mathrm{SPATIAL},\mathrm{POINT}\}$.

The primitive relation 'x<y' is axiomatised as:

(A32) $\forall x\ \neg x<x$
(A33) $\forall xyz[[x<y \wedge y<z] \rightarrow x<z]$
$x,y,z:\mathrm{NUMBER}$,

and the axiom:

(A34) $\forall xy[\mathrm{EQUAL}(x,y) \vee x<y \vee y>x]$[26],
$x,y:\mathrm{NUMBER}$

is added.

(D70) $x\geq y \equiv\mathrm{def.}\ \neg[x<y]$
(D71) $x>y \equiv\mathrm{def.}\ x\geq y\ \&\ \neg\mathrm{EQUAL}(x,y)$
(D72) $x\leq y \equiv\mathrm{def.}\ \neg[x>y]$
$x,y:\mathrm{NUMBER}$

The standard set of axioms that define a metric space with the distance function is not given here. In particular the axiom: $\forall xyz[d(x,z) \leq [d(x,y)+d(y,z)]]$, and the function $x+y$, is missing. However, the constant 0 and a total ordering on (symbolic) numbers is all that is required to illustrate the formalism.

2.21: The temporal part

Interval logics for reasoning about action and time have been much researched e.g. Hamblin 1967,1971, Allen 1981,1984, Allen and Koomen 1983, Allen and Kautz 1985, and Allen and Hayes 1985,1987. This being so, the pure temporal part of the formalism is given much less attention in this thesis, than that part used to model space.

For the temporal part of the theory, the ontology is restricted so that only open regions are admitted to the status of temporal regions, which we call periods. The justification for this is largely motivated by questions of ontological and technical simplicity. Firstly, there is no immediate practical gain to be made by allowing periods to be either open, closed or clopen. One can define the standard 13 mutually exclusive and exhaustive interval relations [see e.g. Allen and Hayes 1985], in the theory by keeping periods open and adding to the theory a relation of precedence defined on pairs of periods, namely the relation $B(x,y)$ cited earlier [27]. Secondly, by dividing periods into moments (understood to be arbitrarily small periods of time - distinct from points) and intervals, and stipulating that propositions are indexed to periods only, the set of moments can be made discrete and totally ordered, avoiding the classic problem of the "divided instant", since meeting periods do not have a shared boundary moment [28].

Using the precedence relation B, a set of definitions functioning as analogues to Allen's set of defined interval relations on pairs of periods are given. For reasons of the general familiarity of Allen's work in AI circles, I adopt Allen's set of names for the defined relations. First I repeat the set of defining axioms for the relation 'B(x,y)' read as 'x is before y', then give the definitions:

(A4) $\forall x\ \neg B(x,x)$

(A5) $\forall xyz[[B(x,y) \wedge B(y,z)] \rightarrow B(x,z)]$

(A6) $\forall xy[B(x,y) \rightarrow \forall zu[P(z,x) \wedge P(u,y)] \rightarrow B(z,u)]$

(D73) $\mathrm{MEETS}(x,y) \equiv\mathrm{def.}\ B(x,y) \wedge \neg\exists z[B(z,y) \wedge B(x,z)]$

(D74) $\mathrm{BEFORE}(x,y) \equiv\mathrm{def.}\ B(x,y) \wedge \exists z[B(z,y) \wedge B(x,z)]$

(D75) $\mathrm{OVERLAPS}(x,y) \equiv\mathrm{def.}\ PO(x,y) \wedge \exists z[P(z,x) \wedge B(z,y)]$

(D76) $\mathrm{STARTS}(x,y) \equiv\mathrm{def.}\ PP(x,y) \wedge \exists z[\mathrm{MEETS}(z,x) \wedge \mathrm{MEETS}(z,y)]$

(D77) $\mathrm{FINISHES}(x,y) \equiv\mathrm{def.}\ PP(x,y) \wedge \exists z[\mathrm{MEETS}(x,z) \wedge \mathrm{MEETS}(y,z)]$

(D78) $\mathrm{DURING}(x,y) \equiv\mathrm{def.}\ PP(x,y) \wedge \exists zu[PP(z,y) \wedge B(z,x) \wedge PP(u,y) \wedge B(x,u)]$

(D79) $B^{-1}(x,y) \equiv \mathrm{def.}\ \neg B(y,x)$

(D80) $\mathrm{MEETS}^{-1}(x,y) \equiv\mathrm{def.}\ \mathrm{MEETS}(y,x)$

(D81) $\mathrm{BEFORE}^{-1}(x,y) \equiv\mathrm{def.}\ \mathrm{BEFORE}(y,x)$

(D82) $\mathrm{OVERLAPS}^{-1}(x,y) \equiv\mathrm{def.}\ \mathrm{OVERLAPS}(y,x)$

(D83) $\mathrm{STARTS}^{-1}(x,y) \equiv\mathrm{def.}\ \mathrm{STARTS}(y,x)$

(D84) $\mathrm{FINISHES}^{-1}(x,y) \equiv\mathrm{def.}\ \mathrm{FINISHES}(y,x)$

(D85) $\mathrm{DURING}^{-1}(x,y) \equiv\mathrm{def.}\ \mathrm{DURING}(y,x)$

(D3) $\mathrm{EQUAL}(x,y) \equiv\mathrm{def.}\ P(x,y) \wedge P(y,x)$

type $\Phi(\tau,\tau)$:UU, τ = PERIOD, $\Phi \in \{\mathrm{B,MEETS,BEFORE,\ldots,DURING}^{-1}\}$

Several points are worth raising here. The first is that this set of interval relations is quite different to those developed by Clarke [Clarke 1985]. A major difference is the transitivity of EQUAL, where the comparable relation used by Clarke, namely that of contemporaneous related spatio-temporal regions is carefully defined to be non-transitive [29]. The second point is that unlike Allen's relations, the above set of relations are not mutually exclusive. This surprising fact is made clearer once it is realised that temporal regions, like their spatial counterparts do not have to be connected (i.e. in one piece). Thus e.g. just because two intervals x and y are discrete, it does not follow that one must be before the other or vice versa - given EQUAL(sum(x1,x2),x), we could have B(x1,y) $\wedge$ B(y,x2) for example. Given an intended model where all periods are individually connected, and with additional axioms required to axiomatise a standard interval logic, one would expect the above set of relations to become mutually exclusive and exhaustive for pairs of periods. This is discussed in more detail below.

For the modelling problems described in this thesis, only the B and MEETS relations are actually used. MEETS is irreflexive (T78), asymmetrical (T79) and intransitive (T80):

(T78) $\forall x\ \neg \mathrm{MEETS}(x,x)$
(T79) $\forall xy[\mathrm{MEETS}(x,y) \rightarrow \mathrm{MEETS}(y,x)]$
(T80) $\forall xyz[\mathrm{MEETS}(x,y) \wedge \mathrm{MEETS}(y,z)] \rightarrow \neg \mathrm{MEETS}(x,z)]$
x,y,z:PERIOD

Periods are split into moments and intervals. Moments are simply periods with no proper parts, and intervals are periods that not moments:

(D86) $\mathrm{MOMENT}(x) \equiv$def. $\forall yz[[P(y,x) \wedge P(z,x)] \rightarrow \neg B(y,z)]$
(D87) $\mathrm{INTERVAL}(x) \equiv$def. $\mathrm{PERIOD}(x) \wedge \neg \mathrm{MOMENT}(x)$

Together with the information encoded in the sort lattice L_s, the following axiomatisation ensures every period has a moment as a part and that periods are open. Periods are either moments or intervals, and moments and intervals are periods. Moreover, the precedence relation B is connected when defined on moments, and the time line is unbounded (infinite, assuming a metric) in both temporal directions:

(A35) $\forall x[\mathrm{PERIOD}(x) \rightarrow [\mathrm{MOMENT}(x) \vee \mathrm{INTERVAL}(x)]]$
(A36) $\forall x[\mathrm{PERIOD}(x) \rightarrow \mathrm{OPEN}(x)]$

(A37) $\forall xy[[MOMENT(x) \wedge MOMENT(y)] \rightarrow [EQUAL(x,y) \vee B(x,y) \vee B(y,x)]]$

(A38) $\forall x[MOMENT(x) \rightarrow \exists y[MOMENT(y) \wedge MEETS(x,y)]]$

(A39) $\forall x[MOMENT(x) \rightarrow \exists y[MOMENT(y) \wedge MEETS(y,x)]]$

x,y:PERIOD

The following theorems are forthcoming:

(T81) $\forall x[PERIOD(x) \rightarrow \exists y[MOMENT(y) \wedge P(y,x)]]$

(C82) $\forall x[MOMENT(x) \leftrightarrow [Atom(x) \wedge PERIOD(x)]]$

x,y:PERIOD

Although the ontology of time presented here is very similar to that of Allen and Hayes [1985,1987] material from Carnap [1958] and Woodger [1937] was actually used when building the formalism. One difference between this formalism and that of Allen and Hayes, is that periods are explicitly axiomatised as open regions. There is also a difference between the interpretations given to both formalisms. While Allen and Hayes consider beginnings and endings of their moments [Allen and Hayes 1985, p531], moments within this formalism are not construed as having beginnings and endings but rather that beginings and endings *are* taken as moments; and that moments are only individuated with respect to other periods that meet it and it meets, not by points.

Given the model of time used here is discrete at the level of moments, three temporal functions are added which generate the initial and final moments for any interval and the next moment (in time) for any period. Note that restrictions are needed for these functions, e.g. given a much richer sort structure, the function final(x) would be only well sorted for periods bounded above. 'initial(x)' is read as 'the initial moment of x, 'final(x)' as 'the final moment of x' and 'next(x)' as 'the next moment (in time) after x':

(D88) initial(x) =def. $\iota y[\mathrm{INTERVAL}(x) \wedge \mathrm{MOMENT}(y) \wedge \mathrm{PP}(y,x) \wedge \neg\exists z[\mathrm{MOMENT}(z) \wedge \mathrm{PP}(z,x) \wedge \mathrm{B}(z,y)]]$ [30]

(D89) final(x) =def. $\iota y[\mathrm{INTERVAL}(x) \wedge \mathrm{MOMENT}(y) \wedge \mathrm{PP}(y,x) \wedge \neg\exists z[\mathrm{MOMENT}(z) \wedge \mathrm{PP}(z,x) \wedge \mathrm{B}(y,z)]]$ [30]

(D90) next(x) =def. $\iota y[\mathrm{PERIOD}(x) \wedge \mathrm{MOMENT}(y) \wedge \mathrm{MEETS}(x,y)]$ [30]

type initial(INTERVAL):MOMENT

type final(INTERVAL):MOMENT

type next(PERIOD):MOMENT

Added to the axioms of the theory, these definitions imply each interval has as least two momentary parts (i.e. an initial and final moment). This choice is motivated by a desire to provide a formal semantics for the intuitive temporal locution "...the next moment...".

Often states, events and processes occur over periods of time that are punctuated by periods of rest. For example, the activity described as reading a book is rarely done continuously without having some form of break. In order to allow intervals to have this property, a new predicate and its dual are introduced then defined, 'Disconnected_Period(x)' is read as 'x is a disconnected period' and 'Connected_Period(x)' 'x is a connected period':

(D91) Disconnected_Period(x) $\equiv$def. $\exists yz[\mathrm{EQUAL}(\mathrm{sum}(y,z),x) \wedge \mathrm{BEFORE}(y,z)]$ [31]

(D92) Connected_Period(x) $\equiv$def. Period(x) $\wedge$ $\neg$Disconnected_Period(x)

type Disconnected_Period(INTERVAL):UU

type Connected_Period(PERIOD):UU

The idea of allowing arbitrary unions of periods and defining a set of interval relations defined on sets of disconnected (or "non-convex") periods has been explored by Ladkin [1986a,b]. Ladkin [1986b] shows that an exhaustive enumeration of such relations is infeasible simply because the number of possible relations grows exponentially. For this reason, while a use is found for reasoning using disconnected periods, no attempt is made to construct a temporal lattice (in the manner of lattice L_c) for an extended set of interval relations.

A desirable result for this part of the theory would be to show that given all intervals are connected, then a model used to interpret the set of defining axioms of this theory, would also be a model in Allen's interval logic. However, this must remain a conjecture, since no proof has been secured to show that with the condition that all periods are individually connected, the defined set of relations become mutually exclusive and exhaustive for pairs of periods. In other words, one would need to prove the following set of *theorems*:

$$\forall xy[[\mathrm{Connected}(x) \wedge \mathrm{Connected}(y)] \rightarrow [O(x,y) \leftrightarrow [B(x,y) \vee B^{-1}(x,y)]]]$$
$$\forall xy[[\mathrm{Connected}(x) \wedge \mathrm{Connected}(y)] \rightarrow [PO(x,y) \leftrightarrow [\mathrm{OVERLAPS}(x,y) \vee \mathrm{OVERLAPS}^{-1}(x,y)]]]$$
$$\forall xy[[\mathrm{Connected}(x) \wedge \mathrm{Connected}(y)] \rightarrow [PP(x,y) \leftrightarrow [\mathrm{STARTS}(x,y) \vee \mathrm{FINISHES}(x,y) \vee \mathrm{DURING}(x,y)]]]$$
$$\forall xy[[\mathrm{Connected}(x) \wedge \mathrm{Connected}(y)] \rightarrow [PP^{-1}(x,y) \leftrightarrow [\mathrm{STARTS}^{-1}(x,y) \vee \mathrm{FINISHES}^{-1}(x,y) \vee \mathrm{DURING}^{-1}(x,y)]]]$$
$$\forall xy[[\mathrm{Connected}(x) \wedge \mathrm{Connected}(y)] \rightarrow [\mathrm{MEETS}(x,y) \bullet ,\ldots, \bullet\ \mathrm{DURING}^{-1}(x,y)]]$$

where '$\bullet$' means exactly one literal of the consequent is true, and where the ellipses ' ,..., ' include the missing relations defined by (D74) to (D84) (including EQUAL) defined above. In other words exactly one relation will hold given the condition that intervals are connected.

Finally, some ordering axioms are required: the first states that if moment meets moments y and z, then y and z are identical. Similarly for the second axiom: if moments x and y meet moment z, x and y are identical.

The last axiom states that for any two pairs of moments x and y, and z and u, either x meets u (in which case they will be identical), or x will be before and separated from u, and the same for z and y.

(A40) $\forall xyz[[\mathrm{MOMENT}(x) \wedge \mathrm{MOMENT}(y) \wedge \mathrm{MOMENT}(z)] \rightarrow$
$[\mathrm{MEETS}(x,y) \wedge \mathrm{MEETS}(x,z)] \rightarrow \mathrm{EQUAL}(y,z)]$

(A41) $\forall xy[[\mathrm{MOMENT}(x) \wedge \mathrm{MOMENT}(y) \wedge \mathrm{MOMENT}(z)] \rightarrow$
$[\mathrm{MEETS}(x,z) \wedge \mathrm{MEETS}(y,z)] \rightarrow \mathrm{EQUAL}(x,z)]$

(A42) $\forall xyzu[\mathrm{MOMENT}(x) \wedge \mathrm{MOMENT}(y) \wedge \mathrm{MOMENT}(z) \wedge \mathrm{MOMENT}(u) \wedge$
$\mathrm{MEETS}(x,y) \wedge \mathrm{MEETS}(z,u)] \rightarrow [\mathrm{MEETS}(x,u) \vee \mathrm{BEFORE}(x,u) \vee$
$\mathrm{BEFORE}(z,y)]]$

x,y,z,u:MOMENT

Finally, it must be pointed out that if a standard interval logic is all that is required, then this can be easily accomodated in this formalism. Given the defined MEETS relation one could import much of Allen and Hayes [1985,1987] axiomatisation into this theory. In this case, the sorting functions for the set of Boolean operators defined on regions would need strengthening: i.e. sum(x,y) and prod(x,y) would need to be restricted to spatial regions only, since without this restriction, arbitrary combinations of regions will be sanctioned, thus building in contradictory consequences.

2.22: Summary

This chapter describes the bulk of the formal theory that is used to describe space and time. First the sorts of the theory were outlined then embedded in a sort lattice ($\mathcal{L}_{\mathrm{s}}$), then from the two primitive relations, C and B, a set of dyadic relations were added and defined on the sort REGION. A subset of these relations were singled out and embedded in a relational lattice ($\mathcal{L}_{\mathrm{C}}$). A set of constants and functions were added, and worked into a set of further definitions. In particular, the function conv(x) was used to define a set of relations and functions that characterised notions of being inside and outside.

So far no attention has been given to how physical bodies are to be integrated into the formal theory, and how states, events and processes are represented and reasoned with. This is the subject matter of the following chapter.

Chapter 3: Reasoning about physical domains over time

3.1: Introduction

There is a considerable body of literature that has been written on the subject of time. For good introductory texts which concentrate upon the formal aspect of time, see Rescher and Urquhart [1971] and van Benthem [1982]. In general, the term "temporal logic" covers formal theories that include reasoning about states, events and processes, agency, planning and aspect, as well as being used in the formal specification of programs (see e.g. the collection of articles in Galton [1987]).

This chapter focuses on the changing world and the formal means to describe it. Firstly, states of affairs, events and processes are introduced and then incorporated into the theory developed so far. Secondly, I show how physical bodies and their properties are assigned to spatial and temporal regions. Finally, I show how by exploiting sortal and other empirical information (abstracted out from the modelled domain), problems associated with temporally projected inference in the theory can be effectively constrained.

3.2: States of affairs, events and processes

States (of affairs) and events are characterised along the lines of Galton [Galton 1984]. According to Galton [1984 p24], the distinction between a state and an event is decided by the way we choose to report happenings, rather than by what as a matter of fact goes on in the world. In this respect, states turn out to be reports that being true for some period of time, continue to be true for any subperiod of that time. States also obtain at moments, are measureable, can be negated and are homogeneous. In contrast, events are unitary (in the sense that the specified event description does not remain true over any sub-period), have individual occurences, can be counted and have no negation.

Thus for example, a description of a relation between two bodies that preserves some degree of topological invariance over a period of time can be associated with a state, while two temporally linked states incorporating an explicit description of change in that topological property can be regarded as an event. In general, processes are assumed be a special kind of event, where the event can decomposed into a specified temporally ordered sequence of state descriptions. As Galton [Galton 1987 p194] concedes, it is unlikely that every event can be simply reduced into a sequence of states of the form: 'first S1, then S2, ..., then Sn'. However, many events can be effectively treated as such. This decomposition of events (or processes) into state descriptions is subsequently adopted.

To make the distinctions mentioned above clearer, consider the concrete example of a working pump which has a piston rising and falling in an inner chamber. To say the pump's piston is in contact with the wall of the inner chamber during some period of time, reports a state (since the relationship will remain constant over any sub period of that time in which the state obtains). In contrast, a cycle of the pump coincides with the report of an event where although parts of the cycle may be identified as phases, the cycle cannot be correctly said to be true over any subperiod of time in which the event occurs. Given the fact that the rising of the piston can be also be construed as a state (since again for any subperiod the rising continues to take place), this leads to a distinction between states of change (as in the case where the piston is said to be rising) and states of no change (as in the case where the piston is continually in in contact with the wall of the inner chamber).

In general, reports of events are only mapped to intervals, while reports of states can be mapped to periods of any duration - i.e. unlike events, states can clearly be momentary. An event cannot be *captured* at a moment (even though one can have momentary events e.g. a flash of light, and punctual events e.g. switching a light off). Reports of events require some notion of completion to take place before one can identitify the event qua event. And given that an event entails a change in the truth value for some proposition over time, if moments are taken to be the minimal periods over which propositions are indexed, then the description of change having occurred must be related to an interval.

The fact that one can have in addition to momentary states, momentary and punctual events is explained by Galton as follows. Consider time to be discrete. An event is momentary if up to some moment in time proposition $\neg\varphi$ holds, for the next moment φ holds, then $\neg\varphi$ holds for the following moment. In contrast, a punctual event arises if up to some moment in time $\neg\varphi$ holds then φ holds at the next and following moments. The momentary event occurs within the interval containing the moment where φ holds, but cannot be located at the moment φ holds since additional information of the duration of the event is required before its momentary status can be decided. The moment where φ holds is a momentary state on this analysis, but again like the case of an event the duration of the description over an interval is required before it can be so described. Similarly given a punctual event the event cannot be described as such at any moment, but only within an interval.

A common source of difficulty and lingustic confusion appears to arise when indexing propositions to moments or points in time where change arises. Take the frequently cited case of a ball following a parabolic path where it rises then falls. There is a temptation to say that that at the nadir of a ball's parabolic flight path, the ball is stationary; but at each moment (whether understood to be periods, or points) the world can only be described atemporally. Loosely speaking one might say that no change arises within a moment, but strictly speaking this is either vacuously true or meaningless. All one can say is that up to a particular moment in time the ball rises and after that moment the ball falls - one simply cannot *say* the ball is stationary at that or any moment, pace Allen and Hayes' comment "... the ball is stationary only for a *time of zero duration*, which in fact is the point where the ball is rising meets the interval in which it is falling." [Allen and Hayes 1987, p2] - my italics. A similar point arises when moments are taken as having duration, for again it is not correct to say "the ball ... rising meets the moment where it is stationary, which in turn meets the interval where it is falling" op cit. Unfortunately for Allen, the type of confusion identified here, reflects a failure to recognise the importance of maintaining a clear distinction between reports of states and events. This leads him into other semantic difficulties, as witnessed in the comment "an event such as 'remaining in the same position' could never occur except at a time point"! [Allen 1981, p8].

While states and events seem to provide seem reasonably clear identifying characteristics, processes straddle awkwardly between the two. Again, one can characterise processes in the way we choose to describe the world. Consider the case where a protozoan surrounds and engulfs some item of food in order to digest it. This process is called phagocytosis.

Part of this process involves another sub-process where the protozoan engulfs the food. Now at one level of description this particular process could readily be described as a state of change, if the protozoan is surrounding the food during some period, it's still true to say it is surrounding the food at some sub-period. But other named processes are not. For example phagocytosis is a case in point which has a unitary quality with identifiable sub-processes as phases. For this reason, although definitions of processes developed in this formalism are typically unpacked in terms of a specified sequence of state descriptions that change over the duration of the process, named processes are not formally identified with either states of affairs or events since in either case the referent can remain the same [32].

The notion of a process is central to Forbus's Qualitative Process Theory (QPT) although it is difficult to see how Forbus's processes differ from events. Forbus [Hobbs and Moore 1985, p185] characterises processes as "something that causes changes through time" where the explicit description of processes operating on a given state are said to facilitate a prediction of how situations will change over time. No formal analysis is offered, however. The *Sole Mechanism* assumption used in QPT: that all changes in physical systems are caused directly or indirectly by processes, clearly brings out the view that causation is an essential component, although without the clear distinction between processes and events, the Sole Mechanism assumption looks rather uninformative - i.e. only events bring about change. The explicit representation of causation is not covered in this thesis, although Allen [1981] shows one method how causally linked events can be formally described using an interval logic.

The decision how best to index propositions to periods of time in a first order interval-based theory is a vexing one. The simplest strategy is to transform every n-place predicate of the theory into an n+1 place predicate. In this case the extra term (marking the adverbial modifier) is used to index the period of time over which some state or event expressed in the predicate, obtains or occurs. Reichgelt [Reichgelt 1987] calls this the naive first order treatment of time.

The advantage of the naive approach is simplicity and a clear linguistic reading, e.g. 'Connects(x,y,z)' as ' x connects with y at or throughout period z, and 'Engulfs(x,y,z)' as 'x engulfs y during period z', for a description of a state and an event respectively. Against this approach is a certain lack of expressiveness. For example, one cannot explicitly state that in general, causes precede their effects, and changing ontologies over time are not readily accomodated [see Reichgelt 1987].

In Allen's [1981,1984] theory, states, events and processes are reasoned about explicitly by using three distinguished relations, 'Holds(p,t)', 'Occurs(e,t)' and 'Occurring(p,t)'. The relations 'Holds(p,t)' and 'Occurs(e,t)' link what he calls "properties" and events to the times they obtain or endure for respectively, the latter links processes to the time they are occurring for. In addition, he includes a set of functions: 'and(p,q)', 'or(p,q)', 'not(p)', 'all(x,p)' and 'exist(x,p)' which corresponding to the familiar logical operators, enable him to use his properties to name complex logical expressions, e.g. 'Holds(and(p,q),t)'.

A certain uneasiness concerning the semantic foundation of Allen's formalism can be recognised in Turner [1984, p87-88], Reichgelt [1987] and Shoham [1988, p39]. According to Allen, the holds relation binds "properties" to intervals of time. However, given the intended linguistic reading, 'p' is mentioned and 't' used. For this expression to be well-formed, the 'p' should be in quotation marks, i.e. Holds("p",t). But this immediately requires some extension to the normal recursively defined set of formation rules used to construct wff in FOL. On the other hand, one cannot naively identify sentences with singular terms without incurring deep problems. Davidson [1984 p19] (citing Frege) shows how if the meaning of a singular term is identified with its reference, all sentences alike in truth value can be shown to be synonomous!

It is possible to avoid such problems by nominalising sentences, although Allen does not go this route. In this case, the 'p' functions as a genuine term and a place holder for the nominalised sentence, e.g. the nominalisation of the (open) sentence 'x is connected with y' would be 'x's being connected with y'. By doing this, the wff 'Holds(c(x,y),z)' would now read as 'x's being connected with y holds throughout period z', which is perfectly acceptable. The advantage of nominalisation is a gain in expressiveness. For example, the means to talk explicitly about states, events and processes become available whereas before the distinctions were embedded in the meaning given to specific predicates. However, there are certain problems going down this route. The first is that nominalised expressions frequently require complex paraphrasing, while the second simply arises from the introduction of a new set of functional expressions into the formal language [**]

In general I use the naive first order theory of time to demonstrate the theory. However, where it is expedient to talk about states, events and processes *explicitly*, I choose the nominalisation route mentioned above. The latter part of the theory is developed as follows:

Two additional primitive sort symbols 'STATE' and 'EVENT' are added to the sort lattice:

sort STATE $\sqsubset$ T
sort EVENT $\sqsubset$ T,

where the set of primitive sort symbols extend to the set: {NULL,REGION,POINT,PHYSOB,NUMBER,STATE,EVENT}.

Next two relations are added to the formal language: 'OBTAINS(x,y)' read as 'x obtains throughout or at period y', and OCCURS(x,y)' as 'x occurs during period y'. OCCURS(x,y) is identical to Allen's relation OCCURS(e,t), and the relation OBTAINS(x,y) is identical to Allen's relation HOLDS(p,t), save for the nominalisation of the sentence p. The sorting functions for these relations are:

type OBTAINS(STATE,PERIOD):UU
type OCCURS(EVENT,INTERVAL):UU

Axioms are then added that govern the intended meaning given for these relations:

(A43) $\forall xy[\mathrm{OBTAINS}(x,y) \leftrightarrow \forall z[P(z,y) \rightarrow \mathrm{OBTAINS}(x,z)]]$
(A44) $\forall x[\exists y[\mathrm{OBTAINS}(x,y) \leftrightarrow \mathrm{STATE}(x)]$
(A45) $\forall xy[\mathrm{OCCURS}(x,y) \rightarrow \neg\exists z[PP(z,y) \wedge \mathrm{OCCURS}(x,z)]]$
(A46) $\forall xy[\exists y[\mathrm{OCCURS}(x,y) \leftrightarrow \mathrm{EVENT}(x)]$

In Allen's [1984] theory, an additional relation appears, namely 'OCCURRING(p,t)'. This relation is used to describe what he calls "processes". For Allen, processes "refer to some activity not involving a culmination or anticipated result", while events "describe an activity that involves a *product or outcome [Allen 1984 p132]. While processes and* events are stipulated to be occurrences, Allen notes a problem with his axiom:

$\forall ett'[[\mathrm{OCCUR}(e,t)\ \&\ \mathrm{IN}(t',t)] \rightarrow \neg\mathrm{OCCUR}(e,t')]$

(in words: if event e occurs during period t and t' is a sub-period, then e doesn't occur during t'). This fails to hold for processes, since someone said to be walking for a period of time might stop for a rest. In view of such difficulties, Allen separates out processes and uses the relation OCCURRING(p,t) with a set of defining axioms.

Galton [1990] finds Allen's categorisation of processes both wanting and unnecessary. Identifying narrow and broad senses of locutions such as "I am walking", he argues that in the broad sense one can be said to be walking for a period of time even though one might have a brief rest; while in the narrow sense "I am walking" is simply false if one considers the walk takes place over the same period of time. For Galton, Allen's "processes" can be grouped with Allen's properties. In its place, Galton suggests two ways how reports of processes can be treated in an interval logic. The first is an implicit categorisation which makes use of an extended set of the standard HOLDS and OCCURS relations, i.e. indexing reports of states and events to moments or intervals of time, while the second (drawing off earlier work - see Galton 1984,1987) introduces special progressive operators defined on events.

A simple alternative way to tackle this problem is to make use of individual connected and disconnected periods over which some state of affairs is said to obtain. Thus if the broad sense is intended, then the period is disconnected, and if the narrow sense is intended, then the period is connected: '$OBTAINS_N(x,y)$' and '$OBTAINS_B(x,y)$' are both read (ambiguously) as 'x obtains during y':

(A47) $OBTAINS_N(x,y) \equiv def.\ OBTAINS(x,y) \wedge CONNECTED\_PERIOD(x)$
(A48) $OBTAINS_B(x,y) \equiv def.\ OBTAINS(x,y) \wedge DISCONNECTED\_PERIOD(x)$

type $OBTAINS_N$(STATE,PERIOD):UU
type $OBTAINS_B$(STATE,INTERVAL):UU

3.3: Integrating empirical and spatial information

The ontological distinction between physical objects and (spatial) regions and is made explicit in the formalism by making the sorts PHYSOB and REGION disjoint. However some means must be provided which preserves this ontological distinction without un-necessarily duplicating properties and relations that are correctly ascribed to regions but seem equally applicable to physical entities. For example, in everyday discourse the relation of being inside makes sense whether we are talking about water inside a cup, but equally in a geometrical context when talking about a partitioning of space.

3.4: Mapping physical objects to regions of space.

Physical objects are mapped to regions by means of a transfer function 'space(x,y)' (cf Hayes' [1985b] one place transfer function 'space(x)') read as 'the space of x at (moment) y'. This function either maps a physical object to the spatial region it occupies at a given moment, or is of sort NULL if the physical object does not exist at that moment:

type space(PHYSOB,MOMENT):SPATIAL ⊔ NULL

For brevity and ease in reading the formalism an alternative syntax is now adopted: 'x|y' is now written instead of 'space(x,y)'. The use of this function has the following consequences that should be noted. Firstly, the wff Inside(water1,cup1) becomes illsorted given the normal interpretation for these terms; rather it should be Inside(water1|t,cup1|t): the justification being that talk of being inside relates (in this instance) physical objects to a theory of space with physical objects construed as though they are spatial regions.

Physical objects support a set of empirical properties which spatial objects do not, and spatial objects support a set of geometrical and topological properties which are strictly speaking not properties of physical objects. Within the formalism the wff Hard(steel_ball1) is well sorted, but the wff Hard(skin(steel_ball1))) is not, because the predicate Hard used here does not apply to regions. It might be thought that this complication can be rectified by simply introducing a new transfer function phys(x,y) that maps an arbitrary spatial region x at a moment y to some physical object, but this is not feasible. Given the intended model where space contains a potential infinite number of regions with varying degrees of connectivity, spatial regions (now individuated in terms of a set of co-ordinated points) can map to compositions of physical objects (embedded in that space) which have no clear individuating characteristics, and a fortiori no clear named sortal categories. In view of this, the following is done. Suppose (taking the above example) we want to attribute the property of hardness to the surface of a steel ball, then we express this fact as follows:

$\mathrm{Hard(a)} \wedge \forall t\,[\mathrm{EQUAL(a|t,skin(steel\_ball1|t))}]$.

I.e. one picks out the physical object in question supporting some empirical property (in this example, object a) and relates it to some other physical object (steel_ball1) by mapping both a and steel-ball1 to the space they occupy at a given moment in time, and then stipulating the spatial relationship between them (in this case an identity between the space occupied by a and the skin of the steel ball). The same technique is done for other spatial properties, e.g. to say (loosely speaking) that the interior of some object b is hard, this is expressed as follows:

$\mathrm{Hard(a)} \wedge \forall t\,[\mathrm{EQUAL(a|t,int(b|t)}]$.

Again, in view of such complications it may also be thought that if the sort PHYSOB were simply stipulated to be a sub-sort of the sort REGION, where the sorts SPATIAL, PERIOD and PHYSOB were pairwise disjoint, the cited difficulty could be met. But this introduces further complications. The complications arise once the sorting functions are relaxed so that expressions such as sum(cup,chair), compl(chair) and conv(cup) become legitimate terms. What physical objects (or are they really regions of space?) are the denotations of these terms, and what are their respective sorts? No easy answer seems forthcoming. Indeed if we do allow such expressions to be well formed/sorted, then the result sorts for these terms, if they are to denote physical objects, cannot in general be anything more specific than PHYSOB, except perhaps in the trivial case where an identity has been hitherto established. For example, take the term sum(chair1,chair2). This cannot be of result sort CHAIR, unless of course both chair1 and chair2 are identical. But equally, by the same argument we should allow sum(water1,water2) to be of sort WATER, since we do talk about distinct bodies of the same material as one body, e.g. the blood inside our body, even though quantities may be separated as it passes through distinct chambers in the heart [34].

Given the ontological distinction that exists between physical objects and the spatial abstractions that are commonly used to represent them, it is of paramount importance to recognise that if an abstraction is made, that that abstraction is clearly kept in mind. For some domains, reasoning about physical objects as though they are regions of space can be quite adequate; indeed, parsimony with respect to an abstraction is not only desireable in our everyday understanding and working with complex phenomena, it lies at the very foundation of theory construction. However if some abstraction has been made, for example talking about physical objects as though they are regions of space, care must be exercised not to import properties into the domain that is not supported by the theory being described. Failure to keep the distinction not only disrupts the legitimacy of the theory, it can lead to a muddied ontology, e.g. Hayes' [1985b] use of directed surfaces which can be wet. The difficulties described above, are of course example to this. With this in mind, spatial relations holding between physical objects are subsequently handled as follows.

For the naive treatment of time, an abbreviational schema '$\Phi(x,y,z)$' is used here, which is understood to mean that x is in relation Φ to y at or throughout period z':

(D93) $\Phi(x,y,z) \equiv$def. $\forall u[P(u,z) \rightarrow \Phi(x|u,y|u)]$

type Φ(PHYSOB,PHYSOB,PERIOD):UU, $\Phi \in \{\alpha$: **type** α(SPATIAL,SPATIAL):UU$\}$

e.g. $C(x,y,z) \equiv$def. $\forall u[P(u,z) \rightarrow C(x|t,y|t)]$

type C(PHYSOB,PHYSOB,MOMENT):UU,

In contrast, with the reified approach the metalogical function $\phi(x,y)$ and relation $\Phi(x|v,y|v)$ used below are taken to indicate that the function $\phi(x,y)$ represents the nominalised (open) sentence $\Phi(x|v,y|v)$. Thus, in the example given below, the function $c(x,y)$ ('x's being connected with y') is the nominalisation of the open sentence $C(x,y)$. The metalogical variables indicate that the same principle extends to all other relations defined in the theory that support a spatial interpretation, e.g. $p(x,y)$ with $P(x,y)$, and inside(x,y) with INSIDE(x,y) and so on.

(D94) $\phi(x,y)$ =def. $\iota z[\forall u[\text{OBTAINS}(z,u) \leftrightarrow \forall v[P(v,u) \rightarrow \Phi(x|v,y|v)]]]$

type ϕ(PHYSOB,PHYSOB):STATE $\sqcup$ NULL

e.g. $c(x,y)$ =def. $\iota z[\forall u[\text{OBTAINS}(z,u) \leftrightarrow \forall v[P(v,u) \rightarrow C(x|v,y|v)]]]$

type c(PHYSOB,PHYSOB):STATE $\sqcup$ NULL

type OBTAINS(STATE,PERIOD):UU

One further variant of the reified approach needs to be mentioned here. Just as in the case where the OCCURS(x,y) relation is linked to the relation $\Phi(\bar{x},y)$, e.g. OCCURS(e,t) $\wedge$ ENGULFS(x,y,e) - describing the event e where x engulfs y during period t, the same approach extends to the case where the relations OBTAINS(x,y) and $\Phi(\bar{z},x)$ are linked together, e.g. OBTAINS(s,t) $\wedge$ C(x,y,s), which describes the state s that obtains where x connects with y for period t.

3.5: Reasoning about empty regions of space

It is useful to be able to state explicitly that some region of space is not occupied by any physical object in the domain over a particular period of time. For example we might want to be able to reason that for a given configuration of physical objects, another physical object can only occupy the place of another if the first is moved from the place that object presently occupies. This is easily done given the addition of a new relation - 'Empty(x,y)' read as 'x is empty at or throughout y' (which is functionally equivalent to Hayes' [1985b p.80] 'Free(s)' predicate:

(D95) **Empty**(x,y) $\equiv$def. $\forall z[P(z,y) \rightarrow \neg\exists u[O(u|z,x)]$

type Empty(Spatial,Period):UU

Given spatial relations can now be indexed with a temporal parameter, we could say that if the space occupied by a physical object at time t1 is not empty, but empty at t2, then it is possible for another physical object to occupy it. Moreover one could easily develop the formalism to be able to infer that if the space occupied by a rigid physical object at time t1 is not identical with the space occupied by that object at time t2, (and where t1 and t2 are periods that meet) then that object has not moved.

3.6: Reasoning about increasing and decreasing rates of change

For increasing, decreasing and constancy measures over time, the reified approach allows one to exploit the polymorphisism of the logic. In this case another primitive sort is required, which is named MEASURE. In the following set of definitions the metalogical n-ary function $\varphi(\mathfrak{x})$ is understood as being replaced with an appropriate function, e.g. '$p_{MB}(x)$' read as 'the pressure of x in millibars' and '$d_{CM}(x,y)$' as 'the distance between x and y in centimeters'. The function 'at$(\varphi(x),t)$' read as '$\varphi(x)$ at t' has the intended meaning that $\varphi(x)$ holds at moment t. The symbols '<','>','≤','≥' and '=' carry their standard meaning. The relation 'INCREASE(x,y)' is read as 'x increases over y', 'DECREASE(x,y)' read as 'x decreases over y' and 'CONSTANT(x,y)' as 'x is constant over y'. Each function of the form $\varphi(\mathfrak{x})$ maps a set of specified physical objects to a φ-history; e.g. in the case of temperature of body x, the φ-history is x's temperature/time curve, while the at$(\mathfrak{x},y)$ function picks out a numerical value of a set of measures for some specified moment:

$$
\begin{aligned}
&\text{(D96) } \mathrm{INCREASE}(\varphi(\mathfrak{x}),y) \equiv_{\text{def.}} \mathrm{at}(\varphi(\mathfrak{x}),\mathrm{initial}(y)) < \mathrm{at}(\varphi(\mathfrak{x}),\mathrm{final}(y)) \wedge \\
&\qquad\qquad \forall zu\big[[P(z,y) \wedge P(u,y) \wedge B(z,u)] \rightarrow \\
&\qquad\qquad\qquad \mathrm{at}(\varphi(\mathfrak{x}),z) \leq \mathrm{at}(\varphi(\mathfrak{x}),u)\big]
\end{aligned}
$$

$$
\begin{aligned}
&\text{(D97) } \mathrm{DECREASE}(\varphi(\mathfrak{x}),y) \equiv_{\text{def.}} \mathrm{at}(\varphi(\mathfrak{x}),\mathrm{initial}(y)) > \mathrm{at}(\varphi(\mathfrak{x}),\mathrm{final}(y)) \wedge \\
&\qquad\qquad \forall zu\big[[P(z,y) \wedge P(u,y) \wedge B(z,u)] \rightarrow \\
&\qquad\qquad\qquad \mathrm{at}(\varphi(\mathfrak{x}),z) \geq \mathrm{at}(\varphi(\mathfrak{x}),u)\big]
\end{aligned}
$$

$$
\begin{aligned}
&\text{(D98) } \mathrm{CONSTANT}(\varphi(\mathfrak{x}),y) \equiv_{\text{def.}} \forall zu\big[[P(z,y) \wedge P(u,y) \wedge B(z,u)] \rightarrow \\
&\qquad\qquad\qquad \mathrm{at}(\varphi(\mathfrak{x}),y) = \mathrm{at}(\varphi(\mathfrak{x}),y)\big]
\end{aligned}
$$

type $\Phi(\tau 1,\tau 2)$:UU, $\tau 1$ = MEASURE, $\tau 2$ = INTERVAL,
$\Phi \in$ {INCREASE,DECREASE,CONSTANT}

type at(MEASURE,MOMENT):NUMBER

type p_{MB}(PHYSOB):MEASURE
type d_{CM}(PHYSOB,PHYSOB):MEASURE

In the non-reified approach, different predicate variants for INCREASE, DECREASE and CONSTANT must be used, depending on the type of measure being introduced. Each measure function now takes an extra argument, e.g. $p_{MB}(x,y)$, (read as 'the pressure of x (in millibars) at moment y'). In general these measure functions map physical bodies and moments to numbers. e.g. INCREASE_IN_PRESSURE(x,y) is defined as:

(D99) INCREASE_IN_PRESSURE(x,y) ≡def. $p_{MB}(x,\mathrm{initial}(y)) < p_{MB}(x,\mathrm{final}(y)) \wedge$
$\forall zu[[P(z,y) \wedge P(u,y) \wedge B(z,u)] \rightarrow$
$p_{MB}(x,z) < p_{MB}(x,u)$

type INCREASE_IN_PRESSURE(PHYSOB,INTERVAL):UU
type p_{MB}(PHYSOB,MOMENT):NUMBER

3.7: Extending the $\Phi(x,y,z)$, OBTAINS(x,y) and OCCURS(x,y) relations.

Given the formal distinction made between moments and intervals, we can easily extend the set of ternary relations of the form $\Phi(x,y,z)$ so that body x can be said to be in relation Φ to body y at a moment or within an interval or throughout an interval respectively (see Hamblin [1967] and Galton [1990]). In this case the definitions assume the following form where '$\Phi_{At}(x,y,z)$' is understood to mean that x is in relation Φ to y at moment z, '$\Phi_{Wthn}(x,y,z)$' as x is in relation Φ to y within z, and '$\Phi_{Th}(x,y,z)$' as x is in relation Φ to y throughout z':

(D100) $\Phi_{At}(x,y,z)$ ≡def. $\Phi(x|z,y|z)$
(D101) $\Phi_{Wthn}(x,y,z)$ ≡def. $\exists u[PP(u,z) \wedge \Phi(x|u,y|u)]$
(D102) $\Phi_{Th}(x,y,z)$ ≡def. $\forall u[PP(u,z) \rightarrow \Phi(x|u,y|u)]$

type $\Phi_{At}(\tau 1,\tau 2,\tau 3)$:UU
type $\Phi_{Wthn}(\tau 1,\tau 2,\tau 4)$:UU
type $\Phi_{Th}(\tau 1,\tau 2,\tau 4)$:UU, where $\tau 1,\tau 2$ = PHYSOB, $\tau 3$ = MOMENT, $\tau 4$ = INTERVAL

The same increase of expressiveness can of course be extended to the reified approach, where the OBTAINS(x,y) predicate splits into the following cases: '$OBTAINS_{At}(x,y)$' is read as 'x obtains at y', '$OBTAINS_{Wthn}(x,y)$' as 'x obtains during y' and '$OBTAINS_{Th}(x,y)$' as 'x obtains throughout y':

(D103) $OBTAINS_{At}(x,y) \equiv def.\ OBTAINS(x,y) \wedge MOMENT(y)$
(D104) $OBTAINS_{Wthn}(x,y) \equiv def.\ \exists z[PP(z,y) \wedge OBTAINS_{At}(x,z)$
(D105) $OBTAINS_{Th}(x,y) \equiv def.\ \forall z[PP(z,y) \rightarrow OBTAINS_{At}(x,z)]$

type $OBTAINS_{At}$(STATE,MOMENT):UU
type $OBTAINS_{Wthn}$(STATE,INTERVAL):UU
type $OBTAINS_{Th}$(STATE,INTERVAL):UU

3.8: Generating an Envisionment

The term and notion of an "envisionment" stems from de Kleer's work in Qualitative Physics. An envisionment takes a set of predetermined set of qualitative states, and expresses these in the form of a graph which represents a temporally partially ordered set of all the qualitative states a physical system can evolve into given some indexed state. Envisioning is the process of constructing an envisionment. Envisionments can be *attainable* (starting from some initial state) or *total* (starting from all possible states). Both types of envisionment appear in QP literature - see Forbus [1988a] for further details.

Given the basic set of dyadic relations defined solely in terms of C, a subset of these (being mutually exclusive and exhaustive) can be used to generate an envisionment which describes legitimate transitions two objects can evolve into given some indexed state. The set of base relations for lattice L_C are the relations: DC, EC, PO, TPP, NTPP, TPI, NTPI, TPP^{-1}, and $NTPP^{-1}$. In practical terms, given an ordered pair of named spatial regions <a,b>, exactly one of these relations will hold. This represents a set of qualitative states.

Next the envisionment itself needs to be set up. This is represented in the form of a graph in Figure 9 below:

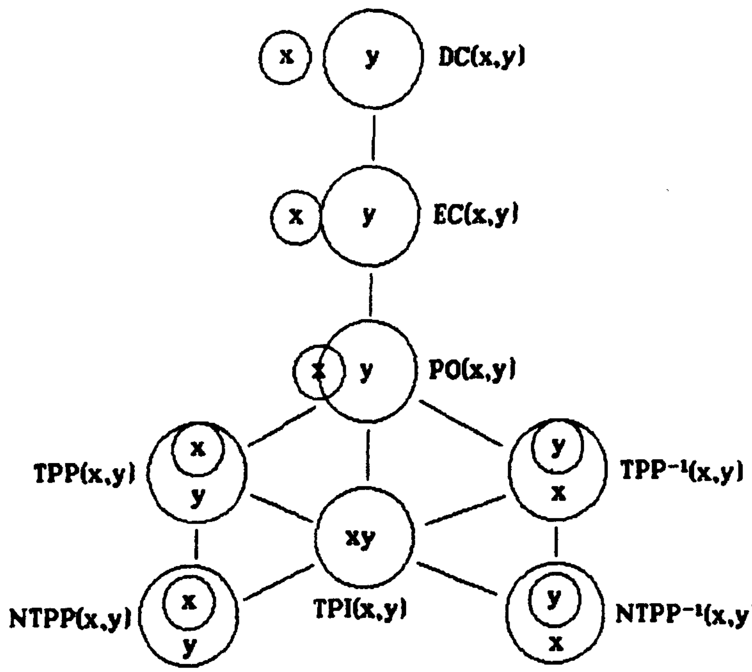


Figure 9: Transition network based on the base relations of lattice L_C. Note that here the relation NTPI does not appear, since this model assumes all the regions to be closed regions.

Legitimate transitions are indicated by edges, thus e.g. given a DC state, this can pass into an EC state (and vice-versa). The guiding intuition behind this network is best illustrated by considering two geometrical solid spheres x and y of different diameters which are initially widely separated, then brought together until their centres coincide. Lets suppose x is smaller than y. The sequence x and y will pass through will pass through will be as follows DC(x,y), EC(x,y), PO(x,y), TPP(x,y) and NTPP(x,y) respectively. The same principle is extended to cover other relations, e.g. the inside and outside relations - depicted in Figure 10.

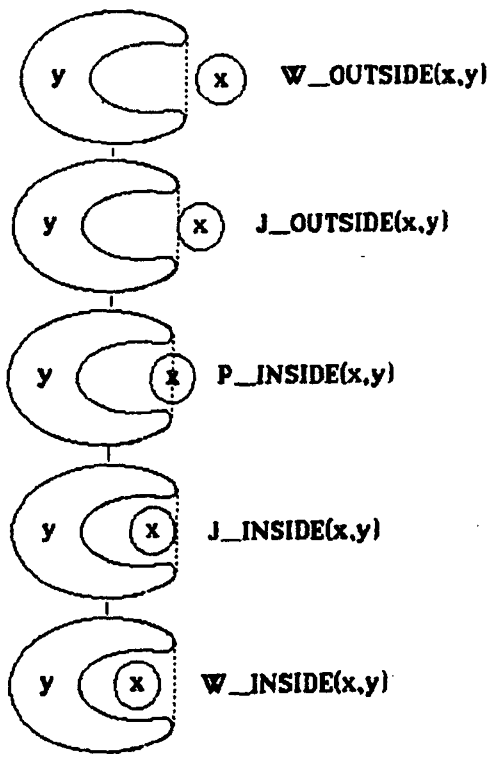


Figure 10: Transition network for the *defined* inside and outside relations. Note that the model used assumes all the pairs of regions to be disconnected. The reader is reminded here, that once DC and EC variants on these relations and inverses are defined, the complete set of relations that define a lattice in the manner of lattice L_C will substantially increase the number of nodes and transitions from those depicted here.

Given different sets of named relations, transition networks and the envisionments constructed from them, some means to 'prune' the number of possible transitions from an indexed state must be made. A cursory glance will soon illustrate that if *no* restriction is made, the number of potential transition states can grow dramatically from some indexed state. In the example used above using two spheres, some pruning had been done implicitly by exploiting metric and geometrical information. For example, the smaller sphere passed inside the latter, but not vice-versa.

In general then, the pruning is done as follows. Initially sortal information is exploited. For example suppose the model uses only open regions, then the number of base relations reduce to the set {DR,PO,PP,PP-',EQUAL}, this is because the EC relation is never satisfied and the relations C and O become equivalent thereby 'collapsing together' many hitherto distinguished relations. With the absence of EC, the tangential relations i.e. TP, TPP TPI and their inverses cease to be satisfied; PP collapses with NTPP, P with NTP and EQUAL with NTPI. This reduces further if only atoms are used, since atoms either remain discrete or are identical: hence the set {DR,EQUAL}.

However, in most cases physical objects will be more naturally associated with closed regions of space which exploit the more expressive set of relations. In this case, empirical information extracted from the domain can be used to good effect. For example, separated solid objects do not normally subsequently overlap, and rigid bodies have constant convex-hulls. Similarly, solid deformable objects will generally change their convex-hulls, and in general only a smaller object will be able to pass inside another. This is discussed in more detail in the following section.

There are several ways an envisionment can be represented and implemented. One way to represent an envisionment uses the next(x) function. In this particular case I assume both objects continue to exist during the temporal projection, and that both regions are closed regions:

(i) $\forall xyz[DC(x,y,z) \rightarrow [DC(x,y,next(z)) \vee EC(x,y,next(z))]]$

(ii) $\forall xyz[EC(x,y,z) \rightarrow [EC(x,y,next(z)) \vee DC(x,y,next(z)) \vee PO(x,y,next(z))]$

(iii) $\forall xyz[PO(x,y,z) \rightarrow [PO(x,y,next(z)) \vee EC(x,y,next(z)) \vee TPP(x,y,next(z)) \vee$
$TPP^{-1}(x,y,next(z)) \vee TPI(x,y,next(z))]$

(iv) $\forall xyz[TPP(x,y,z) \rightarrow [TPP(x,y,next(z)) \vee NTPP(x,y,next(z)) \vee$
$PO(x,y,next(z)) \vee TPI(x,y,next(z))]$

(v) $\forall xyz[NTPP(x,y,z) \rightarrow [NTPP(x,y,next(z)) \vee TPP(x,y,next(z)) \vee$
$TPI(x,y,next(z))]]$

(vi) $\forall xyz[TPI(x,y,z) \rightarrow [PO(x,y,next(z)) \vee TPP(x,y,next(z)) \vee$
$NTPP(x,y,next(z))]]$

(vii) $\forall xyz[TPP^{-1}(x,yz) \rightarrow [TPP^{-1}(x,y,next(z)) \vee NTPP^{-1}(x,y,next(z)) \vee$
$PO(x,y,next(z)) \vee TPI(x,y,next(z))]$

(viii) $\forall xyz[NTPP^{-1}(x,y,z) \rightarrow [NTPP^{-1}(x,y,next(z)) \vee TPP^{-1}(x,y,next(z)) \vee$
$TPI(x,y,next(z))]]$

x,y:PHYSOB, z:PERIOD, $next(x)$:MOMENT

This particular set of axioms for generating an envisionment reflects the graph in Figure 9, excepting that each node would have in addition to the edges shown, a directed edge pointing back to itself. This would apply if no change in the relation between x and y arose throughout period z and for the next moment following z.

A second method introduces the notion of a maximal period of time over which some property Φ holds between regions x and y. (The notion of a maximal period over which properties are true is assumed both by Allen in his temporal logic and in general in QP, where envisionments are used):

$$\Phi_{MAX}(x,y,z) \equiv def. \ \Phi_{TH}(x,y,z) \wedge \forall u[[MOMENT(u) \wedge [MEETS(u,z) \vee MEETS(z,u)] \rightarrow \neg\Phi_{At}(x,y,u)]$$

(in words: z is a maximal period during which x bears Φ to y, if x bears Φ to y for all subperiods of z, and x does not bear Φ to y either for the moment that meets z or the moment that is met by z).

For the envisionment, the transition network follows that given above, except now, the direct transition is given. For this I only give one axiom, since the reader can easily construct the complete set of axioms using the above set of envisionment axioms as a guide:

$\forall xyz\,[\,[DC_{MAX}(x,y,z) \wedge \neg NULL(x,next(z)) \wedge \neg NULL(y,next(z)] \rightarrow$
$\quad EC(x,y,next(z)]$

(Note the *additional conjuncts* which ensures a next state holds only if x and y are not null for the moment immediately following.)

A third way to represent axioms for generating an envisionment introduces a NEXT(x,y,z,u) relation. This relation serves to link successive states. 'NEXT(x,y,z,u)' is read as 'state x is the next state immediately following state y, that obtains between z and u', and a time(x) function read as 'the temporal duration of x'. The sorting functions are:

type $NEXT(\tau 1,\tau 2,\tau 3,\tau 4)$:UU, $\tau 1,\tau 2$ = STATE, $\tau 3,\tau 4$ = PHYSOB
type time(STATE):PERIOD

The following axioms are added:

$\forall xyzu\,[NEXT(x,y,z,u) \rightarrow MEETS(time(x),time(y))]$
x,y:STATE, z,u:PHYSOB
(in words: consecutive states endure for periods of time that meet)

$\forall xyz\,[\exists u[NEXT(u,x,y,z) \rightarrow [\neg NULL(y,time(x)) \wedge \neg NULL(z,time(x))]\,]$

(in words: a next (different) state u exists between y and z only if y and z are not null during the duration of u, where:

$NULL(x,y) \equiv$def. $\forall z[P(z,y) \rightarrow NULL(x|z)]$

type NULL(PHYSOB,PERIOD):UU

The transition network again follows that described above excepting that an explicit way to describe the change of state is now given. In contrast to the ternary relation $\Phi(x,y,z)$ used above, the latter argument is now changed to be of sort STATE and the reading changed accordingly - thus e.g. C(a,b,c) would now read as 'a is connected with b in state c'. For this

I only give one entry, since as before, the reader can easily construct the complete set of envisionment axioms:

$\forall xyzu[DC(x,y,z) \wedge NEXT(u,z,x,y)] \rightarrow EC(x,y,u)]$
$x,y{:}PHYSOB,\ z,u{:}STATE$

The reader is reminded here that named events as well as states can be incorporated into the NEXT(x,y,z,u) relation if required. For this the reading of the relation is changed accordingly, as are the sorting functions for NEXT(x,y,z,u) and time(x) so that the corresponding formule are well sorted when defined on the sort EVENT.

3.9: Adding and exploiting empirical information

In the previous section I mentioned how by exploiting metrical, geometrical and empirical properties of particular bodies, one can restrict the manner in which objects can be spatially related to each other over time. This is covered in more detail here.

In general, physical objects can be adequately modelled by mapping them to closed spatial regions; the exception is perhaps gaseous objects that having no clear identifiable perceptual boundaries might be good candidates to map to open regions. However, given the number of base relations that can be satisfied using closed regions, and given the number of possible relations generated in an envisionment from some given state, additional information uncovered from the model and introduced into the theory, must be seen to cut the potential search space if the theory is to be computationally viable. Fortunately, this does seem to be the case.

One modelling domain used to illustrate this theory describes an amoeba which surrounds and engulfs a food particle so that the food passes inside. I will use this example to show how in principle the reduction might proceed. Initially, as suggested, physical bodies are mapped to closed regions of space:

(i) $\forall xy[\neg NULL(x|y) \rightarrow CLOSED(x|y)]$

This immediately cuts out one base relation from L_C, i.e. NTPI, since this is only satisfied if a spatial region is open. This leaves 8 base relations from L_C. Next, we note that the food and the amoeba in the cited process are treated as distinct bodies and remain so even when the food is inside the cell. This naturally suggests the following axiom:

$$
\begin{aligned}
&(ii)\ \forall xyz[[PHYSOB(x) \wedge PHYSOB(y) \wedge DR(x,y,z)] \rightarrow \\
&\quad \forall u[B(z,u) \rightarrow \\
&\qquad [\neg NULL(x,u) \wedge \neg NULL(y,u)] \rightarrow DR(x,y,u)]]]
\end{aligned}
$$

(in words: if two physical bodies are disjoint for any time, then (on their continued existence) they will always remain disjoint).

Now the set of base relations of L_C reduces from eight to two, i.e. {DC,EC}. However, this set will expand again once the inside and outside relations (and their inverse relations) are included, so additional information is sought.

First note that in general, for one thing to be able to pass inside another, it must be smaller in size. This relative comparison of size immediately suggests an immediate way to constrain the set of possible transitions by introducing the following two axioms:

(iii) $\forall xy[[Amoeba(x) \wedge Food(y)] \rightarrow MUCH\_SMALLER\_THAN(y,x)]$,

and

(iv) $\forall xyz[MUCH\_SMALLER\_THAN(x,y) \rightarrow \neg INSIDE(y|z,x|z)]$.

Hence, given an amoeba (amoeba1) and some amoebal food (food1) we can now deduce that for any moment z, $\neg INSIDE(amoeba1|z, food1|z)$ holds. This immediately reduces the extended set of base relations by excluding all the inverse relations covered by the relation $INSIDE_{-1}(food1|z, amoeba1|z)$.

Next we note that ordinarily, we would want to exclude not only the case where the amoeba is inside the food but partially inside it too. This guiding intuition suggets that some notion of granularity is evident in the model based on the relative sizes and functional relationship we impose on the two objects. Even though the food may well possess negative surface curvature so that it could wrap around part of the amoeba's body (and sanction the wff $P\_INSIDE(amoeba1|z, food1|z)$), it seems innapropriate to model this. Two strategies are suggested, the first by strengthening the axiom cited above, so that the axiom:

(iv) $\forall xyz[MUCH\_SMALLER\_THAN(x,y) \rightarrow \neg INSIDE(y|z,x|z)]$, now becomes:

(v) $\forall xyz[MUCH\_SMALLER\_THAN(x,y) \rightarrow [\neg INSIDE(y|z,x|z) \wedge \neg P\_INSIDE(y|z,x|z)]]$.

or, alternatively we could introduce the element of granularity inherent in the model by stipulating that:

(vi) $\forall x[Food(x) \rightarrow \forall y[\neg NULL(x|y) \rightarrow CONVEX(x|y)]]$.

Given (vii) - the theorem:

(T83) $\forall xy[[P\_INSIDE(x,y) \vee INSIDE(x,y)] \rightarrow \neg Convex(y)]$
x,y: SPATAIL\SPATIAL_UNIVERSE

then, $P\_INSIDE^{-1}(food1|z, amoeba1|z)$, will cease to hold, with the net result that all the inverse relations of the inside and partially inside relations will be pruned out of the set of possible relations given the ordered pair <amoeba1,food1>.

Other empirical information might be possible to exploit. For example, given a either a close proximity between the amoeba and its food, or contact, we would not ordinarily expect the organism to move away, or exhibit oscillatory behaviour. (Perhaps the amoeba responds to some chemical trace in the fluid that surrounds the food?). Assuming this to be true, we could interpret close proximity between the amoeba and its food as J_OUTSIDE(food1 | z,amoeba1 | z). But from this we can state:

$$\forall xyz[[\text{Amoeba}(x) \wedge \text{Food}(y) \wedge \text{J\_OUTSIDE}(y,x,z)] \rightarrow \\ \neg\exists u[B(z,u) \wedge \text{W\_OUTSIDE}(y,x,u)]]$$

(in words: if the food is just outside the amoeba, then no following state will arise where the food is wholly outside the amoeba)

and,

$$\forall xyz[[\text{Amoeba}(x) \wedge \text{Food}(y) \wedge \text{J\_OUTSIDE}(y,x,z)] \rightarrow \\ \exists u[B(z,u) \wedge \text{EC}(y,x,u)]]$$

(in words: if the food is just outside the amoeba, a following time will arise where both the food and the amoeba are contact).

A virtue of this formal theory, is that it is relatively easy to see how to model a domain using less expressive subsets of the full set of defined relations. I will use the same example again, of an amoeba engulfing some food. *For this example I will restrict myself to the basic* set of relations encoded in lattice L_c. Again I use the axiom:

(i) $\forall xy[\neg\text{NULL}(x|y) \rightarrow \text{CLOSED}(x|y)]$

but *not* the axiom:

(ii) $\forall xyz[[\text{PHYSOB}(x) \wedge \text{PHYSOB}(y) \wedge \text{DR}(x,y,z)] \rightarrow \\ \forall u[B(z,u) \rightarrow \\ [\neg\text{NULL}(x,u) \wedge \text{NULL}(y,u)] \rightarrow \text{DR}(x,y,u)]]].$

simply because I now want to allow the the spaces occupied by the food and the amoeba to overlap. (The reason for this is because the passage of the food inside the cell, modelled in terms of the sequence: DC to EC to PO to TPP to NTPP, will require this condition to hold.) Again we add the axiom:

(iii) $\forall xy[[\mathrm{Amoeba}(x) \wedge \mathrm{Food}(y)] \rightarrow \mathrm{MUCH\_SMALLER\_THAN}(y,x)]$,

and now the *new* axiom:

(iv') $\forall xyz[\mathrm{MUCH\_SMALLER\_THAN}(x,y) \rightarrow \neg P(y|z,x|z)]$.

Suppose now we are given the constants amoeba1 and food1. From axiom (i) we eliminate the relation NTPI from the set of base relations as before, which leaves 8, and from axioms (iii) and (iv') all the inverse relations and equality. This leaves the set {DC,EC,PO,TPP,NTPP} from which the reader should be able to see that a unique envisionment can be constructed. Again following the above example, additional axioms could be added, e.g.:

$$\forall xyz[[\mathrm{Amoeba}(x) \wedge \mathrm{Food}(y) \wedge \mathrm{EC}(y,x,z)] \rightarrow \neg\exists u[B(z,u) \wedge \mathrm{DC}(y,x,u)]],$$

$$\forall xyz[[\mathrm{Amoeba}(x) \wedge \mathrm{Food}(y) \wedge \mathrm{PO}(y,x,z)] \rightarrow \neg\exists u[B(z,u) \wedge \mathrm{EC}(y,x,u)]]$$

$$\forall xyz[[\mathrm{Amoeba}(x) \wedge \mathrm{Food}(y) \wedge \mathrm{TPP}(y,x,z)] \rightarrow \neg\exists u[B(z,u) \wedge \mathrm{PO}(y,x,u)]]$$

$$\forall xyz[[\mathrm{Amoeba}(x) \wedge \mathrm{Food}(y) \wedge \mathrm{NTPP}(y,x,z)] \rightarrow \neg\exists u[B(z,u) \wedge \mathrm{TPP}(y,x,z)]]$$

The model satisfying these axioms is one where the process of coming into contact and eventually engulfing the food is monotonic. In general, however, it will prove expedient to clearly separate out process descriptions from the conditions that link and constrain them, suggested here. Otherwise, the model for the theory will be too restrictive and will not be flexible enough to account for legitimate variations in behaviour that are observed in the actual physical system used to interpret the theory.

3.10: Defining continuity in processes

Up until now, the notion of continuity sanctioning direct transitions between states has been remained implicit. The justification made an appeal to intuition by considering specified sequences of pictorial representations of spatial regions. However, not only can the notion of continuity be made explicit, the envisionment can also be generated from an application two simple rules. In this case change is related to a change in the quantity an class of incident points shared between pairs of regions.

Table 1 represents two qualitatively identical closed regions x and y passing from DC through to NTPP. The process is represented below in a tabular form. Under each relation, the quantity of commonly shared boundary and interior points is given. The entries "none", "some" and "all" mean that no, some or all points in a given category are held in common between the ordered pair <x,y>, while "subset" ("superset") means that x's points are a proper subset (superset) of y's. The symbol "... ◄-► ---" can be read as '... can directly change to --- (and vice-versa)':

Table 1:

<x, y>	DC		EC		PO		TPP		NTPP
Boundary	none	◄-►	some		some		some	◄-►	none
Interior	none		none	◄-►	some	◄-►	subset		subset

Excepting the case where regions x and y pass to equality, continuity across adjacent states is fixed by the following two conditions:

a) that for each class of incident points, the change in quantity can change from "none" to "some" (or vice-versa) and from "some" to "all" (or vice-versa) but not from "none" to "some" (or vice-versa). Similarly a change from "some" to "subest" (or vice versa) is allowed, or from "some" to "superset" (or vice-versa), as is either "subset" or "superset" to "all" (or vice-versa), but not from "none" to either "subset" or "superset" (or vice-versa), and

b) only one class of points can change at any one time.

Note that just as in the case where in QP, the value "+" cannot pass to "-" (or "-" to "+") without first passing through "0" the same principle applies in the current theory. Here the analogue of "0" corresponds to states where boundary connection between two regions hold. Thus for example, DC cannot pass to PO (or PO to DC) without first passing through PO, and PO cannot pass to NTPP (or NTPP to PO) without first passing through TPP.

To reveal the explicit characterisation of continuity in terms of changing quantities in the classes of incident points shared between x and y for the inside and outside relations, the relations must first be unpacked in terms of their respective definiens. In this instance the comparison between x and y is taken to be between x and conv(y).

Table 2:

(x, y)	W_Outside		J_Outside		P_Inside		J_Inside		W_Inside
Boundary	none	↔	some		some		some	↔	none
Interior	none		none	↔	some	↔	all		all

e.g. taking the relation J_Outside(x,y), the following equivalence arises:

$\forall xy[\text{J\_Outside}(x,y) \leftrightarrow EC(x,conv(y))]$.

Looking at the entry for Table 1:

⟨x, conv(y)⟩	EC(x, conv(y))
Bondary	some
Interior	none

3.11 Changing universes of discourse

Objects that come into existence at a particular moment of time are created by invoking a new existentially quantified variable or an individual constant; or skolem function or individual constant respectively. For entities that pass out of existence, these are mapped to the sort NULL. For example, the wff:

$$Vacuole(a,t1) \wedge Null(a|next(t1)) \wedge \forall t2[B(next(t1),t2) \rightarrow Null(a,t2)]$$

captures the process of a vacuole passing out of existence and remaining so.

Note that this is a very strong condition for non-existence, since the *bearer* of some property actually passes out of existence (in the sense that it does not occupy (physical space), rather than loose some other defining property.

3.12 Summary

In this chapter I have shown how descriptions of states, events and processes are incorporated into the theory, and how from the simple expedient of mapping physical bodies to the spaces they occupy, complex relations between bodies in space can be easily described. Two methods to incorporate the explicit representation of time in wff were discussed, the simple case where each n-place relation was complemented witha n+1 place relation, and a reified approach that allowed in addition to the explicit representation of time, the explicit representation of named states and events. The notion of an envisionment was introduced, and examples were given. Techniques to reduce the number of projected states from a given state were discussed. This involved both the use of sortal information embodied in the general theory and empirical information extracted from the modelled domain.

In the following chapter I discuss two reasonably complex domains to show in more detail how the theory described so far is used, and how individual process descriptions are constructed, and linked together.

Chapter 4: Sample Modelling Problems

4.1: Introduction

In this chapter I show how simple physical systems can be formally described using the formal apparatus set up in chapters 2 to 3. Processes are defined in terms of specified sequences of state descriptions. Typically, these resolve into descriptions of spatial relations holding between particular objects where the degree of connectivity between them vary over time. I use two examples for this. The first describes phagocytosis and exocytosis of a simple protozoan. The second concentrates upon the series of processes that arise during the cycle of operations associated with a force pump.

A complete axiomatisation describing either domain is beyond the scope of this thesis. The reader will better appreciate the anticipated complexity and scope of such formal theories after reading this chapter, given that this chapter simply sets out to show the adequacy of the formal theory for describing reasonably complex physical domains.

For the following examples, the set of sort symbols defining lattice L_* are increased in number, in particular the number of sort symbols that are subsorts of PHYSOB. In order to help the reader reading the definitions, sort predicates are made explicit. Using LLAMA, these would not normally appear in their clausal translations, but would be absorbed into the sortal machinery supported by the logic.

Earlier workings of both domains can be found in Randell and Cohn [1989a,b] (where the process of phagocytosis is outlined) and in Randell and Cohn [1989c] and in Randell, Cohn and Cui [1991] (where the force pump is described).

4.2: Phagocytosis and Exocytosis

Phagocytosis is the process by which cells surround, engulf and then digest food particles. It is the feeding method used by some unicellular protozoans of which the amoeba is an example and adopted here. The same process is also used by white blood cells in an attempt to deal with invading micro-organisms. Exocytosis refers to a similar 'inverse' process where waste material is expelled from the cell.

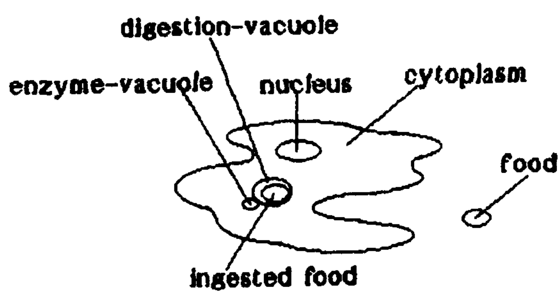


Figure 11: The amoeba

In the proposed model (see Figure 11) an amoeba is depicted living within a fluid environment containing other organisms that are its food. Each amoeba is credited with vacuoles (or fluid filled spaces) containing either enzymes or food which the animal has ingested. The enzymes are used by the amoeba to break down and digest the ingested food into nutrient and waste. This is done by routing the enzymes to the food vacuole. Upon contact the enzyme vacuole and food vacuole fuse together and the enzymes merge into the fluid filled space containing the food particle. The enzymes act upon the food breaking it down into nutrient and waste. The nutrient is absorbed into amoebal protoplasm leaving the waste material in the vacuole ready to be expelled. The latter is achieved by letting the vacuole pass to the exterior of the protozoan's body which opens up, letting the waste material pass into the amoebal environment.

The various stages of phagocytosis and exocytosis are depicted in Figure 12 which should be referred to when reading the formal descriptions given below.

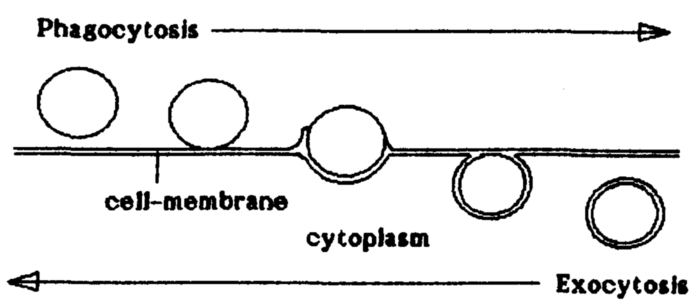


Figure 12: Phagocytosis and exocytosis. In phagocytosis, the cell approaches, contacts and then engulfs the food, eventually forming a food vacuole. In exocytosis the waste or residual material left after digestion passes out toward the cell membrane and then is released into the amoebal environment.

Firstly, I introduce three axioms that hold in the domains I am considering here. These axioms ensure that every named physical object exists (physically) for at least one moment, that any physical object that is null for a moment is null for all time after that moment, and that when a physical object comes into (physical) existence at a particular moment, that object is null for all time before that moment:

$$\forall x[\mathrm{PHYSOB}(x) \rightarrow \exists y[\neg\mathrm{NULL}(x|y)]]$$

$$\forall xy[[\mathrm{PHYSOB}(x) \wedge \mathrm{NULL}(x|y)] \rightarrow \forall z[\mathrm{B}(y,z) \rightarrow \mathrm{NULL}(x,z)]]$$

$$\forall x[[\mathrm{PHYSOB}(x) \rightarrow [\exists yz[\mathrm{MEETS}(y,z) \wedge \mathrm{NULL}(x|y) \wedge \neg\mathrm{NULL}(x|z)] \rightarrow \forall u[\mathrm{B}(u,y) \rightarrow \mathrm{NULL}(x,u)]]]$$

Next I describe the protozoan. The protozoan consists of a nucleus and cytoplasm. The cytoplasm is the liquid body of the cell in which the chemical reactions of life occur. The nucleus is isolated from the cytoplasm by a nuclear membrane; a similar membrane bounds the cell which controls the entry and exit of materials by allowing certain substances through but not others:

$$\mathrm{AMOEBA}(x) \equiv\mathrm{def.}\ \exists yz[\mathrm{NUCLEUS}(y) \wedge \mathrm{CYTOPLASM}(z) \wedge \forall u[[\neg\mathrm{NULL}(y|u) \wedge \neg\mathrm{NULL}(z|u)] \rightarrow \neg\mathrm{O}(y|u,z|u) \wedge \mathrm{NTPP}(y|u,x|u) \wedge \mathrm{EQUAL}(\mathrm{sum}(y|u,z|u),x|u)]]$$

$$\mathrm{NUCLEAR\_MEMBRANE}(x) \equiv\mathrm{def.}\ \exists y[\mathrm{NUCLEUS}(y) \wedge \forall z[\neg\mathrm{NULL}(y|z) \rightarrow \mathrm{EQUAL}(x|z,\mathrm{skin}(y|z))]]$$

$$\mathrm{CELL\_MEMBRANE}(x) \equiv\mathrm{def.}\ \exists y[\mathrm{Cell}(y) \wedge \forall z[\neg\mathrm{NULL}(y|z) \rightarrow \mathrm{EQUAL}(x|z,\mathrm{skin}(y|z))]]$$

sort CELL $\sqsubset$ PHYSOB
sort AMOEBA $\sqsubset$ CELL
sort NUCLEAR_MEMBRANE $\sqsubset$ MEMBRANE
sort CELL_MEMBRANE $\sqsubset$ MEMBRANE
sort NUCLEUS $\sqsubset$ PHYSOB
sort CYTOPLASM $\sqsubset$ PHYSOB
sort MEMBRANE $\sqsubset$ PHYSOB

Each protozoan has at least one enzyme vacuole as a part for every moment of its existence:

$$\forall x[\text{AMOEBA}(x) \rightarrow \forall y[\neg \text{NULL}(x|y) \rightarrow \exists z[\text{ENZYME\_VACUOLE}(z) \wedge \text{NTPP}(z|y,x|y)]]$$

Definitions for different types of vacuoles are constructed as follows. The definition for a vacuole states that it is a a fluid filled space, and that whenever it exists, there also exists a cell of which it is a part. This definition relies on the stipulation that the fluid filled space comprises of a connected body of water, such that no other connected body of water exists of which it is a proper part. (Note the similarity with maximal periods of time for which some property holds, described in section 3.8.)

$$\text{VACUOLE}(x) \equiv\text{def. } \text{WATER}_{\text{MAX}}(x) \wedge \forall y[\neg \text{NULL}(x|y) \rightarrow \exists z[\text{CELL}(z) \wedge \text{PP}(x|y,z|y)]]$$

where:

$$\text{WATER}_{\text{MAX}}(x) \equiv\text{def. } \text{WATER}(x) \wedge \forall y[[\neg \text{NULL}(x|y) \wedge \text{Connected}(x|y)] \rightarrow \neg\exists z[\text{WATER}(z) \wedge \text{Connected}(z|y) \wedge \text{PP}(x|y,z|y)]]$$

The definitions for specialisations of vacuoles follow the same form, excepting the definition for the waste vacuole, which adds the condition that only residual material is contained:

$$\text{ENZYME\_VACUOLE}(x) \equiv\text{def. } \text{VACUOLE}(x) \wedge \forall y[\neg \text{NULL}(x|y) \rightarrow \exists z[\text{ENZYME}(z) \wedge \text{PP}(z|y,x|y)]]$$

$$\text{FOOD\_VACUOLE}(x) \equiv\text{def. } \text{VACUOLE}(x) \wedge \forall y[\neg \text{NULL}(x|y) \rightarrow \exists z[\text{FOOD}(z) \wedge \text{PP}(z|y,x|y)]$$

$$\text{DIGESTION\_VACUOLE}(x) \equiv\text{def. } \text{FOOD\_VACUOLE}(x) \wedge \text{ENZYME\_VACUOLE}(x)$$

$$\text{WASTE\_VACUOLE}(x) \equiv\text{def. } \text{VACUOLE}(x) \wedge \forall y[\neg \text{NULL}(x|y) \rightarrow \exists z[\text{WASTE}(z) \wedge \text{PP}(z|y,x|y) \wedge \neg\exists u[\text{PHYSOB}(u) \wedge \text{PP}(u,x) \wedge \neg \text{WASTE}(u) \wedge \neg \text{WATER}(u)]$$

sort VACUOLE:PHYSOB

sort ENZYME_VACUOLE:VACUOLE

sort FOOD_VACUOLE:VACUOLE

sort DIGESTION_VACUOLE = FOOD_VACUOLE $\sqcap$ ENZYME_VACUOLE

sort WASTE_VACUOLE:VACUOLE

sort ENZYME:PHYSOB

sort WASTE:PHYSOB

sort WATER:PHYSOB

sort $\mathrm{WATER}_{\mathrm{MAX}}$:WATER

Each vacuole is bounded by a membrane (composed of the same material as the cell membrane):

$$\mathrm{VACUOLE\_MEMBRANE}(x) \equiv \mathrm{def.}\ \exists y[\mathrm{VACUOLE}(y) \wedge \forall z[\neg \mathrm{NULL}(y|z) \rightarrow \mathrm{EQUAL}(x|z, \mathrm{skin}(y|z))]$$

sort VACUOLE_MEMBRANE:MEMBRANE

Here, I regard the vacuole membrane as part of the vacuole. However, in actual fact the vacuole is delimited by its membrane - hence the vacuole could also be formally described as being surrounded by the membrane but forming no part of it. (Indeed, although this is not done here, one could easily define a function that picks out the 'layer' of atoms that surround a given region, and map the vacuole membrane to that.) In point of fact, when the food is enveloped by the protozoan and the food vacuole formed, the cell membrane wraps around the food, and detaches itself thus forming the material of the vacuole membrane. (This process is analogous to a soap bubble being blown from a hoop dipped in soapy solution and leaving the hoop with a soap film intact.) Similarly, in exocytosis that same vacuole material is reabsorbed as the vacuole membrane first contacts the cell membrane, fuses together thus expelling the residue material.

Food is regarded as having nutrient which is absorbed by the cell (and undigestible material which is not absorbed and is eventually expelled):

$$\forall x[\mathrm{FOOD}(x) \rightarrow \forall y[\neg\mathrm{NULL}(x|y) \rightarrow \exists z[\mathrm{NUTRIENT}(z) \wedge \mathrm{PP}(z|y,x|y)]$$

sort NUTRIENT:PHYSOB

I now start to describe simple processes. These are later conjoined together to describe more complex processes. I shall describe these processes in the order in which they arise in the informal description given above, so that the linkage between them can be made clear. Firstly, the process where an object moves toward another object:

MOVES_TOWARD(x,y,z) ≡def. DECREASES(d_{CM}(x,y),z).

type MOVES_TOWARD(PHYSOB,PHYSOB,INTERVAL):UU

It should be clear from this definition that as long as x moves toward y, x and y are not connected during period z, although x and y may come into contact at the final moment of z.

Next, the state where one object is in contact with another. This is expressed using the relation EC(x,y,z).

The next process to be defined is rather complex. In this case it is where one object x engulfs another object y. The formalism allows this particular process to be described in different ways according to the level of detail required. For example it may be deemed sufficient to describe this process by allowing x and y to overlap, and stipulating the sequence where the relation between y and x passes from PO to TPP to NTPP over consecutive periods: (Note, immediately below and elsewhere, I express several conjunctions of the MEETS relation in a canonical form. Thus e.g. "MEETS(u,v,w)" abbreviates "MEETS(u,v) $\wedge$ MEETS(v,w)" - where Φ(x1,x2, ,,, xn) requires xn-1 conjunctions of literals using the predicate Φ. The same principle is extended to the summation function. Thus for example, the wff "sum(x,y,z)" abbreviates the wff "sum(sum(x,y),z)").

$$ENGULFS(x,y,z) \equiv_{def.} \exists uvw[PO(x,y,u) \wedge TPP(y,x,v) \wedge NTPP(y,x,w) \wedge MEETS(u,v,w) \wedge EQUAL(sum(u,v,w),z)]$$

type ENGULFS(PHYSOB,PHYSOB,INTERVAL):UU

Alternatively, one can keep x and y discrete (until perhaps some process acts on them so that we would then allow them to overlap). In this case the inside and outside relations can be used. Thus the passage of y into x and being enveloped by x could be formally described by stipulating the sequence from P_INSIDE to J_INSIDE to W_INSIDE for y and x, and then from W_INSIDE to TPP to NTPP.

$$ENGULFS(x,y,z) \equiv_{def.} \exists uvwu'v'w'[P\_INSIDE(y,x,u) \wedge J\_INSIDE(y,x,v) \wedge W\_INSIDE(y,x,w) \wedge TPP(y,x,u') \wedge NTPP(y,x,v') \wedge MEETS(u,v,w,u',v') \wedge EQUAL(sum(u,v,w,u',v'),z)$$

type ENGULFS(PHYSOB,PHYSOB,INTERVAL):UU

Alternatively, the TS and NTS relations could be incorporated, thus:

$$ENGULFS(x,y,z) \equiv_{def.} \exists uvw[P\_INSIDE(y,x,u) \wedge TS(y,x,v) \wedge NTS(y,x,w) \wedge MEETS(u,v,w) \wedge EQUAL(sum(u,v,w),z)]$$

Note too, that just as the proper part relations admit surround duals, an analogue can be defined for the PO relation, thus:

$$PARTIALLY\_SURROUNDED(x,y) \equiv_{def.} P\_INSIDE(x,y) \wedge EC(x,y) \wedge \forall z[[P(z,skin(x)) \wedge P(z,conv(y))] \rightarrow EC(z,y)]$$

type PARTIALLY_SURROUNDED(τ,τ):UU, τ = SPATIAL\SPATIAL_UNIVERSE

(In this case x is in contact with y and partially inside it, and every part of the skin of x which is inside y externally connects with y - in other words y wraps around x, but some of x protrudes). Thus, another alternative description for the process of being engulfed could be expressed as follows:

$$ENGULFS(x,y,z) \equiv def.\ \exists uvw[PARTIALLY\_SURROUNDED(y,x,u) \wedge TS(y,x,v) \wedge NTS(y,x,w) \wedge MEETS(u,v,w) \wedge EQUAL(sum(u,v,w),z)]$$

type ENGULFS(PHYSOB,PHYSOB,INTERVAL):UU

Here one can see Hayes' idea of constantly seeking out the simplest model for a given formal theory and introducing more formal constraints as the intended model is better understood and isolated.

Given we now have the state where the amoeba has the food contained in a vacuole, the next process to be described is where the enzymes, having made contact with the food vacuole, fuse with the food and break down the food into its constituent parts - nutrient and waste. Again, as before there are several ways this process could be described. Here I capture the notion of absorption by explicitly allowing both the food and the enzyme body to overlap:

$$DIGESTS(x,y,z) \equiv def.\ \exists uvwu'v'[FOOD(u) \wedge ENZYME(v) \wedge O(x,y,w) \wedge MEETS(w,u') \wedge EQUAL(sum(w,u'),z) \wedge WASTE(v') \wedge NULL(v',w) \wedge \neg NULL(v',u') \wedge NULL(u,final(z))]$$

type DIGESTS(PHYSOB,PHYSOB,INTERVAL)

For process of absorption itself, this is modelled by letting the nutrient pass out of the digestion vacuole through the vacuole membrane into the surrounding cell material.

$$ABSORBS(x,y,z) \equiv def.\ \exists uvwu'[CELL(x) \wedge NUTRIENT(y) \wedge DIGESTION\_VACUOLE(u) \wedge P(u,x,z) \wedge P(y,u,v) \wedge PO(y,u,w) \wedge \neg O(y,u,u') \wedge P(y,x,u') \wedge MEETS(v,w,u') \wedge EQUAL(sum(v,w,u'),z)]$$

type ABSORBS(PHYSOB,PHYSOB,INTERVAL):UU

The converse process of expulsion reverses the sequence of states described for the engulfing process:

$$\text{EXPELS}(x,y,z) \equiv\text{def. } \exists uvw[\text{P\_INSIDE}(y,x,u) \wedge \text{TS}(y,x,v) \wedge \text{NTS}(y,x,w) \wedge \text{MEETS}(w,v,u) \wedge \text{EQUAL}(\text{sum}(w,v,u),z)]$$

type EXPELS(PHYSOB,PHYSOB,INTERVAL):UU

Finally, I describe the process where one object moves away from another:

$$\text{MOVES\_AWAY\_FROM}(x,y,z) \equiv\text{def. } \text{INCREASES}(d_{CM}(x,y),z)$$

type MOVES_AWAY_FROM(PHYSOB,PHYSOB,INTERVAL):UU

It now remains to link these sub-process descriptions together. The definitions for phagocytosis and exocytosis are consequently defined and drawn together as follows:

$$\begin{aligned}\text{PHAGOCYTOSIS}(x,y) \equiv\text{def. } & \text{CELL}(x) \wedge \text{INTERVAL}(y) \wedge \\ & \exists zuvwz'u'v'[\text{FOOD}(z) \wedge \text{FOOD\_VACUOLE}(u) \wedge \text{NUTRIENT}(v) \wedge \\ & \quad \text{EC}(x,z,w) \wedge \text{ENGULFS}(x,z,z') \wedge \\ & \quad \text{DIGESTS}(x,z,u') \wedge \text{ABSORBS}(x,v,v') \wedge \\ & \quad \text{MEETS}(w,z',u',v') \wedge \\ & \quad \text{EQUAL}(\text{sum}(w,z',u',v'),y)]\end{aligned}$$

$$\begin{aligned}\text{EXOCYTOSIS}(x,y) \equiv\text{def. } & \text{CELL}(x) \wedge \text{INTERVAL}(y) \wedge \\ & \exists z[\text{WASTE}(z) \wedge \text{EXPELS}(x,z,y)]\end{aligned}$$

type PHAGOCYTOSIS(CELL,INTERVAL):UU
type EXOCYTOSIS(CELL,INTERVAL):UU

Note that some of the sort declarations used above could be made more specific than that given. For example, the declaration:

type ENGULFS(PHYSOB,PHYSOB,INTERVAL):UU,

could be declared as type ENGULFS(CELL,FOOD):UU, and

type MOVES_TOWARD(PHYSOB,PHYSOB,INTERVAL) as
type MOVES_TOWARD(CELL,FOOD,INTERVAL):UU.

Obviously, the specificity of sorts embedded in particular process descriptions will depend on the complexity of the model used and the degree of generality required for such process descriptions. Other sortal information could be built into the declarations: e.g. **type** EXPELS(AMOEBA,WASTE):UU and **type** EXPELS(T\AMOEBA,T\WASTE):FF, indicating that only amoebae (in the model) can expel waste matter, and only waste material at that.

The attentive reader will probably note several inadequacies for the process definitions given above. For example, the literal MOVES_TOWARD(amoeba1,food1,t1) is satisfied if the protozoan remains stationary and the food drifts toward the protozoan during time t1. One useful notion missing here is agency, another of location and whether or not a body remains in that same location over time. Both notions could be readily accommodated in the formal theory if required, though this moves outside the scope of the present formalism - remembering that here and throughout this thesis primacy is given to descriptions rooted in naked observations, i.e. eschewing notions of forces, agency and goals. For example, in the case of the former, attributing agency to the protozoan could be linked to its ability to change locations without recourse to some external force acting upon it, and its ability to satisfy simple goals, in this case garnering food and undergoing transformations in shape and topology in order to do so. The food in contrast is taken to be of secondary importance, in the sense that apart from its constituent parts, no further explicit information about its shape is required, although the relative size between the protozoan and its food has a bearing on what the protozoan can in principle engulf. The latter notion appeared in section 3.9. There, empirical information about the relative sizes of bodies was exploited to cut down the number of possible spatial relations associated between particular bodies.

Above, processes have been defined by decomposing each process into specified sequences of consecutive states. This ordering, for the greater part, followed the direct transitions sanctioned by the envisionment axioms constructed for different sets of relations. However, these processes can be defined in a more compact form by simply stipulating preconditions that must hold together with descriptions of the the intial and final stages of the process. The intermediate states are subsequently generated with the envisionment axioms. For example, phagocytosis could be defined as:

$$\begin{aligned} \mathrm{PHAGOCYTOSIS}(x,y) \equiv_{def.}\ & \mathrm{CELL}(x) \wedge \mathrm{INTERVAL}(y) \wedge \\ & \exists z[\mathrm{FOOD}(z) \wedge \mathrm{J\_OUTSIDE}(z|\mathrm{initial}(y),x|\mathrm{initial}(y)) \wedge \\ & \mathrm{NTPP}(z|\mathrm{final}(y),x|\mathrm{final}(y))] \end{aligned}$$

(In this case we see phagocytosis begins with the food just outside the cell, and ends when it appears as a nontangential part of the cell, i.e. as part of the food vacuole so formed.)

4.3: Modelling the force pump

A force pump is illustrated in Figures 13 and 14 below. For simplicity, I have assumed that the pump is primed and that the reservoir feeding the inlet pipe is always full of liquid. The pump has two valves, valve1 and valve2 which open by doors, door1 and door2. The doors are hinged to the pump body closing portals portal1 and portal2 respectively. On the upstroke, valve1 is open while valve2 is shut. This arises because the upthrust pressure of the liquid acting upon door1 is greater than the downthrust forces acting from within the pump and acting on that door.

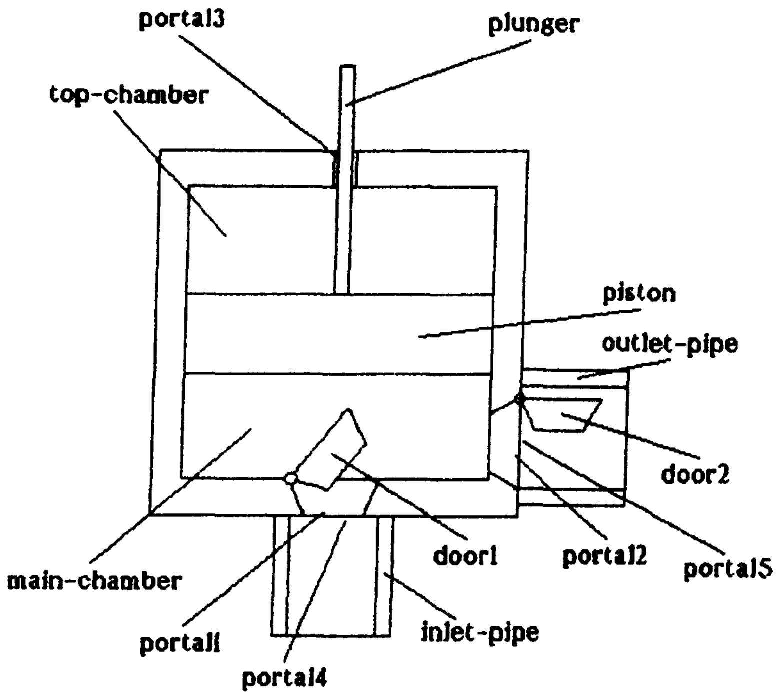


Figure 13: A force pump.

The pressure difference opens the valve door and allows the liquid to pass from the inlet pipe into the main chamber. During this process the door of valve2 remains closed, sealing valve2. In this case atmospheric pressure acting on the door plus that arising from any liquid in the outlet pipe, thrusts the door into the portal effecting a seal. A similar chain of processes arise with the downstroke of the piston. In this case, valve1 shuts and valve2 opens and the liquid passes from inside the pump out into the outlet pipe. The cycle is then repeated.

Three basic states are assumed, where the piston is moving up, is moving down and is stationary. For simplicity I have assumed that when the piston is either at the nadir of its upward or downward motion, the next moment in time coincides with both valves being shut. In actual fact this would not arise in a primed working force pump, e.g. valve1 would almost certainly remain open for a few moments as the piston travelled on its downward path. Other strong assumptions implicit in the description of the working pump are covered below.

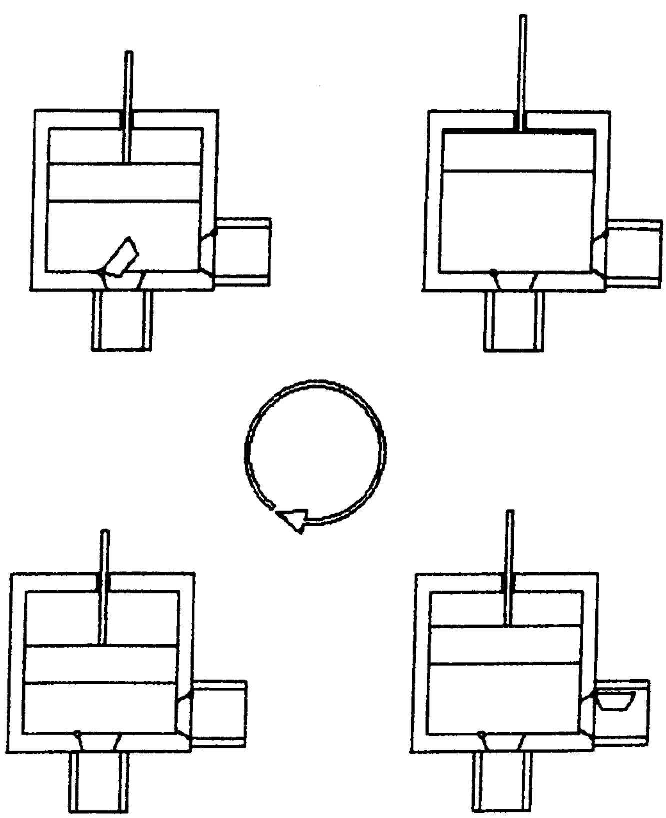


Figure 14: The main cycle of operations of the force pump

Firstly, I build the pump from a library of component parts. The pump body is a multiply connected rigid object with three portals, portal1, portal2 and portal3 which are proper parts of its inside. Note that the portals are represented as *regions* inside the pump body, and that in the model the outer surface of the portals align with exterior surface of the pump body. This makes a portal distinct from any passageway that might link the outside of a body from some inner chamber that might exist (as in this example). Portals are specifically defined not to be surface only or having zero thickness.

The definition of a portal proceeds as follows. A portal x of region y is defined as part of the inside of y such that every closed atom which is part of x, connects with the outside of y. The last conjunct in the definition ensures that the portal/outside interface is not point like. 'PORTAL(x,y)' is read as 'x is a portal of y' and 'Portal(x)' as 'x is a portal':

$$\begin{aligned}\mathrm{PORTAL}(x,y) \equiv_{def.}\ & P(x,\mathrm{inside}(y)) \wedge \\ & \forall z[C(z,x) \leftrightarrow \\ & \quad \exists w[\mathrm{C\_atom}(w) \wedge P(w,\mathrm{inside}(x)) \wedge C(w,\mathrm{outside}(x)) \wedge \\ & \qquad C(z,w)]] \wedge \mathrm{Manifold}(\mathrm{sum}(x,\mathrm{outside}(y)))]]\end{aligned}$$

$$\forall x \exists y[\mathrm{PORTAL}(x,y) \leftrightarrow \mathrm{PORTAL}(x)]$$

type PORTAL(PORTAL,SPATIAL\SPATIAL_UNIVERSE):UU
type PORTAL(SPATIAL\SPATIAL_UNIVERSE):UU

By making portals regions and not part of the boundary interface between the inside and outside of bodies, properties that can be ascribed to regions can also be ascribed to portals. In particular, if the space taken up by a portal's door seals a passageway between the interior (in the non-topological sense) of a pump body and its outside (hence filling in the portal), we can infer that the portal is sealed. A three place predicate 'PORTAL(x,y,z)' read as 'x is a portal of y during time z' is also added and defined as follows:

PORTAL(x,y,z) $\equiv$def. $\forall u[P(u,z) \rightarrow \text{PORTAL}(x,y|u)]$

type PORTAL(PORTAL,PHYSOB,PERIOD):UU

A piston with plunger attached, two pipes, an inlet and outlet pipe, are added. Since the piston always forms a seal with the inner wall of the pup body, adding the piston means that two disconnected chambers are created, the main and top chamber. It is worth pointing out that the formalism makes this relationship explicit.

It would be useful to pick out that region of the pump that functions as the main chamber. Given the particular example of the force pump modelled, this region is delineated by first of all taking the sum region of the pump body and its inside, and then taking the difference between this composite region and the piston and plunger. This results in a disconnected region consisting of the top and bottom chambers. The region connected to portal1 is chosen. Finally the target region is isolated by taking the maximally convex region that fits 'inside' the region is question. In effect this is tantamount to defining a convex kernal (c.f. convex hull), but unlike the convex hull, a convex kernel is not unique (e.g. as with a body with a regular cruciform shaped interior - it would contain two such convex regions) and hence cannot be defined as a function. However, this limitation hides an important fact about pumps of the type given. Given the function of a sliding piston in a pump body (and the fact that pistons and pump bodies are typically rigid objects) some regularity in the interior shape of the inside of the pump body is ensured. The piston always forms a good seal with the inside of the wall of the pump body and one would not expect to find component parts of the pump body acting as protrusions into the work space. Hence despite the fact that no general definition of this region can be given (although in many cases it can be adequately described) a certain utility in picking it out can be argued for. For example, we might want to be able to reason that if the inside of the working pump body got indented, the piston would jam, or the pump would loose its efficiency.

Valves are created by adding hinged doors to the pump body which can seal their respective portals. We represent valves as a two place functor valve(x,y) whose argument sorts are PORTAL, DOOR and whose result sort is VALVE.

A partial formal description of the pump is given below:

∀x Multiply_Connected(pump-body|x)
∀x PP(portal1,inside(pump-body|x))
∀x PP(portal2,inside(pump-body|x))
∀x PP(portal3,inside(pump-body|x))
∀x INSIDE(piston|x,pump-body|x)
∀x EC(plunger|x,pump-body|x)
∀x P_INSIDE(plunger|x,pump-body|x)
∀x Disconnected(diff(inside(pump-body|x,piston|x)
DC(sum(top-chamber,portal3),sum(sum(portal1,portal2),mainchamber))
∀x J_OUTSIDE(inlet-pipe|x,pump-body|x)
∀x EC(inlet-pipe|x,pump-body|x)
∀x J_OUTSIDE(outlet-pipe|x,pump_body|x)
∀x EC(outlet-pipe|x,pump_body|x)
∀x [[SHUT(valve1,x) ∧ SHUT(valve2,x)] →
NTS(main-chamber|x,sum(sum(sum(door1|x,door2|x),pump-body|x),piston|x))
EQUAL(valve1,valve(portal1,door1))
EQUAL(valve2,valve(portal2,door2))

Axioms given below establish a relationship between the pump's valves, the regions that straddle them and the possibility of liquid flow through the valves. The first axiom states that a valve is shut if and only if that valve's portal is filled by (part of) the (solid) valve door. The predicate 'SHUT(x,y)' read as 'x is shut during time y' has the obvious intended meaning the x has a sealed aperture; while 'Solid(x)' read as 'x is solid' denotes the empirical notion of solidity or inpenetrability. The definition for 'SEALED(x,y)' states that a portal is sealed iff it is part of anything solid.

$\forall xyz\,[\mathrm{SHUT}(\mathrm{valve}(x,y),z) \leftrightarrow \forall u[P(u,z) \rightarrow P(x,y|u)]$

$\mathrm{SEALED}(x,y) \equiv \mathrm{def.}\ \forall z[P(z,y) \rightarrow \exists u[P(x,u|z) \wedge \mathrm{SOLID}(u)]]$

type valve(PORTAL,DOOR):VALVE

type SHUT(PHYSOB,PERIOD):UU

type SEALED(PORTAL,PERIOD):UU

sort DOOR $\sqsubset$ SOLID

sort SOLID $\sqsubset$ PHYSOB

For example, given the following description:

EQUAL(valve1,valve(portal1,door1))

we can see that if for some moment in time z, valve1 is shut, door1 seals portal1 making it a solid region and hence SEALED(portal1,z); and that conversely if valve1 is not shut, portal1 is not sealed by door1 and hence (by a closed world assumption), portal1 is open (i.e. not sealed).

Additional axioms give functional definitions of both liquid outflow, inflow and the liquid being static with respect to a portal. Note the use put to the part whole relation "P(w|initial(z),x|final(z))". Here the relation is used to capture the idea of some quantity of a liquid body moving e.g. outside the portal over time.

$$
\begin{aligned}
&\mathrm{OUTFLOWING}(x,y,z) \equiv_{def.} \mathrm{LIQUID}(x) \wedge \\
&\qquad \exists uv\big[\mathrm{PORTAL}(y,u,z) \wedge \neg\mathrm{SEALED}(y,z) \wedge \\
&\qquad\quad \mathrm{P}(w|\,\mathrm{initial}(z),x|\,\mathrm{initial}(z)) \wedge \\
&\qquad\quad \mathrm{P}(w|\,\mathrm{initial}(z),\mathrm{inside}(u,\mathrm{initial}(z))) \wedge \\
&\qquad\quad \mathrm{O}(w|\,\mathrm{initial}(z),y) \wedge \\
&\qquad\quad \mathrm{J\text{-}OUTSIDE}(v|\,\mathrm{final}(z),u|\,\mathrm{final}(z)) \wedge \\
&\qquad\quad \mathrm{C}(v|\,\mathrm{final}(z),y)\big]
\end{aligned}
$$

type OUTFLOWING(LIQUID,PORTAL,PERIOD):UU

sort LIQUID $\sqsubset$ PHYSOB

e.g. given the description:

$$
\begin{aligned}
&\mathrm{OUTFLOWING}(\mathrm{liquid1},\mathrm{portal4},t) \equiv_{def.} \mathrm{LIQUID}(\mathrm{liquid1}) \wedge \\
&\qquad \mathrm{PORTAL}(\mathrm{portal4},\mathrm{inlet\text{-}pipe},t) \wedge \neg\mathrm{SEALED}(\mathrm{portal4},t) \wedge \\
&\qquad \mathrm{P}(\mathrm{liquid2}|\mathrm{initial}(t),\mathrm{liquid1}|\mathrm{initial}(t)) \wedge \\
&\qquad \mathrm{P}(\mathrm{liquid2}|\mathrm{initial}(t),\mathrm{inside}(\mathrm{inlet\text{-}pipe},\mathrm{initial}(t)) \wedge \\
&\qquad \mathrm{O}(\mathrm{liquid2}|\mathrm{initial}(t),\mathrm{portal4}) \wedge \\
&\qquad \mathrm{J\text{-}Outside}(\mathrm{liquid2}|\mathrm{final}(t),\mathrm{inlet\text{-}pipe}|\mathrm{final}(t)) \wedge \\
&\qquad \mathrm{C}(w|\mathrm{final}(t),\mathrm{portal4})\big]
\end{aligned}
$$

we can see that during an outflowing of liquid from portal4, i.e. out of the portal of the inlet pipe (and into portal1) during time t, a quantity of liquid overlapping portal4 moves to be just outside the inlet-pipe and (with the last condition) just outside portal4.

$$
\begin{aligned}
&\mathrm{INFLOWING}(x,y,z) \equiv_{def.} \mathrm{LIQUID}(x) \wedge \\
&\qquad \exists uv\big[\mathrm{PORTAL}(y,u,z) \wedge \neg\mathrm{SEALED}(y,z) \wedge \\
&\qquad\quad \mathrm{P}(w|\mathrm{initial}(z),x|\mathrm{initial}(z)) \wedge \\
&\qquad\quad \mathrm{J\text{-}Outside}(v|\mathrm{initial}(z),u|\mathrm{initial}(z)) \wedge \\
&\qquad\quad \mathrm{C}(v,\mathrm{initial}(z),y) \wedge \\
&\qquad\quad \mathrm{P}(v,\mathrm{final}(z),\mathrm{inside}(u|\mathrm{final}(z)) \wedge \\
&\qquad\quad \mathrm{O}(v|\mathrm{final}(z),y)\big]
\end{aligned}
$$

$$
\mathrm{STATIC}(x,y,z) \equiv_{def.} \forall u\big[\mathrm{P}(u,z) \rightarrow \big[\neg\mathrm{OUTFLOWING}(x,y,z) \wedge \neg\mathrm{INFLOWING}(x,y,z)\big]\big]
$$

type INFLOWING(LIQUID,PORTAL,PERIOD):UU

type STATIC(LIQUID,PORTAL,PERIOD):UU

The processes just defined are not continuous: if OUTFLOWING(x,y,z) is true, it is not necessarily true that OUTFLOWING(x,y,z') is true where z' is a subinterval of z. Continuous versions of these processes are easily defined if required; for example here is a continuous outflowing:

$$\begin{aligned}\text{CONTINUOUS\_OUTFLOWING}(x,y,z) \equiv\text{def. } \forall u\big[[&\text{MOMENT}(u) \wedge \\ &\text{B}(\text{initial}(z),\text{next}(u)) \wedge \\ &\text{B}(u,\text{final}(z))] \rightarrow \\ &\quad \text{OUTFLOWING}(x,y,\text{sum}(u,\text{next}(u)))\big]\end{aligned}$$

type CONTINUOUS_OUTFLOWING(LIQUID,PORTAL,PERIOD):UU

A relation for connected portals (where 'CONNECTED_PORTAL(x,y)' is read as 'x and y are connected (i.e. adjacent) portals' is defined; and an axiom is given that states that for any two connected portals, outflow from one coincides with an inflow into the other:

$$\begin{aligned}\text{CONNECTED\_PORTAL}(x,y) \equiv\text{def. } &\text{PORTAL}(x) \wedge \text{PORTAL}(y) \wedge \neg\text{EQUAL}(x,y) \wedge \\ &\text{Manifold}(\text{sum}(x,y)) \wedge \\ &\forall z\big[[\text{P}(z,x) \wedge \text{C\_Atom}(z)] \rightarrow \text{C}(z,y)\big] \wedge \\ &\forall w\big[[\text{P}(w,x) \wedge \text{C\_Atom}(w)] \rightarrow \text{C}(w,x)\big]\end{aligned}$$

$$\forall xyzu\big[\text{CONNECTED\_PORTAL}(x,y) \rightarrow [\text{OUTFLOWING}(z,x,u) \leftrightarrow \text{INFLOWING}(z,y,u)]\big]$$

type CONNECTED_PORTAL(PORTAL,PORTAL):UU

e.g. $\text{CONNECTED\_PORTAL}(\text{portal1},\text{portal4}) \rightarrow$
$[\text{OUTFLOWING}(\text{liquid1},\text{portal4},t) \leftrightarrow \text{INFLOWING}(\text{liquid1},\text{portal4},t)]$

The definition of connected portals ensures that the connection between them is not point-like (use of Manifold) and that they are totally aligned.

The axioms and definitions given above are sufficient to make the following deductions. Suppose valve1 is shut, and portal1 and portal4 are connected. We can infer that since portal1 is part of door1 (i.e. occupied by the door), the portal is not open (because implicitly the door has been construed as a 'solid' region). We can then deduce that no inflowing or outflowing can arise through either portal (or between the connected portals). Hence the liquid within the pump is static with respect to portal1. With the converse case when valve1 is open (not shut), either an inflowing or outflowing may arise across the connected portals.

Directionality of fluid flow through the valves in the example pump is fixed (eschewing the realistic case where e.g. portal1 would actually experience bi-directionality of fluid flow over time as the piston commenced on its downstroke and the valve was closing). Appropriate axioms fixing the directionality of the flow (actualy fixing the direction in which the valve doors open) could be done as follows:

IN_VALVE(valve(x,y)) $\equiv$def. $\forall$zu $\neg$OUTFLOWING(z,x,u)
OUT_VALVE(valve(x,y)) $\equiv$def. $\forall$zu $\neg$INFLOWING(z,x,u)

type IN_VALVE(VALVE):UU
type OUT_VALVE(VALVE):UU

We can now state that valve1 is an in-valve and valve2 an out-valve, i.e.

IN_VALVE(valve1)
OUT_VALVE(valve2)

There are some strong assumptions underlying the use of these biconditionals used in the axiomatisation, e.g. that at no time does a foreign body block a valve (even though a portal may remain open) and that the liquid doesn't undergo any change of state. This has been done to simplify the example, but this is no indication that such restrictions are a by-product of the formalism and its underlying ontology.

As yet no information has been given covering either causal factors or the initial conditions required for the fluid to flow through the pump. But is is not that difficult to see what could be added and exploited. For example we could state that the inlet-pipe must be filled with liquid in order for the liquid to pass into the pump body on the upstroke of the piston. Given the simple case of a primed pump, this fact is easily expressed in the formalism:

$$\exists x \forall y [\mathrm{LIQUID}(x) \wedge P(\mathrm{inside}(\mathrm{inlet\_pipe}|y),x|y)]$$

i.e. that the inside of the inlet-pipe is part of a liquid body - which is to say that the pipe is (in this instance) always full of liquid. The fact that liquid can be drawn up into the inlet-pipe and into the pump body, i.e. that the condition given above need not hold to get liquid into the pump could be expressed in the formalism reasonably easily. In this case it would be useful to add an axiom abstracting out the inequalities expressed by Boyle's Law which states that at constant temperature the pressure of a given mass of gas is inversely proportional to its volume.

$$\forall xy [[\mathrm{GAS}(x) \ \& \ \mathrm{CONSTANT}(\mathrm{temp}(x),y)] \rightarrow$$
$$[\mathrm{INCREASE}(\mathrm{press}(x),t) \longleftrightarrow \mathrm{DECREASE}(\mathrm{vol}(x),y)]]$$

Given this information we could reason that when the pump is started (and the inlet-pipe placed in a reservoir filled with fluid) the act of pulling up the piston would coincide with the trapped air in the pump (constant mass) increasing in volume. Assuming portal1 was not sealed this would mean that forces aring from the atmospheric pressure acting on the reservoir fluid would propogate through the liquid. This would force the liquid into the inlet-pipe and eventually into the pump body. Indeed we could adopt a naive view of suction by stipulating that at all times a pocket of air exists between the bottom face of the piston and the liquid in the pump body; and that when the piston moves upward, the volume of the air pocket increases, its pressure drops, and the liquid fills the vacuum formed. In the downstroke process, the trapped air would decrease in volme resulting in its internal pressure increasing which would force the liquid down and out through the outlet pipe.

As indicated above a complete axiomatisation of the pump is beyond the scope of this thesis. The purpose of this chapter is to show in principle the formal adequacy of the theory for modelling non-trivial physical domains. However, below, I indicate some directions in which this could be tackled.

In the first place it would be useful to be able to pick out those surfaces of the liquid that come into contact with the surfaces of the valve doors, the piston and the surface of the air/liquid interface (known as the freesurface). By doing this the action of an external force on such bodies (or impressed force of liquid on an object) could be described. The definition picking out the outside 'surface' or 'skin' of an object is already given. This is used as the basis for describing the free surface of a liquid body:

$$
\begin{aligned}
\mathrm{freesurface}(x|y) =\mathrm{def.}\ \ \iota z[&\mathrm{LIQUID}(x) \wedge \\
&\forall u[\mathrm{C\_Atom}(u) \rightarrow \\
&\quad [P(u,z) \leftrightarrow P(u,\mathrm{skin}(x|y)) \wedge \\
&\quad\ \exists v[\mathrm{AIR}(v) \wedge C(u,v)]]]]
\end{aligned}
$$

type freesurface(τ):$\tau \sqcup$ NULL, τ = SPATIAL\SPATIAL_UNIVERSE

With the free surface defined, and an adequate characterisation of one region being above (or below) another, one could then reason that if the freesurface of the liquid did not overlap the bottom portal of the inlet pipe, no additional water from the reservior could be pumped through the pump. Varying volumes of liquid could then be linked to the position of the piston in the main chamber. This would require one to pick out e.g. surfaces of component parts of the ump and parts of the surface of the pump body. Given the formalism has an explicit distance function ('d(x,y)') this could be used here. For example one could simply say that the piston moves up in the pump if the distance between it and portal1 increases (assuming rigidity of the component parts). In turn this would be linked with differences in pressure between bodies of liquid, and whether or not valves were shut. One would need to be able to reason that when the piston is drawn up, the downthrust force of the piston acting on the contained fluid is less than thre atmospheric pressure propogating a force through the liquid and acting on the free surface of the liquid in the reservoir. The downthrust force of the atmospheric pressure propogating a force through the liquid results in an upthrust force on the piston/liquid interface (if we assume no pocket of air between the two, or between the liquid/air and air/piston surface interfaces if we do). Pressure differences serve to force the liquid through the inlet-pipe and into the main chamber; valve1 opens because the external force of the liquid impressed on its underside is less than the sum forces acting on the side of the door.

Additional empirical information can be added and exploited. For example, rigidity in a body would mean deformability could not arise, that physical objects if originally discrete would typically remain so over time. Liquids being construed as deformable incompressible bodies would have constant volume with respect to compressive forces but would be allowed to change their shape and pass into and fill insides of regions. In contrast gaseous have the property of filling and occupying the inside of sealed containers. Below this property is defined, although it is recognised that the definition for a generalised container, and where in actual fact containers vary according to the material contained, questions of gravity, orientation and so on.

$$\text{SEALED\_INSIDE}(x,y,z) \equiv_{def.} \text{CONTAINER}(y,u) \wedge \exists v[P(v,y|z) \wedge \text{INSIDE}(x|z,v) \wedge \forall w[\text{PORTAL}(w,v) \rightarrow \text{SEALED}(w,z)]]$$

$$\text{CONTAINER}(x,y) \equiv_{def.} [\text{SOLID}(x) \wedge \text{HOLLOW}(x|y) \vee \exists z\ \text{PORTAL}(z,x|y)]$$

type SEALED_INSIDE(PHYSOB,PHYSOB,PERIOD):UU
type CONTAINER(PHYSOB,PERIOD):UU
type SEALED(PORTAL,PERIOD):UU

One final point: the notion of being a part has been blurred somewhat. Above "part" is used in the sense of part to whole of *regions*, and secondly in where parts of the pump have been picked out - the "component" parts. The relationship between the two can be made explicit as follows:

$$\forall x[\text{FORCE\_PUMP}(x) \rightarrow \forall y[\text{COMPONENT\_PART}(y,x) \rightarrow \text{RIGID}(y)]]$$
$$\forall xyz[\text{COMPONENT\_PART}(x,y) \rightarrow P(x|z,y|z)]$$

sort RIGID $\subset$ PHYSOB
sort FORCE_PUMP:PHYSOB
type FORCE_PUMP(PHYSOB):UU
type COMPONENT_PART(PHYSOB,PHYSOB):UU

4.4: Summary

Hayes [1979,1985a] indicated that an indication of success in theory development was when one found one had enough concepts to describe the chosen domain - what Hayes called "conceptual closure". Complete closure was considered unlikely. Both points seem vindicated here. The rich partitioning of space, and the emphasis on expressing topological information seems adequate to describe many important properties and relations. As has been shown above, many descriptions of process can be characterised in terms what happens when something else happens. While additional notions of, for example, force and agency are useful, these are not necessary in order to describe information derived directly from our experience of physical space, but appear when explanations are sought why such processes occur when they do.

Chapter 5: Efficiency of inference

5.1: Introduction

Despite the fact one can endorse Hayes' [1979,1985a] point that one should not let implementational questions detract from the primary task of building rich formal theories, there is a comparable danger that decoupling representation from inference will also result in a poor research methodology. At all times in the process of theory construction, it is wise to consider questions of computational cost arising from implemented theories.

The computational cost of using uncontrolled inference for computational logics is well known. This fact has given rise to the recent interest shown in the use of different hybrid representation and reasoning systems (see e.g. Frisch and Cohn 1990 for a fairly recent summary). The basic idea is to abstract or factor out particular knowledge structures embeded in a theory, and then assigning each "factor" to a subsystem in which specialist inference is done. It should be apparent that the theory used in this thesis reflects this. Although the representational language is first order and sorted, the theory includes knowledge about sorts, subsumption relationships (both for sort predicates and relations - see below), transitivity networks and transition/continuity restrictions, all of which are factored out and can be used in different ways.

This chapter concentrates upon one way the theory described in this thesis can be used and implemented - in this case a direct implementation within an automated resolution based reasoning program. Efficient means to secure various forms of control of inference are suggested. However, it should be borne in mind that given the emphasis given to the development of the conceptual apparatus of the theory, sections that discuss efficiency of inference are exploratory in nature.

This section presupposes some familiarity with the machine inference rules known as *resolution* and *paramodulation*. The classic introduction to the former is Robinson [1979]. Good introductory texts to resolution and paramodulation are Chang and Lee [1973] and Wos et al [1984].

5.2: Relating unsorted and sorted logics and axiomatic theories: some problems.

There are well known methods by which sorted logics are mapped to their unsorted counterparts. The translation given is called the *relativisation*. The isomorphism between the sorted and unsorted sorted theory is then established with the *Sort Theorem* that shows (for the model theoretic part), a set of clauses expressed in a sorted logic is unsatisfiable iff its relativisation is unsatisfiable and (for the proof theoretic part) a refutation for a set of clauses in a sorted logic exists iff a refutation for its relativisation exists. [Cohn, 1988]. With respect to the converse case (relating an unsorted logic to its sorted counterpart), general translation rules do not exist (but see Schmidt-Schauss [1988] where a technique is given for his logic).

Converting a first order unsorted theory to a sorted one frequently requires much groundwork establishing the embedded sort structure. In practise this requires proving that for each pair of potential sort symbols, the monadic predicates in the unsorted theory are either disjoint, or form a subsumption relationship. If the sorted logic into which the unsorted theory is being translated requires complete knowledge about the sort structure i.e. having a set of base sorts that are all pairwise disjoint (e.g. as currently required by LLAMA), the difficulties can increase dramatically. If for example 7 non-base sorts are subsorts of some given sort, 2^7 paired sort intersections must be first evaluated before other subsorts are added and the new set of sortal relationships established. Closing such a lattice structure can prove difficult in practice. This becomes particularly problematic if the target unsorted theory uses only a few primitive notions, and employs many definitions (as in Clarke's [1981,1985] theory, and in the theory developed in this thesis). In this case, proofs to secure the relative positions of the potential sort symbols will tend to prove difficult to tease out. Until a set of base sorts are generated and the sortal lattice closed, the theory builder will be required to continually revise the sortal declarations provisionally made. Moreover, given complete knowledge of the sort structure, these will also change if an extant theory is further developed, and where additional base sorts are embedded in the lattice structure.

One other point needs mentioning here. It is well known that using the standard (objectual) interpretation of the quantifiers for FOL, at least one object must be posited in the intended model. But in a sorted logic the minimal model will change, simply because whereas in the unsorted logic only one object may give the minimal model, in a sorted logic each sort must be non-empty too. That is to say, in general as one moves from the unsorted, to the sorted theory, the minimal models will constantly change. In the case of LLAMA, the theory builder needs to be particularly aware of this fact. It is all too easy to fail to recognise that two potential sort symbols must be disjoint, or the one must subsume the other simply because each sort must be non-empty. For example, in an unsorted theory (in this case not a relativisation) there seems no apriori reason to rule out the case where the universal temporal region is atomic i.e. has momentary existence. But once the sorts MOMENT and INTERVAL are added, this interpretation cannot be maintained. The point here is that, given the process of theory contruction, one's intended guiding model may not be the *minimal* one required by the theory.

Further difficulties also arise if the unsorted theory under investigation is incomplete, but not known to be so. In this case the presupposition of completeness will hide the fact that the formalised theory may not be capable of eliciting the desired proof (e.g. as may arise when an axiom is missing) and prevent the relative position of the sort from being factored out. Such difficulties are especially apparent when building large scale Naive Physical theories along Hayesian lines, since a rich theory will support a dense web of inferential connections between a theory's concept tokens and may make it difficult to see what is 'missing'.

Although LLAMA supports some useful computational properties that can be exploited if one has at the outset complete knowledge of the sort structure (which is discussed below), in practice this is unlikely to be given. It would be useful to be able to relax the condition that the sort lattice be closed, since evaluation of disjointness of 'base sorts' would not be required. An outline of such a logic can be found in Cohn [1990]. In this case a sorted logic with the same expressiveness of LLAMA (i.e. allowing ad hoc polymorphic functions and predicates, and overlapping) is envisaged, but the condition for complete knowledge of the sort structure is relaxed. It is perhaps instructive to realise that it was in recognition of the difficulties cited here, that convinced Tony Cohn (my thesis supervisor) to set to and develop such a logic.

In some respects, unsorted axiomatisations that are chosen and converted into sorted ones (to demonstrate efficiency gains in automated theorem proving) are somewhat contrived and ad hoc. In most cases taxonomic information is not deeply embedded in the axiomatisation, but appears at the surface and is easily extracted. This makes the translation of the unsorted axiomatisation into a sorted one fairly straightforward. Although it could be argued that the sort structure encoded in, for example, Schubert's Steamroller challenge problem [Stickel 1985] was intentionally kept at the surface so that it could be easily extracted, exploited and then used as a test-bed for evaluating automated sorted logics, it would be a mistake to think such axiomatisations are always forthcoming or even desirable. While it is true that (given a theory rich in taxonomic information) an implemented sorted logic has well known computational advantages over its unsorted counterpart, it is all too easy to let questions of efficiency dominate one's thinking in the selection or construction of first order formalisms deemed suitable for AI applications.

Most of the interesting axiomatisations that could be used as a foundation for modelling reasoning about aspects of the everyday world are not sorted, or if sorted only support a few sorts, see e.g. Carnap [1958 Chapter's D to H and Appendix]. One can invest a greater degree of confidence in the use of such formal theories than some of those that have appeared in AI literature, since it is reasonable to expect questions of economy, for example of establishing formal independence of the axioms and the desire to use a minimal set of primitives, consistency and completeness (with respect to the formalised theory) have been addressed. Unfortunately the same cannot be said for the latter. See, for example the Commonsense Summer '85 report [Hobbs et al 1985] where questions of consistency were waivered in lieu of expressiveness.

Axiomatisations that have a set of independent postulates (axioms) and primitives are particularly difficult to construct and use. Gains in economy coincide with a gain in complexity in use: both in terms of constructing desired proofs and given an uninterpreted formal system, finding a concrete interpretation. However, in practical terms axiomatisations that support a set of non-independant axioms are frequently used along with the use of lemmas to assist in the derivation of desired proofs. Similarly a sorted logic may be employed to facilitate shorter proofs and thereby render them easier to construct either by hand or mechanised, by machine.

5.3: Using the sorted logic LLAMA

A brief introduction to sorted logics and LLAMA is covered in Chapter 2 and is assumed here.

LLAMA's sort lattice L_{*} and special Boolean sort lattice $L_{\bullet}$ provides the basis for 'building in' theorems or lemmas into the sortal machinery without increasing the number of clauses that serve to define the formalised theory. For example, in the present theory, the theorem $\forall x\ \neg EC(x,u_T)$ (which states that no period externally connects with the universal temporal period) can be embedded in the declaration **type** EC(PERIOD,PERIOD_UNIVERSE):FF. The same can be done for theorems (or lemmas) which incorporate constants or function symbols. For example, the theorem: $\forall xy\,[OPEN(x) \wedge OPEN(y)] \rightarrow OPEN(sum(x,y))]$ is absorbed in the declaration **type** sum(OPEN,OPEN):OPEN. Securing proofs of these theorems just using the main set of defining axioms of the theory, are surprisingly complex. For example, take the first theorem:

Refutation set:

1 ¬C(x,y) v C(y,x) (from A2)

2 ¬O(x,y) v P(f3(x,y),x) (from D5)

3 ¬O(x,y) v P(f3(x,y),y) (from D5)

4 ¬P(z,x) v ¬P(z,y) v O(x,y) (from D5)

5 ¬EC(x,y) v C(x,y) (from D8)

6 ¬EC(x,y) v ¬O(x,y) (from D8)

7 ¬C(x,y) v O(x,y) v EC(x,y) (from D8)

8 ¬NTP(x,y) v ¬EC(z,x) v ¬EC(z,y) (from D10)

9 ¬EQUAL(int(x),x) v NTP(x,x) (lemma)

10 ¬OPEN(x) v EQUAL(int(x),x) (from D31)

11 ¬PERIOD(x) v OPEN(x) (from A36)

12 PERIOD(a)

13 EC(a,b)

Proof:

14 OPEN(a) ancestors: 12,11

15 EQUAL(int(a),a) ancestors: 14,10

16 NTP(a,a) ancestors: 15,9

17 ¬EC(x,a) ancestors: 16,8

18 C(a,b) ancestors: 18,1

19 ¬O(a,b) ancestors: 13,6

20 C(b,a) ancestors: 18,1

21 ¬C(x,a) v O(x,a) ancestors: 17,7

22 O(b,a) ancestors: 21,20

23 ¬P(x,a) v ¬P(x,b) ancestors: 19,4

24 P(f3(b,a),b) ancestors: 22,2

25 P(f3(b,a),a) ancestors: 22,3

26 ¬P(f3(b,a),b) ancestors: 25,23

27 null ancestors: 26,24

This particular proof uses 14 (binary resolution) inference steps - and as the attentive reader will notice it also uses a lemma - clause 9.

It should be reasonably clear that simply adding lemmas to a set of defining axioms and definitions using a simple mechanised unsorted logic runs the risk of dramatically increasing the potential search space. Thus the facility where such information can be 'built in' without increasing the size of the clause set that defines the theory is to be welcomed.

As mentioned above, LLAMA's requirement that the sort lattice L_s be a complete Boolean lattice offers some useful computational properties. The first is that the elements of the sort lattice can be represented as a bit map (see Aït-Kaci et al [1989] for the relevant details). The second advantage gained is that a normal form can be defined so that no term appears as the argument to more than one sort predicate in any clause. For example, the clause: 'φ v INTERVAL(x) v MOMENT(x)' can be normalised to the clause 'φ v PERIOD(x)' - see Cohn [1987]. A third advantage is that reasoning by cases is possible [see Cohn 1989b].

5.3.1: Comparing unsorted and sorted (LLAMA) proofs

Below I show how by exploiting sortal information, the number of inference steps are reduced in LLAMA when compared with the unsorted case. In fact for the following example, the unsorted case is actually sorted. That is to say, I compare two proofs that use the minimal sort lattice for the "unsorted" case, and a richer sort lattice for the sorted case. But in any case the principle should be clear.

In the following example, I introduce OPEN, and CLOSED as additional sort symbols. Declarations for the set of mereological relations (without their inverses), and the topological function int(x) follow. The reader is reminded that here I assume the modelled domain to be space, i.e. the variables range over spatial regions only.

Table 3: Boolean sort declaractions for the mereological relations.

$\Phi(\tau 1, \tau 2)$	C	DC	P	PP	EQUAL	O	DR	PO	EC	TP	NTP	TPP	NTPP	TPI	NTPI
Open Open	UU	UU	UU	UU	UU	UU	UU	UU	FF	FF	UU	FF	UU	FF	UU
Open Closed	UU	UU	UU	UU	FF	UU	UU	UU	FF	FF	UU	FF	UU	FF	FF
Closed Open	UU	UU	UU	UU	FF	UU	UU	UU	FF	FF	UU	FF	UU	FF	FF
Closed Closed	UU	UU	UU	UU	UU	UU	UU	UU	UU	UU	UU	UU	UU	UU	FF

where $\tau 1, \tau 2$ = SPATIAL, $\Phi \in$ {C,DC,P,PP,EQUAL,O,DR,PO,EC,TP,NTP,NTPP,TPI,NTPI}

type OPEN(OPEN):TT

type OPEN(CLOSED):FF

type CLOSED(CLOSED):TT

type CLOSED(OPEN):FF

type int(SPATIAL):OPEN

Example 1: Unsorted proof of the thereom: ∀x NTP(int(x),int(x))

Refutation set:

1 P(x,y) ∨ C(f2(x,y),x) (from D2)

2 ¬C(f2(x,y),y) ∨ P(x,y) (from D2)

3 ¬P(z,x) ∨ ¬P(z,y) ∨ O(x,y) (from D5)

4 ¬EC(x,y) ∨ ¬O(x,y) (from D8)

5 ¬P(x,y) ∨ NTP(x,y) ∨ EC(f5(x,y),x) (from D10)

6 P(x,x) (from T3)

7 ¬NTP(int(a),int(a))

Proof:

8 ¬P(int(a),int(a)) ∨
EC(f5(int(a),int(a)),int(a)) ancestors: 7,5

9 EC(f5(int(a),int(a)),int(a)) ancestors: 8,6

10 ¬O(f5(int(a),int(a))),int(a)) ancestors: 9,4

11 ¬P(x,f5(int(a),int(a))) ∨ ¬ P(x,int(a)) ancestors: 10,3

12 ¬P(int(a),f5(int(a),int(a))) ancestors: 11,6

13 C(f2(int(a),f5(int(a),int(a)) ancestors: 12,1

14 ¬C(f2(int(a),f5(int(a),int(a)) ancestors: 13,2

15 null ancestors: 14,13

Example 1b: Sorted (LLAMA) proof of ∀x NTP(int(x),int(x)

Refutation set:

1 P(x,y) ∨ C(f2(x,y),x) (from D2)

2 ¬C(f2(x,y),y) ∨ P(x,y) (from D2)

3 ¬P(z,x) ∨ ¬P(z,y) ∨ O(x,y) (from D5)

4 ¬EC(x,y) ∨ ¬O(x,y) (from D8)

*5 ¬P(x,y) ∨ NTP(x,y) ∨ EC(f5(x,y),x) (from D10)

*6 P(x,x) (from T3)

*7 ¬NTP(int(a),int(a))

Only clauses marked with an asterisk "*" are actually used in the following proof, but the original set is repeated to show the reduction in the number of clauses used:

Proof:

(7) ¬NTP(int(a),int(a))

(8) ¬P(int(a),int(a)) ancestors: 7,5

(9) null ancestors: 8,6

In this particular example, clause 7: ¬NTP(int(a),int(a)), is resolved with clause 5 producing the resolvent:
¬P(int(a),int(a)) ∨ EC(f5(int(a),int(a)),int(a)). LLAMA then detects that the sort environment for literal EC(f5(int(a),int(a)),int(a)) is FF (as can be verified from the table above) and deletes the literal from the clause, resulting in the simpler clause ¬P(int(a),int(a)).

Both proofs use the same general rules of inference - in this case binary resolution. The LLAMA proof reduces the number of general inference steps from 8 (in the unsorted case) to 2 in the sorted case. It is difficult to evaluate efficiency gains of a sorted logic over its unsorted counterpart from a few examples, however it does seem clear from the literature and initial forrays using complete sub-lattices embedded in the overall sort lattice, that given a non-trivial theory supporting a rich taxonomic structure, the sorted logic will typically score over its unsorted counterpart.

Further work is needed to absorb all the monadic predicates supported by the formalism into the sortal apparatus afforded by LLAMA before the complete theory can be implemented and statistical measures made. There is however need of a cautionary note here.

While it is indeed possible to factor out all the monadic predicates and rework them as sort predicates, the demand made on the translator of the formalism simply reflects the computational complexity that arises in the use of the sorting functions, and in the work undertaken by the sort algorithm. For some applications, it may be more expedient to use a minimal sort lattice and not factor out all the monadic predicates in the implemented theory. An example of this can be seen with the theorem: $\forall x\ \neg EC(x,u_s)$ (i.e. no spatial region externally connects with the universal spatial region). In this theory, no open region can externally connect with another region. If OPEN is included as a subsort of REGION, the sorting function declarations:

type EC(REGION,OPEN):FF
type EC(OPEN,REGION):FF
type u_s:OPEN

would be sufficient (using the entries in Table 1) to immediately detect that the wff $\forall x\ \neg EC(x,Us)$, is a theorem. But the same result can be derived using the following sorting function declarations, *without* making the monadic predicate OPEN(x) a sort predicate.

type EC(SPATIAL_UNIVERSE,SPATIAL):FF
type EC(SPATIAL,SPATIAL_UNIVERSE):FF

type u_s:SPATIAL_UNIVERSE

For a given class of theorems to be derived, there may well be some optimum point after which the conversion of monadic predicates into sort predicates, may actually increase the time for which a desired proof is secured. Further work is needed here, although it may be difficult to generalise the results to other axiomatisations.

5.3.2: Expressing defined functions as identity unit clauses.

Currently formulated, the definitions used for the expanded set of Boolean and topological operators follow Clarke [1981]. However, an alternative set of definitions can be constructed simply by defining the complement operator and one other Boolean operator, and similarly for the topological operators, e.g. given compl(x), sum(x,y) and int(x) as defined, i.e.:

prod(x,y) =df. ιz[EQUAL(z,compl(sum(compl(x),compl(y)))
diff(x,y) =df. ιz[EQUAL(z,prod(x,compl(y))]
cl(x) =df. ιy[EQUAL(y,compl(int(compl(x)))]
ext(x) =df. ιy[EQUAL(y,int(compl(x))]

When seeking mechanised proofs of some theorems, the use of such a set of equality unit clauses combined with paramodulation can lead to a quicker derivation of the null clause than using the default set of definitions. Similarly, one proof run will terminate quicker if equality term rewriting is done instead of unpacking the equality relation in terms of other mereological relations and using normal inference on the set of generated clauses. The appearance of deeply nested functions in a proof run might suggest the use of unit identity definitions coupled with either paramodulation or simply assigning them to a demodulator list. However, as is well known, paramodulation is difficult to control, while the practice of simply assigning unit clauses to a demodulator list can result in an incomplete refutation (proof) strategy (see e.g. Wos 1988).

Having equality sorted in a mechanised logic provides an effective way to constrain the number of potential clauses generated with the unrestricted use of paramodulation. In the case of LLAMA, equality clauses instantiated with incompatible sorts are immediately rendered FF, and can significantly add to the sought refutation.

5.4: Adding further global control strategies

5.4.1: Peeking, 'Old Gazing' and Gazing

Although definitions allow compact expressions to be constructed and used in a formal language, many useless branches in the search space using an automated logic can arise, if the definitions are unpacked without restriction. In addition to the use of a sorted logic, techniques exist to control the proliferation of inference by controlling the manner in which definitions are unfolded in a proof run.

'Old' gazing [Plummer,1987] and 'Gazing' [Giunchiglia and Walsh, 1988,1989] employ efficient global techniques for directing a proof in automated logics. Gazing improves upon earlier local strategies employed in the use of definitions, e.g. peeking [Bledsoe and Tyson, 1975]. Giunchiglia and Walsh take earlier work [Plummer, 1987; Warren, 1987 and Simpson, 1987] and put this in a formal framework.

Old Gazing [Plummer,1987] utilises a heuristic that only unfolds definitions deemed necessary to ensure the set of functions and predicates in the hypothesis and conclusion match. The hypotheses and conclusion are abstracted to give the set of predicate names used. Definitions are abstracted as a set of rewrite rules. In this case the direction of the rewrite is restricted so that e.g. predicates can only be unfolded in terms of more primitive predicates in the theory. Old gazing and gazing use a propositional abstract space in which the abstract solution for some problem is sought. An immediate consequence of this abstraction is that a proof found in the absract space does not guarantee the existence of a proof in the non-abstracted first order case. However Giunchiglia and Walsh prove for gazing that if some wff is a theorem in the original space, a proof exists in its abstraction space. Given, the extensive use of definitions in the current theory and the relatively few number of primitives used, the use of such techniques would seem promising for securing proofs that normally require much unpacking to find literals that clash and eventually secure a proof.

5.4.2 Theory resolution

Stickel's [1985a] Theory Resolution offers a general framework for building in theories into a resolution theorem proving program so that it is not necessary to resolve directly upon the given axioms of a theory. This is a powerful technique since theory resolution related to the set of nodes of the relational lattice (Figure 2) would detect the unsatisfiability of e.g. clause PO(a,b) with clause TPP(a,b) without having to unpack the definitions for both predicates to get the clash. Theory resolution generalises the notion of a clash between literals, since normally only literals with opposite polarity (e.g. $\Phi(x,y)$ and $\neg\Phi(x,y)$) are allowed to clash.

An application of theory resolution called *characteristic resolution* appears in LLAMA [Cohn 1987]. In this case normal resolution is extended to allow two sort literals, $\alpha(x)$ and $\beta(x)$ (e.g. MOMENT(a) and INTERVAL(a)) to clash even if they are not complementary and have different names. In this case the clash is deduced from the relative positions of the sort symbols in the sort lattice $L_{\bullet}$. If, for example $[\alpha \sqcap \beta](x) = \perp$ as in the case where literals MOMENT(a) and INTERVAL(a) occur - sorts MOMENT and INTERVAL - then the resolvent of the two formulae is semantically equivalent to "false" and the clash indicated. Characteristic resolution also allows a partial clash between literals resulting in a residue literal. For example, given the monadic predicate Atom(x) now functioning as a sort symbol, the clauses: $\phi \vee$ ATOM(a), and $\phi \vee$ PERIOD(a) resolve to $\phi \vee$ MOMENT(x).

Characteristic resolution is defined as follows:

(i) $\alpha(x)$ and $\beta(x)$ resolve to give $[\alpha \sqcap \beta](x)$,
(ii) $\alpha(x)$ and $\neg\beta(x)$ resolve to give $[\alpha \setminus \beta](x)$
(iii) $\neg\alpha(x)$ and $\neg\beta(x)$ resolve to give $\neg[\alpha \sqcup \beta](x)$

Further, if respectively either $[\alpha \sqcap \beta] = \perp$, or $[\alpha \setminus \beta] = \perp$, or $[\alpha \sqcup \beta] = \top$, then the resolvent is semantically equivalent to "false".

The rule of characteristic resolution is generalised so that it applies to sets of relations that form a lattice, and not just the monadic sort predicates. As in the case of the sort lattice $L_{\bullet}$, the set of named nodes are complemented with a set of un-named nodes, so that the set of nodes can be embedded in a complete Boolean lattice. This application of theory resolution is illustrated here using the relational lattice L_c (depicted in Figure 3) which covers the set of dyadic relations defined solely in terms of C(x,y). (However, it should be evident that the principle applies for other sets of relations that can be embedded in a lattice.) Thus (for the dyadic case): given two distinct literals $\alpha(x,y)$ and $\beta(x,y)$ belonging to L_c, these resolve to give $[\alpha \sqcap \beta](x,y)$; just as is done in the monadic case.

Cases where one literal is positive and the other negative, or both literals are negative again follow the rule of charateristic resolution for the monadic case.

It is reasonably easy to verify the correctness of this inference procedure. Each literal of the form $\alpha(x,y)$ and belonging to L_C is proved to be equivalent to a finite disjunction: $\alpha 1(x,y) \vee \ldots \alpha n(x,y)$, where $\alpha 1 \ldots \alpha n$ represents the set of base predicates that extend below α and are above $\perp$. Given that each predicate appearing as a node in the lattice can be identified with a set of base predicates, one can simply use the lattice theoretic operations: $\sqcap$, $\backslash$, and $\sqcup$ on the corresponding set of base predicates in this specialised form of resolution. For example, suppose we wished to resolve the literals $P(a,b)$ and $EC(a,b)$; we compute $[P \sqcap EC](a,b)$. This is $\perp$ and a clash is found. This is equivalent to proving that the intersection of the set of base predicates for $P(a,b)$ and $EC(a,b)$ is empty, i.e.

$$P(a,b) = \{TPP(a,b), NTPP(a,b), TPI(a,b), NTPI(a,b)\}$$
$$EC(a,b) = \{EC(a,b)\}$$

which it is.

By parity of reasoning, if $[\alpha \sqcap \beta](x,y) \neq \perp$ then $\alpha(x,y)$ and $\beta(x,y)$ are consistent, if $[\alpha \sqcap \beta] = \beta$, then $\alpha(x,y)$ is more general than $\beta(x,y)$, i.e. $\beta(x,y) \rightarrow \alpha(x,y)$. Finally if $[\alpha \sqcup \beta](x,y) = \top$, a tautology is indicated: appearing within a single clause, the whole clause can be safely deleted in the proof run as it cannot add in any way to the desired refutation.

Simplification of formulae also carries across to the higher arity predicate case: the clause can be normalised so that no argument tuple is predicated by more than one predicate symbol acting as a node in L_C. For example, the literals $TP(a,b)$ and $NTP(a,b)$ normalise to $P(a,b)$. Similarly the definition of subsumption can be changed (as it is in characteristic resolution to take account of characteristic literals appearing in formulae) to take account of redundancy in the predicate case: e.g. where EC(a,b) v ϕ, subsumes DR(a,b) v ϕ.

Obviously properties of relations e.g. the symmetry of the relations C(x,y), O(x,y), DC(x,y), DR(x,y), EC(x,y), EQUAL(x,y), TPI(x,y) and NTPI(x,y) must be taken into account, since without for example, symmetric unification, the clauses P(a,b) and DC(b,a) will fail to resolve using this form of resolution. Similarly, normalisation of formulae will be affected. The clause P(a,b) v TPP(b,a) v EC(a,b) can be normalised as $[P \sqcup EC]$(a,b) v TPP(b,a), or as P(a,b) v $[TPP \sqcup EC]$(b,a). This non-uniqueness does not cause any particular problem (except perhaps for determining which will give rise to the better search space for the problem under consideration).

Further properties of the base theory can be built into this form of resolution. For example, in the base theory the unit clauses: C(x,x), O(x,x), P(x,x) and EQUAL(x,x) are equivalent. The addition of the single axiom: $\forall$x EQUAL(x,x) to the clause set, combined with this rule of resolution is sufficient to prove all the other totally reflexive (and irreflexive) properties of the relations supported by L_C.

5.4.3 Transitivity networks

A transitivity table similar in function to that used by Allen [1983]) is calculated for all combinations of the base relations that appear in L_C. Each entry of the form R1(a,b) and R(b,c) is mapped to a disjunctive set of base predicates, corresponding to a theorem. For example the entry

	DC	EC	PO	TPP	NTPP	TPP-1	NTPP-1	TPI	NTPI
DC	no info	DC,EC, PO,TPP, NTPP	DC,EC, PO,TPP, NTPP	DC,EC, PO,TPP, NTPP	DC,EC PO,TPP, NTPP	DC	DC	DC	DC
EC	DC,EC, PO, TPP-1, NTPP-1	DC,EC, PO,TPP, TPP-1, TPI	DC,EC PO,TPP, NTPP	EC,PO, TPP, NTPP	PO,TPP, NTPP	DC,EC	DC	EC	X
PO	DC,EC, PO, TPP-1 NTPP-1	DC,EC, PO, TPP-1, NTPP-1	no info	PO,TPP, NTPP	PO,TPP NTPP	DC,EC, PO, TPP-1, NTPP-1	DC,EC, PO, TPP-1, NTPP-1	PO	PO
TPP	DC	DC,EC	DC,EC, PO,TPP, NTPP	TPP NTPP	NTPP	DC,EC, PO,TPP, TPP-1 TPI	DC,EC PO TPP-1, NTPP-1	TPP	X
NTPP	DC	DC	DC,EC, PO, TPP, NTPP	NTPP	NTPP	DC,EC, PO, TPP, NTPP	no info	NTPP	NTPP
TPP-1	DC,EC, PO, TPP-1, NTPP-1	EC,PO, TPP-1 NTPP-1	PO TPP-1, NTPP-1	PO,TPP, TPP-1, TPI	PO, TPP, NTPP	TPP-1, NTPP-1	NTPP-1	TPP-1	X
NTPP-1	DC,EC PO,TPP, TPP-1, NTPP-1	PO, TPP-1, NTPP-1	PO, TPP-1, NTPP-1	PO, TPP-1, NTPP-1	PO,TPP, NTPP, TPP-1, NTPP-1, TPI NTPI	NTPP-1	NTPP-1	NTPP-1	NTPP-1
TPI	DC	EC	PO	TPP	NTPP	TPP-1	NTPP-1	TPI	X
NTPI	DC	X	PO	X	NTPP	X	NTPP-1	X	NTPI

Table 4: The transitivity table for the set of base relations of lattice L_C. If R1(a,b) and R2(b,c) are satisfied, then R3(a,c) follows, where R3 is looked up in the table. "No info" means every base relation is possible and "X" means the conjunction R1(a,b) and R2(b,c) cannot be satisfied.

NTPP(a,b) and EC(b,c) is DC(a,c), and corresponds to an instance of the theorem: $\forall xyz[[\mathrm{NTPP}(x,y) \; \& \; \mathrm{EC}(y,z)] \rightarrow \mathrm{DC}(x,z)]$. Cells marked with an "no info." indicate that for the ⟨R1(a,b),R2(b,c)⟩ pair, no base relation as the result is excluded, and those marked with an "X" indicate that the related conjunction cannot be true and thus no deduction is made. For example EC(a,b) and NTPI(b,c) are unsatisfiable: for EC(x,y) to be true both x and y must be non-open regions, but NTPI(b,c) requires both b and c to be open regions - contradiction. Where non-base relations appear in the target set (e.g. PP(a,b) and PP(b,c)), the calculation is done as follows. Firstly, the lattice is used to compute the set of base relations each relation covers (in this case {TPP(a,b),NTPP(a,b)} and {TPP(b,c) and NTPP(b,c)} - remembering that $\forall xy[\mathrm{PP}(x,y) \longleftrightarrow [\mathrm{TPP}(x,y) \vee \mathrm{NTPP}(x,y)]$ is a theorem). Next we take each R1(a,b), R2(b,c) pair where R1(a,b) $\in$ {TPP(a,b), NTPP(a,b)} and R2(a,b) $\in$ {TPP(b,c),NTPP(b,c)} and form the union of all the disjunctive sets of base relations each R1(a,b) and R2(b,c) pair yields using the transitivity table. In this case this would be $[\mathrm{TPP} \sqcup \mathrm{NTPP}](a,c)$ or simply PP(a,c). So given PP(a,b) and PP(b,c) we deduce PP(a,c).

5.4.4: Building in sets of defining axioms

Given an implementation of theory resolution described above combined with the use of a transitivity table, there is a clear indication that most, if not all of the defining axioms of the theory (which correspond to the axioms and definitions that describe the properties of the mereological relations) can be excised.

I put forward as a *conjecture* that in the case of lattice L_C, all the defining clauses that describe the base theory, and which use only free variables, can be excised. The conjecture implies that all such clauses which define the basic theory become theorems in the hybrid reasoning case. In other words given the lattice, the specialised form of resolution performed on that lattice, and building in symmetry and transitivity into the inference mechanism in the manner suggested, all the clauses using free variables that define the formal theory, will be derivable as theorems. Clauses from the same set containing bound variables (i.e. using skolem functions) cannot be similarly proved, since these express the existential conditions required by the theory. However, the reduction is still significant. For example, simply taking axioms (A1) to (A3) and definitions (D1) to (D20), *the number of clauses reduce from* 59 *to* 10.

However, there is reason to believe that all the other clauses that use skolem functions may be in turn absorbed. Take for example the formula: $\forall xy[O(x,y) \rightarrow \exists z[P(z,x) \wedge P(z,y)]]$ which embodies a skolem function. This incorporates two clauses and is in fact one half of the definition for $O(x,y)$. Firstly, we know from the theory that if two regions overlap they share a common part, and that that part can be the product of x and y (i.e. the theorem $\forall xy[O(x,y) \leftrightarrow \neg NULL(prod(x,y))]$). Next we note that $O(x,y)$ is a symmetrical relation, and that $prod(x,y)$ is commutative, i.e. $prod(x,y) = prod(y,x)$. By building these properties into the inference mechanism, the formula could be replaced with the single clause $\forall xy[O(x,y) \rightarrow P(prod(x,y),x)]$ - remembering that if two regions overlap they have a non-null product as a common part. It is then relatively easy to see how this clause could in turn be absorbed. In this case one could extend the transitivity table so that the conjunction $\neg P(prod(x,y),x) \wedge O(x,y)$ is mapped to "false". Further work is needed to see if all the remaining clauses can be similarly absorbed.

5.5: Summary

Although rules exist where sorted logics are translated into unsorted logics, in general, the converse is not available. Translating an unsorted axiomatic theory to a sorted one is a non-trivial task. The translation of a complex theory that uses few primitives and many definitions into a LLAMA translation is especially difficult, in particular it can prove especially difficult proving (in the unsorted logic) the relative positions of potential sort symbols in the sort hierarchy in the sorted case.

Given a rich sort structure, LLAMA can be effectively used to 'build in' theorems and cut the search space during a proof run. Techniques such as Peeking and Gazing seem particularly suited to the theory developed in this thesis. An extension of Cohn's characteristic resolution is outlined. This allows paired literals of any degree of polyadicity and with differing names (but semantically known to be contradictory) to clash. This is used along with a transitivity table for the set of base relations of lattice L_C. I add as a conjecture that using theory resolution, the transitivity table and building in other properties of the theory, that most if not all the defining axioms for the main part of the theory can be effectively absorbed and thereby pruned from the main clause set.

Chapter 6: Ontological and related issues

6.1: Introduction

This chapter introduces ontological and related issues thrown up by the working methodology and chosen formal theory developed in this thesis. Two main parts are discussed in turn. The first is an examination of the formal treatment of mass terms in a first order language. The second part discusses the relationship between epistemic priorities in ones ontology, representation and implementation, and raises the question of a cognitively adequate theory.

6.2: Quantification theory and mass terms.

As there is an extensive literature on how mass terms should (or should not) be treated in a formal theory (see e.g. Pelletier 1979 for comprehensive bibliography) a comprehensive analysis of this subject cannot be undertaken. However, given the appearance of mass term extensions in this theory, the formal adequacy of a language used to represent mass term extensions must be addressed if only to show that a chosen representation has its limitations.

In natural language, the distinction between stuffs and things is preserved in the separation made between mass and count nouns. Grammatically, mass and count nouns are respectively paired with the adjectives "less" and "fewer". Whether a noun is mass or count depends upon which adjective is correctly applied. In some cases syntactic distinctions appear sufficient to distinguish between mass and count nouns, but as Bunt [1979 p249] points out, most nouns can be used both as mass or count nouns. For example, the noun "apple" in the phrase "an apple" functions as a count noun, and in the phrase "the apple in the salad" as mass.

Throughout this thesis I have assumed that standard quantification theory to be adequate for the formal characterisation of stuffs (e.g. liquid and gaseous bodies) as well as things (amoebas, pumps and valves). I have assumed that mass terms can be treated as predicates, and that the domain of quantification for stuffs ranges over physical objects or bodies. But this approach is by no means free from problems. For example, given a body of water, it is not entirely clear whether it has bodies as parts, and if it does, and how these bodies are to be individuated. In general, universals such as *cell* and *pump* provide a principle for distinguishing, counting and reidentifying particulars - saying what sort they are [Lacey 1976]. But by treating physical objects and indeed water as sorts presupposes that bodies can be so distinguished, counted and reidentified, and it is that, that begs the question. For further difficulties associated with this approach see Pelletier [1974] and Bunt [1985].

A good overview of the different approaches that have been used to deal with the formal treatment of mass terms can be found in Pelletier [1974] and Bunt [1985]. These sources are extensively used in the summary given below.

The classical treatment advanced by Quine [1960] allows for two uses of mass nouns, appearing in a subject position it counts as a singular term, and in a predicate position as a general term. In the latter case the extension of the predicate is taken to be true of each portion of the stuff in question, but excluding those parts too small to count. Quine's analysis can capture enough intra-sentential form so that the sentence "This puddle is water, Water is wet, so this puddle is wet' expresses a valid argument, but it fails to capture the analyticity of the sentence 'Water is water', or 'Dirty water is water' - but note the analyticity of: $\forall x[\mathrm{Water}(x) \rightarrow \mathrm{Water}(x)]$ and $\forall x[[\mathrm{Water}(x) \wedge \mathrm{Dirty}(x)] \rightarrow \mathrm{Water}(x)]$ assuming standard quantification theory and where the dual treatment is dropped.

The approach championed by e.g. Strawson [1959] and Clarke [1970] assumes a set-theoretic interpretation. In this case expressions using mass terms are taken to be elliptical expressions that hide an implicit sortal term. Thus for example, 'is water' might be elliptical for 'is a *body* of water', 'is a *kind* of water', or 'is a *puddle* of water'. The context-dependence inherrent in this approach creates problems establishing which sortal to apply. Another difficulty arises given identity claims. Given, for example the sentence: 'The water spilt on the floor is the same water that was in the tank before', we have two individuating standards: say, a puddle and a tank of water respectively. But given the identity claim, neither of the two standards can be used simultaneously; for example puddles cannot be spilt.

As far as context-independent standards go, this too runs into problems. In this case one needs to make sure the standard used is small enough so that nothing is excluded from the intended set of entities, but what then are these implied minimal parts? Water, for example takes the form of drops, but then drops can be further broken into smaller drops. In the case of scientific classifications, water as the set of H_2O molecules seems plausible since its parts are not water, but somewhat artificial. Bunt [1979] argues that the minimal parts hypothesis leads to difficulties on several accounts. In the first instance, if we take the hypothesis to be an assertion about the real world, then the referents of some abstact nouns, e.g. 'time' have no known minimal parts. On the other hand if we then take the hypothesis to be an assertion about the use of mass nouns in English, it is difficult to see how this analysis can be correct, since for example, speakers of English have used (and still use) the term correctly without any knowledge of the chemistry of water. These observations lead Bunt to the view that mass nouns provide a way about talking about things as homogeneous entities, without the presupposition of minimal parts. Bunt's [1979,1985a,b] Ensemble Theory actually builds on this assumption.

An alternative approach taken by Parsons [1970], takes mass nouns to name abstract entities. Parsons theory has been heavily criticised for being too complex - assuming a distinction between physical objects, bits of matter and substances - see Bunt [1985a].

Mereology [Leisniewski 1927-31], reformulated by Leonard and Goodman [Leonard and Goodman, 1940] as *the calculus of individuals* has also been used to deal with mass terms, e.g. by Quine [1960] and Moravcsik [1973]. Mereology uses the part/whole relation to deal with the fact that mass nouns unlike count nouns, divide their reference. That is to say that whereas count nouns possess built in modes of reference that determine what counts as a singular instance, mass nouns do not. Mereology takes, for example, the water on this planet as an *individual*, and lakes, puddles and drops as *parts*. Mereology has been criticised by Bunt [1985a] on the grounds that it does not support an *empty* object, which Bunt takes to mean that mass terms failing to denote cannot be accommodated (but see notes 11,12 and 14). Bunt also notes a uneasy tension that exists between mereology proper (i.e. Lesniewski's sytem which was set up as an alternative to set theory) and Leonard and Goodman's reformulation (which is formally defined in set-theoretic terms) and to what extent the two can be formally wedded together. Bunt's *Ensemble Theory* which employs both the part-whole relation and set-theoretic notions can be seen as an answer to this problem.

Given the appearance of mereology in this thesis (for example I refer to the set of dyadic relations defined in terms of C as "the mereological relations"), some response to Bunt's criticism is required. In the first instance, while I use mereological names for relations, I never the less use set theoretic notions in my metalanguage to interpret the theory. Thus for example, a formal distinction is preserved between bearers of properties and the properties themselves. This does not accord with a full flung mereological system that would treat a yellow ball as the overlap of the individuals *yellow* and *ball*. Secondly, while Goodman allows an unrestricted use of his summation operator, I only allow this for regions of space or time, but not for physical bodies. Goodman's insistence to allow, e.g. the existence of an individual that is the sum of a speck of dust in the Sahara and the Arctic Sea (and the criticism raised by this - see e.g. Eberle 1970) is thereby avoided.

On Goodman's [1951] understanding individuals "need not be organised or uniform, [and] need not be continuous or have regular boundaries". Appeals to minimal notions of coherence or homogeneity of individuals are similary dealt with. Despite Goodman's insistence on allowing bizarre individuals into his ontology, there is reason to believe additional criteria to identify individuals must be secured. It would seem that on Goodman's analysis a cup would be a cup whether it were in one piece, or scattered in bits all over the floor - even though common sense would prevail which one would be the better container. *Pace* Goodman, criteria for individuating, identifying and reidentifying individuals, as well as criteria for coming into and passing out of existence for material bodies must be addressed. Otherwise (given the continued existence of the cup parts) one would be forced to accept the unintuitive consequence that the cup still existed.

As mentioned above, Bunt criticises the mereological approach on several accounts. The first is that mereology does not support an empty object. Although Bunt's criticism can be endorsed (e.g. in the decision to import NULL(x) into the theory described in this thesis), two points must be made. The first is that a virtue of mereology over standard set theory may be claimed in the fact that it does not require one to posit dubious 'general objects' (see note 12). Thus while Bunt implicitly assumes a set-theoretic foundation to standard mereology, the gain in expressiveness in Ensemble Theory is not argued for against the nominalist's stance that the underlying ontology assumed by Bunt is equally not without question. The second is that the formal treatment of objects without extension can be successfully carried out *without* importing an empty object into the domain using Free Logics (see note note 14). However, in actual fact, mereological systems *have* been constructed using a null individual (see note 11).

6.3: Epistemological priority, representation and implementation.

The notion of *epistemological priority* appears in Clarke [1985 p69], in the following context:

"...an atomic definition of points is philosophically questionable, because of what [Mortensen and Nerlich] call "the epistemological priority of intervals over points: Separation and intervals are somehow visible in a way that points are not." [Clarke 1985 p69].

Although (as Clarke correctly points out) the notion is undeveloped, there is an obvious immediacy that regions have over points in our conception of space and time. This observation is shared by Turner [1984 p89] and by van Benthem [1982 p230]. Hamblin [1971 p127] also reiterates the same point, saying the much interest in Philosophy has centred on "supposedly primitive observational or phenomenal languages, languages whose features can be directly related to features of our experience of the world around us".

The notion of epistemological priority also appears in the following. Van Benthem [1982 p230] cites an interesting reflection on the distinction drawn between the order of knowledge - going from points to extended objects, and the order of learning - going from extended objects to points within geometry. Tarski's development of a solid body geometry which only appeared this century is cited as a case in point, but other examples arise, e.g. Whitehead [1978], and Laguna [1922]. The same trend is also apparent in other domains and in AI itself, where extended objects are taken as primitive as opposed to points e.g. for the temporal: Hamblin [1969,1971] and Allen [1981,1984] and Allen and Hayes [1985]; and for the spatial/temporal: Woodger [1937], Clarke [1985] and Blizard [1990].

Given that formal theories exist which take either points or extended bodies as ontological primitives from which the other may be derived, no a priori argument on these grounds alone can be given for an alleged logical priority of points over regions or vice versa. However the notion of a stated epistemological priority is still worthy of some consideration.

There are at least three criteria that can be considered in this respect. The first is in terms of the ease by which a person can use a theory. The more intuitive and immediate the concepts appear to be, the easier it is to understand. The second is related to the question of cognitive validity, for example, which ontology better matches the set of primitives assumed to be at the heart of a theory of cognition, while the the third is related to what degree a theory exhibits desirable computational properties over its rivals.

In Hayes [1985a], Hayes' argued that obvious deductions within a naive physical formalism should coincide with short proofs. On the surface this seems to reflect a reasonable position. Given the apparent ease by which we make certain decisions in our everyday dealings with the physical world, it seems plausible that if some automaton is to be seen to share in our way of life, it too should reflect this same ease in decision making given comparable tasks. But against this must be weighed the point made by van Benthem [1982 p237] that "natural primitives [in a formal theory] often require complex axioms, and vice versa". Hayes also mentioned that a good naive physical formalism should be both broad and dense, meaning that although a clustering should arise in large formalisms, a complex web of inferential connections should be seen to exist, linking the concept tokens of the theory. If computational efficiency is to be upheld as an important criteria in the decision to use a particular theory over any rivals, care must be taken that the chosen axiomatisation is not sparse (cf. Hayes's [1979,1985a] criticisms), otherwise the general expressiveness of the formalism will suffer.

Within automated reasoning, it is well known that a tight coupling exists between the different ways a theory might be represented and processed using distinct general rules of inference and proof strategies. Given a formal theory, and a class of theorems to be proved, it is something of an art to recognise what axiomatisation of a theory, representational language and set of inference rules have the computational edge over any rival. The important point here, is that the question of control (of inference) should be sought using a rich theory. However one proceeds, from natural primitives or from less natural ones, the theory generated will tend to be a rich one. Although different techniques can be used to effectively implement a given theory (for example, by using hybrid reasoning and abstracting out and exploiting topological and emprical information from the modelled domain), in each case the richness in the formal theory is not sacrificed.

The notion of "epitemological priority" discussed above and that of the contrasting ordering of knowledge and learning, carries over quite naturally into establishing whether or not a particular theory can be interpreted as a theory of cognition, or at least supports an ontology that can be grounded in cognition. In this respect, it is interesting to reflect that many of the dyadic relations defined in terms of the relation C, admit intuitive names. This suggests that the primitive notion of connectivity between regions (which is used to describe a rich theory of space) may well be grounded in a theory of cognition. That is to say, given the fact that a comprehensive theory of space can be developed from the use of this primitive relation, connectivity between objects in space, and changes in connectivity may well be an invariant exploited by the human visual system. It is also interesting to note a connection that can be made with Pentland's [1986] work. Pentland [1986] points out that in much vision research, the models tend to be either high-level specific models, e.g. of people or houses, and low level models of image formation, e.g. of edges. The latter approach has grown from models used within the well-developed fields of optics, material science and physics (the order of knowledge), the former from modelling industrial parts and assemblies used in engineering and computer-aided design (CAD) applications (the order of learning). Pentland finds both the pointwise, quantitative models of image formation process and the CAD-like models used in engineering, wanting. In its place, he argues for a model that lies between the two. Pentland's ontology assumes the primitive notion of a part. These macroscopic parts are deformed by stretching, bending twisting or tapering them, and are then combined using Boolean operations to form new complex prototypical objects that themselves can be similarly deformed. Pentland's parts are abstractly conceived to be akin to lumps of clay. The correspondence with the notion of a region, the part/whole relation, and the use given to Boolean combinations of regions for modelling complex physical models in the theory developed in this thesis should be reasonably clear. Pentland shows that a close corresepondence can be made between the way objects are described and represented using his model, and that made by people describing the same domain. This leads him to suggest the primitive predicates used would be a good set to use for commonsense reasoning in the manner of Naive Physics. Given the correspondence drawn here between Pentland's model and that used in the basic theory developed in this thesis, this also provides some support for grounding the theory developed in this thesis in a theory of cognition.

6.4: Summary

Any comprehensive theory that aims to describe the everyday world comes upon the problem how best to represent mass terms. While standard quantification theory is assumed in this theory, problems exist when it comes to formally representing mass terms and stuffs. An indication of the limitation of standard quantification theory is highlighted.

I show how notions of epistemological priority, representation and how a theory might be used and implemented, are intimately related. I also show that some support drawn from Cognitive Science validates the basic ontology used.

Chapter 7: Related work.

7.1: Introduction

This chapter compares and contrasts related work. It presupposes knowledge of Chapters 1 through to 6. Some of the discussion of related work appears throughout the thesis, and in the notes on the text. For ease of expression, throughout this chapter the theory described in this thesis is refered to as AF (from 'Analysing the Familiar'), and Hayes' Naive Physics programme as NP.

7.2: Related work: comparisons and contrasts

Earlier workings of the theory described in this thesis are to be found in Randell and Cohn [1989a,b,c and 1991] and Randell, Cohn and Cui [1991].

In general, the direction of research and methodology used owes much to that outlined in Hayes [1979a,1985a] Naive Physics programme. A fuller outline of Hayes' programme is given in Chapter 1, and is not repeated here. However, below, I summarize the main points of comparison and contrast between the two.

The first point of similarity between AF and NP is the joint use of FOL as the representational language. In both cases a sorted logic is used, but for Hayes [1985a,b] the choice seems simply motivated by the need to have a compact, yet perspicuous notation. The advantage of using a polymorphic sorted logic in automated reasoning is well known both in terms of search and the compactness of the set of defining axioms. But Hayes does not use his sorted logic to explicitly represent and draw attention to the ontology assumed by the formalised theory which has been done in AF. In this respect, the ontological primitives of AF are immediately identifiable as the set of named disjoint sorts that fall immediately below T. A similar point can be made for the set of sorting functions where the ontology is clarified by having to make explicit the sorts a particular constant, function or predicate draws together, and in the case of the transfer functions e.g. the function space(x,y) (abbreviated as x|y), as **type** space(PHYSOB,MOMENT):SPATIAL ⊔ NULL, the mapping between *disparate* sorts.

The second point of similarity between AF and NP is the way the model and proof theoretic aspects of FOL are used as a tool to develop the theory. In Hayes [1985a] the model theory is used to help isolate unintended models and suggest ways to eliminate them by adding distinguishing concepts that had hitherto remained implicit. This of course follows from Hayes' conviction that a good theory should be both broad and dense, and that the modelling should not suffer from premature attempts to implement the theory. In AF the process of theory refinement was automated. Using the automated reasoning programs ITP [Lusk and Overbeek 1984] and OTTER McCune [1988], unintended models were isolated by proving that a set of clauses (thought to be inconsistent) were in fact consistent, and partial consistency results were established by proving the existence of a logical model. An exameinination of clause dumps were also found to be particularly useful. Unexpected theorems often exposed an oversight in the modelling and enabled inadequacies in the set of defining axioms to be quickly isolated.

The next point of similarity between AF and NP is the primary task of building a rich formal theory and not letting implementational questions intrude too much. In Hayes [1979a] the notion that one can clearly separate out representational content and implementational details is taken to be substantially true, though not absolutely. And in Hayes [1985a p3] he also points out that insurmountable problems can arise from inadequate representations. However, in practice, modelling and implementation are more tightly coupled than at first may be thought. That this is so has in fact been said in different ways many times before. For example, Wos [1984 p148] points out that a tight coupling exists between representation and inference using automated reasoning programs, and that certain combinations of a given representation and rules of inference can greatly degrade the performance of a reasoning program. And Marr [1982 p21] makes the general point that the way information is represented greatly affects what can be done with it. This suggests that modelling and implementation should not be kept too isolated in practice but viewed as complementary aspects of theory development. Hayes' NP programme puts the primary emphasis on theory construction, and in general, Qualitative Physics (discussed below) puts implementational questions to the fore, with less attention given to modelling. I argue that modelling, representation and implemention cannot be effectively seperated in theory as well as in practice given the central goals of AI.

An example of the complementary relationship between representation and implementation can be seen in AF with the decision to use functor notation. The decision was based on pragmatic grounds, since functor notation (rather than relational notation) seemed the best alternative of the two given a mechanised sorted logic was to be used to implement the theory. It is also worth pointing out here, that a tight coupling also exists between a given representation and the ontology assumed by a theory. Again in AF this is clearly evident when nominalised sentences are used to reason about objects over time: e.g. the term c(x,y) in OBTAINS(c(x,y),z) requires a new sort STATE needs to be added, as does the metalogical function $\varphi(\bar{x})$ in INCREASE($\varphi(\bar{x}),y$) with the sort MEASURE. Thus, even though implementational questions might be given little attention during theory development, once a representation is decided on, the ontological commitment embedded in that representation sets immediate constraints on how that theory can be implemented.

In NP Hayes uses a prolix ontology but in AF a reductionistic working assumption is argued for. This needs some clarification since on a first reading, one might argue that the conceptual apparatus used in AF actually incorporates a prolix ontology itself. By "reductionistic" I mean the following. In the first instance, care has been exercised to keep the number of primitive sorts, functions and predicates supported by the theory to a minimum. All the functions and predicates described in terms of the dyadic relation C are a case in point. For example, although the function 'sum(x,y)' appears as a term in the object language, the function can be contextually eliminated away using Russell's [1903] theory of descriptions.

The reductionistic methodology is justified as follows. Modelling common sense is argued to be at least as difficult as that expected in formalising any other theory encountered within either the sciences or in philsophy. *If one is to be reasonably sure that one's theory is free from* contradiction, yet is both broad and dense, the fewer primitives in the theory, the better. I find it difficult to believe that axiomatisations constructed by mathematicians and logicians are always motivated by aesthetic considerations. Unlike Hayes, I do not view reductionism in terms of theoretical elegance and having ontological interest only. Establishing a set of primitives and independent axioms (in principle if not in practice) enables the theory to provide a useful foundation for general AI. Having proved the sufficiency of a theory to generate a plausible set of consequences, the search for physical correlates of the theory's underlying set of primitives can be effectively constrained. Pentland's [1986] theory of vision which uses an ontology of macroscopic "parts" that can be deformed by stretching, bending, twisting or tapering, and which can be combined using Boolean operations is a case in point (see section 6.4).

In 'Ontology for Liquids' [Hayes 1985b], Hayes sketches the beginnings of a topological theory of space. This is modified and developed further in Welham and Hayes [1984]. However, despite the importance given to spatial information, and in particular topological information, no further attempt appears to have been made. Hayes' [1985b p80] relation IN(s1,s2), (see also Welham and Hayes, 1984) expresses a partial ordering defined over pieces of space. This has similarities with the relation P(x,y) defined over regions used in AF. However, whereas Hayes clearly intends to include faces as pieces of space [Hayes 1985b p80], and moreover defines IN(s1,s2) over (directed) surfaces, only entities possessing non-zero volume, are admitted as (spatial) regions in AF. Similarly Hayes' characterisation of a portal differs from that developed in AF; for Hayes a portal is surface-like where I have chosen to represent portals as arbitrarily thin regions of space which are not surface only.

The spatial sub-theory used in AF subsumes much of Hayes' earlier work but develops from fewer primitive notions. Hayes' complex ontology of directed surfaces and edges is dispensed with, and concepts such as objects being joined or touching are reworked in terms of externally connected regions.

Hayesian histories (contiguous chunks of space-time) are not used in AF - the spatial and temporal domains are modelled and reasoned about separately. In contrast to Hayes, dynamic descriptions of the world are modelled by stipulating sequences of state descriptions, with physical bodies mapped to the spaces they occupy at a given moment with the transfer space(x,y) function. Although *histories* were originally developed to meet the well known frame problem and poverty associated with the situation calculus, it too is not immune from it [see e.g. Shoham and McDermott 1988 - and the interesting slant by Rayner [1989] who argues that the related "extended prediction problem" is rooted in a mistaken analysis.

As has been mentioned before, little work clearly falling in the Naive Physics mould has appeared in AI literature. Cunningham [1985a] devotes some attention to modelling common sense knowledge, but this is clearly peripheral to the central contribution to his thesis.

An earnest attempt to put common sense knowledge upon a formal footing can be found in the Commonsense Summer workshop, [Hobbs et al, 1985] and in [Hobbs and Moore 1985]. The former contains work by Kautz [1985] who discusses a formalism for the representation of spatial descriptions and concepts, Hager [1985] who tackles the representation of properties of materials, and Shoham [1985b] the representation of kinematics and shape. Unfortunately, many of these papers suffer from a free use of "axioms" and there is little overall evidence of theory development in terms of interesting theorems given. Part of this tendency seems to have arisen from the methodology adopted by the group, for example, putting questions of consistency aside and aiming for conceptual cover. The methodological weakness of this approach and its attendant dangers has been amply discussed elsewhere, and is not repeated here.

A notable attempt to develop a formal theory describing a non-trivial domain problem arises in Davis [1988]. Davis uses a first order formalism for reasoning about solid object behaviour; the whole being illustrated by modelling the possibility of and passage of a die passing through a funnel. Several concepts used in AF, e.g. open and closed regions, and the notion of a convex hull also appear in Davies' work, but the set of primitives differ. The emphasis of the work is representation with no discussion about effective ways to implement the theory. This piece of work is interesting since it does not simply seek to simulate the behaviour of the die, but provides a proof (given certain conditions) that the die will pass through the funnel. Other spatial work can be found in Kuipers and Levitt [1988] where navigation in large scale space is discussed.

Other work in naive physics includes Gardin et al [1986] and Gardin and Meltzer [1989] who concentrate upon simulation using analogical models of fluid flow, strings, flexible and rigid objects. A common criticism of any simulation approach is simply that certain contrafactual scenarios cannot be represented (since the model is purposely constructed to exclude these); moreover the question of drawing inferences from such an approach e.g. as might be required in planning and system diagnosis is not only dependant upon the efficient running of the simulation, it still requires a theory (independent of the program) to interpret it. Di Manzo and Trucco [1987] also select a domain of flexible string-like objects, but choose Forbus's [1984] Qualitative Process Theory (see below) to encode the theory.

In contrast to Naive Physics proper, Qualitative Reasoning literature is well supported but is too extensive to cover in any detail here. Cohn [1989c] reviews the field up to the summer of 1987; for other useful introductory material see the special edition of the journal *Artificial Intelligence* (vol 24, 1984) which covers key contributions to the field, Forbus [1988b] for a particularly clear survey of qualitative physics, and Bonissone and Valavanis [1985] who compare key approaches using a coffee maker as the domain model.

Qualitative Physics takes the physical world as its domain and seeks to model both expert and commonsense reasoning about physical systems; it also seeks the common ground between the models it uses and the more traditional ones associated with physics [Forbus, 1988a,b]. Difficulties are acknowledged exactly what the defining characteristic is by the use of the term "qualitative", but it has been identified with modelling that takes continuous parameters and abstracts out a finite set of discrete values to work with [Forbus, 1988a]. A common example is the use of signed numerical values "+", "0" and "-" - changes in such signs being associated with significant changes in the behaviour of some physical system, e.g. heat flow coinciding with a temperature difference across two connected bodies of water. In AF the finite set of discrete values can be identified with the role given to changes in boundary connection between regions over time, e.g. DC to EC to PO to TPP and to NTPP. As in QP where "+" cannot change directly to "-" without passing through "0", the same restriction arises for pairs of regions undergoing a change in connectivity. In this case the change from e.g. DC to PO must pass through EC, and PO to NTPP must pass through TPP in the normal case.

The notion of an "envisionment" common to QP also appears in AF. The set of finite states assumed by physical systems in QP modelling, is carried across to finite sets of base relations developed in the theory. These sets of base relatins are used both to describe individual states, and events and processes as specified sequences of state transitions.

Restrictions on direct transitions are fixed by transition networks developed in the theory.

There are three generally recognised approaches to qualitative reasoning and modelling: the constraint centred approach, the component centred approach and the process centred approach. These are briefly discussed in turn.

The constraint centred approach, exemplified in the work of Kuipers [1984,1986], models a physical system as a set of constraint equations which are qualitative analogues of differential equations.

In contrast the component centred approach, exemplified in the work of de Kleer and Brown [1984] and Williams [1984], models a physical system by explicitly representing a connected network of components drawn from a library. Each component has an associated description that describes its behaviour. Assumptions about how the components are used in a physical system are carefully avoided so that the generic descriptions associated with the individual components do not import function into the given structure (the *no function in structure* postulate - criticised in Keuneke and Allemang [1989]), although behaviour assumed to hold over a wide class of physical systems (called *classwise assumptions*) are used. The global behaviour of a given physical system is then determined from computing the behaviour of the primitives in the system.

The process centered approach, exemplified by the work of Forbus [1984] follows the latter approach in spirit by modelling components but also by modelling the processes that are said to act upon them. Qualitative Process Theory (QPT) takes as its major assumption that "all changes in physical systems are caused directly or indirectly by processes" (the sole mechanism assumption). QPT describes physical situations in terms of a collection of objects, their properties and relationships arising betwen them. Processes are specified in terms of a set of individuals (those objects stipulated to be affected by a given process), a set of preconditions (descriptions about the objects and their relationships other than quantity conditions), a set of quantity conditions (statements representing either inequalities between quantities or statements about the status of processes and individual views), a set of relations arising between the parameters of the individuals, and a set of influences imposed on the process on the parameters of the individuals. The set of individuals, preconditions, quantity conditions and relations make up what Forbus calls an individual view. Processes acting on these objects have various effects. Some are modelled by using a set of quantities mapping to these objects which have as parts an amount and a derivative both represented as numbers. Forbus relates the values of these numbers to a Quantity Space which consists of a set of elements and a set of orderings. Values of these quantities can either be completely specified or are regarded as incomplete. QPT can be used to determine which processes are active, what changes can happen and deduce possible behaviour in the modelled physical system. Typically, processes begin and end when inequalities change. Influences are regarded as that which can cause a quantity to change.

From an ontological and modelling standpoint, the constraint centred approach is perhaps the simplest. The physical system is completely described in terms of the set of constraints with the emphasis placed on the use of the simulation algorithm employed. The component centred approach offers more in the way of modelling with the use of a component library and with the facility to make causal connections between such components explicit. But of the three the process centred approach is the most sophisticated in terms of the conceptual apparatus used.

It is difficult to compare the work done in QP with that developed in AF since no causal mechanisms appear in the language of the latter. However, AF makes use of composable descriptions that can be used as the basis to build a library of component parts and processes, and AF also uses an envisionment to determine possible changes in a physical system.

It is interesting to note that a detailed representation of time and space does not feature much in literature devoted to qualitative physics. For example, of the three main approaches, only Forbus actually uses Allen's interval logic to explicitly represent partial orderings of processes, and much spatial reasoning in the qualitative physics literature assumes a one-dimensional representation [Cohn 1989c p216].

Some work [Shoham 1985a,1985b, and particularly Forbus et al,1987] has been done on qualitative kinematics. But a purely qualitative spatial representation is not used. Reasoning from their "poverty conjecture", Forbus et al [1987] argue that any qualitative account of kinematics must be backed up with complementary representations using quantitative information. The *poverty conjecture* aims to explain why little progress has been made in developing purely qualitative representations for modelling kinematic domains. This conjecture is based on three points: i) that no one has succeeded despite numerous attempts, ii) that while qualitative dynamics makes much use of partial orderings and monotonic functions, they cannot be gracefully extended to cope with reasoning that exploits higher dimensions, and iii) the fact that people tend to perform poorly at spatial reasoning problems without the help of diagrams. In response to this, numeric or algebraic based representations are advocated.

For example, Forbus et al [1987] use a *Metric Diagram/Place Vocabulary* model to meet such deficits. The metric diagram carries quantitative and symbolic information and is used to answer geometric questions where measurement or calculation is required, the latter reflects a partioning of space, abstracted from the former, e.g. separating out regions of free space from those occupied by certain physical objects. This approach is essentially the same as that outlined in Forbus's [1980] FROB system which reasoned about the behaviour of a bouncing ball. Other work in this area includes Falting's [1987] demonstration of the kinematics of a cog and ratchet mechanism, Stanfill's [1985] program that reasons about pistons and other devices using a library of standard shapes, and Joskowicz [1987] who discusses modelling of rotational devices. Most of the work done in this field uses two-dimensional modelling. Joskowicz, using a three-dimensional model is an exception to this. Kaufmann [1991] cites a major drawback of the approach that uses the Metric Diagram/Place Vocabulary model, namely that if the model is changed only slightly (e.g. if an escapement wheel is added with just one more tooth) the configuration description has to be re-computed. In other words the model does not allow generalised descriptions of the behaviour of (in this case) wheels.

Little work seems to have been done to develop an interval logic for reasoning about spatial configurations of objects, despite the intuitive possibility suggested using Allen's logic and relaxing the ordering imposed on the primitive relation Meets(x,y). Forbus [1988b] seems to have recognised and considered this possibility, but expresses doubts as to why he thinks a spatial interval logic along these lines is unlikely to be productive. Taking R1 and R2 to be spatial analogues of Allen's temporal interval relations, Forbus argues that the degree of constraint obtained when computing the transitive closure for $R_1(a,b)$ and $R_2(b,c)$ rules out fewer cases than the temporal case. However, Forbus does not consider the degree of constraint that results from importing geometrical, metrical and empirical information that can be abstracted from the model, and imported into into the formal theory as is done in AF.

Freksa [1990] considers a one-dimensional spatial interpretation of Allen's 13 interval relations where the asymmetry of Allen's *Meets* relation is preserved in a left-right ordering. The transition networks that appear in AF (which arise from ordering sets of base relations in terms of direct topological transformations allowed, e.g. from DC to EC and vice-versa, reappears in Freksa's work as a "conceptual neighborhood". (See also Nökel 1988 for a temporal interpretation of the same structure, which too uses Allen's 13 interval relations.) The paper is informal with little analysis offered. Freksa uses two fish A and B to illustrate the spatial relations and the conceptual neighbourhood. The model used assumes an observer-based interpretation of the relations, and the fish remain clearly separated, even when identity between A and B is satisfied (compare with note 6 in this thesis). Higher dimensional spaces are considered, but not developed. No attempt to describe composite objects is undertaken (compare with the interpretation given to the relations NTPP, NTS, INSIDE and OUTSIDE in AF).

Work related to Freksa's appears in Guesgen and Fidelak [1990] and Hernández [1990]. In Guesgen and Fidelak's paper Allen's 13 interval relations are given a spatial interpretation. A transivity table is given as are details for constraint satisfaction algorithms to remove inconsistencies. Higher dimensional models are considered using Cartesian tuples of relations. For example, in 3-space, relations between any pair of objects o1 and o2 is represented in canonical form as o1 (R1,R2,R3)o2, where R1, R2 and R3 represent the relations holding between o1 and o2 as viewed along axes x,y and z respectively. The use of a polar coordinate reference frame and general problems choosing adequate reference or orientation frames are discussed. Hernández [1990] also uses Allen's interval relations as the basis for his representation. In doing so he reduces Allen's set of 13 dyadic relations to 5 (corresponding to the dyadic relations DC, EC, PO, NTPP and EQUAL in AF - with TPP missing). The chosen model is a 2D *projection* of a given 3D scene. The modelled domain is restricted to "convex" objects (in AF this restriction is lifted). Projection and orientation are singled out as important abstractions. "Projection" relates to the degree of boundary connection between objects, and "orientation" incorporating 8 notions which include being to the front of, to the back of, to the left of, to the right of. Information derived from projection and orientation is canonically represented in set of abstract maps. Composition of relations and transitivity networks for constant orientation and projection is given and discussed. Further related work can be found in Maddux [1989] where compass algebras are developed.

An alternative way to represent shape and enable useful inferences to be made about processes arises in the work of Leyton [1988]. Leyton develops a process grammar that exploits curvature extrema to infer basic processes. Transformation of shape is covered, but the modelling is restricted to capturing geometrical features of individual homogeneous entities. It is interesting to note the importance Leyton attaches to the notion of tangential connection, and that much of the expressive power of Clarkes's calculus (upon which the present theory is built) derives from the explicit characterisation of boundary connection between regions.

A general theory that aims to model our everyday notions about objects, space and time is given in Blizard [1990]. A three-sorted first order logic (i.e. quantifying over objects, locations and times) is used to encode the theory. Like AF the notion of time is discrete, and objects are indexed to locations. Small and large objects are considered (though the latter require quantification over sets of location points) and some dynamic situations are discussed, e.g. fusion and fission ("splitting"). No attempt is made to describe composite arrangements of objects. The theory is simply presented as textual - computational questions involved in implementing the theory are not addressed. In contrast Kaufman's [1991] theory which aims to model everyday conceptions about physical systems, develops Hayes' [1985] suggestion that commonsense reasoning about space might be better modelled using tolerance spaces rather than assuming a metric. He uses his formalism to describe the behaviour of string like objects (that can pull but not push other objects), and the uni-directional properties of a ratchet wheel. Kaufman's theory is particularly interesting for his use of induction to secure his proofs.

While little published work in AI has concentrated upon topological descriptions for reasoning about space, much work has been devoted to the use of interval logics for reasoning about time. The much cited work of Allen [1981,1983] and Allen and Hayes [1985,1987] develops a theory of time that exploits topological relations between one dimensional objects - although Hamblin [1969,1971] seems to have produced an identical system much earlier [see Galton 1990]. Hamblin's work is presented as a piece of philosophy/logic. An axiomatisation of an interval logic is given, but (understandably) computational questions are not considered, i.e. Allen's [1983] method of constraint propagation and use of a transitivity table.

Allen's ontology supports intervals, moments and points, and the topological relations between sets of intervals are related to states, events and processes; moreover the logic has been implemented. Galton

[1990] discusses the difficulty posed by Allen's logic to deal with continuous change and revises the theory to allow for this. Other work developing the representational side of Allen's basic theory appears in Sadri [1987]. The close correspondence between Allen's logic and the temporal sub-theory described in this thesis is covered elswhere. It is notable that Allen does not incorporate either Boolean or topological operators within his logic (although it is interesting to note that Galton [1990] revision of Allen's theory, introduces and uses a JOIN(S,T) operator which returns that interval which is sum of S and T when S and T meet), nor is any attempt made to develop a unitary formalism that can support both a spatial and/or temporal interpretation.

Moving outside AI proper, Woodger [1937,1939] makes a significant attempt to formalise an empirical science. He develops a formalism including the mereological part/whole relation to capture many concepts central to biology, e.g. cell division and fusion and the description of hierarchical relations. He recognised the importance of using a formal language to describe a chosen domain and regarded his Axiomatic Method in Biology as an experiment in ordering biological knowledge. Although symbolic calculation within a mechanised framework was envisaged [Woodger 1939], computational questions raised within the domain of automated theorem proving were understandably not addressed. Woodger does not introduce any mereological relation weaker than the chosen primitive relation *of part to whole*, and the temporal relation *of one event being before another*. He does not develop a rich theory of space and time but concentrates upon the task of defining hierarchical relations and processes using a spatiotemporal ontology. It should be noted that by the use of the term "spatiotemporal", Woodger does not assume a relativistic physics (and a similar point applies to Hayes [1985b] with his spatiotemporal ontology *of histories*). *Woodger's relation* 'C(x,y)' (*of coincidence in time*) is transitive and symmetrical (hence is an equivalence relation). The underlying physics is consistent with a Newtonian world view that treats time as absolute. In contrast Clarke's [1983,1985] spatiotemporal ontology supports a non-transitive reading for contemporaneous events, and thus aligns with a relativistic physics, although this in turn is a point of difference between Clarke's theory and AF where time is taken to be absolute. Carnap [1958] provides a simplified axiomatisation of Woodger's [1937] work and also includes a comprehensive selection of axiomatic theories covering domains such as set theory and arithmetic, geometry, physics, and biology just cited. These axiom systems provide a good hunting ground for anyone interested in building non-trivial deductive theories.

The use of the part-whole relation in geometry also appears in de Laguna [1922], Tarski's [1956] axiomatisation of solid body geometry, and in Whitehead's [1919,1920,1929,1978] sustained attempt to build a comprehensive deductive theory of geometry. Originally this latter work was to have been incorporated into a projected fourth volume of *Principia* [Russell and Whitehead 1910]. This project had been left to Whitehead, but it was never completed and the books followed. Clarke's [1981,1985] calculus utilises most of Whitehead's mereological definitions, but differs with the inclusion of quasi-Boolean and quasi-topological operators and predicates, and so relaxing the assumption that individuals be continuous. Clarke and Laguna's calculi are also closely related. However, beyond defining the basic set of relations defined on solids (Laguna), and spatiotemporal regions (Clarke), the differences soon become immediately apparent. For example, Laguna does not introduce temporal parameters into his formalism (being a geometrical exposition) while Clarke does (assuming a spatiotemporal ontology); Laguna introduces the concept of distance between solids into his formalism, while Clarke does not but develops a set of Boolean and topological operators, missing in Laguna. Laguna's primitive relation "can connect" compares with Clarke's primitive relation C(x,y), excepting that Laguna introduces modality into the interpretation of his primitive relation, which is absent in Clarke's.

In AF the spatial and temporal elements of the theory are separated out, and an additional axiom (A8) (missing in Clarke 1981,1985) is added. This axiom guarantees that every standard region is embedded in another as its externally connected 'complement'. This avoids an unintuitive result that can arises in Clarke's theory (given a spatial reading) where a model can be constructed that allows a change from NTP(a,b) to TP(a,b) to take place, with no change in the relative positions between regions a and b - as when a is part of b and in boundary connection with b, and when another body c 'bumps' into both b and a as b's part. In this three-body universe the relation between a and b will change from NTP(a,b) to TP(a,b). Of course given the explicit representation of logical possibility as the sentence forming operator $\Diamond$, the definitions could be re-worked as:

$$TP(x,y) \equiv_{def.} P(x,y) \wedge \Diamond\exists z[EC(z,x) \wedge EC(z,y)],$$
$$NTP(x,y) \equiv_{df.} P(x,y) \wedge \neg\Diamond\exists z[EC(z,x) \wedge EC(z,y)].$$

i.e. x is a tangential part of y iff x is a part of y and it is is possible for there to exist a z such that z externally connects with both x and y, and x is a nontangential part of y iff x is a part of y and it is not possible for some z to exist such that z externally connects with both x and y. But this is neither desireable (given the additional formal machinery required to deal with the operator "$\Diamond$" and anticipated computational cost incurred) nor necessary.

Other formal work in geometrical vein includes Tarski's [1959] first order axiomatisation of elementary geometry using points and two relations: a ternary betweeness relation and a quartenary equidistance relation. Tarski's work is also mentioned by van Benthem [1982 Appendix A].

7.3: Summary

It is to be expected that any theory that takes space and time as its subject, will give rise to an extensive body of literature that can be said to be related. While this survey is representative of the main strands of work that appears in the literature, it cannot be regarded as exhaustive.

Up until fairly recently, rich formalisms for describing space have been lacking in AI literature, although as has been shown, this is not so in philosophy. In contrast the formal representation of time has been well researched in both AI and philosophical literature. The theory described in AF develops particular elements suggested in Hayes' Naive Physics programme, but differs in the emphasis given to the relationship between ontological, representational and implementational points. I argue that "reductionism", so prevalent in early axiomatic treatments of subjects is desirable, and that the rigour shown in such work is of great importance in general AI and Cognitive Science.

The following, and final chapter, discusses future work. The thesis is drawn to an end with a summary of the central contributions made.

Chapter 8: Future work and conclusions

8.1: Introduction

This chapter splits into two main parts. The first part suggests ways in which the theory could be forseeably extended, refined and used. The second part concludes the chapter with a summary of the thesis.

8.2: Future Work

8.2.1: Extending the number of subsorts of physical object

It would be useful to increase the number of subsorts of PHYSOB from those suggested and used in Chapter 5's modelled domains. A simple classification hierarchy that might prove useful would be to encode dispositional properties as well as simple taxonomic categories in terms of sorts. Knowing for example, that an object is rigid (and not deformable) or is fluid or gelatinous (and is deformable), immediately suggests ways of linking the bearers of these properties to other parts of the theory. For example given that an amoeba is a cell, which is gelatinous, which can deform, and that deformation implies a change in shape, can be readily accommodated in the theory. In this case AMOEBA, CELL, GELATINOUS_OBJECT and DEFORMABLE_OBJECT would appear as sorts. However, the reader should not be lulled into thinking that dispositional properties (appearing as sorts) can be easily *defined*.

It is well known that dispositional terms commonly used in science resist being given explicit formal definitions. For example, take the property of being soluble (in water). It is not enough to say x is soluble iff, if it is placed in water it will dissolve; since the implicit use of a material conditional in the consequent would mean that not only is the sentence true if x were never placed in water, but also for *any other substance* not placed in water [Flew 1979 p279]. A solution suggested by Carnap [cited in Flew 1979 p279] makes use of *reduction sentences* (being a reduction of the sense of such terms) which is used in place of definitions. Thus, for example, being soluble in water is expressed as 'if a substance is placed in water, then it is soluble iff it dissolves'. In this case, being soluble is not defined, but conditions are given under which something is either soluble or not soluble. The same principle can be extended to cover other dispositional properties where in each case the conditions under which some object either has that property or not, is specified.

Note that while named sort intersections for specialisations of the sort REGION are commonplace, the same does not arise for subsorts of the sort PHYSOB. There are two main difficulties which arise here. Firstly, if, as is suggested above, dispositional properties are to be embedded in a sort lattice, then formal definitions are difficult to secure. Secondly, if the domain being modelled is particularly rich (as would be the case if the real world were used as the model), then the denotation of two sorts may well prove to be a subset of the intended sort domain. For example, while Cohn [1987], takes the sort INSECT to be the sort intersection of the sorts COLD_BLOODIED_ANIMAL and WINGED_ANIMAL, this fails to be true if the domain includes extinct animals and includes 'flying lizards'. Thus although the use of a glb operator may allow multiple inheritance hierarchies or tangled hierarchies to be represented, in practice the Boolean sort lattice forces severe demands on the theory builder. This is especially so if the encoded theory uses the real world as a model, and aims for the coverage suggested either by the Naive Physics programme

[Hayes 1979,1985a] or that aimed for in the *Cyc* project [Guha and Lenat 1990]. The lattice requirement for LLAMA is, as mentioned elsewhere, relaxed in Cohn [1991].

8.2.2: Modelling filtration and osmosis

Filtration and osmosis can be effectively modelled in the extant theory. One example of osmotic diffusion is used by the amoeba where the nutrient having been released from the ingested food is absorbed into the cell body through the vacuole membrane. The model used in chapter 5, treated the nutrient as a bounded region that was allowed to *overlap* the vacuole membrane and pass into the cell body material. But another more detailed model could have been used. In this case the food is represented as consisting of nutrient and waste parts. The nutrient is then characterised as 'small parts' and the waste or residual material left over from digestion 'large parts'. After digestion the nutrient is treated as a disconnected region, where each separated part is attributed a particular size. The vacuole's membrane is then described as a multiply connected skin, whose interstices are large enough for the small parts to pass through but too small for the large parts to pass through. The passage of the nutrient from the vacuole into the cell material follows the usual method of modelling passage of one body through a portal.

Filtration can also be modelled along these lines. 'Small parts' could ether be treated as atomic parts or notions of relative size between nutrient parts and waste material could be introduced and expoited, along the same lines as that described in section 3.9 where an ordering of size between the amoeba and its food was exploited aand related to the inside relations.

8.2.3: Adding ordering relations in space and using strings

Given that time has one dimension and has one ordering relation B, it would seem plausible to extend this to space, which has three dimensions and by analogy, three ordering relations (with their inverses) corresponding to, for example, being in front (to the back) of, being to the left (to the right) of, and being above (below). The use of ordering relations in space appears in [Guesgen and Fidelak 1990] using Cartesian tuples of relations, and in Hernández [1990] using projection and orientation relations - see Chapter 7.

The introduction of ordering relations in space immediately requires one to decide what reference frame should be used. For example, the reference frame may be intrinsic (where the orientation is given by some inherent property of the reference object), extrinsic (where external factors impose an orientation on the reference object) or deictic (where orientation is related to a particular viewpoint) - see Hernández 1990. The relation of being in front of is a case in point. For example, if an intrinsic reference frame is envisaged, the relation is non-transitive (as when cars may be arranged 'nose to tail', and forming a circle, but transitive if extrinsic (as when the cars are engaged in a race).

An indication of the utility of adding ordering relations in space in the current theory can be seen in the following sketched example. First the relation 'ABOVE(x,y)' is introduced read as 'x is above y'. This relation is understood to be irreflexive, asymmetrical and non-transitive. Next a vertical string of atoms is defined, in this case 'V_String(x)' is read as 'x is a vertical string':

$$V\_String(x) \equiv_{def.} String(x) \wedge \forall y[C\_Atom(y) \wedge P(y,x)] \rightarrow \forall z[[C\_Atom(z) \wedge [ABOVE(z,y) \vee ABOVE(y,z)] \rightarrow P(z,x)]$$

A new constant called 'base' is added. The base is a region of space conceived to be a flat region - Convex(base) and Convex(cl(compl(base)))? - such that every region that is discrete from the base is axiomatised to be above the base. Every vertical string overlaps the base, and every region has a string that overlaps it. Distinct vertical strings do not overlap each other, and if either of two regions the one is above the other, some string overlaps them both. No string extends above another string, and some string extends above every 'normal' region. Vertical strings could be used in several ways. Firstly, to indicate possible pathways for lazy fluid flow in free space, and to provide some way to relate pressure variations of liquid with depth.

Strings prove to be particularly useful basic objects for describing many useful properties of space and the motion of bodies through space. This has been noted by Gardin et al [1986] and Gardin and Meltzer [1989] who use strings as basic objects in analogical modelling.

Intuitively, a string naturally suggests a possible pathway along which a body can pass. Moreover, a string provides a useful way to describe a sealed body, since all one needs to say is that body x is sealed (in the sense of a sealed container), if and only if for every atom which is part of the inside of x, and for every atom that is part of the outside of x, then every string which overlaps both atoms, overlaps an atom of x. If now we stipulate that x is solid (and where all its parts are solid), then on the stipulation that all motion in space follows a string, or a string bundle - where a string bundle has a proper region for every part, then it becomes clear how one can express the fact that when a body is sealed, no other body can either pass into it, or out of it. Gardin and Meltzer use strings to find configuration paths through two-dimensional mazes; the idea of using strings to define sealed bodies works on the same principle - in this case, establishing that no connecting path exists.

Given the definition of a vertical string, it should be obvious that definitions for other variants of strings using the other ordering relations for space are forthcoming. Again this has an immediate forseeable use. For example, one can then easily define planes (from which the notion of a flat body or surface is immediately forthcoming) and composite laminar like regions, arranged rather like rock strata.

That lamina are frequently used in Fluid Mechanics to describe and model fluid flow along conduits or across a surface suggests a natural way forward for using lamina to describe the behaviour of liquid in force pump as it is sucked into the main chamber and then expelled. Horizontal lamina provide a powerful framework with which to describe the observational fact that water finds its own level as it fills a container, and filling as the 'filling' of layers of connected lamina.

8.2.4: Incorporating motion and force into the theory

Given the emphasis given to the representation of topological and geometrical information in this theory, the need to incorporate notions of motion and in particular force did not arise. However, once an explanation is sought for why a particular state of affairs has come about, such explanations will tend to make an appeal to such notions as force, influences and dispositional properties of particular bodies. This becomes particularly important if, for example the theory described here is to be used with planning in mind, or for abductive reasoning where one needs to reason restrospectively why some particular state came about, or what dynamic factors are required or were required in order to bring about a particular state of affairs, an event or a process.

Presently constructed, the theory relates material bodies to the spaces they occupy. Any notion of movement is treated by describing a change in the degree of connectivity between bodies (or rather the spaces they occupy) over time. However, a simple theory of motion can be accommodated by the theory, which I outline here.

Firstly, for a stationary body, all one needs to say is that that body occupies the same region of space over time, or over some specified period of time. There are noted complications once rotational motion is envisaged and where the body in question is symmetric about that axis of rotation, since one would need to say that all the parts of that body occupy different locations in space over consecutive moments in time; but the formalism can accommodate such variants.

In order to say some body, object a, is stationary for all time (or for some specified period), this is described as follows:
$\forall x$ EQUAL(a|x,a|next(x)), or EQUAL(a|t1,a|t2) respectively. Conversely, for continuous motion this is expressed as $\forall x$ $\neg$EQUAL(a|x,a|next(x)). In order to get continuity in motion through space (and thereby dissallowing "jumps"), one could then give the global condition $\forall xy$ C(x|y,x|next(y)). This condition actually appears in the assumption of continuity via connection built into the envisionment axioms described in Chapter 3. Note here, that if x is a body that occupies an atomic region of space (and where the condition $\forall xy$ C(x|y,x|next(y)) holds), then that body cannot occupy different regions of space over time. That this is so is forced by

the condition that connectedness between atoms implies their identity (see T53). The same does not apply for closed atoms however, since the closures of atoms can externally connect (T59). But note that if we go on to say that EC(a|x,a|next(x), where object a is atomic, then given the generalised notion of a string (of atoms) we can see the beginnings of describing continuous motion for a closed atom along a string.

For rotational motion, the addition of the ordering relations for space can be effectively exploited. For example, one could describe the motion of door2 of valve2 in the force pump by first defining the relation 'ROTATE_RIGHT_UP(x,y,z)' read as 'x rotates towards the right and upward throughout period z', as:

$$\text{'ROTATE\_RIGHT\_UP}(x,y) \equiv \text{def. } \forall zu\big[[P(z,y) \wedge P(u,y) \wedge B(z,u)] \rightarrow \forall v\big[[\text{C\_Atom}(v) \wedge P(v|z,x|z) \wedge P(v|u,x|u)|] \rightarrow \text{RIGHT}(v|z,v|u) \wedge \text{ABOVE}(w|u,w|v)\big]\big].$$

8.2.5: Factoring out additional lattices

As mentioned in Chapter 5, further work needs to be done in order to translate each monadic predicate into a sort predicate, and close the extended sort lattice. As argued in Chapter 2 and 5, this is not an easy task. The same principle and difficulties carry across to sets of relations that can be factored out and embedded in lattice structures.

The lattice that encodes the set of relations defined solely in terms of the relation C(x,y) has been completed. But others, in particular the set of relations expressing the inside and outside relations will require many more relations to be defined, than the set given. Unlike the set of relations encoded in the L_C lattice, most of the pairs of named relations that are defined have non-empty intersections. This is not helped by the fact that most of the defined relations are not symmetrical, e.g. W_OUTSIDE(x,y), since unlike any symmetrical predicate, an inverse case will need to be defined. Some work developing these relations can be found in Randell and Cohn [1991], but the lattices shown there are strictly only partial, i.e. the set of base relations expand once inverses are taken into account.

8.2.6: Eliminating redundancy in the set of defining axioms

Careful inspection of the set of clauses defining the theory reveals some redundancy. Two examples are given here. For example, given EQUAL is defined, axiom A3 (which appears in Clarke [1981]) can be excised, since it can be proved as a theorem. Similarly, the clause set defining prod(x,y) (numbering four clauses):

(i) ¬C(u,prod(x,y)) v P(f6(x,y,u),x)
(ii) ¬C(u,prod(x,y)) v P(f6(x,y,u),y)
(iii) ¬C(u,prod(x,y)) v C(u,f6(x,y,u))
(iv) ¬P(v,x) v ¬P(v,y) v ¬C(u,v) v C(u,prod(x,y))

can be reduced to the three clauses:

(i') P(prod(x,y),x)
(ii') P(prod(x,y),y)
(iii') ¬P(v,x) v ¬P(v,y) v ¬C(w,v) v C(w,prod(x,y)).

In this case the function prod(x,y) is substituted for the skolem function f6(x,y,u) - where the region prod(x,y) is taken to be *the* part held in common between regions x and y. In that case one can see immediately that clause (iii) is tautologous, and hence can be eliminated. Then we note that the literals, P(prod(x,y),x) and P(prod(x,y),y) are theorems, hence clauses (i) and (ii) can be respectively simplified to clauses (i') and (ii').

It would be useful to reduce the set of defining axioms to a minimum, though establishing independence of clauses in a large axiomatic theory is recognised as a particularly difficult task.

8.2.7: Establishing consistency in large axiomatic theories

Hayes [1985a] acknowledged the difficulty facing any person building large scale formal theories, namely that establishing consistency for large axiomatic theories is a non-trivial task. Cunningham's [1985a] thesis, that of constructing a model bulding program for first order theories, was also motivated by this sense of unease.

In general, the existence of a model that interprets a first order theory guarantees its consistency - at least relative to the model - for the model itself may have deeply embedded inconsistent notions. However, given a first order axiomatic theory, techniques exist to prove the existence of a logical model (if a finite model exists), hence establish its consistency relative to that model. However, just because a finite model can be established, that model may not be the intended one. Hence the importance for insisting that the expressions in a formal language can support clear semantic readings in the metalanguage - for by doing this, the intended model used to interpret the theory establishes the relative consistency of the axiomatic theory for that intended model. Without this condition being satisfied, a logical model may exist, but the theory will fail to be a theory of the intended domain.

Work has already been done to automate model building (see e.g. Cunningham [1985a,b], Manthey and Bry [1988] and Winker [1982]). It would be useful to employ such specialised programs to repeatedly test an expanding theory for consistency, or for demonstrating counterexamples for satisfiable sets of clauses that were originally thought to be unsatisfiable - as may arise when an axiom is missing making the interpreted formal theory, incomplete.

8.3: Conclusions

It is useful to summarise the thesis and highlight the main points and contributions made.

Methodological contributions

I show that a fruitful approach for modelling the everyday world needs to be grounded in an ontology that is directly related to perceptual experience. Topological information is especially singled out. Topological relationships between objects remain relatively stable over useful stretches of time and indicates the nature of the type of regularities and invariants we should attend to and ground in a theory of cognition. I show that the differences in connectivity between regions are a useful abstraction with which to model space, and that changes in the degree of connectivity between objects can be used to explain the manner in which one state changes into another.

I show how naive theories, or theories of the commonsense world must be expected to be as complex as any other theory gainfully used in the sciences, and that a difficulty in the past has stemmed from inadequate time and attention given to the nature and scope of common sense knowledge. In this respect I show that the idea of commonsense knowledge as "deep knowledge" is at best a misleading metaphor, and that the common distinction drawn between an "engineering" and "psychological" solution to program design and validation is not particularly useful.

The important conceptual distinction between a theory and how it might be used and implemented is drawn out. Problems stemming from the popular acceptance of representational theories of mind in AI research are highlighted. A recognition of solipsism in the literature as a working assumption is found to be particularly troublesome. This is considered to be of especial importance for the future of AI, if AI is to make progress either in its attempts to understand cognitive functioning, or in building intelligent program driven machines.

Foundational contributions:

The formal theory described provides the means to describe much intuitive spatial and temporal knowledge associated with the everyday world. This addresses a distinct lacuna in the literature, especially where rich formalisms for describing space are concerned. The theory requires remarkably few primitives in order to lift up a particularly rich theory describing space and time in terms of *regions*. The formal nature of the work makes all the descriptions composable. The theory satisfies Hayes' requirement that the theory be broad and dense, and in this context provides a good test bed for evaluating the computational adequacy of a theory that is representative of the Naive Physics programme.

On the inference side, I show how hybrid reasoning can be gainfully employed using a rich theory in an automated reasoning setting. In particular, the computational benefits that arise from using lattice structures for encoding monadic (sort) and higher arity predicates is demonstrated. An extension of Cohn's [1987] rule of *characteristic resolution* is given for lattices encoding sets of relations of any degree of polyadicity.

The importance of abstracting out differences in the degree of "connectivity" between regions in space is drawn out and demonstrated in the development of a comprehensive theory. Simple process descriptions are shown to be constructable from specifying transitions between state descriptions - where each state is formally described in terms of spatial relations holding between objects for a period of time.

Transition networks governing legitimate changes in the degree of connectivity between regions are developed and used to constrain projected envisionments for a given modelling problem.

Clarke's [1981,1985] theory which lies at the foundation of this thesis is modified and substantially enriched, and is expressed in a sorted logic.

Applied contributions

Partial axiomatisations of two domains are given: modelling cell behaviour of phagocytes, and describing processes associated with a working force pump.

Conclusion

Throughout this thesis, I have shown that a key to understanding the nature and grounds of commonsense knowledge lies in abstracting out useful invariants grounded in perception. Also, the explicit representation of varying topological relationships between objects in space seems particularly important. It is interesting to note, that Clarke's [1981,1985] axiomatic theory upon which the current theory is built, takes Whitehead's theory of EXTENSIVE CONNECTION outlined in *Process and Reality* [1929,1978] as his main source. Clarke expressed the thought that having axiomatised a theory that captured "so much topology ... with such minimal assumptions", boded well for Whitehead's over-all project to found geometry on such a basis [1981 p216]. Whether or not Whitehead had this in mind when the quotation with which this thesis begins was written, I do not know. What I do know, and hope to have shown, is that by using Clarke's [1981,1985] calculus of individuals (which in turn uses Whitehead's mereological definitions), a rich theory can be constructed embodying much commonsense knowledge about the nature of space and time.

Notes

[1] The individual constants include both proper and arbitrary names.

[2] Russell's [1905] theory of descriptions (which provides a general framework for contextually eliminating definite descriptions in terms of bound variables, predicates and identity) is not assumed here. The inverted iota symbol 'ι' used in this formalism, appears in the metalanguage only where definitions are introduced. The metalinguistic schema $\alpha(\bar{x})$ =def. $\iota y[\Phi(y)]$ used in this formalism is translated as $\forall\bar{x}\ \Phi(\alpha(\bar{x}))$: thus e.g. the definition:

(i) $sum(x,y)$ =def. $\iota z[\forall w[C(w,z) \leftrightarrow [C(w,x) \vee C(w,y)]]]$, in the metalanguage, is translated as:

(ii) $\forall xy[\forall w[C(w,sum(x,y)) \leftrightarrow [C(w,x) \vee C(w,y)]]]$

in the object language.

[3] The metalinguistic E-shriek operator '$\exists!$' is defined as follows: $\exists!x[\Phi(x)]$' $\equiv$def. $\exists x[\Phi(x) \wedge \forall y[\Phi(y) \rightarrow EQUAL(y,x)]]$.

The E-shriek operator is simply used as notational shorthand for the expansion expressed by the definitions.

[4] In Clarke [1985 p70] an additional conjunct '$\neg C(x,y)$' appears in what would be the the antecedent of axiom (A6), i.e. $\forall xy[[B(x,y) \wedge \neg C(x,y)] \rightarrow \varphi]$. This is dropped in axiom (A6) because temporal regions are restricted to open regions in the present theory, which makes the additional conjunct redundant. Clarke's reading of the relation $B(x,y)$ as 'x is wholly before y' is similarly changed to 'x is before y'. Given two temporal regions that either abut (i.e. have no region between them) or have a region separating them, in both cases, no incident point is held in common. This justifies the reading given for $B(x,y)$ in this theory as 'x is before y'.

[5] In general, when using LLAMA, the set of constants, functions and predicates supported by the theory are actually defined on the set of base sorts of S, and not on much weaker sorts as has been done here. Thus e.g. taking the predicate C defined on spatial regions, this would actually be expressed in LLAMA as:

type $C(\tau 1,\tau 1)$:TT

type $C(\tau 1,\tau 2)$:TT

type $C(\tau 2,\tau 1)$:TT

type $C(\tau 2,\tau 2)$:UU,

where $\tau 1$ = SPATIAL_UNIVERSE, and $\tau 2$ = SPATIAL\SPATIAL_UNIVERSE.

The weaker sort declarations are used simply to aid the readability of the formal theory, though an implementation of the theory would normally use the more specific declarations, in order to fully exploit the sortal information embedded in the theory. Note the appearance of the sort TT indicating logical truths in the theory - in this case corresponding to the set of theorems: $C(u_s,u_s)$, $\forall x\ C(u_s,x)$ and $\forall x\ C(x,u_s)$.

[6] Although the formalism is used to describe relations between physical objects embedded in 3-space, it is worth pointing out that a part of the calculus may well support a model in 2-space where the the relation of connectivity is understood to be optical, relative to an observer rather than actual, as in the distinction drawn in astronomy between optical and actual binary star sytems. Some modification of the *interpretation* of the EQUAL relation would be required to support the optical interpretation of connectivity, since it would cease to be true that two spatial regions would be identical just because they share the same connectivity with other regions (e.g. when one exactly occludes or superimposes another).

[7] Strictly speaking the relation of being a part used in the formalism is not defined on physical objects but on the regions of space they occupy at any given moment in time (see section 3.3).

[8] Relation R is weakly reflexive iff $\forall xy[R(x,y) \rightarrow R(x,x)]$, and totally reflexive iff $\forall x\ R(x,x)$. Most reflexive relations are in fact weakly reflexive and not totally reflexive. The relation 'is identical with' is clearly totally reflexive, but the relation 'weighs the same as' is not for it does not relate every object in the domain to itself; e.g. numbers [Anderson and Johnstone, 1962 p200]. A weakly reflexive relation is rendered totally reflexive if its domain is sufficiently constrained. Thus e.g. the relation TP would be totally reflexive if the only regions it were defined on were closed.

[9] Strictly speaking the domain referred to here will be sub-domain in practice. The sorted logic assumed in this axiomatisation requires each sort to be non-empty. This means that once additional subsorts of SPATIAL are introduced, e.g. the sorts OPEN AND CLOSED, only a sub-domain will support a model where the intended interpretation is a set of open regions.

[10] Clarke [1985] p68.

[11] The similarity of mereology to complete Boolean Algebras has been commented on by many other authors, e.g. Eberle [1970], Clarke [1981] and Roper [1983]. Tarski [1935] pointed out that the relation of part to whole taken as a primitive in the system had a correlate with the Boolean-algebraic inclusion operator. Grzegorczyk [1955] developed this idea stating that the models of mereology and those for a complete Boolean algebra with the zero (null) element missing were identical. Clay [1974] provides a detailed rebuttal of this. One central point being that Grzegorczyk's system described as mereology is not identical to Lesniewski's of the same name. The question as to whether these other systems also called mereology (now denoted as "*mereology*") can or can not be regarded as having identical models with Boolean algebras having the zero element missing remains an open one. Some *mereological* systems include a correlate of the null element, e.g. Martin [1947] calculus and Bunt's [1985] *Ensemble Theory* includes an empty ensemble (object), although Bunt gives reasons why his system should not be formally identified with mereology (which would include *mereology*). Bunt actually gives ensemble analogues of the classical elements of a complete Boolean algebra.

[12] It is common practice to find among calculi of individuals a distinct refusal to admit an individual that functions not unlike the null element in a Boolean algebra and null set in classical set theory. This metaphysical stance has strong associations with nominalism and contructivism. While it is difficult to find a common ground among the numerous positions now claiming to be nominalistic [Eberle, 1970 p.10] the trend towards parsimony and the clear distrust of certain categories of entities remains constant. The work of Lesniewski [1927-1931] falls into this category. According to Eberle [Eberle 1970 p7], Lesniewski adopted a

stongly nominalistic position, refusing to countenance 'general objects' i.e. those objects having all and only those properties common to several individuals, an empty set, unit sets differing from their elements and sets of individuals that was not itself a individual. It was Lesniewski who constructed the first logical system dealing with the part/whole relation (called by him 'Mereology') that has become mistakenly identified with the calculus of individuals developed independently by Goodman (see Eberle 1970).

It is noteworthy that with standard set theories it is inconsistent to *deny* that there is a such an object as the set without members, hence the null set must be included on pain of contradiction. In contrast it is inconsistent with standard calculi of individuals to affirm the existence of an individual without parts (i.e. without content). A resolute nominalist might well argue that the fact standard set theory allows sets without members without contradiction, while calculi of individuals do not, indicates the exotic nature of the former in contrast with the latter.

[13] Finding a set of criteria which can effectively choose between predicate and function notation has been put forward as an open research problem by Larry Wos in 'Automated reasoning: 33 Basic Research Problems' [Wos 1988, p.160]. A related open problem lies in selecting criteria which can effectively choose between using and avoiding equality predicates [Wos 1988, p.61].

Wos points out that the choice of a particular notation in automated reasoning can have a marked effect on the performance of a program, not unlike the peformance of a person trying to find hand built proofs using alternative notations (cf. witness Anderson and Johnstone's [Anderson and Johnstone 1962, p.241] comment that using a Russellian analysis of descriptive functions "is not altogether practical."). No simple solution to this problem is expected. Wos points out that the effects of representation, inference rule and strategy are tightly coupled and the criteria chosen would most certainly reflect this. This point is discussed further in Chapter 5.

[14] There exist logics known as 'Free logics' e.g. Schock [1968] and Tennant [1978] which allow non denoting singular terms into the formal language. Functional expressions and definite descriptions are treated as

singular terms and manipulated as names. Free logics avoid the pressuposition that singular terms (i.e. functional expressions, definite descriptions or proper names) must denote, found in standard predicate logic. Rules of inference governing the quantifiers are consequently complicated by this move, e.g. in Tennant's logic $\exists xF(x)$ cannot be inferred from $F(t)$ alone but requires the premiss $\exists x[t=x]$. There appears no immediate difficulty incorporating these rules of inference into a resolution based mechanised logic. The practical gain that would arise using an automated free logic remains an open question.

[15] See Geach [1980] who offers a humorous yet instructive point that the temptation to treat "nothing" as a name opens the way to innnmerable fallacies.

[16] Alternative definitions for prod(x,y) and diff(x,y) could be given in terms of sum, compl and EQUAL, i.e.:

prod(x,y) =def. ιx[EQUAL(x,compl(sum(compl(x),compl(y))))]

diff(x,y) =def. ιx[EQUAL(x,prod(x,compl(y)))]

while the definitions are formally equivalent it raises several computational questions e.g. when to select suitable sets of definitions for a given class of theorems to be proved, when to add such identities to the demodulator list and when to add redundancy into the clausal set by including lemmas - see Chapter 5.

[17] Given this theory is meant to reflect a naive theory of the world, the fact that compl(x) is defined on u_s might be taken be be at variance with the the simple intuition that the physical universe has no (obvious) spatial complement - thus it might be thought compl(u_s) should return $\perp$ and not NULL. The decision to declare NULL or $\perp$ for improper functions must in part depend on the number of theorems one wishes to prove in the logic - since declaring $\perp$ rather than NULL for the result sort results in a larger number of ill-sorted terms arising in wff's.

[18] It should be pointed out that the wff:

$\forall x$[EQUAL(sum(x,compl(x)),u_s)

x,compl(x):SPATIAL\SPATIAL_UNIVERSE, sum(x,compl(x)):SPATIAL_UNIVERSE,

u_s:SPATIAL_UNIVERSE

is a theorem in this logic, but that x cannot be instantiated with $u_{\varnothing}$ without the literal being ill-sorted.

[19] See Clarke [1981, p.216, note 3]. From another standpoint the definition for P in the classical calculus of individuals:
$P(x,y) \equiv def.\ \forall z[O(z,x) \rightarrow O(z,y)]$
cannot be used to characterise the interior of a closed region. While it is true that every region that overlaps the closure of a region, overlaps its interior, it doesn't follow that the closure of a region is part of its interior. Hence the cited definition fails.

[20] This interpretation is a little opaque: $\forall xy[C(x,y) \rightarrow O(x,y)]$ is equivalent to $\forall xy[\neg EC(x,y)]$, $\forall x \neg \exists y[EC(y,x)$, $\forall x\ NTP(x,x)$, $\forall x\ EQUAL(int(x),x)$, hence Open(x), i.e. in the absence of external connectedness in the domain, x and y become open regions.

[21] That the converse case is not a theorem can be recognised from the fact that the closure of a region and its interior contains identical atomic parts, but that the closure of a region and its interior are not necessarily identical.

[22] If points are added to the formalism by introducing a new sort POINT (stipulated to be pairwise disjoint with the sorts REGION and NULL) convexity, and the convex hull of a region could be defined along the following lines:

$$\begin{aligned}Convex(x) \equiv def.\ \forall yz[[&POINT(y) \wedge POINT(z) \wedge\\ &IN(y,x) \wedge IN(z,x) \wedge \neg EQUAL(x,y)] \rightarrow\\ &\forall u[[POINT(u) \wedge Coll(y,z,u) \wedge B(u,y,z)] \rightarrow IN(u,x)]]\end{aligned}$$

where, Coll(x,y,z) embodies the notion of three points being collinear, and B(x,y,z) that of one point being between two others.

[23] The definitions for INSIDE(x,y), OUTSIDE(x,y) and P_INSIDE(x,y) used here replace the set given in Randell and Cohn [1989a,b,c]. The original set of definitions (which used the conjunct $\neg P(x,y)$ instead of the conunct $\neg O(x,y)$ used here), fail to exclude unintended models where x overlaps y but is not part of y.

[24] In contrast to the axioms given in Randell and Cohn [1989b] this set admits only solid spheres as a model for the predicate Ball(x), e.g. the previous axioms admitted cubes as a model.

[25] Here I depart from van Benthem's axiomatisation by adding the disjunct C(cl(x),cl(y)). Given two regions that are non-identical, e.g. EC(a,b), it does not follow that b is nearer to a than a is to itself. Without this restriction, a contradiction is immediately derivable within the theory: EC(a,b) implies C(a,b), which implies C(cl(a),cl(b)), which implies EQUAL(d(a,b),0). But given that EC(a,b) clashes with EQUAL(a,b), this resolves into N(a,a,b), which in turn implies d(a,a)<d(a,b), hence d(a,a)<0 and $\neg$[d(a,a)≥0]. However, by (A29) d(a,a)≥0 - contradiction.

[26] Note that LLAMA does not require any restriction to be made here in the way of a conditional statement, i.e. $\forall$xy[[NUMBER(x) $\wedge$ NUMBER(y)] $\rightarrow$ ϕ]. The sorting function for disjunction (the symbol: 'v') is strict, meaning a clause is illsorted (and EE) if any literal in that clause is of sort EE. Given, a sort environment where x,y:T\NUMBER, either of the disjuncts x<y or y>x are evaluated as EE, and the whole clause is illsorted. Thus, the only interpretation allowed is where the variables are place holders for numbers.

[27] Given that temporal regions are stipulated to be open, relations requiring the satisfiability of EC in the domain are excluded from these definitions. One could improve the computational efficiency of the logic by re-defining the NTP relation as follows:

NTP(x,y) ≡def. P(x,y) $\wedge$ $\neg\exists$z[C(z,x) $\wedge$ $\neg$O(z,x) $\wedge$ C(z,y) $\wedge$ $\neg$O(z,y)]

then declaring:

type EC(τ,τ):UU, τ = SPATIAL

Presently defined, Open(x) is defined in terms of the int(x) function, and that in terms of the C and NTP relation. The definition for NTP uses the relations P and EC. Thus the sorting function for EC must be well-sorted when defined on periods (which it is), even though any literal of the form EC(x,y) will be false where the substiuends for x and y are periods.

It is worth mentioning that the formalism could conceivably support either closed (or clopen periods) if required, although this introduces additional complexity in the definitions, and may generate conceptual absurdities given disconnected periods; e.g. for closed regions the definitions for Meets and Starts would assume the following form:

MEETS(x,y) ≡df. B(x,y) ∧ EC(x,y)

STARTS(x,y) ≡df. TPP(x,y) ∧ ∃z[MEETS(z,x) ∧ MEETS(z,y)]

Complications arise with clopen periods, since the ordering, i.e. the sequence •---[ )[ )---• or •---( ]( ]---• would first need to be specified. Additional complications then arise for moments, since initial moments for the first sequence would have to be closed, and open for the final moment, and open for the first, and closed for the latter sequence respectively.

[28] The problem arises when intervals are construed as sets of points and where propositions are indexed to individual points, or sets of these points and where we wish to to model some event where a change arises, e.g. in a light being on then being switched off. This requires either a decision about whether truth value gaps are to be allowed, or what topology intervals should have.

[29] Clarke [1985] uses a different set of relations to describe temporal position than the set used here. Clarke assumes a spatiotemporal interpretation for his regions and points whereas the spatial and temporal elements are clearly separated in this formalism. The equality relation defined above is transitive, which in a temporal setting assumes a model of time that is absolute. In contrast Clarke assumes a relativistic notion of time which is mirrored in the definition he sets up for contemporaneity between spatio-temporal regions which is carefully formulated to be *non-transitive*.

[30] It should be noted here, that in actual fact, the restrictions on x need to be tighter than those stipulated.

Given the fact that this theory admits a universal period of which every period is a part, and that temporal functions corresponding to the initial, final and next moment of a given period are subsequently defined, some additional restriction on the type of periods supported by the theory is required - namely that periods be either bounded above, bounded below or both bounded above and below. For example, given the universal period is unbounded in the intended model, it does not necessarily follow that if x is not identical to the period universe, that x has either an upper or lower bound, and a fortiori a final or initial moment. Hence the formal theory needs augmenting with defined periods that are bounded above, bounded below, and both bounded above and below. By x being bounded above, I mean that period x has a moment y such that that every other moment of x is before y, and by period x being bounded below, that x has a moment y such that y is before every other moment of x. This restriction also carries across to definitions (D88), (D89) and (D90). In this case, the function initial(x) needs to be restricted so that interval x is bounded above, and for the function final(x), that interval x be bounded below. For the function next, period x must be bounded above. I am indebted to Zhan Cui for pointing out the need for this additional restriction.

[31] Given the closure function (which appears in the definition of the relation SEPARATED(x,y) and implicitly in the definition of Connected(x)) is not defined on periods, these relations cannot be used to define disconnected periods - hence the new defintion.

As mentioned earlier in the text, one could weaken the sortal restrictions to allow either open or closed or clopen regions to be periods, but the increase in expressiveness (without any real practical gain required by an interval logic) would simply reduce computational efficiency by allowing fewer cases of illsorted terms and formulae in a proof run to cut down the search space.

It is also possible to allow a new sort Period* defined on both open or closed regions that is more general than Period, and define Connected(x) on this sort. The implemented logic could then be tailored to only allow the user access to periods proper; the sort Period* only being allowed internally within the system for the purposes of setting up the definition. This option is not used in the interests of a simpler ontology. There remains a separate and open question as to the computational cost of the selected and alternative option.

[32] Given a clear distinction between states, events and processes, each could be named and assigned to a sort in this logic, if required. This possibility is discussed later on in this chapter.

[33] That is to say each function of the form $\varphi(x,y)$ would need to be assigned to an axiom that gives the conditions when $\varphi(x,y)$ holds and when $\varphi(x,y)$ does not hold. That, for example c(x,y) is an improper function,

can seen in the possible case where objects a and b do not connect for all time. One could extend the interpretation of the sort NULL to cover such improper cases.

[34] One could extend the sort PHYSOB to include two new sorts STUFFS and THINGS, and then stipulate that the sort WATER is a subsort of STUFFS. In this case the function sum(x,y) could be defined to be well sorted and well formed when defined on argument sorts of the same stuff type.

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Appendix A: Glossary of special symbols used.

Below are assembled together all the distinguished constants (i.e. constants excluding individual constants such as "amoeba1", "main-chamber" and "valve1"), functions, predicates and specially defined symbols used in the theory. Each symbol is given with its linguistic reading in parentheses (where this is given in the text) and the page number(s) where it is first introduced.

ABOVE(x, y) 207
ABSORBS(x, y, z) 135
AMOEBA or AMOEBA(x) 119, 130
at(φ(x), y) ('φ(x) at y') 110
Atom(x) ('x is an atom') 65

B(x, y) ('x is before y') 42, 89
Ball(x) 85
base 208
BEFORE(x, y) 89
B^{-1}(x, y) 89
$BEFORE^{-1}$(x, y) 89

C(x, y) ('x connects with y') 42
C_Atom(x) ('x is a closed atom') 67
CELL_MEMBRANE or CELL_MEMBRANE(x) 130
cl(x) ('the closure of x') 60
Clopen(x) ('x is neither open nor closed') 60
Closed(x) ('x is closed') 60
compl(x) ('the complement of x') 54
COMPONENT_PART(x, y) 151
Concave(x) ('x is concave') 80
CONCENT_PART(x, y) ('(ball) x is a concentric part of (ball) y') 85
Connected(x) ('x is connected') 63
Connected_Period(x) 92
CONNECTED_PORTAL(x, y) 146

Proofs of all the theorems cited in the text are collated below. First the theorem is cited, and then the proof is given. In the interest of space, most of the proofs list the set of axioms, definitions and lemmas that were used. The general proof method used is proof by contradiction. Full proofs, i.e. where each inference step is made explicit, are reserved for informative theorems only. In this case clausal form is used as the main representational language, and binary resolution, factoring and paramodulation are the rules of inference used. Where paramodulation is used in a proof, this is indicated as follows. In the case of a simple listing, the term "paramod." is added to the stipulated set of axioms, definitions and lemmas used. In the case of a full proof, the same expression appears appended to the set of ancestor clauses used. Although binary resolution has been chosen to make proofs easy to scan, lemmas are frequently used to keep the listings to a reasonable length.

Bracketed entries e.g. (A1), (D1), (T1) and (C1) respectively refer to the main list of axioms, definitions and theorems/lemmas and conjectured theorems that appear in the text. Non bracketed numerals are reserved for the clause sets used and generated in the proof only. In this case, arbitrary constants (or ground terms) used in the proofs are selected from the set $\{a,b,c, \ldots\}$

As a general rule, wff's of the form '$\varphi \equiv$def. ψ' are initially translated as '$\varphi \leftrightarrow \psi$', '$\varphi(\mathfrak{X})$ =def. $\iota y[\Psi(y)]$' as '$\forall x[\Psi(\varphi(\mathfrak{X}))]$', and '$\exists! x[\Phi(x)]$' as '$\exists x[\Phi(x) \wedge \forall y[\Phi(y) \rightarrow \mathrm{EQUAL}(y,x)]]$' *prior* to the translation into clausal form.

THEOREMS:

(T1) $\forall x\ \neg DC(x,x)$

from: A1, D1

(T2) $\forall x[DC(x,y) \rightarrow DC(y,x)]$

from: A2, D1

(T3) $\forall x\ P(x,x)$

from: D2

(T4) $\forall xy[[P(x,y) \wedge P(y,x)] \rightarrow EQUAL(x,y)]$

from: D3

(T5) $\forall xyz[[P(x,y) \wedge P(y,z)] \rightarrow P(x,z)]$

from: D2

(T6) $\forall x\ EQUAL(x,x)$

from: D2, D3

(T7) $\forall xy[EQUAL(x,y) \rightarrow EQUAL(y,x)]$

from: D3

(T8) $\forall xyz[[EQUAL(x,y) \wedge EQUAL(y,z)] \rightarrow EQUAL(x,z)]$

from: D3, T5

(T9) $\forall x\ \neg PP(x,x)$

from: D4

(T10) $\forall xy[PP(x,y) \rightarrow \neg PP(y,x)$

from: D4

(T11) $\forall xyz[[PP(x,y) \wedge PP(y,z)] \rightarrow PP(x,z)]$

from: D4, T5

(T12) $\forall x\ O(x,x)$

from: D2, D5

(T13) $\forall xy[O(x,y) \rightarrow O(y,x)]$

from: D5

(T14) $\forall x\ \neg DR(x,x)$

from: D2, D5, D6

(T15) $\forall xy[DR(x,y) \rightarrow DR(y,x)$

from: D5, D6

(T16) $\forall x\ \neg PO(x,x)$

from: D2, D7

(T17) $\forall xy[PO(x,y) \rightarrow PO(y,x)]$

from: D5, D7

(T18) $\forall x \neg EC(x,x)$

from: D2, D5, D8

(T19) $\forall xy[EC(x,y) \rightarrow EC(y,x)]$

from: A2, D5, D8

(T20) $\forall xy[TP(x,y) \rightarrow TP(x,x)]$

from: D2, D9

(T21) $\forall xy[[TP(x,y) \wedge TP(y,x)] \rightarrow EQUAL(x,y)]$

from: D3, D9

(T22) $\forall xy[[NTP(x,y) \wedge NTP(y,x)] \rightarrow EQUAL(x,y)]$

from: D3, D10

(T23) $\forall xyz[[NTP(x,y) \wedge NTP(y,z)] \rightarrow NTP(x,z)]$

from: D8, D10, T5, T32

(T24) $\forall x \neg TPP(x,x)$

from: D2, D11

(T25) $\forall xy[TPP(x,y) \rightarrow \neg TPP(y,x)]$

from: D11, T21, T24 paramod.

(T26) $\forall x \neg NTPP(x,x)$

from: D2, D12

(T27) $\forall xy[NTPP(x,y) \rightarrow \neg NTPP(y,x)]$

from: D12, T22, T26 paramod.

(T28) $\forall xyz[[NTPP(x,y) \wedge NTPP(y,z)] \rightarrow NTPP(x,z)]$

from: D3, D10, D12, T5, T23 paramod.

(T29) $\forall xy[\neg EC(x,y) \leftrightarrow [C(x,y) \leftrightarrow O(x,y)]]$

(i) $\forall xy[\neg EC(x,y) \rightarrow [C(x,y) \leftrightarrow O(x,y)]]$

Refutation set:
1 C(x,x) (from A1)
2 ¬C(x,y) v C(y,x) (from A2)
3 ¬P(x,y) v ¬C(z,x) v C(z,y) (from D2)
4 ¬O(x,y) v P(f3(x,y),x) (from D5)
5 ¬O(x,y) v P(f3(x,y),y) (from D5)
6 ¬C(x,y) v O(x,y) v EC(x,y) (from D8)
7 ¬EC(a,b)
8 ¬C(a,b) v ¬O(a,b)
9 C(a,b) v O(a,b)

Proof:
10 ¬C(a,b) v O(a,b) ancestors: 7,6
11 O(a,b) ancestors: 10,9
12 ¬C(a,b) ancestors: 11,18
13 P(f3(a,b),a) ancestors: 11,4
14 P(f3(a,b),b) ancestors: 11,5
15 ¬C(z,f3(a,b)) v C(z,b) ancestors: 14,3
16 ¬C(a,f3(a,b)) ancestors: 15,12
17 ¬C(z,f3(a,b)) v C(z,a) ancestors: 13,3
18 ¬C(f3(a,b),a) ancestors: 16,2
19 ¬C(f3(a,b),f3(a,b)) ancestors: 18,17
20 null ancestors: 19,1

(ii) $\forall xy[[C(x,y) \leftrightarrow O(x,y)] \rightarrow \neg EC(x,y)]$

Refutation set:
1 ¬EC(x,y) v C(x,y) (from D8)
2 ¬EC(x,y) - ¬O(x,y) (from D8)
3 ¬C(a,b) v O(a,b)
4 ¬O(a,b) v C(a,b)
5 EC(a,b)

Proof:
6 C(a,b) ancestors: 5,1
7 ¬O(a,b) ancestors: 5,2
8 O(a,b) ancestors: 6,3
9 null ancestors: 8,9

n.b. clause 4 is one clause of the set generated from the negation of (ii) but is not used in the proof - $\forall xy[O(x,y) \rightarrow C(x,y)]$ is a theorem.

(T30) $\forall xy[\neg\exists z[EC(z,x)] \rightarrow [P(x,y) \leftrightarrow \forall u[O(u,x) \rightarrow O(u,y)]]]$

(i) $\forall xy[\neg\exists z[EC(z,x)] \rightarrow [P(x,y) \rightarrow \forall u[O(u,x) \rightarrow O(u,y)]]]$

from: D5, T5

(ii) $\forall xy[\neg\exists z[EC(z,x)] \rightarrow [\forall u[O(u,x) \rightarrow O(u,y)] \rightarrow P(x,y)]]$

from: D8, T30.lemma1

(T30.lemma1) $\forall xy[O(x,y) \rightarrow C(x,y)]$

from: A1, A2, D2, D5

(T31) $\forall x[NTP(x,x) \leftrightarrow \neg\exists y[EC(y,x)]]$

(i) $\forall x[NTP(x,x) \rightarrow \neg\exists y[EC(y,x)]]$

Refutation set:
1 $\neg NTP(x,y) \vee \neg EC(z,x) \vee \neg EC(z,y)$ (from D10)
2 $NTP(a,a)$
3 $EC(b,a)$

Proof:
4 $\neg EC(z,a)$ ancestors: 2,1
5 null ancestors: 4,3

(ii) $\forall x[\neg\exists y[EC(y,x)] \rightarrow NTP(x,x)]$

Refutation set:
1 $\neg P(x,y) \vee NTP(x,y) \vee EC(f5(x,y),x)$ (from D10)
2 $P(x,x)$ (from T3)
3 $\neg EC(y,a)$
4 $\neg NTP(a,a)$

Proof:
5 $\neg P(a,a) \vee EC(f5(a,a),a)$ ancestors: 4,1
6 $EC(f5(a,a),a)$ ancestors: 5,2
7 null 6,3

(T32) $\forall xyz[[NTP(x,y) \wedge C(z,x)] \rightarrow O(z,y)]$

Refutation set:
1 ¬P(x,y) ∨ ¬C(z,x) ∨ C(z,y) (from D2)
2 ¬O(x,y) ∨ P(f3(x,y),x) (from D5)
3 ¬O(x,y) ∨ P(f3(x,y),y) (from D5)
4 ¬P(z,x) ∨ ¬P(z,y) ∨ O(x,y) (from D5)
5 ¬C(x,y) ∨ O(x,y) ∨ EC(x,y) (from D8)
6 ¬NTP(x,y) ∨ P(x,y) (from D10)
7 ¬NTP(x,y) ∨ ¬EC(z,x) ∨ ¬EC(z,y) (from D10)
8 ¬P(x,y) ∨ ¬P(y,z) ∨ P(x,z) (from T5)
8' ¬O(x,y) ∨ O(y,x) (T13)
9 NTP(a,b)
10 C(c,a)
11 ¬O(c,b)

Proof:
12 P(a,b) ancestors: 9,6
13 ¬EC(z,a) ∨ ¬EC(z,b) ancestors: 9,7
14 ¬C(z,a) ∨ C(z,b) ancestors: 12,1
15 C(c,b) ancestors: 14,10
16 O(c,b) ∨ EC(c,b) ancestors: 15,5
17 EC(c,b) ancestors: 16,1
18 ¬EC(c,a) ancestors: 17,13
19 ¬C(c,a) ∨ O(c,a) ancestors: 18,5
20 O(c,a) ancestors: 19,10
21 P(f3(c,a),c) ancestors: 20,2
22 P(f3(c,a),a) ancestors: 20,3
23 ¬P(a,z) ∨ P(f3(c,a),z) ancestors: 22,8
24 P(f3(c,a),b) ancestors: 23,12
25 ¬P(f3(c,a),x) ∨ O(x,c) ancestors: 24,4
25' O(b,c) ancestors: 25,24
26 O(c,b) ancestors: 25',8'
27 null ancestors: 26,11

(T33) $\forall xy[TPI(x,y) \rightarrow TPI(x,x)]$

from: D3, D9, D13, T3 paramod.

(T34) $\forall xy[NTPI(x,y) \rightarrow NTPI(x,x)]$

from: D3, D10, D14, T3

(T35) $\forall xy[TPI(x,y) \rightarrow TPI(y,x)]$

from: D3, D9, D13, paramod.

(T36) $\forall xy[NTPI(x,y) \rightarrow NTPI(y,x)]$

from: D3, D10, D14

(T37) $\forall xyz[[TPI(x,y) \wedge TPI(y,z)] \rightarrow TPI(x,z)]$

from: D3, D9, D13 paramod.

(T38) $\forall xyz[[NTPI(x,y) \wedge NTPI(y,z)] \rightarrow NTPI(x,z)]$

from: D3, D10, D14, paramod.

(T39) $\forall x\ \neg C(compl(x),x)$

from: A2, D22, T3

(T40) $\forall x\ PP(int(x),cl(x))$

from: A8, D4, D3, T40.lemma2, T40.lemma6, paramod.

(T40.lemma1) $\forall x\ P(int(x),x)$

from: D2, D10, D28

(T40.lemma2) $\forall x\ P(x,cl(x))$

from: D2, D29, T39, T40.lemma1

(T40.lemma3) $\forall x\ P(int(x),cl(x))$

from: T5, T40.lemma1, T40.lemma2

(C40.lemma4) $\forall xy[O(x,y) \rightarrow O(int(x),int(y))]$

(T40.lemma5) $\forall xy[O(x,y) \rightarrow O(int(x),y)]$

from: D5, T5, T40.lemma1, C40.lemma4

(T40.lemma6) $\neg EC(int(x),y)$

from: A2, D8, D28, T32, T40.lemma5

(T41) $\forall xy[P(y,cl(x)) \rightarrow O(y,int(x))]$

Refutation set:
1 $EC(cl(x),cl(compl(x)))$ (from A8)
2 $\neg EC(x,y) \vee \neg O(x,y)$ (from D8)
3 $\neg P(z,x) \vee \neg P(z,y) \vee O(x,y)$ (from D5)
4 $O(y,int(x)) \vee P(y,cl(compl(x)))$ (from T42)
5 $P(a,cl(b))$
6 $\neg O(a,int(b))$

Proof:
7 $P(a,cl(compl(b)))$ ancestors: 6,4
8 $\neg P(a,y) \vee O(cl(b),y)$ ancestors: 5,3
9 $O(cl(b),cl(compl(b)))$ ancestors: 8,7
10 $\neg EC(cl(b),cl(compl(b)))$ ancestors: 9,2
11 null ancestors: 10,1

(T42) $\forall xy[\neg O(y,int(x)) \rightarrow P(y,cl(compl(x)))]$

from: D2, D8, D29, T40.lemma6, T42.lemma1

(T42.lemma1) $\forall x$ EQUAL(compl(compl(x)),x)

from: D2, D3, T6, D22, paramod.

(T43) $\forall x[Open(x) \vee Closed(x) \vee Clopen(x)]$

from: D33

(T44) $\forall x[Open(x) \rightarrow \neg Closed(x)]$

from: A8, D32, T19, T49 paramod.

(T45) $\forall x[Closed(x) \rightarrow \neg Clopen(x)]$

from: D33

(T46) $\forall x[Open(x) \rightarrow \neg Clopen(x)]$

from: D33

(T47) $\forall x \exists y[EC(cl(x),y)]$

from: A8

(T48) Open(u_s)

from: D2, D3, D5, D8, D23, D28, D31, T3, T40.lemma1, T49

(T49) $\forall x[Open(x) \leftrightarrow \neg \exists y[EC(y,x)]]$

(i) $\forall x[Open(x) \rightarrow \neg \exists y[EC(y,x)]]$

from: T31, T49.lemma3

(ii) $\forall x[\neg \exists y[EC(y,x)] \rightarrow Open(x)]$

from: T31, T49.lemma4

(T49.lemma1) $\forall xy[NTP(x,y) \rightarrow P(x,int(y))]$

from: D2, D28

(T49.lemma2) $\forall xy[P(x,int(y)) \rightarrow NTP(x,y)]$

from: T49.lemma5, T49.lemma6

(T49.lemma3) $\forall x[Open(x) \rightarrow NTP(x,x)]$

from: D3, D31, T49.lemma2

(T49.lemma4) $\forall x[NTP(x,x) \rightarrow Open(x)]$

from: D3, D31, T40.lemma1, T49.lemma1

(T49.lemma5) $\forall xyz[[P(x,y) \wedge NTP(y,z)] \rightarrow NTP(x,z)]$

from: D2, D5, D8, D10, T5

(T49.lemma6) $NTP(int(x),x)$

from: D8, D10, D28, T32, T40.lemma1

(T50) $\forall x[Closed(x) \rightarrow \exists y[EC(y,x)]]$

from: T44, T49

(T51) $\forall x[Clopen(x) \rightarrow \exists y[EC(y,x)]]$

from: D33, T49

(T52) $\forall x[Atom(x) \rightarrow Open(x)]$

from: D2, D10, D28, D31, D38

(T53) $\forall xy[[Atom(x) \wedge Atom(y) \wedge C(x,y)] \rightarrow EQUAL(x,y)]$

from: D5, D8, D38, T7, T8, T49, T52

(T54) $\forall xy[[Atom(x) \wedge P(x,y)] \rightarrow NTP(x,y)]$

from: D10, T49, T52

(T55) $\forall xy[O(x,y) \leftrightarrow \exists z[Atom(z) \wedge P(z,x) \wedge P(z,y)]]$

(i) $\forall xy[O(x,y) \rightarrow \exists z[Atom(z) \wedge P(z,x) \wedge P(z,y)]]$

from: A10, D5, A10, T5

(ii) $\forall xy[\exists z[Atom(z) \wedge P(z,x) \wedge P(z,y)] \rightarrow O(x,y)]$

from: A10, D5, D38 paramod.

(T56) $\forall xyz[C(z,int(x)) \leftrightarrow \exists u[Atom(u) \wedge P(u,x) \wedge C(u,z)]]$

(i) $\forall xz[C(z,int(x)) \rightarrow \exists u[Atom(u) \wedge P(u,x) \wedge C(z,u)]]$

from: D10, D28

(ii) $\forall xyz[\exists u[Atom(u) \wedge P(u,x) \wedge C(z,u)] \rightarrow C(z,int(x))]$

from: D28, D38, T49.lemma3, T52 paramod.

(T57) $\forall xy[EQUAL(x,y) \rightarrow \forall z[Atom(z) \rightarrow [P(z,x) \leftrightarrow P(z,y)]]]$

from: A10, D38 paramod.

(C58) $\forall xy[EC(x,y) \rightarrow \forall zu[[Atom(z) \wedge Atom(u) \wedge P(z,x) \wedge P(u,y)] \rightarrow \neg EC(z,u)]$

(C59) $\forall xy[[C\_Atom(x) \wedge C\_Atom(y)] \rightarrow [DC(x,y) \vee EC(x,y) \vee EQUAL(x,y)]$

(T60) $\forall xy[[C\_Atom(x) \wedge C\_Atom(y) \wedge O(x,y)] \rightarrow EQUAL(x,y)]$

from: D5, D38, T60.lemma1, paramod.

(T60.lemma1) $\forall x[C\_Atom(x) \rightarrow Atom(int(x))]$

from: D31, D32, D38, D39, T5, T30.lemma1, T32, T41, T40.lemma1, T40.lemma2, T44, T52, C60.lemma2, C60.lemma3, C60.lemma4, paramod.

(C60.lemma2) $\forall xy[O(int(x),int(y) \rightarrow O(x,y)]$

(C60.lemma3) $\forall xy[Atom(x) \wedge O(x,y)] \rightarrow P(x,y)]$

(C60.lemma4) $\forall xy[P(x,int(y) \rightarrow NTP(x,int(y))]$

(T61) $\forall x\exists y[IN(y,x)]$

from: A1, A11

(T62) $\forall xy[DC(x,y) \leftrightarrow \neg\exists z[IN(z,x) \wedge IN(z,y)]]$

(i) $\forall xy[DC(x,y) \rightarrow \neg\exists z[IN(z,x) \wedge IN(z,y)]]$

from: A11, D1

(ii) $\forall xy[\neg\exists z[IN(z,x) \wedge IN(z,y)] \rightarrow DC(x,y)]$

from: A1, A11, D1

(T63) $\forall xy[O(x,y) \leftrightarrow \exists z[IN(z,int(x)) \wedge IN(z,int(y))]]$

(i) $\forall xy[O(x,y) \rightarrow \exists z[IN(z,int(x)) \wedge IN(z,int(y))]]$

from: A11, T30.lemma1, T63.lemma1

(ii) $\forall xy[\exists z[IN(z,int(x)) \wedge IN(z,int(y))] \rightarrow O(x,y)]$

from: A11, D5, D28, T5, T32, T40.lemma1

(T63.lemma1) $\forall xy[O(x,y) \rightarrow O(int(x),int(y))]$

from: A9, D5, T49.lemma1, T63.lemma2

(T63.lemma2) $\forall xyz[[NTP(x,y) \wedge P(y,z)] \rightarrow NTP(x,z)]$

from: D8, D10, T5, T63.lemma3

(T63.lemma3) $\forall xyz[[P(x,y) \wedge O(z,x)] \rightarrow O(z,y)]$

from: D5, T5

(T64) $\forall xy[P(x,y) \leftrightarrow \forall z[IN(z,x) \rightarrow IN(z,y)]]$

(i) $\forall xy[P(x,y) \rightarrow \forall z[IN(z,x) \rightarrow IN(z,y)]]$

from: A12

(ii) $\forall xy[\forall z[IN(z,x) \rightarrow IN(z,y)]] \rightarrow P(x,y)]$

from: A11, D2

(T65) $\forall xy[EQUAL(x,y) \leftrightarrow \forall z[IN(z,x) \leftrightarrow IN(z,y)]]$

(i) $\forall xy[EQUAL(x,y) \rightarrow \forall z[IN(z,x) \leftrightarrow IN(z,y)]]$

from: A12, D3

(ii) $\forall xy[\forall z[IN(z,x) \leftrightarrow IN(z,y)] \rightarrow EQUAL(x,y)]$

from: A11, D2, D3

(C66) $\forall xy[EC(x,y) \leftrightarrow \exists z[IN(z,x) \wedge IN(z,y)] \wedge$
$\neg\exists u[IN(u,int(x)) \wedge IN(u,int(y))]]$

(T67) $\forall x \neg INSIDE(x,x)$

from: D49, T14

(T68) $INSIDE(x,y) \rightarrow \neg INSIDE(y,x)]$

from: A17, D6, D49

(T69) ∀xyz[[DR(x,y) ∧ DR(y,z) ∧ DR(x,z) ∧
INSIDE(x,y) ∧ INSIDE(y,z)] → INSIDE(x,z)]

from: A16, D6, D49

(T70) ∀x ¬OUTSIDE(x,x)

from: D5, D6, T3, T77

(T71) ∀xy[INSIDE(x,y) ↔ [J_INSDE(x,y) v W_INSIDE(x,y)]]

(i) ∀xy[INSIDE(x,y) → [J_INSIDE(x,y) v W_INSIDE(x,y)]]

from: D49, D54, D55, T118

(ii) ∀xy[[J_INSIDE(x,y) v W_INSIDE(x,y)] → INSIDE(x,y)]

from: D54, D55

(T72) ∀xy[OUTSIDE(x,y) ↔ [J_OUTSIDE(x,y) v W_OUTSIDE(x,y)]]

(i) ∀xy[OUTSIDE(x,y) → [J_OUTSIDE(x,y) v W_OUTSIDE(x,y)]]

from: D1, D6, D8, D50, D51, D52

(ii) ∀xy[[J_OUTSIDE(x,y) v W_OUTSIDE(x,y)] → OUTSIDE(x,y)]

from: D1, D6, D8, D50, D51, D52, T30.lemma1

(T73) INSIDE(x,y) → ¬OUTSIDE(x,y)]

from: D5, D6, D49, T3

(T74) ∀xy[INSIDE(x,y) → ¬P_INSIDE(x,y)]

from: D49, D53

(T75) ∀xy[INSIDE(x,y) → P(x,inside(y))]

from: D2, D56

(T76) ∀xy[OUTSIDE(x,y) → P(x,outside(y)]

from: D2, D57

(T77) ∀x P(x,conv(x))

from: A15, D2

(T78) ∀x ¬MEETS(x,x)

from: A4, D74

(T79) ∀xy[MEETS(x,y) → ¬MEETS(y,x)]

from: A4, A5, D75

(T80) $\forall xyz[\text{MEETS}(x,y) \wedge \text{MEETS}(y,z)] \rightarrow \neg\text{MEETS}(x,z)]$

from: A5, D74

(T81) $\forall x[\text{PERIOD}(x) \rightarrow \exists y[\text{MOMENT}(y) \wedge P(y,x)]]$

from: A4, A10, D38, D87, paramod.

(C82) $\forall x[\text{MOMENT}(x) \leftrightarrow [\text{Atom}(x) \wedge \text{PERIOD}(x)]]$

(C83) $\forall x[[\text{P\_INSIDE}(x,y) \vee \text{INSIDE}(x,y)] \rightarrow \neg\text{Convex}(y)]$

Appendix C: The relational lattice L_c (proofs)

Below are assembled together a set of proofs that define the properties of the relational lattice L_c. A diagrammatic representation of lattice L_c is illustrated in Figure 3. The proof method follows that used in Appendix B.

THEOREMS:

(T100) $\forall xy[PP(x,y) \leftrightarrow [TPP(x,y) \vee NTPP(x,y)]]$

(i) $\forall xy[PP(x,y) \rightarrow [TPP(x,y) \vee NTPP(x,y)]]$

from: D4, D9, D10, D11, D12

(ii) $\forall xy[[TPP(x,y) \vee NTPP(x,y)] \rightarrow PP(x,y)]$

from: D4, D9, D10, D11, D12

(T101) $\forall xy[TPP(x,y) \rightarrow \neg NTPP(x,y)]$

from: D9, D10, D11, D12

(T102) $\forall xy[TP(x,y) \leftrightarrow [TPP(x,y) \vee TPI(x,y)]]$

(i) $\forall xy[TP(x,y) \rightarrow [TPP(x,y) \vee TPI(x,y)]]$

from: D11, D13

(ii) $\forall xy[[TPP(x,y) \vee TPI(x,y)] \rightarrow TP(x,y)]$

from: D11, D13

(T103) $\forall xy[TPP(x,y) \rightarrow \neg TPI(x,y)]$

from: D11, D13

(T104) $\forall xy[TP^{-1}(x,y) \leftrightarrow [TPP^{-1}(x,y) \vee TPI(x,y)]]$

(i) $\forall xy[TP^{-1}(x,y) \rightarrow [TPP^{-1}(x,y) \vee TPI(x,y)]]$

from: D9, D11, D13, D17, D19, T35, T100

(ii) $\forall xy[[TPP^{-1}(x,y) \vee TPI(x,y)] \rightarrow TP^{-1}(x,y)]$

from: D11, D13, D17, D19, T35

(T105) $\forall xy[TPP^{-1}(x,y) \rightarrow \neg TPI(x,y)]$

from: D11, D13, D19, T35

(T106) $\forall xy[NTP(x,y) \leftrightarrow [NTPP(x,y) \vee NTPI(x,y)]]$

(i) $\forall xy[NTP(x,y) \leftrightarrow [NTPP(x,y) \vee NTPI(x,y)]]$

from: D12, D14

(ii) $\forall xy[[NTPP(x,y) \vee NTPI(x,y)] \rightarrow NTP(x,y)]$

from: D12, D14

(T107) $\forall xy[NTPP(x,y) \rightarrow \neg NTPI(x,y)]$

from: D12, D14

(T108) $NTP^{-1}(x,y) \leftrightarrow [NTPP^{-1}(x,y) \vee NTPI(x,y)]]$

(i) $NTP^{-1}(x,y) \rightarrow [NTPP^{-1}(x,y) \vee NTPI(x,y)]]$

from: D12, D14, D18, D20

(ii) $\forall xy[[NTPP^{-1}(x,y) \vee NTPI(x,y)] \rightarrow NTP^{-1}(x,y)]$

from: D12, D14, D18, D20, T36

(T109) $\forall xy[NTPP^{-1}(x,y) \rightarrow \neg NTPI(x,y)]$

from: D12, D14, D20, T36

(T110) $\forall xy[PP^{-1}(x,y) \leftrightarrow [TPP^{-1}(x,y) \vee NTPP^{-1}(x,y)]]$

(i) $\forall xy[PP^{-1}(x,y) \rightarrow [TPP^{-1}(x,y) \vee NTPP^{-1}(x,y)]]$

from: D16, D19, D20, T100

(ii) $\forall xy[[TPP^{-1}(x,y) \vee NTPP^{-1}(x,y)] \rightarrow PP^{-1}(x,y)]$

from: D16, D19, D20, T100

(T111) $\forall xy[TPP^{-1}(x,y) \rightarrow \neg NTPP^{-1}(x,y)]$

from: D19, D20, T101

(T112) $\forall xy[EQUAL(x,y) \leftrightarrow [TPI(x,y) \vee NTPI(x,y)]]$

(i) $\forall xy[EQUAL(x,y) \rightarrow [TPI(x,y) \vee NTPI(x,y)]]$

from: D3, D9, D10, D13, D14

(ii) $\forall xy[[TPI(x,y) \vee NTPI(x,y)] \rightarrow EQUAL(x,y)]$

from: D3, D13, D14

(T113) $\forall xy[TPI(x,y) \rightarrow \neg NTPI(x,y)]$

from: D9, D10, D13, D14

(T114) $\forall xy[O(x,y) \leftrightarrow [PO(x,y) \vee P(x,y) \vee P^{-1}(x,y)]$

(i) $\forall xy[O(x,y) \rightarrow [PO(x,y) \vee P(x,y) \vee P^{-1}(x,y)]$

from: D7, D15

(ii) $\forall xy[[PO(x,y) \vee P(x,y) \vee P^{-1}(x,y)] \rightarrow O(x,y)]$

from: D5, D7, D15, T3

(T115) $\forall xy[PO(x,y) \rightarrow \neg P(x,y)]$

from: D7

(T116) $\forall xy[PO(x,y) \rightarrow \neg P\text{-}1(x,y)]$

from: D7, D15

(T116) $\forall xy[DR(x,y) \leftrightarrow [EC(x,y) \vee DC(x,y)]]$

(i) $\forall xy[DR(x,y) \rightarrow [EC(x,y) \vee DC(x,y)]]$

from: D1, D6, D8

(ii) $\forall xy[[EC(x,y) \vee DC(x,y)] \rightarrow DR(x,y)]$

from: D1, D6, D8, T30.lemma1

(T117) $\forall xy[EC(x,y) \rightarrow \neg DC(x,y)]$

from: D1, D8

(T118) $\forall xy[P(x,y) \leftrightarrow [TP(x,y) \vee NTP(x,y)]]$

(i) $\forall xy[P(x,y) \rightarrow [TP(x,y) \vee NTP(x,y)]]$

from: D9, D10

(ii) $\forall xy[[TP(x,y) \vee NTP(x,y)] \rightarrow P(x,y)]$

from: D9, D10

(T119) $\forall xy[TP(x,y) \rightarrow \neg NTP(x,y)]$

from: D9, D9, D10

(T120) $\forall xy[P^{-1}(x,y) \leftrightarrow [TP^{-1}(x,y) \vee NTP^{-1}(x,y)]]$

(i) $\forall xy[P^{-1}(x,y) \rightarrow [TP^{-1}(x,y) \vee NTP^{-1}(x,y)]]$

from: D15, D17, D18, T118

(ii) $\forall xy[[TP^{-1}(x,y) \vee NTP^{-1}(x,y)] \rightarrow P^{-1}(x,y)]$

from: D9, D10, D15, D17, D18

(T121) $\forall xy[TP^{-1}(x,y) \rightarrow \neg NTP^{-1}(x,y)]$

from: D17, D18, T119

(T122) $\forall xy[P(x,y) \leftrightarrow [PP(x,y) \vee EQUAL(x,y)]]$

(i) $\forall xy[P(x,y) \rightarrow [PP(x,y) \vee EQUAL(x,y)]]$

from: D3, D4

(ii) $\forall xy[[PP(x,y) \vee EQUAL(x,y)] \rightarrow P(x,y)]$

from: D3, D4

(T123) $\forall xy[PP(x,y) \rightarrow \neg EQUAL(x,y)]$

from: D3, D4

(T124) $\forall xy[P^{-1}(x,y) \leftrightarrow [PP^{-1}(x,y) \vee EQUAL(x,y)]]$

(i) $\forall xy[P^{-1}(x,y) \rightarrow [PP^{-1}(x,y) \vee EQUAL(x,y)]]$

from: D3, D4, D15, D16

(ii) $\forall xy[[PP^{-1}(x,y) \vee EQUAL(x,y)] \rightarrow P^{-1}(x,y)]$

from: D3, D4, D15, D16

(T125) $\forall xy[PP^{-1}(x,y) \rightarrow \neg EQUAL(x,y)]$

from: D3, D4, D16

(T126) $\forall xy[C(x,y) \leftrightarrow [O(x,y) \vee EC(x,y)]]$

(i) $\forall xy[C(x,y) \rightarrow [O(x,y) \vee EC(x,y)]]$

from: D8

(ii) $\forall xy[[O(x,y) \vee EC(x,y)] \rightarrow C(x,y)]$

from: D8, T30.lemma1

(T127) $\forall xy[O(x,y) \rightarrow \neg EC(x,y)]$

from: D8

(T128) $\forall xy[C(x,y) \vee DR(x,y)]$

from: D6, T30.lemma1

(T129) $\forall xy[DC(x,y) \vee EC(x,y) \vee PO(x,y) \vee TPP(x,y) \vee NTPP(x,y) \vee$
$TPP^{-1}(x,y) \vee NTPP^{-1}(x,y) \vee TPI(x,y) \vee NTPI(x,y)]$

from: D5, D7, T116, T100, T110, T112, T122, T124
